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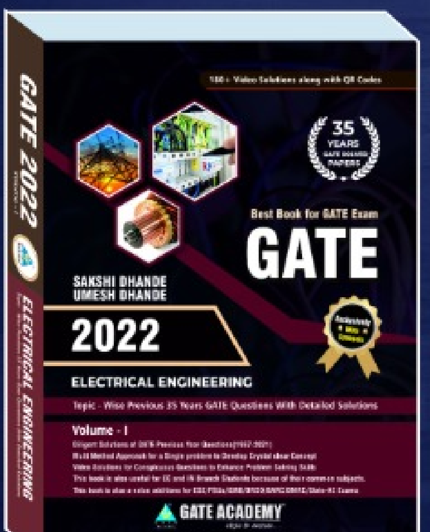

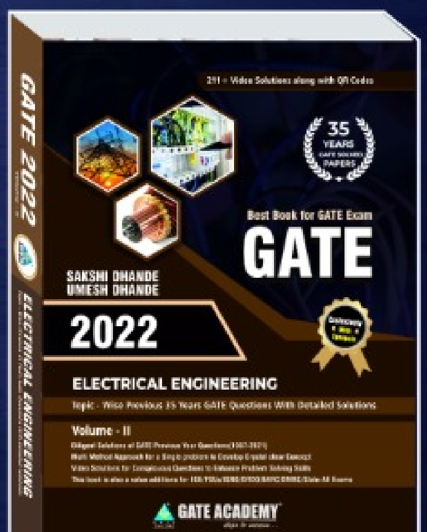

SAMPLE PDF

ELECTRICAL ENGINEERING

VOLUME - 02

This sample PDF of **GATE Previous Years Solution Book** contains randomly selected questions with solutions from some of the chapters of every subject along with part of concept refresher **Synopsis** of those chapters to let the aspirants have an idea about the content, style and appearance of the book.

Previous Years marks distribution analysis is also given in tabular form with index page of every subject, which contains analysis of GATE papers from 2003 onwards as GATE pattern has turned objective since 2003.

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GATE 2022
Electrical Engineering
(Volume - II)

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IMPORTANCE of GATE

GATE examination has been emerging as one of the most prestigious competitive exam for engineers. Earlier it was considered to be an exam just for eligibility for pursuing PG courses, but now GATE exam has gained a lot of attention of students as this exam open an ocean of possibilities like :

1. **Admission into IISc, IITs, IIITs, NITs**

   A good GATE score is helpful for getting admission into IISc, IITs, IIITs, NITs and many other renowned institutions for M.Tech./M.E./M.S. An M.Tech graduate has a number of career opportunities in research fields and education industries. Students get ₹ 12,400 per month as stipend during their course.

2. **Selection in various Public Sector Undertakings** (**PSUs**)

   A good GATE score is helpful for getting job in government-owned corporations termed as **Public Sector Undertakings** (**PSUs**) in India like IOCL, BHEL, NTPC, BARC, ONGC, PGCIL, DVC, HPCL, GAIL, SAIL & many more.

3. Direct recruitment to Group A level posts in Central government, i.e., Senior Field Officer (Tele), Senior Research Officer (Crypto) and Senior Research Officer (S&T) in Cabinet Secretariat, Government of India, is now being carried out on the basis of GATE score.

4. **Foreign universities through GATE**

   GATE has crossed the boundaries to become an international level test for entry into postgraduate engineering programmes in abroad. Some institutes in two countries **Singapore** and **Germany** are known to accept GATE score for admission to their PG engineering programmes.

5. **National Institute of Industrial Engg. (NITIE)**
   - NITIE offers **PGDIE / PGDMM / PGDPM** on the basis of GATE scores. The shortlisted candidates are then called for group Discussion and Personal Interview rounds.
   - NITIE offers a Doctoral Level Fellowship Programme recognized by Ministry of HRD (MHRD) as equivalent to PhD of any Indian University.
   - Regular full time candidates those who will qualify for the financial assistance will receive ₹ 25,000 during 1st and 2nd year of the Fellowship programme and ₹ 28,000 during 3rd, 4th and 5th year of the Fellowship programme as per MHRD guidelines.

6. **Ph.D. in IISc/ IITs**
   - IISc and IITs take admissions for Ph.D. on the basis of GATE score.
   - Earn a Ph.D. degree directly after Bachelor’s degree through integrated programme.
   - A fulltime residential researcher (RR) programme.

7. **Fellowship Program in management (FPM)**
   - Enrolment through GATE score card
   - Stipend of ₹ 22,000 – 30,000 per month + HRA
   - It is a fellowship program
   - Application form is generally available in month of sept. and oct.

Note : **In near future, hopefully GATE exam will become a mandatory exit test for all engineering students, so take this exam seriously. Best of LUCK !**

GATE Exam Pattern

Section	Question No.	No. of Questions	Marks Per Question	Total Marks
General Aptitude	1 to 5	5	1	5
	6 to 10	5	2	10
Technical + Engineering Mathematics	1 to 25	25	1	25
	26 to 55	30	2	60
Total Duration : 3 hours		**Total Questions : 65**		**Total Marks : 100**

Note :

(i) 40 to 45 marks will be allotted to Numerical Answer Type Questions.

(ii) MSQ also added from GATE 2021 for which **no negative** marking.

Pattern of Questions :

GATE 2021 would contain questions of THREE different types in all the papers :

(i) **Multiple Choice Questions (MCQ)** carrying 1 or 2 marks each, in all the papers and sections. These questions are objective in nature, and each will have choice of four answers, out of which ONLY ONE choice is correct.

Negative Marking for Wrong Answers : For a wrong answer chosen in a MCQ, there will be negative marking. For 1-mark MCQ, 1/3 mark will be deducted for a wrong answer. Likewise, for 2-mark MCQ, 2/3 mark will be deducted for a wrong answer.

(ii) **Multiple Select Questions (MSQ)** carrying 1 or 2 marks each in all the papers and sections. These questions are objective in nature, and each will have choice of four answers, out of which ONE or MORE than ONE choice(s) are correct.

Note : There is **NO negative** marking for a wrong answer in MSQ questions. However, there is NO partial credit for choosing partially correct combinations of choices or any single wrong choice.

(iii) **Numerical Answer Type (NAT)** Questions carrying 1 or 2 marks each in most of the papers and sections. For these questions, the answer is a signed real number, which needs to be entered by the candidate using the virtual numeric keypad on the monitor (keyboard of the computer will be disabled). No choices will be shown for these types of questions. The answer can be a number such as 10 or -10 (an integer only). The answer may be in decimals as well, for example, 10.1 (one decimal) or 10.01 (two decimals) or -10.001 (three decimals). These questions will be mentioned with, up to which decimal places, the candidates need to present the answer. Also, for some NAT type problems an appropriate range will be considered while evaluating these questions so that the candidate is not unduly penalized due to the usual round-off errors. Candidates are advised to do the rounding off at the end of the calculation (not in between steps). Wherever required and possible, it is better to give NAT answer up to a maximum of three decimal places.

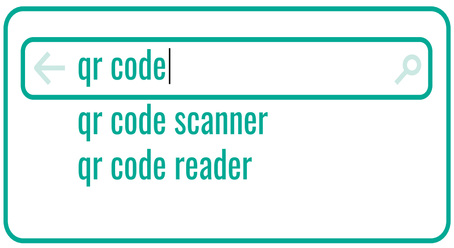
Example : If the wire diameter of a compressive helical spring is increased by 2%, the change in spring stiffness (in %) is _ (correct to two decimal places).

Note : *There is NO negative marking for a wrong answer in NAT questions.*
Also, there is NO partial credit in NAT questions.

What is special about this book ?

GATE ACADEMY Team took several years' to come up with the solutions of GATE examination. It is because we strongly believe in quality. We have significantly prepared each and every solution of the questions appeared in GATE, and many individuals from the community have taken time out to proof read and improve the quality of solutions, so that it becomes very lucid for the readers. Some of the key features of this book are as under :

- This book gives complete analysis of questions chapter wise as well as year wise.
- Video Solution of important conceptual questions has been given in the form of QR code and by scanning QR code one can see the video solution of the given question.
- Solutions has been presented in lucid and understandable language for an average student.
- In addition to the GATE syllabus, the book includes the nomenclature of chapters according to text books for easy reference.
- Last but not the least, author's 10 years experience and devotion in preparation of these solutions.
- Steps to Open Video solution through mobile.



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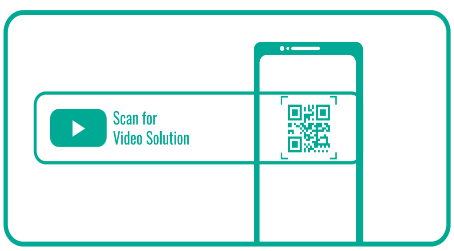

(3) Scan the given QR Code for particular question.

(4) Visit the link generated & you'll be redirect to the video solution.

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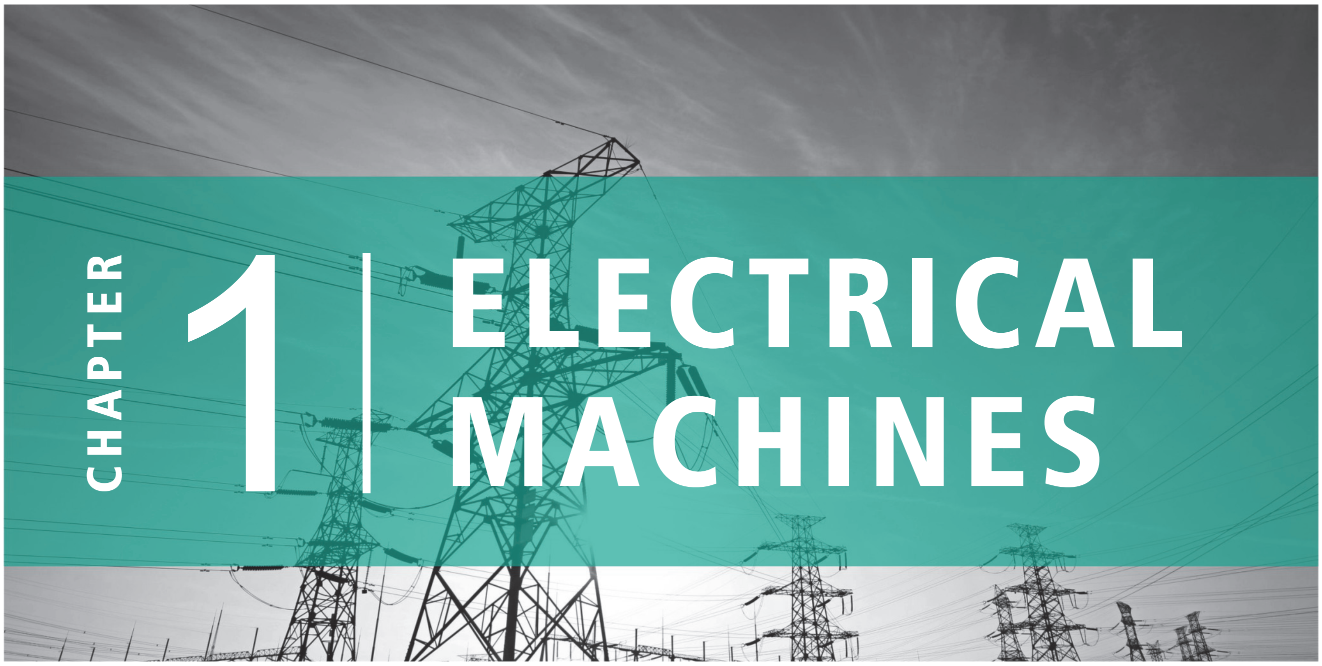


CHAPTER 1 | ELECTRICAL MACHINES

Marks Distribution of Electrical Machines in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	7	11	29
2004	6	13	32
2005	4	10	24
2006	4	12	28
2007	7	7	21
2008	4	10	24
2009	3	8	19
2010	2	4	10
2011	3	3	9
2012	1	3	7
2013	3	2	7
2014 Set-1	3	4	11

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-2	4	4	12
2014 Set-3	4	5	14
2015 Set-1	2	5	12
2015 Set-2	2	0	2
2016 Set-1	4	4	12
2016 Set-2	1	4	9
2017 Set-1	2	5	12
2017 Set-2	2	6	14
2018	3	4	11
2019	3	5	13
2020	3	4	11
2021	2	4	10

Syllabus : Electrical Machines

Single phase transformer: equivalent circuit, phasor diagram, open circuit and short circuit tests, regulation and efficiency; Three-phase transformers: connections, vector groups, parallel operation; Auto-transformer, Electromechanical energy conversion principles; DC machines: separately excited, series and shunt, motoring and generating mode of operation and their characteristics, speed control of dc motors; Three-phase induction machines: principle of operation, types, performance, torque-speed characteristics, no-load and blocked-rotor tests, equivalent circuit, starting and speed control; Operating principle of single-phase induction motors; Synchronous machines: cylindrical and salient pole machines, performance and characteristics, regulation and parallel operation of generators, starting of synchronous motors; Types of losses and efficiency calculations of electric machines.

Contents : Electrical Machines

1 Single Phase Transformer

➢ Partial Synopsis

Voltage regulation of Transformer

It is defined as the change in magnitude of the secondary terminal voltage, expressed as a percentage (or per unit) of the secondary rated voltage, when load at a given power factor is reduced to zero, with primary applied voltage held constant.

If V_2 = secondary terminal voltage at any load, and E_2 = secondary terminal voltage at no load.

Then at a given power factor and specified load, the voltage regulation is given by

$$\text{Voltage regulation} = \frac{E_2 - V_2}{\text{Secondary rated voltage}} \text{ in p.u.}$$

The secondary rated voltage of a transformer is equal to the secondary terminal voltage at no load, i.e. E_2.

$$\therefore \ \textbf{Voltage regulation} = \frac{E_2 - V_2}{E_2} \textbf{ in p.u.} = \frac{E_2 - V_2}{E_2} \times 100 \textbf{ in percentage.}$$

At no load, the primary leakage impedance drop is almost negligible, therefore, the secondary no load voltage $E_2 = V_1 \dfrac{N_2}{N_1}$. The expression for voltage regulation can also be written as

$$\text{Voltage regulation} = \frac{V_1 \frac{N_2}{N_1} - V_2}{V_1 \frac{N_2}{N_1}} \times 100 = \frac{V_1 - V_2 \frac{N_2}{N_1}}{V_1} \times 100 \text{ in percentage.}$$

Here V_1 is the primary applied voltage.

The change in secondary terminal voltage with load current is due to the primary and secondary leakage impedances of the transformer. The magnitude of this change depends on the load pf, load current, total resistance and total leakage reactance of the transformer.

Thus, the voltage drop in the secondary terminal voltage $= E_2 - V_2 = I_2 R_{02} \cos\theta_2 + I_2 X_{02} \sin\theta_2$

Per unit voltage regulation for any load current I_2 is, $\dfrac{E_2 - V_2}{E_2} = \dfrac{I_2 R_{02}}{E_2} \cos\theta_2 \pm \dfrac{I_2 X_{02}}{E_2} \sin\theta_2$

Where + sign is for lagging load and – sign is for leading load.

Condition for Zero Voltage regulation (at leading power factor) :

$$\frac{E_2 - V_2}{E_2} = \frac{I_2 R_{02}}{E_2}\cos\theta_2 - \frac{I_2 X_{02}}{E_2}\sin\theta_2 = 0 \quad \Rightarrow \quad \tan\theta_2 = \frac{R_{02}}{X_{02}}$$

Condition for Maximum Voltage regulation (at lagging power factor) :

$$\frac{d}{d\theta_2}\left[\frac{E_2 - V_2}{E_2}\right] = \frac{d}{d\theta_2}\left[\frac{I_2 R_{02}}{E_2}\cos\theta_2 + \frac{I_2 X_{02}}{E_2}\sin\theta_2\right] = 0 \quad \Rightarrow \quad \tan\theta_2 = \frac{X_{02}}{R_{02}}$$

Losses in Transformer

(1) Copper losses : The copper losses are due to the power wasted in the form of I^2R loss due to the resistances of the primary and secondary windings. The copper loss depends on the magnitude of the currents flowing through the windings.

$$\textbf{Total copper loss, } P_{cu} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} = I_2^2 R_{02} \textbf{ Watts}$$

(2) Iron losses : Hysteresis loss and eddy current loss, both depend upon magnetic properties of the materials used to construct the core of transformer and its design. So these **losses in transformer** are fixed and do not depend upon the load current. So **core loss or iron loss in transformer** can be considered as **constant loss** for all range of load.

Iron loss or Core loss = Hysteresis loss + Eddy current loss

Hysteresis loss is given by, $P_h = K_h B_{max}^{1.6} fv$ **Watts**

Where, K_h : Hysteresis constant depending on the material, f : Frequency, and v : Volume of core.

Hysteresis losses are proportional to area of BH curve and the frequency.

Hysteresis losses are reduced by using material with a thin BH curve.

Eddy current loss

Eddy current loss is given by, $P_e = K_e B_{max}^2 f^2 t^2 v$ **Watts**

Where, K_e : Eddy current constant depending on the material, t : Thickness of lamination, v : Volume of core.

Dielectric loss (Minor loss)

1. It takes place in insulating materials (oils and solid insulation).
2. Dielectric loss depends on voltage and is a constant loss (less than 0.25% of total output so negligible).

Stray loss

1. It takes place in metallic parts of transformer.
2. It occurs due to leakage flux (due to load) and is a variable loss.

(a) Iron stray load loss

1. Iron loss depends on supply voltage which occurs in main iron part or core of transformer and it is constant loss.
2. Iron stray load loss depends on load current and is a variable loss which occurs in auxiliary iron parts like transformer tank or conservators due to leakage flux or stray flux.

(b) Copper stray load loss

1. It is a variable loss which takes place under load condition.
2. Leakage flux links with windings and produces leakage emf which produces circulating current called stray current which produces copper stray load loss.
3. As load increases, load current increases, leakage flux or stray flux increases, leakage emf increases, stray current increases and copper stray load loss increases.
4. Stray loss occurs in metallic parts due to eddy currents induced by leakage fields.

Efficiency of Transformer

The efficiency of a transformer (or any other device) is defined as the ratio of output power to input power.

$$\textbf{Efficiency, } \eta = \frac{\textbf{Output power}}{\textbf{Input power}} = \frac{E_2 I_2 \cos\theta_2}{E_2 I_2 \cos\theta_2 + P_i + I_2^2 r_{e2}}$$

Where P_i = Total core loss, $I_2^2 r_{e2}$ = Total ohmic losses, $E_2 I_2$ = Volt ampere, and $\cos\theta_2$ = Load p.f.

Efficiency of a transformer can also be expressed in per unit parameters.

$$\eta = \frac{\cos\theta_2}{\cos\theta_2 + \dfrac{P_i}{E_2 I_2} + \dfrac{I_2^2 r_{e2}}{E_2 I_2}} = \frac{\text{load pf}}{\text{load pf} + \text{p.u. core loss} + \text{p.u. equivalent resistance}}$$

Condition for maximum efficiency :

$m^2 P_{cufl} = I_2^2 r_{e2} = P_i$ where $m = \dfrac{\text{Given load}}{\text{Rated load}}$ **Variable ohmic loss = Constant core loss**

Load current at maximum efficiency : $I_{2m} = I_{fl} \times \sqrt{\dfrac{P_i}{P_{cufl}}}$

KVA supplied atmaximum efficiency : $\text{kVA}_{\eta(\max)} = (\text{rated kVA}) \times \sqrt{\dfrac{P_i}{P_{cufl}}}$

➢ Sample Questions

1991 IIT Madras

1.1 The hysteresis and eddy current losses of a single phase transformer working on 200 V, 50 Hz supply are P_h and P_e respectively. The percentage decrease in these, when operated at a 160 V, 40 Hz supply is

(A) 32, 36 (B) 20, 36
(C) 25, 20 (D) 40, 80

1995 IIT Kanpur

1.2 A 4 kVA, 50 Hz, single-phase transformer has a ratio 200 V/400 V. The data taken on the L.V. side at rated voltage show that the open circuit input wattage is 80 W. The mutual inductance between the primary and the secondary windings is 1.91 H. (Neglecting the effect of winding resistance and leakage reactances). The value of current (in Amperes) taken by the transformer, if the no-load test is conducted on the H.V. side at rated voltage will be __________.

1999 IIT Bombay

1.3 A 400 V/100 V, 10 kVA two winding transformer is reconnected as an auto transformer across a suitable voltage source. The maximum rating of such an arrangement could be

(A) 50 kVA (B) 15 kVA
(C) 12.5 kVA (D) 8.75 kVA

1.4 Two single-phase transformers *A* and *B* have the following parameters.

Transformer *A* : 400 V/200 V, 10 kVA, percentage resistance and percentage reactance are 3% and 4% respectively.

Transformer *B* : 5 kVA, 400 V/200 V, percentage resistance and percentage reactance are 4% and 3% respectively.

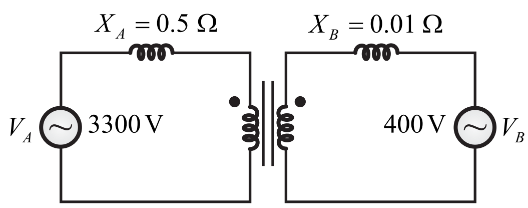
These two transformers are connected in parallel and they share a common load of 12 kW at a power factor of 0.8 lag. Determine the active power (kW) and reactive power (kVAR) delivered by transformer *A*.

(A) 2.5, 5 (B) 7.5, 12.5
(C) 7.5, 6.78 (D) 2.5, 6.78

2002 IISc Bangalore

Common Data for Questions 1.5 & 1.6

A single phase 6300 kVA, 50 Hz, 3300 V/400 V distribution transformer is connected between two 50 Hz supply systems, *A* and *B* as shown in figure, the transformer has 12 and 99 turns in the low and high voltage windings respectively. The magnetizing reactance of the transformer referred to the high voltage side is $500\ \Omega$. The leakage reactance of the high and low voltage windings are $1.0\ \Omega$ and $0.012\ \Omega$ respectively. Neglect the winding resistance and core losses of the transformer. The Thevenin's voltage of system *A* is 3300 V while that of system *B* is 400 V. The short circuit reactance of system *A* and *B* are $0.5\ \Omega$ and $0.010\ \Omega$ respectively. If no power is transferred between *A* and *B*, so that the two system voltage are in phase,



1.5 The magnetizing ampere turns of the transformer will be _________.

1.6 The phase relation between the two system voltages will be

(A) V_A leads V_B

(B) V_A lags V_B

(C) Both V_A and V_B are in phase

(D) None of the above

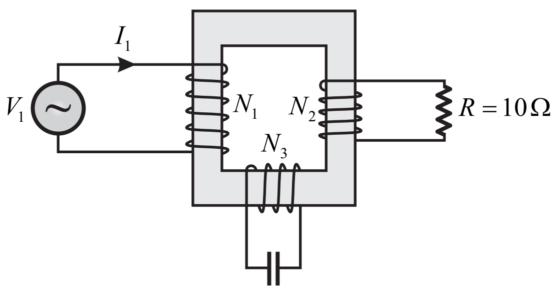
2003 IIT Madras

1.7 A single phase transformer has a maximum efficiency of 90% at full load and unity power factor. Efficiency at half load, at the same power factor is

(A) 86.7% (B) 88.26%

(C) 88.9% (D) 87.8%

1.8 Figure shows an ideal three winding transformer. Windings are wound on the same core as shown. The turns ratio $N_1 : N_2 : N_3$ is 4 : 2 : 1. A resistor of 10 Ω is connected across winding-2. A capacitor of reactance 2.5 Ω is connected across winding-3. Winding-1 is connected across a 400 V, ac supply. If the supply voltage phasor $V_1 = 400\angle 0^0$ V, the supply current phasor I_1 is given by



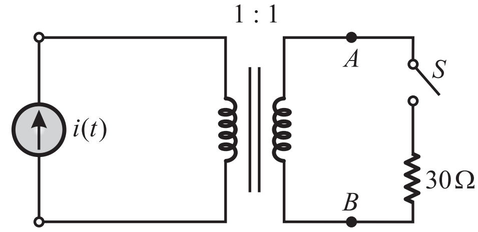
(A) $(-10 + j10)$ A (B) $(-10 - j10)$ A

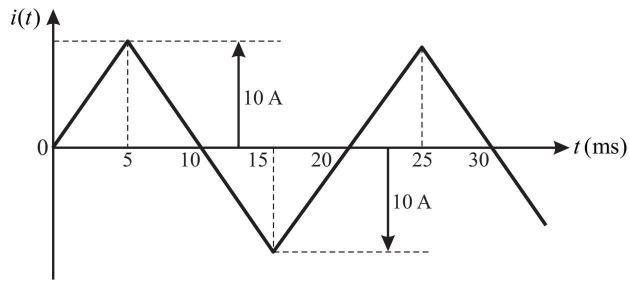
(C) $(10 + j10)$ A (D) $(10 - j10)$ A

2009 IIT Roorkee

Common Data for Questions 1.9 & 1.10

The circuit diagram shows a two-winding, lossless transformer with no leakage flux, excited from a current source $i(t)$, whose waveform is also shown. The transformer has a magnetizing inductance of $400/\pi$ mH.





1.9 The peak voltage across A and B, with S open is

(A) $\frac{400}{\pi}$ V (B) 800 V

(C) $\frac{4000}{\pi}$ V (D) $\frac{800}{\pi}$ V

1.10 If the waveform of $i(t)$ is changed to $i(t) = 10\sin(100\pi t)$ A, the peak voltage across A and B with S closed is

(A) 400 V (B) 240 V

(C) 320 V (D) 160 V

2012 IIT Delhi

1.11 A single phase 10 kVA, 50 Hz transformer with 1 kV primary winding draws 0.5 A and 55 W, at rated voltage and frequency, on no load. A second transformer has a core with all its linear dimensions $\sqrt{2}$ times the corresponding dimensions of the first transformer. The core material and lamination thickness are the same in both transformers. The primary windings of both the transformers have the same number of turns. If a rated voltage of 2 kV at 50 Hz is applied to the primary of the second transformer, then the no load current and power respectively are

(A) 0.7 A, 77.8 W (B) 0.7 A, 155.6 W
(C) 1 A, 110 W (D) 1 A, 220 W

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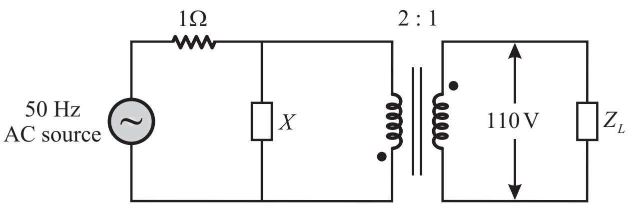
1.12 The load shown in the figure absorbs 4 kW at a power factor of 0.89 lagging.



Assuming the transformer to be ideal, the value of the reactance X to improve the input power factor to unity is ____.

[Set - 03]

2020 IIT Delhi

1.13 A single-phase, 4 kVA, 200 V/100 V, 50 Hz transformer with laminated CRGO steel core has rated no-load loss of 450 W. When the high-voltage winding is excited with 160 V, 40 Hz sinusoidal ac supply, the no-load losses are found to be 320 W. When the high-voltage winding of the same transformer is supplied from a 100 V, 25 Hz sinusoidal ac source, the no-load losses will be __________ W (rounded off to 2 decimal places).

2021 IIT Bombay

1.14 In a single phase transformer, the total iron loss is 2500 W at nominal voltage of 440 V and frequency 50 Hz. The total iron loss is 850 W at 220 V and 25 Hz. Then, at nominal voltage and frequency, the hysteresis loss and eddy current loss respectively are

(A) 1600 W and 900 W

(B) 900 W and 1600 W

(C) 250 W and 600 W

(D) 600 W and 250 W

❖❖❖❖

Explanations | Single Phase Transformer

1.1 (B)

Given :

(i) Single phase transformer
(ii) Rated voltage, $V = 200$ V
(iii) Frequency, $f = 50$ Hz
(iv) Hysteresis losses is P_h
(v) Eddy current losses is P_e
(vi) New operating condition : 160 V, 40 Hz

Here, $\frac{V}{f} = \frac{200}{50} = \frac{160}{40} = 4$ (constant)

Since, given $\frac{V}{f}$ ratio is constant.

$$B_{max} \propto \frac{V}{f}, \text{ so } B_{max} = \text{Constant}$$

Case 1 : Hysteresis loss calculation

$$P_h = K_h B_{max}^x f\, v_{core}$$

$K_h =$ Hysteresis constant

$x =$ Steinmetz coefficient

$f =$ Supply frequency

$v_{core} =$ Volume of core

As K_h, B_m, and v_{core} are constant

Hence, $P_h \propto f$

% change in hysteresis loss is given by,

$$\% \Delta P_h = \frac{P_{h_2} - P_{h_1}}{P_{h_1}} \times 100$$

$$\% \Delta P_h = \frac{f_2 - f_1}{f_1} \times 100 = \frac{40 - 50}{50} \times 100$$

$$\% \Delta P_h = -20\%$$

Here, negative sign indicates decrement.

Case 2 : Eddy current loss calculation

$$P_e = K_e B_{max}^2 f^2 t^2 v_{core}$$

$K_e =$ Eddy current constant

$f =$ Supply frequency

$t =$ Thickness of lamination

$v_{core} =$ Volume of core

As K_e, B_m, t, and v_{core} are constant

Hence, $P_e \propto f^2$

% change in eddy current loss is given by,

$$\% \Delta P_e = \frac{P_{e_2} - P_{e_1}}{P_{e_1}} \times 100$$

$$\% \Delta P_e = \frac{f_2^2 - f_1^2}{f_1^2} \times 100 = \frac{40^2 - 50^2}{50^2} \times 100$$

$$\% \Delta P_e = -36\%$$

Here, negative sign indicates decrement.

Hence, the correct option is (B).

Scan for Video Solution

Key Point

For the problem based on core losses i.e. P_h and P_e, always do check first that (V/f) ratio is constant or not.

(i) If $\frac{V}{f} =$ constant or $B_m \propto \left(\frac{V}{f}\right) =$ constant $P_h \propto f$ and $P_e \propto f^2$

(ii) If $\frac{V}{f} \neq$ constant or $B_m \propto \left(\frac{V}{f}\right) \neq$ constant $P_h \propto \left(\frac{V}{f}\right)^x f \rightarrow P_h \propto V^x f^{1-x}$ and $P_e \propto V^2$

1.2 0.26

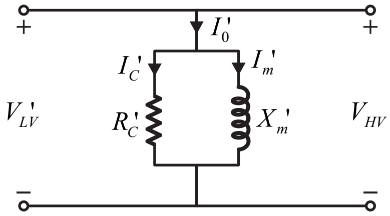
Given :

(i) $1-\phi$ 200 V/400 V transformer
(ii) kVA rating, $S = 4$ kVA
(iii) Supply frequency, $f = 50$ Hz
(iv) Mutual inductance, $M = 1.91$ H
(v) O.C test at L.V. side, $P_{O.C} = 80$ W

(vi) $X_m = 2\pi f M$

Transformation ratio, $k = \dfrac{400}{200} = 2$

The equivalent circuit referred to secondary side or HV side is shown below,



Magnetizing reactance (Referred to secondary) is given by,

$$X_m' = 2\pi f M K^2 = 2\pi \times 50 \times 1.91 \times 2^2 = 2400\ \Omega$$

Magnetizing current is given by,

$$I_m' = \frac{V_{H.V.}}{X_m'} = \frac{400}{2400} = 0.167\ \text{A}$$

Losses must remain constant while measuring from both sides,

$$P_{O.C(HV)} = P_{O.C(LV)} = 80\ \text{W}$$

Core loss component of current,

$$I_C' = \frac{P_{O.C(H.V)}}{V_{H.V}} = \frac{80}{400} = 0.2$$

No load current is given by,

$$I_0' = I_C' + jI_m'$$

$$|I_0'| = \sqrt{I_C'^2 + I_m'^2}$$

$$|I_0'| = \sqrt{0.2^2 + 0.167^2} = 0.26\ \text{A}$$

Hence, the value of current on the H.V. side is **0.26 A.**

1.3 (A)

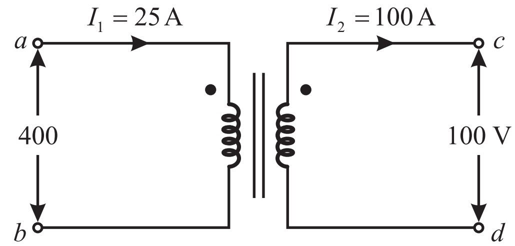
Given :

(i) Single-phase 400 V/100 V transformer

(ii) kVA rating, $S = 10$ kVA

(iii) It is reconnected as auto transformer

The equivalent circuit is shown below,



Primary current of transformer,

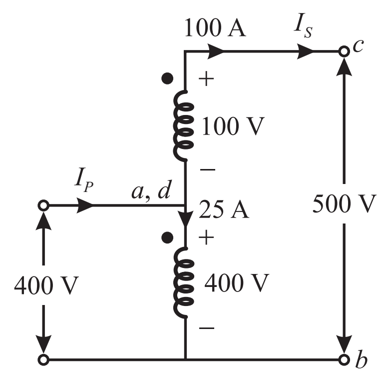
$$I_1 = \frac{10 \times 10^3}{400} = 25\ \text{A}$$

Secondary current of transformer,

$$I_2 = \frac{10 \times 10^3}{100} = 100\ \text{A}$$

Method 1 : Additive polarity concept

For maximum kVA rating of auto transformer the transformation ratio to be near to unity. Therefore, connecting *a* to *d* the auto transformer connection is shown below,



Primary current of auto-transformer,

$$I_P = 100 + 25 = 125\ \text{A}$$

Secondary current of auto-transformer,

$$I_S = 100\ \text{A}$$

Primary voltage, $V_P = 400$ V

Secondary voltage, $V_S = 500$ V

Maximum VA rating $= V_P I_P = V_S I_S$

$$= 500\ \text{V} \times 100\ \text{A} = 50\ \text{kVA}$$

Hence, the correct option is (A).

Method 2 : Direct approach

If *k* is the transformation ratio of auto transformer (in step down format) then,

$$\frac{S_{auto}}{S_{two\ winding}} = \frac{1}{1-k}$$

Here, $k = \frac{400}{500} = 0.8$

Hence, $\frac{S_{auto}}{S_{two\ winding}} = \frac{1}{1-0.8} = 5$

$$S_{auto} = S_{two\ winding} \times 5$$

$$S_{auto} = 10 \times 5 = 50 \text{ kVA}$$

Hence, the correct option is (A).

1.4 (C)

Given :

For transformer *A* :

(i) Single-phase 400 V/200 V transformer

(ii) kVA rating, $S_A = 10$ kVA

(iii) Resistance, $R_{pu} = 0.03$

(iv) Reactance, $X_{pu} = 0.04$

For transformer *B* :

(i) Single-phase 400 V/200 V transformer

(ii) kVA rating, $S_B = 5$ kVA

(iii) Resistance, $R_{pu} = 0.04$

(iv) Reactance, $X_{pu} = 0.03$

For load :

(i) Power, $P_L = 12$ kW

(ii) Power factor, $\cos\phi_L = 0.8$ lag

Load, $S_L (\text{in kVA}) = \frac{P_L}{\cos\phi_L} = \frac{12}{0.8} = 15 \text{ kVA}$

Let base kVA = 10 kVA

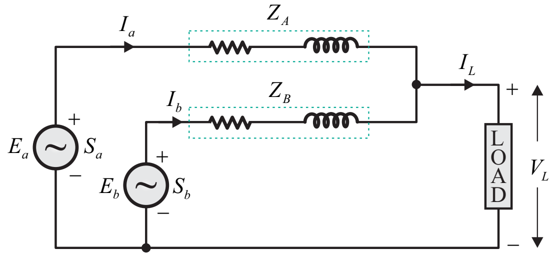
$$Z_{A(pu)} = (0.03 + j0.04)$$

[At common base of 10 kVA]

$$Z_{B(pu)_{new}} = \frac{\text{MVA}_{new}}{\text{MVA}_{old}} \times Z_{B(pu)_{old}}$$

$$Z_{B(pu)_{new}} = \frac{10}{5}(0.04 + j0.03) = 0.08 + j0.06$$

The equivalent circuit of parallel operation is shown below,



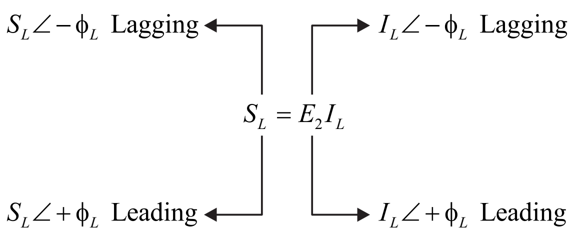
Load shared by transformer *A* is given by,

$$S_A = \left(\frac{Z_B}{Z_A + Z_B}\right)^* \times S_L \qquad \dots \text{(i)}$$

Load shared by transformer *B* is given by,

$$S_B = \left(\frac{Z_A}{Z_A + Z_B}\right)^* \times S_L \qquad \dots \text{(ii)}$$

$$S_L = \text{Load kVA}$$



Put the values in equation (i),

$$S_A = \left(\frac{0.08 + j0.06}{0.03 + j0.04 + 0.08 + j0.06}\right)^* \times 15\angle -36.86^0$$

$$S_A = (7.46 + j6.78) \text{ kVA}$$

Thus, Active power = 7.5 kW

Reactive power = 6.78 kVAR

Hence, the correct option is (C).

Scan for Video Solution

Due to slip of tongue induced emf was pronounced as terminal voltage, actually it is induced emf (voltage).

1.5 652.806

Given :

(i) 3300 V/ 400 V single phase transformer

(ii) Supply frequency, $f = 50$ Hz

(iii) Magnetizing reactance, $X_m = 500\ \Omega$

(iv) Leakage reactances :

Primary, $X_1 = 1\ \Omega$

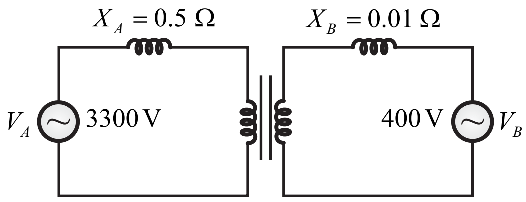
Secondary, $X_2 = 0.012\ \Omega$

(v) Turns :

Primary, $N_1 = 99$

Secondary, $N_2 = 12$

The system is shown below,



where, transformation ratio

$$k = \frac{N_2}{N_1} = \frac{12}{99} = \frac{4}{33} = 0.1212$$

The secondary reactance refer to primary,

$$X_2' = \frac{X_2}{k^2} = \frac{0.012}{(4/33)^2} = 0.82\ \Omega$$

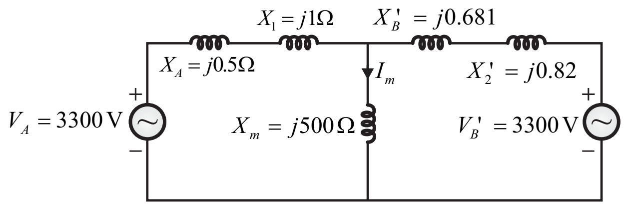
The secondary reactance $X_B = 0.01\ \Omega$ referred to primary,

$$X_B' = \frac{X_B}{k^2} = \frac{0.01}{(4/33)^2} = 0.681\ \Omega$$

The secondary voltage V_B referred to primary,

$$V_B' = \frac{V_B}{k} = \frac{400}{(4/33)} = 3300\ \text{V}$$

The equivalent circuit of above system referred to primary is given by,



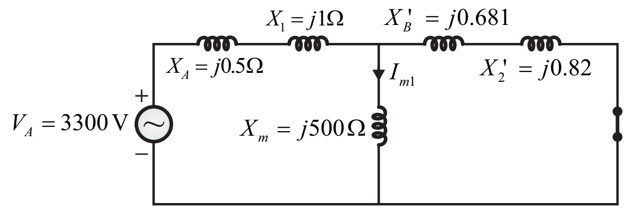
To get the magnetizing MMF or ampere turn, first calculate the value of I_m. To get the value of I_m, we can opt different circuit analysis procedures as given below,

Using superposition theorem,

$$I_m = I_{m_1} + I_{m_2}$$

Case I :

When V_A is active. $V_B' \to SC$



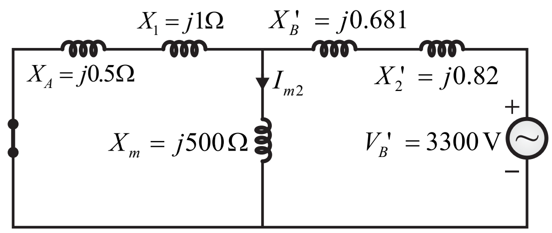
$$I_{m_1} = \frac{V_A}{(jX_m \parallel j1.49) + j0.6} \times \frac{1.49}{501.49}$$

$$I_{m_1} = \frac{3300}{(j500 \parallel j1.49) + j0.6} \times \frac{1.49}{501.49}$$

$$I_{m_1} = 4.701\ \text{A}$$

Case II :

When V_B' is active. $V_A \to SC$



$$I_{m_2} = \frac{V_B'}{(jX_m \parallel j0.6) + j1.49} \times \frac{0.6}{500.6}$$

$$I_{m_2} = \frac{3300}{(j500 \parallel j0.6) + j1.49} \times \frac{0.6}{500.6}$$

$$I_{m_2} = 1.893\ \text{A}$$

Hence, $I_m = I_{m_1} + I_{m_2}$

$$I_m = 4.701 + 1.893 = 6.594\ \text{A}$$

Hence, magnetizing Ampere turns,

$$AT = N_1 I_m = 99 \times 6.594 = 652.806\ \text{AT}$$

Hence, the magnetizing ampere turns is **652.806 AT.**

1.6 (C)

Power transfer from system A to B is given by,

$$P = \frac{V_A V_B}{X_{eq}} \sin\delta$$

For $P = 0$,

$$\frac{V_A V_B}{X_{eq}} \sin\delta = 0$$

i.e. $\sin\delta = 0$

$\delta = 0^0$

Thus, V_A is in phase with V_B.

Hence, the correct option is (C).

1.7 (D)

Given :

(i) Single phase transformer

(ii) Maximum efficiency, $\eta_{max} = 90\%$ at rated load and unity power factor.

Efficiency of single phase transformer is given by,

$$\eta = \frac{m\times(\text{kVA})\times\cos\phi}{m\times(\text{kVA})\cos\phi + P_i + m^2 P_{cu_{fl}}} \quad \ldots\text{(i)}$$

where, kVA = Rating of transformer

$\cos\phi$ = Power factor

P_i = Iron or core loss

$P_{cu_{fl}}$ = Full load copper loss

m = Fraction of load

Case I : At full load

Given :

$\eta_{max} = 0.9$

$m = 1.0$

$\cos\phi = 1.0$

$P_i = P_{cu_{fl}}$ for $m = 1.0$

From equation (i),

$$\frac{1\times(\text{kVA})\times 1}{1\times(\text{kVA})\times 1 + P_i + P_{cu_{fl}}} = 0.9$$

$$\frac{(\text{kVA})}{(\text{kVA}) + 2P_i} = 0.9$$

$$P_i = 0.055\,\text{kVA} \quad \ldots\text{(ii)}$$

$$P_i = P_{cu_{fl}} = 0.055\text{ kVA}$$

Case II : At half load

Given : $m = \frac{1}{2}$, $\cos\phi = 1$

Efficiency is given by,

$$\eta = \frac{\frac{1}{2}\times(\text{kVA})\times 1}{\frac{1}{2}\times(\text{kVA})\times 1 + P_i + \left(\frac{1}{2}\right)^2 P_{cu_{fl}}}$$

[From equation (i)]

$$\eta = \frac{0.5(\text{kVA})}{0.5(\text{kVA}) + P_i + 0.25P_{cu_{fl}}}$$

From equation (ii),

$$\eta = \frac{0.5(\text{kVA})}{\left[\begin{array}{r}0.5(\text{kVA}) + 0.055\ (\text{kVA}) + 0.25 \\ \times\, 0.055(\text{kVA})\end{array}\right]}$$

$$\eta = \frac{0.5}{0.5 + 0.055 + 0.25\times 0.055} = 0.878$$

$\%\ \eta = 87.8\%$

Hence, the correct option is (D).

1.8 (C)

Given :

(i) Three winding transformer

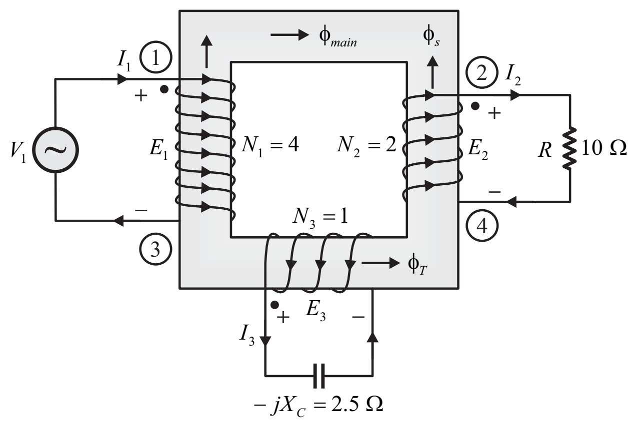
(i) Turns ratio, $N_1 : N_2 : N_3 = 4 : 2 : 1$

(iii) Resistance at secondary, $R = 10\ \Omega$

(iv) Capacitor at tertiary, $X_C = 2.5\ \Omega$

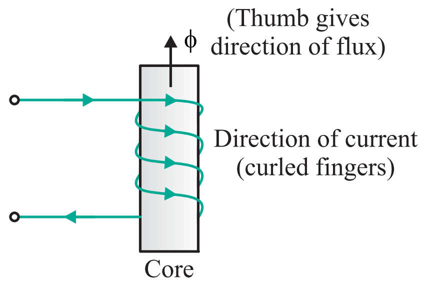
(v) Primary voltage, $V_1 = 400$ V

Given figure is shown below,



1. In primary, current flows from dot to coil.
2. In secondary and tertiary current flows from coil to dot.

3. Primary current produces main flux ϕ_{main} according to right hand thumb rule.



4. Secondary and tertiary currents produce flux ϕ_S and ϕ_T which oppose ϕ_{main}, ϕ_P compensate ϕ_S and ϕ_T.

In a transformer, emf per turn is constant.

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = \frac{E_3}{N_3}$$

$$\frac{400\angle 0^0}{4} = \frac{E_2}{2} = \frac{E_3}{1}$$

$$E_2 = 200\angle 0^0 \text{ V}, \; E_3 = 100\angle 0^0 \text{ V}$$

Load current in secondary winding is given by,

$$I_2 = \frac{E_2}{R} = \frac{200\angle 0^0}{10} = 20\angle 0^0 = 20 \text{ A}$$

Load current in tertiary winding is given by,

$$I_3 = \frac{E_3}{-jX_C} = \frac{100\angle 0^0}{-j2.5} = 40\angle 90^0 = j40 \text{ A}$$

Method 1 : MMF Balance Equation

MMF balance equation is given by,

$$I_1 N_1 = I_2 N_2 + I_3 N_3$$

$$I_1 = I_2 \frac{N_2}{N_1} + I_3 \frac{N_3}{N_1} = 20 \times \frac{2}{4} + j40 \times \frac{1}{4}$$

$$I_1 = (10 + j10) \text{ A}$$

Hence, the correct option is (C).

Method 2 : Power Balance Equation

Power balance equation is given by,

$$S_1 = S_2 + S_3$$

$$E_1 I_1^* = E_2 I_2^* + E_3 I_3^*$$

$$I_1^* = \frac{E_2}{E_1} I_2^* + \frac{E_3}{E_1} I_3^*$$

$$I_1^* = \frac{200\angle 0^0}{400\angle 0^0} \times 20\angle 0^0 + \frac{100\angle 0^0}{400\angle 0^0} \times 40\angle -90^0$$

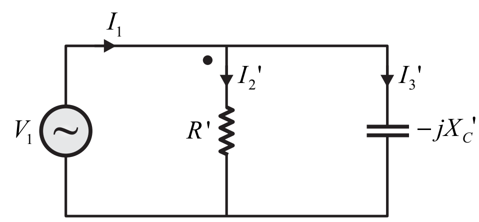
$$I_1^* = 14.14\angle -45^0$$

Hence, $I_1 = 14.14\angle 45^0 = (10 + j10)$ A

Hence, the correct option is (C).

Method 3 : Concept of Equivalent Circuit

Refer the secondary and tertiary circuit on primary side as shown below. While referring, it must be taken care of that the dotted terminal of secondary and tertiary must be connected to the dotted terminal of primary.



I_2' = Secondary current referred to primary side.

$$I_2' = I_2 \times \frac{N_2}{N_1} = 20\angle 0^0 \times \frac{2}{4} = 10\angle 0^0 \text{ A}$$

$$I_3' = I_3 \times \frac{N_3}{N_1} = 40\angle 90^0 \times \frac{1}{4} = 10\angle 90^0 \text{ A}$$

Hence, $I_1 = I_2' + I_3' = (10\angle 0^0) + (10\angle 90^0)$

$$I_1 = 14.14\angle 45^0 = (10 + j10) \text{ A}$$

Hence, the correct option is (C).

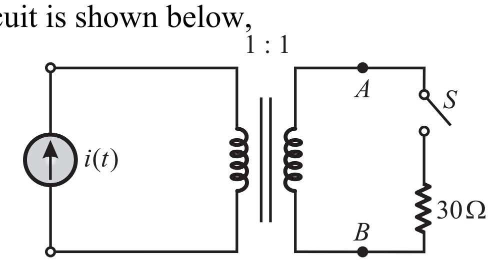
Scan for Video Solution

1.9 (D)

Given :

Magnetizing inductance, $L_m = \frac{400}{\pi}$ mH

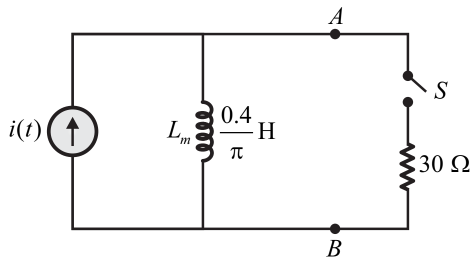
Circuit is shown below,



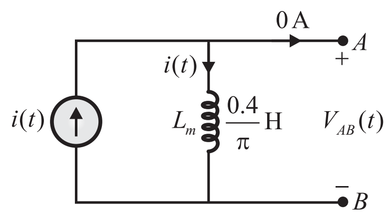
Magnetizing inductance,

$$L_m = \frac{400}{\pi}\text{ mH} = \frac{0.4}{\pi}\text{ H}$$

Equivalent circuit referred to primary of (1 : 1) transformer

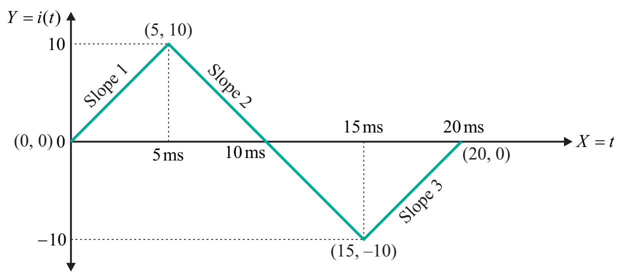


When switch is open, no current flows in $30\,\Omega$ resistor and $i(t)$ flows through L_m.



$$V_{AB}(t) = L_m \frac{d}{dt} i(t) = \frac{0.4}{\pi}\frac{di(t)}{dt}$$

Current waveform is shown below,



From equation of slope,

$$\frac{Y - y_1}{X - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

For slope-1 $(0 < t < 5 \text{ ms})$,

$$\frac{i(t) - 0}{t - 0} = \frac{10 - 0}{(5 - 0)\times 10^{-3}}$$

$$i(t) = 2000t$$

For slope-2 $(5 \text{ ms} < t < 15 \text{ ms})$,

$$\frac{i(t) - 10}{t - (5\times 10^{-3})} = \frac{-10 - 10}{(15 - 5)\times 10^{-3}}$$

$$i(t) = -2000t + 20$$

For slope-3 $(15 \text{ ms} < t < 20 \text{ ms})$,

$$\frac{i(t) - (-10)}{t - (15\times 10^{-3})} = \frac{0 - (-10)}{(20 - 15)\times 10^{-3}}$$

$$i(t) = 2000t - 40$$

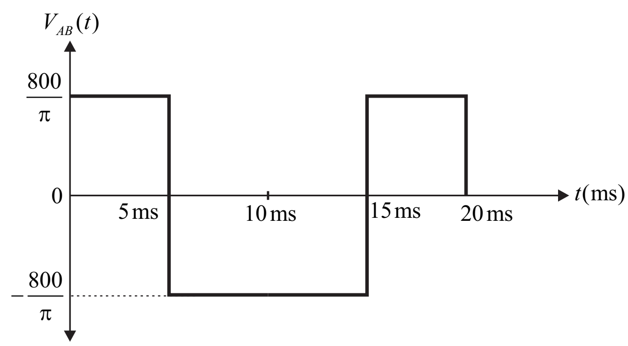
$$i(t) = \begin{cases} 2000t & ; \quad 0 < t < 5 \text{ ms} \\ -2000t + 20 & ; \quad 5 \text{ ms} < t < 15 \text{ ms} \\ 2000t - 40 & ; \quad 15 \text{ ms} < t < 20 \text{ ms} \end{cases}$$

$$V_{AB}(t) = \frac{0.4}{\pi}\frac{d}{dt} i(t)$$

$$V_{AB}(t) = \frac{0.4}{\pi}\begin{cases} \frac{d}{dt}(2000t) & ; \quad 0 < t < 5 \text{ ms} \\ \frac{d}{dt}(-2000t + 20) & ; \quad 5 \text{ ms} < t > 15 \text{ ms} \\ \frac{d}{dt}(2000t - 40) & ; \quad 15 \text{ ms} < t < 20 \text{ ms} \end{cases}$$

$$V_{AB}(t) = \begin{cases} \frac{800}{\pi} & ; \quad 0 < t < 5 \text{ ms} \\ \frac{-800}{\pi} & ; \quad 5 \text{ ms} < t < 15 \text{ ms} \\ \frac{800}{\pi} & ; \quad 15 \text{ ms} < t < 20 \text{ ms} \end{cases}$$

Waveform of V_{AB} is shown below,



Therefore, Peak value of V_{AB} is $\frac{800}{\pi}$ V.

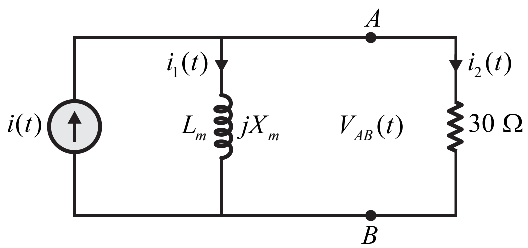
Hence, the correct option is (D).

Scan for Video Solution

1.10 (B)

Given : $i(t) = 10\sin(100\pi t)$ A

When the switch is closed the circuit is shown below,



$$jX_m = j\omega L_m = j100\pi \times \frac{0.4}{\pi} = j40\ \Omega$$

$$V_{AB}(t) = i_1(t) \times j40 = i_2(t) \times 30$$

$$i_1(t) = \frac{30 \times i(t)}{30 + j40} \qquad \text{[By CDR]}$$

$$i_2(t) = \frac{j40 \times i(t)}{30 + j40} \qquad \text{[By CDR]}$$

Voltage across A and B is given by,

$$V_{AB}(t) = i_2(t) \times 30$$

$$V_{AB}(t) = \frac{j40 \times 30\, i(t)}{30 + j40} = \frac{j1200\, i(t)}{30 + j40}$$

$$V_{AB}(t) = \frac{1200\angle 90^0}{50\angle 53.13^0} \times i(t)$$

$$V_{AB}(t) = 24\angle 36.87^0 \times i(t)$$

$$V_{AB}(t) = (24\angle 36.87^0) \times 10\sin(100\pi t)$$

$$V_{AB}(t) = 240\sin(100\pi t + 36.87^0)$$

Hence, the correct option is (B).

Scan for Video Solution

1.11 (B)

Given :

(i) Transformer rating, S = 10 kVA

(ii) Frequency, f = 50 Hz,

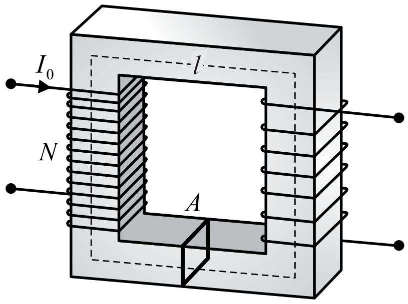
(iii) Primary supply, E_1 = 1 kV,

(iv) No load loss, P_{0_1} = 55 W

(v) No load current, $I_{0_1} = 0.5$ A

(vi) Linear dimension of second transformer is $\sqrt{2}$ times of first transformer.

(vii) 2 kV, 50 Hz is applied to primary of second transformer.



Method 1

Emf equation is given by,

$$E = 4.44\, fN\phi_m$$

Since, N and f is constant

$$E \propto \phi_m \qquad \text{...(i)}$$

$$\phi_m = B_m \times A = \mu H \times A = \frac{\mu N I_0 A}{l}$$

Hence, $\phi_m \propto \dfrac{I_0 A}{l}$ ...(ii)

From equations (i) and (ii)

$$E \propto \frac{I_0 A}{l}$$

where, A = Cross sectional area of core,

l = Length of core,

I_0 = No load current

$$\frac{E_2}{E_1} = \frac{I_{0_2} A_2}{I_{0_1} A_1} \times \frac{l_1}{l_2} \qquad \text{...(iii)}$$

Since, linear dimensions of second transformer is $\sqrt{2}$ times of the first transformer (given).

So, $A_2 = (\sqrt{2})^2 A_1$ and $l_2 = \sqrt{2}\, l_1$

$$E_2 = 2 \text{ kV}$$

$$E_1 = 1 \text{ kV} \quad \text{(given)}$$

Put the values in equation (iii),

$$\frac{2}{1}=\frac{I_{0_2}}{I_{0_1}}\frac{(\sqrt{2})^2 A_1}{A_1}\times\frac{l_1}{\sqrt{2}\,l_1}$$

$$I_{0_2}=\sqrt{2}\,I_{0_1}\quad (\text{Given, } I_{0_1}=0.5\text{ A})$$

$$I_{0_2}=\sqrt{2}\times 0.5=0.707\text{ A}$$

Maximum flux density of core is given by,

$$B_{\max}=\frac{\phi_{\max}}{A}$$

$$\phi_{\max}=\frac{E}{4.44\,N\times f}$$

$$\frac{B_{\max 2}}{B_{\max 1}}=\frac{\phi_{\max 2}}{A_2}\times\frac{A_1}{\phi_{\max 1}}$$

$$\frac{B_{\max 2}}{B_{\max 1}}=\frac{E_2}{4.44N\times f\times A_2}\times\frac{4.44N\times f\times A_1}{E_1}$$

$$\frac{B_{\max 2}}{B_{\max 1}}=\frac{2000}{2A_1}\times\frac{A_1}{1000}$$

$$B_{\max 2}=B_{\max 1}$$

Hence, core loss $\propto$ volume of core

$$\frac{P_{o_2}}{P_{o_1}}=\frac{v_2}{v_1}=\frac{A_2 l_2}{A_1 l_1}=\frac{(\sqrt{2})^2\times\sqrt{2}A_1 l_1}{A_1 l_1}$$

$$\frac{P_{o_2}}{55}=\frac{(\sqrt{2})^3}{1}$$

$$P_{o_2}=\frac{(\sqrt{2})^3}{1}\times 55=155.6\text{ W}$$

Hence, No load current is 0.7 A and no load power is 155.6 W

Hence, the correct option is (B).

Method 2

No load current $\propto$ Linear dimension

$$\frac{I_1}{I_2}=\frac{\text{dimension}_1}{\text{dimension}_2}$$

$$\frac{0.5}{I_2}=\frac{1}{\sqrt{2}}$$

$$I_2=\sqrt{2}\times 0.5=0.7\text{ A}$$

$$\text{Power}\propto(\text{Linear dimension})^3$$

$$\frac{P_1}{P_2}=\left(\frac{\text{dimension}_1}{\text{dimension}_2}\right)^3$$

$$\frac{55}{P_2}=\left(\frac{1}{\sqrt{2}}\right)^3$$

$$P_2=2\sqrt{2}\times 55$$

$$P_2=1.414\times 110=155.6\text{ W}$$

Hence, the correct option is (B).

Key Point

Reluctance : Reluctance is basically inversely proportional to flux.

$$\text{Reluctance, } S=\frac{l}{\mu A}=\frac{NI}{\mu HA}=\frac{NI}{BA}=\frac{NI}{\phi}$$

$$\phi=\frac{NI}{S}\qquad\left[\text{Flux}=\frac{\text{MMF}}{\text{Reluctance}}\right]$$

Ohm's law in magnetic field.

1.12 | 23.62

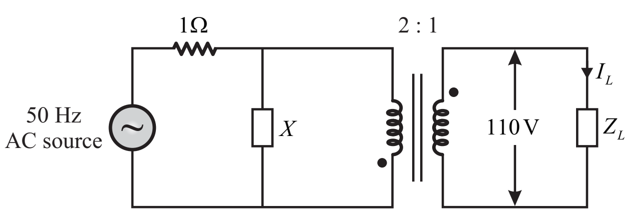
Given :

(i) Load : Power, $P = 4$ kW,

Power factor $=\cos\phi_1=0.89$ lag

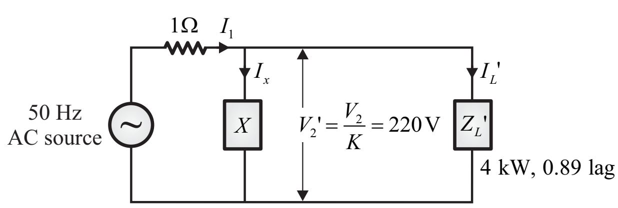
(ii) Transformation ratio, $k=\frac{1}{2}$

Circuit is shown below,



Method 1

The equivalent circuit refer to primary is shown below,



Load current, $I_L' = \dfrac{P}{V_2'\cos\phi} = \dfrac{4\times10^3}{220\times0.89}$

$$I_L' = 20.43\angle -27.13^0 \text{ A}$$

$$(\text{As } \phi = \cos^{-1} 0.89 = 27.13^0)$$

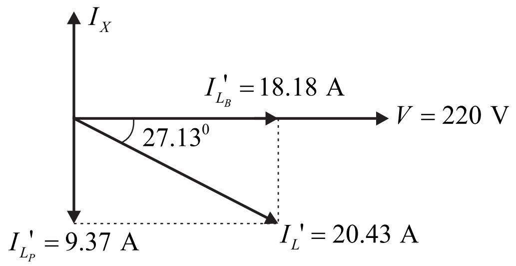
Case 1 : When reactance X is not connected

$$I_1 = I_L' = 20.43\angle -27.13^0$$

$$= 18.18 - j9.37 \text{ A}$$

Case 2 : When reactance X is connected,

Phasor diagram is given by,



Now, to make input power factor unity, the current through reactance is given by,

$$I_X = I_{L_p}' = I_L' \sin(27.13) = 9.37 \text{ A}$$

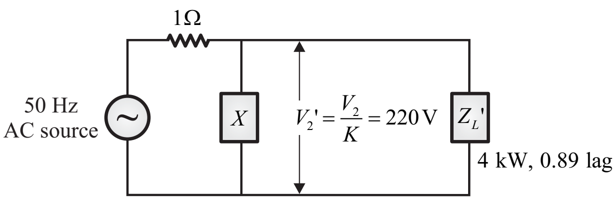
Also, $I_X = \dfrac{V_2'}{X}$

Thus, $X = \dfrac{220}{9.37} = 23.62\ \Omega$

Hence, the value of the reactance X to improve the input power factor to unity is **23.62 Ω .**

Method 2

The equivalent circuit refer to primary is shown below,



Now, the reactance power supplied by the capacitance is given by,

$$Q_C = P[\tan\phi_1 - \tan\phi_2]$$

$$\phi_1 = \cos^{-1} 0.89 = 27.12^0$$

$$\phi_2 = \cos^{-1} 1 = 0^0$$

Hence, $Q_C = 4\times10^3\left[\tan(27.12^0) - \tan(0^0)\right]$

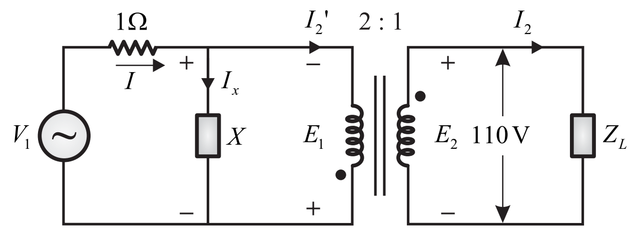
$$Q_C = 2.05 \text{ kVAR}$$

Also, $Q_C = \dfrac{(V_2')^2}{X}$

$$X = \frac{220^2}{2.05\times10^3} = 23.62\ \Omega$$

Hence, the value of the reactance X to improve the input power factor to unity is **23.62 Ω .**

Method 3



X should be capacitive in order to compensate for reactive power and hence bring the pf to unity.

So, I_x will lead $-E_1$ by 90^0 and lag $+E_1$ by 90^0.

As per dot convention,

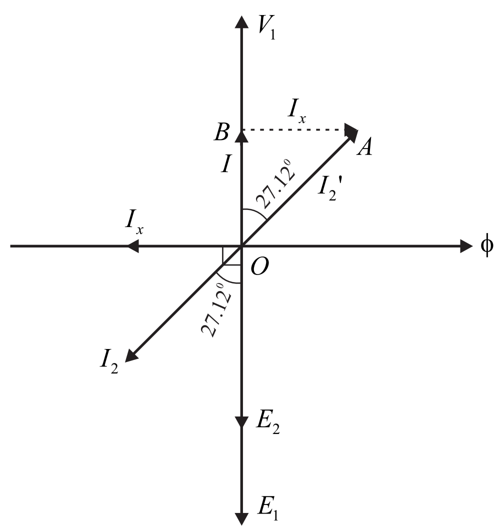
$$I_2' + I_x = I$$

I and V_1 are in phase (upf),

$$\phi_1 = \cos^{-1} 0.89 = 27.12^0$$

$$I_2 = \frac{4000}{110\times0.89}\angle -27.12^0 = 40.85\angle -27.12$$

$$I_2' = kI_2 = \frac{I_2}{2} = 20.43\angle -27.12$$



So from ΔOAB,

$$\frac{AB}{OA} = \sin(27.12^0)$$

$$\frac{I_x}{I_2'} = \sin(27.12^0)$$

Therefore, $I_x = I_2' \sin 27.12$

$$I_x = 20.43 \times \sin 27.12 = 9.31 \text{ A}$$

$$\frac{E_1}{I_x} = \frac{220}{9.31} = 23.62\ \Omega$$

Hence, the value of the reactance X to improve the input power factor to unity is **23.62 Ω.**

1.13 162.5

Given : 4 kVA, 200 V/100 V single phase transformer

(i) $V = 200\text{ V}$

(ii) $f = 50\text{ Hz}$

(iii) $P_{(core)} = 450\text{ W}$

Case-I : $V_1 = 200\text{ V}$, $f_1 = 50\text{ Hz}$,

$P_{(core)1} = 450\text{ W}$

$$\frac{V_1}{f_1} = \frac{200}{50} = 4$$

$$P_{(core)_1} = Af_1 + Bf_1^2$$

$$450 = 50A + 2500B \qquad \ldots(i)$$

Case-II : $V_2 = 160\text{ V}$, $f_2 = 40\text{ Hz}$,

$P_{(core)_2} = 320\text{ W}$

$$\frac{V_2}{f_2} = \frac{160}{40} = 4$$

$\frac{V}{f}$ is constant

$$P_{(core)2} = Af_2 + Bf_2^2$$

$$320 = 40A + 1600B \qquad \ldots(ii)$$

Solving equation (i) and (ii)

$$A = 4,\ B = 0.1$$

Case-III : $V_3 = 100\text{ V}$, $f_3 = 25\text{ Hz}$, $P_{(core)3} = ?$

$$P_{(core)3} = Af_3 + Bf_3^2$$

$$P_{(core)3} = 162.5\text{ W}$$

1.14 B

Given : Single phase transformer

Case 1 : $V_1 = 440\text{ V}, f_1 = 50\text{ Hz}$

Total iron loss, $P_1 = 2500\text{ W}$

$$\frac{V}{f} = \frac{V_1}{f_1} = \frac{440}{50} = 8.8$$

Case 2 : $V_2 = 220\text{ V}, f_2 = 25\text{ Hz}$

Total iron loss, $P_2 = 850\text{ W}$

$$\frac{V}{f} = \frac{V_2}{f_2} = \frac{220}{25} = 8.8$$

Here, $\frac{V}{f} =$ constant

Hysteresis losses is given by

$$P_h = K_h (B_m)^x f\, v$$

$K_h =$ Hysteresis constant

$x =$ Steinmetz constant

$B_m =$ Maximum flux density

$f =$ Supply frequency

$v =$ volume of the core

Eddy current loss is given by

$$P_e = K_e (B_m)^2 f^2 v$$

$K_e =$ Eddy current constant

$x =$ Steinmetz constant

$B_m =$ Maximum flux density

$f =$ Supply frequency

$v =$ volume of the core

Core loss is defined as,

$$P_i = P_h + P_e$$

Hysteresis loss, $P_h = K_h B_m^{1.6} f$

Eddy current loss, $P_e = K_e B_m^2 f^2$

$$P_e = K_e f^2$$

$$P_h = K_h f$$

and at $V_2 = 220\text{ V}$,

$f_2 = 25\,\text{Hz}, P_2 = 850\,\text{W}$

As ratio of $\dfrac{V}{f}$ is constant in both case.

So, $\left(B_m \propto \dfrac{V}{f}\right)$ is also constant.

$\because \quad V \approx E = 4.44\ f\phi_m N$

$$B_m = \frac{\phi_m}{A} = \frac{V}{4.44\,fNA}$$

[N and A = Constant]

Case 1 : At normal voltage

$V_1 = 440\,\text{V},\ f_1 = 50\,\text{Hz},\ P_i = 2500\,\text{W}$

$P_1 = K_h f + K_e f^2$

$2500 = 50K_h + (50)^2 K_e$

$K_h + 50K_e = 50 \quad \ldots\text{(i)}$

Case 2 : At $V_2 = 220\,\text{V},\ f_2 = 25\,\text{Hz}, P_2 = 850\,\text{W}$

$P_2 = K_h f_2 + K_e f_2^2$

$850 = 25K_h + (25)^2 K_e$

$K_h + 25K_e = 34 \quad \ldots\text{(ii)}$

By equation (i) and equation (ii),

$$\begin{array}{r} K_h + 50K_e = 50 \\ -K_h + 25K_e = 50 \\ \hline 25K_e = 16 \end{array}$$

$K_e = 0.64$

Putting the value of K_e in equation (i),

$K_h + 50(0.64) = 50$

$K_h = 18$

At nominal voltage $V = 440,\ f = 50\,\text{Hz}$

Hysteresis loss, $P_h = K_h f$

$P_h = 18 \times 50 = 900\,\text{W}$

Eddy current loss, $P_e = K_e\ f^2$

$P_e = (0.64) \times 50^2 = 1600\,\text{W}$

Hence, the correct option is (B).

❖❖❖❖

2 Three Phase Transformer

➢ Partial Synopsis

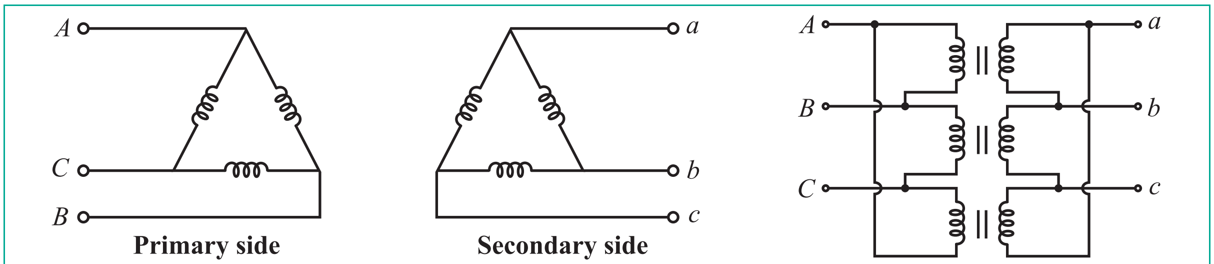
Three phase transformer connections

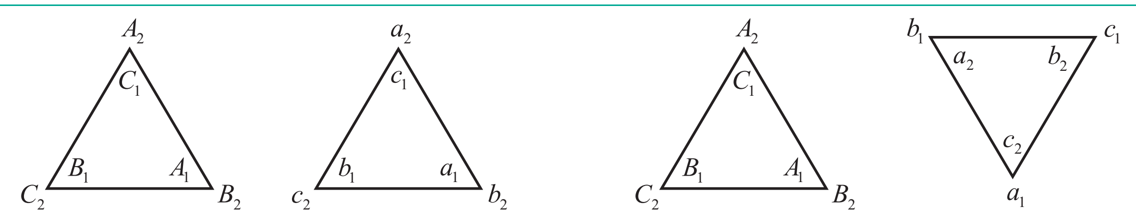
The primary and secondary windings of three phase transformers as three phase windings can be connected in different ways such as in star or in delta. With suitable windings the voltage can be raised or lowered. The most useful connections are :

(1) Star-star $(Y-Y)$ connection
(2) Delta-delta $(\Delta-\Delta)$ connection
(3) Star-delta $(Y-\Delta)$ connection
(4) Delta-star $(\Delta-Y)$ connection
(5) Open delta or V connection
(6) Scott connection or T-T connection

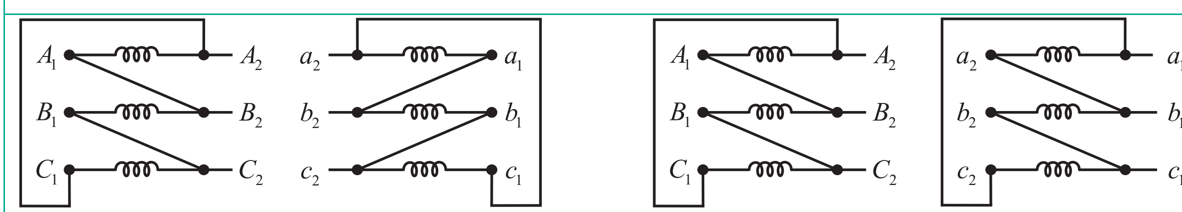
Delta-Delta Connection

In this type of connection, both the three phase primary and secondary windings are connected in delta.





$Dd0$ (0^0) connection $\quad$ $Dd6$ (180^0) connection



$Dd0$ (0^0) connection $\quad$ $Dd6$ (180^0) connection

There is no phase shift between primary and secondary voltages.

V_{L1} / I_{L1} = Line voltage / Line current on primary side

V_{L2} / I_{L2} = Line voltage / Line current on secondary side

V_{ph1} / I_{ph1} = Phase voltage / Phase current on primary side

V_{ph2} / I_{ph2} = Phase voltage / Phase current on secondary side

K = Transformation ratio

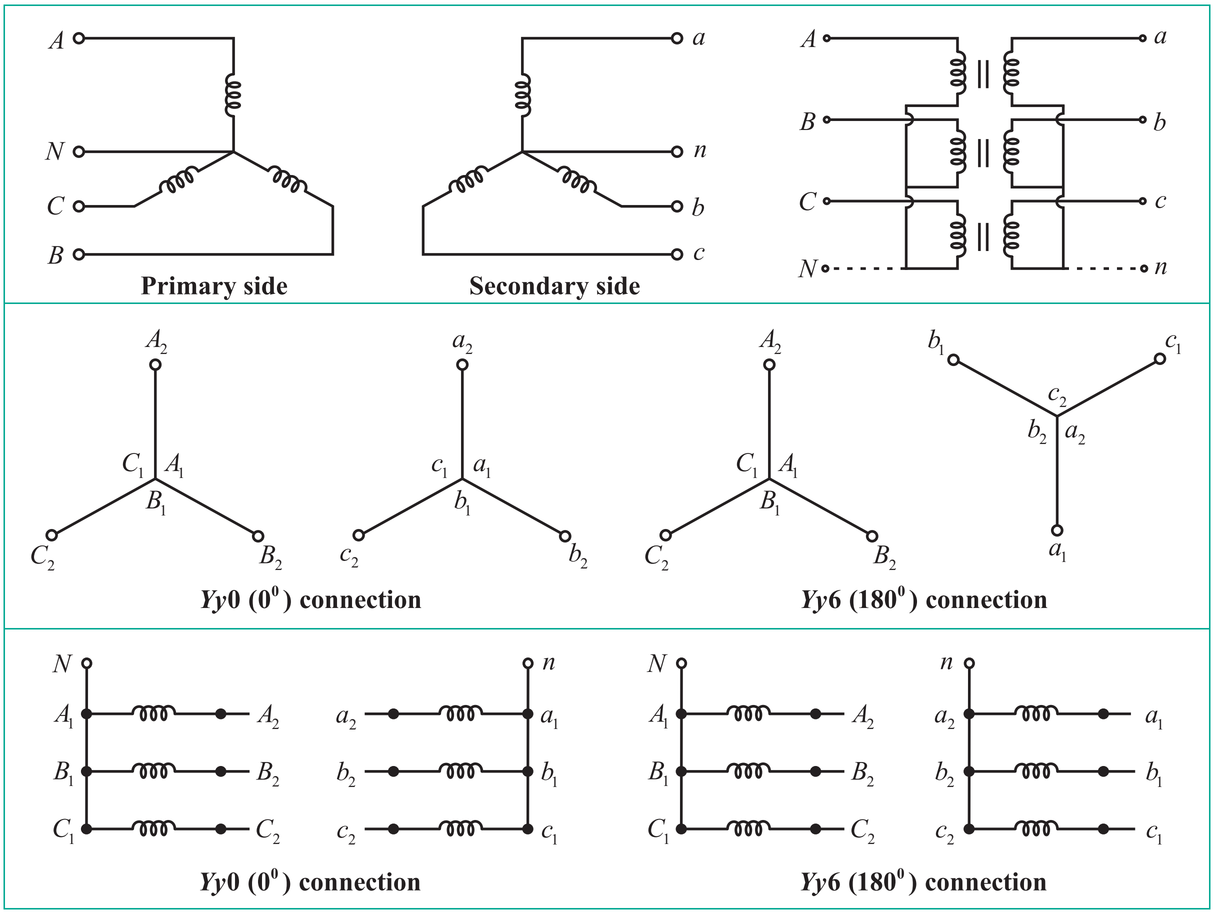
For delta connection, $V_{L1} = V_{ph1}$ $\quad V_{ph2} = K V_{ph1}$ $\quad V_{L2} = V_{ph2}$

$$I_{L1} = \sqrt{3} I_{ph1} \quad I_{ph1} = K I_{ph2} \quad I_{L2} = \sqrt{3} I_{ph2}$$

The delta-delta connection is commonly used for large low voltage transformers.

Star-Star Connection

In this type of connection, both the three phase primary and secondary windings are connected in star.



There is no phase shift between primary and secondary voltages.

For star connection, $V_{L1} = \sqrt{3} V_{ph1}$ $\quad V_{ph2} = K V_{ph1}$ $\quad V_{L2} = \sqrt{3} V_{ph2}$

$$I_{L1} = I_{ph1} \quad I_{ph1} = K I_{ph2} \quad I_{L2} = I_{ph2}$$

The star-star connection is commonly used for small high voltage transformers.

➤ Sample Questions

1999 IIT Bombay

2.1 The percentage resistance and percentage reactance of a 10 kVA, 400 V/200 V, 3-phase transformer are 2% and 10% respectively. If the constant losses in the machine are 1%, the maximum possible percentage efficiency of the transformer is

(A) 98.32

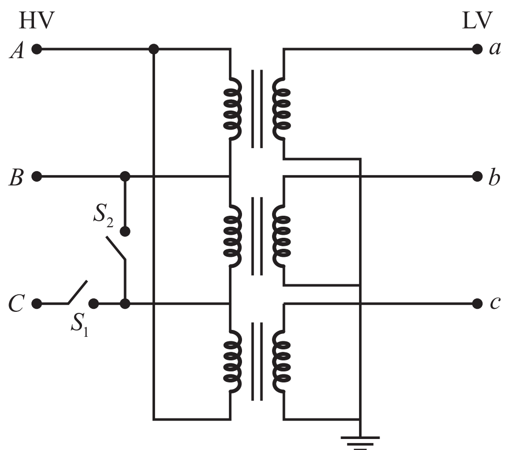
(B) 97.25

(C) 96.85

(D) 96.12

2003 IIT Madras

2.2 Figure shows a Δ-Y connected 3-phase distribution transformer used to step down the voltage from 11000 V to 415 V line-to-line. It has two switches S_1 and S_2. Under normal conditions S_1 is closed and S_2 is open. Under certain superior conditions S_1 is open and S_2 is closed. In such a case the magnitude of the voltage across the LV terminals *a* and *c* is



(A) 240 V　(B) 480 V

(C) 415 V　(D) 0 V

2017 IIT Roorkee

2.3 A three-phase, three winding $\Delta/\Delta/Y$ (1.1 kV/6.6 kV/400 V) transformer is energized from AC mains at the 1.1 kV side. It supplies 900 kVA load at 0.8 power factor lag from the 6.6 kV winding and 300 kVA load at 0.6 power factor lag from the 400 V winding. The RMS line current in ampere drawn by the 1.1 kV winding from the mains is ______. (Give the answer up to one decimal place.)

[Set - 01]

2018 IIT Guwahati

2.4 A 3-phase 900 kVA, 3 kV/$\sqrt{3}$ kV (Δ/Y) 50 Hz transformer has primary (high voltage side) resistance per phase of 0.3 Ω and secondary (low voltage side) resistance per phase of 0.02 Ω. Iron loss of the transformer is 10 kW. The full load % efficiency of the transformer operated at unity power factor is ______ (up to 2 decimal places).

❖❖❖❖

Explanations **Three Phase Transformer**

2.1 (B)

Given :

(i) 400 V/200 V, 3-phase transformer

(ii) kVA rating, $S = 10$ kVA

(iii) % reactance, $\%X = 10\%$

(iv) % resistance, $\%R = 2\%$

(v) Constant loss, $P_i = 1\%$

% Full load copper loss is given by,

$$P_{cu_{fl}} = \%R = 2\%$$

The fraction of loading at which the maximum efficiency occurs, is given by,

$$m = \sqrt{\frac{P_i}{P_{cu_{fl}}}} = \sqrt{\frac{0.01}{0.02}} = 0.707$$

For maximum efficiency, $P_{cu} = P_i = 0.01$ pu and take unity power factor i.e. $\cos\phi = 1.0$ (since power factor is not given).

Maximum efficiency,

$$\eta_{\max} = \frac{m\cos\phi \times 100}{m\cos\phi + 2P_{i\,pu}}$$

where, $P_{i_{pu}} = P_{cu_{pu}}$, m = % of loading

Hence, $$\eta_{\max} = \frac{0.707 \times 1}{0.707 \times 1 + 2 \times 0.01} \times 100$$

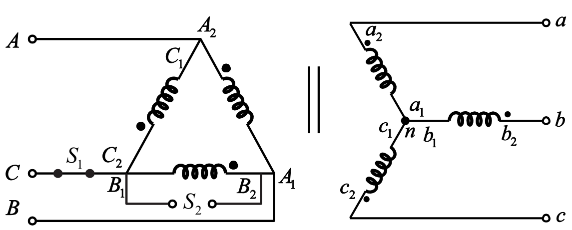
$$\eta_{\max} = 97.25\%$$

Hence, the correct option is (B).

2.2 (B)

Given : 11000/415 V Δ-Y 3-phase transformer

Case I : S_1 is closed and S_2 is open (normal operating condition). The phasor diagram is shown below,



(i) At primary side :

$$V_{AB} = V_{1(\text{line})} = V_{1(\text{phase})} = 11\angle 0^0 \text{ kV}$$

$$V_{BC} = 11\angle -120^0 \text{ kV}$$

$$V_{CA} = 11\angle +120^0 \text{ kV}$$

And, $$V_{AB} + V_{BC} + V_{CA} = 0 \text{ V}$$

$$11\angle 0^0 + 11\angle -120^0 + 11\angle +120^0 = 0 \text{ V}$$

(ii) At Secondary side :

$$V_{ab} = V_{2(\text{line})} = 415\angle 0^0 \text{ V}$$

$$V_{bc} = 415\angle -120^0 \text{ V}$$

$$V_{ca} = 415\angle +120^0 \text{ V}$$

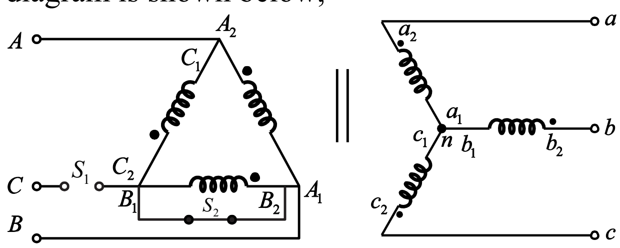
and, $$V_{ab} + V_{bc} + V_{ca} = 0 \text{ V}$$

$$415\angle 0^0 + 415\angle -120^0 + 415\angle +120^0 = 0 \text{ V}$$

Hence, the transformation ratio,

$$(k) = \frac{V_{2(\text{phase})}}{V_{1(\text{phase})}} = \frac{415/\sqrt{3}}{11 \times 10^3} = 0.0217$$

Case II : S_1 is open and S_2 is close (at certain superior operating condition). The phasor diagram is shown below,



When S_2 is closed, primary winding B_2B_1 is short circuited as shown in figure. Here, we will have three cases :

(i) In primary 11 kV applied on AB, that is $V_{AB} = V_{A_2A_1}$. Hence, the induced voltage in a_2a_1 in secondary is given by,

$$V_{an} = V_{a_2a_1} = k \times V_{A_2A_1}$$

$$V_{an} = V_{a_2a_1} = \frac{415}{\sqrt{3} \times 11} \times 11 = 239.6 \text{ V}$$

And, $V_{A_2A_1}$ and $V_{a_2a_1}$ will be in same phase.

(ii) As the winding B_2B_1 is shorted, hence,

$V_{BC} = V_{B_2B_1} = 0\,\text{V}$

The induced voltage in winding b_2b_1 is given by,

$V_{bn} = V_{b_2b_1} = k \times V_{B_2B_1} = 0\ \text{V}$

(iii) As, $V_{AB} + V_{BC} + V_{CA} = 0\,\text{V}$

Hence, $V_{CA} = -V_{AB} - V_{BC} = V_{C_2C_1}$

$V_{CA} = -11 \times 10^3 \angle 0^0\ \text{V}$

The induced voltage in c_2c_1 is given by,

$V_{cn} = V_{c_2c_1} = k \times V_{C_2C_1}$

$$V_{cn} = -\frac{415}{\sqrt{3}} \times \frac{1}{11} \times 11 = -239.6\,\text{V}$$

Hence, $V_{ac} = V_{an} - V_{cn} = 239.6\angle 0^0 + 239.6\angle 0^0$

$V_{ac} \approx 480\ \text{V}$

Hence, the correct option is (B).

Scan for Video Solution

2.3 **625.09**

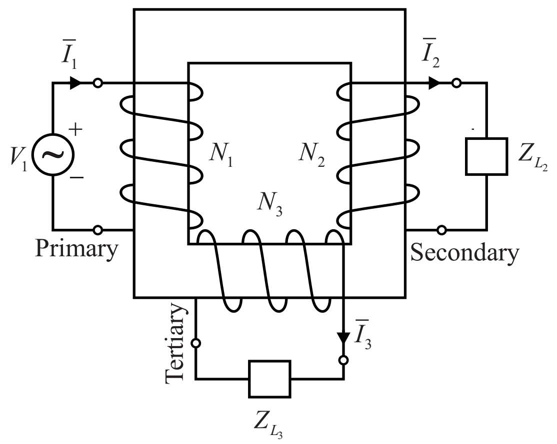
Given :

(i) 1.1 kV/6.6 kV/400 V, 3-phase, 3-winding $\Delta/\Delta/Y$ transformer

(ii) Supply at 1.1 kV side

(iii) Load :
900 kVA, 0.8 *pf* lag at 6.6 kV side
300 kVA, 0.6 *pf* lag at 400 V side

The equivalent is shown below,



Method 1

Power balance equation :

From transformer action,

Input kVA = Output kVA

$$\text{kVA}_1 = \text{kVA}_2 + \text{kVA}_3$$

$$\text{kVA}_1 = 900\angle -36.87^0 + 300\angle -53.13^0$$

$$\text{kVA}_1 = 720 - j540 + 180 - j240$$

$$\text{kVA}_1 = 900 + j780 = 1190.9\angle -40.9^0\ \text{kVA} \quad \ldots\text{(i)}$$

Also, $\text{kVA}_1 = \sqrt{3}V_1 \times I_1$ …(ii)

Where, V_1 = line voltage of supply

I_1 = line current of supply

$$I_1 = \frac{\text{kVA}_1}{\sqrt{3} \times V_1} = \frac{1190.9\angle 40.9^0}{\sqrt{3} \times 1.1} \times \frac{\text{kVA}}{\text{kV}}$$

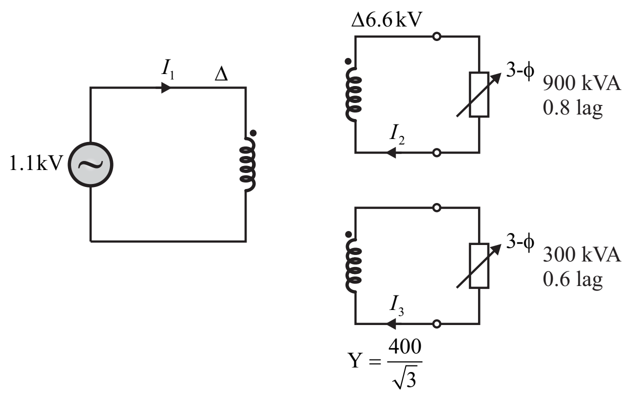
$$I_1 = 625.09\angle -40.9^0\ \text{A}$$

Hence, the RMS line current drawn by the 1.1 kV winding from the mains is **625.09 A.**

Method 2

MMF balance equation :

The equivalent circuit is shown below,



Load 01 :

Load on 6.6 kV winding is 900 kVA at 0.8 power factor lagging,

Hence,

$$I_{2(\text{line})} = \frac{VA}{\sqrt{3} \times V_2} = \frac{900 \times 10^3}{\sqrt{3} \times 6.6 \times 10^3} = 78.73\angle -36.86^0$$

Hence, $I_{2(\text{phase})} = \frac{I_{2(\text{line})}}{\sqrt{3}} = \frac{78.73\angle -36.86^0}{\sqrt{3}}$

$I_{2(\text{phase})} = 45.45\angle -36.86^0 \text{ A}$

The load current $I_{2(\text{phase})}$ on primary side is given by,

$I'_{2(\text{phase})} = k_1 \cdot I_{2(\text{phase})} = 6\times 45.45\angle -36.86^0$

$I'_{2(\text{phase})} = 272.7\angle -36.86^0 \text{ A}$

$\left[\text{Where, } k_1 = \frac{6.6}{1.1} = 6 = \text{transformation ratio}\right]$

Hence, $I_2'{}_{(\text{line})} = \sqrt{3}\times I_2'{}_{(\text{phase})} = 472.38\angle -36.86^0 \text{ A}$

Load 2 : Load on 400 V winding is 300 kVA at 0.6 power factor lagging,

$I_{3(\text{line})} = \frac{VA}{\sqrt{3}\times V_3} = \frac{300\times 10^3}{\sqrt{3}\times 400} = 433.013\angle -53.13^0 \text{ A}$

Hence, $I_{3(\text{phase})} = I_{3(\text{line})} = 433.013\angle -53.13^0 \text{ A}$

The load current $I_{3(\text{phase})}$ on primary side is given by,

$I'_{3(\text{phase})} = k_2 \cdot I_{3(\text{phase})} = 0.2099\times 433.025\angle -53.13^0$

$I'_{3(\text{phase})} = 90.90\angle -53.13^0 \text{ A}$

$\left[\text{Where, } k_2 = \frac{400/\sqrt{3}}{1.1\times 10^3} = 0.2099 = \text{transformation ratio}\right]$

Hence, $I_3'{}_{(\text{line})} = \sqrt{3}\times I_3'{}_{(\text{phase})} = 157.46\angle -53.13^0 \text{ A}$

Hence, current drawn by 1.1kV winding will be,

$I_{1(\text{line})} = I_2'{}_{(\text{line})} + I_3'{}_{(\text{line})}$

$I_{1(\text{line})} = 472.4\angle -36.86^0 + 157.56\angle -53.13^0$

$I_{1(\text{line})} = 625.09\angle -40.9^0$

Hence, the required RMS line current is **625.19 A.**

Scan for Video Solution

2.4 97.36

Given :

(i) Three phase 50 Hz transformer

(ii) Rating, $S = 900$ kVA, pf, $\cos\phi = 1$

(iii) Primary line voltage $= V_l(\Delta) = 3$ kV

(iv) Secondary line voltage $= V_l(\text{Y}) = \sqrt{3}$ kV

(v) Primary resistance per phase $= R_{ph}(\Delta) = 0.3\ \Omega$

(vi) Secondary resistance per phase $= R_{ph}(\text{Y}) = 0.02\ \Omega$

(vii) Iron loss, $P_i = 10$ kW

The percentage efficiency of the transformer is given by,

$$\%\eta = \frac{x\times(\text{kVA})\times\cos\phi}{x\times(\text{kVA})\times\cos\phi + P_i + x^2 P_{cufl}}\times 100 \quad \ldots\text{(i)}$$

At full load, fraction of loading $(x) = 1$, $pf = 1$

$$P_{cufl} = 3I_{ph\Delta}^2 R_{ph}(\Delta) + 3I_{phY}^2 R_{ph}(\text{Y}) \quad \ldots\text{(ii)}$$

where, $I_{ph\Delta} = \frac{900}{3\times V_{l\Delta}} = \frac{900}{3\times 3} = 100\text{ A}$

and $I_{phY} = \frac{900}{\sqrt{3}V_{lY}} = \frac{900}{\sqrt{3}\times\sqrt{3}} = 300\text{ A}$

From equation (ii),

$P_{cufl} = 3\times 100^2\times 0.3 + 3\times 300^2\times 0.02$

$P_{cufl} = 14.4\text{ kW}$

Hence, percentage efficiency is given by,

$$\%\eta = \frac{1\times 900\times 1}{1\times 900\times 1 + 10 + 14.4}\times 100$$

$\%\eta = 97.36$

Hence, the percentage full load efficiency of the transformer is **97.36 %.**

Scan for Video Solution

❖❖❖❖

3 DC Machine

➢ Partial Synopsis

Torque of a DC Machine

When the machine operates as a generator at constant speed, this torque is equal and opposite to that provided by the prime-mover. When the machine is operating as a motor, the torque is transferred to the shaft of the rotor and drives the mechanical load. The expression for the torque is the same for the generator and the motor.

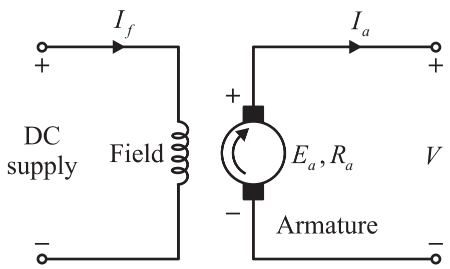
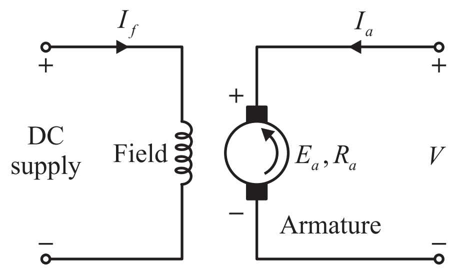
Mechanical power developed by the armature, $P_m = \omega_m T_{av} = \frac{2\pi N}{60} \times T_{av} = E_a I_a$

Average electromagnetic torque developed by the armature in (N-m) is,

$\therefore$ $$\boldsymbol{T_{av} = \frac{PZ}{2\pi A}\phi I_a = K\phi I_a}$$ where, $K = \frac{PZ}{2\pi A}$

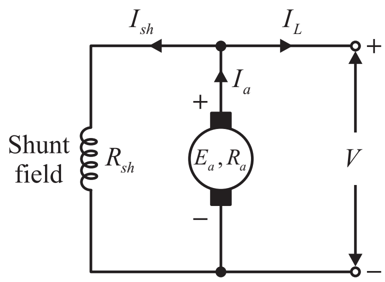
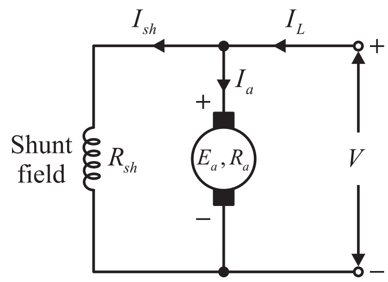
Separately Excited DC Machine

In separate excitation, the field coils are energized by a separate DC source.

Separately excited DC generator	Separately excited DC motor
	
$I_a = I_L$ $\quad$ $E_a = V + I_a R_a$	$I_a = I_L$ $\quad$ $E_a = V - I_a R_a$
$E_a I_a = VI_a + I_a^2 R_a = VI_L + I_a^2 R_a$ Armature power = Output power + Armature copper loss	$E_a I_a = VI_a - I_a^2 R_a = VI_L - I_a^2 R_a$ Armature power = Input power – Armature copper loss

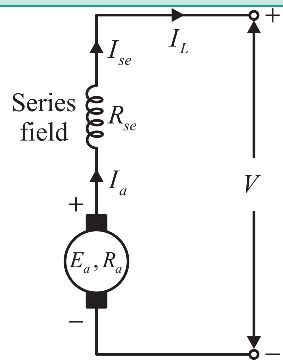
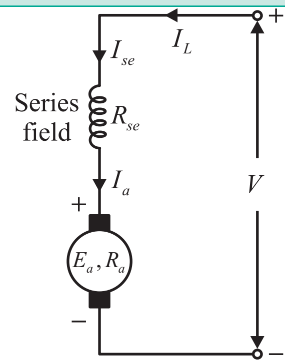
Shunt Wound DC Machine

A machine in which the field coils are connected in parallel with the armature is called a shunt machine. Since, the shunt field receives the full output voltage of a generator or the supply voltage of a motor, it is generally made of large number of turns of fine wire carrying a small field current. The voltage across the armature terminals and the shunt field is the same and it is for this reason that a shunt field may be called a voltage-operated field.

Shunt wound DC generator	**Shunt wound DC motor**
	
$I_a = I_L + I_{sh}$ $I_{sh} = \frac{V}{R_{sh}}$ $E_a = V + I_a R_a$	$I_a = I_L - I_{sh}$ $I_{sh} = \frac{V}{R_{sh}}$ $E_a = V - I_a R_a$
$E_a I_a = VI_a + I_a^2 R_a = V(I_L + I_{sh}) + I_a^2 R_a$ $= VI_L + VI_{sh} + I_a^2 R_a$ $E_a I_a = VI_L + I_{sh}^2 R_{sh} + I_a^2 R_a$ Armature power = Output power + Shunt field copper loss + Armature copper loss	$E_a I_a = VI_a - I_a^2 R_a = V(I_L - I_{sh}) - I_a^2 R_a = VI_L - VI_{sh} - I_a^2 R_a$ $E_a I_a = VI_L - I_{sh}^2 R_{sh} - I_a^2 R_a$ Armature power = Input power – Shunt field copper loss – Armature copper loss

Series Wound DC Machine :

A DC machine in which the field coils are connected in series with the armature is called a series machine. The series field winding carries the armature current and since, the armature current is large, the series field winding consists of few turns of wire of large cross-sectional area. In other words, the series field current depends on the armature current and it is for this reason that a series field may be called a current-operated field.

DC series generator	**DC series motor**
	
$I_a = I_{se} = I_L$ $E_a = V + I_a(R_a + R_{se})$	$I_a = I_{se} = I_L$ $E_a = V - I_a(R_a + R_{se})$
$E_a I_a = VI_a + I_a^2(R_a + R_{se}) = VI_L + I_{se}^2 R_{se} + I_a^2 R_a$ Armature power = Output power + Series field copper loss + Armature copper loss	$E_a I_a = VI_a - I_a^2 R_a - I_a^2 R_{se} = VI_L - I_{se}^2 R_{se} - I_a^2 R_a$ Armature power = Input power – Series field copper loss – Armature copper loss

➢ Sample Questions

1992 IIT Delhi

3.1 A separately excited DC motor has an armature resistance of 0.5 ohms. It runs from a 250 V DC supply drawing an armature current of 20 A at 1500 rpm. For the same field current, the torque developed (in Nm) for an armature current of 10 A will be _________.

1995 IIT Kanpur

3.2 A differentially compounded DC motor with interpoles and with brushes on the neutral axis is to be driven as a generator in the direction with the same polarity of the terminal voltage. It will then

(A) be a cumulatively compounded generator but the interpole coil connections are to be reversed.

(B) be a cumulatively compounded generator without reversing the interpole coil connections.

(C) be a differentially compounded generator without reversing the interpole coil connections.

(D) be a differentially compounded generator but the interpole coil connections are to be reversed.

1997 IIT Madras

3.3 At 50% of full load, the armature current drawn by a DC shunt motor is 40 A when connected to a 200 V DC mains. By decreasing the field flux, its speed is raised by 20%, this also causes a 10% increase in load torque. The armature resistance including the brushes is 1 Ohm. Neglecting saturation, the percentage change in field current is _______.

3.4 The field coil of a 2 pole dc series motor is made up of 2 identical sections. In case (i) the 2 sections are connected in series and in case (ii) the two sections are connected in parallel. If the motor takes the rated current in both the cases, then ratio of torque (i) : torque (ii) and speed (i) : speed (ii) respectively are

(A) 2:1, 1:2 (B) 2:1, 2:1

(C) 1:2, 1:2 (D) 1:2, 2:1

1999 IIT Bombay

3.5 A 4-pole lap-wound DC generator has a developed power of *P* watts and voltage of *E* volts. Two adjacent brushes of the machine are removed as they are worn out. If the machine operates with the remaining brushes, the developed voltage and power that can be obtained from the machine are

(A) E, P (B) $\frac{E}{2}, \frac{P}{2}$

(C) $E, \frac{P}{4}$ (D) $E, \frac{P}{2}$

2001 IIT Kanpur

3.6 In a dc motor running at 2000 rpm, the hysteresis and eddy current losses are 500 W and 200 W respectively. If the flux remains constant, the speed (in rpm) at which the total iron losses are halved will be ___________.

2014 IIT Kharagpur

3.7 A 250 V DC shunt machine has armature circuit resistance of 0.6 Ω and field circuit resistance of 125 Ω. The machine is connected to 250 V supply mains. The machine is operated as a generator and then as a motor separately. The line current of the machine in both the cases is 50 A. The ratio of the speed as a generator to the speed as a motor is ______. **[Set - 02]**

2020 IIT Delhi

3.8 A 250 V dc shunt motor has an armature resistance of 0.2 Ω and a field resistance of 100 Ω. When the motor is operated on no-load at rated voltage, it draws an armature current of 5 A and runs at 1200 rpm. When a load is coupled to the motor, it draws total line current of 50 A at rated voltage, with a 5 % reduction in the air-gap flux due to armature reaction. Voltage drop across the brushes can be taken as 1 V per brush under all operating conditions. The speed of the motor, in rpm, under this loaded condition, is closest to

(A) 900 (B) 1200

(C) 1000 (D) 1220

2021 IIT Bombay

3.9 A belt-driven DC shunt generator running at 300 rpm delivers 100 kW to a 200 V DC grid. It continues to run as a motor when the belt breaks, taking 10 kW from the DC grid. The armature resistance is 0.025 Ω, field resistance is 50 Ω and brush drop is 2 V. Ignoring armature reaction, the speed of the motor is _____ rpm. (Round off to 2 decimal places.)

❖❖❖❖

Explanations DC Machine

3.1 15.28

Given :

(i) Separately excited DC motor,

(ii) Terminal voltage, $V = 250$ V

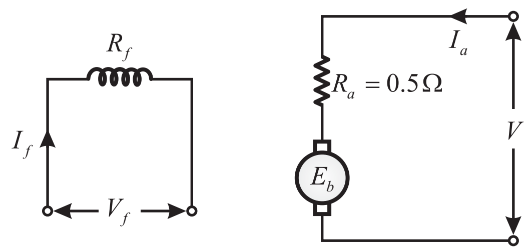
(iii) Armature resistance, $R_a = 0.5\ \Omega$

(iv) Armature current, $I_a = 20$ A

(iv) Speed, $N = 1500$ rpm

(v) Field current = constant

The equivalent circuit of separately excited DC motor is shown below,



Torque equation for a DC machine is given by,

$$T = \frac{Z\phi P I_a}{2\pi A},$$

$$\frac{ZP}{2\pi A} = \text{Constant}$$

Since, for separately excited DC motor flux remains constant. Therefore,

$$T \propto I_a$$

Hence, $$\frac{T_2}{T_1} = \frac{I_{a_2}}{I_{a_1}} \quad \ldots\text{(i)}$$

Also, Torque $= \dfrac{\text{Power}}{\text{Speed (rad/sec)}}$

$$T_1 = \frac{E_{a_1} I_{a_1}}{\omega}$$

EMF equation is given by,

$$E_{a_1} = V - I_{a_1} R_a = 250 - 20 \times 0.5 = 240 \text{ V}$$

Therefore, $$T_1 = \frac{240 \times 20}{2\pi \times \frac{1500}{60}} = 30.557 \text{ Nm}$$

From equation (i),

$$T_2 = \frac{I_{a_2}}{I_{a_1}} \times T_1 = \frac{10}{20} \times 30.557 = 15.278 \text{ Nm}$$

Hence, the developed torque is **15.28 Nm.**

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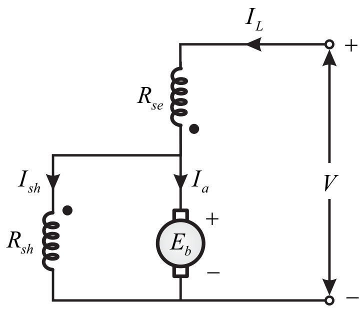
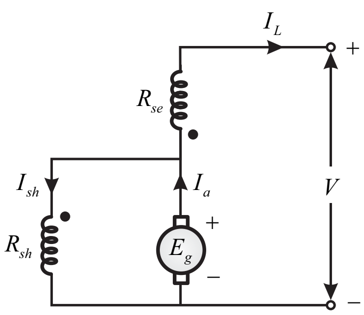
3.2 (B)

Given : Differentially compounded DC motor.

Its equivalent circuit (considering short circuit) is shown below,



When motor working as a generator then the equivalent circuit is,



As, the flux direction and rotating direction remain unchanged generated EMF polarity in case of generator is same as back EMF polarity as in case of motor.

Since, the direction of armature current is reversed the nature will be cumulative.

This reversed current in armature will also change the nature of interpole flux, therefore no need to reverse connection.

Hence, the correct option is (B).

Key Point

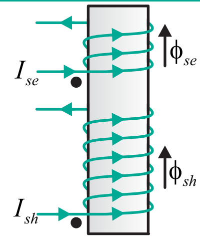
Dot concept

Cumulatively compound :

Series field assists the shunt field.

$I_{sh} \rightarrow$ Dot to coil

$I_{se} \rightarrow$ Dot to coil



$$\phi_{sh} > \phi_{se}$$

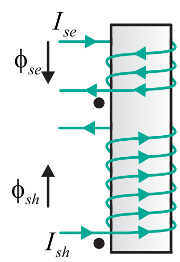
$$\phi_{cum} = \phi_{sh} + \phi_{se}$$

Differentially compound :

Series field opposes the shunt field.

$I_{sh} \rightarrow$ Dot to coil

$I_{se} \rightarrow$ Coil to dot



$$\phi_{diff} = \phi_{sh} - \phi_{se}$$

3.3 – 28

Given :

(i) DC shunt motor

(ii) Terminal voltage, $V = 200$ V

(iii) Armature current, $I_a = 40$ A

(Armature 50% full load)

(iv) Armature resistance, $R_a = 1\Omega$

(v) Load conditions : Speed, $\omega_2 = 1.2\omega_1$

(vi) Torque, $T_2 = 1.1T_1$

Torque equation for a dc machine is given by,

$$T = \frac{ZP}{2\pi A}\phi I_a = k\phi I_a \quad \ldots(i)$$

$$\frac{T_2}{T_1} = \frac{\phi_2 I_{a_2}}{\phi_1 I_{a_1}}$$

$$\frac{I_{a_2}}{I_{a_1}} = \frac{T_2\phi_1}{T_1\phi_2}$$

$$I_{a_2} = 40 \times 1.1\frac{\phi_1}{\phi_2} = 44\frac{\phi_1}{\phi_2} \quad \ldots(ii)$$

EMF equation for a dc machine is given by,

$$E_b = \frac{ZQ}{2\pi A}\phi\omega = k\phi\omega \qquad \ldots(iii)$$

Also, $E_b = V - I_a R_a$

$$\frac{E_{b_2}}{E_{b_1}} = \frac{V - I_{a_2} R_a}{V - I_{a_1} R_a} = \frac{\phi_2\omega_2}{\phi_1\omega_1}$$

From equation (ii),

$$\frac{200 - 44\dfrac{\phi_1}{\phi_2}\times 1}{200 - 40\times 1} = \frac{\phi_2}{\phi_1}\times 1.2$$

$$200 - 44\frac{\phi_1}{\phi_2} = \frac{\phi_2}{\phi_1}\times 192$$

Let, $\dfrac{\phi_2}{\phi_1} = x$

$$200 - \frac{44}{x} = 192x$$

$$192x^2 - 200x + 44 = 0$$

$$48x^2 - 50x + 11 = 0$$

$$x = \frac{50 \pm \sqrt{50^2 - 4\times 11\times 48}}{96} = \frac{50\pm 19.7}{96}$$

$$x = 0.72 \quad \text{or} \quad x = 0.32$$

For $\dfrac{\phi_2}{\phi_1} = 0.72,\ I_{a_2} = \dfrac{44}{0.72} = 61.11\text{ A} \ (< I_{rated})$

For, $\dfrac{\phi_2}{\phi_1} = 0.32,\ I_{a_2} = \dfrac{44}{0.32} = 137.5\text{ A} \ (> I_{rated})$

Hence, $\dfrac{\phi_2}{\phi_1} = 0.72$

% Change in field current is given by,

$$\%\Delta I_{sh} = \frac{I_{sh_2} - I_{sh_1}}{I_{sh_1}}\times 100 = \frac{\phi_2 - \phi_1}{\phi_1}\times 100$$

$$\%\Delta I_{sh} = \frac{0.72\phi_1 - \phi_1}{\phi_1}\times 100 = -28\%$$

Here negative sign indicates decrement.

Hence, percentage change in field current is **−28%**.

3.4 (A)

Given :

(i) DC series motor

(ii) Number of poles, $P = 2$

(iii) Current rating $=$ Constant

(iv) Assuming induced EMF is constant in both cases

Induced EMF for a DC machine is given by,

$$E = \frac{ZP}{2\pi A}\phi\omega = k\phi\omega$$

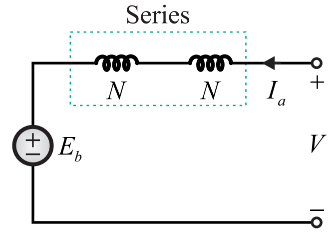
i.e. $E \propto \phi\omega$

Developed torque for a DC machine is given by,

$$T = \frac{ZP}{2\pi A}\phi I_a = k\phi I_a$$

i.e. $T \propto \phi I_a$

Case I : Series connection



Generated MMF is given by,

$$\text{MMF}_S = NI_a + NI_a = 2NI_a$$

Where, $N =$ Number of turns in each section

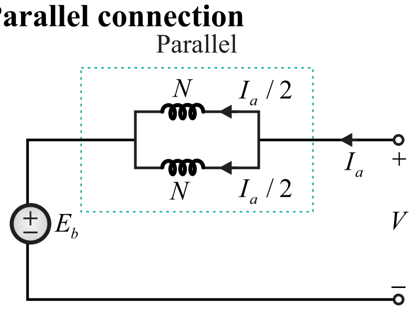
Flux, $\phi_S = \dfrac{\text{MMF}}{\text{Reluctance}} = \dfrac{2NI_a}{S}$

Torque, $T_S \propto \phi_S I_a \propto \dfrac{2NI_a^2}{S} \qquad \ldots(i)$

Back EMF,

$$E_b \propto \phi_S\omega_S \propto \frac{2NI_a}{S}\omega_S \qquad \ldots(ii)$$

Case II : Parallel connection



Generated MMF is given by,

$$\text{MMF}_P = \frac{NI_a}{2} + \frac{NI_a}{2} = NI_a$$

Flux, $\phi_P = \dfrac{NI_a}{S}$

Torque, $T_P \propto \phi_P I_a \propto N\dfrac{I_a^2}{S}$ …(iii)

Induced EMF,

$$E_b \propto \phi_P \omega_P \propto \frac{NI_a}{S}\omega_P \qquad \ldots\text{(iv)}$$

Dividing equation (i) by (iii),

$$\frac{T_S}{T_P} = \frac{\dfrac{2NI_a^2}{S}}{\dfrac{NI_a^2}{S}} = 2$$

$$\frac{\text{Torque (i)}}{\text{Torque (ii)}} = 2$$

Dividing equation (ii) by (iv),

$$\frac{E_b}{E_b} = \frac{\phi_S \omega_S}{\phi_P \omega_P}$$

$$\frac{\omega_S}{\omega_P} = \frac{\phi_P}{\phi_S} = \frac{\dfrac{NI_a}{S}}{\dfrac{2NI_a}{S}} = \frac{1}{2}$$

$$\frac{\text{Speed (i)}}{\text{Speed (ii)}} = \frac{1}{2}$$

Hence, the correct option is (A).

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3.5 **(C)**

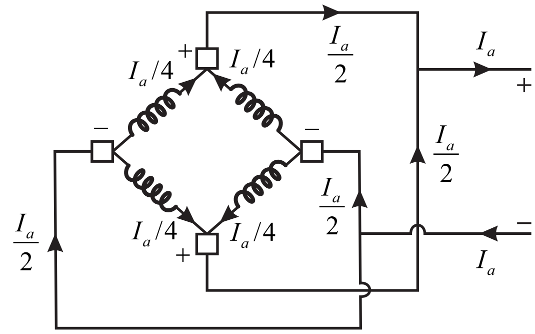
Given :

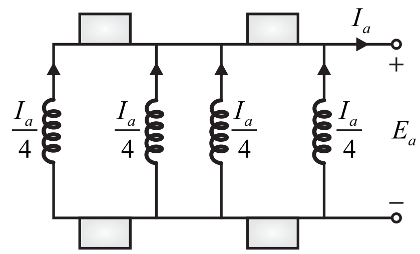
(i) Lap-wound DC generator

(ii) Number of poles, $P = 4$

Case I : At normal operation

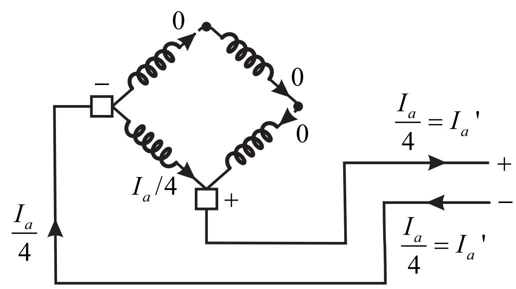
The equivalent circuit is shown below,

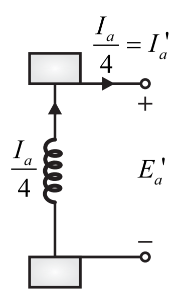




Power, $P = EI_a$

Case II : When two adjacent brushes are removed then the equivalent circuit becomes.





From figure,

Current, $I_a' = \dfrac{I_a}{4}$

Voltage, $E' = E$

Power, $P' = E' \times I' = E \times \dfrac{I_a}{4} = \dfrac{P}{4}$

Hence, the correct option is (C).

3.6 **1140**

Given :

(i) DC motor

(ii) Hysteresis loss, $P_{h_1} = 500\text{ W}$

(iii) Eddy current loss, $P_{e_1} = 200\text{ W}$

(iv) Flux remains constant, $\phi =$ Constant

(v) Speed, $N_1 = 2000\text{ rpm}$

Hysteresis loss is given by,

$$P_h = k_h B_{\max}^x f v_{core}$$

Eddy current loss is given by,

$$P_e = K_e B_{\max}^2 f^2 v_{core}$$

Since, ϕ is constant,

$$B_{\max} = \frac{\phi}{\text{Area}} = \text{Constant}$$

Also, Speed is given by,

$$N = \frac{120 f}{P} \text{ i.e., } N \propto f$$

Thus, $P_h \propto f \propto N = K_1 N$

$$P_e \propto f^2 \propto N^2 = K_2 N^2$$

$$P_{h_1} = 500\text{ W} = K_1 N$$

$$K_1 = \frac{500}{2000} = 0.25 \text{ W/rpm}$$

$$P_e = 200\text{ W} = K_2 N^2$$

$$K_2 \times (2000)^2 = 200$$

$$K_2 = 5\times 10^{-5} \text{ W/rpm}^2$$

Total iron loss,

$$P_{i_1} = P_{h_1} + P_{e_1}$$

$$P_{i_1} = 500 + 200 = 700\text{ W}$$

If iron losses are halved,

$$P_{i_2} = \frac{P_{i_1}}{2} = 350$$

i.e., $P_{h_2} + P_{e_2} = 350$

$$K_1 N_2 + K_2 N_2^2 = 350$$

$$5\times 10^{-5} N_2^2 + 0.25 N_2 = 350$$

Solving above quadratic equation,

$$N_2 = 1140\text{ rpm or } -6140\text{ rpm}$$

Ignoring negative value of speed,

$$N_2 = 1140\text{ rpm}$$

Hence, the speed at which the total iron losses are halved is **1140 rpm.**

Scan for Video Solution

3.7 **1.27**

Given :

(i) DC shunt machine

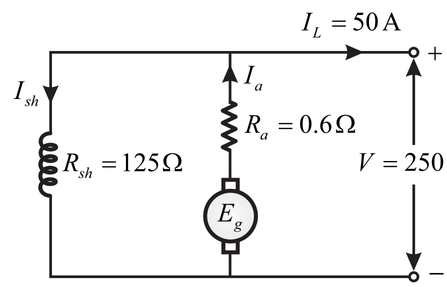
(ii) Armature resistance, $R_a = 0.6\ \Omega$

(iii) Field resistance, $R_{sh} = 125\ \Omega$

(iv) Supply voltage, $V = 250$ V

(v) Line current, $I_L = 50$ A

Case I : DC machine operating as generator then the equivalent circuit is shown below,



From figure,
Field current,

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{125} = 2\text{ A}$$

Armature current, $I_a = I_L + I_f$

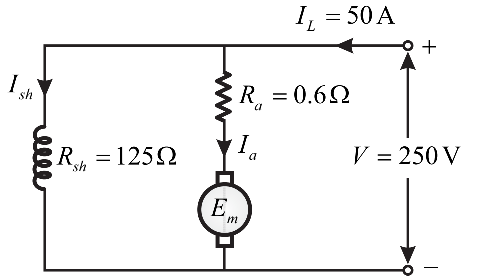
$$I_a = 50 + 2 = 52 \text{ A}$$

Induced EMF equation of DC shunt generator is given by,

$$E_g = V + I_a R_a$$

$$E_g = 250 + 52\times 0.6 = 281.2 \text{ V}$$

Case II : DC machine operating as motor then the equivalent circuit is shown below,



From figure,

Armature current, $I_a = I_L - I_{sh}$

$$I_a = 50 - 2 = 48\text{ A}$$

Back EMF equation of DC shunt motor is given by, $E_m = V - I_a R_a$

$$E_m = 250 - 48 \times 0.6 = 221.2\text{ V}$$

Since, field current is constant.

Hence, $\phi = \text{Constant}$

and $E \propto N$

Hence, $\dfrac{E_g}{E_m} = \dfrac{N_g}{N_m}$ …(i)

where, N_g = Speed of generator

N_m = Speed of motor

Put the values of E_g and E_m in equation (i), we have

$$\frac{N_g}{N_m} = \frac{281.2}{221.2}$$

$$\frac{N_g}{N_m} = 1.271$$

Hence, the ratio of the speed as a generator to the speed as a motor is **1.27.**

Scan for Video Solution

3.8 (D)

Given : DC shunt motor

(i) Terminal voltage, $V_t = 250\,V$

(ii) Armature resistance, $R_a = 0.2\,\Omega$

(iii) Field resistance, $R_f = 100\,\Omega$

(iv) Armature current at no load, $(I_a)_{nl} = 5\,A$

(v) Rotor speed at no load, $N_{nl} = 1200\,\text{rpm}$

(vi) Armature current at load $(I_a)_l = 50\,A$

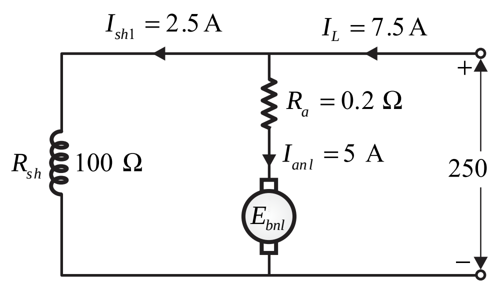
(vii) Flux at loaded condition $\phi_2 = 0.95\phi_1$

(viii) Voltage drop due to brush, $V_b = 1\,V$ per brush

Field current of DC shunt motor is given by,

$$I_f = \frac{V_t}{R_{sh}} = \frac{250}{100} = 2.5\,A$$

Case I : No load condition



Terminal voltage for DC shunt motor is given by,

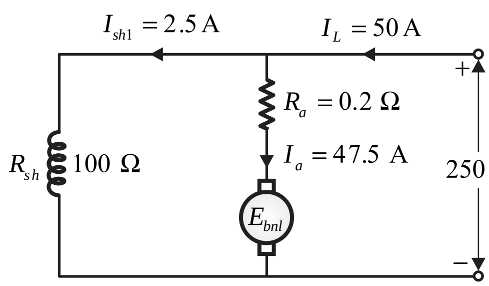
$$V_t = (E_b)_{nl} + (I_a)_{nl} r_a + V_b \times 2$$

$$250 = (E_b)_{nl} + 5 \times 0.2 + 2 \times 1$$

$$250 = (E_b)_{nl} + 1 + 2$$

$$(E_b)_{nl} = 247\,V \quad \text{…(i)}$$

Case II : Loaded condition



Line current is given by

$$I_L = I_a + I_{sh}$$

$$I_a = 50 - 2.5 = 47.5\,A$$

$$V_t = (E_b)_l + (I_a)_l \times r_a + 2 \times V_b$$

$$250 = (E_b)_l + 47.5 \times 0.2 + 2 \times 1$$

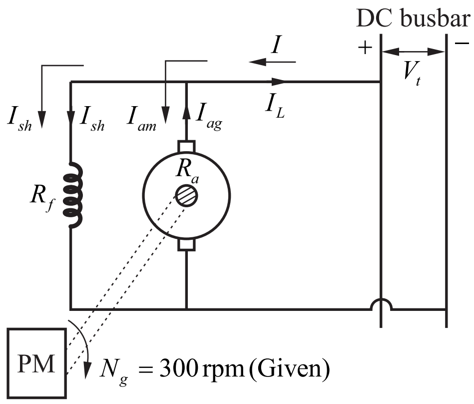
$(E_b)_l = 238.5\,\text{V}$

$$\frac{(E_b)_n}{(E_b)_l} = \frac{\phi_1 \times N_{nl}}{\phi_2 \times N_l}$$

$$\frac{247}{238.5} = \frac{\phi_1 \times 1200}{0.95\phi_1 \times N_2}$$

$N_{fl} = 1220\,\text{rpm}$

3.9 **275.186**



Case 1 : When machine acts as generator,

(i) Power output, $P_0 = 100\,\text{kW}$

(ii) Terminal voltage, $V_t = 200\,\text{V}$

Load current, $I_L = \dfrac{P_0}{V_t} = \dfrac{100\,\text{kW}}{200} = 500\,\text{A}$

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{200}{50} = 4\,\text{A}$$

$$I_a = I_L + I_{sh} = 500 + 4 = 504\,\text{A}$$

Generated voltage, $E_g = V_t + I_a R_a + V_{BD}$

$$E_g = 200 + (504 \times 0.025) + 2$$

$$E_g = 214.6\,\text{V}$$

Case 2 : When machine continue to run as a motor,

(i) Power input, $P_{in} = 10\,\text{kW}$

(ii) Terminal voltage of busbar, $V_t = 200\,\text{V}$

Supply current, $I_L = \dfrac{P_{in}}{V_t} = \dfrac{10\,\text{kW}}{200} = 50\,\text{A}$

$$I_a = I_L - I_{sh} = 50 - 4 = 46$$

∴ Back emf of motor,

$$E_b = V - I_a R_a - V_b$$

$$E_b = 200 - (46 \times 0.025) - 2 = 196.85\,\text{V}$$

When machine acts as generator, $E_g \propto \phi_{fg} N_g$

∴ Speed of generator, $N_g \propto \dfrac{E_g}{\phi_{fg}}$ …(i)

When machine acts as motor, $E_b \propto \phi_{fm} N_m$

∴ Speed of motor, $N_m \propto \dfrac{E_b}{\phi_{fm}}$ …(ii)

From equation (i) and (ii), we get

$$\frac{N_m}{N_g} = \frac{E_b}{E_g} \times \frac{\phi_{fg}}{\phi_{fm}}$$

$$\frac{N_m}{N_g} = \frac{E_b}{E_g} \times \frac{I_{fg}}{I_{fm}} = \frac{E_b}{E_g} \times \frac{\frac{V}{R_f}}{\frac{V}{R_f}}$$

∴ $$\frac{N_m}{N_g} = \frac{E_b}{E_g}$$

∴ $$N_m = N_g \times \frac{E_b}{E_g}$$

$$N_m = 300 \times \frac{196.85}{214.6} = 275.186$$

∴ Speed of motor, $N_m = 275.186\,\text{rpm}$

Hence, the correct answer is 275.186

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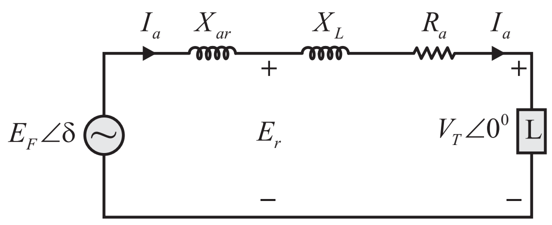
4 Synchronous Machine

➢ Partial Synopsis

Equivalent Circuit for Non-salient or Cylindrical rotor Synchronous Generator



E_F : Excitation emf per phase

V_T : Terminal voltage per phase

I_a : Armature current per phase

R_a : Armature resistance per phase

X_L : Armature leakage reactance per phase

X_{ar} : Armature reactance (or fictitious reactance per phase)

E_r : Voltage behind leakage impedance $(R_a + jX_L)$

Applying KVL in the circuit,

$$E_r = V_T\angle 0^0 + jI_aX_L + I_aR_a$$

$$E_F\angle\delta = V_T\angle 0^0 + I_aR_a + jI_a(X_L + X_{ar}) = V_T\angle 0^0 + I_aR_a + jI_aX_s$$ Synchronous reactance $\boldsymbol{X_s = X_L + X_{ar}}$

$$E_F\angle\delta = V_T\angle 0^0 + I_a(R_a + jX_s) = V_T\angle 0^0 + I_aZ_s$$ Synchronous impedance $\boldsymbol{Z_s = R_a + jX_s}$

At unity power factor, $\boldsymbol{E_F = \sqrt{(V_T + I_aR_a)^2 + (I_aX_s)^2}}$

At lagging power factor, $\boldsymbol{E_F = \sqrt{(V_T\cos\phi + I_aR_a)^2 + (V_T\sin\phi + I_aX_s)^2}}$

At leading power factor, $\boldsymbol{E_F = \sqrt{(V_T\cos\phi + I_aR_a)^2 + (V_T\sin\phi - I_aX_s)^2}}$

Voltage Regulation of an Alternator

The **voltage regulation** of an alternator is defined as the change in terminal voltage expressed as the percentage of rated voltage, when the load of given power factor is removed, with field current and speed remaining constant.

The **voltage regulation** of an alternator is defined as the change in its terminal voltage when full load is removed, keeping field excitation and speed constant, divided by the rated terminal voltage.

$$\text{\% Voltage regulation} = \frac{\text{Change in terminal voltage from no load to full load}}{\text{Full load terminal voltage}} \times 100\%$$

$$\textbf{Percentage voltage regulation} = \frac{|E_F| - |V_T|}{|V_T|} \times 100\%$$

$|E_F|$: Magnitude of generated voltage per phase, and $|V_T|$: Magnitude of rated or full load terminal voltage per phase, at the same speed and field excitation

Condition for maximum voltage regulation : $\theta_z = \phi$

$\theta_z = \tan^{-1}(X_s / R_a)$: Impedance angle

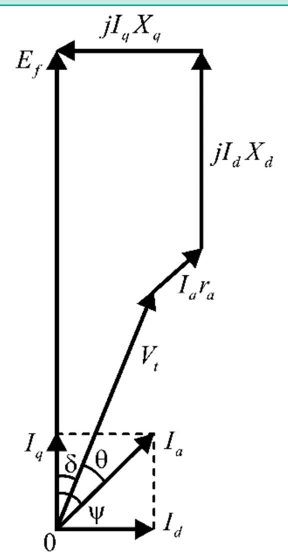
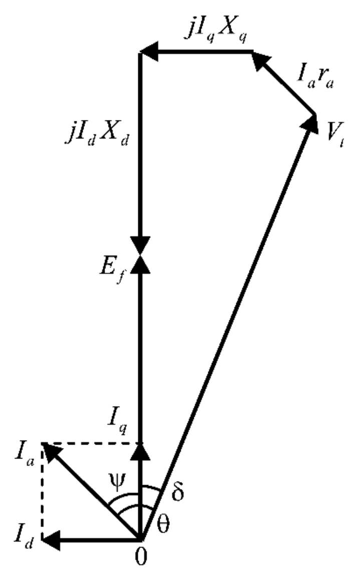
Condition for zero voltage regulation : $\cos(\theta_z + \phi) = -\dfrac{I_a Z_s}{2V_T}$

ϕ : Load power factor angle

Salient pole synchronous generator

In a salient pole synchronous generator, $E_f = V_t + r_a I_a + jX_d I_d + jX_q I_q$

$E_f' = V_t + r_a I_a + jI_a X_q$ $\qquad$ $E_f = E_f' + jI_d(X_d - X_q)$

Lagging power factor $\cos\theta$	Leading power factor $\cos\theta$
	
$E_f = V_t\cos\delta + I_q r_a + I_d X_d$ $I_q X_q = V_t \sin\delta + I_d r_a$ $\psi = \delta + \theta$ $\tan\psi = \tan(\theta+\delta) = \dfrac{V_t\sin\theta + I_a X_q}{V_t\cos\theta + I_a r_a}$	$E_f = V_t\cos\delta + I_q r_a - I_d X_d$ $I_q X_q = V_t \sin\delta - I_d r_a$ $\psi = \theta - \delta$ $\tan\psi = \tan(\theta-\delta) = \dfrac{V_t\sin\theta - I_a X_q}{V_t\cos\theta + I_a r_a}$

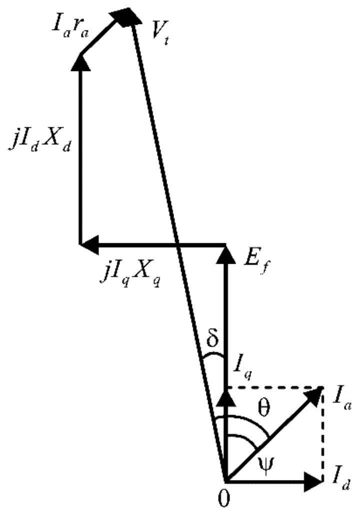
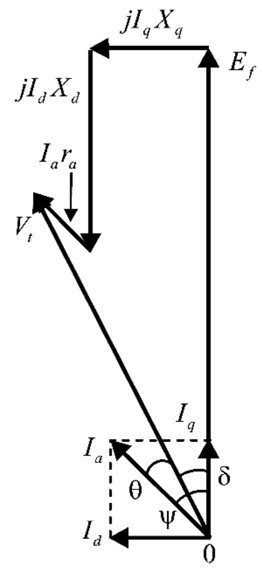
Salient pole synchronous motor

In a salient pole synchronous motor,

$V_t = E_f + r_a I_a + jX_d I_d + jX_q I_q$

$E_f' = V_t - r_a I_a - jI_a X_q$

$E_f = E_f' - jI_d(X_d - X_q)$

Lagging power factor $\cos\theta$	Leading power factor $\cos\theta$
	
$E_f = V_t\cos\delta - I_q r_a - I_d X_d$ $I_q X_q = V_t\sin\delta + I_d r_a$ $\psi = \theta - \delta$ $\tan\psi = \tan(\theta-\delta) = \dfrac{V_t\sin\theta - I_a X_q}{V_t\cos\theta - I_a r_a}$	$E_f = V_t\cos\delta - I_q r_a + I_d X_d$ $I_q X_q = V_t\sin\delta - I_d r_a$ $\psi = \theta + \delta$ $\tan\psi = \tan(\theta+\delta) = \dfrac{V_t\sin\theta + I_a X_q}{V_t\cos\theta - I_a r_a}$

➢ Sample Questions

2001 IIT Kanpur

4.1 A 50 kW synchronous motor is tested by driving it by another motor. When the excitation is not switched on, the driving motor takes 800 W. When the armature is short-circuited and the rated armature current of 10 A is passed through it, the driving motor required 2500 W. On open circuiting the armature with rated excitation, the driving motor takes 1800 W (Neglect the losses in the driving motor). The percentage efficiency of the synchronous motor at 50% load will be _________.

2006 IIT Kharagpur

Common Data for Questions 4.2 to 4.4

A 4 pole, 50 Hz, synchronous generator has 48 slots in which a double layer winding is used. Each coil has 10 turns and is short pitched by an angle to 36^0 electrical. The fundamental flux per pole is 0.025 Wb.

4.2 The line-to-line induced emf (in volts), for a three phase star connection is approximately

(A) 808 (B) 888

(C) 1400 (D) 1538

4.3 The line-to-line induced emf (in volts), for a two phase connection is approximately

(A) 1143 (B) 1332

(C) 1617 (D) 1791

4.4 The fifth harmonic component of phase emf (in volts), for a three phase star connection is,

(A) 0 (B) 269

(C) 281 (D) 808

2011 IIT Madras

4.5 The direct axis and quadrature axis reactances of a salient pole alternator are 1.2 p.u. and 1.0 p.u. respectively. The armature resistance is negligible. If this alternator is delivering rated kVA at upf and at rated voltage then its power angle is

(A) 30^0 (B) 45^0

(C) 60^0 (D) 90^0

2014 IIT Kharagpur

4.6 A non-salient pole synchronous generator having synchronous reactance of 0.8 pu is supplying 1 pu power to a unity power factor load at a terminal voltage of 1.1 pu. Neglecting the armature resistance, the angle of the voltage behind the synchronous reactance with respect to the angle of the terminal voltage in degrees is _______.

[Set - 03]

2017 IIT Roorkee

4.7 A 25 kVA, 400 V, Δ - connected, 3-phase, cylindrical rotor synchronous generator requires a field current of 5 A to maintain the rated armature current under short-circuit condition. For the same field current, the open-circuit voltage is 360 V. Neglecting the armature resistance and magnetic saturation, its voltage regulation (in % with respect to terminal voltage), when the generator delivers the rated load at 0.8 *pf* leading, at rated terminal voltage is ______. **[Set - 02]**

2020 IIT Delhi

4.8 A single 50 Hz synchronous generator on droop control was delivering 100 MW power to a system. Due to increase in load, generator power had to be increased by 10 MW, as a result of which, system frequency dropped to 49.75 Hz. Further increase in load in the system resulted in a frequency of 49.25 Hz. At this condition, the power in MW supplied by the generator is_____ (rounded off to two decimal places).

❖❖❖❖

Explanations Synchronous Machine

4.1 91.827

Given : Star connected synchronous motor

(i) 50 kW synchronous motor is tested by driving it by another motor.

(ii) 50% load

(iii) Short circuit current, $I_{sc} = 10$ A

DC motor - AC Synchronous efficiency Synchronous machine (SM) driven at rated synchronous speed by a dc motor			
1.	**No field excitation**	DC output = Friction windage loss of SM	$P_1 = P_{FW}$
2.	**Open circuit armature at rated excitation**	DC output = Iron loss of SM + Friction windage loss	$P_2 = P_{core} + P_{FW}$ $P_{core} = P_2 - P_1$
3.	**Short circuit armature at rated armature current**	DC output = Copper loss of SM + Friction windage loss	$P_3 = P_{cu(FL)} + P_{FW}$ $P_{cu(FL)} = P_3 - P_1$

This motor is driven by another lossless motor i.e. the power input of driving motor is completely transferred to the synchronous motor.

1. **When the excitation is not switched on, the driving motor takes 800 W.**

   Friction and windage loss,

   $$P_1 = P_{FW} = 800 \text{ W}$$

2. **On open circuiting the armature with rated excitation, the driving motor takes 1800 W.**

   $$P_2 = 1800 = P_{core} + P_{FW}$$

   $$P_{core} = 1800 - 800 = 1000 \text{ W}$$

3. **When the armature is short circuited and the rated armature current at 10 A is passed through it, the driving motor requires 2500 W.**

   $$P_3 = 2500 = P_{cu(FL)} + P_{FW}$$

   $$P_{cu(FL)} = 2500 - 800 = 1700 \text{ W}$$

   At rated or full load current of 10 A, full load copper loss is 1700 W.

At half load, copper loss will be

$$P_{cu(HL)} = (0.5)^2 P_{cu(FL)} = 0.25 \times 1700 = 425 \text{ W}$$

Efficiency at half load,

$$\%\eta = \frac{P_{out}}{P_{out} + P_{loss}}$$

$$\%\eta = \frac{25000}{25000 + 800 + 1000 + 425} = 91.827\%$$

Hence, the percentage efficiency is **91.827%**.

Scan for Video Solution

4.2 (C)

Given : synchronous generator

(i) Number of poles, $P = 4$

(ii) Double layer lap winding.

(iii) Supply frequency, $f = 50$ Hz,

(iv) Slots = 48

(v) Flux per pole, $\phi = 0.025$ Wb

(vi) Number of turns in each coil $= 10$

(vii) Short pitch angle, $\alpha = 36^0$

For double layer winding,

Number of coils = Number of slots = 48,

So, total number of turns,

$$T = 48 \times 10 = 480$$

Short pitch angle, $\alpha = 36^0$

Fundamental flux per pole,

$$\phi = 0.025 \text{ Wb}$$

Turns per phase,

$$T_{ph} = \frac{\text{Number of turns}}{\text{phase}} = \frac{480}{3}$$

$$T_{ph} = 160$$

Slots/pole/phase is given by,

$$m = \frac{48}{4 \times 3} = 4$$

Slot angle, $\gamma = \frac{180 \times \text{poles}}{\text{slots}} = \frac{180 \times 4}{48}$

$$\gamma = 15$$

For n^{th} Harmonic, distribution factor is given by,

$$k_{dn} = \frac{\sin \frac{mn\gamma}{2}}{m \sin \frac{n\gamma}{2}}$$

Hence, the fundamental distribution factor is given by,

$$k_{d_1} = \frac{\sin \frac{m\gamma}{2}}{m \sin \frac{\gamma}{2}} = \frac{\sin \frac{4 \times 15}{2}}{4 \sin \frac{15}{2}} = 0.957$$

For n^{th} Harmonic, pitch factor is given by,

$$k_{pn} = \cos \frac{n\alpha}{2}$$

Hence, the fundamental pitch factor is given by,

$$k_{p_1} = \cos \frac{\alpha}{2} = \cos 18^0 = 0.951$$

Winding factor is given by,

$$k_{w_1} = k_{p_1} k_{d_1} = 0.951 \times 0.957 = 0.91$$

Induced emf per phase is given by,

$$E_{ph} = 4.44\, k_{w_1} f\, T_{ph} \phi$$

$$E_{ph} = 4.44 \times 0.91 \times 50 \times 160 \times 0.025$$

$$E_{ph} = 808 \text{ V}$$

$$E_{line} = \sqrt{3} E_{ph} = \sqrt{3} \times 808 = 1400 \text{ V}$$

Hence, the correct option is (C).

Scan for Video Solution

4.3 (C)

For 2-phase,

Slots/pole/phase, $m = \frac{48}{2 \times 4} = 6$

$$T_{ph} = \frac{480}{2} = 240$$

Since, $E_{ph} \propto k_d T_{ph}$

$$\frac{E_{ph}(3\text{-}\phi)}{E_{ph}(2\text{-}\phi)} = \frac{k_d(3\text{-}\phi) T_{ph}(3\text{-}\phi)}{k_d(2\text{-}\phi) T_{ph}(2\text{-}\phi)} \quad \ldots\text{(i)}$$

$$k_d(2\text{-}\phi) = \frac{\sin\left(\frac{m\gamma}{2}\right)}{m \sin\left(\frac{\gamma}{2}\right)} = \frac{\sin \frac{6 \times 15}{2}}{6 \sin \frac{15}{2}} = 0.903$$

From equation (i),

$$\frac{E_{ph}(3-\phi)}{E_{ph}(2-\phi)} = \frac{0.957}{0.903} \times \frac{160}{240} = 0.707$$

$$E_{ph}(2-\phi) = \frac{E_{ph}(3\text{-}\phi)}{0.707}$$

$$E_{ph}(2\text{-}\phi) = \frac{808}{0.707} = 1142.85 \text{ V}$$

Hence, $E_{line}(2\text{-}\phi) = \sqrt{2} \times 1142.85 = 1617 \text{ V}$

Hence, the correct option is (C).

Scan for Video Solution

Key Point

(i) Single layer winding : In a single layer winding one coil side occupies the total slot area or a slot consists of only one coil side.

$$\text{Number of coils} = \frac{1}{2} \times \text{Number of slots}$$

(ii) Double layer winding : In a double layer winding the slot contains even number of coil side in two layers or a slot consists of at least two coil side.

Number of coil = Number of slots

4.4 (A)

For n^{th} Harmonic, pitch factor is given by,

$$k_{pn} = \cos\frac{n\alpha}{2}$$

Hence, pitch factor due to 5^{th} harmonic component is given by,

$$k_{p_5} = \cos\left(\frac{5\times 36}{2}\right) = \cos(90^0) = 0$$

As k_{p_5} is zero, induced emf due to 5^{th} harmonic component is zero.

Hence, the correct option is (A).

4.5 (B)

Given :

(i) Salient pole alternator

(ii) Rated voltage, $V_t = 1$ pu

(iii) Rated armature current, $I_a = 1$ pu

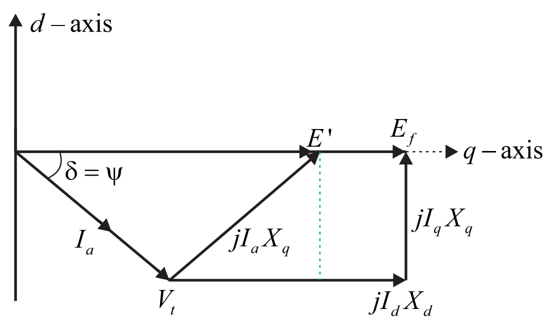
(iv) Direct axis reactance, $X_d = 1.2$ pu

(v) Quadrature axis reactance, $X_q = 1.0$ pu

(vi) Armature resistance, $R_a = 0$

Method 1

(iv) The phasor representation of salient pole alternator at upf is shown below,



As rated power is delivered at unity power factor i.e. I_a and V_t are in phase,

$$I_a = 1\angle 0 \text{ pu}$$

$$E' = V_t + jI_aX_q = 1 + j1\times 1$$

$$E' = \frac{1}{\sqrt{2}}\angle 45^0$$

Since, E' and E_f are in phase,

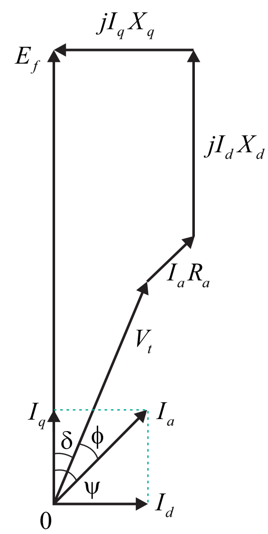
Angle between E' and V_t is δ, and E_f and V_t is also 'δ'.

$$\delta = 45^0$$

Hence, the correct option is (B).

Method 2

(v) The phasor representation of salient pole alternator is shown below,



For a salient pole alternator,

$$\tan\psi = \tan(\phi+\delta) = \frac{V_t\sin\phi \pm I_aX_q}{V_t\cos\phi + I_aR_a}$$

+ for lagging, upf
– for leading, pf

Where, $\psi =$ internal power factor angle

$\delta =$ load angle

$$\tan\psi = \frac{V_t\times 0 + I_aX_q}{V_t\times 1 + I_a\times 0} = \frac{I_aX_q}{V_t}$$

$$\tan\psi = \frac{1\times 1}{1} = 1$$

$$\psi = \tan^{-1}(1) = 45^0$$

Hence, $\delta = \psi - \phi = 45^0 - 0$

$$\psi = \delta = 45^0$$

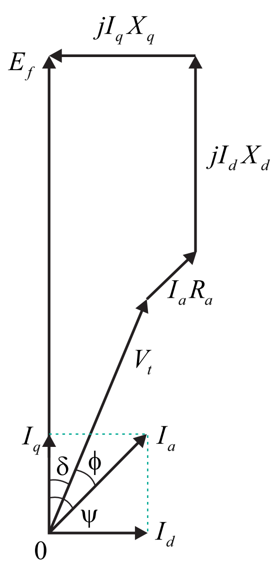
Hence, the correct option is (B).

Scan for Video Solution

Key Point

(i) Generator :

Generator at lagging power factor :



From phasor diagram :

1. $\psi = \delta + \phi$
2. $\tan\psi = \dfrac{V_t\sin\phi + I_aX_q}{V_t\cos\phi + I_aR_a}$
3. $E_f = V_t\cos\delta + I_qR_a + I_dX_d$
4. $I_qX_q = V_t\sin\delta + I_dR_a$

Generator at leading power factor :

1. $\psi = \delta - \phi$
2. $\tan\psi = \dfrac{V_t\sin\phi - \mathbf{I_aX_q}}{V_t\cos\phi + I_aR_a}$
3. $E_f = V_t\cos\delta + I_qR_a - \mathbf{I_dX_d}$
4. $I_qX_q = V_t\sin\delta - \mathbf{I_dR_a}$

(ii) Motor :

Motor at lagging power factor :

1. $\psi = \delta - \phi$
2. $\tan\psi = \dfrac{V_t\sin\phi - \mathbf{I_aX_q}}{V_t\cos\phi - \mathbf{I_aR_a}}$
3. $E_f = V_t\cos\delta - \mathbf{I_qR_a} - \mathbf{I_dX_d}$
4. $I_qX_q = V_t\sin\delta + I_dR_a$

Motor at leading power factor :

1. $\psi = \delta + \phi$
2. $\tan\psi = \dfrac{V_t\sin\phi + I_aX_q}{V_t\cos\phi - \mathbf{I_aR_a}}$
3. $E_f = V_t\cos\delta - \mathbf{I_aR_a} + I_dX_d$
4. $I_qX_q = V_t\sin\delta - \mathbf{I_dR_a}$

Note : Bold letters represents deviation from formula of generator at lagging power factor.

4.6 | 33.49

Given :

(i) A non-salient pole synchronous generator

(ii) Synchronous reactance, $X_s = 0.8$ pu

(iii) Power factor, $\cos\phi = 1.0$

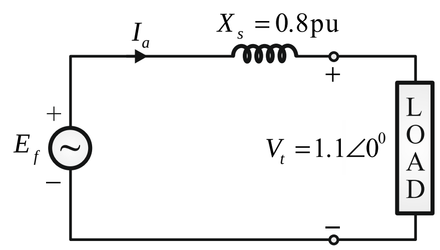
(iv) Power supplied, $P = 1$ pu

(v) Load terminal voltage, $V_t = 1.1$ pu

(vi) Armature resistance, $R_a = 0$

Method 1

The equivalent circuit is shown below,



Since, $P=1$

$$P=V_t I_a \cos\phi = 1.0$$

$$I_a = \frac{1}{V_t \cos\phi} = \frac{1.0}{1.1\times 1} = 0.91 \text{ pu}$$

$$I_a = 0.91\angle 0^0$$

From figure the excitation emf is given by,

$$E_f = V_t + I_a jX_s$$

$$E_f = 1.1\angle 0 + (0.91\angle 0^0)(j0.8)$$

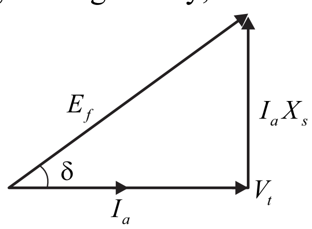
$$E_f = |E_f|\angle\delta = 1.319\angle 33.49^0$$

$$\delta = 33.49^0$$

Hence, the angle of the voltage behind the synchronous reactance with respect to the angle of the terminal voltage is $\mathbf{33.49^0}$.

Method 2

Phasor diagram is given by,



From phasor diagram torque angle is given by,

$$\tan\delta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{I_a X_s}{V_t} \quad \dots \text{(i)}$$

Since, $P=1$

$$P = V_t I_a \cos\phi$$

$$V_t I_a \cos\phi = 1.0$$

$$I_a = \frac{1}{V_t\cos\phi}$$

$$I_a = \frac{1.0}{1.1\times 1} = 0.91 \text{ pu}$$

$$I_a = 0.91\angle 0^0$$

From equation (i),

$$\tan\delta = \frac{I_a X_s}{V_t} = \frac{0.91\times 0.8}{1.1} = 0.6618$$

$$\delta = \tan^{-1} 0.6618 = 33.49^0$$

Hence, the angle of the voltage behind the synchronous reactance with respect to the angle of the terminal voltage is $\mathbf{33.49^0}$.

4.7 **–14.55**

Given : 3-phase, Δ-connected synchronous generator

(i) Terminal voltage,

$$V_{t(line)} = V_{t(phase)} = 400\text{ V}$$

(ii) kVA rating, $S = 25$ kVA

(iii) Field current, $I_f = 5\text{A}$

(iv) Armature resistance, $R_a = 0$

(v) Open circuit voltage,

$$V_{OC(line)} = V_{OC(phase)} = 360\text{ V}$$

Short circuit current, $I_{SC} = I_{rated}$

$$I_{rated(line)} = I_{SC} = \frac{25\times 10^3}{\sqrt{3}\times 400} = 36.08\text{ A}$$

$$I_{rated(phase)} = \frac{36.08}{\sqrt{3}} = 20.83\text{ A}$$

$$X_S = \left.\frac{V_{OC}}{I_{SC}}\right|_{I_f=5\text{A}} = \frac{360}{20.83} = 17.28\,\Omega$$

Method 1

Since, machine is delivering rated load at 0.8 *pf* leading at rated voltage, hence

$$E_f = V_t + jI_a X_S$$

$$E_f = 400 + 20.83\angle 36.86\times 17.28\angle 90^0$$

$$E_f = 341.8\angle 57.41$$

$$\%\text{ Voltage regulation } = \frac{|E_f| - |V_t|}{|V_t|}\times 100$$

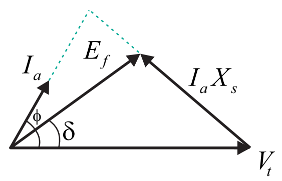
$$\%\text{ V.R. } = \frac{341.8-400}{400}\times 100$$

$$\%\text{ V.R.} = -14.55\%$$

Hence, the voltage regulation is **–14.55%.**

Method 2

The phasor diagram under leading *pf* is shown below,



Magnitude of excitation voltage for lagging power factor is given by,

$$E_f = \sqrt{(V_t \cos\phi + I_a R_a)^2 + (V_t \sin\phi - I_a X_s)^2}$$

$$E_f = \sqrt{(400\times 0.8)^2 + (400\times 0.6 - 20.83\times 17.28)^2}$$

$$E_f = 341.8 \text{ V}$$

$$\% \text{ Voltage regulation } = \frac{|E_f| - |V_t|}{|V_t|}\times 100$$

$$\% \text{ V.R. } = \frac{341.8 - 400}{400}\times 100$$

$$\% \text{ V.R. } = -14.55\%$$

Hence, the voltage regulation is **–14.55%.**

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4.8 **130**

Given : Synchronous generator

(i) Power deliverd to load, $P_1 = 100$ MW

(ii) Power deliverd to load due to increase in load by 10 Mw, $P_2 = 110$ MW

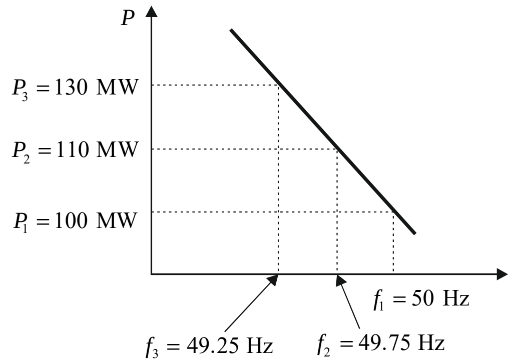
(iii) frequency when 100 Mw load is being delivered, $f_1 = 50$ Hz

(iv) frequency when 110 Mw load is being delivered, $f_2 = 49.75$ Hz

Method 1

Line constant :

$$Y = mX + C$$



$$m = \text{Slope} = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{110-100}{49.75-50} = -40\frac{\text{MW}}{\text{Hz}}$$

$$100 = -40\times 50 + C$$

$$C = 2100 \text{ MW}$$

At 49.25 Hz

$$P_3 = Y = mX + C = -40\times 49.25 + 2100$$

$$P_3 = 130 \text{ MW}$$

Method 2

We know that,

$$s_P = \frac{-\Delta P}{\Delta f} = \frac{-10 \text{ MW}}{(49.75-50) \text{ Hz}}$$

$$\therefore \quad s_P = 40$$

$$\text{Now,} \quad s_P = \frac{-\Delta P'}{49.25 - 49.75}$$

$$\Delta P' = 20$$

$$\therefore \quad \text{Total change } = \Delta P + \Delta P' = 30 \text{ MW}$$

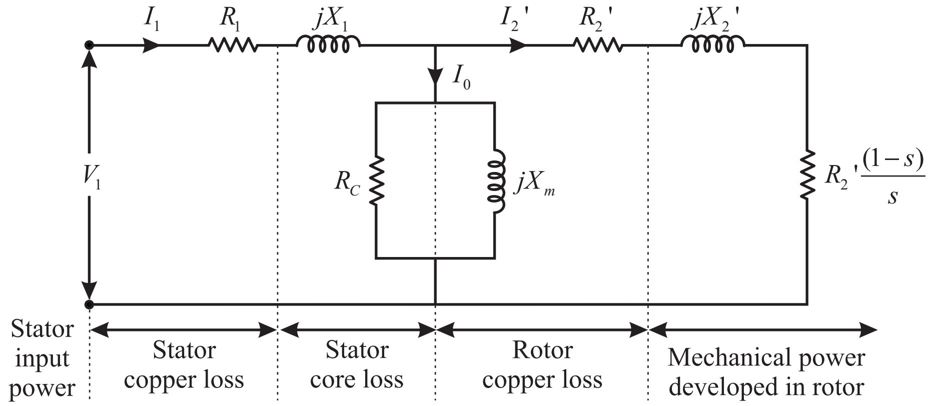
Power delivered $= 100 + 30 = 130$ MW

❖❖❖❖

5 Three Phase Induction Machine

➢ Partial Synopsis

Rotor Power Flow In Three Phase Induction motor



Rotor copper losses, $P_{cur} = 3I_2^2R_2$ Gross Mechanical power developed in rotor, $P_m = 3I_2^2R_2\left[\frac{1-s}{s}\right]$

Air gap power, $P_g = P_{cu} + P_m = \frac{3I_2^2R_2}{s}$ $\therefore$ $P_{cur} = sP_g$ and $P_m = P_g(1-s)$

The relationship can be expressed in the ratio form as, $\boldsymbol{P_g : P_c : P_m = 1 : s : 1-s}$

Induced torque, $T_I = \frac{P_m}{\omega_m} = \frac{P_g}{\omega_s}$ Shaft torque, $T_{sh} = \frac{P_{out}}{\omega_m} = \frac{P_m - P_{FW}}{2\pi N_m/60}$

Efficiency of induction motor, $\eta_{\text{external}} = \frac{\text{Net Mechanical power devloped } (P_{sh})}{\text{Stator input power } (P_{in})}$

Torque in Induction motor

Torque	When stator impedance and magnetizing reactance are neglected	When stator impedance and magnetizing reactance are not neglected $Z_{TH} = R_{TH} + jX_{TH} = [R_1 + jX_1] \parallel jX_m$ $V_{TH} = \frac{V_1 \cdot jX_m}{R_1 + jX_1 + jX_m}$
Electromagnetic torque	$T_e = \frac{3}{\omega_s} \cdot \frac{sV^2R_2}{R_2^2 + (sX_2)^2}$	$T_e = \frac{3}{\omega_s} \cdot \frac{V_{TH}^2}{\left[R_{TH} + \frac{R_2}{s}\right]^2 + [X_{TH} + X_2]^2} \cdot \frac{R_2}{s}$

Full load torque	$T_{eFL} = \frac{3}{\omega_s} \cdot \frac{s_{FL} V^2 R_2}{R_2^2 + (s_{FL} X_2)^2}$	$T_{eFL} = \frac{3}{\omega_s} \cdot \frac{V_{TH}^2}{\left[R_{TH} + \frac{R_2}{s_{FL}}\right]^2 + [X_{TH} + X_2]^2} \cdot \frac{R_2}{s_{FL}}$
Starting torque	$T_{eST} = \frac{3}{\omega_s} \cdot \frac{V^2 R_2}{R_2^2 + X_2^2}$ $s = 1$	$T_{eST} = \frac{3}{\omega_s} \cdot \frac{V_{TH}^2}{[R_{TH} + R_2]^2 + [X_{TH} + X_2]^2} \cdot R_2$
Maximum torque	$T_{eMax} = \frac{3}{\omega_s} \cdot \frac{V^2}{2X_2}$ $s_m = \frac{R_2}{X_2}$	$T_{eMax} = \frac{3}{\omega_s} \cdot \frac{V_{TH}^2}{2\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]}$ $s_m = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$

Ratio of Torques

When stator impedance and magnetizing reactance are neglected

Full load torque to Maximum torque	$\frac{T_{FL}}{T_{\max}} = \frac{2s_{FL}s_m}{s_{FL}^2 + s_m^2}$	$\frac{T_{FL}}{T_{\max}} = \left(\frac{I_{FL}}{I_{\max}}\right)^2 . \frac{s_m}{s_{FL}}$	$\left(\frac{I_{FL}}{I_{\max}}\right)^2 = \frac{2s_{FL}^2}{s_{FL}^2 + s_m^2}$
Starting torque to Maximum torque	$\frac{T_{st}}{T_{\max}} = \frac{2s_m}{s_m^2 + 1}$	$\frac{T_{st}}{T_{\max}} = \left(\frac{I_{st}}{I_{\max}}\right)^2 .s_m$	$\left(\frac{I_{st}}{I_{\max}}\right)^2 = \frac{2}{s_m^2 + 1}$
Starting torque to Full load torque	$\frac{T_{st}}{T_{FL}} = \frac{s_{FL}^2 + s_m^2}{(s_m^2 + 1)s_{FL}}$	$\frac{T_{st}}{T_{FL}} = \left(\frac{I_{st}}{I_{FL}}\right)^2 .s_{FL}$	$\left(\frac{I_{st}}{I_{FL}}\right)^2 = \frac{s_{FL}^2 + s_m^2}{(s_m^2 + 1)s_{FL}^2}$

➢ Sample Questions

1997 IIT Madras

5.1 A 3-phase, 20 kW, 400 V, 1470 rpm, 50 Hz squirrel cage induction motor develops a torque of 100 N-m at a speed of 1400 rpm. If the motor is connected to a 30 Hz supply, for keeping the same air-gap flux, the supply voltage should be _____V and for the same load torque, the new speed will be _________ rpm.

(A) 240 V and 864 rpm

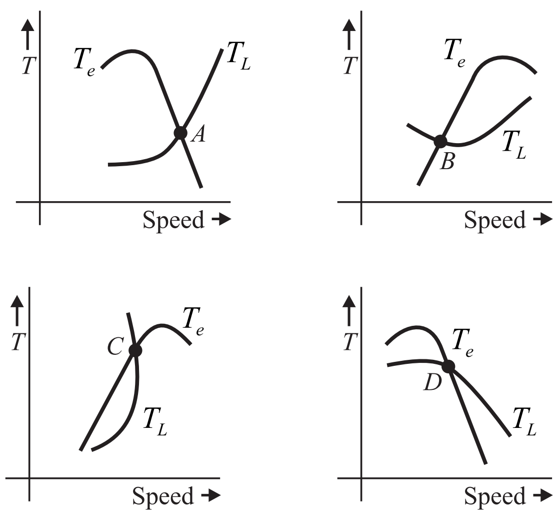
(B) 400 V and 800 rpm

(C) 240 V and 1400 rpm

(D) 400 V and 1400 rpm

2007 IIT Kanpur

5.2 The electromagnetic torque T_e of a drive and its connected load torque T_L are as shown below. Out of the operating points *A*, *B*, *C* and *D*, the stable ones are



(A) *A*, *C*, *D*   (B) *B*, *C*

(C) *A*, *D*   (D) *B*, *C*, *D*

2008 IISc Bangalore

5.3 A 400 V, 50 Hz, 30 hp, three-phase induction motor is drawing 50 A current at 0.8 power factor lagging. The stator and rotor copper losses are 1.5 kW and 900 W respectively. The friction and windage losses are 1050 W and the core losses are 1200 W. The air-gap power of the motor will be :

(A) 23.06 kW   (B) 24.11 kW

(C) 25.01 kW   (D) 26.21 kW

5.4 A 400 V, 50 Hz, 4 pole, 1400 rpm, star connected squirrel cage induction motor has the following parameters referred to the stator :

$$R_r' = 1.0\ \Omega,\ X_s = X_1' = 1.5\ \Omega$$

Neglect stator resistance and core and rotational losses of the motor. The motor is controlled from a three-phase voltage source inverter with constant $\frac{V}{f}$ control. The stator line-to-line voltage (rms) and frequency to obtain the maximum torque at starting will be

(A) 20.6 V, 2.7 Hz

(B) 133.3 V, 16.7 Hz

(C) 266.6 V, 33.3 Hz

(D) 323.3 V, 40.3 Hz

2014 IIT Kharagpur

5.5 A 3-phase, 50 Hz, 6-pole induction motor has a rotor resistance of $0.1\,\Omega$ and reactance of $0.92\,\Omega$. Neglect the voltage drop in stator and assume that the rotor resistance is constant. Given that, the full load slip is 3%, the ratio of maximum torque to full load torque is

[Set - 01]

(A) 1.567   (B) 1.712

(C) 1.948   (D) 2.134

2020 IIT Delhi

5.6 A three-phase, 50 Hz, 4-pole induction motor runs at no-load with a slip of 1%. With full load, the slip increases to 5%. The % speed regulation of the motor (rounded off to two decimal places) is ______.

2021 IIT Bombay

5.7 The power input to a 500 V, 50 Hz, 6 pole, 3 phase induction motor running at 975 rpm is 40 kW. The stator losses are 1 kW. If the total friction and windage losses are 2.025 kW, then the efficiency is ______%.

❖❖❖❖

Explanations Three Phase Induction Machine

5.1 (A)

Given : Three phase squirrel cage induction motor

(i) Terminal voltage, $V_{t(line)} = 400\text{ V}$

(ii) Supply frequency, $f_1 = 50$ H

(iii) Power output, P_{out} = 20 kW

(iv) Load torque,

T_L = 100 Nm = Constant

(v) Rated speed = 1470 rpm

(vi) Rotor speed = 1400 rpm

(vii) Supply frequency, $f_2 = 30$ Hz

Since, synchronous speed is not given we will assume the nearest possible synchronous speed i.e. 1500 rpm (for 4 poles)

Now, for constant air gap flux.

Assume same flux i.e. $V/f = k$

$$\frac{V_1}{f_1} = \frac{V_2}{f_2}$$

$$\frac{400}{50} = \frac{V_2}{30}$$

$$V_2 = \frac{400 \times 30}{50} = 240\text{ V}$$

Slip, $$s_1 = \frac{1500 - 1400}{1500} = 0.0667$$

For same load torque $(T_1 = T_2)$,

$$T_e = \frac{3}{\omega_s} \times \frac{sE_2^2}{R_2^2 + (sX_2)^2} \times R_2$$

$$T_e \propto \frac{1}{f} sV_1^2$$

Since, $$\frac{s_1 V_1^2}{f_1} = \frac{s_2 V_2^2}{f_2}$$

$$\frac{s_1}{f_1} = \frac{s_2}{f_2}$$

$$\frac{0.0667}{50} = \frac{s_2}{30}$$

$$s_2 = 0.04$$

Slip, $$s_2 = 0.04 = \frac{N_{s2} - N_m}{N_{s2}}$$

New speed can be calculated as,

$$N_{s2} = \frac{120 \times f_2}{P} = \frac{120 \times 30}{4}$$

$$N_{s2} = 900\text{ rpm}$$

$$N_m = N_{s2}(1 - 0.04) = 900(1 - 0.04)$$

$$N_m = 864\text{ rpm}$$

Hence, the correct option is (A).

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5.2 (C)

Given : Three phase induction motor

(i) T_e = Electromagnetic torque

(ii) T_L = Load torque

For any motor,

$T_L > T_e \xrightarrow{\text{Output > Input}}$	Deceleration
$T_L < T_e \xrightarrow{\text{Output < Input}}$	Acceleration
$T_L = T_e \xrightarrow{\text{Output = Input}}$	Steady state operation

(i) At point *A*, if speed is slightly increased, the load torque (T_L) becomes more than electromagnetic torque (T_e), then there will be retardation and it will come back to point *A*. If speed is slightly decreased, the load torque (T_L) becomes less than electromagnetic torque (T_e), then there will be acceleration and it will come back to point *A*.
Therefore at point *A* operation is stable.

(ii) At point *B*, if speed is slightly increased, the load torque (T_L) becomes less than electromagnetic torque (T_e), then there will be acceleration and the speed will further increase. If speed is slightly decreased, the load torque (T_L) becomes more than electromagnetic torque (T_e), then there will be retardation and the speed will decrease further.
Therefore at point *B* operation is unstable.

(iii) At point *C*, if speed is slightly increased, the load torque (T_L) becomes less than electromagnetic torque (T_e), then there will be acceleration and the speed will further increase. If speed is slightly decreased, the load torque (T_L) becomes more than electromagnetic torque (T_e), then there will be retardation and the speed will decrease further.
Therefore at point *C* operation is unstable.

(iv) At point *D*, if speed is slightly increased, the load torque (T_L) becomes more than electromagnetic torque (T_e), then there will be retardation and it will come back to point *D*. If speed is slightly decreased, the load torque (T_L) becomes less than electromagnetic torque (T_e), then there will be acceleration and it will come back to point *D*.
Therefore at point *D* operation is stable.

Hence, the correct option is (C).

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Key Point

The stability means the ability of the system to regain equilibrium after disturbance.

5.3 (C)

Given : Three phase induction motor

(i) Terminal voltage, $V_{t(line)} = 400\text{ V}$

(ii) Supply frequency, $f = 50\text{ Hz}$

(iii) Power output, $P_{out} = 30\text{ hp}$

(iv) Current, $I_L = 50\text{ A}$ at 0.8 power factor lagging

(v) Stator copper loss, $P_{stator} = 1.5\text{ kW}$

(vi) Rotor copper loss, $P_{cu} = 900\text{ kW}$

(vii) Friction and windage losses, $P_{wf} = 1050\text{ W}$

(viii) Stator core loss $P_i = 1200\text{ W}$

Stator input power is given by,

$$P_{in} = \sqrt{3} \times V_L I_L \cos\phi$$

$$P_{in} = \sqrt{3} \times 400 \times 50 \times 0.8$$

$$P_{in} = 27712.8 \text{ W}$$

Rotor input = Air gap power

= Stator input – (Stator Cu loss + Stator core losses)

$= 27712.8 - (1500 + 1200)$

$= 25012.8 \text{ W} = 25.012 \text{ kW}$

Hence, the correct option is (C).

5.4 (B)

Given : Three phase squirrel cage induction machine

(i) Terminal voltage, $V_{t(line)} = 400\text{ V}$

(ii) Supply frequency, $f = 50$ Hz

(iii) Number of poles, $P = 4$

(iv) Rotor speed, $N_r = 1400\text{ rpm}$

(v) Circuit parameters :

Rotor resistance, $R_r' = 1.0\ \Omega$,

Stator and rotor reactance, $X_s = X_r' = 1.5\ \Omega$

Stator resistance, $R_1 =$ Stator resistance $= 0$

Stator reactance is given by,

$$X_s = 2\pi f L_s$$

$$L_s = \frac{X_s}{2\pi \times f} = \frac{1.5}{2\pi \times 50}$$

$$L_s = 4.77 \text{ mH}$$

Since, $X_s = X_r' = 1.5\ \Omega$

Hence, $L_r' = 4.77$ mH

Slip corresponding to maximum torque,

$$s_m = \frac{R_r'}{\sqrt{(X_s + X_r')^2 + R_1^2}}$$

$$s_m = \frac{R'}{X_s + X_r'} \qquad [\text{As } R_1 = 0]$$

$$s_m = \frac{R'}{2\pi f_m L_s + 2\pi f_m L_r'}$$

$$s_m = \frac{R'}{2\pi f_m (L_s + L_r')} \qquad \ldots\text{(i)}$$

where, $f_m =$ Frequency to obtain maximum torque.

For maximum torque at starting, $s_m = 1$

From equation (i),

$$\frac{R'}{2\pi f_m (L_s + L_r')} = 1$$

$$f_m = \frac{R'}{2\pi (L_s + L_r')}$$

$$f_m = \frac{1.0}{2\pi \times 2 \times 4.77 \times 10^{-3}}$$

$$f_m = 16.68 \simeq 16.7 \text{ Hz}$$

In $\left(\frac{V}{f}\right)$ control, $\frac{V}{f}$ is constant.

Hence, $\frac{V_2}{f_2} = \frac{V_1}{f_1}$

$$\frac{V_2}{f_2} = \frac{400}{50}$$

Since, $f_m = f_2 = 16.7$ Hz

$$V_{2(line)} = \frac{400}{50} \times 16.7 = 133.3\text{ V}$$

Hence, the correct option is (B).

Scan for Video Solution

5.5 (C)

Given :

(i) Three phase induction motor

(ii) Supply frequency, $f = 50\text{ Hz}$

(iii) Number of poles, $P = 6$

(iv) Full load slip, $s_{fl} = 0.03$

(v) Rotor resistance, $R_2 = 0.1\ \Omega$

(vi) Rotor reactance, $X_2 = 0.92\Omega$

(vii) Stator voltage drop is negligible i.e.

$R_1 = 0,\ X_1 = 0$

Method 1

Slip at maximum torque is given by,

$$s_m = \frac{R_2}{X_2} = \frac{0.1}{0.92} = 0.1086$$

Ratio of full load torque to maximum torque is given by,

$$\frac{T_{fl}}{T_m} = \frac{2s_{fl}s_m}{s_{fl}^2 + s_m^2}$$

$$\frac{T_{fl}}{T_m} = \frac{2\times 0.1086\times 0.03}{(0.03)^2+(0.1086)^2} = 0.5133$$

So, $\frac{T_m}{T_{fl}} = 1.948$

Hence, the correct option is (C).

Method 2

The expression for maximum torque,

$$T_m = \frac{3}{2\omega_s}\times\frac{V^2}{X_2}$$

The general expression for developed torque in 3-phase induction motor is given below,

$$T = \frac{3}{\omega_s}\times\frac{V^2\frac{R_2}{s}}{\left(R_1+\frac{R_2}{s}\right)^2+(X_2)^2}$$

As stator voltage drop is negligible hence the full load torque can be expressed as given below,

$$T_{fl} = \frac{3}{\omega_s}\times\frac{V^2\frac{R_2}{s_{fl}}}{\left(\frac{R_2}{s_{fl}}\right)^2+(X_2)^2}$$

$$T_{fl} = \frac{3}{\omega_s}\times\frac{V^2\frac{R_2}{0.03}}{\left(\frac{R_2}{0.03}\right)^2+(X_2)^2}$$

$$T_{fl} = \frac{3V^2}{\omega_s}\times\frac{0.03R_2}{R_2^2+(0.03X_2)^2}$$

Now, the ratio of maximum torque to full load torque is given below,

$$\frac{T_m}{T_{fl}} = \frac{1}{2X_2}\times\frac{R_2^2+(0.03X_2)^2}{0.03R_2}$$

$$\frac{T_m}{T_{fl}} = \frac{1}{0.06}\left[\frac{R_2}{X_2}+(0.03)^2\left(\frac{X_2}{R_2}\right)\right]$$

$$\frac{T_m}{T_{fl}} = \frac{1}{0.06}\left[\frac{0.1}{0.92}+(0.03)^2\times\frac{0.92}{0.1}\right] = 1.95$$

Hence, the correct option is (C).

5.6 **4.21**

Given : Three phase induction machine

(i) Frequency, $f = 50$ Hz

(ii) Number of poles, $P = 4$

(iii) No load slip, $s_{nl} = 1\%$

(iv) Full load slip, $s_{fl} = 5\%$

$$\%\text{ speed regulation } = \left(\frac{N_{nL}-N_{fL}}{N_{fL}}\right)\times 100$$

$$N_s = \frac{120f}{P} = \frac{120\times 50}{4} = 1500\text{ rpm}$$

$$N_{r(nl)} = N_s(1-s_{nl})$$

$$N_{r_{(nL)}} = 1500[1-0.01]$$

$$N_{r(fl)} = N_s(1-s_{fl})$$

$$N_{r_{(fL)}} = 1500[1-0.05]$$

% speed regulation

$$= \left(\frac{1485-1425}{1425}\right)\times 100 = 4.21\%$$

Key Point

$$\%\text{ Speed regulation } = \left(\frac{N_{nL}-N_{fL}}{N_{fL}}\right)\times 100$$

As, $N_{\text{rated}} = N_{fl}$

$$\therefore\ \text{Speed regulation} = \left(\frac{N_{nL}-N_{fL}}{N_{fL}}\right)\times 100$$

5.7 **90**

Given :

(i) Input power, $P_{in} = 40\,\text{kW}$

(ii) Voltage, $V = 500\,\text{V}$

(iii) Frequency $f = 50\,\text{Hz}$

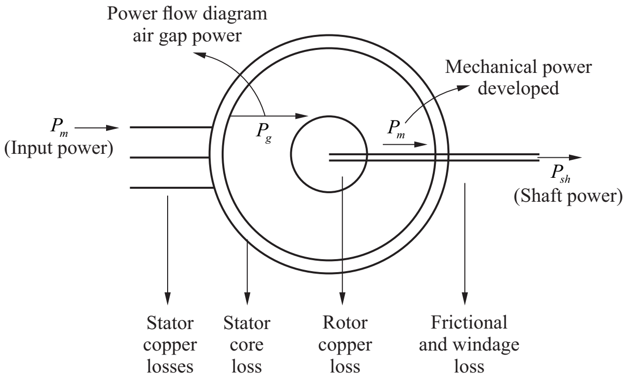
(iv) Pole $P = 6$

(v) Rotor speed, $N_r = 975$ rpm

(vi) The total stator losses are $1\,\text{kW} = 1000\,\text{W}$

(vii) Total friction and windage loss

$= 2.025\,\text{kW} = 2025\,\text{W}$



Air gap power, $P_g = P_{in} -$ Stator losses

$$P_g = 40000 - 1000 = 39000\,\text{W}$$

Rotor copper losses, $P_{cu} = sP_g$

So, $P_m = P_g - sP_g$

$$P_m = (1-s)P_g$$

Synchronous speed, $N_s = \dfrac{120\,f}{P}$

$$N_s = \frac{120 \times 50}{6} = 1000\,\text{rpm}$$

So, slip $s = \dfrac{N_s - N_r}{N_s} = \dfrac{1000 - 975}{1000} = 0.025$

So, $P_m = (1 - 0.025) \times 39000 = 38025\,\text{W}$

Shaft power,

$P_{sh} = P_m -$ Frictional and windage losses

$P_{sh} = 38025 - 2025 = 36000\,\text{W}$

Efficiency, $\eta = \dfrac{P_0}{P_{in}} = \dfrac{P_{sh}}{P_{in}}$

$$\%\eta = \frac{36000}{40000} \times 100 = 90\%$$

Hence, the correct answer is 90.

❖❖❖❖

6 Single Phase Induction Motor

➢ Sample Questions

2004 IIT Delhi

6.1 A single-phase, 230 V, 50 Hz, 4 pole capacitor-start induction motor has the following stand-still impedances

Main winding, $Z_m = 6.0 + j4.0\ \Omega$

Auxiliary winding, $Z_a = 8.0 + j6.0\ \Omega$

The value of the starting capacitor required to produce 90^0 phase difference between the currents in the main and auxiliary windings will be

(A) 176.84 μF (B) 187.24 μF

(C) 265.26 μF (D) 280.86 μF

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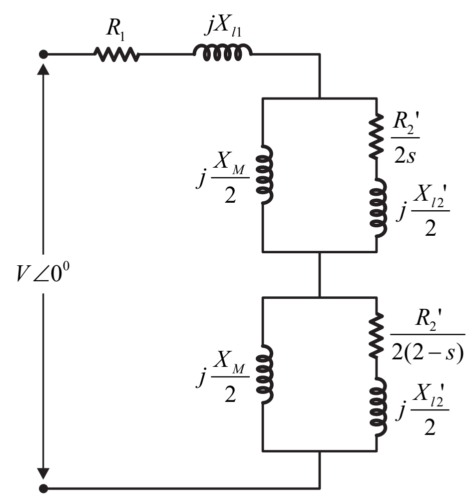
6.2 A 375 W, 230 V, 50 Hz, capacitor start single-phase induction motor has the following constants for the main and auxiliary windings (at starting) :

$Z_m = (12.50 + j15.75)\Omega$ (main winding),

$Z_a = (24.50 + j12.75)\Omega$ (auxiliary winding). Neglecting the magnetizing branch, the value of the capacitance (in μF) to be added in series with the auxiliary winding to obtain maximum torque at starting is ______. **[Set - 01]**



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Explanations **Single Phase Induction Motors**

6.1 (A)

Given : Single phase capacitor-start induction motor

(i) Rated voltage, V_s = 230 V,

(ii) Frequency, f = 50 Hz

(iii) Number of poles, $P = 4$

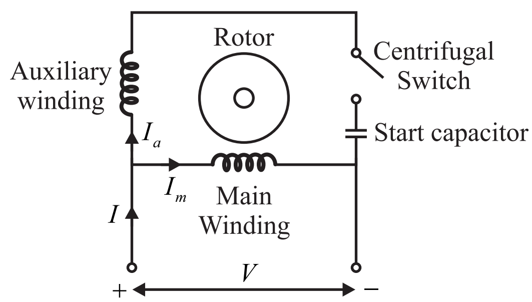
(iv) Stand still impedances :

Main winding, $Z_m = 6.0 + j4.0\ \Omega$

Auxiliary winding, $Z_a = 8.0 + j6.0\ \Omega$

Method 1

The equivalent circuit of capacitor start induction motor is shown below,

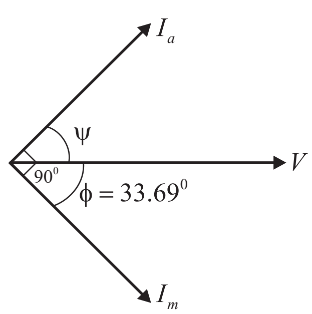


Current in main winding is given by,

$$I_m = \frac{V}{Z_m} = \frac{230}{6 + j4}$$

$$I_m = 31.89\angle -33.69^0 \text{ A}$$

The phasor representation is shown below,



When the starting capacitor is added in series with auxiliary winding, the angle $(\psi + \phi)$ between I_m and I_a is 90^0.

i.e. $\psi + \phi = 90^0$

$$\psi = 90^0 - 33.69^0 = 56.31^0 \quad \ldots(i)$$

Modified impedance of auxiliary winding is given by,

$$Z_a' = Z_a - jX_C = 8 + j6 - jX_C$$

$$Z_a' = 8 + j(6 - X_C)$$

As the angle $\psi > \phi$, hence, the nature of auxiliary winding impedance will be capacitive then the modified value Z_a' will be

$$Z_a' = 8 - j(X_C - 6)$$

where, $\omega = 2\pi \times 50$

Angle between I_a and V is given by,

$$\psi = \tan^{-1}\left(\frac{\text{Img}(Z_a')}{\text{Real}(Z_a')}\right)$$

$$56.31^0 = \tan^{-1}\left(\frac{X_C - 6}{8}\right)$$

[From equation (i)]

$$\tan 56.31^0 = \frac{X_C - 6}{8}$$

$$X_C = \frac{1}{2\pi \times 50 \times C} = 18$$

$$C = 176.84\ \mu\text{F}$$

Hence, the correct option is (A).

Method 2

The value of starting capacitor required to produce 90^0 phase difference between the currents in the main and auxiliary winding is

$$X_C = X_a + \frac{R_a R_m}{X_m} \quad \ldots(i)$$

Main winding, $Z_m = 6.0 + j4.0\ \Omega$

Here, $R_m = 6\ \Omega$, $X_m = 4\ \Omega$

Auxiliary winding, $Z_a = 8.0 + j6.0\ \Omega$

Here, $R_a = 8\ \Omega$, $X_a = 6\ \Omega$

Put the values in equation (i),

$$X_C = 6 + \frac{8\times 6}{4} = 18$$

Therefore, the starting capacitor is given by,

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi\times 50\times 18} = 176.84\ \mu\text{F}$$

Hence, the correct option is (A).

📖 Key Point

(i) For capacitive impedance, $Z = a - jb$
i.e. I leads V.

(ii) For inductive impedance, $Z = a + jb$
i.e. I lags V.

6.2 **149.5**

Given : Capacitor start, single-phase induction motor

(i) Rated voltage, $V = 230$ V

(ii) Output power, $P = 375$ W

(iii) Supply frequency, $f = 50$ Hz

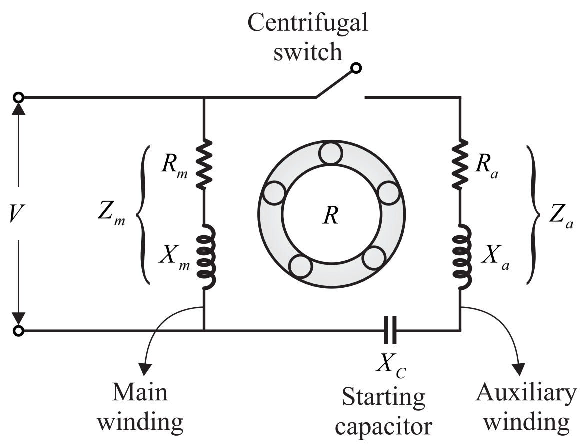
(iv) Stand still impedances :

Main winding, $Z_m = (12.50 + j15.75)\ \Omega$

Auxiliary winding,
$Z_a = (24.50 + j12.75)\ \Omega$

Maximum starting torque condition is single phase induction motor is capacitor start.

$$X_C = X_a + \frac{R_a R_m}{Z_m + X_m}$$



Auxiliary winding,

$$Z_a = R_a + jX_a = 24.5 + j12.75\ \Omega$$

Since, $R_a = 24.5\ \Omega$, $X_a = 12.75\ \Omega$

Main winding,

$$Z_m = R_m + jX_m = 12.5 + j15.75\ \Omega$$

$$R_m = 12.5\ \Omega \quad X_m = 75.75\ \Omega$$

$$Z_m = \sqrt{R_m^2 + X_m^2} = \sqrt{12.5^2 + 15.75^2}$$

$$Z_m = 20.1\ \Omega$$

Therefore,

$$X_C = X_a + \frac{R_a R_m}{Z_m + X_m} = 12.75 + \frac{24.5\times 12.5}{20.1 + 15.75}$$

$$X_C = 12.2925\ \Omega$$

The value of capacitance to be injected in series with the auxiliary winding for obtaining maximum torque at starting is given by,

$$C = \frac{1}{\omega X_C}$$

$$\omega = 2\pi f = 2\pi\times 50 \text{ rad/sec}$$

Therefore,

$$C = \frac{1}{100\pi\times 21.2925} = 1.495\times 10^{-4}\ \text{F}$$

$$C = 149.5\ \mu\text{F}$$

Hence, the value of capacitance required is **149.5 μF.**

📖 Key Point

(i) For capacitive impedance, $Z = a - jb$
i.e. I leads V.

(ii) For inductive impedance, $Z = a + jb$
i.e. I lags V.

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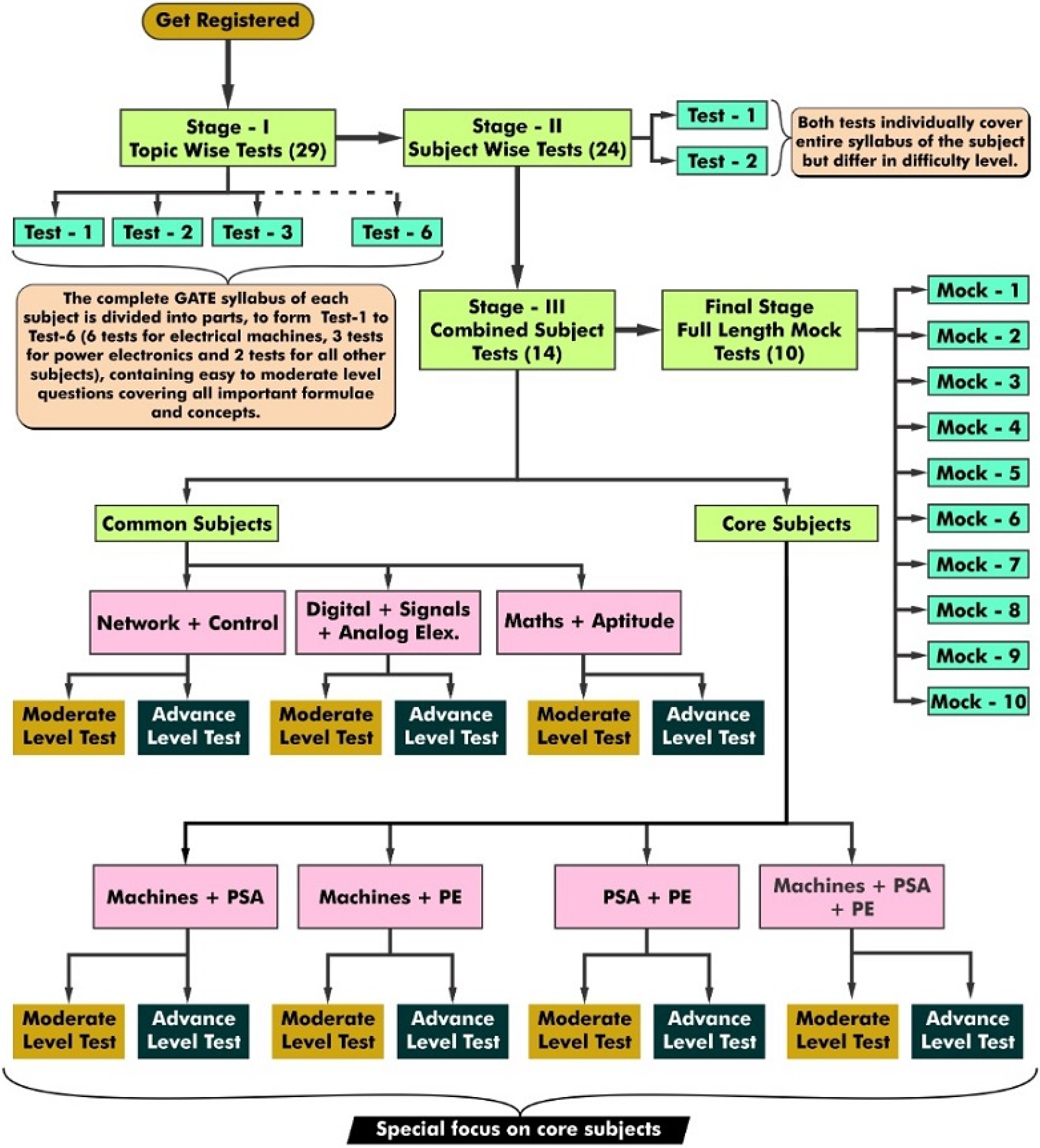

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Ajay Kumar Gupta


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Pritam Saurav


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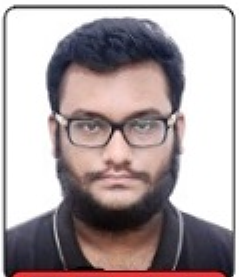
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Divyansh Trivedi


EE-146
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EE-170
Hritik Jaiswal


EE-170
Aman Kumar


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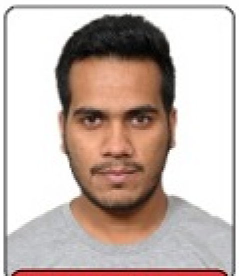
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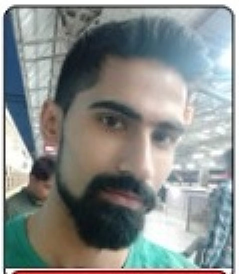
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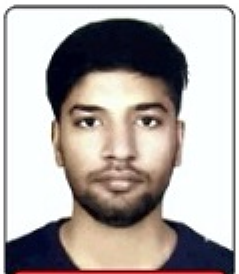
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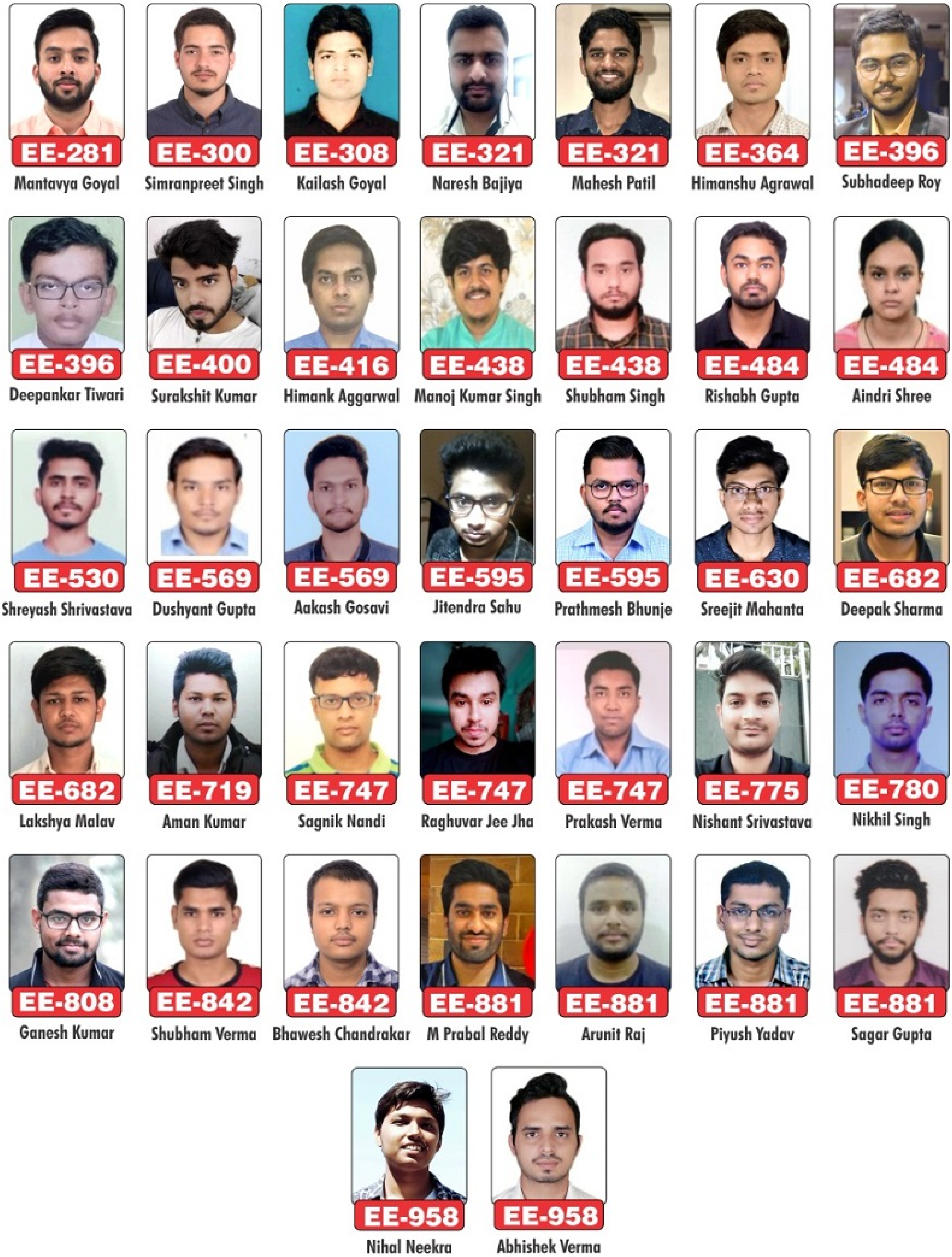


CHAPTER 2 | POWER ELECTRONICS

Marks Distribution of Power Electronics in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	3	4	11
2004	4	6	16
2005	3	5	13
2006	1	8	17
2007	4	7	18
2008	2	6	14
2009	1	4	9
2010	2	–	2
2011	2	3	8
2012	2	3	8
2013	–	7	14
2014 Set-1	2	3	8

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-2	1	3	7
2014 Set-3	–	3	6
2015 Set-1	2	3	8
2015 Set-2	2	3	8
2016 Set-1	2	4	10
2016 Set-2	3	4	11
2017 Set-1	4	2	8
2017 Set-2	4	3	10
2018	3	3	9
2019	2	4	10
2020	4	2	8
2021	ρ	σ	χ

Syllabus : Power Electronics

Static V-I characteristics and firing/gating circuits for Thyristor, MOSFET, IGBT; DC to DC conversion: Buck, Boost and Buck-Boost Converters; Single and three-phase configuration of uncontrolled rectifiers; Voltage and Current commutated Thyristor based converters; Bidirectional ac to dc voltage source converters; Magnitude and Phase of line current harmonics for uncontrolled and thyristor based converters; Power factor and Distortion Factor of ac to dc converters; Single-phase and three-phase voltage and current source inverters, sinusoidal pulse width modulation.

Contents : Power Electronics

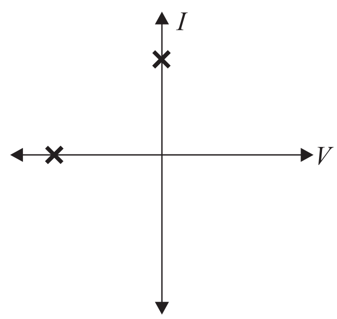
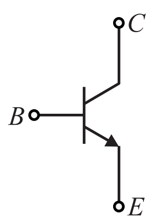
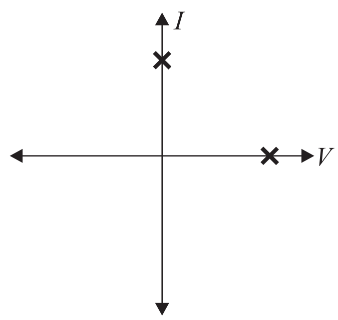
1 Power Semiconductor Devices

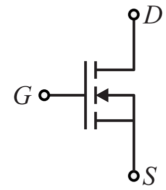
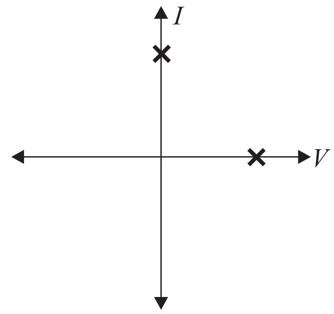
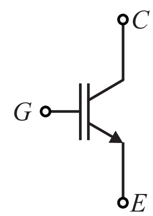
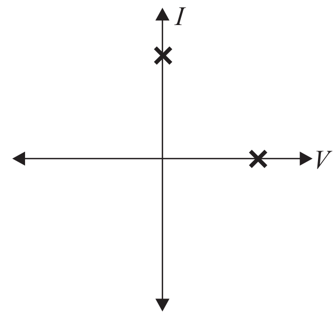
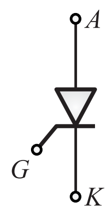
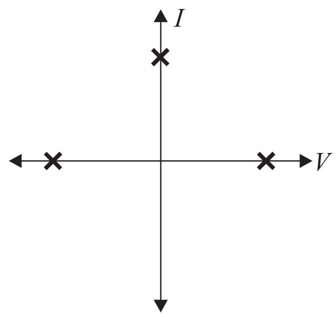
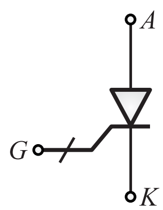
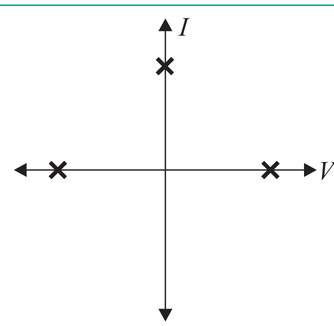
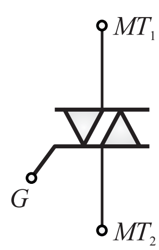
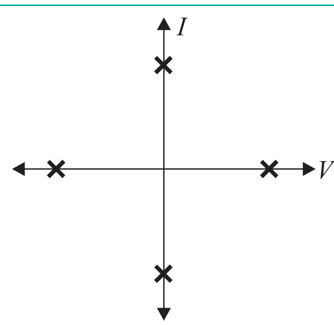
➢ Partial Synopsis

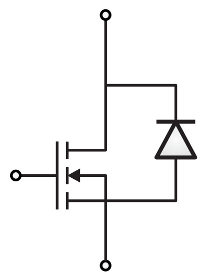
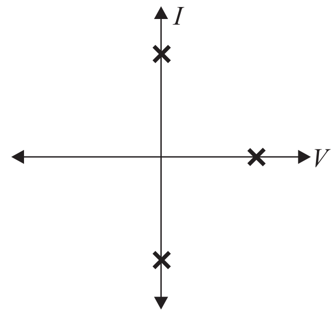
Classification of switches :

1. **Unipolar switch :** This switch can block only one polarity of voltage when it is in OFF state.
2. **Bipolar switch :** This switch can block both polarity of voltage when it is in blocking state.
3. **Unidirectional switch :** This switch can carry current in only one direction when it is in conduction state.
4. **Bidirectional switch :** This switch can carry current in both the directions when it is in conduction state.

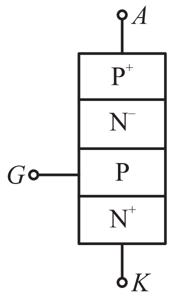
Ideal characteristics of power semiconductor switches :

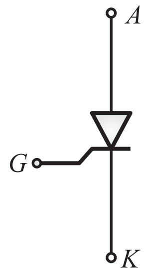
Device	Symbol	Characteristic
Diode		
BJT		

MOSFET		
IGBT		
SCR		
GTO		
TRIAC		

MOSFET with body diode		

Silicon controlled rectifier (SCR) :



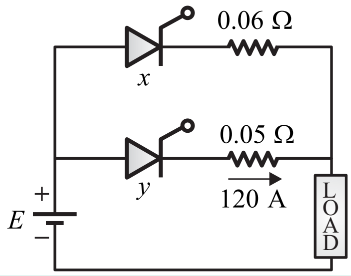


1. In forward blocking mode, J_1, J_3 are forward biased and J_2 is reverse biased.
2. In forward conduction mode, J_2 breakdown J_1, J_3 are forward biased.
3. In reverse blocking mode, J_1, J_3 are reverse biased and J_2 is forward biased.

➢ Sample Questions

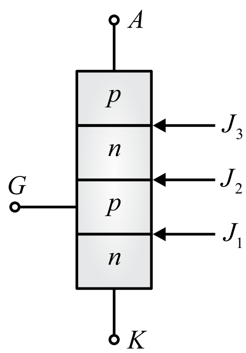
1995 IIT Kanpur

1.1 Figure show two thyristor, each rated 500 A (continuous) sharing a load current. Current through thyristor y is 120 A. The current through thyristor x will be nearly ________ A.



2003 IIT Madras

1.2 Figure shows a thyristor with the standard terminations of anode (A), cathode (K), gate (G) and the different junctions named J_1, J_2 and J_3. When the thyristor is turned on and conducting



(A) J_1 and J_2 are forward biased and J_3 is reverse biased

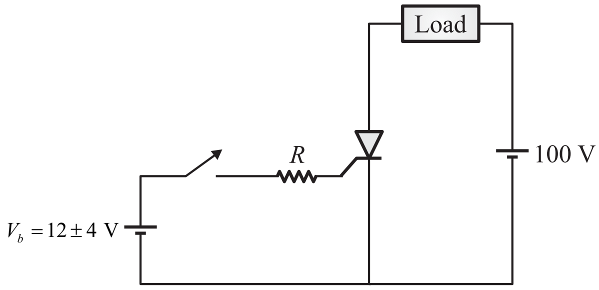
(B) J_1 and J_3 are forward biased and J_2 is reverse biased

(C) J_1 is forward biased and J_2 and J_3 are reverse biased.

(D) J_1, J_2 and J_3 are all forward biased

2004 IIT Delhi

1.3 The triggering circuit of a thyristor is shown in figure. The thyristor requires a gate current of 10 mA, for guaranteed turn-on. The value of R required for the thyristor to turn on reliably under all conditions of V_b variation is

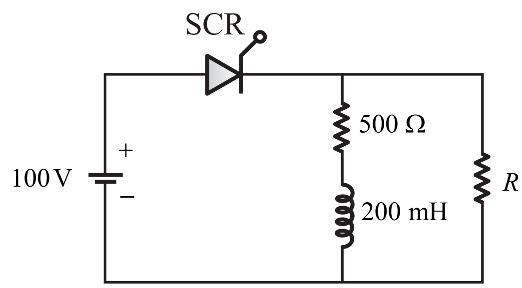


(A) 10000 Ω (B) 1600 Ω

(C) 1200 Ω (D) 800 Ω

2014 IIT Kharagpur

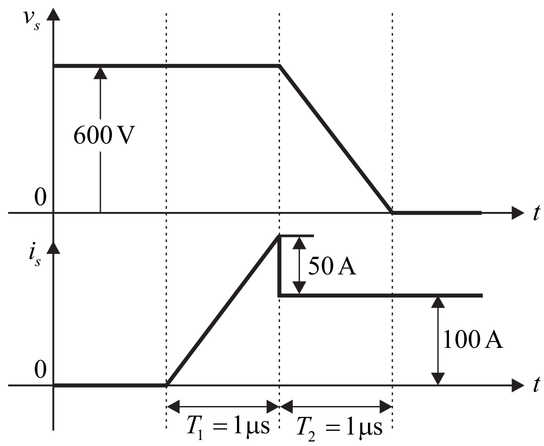
1.4 The SCR in the circuit shown has a latching current of 40 mA. A gate pulse of 50 μs is applied to the SCR. The maximum value of R in Ω to ensure successful firing of the SCR is __________. **[Set - 02]**



2016 IISc Bangalore

1.5 The voltage (v_s) across and the current (i_s) through a semiconductor during a turn-ON transition are shown in figure. The energy dissipated during the turn-ON transition, in mJ, is __________.

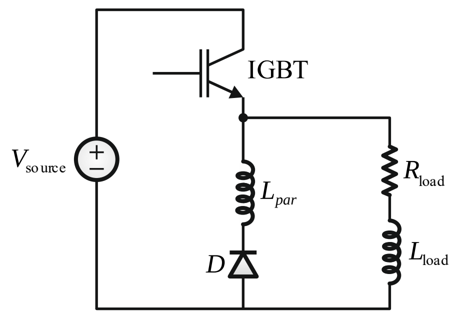
[Set - 01]

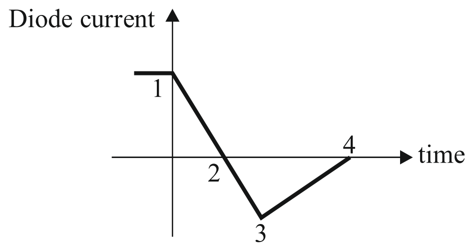


2020　IIT Delhi

1.6 A double pulse measurement for an inductively loaded circuit controlled by the IGBT switch is carried out to evaluate the reverse recovery characteristic of the diode, D, represented approximately as a piecewise linear plot of current vs time at diode turn-off. L_{par} is a parasitic inductance due to the wiring of the circuit and is in series with the diode.

The point on the plot (indicate your choice by entering 1, 2, 3 and 4) at which IGBT experiences the highest current stress is ________.





❖❖❖❖

Explanations　Power Semiconductor Devices

1.1　100

The thyristor x and thyristor y are connected in parallel. Hence, the voltage across both the thyristors is same.

Let, the current through thyristor x and y be I_x and I_y respectively, when they are ON.

As, voltage drop across both the thyristors,

$$I_x \times 0.06 = I_y \times 0.05$$

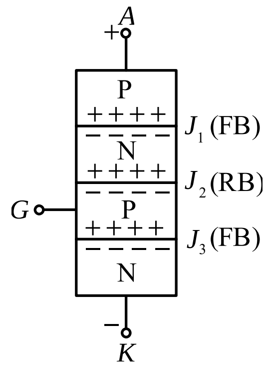
$$I_x = \frac{I_y \times 0.05}{0.06} = \frac{120 \times 0.05}{0.06} = 100\text{ A}$$

Hence, the current through thyristor x will be nearly **100 A.**

1.2　(B)

Given : Forward conduction mode

Anode is positive with respect to cathode. Under forward conduction mode J_1 and J_3 are in forward bias and J_2 will always be in reverse biased state. In FCM J_2 junction breaks down due to gate current.

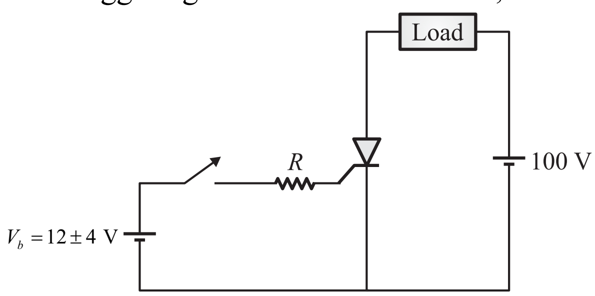


Hence, the correct option is (B).

1.3 (D)

Given : $I_{g\,\min} = 10$ mA ,

Given triggering circuit is shown below,



The variation in triggering voltage is,

$$V_b = 12 \pm 4 \text{ V}$$

Thus, the minimum and maximum triggering voltages are, $(V_b)_{\min} = 8$ V and $(V_b)_{\max} = 16$ V .

Gate current required by thyristor to turn ON,

$$(I_g)_{\min} = 10 \text{ mA}$$

Neglecting the gate to source voltage drop

$$(I_g)_{\min} = \frac{(V_b)_{\min}}{R}$$

$$10 \times 10^{-3} = \frac{12 - 4}{R}$$

$$R = 800\ \Omega$$

Hence, the correct option is (D).

📖 Key point

Both the voltage (maximum and minimum) can be applied to turn ON thyristor, but it must turn ON even with minimum value of applied voltages.

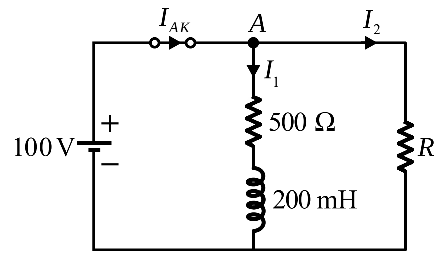
1.4 6060

Given :

Given :

(i) Laching current, $I_L = 40$ mA

(ii) Gate pulse width, $t = 50$ μsec



Applying KCL at point A,

$$I_{AK} = I_1 + I_2 \quad \ldots \text{(i)}$$

$$I_2 = \frac{100}{R} \quad \ldots \text{(ii)}$$

$$I_1 = I_L(t) = I_L(\infty) + \left(I_C(0^+) - I_C(\infty)\right)e^{\frac{-t}{\tau}}; t \geq 0$$

$$I_L(0^-) = I_L(0^+) = 0 \text{ A}$$

$$I_L(\infty) = \frac{100}{500} = 0.2 \text{ A}$$

$$\tau = \frac{L}{R} = \frac{200 \times 10^{-3}}{500} = 400 \times 10^{-6} \text{ sec} \quad \ldots \text{(iii)}$$

Putting the value of equation (ii) and (iii) in equation (i)

$$\therefore \quad 40 \times 10^{-3} = 0.2\left(1 - e^{\frac{-50 \times 10^{-6}}{400 \times 10^{-6}}}\right) + \frac{100}{R}$$

$$R = 6060\ \Omega$$

Hence, the maximum value of R is **6060 Ω .**

📖 Key point

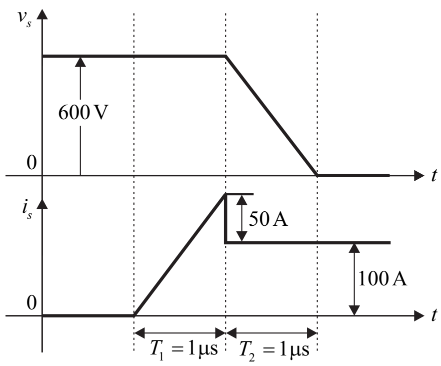
If there is no information about holding current, then we can assume holding current as 0 ampere and then we will proceed for the solution.

Latching current : It is the minimum amount of current required to maintain the thyristor in on-state immediately after a thyristor is turned ON.

Holding current : It is a minimum current that is required to maintain the thyristor in on-state not allowing it to turn OFF.

1.5 75

Method 1



Power loss $P_s = v_s i_s$ is

Energy loss during time T_1,

$$E_1 = \int_0^{T_1} vi\,dt = 600 \times \int_0^{T_1} i\,dt$$

$$E_1 = 600 \times \text{Area under } T_1$$

$$E_1 = 600 \times \frac{1}{2} \times 150 \times 1 \times 10^{-6} = 45 \text{ mJ}$$

Energy loss during time T_2,

$$E_2 = \int_0^{T_2} vi\,dt = 100 \times \int_0^{T_2} v\,dt$$

$$E_2 = 100 \times \text{Area under } T_2$$

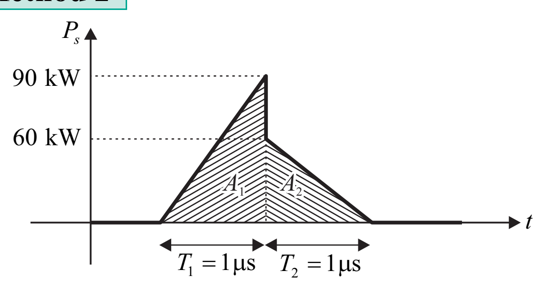
$$E_2 = 100 \times \frac{1}{2} \times 600 \times 10^{-6} \times 1 = 30 \text{ mJ}$$

Total energy loss

$$E_1 + E_2 = 45 + 30 = 75 \text{ mJ}$$

Hence, the energy dissipated during the turn-ON transition is **75 mJ.**

Method 2



Energy loss E_1 corresponding to T_1 is area A_1 and energy loss E_2 corresponding to T_2 is area A_2.

$$A_1 = \frac{1}{2} \times 90 \times 10^3 \times 10^{-6} \text{ J}$$

$$A_1 = 45 \text{ mJ}$$

$$A_2 = \frac{1}{2} \times 60 \times 10^3 \times 10^{-6} \text{ J}$$

$$A_2 = 30 \text{ mJ}$$

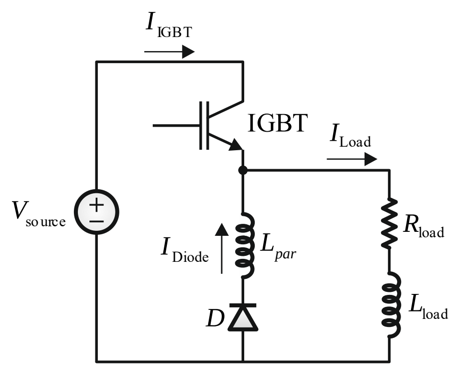
Total energy loss

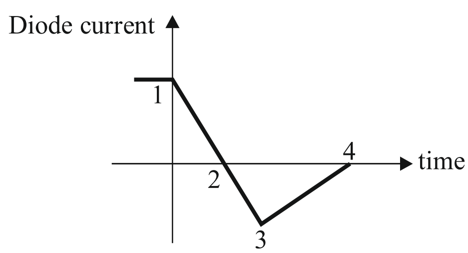
$$E_1 + E_2 = A_1 + A_2 = 45 + 30 = 75 \text{ mJ}$$

Hence, the energy dissipated during the turn-ON transition is **75 mJ.**

1.6 3

Given plot of diode current versus time and inductively loaded circuit is shown below,





Considering the load to be highly inductive

$\therefore$ Load current will be constant

Load current = Switch current + Diode current (diode current is considered to evaluate the reverse recovery characteristics of the diode)

So, $I_{\text{IGBT}} = I_{\text{Load}} - I_{\text{Diode}}$

I_{IGBT} will experience the highest current stress when the switch current will be maximum

Switch current = (Load current – Diode current) to be maximum diode current has to be minimum.

Diode current is minimum at point 3.

Hence, the correct point is (3).

❖❖❖❖

2 Single Phase Rectifier

➢ Partial Synopsis

Single phase controlled rectifier with RLE load with FD (Assuming continuous conduction ripple free load current)

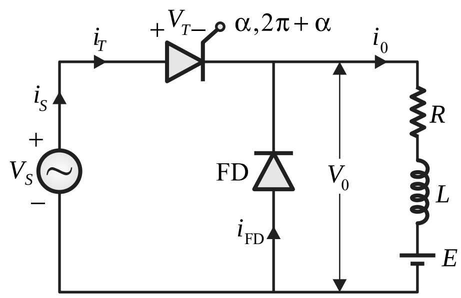


Fig. Circuit diagram

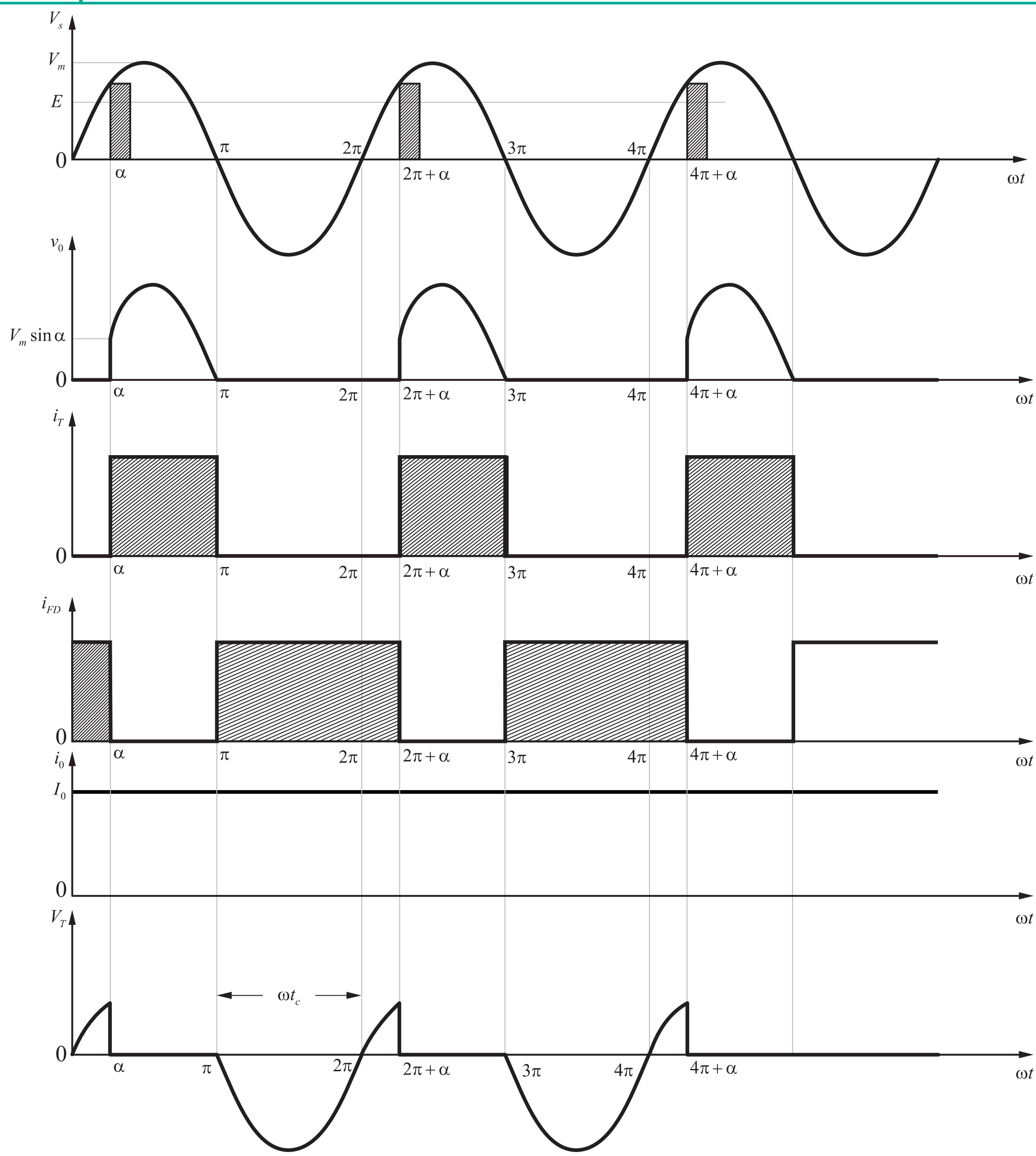


Fig. Voltage and current waveform

Performance parameter :

(i) Average output voltage,

$$V_0 = \frac{V_m}{2\pi}[1+\cos\alpha]$$

(ii) Average value of thyristor current,

$$I_T = I_0\left[\frac{\pi-\alpha}{2\pi}\right]$$

(iii) Average value of FD current,

$$I_{FD} = I_0\left[\frac{2\pi+\alpha-\pi}{2\pi}\right] = I_0\left[\frac{\pi+\alpha}{2\pi}\right]$$

(iv) Circuit turn off time,

$$t_c = \frac{\pi}{\omega}\ \text{sec}$$

Single phase full wave controlled rectifier (B – 2)

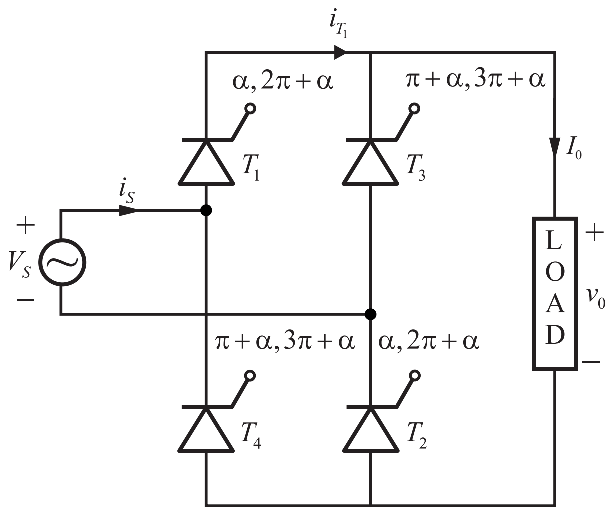


Fig. Circuit diagram

Single phase full wave controlled rectifier (B – 2) with continuous conduction and ripple free load current (I_0)

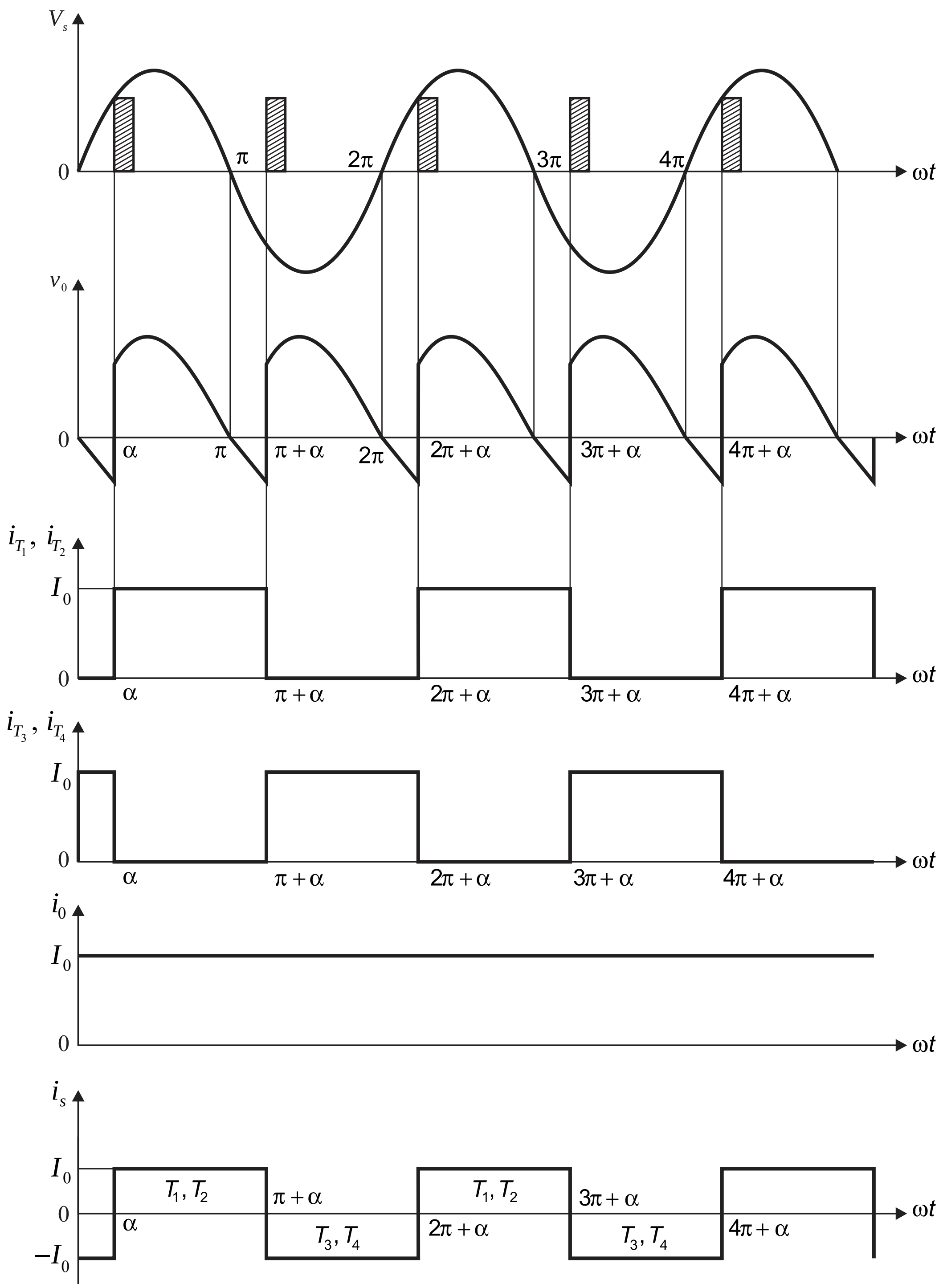


Fig. Voltage and current waveform

Performance Parameter for Continuous Conduction and Ripple Free Load current :

(i) Average output voltage, $V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t\, d\omega t \;= \frac{2V_m}{\pi}\cos\alpha$

(ii) Average output current,

$$I_0 = \frac{V_0}{R} \qquad \text{(For RL load)}$$

$$I_0 = \frac{V_0 - E}{R} \qquad \text{(For RLE load)}$$

(iii) RMS output voltage, $V_{or} = \frac{V_m}{\sqrt{2}}$

(iv) RMS output current, $I_{or} = I_0$

(v) Supply power factor, $\text{SPF} = \frac{V_o I_o}{V_s I_s}$ (For lossless converter)

Performance Parameters of Full Wave Controlled Rectifier with continuous conduction and ripple free load current

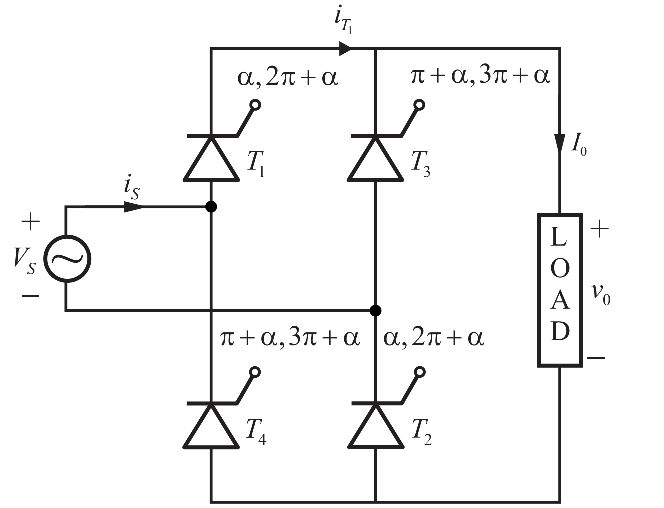
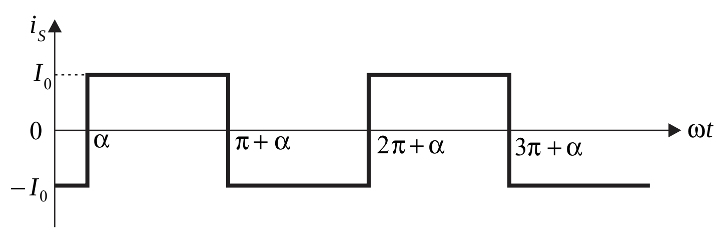


Fig. Circuit diagram

(i) The waveform of source current is



(ii) The Fourier series representation is

$$i_{Sn} = \sum_{n=1,3,5}^{\infty} \frac{4I_0}{n\pi}\sin(n\omega t + \phi_n)$$

where, $\phi_n = -n\alpha$

(iii) RMS value of n^{th} harmonic component

$$i_{Sn} = \frac{4I_0}{n\pi\sqrt{2}} = \frac{2\sqrt{2}\,I_0}{n\,\pi} = \frac{0.9\,I_0}{n}$$

(iv) RMS value of fundamental component

$$i_{S1} = \frac{4\,I_0}{\pi\sqrt{2}} = \frac{2\sqrt{2}\,I_0}{\pi} = 0.9\,I_0$$

(v) Displacement factor

$$\text{DF} = \cos\phi_n$$

$$\text{DF} = \cos(-n\alpha) = \cos(n\alpha)$$

(vi) Fundamental displacement factor, $\text{FDF} = \cos\,\phi_1 = \cos(-\alpha) = \cos(\alpha)$

(vii) Current distortion factor

$$\text{CDF} = \frac{i_{S1}}{i_{S(rms)}} = \frac{\frac{2\sqrt{2}I_0}{\pi}}{I_0} = \frac{2\sqrt{2}}{\pi} = 0.9$$

(viii) Total harmonic distortion

$$\text{THD} = \sqrt{\left(\frac{i_{S(rms)}}{i_{S1}}\right)^2 - 1} = 0.484$$

Total harmonic distortion is also called as Harmonic factor.

(ix) Supply power factor, $\text{SPF} = \frac{V_0\,I_0}{V_S\,I_S}$

$$\text{SPF} = \frac{2\sqrt{2}}{\pi}\cos\alpha$$

$\text{SPF} = \text{CDF} \times \text{FDF}$ (For lossless converter)

(x) Active input power or Active power transferred to load

$$P_i = V_S I_{S1}\cos\alpha$$

$$P_i = V_0\,I_0$$

(xi) Reactive input power or Reactive power transferred to load

$$Q_i = V_S I_{S1}\sin\alpha$$

$$Q_i = V_0\;I_0\;\tan\alpha$$

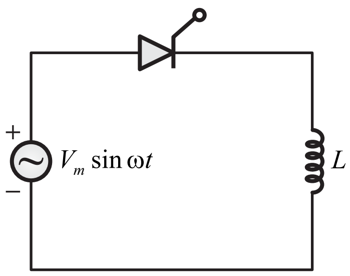
(xii) Apparent power, $S_i = \sqrt{P_i^2 + Q_i^2}$

$$S_i = V_0\,I_0\sec\alpha$$

➢ Sample Questions

2001 IIT Kanpur

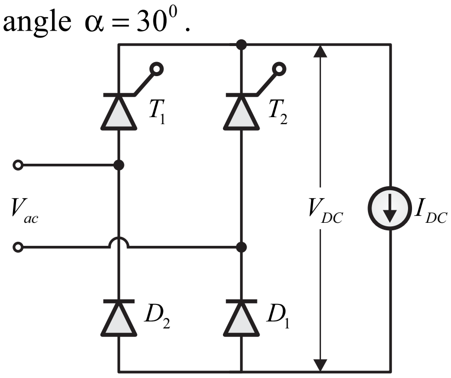
2.1 A half-wave thyristor converter supplies a purely inductive load, as shown in figure. If the triggering angle of the thyristor is 120^0, the extinction angle will be



(A) 240^0 (B) 180^0
(C) 200^0 (D) 120^0

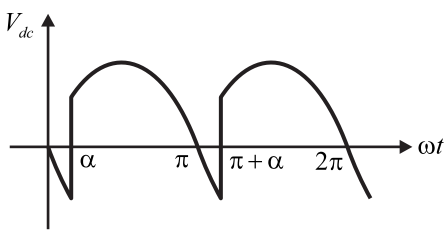
2003 IIT Madras

2.2 A half-controlled single-phase converter is shown in below figure. The control angle $\alpha = 30^0$.

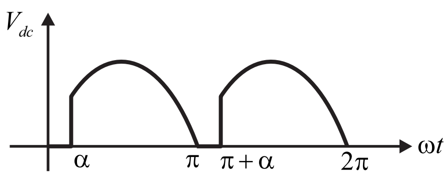


The output dc voltage wave shape will be as shown in

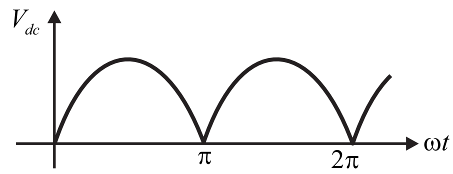
(A)



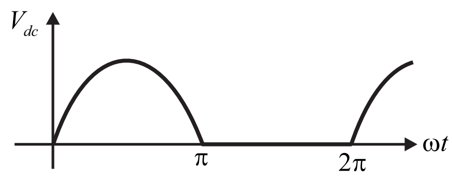
(B)



(C)

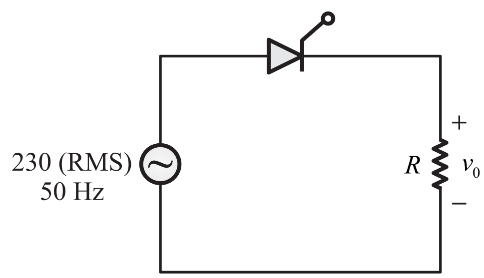


(D)



2005 IIT Bombay

2.3 Consider a phase-controlled converter shown in the figure. The thyristor is fired at an angle α in every positive half cycle of the input voltage. If the peak value of the instantaneous output voltage equals 230 V, the firing angle α is close to



(A) 45^0 (B) 135^0
(C) 90^0 (D) 83.6^0

2007 IIT Kanpur

2.4 A single phase full-wave half-controlled bridge converter feeds an inductive load. The two SCRs in the converter are connected to a common DC bus. The converter has to have a freewheeling diode

(A) because the converter inherently does not provide for free-wheeling.

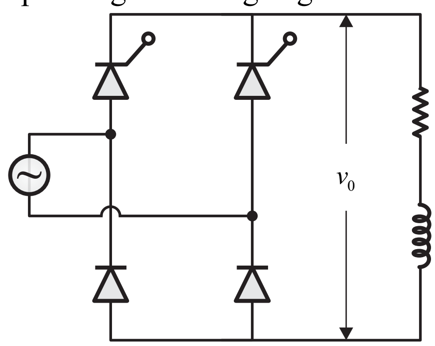
(B) because the converter does not provide for free-wheeling for high values of triggering angles.

(C) or else the free-wheeling action of the converter will cause shorting of the AC supply.

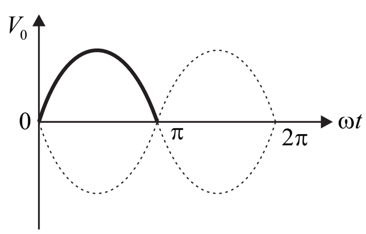
(D) or else if a gate pulse to one of the SCRs is missed, it will subsequently cause a high load current in the other SCR.

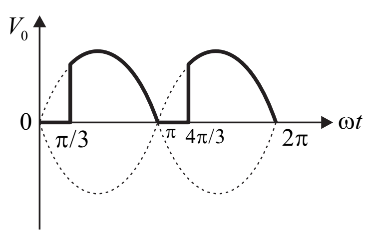
2008 IISc Bangalore

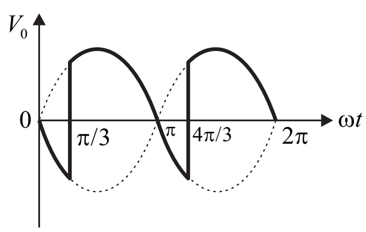
2.5 A single-phase half controlled converter shown in the figure feeding power to highly inductive load. The converter is operating at a firing angle of 60^0.

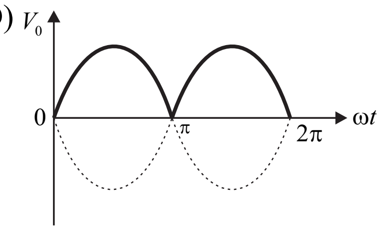


If the firing pulses are suddenly removed, the steady state voltage V_0 waveform of the converter will become

(A)


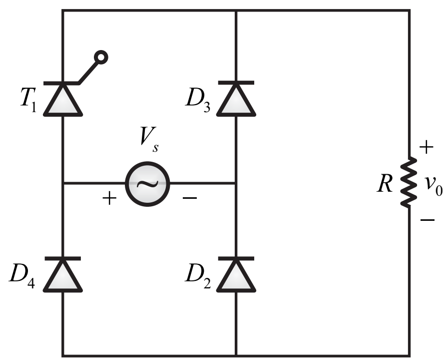
(B)


(C)


(D)


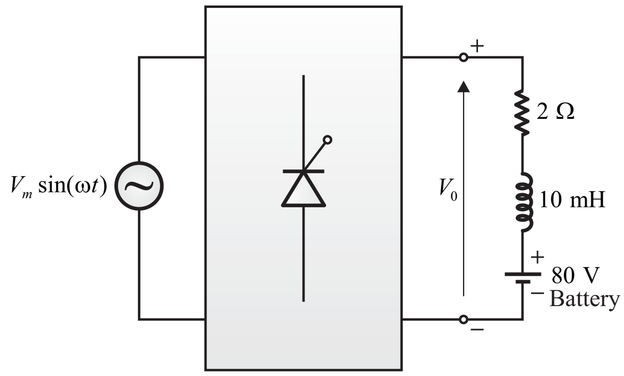
2015 IIT Kanpur

2.6 In the given rectifier, the delay angle of the thyristor T_1 measured from the positive going zero crossing of V_s is 30^0. If the input voltage V_s is $100\sin(100\pi t)\,\text{V}$, the average voltage across R (in Volt) under steady-state is ________. **[Set - 02]**



2018 IIT Guwahati

2.7 A phase controlled single phase rectifier, supplied by an AC source, feeds power to an *R-L-E* load as shown in the figure. The rectifier output voltage has an average value given $V_0 = \dfrac{V_m}{2\pi}(3+\cos\alpha)$, where $V_m = 80\pi$ volts and α is the firing angle. If the power delivered to the lossless battery is 1600 W, α in degree is __________ (up to 2 decimal places).



2019 IIT Madras

2.8 A single-phase fully-controlled thyristor converter is used to obtain an average voltage of 180 V with 10 A constant current to feed a DC load. It is fed from single-phase AC supply of 230 V, 50 Hz. Neglect the source impedance. The power factor (round off to two decimal places) of AC mains is _______.

2020 IIT Delhi

2.9 A single-phase, full-bridge diode rectifier fed from a 230 V, 50 Hz sinusoidal source supplies a series combination of finite resistance, R, and a very large inductance, L. The two most dominant frequency components in the source current are

(A) 50 Hz, 0 Hz

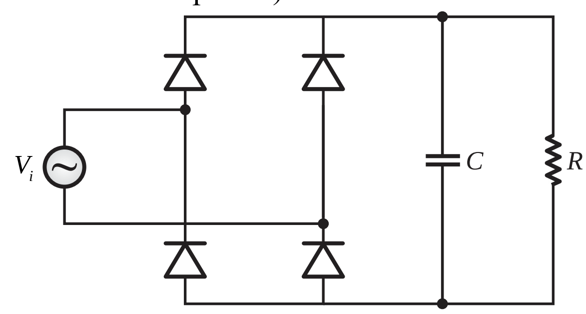
(B) 50 Hz, 100 Hz

(C) 50 Hz, 150 Hz

(D) 150 Hz, 250 Hz

2021 IIT Bombay

2.10 In the circuit shown, the input V_i is a sinusoidal AC voltage having an rms value of 230 V$\pm$ 20%. The worst case peak-inverse voltage seen across any diode is _______ V. (Round off to 2 decimal places)



Explanations Single Phase Rectifier

2.1 (A)

Given :

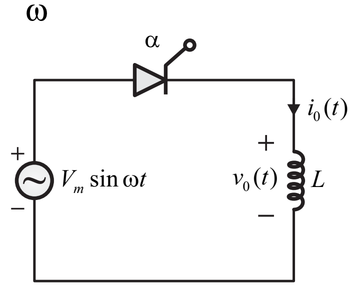
(i) Single phase Half wave thyristor converter

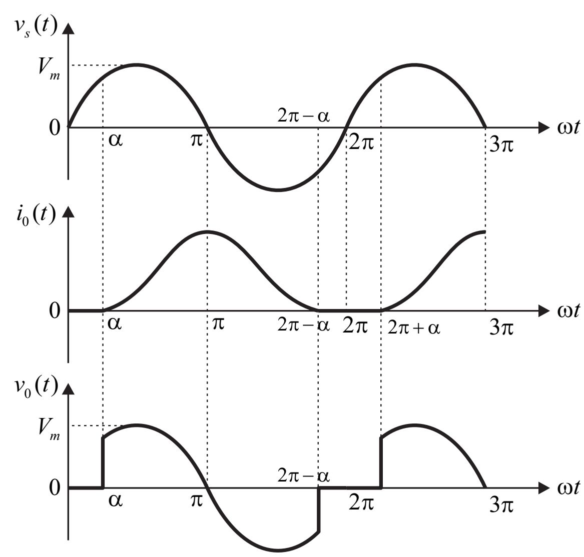
(ii) purely inductive load

(iii) Firing angle, $\alpha = 120^0$

$$\omega t_1 = \alpha$$

$$t_1 = \frac{\alpha}{\omega}$$





Device Working Status :

ωt	$i_0(t)$
α	0
$\frac{\pi}{2}$	$\frac{V_m}{\omega L}[\cos\alpha]$
π	$\frac{V_m}{\omega L}[\cos\alpha+1]$
$2\pi-\alpha$	0

When SCR in ON,

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$\int_0^i di = \frac{V_m}{L}\int_{t_1}^{t}\sin\omega t\, dt$$

$$\int_0^i di = \frac{V_m}{L}\int_{\alpha/\omega}^{t}\sin\omega t\, dt$$

$$i = \frac{-V_m}{\omega L}[\cos\omega t]_{\alpha/\omega}^{t}$$

$$i = \frac{V_m}{\omega L}[\cos\alpha - \cos\omega t]$$

At extinction angle $\omega t = \beta$, $i = 0$

$$0 = \frac{V_m}{\omega L}(\cos\alpha - \cos\beta)$$

$$\cos\alpha = \cos\beta$$

$$\beta = 2\pi - \alpha = 360^0 - 120^0$$

$$\beta = 240^0$$

Hence, the correct option is (A).

Key point

Firing angle (α) : At which the SCR starts its conduction.

Extinction angle (β) : At which SCR stops its conduction.

At firing angle (α) and extinction angle (β) the SCR current is always zero.

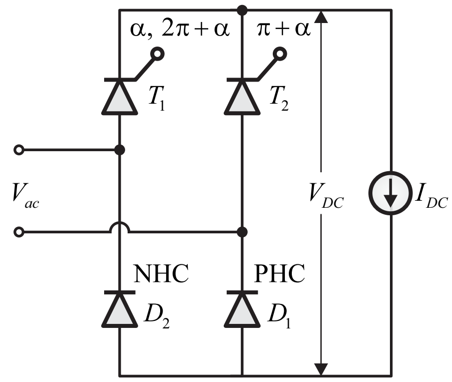
Conduction angle (γ) : Upto which SCR conducts.

2.2 (B)

Given :

(i) A half-controlled single-phase converter.

(ii) Firing angle, $\alpha = 30^0$



The current source is constant and unidirectional. This implies highly inductive load.

(i) At $\omega t = 30^0$, T_1 is triggered in the positive half cycle of the supply voltage. Hence I_{DC} is directly connected across the load through T_1 and D_1. D_1 is forward biased in the positive half cycle. From α to π, energy is stored in the highly inductive load.

(ii) At $\omega t = \pi$, the supply voltage becomes negative. So, the inductance tries to release its energy to the source but cannot do so as D_2 is forward biased and current freewheels through T_1 and D_2. Hence, the output voltage is zero from π to $\pi+\alpha$.

(iii) At $\omega t = \pi+\alpha$, T_2 is triggered. In the negative half cycle of the supply voltage D_2 is forward biased and T_2 & D_2 conduct. Hence, the load is reversely connected to the supply voltage.

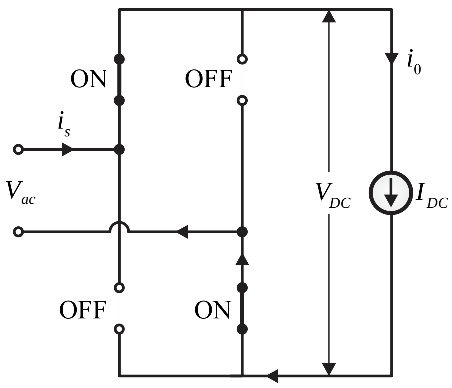
(iv) At $\omega t = 2\pi$, D_2 is reverse biased and the current freewheels through T_2, I_{DC} and

D_1, thus given zero output voltage from 2π to $2\pi+\alpha$.

During positive half cycle $(\alpha<\omega t<\pi)$:

Device	States
T_1	ON
T_2	OFF
D_1	ON
D_2	OFF

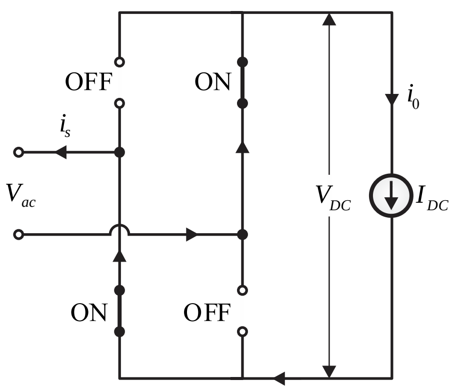
The equivalent circuit is shown below,



During negative half cycle $(\pi+\alpha<\omega t<2\pi)$:

Device	States
T_1	OFF
T_2	ON
D_1	OFF
D_2	ON

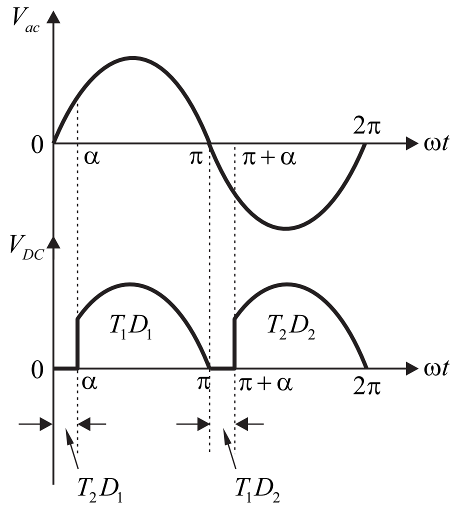
The equivalent circuit is shown below,



Device Working Status :

Duration	Device working	Output V_{DC}	Free-wheeling action
$0<\omega t<\alpha$	T_2D_1	0	T_2D_1
$\alpha<\omega t<\pi$	T_1D_1	V_s	×
$\pi<\omega t<\pi+\alpha$	T_1D_2	0	T_1D_2
$\pi+\alpha<\omega t<2\pi$	T_2D_2	V_s	×
$2\pi<\omega t<2\pi+\alpha$	T_1D_2	0	T_1D_2

The output dc voltage wave-shape is shown below,



Hence, the correct option is (B).

Scan for Video Solution

2.3 (B)

Given :

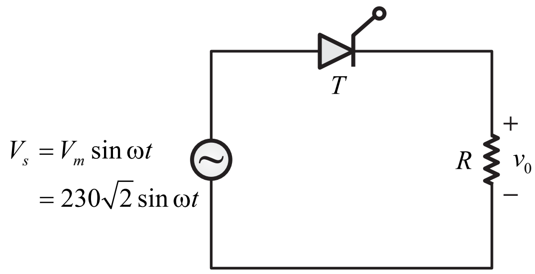
(i) Single phase half wave phase-controlled converter.

(ii) Supply voltage, $V_s = 230$ V (RMS value)

Maximum value of supply voltage,

$$V_m = 230\sqrt{2}\ \text{V}$$

(iii) Peak value of the instantaneous output voltage is equal to 230 V.



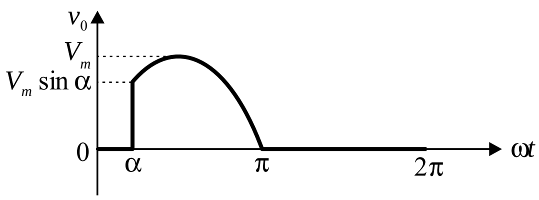
$$V_m \sin \alpha = 230 \text{ V}$$

$$230\sqrt{2} \sin \alpha = 230$$

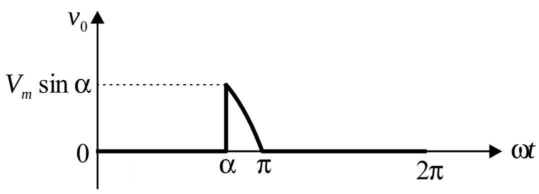
$$\sin \alpha = \frac{1}{\sqrt{2}}$$

$$\alpha = 45^0, 135^0$$

Case 1 : $\alpha = 45^0$



Case 2 : $\alpha = 135^0$



Since, instantaneous peak voltage given is less then V_m hence,

$$V_m \sin \alpha = 230 \text{ V}$$

$$230\sqrt{2} \sin \alpha = 230$$

$$\alpha = 135^0 > 90^0$$

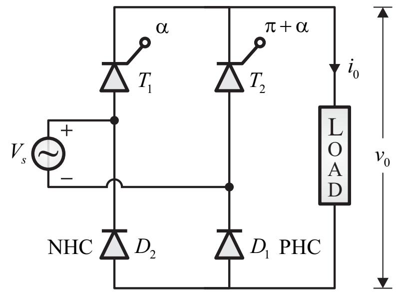
Hence, the correct option is (B).

Scan for Video Solution

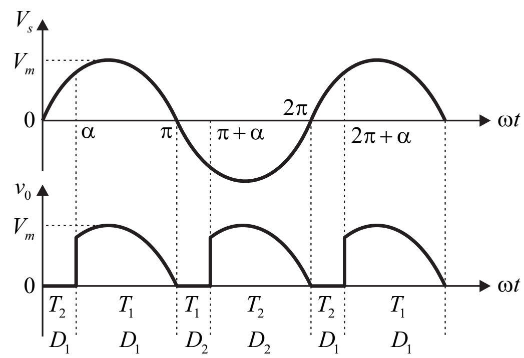
2.4 (C)

Given : Single phase full wave half controlled bridge converter.

Single phase full wave half controlled bridge converter is shown in below figure,



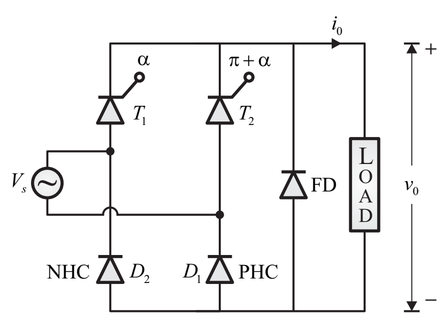
Output voltage waveform of the given converter is shown below,



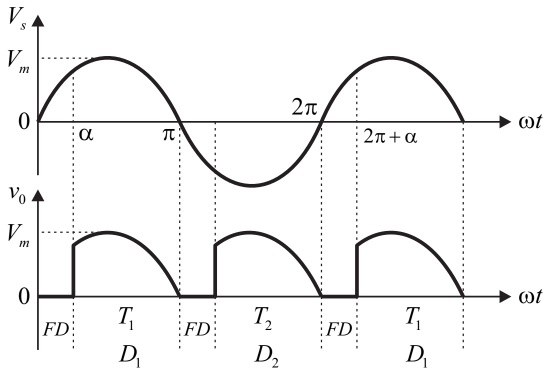
Device Working Status :

Duration	Device working	Output V_{DC}	Free-wheeling action
$0 < \omega t < \alpha$	$T_2 D_1$	0	$T_2 D_1$
$\alpha < \omega t < \pi$	$T_1 D_1$	V_s	×
$\pi < \omega t < \pi + \alpha$	$T_1 D_2$	0	$T_1 D_2$
$\pi + \alpha < \omega t < 2\pi$	$T_2 D_2$	V_s	×
$2\pi < \omega t < 2\pi + \alpha$	$T_2 D_1$	0	$T_2 D_1$

It is possible that due to commutation delay outgoing and incoming thyristor (at α, $\pi + \alpha$, $2\pi + \alpha$ etc.) both are on for some time which can lead to short circuit of source voltage that is dangerous.



Output voltage waveform of the given converter is shown below,



Device Working Status :

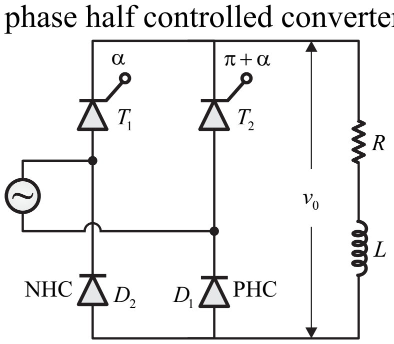
Duration	Device working	Output V_{DC}	Free-wheeling action
$0 < \omega t < \alpha$	FD	0	ON
$\alpha < \omega t < \pi$	T_1D_1	V_s	OFF
$\pi < \omega t < \pi + \alpha$	FD	0	ON
$\pi + \alpha < \omega t < 2\pi$	T_2D_2	V_s	OFF
$2\pi < \omega t < 2\pi + \alpha$	FD	0	ON
$2\pi + \alpha < \omega t < 3\pi$	T_1D_1	V_s	OFF

Hence, when FD is used it reduces the chances of short circuit of supply because from output waveform it is clear that FD conducts between T_1 and T_2 hence there is less chances of conducting T_1 and T_2 simultaneously.

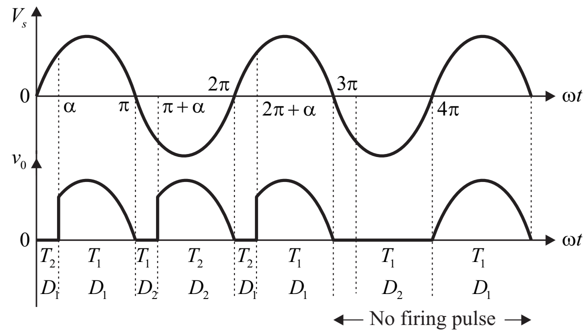
Hence, the correct option is (C).

2.5 (A)

A single phase half controlled converter.



Output voltage waveform of the given converters is shown below,



Device Working Status :

Duration	Device working	Output V_0	Free-wheeling action
$0 < \omega t < \alpha$	T_2D_1	0	T_2D_1
$\alpha < \omega t < \pi$	T_1D_1	V_s	×
$\pi < \omega t < \pi + \alpha$	T_1D_2	0	T_1D_2
$\pi + \alpha < \omega t < 2\pi$	T_2D_2	V_s	×
$2\pi < \omega t < 2\pi + \alpha$	T_2D_1	0	T_2D_1
$2\pi + \alpha < \omega t < 3\pi$	T_1D_1	V_s	×

Let the firing pulse is suddenly removed at $\omega t = 3\pi$ i.e. T_2 and D_2 are forward biased but as firing angle pulse is removed, hence T_2 can never get ON again. But before removing the gate pulse T_1 was conducting and as load current is continuous the output current will flow through T_1, D_2 and makes output voltage zero.

At $\omega t = 4\pi$, D_1 is forward bias and load current will flow through T_1 and D_1 an output voltage is equal to input voltage as shown in waveform.

Hence, the correct option is (A).

Scan for Video Solution

Key point

If we suddenly remove firing pulse of a SCR already in conduction state i.e. the SCR will remain in its conduction state as it has been latched. It does not mean that the conducting SCR will be OFF due to removal of firing pulse.

2.6 **61.53**

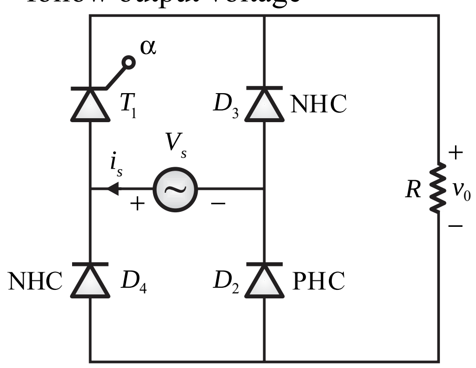
Given :

(i) Single phase special controlled rectifier

(ii) Supply voltage, $V_s = 100\sin(100\pi t)$

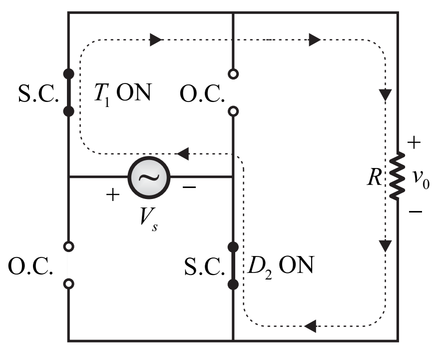
(iii) Firing angle, $\alpha = 30^0$

(iv) For resistive load output current will follow output voltage



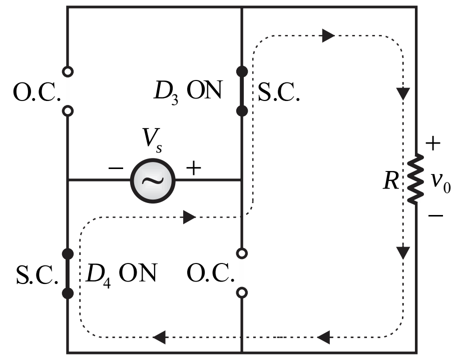
During positive half cycle $(\alpha < \omega t < \pi)$:

Thyristor T_1 and diode D_2 are in forward bias condition, at $\alpha = 30^0$, thyristor T_1 is fired so output voltage $V_0 = V_s$.



During negative half cycle $(\pi < \omega t < 2\pi)$:

Diode D_3 and D_4 are forward biased so output voltage $V_0 = -V_s$.



Device Working Status :

Duration	Device working	Output V_0
$0 < \omega t < \alpha$	×	0
$\alpha < \omega t < \pi$	$T_1 D_2$	V_s
$\pi < \omega t < 2\pi$	$D_3 D_4$	$-V_s$

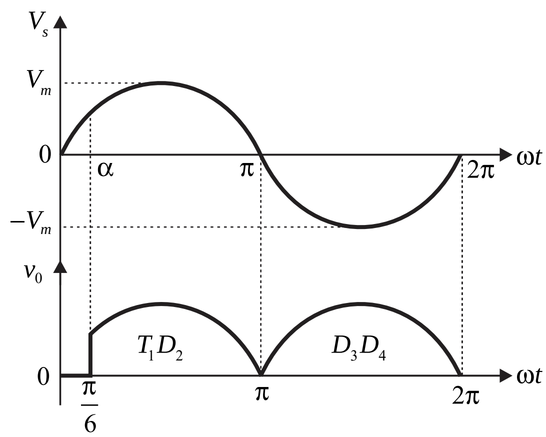


Fig. Output waveform of ....

The average output voltage is given by,

$$V_0 = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi} V_m \sin\omega t\, d(\omega t) + \int_{\pi}^{2\pi} -V_m \sin\omega t\, d(\omega t)\right]$$

$$V_0 = \frac{V_m}{2\pi}[-(\cos\pi - \cos\alpha) + (\cos 2\pi - \cos\pi)]$$

$$V_0 = \frac{V_m}{2\pi}(3 + \cos\alpha)$$

Hence,

$$V_0 = \frac{100}{2\pi}(3 + \cos 30^0) = 61.53 \text{ V}$$

$$V_0 = 61.53 \text{ V}$$

Hence, the average voltage under steady state across R is **61.53 V.**

Scan for
Video Solution

2.7 **90**

Given :

(i) Phase controlled rectifier

(ii) RLE load having

$$R = 2\ \Omega,\ L = 10 \text{ mH},\ E = 80 \text{ V}$$

(iii) Average output voltage,

$$V_0 = \frac{V_m}{2\pi}(3 + \cos\alpha)$$

(iv) $V_m = 80\pi$ V

(v) Power delivered to the lossless battery,

$$P_0 = 1600 \text{ W}$$

Power transfer to the 80 V battery,

$$P_0 = EI_0 = 1600 \text{ W}$$

Hence, $I_0 = \dfrac{1600}{80} = 20 \text{ A}$

Average output voltage,

$$V_0 = \frac{V_m}{2\pi}(3 + \cos\alpha) = \frac{80\pi}{2\pi}(3 + \cos\alpha)$$

$$V_0 = 40(3 + \cos\alpha) \qquad \ldots\text{(i)}$$

The relation between V_0 and E is given by,

$$V_0 = I_0 R + E = 20 \times 2 + 80$$

$$V_0 = 40 + 80 = 120 \text{ V}$$

From equation (i),

$$V_0 = 120 = 40(3 + \cos\alpha)$$

$$\cos\alpha = \frac{120}{40} - 3 = 0$$

Hence, $\alpha = \cos^{-1}(0) = 90^0$

Hence, the firing angle (α) is $\mathbf{90^0}$.

2.8 **0.78**

Given :

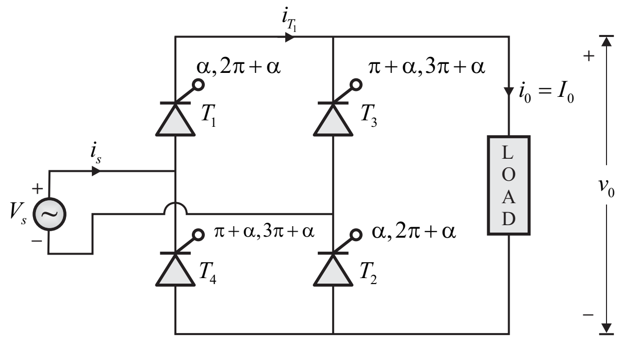
(i) Single-phase fully-controlled thyristor converter

(ii) DC-load

(iii) Supply voltage, $V_s = 230$ V

(iv) Average output voltage, $V_0 = 180$ V

(v) Load current is constant ripple free, $I_0 = 10$ A



Method 1

Continuous conduction mode

The average output voltage of the converter for firing angle α is given by,

$$V_0 = \frac{2V_m}{\pi}\cos\alpha$$

$$180 = \frac{2 \times 230\sqrt{2}}{\pi}\cos\alpha$$

$$\cos\alpha = 0.869$$

$$\text{IPF} = \frac{2\sqrt{2}}{\pi}\cos\alpha = \frac{2\sqrt{2}}{\pi} \times 0.869 = 0.786 \text{ lag}$$

Method 2

Input Power = Output Power

$$(V_S)_{rms}(i_s)_{rms}\cos\phi_S = V_0 I_0$$

$\cos\phi_S$ = Supply (Input) Power factor

For a single phase fully controlled converter

$$(i_s)_{rms} = I_0 = 10 \text{ A}$$

$$230 \times 10 \times \cos\phi_S = 180 \times 10$$

$$\cos\phi_S = \frac{180}{230}$$

$$\cos\phi_S = 0.786 \text{ lag}$$

Hence, the power factor of AC mains is **0.786 lag.**

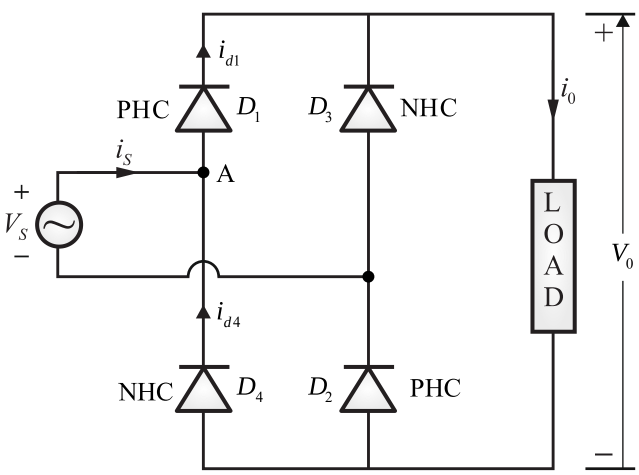
2.9 (C)

Given :

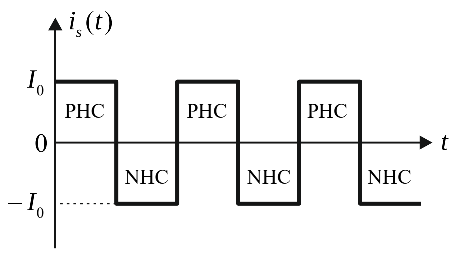
(i) Single-phase full-bridge diode rectifier.

(ii) Constant ripple free load current due to highly inductive load.

Single-phase, full-bridge diode rectifier is shown below,



Source current waveform is shown below,



Fourier series expression of source current is given by,

$$i_s(t) = \sum_{n=1,3,5}^{\infty} \frac{4I_0}{n\pi}\sin(n\omega_0 t)$$

$\therefore$ Most dominant frequency components are,

$$\frac{4I_0}{\pi}, \frac{4I_0}{3\pi}, \frac{4I_0}{5\pi}$$

$$\therefore \quad f = 50\text{ Hz}, 50\times 3\text{ Hz}, 50\times 5\text{ Hz}$$

Most dominant frequency components are 50 Hz, 150 Hz, 250 Hz.

Hence, the correct option is (C).

2.10 390.32

Given :

Supply voltage, $V_{s(rms)} = (230 \pm 20\%)\text{ V}$

$$\Delta V = \pm\frac{20\times 230}{100} = \pm 46$$

$$V_{s(rms)} = (230 \pm 46)\text{ V}$$

$$V_{s(rms)} = (184 \text{ to } 276)\text{ V}$$

Supply voltage will vary in between $184\sqrt{2}\sin\omega t$ to $276\sqrt{2}\sin\omega t$.

In given converter, the discharging time of capacitor will be 4τ and capacitor will charge for every less time than $\frac{\pi}{2\omega}$ sec.

Let us assume, in the range of $0-\pi/2$ D_1D_2 are ON, so D_3D_3 will be OFF.

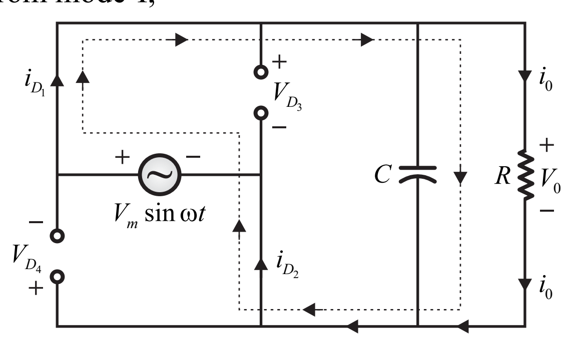
Mode I : $0 < \omega t < \pi$

Diode D_1, D_2 will be forward biased and diode D_3, D_4 will be reversed biased.

Mode II : $\pi < \omega t < 2\pi$

Diode D_3, D_4 will be forward biased and diode D_1, D_2 will be reversed biased.

From mode-I,



Applying KVL in the above circuit,

$$V_m \sin \omega t - V_{D_3} = 0$$

$$V_{D_3} = V_m \sin \omega t$$

The peak inverse voltage across each diode is given by,

$$V_D\big|_{max} = 276\sqrt{2}\ \text{V} = 390.32\ \text{V}$$

Hence, the correct answer is 390.32.

❖❖❖❖

3 Three Phase Rectifier

➢ Partial Synopsis

Three-phase full wave controlled rectifier :

A three-phase full controlled converter works as a three phase ac to dc converter for delay angle (α) $0^0 < \alpha < 90^0$ and as a three-phase line-commutated inverter for $90^0 < \alpha < 180^0$.

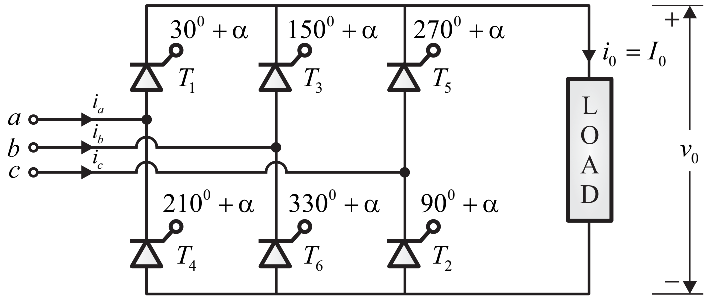


Fig. Circuit diagram

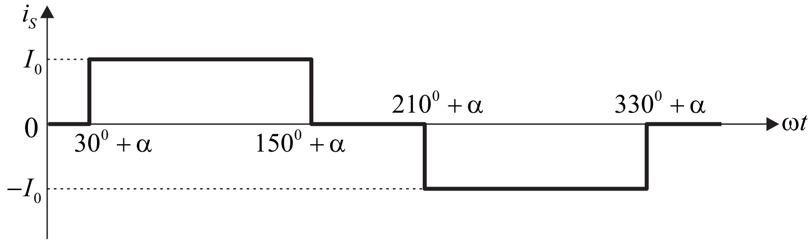
(i) Average output voltage, $V_0 = \frac{1}{\pi/3}\int_{\alpha+\pi/3}^{\alpha+2\pi/3}(V_{ml}\sin\omega t)d\omega t = \frac{3V_{ml}}{\pi}\cos\alpha = \frac{3\sqrt{3}V_{mp}}{\pi}\cos\alpha$

(ii) RMS output voltage, $V_{or} = \sqrt{\frac{1}{\pi/3}\int_{\alpha+\pi/3}^{\alpha+2\pi/3}(V_{ml}\sin\omega t)^2 d\omega t}$

$$V_{or} = \frac{\sqrt{3}V_{ml}}{\sqrt{2}}\left[\frac{\pi}{3}+\frac{1}{2}\left[\sin 2\left(\alpha+\frac{\pi}{3}\right)-\sin 2\left(\alpha+\frac{2\pi}{3}\right)\right]\right]^{1/2}$$

Performance Parameter of three-phase controlled rectifier with continuous conduction and ripple free load current

Source current waveform :



The Fourier series representation of source current is given by,

$$i_{sn} = \sum_{n=1,3,5}^{\infty} \frac{4I_0}{n\pi} \sin\left(\frac{n\pi}{3}\right) \sin(n\omega t - n\alpha)$$

(i) RMS value of n^{th} harmonic component is

$$i_{srn} = \frac{4I_0}{n\pi\times\sqrt{2}} \sin\left(\frac{n\pi}{3}\right)$$

(ii) RMS value of fundamental component of source current,

$$i_{sr1} = \frac{\frac{4I_0}{\pi}\sin\frac{\pi}{3}}{\sqrt{2}} = \frac{\sqrt{6}}{\pi} I_0 = 0.78\, I_0$$

Odd triple harmonics will be absent.

(iii) The order of harmonics exist in M-6 converter is $(6n \pm 1)\, f$

where, $n = 0,1,2, \dots\dots\dots \infty$

f = fundamental input frequency.

The frequency components which exist are f, $5f$, $7\,f$, $11\,f$, $13f$ ....

(iv) Fundamental displacement angle $(\phi_1) = -\alpha$

(v) Fundamental displacement factor

$$\text{FDF} = \cos\alpha$$

(vi) Current displacement factor

$$\text{CDF} = \frac{i_{s1}}{i_{sr}} = \frac{0.78 I_0}{I_0\sqrt{\frac{2}{3}}} = 0.955$$

(vii) Total harmonic distortion

$$\text{THD} = \sqrt{\left(\frac{1}{\text{CDF}}\right)^2 - 1} = \sqrt{\left(\frac{i_{s(rms)}}{i_{s1}}\right)^2 - 1} = \sqrt{\frac{i_{s(rms)}^2 - i_{s1}^2}{i_{s1}^2}}$$

$$\text{THD} = \sqrt{\left(\frac{1}{0.955}\right)^2 - 1} = 31.05\%$$

(viii) Supply power factor (SPF/IPF)

$$\text{SPF} = \text{CDF}\times\text{FDF} = 0.955\times\cos\alpha$$

$$\text{SPF} = \frac{V_0 I_0}{3V_S i_S}$$ (where, V_{ml} = phase maximum voltage)

(ix) Active input power or active power delivered to load

$$P = \sqrt{3} V_{S(rms)} \times i_{s1} \cos\alpha$$

$$P = V_0 I_0 \text{ Watts}$$

(x) Reactive input power or reactive power delivered to load

$$Q = \sqrt{3} V_{S(rms)} \times i_{s1} \sin\alpha$$

$$Q = \sqrt{3}\frac{V_{ml}}{\sqrt{2}} \times \frac{\sqrt{6}}{\pi} \times I_0 \times \sin\alpha$$

$$Q = \frac{3V_{ml}}{\pi}\cos\alpha \times I_0 \frac{\sin\alpha}{\cos\alpha}$$

$$Q = V_0 I_0 \tan\alpha \text{ VAR}$$

➢ Sample Questions

1998 | IIT Delhi

3.1 A 3-phase fully controlled converter is feeding power into a dc load at a constant current of 150 A. The rms current through each thyristor of the converter is

(A) 50 A (B) 100 A

(C) $\frac{150\sqrt{2}}{\sqrt{3}}$ A (D) $\frac{150}{\sqrt{3}}$ A

2003 | IIT Madras

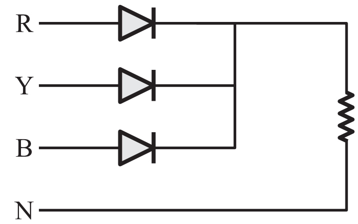
3.2 A fully controlled natural commutated 3-phase bridge rectifier is operating with a firing angle $\alpha = 30^0$. The peak to peak voltage ripple expressed as a ratio of the peak output dc voltage at the output of the converter bridge is

(A) 0.5 (B) $\sqrt{3}/2$

(C) $\left(1-\frac{\sqrt{3}}{2}\right)$ (D) $\sqrt{3}-1$

2004 | IIT Delhi

3.3 The circuit in figure shows a 3-phase half-wave rectifier. The source is a symmetrical, 3-phase four-wire system. The line-to-line voltage of the source is 100 V. The supply frequency is 400 Hz. The ripple frequency at the output is



(A) 400 Hz (B) 800 Hz

(C) 1200 Hz (D) 2400 Hz

2006 | IIT Kharagpur

3.4 A 3-phase fully controlled bridge converter with freewheeling diode is fed from 400 V, 50 Hz AC source and is operating at a firing angle of 60^0. The load current is assumed constant at 10 A due to high load inductance. The input displacement factor (IDF) and the input power factor (IPF) of the converter will be

(A) IDF = 0.867; IPF = 0.828

(B) IDF = 0.867; IPF = 0.552

(C) IDF = 0.5; IPF = 0.478

(D) IDF = 0.5; IPF = 0.318

2008 | IISc Bangalore

3.5 A three phase fully controlled bridge converter is feeding a load drawing a constant and ripple free load current of 10 A at a firing angle of 30^0. The approximate Total harmonic Distortion

(%THD) and the rms value of fundamental component of the input current will respectively be

(A) 31 % and 6.8 A

(B) 31% and 7.8 A

(C) 66% and 6.8 A

(D) 66% and 7.8 A

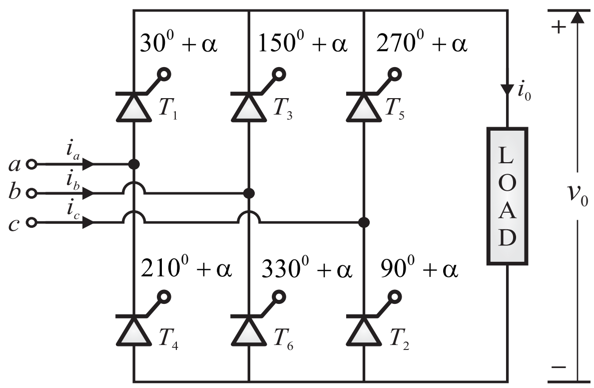
Explanations — Three Phase Rectifier

3.1 (D)

Given :

(i) 3-phase fully controlled converter

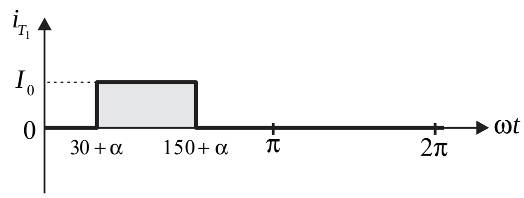
(ii) Ripple free and constant output current, $I_0 = 150$ A



Device Working Status :

ωt	Conducting Thyristor	Output Voltage V_0 (line)
$0<\omega t<30^0+\alpha$	T_5T_6	V_{cb}
$30^0+\alpha<\omega t<90^0+\alpha$	T_1T_6	V_{ab}
$90^0+\alpha<\omega t<150^0+\alpha$	T_1T_2	V_{ac}
$150^0+\alpha<\omega t<210^0+\alpha$	T_3T_2	V_{bc}
$210^0+\alpha<\omega t<270^0+\alpha$	T_3T_4	V_{ba}
$270^0+\alpha<\omega t<330^0+\alpha$	T_5T_4	V_{ca}
$330^0+\alpha<\omega t<390^0+\alpha$	T_5T_6	V_{cb}

In 3-phase fully controlled converter each thyristor conducts for $\frac{2\pi}{3}$ radian (120^0) in each cycle.



The rms current through each thyristor,

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i_T^2\, d(\omega t)}$$

[From device working table]

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{30^0+\alpha}^{150^0+\alpha} I_0^2\, d(\omega t)} = \frac{I_0}{\sqrt{3}}$$

$$I_{rms} = \frac{150}{\sqrt{3}} \text{ A}$$

Hence, the correct option is (D).

Key point

The output voltage expression of 3-phase fully controlled converter with voltage drop across the thyristor is given by,

Then, $V_0 = \frac{3V_{ml}}{\pi}\cos\alpha - 2V_T$

where, V_T = Voltage drop across the thyristor.

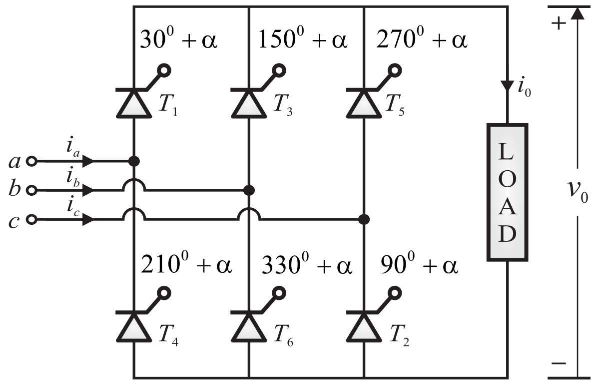
Voltage drop is taken as $2V_T$ because in B-6 converter two thyristor conducts at a time.

3.2 (A)

Given :

(i) 3-phase fully wave controlled converter

(ii) Firing angle, $\alpha = 30^0$

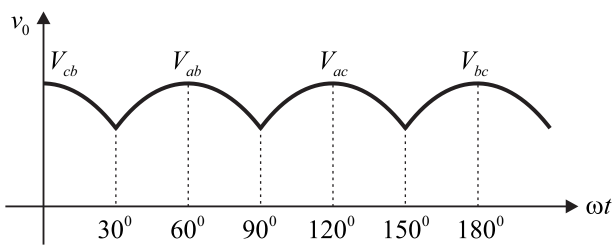


Device Working Status :

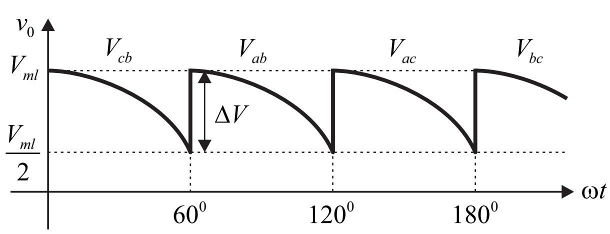
ωt	Conducting Thyristor	Output Voltage V_0 (line)
$0 < \omega t < 30^0 + \alpha$	$T_5\,T_6$	V_{cb}
$30^0 + \alpha < \omega t < 90^0 + \alpha$	$T_1\,T_6$	V_{ab}
$90^0 + \alpha < \omega t < 150^0 + \alpha$	$T_1\,T_2$	V_{ac}
$150^0 + \alpha < \omega t < 210^0 + \alpha$	$T_3\,T_2$	V_{bc}
$210^0 + \alpha < \omega t < 270^0 + \alpha$	$T_3\,T_4$	V_{ba}
$270^0 + \alpha < \omega t < 330^0 + \alpha$	$T_5\,T_4$	V_{ca}
$330^0 + \alpha < \omega t < 390^0 + \alpha$	$T_5\,T_6$	V_{cb}

For 3-phase full wave bridge rectifier output line voltage waveform is,

(i) For $\alpha = 0^0$



(ii) For $\alpha = 30^0$



$$V_{ab} = V_{ml}\cos(\omega t - 60^0) \text{ or}$$

$$V_{ab} = V_{ml}\sin(\omega t + 30^0)$$

Peak to peak ripple voltage is given by,

$$\Delta V = V_{ml} - V_{ml}\cos(\omega t - 60^0)$$

$$\Delta V = V_{ml} - V_{ml}\cos(120^0 - 60^0)$$

$$\Delta V = V_{ml}(0.5)$$

$$\frac{\Delta V}{V_{ml}} = 0.5$$

Hence, the correct option is (A).

3.3 (C)

Given :

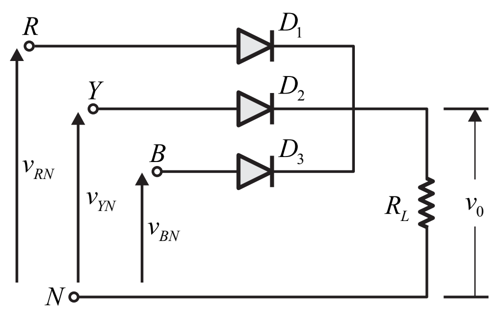
(i) 3-phase half wave rectifier

(ii) Supply voltage, $V_s = 100$ V

$$V_s(\text{maximum line voltage}) = 100\sqrt{2}\ \text{V}$$

(iii) Supply frequency, $f = 400$ Hz

Given 3-phase half wave rectifier is shown below,



Among D_1, D_2, D_3 only one diode conducts at a time corresponding to which voltage is more positive than other two. For e.g. if $v_R = +60$ V, $v_Y = +20$ V and $v_B = -80$ V, only D_1, will conduct.

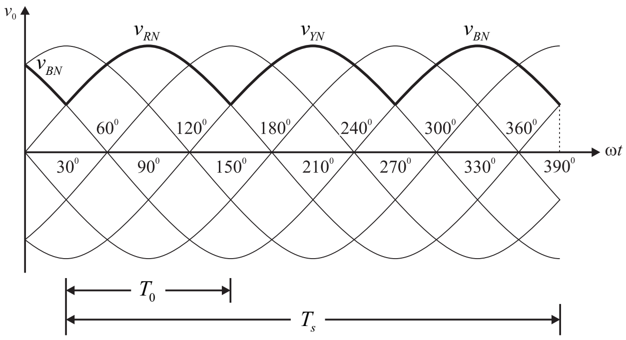
Each diode remains forward biased for 120^0 i.e., one-third of the cycle.

If D_1 is ON, $v_0 = v_R$

If D_2 is ON, $v_0 = v_Y$

If D_3 is ON, $v_0 = v_B$

The output voltage waveform will be,



From figure, $T_s = 360^0$ and $T_0 = 120^0$ hence,

$$T_0 = \frac{T_s}{3}$$

Means, $f_0 = 3f = 3 \times 400 = 1200$ Hz

Hence, the correct option is (C).

3.4 (C)

Given :

(i) 3-phase fully controlled bridge converter

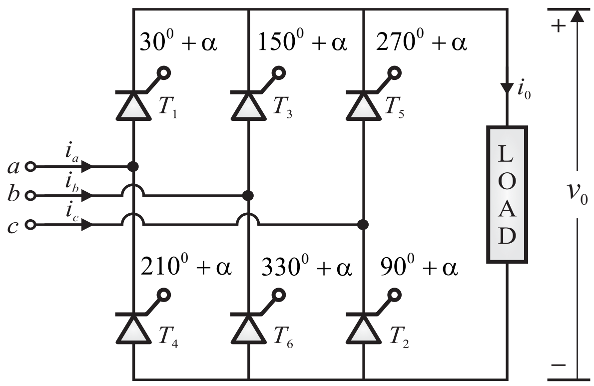
(ii) Supply voltage, $V_s = 400$ V

$V_s(\text{maximum line voltage}) = 400\sqrt{2}$ V

(iii) Supply frequency, $f = 50$ Hz

(iv) Ripple free and constant output current, $I_0 = 10\,\text{A}$

(v) Firing angle, $\alpha = 60^0$



Device Working Status :

ωt	**Conducting Thyristor**	**Output Voltage V_0 (line)**
$0 < \omega t < 30^0 + \alpha$	$T_5 T_6$	V_{cb}
$30^0 + \alpha < \omega t < 90^0 + \alpha$	$T_1 T_6$	V_{ab}
$90^0 + \alpha < \omega t < 150^0 + \alpha$	$T_1 T_2$	V_{ac}
$150^0 + \alpha < \omega t < 210^0 + \alpha$	$T_3 T_2$	V_{bc}
$210^0 + \alpha < \omega t < 270^0 + \alpha$	$T_3 T_4$	V_{ba}
$270^0 + \alpha < \omega t < 330^0 + \alpha$	$T_5 T_4$	V_{ca}
$330^0 + \alpha < \omega t < 390^0 + \alpha$	$T_5 T_6$	V_{cb}

Input displacement factor (IDF)

$$\text{IDF} = \text{FDF} = \cos\alpha$$

$$\text{IDF} = \cos 60^0 = 0.5$$

RMS value of source current,

$$i_s = I_0\sqrt{\frac{2}{3}} = 10 \times \sqrt{\frac{2}{3}} = 8.16\,\text{A}$$

RMS value of source current for n^{th} harmonic is given by,

$$i_{sn} = \frac{4I_0}{\sqrt{2}n\pi}\sin\frac{n\pi}{3}$$

RMS value of fundamental current is,

$$i_{s1} = \frac{2\sqrt{2}}{\pi} I_0 \sin 60 = \frac{2\sqrt{2}}{\pi} \times 10 \times \frac{\sqrt{3}}{2}$$

$$i_{s1} = 7.796 \text{ A}$$

Current Distortion factor CDF,

$$\text{CDF} = \frac{i_{s1}}{i_s} = \frac{7.796}{8.16} = 0.955$$

Input power factor (IPF),

$$\text{IPF} = \text{CDF} \times \text{IDF}$$

$$\text{IPF} = 0.955 \times 0.5 = 0.478$$

Hence, the correct option is (C).

Key point

Performance Parameters of 3-phase fully controlled bridge converter are :

(i) Average value of thyristor current,

$$I_{T(avg)} = \frac{I_0}{3}$$

(ii) Average value of source current/phase current, $I_{a0} = 0$

(iii) RMS value of source current/phase current, $I_{a\,(rms)} / i_{s(rms)} = I_0\sqrt{\frac{2}{3}}$

(iv) RMS value of fundamental component of source current,

$$i_{s1(rms)} = \frac{2\sqrt{2}I_0}{\pi}\sin\left(\frac{\pi}{3}\right)$$

(v) Fundamental displacement factor,

$$\text{FDF} = \cos(-\alpha) = \cos\alpha$$

(vi) Current distortion factor (CDF),

$$g = \frac{i_{S1}}{i_S} = \frac{3}{\pi}$$

(vii) Supply power factor (SPF)

$$= \text{CDF} \times \text{FDF} = 0.955\cos\alpha$$

(viii) Total harmonic distortion

$$= \sqrt{\frac{1}{g^2} - 1} = 31.1\%$$

(ix) Active input power

$$P_i = 3V_s i_{S1}\cos\alpha$$

$$P_i = \frac{3V_{ml}}{\pi} I_0 \cos\alpha = V_0 I_0$$

(x) Reactive input power

$$Q_i = 3V_s i_{s1}\sin\alpha = 3V_s i_{s1}\cos\alpha\tan\alpha$$

$$Q_i = V_0 I_0 \tan\alpha$$

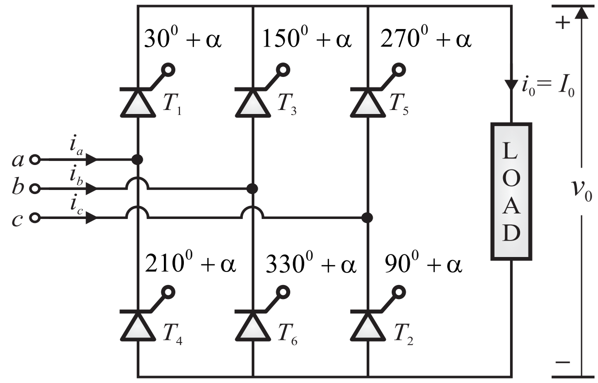
3.5 (B)

Given :

(i) 3-phase fully controlled bridge converter

(ii) Ripple free and constant output current, $I_0 = 10$ A.

(iii) Firing angle, $\alpha = 30^0$



Device Working Status :

ωt	Conducting Thyristor	Output Voltage V_0 (line)
$0 < \omega t < 30^0 + \alpha$	$T_5\,T_6$	V_{cb}
$30^0 + \alpha < \omega t < 90^0 + \alpha$	$T_1\,T_6$	V_{ab}
$90^0 + \alpha < \omega t < 150^0 + \alpha$	$T_1\,T_2$	V_{ac}
$150^0 + \alpha < \omega t < 210^0 + \alpha$	$T_3\,T_2$	V_{bc}
$210^0 + \alpha < \omega t < 270^0 + \alpha$	$T_3\,T_4$	V_{ba}
$270^0 + \alpha < \omega t < 330^0 + \alpha$	$T_5\,T_4$	V_{ca}
$330^0 + \alpha < \omega t < 390^0 + \alpha$	$T_5\,T_6$	V_{cb}

rms value of supply current is given by,

$$i_s = I_0\sqrt{\frac{2}{3}} = 10\sqrt{\frac{2}{3}} = 8.165 \text{ A}$$

Supply current I_s can be expressed by Fourier series,

$$i_s(t) = \sum_{n=1,3,5}^{\infty} \frac{4I_0}{n\pi}\sin\frac{n\pi}{3}\sin(n\omega t - n\alpha)$$

rms value of fundamental current is given by,

$$i_{sr_1} = \frac{4I_0}{\sqrt{2}\pi}\sin\frac{\pi}{3}$$

$$i_{sr_1} = \frac{4\times 10}{\sqrt{2}\times\pi}\times\frac{\sqrt{3}}{2} = 7.8 \text{ A}$$

Current distortion factor, CDF $= \frac{i_{sr_1}}{i_{sr}}$

Total Harmonic distortion,

$$\text{THD} = \sqrt{\left[\left(\frac{i_{sr}}{i_{sr_1}}\right)^2 - 1\right]} \times 100$$

$$\text{THD} = \sqrt{\left[\left(\frac{8.165}{7.8}\right)^2 - 1\right]} \times 100 = 31\%$$

Hence, the correct option is (B).

Key Point

1. Total harmonic distortion for 3 phase fully controlled bridge converter in case of continuous conduction is always 31%.
2. Total harmonic distortion for single phase fully controlled bridge converter in case of continuous conduction is 48.3%.
3. Total Harmonic distortion,

$$\text{THD} = \sqrt{\left(\frac{1}{\text{CDF}}\right)^2 - 1}$$

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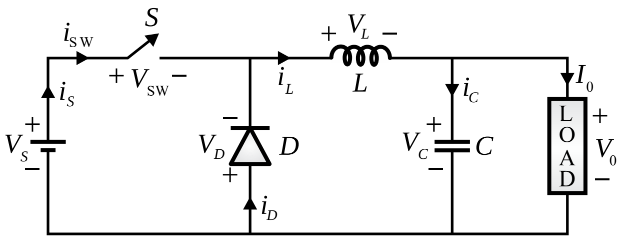
4 Choppers & Commutation Techniques

➤ Partial Synopsis

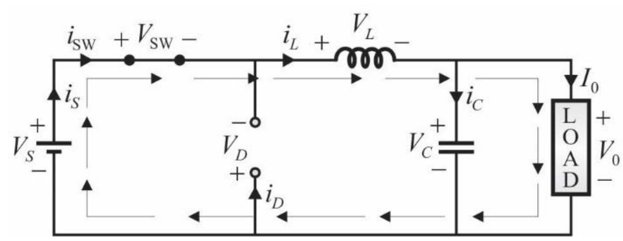
Buck Converter :



Operation :

Mode 1 : During, $0 < t < \delta T$ or $0 < t < T_{ON}$,

When the switch is ON, diode will be OFF



Applying KVL in the above loop,

$$-V_s + V_{L(\text{ON})} + V_0 = 0$$

$$\frac{di_L}{dt} = \frac{V_s - V_0}{L} = \text{ constant}$$

$$V_{\text{SW}} = 0\,\text{V},\ V_D = -V_S,\ V_{C(\text{ON})} = V_0$$

$$i_{S(\text{ON})} = i_{L(\text{ON})},\ i_D = 0\,\text{A}$$

$$i_{C(\text{ON})} = i_{L(\text{ON})} - I_0$$

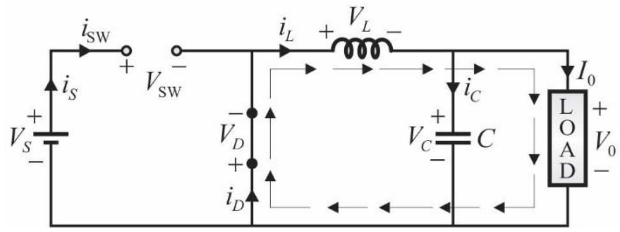
$$\frac{di_L}{dt} = \frac{V_s - V_0}{L}$$

$$\int di_L = \int\left(\frac{V_s - V_0}{L}\right)dt$$

$$i_L = \left(\frac{V_s - V_0}{L}\right)t + I_{L(\min)} \qquad \text{[Charging current]}$$

Mode 2 : During, $\delta T < t < T$ or $T_{ON} < t < T$

When the switch is OFF, diode will be ON



Applying KVL in the above loop,

$$-V_0 + V_{L(\text{OFF})} = 0$$

$$V_{L(\text{OFF})} = V_s = L\frac{di_L}{dt} = \text{ constant}$$

$$\frac{di_L}{dt} = \frac{V_s}{L} = \text{ constant}$$

$$V_{\text{SW}} = V_s,\ V_D = 0\,\text{V},\ V_{C(\text{OFF})} = V_0$$

$$i_{C(\text{OFF})} = i_{L(\text{OFF})} - I_0$$

$$\frac{di_L}{dt} = \frac{-V_0}{L},$$

$$di_L = \left(\frac{-V_0}{L}\right)dt$$

$$i_L = I_{L(\max)} - \frac{V_0}{L}(t - \delta T) \qquad \text{[Discharging current]}$$

Step 3 : Applying volt-sec balance equation across inductor,

$$V_{L(\text{ON})}T_{\text{ON}} + V_{L(\text{OFF})}T_{\text{OFF}} = 0$$

$$(V_s - V_0)\delta T + (-V_0)(1-\delta)T = 0$$

$$V_0 = \delta V_s$$

Step 4 : Applying ampere-sec balance equation across capacitor,

$$i_{C(\text{ON})}T_{\text{ON}} + i_{C(\text{OFF})}T_{\text{OFF}} = 0$$

$$(I_L - I_0)\delta T + (I_L - I_0)(1-\delta)T = 0$$

$$(I_L - I_0)(\delta T + T - \delta T) = 0$$

$$i_L = I_{L(avg)} = I_0$$

Step 5 : Expression for peak to peak ripple in inductor current,

$$\Delta I_L = I_{L(\max)} - I_{L(\min)}$$

During T_{ON},

$$V_{L(ON)} = V_s - V_0$$

$$L\frac{d}{dt}i_{L(ON)} = V_s - V_0$$

$$L\frac{\Delta I_L}{T_{ON}} = L\frac{\Delta I_L}{\delta T} = V_s - V_0$$

$$\Delta I_L = \frac{(V_s - V_0)\delta T}{L} = \frac{(V_s - V_0)\delta}{fL} \qquad \left[\text{As, } T = \frac{1}{f}\right]$$

$$\Delta I_L = \frac{\delta(1-\delta)V_s}{fL} \qquad [\text{As, } V_0 = \delta V_s]$$

Step 6 :

$$I_{L(\max)} = I_{L(avg)} + \frac{\Delta I_L}{2} = I_0 + \frac{\Delta I_L}{2}$$

$$I_{L(\min)} = I_{L(avg)} - \frac{\Delta I_L}{2} = I_0 - \frac{\Delta I_L}{2}$$

Step 7 : As the power loss in switches is zero.

Hence, Input power (P_{in}) = Output power (P_{out}) [For lossless converter]

$$V_s\, i_{s(avg)} = V_0\, I_0$$

$$i_{s(avg)} = \frac{V_0}{V_s} \times I_0 = \delta I_0$$

From the source current waveform,

$$i_{s(avg)} = i_{SW(avg)} = \frac{1}{2} \times \frac{\Delta I_L \times \delta T}{T} + I_{L\min}$$

$$i_{s(rms)} = i_{SW\,(rms)} = \frac{\Delta I_L \delta}{2} + I_{L\min}$$

From the diode current waveform,

$$I_{D(avg)} = \frac{1}{2} \times \frac{\Delta I_L \times (1-\delta T)}{T} + I_{L\min}$$

$$I_{D(rms)} = \frac{1}{2} \times \Delta I_L \times (1-\delta) + I_{L\min}$$

Step 8 : Expression for peak to peak ripple voltage in the capacitor.

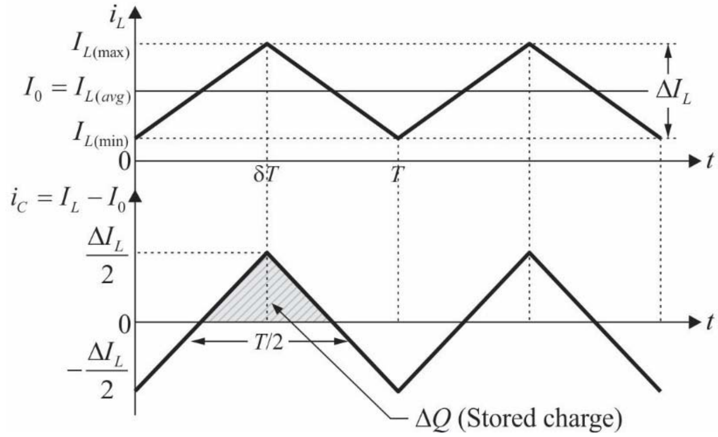
$$\Delta V_C = V_{C(\max)} - V_{C(\min)}$$

For a capacitor,

Charge, $Q = C \times V_C$

$$\Delta Q = C \times \Delta V_C$$

$$\Delta V_C = \frac{\Delta Q}{C}$$

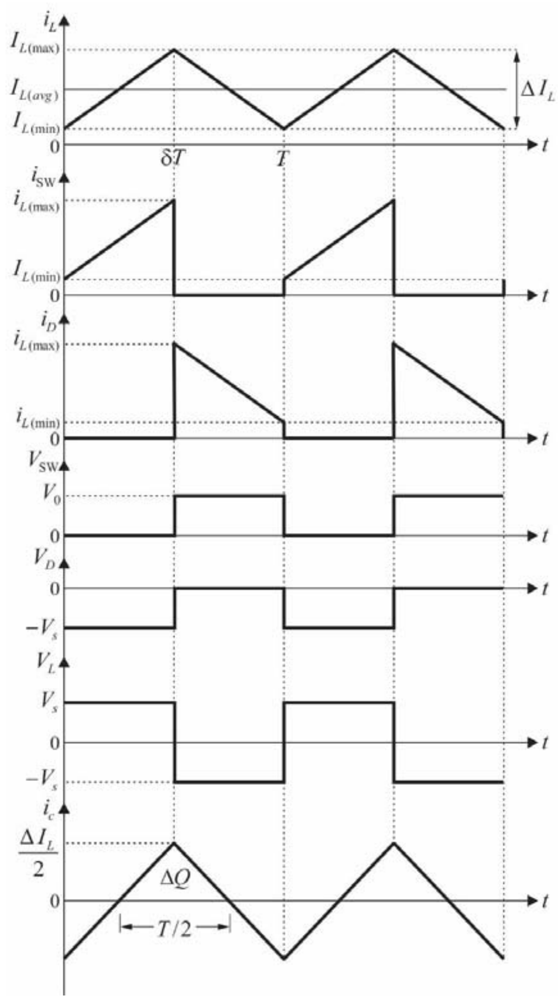


From the given waveform of capacitor current during one time period of T,

$$\Delta Q = \frac{1}{2} \times \frac{\Delta I_L}{2} \times \frac{T}{2} = \frac{\Delta I_L}{8f} \qquad \left[\text{As, } T = \frac{1}{f}\right]$$

$$\Delta V_C = \frac{\Delta Q}{C} = \frac{\Delta I_L}{8fC} = \frac{\delta(1-\delta)V_s}{8f^2 LC}$$

- The complete waveform is shown below,

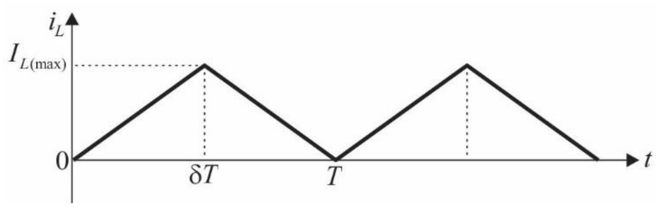


Step 9 : $$V_{C(\max)} = V_{C(avg)} + \frac{\Delta V_C}{2} = V_0 + \frac{\Delta V_C}{2} \qquad \left[\text{As, } V_{C(avg)} = V_0\right]$$

$$V_{C(\min)} = V_{C(avg)} - \frac{\Delta V_C}{2} = V_0 - \frac{\Delta V_C}{2}$$

Step 10 : Calculation of critical value of inductor (L_c).

The minimum value of 'L' for which inductor current is just continuous or about to become discontinuous. Hence, at critical value of L_c,



$$I_{L(\min)} = 0$$

$$I_{L(avg)} - \frac{\Delta I_L}{2} = 0$$

$$I_0 - \frac{\Delta I_L}{2} = 0$$

$$I_0 = \frac{\Delta I_L}{2} = \frac{\delta(1-\delta)V_s}{2fL_c}$$

$$L_c = \frac{\delta(1-\delta)V_s}{2fI_0}$$

Step 11 : Calculation of critical value of capacitor (C_c).

The minimum value of C for which output or capacitor voltage is just continuous or about to becomes discontinuous. Hence, at critical value of C_c,

$$V_{C(\min)} = 0$$

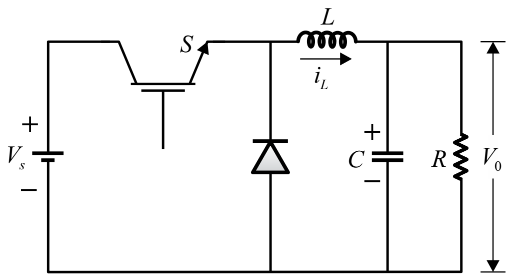
$$V_{C(avg)} - \frac{\Delta V_C}{2} = 0$$

$$C_c = \frac{\delta(1-\delta)V_s}{16f^2L_cV_0} = \frac{(1-\delta)}{16f^2L_c} \qquad \left[\text{As, } V_0 = \delta V_s\right]$$

➢ Sample Questions

2002 IISc Bangalore

4.1 In the chopper circuit shown in figure, the input DC voltage has a constant value V_s. The output voltage V_0 is assumed ripple-free. The switch S is operated with a switching time period T and a duty ratio D. What is the value of D at the boundary of continuous and discontinuous conduction of inductor current i_L ?

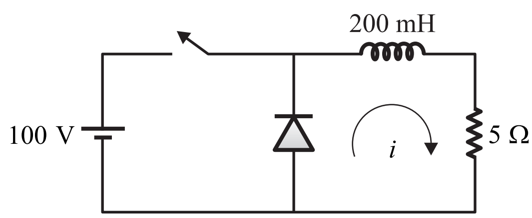


(A) $D = 1 - \frac{V_s}{V_0}$ (B) $D = \frac{2L}{RT}$

(C) $D = 1 - \frac{2L}{RT}$ (D) $D = \frac{RT}{L}$

2005 IIT Bombay

4.2 The given figure shows a step-down chopper switched at 1 kHz with a duty ratio $D = 0.5$. The peak-peak ripple in the load current is close to



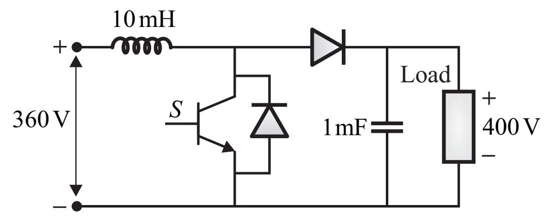
(A) 10 A (B) 0.5 A

(C) 0.125 A (D) 0.25 A

2016 IISc Bangalore

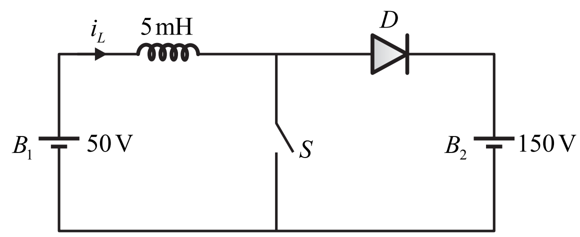
4.3 A DC-DC boost converter, as shown in the figure below, is used to boost 360V to 400 V, at a power of 4 kW. All devices are ideal. Considering continuous inductor current, the rms current in the solid state switch (S), in ampere, is ________.

[Set - 02]



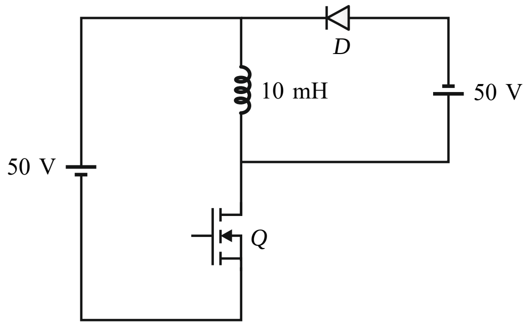
2018 IIT Guwahati

4.4 A dc to dc converter shown in the figure is charging a battery bank, B_2 whose voltage is constant at 150 V. B_1 is another battery bank whose voltage is constant at 50 V. The value of the inductor. L is 5 mH and the ideal switch S is operated with a switching frequency of 5 kHz with a duty ratio of 0.4. Once the circuit has attained steady state and assuming the diode D to be ideal, the power transferred from B_1 to B_2 (in Watt) is ________ (up to 2 decimal places).



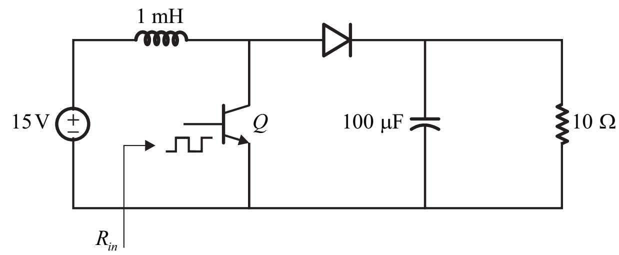
2020 IIT Delhi

4.5 In the dc-dc converter circuit shown, switch Q is switched at a frequency of 10 kHz with a duty ratio of 0.6. All components of circuit are ideal and the initial current in the inductor is zero. Energy stored in the inductor in mJ (rounded off to 2 decimal places) at the end of 10 complete switching cycles is ______.



2021 IIT Bombay

4.6 Consider the boost converter shown. Switch Q is operating at 25 kHz with a duty cycle of 0.6. Assume the diode and switch to be ideal. Under steady-state condition, the average resistance R_{in} as seen by the source is ________ Ω. (Round off to 2 decimal places)



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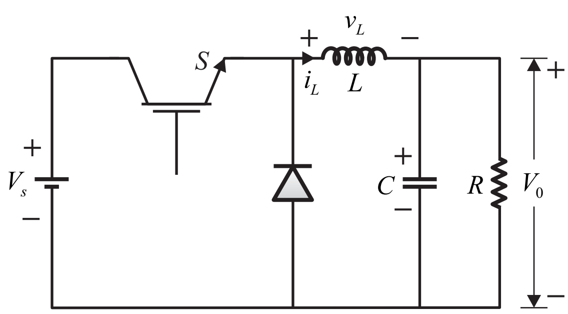
Explanations — Choppers & Commutation Techniques

4.1 (C)

Given : A Buck converter



Waveform of inductor current i_L for boundary condition between continuous and discontinuous conduction is shown below,

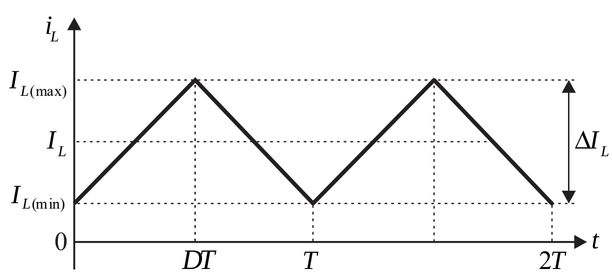


Fig. (a) Continuous conduction mode

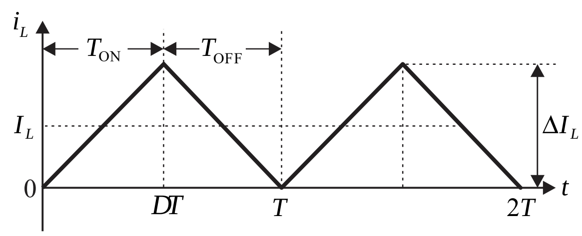


Fig. (b) Boundary of continuous and discontinuous conduction mode

For continuous conduction [fig. (a)]

$$I_{L(max)} = I_L + \frac{\Delta I_L}{2}$$

$$I_{L(min)} = I_L - \frac{\Delta I_L}{2}$$

At the boundary of continuous conduction $I_{L(min)} = 0$ [fig. (b)]

Hence, $0 = I_L - \frac{\Delta I_L}{2}$ $\Rightarrow$ $I_L = \frac{\Delta I_L}{2}$

$$\frac{V_0}{R} = \frac{\Delta I_L}{2} \quad \left(\text{Since, } I_L = \frac{V_0}{R}\right) \quad \ldots(i)$$

Also, $\Delta I_L = \dfrac{V_s D(1-D)}{fL}$ ...(ii)

$I_L = \dfrac{V_0}{R} = \dfrac{DV_s}{R}$ ...(iii)

Putting equation (ii), (iii) in equation (i),

$$\frac{V_s D(1-D)}{fL} = \frac{2DV_s}{R}$$

$$1-D = \frac{2fL}{R}$$

$$1-D = \frac{2L}{RT} \qquad \left(f = \frac{1}{T}\right)$$

$$D = 1-\frac{2L}{RT}$$

Hence, the correct option is (C).

4.2 (C)

Given : Buck converter

(i) Supply voltage, $V_s = 100$ V

(ii) Load resistance, $R = 5\ \Omega$

(iii) Inductance, $L = 200$ mH

(iv) Chopping frequency, $f = 1$ MHz

(v) Duty cycle, $D = 0.5$

Method 1

Mode 1 : During, $0 < t < DT$ or $0 < t < T_{ON}$

When the switch is ON, thyristor will be OFF

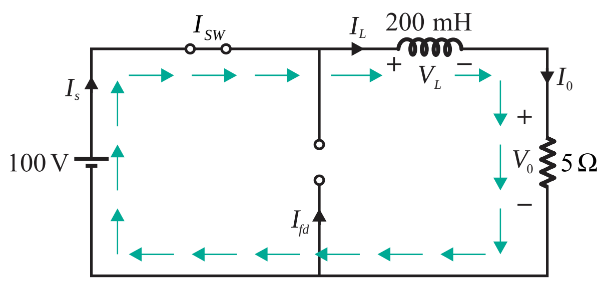


Fig. $0 < t < T_{ON}$

For type A chopper output voltage is given by,

$$V_0 = DV_s = 0.5\times 100$$

$$V_0 = 50\text{ V}$$

Applying KVL in the above circuit,

$$V_s - L\frac{di}{dt} - V_0 = 0$$

$$L\frac{di}{dt} = V_s - V_0$$

$$\Delta I_L = \frac{(V_s - V_0)DT}{L} = \frac{(V_s - DV_s)D}{fL}$$

$$\Delta I_L = \frac{\left(V_s - \frac{V_s}{2}\right)0.5}{10^6\times 200\times 10^{-3}} = 0.125\text{ A}$$

Hence, the correct option is (C).

Method 2

Mode 2 : During, $DT < t < T$ or $T_{ON} < t < T$

When the thyristor is OFF, diode will be ON

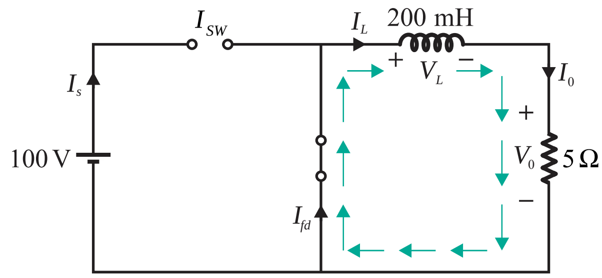


Fig. $T_{ON} < t < T_0$

Applying KVL in above circuit,

$$V_L + V_0 = 0$$

$$V_L = -V_0$$

$$V_L = V_0$$

$$\therefore \quad L\frac{dI_L}{dt} = V_0$$

$$\Delta I_L = \frac{V_0(1-D)T}{L}$$

$$\Delta I_L = \frac{50\times(1-0.5)}{10^6\times 200\times 10^{-3}} = 0.125\text{ A}$$

Hence, the correct option is (C).

Method 3

Ripple current for step down / type A chopper is given by,

$$\Delta I = \frac{V_S}{R}\left[\frac{(1-e^{-T_{ON}/\tau})(1-e^{-T_{OFF}/\tau})}{1-e^{-T/\tau}}\right]$$

Where, $\tau = \frac{L}{R} = \frac{200\times10^{-3}}{5} = 40\times10^{-3}$

$$T = \frac{1}{1000} = 1\times10^{-3}$$

$$T_{ON} = DT = 0.5\times10^{-3}$$

$$T_{OFF} = (1 \cdot D)T = 0.5\times10^{-3}$$

Therefore,

$$\Delta I = \left[\frac{\left(1-e^{-\frac{0.5\times10^{-3}}{40\times10^{-3}}}\right)\left(1-e^{-\frac{0.5\times10^{-3}}{40\times10^{-3}}}\right)}{\left(1-e^{-\frac{1\times10^{-3}}{40\times10^{-3}}}\right)}\right]$$

$$\Delta I = 0.125 \text{ A}$$

Hence, the correct option is (C).

Method 4

For step down chopper, maximum value of inductor / load current is given by,

$$I_{max} = \frac{V_S}{R}\left[\frac{1-e^{-T_{ON}/\tau}}{1-e^{-T/\tau}}\right] = \frac{100}{5}\left[\frac{1-e^{-\frac{0.5\times10^{-3}}{40\times10^{-3}}}}{1-e^{-\frac{1\times10^{-3}}{40\times10^{-3}}}}\right]$$

$$I_{max} = 10.062 \text{ A}$$

Minimum value of inductor / load current is given by,

$$I_{min} = \frac{V_S}{R}\left[\frac{1-e^{T_{ON}/\tau}}{1-e^{T/\tau}}\right] = \frac{100}{5}\left[\frac{1-e^{\frac{0.5\times10^{-3}}{40\times10^{-3}}}}{1-e^{\frac{1\times10^{-3}}{40\times10^{-3}}}}\right]$$

$$I_{min} = 9.937 \text{ A}$$

Therefore, ripple current is given by,

$$\Delta I = I_{max} - I_{min} = 10.062 - 9.937$$

$$\Delta I = 0.125 \text{ A}$$

Hence, the correct option is (C).

Method 5

For type A chopper

When D = 0.5 maximum value of ripple current occurs.

$$\Delta I_{max} = \frac{V_S}{4fL} = \frac{100}{4\times1\times10^{3}\times200\times10^{-3}}$$

$$\Delta I_{max} = 0.125 \text{ A}$$

Hence, the correct option is (C).

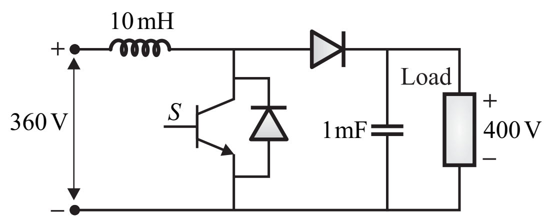
4.3 **3.5**

Given : A DC to DC boost converter.

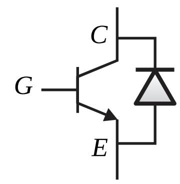
(i) $V_s = 360$ V

(ii) $V_0 = 400$ V

(iii) Load power = 4 kW



📖 Key Point



The given switch is combination of IGBT with parallel diode, hence it is composite switch.

When V_{CE} is positive

Switch is ON and diode is OFF

$$V_{CE} = 0 \text{ V}$$

and current flows in downward direction.

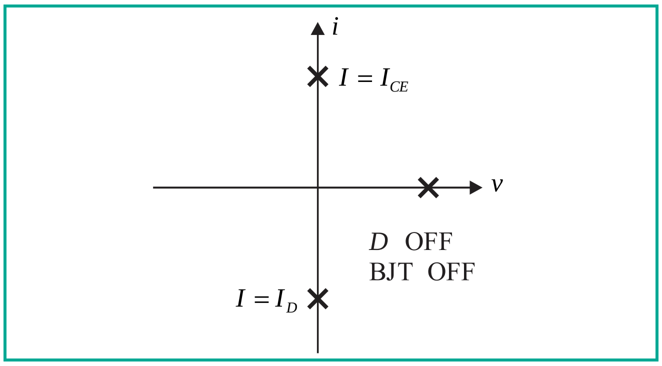
When V_{CE} is negative

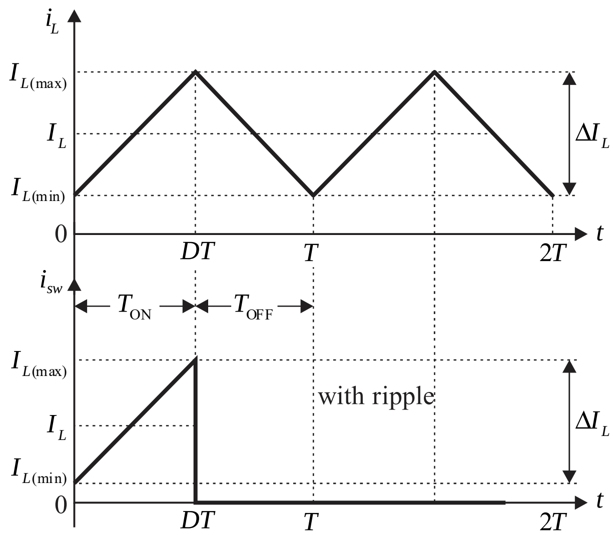
Switch is OFF and diode is ON

$$V_D = 0 \text{ V}$$

and current flows in upward direction.

As the given composite switch allows the bidirectional current flow and unipolar voltage blocking capability, it gives the two quadrant operation of switch.





Output voltage expression for boost converter is given by,

$$V_0 = \frac{V_s}{1-\delta}$$

Where, $\delta = \frac{T_{ON}}{T}$ is duty ratio

$$400 = \frac{360}{1-\delta}$$

$$\delta = 0.1$$

As the power loss in the switch is zero.

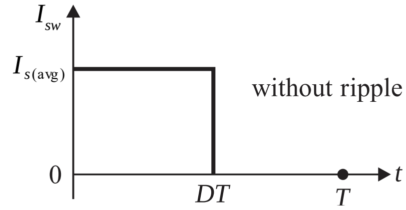
Applying power balance equation,

$$P_{in} = P_0$$

$$V_s I_s = V_0 I_0$$

$$360 \times I_s = 4000$$

$$I_s = I_{s(avg)} = 11.1 \text{ A}$$



$$I_{switch\,(rms)} = I_s \left(\frac{T_{ON}}{T}\right)^{\frac{1}{2}} = I_s (D)^{\frac{1}{2}}$$

$$I_{switch\,(rms)} = I_s\sqrt{\delta} = 11.1\sqrt{0.1} = 3.5\,\text{A}$$

Hence, the rms current in the solid state switch (S) is **3.5 A.**

4.4 **12**

Given :

(i) A DC to DC converter charging a battery bank.

(ii) Battery (B_2) voltage, $V_{B_2} = 150$ V

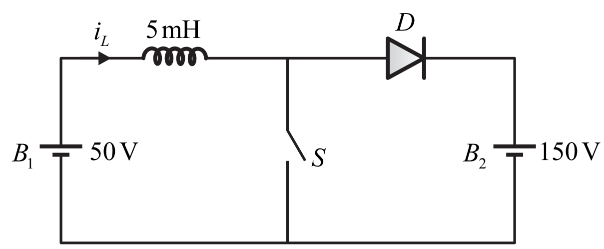
(iii) Battery (B_1) voltage, $V_{B_1} = 50$ V

(iv) $L = 5$ mH

(v) Switching frequency, $f = 5$ kHz

(vi) Duty ratio, $\delta = 0.4$

Given DC to DC converter is shown below,



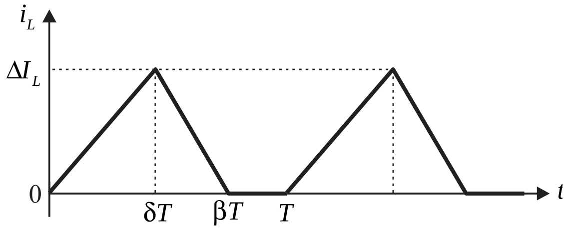
The output voltage expression of a boost converter for continuous conduction is given by,

$$V_0 = \frac{V_S}{1-\delta} = \frac{50}{1-0.5}$$

$$V_0 = 83.33 \text{ V}$$

As the calculated output voltage for continuous conduction is less than the given output voltage, then the converter operates in discontinuous conduction mode.

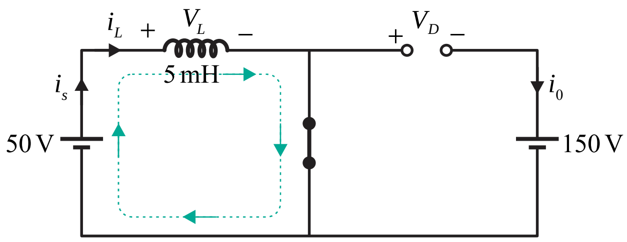
Inductor current waveform for discontinuous conduction is as shown below,



Working :

Mode 1 : During, $0 < t < \delta T$ or $0 < t < T_{ON}$

When the switch is ON, diode will be OFF



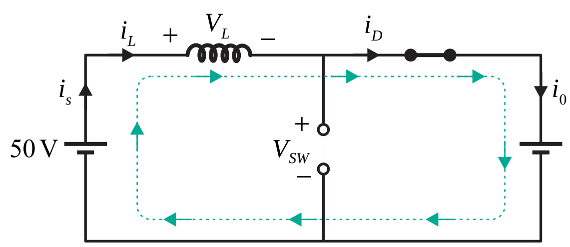
Applying KVL in the above circuit,

$$V_s - V_L = 0$$

$$V_{L(\text{ON})} = V_S$$

Mode 2 : During, $\delta T < t < \beta T$ or $T_{ON} < t < \beta T$

When the thyristor is OFF, diode will be ON



Applying KVL in the above circuit,

$$V_S - V_L - V_0 = 0$$

$$V_S - V_0 = V_{L(\text{OFF})}$$

Applying volt second between equation across inductor voltage,

$$V_{L(\text{ON})}\delta T + V_{L(\text{OFF})}(\beta - \delta)T = 0$$

$$V_S \delta T + (V_S - V_0)(\beta - \delta)T = 0$$

$$V_S \delta + V_S \beta - \delta V_S - \beta V_0 + \delta V_0 = 0$$

$$V_0 = \frac{\beta V_S}{\beta - \delta} \qquad \ldots \text{(i)}$$

Putting the values of V_0, V_S, δ in equation (i),

$$150 = \frac{\beta \times 50}{\beta - 0.4}$$

$$150\beta - 60 = 50\beta$$

$$100\beta = 60$$

$$\beta = \frac{60}{100} = 0.6$$

Calculation of inductor current

When the switch is ON

$$V_{L(\text{ON})} = V_S$$

$$V_S = L\frac{di_L}{dt}$$

$$V_S = L\frac{\Delta I_L}{\delta T}$$

$$\Delta I_L = \frac{V_S \delta T}{L}$$

$$\Delta I_L = \frac{V_S \delta}{f_L}$$

$$\Delta I_L = \frac{150 \times 0.4}{5 \times 10^3 \times 5 \times 10^{-4}}$$

$$\Delta I_L = 0.8 \text{ A}$$

Calculation of output current :

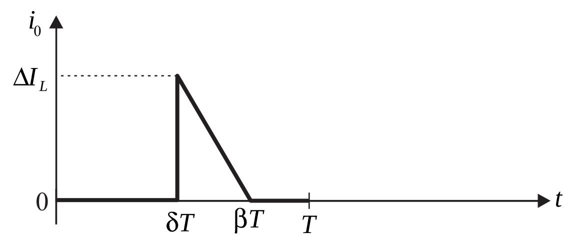


Fig. Output current waveform

$$I_0 = \frac{1}{2} \times \frac{\Delta I_L \times (\beta - \delta)T}{T}$$

$$I_0 = \frac{1}{2} \times 0.8(0.6 - 0.4)$$

$$I_0 = 0.08 \text{ A}$$

Output power,

$$P_0 = V_0 I_0$$

$$P_0 = 150 \times 0.08 = 12 \text{ W}$$

Hence, the power transferred from B_1 to B_2 is **12 W.**

Key Point

Results of output voltages for discontinuous conduction.

Buck Converter :

$$V_0 = \frac{\delta V_s}{\beta}$$

$[\beta = 1$ for continuous conduction$]$

Boost Converter :

$$V_0 = \frac{\beta V_s}{(\beta - \delta)}$$

$[\beta = 1$ for continuous conduction$]$

Buck-Boost Converter :

$$V_0 = \frac{\delta V_S}{(\beta - \delta)}$$

$[\beta = 1$ for continuous conduction$]$

4.5 **5**

Given : Buck boost converter

(i) $f_s = 10$ kHz

(ii) $T = 0.1$ ms

(iii) $\delta = 0.6$

When the switch is ON from 0 to δT

$$\therefore \quad V_L = L\frac{di}{dt}$$

$$\frac{V_L}{L} = \frac{di}{dt}$$

$$I_L = \frac{V_L}{L}\cdot \delta T$$

$$I_L = \frac{50}{10\times 10^{-3}} \times 0.6 \times \frac{1}{10\times 10^{3}}$$

$$I_L = 0.3 \text{ A}$$

$\therefore$ When the switch is closed from δT to T

$$V_L = L\frac{di}{dt}$$

$$\Delta I_L = \frac{V_L \times (1-\delta)T}{L}$$

$$\Delta I_L = \frac{50\times(1-0.6)}{10\times 10^{-3}} \times \frac{1}{10\times 10^{3}}$$

$$\Delta I_L = 0.2 \text{ A}$$

$\therefore$ In first on period inductor charges from 0 A to 0.3 A, and in first of period inductor discharge from 0.3 A to 0.1 A. As it's discharged to 0.1 A it has 0.1 A as initial value for next cycle.

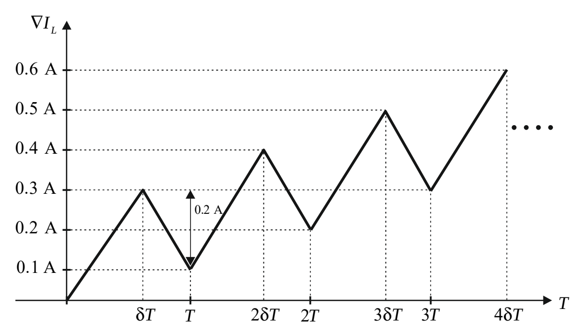
$\therefore$ In second cycle it will charge from 0.1 A to 0.4 A, (charge by 0.3 A and discharge by 0.2 A) and discharge to 0.2A from 0.4 A.

$\therefore$ After each cycle inductor current increases by 0.1 A from the value of its previous cycle.

After 10[th] cycle remaining inductor current will be $10 \times 0.1 \text{ A} = 1 \text{ A}$.

$\therefore$ Energy stored by inductor at the end of 10 complete switching cycles

$$= \frac{1}{2}LI^2 = \frac{1}{2}\times 10\times 10^{-3} \times 1^2 = 5 \text{ mJ}$$



4.6 **1.6**

Given : Boost convertor

(i) Switching frequency, $f_s = 25$ kHz

(ii) Duty cycle, $\delta = 0.6$

Method 1

As the switches are lossless,

$$P_{in} = P_{out}$$

$$V_s i_s = V_0 i_0$$

$$i_s = \frac{i_0}{(1-\delta)}$$

$$\therefore \quad V_{0_{(avg)}} = \frac{15}{(1-0.6)} = 37.5\,\text{V}$$

Average value of load current is given by,

$$i_{0(avg)} = \frac{V_0}{R} = \frac{37.5}{10} = 3.75\,\text{Amp}$$

Average value of source current,

$$i_{s(avg)} = i_{L(avg)} = \frac{10}{(1-\delta)} = 9.375\ \text{Amp}$$

Ripple component of inductor current is given by,

$$\Delta i_L = \frac{\delta V_s}{fL} = \frac{0.6\times 15}{25\,\text{k}\times 1\,\text{m}} = 0.36\,\text{Amp}$$

$\therefore$ Minimum current through inductor,

$$i_{L_{\min}} = i_{L_{(avg)}} - \frac{\Delta i_L}{2}$$

$$i_{L_{\min}} = 9.375 - \frac{0.36}{2} = 9.195\,\text{Amp}$$

As $i_{L_{\min}}$ is not zero, hence inductor current will be continuous

Input resistance seen from supply side is given by,

$$R_{in} = \frac{V_s}{i_s} = \frac{15}{9.375} = 1.6\ \Omega$$

Method 2

$$R_{in} = \frac{V_s}{i_s} = \frac{\dfrac{V_0}{(1-\delta)}}{\dfrac{i_0}{(1-\delta)}} = \left(\frac{V_0}{i_0}\right)(1-\delta)^2$$

$$R_{in} = R_0(1-\delta)^2 = 10(1-0.6)^2 = 1.6\,\Omega$$

Hence, the correct answer is 1.6.

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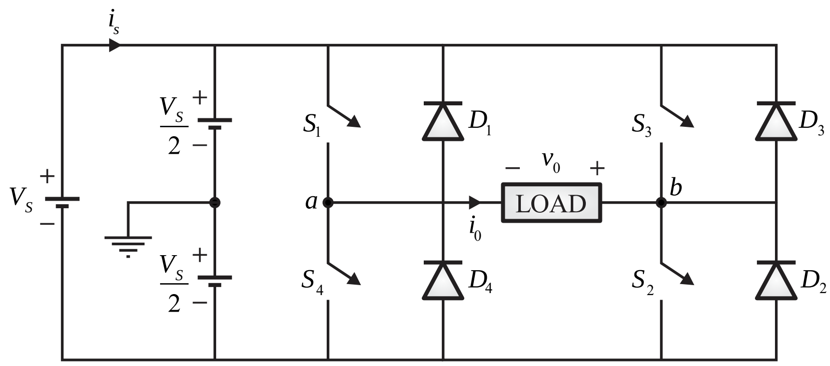
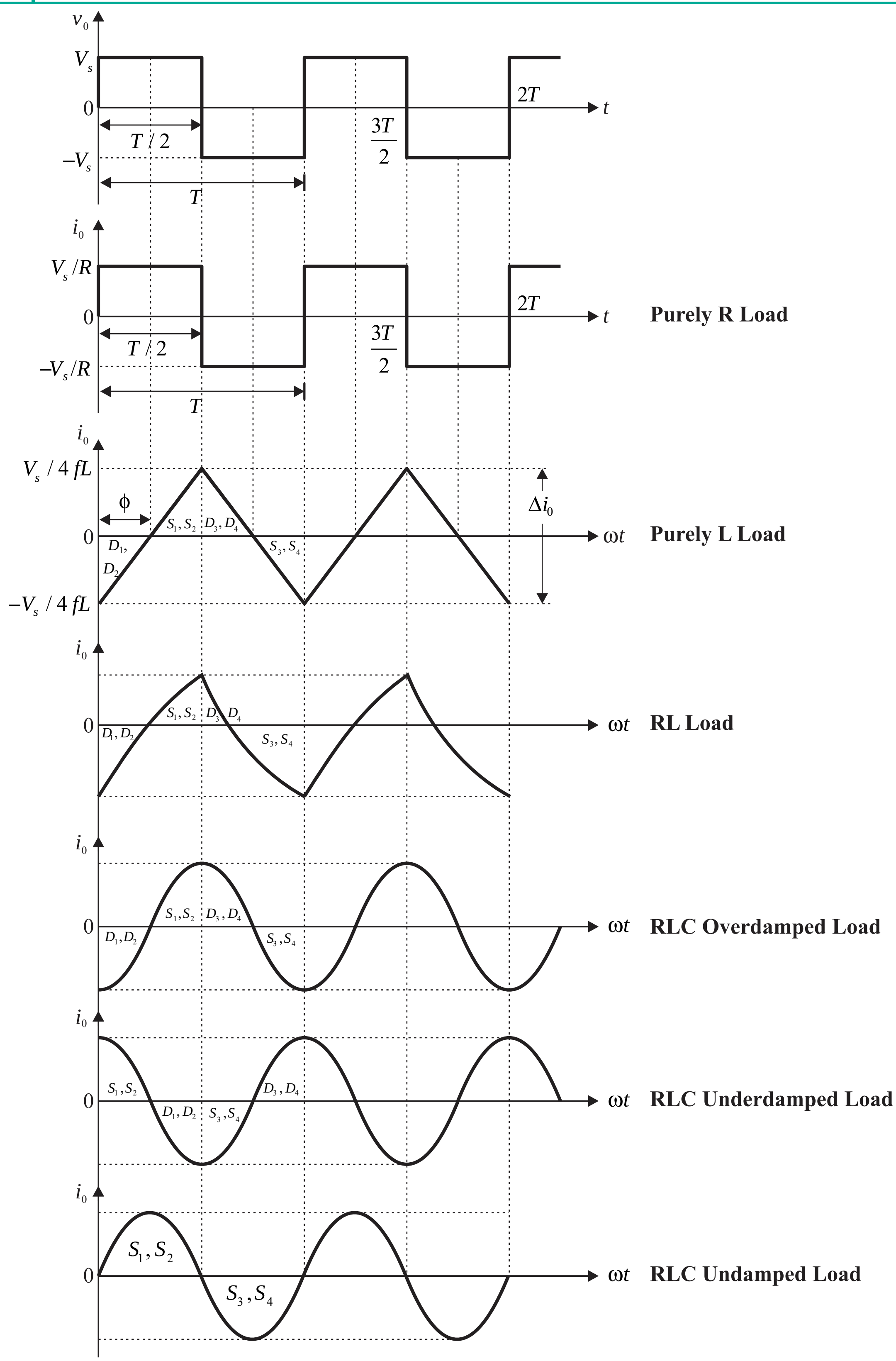
5 Inverters

➢ Partial Synopsis

Single phase full bridge inverter circuit



Voltage and Current waveforms for different types of Loads are as shown below,



v_0
V_s
0
$-V_s$
$T/2$
T
$3T/2$
$2T$
t
i_0
V_s/R
0
$-V_s/R$
$T/2$
T
$3T/2$
$2T$
t
Purely R Load
i_0
$V_s/4fL$
0
$-V_s/4fL$
ϕ
D_1, D_2
S_1, S_2
D_3, D_4
S_3, S_4
Δi_0
ωt
Purely L Load
i_0
0
D_1, D_2
S_1, S_2
D_3, D_4
S_3, S_4
ωt
RL Load
i_0
0
D_1, D_2
S_1, S_2
D_3, D_4
S_3, S_4
ωt
RLC Overdamped Load
i_0
0
S_1, S_2
D_1, D_2
S_3, S_4
D_3, D_4
ωt
RLC Underdamped Load
i_0
0
S_1, S_2
S_3, S_4
ωt
RLC Undamped Load

Performance parameters of single phase full bridge inverter

(i) RMS value of output voltage will be, $V_{0r} = V_s$

(ii) RMS value of output current, $I_{0r} = \dfrac{V_s}{R}$ [For R load]

(iii)Power delivered to load, $P_0 = I_{0r}^2 R = \dfrac{V_s^2}{R}$ [For R load]

(iv)Fourier analysis of single phase full bridge inverter,

$$V_0 = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi}\sin(n\omega t) \text{ Volts}$$

(v) n^{th} harmonic component of output voltage, $V_{0n} = \dfrac{4V_s}{n\pi}\sin n\omega t$

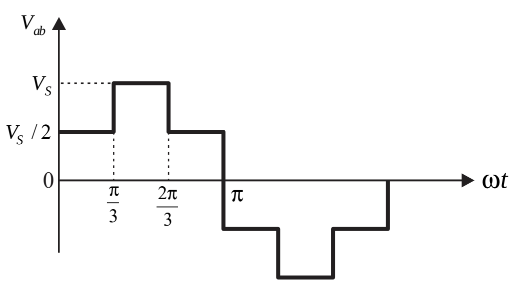
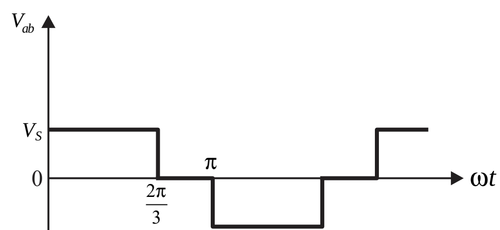
(vi)R.M.S. value of n^{th} harmonic, $V_{0n(rms)} = \dfrac{2\sqrt{2}V_s}{n\pi}$

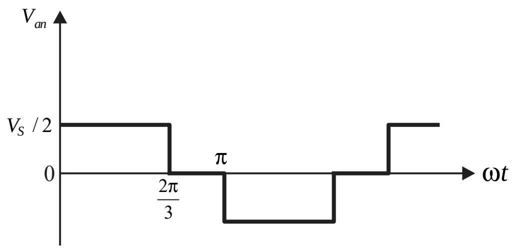
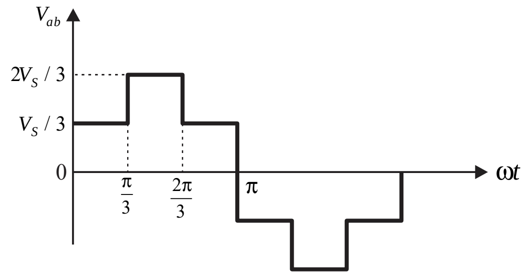
(vii) Fundamental component of output voltage, $V_{01} = \dfrac{2\sqrt{2}V_s}{\pi}$

(viii) Distortion factor, $g = \dfrac{V_{01}}{V_{0r}} = \dfrac{2\sqrt{2}V_s/\pi}{V_s} = \dfrac{2\sqrt{2}}{\pi} = 0.9003$

(ix)Total harmonic distortion, THD $= \sqrt{\dfrac{V_{0r}^2 - V_{01}^2}{V_{01}^2}} = \sqrt{\dfrac{1}{g^2} - 1} = 48.34\%$

3 - Phase Voltage Source Inverter :

Sr.	120^0 - Conduction	180^0 - Conduction
1.	Each Switch conducts for 120^0	Each Switch conducts for 180^0
2.	Line voltage and phase voltage is free from triplen and even harmonics.	Line voltage and phase voltage is free from triplen and even harmonics.
3.	**Line voltage**  $(V_l)_{\text{avg}} = 0$ V $(V_l)_{\text{rms}} = \sqrt{\frac{1}{\pi}\left[\int_0^{2\pi/3}\left(\frac{V_S}{2}\right)^2 d\omega t + \int_0^{\pi/3}(V_S^2)d\omega t\right]}$ $(V_l)_{\text{rms}} = \frac{V_S}{\sqrt{2}}$	**Line voltage waveform**  $(V_l)_{\text{avg}} = 0$ V $(V_l)_{\text{rms}} = \sqrt{\frac{1}{\pi}\left[\int_0^{2\pi/3} V_S^2 d\omega t\right]}$ $(V_l)_{\text{rms}} = V_S\sqrt{\frac{2}{3}}$

4.	**Phase Voltage** 	**Phase Voltage** 
5.	$(V_{ph})_{\text{avg}} = 0$	$(V_{ph})_{\text{avg}} = 0$
6.	$(V_{ph})_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi/5}\left(\frac{V_S}{2}\right)^2 d\omega t}$ $(V_{ph})_{\text{rms}} = \frac{V_S}{2}\sqrt{\frac{2}{3}} = \frac{V_S}{\sqrt{6}}$	$(V_{ph})_{\text{rms}} = \frac{\sqrt{2}V_S}{3}$
7.	$(I_{ph})_{\text{avg}} = 0$	$(I_{ph})_{\text{avg}} = 0$
8.	$(I_{ph})_{\text{rms}} = \frac{V_S}{\sqrt{6}R}$	$(I_{ph})_{\text{rms}} = \frac{\sqrt{2}V_S}{3R}$
9.	Power delivered to load (for R load) $P_0 = \frac{3(V_{ph})^2_{rms}}{R_L} = \frac{3\left(\frac{V_S}{\sqrt{6}}\right)^2}{R_L}$ $P_0 = \frac{V_S^2}{2R}$	Power delivered to load (for R load) $P_0 = 3\left(\frac{\sqrt{2}V_S}{3R}\right)^2 \times R = \frac{2V_S^{\ 2}}{3R}$

➢ Sample Questions

2000 IIT Kharagpur

5.1 A three-phase voltage source inverter supplies a purely inductive three-phase load. Upon Fourier analysis the output voltage waveform is found to have n^{th} order harmonics of magnitude a_n times that of the fundamental frequency component $(a_n < 1)$, the load current would then have an n^{th} order harmonic of magnitude

(A) zero

(B) a_n times the fundamental frequency component.

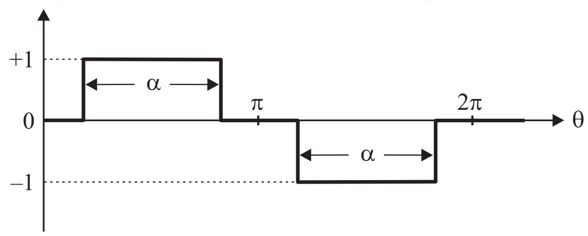
(C) na_n times the fundamental frequency component.

(D) a_n/n times the fundamental frequency component.

2003 IIT Madras

Statement for Linked Answer Questions 5.2 & 5.3

An inverter has a periodic output voltage with the output waveform as shown in figure.



5.2 When the conduction angle $\alpha = 120^0$, the rms fundamental component of the output voltage is

(A) 0.78 V (B) 1.10 V

(C) 0.90 V (D) 1.27 V

5.3 With reference to the output waveform given in figure, the output of the converter will be free from 5^{th} harmonic when

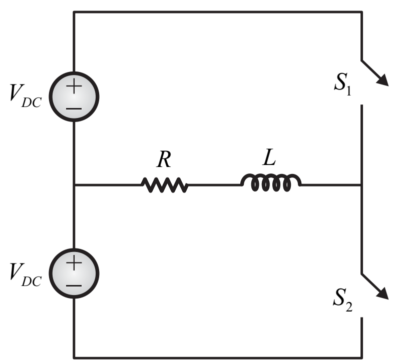
(A) $\alpha = 72^0$ (B) $\alpha = 36^0$

(C) $\alpha = 120^0$ (D) $\alpha = 150^0$

2017 IIT Roorkee

5.4 The figure below shows a half-bridge voltage source inverter supplying an RL-load with $R = 40\,\Omega$ and $L = \left(\frac{0.3}{\pi}\right)$H. The desired fundamental frequency of the load voltage is 50 Hz. The switch control signals of the converter are generated using sinusoidal pulse width modulation with modulation index $M = 0.6$. At 50 Hz, the RL-load draws an active power of 1.44 kW. The value of DC source voltage V_{DC} in volts is

[Set - 02]

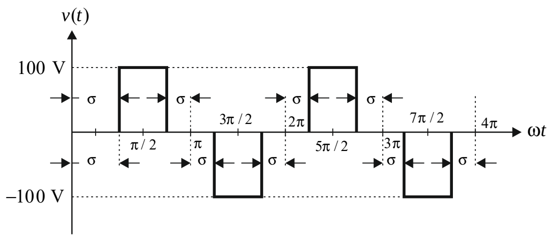


(A) $300\sqrt{2}$ (B) 500

(C) $500\sqrt{2}$ (D) $1000\sqrt{2}$

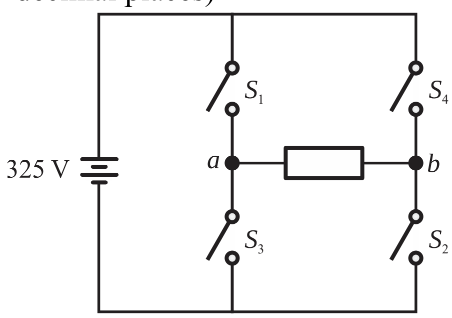
2020 IIT Delhi

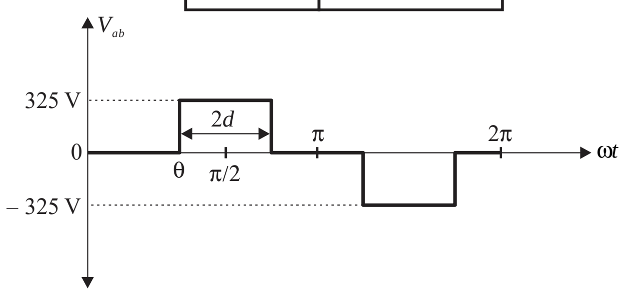
5.5 A single-phase inverter is fed from a 100 V dc source and is controlled using a quasi square wave modulation scheme to produce an output waveform, $v(t)$ as shown. The angle σ is adjusted to entirely eliminate the 3^{rd} harmonic component from the output voltage. Under this condition, for $v(t)$, the magnitude of the 5^{th} harmonic component as a percentage of magnitude of the fundamental component is ________ (rounded off to two decimal places).



2021 IIT Bombay

5.6 A single-phase full bridge inverter fed by a 325 V DC produces a symmetric quasi-square waveform across "*ab*" as shown. To achieve a modulation index of 0.8, the angle θ expressed in degrees should be ________. (Round off to 2 decimal places)





(Modulation index is defined as the ratio of the peak of the fundamental component of V_{ab} to the applied DC value)

Explanations Inverters

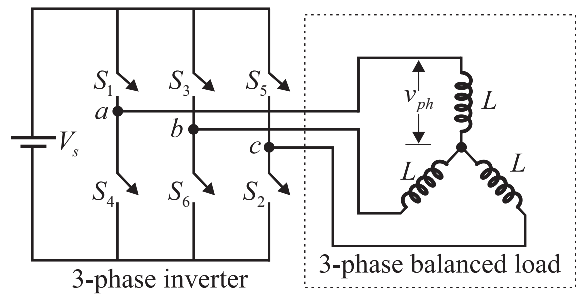
5.7 (D)

Given : A three phase voltage source inverter operating in 180^0 conduction mode.

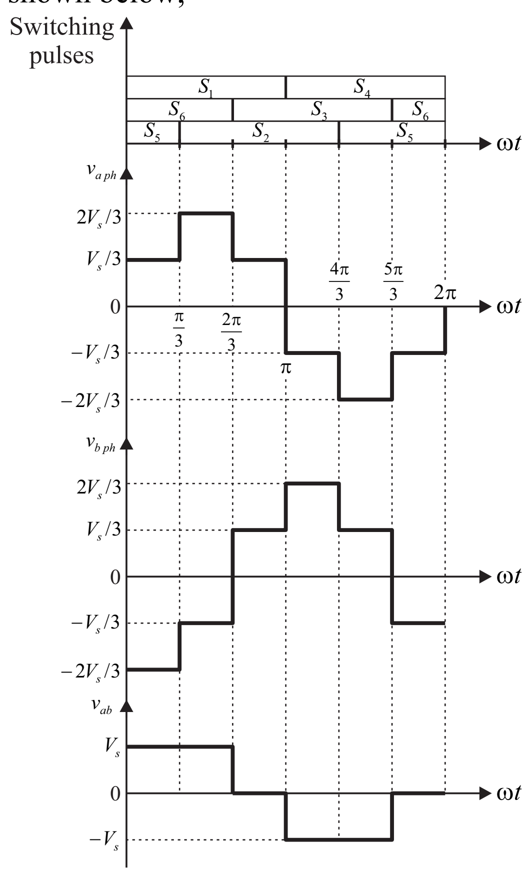
(i) Supply voltage $= V_s$

(ii) Per phase load inductance $= L_{phase}$

A three phase voltage source inverter operating in 180^0 conduction mode feeding pure inductive load is shown below,



The phase voltage and line voltage waveforms are shown below,



For 180^0 mode inverter, the Fourier series expansion of line to neutral voltage for phase "*a*" is,

$$v_{a0} = \sum_{n=6k\pm1}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t$$

Where $k = 0, 1, 2, \ldots$

Peak amplitude of n^{th} harmonic component of phase voltage is,

$$V_{a0,\,n} = \frac{2V_s}{n\pi}$$

Peak amplitude of fundamental frequency component of phase voltage is,

$$V_{a0,\,1} = \frac{2V_s}{\pi}$$

$$\frac{V_{a0,\,n}}{V_{a0,\,1}} = \frac{1}{n} = a_n$$

Phase current,

$$i_{a0} = \frac{V_{a0}}{Z_n} = \sum_{n=6k\pm1}^{\infty} \frac{2V_s}{n\pi Z_n} \sin(n\omega t \pm \theta_n)$$

where, $Z_n = \sqrt{R^2 + \left(n\omega L - \frac{1}{n\omega C}\right)^2}$ and

$$\theta_n = \tan^{-1}\left(\frac{n\omega L - \frac{1}{n\omega C}}{R}\right)$$

Peak amplitude of n^{th} harmonic component of phase current is,

$$I_{a0,\,n} = \frac{2V_s}{n\pi Z_n}$$

where, $Z_n = 2\pi nfL$ as load is purely inductive.

$$I_{a0,\,n} = \frac{2V_s}{n\pi\, 2\pi nfL}$$

$$\frac{I_{a0,\,n}}{I_{a0,\,1}} = \frac{\dfrac{2V_s}{n\pi\, 2\pi nfL}}{\dfrac{2V_s}{\pi 2\pi fL}}$$

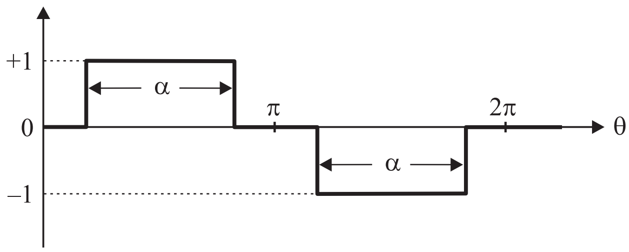
$$\frac{I_{a0,n}}{I_{a0,1}} = \frac{1}{n^2} = \left(\frac{1}{n}\right)\frac{1}{n} = \frac{a_n}{n}$$

Hence, the correct option is (D).

5.10 (A)

Given : Single phase pulse width modulated inverter.

(i) Pulse width, $2d = \alpha = 120^0$



The above output voltage wave is the output voltage of single phase inverter with single pulse modulation.

The Fourier analysis expression for output voltage is

$$v_{on}(t) = \sum_{n=1,3,5}^{\infty}\left(\frac{4V_s}{n\pi}\sin\frac{n\pi}{2}\sin nd\sin n\omega t\right)$$

Where, $d = \frac{\alpha}{2}$

r.m.s. value of fundamental component is,

$$V_{01} = \frac{4V_s}{\pi\sqrt{2}}\sin\frac{\pi}{2}\sin d = \frac{4V_s}{\pi\sqrt{2}}\sin\frac{\alpha}{2}$$

If $\alpha = 120^0$, then

$$V_{rms} = \frac{4\times1}{\pi\sqrt{2}}\sin\frac{\pi}{2}\sin\frac{120}{2} = 0.78\text{V}$$

Hence, the correct option is (A).

5.11 (A)

For eliminating any harmonic from output waveform, amplitude of the waveform should be equal to zero. From the Fourier analysis expression,

$$\frac{4V_s}{n\pi}\sin\frac{\pi}{2}\sin nd = 0$$

(where, $2d$ = pulse width)

$$\sin nd = 0$$

$$nd = \pi$$

$$d = \frac{\pi}{n}$$

Here, Pulse width, $\alpha = 2d = \frac{2\pi}{n} = \frac{2\pi}{5} = 72^0$

Hence, the correct option is (A).

5.33 (C)

Given : Single phase half voltage source inverter.

(i) Fundamental frequency, $f = 50\,\text{Hz}$

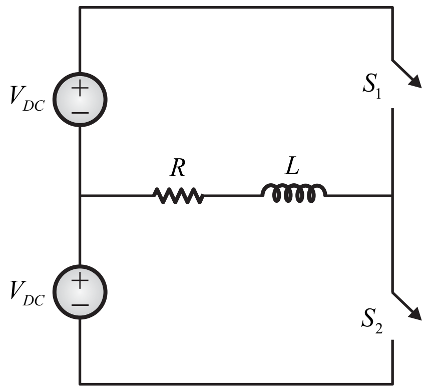
(ii) Load power, $P = 1.44$ kW

(iii) Load resistance, $R = 40\,\Omega$

(iv) Load inductance, $L = \frac{0.3}{\pi}$ H

(v) Modulation index, $m_a = 0.6$

Single phase half voltage source inverter is given by,



For sinusoidal PWM control, the peak value of fundamental voltage of half bridge inverter is given by,

$$V_{m1} = m_a \times V_{DC} = 0.6V_{DC}$$

[From sinusoidal PWM control]

Load reactance at 50 Hz is given by,

$$X_L = 2\pi fL = 100\pi\times\frac{0.3}{\pi} = 30\ \Omega$$

Load impedance is given by

$$Z = R + jX_L = 40 + j30 = 50\angle 36.86^0\ \Omega$$

Since, active power drawn, $P = 1.44$ kW

$$I_{rms}^2 R = 1440$$

$$I_{rms}^2 = \frac{1440}{40} = 36$$

$$I_{rms} = 6 \text{ A}$$

$$V_{rms} = I_{rms} \times Z = 6 \times 50 = 300 \text{ V}$$

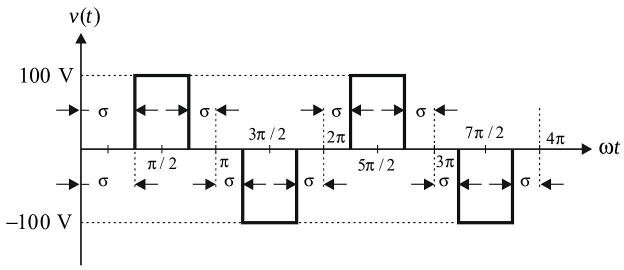
Peak value, $V_m = 300\sqrt{2} = 0.6 V_{DC}$

$$V_{DC} = \frac{300}{0.6}\sqrt{2} = 500\sqrt{2} \text{ V}$$

Hence, the correct option is (C).

5.35 20

Given output voltage waveform is shown below,



For single phase inverter, Fourier series expression of single pulse modulation is given by,

$$v_{on}(t) = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin n\frac{\pi}{2} \sin nd \sin n\omega t$$

To eliminate third harmonics pulse width required is given by,

$$nd = 180^0$$

$$3d = 180^0 \qquad (n = \text{order of harmonics})$$

$$d = 60^0$$

$\therefore$ Fundamental harmonic component is given by,

$$I_1 = \frac{4I_0}{\pi} \sin n\frac{\pi}{2} \sin 60^0$$

$$I_1 = \frac{4I_0}{\pi} \times \frac{\sqrt{3}}{2}$$

5^{th} Harmonic component is given by,

$$I_5 = \frac{4I_0}{5\pi} \sin\left(\frac{5\pi}{2}\right) . \sin 5 \times 60^0$$

$$I_5 = \frac{4I_0}{5\pi} \times \frac{\sqrt{3}}{2}$$

$$\frac{I_5}{I_1} = 20\%$$

5.36 51.07

Fourier series expression for single pulse width modulation is given by,

$$V_0(nt) = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin(nd) \sin\left(\frac{n\pi}{2}\right) \sin n\omega t$$

Fundamental component of single pulse width modulation is given by,

$$V_{0_1}(t) = \frac{4V_s}{\pi} \sin(d) \qquad \ldots \text{(i)}$$

Peak amplitude of fundamental component of single pulse with a modulation index of 0.8 is

$$V_{0_1} = mV_s$$

$$V_{0_1} = 0.8 V_s \qquad \ldots \text{(ii)}$$

Comparing equation (i) and (ii),

$$\frac{4V_s}{\pi} \sin(d) = 0.8 V_s$$

$$\sin d = 0.62832$$

Therefore, $\theta = \left(\frac{\pi}{2} - d\right) = 51.07^0$

Hence, the correct answer is 51.07.

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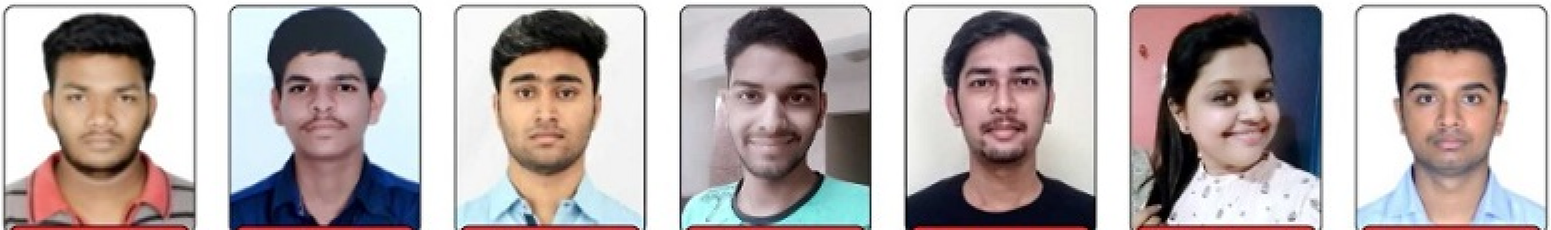

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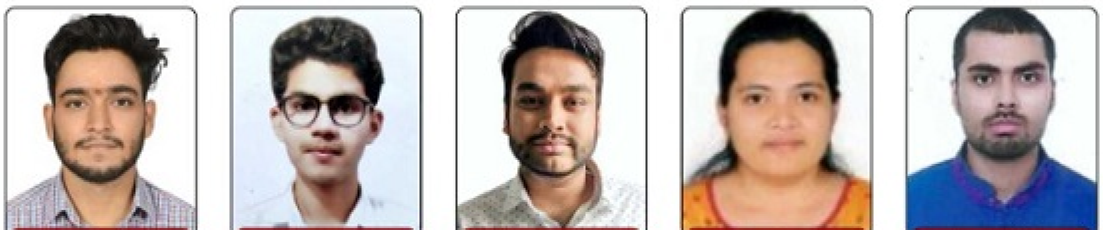

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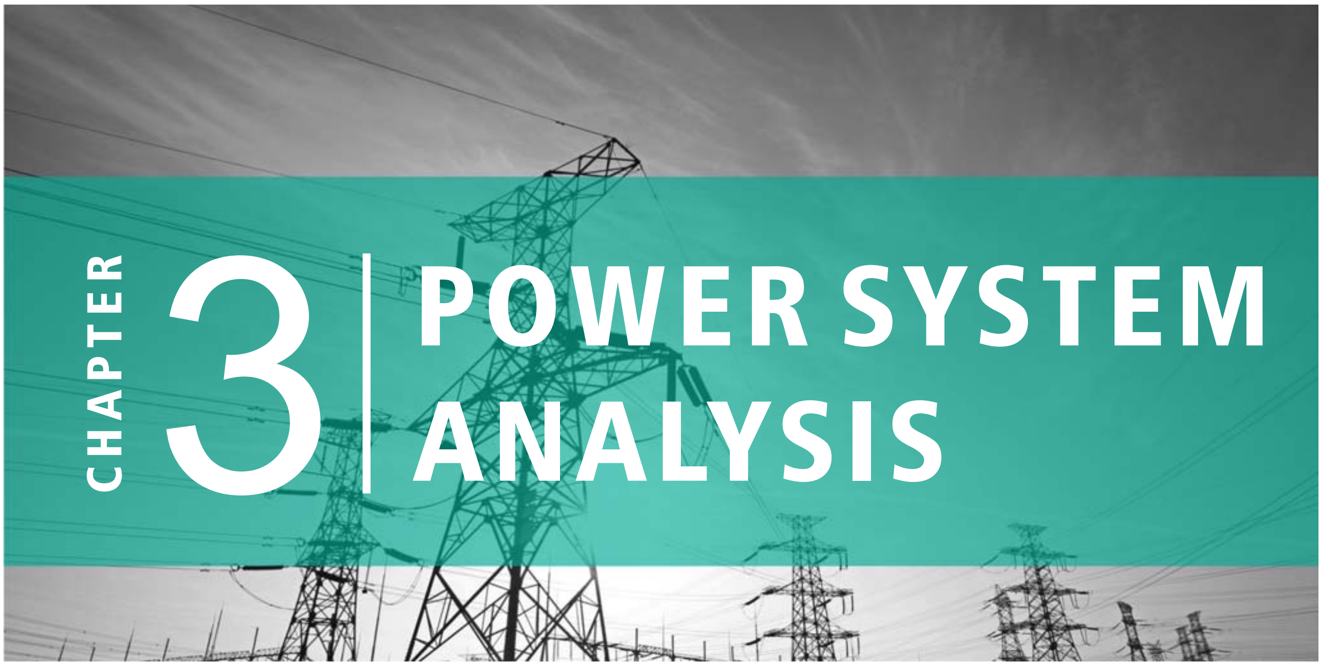


CHAPTER 3 POWER SYSTEM ANALYSIS

Marks Distribution of Power System Analysis in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	4	11	26
2004	6	9	24
2005	4	7	18
2006	4	10	24
2007	4	9	22
2008	3	8	19
2009	2	5	12
2010	3	5	13
2011	5	4	13
2012	3	2	7
2013	1	3	7
2014 Set-1	3	5	13

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-2	2	4	10
2014 Set-3	3	3	9
2015 Set-1	1	4	9
2015 Set-2	2	3	8
2016 Set-1	2	4	10
2016 Set-2	4	3	10
2017 Set-1	3	4	11
2017 Set-2	3	3	9
2018	4	4	12
2019	4	4	12
2020	3	3	9
2021	τ	σ	ρπ

Syllabus : Power System Analysis

Basic concepts of electrical power generation, ac and dc transmission concepts, Models and performance of transmission lines and cables, Economic Load Dispatch (with and without considering transmission losses), Series and shunt compensation, Electric field distribution and insulators, Distribution systems, Per-unit quantities, Bus admittance matrix, Gauss- Seidel and Newton-Raphson load flow methods, Voltage and Frequency control, Power factor correction, Symmetrical components, Symmetrical and unsymmetrical fault analysis, Principles of over-current, differential, directional and distance protection; Circuit breakers, System stability concepts, Equal area criterion.

Contents : Power System Analysis

2 Performance of Transmission Lines

➢ Partial Synopsis

Voltage regulation :

$$VR = \frac{|V_R|_{NL} - |V_R|_{FL}}{|V_R|_{FL}}$$

$$V_S = AV_R + BI_R$$

At no load $I_R = 0$

$$V_{R_{NL}} = \frac{V_S}{A}$$

So, $$VR = \frac{\left|\frac{V_S}{A}\right| - |V_R|}{|V_R|} \times 100$$

For short TL, $|A| = 1$

$$VR = \frac{|V_S| - |V_R|}{|V_R|} \times 100$$

- Voltage regulation when load and power factor is given,

  Voltage regulation, $VR = \frac{|V_S| - |V_R|}{|V_R|} = \frac{I_R[R\cos\phi + X\sin\phi]}{|V_R|}$ (For lagging power factor)

  Voltage regulation, $VR = \frac{I_R(R\sin\phi - X\sin\phi)}{|V_R|}$ (For leading power factor)

- When line is assumed to be resistance free i.e. $R = 0$,

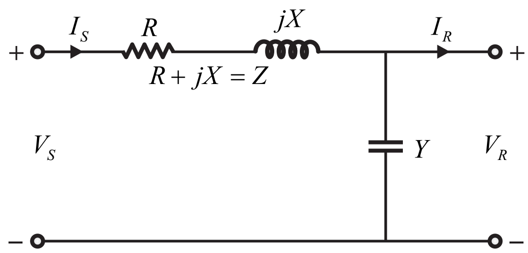
  $V_R = 0$ for unity power factor

  $V_R \neq 0$ for other than unity power factor

  That is why its become necessary to improve the power factor to reduce voltage regulation.

Medium transmission line :

End condenser model,

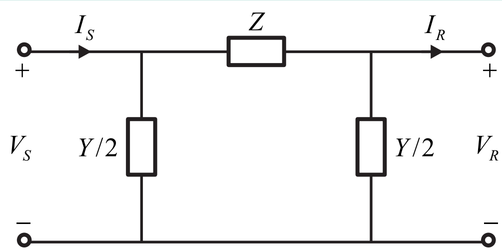


$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1+YZ & Z \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Equating medium transmission line parameters with A B C D parameters

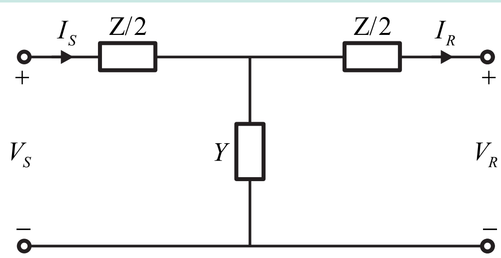
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+YZ & Z \\ Y & 1 \end{bmatrix}$$

π- Model :



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+\dfrac{YZ}{2} & Z \\ Y\left[1+\dfrac{YZ}{4}\right] & 1+\dfrac{YZ}{2} \end{bmatrix}$$

T-Model :



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+\dfrac{ZY}{2} & Z\left(1+\dfrac{YZ}{4}\right) \\ Y & 1+\dfrac{YZ}{2} \end{bmatrix}$$

- Out of T and π model, π model is preferred as there are only two nodes.
- In all transmission lines $AD-BC=1$ i.e. if follows reciprocity theorem.

- *ABCD* parameters for two cascaded TL is given by :

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \times \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

- *ABCD* parameters for two parallel TL :

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{A_1B_2 + A_2B_1}{B_1 + B_2} & \dfrac{B_1B_2}{B_1 + B_2} \\ C_1 + C_2 + \dfrac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2} & \dfrac{D_1B_2 + D_2B_1}{B_1 + B_2} \end{bmatrix}$$

➢ Sample Questions

2016　IISc Bangalore

2.1 At no load condition, a 3-phase, 50 Hz, lossless power transmission line has sending-end and receiving-end voltages of 400 kV and 420 kV respectively. Assuming the velocity of traveling wave to be the velocity of light, the length of the line, in km, is ______.

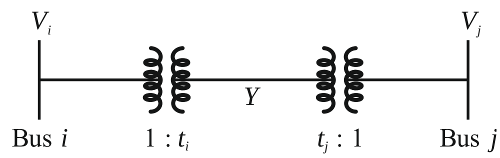
[Set - 02]

2019　IIT Madras

2.2 A three-phase 50 Hz, 400 kV transmission line is 300 km long. The line inductance is 1 mH/km per phase and the capacitance is 0.01 μF/km per phase. The line is under open circuit condition at the receiving end and energized with 400 kV at the sending end. The receiving end line voltage in kV (round off to two decimal places) will be _______.

2020　IIT Delhi

2.3 Two buses, *i* and *j*, are connected with a transmission line of admittance *Y*, at the two ends of which there are ideal transformers with turns ratio as shown below. Bus admittance matrix for the system is



(A) $\begin{bmatrix} t_it_jY & -t_j^2Y \\ -t_i^2Y & t_it_jY \end{bmatrix}$

(B) $\begin{bmatrix} -t_it_jY & t_j^2Y \\ t_i^2Y & -t_it_jY \end{bmatrix}$

(C) $\begin{bmatrix} t_i^2Y & -t_it_jY \\ -t_it_jY & t_j^2Y \end{bmatrix}$

(D) $\begin{bmatrix} t_it_jY & -(t_i - t_j)^2Y \\ -(t_i - t_j)^2Y & t_it_jY \end{bmatrix}$

❖❖❖❖

Explanations | **Performance of Transmission Lines**

2.1 | **295.88**

Given :

(i) 3-ϕ lossless transmission line.

(ii) Supply frequency, $f = 50$ Hz at no load,

(iii) Sending end voltage, $V_s = 400\text{kV}$

(iv) Receiving end voltage, $V_R = 420\,\text{kV}$

(v) Velocity of traveling wave,

$v_p = 3\times10^8$ m/sec

To find : Length of line.

Method 1

Assuming that the transmission line is long.

So, for a long lossless transmission line,

$$V_S = V_R\cos(\beta l) + jZ_0 I_R \sin(\beta l) \quad \ldots\text{(i)}$$

At no load condition, $I_R = 0$

So, from equation (i),

$$V_S = V_R\cos(\beta l)$$

$$\cos(\beta l) = \frac{V_S}{V_R} \quad \ldots\text{(ii)}$$

where, $\beta = \frac{\omega}{v_p} = 2\pi f\sqrt{LC}$

and $\omega = 2\pi f$ = Angular frequency.

From equation (ii),

$$\cos(\beta l) = \frac{V_S}{V_R} = \frac{400}{420} = 0.9524$$

$$\beta l = 17.749^0 = 17.749^0\times\frac{\pi}{180^0}$$

$$\beta l = 0.3098\,\text{rad}$$

Length of line, $l = \frac{0.3098}{\beta}$

Since, $\beta = \frac{\omega}{v_p}$

Thus, $l = 0.3098\times\frac{v_p}{\omega}$

$$l = 0.3098\times\frac{3\times10^8}{2\pi\times50} = 295837.2\,\text{m}$$

Hence, the length of line is **295.83 km**.

Method 2

During no load, $V_s = AV_{R(NL)} + BI_{R(NL)}$

$$\left[I_{R(NL)} = 0\right]$$

$$V_s = AV_{R(NL)} \Rightarrow A = V_s / V_{R(NL)}$$

$$A = \frac{400/\sqrt{3}}{420/\sqrt{3}} = \frac{400}{420} = \frac{20}{21}$$

A parameter of long lossless line is,

$$A = 1 + \frac{Z_T Y_T}{2}$$

where, $Z_T = j\omega L_T = j\omega L\cdot l$ in Ω

$Y_T = j\omega C_T = j\omega C\cdot l$ in Ω^{-1}

Therefore,

$$A = 1 + \frac{(j\omega L\cdot l)(j\omega C\cdot l)}{2} = 1 - \frac{\omega^2 l^2 LC}{2}$$

$$A = 1 - \frac{\omega^2 l^2}{2v_p^2} \qquad \left[v_p = \frac{1}{\sqrt{LC}}\right]$$

where, $v_p = 3\times10^8\,\text{m/sec} = 3\times10^5$ km/sec

$l \rightarrow$ Length of line in km

$\omega = 2\pi f = 2\pi\times50 = 314.2$ rad/sec

Therefore, $A = 1 - \frac{\omega^2 l^2}{2v_p^2}$

$$\frac{1}{2}\left(\frac{\omega l}{v_p}\right)^2 = 1 - A$$

Or $\quad V_S = V_R\cos\beta l$

$$\cos^{-1}\left(\frac{V_S}{V_R}\right) = \beta l$$

$$\beta = \frac{2\pi f}{v}$$

$$l = \frac{1}{\beta}\cos^{-1}\left(\frac{V_S}{V_R}\right)$$

$$\left[\frac{314.2\times l}{3\times 10^5}\right]^2 = 2\left(1-\frac{20}{21}\right)$$

$$1.047\times 10^{-3} l = \sqrt{\frac{2}{21}}$$

$$l = 294.66 \text{ km}$$

Hence, the length of line is **294.66 km**.

2.2 418.59

Given :

(i) Sending end voltage, $V_s = 400$ kV

(ii) Line inductance, $L = 1$ mH/km/ph

(iii) Line capacitance, $C = 0.01$ μF/km/ph

(iv) Supply frequency, $f = 50$ Hz

(v) Transmission line length, $l = 300$ km

By using exact method,

$$\beta = \omega\sqrt{LC} = 100\pi\sqrt{10^{-3}\times 10^{-8}}$$

$$\beta = 9.9345\times 10^{-4} \text{ rad/km}$$

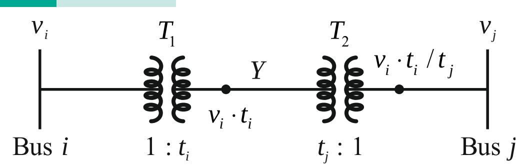
$$A = \cos\beta l = \cos(9.9345\times 10^{-4}\times 300)$$

$$A = \cos(0.2980 \text{ rad}) = 0.9559$$

$$V_R = \frac{V_s}{A} = \frac{400}{0.9559} = 418.59 \text{ kV}$$

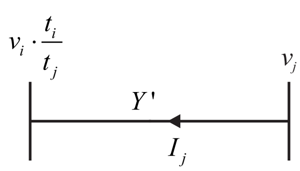
Hence, the receiving end line voltage is **418.59 kV.**

2.3 (C)



Admittance refered to primary side of $T_{1t} = Y.t_i^2$

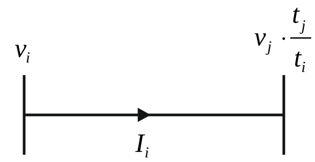
Admittance refered to secondary side of $T_2 = Y.t_j^2$



$$I_j = \left(V_j - v_i\frac{t_i}{t_j}\right)Y'$$

$$I_j = \left[V_j - V_i.\frac{v_i}{t_j}\right]Yt_j^2 = v_j.Y.t_j^2 - v_i.t_i t_j Y$$

$$I_j = -t_i t_j Y v_i + Y t_j^2 v_j$$



$$I_i = \left(v_i - v_j\frac{t_j}{t_i}\right)Yt_i^2$$

$$I_i = Yt_i^2 v_i - v_j Y t_i t_j$$

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} t_i^2 Y & -t_i t_j Y \\ -t_i t_j Y & t_j^2 Y \end{bmatrix}\begin{bmatrix} v_i \\ v_j \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} t_i^2 Y & -t_i t_j Y \\ -t_i t_j Y & t_j^2 Y \end{bmatrix}$$

Hence, the correct option is (C).

❖❖❖❖

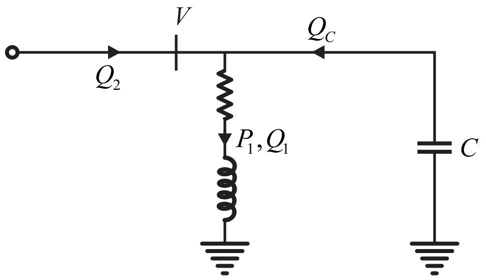
3 Voltage Control

➤ Partial Synopsis

- The voltage is controlled by controlling the net reactive power flow of the system. As voltage is directly related to reactive power.

$$Q = \frac{V^2}{X}$$

Example :

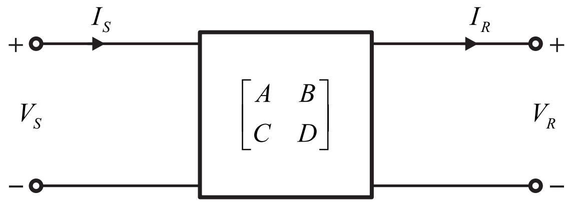


If reactive power demand of the load Q_1 is greater than supplied reactive power Q_2, the voltage at bus 1 will decrease. To increase the voltage a lagging reactive power generator i.e. a capacitor is applied to supply total reactive power demand of the load.

$$Q_1 = Q_2 + Q_C$$

- Capacitor supplies lagging reactive power and absorbs leading reactive power.
- Inductor supplies leading reactive power and absorb lagging reactive power.
- If nature of reactive power is not mentioned, assume lagging reactive power always.

Power flow of transmission line :



Receiving end active and reactive power,

$$P_R = \frac{|V_S||V_R|}{|B|}\cos(\beta-\delta) - \frac{|A||V_R|^2}{|B|}\cos(\beta-\alpha)$$

$$Q_R = \frac{|V_S||V_R|}{|B|}\sin(\beta-\delta) - \frac{|A||V_R|^2}{|B|}\sin(\beta-\alpha)$$

- δ is called power angle and is variable.
- Locus of active and reactive power is a circle of radius $\frac{|V_S||V_R|}{|B|}$.
- In all the problem based on voltage control and power, one information will be given, calculate δ from the given parameters and put into the required formula.

➢ Sample Questions

2003 IIT Madras

3.1 The ABCD parameters of a 3-phase overhead transmission line are,

$A = D = 0.9\angle 0^0$, $B = 200\angle 90^0\ \Omega$

and $C = 0.95\times 10^{-3}\angle 90^0$ S.

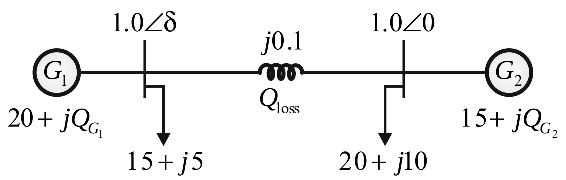
At no-load condition a shunt inductive reactor is connected at the receiving end of the line to limit the receiving-end voltage to be equal to the sending-end voltage. The ohmic value of the reactor is

(A) $\infty\ \Omega$ (B) $2000\ \Omega$

(C) $105.26\ \Omega$ (D) $1052.6\ \Omega$

2018 IIT Guwahati

3.2 Consider the two bus power system network with given loads as shown in the figure. All the values shown in the figure are in per unit. The reactive power supplied by generator G_1 and G_2 are Q_{G_1} and Q_{G_2} respectively. The per unit values of Q_{G_1}, Q_{G_2} and line reactive power loss (Q_{loss}) respectively are



(A) 5.00, 12.68, 2.68

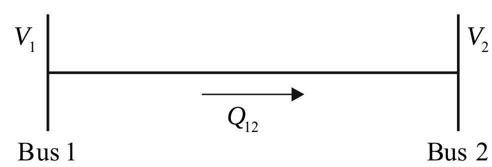
(B) 6.34, 10.00, 1.34

(C) 6.34, 11.34, 2.68

(D) 5.00, 11.34, 1.34

2020 IIT Delhi

3.3 Bus 1 with voltage magnitude $V_1 = 1.1$ pu is sending reactive power Q_{12} towards bus 2 with voltage magnitude $V_2 = 1$ pu through a lossless transmission line of reactance X. Keeping the voltage at bus 2 fixed at 1 pu, magnitude of voltage at bus 1 is changed, so that the reactive power Q_{12} sent from bus 1 is increased by 20%. Real power flow through the line under both the condition is zero. The new value of the voltage magnitude, V_1, in pu (rounded off to 2 decimal places), at bus 1 is __________.



❖❖❖❖

Explanations **Voltage Control**

3.1 **(B)**

Given :

(i) 3-phase overhead transmission line

(ii) ABCD parameters :

$$A = D = 0.9\angle 0^0$$

$$B = 200\angle 90^0\ \Omega$$

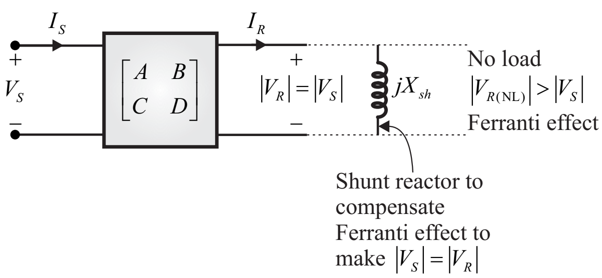
$$C = 0.95\times 10^{-3}\angle 90^0\ \text{S}$$

To find : Ohmic value of the reactor.

Method 1

In case of transmission line,

$$V_s = AV_R + BI_R$$



From figure, $V_R = jX_{sh}.I_R \Rightarrow I_R = \dfrac{V_R}{jX_{sh}}$

If $\overline{V}_R = |V_R|\angle 0^0$ then $\overline{I}_R = \left|\dfrac{V_R}{X_{sh}}\right|\angle -90^0$

Since, $\overline{V}_s = A\overline{V}_R + B\overline{I}_R$

$$|V_s|\angle\delta = 0.9\angle 0^0 \times |V_R|\angle 0^0$$

$$+200\angle 90^0 \times \left|\frac{V_R}{X_{sh}}\right|\angle -90^0$$

$$|V_s|\angle\delta = 0.9|V_R| + 200\left|\frac{V_R}{X_{sh}}\right|$$

Given : $|V_s| = |V_R|$ and right side term is real so $\delta = 0^0$.

$$|V_R| = 0.9|V_R| + 200\left|\frac{V_R}{X_{sh}}\right|$$

$$1 = 0.9 + \frac{200}{X_{sh}}$$

$$X_{sh} = 2000\,\Omega$$

Hence, the correct option is (B).

Method 2

Apparent power at receiving is given by,

$$S_R = P_R + jQ_R$$

During no load, $P_R = 0$

Active power at receiving end is given by,

$$P_R = \frac{V_S V_R}{B}\cos(\beta - \delta) - \frac{AV_R^2}{B}\cos(\beta - \alpha) = 0$$

$$P_R = \frac{V_S V_R}{B}\cos(90^0 - \delta) - \frac{AV_R^2}{B}\cos(90^0 - 0^0) = 0$$

$$\frac{V_S V_R}{B}\sin\delta = 0$$

$$\delta = 0^0$$

Reactive power at receiving end is given by,

$$Q_R = \frac{V_S V_R}{B}\sin(\beta - \delta) - \frac{AV_R^2}{B}\sin(\beta - \alpha)$$

$$Q_R = \frac{V_S V_R}{B}\sin(90^0 - 0^0) - \frac{AV_R^2}{B}\sin(90^0 - 0^0)$$

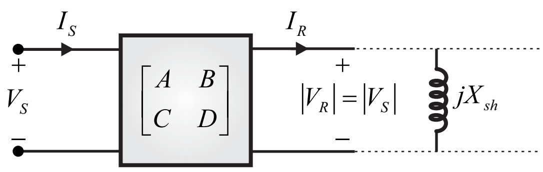
$$Q_R = \frac{V_R^2}{200}\sin 90^0 - \frac{0.9\times V_R^2}{200}\sin 90^0$$

$$Q_R = \frac{V_R^2}{200} - \frac{0.9V_R^2}{200} = \frac{V_R^2}{200}(1 - 0.9)$$

$$Q_R = \frac{V_R^2}{2000} \qquad \ldots(i)$$

Receiving end voltage is greater than sending end voltage due to Ferranti effect caused by capacitance of transmission line.

To compensate for this, shunt inductor is connected at receiving end as shown below,



Reactive power absorbed by inductor is given by,

$$Q_L = \frac{V_R^2}{X_{sh}} = Q_R$$

From equation (i),

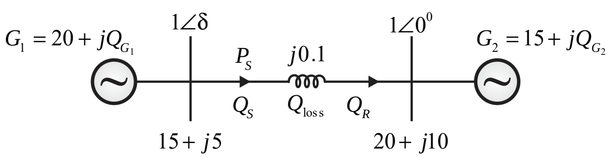
$$\frac{V_R^2}{X_{sh}} = \frac{V_R^2}{2000}$$

$$X_{sh} = 2000\,\Omega$$

Hence, the correct option is (B).

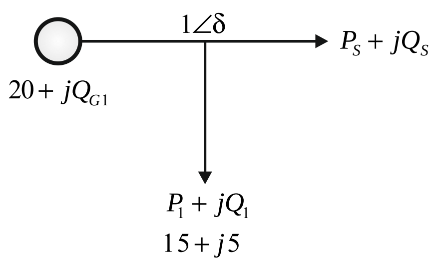
3.2 (C)

Given :



Method 1

At Generator 1 (G_1) :



Active power balance at input side is,

$$P_S = 20 - 15 = 5 = \frac{V_S V_R}{X_L}\sin\delta$$

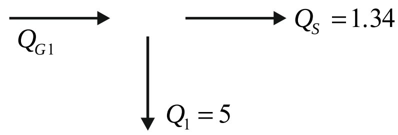
$$5 = \frac{1 \times 1}{0.2}\sin\delta \quad \Rightarrow \quad \delta = 30^0$$

Sending end reactive power is given by,

$$Q_S = \frac{V_S^2}{X} - \frac{V_S V_R}{X}\cos\delta$$

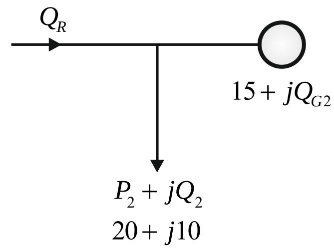
$$Q_S = \frac{1^2}{0.2} - \frac{1 \times 1}{0.2}\cos 30^0 = 1.34 \text{ pu}$$

So, reactive power balance at input side is,



$$Q_{G1} = 5 + 1.34 = 6.34 \text{ pu}$$

At Generator 2 (G_2) :

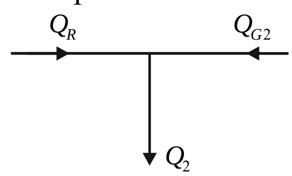


Receiving end reactive power is given by,

$$Q_R = \frac{V_R V_S}{X}\cos\delta - \frac{V_R^2}{X}$$

$$Q_R = \frac{1 \times 1}{0.1}\cos 30^0 - \frac{1^2}{0.1} = -1.34$$

So, from reactive power balance at output side,



$$Q_{G2} + Q_R = Q_2$$

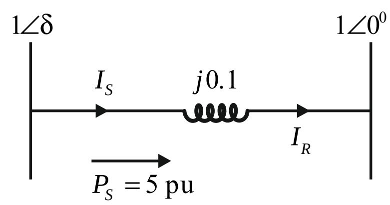
$$Q_{G2} = Q_2 - Q_R = 10 - (-1.34)$$

$$Q_{G2} = 11.34 \text{ pu}$$

Hence, the correct option is (C).

Note : Formula of Q_R is designed for towards receiving end but we are getting negative values so to make it positive we revered the arrow to calculate Q_{G2}.

Method 2



Active power flow in the line is given by,

$$P_S = P_R = \frac{V_R V_S}{X}\sin\delta$$

$$5 = \frac{1\times 1}{0.1}\sin\delta \quad \Rightarrow \quad \delta = 30^0$$

So, current flowing through transmission line will be,

$$I_S = \frac{V_S - V_R}{X_S} = \frac{1\angle 30^0 - 1\angle 0^0}{0.1\angle 90^0}$$

$$I_S = 5.1763\angle 15^0 \text{ pu}$$

Sending end apparent power $= V_S I_S^*$

$$= 1\angle 30^0\left[5.1763\angle -15^0\right] = 5.1763\angle 15^0$$

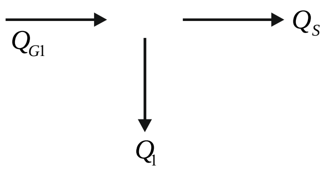
$$P_S = 5 \text{ pu}, \; Q_S = 1.34 \text{ pu}$$

Receiving end apparent power $= V_R I_R^*$

$$= 1\angle 0^0\left[5.1763\angle -15^0\right] = 5.1763\angle -15^0$$

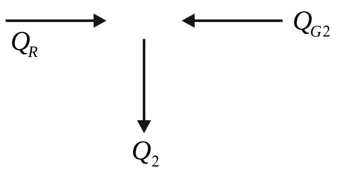
$$P_R = 5 \text{ pu}, \; Q_R = -1.34 \text{ pu}$$

(i)



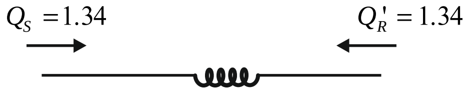
$$Q_{G1} = 5 + 1.334 = 6.334$$

(ii)



$$Q_{G2} = 10 + 1.34 = 11.34$$

(iii)



$$Q_{\text{Loss}} = Q_{\text{absorbed}}$$

$$Q_S + Q_R = 1.34 + 1.34 = 2.68 \text{ pu}$$

Hence, the correct option is (C).

3.3 1.12

Given :

(i) $V_1 = 1.1$ pu

(ii) $V_2 = 1$ pu

Reactive power, $Q_{12} = \frac{V_1 V_2}{X}\cos(\delta_1 - \delta_2) - \frac{V_2^2}{X}$

Since active power flow is zero hence, $\delta_1 = \delta_2$

$$Q_{12} = \frac{V_1 V_2}{X} - \frac{V_2^2}{X} = \frac{1.1\times 1}{X} - \frac{1}{X}$$

$$Q_{12} = \frac{0.1}{X}$$

From given condition new reactive is given by,

$$Q_{12}' = 1.2 Q_{12}$$

Let new bus 1 voltage $= V_1'$

$$1.2\times\frac{0.1}{X} = \frac{V_1' \cdot V_2}{X}\cos(\delta_1 - \delta_2) - \frac{V_2^2}{X}$$

$$\frac{0.12}{X} = \frac{V_1'}{X} - \frac{1^2}{X}$$

$$V_1' = 1 + 0.12 = 1.12 \text{ pu}$$

❖❖❖❖

7 Cables & Insulators

➢ Partial Synopsis

Cable :

Cable is a combination of conductors and insulators. Insulations are provided throughout the conductors.

1. Due to less spacing between the conductors, inductance of underground cable is less than overhead line.

   Inductance/phase $L = 2\times10^{-7}\ \text{In}\left(\frac{\text{GMD}}{\text{GMR}}\right)$ H/m $(L_{UG\,cable} < L_{OH\,line})$

   Capacitance/phase $C = \frac{2\pi\varepsilon_0\varepsilon_r}{\text{In}\left(\frac{\text{GMD}}{\text{GMR}}\right)}$ F/m, $(C_{UG\,cable} > C_{OH\,line})$

2. Surge impedance, $Z_S = \sqrt{\frac{L}{C}}\ \Omega$

   $Z_{S(UG)cable} < Z_{C(OH)\text{line}}$

3. The loading in case of UG cable is done on the basis of thermal limit.

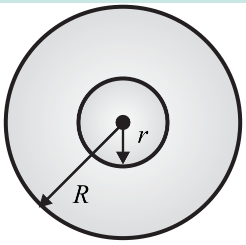
4. $R_{Insulation} = \frac{\rho}{2\pi l}\ln\left(\frac{R}{r}\right)\Omega$

5. $R_{Insulation}$ is very high and $R_{Insulation} \propto \frac{1}{\text{Length}}$

6. $C_{Insulation} = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{R}{r}\right)}$ F/m

7. Charging current $I_C = \frac{V_{ph}}{X_c} = \omega C V_{ph}$ Amp .

Electrostatic stress in a single core cable :



Electric field stress, $g(x)$ at a distance x is given by,

$$g(x) = \frac{V}{x \ln\left(\frac{R}{r}\right)} \text{ V/m}$$

Electric field stress is maximum at the surface of conductor and minimum at the surface of insulation.

Most economical size of conductor :

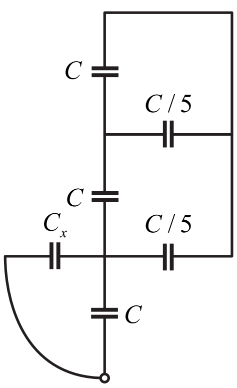
$$r = \frac{R}{2.71}$$

Sample Questions

1999 IIT Bombay

Common Data for Questions 7.1 & 7.2

In a transmission line each conductor is at 20 kV and is supported by a string of 3 suspension insulators. The air capacitance between each cap-pin junction and tower is one-fifth of the capacitance C of each insulation unit. A guard ring, effective only over the line-end insulator unit is fitted so that the voltages on the two unit nearest the line-end are equal.



7.1 The voltage (in kV) at the line end of the unit will be ______.

7.2 If the value of capacitance $C_x = pC$.

then the value of p will be ______.

2021 IIT Bombay

7.3 Two single-core power cables have total conductor resistances of 0.7 Ω and 0.5 Ω respectively and their insulation resistances (between core and sheath) are 600 MΩ and 900 MΩ respectively. When the two cables are joined in series, the ratio of insulation resistance to conductor resistance is ______ $\times 10^6$

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Explanations　Cables & Insulators

7.1　7.059

Given :

(i) Suspension insulator strings

(ii) Number of units, $n = 3$

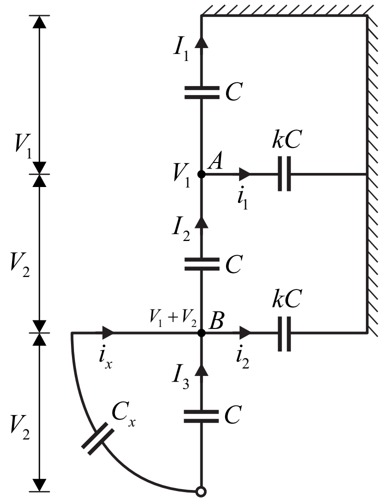
(iii) Self capacitance, $C_S = C$

(iv) Mutual capacitance, $C_M = kC_S = \frac{C}{5}$

i.e. $k = 0.2$

(v) Conductor voltage, $V_r = 20\,\text{kV}$

To find : Voltage (in kV) on the line end.



Applying KCL at node A,

$$I_2 = I_1 + i_1$$

$$V_2\omega C = V_1\omega C + V_1\omega kC$$

$$V_2 = (1+k)V_1 = (1+0.2)V_1$$

$$V_2 = 1.2V_1 \quad \ldots\text{(i)}$$

Conductor voltage, $V_r = V_1 + V_2 + V_2$

$$V_r = V_1 + 2V_2 = V_1 + 2(1+k)V_1$$

$$V_r = (2k+3)V_1 = (3.4)V_1$$

$$V_1 = \frac{20}{3.4} = 5.88\,\text{kV}$$

From equation (i),

$$V_2 = 1.2V_1 = 1.2\times 5.88 = 7.059\text{ kV}$$

Hence, the voltage on the line end is **7.059 kV**.

7.2　0.3665

To find : Value of p.

Applying KCL at node B,

$$i_x + I_3 = I_2 + i_2$$

$$V_2\omega C_x + V_2\omega C = V_2\omega C + (V_1+V_2)\omega kC$$

$$V_2C_x = (V_1+V_2)\times 0.2\times C$$

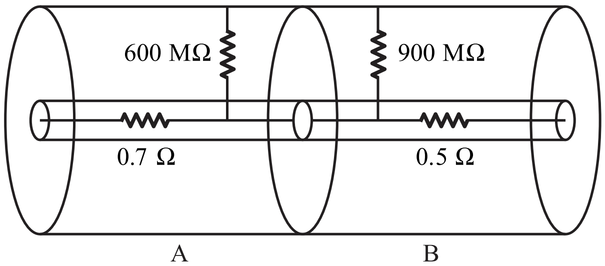
$$C_x = \frac{0.2(5.88+7.059)C}{7.059}$$

$$C_x = 0.3665\,C = pC$$

Hence, the value of p is **0.3665**.

7.3　300

Note : When two cables are connected in series, conductor resistances will be in series and insulation resistances will be in parallel.



$$R_{conductor} = 0.7 + 0.5 = 1.2\,\Omega$$

$$R_{insulation} = \frac{600\times 900}{600+900} = 360\text{ M}\Omega$$

$$\frac{R_{insulation}}{R_{conductor}} = \frac{360\,\text{M}\Omega}{1.2\,\Omega} = 300\times 10^6$$

Hence, the correct answer is 300.

❖❖❖❖

8 Per Unit System

Partial Synopsis

- $\text{pu value} = \dfrac{\text{Actual value in some unit}}{\text{Base value in same unit}}$
- pu value can be less than, greater than or equal to unity.
- $V_{pu} = \dfrac{V}{V_{base}}$

  $I_{pu} = \dfrac{I}{I_{base}}$

  $Z_{pu} = \dfrac{Z}{Z_{base}}$
- To avoid complication generally rating are taken as base values.
- $Z_{base} = \dfrac{(\text{kV})^2}{\text{MVA}} \dfrac{\rightarrow \text{Voltage rating}}{\rightarrow \text{Power rating}}$
- Base value of voltage or current on two sides of transformer are not same, it will steps up or down according to turn ratio, but base MVA on both the sides will remain same.
- pu value of equivalent impedance on both the sides of transformer are same in single phase as well as in three-phase.
- pu value of line voltage is same as phase voltage in balanced star as well as in delta connections.
- pu value of line current is same as phase current in balanced star as well as in delta connections.
- $S = P + jQ$

  $\dfrac{S}{S_b} = \dfrac{P}{S_b} + j\dfrac{Q}{S_b}$ where, S_b is base power in MVA/KVA

  $S_{(pu)} = P_{(pu)} + jQ_{(pu)}$

  To convert active and reactive power in pu we use same base MVA.

- $P_{3\phi} = \sqrt{3}VI\cos\phi$

  $S_b = \sqrt{3}V_b I_b$

  $P_{pu} = \dfrac{\sqrt{3}VI\cos\phi}{\sqrt{3}V_b I_b} = \left(\dfrac{V}{V_b}\right)\left(\dfrac{I}{I_b}\right)\cos\phi$

  $P_{pu} = V_{pu} I_{pu}\cos\phi$ (per unit 3-phase active power).

- Pu impedance when base values are changed

$$Z_{pu\,new} = Z_{pu.old} \times \frac{\text{MVA}_{(new)}}{\text{MVA}_{(old)}} \times \frac{\text{kV}^2_{(old)}}{\text{kV}^2_{(new)}}$$

➢ Sample Questions

2010 IIT Kanpur

8.1 For the power system shown in the figure below, the specifications of the components are the following :

G_1 : 25 kV, 100 MVA, $X_{G_1} = 9\%$

G_2 : 25 kV, 100 MVA, $X_{G_2} = 9\%$

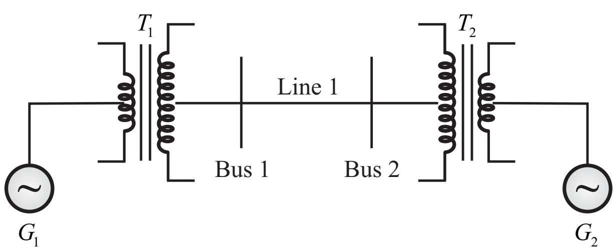
T_1 : 25 kV/ 220 kV, 90 MVA

$X_{T_1} = 12\%$

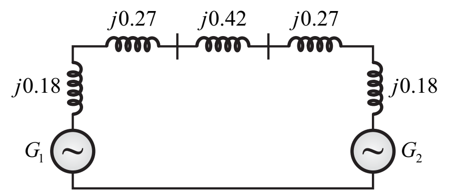
T_2 : 220 kV/ 25 kV, 90 MVA

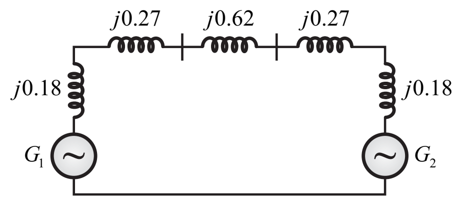
$X_{T_2} = 12\%$

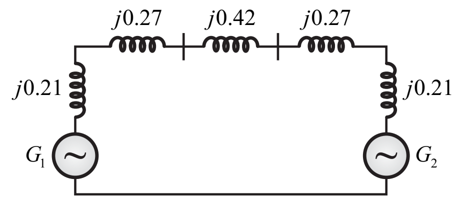
line_1 : 220 kV, $X_l = 150$ ohms

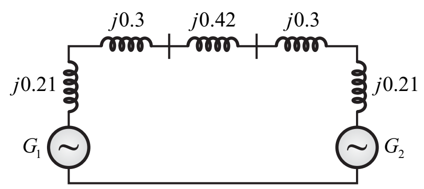


Choose 25 kV as the base voltage at the generator G_1 and 200 MVA as the MVA base. The impedance diagram is

(A)


(B)


(C)


(D)


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Explanations **Per Unit System**

8.1 (B)

Given :

(i) Components specification :

G_1 : 25 kV, 100 MVA, $X_{G_1} = 9\%$

G_2 : 25 kV, 100 MVA, $X_{G_2} = 9\%$

T_1 : 25 kV/ 220 kV, 90 MVA

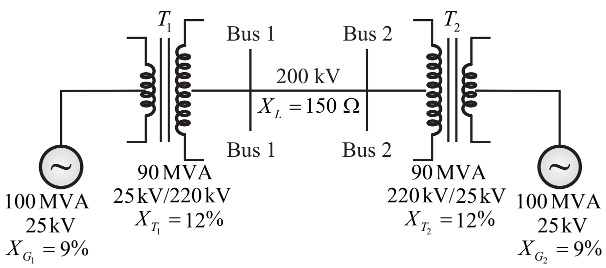
$X_{T_1} = 12\%$

T_2 : 220 kV/ 25 kV, 90 MVA

$X_{T_2} = 12\%$

Line$_1$: 220 kV, $X_l = 150$ ohms

The one line diagram of the given power system network is shown below,

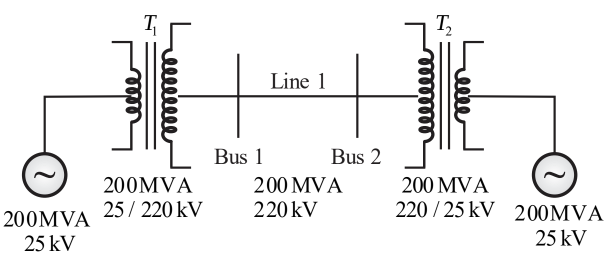


(ii) Base values :

$V_b = 25\,\text{kV}$, $S_b = 200\,\text{MVA}$

To find : Impedance diagram.

The new base value of different components is as shown below,



New per unit impedance is given by,

$$Z_{pu(\text{New})} = Z_{pu(\text{Old})} \times \frac{S_{b(\text{New})}}{S_{b(\text{Old})}} \times \left[\frac{V_{b(\text{Old})}}{V_{b(\text{New})}}\right]^2$$

Reactance of generator G_1 at new base is given by,

$$X_{G_1} = 0.09 \times \frac{200}{100} \times \left(\frac{25}{25}\right)^2 = 0.18 \text{ pu}$$

Reactance of generator G_2 at new base is given by,

$$X_{G_2} = 0.09 \times \frac{200}{100} \times \left(\frac{25}{25}\right)^2 = 0.18 \text{ pu}$$

Reactance of transformer T_1 at new base is given by,

$$X_{T_1} = 0.12 \times \frac{200}{90} \times \left(\frac{25}{25}\right)^2 = 0.27 \text{ pu}$$

Reactance of transformer T_2 at new base is given by,

$$X_{T_2} = 0.12 \times \frac{200}{90} \times \left(\frac{220}{220}\right)^2 = 0.27 \text{ pu}$$

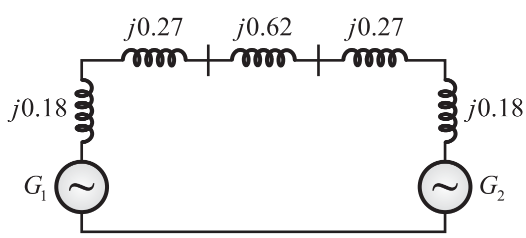
Reactance of transmission line in per unit is given by,

$$X_{l(\text{p.u.})} = X_l \times \frac{S_b}{V_b^2} = 150 \times \frac{200}{(220)^2}$$

[kV in line = 220 kV]

$$X_l = 0.62 \text{ p.u.}$$

The impedance diagram of given power system is given by,



Hence, the correct option is (B).

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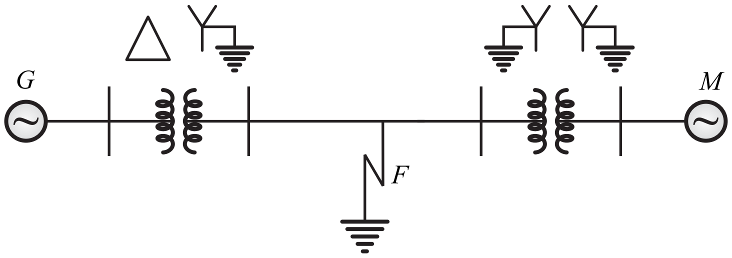
9 Symmetrical Faults

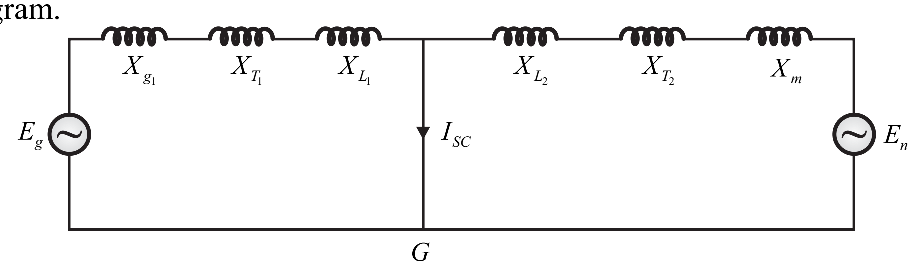
➢ Partial Synopsis



Steps to solve questions based on symmetrical faults.

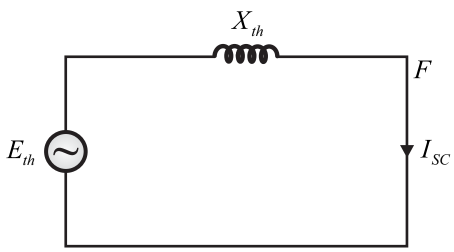
Let a 3-phase symmetrical fault occurs at midpoint of transmission line.

1. To calculate fault current, convert given single line diagram into equivalent pu reactance diagram.



$$X_{L_1} = X_{L_2} = \frac{X_L}{2} \qquad \text{(As fault occurs exactly midpoint of TL)}$$

2. Draw the thevenin equivalent circuit across F and G,



Thevenin's equivalent current (short circuit current) is given by,

$$I_{SC} = \frac{E_{th}}{X_{th}}$$

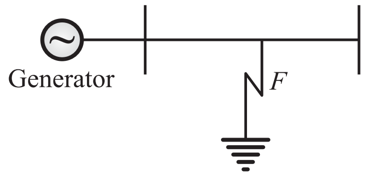
In case of no load, $E_{th} = 1\text{pu}$

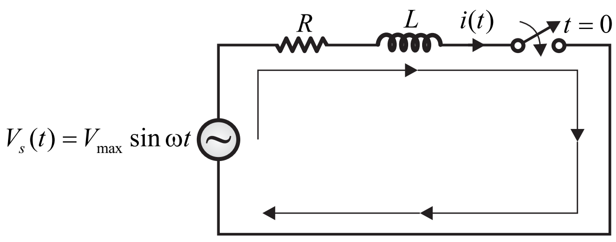
$$I_{SC} = \frac{1}{X_{th}}$$

To calculate I_{SC},

- Short circuit all the voltage sources and calculate X_{th} across the fault point.
- In case of loaded condition calculate E_g, E_m from the initial condition. (in case of loaded condition, $E_{th} \neq 1$).

Short circuit transient in a transmission line :





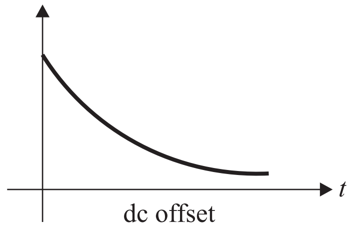
Applying KVL in the above circuit,

$$V_s(t) - Ri(t) + L\frac{di}{dt} = 0$$

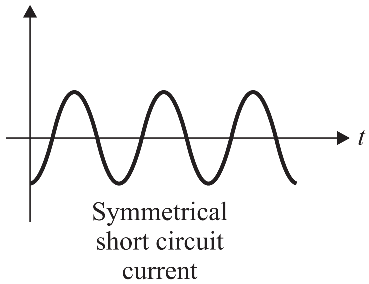
$$i(t) = \frac{V_{max}}{|Z|} e^{-\frac{Rt}{L}} \sin(\theta - \alpha) + \frac{V_{max}}{|Z|} \sin(\omega t + \alpha - \theta)$$

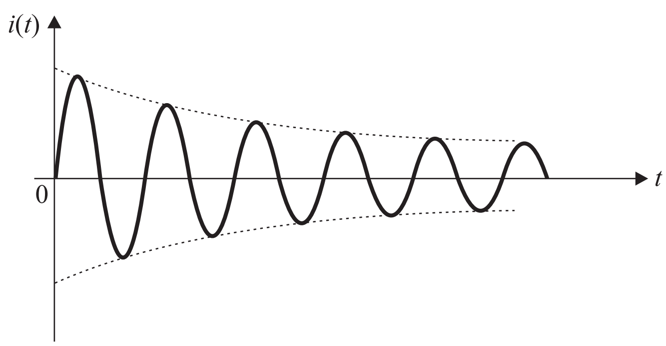
$i(t)$ consist of DC offset current and steady state component circuit.

$$\frac{V_{max}}{|Z|} e^{-\frac{Rt}{L}} \sin(\theta - \alpha)$$



$$\frac{V_{max}}{|Z|} \sin(\omega t + \alpha - \theta)$$



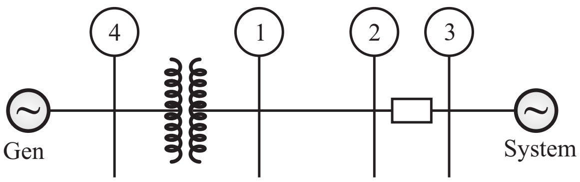


- For maximum value of dc offset current, $\theta - \alpha = 90^0$
- For zero dc offset current, $\theta - \alpha = 0$ in case of sinusoidal excitation.

➢ Sample Questions

2000 IIT Kharagpur

9.1 For the configuration shown in below, the breaker connecting a large system to BUS2 is initially open. The system 3-phase fault level at BUS3 under this condition is not known. After closing the system breaker, the 3-phase fault level at BUS1 was found to be 5.0 pu All per unit values are on common bases, prefault load currents are neglected and prefault voltages are assumed to be 1.0 pu at all buses. What will be new 3-phase fault level (in pu) at systems BUS3 after the interconnection will be _______.

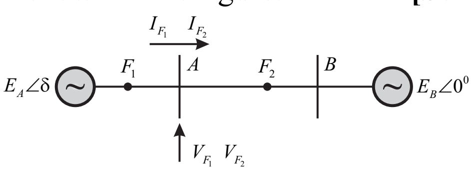


$E_g = 1.0\,\text{pu},\ X_T = 0.2\,\text{pu}$

$X_{Line} = 0.3\,\text{pu},\ X_{dg} = 0.2\,\text{pu}$

2014 IIT Kharagpur

9.2 3-phase to ground fault takes place at locations F_1 and F_2 in the system shown in the figure. **[Set - 01]**



If the fault takes place at location F_1, then the voltage and the current at BUS A are V_{F_1} and I_{F_1} respectively. If the fault takes place at location F_2, then the voltage and the current at BUS A are V_{F_2} and I_{F_2} respectively. The correct statement about voltage and currents during faults at F_1 and F_2 is

(A) V_{F_1} leads I_{F_1} and V_{F_2} leads I_{F_2}.

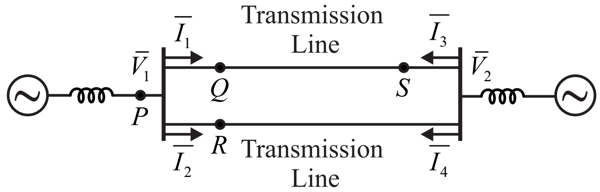
(B) V_{F_1} leads I_{F_1} and V_{F_2} leads I_{F_2}.

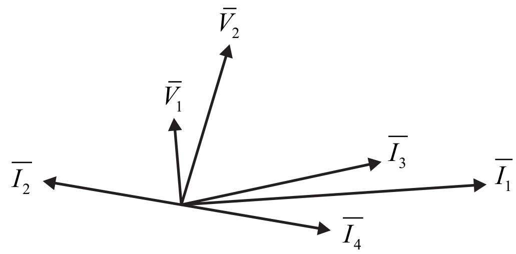
(C) V_{F_1} lags I_{F_1} and V_{F_2} leads I_{F_2}.

(D) V_{F_1} lags I_{F_1} and V_{F_2} lags I_{F_2}.

2015 IIT Kanpur

9.3 A sustained three-phase fault occurs in the power system shown in the figure. The current and voltage phasors during the fault (on a common reference), after the natural transients have died down, are also shown. Where is the fault located? **[Set - 01]**





(A) Location P (B) Location Q

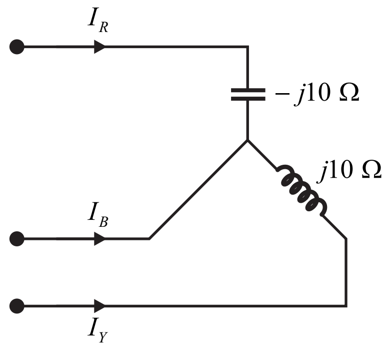
(C) Location R (D) Location S

2020 IIT Delhi

9.4 A cylindrical rotor synchronous generator has steady state synchronous reactance of 0.7 pu and sub transient reactance of 0.2 pu. It is operating at $(1+j0)$ pu terminal voltage with an internal emf of $(1+j0.7)$ pu. Following a 3-ϕ solid short circuit fault at the terminal of the generator, the magnitude of the subtransient internal emf (rounded off to 2 decimal places) is________pu.

2021 IIT Bombay

9.5 A three-phase balanced voltage is applied to the load shown. The phase sequence is RYB. The ratio $\left|\frac{I_B}{I_R}\right|$ is ______.



Explanations **Symmetrical Faults**

9.1 **11.42**

Given : Alternator

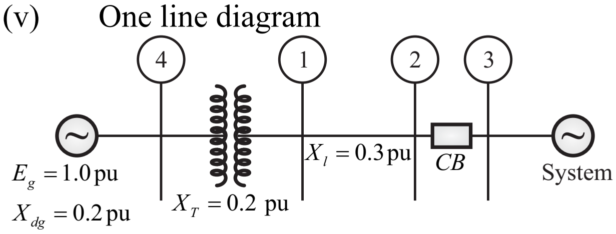
(i) Internal emf, $E_g = 1.0\,\text{pu}$,

(ii) Generator reactance, $X_{dg} = 0.2\,\text{pu}$

(iii) Line : $X_l = X_{Line} = 0.3\,\text{pu}$

(iv) Transformer : $X_T = 0.2\,\text{pu}$

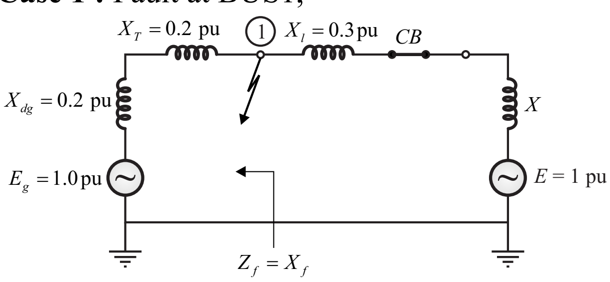
(v) One line diagram



(vi) Fault level at BUS1, $I_f = 5\,\text{pu}$

To find : Fault level at BUS3.

Case 1 : Fault at BUS1,



Fault impedance at BUS1 is given by,

$$Z_f = X_f = (X_{dg} + X_T) \| (X_l + X)$$

$$X_f = (0.2 + 0.2) \| (0.3 + X)$$

$$X_f = 0.4 \| (0.3 + X) = \frac{0.4 \times (0.3 + X)}{0.7 + X}$$

Fault level at BUS1 is given by,

$$I_f = \frac{E_f}{Z_f} = \frac{1}{X_f} = 5\,\text{pu}$$

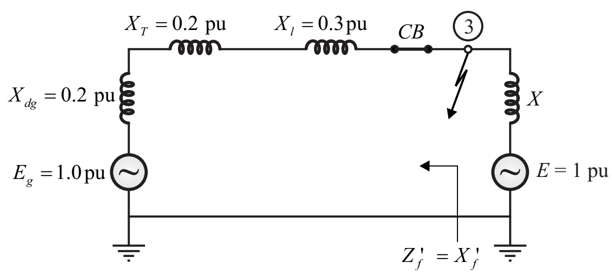
$$X_f = \frac{0.4 \times (0.3 + X)}{0.7 + X} = \frac{1}{5}$$

$$2.0(0.3 + X) = 0.7 + X$$

$$0.6 + 2X = 0.7 + X$$

$$X = 0.1\,\text{pu}$$

Case 2 : Fault at BUS3,



Fault impedance at BUS3 is given by,

$$Z_f' = X_f' = (X_{dg} + X_T + X_l) \| X$$

$$X_f' = (0.2 + 0.2 + 0.3) \| (0.1)$$

$$X_f' = 0.7 \| 0.1$$

$$X_f' = \frac{0.7 \times 0.1}{0.7 + 0.1} = 0.0875 \text{ pu}$$

Fault level at BUS3 is given by,

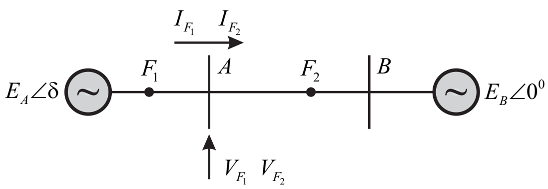
$$I_f = \frac{E_f}{X_f'} = \frac{1}{0.0875} = 11.4285 \text{ pu}$$

Hence, the new 3-phase fault level at systems BUS3 after the interconnection is **11.42 pu**.

9.2 (C)

Given :

(i) 3-phase to ground fault takes place at locations F_1 and F_2 in the system as,

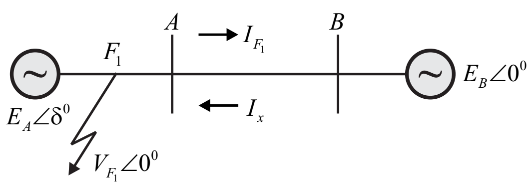


(ii) When fault takes place at location F_1, then the voltage and the current at BUS A are V_{F_1} and I_{F_1} respectively.

(iii) When fault takes place at location F_2, then the voltage and the current at BUS B are V_{F_2} and I_{F_2} respectively.

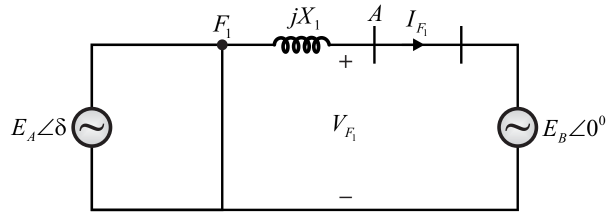
Case 1 :

When fault takes place at location F_1, then the equivalent circuit can be represented as,



For fault at F_1 :

Equivalent diagram is given by,



Assume reactance between F_1 and A is jX_1.

Applying KVL, $\vec{V}_{F_1} = -jX_1\vec{I}_{F_1}$

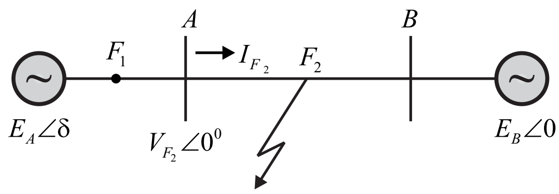
If $\vec{I}_{F_1}$ is $I_{F_1}\angle 0^0$, then

$$\vec{V}_{F_1} = XI_{F_1}\angle -90^0 = V_{F_1}\angle -90^0$$

Therefore, $\vec{V}_{F_1}$ lags $\vec{I}_{F_1}$ by 90^0.

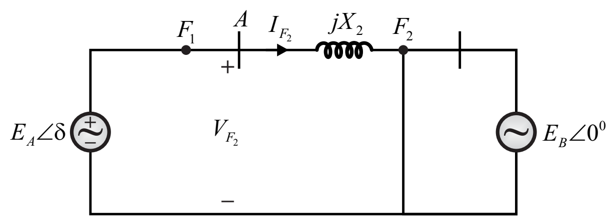
Case 2 :

When fault takes place at location F_2, then the equivalent circuit can be represented as,



For a fault at F_2 :

Equivalent diagram is given by,



Assume reactance between F_2 and A is jX_2.

Applying KVL, $\vec{V}_{F_2} = jX_2\vec{I}_{F_2}$

If $\vec{I}_{F_2}$ is $I_{F_2}\angle 0^0$, then

$$\vec{V}_{F_2} = XI_{F_2}\angle 90^0 = V_{F_2}\angle 90^0$$

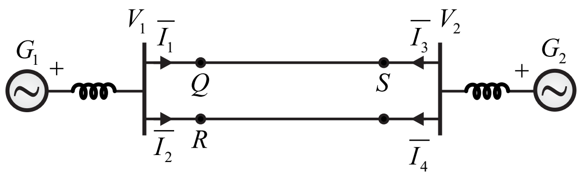
Therefore, $\vec{V}_{F_2}$ leads $\vec{I}_{F_2}$ by 90^0.

Hence, the correct option is (C).

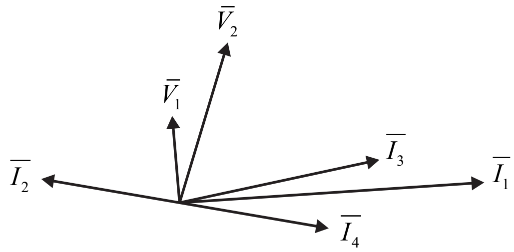
9.3 **(B)**

Given :

(i) A sustained three-phase fault occurs in the power system shown in the figure,



(ii) For a 3-ϕ fault, the voltage and current phasor are given as,



Fault current must always lag voltage due to inductive reactance of the system and if any current leads the voltage, it is only possible if direction of current is assumed opposite to actual direction.

According to the phasor diagram, $\overline{I}_2$ and & $\overline{I}_4$ are in the opposite direction $(\overline{I}_2 = -\overline{I}_4)$ as consistent with circuit. So, there can be no fault between $\overline{I}_2$ & $\overline{I}_4$.

But, $\overline{I}_1$ & $\overline{I}_3$ are not equal & opposite. So, fault lies between $\overline{I}_1$ & $\overline{I}_3$ but since $\overline{I}_1$ is more than $\overline{I}_3$, fault is closer to $\overline{V}_1$ than $\overline{V}_2$. Thus fault is located at Q.

Hence, the correct option is (B).

9.4 **1.02**

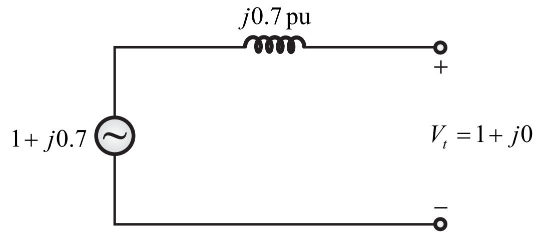
Given : Cylindrical rotor synchronous generator

(i) Internal emf, $E_g = 1 + j0.7$ pu

(ii) Synchronous reactance, $X_d = j0.7$ pu

(iii) Sub transient reactance, $X_d^{''} = j0.2$ pu

Case-1 : At pre-fault condition,

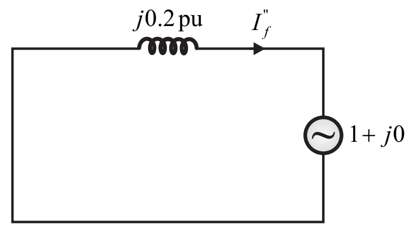


Generator current,

$$I_g = \frac{E_g - V_t}{X_d}$$

$$I_g = \frac{1 + j0.7 - 1}{j0.7} = 1\angle 0^0 \text{ pu} = 1 \text{ pu}$$

Case-2 : At post-fault condition,



Fault current,

$$I_f^{''} = \frac{1\angle 0^0}{0.2\angle 90^0} = 5\angle -90^0$$

Total generator current,

$$(I_g)_{total} = I_f + I_f^{''} = 1\angle 0^0 + 5\angle -90^0$$

$$(I_g)_{total} = 1 - j5$$

Sub transient emf $= (I_g)_{total} \times X_d^{''}$

$$E_f^{''} = (1 - j5) \times j0.2$$

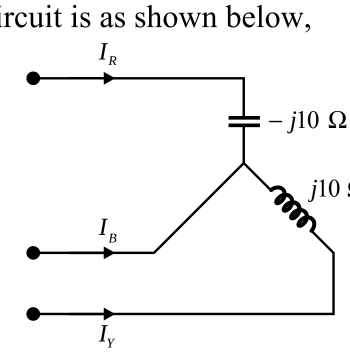
$$E_f^{''} = 1 + j0.2$$

Magnitude, $|E_f^{''}| = 1.0198 \text{ pu} \approx 1.02$ pu

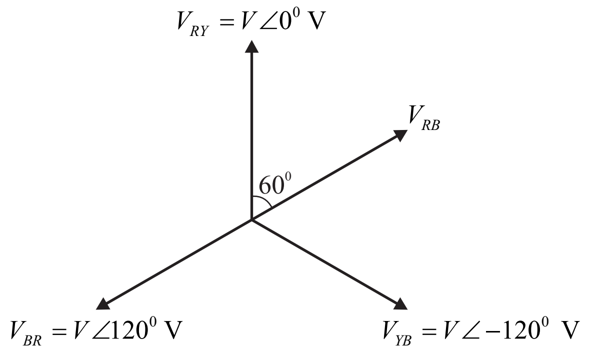
Hence, the magnitude of the sub-transient internal emf is **1.02 pu**.

9.5 **1**

Given circuit is as shown below,



Voltage across capacitor is V_{RB} and voltage across inductor is V_{YB} (where, V_{RB}, V_{YB} are line voltages)



$$V_{RY} = V\angle 0^0 \text{V}$$

$$V_{YB} = V\angle -120^0 \text{V}$$

$$V_{BR} = V\angle 120^0 \text{V}$$

$$I_R = \frac{V_{BR}}{-j10\Omega} = \frac{V\angle -60^0}{-j10} = \frac{V}{10}\angle 30^0 \text{A}$$

$$I_Y = \frac{V_{YB}}{j10} = \frac{V\angle -120^0}{j10} = \frac{V}{10}\angle -210^0 \text{A}$$

From the above circuit the value of I_B is given by,

$$I_B = -(I_R + I_y)$$

$$I_B = -\left[\frac{V}{10}\angle 30^0 + \frac{V}{10}\angle -210^0\right] \text{A}$$

$$I_B = -\frac{V}{10}\angle 90^0 \text{A}$$

$$|I_R| = \frac{V}{10}$$

$$|I_B| = \frac{V}{10}$$

$$\left|\frac{I_B}{I_Y}\right| = 1$$

Hence, the correct answer is 1.

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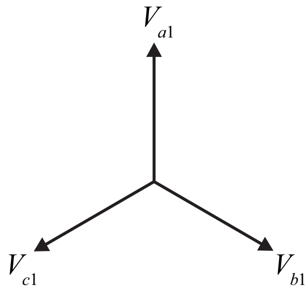
10 Symmetrical Components

➢ Partial Synopsis

1. ***a* operator :** The operator a is one, which when multiplied to a vector rotates the vector through 120^0 in the anticlockwise direction without changing the magnitude of the phasor upon which it operates.

$$a = 1 \cdot e^{j2\pi/3} \qquad a = 1\angle 120^0 \qquad a^2 = 1\angle 240^0 \qquad a^3 = 1\angle 360^0 = 1$$

2. **Positive sequence components :** It consists of three phasors equal in magnitude displaced from each other by 120^0 in phase and having the same phase sequence as the original phasors. It is denoted by suffix 1. The sequence at which the system was working before fault is the original sequence.

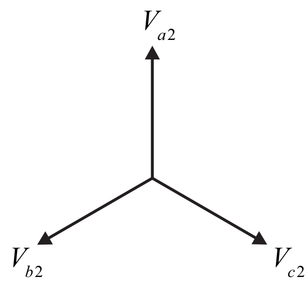


$$|V_{a1}| = |V_{b1}| = |V_{c1}|$$

$$V_{a1} + V_{b1} + V_{c1} = 0 \qquad \text{(Balanced)}$$

CW direction	$V_{a1} = V_{a1}\angle 0^0 = V_{a1}$	$V_{b1} = V_{a1}\angle -120^0$	$V_{c1} = V_{a1}\angle -240^0$
ACW direction	$V_{a1} = V_{a1}\angle 0^0 = V_{a1}$	$V_{b1} = V_{a1}\angle +240^0 = a^2 V_{a1}$	$V_{c1} = V_{a1}\angle +120^0 = aV_{a1}$

3. **Negative sequence components :** It consists of three phasors equal in magnitude displaced from each other by 120^0 in phase and having the phase sequence opposite to that of the original phasors. It is denoted by suffix 2.



$$|V_{a2}| = |V_{b2}| = |V_{c2}|$$

$$V_{a2} + V_{b2} + V_{c2} = 0 \qquad \text{(Balanced)}$$

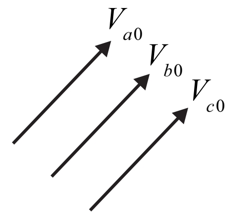
CW direction	$V_{a2} = V_{a2}\angle 0^0 = V_{a2}$	$V_{b2} = V_{a2}\angle -240^0$	$V_{c2} = V_{a2}\angle -120^0$
ACW direction	$V_{a2} = V_{a2}\angle 0^0 = V_{a2}$	$V_{b2} = V_{a2}\angle +120^0 = aV_{a2}$	$V_{c2} = V_{a2}\angle +240^0 = a^2 V_{a2}$

4. **Zero sequence components :** It consists of three phasors equal in magnitude and with zero phase displacement from each other. It is denoted by suffix 0.

$$|V_{a0}| = |V_{b0}| = |V_{c0}|$$

$$V_{a0} + V_{b0} + V_{c0} = 3V_{a0} \quad \text{(Unbalanced)}$$

$$V_{a0}\angle 0^0 \qquad V_{b0}\angle 0^0 \qquad V_{c0}\angle 0^0$$



5. Relationship between symmetrical component and unsymmetrical phasor :

$$[V_{abc}] = [A][V_a^{012}] \qquad [I_{abc}] = [A][I_a^{012}]$$

$$[V_a^{012}] = [A]^{-1}[V_{abc}] \qquad [I_a^{012}] = [A]^{-1}[I_{abc}]$$

where, $[V_{abc}]$: Unsymmetrical voltage phasor, $[I_{abc}]$: Unsymmetrical current phasor,

$[V_a^{012}]$: Symmetrical voltage component, $[I_a^{012}]$: Symmetrical current component.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \text{ and } A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

6. Relationship between line and phase base values in 3-ϕ transformers :

(a) Y -connection :

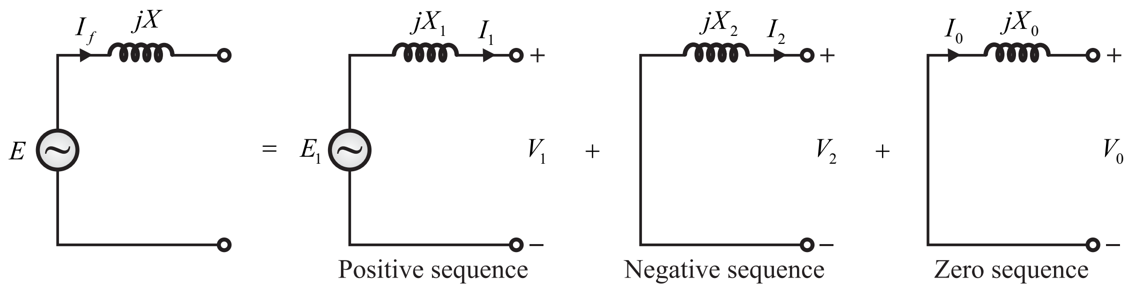
$$V_{b\,(line)} = \sqrt{3}V_{b\,(phase)} \qquad I_{b\,(line)} = I_{b\,(phase)}$$

(b) Δ - Connection :

$$V_{b\,(line)} = V_{b\,(phase)} \qquad I_{b\,(line)} = \sqrt{3}\,I_{b\,(phase)}$$

But, when per unit values are specified, then $V_{b\,(line)\text{pu}} = V_{b\,(phase)\text{pu}}$ and $I_{b\,(line)\text{pu}} = I_{b\,(phase)\text{pu}}$ irrespective of transformer connection.

Sequence diagram :



$V_1 = E_1 - jI_1X_1$ $\qquad$ $V_2 = -jI_2X_2$ $\qquad$ $V_0 = -jI_0X$

- Voltage is induced in positive sequence only.

Zero sequence diagram for 3-phase transformer is given by,

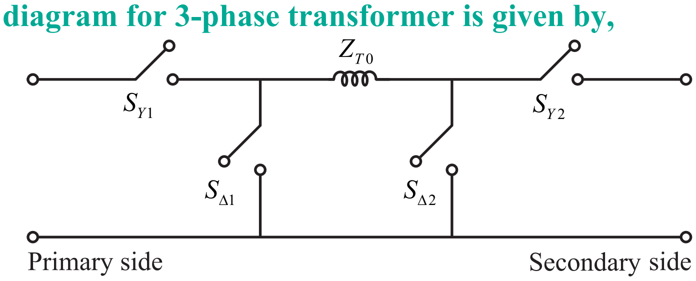


Fig. Switch diagram

1. S_1 or S_2 is closed depending upon which side star connected transformer with neutral grounded is connected ( ).
2. P_1 or P_2 is closed depending upon which side (Δ) connected transformer is connected.

➤ Sample Questions

2007 IIT Kanpur

10.1 A 230 V (phase), 50 Hz, three-phase, 4-wire system has a phase sequence *ABC*. A unity power- factor load of 4 kW is connected between phase A and neutral *N*. It is desired to achieve zero neutral current through the use of a pure inductor and a pure capacitor in the other two phase. The value of inductor and capacitor is

(A) 72.95 mH in phase *C* and 139.02 μF in phase *B*

(B) 72.95 mH in phase *B* and 139.02 μF in phase *C*

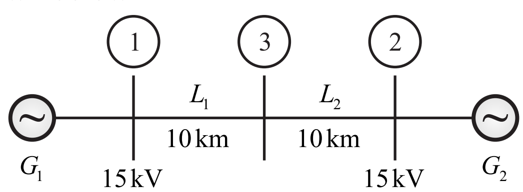
(C) 42.12 mH in phase *C* and 240.79 μF in phase *B*

(D) 42.12 mH in phase *B* and 240.79 μF in phase *C*

2011 IIT Madras

Common Data for Questions 10.2 & 10.3

Two generator units G_1 and G_2 are connected by 15 kV line with a bus at the mid-point as shown below



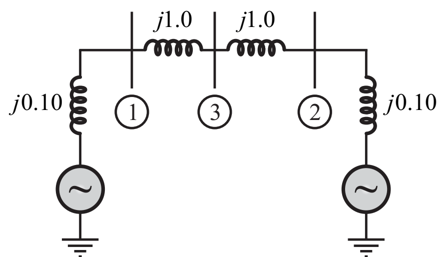
$G_1 = 250$ MVA, 15 kV, Positive sequence reactance $(X) = 25\%$ on its own base.

$G_2 = 100$ MVA, 15 kV, Positive sequence reactance $(X) = 10\%$ on its own base.

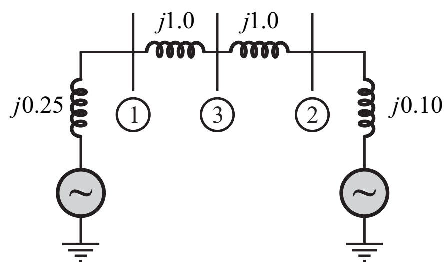
L_1 and $L_2 = 10$ km, positive sequence reactance $X = 0.225\,\Omega/\text{km}$.

10.2 For the above system, the positive sequence diagram with the pu value on the 100 MVA common base is

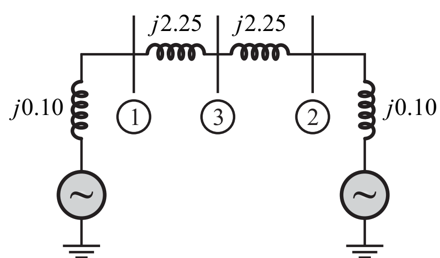
(A)



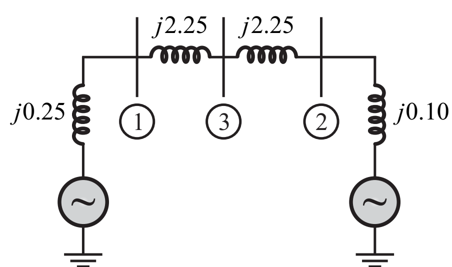
(B)



(C)



(D)



10.3 In the above system, the three phase fault MVA at the bus 3 is

(A) 82.55 MVA (B) 85.11 MVA

(C) 170.91 MVA (D) 181.82 MVA

2014 IIT Kharagpur

10.4 In an unbalanced 3-phase system, phase current $I_a = 1\angle -90^0$ pu, negative sequence current $I_{b_2} = 4\angle -150^0$ pu, zero sequence current $I_{c_0} = 3\angle 90^0$ pu. The magnitude of phase current I_b in pu is **[Set - 01]**

(A) 1.00 (B) 7.81

(C) 11.53 (D) 13.00

2021 IIT Bombay

10.5 Suppose I_A, I_B and I_C are a set of unbalanced current phasors in a three-phase system. The phase-B zero-sequence current $I_{B0} = 0.1\angle 0^0$ p.u. If phase-A current $I_A = 1.1\angle 0^0$ p.u and phase-C current $I_C = (1\angle 120^0 + 0.1)$ p.u., then I_B in p.u is

(A) $1\angle 240^0 - 0.1\angle 0^0$

(B) $1.1\angle 240^0 - 0.1\angle 0^0$

(C) $1.1\angle -120^0 + 0.1\angle 0^0$

(D) $1\angle -120^0 + 0.1\angle 0^0$

❖❖❖❖

Explanations Symmetrical Components

10.1 (B)

Given :

(i) ABC phase sequence 4-wire system :

(ii) Phase voltage, $V_{b(ph)} = 230\text{ V}$

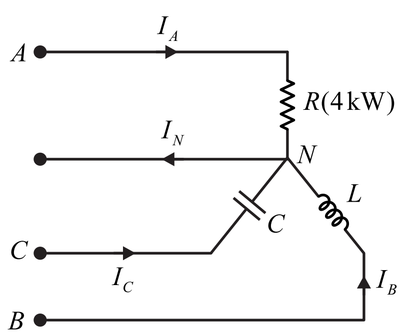
(iii) Frequency, $f = 50\text{ Hz}$

(iv) Load : $P_L = 4$ kW, upf

(v) Neutral current, $I_N = 0$.

To find : Value of inductor and capacitor to achieve zero neutral current.

The equivalent circuit is represented as,



Since, phase sequence is ABC,

$$\bar{V}_A = 230\angle 0^0 \text{ V}, \ \bar{V}_B = 230\angle -120^0 \text{ V},$$

$$\bar{V}_C = 230\angle +120^0 \text{ V}$$

Between phase A and neutral load connected 4 kW, upf i.e. pure resisitive load so

$$P_A = V_A I_A$$

$$4\times 10^3 = 230\times I_A$$

$$I_A = 17.4\text{ A}$$

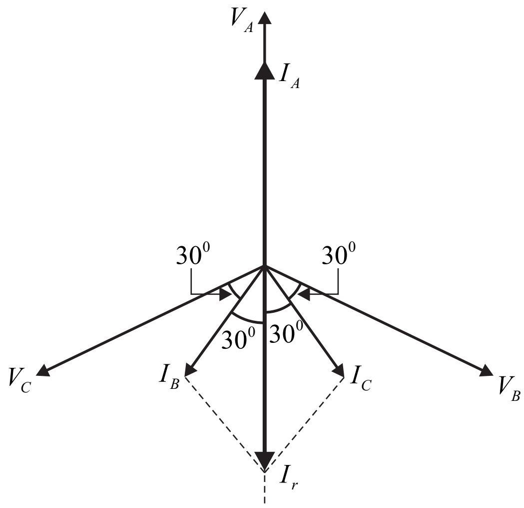
$$\bar{I}_A = 17.4\angle 0^0 \text{ along } \bar{V}_A$$

Assuming inductor in phase 'B' and capacitor in phase 'C'.

So, $$I_B = \frac{V_B}{j\omega L}$$

I_B lags V_B by 90^0 and $I_C = j\omega C V_C$

I_C leads V_C by 90^0



The current through neutral is given by,

$$I_N = I_A + I_B + I_C = 0$$

$$I_B + I_C = -I_A$$

So, resultant of I_B and I_C should be in the opposite direction of I_A i.e., resultant of I_B and I_C should be in between I_B and I_C.

$$|I_B| = |I_C| = I$$

$$\frac{230}{\omega L} = \omega C \times 230$$

$$\omega L = \frac{1}{\omega C}$$

Resultant current, $I_r = I_B + I_C$

$$|I_r| = 2I\cos 30^0 = I\sqrt{3}$$

As, $I_r = -I_A$

$$|I_r| = |I_A|$$

$$I\sqrt{3} = 17.4$$

$$I = 10.05 \text{ A}$$

$$|I_B| = \frac{230}{\omega L} = I$$

$$L = \frac{230}{10.05 \times 2\pi \times 50} = 72.85 \text{ mH}$$

$$|I_C| = I = 10.05 \text{ A}$$

$$\omega C \times 230 = 10.05 \text{ A}$$

$$C = \frac{10.05}{230 \times 2\pi \times 50} = 139.1 \mu\text{F}$$

Hence, the correct option is (B).

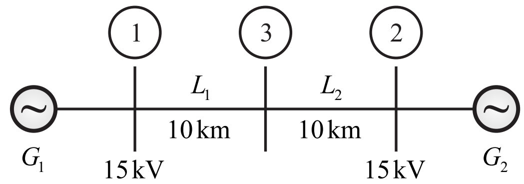
Key Point

If C and L would have been placed in Y and B phases respectively, then the required condition of zero neutral current is not achieved. Hence C and L are placed in B and Y phases respectively.

10.2 (A)

Given :

(i) One line diagram :



(ii) Generator G_1 :

$$S_{b1} = G_1 = 250 \text{ MVA},$$

$$X_{g1} = 25\% = 0.25 \text{ pu}$$

(iii) Generator G_2 :

$$S_{b2} = G_2 = 100 \text{ MVA},$$

$$X_{g2} = 10\% = 0.1 \text{ pu}$$

(iv) Lines L_1 and L_2 :

$$X_{l1} = \phi = X_{l2} = 0.225 \Omega/\text{km}$$

(v) Base MVA, $S_b = 100 \text{ MVA}$

To find : Positive sequence diagram.

At new base the positive sequence reactance of different components is given by,

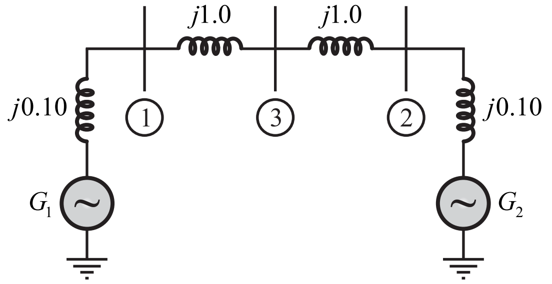
$$X_{g1} = 0.25 \times \frac{100}{250} = 0.1$$

$$X_{g2} = 0.10 \times \frac{100}{100} = 0.1$$

$$X_{l1} = 0.225 \times 10 \times \frac{100}{(15)^2} = 1$$

$$X_{l2} = 0.225 \times 10 \times \frac{100}{(15)^2} = 1$$

The positive sequence diagram with the pu value on the 100 MVA common base is given as,



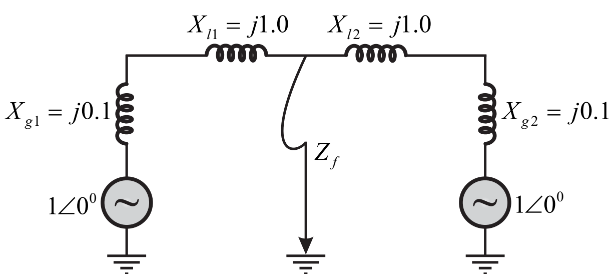
Hence, the correct option is (A).

10.3 (D)

Given : Three phase fault occurs at bus 3.

To find : 3-ϕ fault MVA at BUS 3.

The equivalent reactance diagram is represented as,



The fault impedance at BUS 3 is given by,

$$Z_f = X_f = (X_{g1} + X_{l1}) \,||\, (X_{g2} + X_{l2})$$

$$Z_f = (j0.1 + j1.0) \,||\, (j0.1 + j1.0)$$

$$Z_f = j1.1 \,||\, j1.1 = \frac{j1.1 \times j1.1}{j1.1 + j1.1} = j0.55$$

Per unit fault MVA is given by,

$$S_{pu} = \frac{1}{Z_f} = \frac{1}{0.55} = 1.81818\,\text{pu}$$

Actual fault MVA is given by,

$$S_{actual} = S_{pu} \times S_{base}$$

$$S_{actual} = 1.81818 \times 100 = 181.81\,\text{MVA}$$

Hence, the correct option is (D).

10.4 (C)

Given :

(i) $I_a = 1\angle -90^0$ pu,

(ii) $I_{b_2} = 4\angle -150^0$ pu,

(iii) $I_{c_0} = 3\angle 90^0$ pu

To find : Magnitude of phase current I_b.

Zero sequence current in all the three phasor of unbalanced 3-ϕ system are equal hence,

$$I_{a_0} = I_{b_0} = I_{c_0} = 3\angle 90^0 \qquad \ldots(i)$$

For unbalance system phase currents are related to the sequence current as

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

From the above relation,

$$I_a = I_{a0} + I_{a1} + I_{a2} \qquad \ldots(ii)$$

$$I_b = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2} = I_{b0} + I_{b1} + I_{b2} \qquad \ldots(iii)$$

$$I_c = I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2} \qquad \ldots(iv)$$

From equation (ii),

$$I_{a1} = I_a - I_{a0} - I_{a2}$$

where, $I_{b2} = aI_{a2}$.

$$I_{a2} = \frac{4\angle -150^0}{1\angle 120^0} = 4\angle -270^0 \text{ A}$$

By substituting the values,

$$I_{a_1} = (1\angle -90^0) - (3\angle 90^0) - (4\angle -270^0)$$

$$I_{a_1} = 8\angle -90^0 \text{ pu} \qquad \ldots(v)$$

From equation (iii),

$$I_b = I_{a_0} + \alpha^2 I_{a_1} + \alpha I_{a_2}$$

$$I_b = (3\angle 90^0) + (1\angle -120^0)(8\angle -90^0) + (1\angle 120^0)(4\angle -270^0)$$

$$I_b = 11.53\angle 154.3^0 \text{ pu}$$

Magnitude of I_b is given by,

$$|I_b| = 11.53\,\text{pu}$$

Hence, the correct option is (C).

10.5 **(D)**

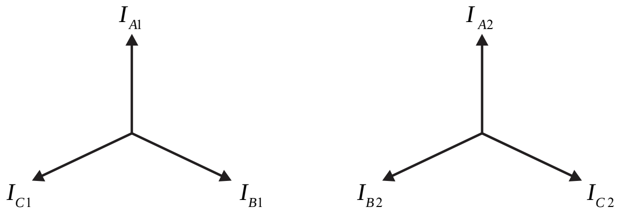
Given : $I_{B0} = 0.1\angle 0^0$

$I_A = 1.1\angle 0^0$

$I_C = (1\angle 120^0 + 0.1)$ pu

$I_B = ?$

Breaking line currents into sequence currents



$I_B = I_{B0} + I_{B1} + I_{B2}$

$I_B = 0.1\angle 0^0 + I_{B1} + I_{B2}$

$I_A = I_{A0} + I_{A1} + I_{A2}$

$1.1\angle 0^0 = 0.1\angle 0^0 + I_{B1}\angle 120^0 + I_{B2}\angle 240^0$ … (i)

$I_C = I_{C0} + I_{C1} + I_{C2}$

$1\angle 120^0 + 0.1 = 0.1\angle 0^0 + I_{B1}\angle 240^0 + I_{B2}\angle 120^0$

$1\angle 120^0 = I_{B1}\angle 240^0 + I_{B2}\angle 120^0$ … (ii)

From equation (i),

$1\angle 0^0 = I_{B1}\angle 120^0 + I_{B2}\angle 240^0$ … (iii)

Adding equation (ii) and (iii),

$$1\angle 0^0 + 1\angle 120^0 = I_{B1}(\angle 120^0 + \angle 240^0) + I_{B2}(\angle 120^0 + \angle 240^0)$$

$$(I_{B1} + I_{B2})\angle 180^0 = 1\angle 0^0 + 1\angle 120^0$$

$$I_{B1} + I_{B2} = 1\angle -180^0 + 1\angle -60^0$$

So, $I_B = I_{B0} + I_{B1} + I_{B2}$

$$I_B = 0.1\angle 0^0 + 1\angle -180^0 + 1\angle -60^0$$

$$I_B = 0.1\angle 0^0 + 1\angle -120^0$$

Hence, the correct option is (D).

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12 Power System Stability

➢ Partial Synopsis

In power system, we deal with three different types of stabilities.

1. Voltage stability, $V = V_{rated} \pm 5\%$
2. Frequency stability, $f = f_{rated} \pm 1\%\ (49.5 - 50.5\,\text{Hz})$
3. Rotor angular stability : Machine should not loose synchronism.

Rotor angular stability :

- The objective of this chapter is to study rotor angular stability i.e., if rotor speed increases continuously, synchronous machine will loose synchronism, and system may becomes unstable.
- Kinetic energy of rotor $= \dfrac{1}{2} J \omega_{sm}^2 \times 10^{-6}$ MJ

  Relation between electrical and mechanical speed is given by,

  (Electrical speed) $\omega_s = \dfrac{2}{p} \omega_{sm}$ (Mechanical speed)

  Kinetic energy of rotor with electrical synchronous speed is given by,

  Kinetic energy $= \dfrac{1}{2} J \left(\dfrac{2}{P}\right)^2 \omega_s \times 10^{-6} = \dfrac{1}{2} \text{M}\omega_s$ MJ

- Inertia constant, $H = \dfrac{\text{KE stored in MJ}}{\text{Rating of the machine in MVA}}$

$$H = \frac{\text{KE}}{G} \text{ (MJ/MVA)}$$

$$\text{KE} = GH$$

$$\frac{1}{2}\text{M}\omega_s = GH$$

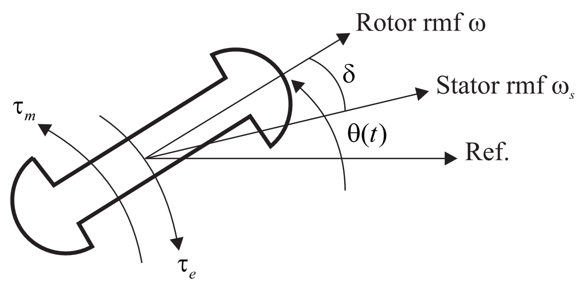
$$\text{M} = \frac{2GH}{\omega_s} = \frac{2GH}{2\pi f} = \frac{GH}{\pi f} \text{ MJ-sec/elec rad}$$

Swing equation :

$$\theta(t) = \omega_s t + \delta$$

$$\frac{d\theta(t)}{dt} = \omega_s + \frac{d\delta}{dt}$$

$$\frac{d\delta}{dt} = \omega_r - \omega_s$$



According to dynamics of the rotor,

$$\tau_m - \tau_e = J\frac{d^2\theta(t)}{dt^2}$$

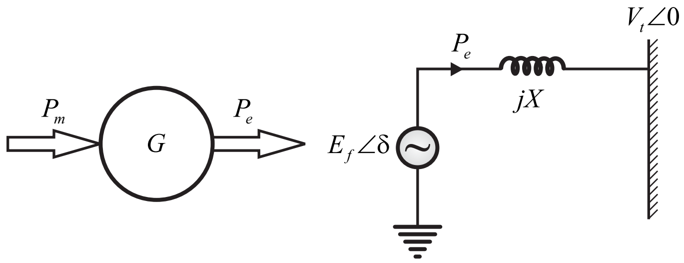
$$\tau_m\omega_{sm} - \tau_e\omega_{sm} = J\omega_{sm}\frac{d^2\theta(t)}{dt^2}$$

$$P_m - P_e = \text{M}\frac{d^2\theta(t)}{dt^2}$$

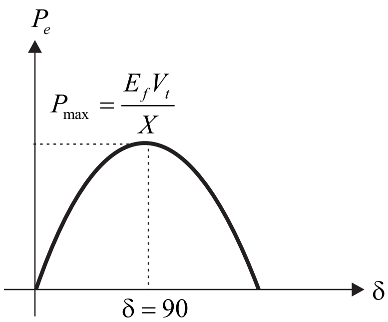
$$P_m - P_e = \text{M}\frac{d^2\delta}{dt^2}$$

Steady state stability :

If the synchronous machine regains its synchronism even after small disturbance, the machine is said to be steady state stable.



$$P_e = \frac{E_f V_t}{X}\sin\delta$$



For $\delta < 90^0$, system is steady state stable.

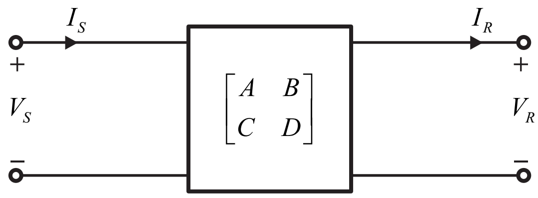
For $\delta = 90^0$, steady state stability limit.

So, for steady state stability.

$$\frac{dP_e}{d\delta} > 0$$

The maximum power that can be transferred without losing stability is known as steady state stability unit.

Steady state stability limit for transmission line :



$$P_R = \frac{|V_S||V_R|}{|B|}\cos(\beta - \delta) - \frac{|A||V_R|^2}{|B|}\cos(\beta - \alpha)$$

$$Q_R = \frac{|V_S||V_R|}{|B|}\sin(\beta - \delta) - \frac{|A||V_R|^2}{|B|}\sin(\beta - \alpha)$$

So, for $\delta = \beta$, steady state stability limit is achieved.

Transient stability :

For the synchronous machine to be transient stable, the kinetic energy stored during accelerating area must be released during decelerating area (For synchronous machine to be transient stable, $\frac{d\delta}{dt} = 0$), which results $\int_{\delta_0}^{\delta_c} P_a d\delta = 0$ which is known as equal area criteria.

➢ Sample Questions

1995 IIT Kanpur

12.1 A lossless alternator supplies 50 MW to an infinite bus, steady state stability limit being 100 MW. If the input to the prime mover of the alternator is abruptly increased by 40 MW. Alternator will

(A) remain in synchronism

(B) will run away

(C) cannot be said

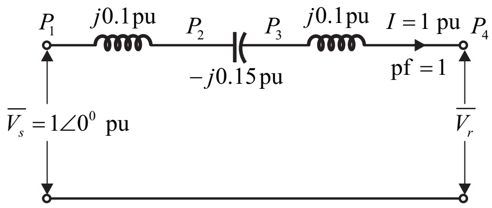
(D) stop

2000 IIT Kharagpur

12.2 A 50 Hz synchronous generator, having a reactance of 0.15 pu, is connected to an infinite bus through two identical parallel transmission lines having reactance of 0.3 pu each. In steady state, generator is delivering 1 pu power to the infinite bus. For a three phase fault at the receiving end of one line, calculate the rotor angle at the end of first time step of 0.05 sec. Assume the voltage behind transient reactance for the generator as 1.1 pu, infinite bus voltage as 1 pu and inertia constant 1 MJ/MVA.

2001 IIT Kanpur

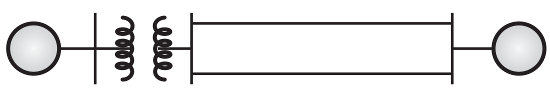
12.3 Consider the model shown in figure of a transmission line with a series capacitor at its mid-point. The maximum voltage on the line is at the location.



(A) P_1 (B) P_2
(C) P_3 (D) P_4

2017 IIT Roorkee

12.4 The figure shows the single line diagram of a power system with a double circuit transmission line. The expression for electrical power is $1.5\sin\delta$, where δ is the rotor angle. The system is operating at the stable equilibrium point with mechanical power equal to $1\,\text{pu}$. If one of the transmission line circuits is removed, the maximum value of δ, as the rotor swings, is 1.221 radian. If the expression for electrical power with one transmission line circuit removed is $P_{max}\sin\delta$, the value of P_{max}, in pu is ______. (Give the answer up to three decimal places.) **[Set - 01]**



2021 IIT Bombay

12.5 An alternator with an internal voltage of $1\angle\delta_1$ pu and synchronous reactance of 0.4 pu is connected by a transmission line of reactance 0.1 pu to a synchronous motor having synchronous reactance 0.35 pu and internal voltage of $0.85\angle\delta_2$ pu. If the real power supplied by the alternator is 0.866 pu, then $(\delta_1 - \delta_2)$ is ______ degrees (Round off to 2 decimal places). (Machines are of non-salient type. Neglect resistances)

Explanations Power System Stability

12.1 (D)

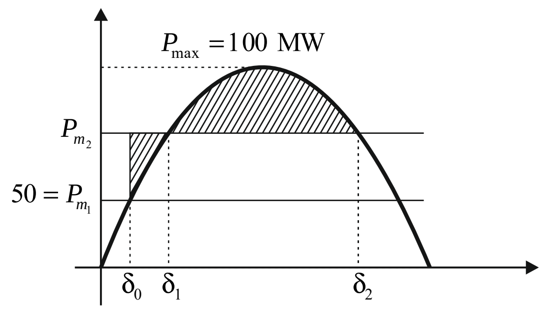
Given :

(i) Lossless alternator

(ii) $P_{max} = 100$ MW

(iii) $P_{mech_0} = 50$ MW

(iv) $P_{mech_1} = 40$ MW



Initially $P_{m_1} = 50$ MW

$$P_e = 100\sin\delta$$

$$P_{m_1} = P_e$$

$$50 = 100 \sin \delta_0$$

$$\delta_0 = 30^0 \text{ or } 0.523 \text{ rad}$$

Now the prime mover abruptly increased by 40 MW so now,

$$P_{m_2} = 50 + 40 = 90 \text{ MW}$$

$$P_{m_2} = P_e$$

$$90 = 100 \sin \delta_1$$

$$\delta_1 = \sin^{-1}(0.9)$$

$$\delta_1 = 64.158^0 \text{ or } 1.1197 \text{ rad}$$

$$\delta_2 = \pi - \delta_1 = \pi - 1.1197$$

$$\delta_2 = 115.842^0 \text{ or } 2.0202 \text{ rad}$$

Accelerating area is given by,

$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m_2} - P_e)\, d\delta$$

$$A_1 = 90[\delta_1 - \delta_0] - P_{\max}[\cos\delta_0 - \cos\delta_1]$$

$$A_1 = 53.703 - 43.0134 = 10.68955$$

Decelerating area is given by,

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_{m_2})\, d\delta$$

$$A_2 = P_{\max}[\cos\delta_1 - \cos\delta_2] - 90[\delta_2 - \delta_1]$$

$$A_2 = 100[\cos 64.158 - \cos 115.842] - 90[2.0202 - 1.1197]$$

$$A_2 = 87.178 - 81.045 = 6.133$$

$$A_2 < A_1$$

Accelerating area is greater than decelerating area, so synchronism of alternator will be lost. Hence the correct option is (D).

12.2 **27**

Given :

(i) Synchronous generator :
$f = 50\,\text{Hz}$, $X_S = 0.15\,\text{pu}$,
$E_f = 1.1\,\text{pu}$, $H = 1\,\text{MJ/MVA}$

(ii) Two identical parallel transmission lines : $X_l = 0.3$ pu.

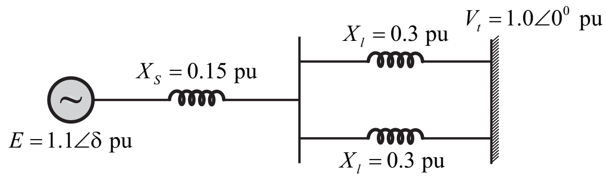
(iii) Bus voltage, $V_t = 1\,\text{pu}$

(iv) First time step = 0.05 sec.

(v) In steady state, generator is delivering 1 pu power to the infinite bus.

To find : The rotor angle at the end of first step.

The one line diagram of system is shown below,



The power output of synchronous generator is given by,

$$P_{e0} = \frac{E_f V_t}{X_{eq}} \sin\delta_0$$

$$1 = \frac{1.1\times 1}{0.15 + \frac{0.3}{2}} \sin\delta_0 = \frac{1.1}{0.15+0.15}\sin\delta_0$$

$$1 = \frac{1.1}{0.3}\sin\delta_0$$

$$\sin\delta_0 = 0.2727$$

$$\delta_0 = \sin^{-1}(0.2727) = 15.82^0$$

Swing equation is given by,

$$M\alpha = P_{accelerating}$$

$$\frac{H}{180\times f}\times\alpha = P_{e0} - P_{e1}$$

$$\frac{H}{180\times 50}\times\alpha = 1 - 0$$

$$\frac{1}{9000}\times\alpha = 1$$

$$\alpha = 9000\ \frac{\text{elec degree}}{\text{sec}^2}$$

Rotor angle is given by,

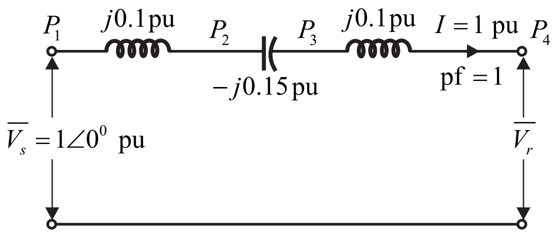
$$\delta = \alpha\frac{T^2}{2} + \delta_0 = 9000 \times \frac{(0.05)^2}{2} + 15.82$$

$$\delta = 27^0$$

Hence, the rotor angle at the end of first time step of 0.05 sec is $\mathbf{27^0}$.

12.3 (C)

Given figure is shown below,



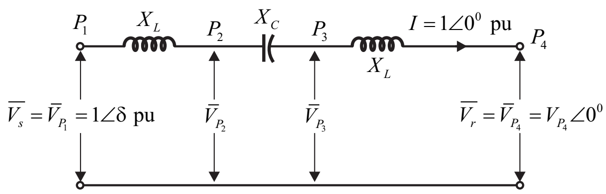
Assume $\bar{V}_r$ as reference phasor,

$$\bar{V}_r = \bar{V}_{P_4} = V_{P_4}\angle 0^0$$

So, with respect to $\bar{V}_r$, $\bar{V}_s$ will become

$$\bar{V}_s = \bar{V}_{P_1} = 1\angle\delta$$

Then the equivalent figure will be as shown below,

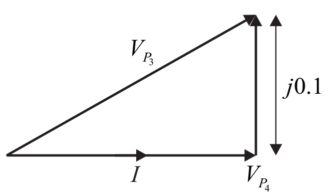


Case 1 : Point P_3,

Applying KVL in above figure,

$$\bar{V}_{P_3} = \bar{V}_{P_4} + \bar{I} \times jX_L$$

Drawing phasor diagram,



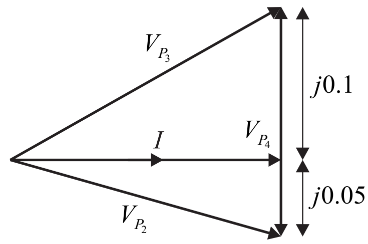
From phasor diagram, $|\bar{V}_{P_3}| > |\bar{V}_{P_4}|$

Case 2 : Point P_2,

Applying KVL in above figure,

$$\bar{V}_{P_2} = \bar{V}_{P_4} + \bar{I} \times jX_L + \bar{I} \times (-jX_C)$$

Drawing phasor diagram,



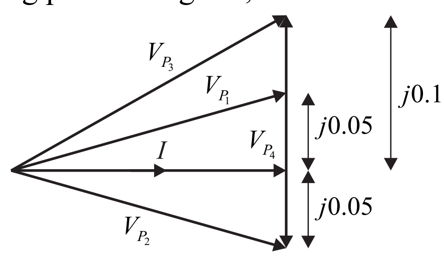
From phasor diagram, $|\bar{V}_{P_3}| > |\bar{V}_{P_2}| > |\bar{V}_{P_4}|$

Case 3 : Point P_1,

Applying KVL in above figure,

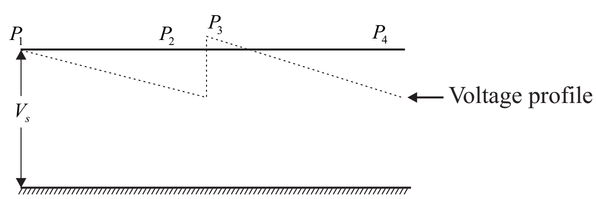
$$\bar{V}_{P_1} = \bar{V}_{P_4} + \bar{I} \times jX_L + \bar{I} \times (-jX_C) + \bar{I} \times jX_L$$

Drawing phasor diagram,



From phasor diagram, $|\bar{V}_{P_3}| > |\bar{V}_{P_2}| = |\bar{V}_{P_1}| > |\bar{V}_{P_4}|$

The voltage profile for the given line is represented as,



Hence, the correct option is (C).

12.4 1.22

Given :

(i) Power system : $P_{mech} = 1$pu

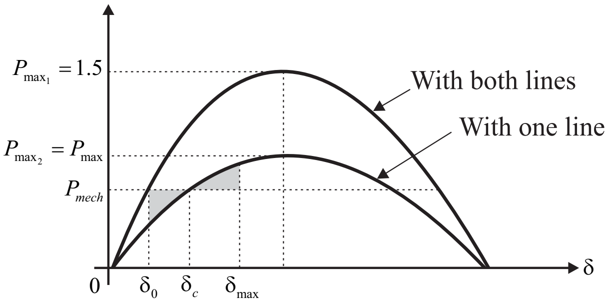
(ii) Normal operation : $P_e = 1.5\sin(\delta_0)$,

$$\delta_0 = 41.8^0 = 0.7295 \text{rad}$$

(iii) One line removed :

$$\delta_{max} = 1.221\,\text{rad} = 69.95^0$$

To find : P_{max} .



Using equal area criteria,

Acceleration area = Deceleration area

$$\int_{\delta_0}^{\delta_c} (P_{mech} - P_{max_2}\sin\delta)\,d\delta = \int_{\delta_c}^{\delta_{max}} (P_{max_2}\sin\delta - P_{mech})\,d\delta$$

$$P_{mech}(\delta_c - \delta_0) + P_{max_2}[\cos\delta_c - \cos\delta_0]$$

$$= P_{max_2}[\cos\delta_c - \cos\delta_{max}] + P_{mech}[\delta_c - \delta_{max}]$$

$$P_{max_2} = \frac{P_{mech}(\delta_{max} - \delta_0)}{\cos\delta_0 - \cos\delta_{max}}$$

Substituting the required value in above equation,

$$P_{max_2} = P_{max} = \frac{1(1.221 - 0.7295)}{\cos 41.8 - \cos 69.95}$$

$$P_{max_2} = P_{max} = \frac{0.4915}{0.4026} = 1.22$$

Hence, the value of P_{max} is **1.22 pu**.

12.5 **60**

Given :

(i) $E_g\angle\delta_1 = 1\angle\delta_1$

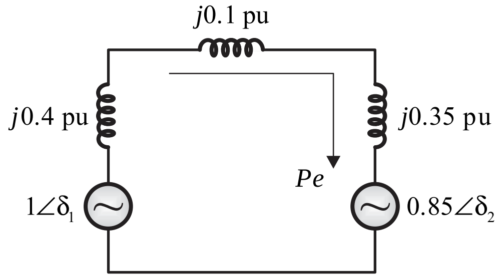
(ii) $X_g = j0.4\,\text{pu}$

(iii) $X_l = j0.1\,\text{pu}$

(iv) $X_m = j0.35\,\text{pu}$

(v) $E_m\angle\delta_2 = 0.85\angle\delta_2$ pu

Active power transferred from generator to motor is 0.866 pu.



$$P_e = \frac{E_g \times V_t}{X_T}\sin(\delta_1 - \delta_2)$$

$$0.866 = \frac{1\times 0.85}{(0.4 + 0.1 + 0.35)}\sin(\delta_1 - \delta_2)$$

$$0.866 = \frac{0.85}{0.85}\sin(\delta_1 - \delta_2)$$

$$\delta_1 - \delta_2 = \sin^{-1}(0.866) = 60^0$$

Hence, the correct answer is 60.

❖❖❖❖

13 Load Flow Studies

➢ Partial Synopsis

Bus Classification :

A load flow study is the one which is able to provide the solution for unknown electrical quantity of the system (where the system is working in the steady state condition along with certain constraints). Depending upon which quantities have been specified, the buses are classified in the following three categories :

1. **Load bus or Motor bus or PQ bus :** At this Bus injected active power and reactive power are specific and the magnitude of voltage and phase angle are unspecified. At a load bus voltage can be allowed to vary within the permissible values e.g., 5%.

   If the value of V_i lies in the range of $V_{i\,\min}$ to $V_{i\,\max}$ than it is considered as PQ bus. If it violates then it will considered as PV bus and the value of V_i will be either $V_{i\,\min}$ or $V_{i\,\max}$ depending upon the calculated value of V_i .

2. **Generator bus or Voltage Controlled bus or PV bus :** At this bus the voltage magnitude of the bus and the injected active power are specified. It is required to find out the injected reactive power generation and the phase angle of the bus voltage.

   If the value of Q_i lies in the range of $Q_{i\,\min}$ to $Q_{i\,\max}$ than it is considered as PV bus. If it violates then it will considered as PQ bus and the value of Q_i will be either $Q_{i\,\min}$ or $Q_{i\,\max}$ depending upon the calculated value of Q_i .

3. **Slack bus or Reference bus :** In load flow studies the network equations are solved by considering the generation and the demand at each without including the line losses because the losses are very less, so that the time taken to get the solution of the network equations are reduced.

   When the losses are included, they are assumed to be supplied by a single generator instead of sharing by all the generators.

   Generally, the largest generating station is used as reference bus and any change in this bus changes the operating condition of the system with power loss remaining as it is.

The load flow equations which are used to calculate the operating points of a system are power flow $(P_i + jQ_i)$, voltage $(V_i\angle\delta_i^0)$ and power losses occurring in the system, out of which 2 variables are specified at any given bus and remaining 2 variables are calculated from load flow equations. Since, power losses cannot be specified and can be calculated only at the end of load flow solution, a reference bus is used to supply these losses.

Types of bus	Known parameters	Unknown parameters
Generator bus	P_i, V_i	δ_i, Q_i
Load bus	P_i, Q_i	V_i, δ_i
Slack bus	V_i, δ_i	P_i, Q_i

Newton Raphson Method :

In Newton Raphson method we calculate the magnitude and load angle of voltage at different buses, i.e. the number of simultaneous equation required are equal to the number of unknown voltages and load angles at different buses which is equal to the order to Jacobian matrix.

Size of Jacobian matrix is given by,

$$J = (2n - m - 2) \times (2n - m - 2)$$

where, $n =$ Total number of buses, $m =$ Number of PV buses excluding slack bus

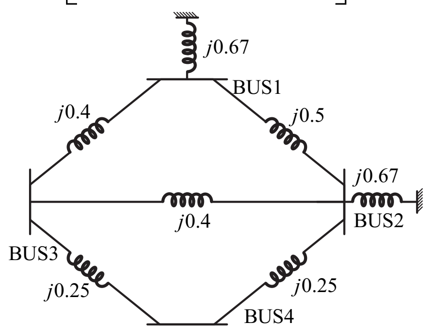
➤ Sample Questions

2001 IIT Kanpur

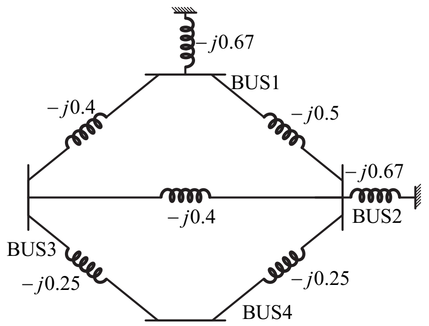
13.1 For the Y-bus matrix given in per unit values, where the first, second, third and fourth row refers to bus 1, 2, 3 and 4 respectively, the reactance diagram is

$$Y_{bus} = j\begin{bmatrix} -6 & 2 & 2.5 & 0 \\ 2 & -10 & 2.5 & 4 \\ 2.5 & 2.5 & -9 & 4 \\ 0 & 4 & 4 & -8 \end{bmatrix}$$

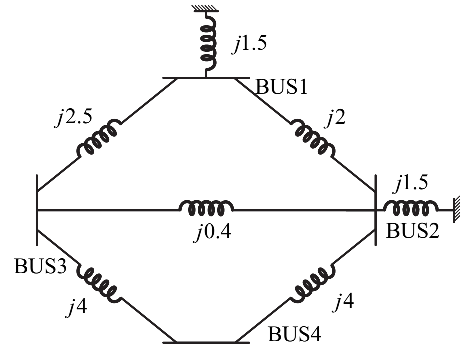
(A)



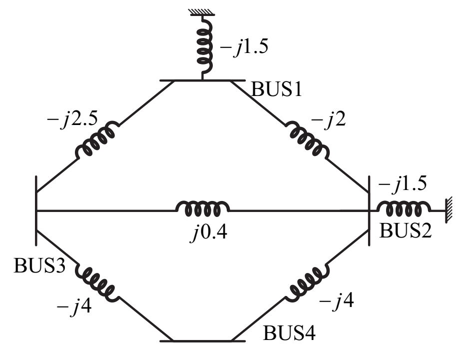
(B)



(C)



(D)



2003 IIT Madras

13.2 A power system consist of 300 buses out of which 20 buses are generator bus, 25 buses are ones with reactive power support and 15 buses are the ones with fixed shunt capacitors. All the other buses are load buses. It is proposed to perform a load flow analysis in the system using Newton-Raphson method. The size of the Newton Raphson Jacobian matrix is

(A) 553×553

(B) 540×540

(C) 555×555

(D) 554×554

2015 IIT Kanpur

13.3 A 3-bus power system network consists of 3 transmission lines. The bus admittance matrix of the uncompensated system is **[Set - 02]**

$$\begin{bmatrix} -j6 & j3 & j4 \\ j3 & -j7 & j5 \\ j4 & j5 & -j8 \end{bmatrix} pu.$$

If the shunt capacitance of all transmission lines is 50% compensated, the imaginary part of the 3^{rd} row 3^{rd} column element (in per unit) of the bus admittance matrix after compensation is

(A) $-j7.0$

(B) $-j8.5$

(C) $-j7.5$

(D) $-j9.0$

2016 IISc Bangalore

13.4 In a 100 bus power system, there are 10 generators. In a particular iteration of Newton Raphson load flow technique (in polar coordinates), two of the PV buses are converted to PQ type. In this iteration, **[Set - 01]**

(A) the number of unknown voltage angles increases by two and the number of unknown voltage magnitudes increases by two.

(B) the number of unknown voltage angles remains unchanged and the number of unknown voltage magnitudes increases by two.

(C) the number of unknown voltage angles increases by two and the number of unknown voltage magnitudes decreases by two.

(D) the number of unknown voltage angles remains unchanged and the number of unknown voltage magnitudes decreases by two.

2017 IIT Roorkee

13.5 A 10-bus power system consists of four generator buses indexed as G1, G2, G3, G4 and six load buses indexed as L1, L2, L3, L4, L5, L6. The generator-bus G1 is considered as slack bus, and the load buses L3 and L4 are voltage controlled buses. The generator at bus G2 cannot supply the required reactive power demand, and hence it is operating at its maximum reactive power limit. The number of non-linear equations required for solving the load flow problem using Newton-Raphson method in polar form is ______. **[Set - 01]**

2020 IIT Delhi

13.6 Out of the following options, the most relevant information needed to specify the real power (P) at the PV buses in a load flow analysis is.

(A) base power of the generator

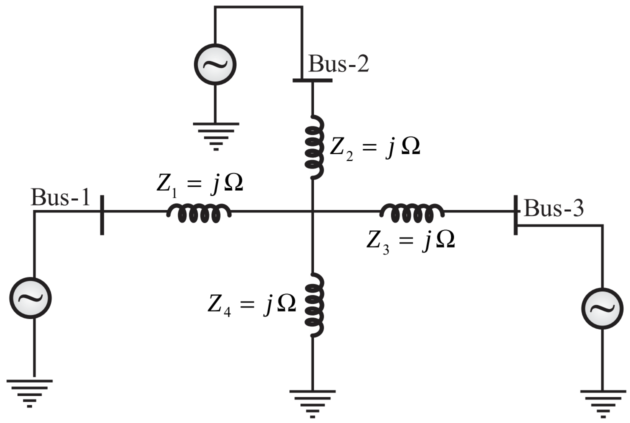
(B) solution of economic load dispatch.

(C) rated power output of the generator

(D) rated voltage of the generator.

2021 IIT Bombay

13.7 A 3-bus network is shown. Consider generators as ideal voltage sources. If rows 1, 2 and 3 of the Y_{bus} matrix correspond to bus 1, 2 and 3 respectively, then Y_{bus} of the network is



(A) $\begin{bmatrix} -4j & j & j \\ j & -4j & j \\ j & j & -4j \end{bmatrix}$

(B) $\begin{bmatrix} -4j & 2j & 2j \\ 2j & -4j & 2j \\ 2j & 2j & -4j \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{3}{4}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{3}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & \frac{-3}{4}j \end{bmatrix}$

(D) $\begin{bmatrix} \frac{-1}{2}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{1}{2}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & \frac{-1}{2}j \end{bmatrix}$

❖❖❖❖

Explanations **Load Flow Studies**

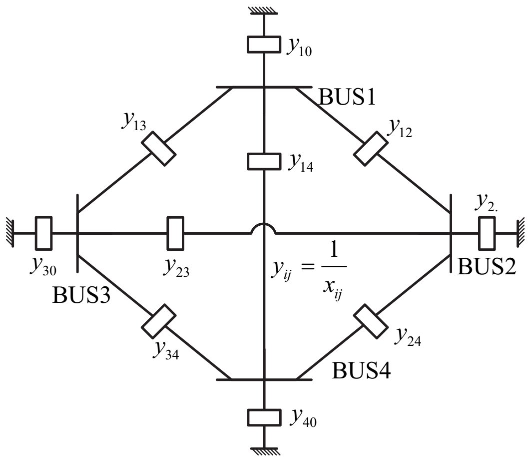
13.1 (A)

Given : $Y_{bus} = j\begin{bmatrix} -6 & 2 & 2.5 & 0 \\ 2 & -10 & 2.5 & 4 \\ 2.5 & 2.5 & -9 & 4 \\ 0 & 4 & 4 & -8 \end{bmatrix}$

Admittance matrix of a 4 bus network is given by,

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

The admittance network is given by,



Key Point

Y-Bus Matrix :

Non diagonal elements : $Y_{ij} = -y_{ij}$

Diagonal elements :

Sum of all elements connected to bus

$$Y_{ii} = y_{i0} + y_{i1} + y_{i2} ...$$

Non diagonal elements :

$$Y_{12} = Y_{21} = -y_{12} = j2$$

$$y_{12} = -j2$$

$$x_{12} = \frac{1}{y_{12}} = \frac{1}{-j2} = j0.5$$

$$Y_{13} = Y_{31} = -y_{13} = j2.5$$

$$y_{13} = -j2.5$$

$$x_{13} = \frac{1}{y_{13}} = \frac{1}{-j2.5} = j0.4$$

$$Y_{14} = Y_{41} = -y_{14} = 0$$

$$y_{14} = 0$$

$$x_{14} = \frac{1}{y_{14}} = \infty \qquad \text{[Open circuit]}$$

$$Y_{23} = Y_{32} = -y_{23} = j2.5$$

$$y_{23} = -j2.5$$

$$x_{23} = \frac{1}{y_{23}} = \frac{1}{-j2.5} = j0.4$$

$$Y_{24} = Y_{42} = -y_{24} = j4$$

$$y_{24} = -j4$$

$$x_{24} = \frac{1}{y_{24}} = \frac{1}{-j4} = j0.25$$

$$Y_{34} = Y_{43} = -y_{34} = j4$$

$$y_{34} = -j4$$

$$x_{34} = \frac{1}{y_{34}} = \frac{1}{-j4} = j0.25\,\text{pu}$$

Diagonal elements :

$$Y_{11} = y_{10} + y_{12} + y_{13} + y_{14} = -j6$$

$$y_{10} - j2 - j2.5 + 0 = -j6$$

$$y_{10} = -j1.5\,\text{pu}$$

$$x_{10} = \frac{1}{y_{10}} = \frac{1}{-j1.5} = j\frac{2}{3}\text{pu} = j0.677\,\text{pu}$$

$$Y_{22} = y_{20} + y_{21} + y_{23} + y_{24} = -j10$$

$$y_{20} - j2 - j2.5 - j4 = -j10$$

$$y_{20} = -j1.5\,\text{pu}$$

$$x_{20} = \frac{1}{y_{20}} = \frac{1}{-j1.5} = j\frac{2}{3} = j0.677\,\text{pu}$$

$$Y_{33} = y_{30} + y_{31} + y_{32} + y_{34} = -j9$$

$$y_{30} - j2.5 - j2.5 - j4 = -j9$$

$$y_{30} = 0$$

$$x_{30} = \frac{1}{y_{30}} = \frac{1}{0} = \infty \quad \text{[Open circuit]}$$

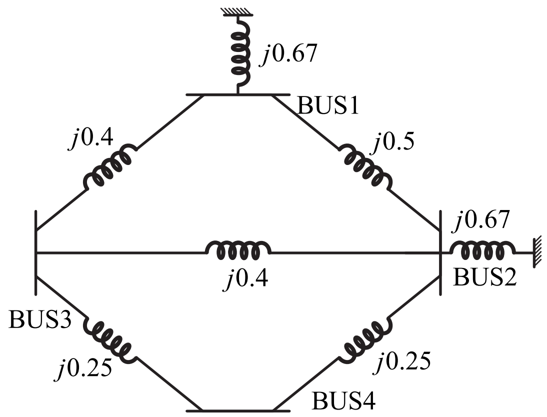
$$Y_{44} = y_{40} + y_{41} + y_{42} + y_{43} = -j8$$

$$y_{40} + 0 - j4 - j4 = -j8$$

$$y_{40} = 0$$

$$x_{40} = \frac{1}{y_{40}} = \frac{1}{0} = \infty \quad \text{[Open circuit]}$$

Thus, the reactance diagram is given by,



Hence, the correct option is (A).

13.2 (D)

Given :

(i) Number of buses, $n = 300$

(ii) Slack bus specified : NO

(iii) Number of generator buses = 20 –1 = 19

(iv) Number of buses with reactive power support = 25

(iv) Buses with fixed shunt capacitors = 15

(v) Number of load buses
= $300 - 20 - 25 - 15 = 240$

Method 1

Bus terminology	Number of simultaneous equations using NR method
Generator $\times 19$	$19 \times 1 = 19$
Reactive power support$\times 25$	$25 \times 1 = 25$
Fixed shunt capacitor$\times 15$	$15 \times 2 = 30$
Load bus $\times 240$	$240 \times 2 = 480$
Total	554

The size of Jacobian matrix is 554×554

Hence, the correct option is (D).

Method 2

Size of Jacobian matrix is given by,

$$J = (2n - m - 2) \times (2n - m - 2)$$

where, $n =$ Total number of buses = 300

$m =$ Number of PV buses excluding slack bus,

$m = 20 + 25 - 1 = 44$

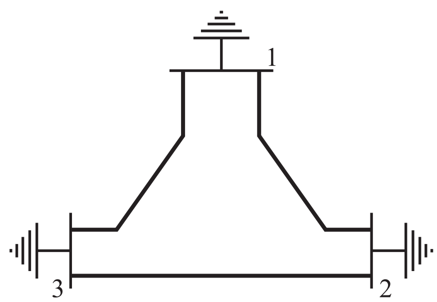
$$J = (2 \times 300 - 44 - 2) \times (2 \times 300 - 44 - 2)$$

$$J = 554 \times 554$$

Hence, the correct option is (D).

13.3 (B)

Given : $Y = \begin{bmatrix} -j6 & j3 & j4 \\ j3 & -j7 & j5 \\ j4 & j5 & -j8 \end{bmatrix}$ pu



Also,

$$Y_{bus} = \begin{bmatrix} Y_{10} + Y_{12} + Y_{13} & -Y_{12} & -Y_{13} \\ -Y_{21} & Y_{20} + Y_{21} + Y_{23} & -Y_{23} \\ Y_{31} & -Y_{32} & Y_{30} + Y_{31} + Y_{32} \end{bmatrix}$$

Comparing given bus admittance matrix with its general form,

$$Y_{13} = Y_{31} = -j4$$

$$Y_{32} = Y_{23} = -j5$$

$$Y_{30} + Y_{31} + Y_{32} = -j8$$

$$Y_{30} + (-j4) + (-j5) = -j8$$

$$Y_{30} = j1$$

After 50% compensation in shunt capacitance Y_{30} changes as,

$$Y_{30}'=0.5\times Y_{30}$$

$$Y_{30}'=0.5\times j1$$

$$Y_{30}'=0.5j$$

The 3rd row and 3rd column element of matrix Y after compensation will be

$$Y_{30}'+Y_{31}+Y_{32}=j0.5+(-j4)+(-j5)$$

$$Y_{30}'+Y_{31}+Y_{32}=-j8.5$$

Hence, the correct option is (B).

13.4 (B)

Given :

(i) Total number of buses = 100

Number of generators = 10

(ii) For a particular iteration of Newton Raphson load flow technique (in polar coordinates), two of the PV buses are converted to PQ type

Since, one of the generator bus is consider as a slack bus.

So, number of PV bus $=10-1=9$

Number of Load buses $=100-10=90$

Slack bus $=1$

If two of the PV buses are converted to PQ it will add 2 unknown voltages to iteration but unknown angles remains constant.

Hence, correct option is (B).

13.5 14

Given :

(i) Number of buses, $n=10$

(ii) Slack bus specified : YES (G_1)

(iii) Number of generator buses $=4-1-1=2(G_3,G_4)$

(iii) Number of buses with reactive power support = 1 (G_2)

(vii) Number of load buses = 4 (L_1,L_2,L_5,L_6)

(viii) Number of voltage controlled bus = 2 (L_3,L_4).

Method 1

Bus terminology	Number of simultaneous equations
Generator $\times 2$	$2\times 1=2$
Reactive power support $\times 1$	$1\times 2=2$
Load bus $\times 4$	$4\times 2=8$
Voltage controlled bus $\times 2$	$2\times 1=2$
Total	14

The size of Jacobian matrix is 14×14.

Hence, the number of non-linear equations required are **14**.

Method 2

Size of Jacobian matrix is given by,

$$J=(2n-m-2)\times(2n-m-2)$$

where, $n=$ Total number of buses = 10

$m=$ Number of PV buses excluding slack bus.

$$m=2+2=4$$

$$J=(2\times 10-4-2)\times(2\times 10-4-2)$$

$$J=14\times 14$$

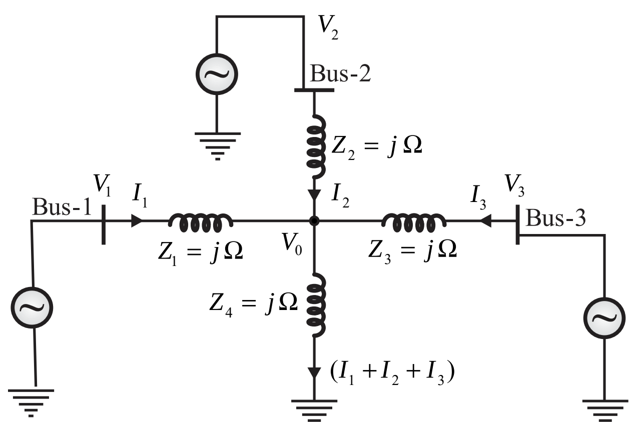
Hence, the number of non-linear equations required are **14**.

13.6 (B)

Solution of economic load dispatch is the most relevant information needed to specify the real power P at PV buses in load flow analysis.

Hence, the correct option is (B).

13.7 (C)



From the above circuit diagram, output voltage is given by,

$$V_0 = j[I_1 + I_2 + I_3]$$

Applying KVL at bus 1,

$$V_1 - I_1[j] = j[I_1 + I_2 + I_3]$$

$$j2I_1 + jI_2 + jI_3 = V_1 \quad \ldots(i)$$

Similarly at bus 2,

$$jI_1 + j2I_2 + jI_3 = V_2 \quad \ldots(ii)$$

Similarly at bus 3,

$$jI_1 + jI_2 + j2I_3 = V_3 \quad \ldots(iii)$$

Solving equation (i), (ii) and (iii),

$$I_1 = -\frac{3}{4}jV_1 + \frac{1}{4}jV_2 + \frac{1}{4}jV_3$$

$$I_2 = \frac{1}{4}jV_1 - \frac{3}{4}jV_2 + \frac{1}{4}jV_3$$

$$I_3 = \frac{1}{4}jV_1 + \frac{1}{4}jV_2 - \frac{3}{4}jV_3$$

$$\text{Hence, } Y\text{ bus} = \begin{bmatrix} -\frac{3}{4}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{-3}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & \frac{-3}{4}j \end{bmatrix}$$

Hence, the correct option is (C).

❖❖❖❖

14 Switch Gear & Protection

➤ Partial Synopsis

Relay :

A device which senses the fault and gives tripping signal to a circuit breaker is called relay.

Overcurrent relay :

When, $I_f > I_{\text{pickup}}$ (the value after which relay starts to operate).

$$\text{Plug setting multiplier (PSM)} = \frac{\text{Secondary fault current}}{\text{Current setting}}$$

Time multiplier setting (TMS) :

$$\text{Operating time of relay} = \text{TMS} \times \text{Relay operating time for (TMS} = 1)$$

Type of distance relay :

1. Impedance relay : It is used for protection of medium line.
2. Reactance relay : It is used for protection of short line.
3. MHO relay : It is used for protection of long line.

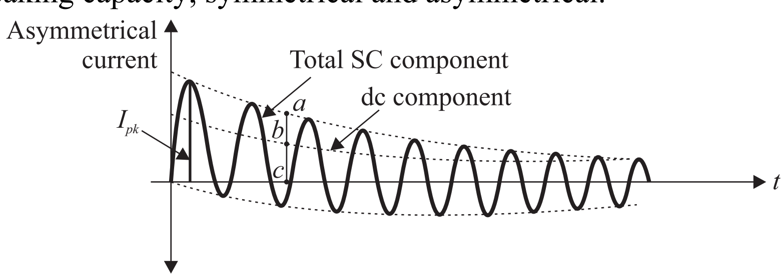
Differential relay : It is used to sense a fault in alternator winding, 3-phase transformer (by merz-price differential protection).

Circuit breaker : The duty of a CB is to switch ON and switch OFF, once or repeatedly several times different electrical circuits during normal as well as abnormal operating conditions.

Due to high voltage gradient between the two contacts of the CB, an arc in the form of plasma, appears across the contacts. For the successful operation of CB, an arc must be extinguished.

Rating of circuit breaker :

1. Rated breaking capacity, symmetrical and asymmetrical.



The actual current interrupted by CB is less than the initial value of short circuit current.

(i) Symmetrical breaking current, $I_{sym} = \dfrac{ab}{\sqrt{2}}$

(ii) Asymmetrical breaking current, $I_{asy} = \sqrt{\left(\dfrac{ab}{\sqrt{2}}\right)^2 + (bc)^2}$

2. Rated making current capacity :

The making current of a CB, when closed on a short circuit, is the rms value of total current (including both ac and dc) at the first major peak.

Making current = RMS value of I_{pk} .

Rated making capacity $= 2.55\times$ Symmetrical breaker capacity.

Restriking voltage :

Transient voltage that exist at or in close proximity to reach zero current during arcing time is called as restriking voltage. It is defined at system frequency.

Restriking voltage is given by,

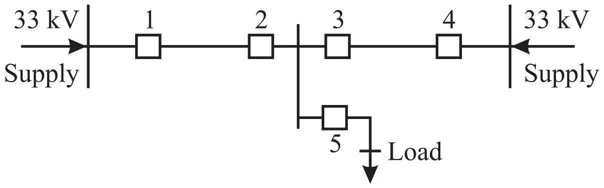
$$V_{RS} = V_m\left(1-\cos\omega_n t\right)$$

Where, $\omega_n = \dfrac{1}{\sqrt{LC}}$

➢ Sample Questions

1993 IIT Bombay

14.1 The distribution system shown in figure is to be protected by over current system of protection. For proper fault discrimination directional over current relays will be required at locations



(A) 1 and 4　　(B) 2 and 3

(C) 1, 4 and 5　　(D) 2, 3 and 5

1995 IIT Kanpur

14.2 The inductance and capacitance of a power system network up to a circuit breaker location are 1 H and 0.01μF respectively, the value of the shunt resistor across the circuit breaker, required for critical damping of the restriking voltage is ______ kΩ.

1998 IIT Delhi

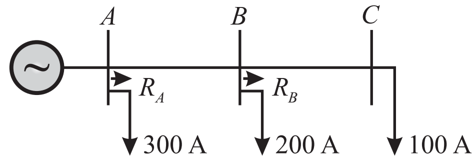
14.3 The neutral of 10 MVA, 11 kV alternator is earthed through a resistance of 5 ohms. The earth fault relay is set to operate at 0.75 A. The CT's have a ratio of 1000/5. What percentage of the alternator winding is protected?

(A) 85%　　(B) 88.2%

(C) 15%　　(D) 11.8%

2014 IIT Kharagpur

14.4 The over current relays for the line protection and loads connected at the buses are shown in the figure.



The relays are IDMT in nature having the characteristic

$$t_{op} = \frac{0.14 \times \text{Time multiplier setting}}{(\text{Plug setting multiplier})^{0.02} - 1}$$

The maximum and minimum fault currents at bus B are 2000 A and 500 A respectively. Assuming the time multiplier setting and plug setting for relay R_B to be 0.1 and 5 A respectively, the operating time of R_B (in seconds) is _______ **[Set - 01]**

2020 | IIT Delhi

14.5 A lossless transmission line with 0.2 pu reactance per phase uniformly distributed along the length of the line, connecting a generator bus to a load bus. is protected up to 80 % of its length by a distance relay placed at the generator bus. The generator terminal voltage is 1 pu. There is no generation at the load bus. The threshold pu current for operation of the distance relay for a solid three phase-to-ground fault on the transmission line is closest to

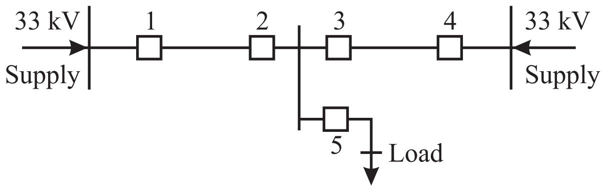
(A) 3.61 (B) 1.00

(C) 6.25 (D) 5.00

Explanations | Switch Gear & Protection

14.1 (B)

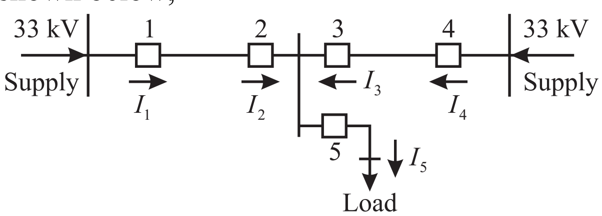
Given :



Directional over current relays are actuated when the direction of current is reversed or phase of the current becomes more than the reference value.

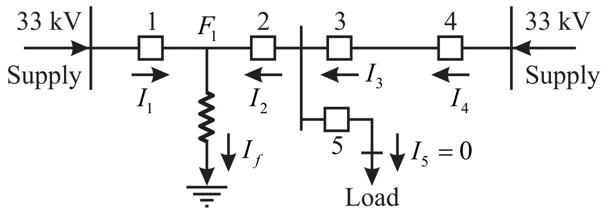
Case 1 :

During normal operation current direction is shown below,



Case 2 :

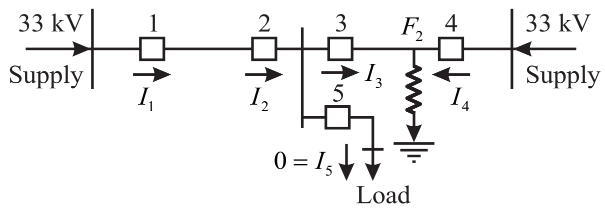
When fault occurs at F_1 (between 1 and 2), the current direction is shown below,



From this we can say that the relay 2 must operate for the above current direction.

Case 3 :

When fault occurs at F_2 (between 3 and 4) the current direction is shown below,



From this we can say that relay 3 must operate for the above current direction.

Hence, relays 2 and 3 must be directional in nature, remaining we can use non-directional in regard or economy.

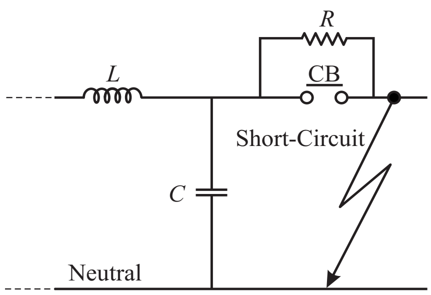
Hence, the correct option is (B).

14.2 **5**

Given :

(i) Line capacitance, $C = 0.01\ \mu F$

(ii) Line inductance, $L = 1\,H$



Method 1

Natural frequency of oscillation is given by,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{1}{4(RC)^2}}$$

During critical damping, $f_n = 0$

i.e., $$\frac{1}{LC} - \frac{1}{4(RC)^2} = 0$$

$$R = \frac{1}{2}\sqrt{\frac{L}{C}} = \frac{1}{2\times 1}\sqrt{\frac{1}{0.01\times 10^{-6}}}$$

$$R = 5000\ \Omega = 5\ k\Omega$$

Hence, the shunt resistor across the circuit breaker is **5 kΩ.**

Method 2

For parallel circuit damping ratio (ξ) is given by,

$$\xi = \frac{1}{2Q_P}$$

where, Q_P is quality factor for parallel RLC circuit.

$$Q_P = R\sqrt{\frac{C}{L}}$$

Thus, $$\xi = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

$$R = \frac{1}{2\xi}\sqrt{\frac{L}{C}}$$

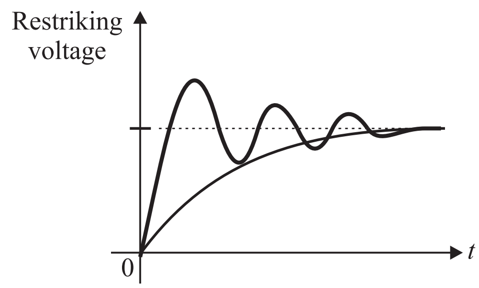
For critical damping of restriking voltage, $\xi = 1$

$$R = \frac{1}{2\times 1}\sqrt{\frac{1}{0.01\times 10^{-6}}} = 5000\ \Omega = 5\ k\Omega$$

Hence, the shunt resistor across the circuit breaker is **5 kΩ.**

Key Point

During fault, the resistance gets automatically connected across the arc (between circuit breaker contacts), this ensures effective damping of the high frequency restriking voltage.



14.3 **(B)**

Given :

(i) 11 kV alternator

(ii) Power rating, $S = 10\,MVA$

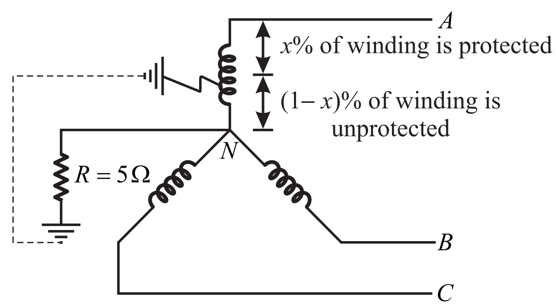
(iii) Neutral resistance, $R = 5\,\Omega$

(iv) C.T. ratio, $n = 1000/5$

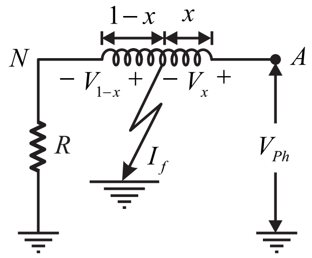
(v) Earth fault relay setting, $I_s = 0.75$ A.

Let, x% of winding is protected Fault current then,

The equivalent figure is represented as,



The equivalent figure of unprotected winding is given as,



Method 1

% unprotected winding is given by,

$$x = \frac{I_{PN} R}{V_{PN}} \times 100$$

where, I_{PN} = Minimum pickup current of relay

$$I_{PN} = n \times I_s = \frac{1000}{5} \times 0.75 = 150\text{ A}$$

$$x = \frac{I_{PN} R}{V_{PN}} \times 100 = \frac{150 \times 5}{\frac{11 \times 10^3}{\sqrt{3}}} \times 100 = 11.8\%$$

Thus, unprotected winding is given by,

$$100 - x = 100 - 11.8 = 88.2\%$$

Hence, the correct option is (B).

Method 2

The fault current is given by,

$$I_f = (1 - x)\frac{V_{ph}}{R}$$

Primary fault setting current is given by,

$$I_p = I_f (1 - x)\frac{V_{ph}}{R} \quad \ldots(i)$$

Primary resistor current setting is given by,

$$I_R = \frac{V_{ph}}{R} \quad \ldots(ii)$$

From equation (i) and (ii),

$$I_p = (1 - x) I_R$$

$$x = 1 - \frac{I_p}{I_R}$$

% of winding protected is given by,

$$x = \left(1 - \frac{I_p}{I_R}\right) \times 100$$

Primary fault setting current is given by,

$$I_p = 0.75 \times \frac{1000}{5} = 150\text{ A}$$

Earthing resistor current setting is given by,

$$I_R = \frac{V_{ph}}{R} = \frac{11000}{\sqrt{3}} \times \frac{1}{5} = 1270.17\text{ A}$$

% of winding protected,

$$x = \left(1 - \frac{150}{1270.17}\right) \times 100 = 88.2\%$$

Hence, the correct option is (B).

Method 3

Voltage across unprotected winding is given by,

$$V_{1-x} = V_{Ph} \times \frac{1 - x}{(1 - x) + x} = V_{Ph}(1 - x)$$

Fault current through earth fault relay is given by,

$$I_f = \frac{\text{Voltage of unprotected winding}}{\text{Neutral resistance}}$$

$$I_f = \frac{V_{ph}(1 - x)}{R} = \frac{11 \times 10^3}{\sqrt{3} \times 5}(1 - x)$$

$$I_f = 1270.17(1 - x) \quad \ldots(i)$$

Primary fault setting current is given by,

$$I_p = I_s \times n = 0.75 \times \frac{1000}{5} = 150\text{ A} \quad \ldots(ii)$$

The fault current through unprotected winding should not exceed primary fault current setting,

$$I_f \le I_p$$

From equations (i) and (ii)

$$1270.1(1 - x) = 150$$

$$1 - x = 0.118$$

$$x = 0.882$$

Thus, % of protected winding is 88.2%.

Hence, the correct option is (B).

14.4 0.227

Given :

(i) Operating time,

$$t_{op} = \frac{0.14 \times \text{Time multiplier setting}}{(\text{Plug setting multiplier})^{0.02} - 1}$$

(ii) Maximum fault current, $I_{f_{max}} = 2000$ A

(iii) Minimum fault current, $I_{f_{min}} = 500$ A

(iv) Time multiplier setting for relay R_B, $TSM = 0.1$

(v) Pick up current (plug setting for relay R_B), $I_{PR} = 5$ A.

Since, minimum current to be sensed by C.T. is 500 A, C.T. ratio is given by,

$$n = \frac{I_{f_{min}}}{\text{Plug setting}} = \frac{500}{5} = 100$$

Assuming 100% current setting.
Plug setting multiplier is given by,

$$\text{PSM} = \frac{I_{f_{max}}}{n \times I_{PR}} = \frac{2000}{5 \times \frac{100}{5}} = 20$$

Operating time of relay R_B is given by,

$$t_{op} = \frac{0.14 \times \text{Time multiplier setting}}{(\text{Plug setting multiplier})^{0.02} - 1}$$

$$t_{op} = \frac{0.14 \times 0.1}{(20)^{0.02} - 1} = 0.227 \text{ sec}$$

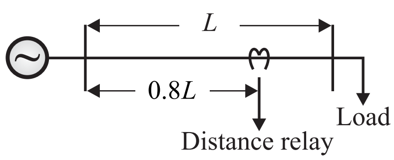
Hence, the operating time of R_B is **0.227 seconds**.

14.5 (C)

Given :

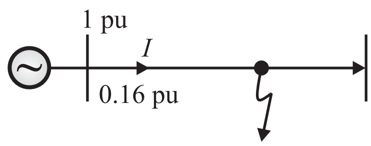
(i) $X_L = 0.2$ pu

(ii) $V_t = 1$ pu



The line to be protected is 80% of this it's length by a distance relay at the generator bus.

The impedance seen by the distance relay $= 0.8 \times 0.2$



$$I = \frac{1}{0.16} = 6.25 \text{ pu}$$

Hence, the correct option is (C).

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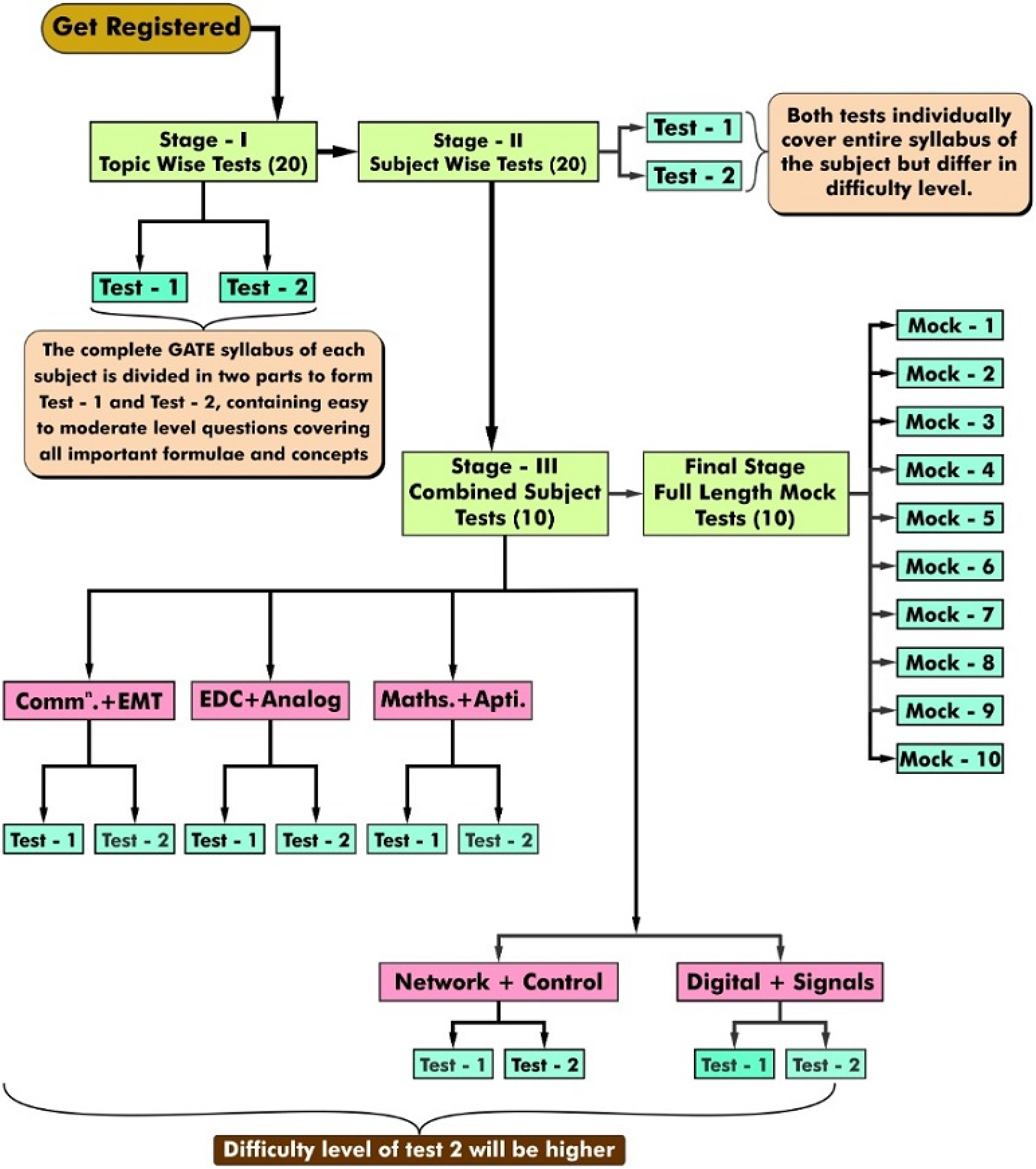

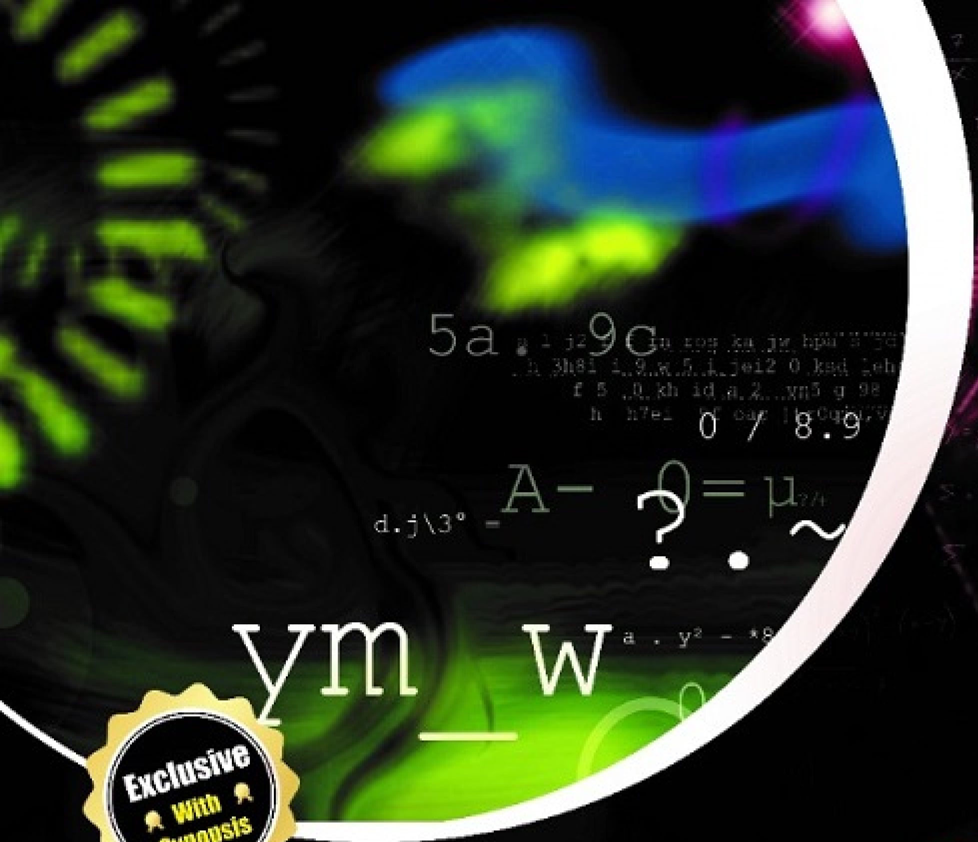

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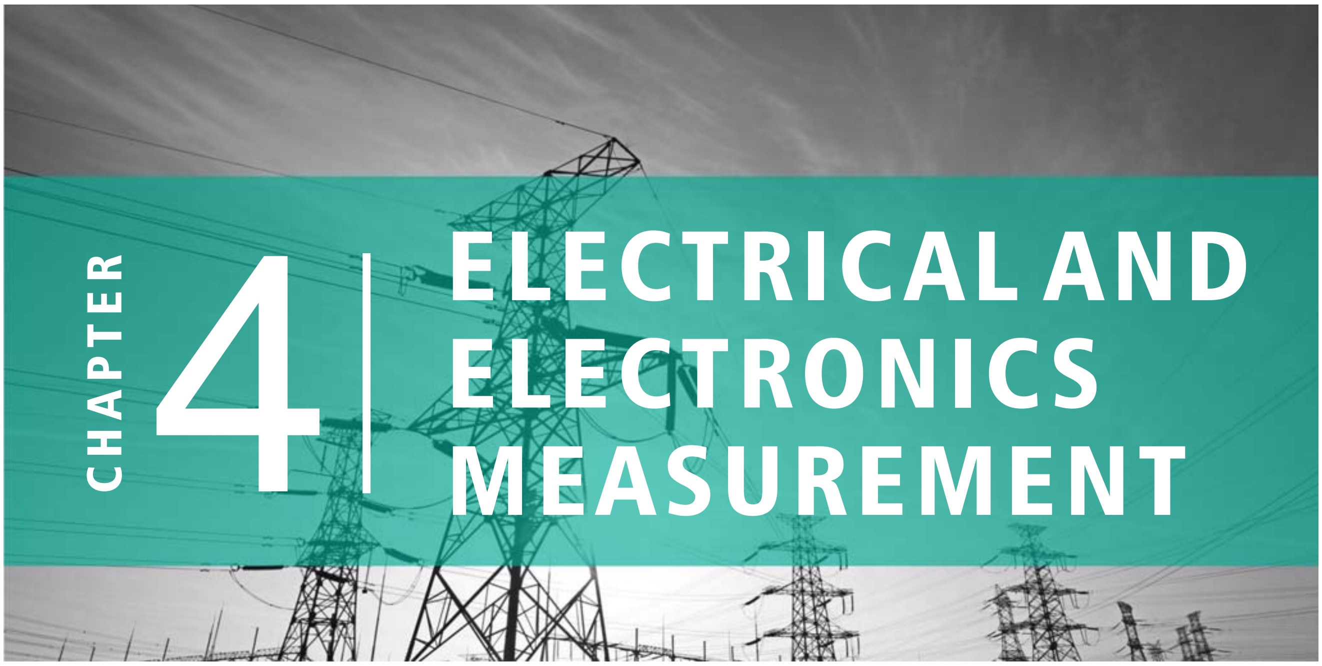


CHAPTER 4 | ELECTRICAL AND ELECTRONICS MEASUREMENT

Marks Distribution of Electrical and Electronic Measurements in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	3	8	19
2004	3	7	17
2005	3	5	13
2006	2	4	10
2007	1	1	3
2008	1	2	5
2009	2	2	6
2010	2	1	4
2011	3	1	5
2012	3	1	5
2013	2	1	4
2014 Set-1	2	2	6

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-2	2	2	6
2014 Set-3	2	2	6
2015 Set-1	2	1	4
2015 Set-2	3	7	17
2016 Set-1	0	0	0
2016 Set-2	1	2	5
2017 Set-1	2	0	2
2017 Set-2	2	2	6
2018	1	1	3
2019	–	2	4
2020	–	2	4
2021	1	–	1

Syllabus : Electrical and Electronics Measurement

Bridges and Potentiometers, Measurement of voltage, current, power, energy and power factor; Instrument transformers, Digital voltmeters and multimeters, Phase, Time and Frequency measurement; Oscilloscopes, Error analysis.

Contents : Electrical and Electronics Measurement

1 Error Analysis & Measurement

➢ Partial Synopsis

Classification of error :

Types of Error

- Absolute error/Static error
- Relative error

Absolute error/Static error	Relative error
Deviation between measured value and true value of quantity under measurement is called absolute error it is also known as limiting or guarantee error, $\delta = A_m - A_T$ Where, δ_A = absolute error, A_m = measured value, A_T = True value.	$\%\varepsilon_r = \frac{A_m - A_T}{A_T} \times 100\%$ $\%\varepsilon_r$ in measured scale $= \frac{\text{Full scale value} \times \%\text{Full scale error}}{\text{Measured scale}}$

Mathematical operations with errors :

Mathematical operations	Results with error
Sum of quantities	Let, $X_1 = a + \varepsilon_{r_1}$ $X_2 = b + \varepsilon_{r_2}$ $X_3 = c + \varepsilon_{r_3}$ For summation or difference $= X_1 \pm X_2 \pm X_3$ $\%\varepsilon_r = \pm\left[\frac{a}{a+b+c}\varepsilon_{r_1} + \frac{b}{a+b+c}\varepsilon_{r_2} + \frac{c}{a+b+c}\varepsilon_{r_3}\right]$
Multiplication of quantities	$X_1 = a + \varepsilon_{r_1}$ $X_2 = b + \varepsilon_{r_2}$ $X_3 = \varepsilon_{r_3}$ Let, $X = X_1 \times X_2 \times X_3$ $\%\varepsilon_r = \pm(\varepsilon_{r_1} + \varepsilon_{r_2} + \varepsilon_{r_3})\%$

Division of quantities	$X_1 = a + \varepsilon_{r_1}$ $X_2 = b + \varepsilon_{r_2}$ $X_3 = c + \varepsilon_{r_3}$ $X = \frac{X_1 X_2}{X_3}$ $\%\varepsilon_r = \pm(\varepsilon_{r_1} + \varepsilon_{r_2} + \varepsilon_{r_3})\%$
Error due to composite factor	Let, $X = \frac{X_1^m X_2^n}{X_3^p}$ or $\frac{X_1^m X_3^p}{X_2^n}$ $\%\varepsilon_r = \pm(m\varepsilon_{r_1} + n\varepsilon_{r_2} + p\varepsilon_{r_3})\%$
Error due to power of the quantity	$X^n = (a + \varepsilon_r)^n$ $\%\varepsilon_r = \pm(n\varepsilon_r)\%$

➢ Sample Questions

2001 IIT Kanpur

1.1 Resistance R_1 and R_2 have, respectively, nominal value of 10 Ω and 5 Ω and tolerances of ±5% and 10%. The range of values for the parallel combination of R_1 and R_2 is

(A) 3.077 Ω to 3.636 Ω

(B) 2.805 Ω to 3.371 Ω

(C) 3.237 Ω to 3.678 Ω

(D) 3.192 Ω to 3.435 Ω

2006 IIT Kharagpur

1.2 A variable w is related to three other variables x, y, z as $w = \frac{xy}{z}$. The variables are measured with meters of accuracy ±0.5% reading, ±1% of full scale value and ±1.5% reading respectively. The actual readings of the three meters are 80, 20 and 50 with 100 being the full scale value for all three. The maximum uncertainty in the measurement of w will be

(A) ±0.5% rdg (B) ±5.5% rdg

(C) ±6.7% rdg (D) ±7.0% rdg

2020 IIT Delhi

1.3 A non-ideal Si-based pn junction diode is tested by sweeping the bias applied across its terminals from –5 V to +5 V. The effective thermal voltage V_T, for the diode is measured to be (29 ± 2) mV. The resolution of voltage source in the measurement range is 1mV. The percentage uncertainty (rounded off to 2 decimal places) in the measured current at a bias voltage of 0.02 V is _____.

❖❖❖❖

Explanations **Error Analysis & Measurement**

1.1 (A)

Given :

(i) Nominal value of resistance

$R_1 = 10\ \Omega$ with tolerances of $\pm 5\%$ i.e.

$R_1 = 10 \pm 5\%$

So, range of $R_1 = 10 \pm 10 \times \frac{5}{100}$

Thus, range of $R_1 = 9.5\ \Omega$ to $10.5\ \Omega$.

(ii) Nominal value of resistance

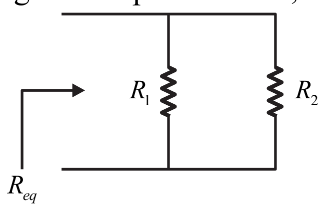
$R_2 = 5\ \Omega$ with tolerances of $\pm 10\%$ i.e.

$R_2 = 5 \pm 10\%$

So, range of $R_2 = 5 \pm 5 \times \frac{10}{100}$

Thus, range of $R_2 = 4.5\ \Omega$ to $5.5\ \Omega$.

For parallel combination of R_1 and R_2, the equivalent figure is represented as,



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

The value of R_{eq} when $R_1 = 9.5\ \Omega$ and $R_2 = 4.5\ \Omega$ is given by,

$$R_{eq} = \frac{9.5 \times 4.5}{9.5 + 4.5} = 3.05\,\Omega$$

The value of R_{eq} when $R_1 = 10.5\ \Omega$ and $R_2 = 5.5\ \Omega$ is given by,

$$R_{eq} = \frac{10.5 \times 5.5}{10.5 + 5.5} = 3.61\ \Omega$$

Thus, the range of values for the parallel combination of R_1 and R_2 is $3.05\ \Omega$ to $3.61\ \Omega$

Hence, the correct option is (A).

1.2 (D)

Given :

(i) Four variables w, x, y and z are related as, $w = \frac{xy}{z}$

(ii) Accuracy of meter $x = \pm 0.5\%$,

Accuracy of meter $y = \pm 1\%$ of full scale,

Accuracy of meter $z = \pm 1.5\%$

(iii) Actual readings of meter x, y and z are 80, 20 and 50 respectively.

(iv) Full scale value $= 100$

Method 1

Since, uncertainty is given in reading values in options.

Converting all the uncertainties in reading value,

Error at reading value

$$= \frac{\text{Error at full scale} \times \text{Full scale value}}{\text{Reading value}}$$

For *x*, $\delta x = \pm 0.5\ \%$

For *y*, $\delta y = \pm 1\% \times \frac{100}{20} = \pm 5\%$

For z, $\delta z = \pm 1.5\ \%$

Therefore,

$$\delta w = \delta x + \delta y + \delta z$$

$$\delta w = \pm 0.5\% + \pm 5\% + \pm 1.5\%$$

$$\delta w = \pm\ 7\%$$

Hence, the correct option is (D).

Method 2

$$w = \frac{xy}{z} \qquad \dots \text{(i)}$$

$\delta x = \pm 0.5\ \%$ of 80 (reading)

$$\delta x = \pm \frac{0.5}{100} \times 80 = \pm 0.4$$

$$\delta y = \pm 1\ \%\ \text{of}\ 100\ \text{(Full scale)}$$

$$\delta y = \frac{\pm 1}{100} \times 100 = 1$$

$$\delta z = \pm 1.5\ \%\ \text{of}\ 50\ \text{(reading)}$$

$$\delta z = \frac{\pm 1.5 \times 50}{100} = \pm 0.75$$

Taking log on both sides of equation (i),

$$\log w = \log x + \log y - \log z$$

$$\frac{\delta w}{w} = \frac{\delta x}{x} + \frac{\delta y}{y} - \frac{\delta z}{z}$$

For maximum uncertainty

$$\frac{\delta w}{w} = \pm\left(\frac{0.4}{80} + \frac{1}{20} + \frac{0.75}{50}\right) \times 100$$

$$\frac{\delta w}{w} = \pm 7\%$$

Hence, the correct option is (D).

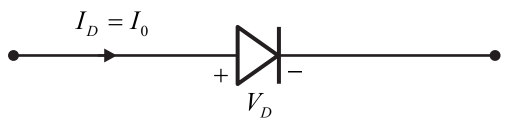
1.3 **4.75**

Given : $V_D = 0.02\ \text{V}$

$$V_T = (29 \pm 2)\ \text{mV} = (0.029 \pm 0.002)$$

$$I_D = I \cong I_0 e^{V_D/\eta V_T}$$

Non ideal silicon diode is as shown below



Applying log on both sides on diode current equation

$$\ln(I) = \ln(I_0) + \frac{V_D}{\eta V_T}$$

Differentiating partially with respect to V_T,

$$\frac{\partial I}{I} = 0 + \frac{V_D}{\eta} \times \left(-\frac{1}{V_T^2}\right) \partial V_T$$

$$\frac{\partial I}{I} = -\frac{I V_D}{\eta V_T^2} \times \partial V_T$$

For $[\eta = 1]$,

$$\frac{\partial I}{\partial V_T} = -\frac{I.V_D}{V_T^2}$$

The expression of uncertainty for diode current equation is given below,

$$W_{res} = W_I = \pm\sqrt{\left(\frac{\partial I}{\partial V_T}\right)^2 .W_v^2} = \pm\frac{\partial I}{\partial V_T} \times W_V$$

$$W_{res} = W_I = \pm\frac{I.V_D}{V_T^2}.W_V = \pm\frac{I \times 0.02}{(0.029)^2} \times 0.002$$

$$= \pm 0.0475\, I$$

$$\%\frac{W_I}{I} = \pm 0.0475 \times 100 = \pm 4.75$$

Percentage uncertainty $= \pm 4.75\%$

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2 Basic Instruments

➢ Partial Synopsis

Permanent Magnet Moving Coil (PMMC) instrument

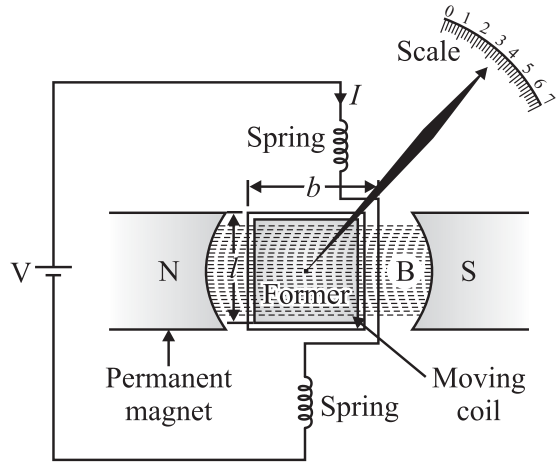


Fig. Circuit diagram

1. It is similar to a permanent magnet DC motor with limited rotation.
2. It works whenever a current carrying conductor is placed in a magnetic field experiences a mechanical force according to Lorentz principle.

Force exerted on the coil is,

$$F = nBIl\sin\theta \qquad [\because \theta = 90]$$

Deflecting torque will be,

$$T_d = F \times b = nBIlb$$

$$T_d = (nBA)I = GI \qquad [G = nBA]$$

Where, $n =$ number of turns of coil

$B =$ Magnetic flux density,

$A =$ Area of coil $= l \times b$

$K =$ Spring constant

At balance condition :

$$T_d = T_c \Rightarrow K\theta = GI$$

$$\theta = \frac{GI}{K} \Rightarrow \theta \propto I$$

Deflection ∝ current :

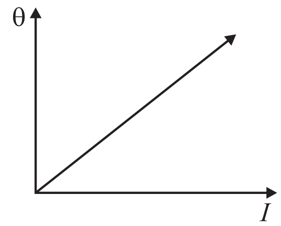


Fig. Relation between deflection and current

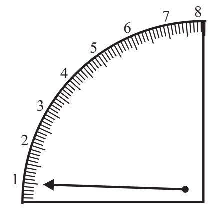


Fig. Measuring Scale

1. $\theta \propto I$ so that scale is uniform and linear.
2. $\theta \propto I$ so that PMMC measures average or DC voltage.
3. If a pure AC signal with equal value of positive and negative, then pointer vibrates near to zero position of the measuring scale.
4. If the output of half wave and full wave rectifier signal passing through PMMC it reads $\frac{I_m}{\pi}$ and $\frac{2I_m}{\pi}$
5. The deflecting torque produced by the permanent magnet is higher due to higher flux density and hence torque to weight (τ/w) ratio of the PMMC is higher so that accuracy is more and sensitivity also high because frictional error is less.

Disadvantages :

1. Permanent magnet flux density changes with surrounding temperature.
2. Spring tension changes with temperature.
3. It has higher cost compared to the moving iron instrument.
4. It measures up to 100 mA and voltage up to 50 mV.

Extension of meters

Ammeter

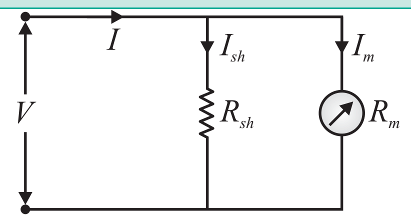


Fig. Circuit diagram

where, I_m = Full scale deflection of meter,

I_{sh} = Current through shunt resistance,

R_{sh} = Shunt resistance, Manganin/Eureka,

$(\alpha_m = 0.00015\ /^0C)$

Voltmeter

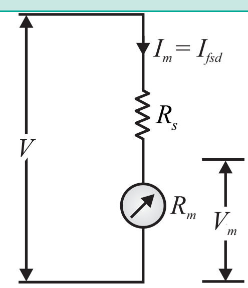


Fig. Circuit diagram

Where, V = Voltage to be measured

$$I_{fsd} = I_m = \text{Full scale deflection current} = \frac{V}{R_m + R_s}$$

R_m = Meter resistance, copper ($\alpha_{cu} = 0.004\ /^0C$) Using current divider rule, $I_m = \frac{R_{sh} I}{R_m + R_{sh}} \Rightarrow \frac{I}{I_m} = \frac{R_m + R_{sh}}{R_{sh}}$ $m = \frac{R_m + R_{sh}}{R_{sh}} \Rightarrow R_{sh} = \frac{R_m}{m-1}$ $[R_{sh} << R_m]$ Where, $m = \frac{I}{I_m}$ = Multiplying actor/Multiplication factor.	R_S = Series resistance, R_m = Meter resistance. Apply voltage division rule, $R_s = R_m(m-1)$ [$\because m =$ Multiplication/Multiplying factor] Voltage sensitivity, $S_V = \frac{1}{I_{fsd}} = \frac{R_m + R_s}{V}\ \Omega/\text{V}$

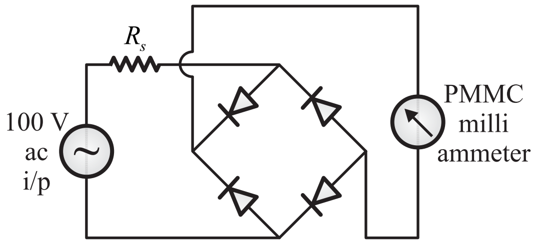
➢ Sample Questions

1996 IISc Bangalore

2.1 The scale of a voltmeter is uniform. Its type is
(A) moving iron
(B) induction
(C) moving coil permanent magnet
(D) moving coil dynamometer

2003 IIT Madras

2.2 A rectifier type ac voltmeter consists of a series resistance R_s, an ideal full –wave rectifier bridge and a PMMC instrument as shown in the figure. The internal resistance of the instrument is 100 Ω and a full scale deflection is produced by a dc current of 1 mA. The value of R_s required to obtain full scale deflection with an ac voltage of 100 V (rms) applied to the input terminals is



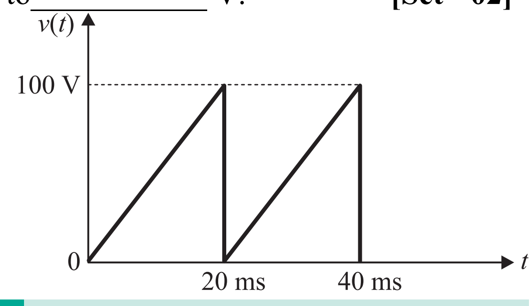
(A) 63.56 Ω (B) 89.93 Ω
(C) 89.93 kΩ (D) 141.3 kΩ

2005 IIT Bombay

2.3 A DC ammeter has a resistance of 0.1 Ω and its current range is 0 - 100 A. If the range is to be extended to 0-500 A, then meter requires the following shunt resistance
(A) 0.010 Ω (B) 0.011 Ω
(C) 0.025 Ω (D) 1.0 Ω

2014 IIT Kharagpur

2.4 The saw-tooth voltage waveform shown in the figure is fed to a moving iron voltmeter. Its reading would be close to____________ V. **[Set - 02]**



2015 IIT Kanpur

2.5 A (0-50 A) moving coil ammeter has a voltage drop of 0.1 V across its terminals at full scale deflection. The external shunt resistance (in milliohms) needed to extend its range to (0 - 500 A) is ___________. **[Set - 01]**

Explanations — Basic Instruments

2.1 (C)

In permanent magnet moving coil (PMMC) instruments the deflection (θ) is given by,

$$\theta = \frac{NBIA}{K} \qquad \ldots\text{(i)}$$

where, $N =$ Number of turns, $B =$ Flux density

$I =$ Current, $A =$ Area

$K =$ Spring constant

From equation (i),

$$\theta \propto I$$

i.e. scale is linear.

Hence, the correct option is (C).

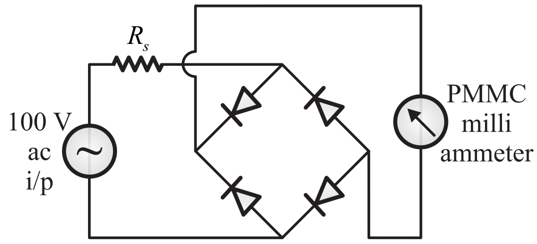
Key Point

S. No.	Types of instrument	Scale
1.	Moving iron	Square $(\theta \propto I^2)$
2.	Induction type instrument	Square $(\theta \propto I^2)$
3.	PMMC	Linear $(\theta \propto I)$
4.	Moving coil dynamometer	Square $(\theta \propto I^2)$

2.2 (C)

Given :

(i) A rectifier type ac voltmeter consists of a series resistance R_s, an ideal full –wave rectifier bridge and a PMMC instrument as shown in the figure,



(ii) $I_{fsd} =$ Current required to produce full scale deflection $= 1$ mA

(iii) $R_m = 100\ \Omega$

(iv) $V_{in} = 100$ V (rms)

(v) Diodes are ideal i.e. forward resistance, $R_f = 0$

Method 1

The dc sensitivity (S_{dc}) is given by,

$$S_{dc} = \frac{1}{I_{fsd}} = \frac{1}{1\text{ mA}} = 10^3\ \Omega/\text{V}$$

For full wave rectifier ac sensitivity (S_{ac}) is given by,

$$S_{ac} = 0.9\, S_{dc}$$

$$S_{ac} = 0.9 \times 10^3\ \Omega/\text{V} = 900\ \Omega/\text{V}$$

Resistance of multiplier (R_s) is given by,

$$R_s = S_{ac} V_{ac} - R_m - 2R_f$$

$$R_s = (900 \times 100) - 100 - (2 \times 0)$$

$$R_s = 89.9\ \text{k}\Omega$$

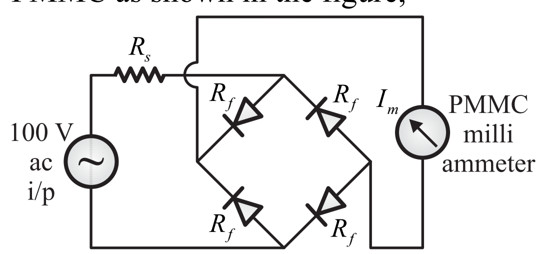
Hence, the correct option is (C).

Method 2

$$I_{fsd} = 1\,\text{mA},\ R_m = 100\,\Omega$$

$$V_{in} = 100\,\text{V (rms)},\ R_f = 0$$

For a full wave bridge rectifier connected with PMMC as shown in the figure,



The value of meter current (I_m) is given by,

$$I_m = I_{average} = \frac{0.9V_{rms}}{R_s + 2R_f + R_m}$$

$$1\times10^{-3} = \frac{100\times0.9}{R_s + (2\times0) + 100}$$

$$R_s = \frac{100\times0.9}{1\times10^{-3}} - 100 = 89.9 \text{ k}\Omega$$

Hence, the correct option is (C).

Key Point

Full wave rectifier type meter :

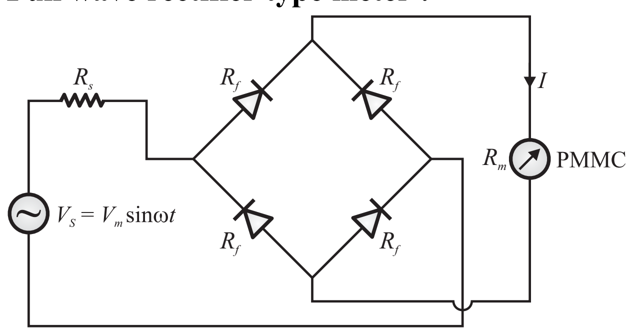


Fig. Circuit diagram

For full wave rectifier,

$$I_{average} = \frac{2I_m}{\pi} = \frac{2}{\pi}\left[\frac{V_m}{R_s + 2R_f + R_m}\right]$$

$$I_{average} = \frac{2\sqrt{2}V_{rms}}{\pi[R_s + 2R_f + R_m]}$$

$$I_{average} = \frac{0.9V_{rms}}{R_s + 2R_f + R_m}$$

For ideal diode, $R_f = 0$

So, $$I_{average} = \frac{0.9V_{rms}}{R_s + R_m}$$

For DC input,

$$I_{m\,dc} = \frac{V_{dc}}{R_s + 2R_f + R_m}$$

where, $|V_{dc}| = |V_{rms}|$

$$\boldsymbol{I_{average} = 0.9\, I_{m\,dc}}$$

Since, average current through the meter is 0.9 times the current for dc input. Therefore a.c. sensitivity for full wave rectifier type instrument is 0.9 times the d.c. sensitivity.

2.3 (C)

Given :

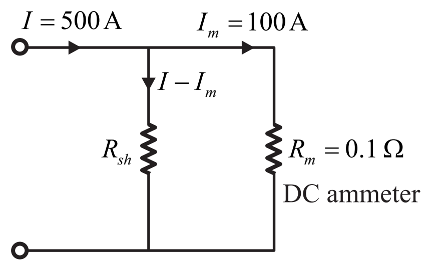
(i) Meter resistance, $R_m = 0.1\ \Omega$

(ii) Meter current, $I_m = 100$ A

(iii) Extended range of meter, $I = 500$ A

Method 1

In enhancement of ammeter, we have to connect a shunt resistance (R_{sh}) across the meter as,



Since, R_{sh} and R_m are connected in parallel. Hence, the voltage drop across the R_{sh} is same as the voltage drop across the R_m i.e.

$$(I - I_m)R_{sh} = R_m \times I_m$$

$$R_{sh} = \frac{R_m I_m}{I - I_m}$$

$$R_{sh} = \frac{0.1\times100}{500-100} = 0.025\ \Omega$$

Hence, the correct option is (C)

Method 2

$$R_m = 0.1\ \Omega,\ I_m = 100 \text{ A},\ I = 500 \text{ A}$$

In enhancement of ammeter, we have to connect a shunt resistance (R_{sh}) across the meter and its value is given as, $R_{sh} = \frac{R_m}{m-1}$ .... (i)

where, $m =$ multiplying factor

Calculation of m :

Multiplying factor (m) is given by,

$$m = \frac{I}{I_m} = \frac{500}{100} = 5$$

From equation (i),

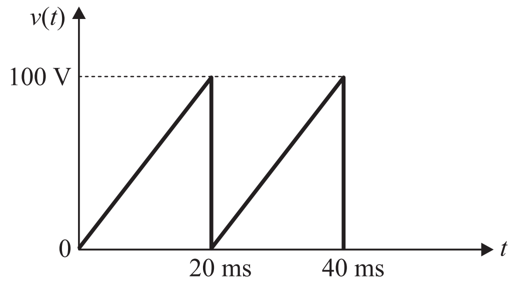
Shunt resistance $(R_{sh}) = \dfrac{R_m}{m-1}$

$$R_{sh} = \frac{0.1}{5-1} = 0.025\ \Omega$$

Hence, the correct option is (C).

2.4 57.73

Given :



From given waveform,

$$v(t) = \begin{cases} 5000t & ; \quad 0 < t < 0.02 \text{ sec} \\ 5000(t-0.02); & 0.02 < t < 0.04 \text{ sec} \end{cases}$$

Time period $(T) = 20$ ms

Since, moving iron voltmeter reads rms value of voltage.

Calculation of V_{rms} :

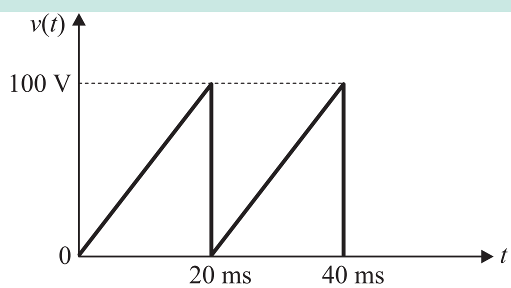
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T [v(t)]^2\, dt}$$

$$V_{rms} = \sqrt{\frac{1}{0.02}\int_0^{0.02} [5000t]^2\, dt}$$

$$V_{rms} = \sqrt{\frac{1}{0.02}\left[5000^2 \times \left(\frac{t^3}{3}\right)_0^{0.02}\right]}$$

$$V_{rms} = 57.73 \text{ V}$$

OR



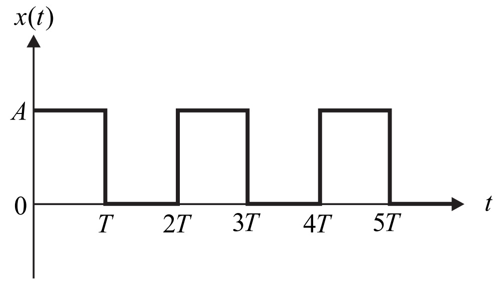
Given waveform is sawtooth waveform with amplitude of 100 volt,
The rms value of $v(t)$ is given by,

$$V_{rms} = \frac{V_m}{\sqrt{3}} = \frac{100}{\sqrt{3}} = 57.73 \text{ V}$$

Hence, the reading by moving iron voltmeter is **57.73 V.**

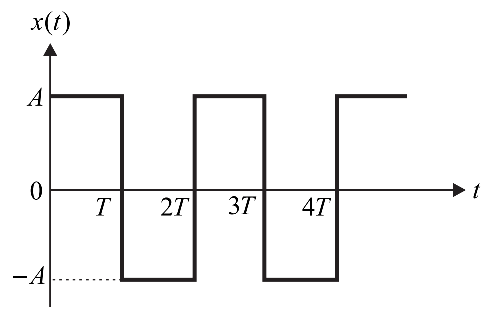
Key Point

(A) If the waveform is in the form as shown in figure :



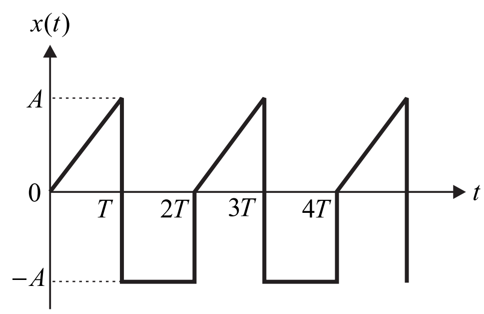
Then, the rms value of $x(t) = \dfrac{A}{\sqrt{2}}$

(B) If the waveform is in the form as shown in figure :



Then, the rms value of $x(t) = A$

(C) If the waveform is in the form as shown in figure :



Then, the rms value of $x(t) = \sqrt{\dfrac{2}{3}}A$

2.5 **0.22**

Given :

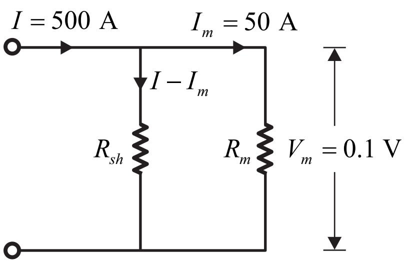
(i) Voltage drop across the meter $(V_m) = 0.1 \text{ V}$

(ii) Full scale meter current $(I_m) = 50 \text{ A}$

(iii) Extended range of meter, $(I) = 500 \text{ A}$

Method 1

In enhancement of ammeter, we have to connect a shunt resistance (R_{sh}) across the meter as,



Since, R_{sh} and R_m are connected in parallel. Hence, the voltage drop across the R_{sh} is same as the voltage drop across the R_m i.e.

$$(I - I_m)R_{sh} = R_m \times I_m = 0.1 \text{ V}$$

$$R_{sh} = \frac{0.1}{I - I_m}$$

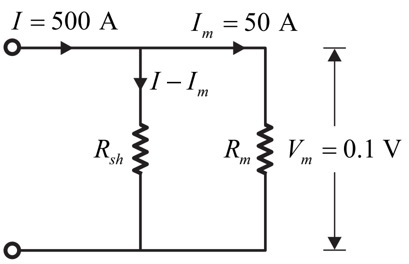
$$R_{sh} = \frac{0.1}{500 - 50} = 0.22 \text{ m}\Omega$$

Hence, the resistance needed is **0.22 mΩ.**

Method 2

$$V_m = 0.1 \text{ V}, \ I_m = 50 \text{ A}, \ I = 500 \text{ A}$$

In enhancement of ammeter, we have to connect a shunt resistance (R_{sh}) across the meter as,



From figure,

$$I_m \times R_m = 0.1$$

$$R_m = \frac{0.1}{50} = 2 \text{ m}\Omega$$

The value of shunt resistance (R_{sh}) is given by,

$$R_{sh} = \frac{R_m}{m - 1} \quad \dots \text{(i)}$$

where, $m =$ multiplying factor

Calculation of *m* :

Multiplying factor (m) is given by,

$$m = \frac{I}{I_m} = \frac{500}{50} = 10$$

From equation (i),

$$\text{Shunt resistance } (R_{sh}) = \frac{R_m}{m - 1}$$

$$R_{sh} = \frac{2 \times 10^{-3}}{10 - 1} = 0.22 \text{ m}\Omega$$

Hence, the resistance needed is **0.22 mΩ.**

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5 Measurement of Energy & Power

➢ Partial Synopsis

Power in D.C. Circuits

<table>
<tr>
<th>Ammeter connected near the load
(Ammeter-voltmeter method)</th>
<th>Voltmeter connected near the load
(Voltmeter-Ammeter method)</th>
</tr>
<tr>
<td>

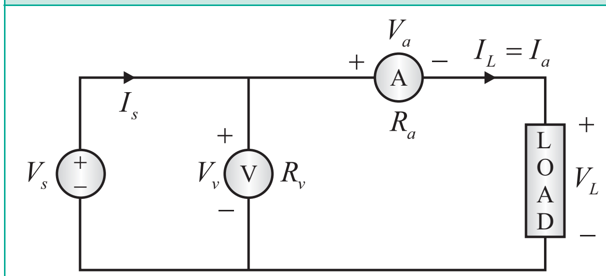


Fig. Circuit Diagram

$$\% \text{Error} = \frac{P_m - P_T}{P_T} \times 100\%$$

$$\% \text{Error} = \frac{(P_L + P_a) - P_L}{P_L} \times 100\%$$

$$\% \text{Error} = \frac{P_a}{P_L} \times 100\% = \frac{V_a I_L}{V_L I_L} \times 100\% = \frac{V_a}{V_L} \times 100\%$$

$$\% \text{Error} = \frac{V_a}{V_L} \times 100\% = \frac{I_L . R_a}{I_L . R_L} \times 100\%$$

$$\% \text{Error} = \frac{R_a}{R_L} \times 100\%$$

From the above equation it is clear that the error in the reading will be small when load resistance is large. Therefore, the above should be preferred for large resistance.

</td>
<td>

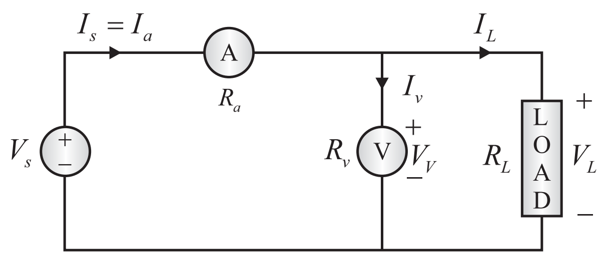


Fig. Circuit Diagram

$$\% \text{Error} = \frac{P_m - P_T}{P_T} \times 100\%$$

$$\% \text{Error} = \frac{(P_L + P_v) - P_L}{P_L} \times 100\%$$

$$\% \text{Error} = \frac{P_v}{P_L} \times 100 = \frac{\frac{V_L^2}{R_v}}{\frac{V_L^2}{R_L}} \times 100\%$$

$$\% \text{Error} = \frac{R_L}{R_v} \times 100\%$$

From the above equation it is clear that the error in the reading will be small when load resistance is small. Therefore, above circuit should be preferred for measurement of power consumed by small resistance.

</td>
</tr>
</table>

Remember

In measurement of power with voltmeter-ammeter method and ammeter-voltmeter method, the measured power is equal to the true power consumed by load and power consumed by the meter connected near the load.

Case I : Ammeter near the load, $P_m = P_T + P_a$

Case II : Voltmeter near the load, $P_m = P_T + P_v$

Harmonic signal :

PC Circuit is energized by instantaneous voltage,

$$v = V_0 + V_1 \sin(\omega t + \theta_1) + V_2 \sin(2\omega t + \theta_2) + ...$$

CC Circuit is energized by instantaneous current,

$$i = I_0 + I_1 \sin(\omega t + \phi_1) + I_2 \sin(2\omega t + \phi_2) + ...$$

$$\therefore \qquad P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} vi\, d(\omega t)$$

Wattmeter Reading :

$$P_{avg} = V_0 I_0 + \frac{1}{2}[V_1 I_1 \cos(\theta_1 - \phi_1) + V_2 I_2 \cos(\theta_2 - \phi_2) + ...]$$

where $V_1, V_2 .... I_1, I_2 ....$ are peak values

Sample Questions

1997　IIT Madras

5.1 A dynamometer type wattmeter responds to the

(A) average value of active power

(B) average value of reactive power

(C) peak value of active power

(D) peak value of reactive power

2001　IIT Kanpur

5.2 If an energy meter disc makes 10 revolutions in 100 seconds when a load of 450 W is connected to it, the meter constant (in rev/kWh) is

(A) 1000　　(B) 500

(C) 1600　　(D) 800

2003　IIT Madras

5.3 A wattmeter reads 400 W when its current coil is connected in the *R* phase and its pressure coil is connected between this phase and the neutral of a symmetrical 3-phase system supplying a balanced star connected 0.8 p.f. inductive load. This phase sequence is *RYB*. What will be the reading of this wattmeter if its pressure coil alone is reconnected between the *B* and *Y* phases, all other connections remaining as before?

(A) 400.0　　(B) 519.6

(C) 300.0　　(D) 692.8

❖❖❖❖

Explanations Measurement of Energy & Power

5.1 (A)

Average deflecting torque in electrodynamometer type wattmeter,

$$T_d = \frac{VI}{R_P}\cos\phi\frac{dM}{d\theta}$$

where, R_P = resistance of pressure coil circuit

controlling torque,

$$T_c = K\theta$$

At balance, $T_c = T_d$

Therefore, $K\theta = \frac{VI}{R_P}\cos\phi\frac{dM}{d\theta}$

$$\theta = VI\cos\phi\cdot\frac{1}{KR_P}\cdot\frac{dM}{d\theta}$$

$$\theta \propto VI\cos\phi$$

$\theta \propto$ Average active power.

Thus, electrodynamometer type wattmeter responds to the average active power.

Hence, the correct option is (A).

Key Point

For electrodynamometer type instruments :

$$T_{d\,avg} = I_1 I_2\cos\phi\frac{dM}{d\theta}$$

where, I_2 = Current flowing through the potential coil.

I_1 = Current flowing through the current coil.

Potential coils are highly resistive.

$$I_2 = \frac{V}{R_s}$$

$$T_{d\,avg} = \frac{I_1 V}{R_s}\cos\phi\frac{dM}{d\theta}$$

$$T_{d\,avg} = \frac{P_{avg}}{R_s}\frac{dM}{d\theta}$$

$$T_{d\,avg} \propto P_{avg}$$

Therefore, wattmeter gives average output active power.

5.2 (D)

Given : An energy meter disc makes 10 revolutions in 100 seconds when a load of 450 W is connected.

Thus, revolution per second $= \frac{10}{100} = 0.1$

Meter constant K is given by,

$$K\left(\frac{\text{rev}}{\text{kWh}}\right) = \frac{10\times3600}{100\times450\times10^{-3}}$$

$$K\left(\frac{\text{rev}}{\text{kWh}}\right) = 800 \text{ rev/kWh}$$

Hence, the correct option is (D).

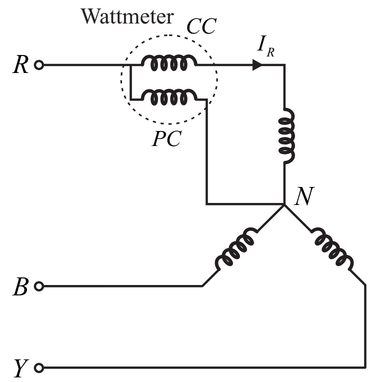
Key Point

Meter constant, $K = \frac{\text{Revolution}}{\text{Energy consumed in kWh}}$

5.3 (B)

Given :

Case I :



Let, V_R = Reference voltage

V = Phase to neutral voltage

$V_R = V\angle 0^0$

$V_Y = V\angle -120^0$

And $V_B = V\angle 120^0$ or $V\angle -240^0$

$V_R I_R\cos\phi = 400$ W (Given)

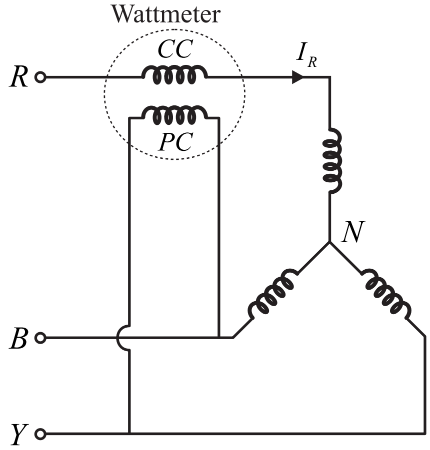
$$VI_R \times 0.8 = 400 \text{ W}$$

$(\cos\phi = \text{Power factor} = 0.8 \text{ lagging})$

$$VI_R = 500 \qquad \ldots \text{(i)}$$

$$I_R = I_R \angle -36.86^0$$

Case II :



$$V_{YB} = V_Y - V_B$$

$$V_{YB} = V\angle -120^0 - V\angle -240^0$$

$$V_{YB} = \sqrt{3}V\angle -90^0 V$$

Angle between V_{YB} and I_R is given by,

$$\theta = -90 - (-36.86) = -53.130$$

As pressure coil is connected between Y and B phases.

Reading of wattmeter $= V_{YB} I_R \cos\theta$

$$P = \sqrt{3} V I_R \cos(-53.13)$$

$$P = \sqrt{3} \times 500 \times 0.6 \text{ \{From equation (i)\}}$$

$$P = 519.6 \text{ W}$$

Hence, the correct option is (B).

❖❖❖❖

6 CRO & Electronic Measurement

➢ Partial Synopsis

Lissajous Pattern :

1. If the horizontal and vertical deflecting plates both are applied with sinusoidal waveforms then waveform pattern appear on the screen is called Lissajous pattern.
2. By using Lissajous pattern
   (i) Unknown phase difference between two signal can be calculated.
   (ii) unknown frequency of signal from the known frequency can be calculated.

Key Point

At any point of time electron beam appearing on the screen is the vector sum of the voltages applied to the horizontal and vertical deflecting plates vector sum consisting of both magnitude and phase angle.

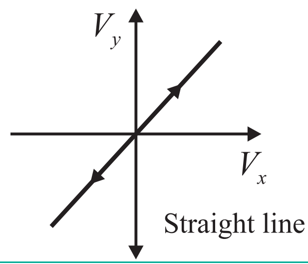
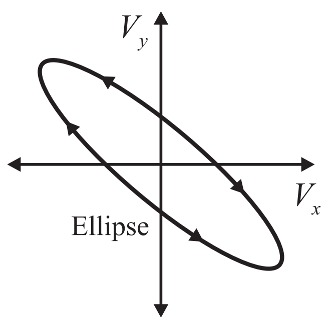
Let $V_x = V_m \sin \omega t$

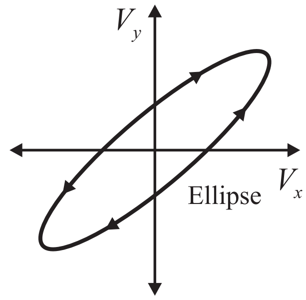
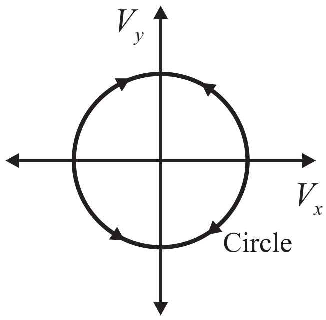
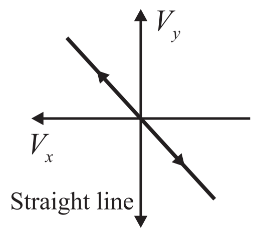
$V_y = V_m \sin(\omega t + \phi)$

Case I : $\omega_x = \omega_y$, $\phi =$ varying

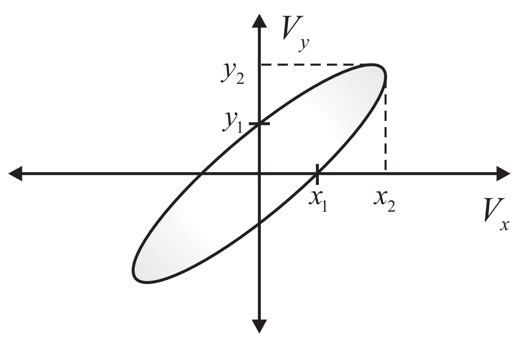
$v_x = v_m \sin \omega t$ (Horizontal plate)

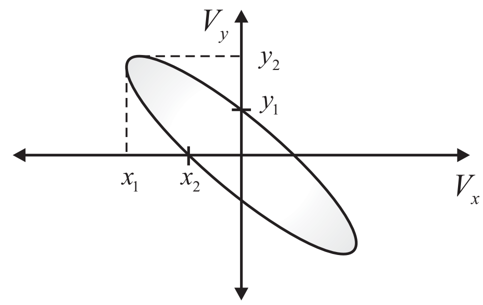
$v_y = v_m \sin(\omega t + \phi)$ (Vertical plate)

Phase angle	Lissajous Pattern
$\phi = 0 = 360^0$	
$90^0 < \phi < 180^0$ $180^0 < \phi < 270^0$	

$\phi = 0 < \phi < 90^0$ $270 < \phi < 360^0$	
$\phi = 90^0 \text{ or } 270^0$	
$\phi = 180^0$	

Finding ϕ If Lissajous pattern is given

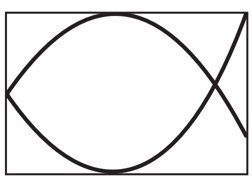
Quadrant of lissajous pattern	Lissajous pattern & phase angle (ϕ)
If LP is in 1st and 3rd quadrant	 $\phi = \sin^{-1}\left(\frac{x_1}{x_2}\right) \text{ or } \sin^{-1}\left(\frac{y_1}{y_2}\right)$ or $360^0 - \phi$

If LP is in the 2^{nd} and 4^{th} quadrant	
	$\phi = 180^0 - \sin^{-}\left(\frac{x_1}{x_2}\right) \Big/ 180^0 - \sin^{-}\left(\frac{y_1}{y_2}\right)$

➢ Sample Questions

1994 IIT Kharagpur

6.1 A Lissajous pattern, as shown in figure, is observed on the screen of a CRO when voltage of frequencies f_x and f_y are applied to the X and Y plates respectively. $f_x : f_y$ is then equal to



(A) 3 : 2

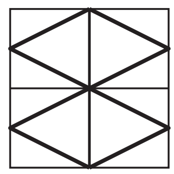
(B) 1 : 2

(C) 2 : 3

(D) 2 : 1

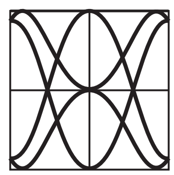
1996 IIT Kharagpur

6.2 In an oscilloscope the input to the horizontal plates is a 100 Hz voltage signal. The lissajous pattern (A), (B) and (C) will be generated when different frequency voltage signals are applied to vertical plates. Match each lissajous pattern to the corresponding frequency by

(A) 

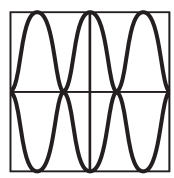
P. $f_y = 50$ Hz

Q. $f_y = 66.66$ Hz

(B) 

R. $f_y = 50$ Hz

S. $f_y = 150$ Hz

(C) 

T. $f_y = 200$ Hz

U. $f_y = 300$ Hz

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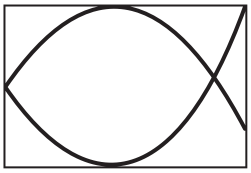
Explanations   CRO & Electronic Measurement

6.1 (C)

Given Lissajous pattern is shown below,



Method 1 : By Intersection Method

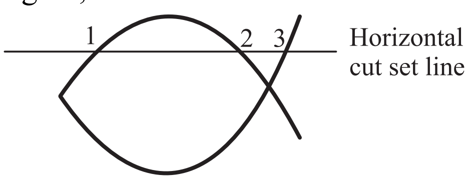
In $X-Y$ plot of CRO frequency of Y input signal is given by,

$$f_y = f_x \times \frac{\text{Horizontal cuts}}{\text{Vertical cuts}}$$

$$\frac{f_x}{f_y} = \frac{\text{Vertical cuts}}{\text{Horizontal cuts}} \quad \ldots \text{(i)}$$

(i) Calculation of horizontal cuts :

Cut the given Lissajous pattern by a horizontal line in such a way that it gives the maximum cuts as shown in figure,

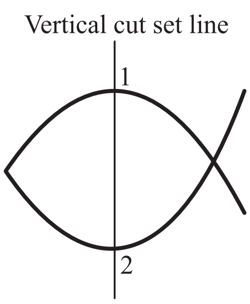


From the above figure,

Number of horizontal cuts = 3

(ii) Calculation of vertical cuts :

Cut the given Lissajous pattern by a vertical line in such a way that they gives the maximum cuts as shown in figure,



Number of vertical cuts = 2

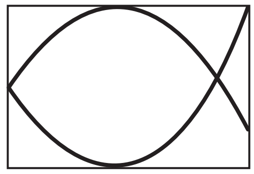
From equation (i),

$$\frac{f_x}{f_y} = \frac{\text{Vertical cuts}}{\text{Horizontal cuts}} = \frac{2}{3}$$

Hence, the correct option is (C).

Method 2 : By Tangent Method

The Lissajous pattern is shown below,

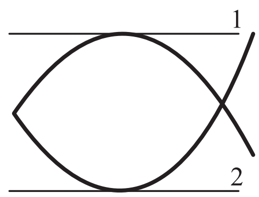


In $X-Y$ plot of CRO frequency of Y input signal is given by,

$$f_y = f_x \times \frac{\text{No. of Horizontal tangencies}}{\text{No. of Vertical tangencies}}$$

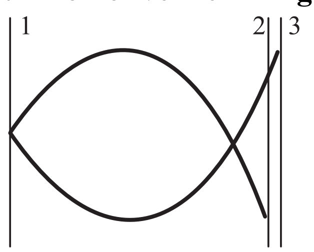
$$\frac{f_y}{f_x} = \frac{\text{No. of Horizontal tangencies}}{\text{No. of Vertical tangencies}}$$

(i) Calculation of horizontal tangencies :



From figure, the number of horizontal tangent = 2

(ii) Calculation of vertical tangencies :



From figure, the number of vertical tangent = 3

The ratio of frequency $f_x : f_y$ is given by,

$$\frac{f_x}{f_y} = \frac{\text{No. of Vertical tangencies}}{\text{No. of Horizontal tangencies}} = \frac{2}{3}$$

Hence, the correct option is (C).

6.2 **A-(P), B-(S), C-(U)**

Given : $f_x = 100$ Hz

(A) $\frac{f_y}{f_x} = \frac{\text{No. of Horizontal tangencies}}{\text{No. of Vertical tangencies}}$

$\frac{f_y}{f_x} = \frac{1}{2}$

$f_y = \frac{1}{2} \times f_x = \frac{1}{2} \times 100 = 50 \text{ Hz i.e. } (P)$

(B) $\frac{f_y}{f_x} = \frac{3}{2}$

$f_y = \frac{3}{2} \times 100 = 150 \text{ Hz i.e. } (S)$

(C) $\frac{f_y}{f_x} = \frac{3}{1}$

$f_y = 3 \times 100 = 300 \text{ Hz i.e. (U)}$

Hence, the correct answer is **(A)-(P), (B)-(S), (C)-(U).**

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CHAPTER 5 ELECTROMAGNETIC FIELDS

Marks Distribution of Electromagnetic Field in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	2	3	8
2004	1	–	1
2005	1	–	1
2006	–	1	2
2007	1	4	9
2008	1	3	7
2009	–	–	–
2010	–	–	–
2011	–	1	2
2012	–	1	2
2013	1	1	3
2014 Set-1	1	1	3

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-2	1	1	3
2014 Set-3	1	1	3
2015 Set-1	3	1	5
2015 Set-2	3	1	5
2016 Set-1	2	1	4
2016 Set-2	2	2	6
2017 Set-1	2	1	4
2017 Set-2	2	1	4
2018	1	1	3
2019	1	–	1
2020	–	3	6
2021	2	2	6

Syllabus : Electromagnetic Fields

Coulomb's Law, Electric Field Intensity, Electric Flux Density, Gauss's Law, Divergence, Electric field and potential due to point, line, plane and spherical charge distributions, Effect of dielectric medium, Capacitance of simple configurations, Biot-Savart's law, Ampere's law,Curl, Faraday's law, Lorentz force, Inductance, Magnetomotive force, Reluctance, Magnetic circuits, Self and Mutual inductance of simple configurations.

Contents : Electromagnetic Fields

1 Electrostatics

➢ Partial Synopsis

Coordinate Systems and their Related Parameters

	Parameters	Cartesian	Cylindrical	Spherical
1.	Variables	x, y, z	ρ, ϕ, z	r, θ, ϕ
2.	Range of variables	$-\infty < x < \infty$ $-\infty < y < \infty$ $-\infty < z < \infty$	$0 \le \rho < \infty$ $0 \le \phi \le 2\pi$ $-\infty < z < \infty$	$0 \le r < \infty$ $0 \le \theta \le \pi$ $0 \le \phi \le 2\pi$
3.	Representation	$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$	$\vec{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$	$\vec{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$
4.	Differential length parameter	$dx =$ differential length in x direction $dy =$ differential length in y direction $dz =$ differential length in z direction	$d\rho =$ differential length in ρ direction $\rho d\phi =$ differential length in ϕ direction $dz =$ differential length in z direction	$dr =$ differential length in r direction $rd\theta =$ differential length in θ direction $r \sin\theta\, d\phi =$ differential length in ϕ direction
5.	Differential displacement	$d\vec{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}$	$d\vec{l} = d\rho\,\hat{a}_\rho + \rho d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$	$d\vec{l} = dr\,\hat{a}_r + rd\theta\,\hat{a}_\theta + r\sin\theta\, d\phi\,\hat{a}_\phi$
6.	Differential surface area (With unit normal)	$d\vec{S}_x = dy\,dz\,\hat{a}_x$ $d\vec{S}_y = dx\,dz\,\hat{a}_y$ $d\vec{S}_z = dx\,dy\,\hat{a}_z$	$d\vec{S}_\rho = \rho\, d\phi\, dz\,\hat{a}_\rho$ $d\vec{S}_\phi = d\rho\, dz\,\hat{a}_\phi$ $d\vec{S}_z = \rho d\rho\, d\phi\,\hat{a}_z$	$d\vec{S}_r = r^2 \sin\theta\, d\theta\, d\phi\,\hat{a}_r$ $d\vec{S}_\theta = r\sin\theta\, d\phi\, dr\,\hat{a}_\theta$ $d\vec{S}_\phi = r\, dr\, d\theta\,\hat{a}_\phi$
7.	Differential volume	$dV = dx\,dy\,dz$	$dV = \rho d\rho\; d\phi\; dz$	$dV = r^2 \sin\theta\; dr\, d\theta\, d\phi$

8.	Relation between the variables	-----	$\rho=\sqrt{x^2+y^2}$, $\phi=\tan^{-1}\frac{y}{x}$, $z=z$	$r=\sqrt{x^2+y^2+z^2}$, $\theta=\tan^{-1}\frac{\sqrt{x^2+y^2}}{z}$, $z=r\cos\theta$
9.	Transformation matrix of $\vec{A}$	$\begin{bmatrix}A_x\\A_y\\A_z\end{bmatrix}=\begin{bmatrix}\cos\phi & -\sin\phi & 0\\ \sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}A_\rho\\A_\phi\\A_z\end{bmatrix}=\begin{bmatrix}\sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi\\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi\\ \cos\theta & -\sin\theta & 0\end{bmatrix}\begin{bmatrix}A_r\\A_\theta\\A_\phi\end{bmatrix}$		

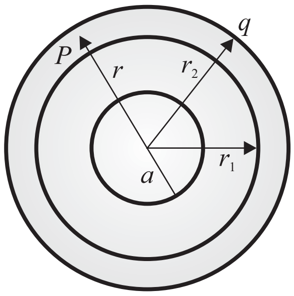
Important Differential Operation

	Operator	Cartesian coordinate	Cylindrical coordinate	Spherical coordinate
		$V=f(x,y,z)$ $\vec{A}=A_x\hat{a}_x+A_y\hat{a}_y+A_z\hat{a}_z$	$V=f(\rho,\phi,z)$ $\vec{A}=A_\rho\hat{a}_\rho+A_\phi\hat{a}_\phi+A_z\hat{a}_z$	$V=f(x,y,z)$ $\vec{A}=A_r\hat{a}_r+A_\theta\hat{a}_\theta+A_\phi\hat{a}_\phi$
1.	Gradient	$\nabla V=\frac{\partial V}{\partial x}\hat{a}_x+\frac{\partial V}{\partial y}\hat{a}_y+\frac{\partial V}{\partial z}\hat{a}_z$	$\nabla V=\frac{\partial V}{\partial \rho}\hat{a}_\rho+\frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi+\frac{\partial V}{\partial z}\hat{a}_z$	$\nabla V=\frac{\partial V}{\partial r}\hat{a}_r+\frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta+\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi$
2.	Divergence	$\nabla\cdot\vec{A}=\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$	$\nabla\cdot\vec{A}=\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho)+\frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi}+\frac{\partial A_z}{\partial z}$	$\nabla\cdot\vec{A}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$
3.	Curl	$\nabla\times\vec{A}=\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_x & A_y & A_z\end{vmatrix}$	$\nabla\times\vec{A}=\frac{1}{\rho}\begin{vmatrix}\hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z\\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z}\\ A_\rho & \rho A_\phi & A_z\end{vmatrix}$	$\nabla\times\vec{A}=\frac{1}{r^2\sin\theta}\begin{vmatrix}\hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi}\\ A_r & rA_\theta & r\sin\theta A_\phi\end{vmatrix}$
4.	Laplacian	$\nabla^2V=\frac{\partial^2V}{\partial x^2}+\frac{\partial^2V}{\partial y^2}+\frac{\partial^2V}{\partial z^2}$	$\nabla^2V=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2V}{\partial\phi^2}+\frac{\partial^2V}{\partial z^2}$	$\nabla^2V=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2V}{\partial\phi^2}$

➤ Sample Questions

1995 IIT Kanpur

1.1 A spherical conductor of radius 'a' with charge 'q' is placed concentrically inside an uncharged and unearthed spherical conducting shell of inner and outer radii r_1 and r_2 respectively. Taking potential to be zero at infinity, the potential at any point P within the shell $(r_1 < r < r_2)$ will be



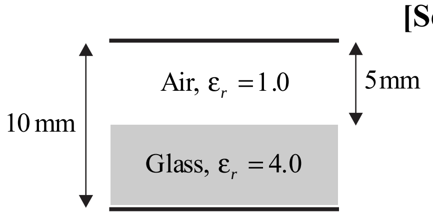
(A) $\dfrac{q}{4\pi\varepsilon_0 r}$　　(B) $\dfrac{q}{4\pi\varepsilon_0 a}$

(C) $\dfrac{q}{4\pi\varepsilon_0 r_2}$　　(D) $\dfrac{q}{4\pi\varepsilon_0 r_1}$

2015 IIT Kanpur

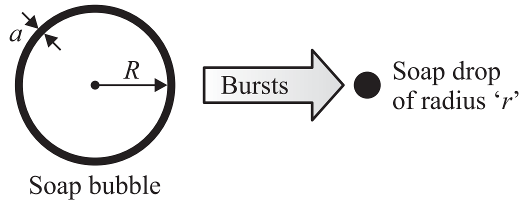
1.2 A parallel plate capacitor is partially filled with glass of dielectric constant 4.0 as shown below. The dielectric strengths of air and glass are 30 kV/cm and 300 kV/cm, respectively. The maximum voltage (in kilovolts), which can be applied across the capacitor without any breakdown, is ______.

[Set - 01]



2017 IIT Roorkee

1.3 A thin soap bubble of radius, $R = 1$ cm and thickness $a = 3.3$ μm $(a << R)$, is at a potential of 1 V with respect to a reference point at infinity. The bubble bursts and becomes a single spherical drop of soap (assuming all the soap is contained in the drop) of radius r. The volume of the soap in the thin bubble is $4\pi R^2 a$ and that of the drop is $\frac{4}{3}\pi r^3$. The potential in volts, of the resulting single spherical drop with respect to the same reference point at infinity is ________. (Give the answer up to two decimal places). **[Set - 02]**



2020 IIT Delhi

1.4 Let $\hat{a}_r$, $\hat{a}_\phi$, $\hat{a}_z$ be unit vector along r, ϕ and z direction respectively in the cylindrical coordinate system. For the electric flux density given by $\vec{D} = (15\,\hat{a}_r + 2r\hat{a}_\phi - 3rz\,\hat{a}_z)$ C/m^2, the total electric flux, in Coulomb emanating from the volume enclosed by solid cylinder of radius 3 m and height 5 m oriented along the z-axis with its base at the origin is

(A) 54π　　(B) 180π

(C) 90π　　(D) 108π

2021 IIT Bombay

1.5 A $1\,\mu C$ point charge is held at the origin of a cartesian coordinate system. If a second point charge of $10\,\mu C$ is moved from (0, 10, 0) to (5, 5, 5) and subsequently to (5, 0, 0), then the total work done is ___mJ. (Round off to 2 decimal places). Take $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ in SI units. All coordinates are in meters.

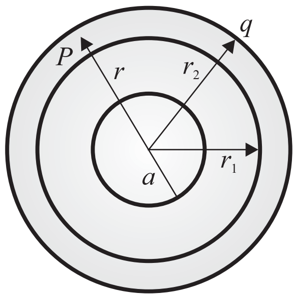
❖❖❖❖

Explanations Electrostatics

1.1 (C)

Given :

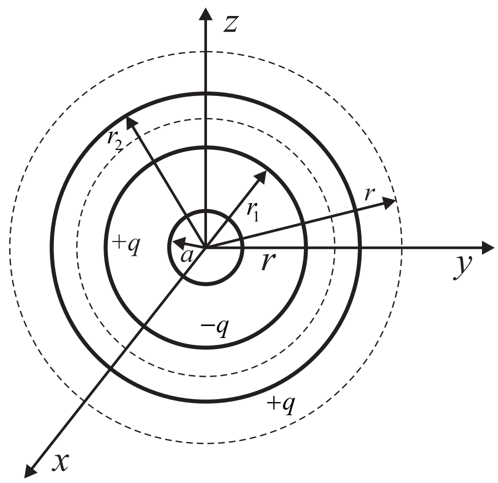
(i) A spherical conductor of radius $'a'$ with charge $'q'$ is placed concentrically inside an uncharged and unearthed spherical conducting shell of inner and outer radii r_1 and r_2 respectively,



(ii) The potential is zero at infinity

When charge on inner conductor is $+q$. The inner surface of conducting shell has induced charge equal to $-q$ and outer surface equal to $+q$. The total charge enclosed in a spherical Gaussian surface outside the shell is $+q$. The potential at distance 'r' from centre, outside the shell, can be given by,

$$V = -\int_{\infty}^{r} \bar{E}\cdot \bar{dr}$$



The electric field at distance 'r' is obtained by applying Gauss's Law as,

$$\bar{E} = \frac{1}{4\pi\epsilon_0}\times\frac{q}{r^2}\vec{a}_r$$

$$V = -\int_{\infty}^{r}\frac{1}{4\pi\varepsilon_0}\times\frac{q}{r^2}\vec{a}_r\,dr\,\vec{a}_r$$

$$V = -\int_{\infty}^{r}\frac{1}{4\pi\varepsilon_0}\times\frac{q}{r^2}dr = -\int_{\infty}^{r}\frac{1}{4\pi\varepsilon_0}\times\frac{q}{r^2}dr$$

$$V = \frac{q}{4\pi\varepsilon_0}\times\frac{1}{r}$$

At $r = r_2$, the value of V is given by,

$$V = \frac{q}{4\pi\varepsilon_0} \times \frac{1}{r_2}$$

Electric field inside the conducting shell for $r_1 < r < r_2$, is zero because the charge enclosed by a Gaussian surface with $r_1 < r < r_2$ is zero. So the electric potential is constant.

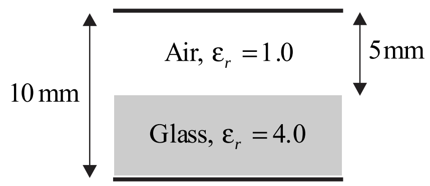
$$V = \frac{q}{4\pi\varepsilon_0} \times \frac{1}{r_2}$$

Hence, the correct option is (C).

1.2 18.75

Given :

(i) A parallel plate capacitor is partially filled with glass of dielectric constant 4.0 as shown below,



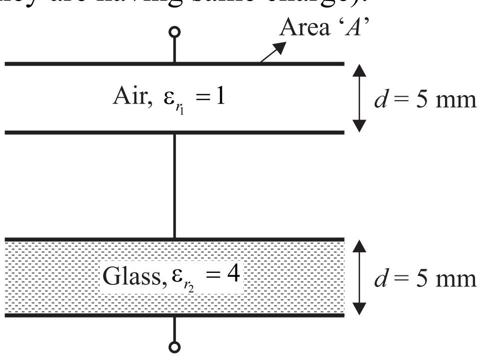
(ii) Dielectric strength of air,

$E_{10} = 30$ kV/cm $= 3$ kV/mm

(iii) Dielectric strength of glass,

$E_{20} = 300$ kV/cm $= 30$ kV/mm

It can be assumed as two capacitors are connected in series (as they carry same current and they are having same charge).



$$C_1 = \frac{\varepsilon_0 \varepsilon_{r_1} A}{d} = \frac{\varepsilon_0 A}{d} = C_0 \text{ let}$$

$$C_2 = \frac{\varepsilon_0 \varepsilon_{r_1} A}{d} = \frac{4\varepsilon_0 A}{d} = 4C_0$$

The equivalent capacitance is given by,

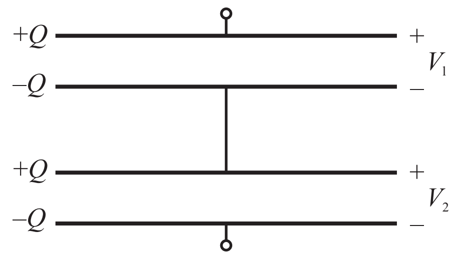
$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{C_0 \times 4C_0}{C_0 + 4C_0} = \frac{4C_0}{5}$$

$$Q = C_1 V_1 = C_2 V_2 = CV$$

where $V = V_1 + V_2$

$$C_0 V_1 = 4C_0 V_2$$

$$V_1 = 4V_2$$



Dielectric strength of air $E_{10} = 3\,\text{kV/mm}$

For a distance $d = 5\,\text{mm}$

So, maximum voltage allowed across C_1 is given by,

$$V_{10} = E_{10} \times d = 15\,\text{kV}$$

For $V_{10} = 15\,\text{kV}$,

$$V_2 = \frac{V_{10}}{4} = \frac{15}{4} = 3.75\,\text{kV}$$

Electric field across C_2 is given by,

$$E_2 = \frac{V_2}{d} = \frac{3.75}{5} = 0.75\,\text{kV/mm}$$

$$E_2 = 7.5\,\text{kV/cm}$$

Dielectric strength of glass,

$$E_{20} = 300\,\text{kV/cm} = 30\,\text{kV/mm}$$

As, $E_2 < E_{20}$

So, it is allowed.

Total voltage across (equivalent) capacitor is given by,

$$V = V_1 + V_2 = 15 + 3.75 = 18.75\,\text{kV}$$

For $E_2 = E_{20} = 30\,\text{kV/mm}$ (Maximum allowed)

$$V_{20} = E_{20} \times d = 30 \times 5 = 150\,\text{kV}$$

$$V_1 = 4V_{20} = 4 \times 150 = 600\text{ kV}$$

Electric field across C_1 is given by,

$$E_1 = \frac{V_1}{d} = \frac{600}{5} = 120\,\text{kV/mm}$$

As $\quad E_1 > E_{10}\ (=3\,\text{kV/mm})$

In this condition, there will be breakdown in C_1 so it is not allowed.

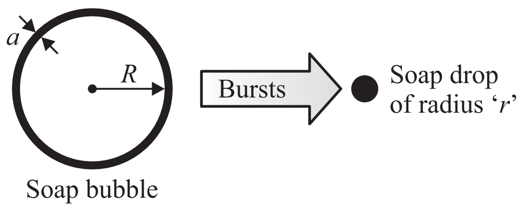
Hence, the maximum voltage is **18.75 kV.**

Scan for Video Solution

1.3 **10.04**

Given :

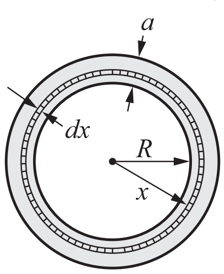
(i) A thin soap bubble of radius, $R = 1$ cm and thickness $a = 3.3\ \mu\text{m}$ $(a << R)$, is at a potential of 1 V with respect to a reference point at infinity.



(ii) Potential of soap bubble = 1 V

(iii) Volume of soap bubble $= 4\pi R^2 a$

(iv) Volume of soap drop $= \frac{4}{3}\pi r^3$



Method 1

When soap bubble burst, then it becomes single spherical drop. So, volume will not change.

Volume of soap bubble = Volume of soap drop

$$4\pi R^2 a = \frac{4}{3}\pi r^3$$

$$r^3 = 3R^2 a$$

$$r^3 = 3\times 1\times 3.3\times 10^{-4}$$

$$r = (0.99)^{1/3}\times 10^{-1}\ \text{cm}$$

At distance x from center we take strip of width of dx. The differential volume of that strip is given by,

$$V = 4\pi x^2 dx$$

Let, total charge on soap bubble = Q

Charge present in $\frac{4}{3}\pi\left[(a+R)^3 - R^3\right]$ is Q.

Then charge present in differential volume $4\pi x^2 dx$ is

$$dq = \frac{Q}{\frac{4}{3}\pi\left[(a+R)^3 - R^3\right]}\times 4\pi x^2 dx$$

$$dq = \frac{3Q}{(a+R)^3 - R^3}\times x^2 dx$$

Potential of soap bubble

$$V = \int_{x=R}^{a+R}\frac{k\,dq}{x} = \int_{R}^{a+R}\frac{k\cdot 3Q\cdot x^2 dx}{\left[(a+R)^3 - R^3\right]x}$$

$$V = \frac{3kQ}{\left[(a+R)^3 - R^3\right]}\left[\frac{x^2}{2}\right]_R^{a+R}$$

$$V = \frac{3kQ}{2\left[(a+R)^3 - R^3\right]}\left[(a+R)^2 - R^2\right]$$

From, $a^2 - b^2 = (a+b)(a-b)$ and

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$V = \frac{3kQ}{2}\frac{(a+R-R)(a+R+R)}{(a+R-R)\left[(a+R)^2 + R^2 + (a+R)R\right]}$$

$$V = \frac{3kQ}{2}\frac{(a+2R)}{\left[(a+R)^2 + R^2 + (a+R)R\right]}$$

[Since $a << R$]

$$V = \frac{3}{2}\frac{kQ\times 2R}{3R^2} = \frac{kQ}{R}$$

For $V = 1$ V, $R = 1$ cm

$$kQ = 1\text{ V}\times 1\text{ cm} = 1\text{ V-cm}$$

Electric potential of solid sphere is given by,

$$V = \frac{kQ}{r} = \frac{1}{(0.99)^{1/3}\times 10^{-1}}$$

$$V = \frac{10}{(0.99)^{1/3}} = 10.04\text{ V}$$

Hence, the potential in volts, of the resulting single spherical drop with respect to the same reference point at infinity is **10.04 V.**

Method 2

Since, $Q_1 = Q_2$ and charge density 'ρ_v' must be same in both cases.

Volume occupied in the two cases must be same.

$$\frac{4}{3}\pi r^3 = 4\pi R^2.a$$

$$r = (3R^2 a)^{1/3}$$

$$r = \left[3\times\left(10^{-2}\right)^2\times\left(3.3\times 10^{-6}\right)\right]^{1/3}$$

$$r = 0.996\times 10^{-3}\text{ m } = 0.996\text{ mm}$$

Let us consider the bigger bubble,

$$R >> a$$

$$V_1 = \frac{Q_1}{4\pi\varepsilon R}$$

$$Q_1 = 4\pi\varepsilon R V_1$$

For smaller bubble,

$$V_2 = \frac{Q_2}{4\pi\varepsilon r}$$

$$Q_2 = 4\pi\varepsilon r V_2$$

$$Q_1 = Q_2$$

$$4\pi\varepsilon r V_2 = 4\pi\varepsilon R V_1$$

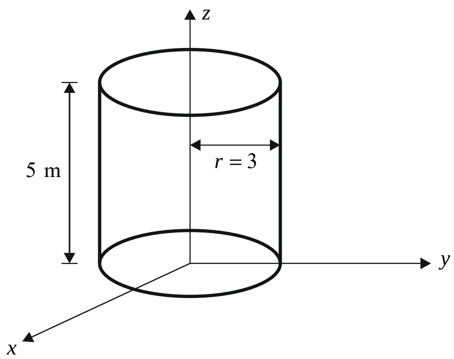
$$V_2 = \frac{R}{r}V_1$$

$$V_2 = \frac{10^{-2}}{0.996\times 10^{-3}}\times 1 = 10.04\text{ V}$$

Hence, the potential in volts, of the resulting single spherical drop with respect to the same reference point at infinity is **10.04 V.**

1.4 (B)

Given : $\vec{D} = 15\hat{a}_r + 2r\hat{a}_\phi - 3rz\hat{a}_z$ C/m^2



Flux passing through closed surface is given by,

$$\phi = \oint_s \vec{D}\cdot d\vec{s}$$

From divergence theorem,

$$\oint_s \vec{D}\cdot d\vec{s} = \int_v (\nabla\cdot\vec{D})\,dv$$

In cylindrical co-ordinate system,

$$\nabla\cdot\vec{D} = \frac{1}{r}\frac{\partial}{\partial r}(r D_r) + \frac{1}{r}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$$

$$\nabla\cdot\vec{D} = \frac{1}{r}\frac{\partial}{\partial r}(15r) + \frac{1}{r}\frac{\partial}{\partial\phi}(2r) + \frac{\partial}{\partial z}(-3rz)$$

$$\nabla\cdot\vec{D} = \frac{15}{r} + 0 - 3r = \frac{15}{r} - 3r$$

$$\oint \vec{D}\cdot d\vec{s} = \int_{z=0}^{5}\int_{\phi=0}^{2\pi}\int_{r=0}^{3}\left(\frac{15}{r} - 3r\right) r\,dr\,d\phi\,dz$$

$$\oint_s \vec{D}\cdot d\vec{s} = \int_{r=0}^{3}(15 - 3r^2)\,dr\int_{\phi=0}^{2\pi}d\phi\int_{z=0}^{5}dz$$

$$\oint_s \vec{D}\cdot d\vec{s} = \left[15r - r^3\right]_0^3(2\pi)(5)$$

$$\oint_s \vec{D}\cdot d\vec{s} = (45 - 27)10\pi$$

$$\oint_s \vec{D}\cdot d\vec{s} = 180\pi$$

Hence, the correct option is (B).

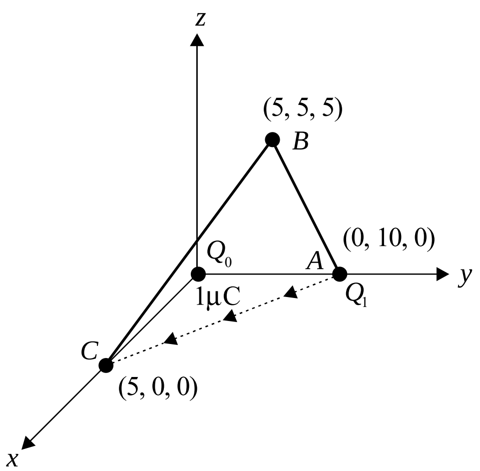
1.5 **9**

Given :

(i) First point charge $Q_0 = 1\,\mu c$

(ii) Second point charge $Q_1 = 10\,\mu c$

Method 1



Now electric field due to $1\mu C$ is,

$$\vec{E} = \frac{1\times10^{-6}}{4\pi\varepsilon_0 \times r^2}\hat{a}_r$$

Only radial component of electric field $\vec{E}$ is exist, so checking for electric field is conservative or not. i.e. $\vec{\nabla}\times\vec{E} = 0$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{a}_r & \hat{a}_\theta & a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ E_r & 0 & 0 \end{vmatrix}$$

Here, $\vec{\nabla}\times\vec{E} = 0$ so, electric field is curl free, path independent, irrotational.

As electric field is path independent so instead of going from A to B to C.

We can go from A to C directly.

So, $$V_{AC} = \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times10^{-6}}{10} - \frac{1\times10^{-6}}{5}\right]$$

$$V_{AC} = \frac{1\times10^{-6}}{4\pi\varepsilon_0}\left\{\frac{1}{10} - \frac{1}{5}\right\} = \frac{1\times10^{-6}}{4\pi\varepsilon_0}\times\frac{1}{10}\text{V}$$

So, total work done is $W = Q_1 \cdot V_{AC}$

$$W = 10\times10^{-6}\times\frac{1\times10^{-6}}{4\pi\varepsilon_0\times10}$$

$$W = 10\times10^{-12}\times9\times10^{9}\times10^{-1} = 9\,\text{mJ}$$

Hence, the total work done is 9 mJ.

Method 2

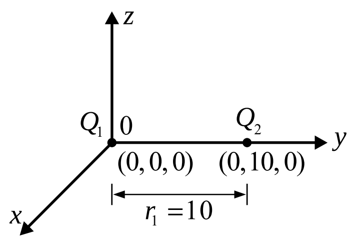
Given :

First point charge $(Q_1) = 1\mu C$

Second point charge $(Q_2) = 10\mu C$

For free space, $K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\text{m/F}$

At initial states, charge Q_1 at origin (0,0,0) and charge Q_2 at (0,10,0) as shown below,

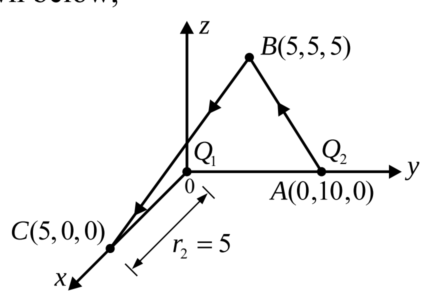


So, initial potential energy (U_I) of given system due to Q_1 and Q_2 is

$$U_I = K\frac{Q_1 Q_2}{r_1}$$

$$U_I = K\left[\frac{1\times10^{-6}\times10\times10^{-6}}{10}\right] = K\left[10^{-12}\right]\text{J}$$

When charge Q_2 is moved from point $A(0,10,0)$ to point $B(5,5,5)$ and reached to point $C(5,0,0)$ as shown below,



So, final potential energy (U_F) of given system due to Q_1 and Q_2 is

$$U_F = K\frac{Q_1 Q_2}{r_2}$$

$$U_F = K\left[\frac{1\times10^{-6}\times10\times10^{-6}}{5}\right] = K\left[2\times10^{-12}\right]\text{J}$$

Thus total work done (W) is,

$$W = U_F - U_I = K\left[2\times10^{-12}\right] - K\left[10^{-12}\right]$$

$$W = K\times10^{-12} = 9\times10^{9}\times10^{-12}$$

$$W = 9\times10^{-3}\,\text{J} = 9\,\text{mJ}$$

Hence, the total work done is 9 mJ.

❖❖❖❖

2 Magnetostatics

➢ Partial Synopsis

Ampere's Circuital Law

- It states that the integral of $\vec{H}$ around a closed path is the same as the net current I_{enc} enclosed by the path.

$$\oint_c \vec{H} \cdot d\vec{l} = I_{enc}$$

- By applying stokes theorem we get

$$\nabla \times \vec{H} = \vec{J}$$

Magnetic Flux and Flux Density

- The magnetic flux density is related to the magnetic field intensity as,

$$\vec{B} = \mu_0 \vec{H}$$

- The magnetic flux through a surface S is given by,

$$\psi = \int_s \vec{B} \cdot d\vec{S}$$

- An isolated magnetic charge does not exist.Thus, the total flux through a closed surface in a magnetic field must be zero.

$$\int_s \vec{B} \cdot d\vec{S} = 0$$

- By applying divergence theorem we get

$$\nabla . \vec{B} = 0$$

This equation is referred to as the **law of conservation of magnetic flux** or Gauss's law for magneto-static fields.

Lorentz Force

- The Lorentz force (or electromagnetic force) is the combination of electric and magnetic force on a point charge due to electromagnetic fields. A particle of charge Q moving with a velocity $\vec{V}$ in an electric field $\vec{E}$ and a magnetic field $\vec{B}$ experiences a force of

$$\vec{F} = \vec{F}_e + \vec{F}_m = Q\vec{E} + Q\vec{V} \times \vec{B}$$

Magnetic Boundary conditions

Boundary conditions for normal components :

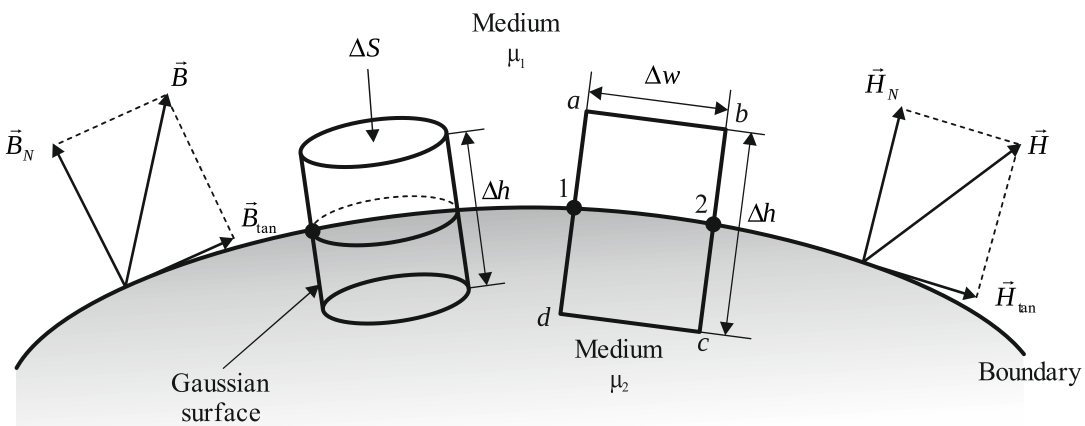
The conditions of the magnetic field existing at the boundary of the two media, when the magnetic field passes from one medium to other are called boundary condition for magnetic field or simply magnetic boundary conditions.

The boundary between the two different magnetic materials is considered. To study conditions of $\vec{B}$ and $\vec{H}$ at the boundary both the vectors are resolved into two components;

(a) Tangential to boundary

(b) Normal (perpendicular) to boundary

Consider a boundary between two isotropic, homogeneous linear materials with different permeability μ_1 and μ_2 as shown in the figure. To determine the boundary conditions, let us use the closed path and the Gaussian surface.



Boundary conditions for normal components :

$$B_{N_1} = B_{N_2}$$

$$\frac{H_{N_1}}{H_{N_2}} = \frac{\mu_2}{\mu_1} = \frac{\mu_{r2}}{\mu_{r1}}$$

Boundary conditions for tangential components :

$$(H_{1t} - H_{2t}) = K \quad \ldots(i)$$

Tangential component of H is discontinuous,

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K$$

$$\boxed{B_{n1} = B_{n2}}$$

$$\frac{H_{n1}}{H_{n1}} = \frac{\mu_2}{\mu_1} = \frac{\mu_{r2}}{\mu_{r1}}$$

- The normal component of $\vec{B}$ is continuous while that of $\vec{H}$ is discontinues at the boundary.

Boundary conditions for tangential components :

If the boundary is free of current or the media are not conductors, $K = 0$.

$$H_{t1} = H_{t2}$$

$$\frac{\vec{B}_{t1}}{\vec{B}_{t2}} = \frac{\mu_1}{\mu_2} = \frac{\mu_{r1}}{\mu_{r2}}$$

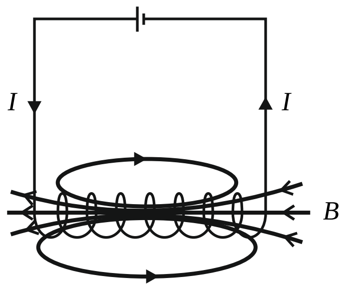
$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{B_{\tan_1}}{B_{\tan 2}} = \frac{\mu_{r1}}{\mu_{r2}}$$

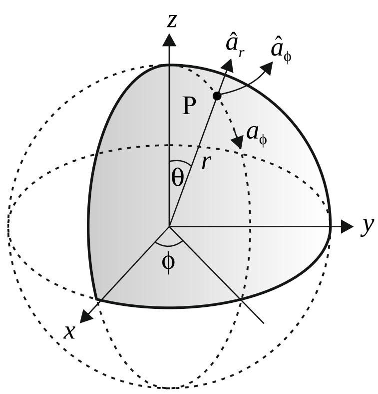
- The tangential component of $\vec{H}$ is continuous while that of $\vec{B}$ is discontinues at the boundary.

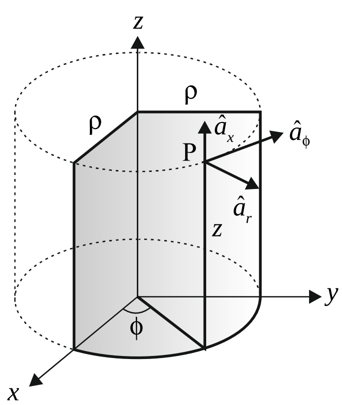
In general,

$$(\vec{H}_1 - \vec{H}_2)\times\hat{a}_{n12} = \vec{K}$$

where $\hat{a}_{n12}$ is a unit vector normal to the interface and is directed from medium 1 to medium 2.



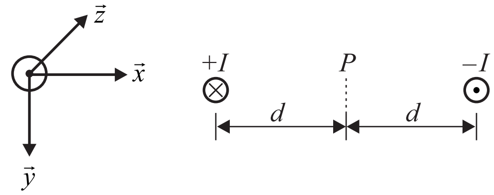




➤ Sample Questions

2003 IIT Madras

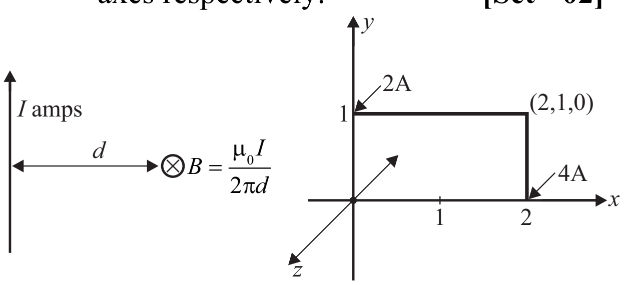
2.1 Two conductors are carrying forward and return current of $+I$ and $-I$ as shown in figure. The magnetic field intensity $\vec{H}$ at point P is



(A) $\frac{I}{\pi d}\vec{y}$ (B) $\frac{I}{\pi d}\vec{x}$

(C) $\frac{I}{2\pi d}\vec{y}$ (D) $\frac{I}{2\pi d}\vec{x}$

2.2 The magnitude of magnetic flux density $(\vec{B})$ at a point having normal distance d meters from an infinitely extended wire carrying current of I A is $\frac{\mu_0 I}{2\pi d}$ (in SI units). An infinitely extended wire is laid along the x-axis and is carrying current of 4 A in the positive x direction. Another infinitely extended wire is laid along the y-axis and is carrying 2 A current in the positive y direction. μ_0 is permeability of free space. Assume $\hat{i}, \hat{j}, \hat{k}$ to be unit vectors along x, y and z axes respectively. **[Set - 02]**



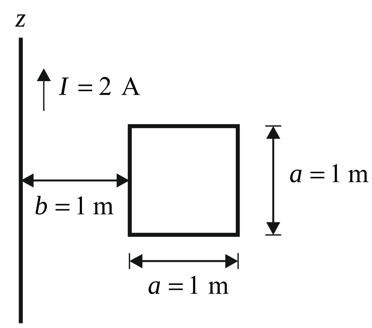
Assuming right handed co-ordinate system, magnetic field intensity, $\vec{H}$ at co-ordinate (2, 1, 0) will be

(A) $\frac{3}{2\pi}\hat{k}$ weber/m^2 (B) $\frac{4}{3\pi}\hat{i}$ A/m^2

(C) $\frac{3}{2\pi}\hat{k}$ A/m (D) 0 A/m

2020 IIT Delhi

2.3 A conducting square loop of side length 1 m is placed at a distance of 1 m from a long straight wire carrying a current $I = 2$ A as shown below. The mutual inductance, in nH (rounded off to 2 decimal places), between conducting loop and the long wire is __________.



2021 IIT Bombay

2.4 Which one of the following vector functions represents a magnetic field $\vec{B}$?

($\hat{x}, \hat{y}$, and $\hat{z}$ are unit vectors along x-axis, y-axis and z-axis respectively)

(A) $10x\hat{x} + 20y\hat{y} - 30z\hat{z}$

(B) $10y\hat{x} + 20x\hat{y} - 10z\hat{z}$

(C) $10z\hat{x} + 20y\hat{y} - 30x\hat{z}$

(D) $10x\hat{x} - 30z\hat{y} + 20y\hat{z}$

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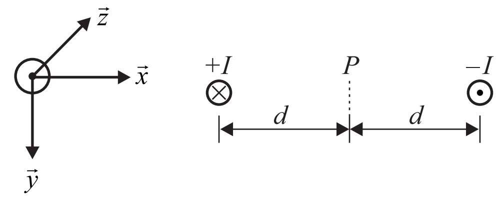
Explanations — Magnetostatics

2.1 (A)

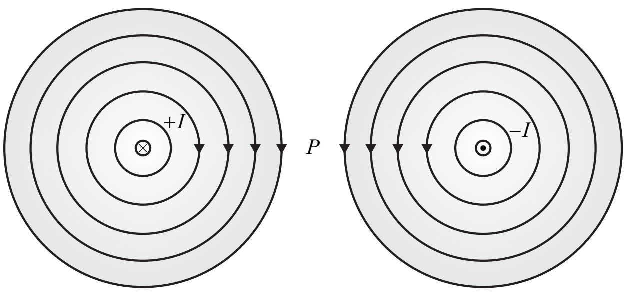
Given : Two conductors are carrying forward and return current of $+I$ and $-I$ as shown in figure,



Method 1

Magnetic field intensity $\vec{H}$ at point P is sum of magnetic field intensity due to both the conductors is given by,

$$\vec{H} = \vec{H}_1 + \vec{H}_2 = \frac{I}{2\pi d}\vec{a}_y + \frac{I}{2\pi d}\vec{a}_y = \frac{I}{\pi d}\vec{a}_y$$



Hence, the correct option is (A).

Method 2

Magnetic field intensity due to infinite long current carrying conductor is given by,

$$\vec{H} = \frac{I}{2\pi\rho}\vec{a}_\phi$$

where, $\rho =$ Perpendicular distance from conductor to point of interest

The direction of magnetic field is given by,

$$\vec{a}_\phi = \vec{a}_l \times \vec{a}_\rho$$

where, $\vec{a}_l =$ Unit vector on the direction of flow of current

$\vec{a}_\rho =$ Unit vector in direction of conductor to point of interest.

The magnetic field at point P due to conductor carrying forward current $+I$ is given by,

$$\vec{H}_1 = \frac{I}{2\pi d}\vec{a}_y \qquad [\vec{a}_\phi = \vec{a}_z \times \vec{a}_x = \vec{a}_y]$$

The magnetic field at point P due to conductor carrying forward current $-I$ is given by,

$$\vec{H}_2 = \frac{I}{2\pi d}\vec{a}_y \qquad [\vec{a}_\phi = -\vec{a}_z \times -\vec{a}_x = \vec{a}_y]$$

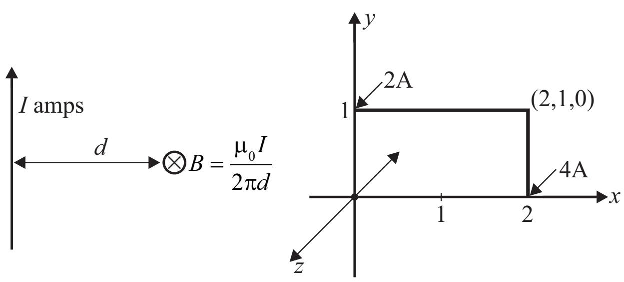
Magnetic field intensity $\vec{H}$ at point P is sum of magnetic field intensity due to both the conductors is given by,

$$\vec{H} = \vec{H}_1 + \vec{H}_2 = \frac{I}{2\pi d}\vec{a}_y + \frac{I}{2\pi d}\vec{a}_y = \frac{I}{\pi d}\vec{a}_y$$

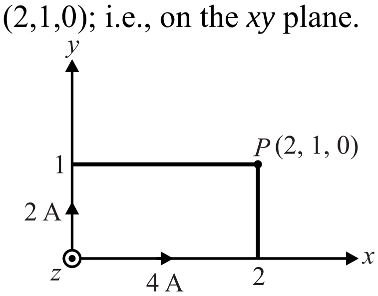
Hence, the correct option is (A).

2.2 (C)

Given : $\vec{B} = \mu_0 \vec{H}$ So, $\vec{H} = \dfrac{I}{2\pi d}$



The given currents are along the x-axis, and the y-axis respectively. The point where $\vec{H}$ is to be found is (2,1,0); i.e., on the xy plane.



Magnetic field intensity at point P due to conductor carrying current 4 A is given by,

$$\vec{H}_1 = \frac{4}{2\pi(1)}\vec{a}_z \text{ A/m}$$

(direction decided by right hand rule)

Magnetic field intensity at point P due to conductor carrying current 2 A is given by,

$$\vec{H}_2 = \frac{2}{2\pi(2)}(-\vec{a}_z) \text{ A/m}$$

(direction decided by right hand rule)

Total magnetic field intensity at point $P(2, 1, 0)$

$$\vec{H} = \frac{2}{\pi}\vec{a}_z - \frac{1}{2\pi}\vec{a}_z = \frac{3}{2\pi}\vec{a}_z \text{A/m}$$

Hence, the correct option is (C).

Scan for Video Solution

Key Point

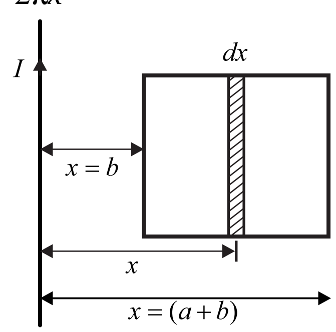
The paper gives as the unit vectors along x, y and z-axis respectively. But since we have to deal not only with rectangular co-ordinate system, but also cylindrical and spherical coordinate systems, it would be better to use the symbols $\vec{a}_x, \vec{a}_y$ and $\vec{a}_z$ of course $\vec{a}_x = \hat{i}; \vec{a}_y = \hat{j}$ and $\vec{a}_z = \hat{k}$.

2.3 **138.63**

Given : $I = 2A,\ a = 1\text{m},\ b = 1\text{m}$

Consider a strip of width dx (of the square loop) at a distance x from the wire carrying current. Magnetic field due to current carrying wire at a distance x from the wire is given by,

$$B = \frac{\mu_0 I}{2\pi x} \qquad \ldots\text{(i)}$$



Small amount of magnetic flux associated with the strip,

$$d\phi = BdA = \frac{\mu_0 I}{2\pi x}(adx)$$

Magnetic flux linked with the square loop is

$$\phi = \frac{\mu_0 Ia}{2\pi}\int_{x=b}^{a+b}\frac{dx}{x}$$

$$\phi = \frac{\mu_0 Ia}{2\pi x}[\ln(x)]_b^{a+b} = \frac{\mu_0 Ia}{2\pi}\ln\left[\frac{a+b}{b}\right]$$

Since, $\phi = MI$ where, M = Mutual Inductance

$$MI = \frac{\mu_0 Ia}{2\pi}\ln\left(\frac{a}{b}+1\right)$$

$$M = \frac{\mu_0 a}{2\pi}\ln\left(\frac{a}{b}+1\right) \qquad \ldots\text{(iii)}$$

From equation (iii),

$$M = \frac{4\pi\times10^{-7}\times1}{2\pi}\ln(2)$$

$$M = 138.63 \text{ nH}$$

2.4 **(A)**

The vector functions represents a magnetic field $\vec{B}$ if $\oiint \vec{B}\cdot ds = 0$

Where, $\vec{B} =$ Magnetic flux density

By applying Divergence theorem,

$$\iiint(\nabla\cdot\vec{B})\,dv = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

It means, no unipole of magnetic field is exist.

So, checking from options,

Option (A) : $\vec{B} = 10x\,\hat{x} + 20y\,\hat{y} - 30z\,\hat{z}$

So, $B_x = 10x,\ B_y = 20y,\ B_z = -30z$

Thus, $$\vec{\nabla}\cdot\vec{B} = \frac{\partial}{\partial x}B_x + \frac{\partial}{\partial y}B_y + \frac{\partial}{\partial z}B_z$$

$$\vec{\nabla}\cdot\vec{B} = \frac{\partial}{\partial x}10x + \frac{\partial}{\partial y}20y + \frac{\partial}{\partial z}(-30z)$$

$$\vec{\nabla}\cdot\vec{B} = 10 + 20 - 30 = 0$$

Hence, Option (A) is correct.

Option (B) : $\vec{B} = 10y\,\hat{x} + 20x\,\hat{y} - 10z\,\hat{z}$

So, $B_x = 10y,\ B_y = 20x,\ B_z = -10z$

Thus, $\vec{\nabla}\cdot\vec{B}=\frac{\partial}{\partial x}B_x+\frac{\partial}{\partial y}B_y+\frac{\partial}{\partial z}B_z$

$$\vec{\nabla}\cdot\vec{B}=\frac{\partial}{\partial x}10y+\frac{\partial}{\partial y}20x+\frac{\partial}{\partial z}(-10z)$$

$$\vec{\nabla}\cdot\vec{B}=0+0-10=-10$$

Hence, Option (B) is incorrect.

Option (C) : $\vec{B}=10z\,\hat{x}+20y\,\hat{y}-30x\,\hat{z}$

So, $B_x=10z,\ B_y=20y,\ B_z=-30x$

Thus, $\vec{\nabla}\cdot\vec{B}=\frac{\partial}{\partial x}B_x+\frac{\partial}{\partial y}B_y+\frac{\partial}{\partial z}B_z$

$$\vec{\nabla}\cdot\vec{B}=\frac{\partial}{\partial x}10z+\frac{\partial}{\partial y}20y+\frac{\partial}{\partial z}(-30x)$$

$$\vec{\nabla}\cdot\vec{B}=0+20-0=20$$

Hence, Option (C) is incorrect.

Option (D) : $\vec{B}=10x\,\hat{x}-30z\,\hat{y}+20y\,\hat{z}$

So, $B_x=10x,\ B_y=-30z,\ B_z=20y$

Thus, $\vec{\nabla}\cdot\vec{B}=\frac{\partial}{\partial x}B_x+\frac{\partial}{\partial y}B_y+\frac{\partial}{\partial z}B_z$

$$\vec{\nabla}\cdot\vec{B}=\frac{\partial}{\partial x}10x+\frac{\partial}{\partial y}(-30z)+\frac{\partial}{\partial z}20y$$

$$\vec{\nabla}\cdot\vec{B}=10-0+0=10$$

Hence, Option (D) is incorrect.

Hence, the correct option is (A).

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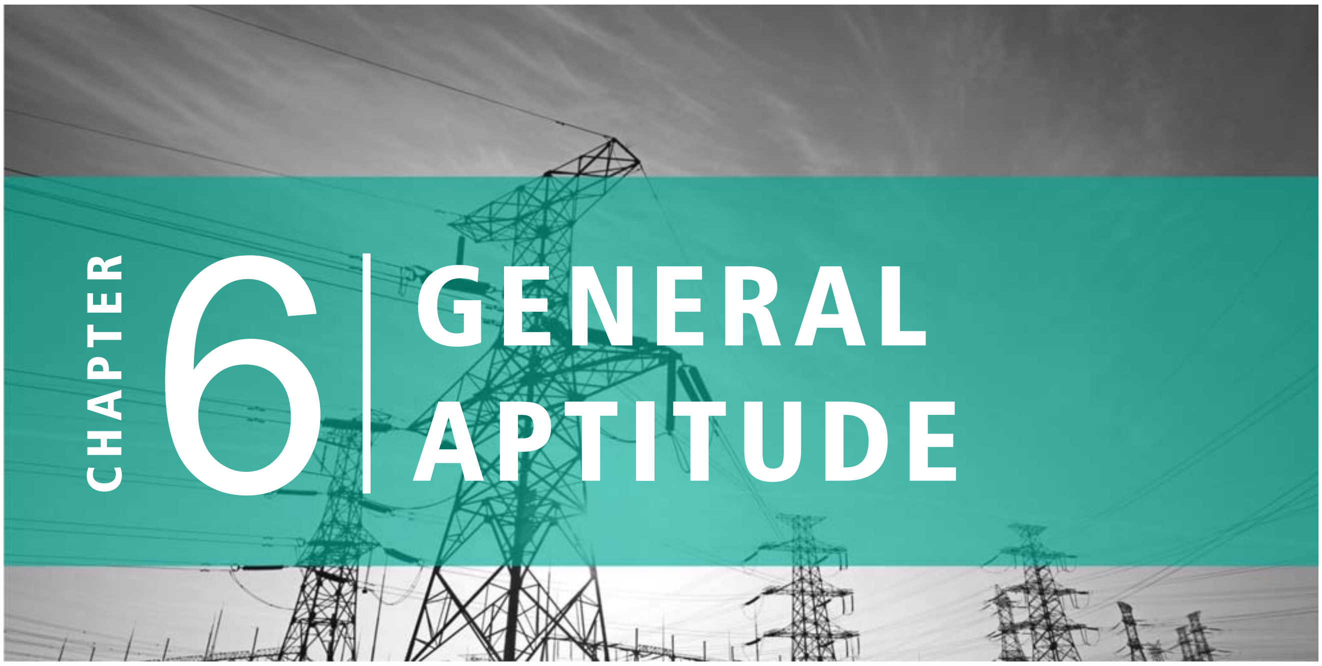


Marks Distribution of General Aptitude in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2010	5	5	15
2011	5	5	15
2012	5	5	15
2013	5	5	15
2014 Set-1	5	5	15
2014 Set-2	5	5	15
2014 Set-3	5	5	15
2015 Set-1	5	5	15
2015 Set-2	5	5	15

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2016 Set-1	5	5	15
2016 Set-2	5	5	15
2017 Set-1	10	–	10
2017 Set-2	10	–	10
2018	5	5	15
2019	6	4	14
2020	5	5	15
2021	5	5	15

Syllabus : General Aptitude (GA)

Verbal Aptitude :

Basic English grammar: tenses, articles, adjectives, prepositions, conjunctions, verb-noun agreement, and other parts of speech Basic vocabulary: words, idioms, and phrases in context Reading and comprehension Narrative sequencing

Quantitative Aptitude :

Data interpretation: data graphs (bar graphs, pie charts, and other graphs representing data), 2- and 3-dimensional plots, maps, and tables Numerical computation and estimation: ratios, percentages, powers, exponents and logarithms, permutations and combinations, and series Mensuration and geometry Elementary statistics and probability

Analytical Aptitude :

Logic: deduction and induction Analogy

Numerical relations and reasoning

Spatial Aptitude :

Transformation of shapes: translation, rotation, scaling, mirroring, assembling, and grouping Paper folding, cutting, and patterns in 2 and 3 dimensions

Contents : General Aptitude (GA)

1 Verbal Ability

2010 IIT Guwahati

1.1 Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their cause.

Which of the following statements best sums up the meaning of the above passage :

(A) Modern warfare has resulted in civil strife.

(B) Chemical agents are useful in modern warfare.

(C) Use of chemical agents in warfare would be undesirable.

(D) People in military establishments like to use chemical agents in war.

2012 IIT Delhi

1.2 One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.

Which one of the following statements best sums up the meaning of the above passage?

(A) Thorough regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.

(B) The legions were treated inhumanly as if the men were animals.

(C) Discipline was the armies' inheritance from their seniors.

(D) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

2014 IIT Kharagpur

1.3 If she _______ how to calibrate the instrument, she _______ done the experiment.

(A) knows, will have

(B) knew, had

(C) had known, could have

(D) should have known, would have

2017 IIT Roorkee

1.4 There are three boxes. One contains apples, another contains oranges and the last one contains both apples and oranges. All three are known to be incorrectly labelled. If you are permitted to open just one box and then pull out and inspect only one fruit, which box would you open to determine the contents of all three boxes?

(A) The box labelled 'Apples'

(B) The box labelled 'Apples and Oranges'

(C) The box labelled 'Oranges'

(D) Cannot be determined

2020 IIT Delhi

1.5 This book, including all its chapters, _____ interesting. The students as well as instructor ______ in agreement about it.

(A) is, was (B) is, are

(C) were, was (D) are, are

2021 IIT Bombay

1.6 The people __________ were at the demonstration were from all sections of society.

(A) whom (B) who

(C) which (D) whose

❖❖❖❖

Explanations Verbal Ability

1.1 (D)

(A) Modern warfare has resulted in civil strife : There is no direct consequence of warfare given, so it is not appropriate.

(B) Chemical agents are useful in modern warfare : Passage does not say whether chemical agents are useful or not, so it is not appropriate.

(C) Use of chemical agents in warfare would be undesirable : Given that people in military think these are useful, undesirable is wrong.

(D) People in military establishment like to use chemical agents in war : Correct choice as last statement tells that military people think that chemical agents are useful tools for their cause (work silently in warfare).

Hence, the correct option is (D).

1.2 (A)

Here, there are three key words "legacy", "legions" and "Discipline". Even though the paragraph starts with "legacy" but the basic idea revolves around "Strict Discipline", the author has mentioned that the main reason because of which the legion is "obedient, intact and fighting" is "Discipline". In option (A), we can see that "Strict Discipline" has been featured as "Regimentation."

Hence, correct option is (A).

Key Point

Legion = *Noun*

(i) A division of the Roman army, usually comprising 3000 to 6000 soldiers.

(ii) A military or semi-military unit.

Regimentation = *Noun*

(i) The act of regimenting or the state of being regimented.

(ii) The strict discipline and enforced uniformity characteristic of military groups.

1.3 (C)

Option (A) : Knows, will have

Present tense, future tense

Option (B) : Knew, had

Past tense, past tense

Option (C) : Had known, could have

Past perfect tense, perfect conditional

Option (D) : Should have known, would have

Present perfect tense, future tense

In a type three conditional sentence, the tense in 'If' clause is past prefect and the tense in the main clause is prefect continuous conditional.

Hence, the correct option is (C).

1.4 (B)

Given : All boxes have been labelled incorrectly.

So,

Statement 1 : Box labelled "Apples" is either "Oranges" or "Apples and Oranges".

Statement 2 : Box labelled "Oranges" is either "Apples" or "Apples and Oranges".

Statement 3 : Box labelled "Apples and Oranges" is either "Apples" or "Oranges".

After opening the box labelled "Apples and Oranges", if we get Apple in it then it is sure that the box labelled "Apples and Oranges" is actually Apple.

So, Box labelled "Oranges" can have either only 'oranges' or 'apples and oranges'. Since, all boxes are incorrectly labelled, box labelled as oranges can not have only oranges. Therefore, it must be having oranges and apple and box labelled "Apples" has to be "Oranges".

Hence, the correct option is (B).

1.5 (B)

'Book' is a singular noun, so it will take singular verb 'is'

When 'as well as' is part of the sentence then the verb must agree with the noun before 'as well as'.

So "the students as well as" will take the verb 'are'

Hence the correct option is (B).

1.6 (B)

Whom : Used to refer to the object of a verb or preposition.

Who : Used to refer to the subject of a sentence.

Which : Used in relative clause or when you have limited field to choose from.

Whose : Used when you're asking to whom something belongs.

Ex. Whose car is that?

Since, who refers to the subject.

Hence, the correct option is (B).

❖❖❖❖

2 Numerical Ability

2010 IIT Guwahati

2.1 25 persons are in a room. 15 of them play hockey, 17 of them play football and 10 of them play both hockey and football. Then the number of persons playing neither hockey nor football is :

(A) 2 (B) 17
(C) 13 (D) 3

2011 IIT Madras

2.2 The sum of n terms of the series $4+44+444+\ldots\ldots$ is

(A) $\left(\frac{4}{81}\right)\left[10^{n+1}-9n-1\right]$

(B) $\left(\frac{4}{81}\right)\left[10^{n-1}-9n-1\right]$

(C) $\left(\frac{4}{81}\right)\left[10^{n+1}-9n-10\right]$

(D) $\left(\frac{4}{81}\right)\left[10^{n}-9n-10\right]$

2.3 Three friends R, S and T shares toffee from a bowl. R took 1/3rd of the toffees, but returned four to the bowl. S took 1/4th of what was left but returned three toffees to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl?

(A) 38 (B) 31
(C) 48 (D) 41

2012 IIT Delhi

2.4 A and B are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is

(A) 1/4 (B) 1/16
(C) 7/16 (D) 9/16

2020 IIT Delhi

2.5 In four-digit integer numbers from 1001 to 9999, the digit group "37" (in the same sequence) appears ________ times.

(A) 270 (B) 299
(C) 279 (D) 280

2021 IIT Bombay

2.6 For a regular polygon having 10 sides, the interior angle between the sides of the polygon, in degree is

(A) 324 (B) 216
(C) 144 (D) 396

❖❖❖❖

Explanations Numerical Ability

2.1 (D)

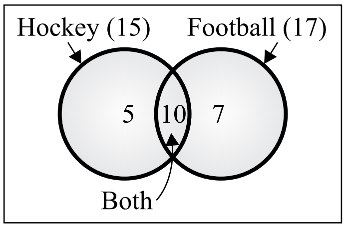
Given : There are 25 persons in a room.

Method 1

Total numbers of players

= Hockey only + Football only

+ Both hockey and football



From Venn diagram,

Total number of players $= 5+7+10 = 22$

Therefore, number of persons playing neither hockey nor football $= 25-22 = 3$

Hence, the correct option is (D).

Method 2

From set theory,

$n(A)$: Number of people who play hockey $=15$

$n(B)$: Number of people who play football $=17$

$n(A\cap B)$: Persons who play both hockey and football = 10

$n(A\cup B)$: Persons who play either hockey or football or both

From set theory,

$$n(A\cup B) = n(A)+n(B)-n(A\cap B)$$

$$n(A\cup B) = 15+17-10$$

$$n(A\cup B) = 22$$

Total number of person $= 25$

Therefore, people who play neither hockey nor football $= 25-22 = 3$

Hence, the correct option is (D).

2.2 (C)

Given : n^{th} series is,

$$4+44+444+\ldots\ldots$$

Method 1

First term of series $=4$

So, sum upto 1st term $=4$

Put, $n=1$ (first term) only option (C) satisfies.

$$S = \frac{4}{81}\left[10^{n+1}-9n-10\right]$$

At $n=1$,

$$S = \frac{4}{81}\left[10^{1+1}-(9\times 1)-10\right]$$

$$S = \frac{4}{81}\times 81 = 4$$

Hence, the correct option is (C).

Method 2

$$S = 4[1+11+111+\ldots\ldots]$$

$$S = \frac{4}{9}[9+99+999+\ldots\ldots.]$$

$$S = \frac{4}{9}\left[(10-1)+(10^2-1)+(10^3-1)+\ldots..\right]$$

$$S = \frac{4}{9}\left[(10+10^2+10^3\ldots\ldots 10^n)-(1+1+1\ldots.)\right]$$

$$S = \frac{4}{9}\left[10(1+10+10^2\ldots\ldots 10^{n-1})-(1+1+1\ldots.)\right]$$

$$S = \frac{4}{9}\left[10\left(\frac{10^n-1}{10-1}\right)-n\right]$$

$$S = \frac{4}{9}\left[\frac{10^{n+1}-10-9n}{9}\right]$$

$$S = \frac{4}{81}\left[10^{n+1}-9n-10\right]$$

Hence, the correct option is (C).

2.3 (C)

Given : Three friends R, S and T share toffees as below,

Firstly, R took 1/3rd and returned 4.

Then, S took 1/4th and returned 3.

Then, T took 1/2nd and returned 2.

Now, the bowl has 17 toffees left.

Method 1

The question can be done orally as R took 1/3rd of toffees initially. So, the total number of toffees have to be a multiple of 3.

Only the number 48 is multiple of 3, from the options.

Hence, the correct option is (C).

Method 2

Let x be the total number of toffees in bowl initially.

And r, s and t are toffees taken by R, S and T respectively.

According to question,

$$r = \frac{x}{3} - 4$$

Number of toffees left,

$$y = x - \left(\frac{x}{3} - 4\right) = \frac{2x}{3} + 4 \quad \ldots\text{(i)}$$

S took 1/4th of y and returned 3

So, $$s = \frac{y}{4} - 3$$

Number of toffees left,

$$z = y - \left(\frac{y}{4} - 3\right) = \frac{3y}{4} + 3$$

$$z = \frac{3}{4}y + 3$$

From equation (i),

$$z = \frac{3}{4}\left(\frac{2x}{3} + 4\right) + 3$$

$$z = \frac{x}{2} + 6 \quad \ldots\text{(ii)}$$

T took 1/2 of z and returned 2 back,

So, $$t = \frac{z}{2} - 2$$

Finally, number of toffees left

$$17 = z - \left(\frac{z}{2} - 2\right) = \frac{z}{2} + 2$$

$$17 = \frac{1}{2}\left(\frac{x}{2} + 6\right) + 2 \quad \text{[From equation (ii)]}$$

$$17 = \frac{x}{4} + 5$$

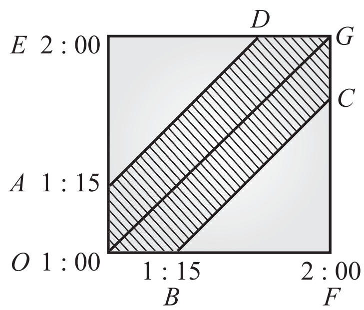
$$x = 48$$

Hence, the correct option is (C).

2.4 (C)

Method 1

OG is the line when both A and B arrive at same time.



Total sample space (area) = 60 × 60 = 3600

Favorable cases = Area of OEGF

– (Area of ADE + Area of BCF)

$$= 3600 - 2\times\left(\frac{1}{2}\times 45\times 45\right) = 1575$$

Therefore, the required probability

$$= \frac{1575}{3600} = \frac{7}{16}$$

Hence, the correct option is (C).

Method 2

Total number of ways to arrive for both friends

$$= 60\times 60 = 3600$$

If one friend comes at time 'x' and other comes after 15 min i.e. '$x+15$', then they will meet.

If other reaches in $x+15$ to $x+60$ i.e. 45 min, then they will not meet.

Therefore, number of ways for them not to meet

$$= 45\times 45 = 2025$$

Number of ways for meeting

$$= 3600 - 2025 = 1575$$

Thus, required probability $= \frac{1575}{3600} = \frac{7}{16}$

Hence, the correct option is (C).

2.5　(D)

Number of ways by which 37 can appear in numbers 1000 to 9999 are shown below,

9 ways (1 to 9)	10 ways (0 to 9)	3	7	$\Rightarrow 9\times10=90$	...(i)
9 ways (1 to 9)	3	7	10 ways (0 to 9)	$\Rightarrow 9\times10=90$	...(ii)
3	7	10 ways (0 to 9)	10 ways (0 to 9)	$\Rightarrow 10\times10=100$	...(iii)

There is no common numbers in case (i) - case (ii) or in case (ii) - case (iii) but there exist a common number between case (iii) and case (i), that is 3737.

So, if the total number of numbers is asked in which 37 appears in this sequence, then the answer would be,

$$90 + 90 + 100 - 1 = 279$$

Which is as given in IIT answer key but the required value is asked only for repetition of 37, not the numbers which contains 37, so in the number "3737", 37 must be counted 2 times, then the correct answer would be,

$$90 + 90 + 100 = 280$$

IIT has changed their answer for this question from option (C) to option (D) in the final answer key, but they do not considered it as MTA.

So, students must read the statement given in the question, before going to select the correct option in such type of problems.

2.6　(C)

Given :

The length of all segments PR, PS, QS, TR and TQ are of equal length,

$$= \frac{(n-2)}{n}\times180^0$$

$$= \frac{(10-2)}{10}\times180^0 = 144^0$$

Thus, the interior angle between the sides of the polygon, in degree is 144^0.

Hence, the correct option is (C).

❖❖❖❖

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