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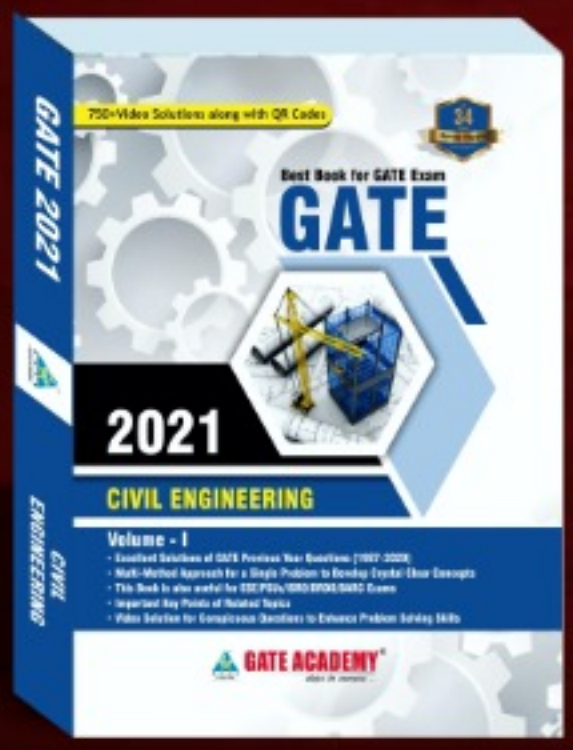

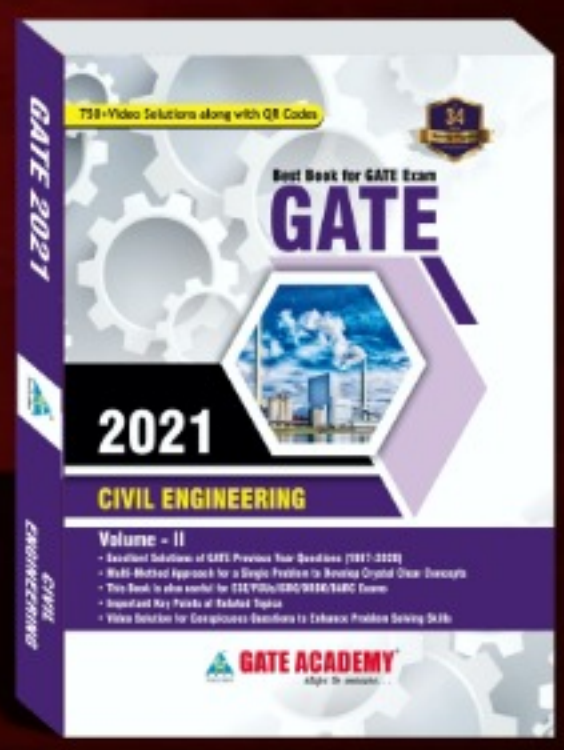

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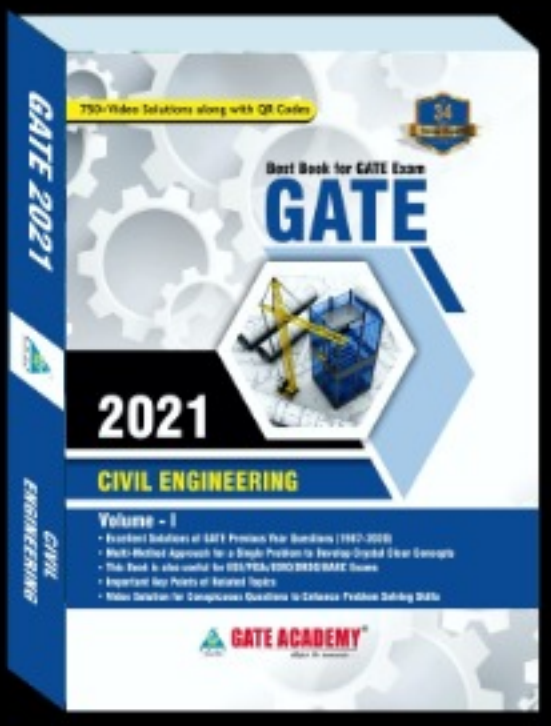

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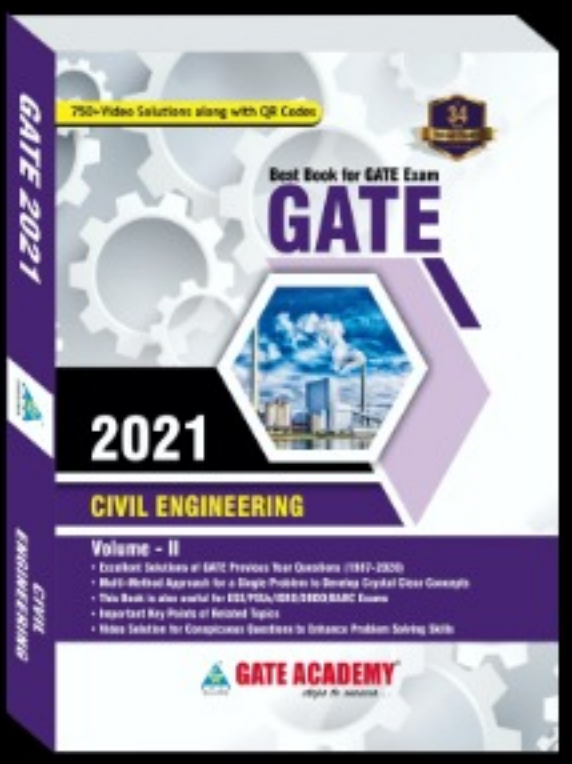

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Volume - I

- Excellent Solutions of GATE Previous Year Questions (1987-2020)
- Multi-Method Approach for a Single Problem to Develop Crystal Clear Concepts
- This Book is also useful for ESE/PSUs/ISRO/DRDO/BARC Exams
- Important Key Points of Related Topics
- Video Solution for Conspicuous Questions to Enhance Problem Solving Skills

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GATE 2021

Civil Engineering

(Volume - I)

TOPIC WISE GATE SOLUTIONS
1987 - 2020

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IMPORTANCE of GATE

GATE examination has been emerging as one of the most prestigious competitive exam for engineers. Earlier it was considered to be an exam just for eligibility for pursuing PG courses, but now GATE exam has gained a lot of attention of students as this exam open an ocean of possibilities like :

1. Admission into IISc, IITs, IIITs, NITs

A good GATE score is helpful for getting admission into IISc, IITs, IIITs, NITs and many other renowned institutions for M.Tech./M.E./M.S. An M.Tech. graduate has a number of career opportunities in research fields and education industries. Students get ₹ 12,400 per month as stipend during their course.

2. Selection in various Public Sector Undertakings (PSUs)

A good GATE score is helpful for getting job in government-owned corporations termed as **Public Sector Undertakings** (**PSUs**) in India like IOCL, BHEL, NTPC, BARC, ONGC, PGCIL, DVC, HPCL, GAIL, SAIL & many more.

3. Direct recruitment to Group A level posts in Central government, i.e., Senior Field Officer (Tele), Senior Research Officer (Crypto) and Senior Research Officer (S&T) in Cabinet Secretariat, Government of India, is now being carried out on the basis of GATE score.

4. Foreign universities through GATE

GATE has crossed the boundaries to become an international level test for entry into postgraduate engineering programmes in abroad. Some institutes in two countries **Singapore** and **Germany** are known to accept GATE score for admission to their PG engineering programmes.

5. National Institute of Industrial Engg. (NITIE)

- NITIE offers **PGDIE / PGDMM / PGDPM** on the basis of GATE scores. The shortlisted candidates are then called for Group Discussion and Personal Interview rounds.
- NITIE offers a Doctoral Level Fellowship Programme recognized by Ministry of HRD (MHRD) as equivalent to Ph.D. of any Indian University.
- Regular full time candidates those who will qualify for the financial assistance will receive ₹ 25,000 during 1st and 2nd year of the Fellowship programme and ₹ 28,000 during 3rd, 4th and 5th year of the Fellowship programme as per MHRD guidelines.

6. Ph.D. in IISc/ IITs

- IISc and IITs take admissions for Ph.D. on the basis of GATE score.
- Earn a Ph.D. degree directly after Bachelor's degree through integrated programme.
- A fulltime residential researcher (RR) programme.

7. Fellowship Program in management (FPM)

- Enrolment through GATE score card
- Stipend of ₹ 22,000 - 30,000 per month + HRA
- It is a fellowship program
- Application form is generally available in month of Sept. and Oct.

Note : In near future, hopefully GATE exam will become a mandatory exit test for all engineering students, so take this exam seriously. Best of LUCK !

GATE Exam Pattern

Section	Question No.	No. of Questions	Marks Per Question	Total Marks
General Aptitude	1 to 5	5	1	5
	6 to 10	5	2	10
Technical + Engineering Mathematics	1 to 25	25	1	25
	26 to 55	30	2	60
Total Duration : 3 hours		**Total Questions : 65**		**Total Marks : 100**
Note : 40 to 45 marks will be allotted to Numerical Answer Type Questions				

Pattern of Questions :

(i) **Multiple Choice Questions (MCQ)** carrying 1 or 2 marks each in all the papers and sections. These questions are objective in nature, and each will have a choice of four answers, out of which the candidate has to select (mark) the correct answer.

Negative Marking for Wrong Answers : For a wrong answer chosen in a MCQ, there will be negative marking. For **1-mark** MCQ, **1/3 mark** will be deducted for a wrong answer. Likewise for, **2-marks** MCQ, **2/3 mark** will be deducted for a wrong answer.

(ii) **Numerical Answer Type (NAT)** Questions carrying **1** or **2** marks each in all the papers and sections. For these questions, the answer is a signed real number, which needs to be entered by the candidate using the virtual numeric keypad on the monitor (Keyboard of the computer will be disabled). No choices will be shown for these type of questions. The answer can be a number such as **10** or **– 10** (an integer only). The answer may be in decimals as well, for example, **10.1** (one decimal) or **10.01** (two decimal) or **–10.001** (three decimal). These questions will be mentioned with, up to which decimal places, the candidates need to make an answer. Also, an appropriate range will be considered while evaluating the numerical answer type questions so that the candidate is not penalized due to the usual round-off errors. Wherever required and possible, it is better to give NAT answer up to a maximum of three decimal places.

Note : There is NO negative marking for a wrong answer in NAT questions.

PREFACE

It is our pleasure, that we insist on presenting **"GATE 2021 Civil Engineering (Volume - I)"** authored for **Civil Engineering** to all of the aspirants and career seekers. The prime objective of this book is to respond to tremendous amount of ever growing demand for error free, flawless and succinct but conceptually empowered solutions to all the question over the period 1987 - 2020.

This book serves to the best supplement for GATE 2021 (CE) but shall be useful to a larger extent for other discipline as well.

Simultaneously having its salient features the book comprises :

- Step by step solution to all questions.
- Complete analysis of questions, i.e. chapter wise as well as year wise.
- Detailed explanation of all the questions.
- Solutions are presented in simple and easily understandable language.
- It covers all GATE questions from 1987 to 2020 (34 years).

The authors do not sense any deficit in believing that this title will in many aspects, be different from the similar titles within the search of student.

We would like to express our sincere appreciation to **Mrs. Sakshi Dhande Mam** (Co-Director, GATE ACADEMY Learning Pvt. Ltd.) for her constant support and constructive suggestions and comments in reviewing the script.

In particular, we wish to thank **GATE ACADEMY** expert team members for their hard work and consistency while designing the script.

The final manuscript has been prepared with utmost care. However, going a line that, there is always room for improvement in anything done, we would welcome and greatly appreciate the suggestions and corrections for further improvement.

Umesh Dhande

(Director, GATE ACADEMY Learning Pvt. Ltd.)

ACKNOWLEDGEMENT

We are glad of this opportunity to acknowledge the views and to express with all the weaknesses of mere words the gratitude that we must always feel for the generosity of them.

We now express our gracious gratitude to the persons who have contributed a lot in order to put forth this into device. They are to be mentioned here and they are Rehan Ahmed, Anurag Verma, Akshay Mishra, Aman Kumar, Umendra Sahu, Ranjeet Kumar, Anuj, Shivam, Dusyant Sahu, Sujeet Pandey, Prashant Pandey, Khomesh Sahu, and Pratima Patel.

We would also like to express our gracious gratitude to the faculty members of Gate Academy who have contributed a lot in order to put forth this into device. They are to be mentioned here and they are **Mrs. Richa Gupta, Mr. Prateek Mishra, Mr. Venugopal Sharma, Mr. Deepraj Chandrakar** and **Mr. Yogesh Tyagi.**

Special thanks to Head of Department **Mr. Amit Zarola Sir** who has been involved in this project from the beginning and has given his best effort on his part. This book was not possible without his unconditional effort. His expertise and profound knowledge in many subjects helped in producing this tremendous book for GATE aspirants. Sir has suggested that this book will help in building the knowledge & concept and will motivate students to stay “on track” of GATE preparation.

Lastly, we take this opportunity to acknowledge the service of the total team of publication and everyone who collaborated in producing this work.

GATE ACADEMY

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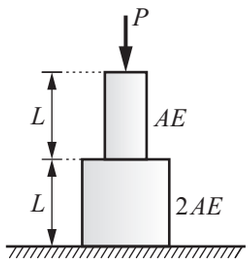
1. Numerical Ability
2. Logical Reasoning
3. Verbal Ability

1 Properties of Metals, Stress and Strain

1989 IIT Kanpur

1.1 The axial movement of top surface of stepped column as shown in figure is

[2 Marks]



(A) 2.5 PL/AE (B) 3 PL/AE

(C) 1.5 PL/AE (D) 2 PL/AE

1990 IISc Bangalore

1.2 The principle of superposition is made use of in structural computations when :

[1 Mark]

(A) The geometry of the structure changes by a finite amount during the application of the loads.

(B) The changes in the geometry of the structure during the application of the loads is too small and the strains in the structure are directly proportional to corresponding stresses.

(C) The strains in the structure are not directly proportional to the corresponding stresses, even though the effect of changes in geometry can be neglected.

(D) None of the above conditions are met.

1991 IIT Madras

1.3 The maximum value of Poisson's ratio for an elastic material is **[1 Mark]**

(A) 0.25 (B) 0.5

(C) 0.75 (D) 0.1

1.4 In a linear elastic structural element

[1 Mark]

(A) Stiffness is directly proportional to flexibility

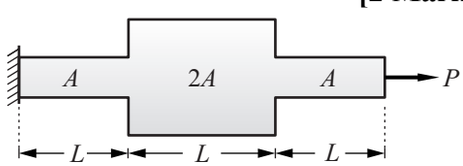
(B) Stiffness is inversely proportional to flexibility

(C) Stiffness is equal to flexibility

(D) Stiffness and flexibility are not related

1.5 The total elongation of the structural element fixed, at one end, free at the other end, and of varying cross-section as shown in the figure when subjected to a force P at free end is given by

[2 Mark]



(A) PL/AE (B) $3PL/AE$

(C) 2.5 PL/AE (D) 2 PL/AE

1.6 A cantilever beam of tubular section consists of two materials, copper as outer cylinder and steel as inner cylinder. It is subjected to a temperature rise of 20^0C and $\alpha_{copper} > \alpha_{steel}$. The stresses developed in the tubes will be **[1 Mark]**

(A) Compression in steel and tension in copper.
(B) Tension in steel and compression in copper.
(C) No stress in both.
(D) Tension in both the materials.

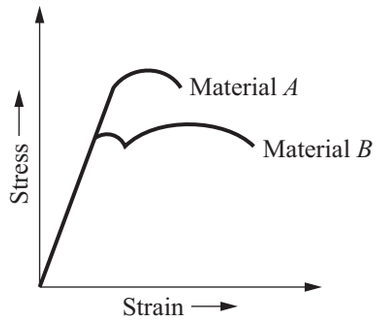
1995 IIT Kanpur

1.7 The material that exhibits the same elastic properties in all directions at a point is said to be **[1 Mark]**

(A) Homogeneous (B) Orthotropic
(C) Viscoelastic (D) Isotropic

2000 IIT Kharagpur

1.8 The stress-strain diagram for two materials A and B is shown below :



The following statements are made based on this diagram :

I. Material A is more brittle than material B.
II. The ultimate strength of material B is more than that of A.

With reference to the above statements, which of the following applies? **[1 Mark]**

(A) Both the statements are false.
(B) Both the statements are true.
(C) I is true but II is false.
(D) I is false but II is true.

1.9 The dimensions for the flexural rigidity of a beam element in mass (M), length (L) and time (T) is given by **[1 Mark]**

(A) MT^{-2} (B) ML^3T^{-2}
(C) $ML^{-1}T^{-2}$ (D) $ML^{-1}T^2$

2002 IISc Bangalore

1.10 The shear modulus (G), modulus of elasticity (E) and the Poisson's ratio (μ) of a material are related as **[1 Mark]**

(A) $G = \dfrac{E}{[2(1+\mu)]}$

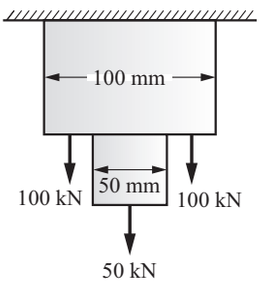
(B) $E = \dfrac{G}{[2(1+\mu)]}$

(C) $G = \dfrac{E}{[2(1-\mu)]}$

(D) $G = \dfrac{E}{[2(\mu-1)]}$

2003 IIT Madras

1.11 A bar of varying square cross-section is loaded symmetrically as shown in the figure. Loads shown are placed on one of the axes of symmetry of cross-section. Ignoring self-weight, the maximum tensile stress in N/mm^2 anywhere is **[1 Mark]**



(A) 16.0 (B) 20.0
(C) 25.0 (D) 30.0

2004 IIT Delhi

1.12 For linear elastic systems, the type of displacement function for the strain energy is **[1 Mark]**

(A) Linear (B) Quadratic

(C) Cubic (D) Quartic

2005 IIT Bombay

1.13 The symmetry of stress tensor at a point in the body under equilibrium is obtained from **[1 Mark]**

(A) Conservation of mass

(B) Force equilibrium equations

(C) Moment equilibrium equations

(D) Conservation of energy

1.14 The components of strain tensor at a point in the plane strain case can be obtained by measuring longitudinal strain in following directions : **[1 Mark]**

(A) Along any two arbitrary directions.

(B) Along any three arbitrary directions.

(C) Along two mutually orthogonal directions.

(D) Along any arbitrary direction.

2007 IIT Kanpur

1.15 For an isotropic materials, the relationship between the Young's modulus (E), shear modulus (G) and Poisson's ratio (μ) is given by **[1 Mark]**

(A) $G = \dfrac{E}{2(1+\mu)}$ (B) $E = \dfrac{E}{2(1+\mu)}$

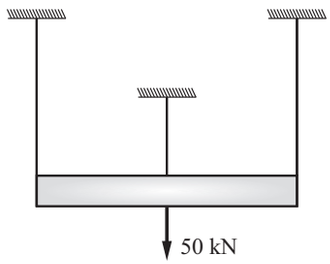
(C) $G = \dfrac{E}{(1+2\mu)}$ (D $G = \dfrac{E}{2(1-2\mu)}$

1.16 A metal bar of length 100 mm is inserted between two rigid supports and its temperature is increased by 10^0C. If the coefficient of thermal expansion is 12×10^{-6} per ^{0}C and the Young's modulus is 2×10^5 MPa, the stress in the bar is **[2 Marks]**

(A) Zero (B) 12 MPa

(C) 24 MPa (D) 2400 MPa

1.17 A rigid bar is suspended by three rods made of the same materials. The area and length of the central rod are $3A$ and L, respectively while that of the two outer rods are $2A$ and $2L$, respectively. If a downward force of 50 kN is applied to the rigid bar, the forces in the central and each of the outer rods will be **[2 Marks]**



(A) 16.67 kN each

(B) 30 kN and 15 kN

(C) 30 kN and 10 kN

(D) 21.4 kN and 14.3 kN

1.18 U_1 and U_2 are the strain energies stored in a prismatic bar due to axial tensile forces P_1 and P_2, respectively. The strain energy U stored in the same bar due to combined action of P_1 and P_2 will be **[2 Marks]**

(A) $U = U_1 + U_2$ (B) $U = U_1 U_2$

(C) $U < U_1 + U_2$ (D) $U > U_1 + U_2$

2008 IISc Bangalore

1.19 A mild steel specimen is under uniaxial tensile stress. Young's modulus and yield stress for mild steel are 2×10^5

MPa and 250 MPa respectively. The maximum amount of strain energy per unit volume that can be stored in this specimen without permanent set is **[1 Mark]**

(A) 156 N-mm/mm^3

(B) 15.6 N-mm/mm^3

(C) 1.56 N-mm/mm^3

(D) 0.156 N-mm/mm^3

1.20 A vertical rod PQ of length L is fixed at its top end P and has a flange fixed to the bottom end Q. A weight W is dropped vertically from a height h ($< L$) on to the flange. The axial stress in the rod can be reduced by **[2 Marks]**

(A) Increasing the length of rod.

(B) Decreasing the length of the rod.

(C) Decreasing the area of cross-section of the rod.

(D) Increasing the modulus of elasticity of the material.

2010 IIT Guwahati

1.21 The number of independent elastic constants for a linear elastic isotropic and homogeneous material is **[1 Mark]**

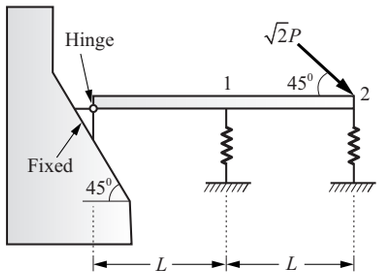
(A) 4 (B) 3

(C) 2 (D) 1

2011 IIT Madras

Common Data for Questions 1.22 & 1.23

A rigid beam is hinged at one end and supported on linear elastic spring (both having a stiffness of 'k') at points '1' and '2' an inclined load acts at '2' as shown



1.22 Which of the following options represents the deflections δ_1 and δ_2 at points '1' and '2'? **[2 Marks]**

(A) $\delta_1 = \frac{2}{5}\left(\frac{2P}{k}\right)$ and $\delta_2 = \frac{4}{5}\left(\frac{2P}{k}\right)$

(B) $\delta_1 = \frac{2}{5}\left(\frac{P}{k}\right)$ and $\delta_2 = \frac{4}{5}\left(\frac{P}{k}\right)$

(C) $\delta_1 = \frac{2}{5}\left(\frac{P}{\sqrt{2}k}\right)$ and $\delta_2 = \frac{4}{5}\left(\frac{P}{\sqrt{2}k}\right)$

(D) $\delta_1 = \frac{2}{5}\left(\frac{\sqrt{2}P}{k}\right)$ and $\delta_2 = \frac{4}{6}\left(\frac{\sqrt{2}P}{k}\right)$

1.23 If the load P equals 100 kN, which of the following options represents forces R_1 and R_2 in the springs at points '1' and '2' ? **[2 Marks]**

(A) $R_1 = 20\,\text{kN}$ and $R_2 = 40\,\text{kN}$

(B) $R_1 = 50\,\text{kN}$ and $R_2 = 50\,\text{kN}$

(C) $R_1 = 30\,\text{kN}$ and $R_2 = 60\,\text{kN}$

(D) $R_1 = 40\,\text{kN}$ and $R_2 = 80\,\text{kN}$

2012 IIT Delhi

1.24 The Poisson's ratio is defined as **[1 Mark]**

(A) $\left|\frac{\text{Axial stress}}{\text{Lateral stress}}\right|$

(B) $\left|\frac{\text{Lateral strain}}{\text{Axial strain}}\right|$

(C) $\left|\frac{\text{Lateral stress}}{\text{Axial stress}}\right|$

(D) $\left|\frac{\text{Axial strain}}{\text{Lateral strain}}\right|$

2013 IIT Bombay

1.25 Creep strains are **[1 Mark]**

(A) caused due to dead load only.

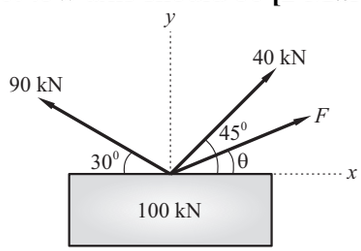
(B) caused due to live load only.

(C) caused due to cyclic load only.

(D) independent of load.

2014 IIT Kharagpur

1.26 A box of weight 100 kN shown in the figure is to be lifted without swinging. If all forces are coplanar, the magnitude and direction (θ) of the forces (F) with respect to x-axis should be **[2 Marks]**



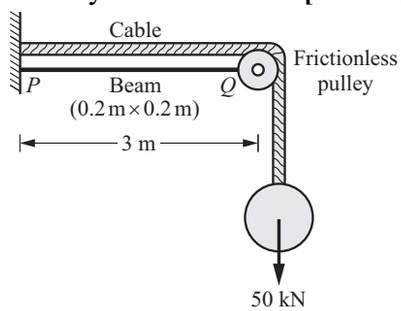
(A) $F = 56.389\,\text{kN}$ and $\theta = 28.28^0$

(B) $F = -56.389\,\text{kN}$ and $\theta = -28.28^0$

(C) $F = 9.055\,\text{kN}$ and $\theta = 1.414^0$

(D) $F = -9.055\,\text{kN}$ and $\theta = -1.414^0$

1.27 The values of axial stress (σ) in kN/m^2, bending moment (M) in kNm, and shear force (V) in kN acting at point P for the arrangement shown in the figure are respectively. **[2 Marks]**



(A) 1000, 75 and 25

(B) 1250, 150 and 50

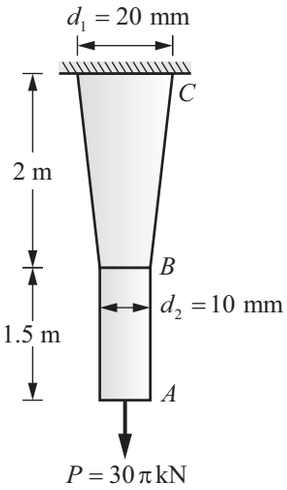
(C) 1500, 225 and 75

(D) 1750, 300 and 100

2015 IIT Kanpur

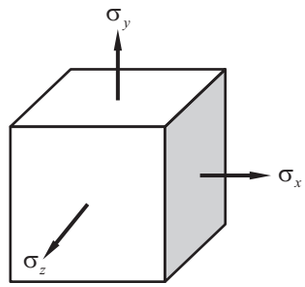
1.28 A tapered circular rod of diameter varying from 20 mm to 10 mm is connected to another uniform circular rod of diameter 10 mm as shown in the following figure. Both bars are made of same material with modulus of elasticity, $E = 2\times10^5$ MPa. When subjected to a load $P = 30\pi$ kN, the deflection at point A is _____ mm.

[2 Marks]



2016 IISc Bangalore

1.29 An elastic isotropic body is in a hydrostatic state of stress as shown in the figure. For no change in the volume to occur, what should be its Poisson's ratio? **[2 Marks]**



(A) 0.00 (B) 0.25

(C) 0.50 (D) 1.00

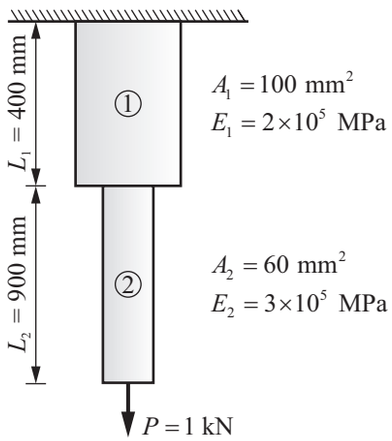
2017 IIT Roorkee

1.30 An elastic bar of length L, uniform cross-sectional area A, coefficient of thermal expansion α, and Young's modulus E is fixed at the two ends. The temperature of the bar is increased by T, resulting in an axial stress σ. Keeping all, other parameters unchanged, if the length of the bar is doubled, the axial stress would be **[1 Mark]**

(A) σ (B) 2σ

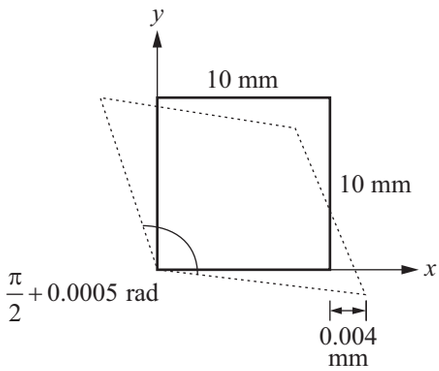
(C) 0.5σ (D) 0.25σ

1.31 Consider the stepped bar made with a linear elastic material and subjected to an axial load of 1 kN, as shown in the figure. **[2 Marks]**



Segments 1 and 2 have cross-sectional area of $100\,\text{mm}^2$ and $60\,\text{mm}^2$. Young's modulus of 2×10^5 MPa and 3×10^5 MPa, and length of 400 mm and 900 mm, respectively. The strain energy (in Nmm up to one decimal place) in the bar due to the axial load is _____.

1.32 In a material under a state of plane strain, a $10\times10\,\text{mm}$ square centered at a point gets deformed as shown in the figure.

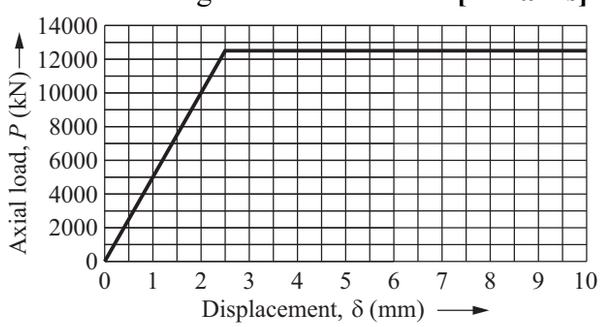


If the shear strain γ_{xy} at this point is expressed as $0.001\ k$ (in rad), the value of k is **[1 Mark]**

(A) 0.50 (B) 0.25

(C) – 0.25 (D) – 0.50

1.33 A 2 m long, axially loaded mild steel rod of 8 mm diameter exhibits the load-displacement (P-δ) behavior as shown in the figure. **[2 Marks]**



Assume the yield stress of steel as 250 MPa. The complementary strain energy (in Nmm) stored in the bar up to its linear elastic behaviour will be _____.

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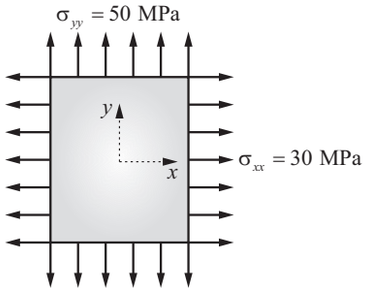
1.34 A plate in equilibrium is subjected to uniform stresses along its edges with magnitude $\sigma_{xx} = 30$ MPa and $\sigma_{yy} = 50$ MPa as shown in the figure. **[2 Marks]**



The Young's modulus of the material is 2×10^{11} N/m^2 and the Poisson's ratio is 0.3. If σ_{zz} is negligibly small and assumed to be zero, then the strain ε_{zz} is

(A) -120×10^{-6} (B) -60×10^{-6}

(C) 0.0 (D) 120×10^{-6}

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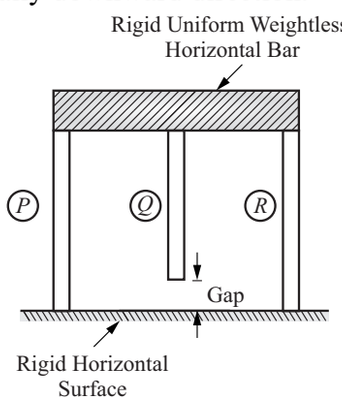
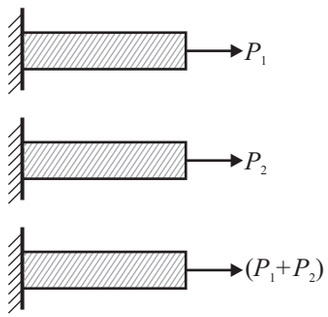
1.35 A rigid, uniform, weightless, horizontal bar is connected to three vertical members P, Q and R as shown in the figure (*not drawn to the scale*). All three members have identical axial stiffness of 10 kN/mm. The lower end of bars P and R rest on a rigid horizontal surface. When **NO** load is applied, a gap of 2 mm exists between the lower end of the bar Q and the rigid horizontal surface. When a vertical load W is placed on the horizontal bar in the downward direction, the bar still remains horizontal and gets displaced by 5 mm in the vertically downward direction.



The magnitude of the load W (in kN, round off to the nearest integer), is _____. **[2 Marks]**

1.36 A prismatic linearly elastic bar of length L, cross-sectional area A, and made up of a material with Young's modulus E, is subjected to axial tensile force as shown in the figures. When the bar is subjected to axial tensile forces P_1 and P_2, the strain energies stored in the bar are U_1 and U_2 respectively. **[2 Marks]**



If U is the strain energy stored in the same bar when subjected to an axial tensile force (P_1+P_2) the correct relationship is

(A) $U=U_1+U_2$ (B) $U<U_1+U_2$

(C) $U=U_1-U_2$ (D) $U>U_1+U_2$

Answers — Properties of Metals, Stress & Strain

1.1	C	1.2	B	1.3	B	1.4	B	1.5	C
1.6	B	1.7	D	1.8	C	1.9	B	1.10	A
1.11	C	1.12	B	1.13	C	1.14	B	1.15	A
1.16	C	1.17	C	1.18	D	1.19	D	1.20	A
1.21	C	1.22	B	1.23	D	1.24	B	1.25	A
1.26	A	1.27	B	1.28	15	1.29	C	1.30	A
1.31	35	1.32	D	1.33	15625000	1.34	A	1.35	130
1.36	D								

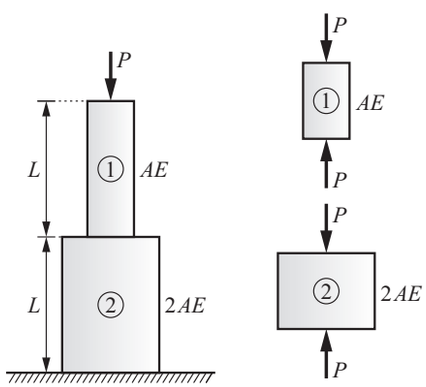
Explanations — Properties of Metals, Stress & Strain

1.1 (C)

Given : Axial rigidity of section (1) $= AE$

Axial rigidity of section (2) $= 2AE$

According questions, FBD of both section is given as,



Deflection of top surface = Deflection of Segment (1) + Deflection of segment (2),

$$\delta l = \delta l_1 + \delta l_2$$

$$\delta l = \frac{PL}{AE} + \frac{PL}{2AE} = 1.5\frac{PL}{AE}$$

Hence, the correct option is (C).

Key Point

For solid bar,

$$\sigma_a = \frac{P}{A} = \frac{4P}{\pi D^2},\ \delta L = \frac{PL}{AE} = \frac{4PL}{\pi D^2 E} \quad \ldots(i)$$

$$\delta L = \frac{P}{EA/L} = \frac{P}{K}$$

Where, K = Axial stiffness of bar,

AE = Axial rigidity,

EI/L = Flexural stiffness.

For hollow bar,

$$\sigma_a = \frac{4P}{\pi D^2(1-k^2)},\ \delta L = \frac{4PL}{\pi D^2(1-k^2)E} \quad \ldots(ii)$$

Where, $k = d/D$.

Equation (i) and (ii) valid under pure axial loading, prismatic bar and same material.

Case 1 : Bars in series :

1. $(\delta L)_{\text{total}} = \delta L_1 + \delta L_2 + \delta L_3 + \ldots.$

   (Axial deformation are cumulative)

2. $P_1 = P_2 = P_3 = P_n = P$

   (Equal in magnitude and like in nature)

Case 2 : Bars in parallel :

1. $P_1 + P_2 = P$ (Axial loads are cumulative)
2. $(\delta L)_1 = (\delta L_2) = \delta L$

1.2 (B)

The principle of super position is a central concept in the analysis of structures. The principle of superposition may be stated as *the deflection at a given point in a structure*

produced by several loads acting simultaneously on the structure can be found by superposing the deflections at the same point produced by loads acting individually.

This is applicable when :

1. There exist a linear relationship between external forces and corresponding structural displacements.
2. The structural members subjected to negligible deformation i.e., strain is small.

Hence, the correct option (B).

Key Point

The deflection, slope, bending moment and the shear force are all directly proportional to the concentrated load P. Therefore, if two or more concentrated loads act on the beam the above quantities due to the action of any one of the loads may be assumed to be independent of all the other loads. The value of the resulting quantity from all the concentrated loads may be obtained at any section of the beam by obtaining the algebraic sum of these quantities.

1.3 (B)

Poisson's ratio is defined as the negative ratio of lateral strain to longitudinal strain, and is constant for a given material, when the material is stressed within the elastic limit. It is generally denoted by μ. Hence mathematically,

$$\text{Poisson's ratio} = -\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

The Poisson's ratio of a stable, isotropic linear elastic material must be between -1.0 to 0.5.

Hence, the maximum value of Poisson's ratio for an elastic material is 0.5.

Key Point

- Poisson's ratio of rubber is nearly 0.5 and same for cork is nearly 0.0.
- Certain material exhibit negative Poisson's ratio, and are known as **auxetic materials**. If these materials are stretched in one direction, they become thicker in the perpendicular direction.
- $\mu = 0.5$, represents a perfectly plastic, incompressible material.

1.4 (B)

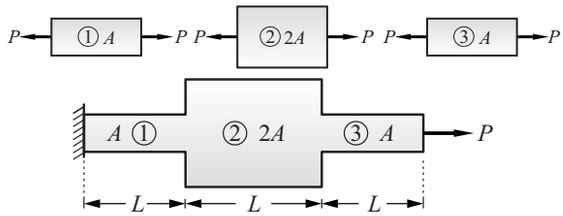
Resistance against deformation is known as **stiffness**, and ease to deformation is known as flexibility. Hence, for any elastic material stiffness is reciprocal of flexibility i.e.,

$$\text{Stiffness} \propto \frac{1}{\text{Flexibility}}$$

Hence, the correct option is (B).

1.5 (C)

FBD of problem is given as,



Total elongation of structural element
= Elongation of segment (1)
+ Elongation of segment (2)
+ Elongation of segment (3)

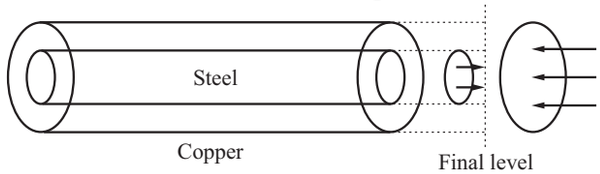
$$\delta l_{total} = \delta l_1 + \delta l_2 + \delta l_3$$

$$\delta l_{total} = \frac{PL}{AE} + \frac{PL}{2AE} + \frac{PL}{AE} = \frac{2.5PL}{AE}$$

Hence, the correct option is (C).

1.6 (B)

In this question, it is not mentioned whether the two material will form a composite section or not, so there will be two case possible.



Because of thermal expansion, the extension in the beam due to change in temperature is given by,

$$\Delta L = L\alpha\Delta T$$

Where, L = Length of tube.

α = Coefficient of thermal expansion

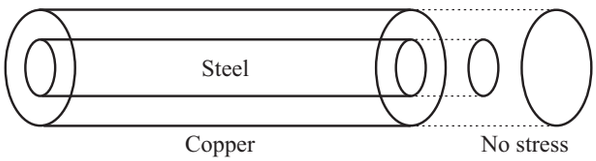
ΔT = Change in temperature

as $\alpha_{copper} > \alpha_{steel}$

$\therefore$ $(\Delta l)_{copper} > (\Delta l)_{steel}$

Thus, free expansion in copper will be more than steel, so equilibrium will develop compressive stress in copper and tensile stress in steel.

If assumed independent, when two materials are independent they do not form composite section and are free to expand. Thus, no stress will generate in both



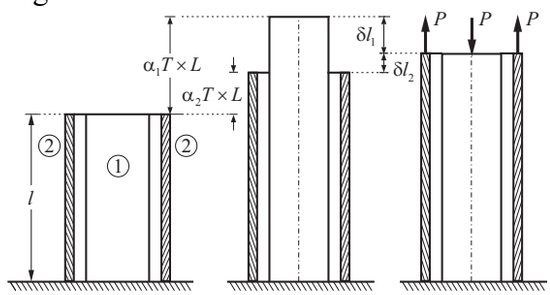
Hence, the correct option is (B).

Key Point

Members in parallel subjected to change in temperature :

- When a composite section (member in parallel) is subjected to a change in temperature then different components will tend to change length by different amount due to their different coefficient of linear expansions.
- If the members are not allowed to move relative with each other than this prevention in motion will cause compressive stress in member with higher coefficient of expansion and tension a member of lower coefficient of expansion.
- If no external force is applied then the forces on both the material will be same.

Consider composite system having two tubes of equal length l but of different material as shown in figure



Compatibility equation :

Final length of bar (1) = Final length of bar (2)

$$\left(\text{Free expansion}\right)_1 - \left(\text{Prevention in expansion}\right)_1 = \left(\text{Free expansion}\right)_2 + \left(\text{Extra expansion}\right)_2$$

$$\delta l_{th_1} - \delta l_1 = \delta l_{th_2} + \delta l_2$$

$$\alpha_1 \Delta TL - \frac{PL}{A_1E_1} = \alpha_2 \Delta TL + \frac{PL}{A_2E_2}$$

$$(\alpha_1 - \alpha_2)\Delta TL = \frac{PL}{A_1E_1} + \frac{PL}{A_2E_2}$$

1.7 (D)

The material that exhibits the same elastic properties in all directions at a point is said to be **isotropic**.

Example : Glass and metals.

Hence, the correct option is (D).

Key Point

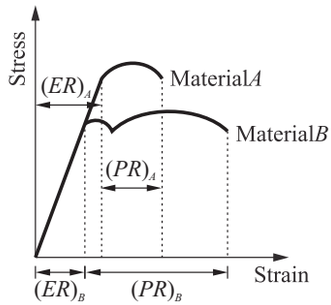
- **Homogeneous materials :** These are the material of uniform composition throughout that cannot be separated mechanically into different materials.
  Examples : Metal, alloys, plastic etc.
- **Anisotropic materials :** These materials have certain properties that vary with the direction from which they are measured i.e. properties are direction dependent.

- **Orthographic materials :** They have properties that are different in three mutually perpendicular direction. They are a subset of antistrophic materials, because their properties change when measured form different directions.
- **Viscoelastic materials :** They have elements of both viscous and elastic properties and, as such exhibit time-dependent strain.

  Examples : Ligaments, Tendons

1.8 (C)

According to question,



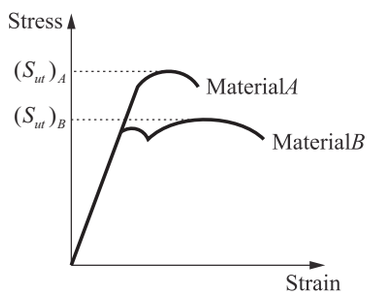
Material A :

Elastic range for A $(ER)_A >$ Plastic range $(PR)_A$.

Material B :

Elastic range for B $(ER)_B <$ Plastic range $(PR)_B$.

Due to brittleness material undergoes less plastic deformation and from above observation it is clear that material A is undergoing less plastic deformation. Hence, A is more brittle than B.



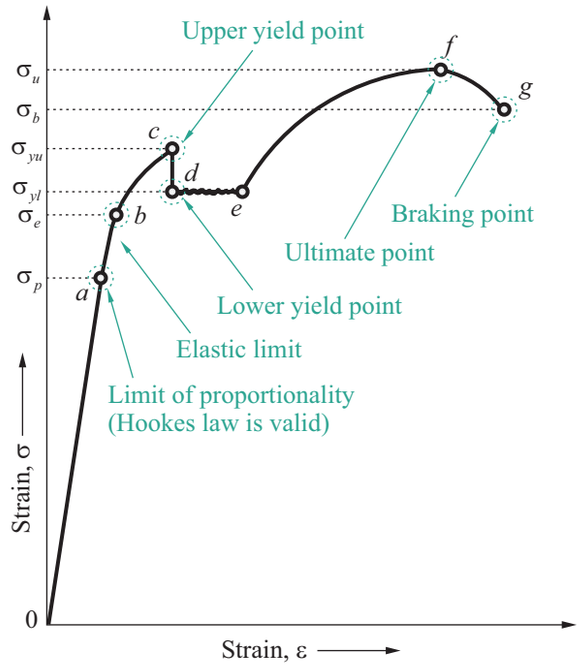
From above figure it is clear that the ultimate strength of material A is more than that of B.

Therefore, statement I is true and statement II is false.

Hence, the correct option is (C).

Key Point

For ductile material, example mild steel.



Where, σ_p = Proportional limit stress,

σ_e = Elastic limit stress,

σ_{yu} = Upper yield point stress,

σ_{yl} = Lower yield point stress,

σ_u = Ultimate stress,

σ_b = Breaking stress.

1.9 (B)

Flexural rigidity : Bending moment required for unit radius of curvature (EI). Mathematically, it is the product of Young's modulus and second moment of area cross section of beam.

Dimensional formula : ML^3T^{-2}.

Hence, the correct option is (B)

Key Point

Flexural rigidity is a measure of the resistance of a beam to bending i.e., the larger the flexural rigidity, the smaller the curvature for a given bending moment.

1.10 (A)

$$E = 2G(1+\mu)$$

$$G = \frac{E}{2(1+\mu)}$$

Hence, the correct option is (A).

Key Point

Relation between *E*, *G*, *K*, *v* :

1. $E = 2G(1+\mu)$
2. $E = 3K(1-2\mu)$
3. $E = \frac{9KG}{3K+G}$
4. $\mu = \frac{3K-2G}{6K+2G}$

Where, E = Young's modulus,

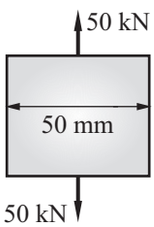
G = Shear modulus, K = Bulk modulus,

μ = Poisson ratio.

- Theoretical limit of Poisson ratio $-1 \le \mu \le 1/2$

1.11 (C)

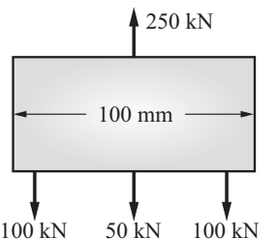
According to the question, FBD of lower portion of bar is,



Stress in lower portion of bar,

$$= \frac{P}{A} = \frac{50\times10^3}{50\times50} = 20\ \text{N/mm}^2$$

FBD of upper portion of bar is,



Stress in upper portion of bar,

$$= \frac{P}{A} = \frac{250\times10^3}{100\times100} = 25\ \text{N/mm}^2$$

So, maximum tensile stress $= 25\,\text{kN/mm}^2$.

Hence, the correct option is (C).

1.12 (B)

Let, δ = Extension of bar.

$$\delta = \frac{PL}{AE} = \frac{\sigma L}{E} \qquad \left(\sigma = \frac{P}{A}\right)$$

Work done in straining or strain energy stored,

$$W = U = \frac{1}{2}\frac{\sigma^2}{E}V \qquad \left(V = \frac{\pi}{4}d^2L\right)$$

Or, $$U = \frac{1}{2}E\varepsilon^2 V$$

i.e., strain energy $\propto \varepsilon^2$

And, we know that strain is directly proportional to deflection or displacement. So, strain energy is quadratic function of displacement.

Hence, the correct option is (B).

Key Point

Linear elastic body is one which satisfies the condition of linearity and elasticity.

Condition of linearity : It must obey Hooke's law which states that stress is directly proportional to strain.

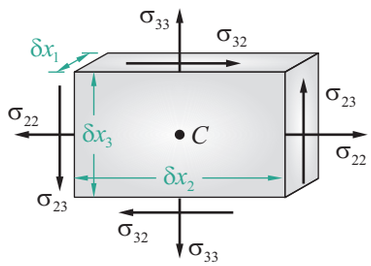
Condition of elasticity : It satisfies only when a material has region its original dimensions after the removal of applied loading.

Example : Steel is an linear elastic material up to proportionality limit.

1.13 (C)

Symmetry of stress tensor :

Consider moment equilibrium of differential element :



Taking moments about x_1 axis (i.e., point C) :

$\sum M_1 = 0$,

$$2\left[\sigma_{23}\underbrace{(\delta x_3 \delta x_1)}_{\text{Area of face}}\frac{\delta x_2}{2}\right] - 2\left[\sigma_{32}(\delta x_2 \delta x_1)\frac{\delta x_3}{2}\right] = 0$$

Moment arm

$$\Rightarrow \quad \sigma_{23} = \sigma_{32}$$

Thus, in general $\sigma_{mn} = \sigma_{nm}$

Stress tensor is symmetric. Six independent components of the stress tensor.

σ_{11} $\quad \sigma_{12} = (\sigma_{21})$

σ_{22} $\quad \sigma_{23} = (\sigma_{32})$

σ_{33} $\quad \sigma_{31} = (\sigma_{13})$

Hence, the correct option is (C).

🕮 Key Point

A positive (tensile) component of stress acts on a face a positive normal in a positive direction. Thus a stress acting on a negative normal face, in a negative direction is also positive. If the stresses do not vary over the infinitesimal element, τ_{ij} acts on opposite faces, in opposite directions but with equal magnitude.

Stress tensor (σ) consists of nine components σ_{ij} that completely define the state of stress at a point inside a material in the deformed state. This is because each component of the stress must be defined not only by the direction in which it acts but also the orientation of the surface upon which it is acting.

The first index i specifies the direction in which the stress component acts, and the second index j identifies the orientation of the surface upon which it is acting stress tensor is given as,

$$\tau_{ij} = \begin{bmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{bmatrix}$$

Here, $\tau_{11} \equiv \sigma_1$, $\tau_{22} \equiv \sigma_2$, $\tau_{33} = \sigma_3$.

Hence, $\tau_{ij} = \begin{bmatrix} \sigma_1 & \tau_{12} & \tau_{13} \\ \tau_{21} & \sigma_2 & \tau_{23} \\ \tau_{31} & \tau_{32} & \sigma_3 \end{bmatrix}$

1.14 (B)

If the only deformations are those in the xy-plane, then three main components may exist, the normal strain ε_x in the x-direction, the normal strain ε_y in the y-direction, and the shear strain γ_{xy}. An element of material subjected to only these strains is said to be in a state of **plane strain**.

Here, we have three unknown, so we need 3 equations to find them. These unknowns are related to longitudinal strain by the equations,

$$\varepsilon_1 = \varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \frac{\gamma_{xy}}{2}\sin 2\theta_1$$

$$\varepsilon_2 = \varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \frac{\gamma_{xy}}{2}\sin 2\theta_2$$

$$\varepsilon_3 = \varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \frac{\gamma_{xy}}{2}\sin 2\theta_3$$

Thus, to find out component of strain tensor at a point, in case of plain strain case, we need to measure the longitudinal strain $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ along any three arbitrary equation.

Hence, the correct option is (B).

Key Point

- In case of 3 dimensional coordinate system the components of strain are expressed by the strain tensor,

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_z \end{bmatrix} = \begin{bmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{xy}}{2} & \varepsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{xz}}{2} & \frac{\gamma_{yz}}{2} & \varepsilon_z \end{bmatrix}$$

- **Principal strains** can be calculated using equation :

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} + \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right) + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

- **Principal angle**, θ_p is calculated using equation :

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

- **Maximum shear strain** (γ_{max}),

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

1.15 (A)

$$E = 2G(1+\mu)$$

$$G = \frac{E}{2(1+\mu)}$$

Hence, the correct option is (A).

1.16 (C)

Given : Length of the bar $(L) = 100\,\text{mm}$

Increase in temperature $(\Delta T) = 10^0\,\text{C}$

Coefficient of thermal expansion

$(\alpha) = 12\times10^{-6}/\,^0\text{C}$

Young's modulus, $(E) = 2\times10^5$ MPa

Since support is fixed so, compressive stress is developed in the bar due to rise in temperature.

So, thermal stress $= \alpha\Delta TE$

$= 12\times10^{-6}\times10\times2\times10^5$

$= 24\,\text{MPa}$

Hence, the stress in the bar is **24 MPa**.

Key Point

Thermal stresses are set up in a body, when the temperature of the body is changed and the change in length is restricted either fully or partially.

Consider a body which is heated to a certain temperature.

Let, L = Original length of the body

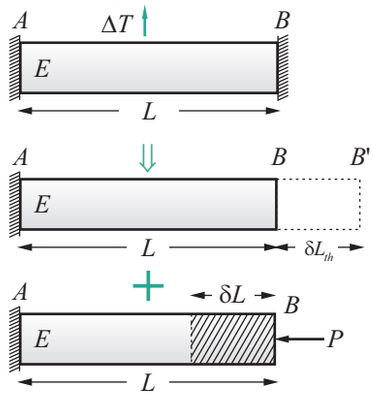
ΔT = Rise in temperature

E = Young's modulus

α = Coefficient of linear expansion

δL = Extension of rod due to rise of temperature

Expansion is fully restricted :



If the rod is free to expand, then extension of the rod is given by,

$$\delta L_{th} = \alpha\Delta TL$$

As the ends of the body are fixed to rigid supports, so that extension is completely prevented, then compressive stress will be set up in the rod.

Hence, net change in length equal to zero. This situation can be seen as,

Complementary equation :

δL_{th} (Free expansion due to change in temp.)

$= \delta L$ (Change in length prevented)

Or $\delta L_{th} - \delta L = 0$

$$\alpha \Delta T L - \frac{PL}{AE} = 0$$

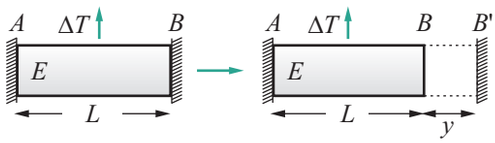
$$\alpha \Delta T L = \frac{PL}{AE}$$

$$\alpha \Delta T L = \frac{\sigma_{th}}{E}$$

$$\sigma_{th} = \alpha \Delta T E$$

Thermal stress and strain when supported yield (i.e. there is a gap) :

Suppose support yield by an amount equal to δ,



Complementary equation :

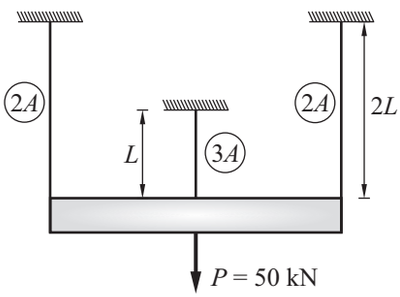
Free expansion – Expansion prevented = y

$$\delta L_m - \delta L = y$$

$$\alpha \Delta T L - \left(\frac{PL}{AE}\right) = y$$

1.17 (C)

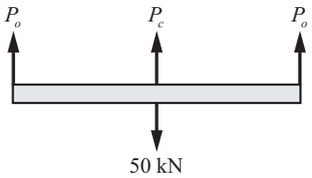
Given : Applied downward force, $(P) = 50\,\text{kN}$



Let, P_o and P_c be the force on the outer rod and central rod, respectively.

From the symmetry, the force in each of outer rods plus force in central rod is same and equal to P_o.

So, $2P_o + P_c = 50$ …(i)



Since all rods are connected to a rigid bar, the deflection in each rod will be same.

$$\therefore \quad (\delta L)_{outer} = (\delta L)_{central}$$

$$\frac{P_o L_o}{A_o E} = \frac{P_c L_c}{A_c E}$$

$$\frac{P_o \times 2L}{2A \times E} = \frac{P_c \times L}{3A \times E}$$

$$P_c = 3P_o \quad \text{…(ii)}$$

From equation (i) and (ii),

$$2P_o + 3P_o = 50$$

$$P_o = 10\,\text{kN}$$

$$P_c = 30\,\text{kN}$$

Hence, the correct option is (C).

1.18 (D)

Work done in straining the bar or strain energy stored in a prismatic bar due to axial load P_1,

$$U_1 = \frac{P_1^2 L}{2AE}$$

Strain energy stored in a prismatic bar due to axial load P_2,

$$U_2 = \frac{P_2^2 L}{2AE}$$

Strain energy stored in a prismatic bar due to combined action of P_1 and P_2,

$$U = \frac{(P_1 + P_2)^2 L}{2AE}$$

$$U = \frac{P_1^2 L}{2AE} + \frac{P_2^2 L}{2AE} + \frac{P_1 P_2 L}{AE}$$

$$U = U_1 + U_2 + \frac{P_1 P_2 L}{AE}$$

i.e. $U > U_1 + U_2$

Hence, the correct option is (D).

1.19 (D)

Given : Young's modulus, $(E) = 2\times10^5$ MPa

Yield stress, $(\sigma_y) = 250$ MPa

For uniaxial tensile stress, the strain energy per unit volume may be given as,

$$u = \frac{U}{V} = \frac{1}{2}\times\text{Stress}\times\text{Strain}$$

$$u = \frac{1}{2}\sigma_y\times\varepsilon = \frac{1}{2}\times\sigma_y\times\frac{\sigma_y}{E}$$

$$u = \frac{1}{2}\times\frac{\sigma_y^2}{E} = \frac{1}{2}\times\frac{(250)^2}{2\times10^5}$$

$$u = 0.156250 \text{ N-mm/mm}^3$$

$$u = 0.156 \text{ N-mm/mm}^3$$

Hence, the correct option is (D).

Key Point

- **Axial loading :**

1. Strain energy per unit volume (Also known as energy density), $u = \frac{1}{2}\frac{\sigma^2}{E}$
2. Total strain energy, $U = \frac{1}{2}\frac{\sigma^2}{E}\times\text{Volume}$
3. Strain energy per unit volume under three- dimensional principal stresses $(\sigma_1, \sigma_2, \sigma_3)$

$$u = \frac{1}{2E}[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)]$$

- **Pure shear stress :**

Strain energy per unit volume, $u = \frac{\tau^2}{2G}$

1.20 (A)

Since weight W is dropped from height h on the flange, it causes an impact on the flange and transfer its potential energy to the rod as strain energy.

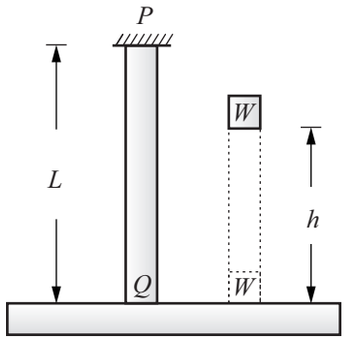
Potential energy of weight $= Wh$,

Strain energy stored in rod

$$= \frac{1}{2}\times\text{Stress}\times\text{strain}\times\text{volume}$$

$$= \frac{1}{2}\times\sigma\times\frac{\sigma}{E}\times A\times L$$

$$= \frac{1}{2}\frac{\sigma^2}{E}\times A\times L$$



Thus, $Wh = \frac{1}{2}\frac{\sigma^2}{E}\times A\times L$

i.e. $\sigma\propto\sqrt{E}$ or $\sigma\propto\frac{1}{\sqrt{A}}$ or $\sigma\propto\frac{1}{\sqrt{L}}$

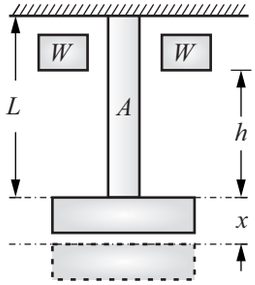
So, to reduce the axial stress in the rod, the length of rod or area of cross - section should be increased or modulus of elasticity should be decreased.

Hence, the correct option is (A).

Key Point

Impact load or shock load :

Let the weight W falls freely on to a collar carried at the lower end of a bar of area of cross section A, length L, fixed at the upper end, through a height h as shown in fig.



Let, x = Extension of the bar

σ = Stress induced in the bar

Potential energy lost by the weight
= Strain energy stored in the bar.

$$\therefore \quad W(h+x)=\frac{1}{2}\sigma Ax \qquad \ldots(i)$$

Now $x=\dfrac{\sigma L}{E}$

So, $W\left(h+\dfrac{\sigma L}{E}\right)=\dfrac{1}{2}\sigma A\left(\dfrac{\sigma L}{E}\right)$

$$\sigma=\frac{W}{A}\left[1\pm\sqrt{1+\frac{2AEh}{WL}}\right] \qquad \ldots(ii)$$

If $h=0$

$$\sigma=\frac{2W}{A} \qquad \text{[From equation (ii)]}$$

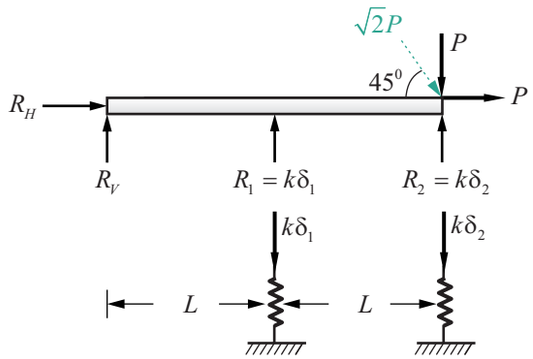
1.21 (C)

Elastic material	No. of elastic constant	No. of independent elastic constant
Homogeneous and isotropic material	4	2
Anisotropic material	∞	21
Orthotropic material (wood)	12	9

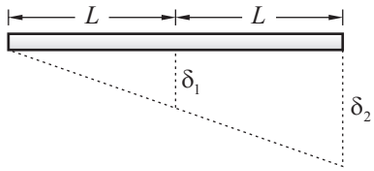
Hence, the correct option is (C).

1.22 (B)

The FBD of the beam is shown below,



Now, since the given beam is rigid and supported on the spring. So, on applying load it gets deflected in downward direction but remains straight. So, deflection can be shown in fig as below,



From similar triangle, we get

$$\frac{L}{\delta_1}=\frac{2L}{\delta_2}$$

$$\therefore \quad \delta_2=2\delta_1 \qquad \ldots(i)$$

Now, taking moment about hinge, we get

$$P\times 2L-k\delta_2\times 2L-k\delta_1\times L=0$$

$$2P-k(2\delta_2+\delta_1)=0$$

$$2P-k(2\times(2\delta_1)+\delta_1)=0 \quad [\because \text{ from (i)}]$$

$$\delta_1=\frac{2P}{5k}$$

So, from equation (i), we get

$$\delta_2=2\times\frac{2P}{5k}$$

$$\delta_2=\frac{4P}{5k}$$

Hence, the correct option is (B).

1.23 (D)

Given : Load $(P)=100\,\text{kN}$

Force in the spring at point 1, $R_1=k\delta_1$,

$$R_1=k\times\frac{2P}{5k}$$

$$R_1=\frac{2P}{5}=\frac{2\times 100}{5}=40\,\text{kN}$$

Force in the spring at point 2, $R_2=k\delta_2$,

$$R_2=k\times\frac{4P}{5k}$$

$$R_2=\frac{4P}{5}=\frac{4\times 100}{5}=80\,\text{kN}$$

Hence, the correct option (D).

Key Point

- A **linear spring** is one with a linear relationship between force and displacement.
- A **nonlinear spring** has a nonlinear relationship between force and displacement.

	Linear springs	Nonlinear springs
Force	$F = kx$	$F = F(x)$
Stiffness	$k = \frac{\Delta F}{\Delta x}$	$k = \frac{dF}{dx}$
Potential energy	$PE = \frac{1}{2}kx^2$	$PE = \int_0^{\delta L} F(u)du$

Where, F = Force in newton (N)

x = Displacement from spring's neutral position in meters (m),

k = Spring stiffness in newton per meter (N/m)

PE = Potential energy in joules (J)

u = Dummy variable used for displacement in the integral for potential energy of a nonlinear spring, in meter (m).

1.24 (B)

Poisson's ratio is defined as the negative ratio of lateral strain to longitudinal strain (or axial strain), and is constant for a given material, when the material is stressed within the elastic limit. It is generally denoted by μ. Hence mathematically,

Poisson's ratio

$$= -\frac{\text{Lateral strain}}{\text{Axial strain (or longitudinal strain)}}$$

$$= \left|\frac{\text{Lateral strain}}{\text{Axial strain (or longitudinal strain)}}\right|$$

Hence, the correct option is (B).

Key Point

When the material behavior is non-linear, the ratio of lateral strain to axial strain is often called the **contraction ratio**. In special case of linearly elastic behavior, the contraction ratio is same as **Poisson's ratio**.

1.25 (A)

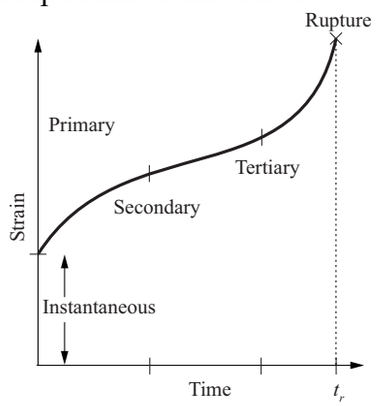
Creep is a time dependent process where a material under an applied stress exhibits deformation at high temperature. So, creep strain occur due to constant load i.e., dead load. Hence, the correct option is (A)

Key Point

- Creep (Sometimes known as cold flow) can occur as a result of long term exposure of high levels of stress that are still below the yield strength of the material.
- Creep is also temperature dependent process, and always increases with temperature.
- **Threshold for creep :** The critical temperature for creep is 40% of the melting temperature.

  If $T > 0.4\,T_m \rightarrow$ creep likely to occur

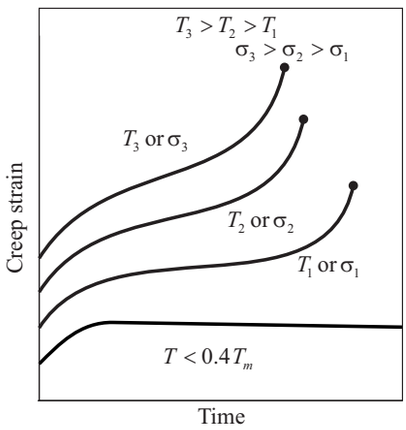
  Here, T_m = Melting temperature
- The rate of deformation is known as creep rate. It is slope of the line in a creep strain vs time curve.



- **Creep stages :**

1. **Primary creep :** Slope of curve (creep rate) decreases with time – work hardening.
2. **Secondary creep :** Constant slope – balance of work hardening and recovery.
3. **Tertiary creep :** Slope increase with time - rapidly accelerating strain rate upto failure. It is associated with formation of internal cracks, voids, grain, boundary, separation, necking, etc.

- Effects of stress and temperature on creep



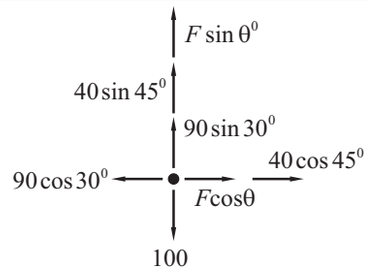
With increase in stress or temperature :

1. The instantaneous strain increases which is mainly because of elasticity.
2. The steady - state creep rate increases.
3. The time to rupture decreases.

1.26 (A)

It is asked in the question that the box should be lifted without swinging, it means that there should be no moment and the forces should be balanced.

The FBD of given question is,



$$\sum F_H = 0,$$

$$90\cos 30^0 = 40\cos 45^0 + F\cos\theta$$

$$\Rightarrow \quad F\cos\theta = 49.66\,\text{kN} \qquad \ldots(\text{i})$$

$$\sum F_V = 0,$$

$$90\sin 30^0 + F\sin\theta + 40\sin 45^0 = 100$$

$$\Rightarrow \quad F\sin\theta = 26.72\,\text{kN} \qquad \ldots(\text{ii})$$

Dividing equation (ii) by equation (i), we get

$$\tan\theta = \frac{26.72}{49.66} = 0.5380$$

$$\theta = 28.28^0$$

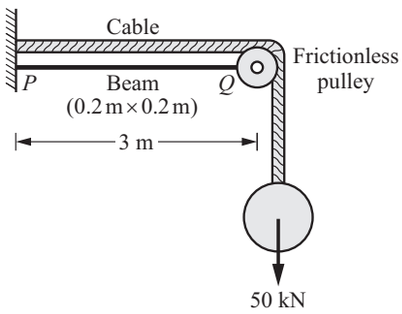
Substituting value of θ in equation (ii), we get

$$F\sin(28.28) = 26.72$$

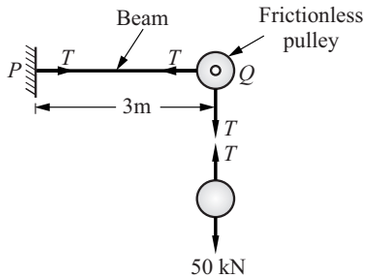
$$F = 56.397\,\text{kN}$$

Hence, the correct option is (A).

1.27 (B)



According to question, the FBD of given problem after removing the cable can be drawn as,



So, $T = 50\,\text{kN}$

$$\text{Axial stress} = \frac{50\,\text{kN}}{0.2\times 0.2} = 1250\,\text{kN/m}^2$$

Bending moment $= 50\times 3 = 150\,\text{kN.m}$

Shear force $= 50\,\text{kN}$

Hence, the correct option is (B).

Key Point

While solving the problem, care should be taken that point P lies on the beam, not on the cable. So, at point P, there are axial stress, bending moment and shear force. If point P lie on the cable, then there will be only axial stress and no shear force and bending moment. This is because cable can with stand only axial tension and no other forces i.e., cable is unable to carry bending moment, shear force, torsion or axial compression.

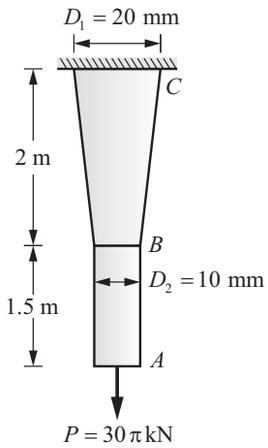
1.28 15

Given : Diameters of tapered circular rod,

$(D_1) = 20\,\text{mm}$ and $(D_2) = 10\,\text{mm}$

Modulus of elasticity $(E) = 2\times 10^5\,\text{MPa}$

Load $(P) = 30\pi\text{kN}$



Total deflection at point A is sum of elongation of uniform circular rod due to axial load P and the elongation of tapered circular rod due to axial load P.

$\therefore$ Deflection of point A = Elongation of uniform circular rod AB + elongation of tapered circular rod BC.

$$\Delta_A = \Delta_{AB} + \Delta_{BC}$$

$$= \frac{PL}{AE} + \frac{PL}{\frac{\pi}{4}D_1D_2E}$$

$$= \frac{(30\pi\times 10^3)\times(1.5\times 10^3)}{\left(\frac{\pi}{4}\times 10^2\right)\times(2\times 10^5)} + \frac{(30\pi\times 10^3)\times(2\times 10^3)}{\frac{\pi}{4}\times 20\times 10\times(2\times 10^5)}$$

$$= (9+6)\,\text{mm}$$

$$B_A = 15\,\text{mm}$$

Hence, the deflection at point A is 15 mm.

Key Point

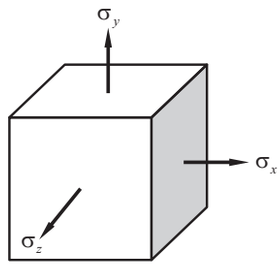
Extension of a tapered bar under axial loading :

1. In circular tapered bar, $\delta L = \dfrac{4PL}{\pi D_1D_2E}$
2. In rectangular tapered bar, $\delta L = \dfrac{PL}{Et(B_2 - B_1)}\ln\dfrac{B_2}{B_1}$

1.29 (C)

Change in volume of an elastic isotropic body subjected to triaxial stress is given by,

$$dV = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{3}\right)(1-2\mu)\times \text{Volume}$$



For hydrostatic stress, $\sigma_x = \sigma_y = \sigma_z = \sigma$

Now, for no change in volume, $dV = 0$

So, $0 = \left(\frac{3\sigma}{3}\right)(1-2\mu) \times \text{Volume}$

$\therefore \quad 1-2\mu = 0$

$\mu = \frac{1}{2} = 0.5$

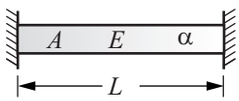
Hence, the correct option is (C)

Key Point

- When an object is submerged in water, it experiences uniform compression because of uniform pressure in all directions i.e. the stress is equal in all direction. This state of stress often known as **hydrostatic stress**.
- Mohr's circle becomes **a point** in case of hydrostatic stress.

1.30 (A)

The given problem can be depicted in figure as shown below,



The axial stress resulting from rise in temperature of the bar is given by

$\sigma = \alpha \Delta T E$

From above equation we can see that length have no effect on thermal stress. Hence, the axial stress will remain same (σ).

Hence, the correct option is (A).

1.31 35

Given : Axial load, $(P) = 1\text{kN}$

Segment 1 :

Length of segment 1, $(L_1) = 400\,\text{mm}$,

Area of segment 1, $(A_1) = 100\,\text{mm}^2$,

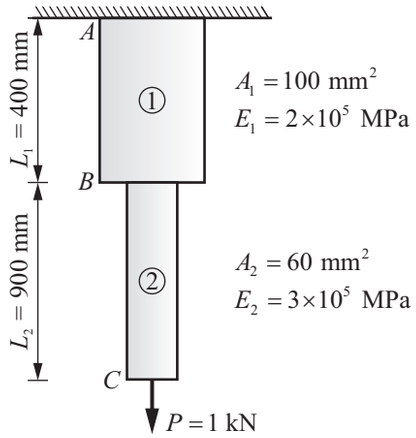
Young's modulus $(E_1) = 2 \times 10^5$ MPa

Segment 2 :

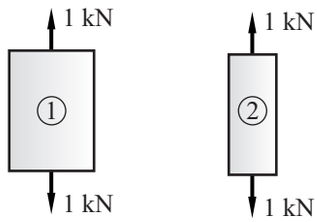
Length of segment 2, $(L_2) = 900\,\text{mm}$

Area of segment 2, $(A_2) = 60\,\text{mm}^2$

Young's modulus $(E_2) = 3 \times 10^5$ MPa



FBD of the given question can be shown as,



Method 1

Elongation of bar = Elongation of segment 1 + Elongation of segment 2

$$\Delta_{AC} = \Delta_{AB} + \Delta_{BC}$$

$$= \frac{PL_1}{A_1E_1} + \frac{PL_2}{A_2E_2}$$

$$= \frac{(1\times10^3)\times(400)}{(100)\times(2\times10^5)} + \frac{(1\times10^3)\times(900)}{(60)\times(3\times10^5)}$$

$$= 0.02 + 0.05 = 0.07\,\text{mm}$$

So, strain energy, $U = \frac{1}{2} \times P \times \Delta_{AC}$

$$= \frac{1}{2} \times (1\times10^3) \times 0.07$$

$$= 35\,\text{N-mm}$$

Hence, the strain energy stored in bar due to axial loading is **35 N-mm**.

Method 2

Strain energy of stepped bar under an axial loading is given by,

$$U = \sum \frac{P^2 L_i}{2 A_i E_i}$$

So, in given problem strain energy is given as,

$$U = \frac{P^2 L_1}{2 A_1 E_1} + \frac{P^2 L_2}{2 A_2 E_2}$$

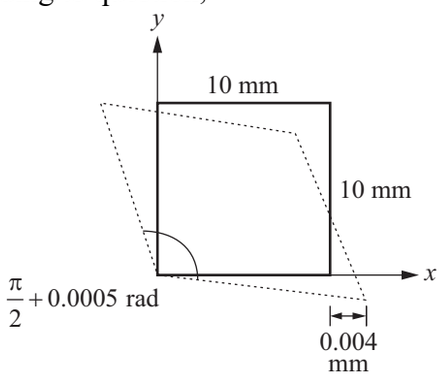
$$= \frac{(1\times10^3)^2 \times (400)}{2\times(100)\times(2\times10^5)} + \frac{(1\times10^3)^2 \times 900}{2\times(60)\times(3\times10^5)}$$

$$= 10 + 25 = 35 \text{ N-mm}$$

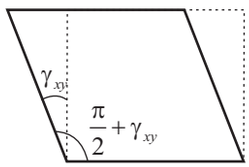
Hence, the strain energy stored in bar due to axial loading is **35 N-mm**.

1.32 (D)

According to question,



The given figure is equivalent to the figure shown below,



–ve Shear strain

From the figure we can see that because of shear strain, the angle has been increased. Therefore, shear strain should be negative.

$\therefore \quad \gamma_{xy} = -0.0005 \text{ rad}$

$\Rightarrow \quad -0.0005 = 0.001k$

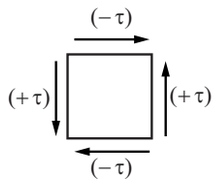
$k = -0.50$

Hence, the correct option is (D).

Key Point

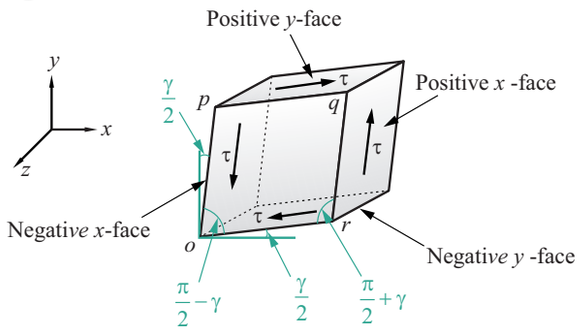
Sign convention for shear stress :

1. The shear stress will be considered positive when a pair of shear stress acting on opposite sides of the element produce a counter clockwise torque (couple).



2. The shear stress will be considered negative when a pair of shear stress acting on opposite sides of the element produce a clockwise torque (couple)

Sign convention for shear strain :



1. If the angle between two positive faces (or two negative faces) are reduced, the shear strain will be considered positive.
2. If the angle between two positive forces (or two negative faces) are increased, the shear strain will be considered negative.

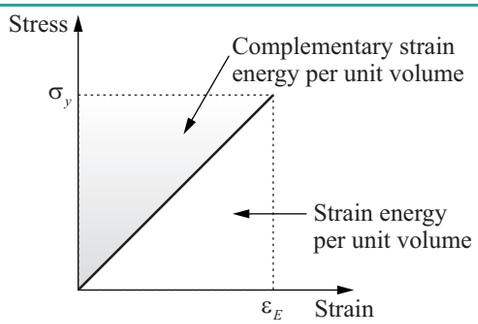
1.33 15625000

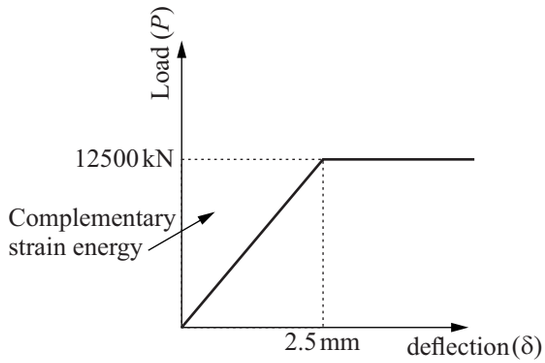
Given : Length of rod $(L) = 2\text{ m} = 2000\text{ mm}$

Diameter of rod $(D) = 8\text{ mm}$

Yield stress $(\sigma_y) = 250\text{ MPa}$

The area of the region above the force-deflection curve is called **complementary energy**.





Complementary strain energy

$$= \text{Strain energy} = \frac{1}{2} P\delta$$

$$= \frac{1}{2} \times (12500 \times 10^3) \times 2.5$$

$$= 15625000 \text{ N-mm}$$

Hence, the complementary strain energy is **15625000 N - mm**.

Key Point

- Strain energy, $U = \int_0^{\delta} P d\delta$

$$dU = Pd\delta \Rightarrow \boxed{\frac{dU}{d\delta} = P} \leftarrow \textbf{Differential}$$

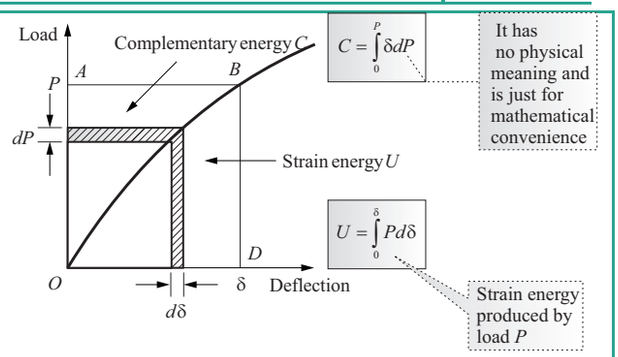
of strain energy is load.

- Complementary strain energy,

$$C = \int_0^{P} \delta dP$$

$$dC = \delta dP \Rightarrow \boxed{\frac{dC}{dP} = \delta} \leftarrow \textbf{Differential}$$

of complementary energy is deflection.



- For linear elastic material the value of complementary energy is equal to that of strain energy.
- The concept of total complementary energy of an elastic system is particularly useful in the solution of statically indeterminate structures, in which infinite number of stresses distribution and reactive forces may be found to satisfy the requirement of equilibrium.

1.34 (A)

Given : Stress in x-direction $(\sigma_{xx}) = 30 \text{ MPa}$

Stress in y-direction $(\sigma_{yy}) = 50 \text{ MPa}$

Young's modulus

$(E) = 2 \times 10^{11} \text{ N/m}^2 = 2 \times 10^5 \text{ N/mm}^2$

Poisson's ratio $(\mu) = 0.3$

Stress in z-direction $(\sigma_{zz}) = 0$

$$\text{Strain, } \varepsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\mu(\sigma_{xx} + \sigma_{yy})}{E}$$

$$= 0 - \frac{0.3(30 + 50)}{2 \times 10^5} = -120 \times 10^{-6}$$

(Negative sign represents contraction)

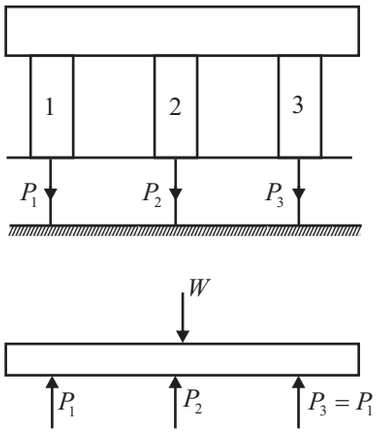
Hence, the correct option is (A).

1.35 130

Given : $\delta_1 = 5 \text{ mm} = \frac{P_1 L}{AE}$

$$\frac{AE}{L} = 10 \text{ kN/mm}$$

$$\delta_2 = 3 \text{ mm} = \frac{P_2 L}{AE}$$



$$P_1 + P_2 + P_1 = W$$

$$P_1 = 10 \times 5 = 50 \text{ kN}$$

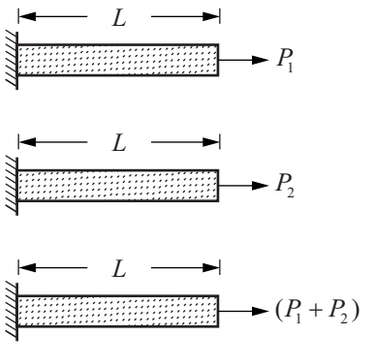
$$P_2 = 10 \times 3 = 30 \text{ kN}$$

$$W = 2 \times (50) + 30 = 130 \text{ kN}$$

Hence, the magnitude of the load W is **130 kN**.

1.36 (D)

According to question,



When bar is subjected to load P_1, the strain energy stored,

$$U_1 = \frac{P_1^2 L}{2AE} \qquad \ldots \text{(i)}$$

Where, L = length of bar

When bar is subjected to load P_2, the strain energy stored,

$$U_2 = \frac{P_2^2 L}{2AE} \qquad \ldots \text{(ii)}$$

Now when bar is subjected to load $(P_1 + P_2)$, the strain energy stored,

$$U = \frac{(P_1 + P_2)^2 L}{2AE}$$

$$U = \frac{P_1^2 L}{2AE} + \frac{P_2^2 L}{2AE} + \frac{2P_1 P_2 L}{2AE} \qquad \ldots \text{(iii)}$$

By equation (i), (ii) and (iii),

We conclude that, $U > U_1 + U_2$

Hence, the correct option is (D).

❖❖❖❖

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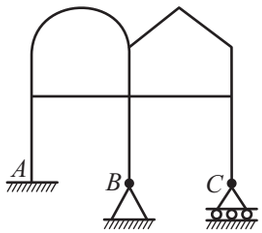
1 Determinacy & Indeterminacy (Static & Kinematic)

1987 IIT Bombay

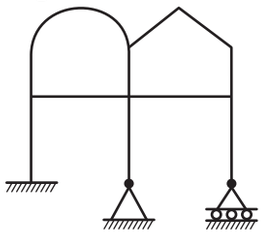
1.1 The static indeterminacy of the structure shown below is **[2 Marks]**



(A) 3 (B) 6

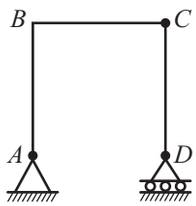
(C) 9 (D) 12

1.2 Determine the degrees of freedom of the following frame. **[2 Marks]**



1988 IIT Kharagpur

1.3 The plane frame shown below is **[1 Mark]**



(A) Statically indeterminate but unstable

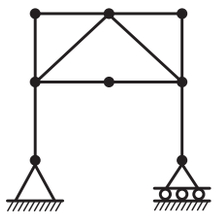
(B) Unstable

(C) Determinate and stable

(D) None of the above

1989 IIT Kanpur

1.4 The structure shown below is **[1 Mark]**



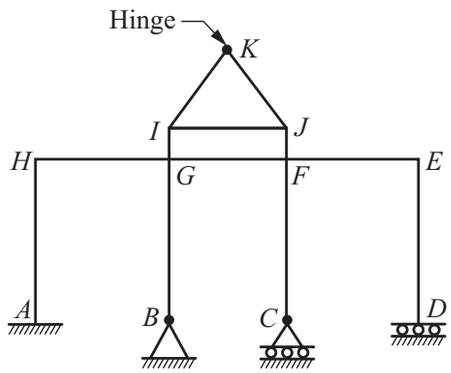
(A) Externally indeterminate

(B) Internally indeterminate

(C) Determinate

(D) Mechanism

1.5 The static indeterminacy of the structure shown below is **[2 Marks]**

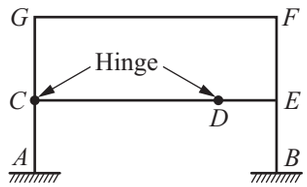


(A) 4 (B) 6

(C) 8 (D) 10

1990 IISc Bangalore

1.6 The static indeterminacy of the structure shown below is **[2 Marks]**



(A) Unstable

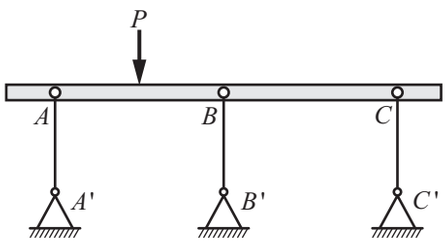
(B) Stable, determinate

(C) Stable, 5^{th} degree indeterminate

(D) Stable, 3^{rd} degree indeterminate

1991 IIT Madras

1.7 The beam supported by 3 links and loaded as shown in the figure is **[1 Mark]**



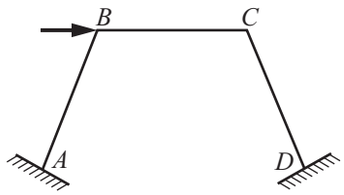
(A) Stable and determinate

(B) Unstable

(C) Stable and indeterminate

(D) Unstable but determinate

1.8 Number of independent displacement modes (sway mechanisms) for the given frame with the load shown in figure is/are **[2 Marks]**

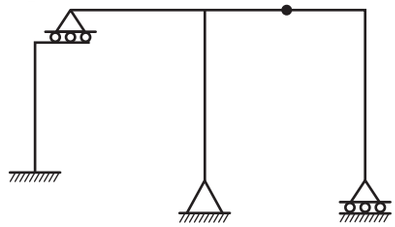


(A) 4 (B) 1

(C) 3 (D) 2

1992 IIT Delhi

1.9 A plane structure shown in the figure is



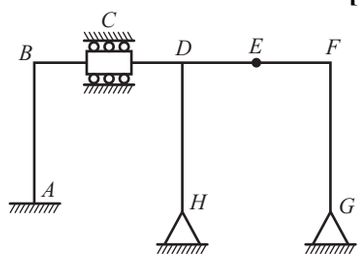
(A) Stable and determinate

(B) Stable and indeterminate

(C) Unstable and determinate

(D) Unstable and indeterminate

1.10 A plane frame *ABCDEFGH* shown in figure has clamp support at *A*, hinge supports at *G* and *H*, axial force release (horizontal sleeve) at *C* and moment release (hinge) at *E*. The static (D_s) and kinematic (D_k) indeterminacies are **[2 Marks]**

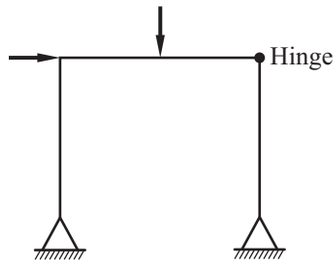


(A) 4, 9 (B) 3, 11

(C) 2, 12 (D) 1, 14

1993 IIT Bombay

1.11 The plane frame shown in figure is **[2 Marks]**



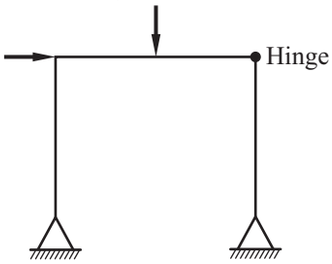
(A) Stable and statically determinate

(B) Unstable and statically determinate

(C) Stable and statically indeterminate

(D) Unstable and statically indeterminate

1.12 The kinematic indeterminacy of the plane frame shown in figure is (disregarding the axial deformation of the members) **[1 Mark]**



(A) 7 (B) 5

(C) 6 (D) 4

1994 IIT Kharagpur

1.13 The kinematic indeterminacy of single bay portal frame fixed at the base is **[1 Mark]**

(A) One (B) Two

(C) Three (D) Zero

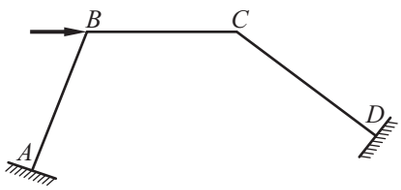
1.14 A beam fixed at the ends and subjected to lateral loads only is statically indeterminate and the degree of indeterminacy is **[1 Mark]**

(A) One (B) Two

(C) Three (D) Four

1997 IIT Madras

1.15 The degree of kinematic indeterminacy of the rigid frame with clamped ends at A and D as shown in the figure is **[2 Marks]**

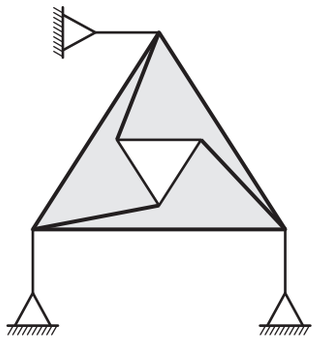


(A) 4 (B) 3

(C) 2 (D) Zero

2000 IIT Kharagpur

1.16 The following two statements are made with reference to the plane truss shown below **[1 Mark]**



I. The truss is statically determinate.

II. The truss is kinematically determinate.

With reference to the above statements, which of the following applies?

(A) Both statements are true

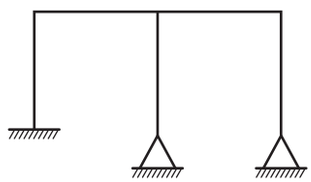
(B) Both statements are false

(C) II is true but I is false

(D) I is true but II is false

2001 IIT Kanpur

1.17 The degree of static indeterminacy (D_s) and the degree of kinematic indeterminacy (D_k) for the plane frame shown below, assuming axial deformations to be negligible, are given by **[2 Marks]**



(A) $D_s = 6$ and $D_k = 11$

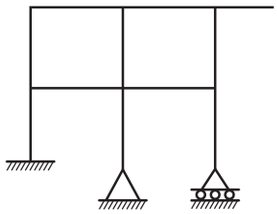
(B) $D_s = 6$ and $D_k = 6$

(C) $D_s = 4$ and $D_k = 6$

(D) $D_s = 4$ and $D_k = 4$

2004 IIT Delhi

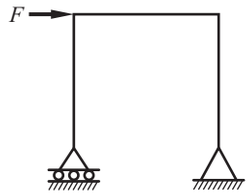
1.18 For the plane frame with an overhang as shown below, assuming negligible axial deformation, the degree of static indeterminacy, d, and the degree of kinematic indeterminacy, k are **[2 Marks]**



(A) $d = 3$ and $k = 10$
(B) $d = 3$ and $k = 13$
(C) $d = 9$ and $k = 10$
(D) $d = 9$ and $k = 13$

2005 IIT Bombay

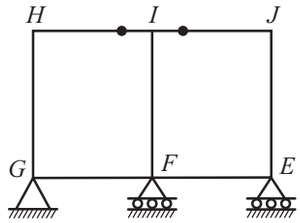
1.19 Considering beam as axially rigid, the degree of freedom of a plane frame shown below is **[2 Marks]**



(A) 9 (B) 8
(C) 7 (D) 6

2008 IISc Bangalore

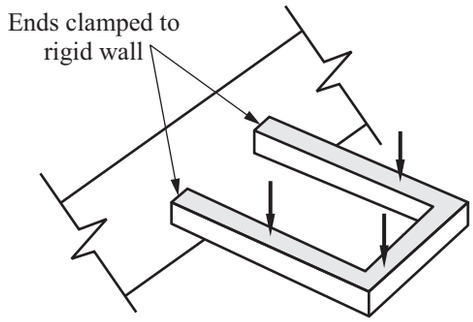
1.20 The degree of static indeterminacy of the rigid frame having two internal hinges as shown in the figure below, is **[2 Marks]**



(A) 8 (B) 7
(C) 6 (D) 5

2009 IIT Roorkee

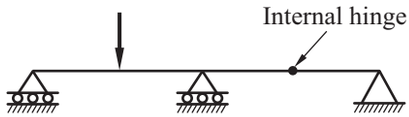
1.21 The degree of static indeterminacy of a rigidly jointed frame in a horizontal plane and subjected to vertical loads only, as shown in figure below, is **[2 Marks]**



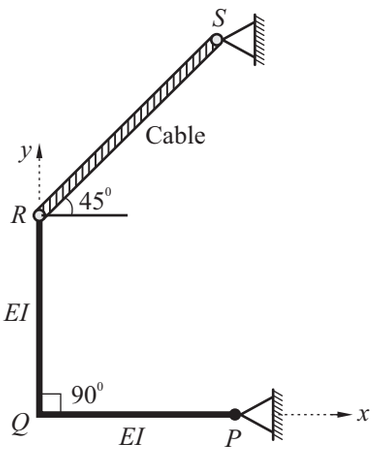
(A) 6 (B) 4
(C) 3 (D) 1

2014 IIT Kharagpur

1.22 The static indeterminacy of the two-span continuous beam with an internal hinge, shown below is _____. **[1 Mark]**



1.23 The degree of static indeterminacy of a rigid jointed frame PQR supported as shown in figure is **[1 Mark]**



(A) Zero (B) One
(C) Two (D) Unstable

2015 IIT Kharagpur

1.24 A guided support as shown in the figure below is represented by three springs (horizontal, vertical and rotational) with stiffness k_x, k_y and k_θ respectively. The limiting values of k_x, k_y and k_θ are

[1 Mark]

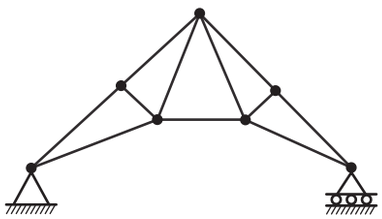
(A) $\infty, 0, \infty$ (B) ∞, ∞, ∞

(C) $0, \infty, \infty$ (D) $\infty, \infty, 0$

2016 IISc Bangalore

1.25 The kinematic indeterminacy of the plane truss shown in the figure is

[1 Mark]

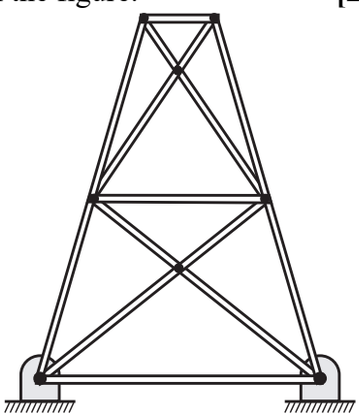


(A) 11 (B) 8

(C) 3 (D) 0

2017 IIT Roorkee

1.26 A planar truss tower structure is shown in the figure. **[2 Marks]**



Consider the following statements about the external and internal determinacies of the truss.

P. Externally determinate

Q. External static indeterminacy = 1

R. External static indeterminacy = 2

S. Internally determinate

T. Internal static indeterminacy = 1

U. Internal static indeterminacy = 2

Which one of the following options is correct?

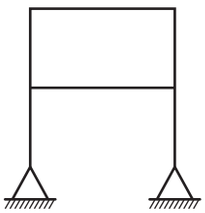
(A) P-False, Q-True; R-False; S-False; T-False; U-True

(B) P-False, Q-True; R-False; S-False; T-True; U-False

(C) P-False, Q-False; R-True; S-False; T-False; U-True

(D) P-True, Q-True; R-False; S-True; T-False; U-True

1.27 Consider the frame shown in the figure



If the axial and shear deformations in different members of the frame are assumed to be negligible, the reduction in the degree of kinematic indeterminacy would be equal to

[1 Mark]

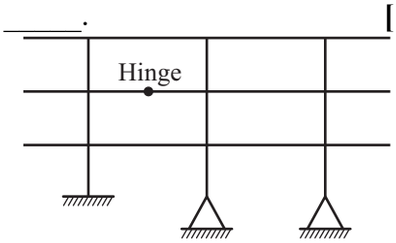
(A) 5 (B) 6

(C) 7 (D) 8

2019 IIT Madras

1.28 The degree of static indeterminacy of the plane frame as shown in the figure is _____.

[1 Mark]

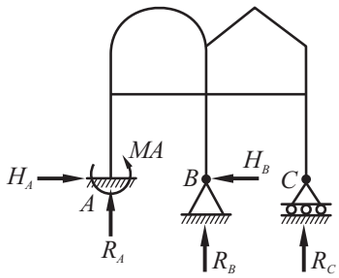


Answers — Determinacy & Indeterminacy (Static & Kinematic)

1.1	C	1.2	13	1.3	B	1.4	D	1.5	D
1.6	D	1.7	C	1.8	C	1.9	A	1.10	C
1.11	A	1.12	C	1.13	C	1.14	B	1.15	B
1.16	D	1.17	C	1.18	D	1.19	B	1.20	D
1.21	C	1.22	0	1.23	A	1.24	A	1.25	A
1.26	A	1.27	B	1.28	15				

Explanations — Determinacy & Indeterminacy

1.1 (C)



From given diagram,
Number of members, $m = 11$
Number of joints, $j = 10$
Total number of external reaction, $R = 6$

Method 1

D_{se} = External indeterminacy

$D_{se} = R - 3 = 6 - 3 = 3$

D_{si} = Internal indeterminacy

$D_{si} = 3C - r_r$ (for plane frame)

Where, C = Number of closed loop = 2,

$r_r = 0$ (r_r = Number of force released)

$D_{si} = 3 \times 2 - 0 = 6$

Static indeterminacy, $D_s = D_{si} + D_{se}$

$D_s = 6 + 3 = 9$

Hence, the correct option is (C).

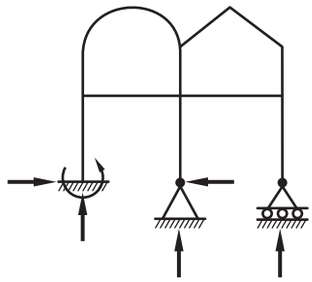
Method 2

For plane frame, $D_s = 3m - (3j - R)$

$D_s = 3 \times 11 - (3 \times 10 - 6) = 3 + 6 = 9$

Hence, the correct option is (C).

1.2 13



From given diagram, to find D_k (degree of freedom)
Number of members = 11
Number of joint = 10
Total external reaction, $R = 6$

Method 1

For plane frame $D_k = 3 \times$ Number of joints $- R$

Note : In case of plane frame number of force acting at joint = 3

$D_k = 3 \times 10 - 6 = 24$

[$D_k = 3j - R$ is for extensible members]

For inextensible member we have to surbstract the total number of member in structure. Then,

$D_k = 24 - 11 = 13$

Hence, the degrees of freedom of the frame is **13**.

Method 2

By counting,

At fixed support, $D_k = 0$

At hinge support, $D_k = 1$ $(M = 0)$

At roller support, $D_k = 2$ $(M = 0$ and $F_X = 0)$

Number of rigid joints = 7

Then total, $D_k = 3 \times j + 0 + 1 + 2$

$$D_k = 3 \times 7 + 3 = 24$$

For inextensible member,

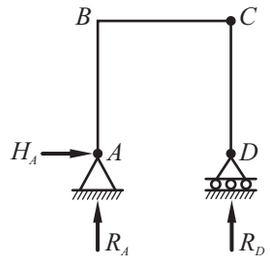
$$D_k = 24 - 11$$

(Number of member $m'' = 11$)

$$D_k = 13$$

Hence, the degrees of freedom of the frame is **13**.

1.3 (B)



From given diagram,

Number of member, $m = 3$

Number of joints, $j = 4$

Number of reaction acting, $R = 3$

Available equilibrium equation = 3

D_{se} = Degree of external indeterminacy

D_{se} = Total external reaction

– Available equilibrium equation

$$D_{se} = 2 + 1 - 3 = 0$$

D_{si} = Degree of internal indeterminacy

$$D_{si} = 3C - r_r$$

Where, C = Number of closed loop in plane frame.

$$r_r = \Sigma(m' - 1) = (2 - 1) = 1$$

$$D_{si} = 3 \times 0 - 1 = -1$$

m' = Number of remember meeting at hinge internal

$$D_s = D_{se} + D_{si} = 0 - 1 = -1$$

Here, $D_s < 0 \Rightarrow$ unstable.

OR

$$D_s = 3m - (3j - R) - r_r$$

Where, $r_r = \Sigma(m' - 1)$

$$D_s = 3 \times 3 - (3 \times 4 - 3) - (2 - 1)$$

$$D_s = -1 \text{ (unstable)}$$

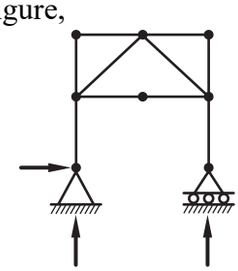
Hence, the correct option is (B).

Key Point

Without the hinge at 'C' the structure is stable and determinate. With the hinge at 'C', static indeterminacy is negative, column CD will have failure. Hence the structure is unstable.

1.4 (D)

From given figure,



$$D_{se} = R - 3 = 3 - 3 = 0$$

$D_{se} = 0$ = Externally determinate.

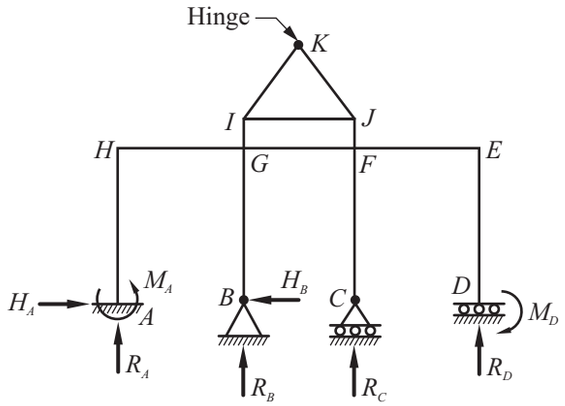
$D_{si} = m - (2j - R)$ (For $2D$ truss)

$$D_{si} = 10 - (2 \times 8 - 3) = -3$$

Hence, structure is unstable and unstable structure are called **'Mechanism'**.

Hence, the correct option is (D).

1.5 (D)



Method 1

D_{se} = External indeterminacy

$D_{se} = R$ – Available equilibrium equation.

From given figure $R = 8$

$$D_{se} = 8 - 3 = 5$$

$$D_{si} = 3C - r_r$$

Where, C = Number of closed loop = 2.

$$r_r = \Sigma(m' - 1) = (2 - 1) = 1$$

$$r_r = 3 \times 2 - 1 = 5$$

$$D_s = D_{se} + D_{si} = 5 + 5 = 10$$

Hence, the correct option is (D).

Method 2

We know that,

$$D_s = 3m - (3j - R) - r_r \text{ (for plane frame)}$$

Number of members, $m = 12$

Number of joints, $j = 11$

External reaction, $R = 8$

Number of force released $= r_r = 1$

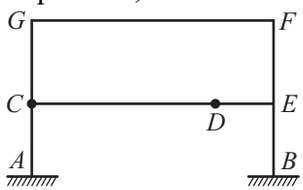
$$D_s = 3 \times 12 - 3 \times 11 + 8 - 1$$

$$D_s = 11 - 1 = 10$$

Hence, the correct option is (D).

1.6 (D)

According to question,



From given figure,

Number of members, $m = 7$

Number of joint, $j = 7$

External reaction, $R = 6$

$$r_r = \Sigma(m' - 1)$$

Where, m' = Number of members connected to internal hinge.

At C, $r_r = 3 - 1 = 2$

At D, $r_r = 2 - 1 = 1$

Total, $r_r = 2 + 1 = 3$

Method 1

$$D_s = 3m - (3j - R) - r_r \text{ (for plane frame)}$$

$$D_s = 3 \times 7 - (3 \times 7 - 6) - 3$$

$$D_s = 3$$

i.e. stable and 3rd degree inderminate.

Hence, the correct option is (D).

Method 2

External indeterminacy,

$$D_{se} = R - \text{Available equilibrium equation}$$

$$D_{se} = 6 - 3 = 3$$

Internal indeterminacy,

$$D_{si} = 3C - r_r$$

Where, C = Closed loop = 1

$$r_r = 2 + 1 = 3$$

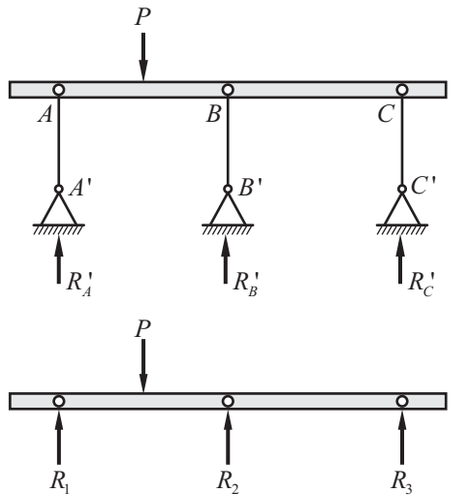
$$D_{si} = 3 \times 1 - 3 = 0$$

$$D_s = D_{se} + D_{si} = 3 + 0 = 3$$

Hence, the correct option is (D).

1.7 (C)

According to question,



Link will carry only axial force.

External indeterminacy,

$$D_{se} = R - \text{Available equilibrium equation}$$

$$D_{se} = 3 - 2 = 1 \text{ (only vertical loading)}$$

Internal indeterminacy,

$$D_{si} = 3C - r_r$$

Where, $C = 0$ and $r_r = 0$

$$D_{si} = 0$$

$$D_s = D_{se} + D_{si} = 1 + 0 = 1$$

Hence, the given structure is stable and indeterminate only for vertical loading.

Hence, the correct option is (C).

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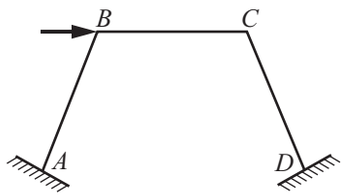
1.8 (C)



Total number of member, $m = 3$

Number of joint, $j = 4$

External reaction, $R = 6$

Method 1

For plane frame,

$$D_k = 3j - R = 3 \times 4 - 6 = 6$$

(for extensible member)

For rigid frame, number of members will be subtracted, then

$$D_k = 6 - 3 = 3$$

Hence, the correct option is (C).

Method 2

We know that, at fixed end, $D_k = 0$

At joint B and C, $D_k = 3 + 3 = 6$

Note : At joint three degree of freedoms are available Δx, Δy, θz.

For rigid member, number of member is substracted from D_k.

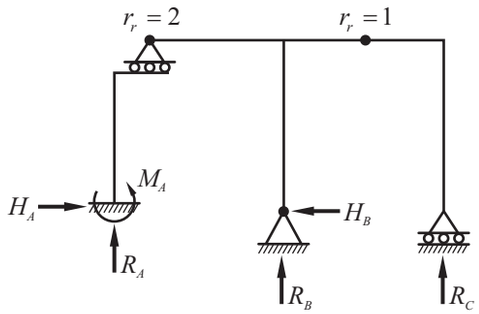
Hence, final $D_k = 6 - 3 = 3$

Hence, the correct option is (C).

Key Point

S. No.	Type of joint	Possible degree of freedom
1.	2-D Truss Joint	Two degree of freedoms $\Delta x, \Delta y$
2.	3-D Truss Joint	Three degree of freedoms $\Delta x, \Delta y, \Delta z$
3.	2-D Rigid Joint	Three degree of freedoms $\Delta x, \Delta y, \theta z$
4.	3-D Rigid Joint	Six degree of freedoms $\Delta x, \Delta y, \Delta z$ θx, θy, θz

1.9 (A)



Method 1

For plane frame, $D_s = 3m - (3j - R) - r_r$

Number of members, $m = 7$

Number of joints, $j = 8$

External reaction, $R = 6$

Force released, $r_r = 2 + 1 = 3$

$$D_s = 3 \times 7 - 3 \times 8 + 6 - 3 = 0$$

Hence, the correct option is (A).

Method 2

$$D_{se} = R - \text{Available equilibrium equation}$$

$$D_{se} = 6 - 3 = 3$$

$$D_{si} = 3C - r_r = 3 \times 0 - (2 + 1) = -3$$

$$D_s = D_{se} + D_{si} = 3 - 3 = 0$$

Stable and determinate.

Hence, the correct option is (A).

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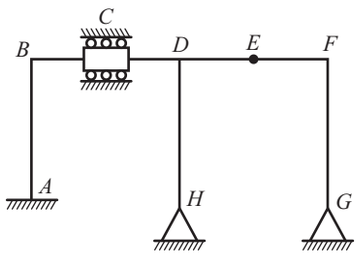
1.10 (C)



Number of member, m = 7

Number of joints, $j = 8$

Number of external reaction, $R = 7$

Force released, $r_r = 1+1 = 2$

$D_s = 3m-(3j-R)-r_r$ (For plane frame)

$D_s = 3\times7-(3\times8-7)-2$

$D_s = 2$

Kinematic indeterminacy,

$D_k = 3j-R+r_r$

$= 3\times8-7+2$

$= 19$ (For extensible member)

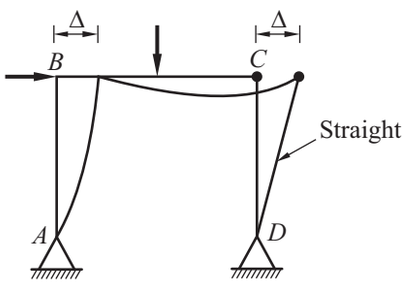
$D_k = 19-m''$

$D_k = 19-7 = 12$ (For rigid members)

m'' = Number of rigid members

Hence, the correct option is (C)

1.11 (A)



Number of member, $m = 3$

Number of joints, $j = 4$

Number of external reaction, $R = 4$

r_r = Number of internal reaction released

$r_r = \Sigma(m'-1)$

Where, m' = Number of member meeting at hybrid joint.

$r_r = 2-1 = 1$

Method 1

$D_s = 3m-(3j-R)-r_r$

$D_s = 3\times3-(3\times4-4)-1 = 0$

$\therefore$ The frame is stable and statically determinate.

Hence, the correct option is (A).

Method 2

External indeterminacy,

D_{se} = R – Available equilibrium equation

$D_{se} = 4-3 = 1$

Internal indeterminacy,

$D_{si} = 3C-r_r$

Where, C = Number of closed loop.

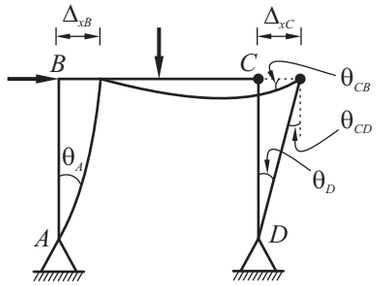
$r_r = \Sigma(m'-1) = 1$

$D_{si} = 3\times0-1 = -1$

$D_s = D_{se}+D_{si} = 1-1 = 0$

Hence, the correct option is (A).

1.12 (C)



Method 1

$$D_k = 3j - R + r_r$$

Where, number of joints, $j = 4$

Number of external reaction, $R = 4$

Number of internal reaction released, $r_r = \Sigma(m' - 1)$

m' = Number of member meeting at hybrid joint.

$r_r = 2 - 1 = 1$

$D_k = 3 \times 4 - 4 + 1 = 9$

For inextensible member substract rigid members, number of member $m'' = 3$.

Where, m'' = Number of rigid members.

$D_k = 9 - 3 = 6$

Hence, the correct option is (C).

Method 2

Degree of freedom,

At $A \rightarrow \theta_A$ (1)

At $B \rightarrow \Delta_{Bx}, \Delta_{By}, \theta_B$

At $C \rightarrow \theta_{BA}, \theta_{BC}, \Delta_{x_2}, \Delta_{y_2}$

At $D \rightarrow \theta_D$

Total = 9

For rigid member : $\Delta_{By} = 0$, $\Delta_{Bx} = \Delta_{Cx}$, $\Delta_{Cy} = 0$

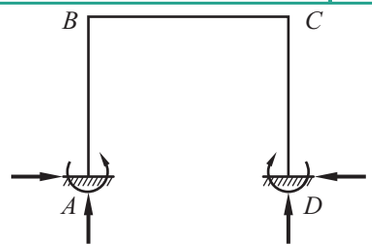
Total = 3

Hence, $D_k = 9 - 3 = 6$

Hence, the correct option is (C).

1.13 (C)

Figure for single bay portal frame fixed at the base is,



Method 1

By formula, $D_k = 3j - R$

Number of joints, $j = 4$

Number of external reaction, $R = 6$

Number of member, $m = 3$

$D_k = 3 \times 4 - 6 = 6$

For inextensible member substracting members,

$D_k = 6 - 3 = 3$

Hence, the correct option is (C).

Method 2

From figure various determinacy are possible

Δx = Displacement in x-direction

Δy = Displacement in y-direction

θ = Rotation

At $A \rightarrow 0$

At $B \rightarrow 3$ $(\Delta_{Bx}, \Delta_{By}, \theta_B)$

At $C \rightarrow 3$ $(\Delta_{Cx}, \Delta_{Cy}, \theta_C)$

At $D \rightarrow 0$

Total = 20

However if members are considered to be inextensible (rigid) then,

$\Delta_{By} = 0$, $\Delta_{Bx} = \Delta_{C_x}$ and $\Delta_{Cy} = 0$

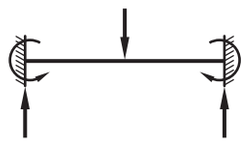
Total = 3

Hence, degree of kinematic indeterminacy,

$D_k = 6 - 3 = 3$

Hence, the correct option is (C).

1.14 (B)



For figure, due to loading there will be no horizontal reaction, $R = 4$

Similarly available equilibrium equation $e = 2$

$\therefore \quad D_{se} = R - e = 4 - 2 = 2$

$D_{si} = 3C - r_r$

Where, $C =$ Number of closed loop = 0

$r_r =$ Internal reaction release = 0

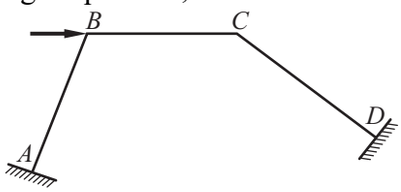
Note : Internal indeterminacy $D_{si} = 0$ (Beam are always internally determinate).

$$D_s = D_{si} + D_{se} = 2 + 0 = 2$$

Hence, the correct option is (B).

1.15 (B)

According to question,



By formula, $D_k = 3j - R$

Number of joints, $j = 4$

Number of external reaction, $R = 6$

Number of members, $m = 3$

$$D_k = 3 \times 4 - 6 = 6$$

It is given that frame is rigid frame.

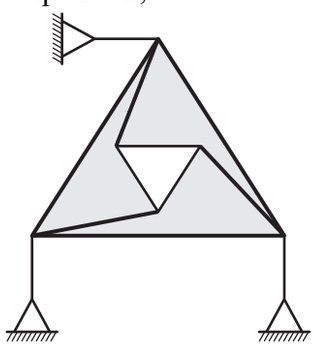
Hence, substracting number of rigid member,

$$D_k = 6 - 3 = 3$$

Hence, the correct option is (B).

1.16 (D)

According to question,



Degree of static indeterminacy,

$$D_s = m - (2j - R) \text{ (for } 2D \text{ truss)}$$

Where, Number of members, $m = 12$

Number of joints, $j = 9$

Number of external reaction, $R = 6$

$\therefore \quad D_s = 12 - (2 \times 9 - 6) = 12 - 18 + 6 = 0$

Hence, the truss is statically determinate. (statement I is true).

Degree of kinematic indeterminacy,

$$D_k = 2j - R = 2 \times 9 - 6 = 12$$

Since, it is not mentioned that members are extensible or inextensible, we will consider it to be extensible.

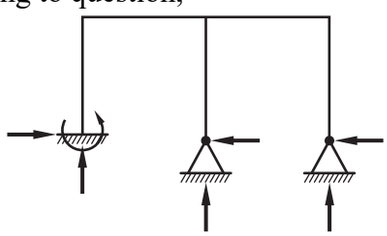
Hence, $D_k = 12$

$\therefore \quad$ Thus, truss is statically determinate and kinematically indeterminate.

Hence, the correct option is (D).

1.17 (C)

According to question,



Degree of static indeterminacy,

$$D_s = 3m - (3j - R) \text{ (for plane frame)}$$

Where, number of members, $m = 5$

Number of joints, $j = 6$

Number of external reaction, $R = 7$

$D_s = 3 \times 5 - (3 \times 6 - 7)$

$D_s = 15 - 18 + 7 = 4$

Degree of kinematic indeterminacy,

$D_k = 3j - R$ (for plane frame)

$D_k = 3 \times 6 - 7 = 11$

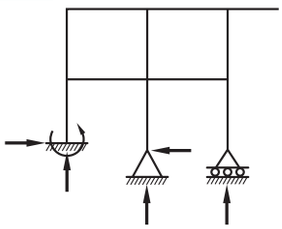
But in question it is given that axial determination is negligible i.e., rigid member, hence substracting the total number of rigid members in above D_k.

$$D_k = 11 - m = 11 - 5 = 6$$

Hence, $D_s = 4$ and $D_k = 6$.

Hence, the correct option is (C).

1.18 (D)



Degree of static indeterminacy,

$$D_s = 3m - (3j - R) \text{ (for plane frame)}$$

Where, Number of members, $m = 11$

Number of joints, $j = 10$

Note : Free end is also taken as joint.

Number of external reaction, $R = 6$

Hence, $D_s = 3\times11 - (3\times10 - 6)$

$$D_s = 33 - 30 + 6 = 9$$

Degree of kinematic indeterminacy (k),

$$D_k = 3j - R = 3\times10 - 6 = 24$$

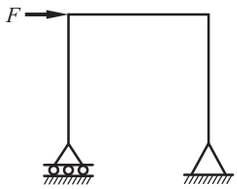
Lets assume member is rigid, then substracting total number of rigid members.

$$D_k = 24 - 11 = 13$$

Hence, $D_s = 9$ and $D_k = 13$.

Hence, the correct option is (D).

1.19 (B)



Degree of freedom,

$D_k = 3j - R$ (for plane frame)

$D_k = 3j - R - m$ (for rigid plane frame)

Number of joints, $j = 4$

Number of external reaction, $R = 3$

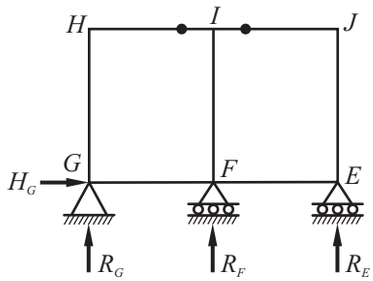
Number of rigid member, $m = 1$

Note : Only beam is given as axially rigid not column.

$$D_k = 3\times4 - 3 - 1 = 8$$

Hence, the correct option is (B).

1.20 (D)



Method 1

Degree of external indeterminacy,

$$D_{se} = R - \text{Available equilibrium equation}$$

Number of external reaction, $R = 4$

$$D_{se} = 4 - 3 = 1$$

Degree of internal indeterminacy,

$$D_{si} = 3C - r_r$$

Where, C = Number of closed loop = 2.

r_r = Number of internal reaction released

$$r_r = \Sigma(m' - 1)$$

Where, m' = Number of members at hybrid joint.

$$r_r = (2-1) + (2-1) = 2$$

$$D_{si} = 3\times2 - 2 = 4$$

$$D_s = D_{se} + D_{si} = 1 + 4 = 5$$

Hence, the correct option is (D).

Method 2

$$D_s = 3m - (3j - R) - r_r$$

Number of members, $m = 9$

$$D_s = 3\times9 - (3\times8 - 4) - 2$$

$$D_s = 3 + 4 - 2 = 7 - 2 = 5$$

Hence, the correct option is (D).

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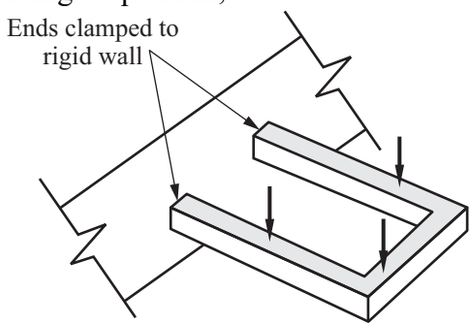
1.21 (C)

According to question,



From given figure,

Number of external reaction, $R = 6$

Number of member, $m = 3$

Number of joints, $j = 4$

$$D_s = 3m - (3j - R)$$

$$D_s = 3\times3 - (3\times4 - 6) = 9 - 12 + 6 = 3$$

Note : If a horizontal grid frame is subjected to vertical loading, then there will be only 3 member forces/support reactions.

Hence, the correct option is (C).

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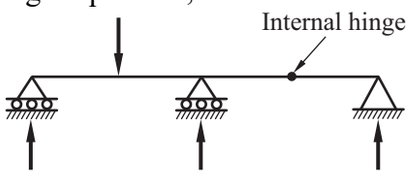
1.22 0

According to question,



Note : The beam given is having only vertical load hence, there will be no horizontal reaction.

$\therefore$ The number of equilibrium equation = 2

Number of external reaction, $R = 3$

Degree of external indeterminacy,

$$D_{se} = R - 2 = 3 - 2 = 1$$

Degree of internal indeterminacy,

$$D_{si} = 3C - r_r$$

Where, C = Number of closed loop = 0.

$$r_r = (m'-1) = (2-1) = 1$$

$$D_{si} = 3\times0 - 1 = -1$$

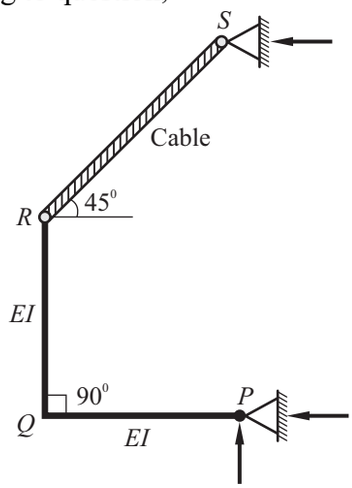
Hence, $D_s = D_{se} + D_{si} = 1 - 1 = 0$

Note : Stable for vertical loading and unstable for horizontal loading.

Hence, the static indeterminacy of the two-span continuous beam with an internal hinge, shown below is **0**.

1.23 (A)

According to question,



$$D_s = D_{se} + D_{si}$$

$$D_{se} = R - 3$$

Number of external reaction = 4

$$D_{se} = 4 - 3 = 1$$

$$D_{si} = 3C - r_r$$

Where, C = Closed loop = 0

$$r_r = \Sigma(m'-1) = 2 - 1 = 1$$

$$D_{si} = 3\times0 - 1 = -1$$

$$D_s = D_{se} + D_{si} = 1 - 1 = 0$$

Hence, the correct option is (A).

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1.24 (A)

According to question,

The give support is guided roller support which has 2 reaction.

1. Horizontal $\Sigma F_x = 0$ and
2. Moment i.e. $\theta = 0$

Stiffness in x-direction,

$$K_x = \frac{\text{Load}}{\text{Deflection}} = \frac{R_x}{\Delta}$$

$\therefore$ There is restriction in x-direction $\Delta = 0$.

$$K_x = \frac{R_x}{0} = \infty$$

Stiffness in y-direction,

$$K_y = \frac{\text{Load}}{\text{Deflection}} = \frac{R_y}{\Delta} = \frac{R_y}{\infty}$$

$\because$ There is no restriction in y-direction $\Delta \to \infty$.

$$K_y = \frac{R_y}{\infty} = 0$$

Similarly, stiffness in θ direction,

$$K_\theta = \frac{\text{Moment}}{\text{Rotation}} = \frac{M_\theta}{\theta}$$

$\therefore$ θ is restricted i.e. $\theta = 0$.

$$K_\theta = \frac{M_\theta}{0} = \infty$$

Hence, K_x, K_y, K_θ are respectively ∞, 0, ∞.

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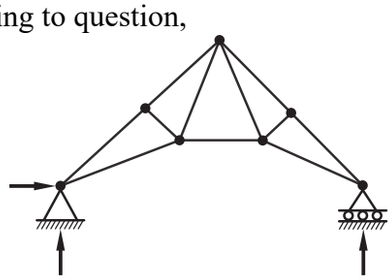
1.25 (A)

According to question,



We know that, for plane truss, kinematic indeterminacy,

$$D_k = 2j - R$$

Where, total number of joint, $j = 7$

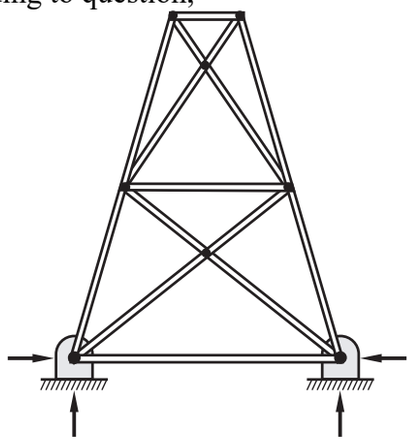
Total number of external reaction, $R = 3$

$\therefore$ $D_k = 2 \times 7 - 3 = 14 - 3 = 11$

Hence, the correct option is (A).

1.26 (A)

According to question,



Degree of external static indeterminacy,

$$D_{se} = R - \text{Equilibrium equation}$$

Where, R = Number of support reaction = 4

Equilibrium equation = 3

$$D_{se} = 4 - 3 = 1$$

(Externally indeterminacy)

$\therefore$ External static indeterminacy is equal to 1.

Degree of internal static indeterminacy,

$$D_{si} = m - (2j - 3) \qquad \text{(for plane truss)}$$

Where, Number of members in the structures $m = 15$

Number of joints in the structure $j = 8$

$$D_{si} = 15 - (2 \times 8 - 3) = 2$$

(Internally indeterminate)

And internal static indeterminacy is 2.

Hence, the correct option is (A).

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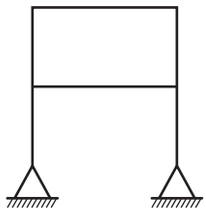
1.27 (B)

According to question,



Given that, the frame shown in figure is rigid.

Hence, the degree of kinematic indeterminacy,

$$D_k = (3j - R) - m''$$

Where, number of joints in the structure, $j = 6$

Total support reaction, $R = 4$

Number of inextensible (rigid) members $m'' = 6$

Note : In question reduction of kinematic indeterminacy is asked when member are assumed to be rigid.

Case 1 : When member are extensible,

$$D_{k_1} = 3j - R = 3\times 6 - 4 = 14$$

(members are extensible)

Case 2 : When member are inextensible (rigid), then

$$D_{k_2} = 3j - R - m''$$

$$D_{k_2} = 3\times 6 - 4 - 6 = 8$$

Reduction in $D_k = D_{k_1} - D_{k_2} = 14 - 8 = 6$

Hence, the correct option is (B).

1.28 15

Method 1

Degree of external indeterminacy,

$$D_{se} = R - \text{Available equilibrium equation}$$

Total external reaction at support,

$$R = 3 + 2 + 2 = 7$$

$$D_{se} = 7 - 3 = 4$$

Degree of internal indeterminacy,

$$D_{si} = 3C - r_r$$

Where, C = Number of closed loop = 4.

$$r_r = m' - 1$$

Where, m' = Number of member meeting at hybrid joint

$$r_r = 2 - 1 = 1$$

$$D_{si} = 3\times 4 - 1 = 11$$

$$D_s = D_{se} + D_{si} = 4 + 11 = 15$$

Hence, the degree of static indeterminacy of the plane frame as shown in the figure is **15**.

Method 2

By formula, $D_s = 3m - (3j - R) - r_r$

(for plane frame)

Where, number of member in structure, $m = 22$

Number of joints in structure, $j = 19$

Total reaction acting at support, $R = 7$

$$r_r = \Sigma(m' - 1)$$

Where, m' = Number of member meeting at hybrid joint.

$$r_r = 2 - 1 = 1$$

Hence, $D_s = 3\times 22 - (3\times 19 - 7) - 1 = 15$

Hence, the degree of static indeterminacy of the plane frame as shown in the figure is **15**.

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A DEDICATED TECH TEAM TO SOLVE TECH QUERIES AND HANDLE TECHNICAL ISSUES.

1 IS Code Recommendations & Fundamentals

1990 IISc Bangalore

1.1 The modulus of elasticity of concrete mix M25 is **[1 Mark]**

(A) $2,5000\ N/mm^2$

(B) $28,500\ N/mm^2$

(C) $15,000\ N/mm^2$

(D) $5,700\ N/mm^2$

1994 IIT Kharagpur

1.2 Permissible bending tensile stress in high yield strength deformed bars of grade 415 N/mm^2 in a beam is **[1 Mark]**

(A) 190 N/mm^2

(B) 230 N/mm^2

(C) 140 N/mm^2

(D) None of the above

1995 IIT Kanpur

1.3 The modulus of rupture of concrete gives **[1 Mark]**

(A) The direct tensile strength of the concrete.

(B) The direct compressive strength of the concrete.

(C) The tensile strength of the concrete under bending.

(D) The characteristic strength of the concrete.

1.4 Workability of concrete is influenced most by its **[1 Mark]**

(A) Water-cement ratio

(B) Aggregate-cement ratio

(C) Cement content

(D) Water content

1.5 Which one of the following set of values give the minimum clear cover (in mm) for the main reinforcements in the slab, beam, column and footing respectively, according to **IS 456 : 1978**? **[1 Mark]**

(A) 20, 25, 30, 75 (B) 5, 15, 25, 50

(C) 15, 25, 40, 50 (D) None of these

1997 IIT Madras

1.6 The cylinder strength of the concrete is less than the cube strength because of **[1 Mark]**

(A) The difference in the shape of the cross-section of the specimens.

(B) The difference in the slenderness ratio of the specimens.

(C) The friction between the concrete specimens and the steel plate of the testing machine.

(D) The cubes are tested without capping but the cylinders are tested with capping.

1.7 The permissible tensile stress in concrete made of M 25 concrete is **[1 Mark]**

(A) 3.5 N/mm^2
(B) 60 N/mm^2
(C) 2.5 N/mm^2
(D) None of the above

1.8 The permissible bending tensile stress in concrete for the vertical wall of an R.C. water tank made of M 25 concrete is **[1 Mark]**

(A) 8.5 N/mm^2 (B) 6.0 N/mm^2
(C) 2.5 N/mm^2 (D) 1.8 N/mm^2

1999 IIT Bombay

1.9 The characteristic strength of concrete is defined as that compressive strength below which not more than **[1 Mark]**

(A) 10% of results fall.
(B) 5% of results fall.
(C) 2% of results fall.
(D) None of the above.

2002 IISc Bangalore

1.10 As per the provisions of **IS 456 : 2000**, the (short term) modulus of elasticity of M 25 grade concrete (in N/mm^2) can be assumed to be **[1 Mark]**

(A) 25,000 (B) 28,500
(C) 30,000 (D) 36,000

2004 IIT Delhi

1.11 In the limit state design method of concrete structures, the recommended partial material safety factor (γ_m) for steel according to **IS 456 : 2000** is **[1 Mark]**

(A) 1.5 (B) 1.15
(C) 1.00 (D) 0.87

2005 IIT Bombay

1.12 The partial factor of safety for concrete as per **IS 456: 2000** is **[1 Mark]**

(A) 1.50 (B) 1.15
(C) 0.87 (D) 0.446

1.13 The flexural strength of M 30 concrete as per **IS 456: 2000** is **[2 Marks]**

(A) 3.83 MPa (B) 5.47 MPa
(C) 21.23 MPa (D) 30.0 MPa

1.14 In a random sampling procedure for cube strength of concrete, one sample consists of X number of specimens. These specimens are tested at 28 days and average strength of these X specimens is considered as test result of the sample, provided the individual variation in the strength of specimens is not more than $\pm Y$ per cent of the average strength. The values of X and Y as per **IS 456 : 2000** are **[2 Marks]**

(A) 4 and 10 respectively
(B) 3 and 10 respectively
(C) 4 and 15 respectively
(D) 3 and 15 respectively

2006 IIT Kharagpur

1.15 If the characteristic strength of concrete f_{ck} is defined as the strength below which not more than 50% of the test results are expected to fall, the expression for f_{ck} in terms of mean strength f_m and standard deviation σ would be **[1 Mark]**

(A) $f_m - 0.1645\sigma$ (B) $f_m - 1.645\sigma$
(C) f_m (D) $f_m + 1.645\sigma$

2007 IIT Kanpur

1.16 Consider the following statements:

1. The compressive strength of concrete decreases with increase in water-cement ratio of the concrete mix.
2. Water is added to the concrete mix for hydration of cement and workability.
3. Creep and shrinkage of concrete are independent of the water-cement ratio in the concrete mix.

The TRUE statements are **[2 Marks]**

(A) 1 and 2 (B) 1, 2 and 3

(C) 2 and 3 (D) 2 only

1.17 Consider the following statements :

1. Modulus of elasticity of concrete increases with increases in compressive strength of concrete.
2. Brittleness of concrete increases with decrease in compressive strength of concrete.
3. Shear strength of concrete increases with increase in compressive strength of concrete.

The true statements are **[2 Marks]**

(A) 2 and 3 (B) 1, 2 and 3

(C) 1 and 2 (D) 1 and 3

2008 IISc Bangalore

1.18 A reinforced concrete structure has to be constructed along a sea coast. The minimum grade of concrete to be used as per **IS 456 : 2000** is **[1 Mark]**

(A) M 15 (B) M 20

(C) M 25 (D) M 30

2009 IIT Roorkee

1.19 For limit state of collapse, the partial safety factors recommended by **IS 456 : 2000** for estimating the design strength of concrete and reinforcing steel are respectively **[1 Mark]**

(A) 1.15 and 1.5 (B) 1.0 and 1.0

(C) 1.5 and 1.15 (D) 1.5 and 1.0

1.20 The modulus of rupture of concrete in terms of its characteristic cube compressive strength (f_{ck}) in MPa according to **IS 456 : 2000** is **[1 Mark]**

(A) $5000 f_{ck}$ (B) $0.7 f_{ck}$

(C) $5000\sqrt{f_{ck}}$ (D) $0.7\sqrt{f_{ck}}$

1.21 Consider a reinforcing bar embedded in concrete. In a marine environment this bar undergoes uniform corrosion, which leads to the deposition of corrosion products on its surface and an increase in the apparent volume of the bar. This subjects the surrounding concrete to expansive pressure. As a result, corrosion induced cracks appear at the surface of concrete. Which of the following statement is TRUE?

[2 Marks]

(A) Corrosion causes circumferential tensile stresses in concrete and the cracks will be parallel to the corroded reinforcing bar.

(B) Corrosion causes radial tensile stress in concrete and the cracks will be parallel to the corroded reinforcing bar.

(C) Corrosion causes circumferential tensile stresses in concrete and the cracks will be perpendicular to the direction of the corroded reinforcing bar.

(D) Corrosion causes radial tensile stresses in concrete and the cracks will be perpendicular to the direction of the corroded reinforcing bar.

2011 IIT Madras

1.22 The cross-section of a thermo-mechanically treated (TMT) reinforcing bar has **[1 Mark]**

(A) Soft ferrite-pearlite throughout.

(B) Hard martensite throughout.

(C) A soft ferrite-pearlite core with a hard martensitic rim.

(D) A hard martensitic core with a soft pearlite-bainitic rim.

2013 IIT Bombay

1.23 Maximum possible value of compaction factor for fresh (green) concrete is **[1 Mark]**

(A) 0.5 (B) 1.0

(C) 1.5 (D) 2.0

1.24 The creep strains are **[1 Mark]**

(A) Caused due to dead loads only

(B) Caused due to live loads only

(C) Caused due to cyclic loads only

(D) Independent of loads

2014 IIT Kharagpur

1.25 The target mean strength f_m for concrete mix design obtained from the characteristic strength f_{ck} and standard deviation σ, as defined in **IS 456 : 2000** is **[1 Mark]**

(A) $f_{ck}+1.35\sigma$ (B) $f_{ck}+1.45\sigma$

(C) $f_{ck}+1.55\sigma$ (D) $f_{ck}+1.65\sigma$

1.26 The flexural tensile strength of M 25 grade of concrete, in N/mm^2, as per **IS 456 : 2000** is _____. **[1 Mark]**

1.27 The modulus of elasticity, $E=5000\sqrt{f_{ck}}$ where, f_{ck} is the characteristic compressive strength of concrete specified in **IS 456: 2000** is based on **[1 Mark]**

(A) Tangent modulus

(B) Initial tangent modulus

(C) Secant modulus

(D) Chord modulus

2015 IIT Kanpur

1.28 According to the concept of Limit State Design as per **IS 456 : 2000**, the probability of failure of a structure is _____. **[2 Marks]**

2016 IISc Bangalore

1.29 For M25 concrete with creep coefficient of 1.5, the long-term static modulus of elasticity (expressed in MPa) as per the provisions of **IS 456 : 2000** is _____. **[2 Marks]**

2017 IIT Roorkee

1.30 Let the characteristic strength be defined as that value, below which not more than 50% of the result are expected to fall. Assuming a standard deviation of 4 MPa, the target mean strength (in MPa) to be considered in the mix design of a M 25 concrete would be **[1 Mark]**

(A) 18.42 (B) 21.00

(C) 25.00 (D) 31.58

2018 IIT Guwahati

1.31 The deformation in concrete due to sustained loading is **[1 Mark]**

(A) Creep (B) Hydration
(C) Segregation (D) Shrinkage

1.32 The frequency distribution of the compressive strength (f) of 20 concrete cube specimens is given in the table.

f (MPa)	Number of specimens with compressive strength equal to f
23	4
28	2
22.5	5
31	5
29	4

If μ is the mean strength of the specimens and σ is the standard deviation, the number of specimens (out of 20) with compressive strength less than $\mu - 3\sigma$ is _____. **[1 Mark]**

2019 IIT Madras

1.33 The characteristic compressive strength of concrete required in a project is 25 MPa and the standard deviation in the observed compressive strength expected at site is 4 MPa. The average compressive strength of cubes tested at different water-cement (w/c) ratios using the same material as is used for the project is given in the table. **[1 Mark]**

Water-cement [w/c (%)]	45	50	55	60
Average compressive strength of cubes (MPa)	35	25	20	15

The water cement ratio (in percent, round off to the lower integer) to be used in the mix is _____.

1.34 When a specimen of M25 concrete is loaded to a stress level of 12.5 MPa, a strain of 500×10^{-6} is recorded. If this load is allowed to stand for a long time, the strain increases to 1000×10^{-6}. In accordance with the provisions of IS : 456 – 2000, considering the long – term effects, the effective modulus of elasticity of the concrete (in MPa) is _____. **[2 Marks]**

2020 IIT Delhi

1.35 The maximum applied load on a cylindrical concrete specimen of diameter 150 mm and length 300 mm tested as per the split tensile strength test guidelines of **IS 5816 : 1999** is 157 kN. The split tensile strength (in MPa, round off to one decimal place) of the specimen is _____. **[1 Mark]**

1.36 As per IS 456 : 2000, the pH value of water for concrete mix shall NOT be less than **[1 Mark]**

(A) 4.5 (B) 6.0
(C) 5.5 (D) 5.0

Answers — IS Recommendations & Fundamentals

1.1	A	1.2	B	1.3	C	1.4	D	1.5	C
1.6	C	1.7	A	1.8	D	1.9	B	1.10	A
1.11	B	1.12	A	1.13	A	1.14	D	1.15	C
1.16	A	1.17	D	1.18	D	1.19	C	1.20	D
1.21	A	1.22	C	1.23	B	1.24	A	1.25	D
1.26	3.5	1.27	C	1.28	0.097	1.29	10000	1.30	C
1.31	A	1.32	zero	1.33	46	1.34	12500	1.35	2.2
1.36	B								

Explanations — IS Recommendations & Fundamentals

1.1 (A)

Given : Grade of concrete = M25

Characteristic compressive strength of M25 grade of concrete, $f_{ck} = 25\ \text{N/mm}^2$

To find : Modulus of elasticity of concrete mix M 25 =?

Solution :

As per **clause 6.2.3.1** mentioned in **IS 456:2000**,

Modulus of elasticity of concrete,

$$E_C = 5000\sqrt{f_{ck}}$$

$$E_C = 5000 \times \sqrt{25} = 25000\ \text{N/mm}^2$$

Hence, the correct option is (A).

Note : E_c is short term static modulus of elasticity. Actual measured values may differ by $\pm 20\%$ from the values obtained from the above expression.

1.2 (B)

Allowable stresses for bars have been standardized by IS-1786

For Fe – 415 grade

Permissible tensile stress $= 230\ \text{N/mm}^2$

Permissible compressive stress $= 190\ \text{N/mm}^2$

Hence, the correct option is (B).

Key Point

Following is the table given in **IS456:2000: Permissible stress in reinforcement**

$$\sigma_{perm} = \frac{f_y}{1.8} \simeq 0.55 f_y = \frac{415}{1.8} \simeq 230\ \text{MPa}$$

Types of stresses in steel reinforcement	Permissible stress in steel (in N/mm²)		
	Mild steel bars	Medium tensile steel	HYSD bars (Fe415)
Tension (σ_{st}): • Up to & including 20 mm • Over 20 mm	140 130	Half the guaranteed yield stress subject to a maximum of 190	230 230
Compression in column bars (σ_{sc}):	130	130	190
Compression in bars in a beam or slab when the compressive resistance of the concrete is taken into account	The calculated compressive stress in the surrounding concrete multiplied by 1.5 times the modular ratio or σ_{sc}, whichever in lower		
Compression in bars in a beam or slab where the compressive resistance is not taken into account : • Upto & including 20 mm • Over 20 mm	140 130	Half the guaranteed yield stress subject to a maximum stress 190	190 190

Note :

1. For HYSD bars of grade Fe500 the permissible stress in direct tension and flexure tension shall be 0.55 f_y .

The permissible stressess for shear and compression reinforcement shall be as for grade Fe415.

1.3 (C)

Modulus of rupture of concrete as per clause 6.2.2 mentioned in **IS456: 2000** is given by,

$$f_{cr} = 0.7\sqrt{f_{ck}}$$

Where, f_{ck} = Characteristic compressive strength of concrete in N/mm^2

f_{cr} = Modulus of rupture (or flexural strength of concrete) in N/mm^2

So, tensile strength of concrete in flexure (or bending) in known as flexural strength (or modulus of rupture) of concrete.

Hence, the correct option is (C).

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1.4 (D)

Factors affecting workability of concrete "water content"

- Mix proportion
- Admixture content
- Size of aggregate
- Grading of aggregate
- Shape of aggregate
- Surface texture of aggregate

Placing Condition	Degree of Workability	Slump (mm)
Blinding concrete, shallow sections, pavements using pavers	Very low	Value of compacting factor 0.75 to 0.80
Mass concrete, lightly reinforced sections in slabs, beams, walls, floors, canal lining, columns	Low	25 – 75
• Heavily reinforced sections in slab, beams, walls • Pumped concrete, columns	Medium	50 – 100 75 – 100
• Trench fill-in situ piling • Tremie concrete	High Very high	100 – 150 flow test

Hence, the correct option is (D).

1.5 (C)

As per **IS 456,**

1. For slabs minimum cover to outside of the main bars (main reinforcement) shall not be less than the following:

   (a) 15 mm (b) Diameter of the main bars

2. For beams, at each end of the reinforcing bars, the cover shall not be less than 25 mm or less than twice the diameter of main bars (main reinforcement).
3. The minimum cover to a column reinforcement shall be 40 mm or diameter of bar whichever is greater.
4. The minimum cover to a footing reinforcement shall be 50 mm

Hence, the correct option is (C).

Key Point

Exposure condition	Minimum clear cover (mm)
Mild	20
Moderate	30
Severe	45
Very severe	50
Extreme	75

1.6 (C)

- The compressive strength of concrete cube specimen is determined by applying uniaxial compressive force through UTM (Universal Testing Machine).
- Ideally, in uni-axial compression there is no lateral restraint, but, lateral restraint develops due to the friction between the end faces of the concrete specimen & the platens of the testing machine. This is called platen restraint.
- The contact area of a standard cube mould with the upper platen in the testing machine is more which results in more confinement(or restrainment).
- As the cube have a four confined ends, which means concrete cube takes more load as compared to cylinder having no confined ends and hence is liable for easy failure.
- Thus, it can be concluded that, the platen restraint helps in increasing the compressive strength of the concrete cube specimen.
- Greater the height/width ratio, lesser the effect of this platen restraint on the compressive strength of the concrete cube and vice-versa.
- Therefore, the strength of standard cylindrical concrete specimen (height to width ratio = 2) is lower than that of standard concrete cube specimen (height to width ratio = 1).
- As per Indian standards the ratio between cube strength and cylinder strength is commonly assumed to be 1.25, but it is not constant.

Hence, the correct option is (C).

1.7 (A)

Given : Characteristic compressive strength for M25 grade concrete, $f_{ck} = 25\ \text{N/mm}^2$

To find : Permissible tensile stress in M25 grade concrete = ?

Solution :

As per **clause 6.2.2** mentioned in **IS456 : 2000,**

Tensile stress of concrete in flexure (or modulus of rupture),

$$f_{cr} = 0.7\sqrt{f_{ck}} = 0.7\times\sqrt{25}$$

$$\therefore \quad f_{cr} = 3.5\ \text{N/mm}^2$$

But, as per **IS456: 2000 (ANNEX B.2.1.1)**,

The permissible tensile stress in direct tensile is $3.2\ \text{N/mm}^2$.

Hence, the correct option is (A).

1.8 (D)

As per **IS 3370 : 1965 [(Part - II), (Table - 1)]**, the permissible bending tensile stress in concrete for the vertical wall of a RC water tank made of M 25 concrete is $1.8\ \text{N/mm}^2$.

Hence, the correct option is (D).

Key Point

Following is the **Table-1 from IS 3370 (Part - 2)**

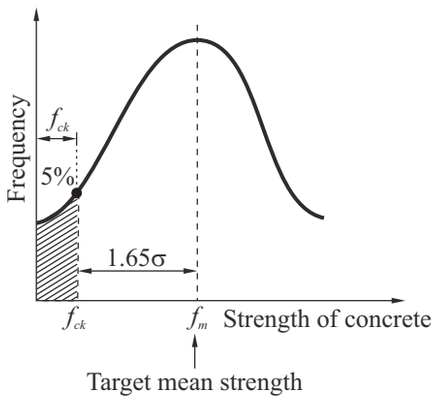
Grade of concrete	Permissible tensile stress in concrete on water face (MPa or N/mm^2)	
	Direct tension	Bending tension
M 15	1.1	1.6
M 20	1.2	1.7
M 25	1.3	1.8
M 30	1.5	2.0
M 35	1.6	2.2
M 40	1.7	2.4

It is notable that **IS 456: 2000 or IS 456: 1978** does not specify the permissible stresses in concrete for its resistance to cracking. However, its earlier version (IS 456: 1964) included the permissible stresses in direct tension, bending tension and shear. These values (excluding shear) are mentioned in above table. The permissible tensile stresses due to bending apply to the face of the member in contact with the liquid.

1.9 (B)

As per clause 6.1.1 mentioned in IS456:2000,

The characteristic strength of a material is that value of strength, below which not more than 5% of the test results are expected to lie (fall). The characteristic strength of concrete is the 28 days cube compressive strength in N/mm^2 and is denoted by f_{ck} .The graph shows the clause defined in the code.



f_{ck} = Characteristic strength of concrete

f_m = Mean strength

σ = Standard deviation

$f_m = f_{ck} + 1.65\bar{\sigma}$

Hence, the correct option is (B).

1.10 (A)

Given : Characteristic compressive strength of M25 grade concrete, $f_{ck} = 25$ N/mm^2

To find : Short term modulus of elasticity of M 25 grade concrete = ?

Solution:

As per **clause 6.2.3.1** mentioned in **IS 456: 2000**,

Short term modulus of elasticity of concrete,

$$E = 5000\sqrt{f_{ck}} = 5000\sqrt{25}$$

$$E = 25000 \text{ N/mm}^2$$

Hence, the correct option is (A).

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1.11 (B)

As per IS456:2000, clause 36.4.2.1,

When assessing the strength of a structure or structural member for the limit stale of collapse, the values of partial safety factor (γ_m) should be taken as 1.5 for concrete and 1.15 for steel.

Hence, the correct option is (B).

Key Point

1. As per **clause 36.3.1, IS456:2000, Design strength of materials,**

$$f_d = \frac{f}{\gamma_m}$$

Where, f = Characteristic strength of the material

γ_m = Partial safety factor appropriate to the material and the limit state being considered.

2. As per **clause 36.3.2, IS456:2000,**

Design loads

$$F_d = F\gamma_f$$

Where, F = Characteristic load

γ_f = Partial safety factor appropriate to the natural loading and the limit state being considered.

1.12 (A)

As per IS456:2000, clause 36.4.2.1,

When assessing the strength of a structure or structural member for the limit stale of collapse, the values of partial safety factor (γ_m) should be taken as 1.5 for concrete and 1.15 for steel.

Hence, the correct option is (A).

Key Point

In working stress method(WSM), factor of safety is applied to stress values only and load is not factored.

The factor of safety is given by,

$$\text{Factor of safety} = \frac{\text{Failure strength}}{\text{Permissible stress}}$$

Where, Failure strength $= f_{ck}$ for concrete and f_y for steel

Permissible stress $= \sigma_{cbc}$ for concrete and σ_{st} for steel

In working stress method (WSM),

Factor of safety of concrete = 3

Factor of safety of steel = 1.8

1.13 (A)

Given : Grade of concrete = M30

Characteristic compressive strength of concrete = 30 N/mm^2

To find : Flexural strength of M30 concrete = ?

As per **clause 6.2.2**, **IS 456: 2000**,

Flexural strength (or modulous of rupture or tensile strength) of concrete,

$$f_{cr} = 0.7\sqrt{f_{ck}} = 0.7\sqrt{30} = 3.83 \text{ MPa}$$

Hence, the correct option is (A).

1.14 (D)

As per **clause 15.3 and 15.4,IS 456 : 2000, ,**

The test results of the sample shall be the average of the strength of three specimens. The individual variation should not be more than ±15% of the average. If more, the test result of the sample are invalid. Hence, three test specimens shall be made for each sample of concrete for testing at 28 days and the individual variation should not be more than ± 15% of the average.

Hence, the correct option is (D).

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1.15 (C)

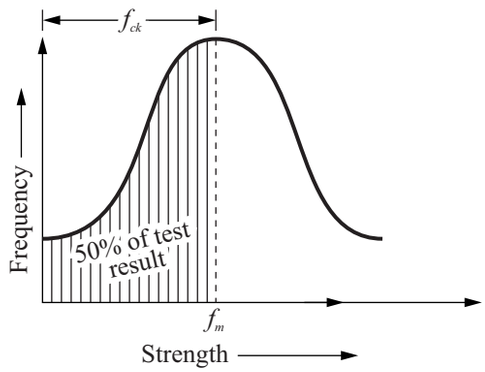

Fig. Normal distribution curve

According to question, f_{ck} is the strength below which not more than 50% of the test result are expected to fall, hence the curve will be like as drawn in the figure and from this curve, the expression for f_{ck} would be like.

Characteristic strength of concrete

= Target mean strength of concrete

i.e., $f_{ck} = f_m$

Hence, the correct option is (C).

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1.16 (A)

(a) As the compressive strength of the coarse aggregate is always very high hence the compressive strength of the concrete is greatly influenced by the strength of cement paste which in terms depends on the "Water-cement ratio". The strength of concrete increases with decrease in water-cement ratio. At an optimum water cement ratio, the workability will be proper and hence there will not be any porosity (or void content). But, as soon as water-cement ratio increases, the porosity (or void content or void ratio) increases and ultimately the strength of concrete decreases.

Hence, compressive strength of concrete decrease with increase in water cement ratio.

(b) The chemical reaction that takes place between the active component (or Bogue compounds) of the cement and water is known as hydration of cement. These compounds have the properties of setting and hardening in presence of water. Hence the strength of concrete depends on the extent of hydration and resultant microstructure of the hydrated cement. Therefore, water is added to the concrete mix for hydration of cement.

Also, the water is added to concrete mix for having good workability. The water when added to the mix lubricates the concrete. This lubrication is required for handling concrete without any segregation, for placing without any loss of homogeneity, for compacting with specified effort and for easy finish which are the indications of a workable concrete mix.

∴ The water content is the greatest single factor that affects the workability of concrete.

(c) **Creep :** It may be defined as the continued deformation with time under applied load. It is also known as plastic flow or time yield. The rate of creep decreases with time. The creep may occur due to closer of internal voids, viscous flow of the cement paste and flow of water out of the cement gel. Hence, creep depends on water cement ratio.

Shrinkage : It may be defined as the contraction of concrete in the absence of load. The rate of shrinkage decreases with time. The shrinkage of concrete increases with high water-cement ratio and high cement content. Shrinkage can be estimated by using Schorer's formula,

Shrinkage strain, $\varepsilon_s = 0.00125(0.90 - h)$

Where, h = Relative humidity (expressed as fraction)

Hence, the correct option is (A).

1.17 (D)

1. As per **clause 6.2.3.1** , **IS 456 : 2000**, Modulus of elasticity of concrete

$E_c = 5000\sqrt{f_{ck}}$

Where, f_{ck} = Characteristic compressive strength

From the above formula, it is evident that $E_c \propto \sqrt{f_{ck}}$

Hence, modulus of elasticity increases with increase in compressive strength.

2. High strength concrete has more brittleness as compared to low strength concrete. Hence, brittleness of concrete increases with the increase in compressive strength of concrete.
3. Shear strength of concrete increases with the increase in compressive strength of concrete, since shear strength is directly proportional to characteristic compressive strength of concrete.

Hence, the correct option is (D).

Scan for Video Solution

1.18 (D)

As per **clause 8.2.8 of IS 456 : 2000,** Concrete in sea water or exposed directly along the sea coast shall be at least M 20 grade in the case of plain concrete and M 30 in case of reinforced concrete. The use of slag or pozzolana cement is advantageous under such conditions.

Hence, the correct option is (D).

1.19 (C)

As per IS456:2000, clause 36.4.2.1,

When assessing the strength of a structure or structural member for the limit stale of collapse, the values of partial safety factor (γ_m) should be taken as 1.5 for concrete and 1.15 for steel.

Hence, the correct option is (C).

Key Point

1. As per **clause 36.3.1, IS456:2000,**

   Design strength of materials,

   $$f_d = \frac{f}{\gamma_m}$$

   Where, f = Characteristic strength of the material

   γ_m = Partial safety factor appropriate to the material and the limit state being considered.

2. As per **clause 36.3.2, IS456:2000,**

   Design loads

   $$F_d = F\gamma_f$$

   Where, F = Characteristic load

   γ_f = Partial safety factor appropriate to the natural loading and the limit state being considered.

1.20 (D)

As per **clause 6.2.2** mentioned in **IS456: 2000,**

Modulus of rupture of concrete in terms of its characteristic cube compressive strength,

$$f_{cr} = 0.7\sqrt{f_{ck}}$$

Where, f_{ck} = Characteristic compressive strength of concrete in N/mm^2 (MPa)

f_{cr} = Modulus of rupture (or flexural strength of concrete) in N/mm^2 (MPa)

Hence, the correct option is (D).

1.21 (A)

Corrosion of steel in RCC structure causes the reduction in cross-section area of steel and exerts radial pressure on adjoining concrete layer which causes cracks and splitting of concrete in parallel to corroded reinforcing bars. Due to reduction of cross-section, the load carrying capacity of reinforcement is reduced.

Hence, the correct option is (A).

1.22 (C)

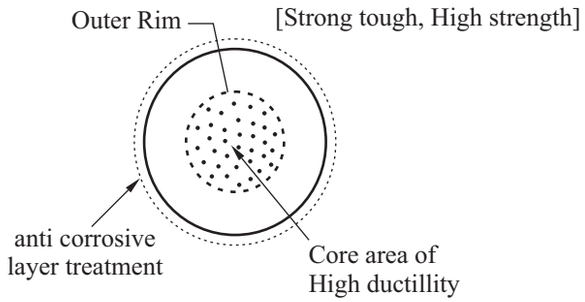


Fig. Cross-section of a TMT bar

Hence, the correct option is (C).

Key Point

Properties of thermo-mechanically treated (TMT) reinforcing bar :

1. High ductility
2. High strength
3. Mostly used for earthquake prone areas.
4. They have soft inner core with very hard and tough outer rim.
5. They are prepared with special heat treatment.
6. They are anti-corrosive in nature.

1.23 (B)

For maximum possible value of compaction factor for fresh (green) concrete,

Weight of partially compacted concrete = Weight of fully compacted concrete

$\therefore$ Compaction factor

$$= \frac{\text{Weight of partially compacted concrete}}{\text{Weight of fully compacted concrete}}$$

$$= 1$$

Hence, maximum possible value of compaction factor for fresh (green) concrete is 1.

Hence, the correct option is (C).

1.24 (A)

The creep may be defined as the increase of strain in concrete with time under sustained stress. This is also known as plastic flow or time yield. Creep is a time dependent phenomenon which is caused by dead loads only.

The rate of strain or creep can be expressed as:

$$\varepsilon_t = \varepsilon_u \left[1 - 10^{-\frac{\sqrt{t}}{4}}\right]$$

Where, ε_t = Creep after time of t months

ε_u = Ultimate strain

Hence, the correct option is (A).

Key Point

The creep increases rapidly with the stress loading at an early age of concrete, broken ballast, soft and porous aggregate, poorly graded and improperly compacted concrete.

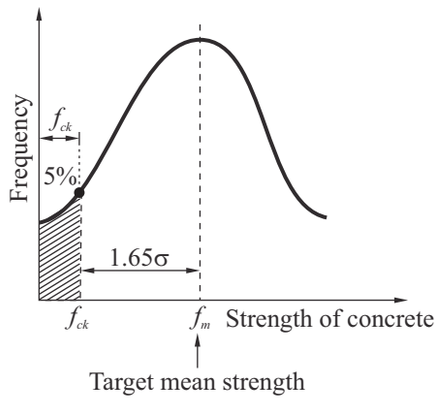
But, the rate of creep decreases with time and the creep strains at five years are taken as terminal values.

1.25 (D)

When the number of cubes are tested for the same sample of concrete, the number of test results that are likely to fall below the target strength (or mean strength or target mean strength) are related to standard deviation (σ) and the relationship is given by the following equation,

Target strength (or target mean strength),

$$f_m = f_{ck} + k(\sigma)$$



Since, as per **clause 6.1.1** mentioned in **IS456:2000,**

The characteristic strength is defined as that value of strength of material, below which not more than 5% of the test results are expected to fall, so in this case, the value of k will be 1.65. Hence the equation will become,

Target mean strength, $f_m = f_{ck} + 1.65(\sigma)$

Where, f_{ck} = Characteristic cube compressive strength of concrete (in N/mm^2)

σ = Standard deviation

k = Constant depending on the probability of certain number of results likely to fall below f_{ck}

Hence, the correct option is (D).

1.26 3.5

Given : Grade of concrete = M25

Characteristic compressive strength for M 25 grade concrete, $f_{ck} = 25\ N/mm^2$

To find : Flexural tensile strength, $f_{cr} = ?$

Solution :

As per **clause 6.2.2** mentioned in **IS 456 : 2000,**

Flexural tensile strength,

$$f_{cr} = 0.7\sqrt{f_{ck}}$$

$$f_{cr} = 0.7 \times \sqrt{25} = 3.5\ N/mm^2$$

Hence, the flexural tensile strength of M 25 grade of concrete, as per **IS 456 : 2000** is **3.5 N/mm²**.

Scan for Video Solution

1.27 (C)

As per **clause 6.2.3.1** mentioned in **IS 456: 2000**,

Modulous of elasticity, $E = 5000\sqrt{f_{ck}}$ is the short term static modulus of elasticity which is also known as secant molulous.

Hence, the correct option is (C).

Scan for Video Solution

1.28 0.097

The structure may fail when :

(i) The load exceeds the design load.

(ii) The strength is less than the characteristic strength.

(iii) Both the load exceeds the design load and strength is less than the characteristic strength. The probability of load exceeding design load = 5 %.

(iv) The probability of strength less than characteristic strength = 5 %.

$\therefore$ The probability of failure

$$= 0.05 \times 0.95 + 0.05 \times 0.95 + 0.05 \times 0.05$$

$$= 0.0975$$

Hence, the probability of failure of a structure is **0.0975**.

1.29 10000

Given : Grade of concrete = M25

Characteristic compressive strength of concrete = 25 MPa

Creep coefficient for M25 concrete, $\theta_c = 1.5$

To find : Long-term static modulus of elasticity, $E_\theta = ?$

Solution :

Long term static modulus of elasticity,

$$E_\theta = \frac{E_c}{1+\theta_c} = \frac{5000\sqrt{f_{ck}}}{1+\theta_c}$$

$$E_\theta = \frac{5000\sqrt{25}}{1+1.5} = 10,000\ MPa$$

Hence, the long-term static modulus of elasticity as per the provisions of **IS 456 : 2000** is **10,000 MPa**.

Key Point

As per **clause 6.2.3.1** mentioned in **IS 456: 2000**,

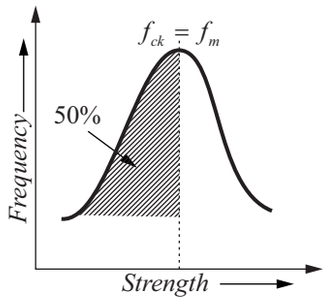
Modulous of elasticity, $E_c = 5000\sqrt{f_{ck}}$

Where, E_c = Short term static modulus of elasticity which is also known as secant molulous.

f_{ck} = Characteristic compressive strength of concrete

Scan for Video Solution

1.30 (C)



According to question, if characteristic strength, f_{ck} is the value below which not more than 50 % of test results are expected then, the normal distribution curve will be like as drawn and from this curve, it can be concluded that, target mean strength will be equal to characteristics strength of concrete i.e.,

$$f_m = f_{ck}$$

So, the target mean strength to be considered in the mix design of M25 concrete will be, characteristic compressive strength of concrete i.e.,

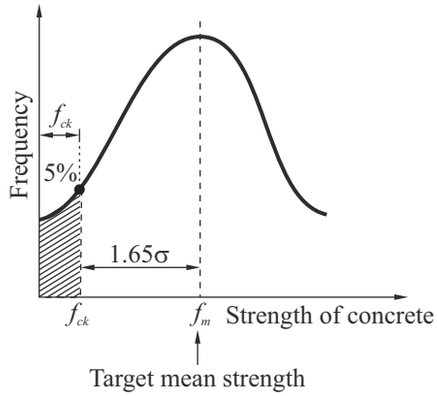
$$f_m = f_{ck} = 25\text{MPa}$$

Hence, the correct option is (C).

Key Point

As per **clause 6.1.1, IS456:2000**,

The characteristic strength is defined as the strength of material below which not more than 5% of the test results are expected to fall.



$\therefore$ Target mean strength,

$$f_m = 1.65\sigma + f_{ck}$$

Where, f_{ck} = Characteristic compressive strength of concrete

σ = Standard deviation

Note : If σ is more than the concrete sample is non-uniform.

If σ is less than the concrete sample is uniform.

Scan for Video Solution

1.31 (A)

- Creep is defined as the deformation of concrete structure with time under sustained load.
- Basically, it is a long term pressure or stress on concrete that can cause the change in its shape.
- This deformation usually occurs in the direction of applied load.
- Creep does not necessarily cause concrete to fail or break apart. Creep is determined when concrete structures are designed.

- Creep may be represented either by a creep coefficient, which is the ratio of total strain to initial strain or by value of strain per unit of concrete.

Hence, the correct option is (A).

Scan for Video Solution

1.32 zero

Mean strength of the specimens,

$$\mu = \frac{(4\times23)+(2\times28)+(5\times22.5)+(5\times31)+(4\times29)}{20}$$

$$\mu = 26.575 \text{ MPa}$$

Standard deviation,

$$\sigma = \sqrt{\frac{\Sigma(\mu - f)^2}{n-1}}$$

$$\sigma = \sqrt{\frac{(26.575-23)^2\times4+(26.575-28)^2\times2+(26.575-22.5)^2\times5+(26.575-31)^2\times5+(26.575-29)^2\times4}{(20-1)}}$$

Standard deviation, $\sigma = 3.69 \simeq 3.7$

$\therefore \quad (\mu - 3\sigma) = 26.575 - 3\times3.7$

$$(\mu - 3\sigma) = 15.475 \simeq 15.5 \text{ N/mm}^2$$

Hence, compressive strength (f) of all 20 specimens are greater than $(\mu - 3\sigma)$.

$\therefore$ The answer will be zero.

Hence, the number of specimens with compressive strength less than $\mu - 3\sigma$ is **zero**.

1.33 (46)

Given : Characteristic compressive strength of concrete $(f_{ck}) = 25$ MPa

Target mean strength $= f_{ck} + 1.65\sigma$

$$= 25 + 1.65\times4.0 = 31.6$$

Note :

Grade of concrete	Standard deviation σ
M 10 – M 15	3.5
M 20 – M 25	4
M 30 – M 40	5

Water content required

$$= 50 - \frac{(50-45)}{(35-25)}\times(31.6-25)$$

$$= 46.7\%$$

Say 46 % (round off to the lower integer)

Hence, the water cement ratio to be used in the mix is **46 %.**

1.34 12500

Given : Grade of concrete = 25

Initial strain $= 500\times10^{-6}$, Stress $= 12.5$ N/mm^2

Method 1

Modulus of elasticity of concrete,

$$E_c = \frac{\text{Stress in concrete}}{\text{Strain}} = \frac{12.5}{500\times10^{-6}}$$

$$= 25000 \text{ N/mm}^2$$

Note : $E_c = 5000\sqrt{f_{ck}}$

$$E_c = 5000\sqrt{25} = 25000 \text{ N/mm}^2$$

Total strain after long time $= 1000\times10^{-6}$

Long term modulus of elasticity, $E_{cl} = \frac{E_c}{1+\theta}$

Where, $\theta =$ Creep coefficient

$$\theta = \frac{\text{Ultimate creep strain}}{\text{Elastic strain}}$$

$$= \frac{(1000-500)\times10^{-6}}{500\times10^{-6}} = 1.0$$

Hence, $E_{cl} = \frac{2500}{1+1.0} = 12500 \text{ N/mm}^2$

Method 2

Long term of effective modulus of elasticity

$$E_{c1} = \frac{\text{Stress on concrete}}{\text{Long term strain}}$$

$$= \frac{12.5}{1000 \times 10^{-6}} = 12500 \text{ MPa}$$

Hence, the long term effective modulus is **12500 MPa.**

1.35 2.2

The measured splitting tensile strength, of the specimen shall be calculated to the nearest 0.05 N/mm^2 using the following formula.

Split tensile strength $(f_{ct}) = \frac{2P}{\pi DL}$

P = maximum load in Newtons applied to the specimen = 157 kN

L = length of the specimen (in mm) = 300 mm

D = cross sectional dimension of the specimen = 150 mm

$$f_{ct} = \frac{2 \times 157 \times 10^3}{\pi \times 150 \times 300} = 2.22 \text{ N/mm}^2$$

$$f_{ct} = 2.22 \text{ MPa}$$

Hence, the spilt tensile strength of the specimen is **2.2 MPa**.

Key Point

Split tensile strength test is the standard test to determine the tensile strength of concrete indirect way in accordance with IS : 5816-1999.

A standard test cylinder of concrete specimen of 300 mm×150 mm diameter is placed horizontally between the loading platens.

The load is applied gradually and increased continuously until failure with in the range of 1.2 MPa/min to 2.4 MPa/min.

1.36 (B)

As per IS 456-2000

The pH value of water used for concrete mix shall not be less than 6.

Hence, the correct option is (B).

❖❖❖❖

1 General Design Requirements

1995 IIT Kanpur

1.1 As per **IS : 800 - 1984**, the minimum pitch of rivets in a row is recommended as the diameter of the rivet times **[1 Mark]**

(A) 2.0 (B) 2.5

(C) 3.0 (D) 4.0

1996 IISc Bangalore

1.2 Generally the maximum deflection/span ratio of a steel member should not exceed.

(A) 1/750 (B) 1/500

(C) 1/325 (D) 1/250

1.3 Generally, fatigue life of welded steel structure to fatigue life of riveted steel structure ratio is **[1 Mark]**

(A) Smaller than 1

(B) Equal to 1

(C) Greater than 1

(D) Greater than 2.1

1997 IIT Madras

1.4 Factor of safety adopted by **IS 800 : 1984** while arriving at the permissible stress in axial compression is **[1 Mark]**

(A) 2.00 (B) 1.00

(C) 1.67 (D) 1.50

2005 IIT Bombay

1.5 Which one of the following is not correct for steel sections as per **IS : 800 - 1984**? **[2 Marks]**

(A) The maximum bending stress in tension or in compression in extreme fiber calculated on the effective section of a beam shall not exceed 0.66 f_y

(B) The bearing stress in any part of a beam when calculated on the net area shall not exceed 0.75 f_y

(C) The direct stress in compression on the cross-sectional area of axially loaded compressive member shall not exceed 0.6 f_y

(D) None of the above

Answers General Design Requirements

1.1	B	1.2	C	1.3	A	1.4	C	1.5	D

Explanations General Design Requirements

1.1 (B)

The minimum spacing between centres of fasteners shall not be less than 2.5 time the nominal diameter of the fasteners.

Hence, the correct option is (B).

1.2 (C)

Maximum deflection in the beam should not exceed $\frac{\text{Span}}{325}$,

$$\delta_{max}\,(\text{in mm}) \not> \frac{\text{Span}}{325}\ (\text{in mm})$$

Hence, the correct option is (C).

1.3 (A)

Welded connection are weak under fatigue loading and prone to cracking. So fatigue life of welded structure is always less than the fatigue life of riveted structure.

$$\frac{\text{Fatigue life of welded structure}}{\text{Fatigue life of riveted structure}} < 1$$

Hence, the correct option is (A).

1.4 (C)

Permissible stress in axial compression as per **IS 800 : 1984**

i.e., $\sigma_{allowable} = 0.6 f_y$

$\therefore$ $\text{Factor of safety} = \left(\frac{1}{0.6}\right) = 1.67$

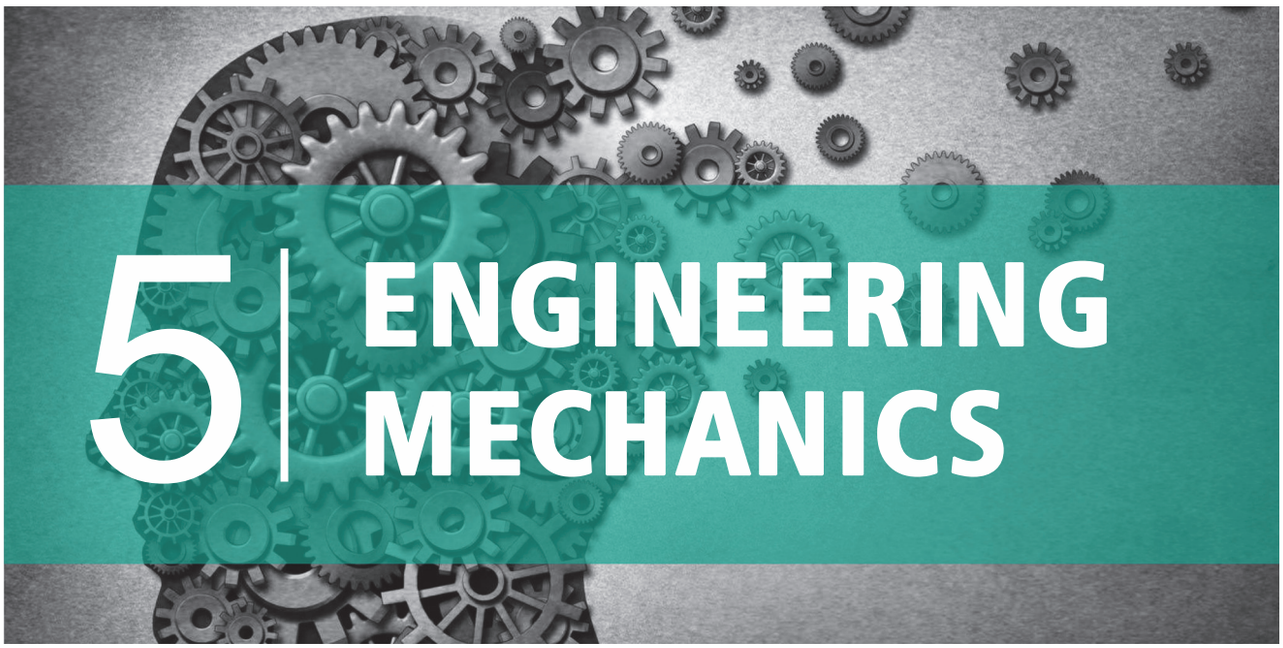
Hence, the correct option is (C).

1.5 (D)

All statements (a), (b) and (c) are correct according to **IS 800 : 2007**.

Hence, the correct option is (D).

❖❖❖❖



5 | ENGINEERING MECHANICS

Syllabus : Engineering Mechanics

System of forces, free-body diagrams, equilibrium equations; Internal forces in structures; Frictions and its applications; Centre of mass; Free Vibrations of undamped SDOF system.

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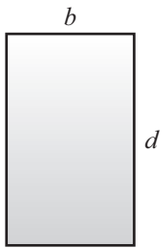
2006 IIT Kharagpur

1. For the section shown below, second moment of the area about an axis $d/4$ distance above the bottom of the area is

[2 Marks]



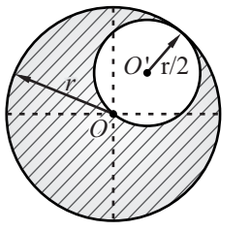
(A) $\frac{bd^3}{48}$ (B) $\frac{bd^3}{12}$

(C) $\frac{7bd^3}{48}$ (D) $\frac{bd^3}{3}$

2010 IIT Guwahati

2. A disc of radius r has a hole of radius r/2 cut-out as shown. The centroid of the remaining disc (shaded portion) at a radial distance from the centre "O" is

[2 Marks]



(A) $\frac{r}{2}$ (B) $\frac{r}{3}$

(C) $\frac{r}{6}$ (D) $\frac{r}{8}$

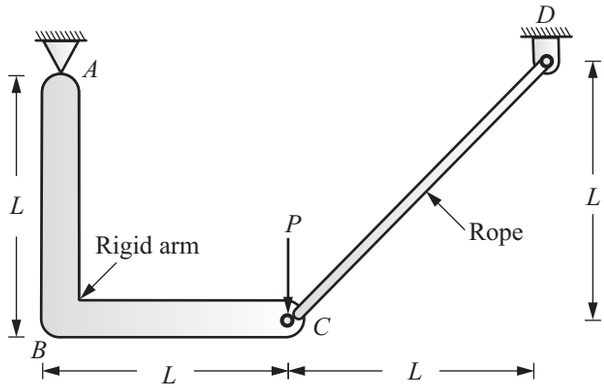
2014 IIT Kharagpur

3. Polar moment of inertia (I_p), in cm^4, of a rectangular section having width, $b = 2\,cm$ and depth, $d = 6\,cm$ is _____.

[1 Mark]

2016 IISc Bangalore

4. An assembly made of a rigid arm ABC hinged at end A and supported by an elastic rope CD at end C as shown in the figure. The members may be assumed to be weightless and the lengths of the respective members are as shown in the figure.



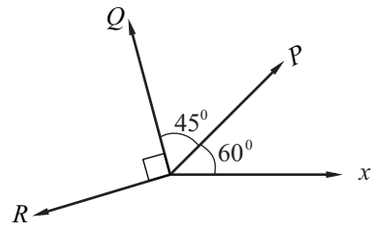
Under the action of a concentrated load P at C as shown, the magnitude of tension developed in the rope is

[1 Mark]

(A) $\frac{3P}{\sqrt{2}}$ (B) $\frac{P}{\sqrt{2}}$

(C) $\frac{3P}{8}$ (D) $\sqrt{2}P$

5. The magnitude of vectors P, Q and R are 100 kN, 250 kN and 150 kN, respectively as shown in the figure



The respective values of the magnitude (in kN) and the direction (with respect to the x - axis) of the resultant vector are

[2 Marks]

(A) 290.9 and 96.0^0

(B) 368.1 and 94.7^0

(C) 330.4 and 118.9^0

(D) 400.1 and 113.5^0

2017 IIT Roorkee

6. A particle of mass 2 kg is travelling at a velocity of 1.5 m/s. A force $f(t) = 3t^2$ (in N) is applied to it in the direction of motion for a duration of 2 seconds, where t denotes time in seconds. The velocity (in m/s, up to one decimal place) of the particle immediately after the removal of the force is _____.

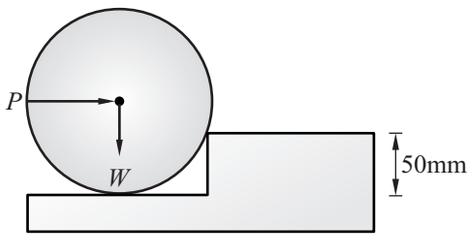
[2 Marks]

2018 IIT Guwahati

7. An aircraft approaches the threshold of a runway strip at a speed of 200 km/h. The pilot decelerates the aircraft at a rate of 1.697 m/s^2 and takes 18 sec to exit the runway strip. If the deceleration after exiting the runway is 1 m/s^2, then the distance (in m, up to one decimal place) of the gate position from the location of exit on the runway is _____m.

[2 Marks]

8. A cylinder of radius 250 mm and weight, $W = 10$ kN is rolled up an obstacle of height 50 mm by applying a horizontal force P at its centre as shown in the figure.



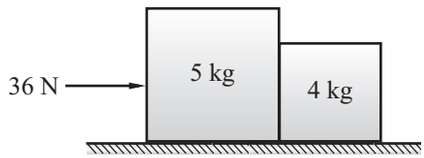
All interfaces are assumed frictionless. The minimum value of P is **[2 Marks]**

(A) 4.5 kN (B) 5.0 kN

(C) 6.0 kN (D) 7.5 kN

9. Two rigid bodies of mass 5 kg and 4 kg are at rest on a frictionless surface until acted upon by a force of 36 N as shown in the figure. The contact force generated between the two bodies is

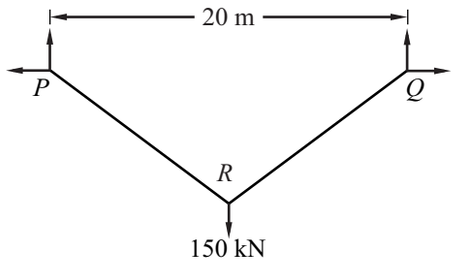
[2 Marks]



(A) 4.0 N (B) 7.2 N

(C) 9.0 N (D) 16.0 N

10. A cable PQ of length 25 m is supported at two ends at the same level as shown in the figure. The horizontal distance between the supports is 20 m. A point load of 150 kN is applied at point R which divides it into two equal parts

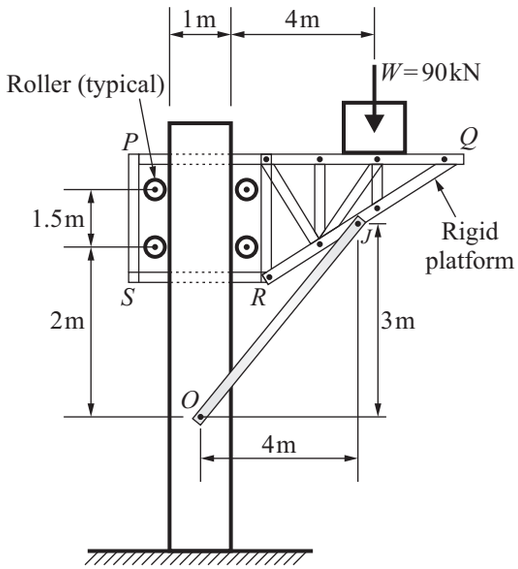


Neglecting the self-weight of the cable, the tension (in kN, integer value) in the cable due to the applied load will be _____. **[2 Marks]**

2020 IIT Delhi

11. A rigid weightless platform **PQRS** shown in the figure (*not drawn to the scale*) can slide freely in the vertical direction. The platform is held in position by the weightless member **OJ**

and four weightless, frictionless rollers, Points **O** and **J** are pin connections. A block of 90 kN rests on the platform as shown in the figure. **[2 Marks]**



The magnitude of horizontal component of the reaction (in kN) at pin **O**, is

(A) 150 (B) 120

(C) 90 (D) 180

Answers Engineering Mechanics

1.	**C**	**2.**	**C**	**3.**	**40**	**4.**	**B**	**5.**	**C**
6.	**5.5**	**7.**	**312.5**	**8.**	**D**	**9.**	**D**	**10.**	**125**
11.	**B**								

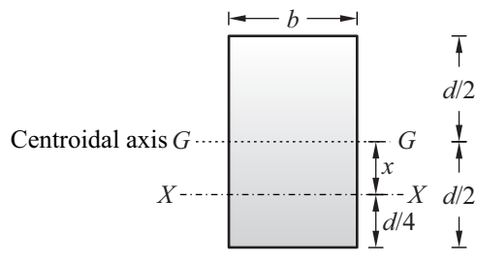
Explanations Engineering Mechanics

1. (C)

According to question,

Area of the section $(A) = bd$

Let, a axis-XX at a distance $\frac{d}{4}$ from bottom of the area.



Therefore, the distance between centroidal axis and XX-axis is,

$$x = \frac{d}{2} - \frac{d}{4} = \frac{d}{4}$$

Now, second moment of the area about centroidal axis is given by,

$$I_{GG} = \frac{bd^3}{12}$$

Therefore, second moment of the area about XX-axis is given by,

$$I_{XX} = I_{GG} + Ax^2$$

(From parallel axis theorem)

$$I_{XX} = \frac{bd^3}{12} + bd \times \left(\frac{d}{4}\right)^2$$

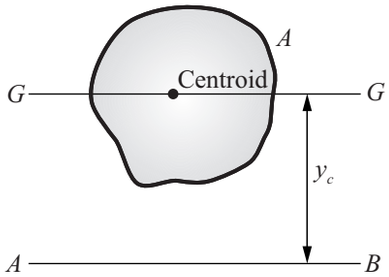
$$I_{XX} = \frac{bd^3}{12} + \frac{bd^3}{16} = \frac{7bd^3}{48}$$

Hence, the correct option is (C).

Scan for Video Solution

Key Point

- **Parallel axis theorem :** Moment of inertia about any axis in the plane of an area is equal to the sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance between the two parallel axes.



Referring to figure the above theorem means,

$I_{AB} = I_{GG} + Ay_c^2$

Where, I_{AB} = Moment of inertia about axis AB

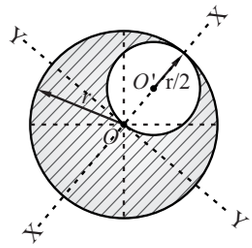
I_{GG} = Moment of inertia about centroidal axis GG parallel to AB,

A = The area of the plane figure given

y_c = The distance between the axis AB and the parallel centroidal axis GG.

- Moment of inertia is also called second moment of the area or area moment of inertia.

2. (C)



Area of the larger circle $(A_L) = \pi r^2$

Area of the smaller circle $(A_S) = \pi\left(\frac{r}{2}\right)^2 = \frac{\pi r^2}{4}$

Distance of the centroid of the larger circle from O, $(x_L) = 0$

Distance of the centroid of the smaller circle from O, $(x_S) = \frac{r}{2}$

Now, centroid of the shaded region,

$$\bar{x} = \frac{A_L \times x_L - A_S x_S}{A_S - A_L} = \frac{\pi r^2 \times 0 - \frac{\pi r^2}{4} \times \frac{r}{2}}{\pi r^2 - \frac{\pi r^2}{4}}$$

$$= \frac{-\frac{\pi r^3}{8}}{\frac{4\pi r^2 - \pi r^2}{4}} = -\frac{\pi r^3}{8} \times \frac{4}{3\pi r^2}$$

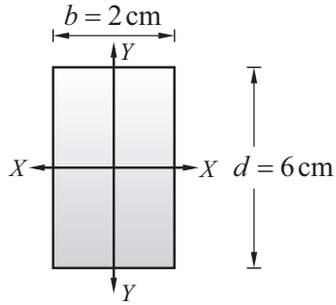
$$\bar{x} = -\frac{1}{6}\frac{\pi r^3}{\pi r^2} = -\frac{r}{6} = \frac{r}{6} \quad \text{(Magnitude)}$$

Hence, the correct option is (C).

Scan for Video Solution

3. 40

According to the question,



Now, polar moment of inertia is given by,

$$I_P = I_X + I_Y = \frac{bd^3}{12} + \frac{db^3}{12}$$

(from perpendicular axis theorem)

$$I_P = \frac{2\times 6^3}{12} + \frac{6\times 2^3}{12} = \frac{432+48}{12}$$

$$I_P = \frac{480}{12} = 40\,\text{cm}^4$$

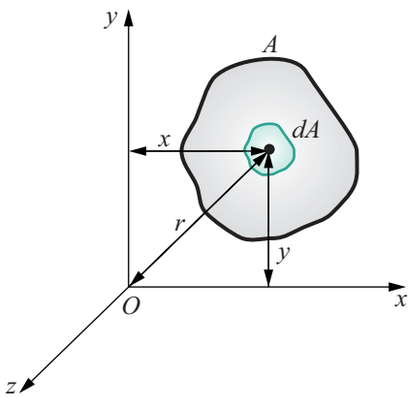
Hence, the correct answer is **40 cm^4**.

Scan for Video Solution

Key Point

Perpendicular axis theorem : The moment of inertia of an area about an axis perpendicular to its plane (polar moment of inertia) at any point O is equal to the sum of moments of inertia about any two mutually perpendicular axes through the same point O and lying in the plane of the area.

Referring to figure, if z-axis is normal to the plane of paper passing through point O, as per theorem, $I_{zz} = I_{xx} + I_{yy}$



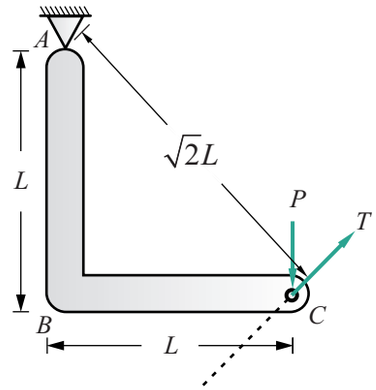
The above theorem can be easily proved. Let us consider an elemental area dA at a distance r from O. Let the coordinates of dA be x and y. Then from definition,

$$I_{zz} = \sum r^2 dA = \sum (x^2 + y^2) dA$$

$$I_{zz} = \sum x^2 dA + \sum y^2 dA = I_{xx} + I_{yy}$$

4. (B)

Let, tension in rope be T



Considering triangle ABC,

$$AC^2 = AB^2 + BC^2$$

$$AC = \sqrt{L^2 + L^2} = \sqrt{2}L$$

Taking moments about A equal to zero.

$\therefore \quad \Sigma M_A = 0,$

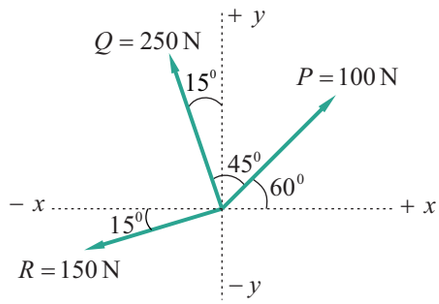
$$-P\times L + T\times\sqrt{2}L = 0$$

$$T = \frac{PL}{\sqrt{2}L} = \frac{P}{\sqrt{2}}$$

Hence, the correct option is (B).

5. (C)

Let resultant of forces P, Q, R be R_{res}.



Now, net horizontal force,

$$\Sigma F_H = P\cos 60 - Q\sin 15 - R\cos 15$$

$$\Sigma F_H = 100\cos 60 - 250\sin 15 - 150\cos 15$$

$$\Sigma F_H = -159.59\,\text{kN}$$

$\therefore$ Net vertical force,

$$\Sigma F_V = P\sin 60 + Q\cos 15 - R\sin 15$$

$$\Sigma F_V = 100\sin 60 + 250\cos 15 - 150\sin 15$$

$$\Sigma F_V = 289.26\,\text{kN}$$

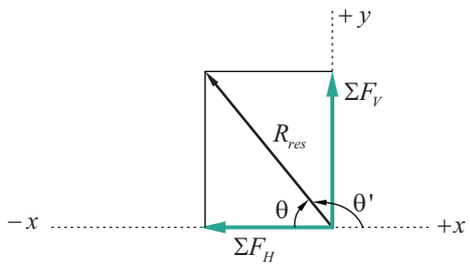
Now, the resultant is given by,

$$R_{res} = \sqrt{(\Sigma F_H)^2 + (\Sigma F_V)^2}$$

$$R_{res} = \sqrt{(-159.59)^2 + (289.26)^2} = 330.4\,\text{kN}$$

(Magnitude)

Since, ΣF_H is negative and ΣF_V is positive then R_{res} is lying in second quadrant.



$$\theta = \tan^{-1}\left(\frac{\Sigma F_V}{\Sigma F_H}\right) = \tan^{-1}\left(\frac{289.26}{-159.59}\right)$$

$$\theta = -61.114^0$$

(w.r.t. $-x$-axis in clockwise direction)

$\therefore$ $\theta' = 180 - 61.114 = 118.886^0$

$\theta' \approx 118.9^0$

(w.r.t. $+x$-axis in counter-clockwise direction).

Hence, the correct option is (C).

Scan for Video Solution

6. 5.5

Given : Mass of the particle $(m) = 2\,\text{kg}$

Initial velocity $(u) = 1.5\,\text{m/s}$

Applied force $F(t) = 3t^2\,\text{N}$

Force duration $(t) = 2\,\text{sec}$

Let, final velocity be v.

Now, we know that,

$$F = ma$$

$$\int_0^t F(t)dt = m(v-u)$$

$$\int_0^2 3t^2 dt = 2(v-1.5)$$

$$\left[t^3\right]_0^2 = 2(v-1.5)$$

$$(8-0) = 2(v-1.5)$$

$$v = 5.5\,\text{m/s}$$

Hence, the velocity of the particle immediately after the removal of the force is **5.5 m/s**.

Scan for Video Solution

Key Point

According to Newton's second law of motion,

Applied force on body

= Rate of change of linear momentum of body

$$F = \frac{d(mV)}{dt}$$

If $m = \text{constant}$, then $F = m\frac{dV}{dt}$

$\therefore$ $\int_0^t F\,dt = m\int_u^v dV = m[v-u]$

7. 312.5

Given :

Initial speed of the aircraft $(u) = 200\,\text{km/h}$

De-acceleration upto the runway exit

$(a_1) = -1.697\,\text{m/s}^2$

Time taken upto the runway exit $(T) = 18\,\text{sec}$

De-acceleration after exiting runway

$(a_2) = -1\,\text{m/s}^2$

Let, v_1 be the velocity at runway exit (after 18 sec), S be the distance travelled by the aircraft from runway exit to the gate position with an acceleration a_2 and final velocity be v_2, where $v_2 = 0$.

Now, velocity at the runway exit,

$$v_1 = u + at$$

$$v_1 = \frac{200\times1000}{3600} + (-1.697\times18) = 25\,\text{m/s}$$

Distance traveled from runway exit to the gate position is,

$$v_2^2 = v_1^2 + 2aS$$

$$0 = 25^2 + 2\times(-1)\times S$$

$$2S = 625$$

$$S = \frac{625}{2} = 312.5\,\text{m}$$

Hence, the distance of the gate position from the location of exit on the runway is **312.5 m**.

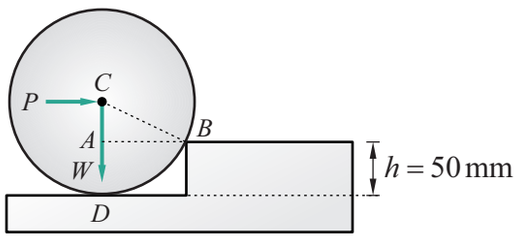
Scan for Video Solution

8. (D)

Given : Radius of the cylinder $(R) = 250$ mm

Weight of the cylinder $(W) = 10\,\text{kN}$

Obstacle height $(h) = 50\,\text{mm}$



$$AC = R - h = 250 - 50 = 200\,\text{mm}$$

$$AB = \sqrt{CB^2 - AC^2}$$

$$AB = \sqrt{250^2 - 200^2} = 150\,\text{mm}$$

At the instant when the cylinder just starts rolling over obstacle, it will be lifted from point D. the contact will only be there with point B and no reaction force takes place at contact point B.

Now, taking the moments about B equal to zero,

$\Sigma M_B = 0$,

$$-P\times AC + W\times AB = 0$$

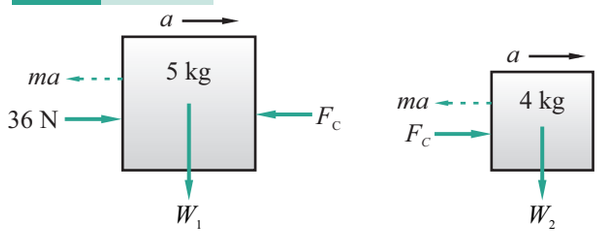
$$-P\times200 + 10\times150 = 0$$

$$P = \frac{1500}{200} = 7.5\ \text{kN}$$

Hence, the correct option is (D).

Scan for Video Solution

9. (D)



The applied load 36 N will cause both the blocks to move together. Let a be the acceleration and F_C be the contact force for both the blocks.

Now, from Newton's second law, $F = ma$

$$a = \frac{F}{m} = \frac{36}{9} = 4\,\text{m/s}^2$$

Similarly, contact force,

$$F_C = ma = 4\times4 = 16\,\text{N}$$

(for smaller block)

Hence, the correct option is (D).

Note :

$F_C = 36 - ma$ can also be used to determine the value of F_C (for bigger block) and value will be same.

Checking, $F_C = 36 - 5\times4 = 36 - 20 = 16$ N

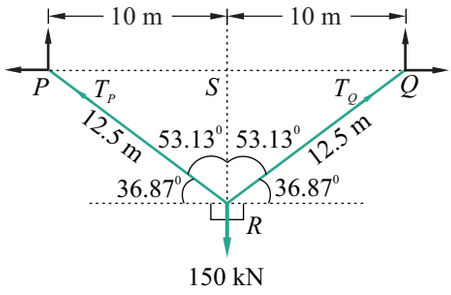
Scan for Video Solution

10. 125

Given : Length of cable $PQ = 25\,\text{m}$

Horizontal distance between $PQ = 20\,\text{m}$

Method 1



Let, tension in cable PR and QR be T_P and T_Q respectively, Now, in triangle PQR,

$$PS = SQ = 10\,\text{m}$$

$$PR = RQ = 12.5\,\text{m}$$

$$\angle PRS = SRQ = \sin^{-1}\left(\frac{10}{12.5}\right) = 53.13^0$$

Using Lami's theorem,

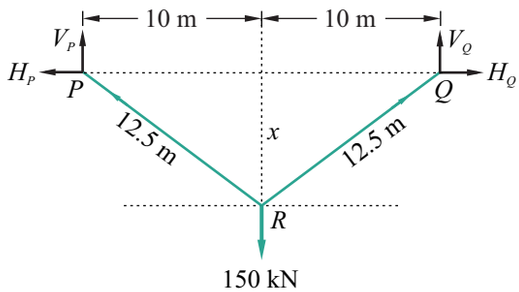
$$\frac{T_P}{\sin(90+36.87)} = \frac{150}{\sin(53.13+53.13)} = \frac{T_Q}{\sin(90+36.87)}$$

$$T_P = \frac{150\times\sin(90+36.87)}{\sin(53.13+53.13)} = 124.98\,\text{kN}$$

$$T_P = T_Q = 124.99 \approx 125\,\text{kN}$$

Hence, the tension in the cable due to the applied load will be **125 kN**.

Method 2



$$V_P + V_Q = 150\ \text{kN}$$

$$V_P = V_Q = 75\ \text{kN}$$

$$12.5^2 = x^2 + 10^2$$

$$x = 7.5\ \text{m}$$

Bending moment at $R = 0$,

$$H_P \times 7.5 = 75 \times 10$$

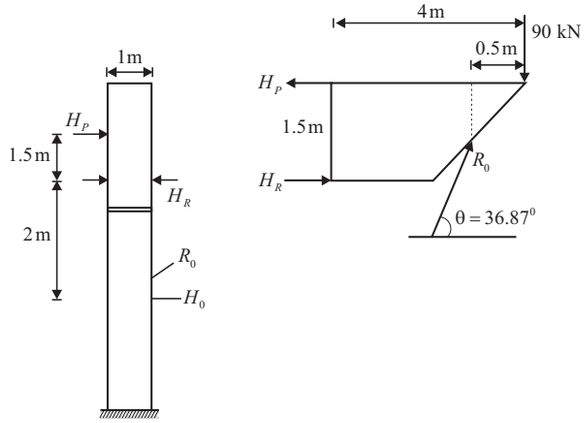
$$H_P = 100\ \text{kN}$$

Tension in cable $= \sqrt{H_P^2 + V_P^2}$

$$= \sqrt{75^2 + 100^2} = 125\ \text{kN}$$

Hence, the tension in the cable due to the applied load will be **125 kN**.

11. (B)



$\Sigma y = 0$,

$$R_0 \sin 36.87 - 90 = 0$$

$$R_0 = \frac{90}{\sin 3687^0} = 150\ \text{kN}$$

Horizontal reaction at $O = H_0$

$$= R_0 \cos 36.87 = 150 \times \cos 36.87$$

$$= 120\ \text{kN}$$

Hence, the correct option is (B).

❖❖❖❖

Space for Notes

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6 CONSTRUCTION MATERIALS AND MANAGEMENT

Syllabus : Construction Materials & Management

Construction Materials: Structural Steel – Composition, material properties and behaviour; Concrete - Constituents, mix design, short-term and long-term properties. Construction Management: Types of construction projects; Project planning and network analysis - PERT and CPM; Cost estimation.

Construction Materials

1990 IISc Bangalore

1. Tensile strength of cement mortar is usually found by **[1 Mark]**

(A) Briquette test
(B) Slump test
(C) Vicat needle test
(D) Split cylinder test

1991 IIT Madras

2. Setting time of cement sample is tested by **[1 Mark]**

(A) Slump test
(B) Vicat needle test
(C) Le-Chatelier's test
(D) Briquette test

1994 IIT Kharagpur

3. Grading of aggregate in a concrete mix is necessary to achieve **[1 Mark]**

(A) Adedquate workability
(B) Higher density
(C) Reduction in voids
(D) Better durability

4. Portland cement is manufactured by burning in a kiln the following materials. **[1 Mark]**

(A) Limestone and alumina
(B) Limestone and clay
(C) Limestone and sand
(D) Lime and clay

1995 IIT Kanpur

5. Workability of concrete is influenced most by its **[1 Mark]**

(A) Water-cement ratio
(B) Aggregate-cement ratio
(C) Cement content
(D) Water- content

2003 IIT Madras

6. **List-I** contains some properties of concrete/cement and **List-II** contains list of some tests on concrete/cement. Match the property with the corresponding test. **[2 Marks]**

List I

(A) Workability of concrete
(B) Direct tensile strength of concrete
(C) Bond between concrete and steel
(D) Fineness of cement

List II

1. Cylinder splitting test
2. Vee-Bee test
3. Surface area test
4. Fineness modulus test
5. Pull out test

Codes :	**A**	**B**	**C**	**D**
(A)	2	1	5	3
(B)	4	5	1	3
(C)	2	1	5	4
(D)	2	5	1	4

2007 IIT Kanpur

7. Consider the following statements :

1. The compressive strength of concrete decreases with increase in water-cement ratio of the concrete mix.
2. Water is added to the concrete mix for hydration of cement and workability.
3. Creep and shrinkage of concrete are independent of the water-cement ratio in the concrete mix.

The true statements are **[1 Mark]**

(A) 1and 2 (B) 1, 2 and 3
(C) 2 and 3 (D) 2 only

8. Consider the following statements :

[2 Marks]

1. Modulus of elasticity of concrete increases with increase in compressive strength of concrete.
2. Brittleness of concrete increases with decrease in compressive strength of concrete.
3. Shear strength of concrete increases with increase in compressive strength of concrete. The true statements are

(A) 2 and 3 (B) 1, 2 and 3
(C) 1 and 2 (D) 1 and 3

2009 IIT Roorkee

9. Match **List-I** (list of test methods for evaluating properties of concrete) with **List-II** (List of properties) and select the correct answer using the codes given below the lists. **[2 Marks]**

List-I

(A) Resonant frequency test
(B) Rebound hammer test
(C) Split cylinder test
(D) Compacting factor test

List-II

1. Tensile strength
2. Dynamic modulus of elasticity
3. Workability
4. Compressive strength

Codes :	A	B	C	D
(A)	2	4	1	3
(B)	2	1	4	3
(C)	2	4	3	1
(D)	4	3	1	2

2013 IIT Bombay

10. Maximum possible value of compaction factor for fresh (green) concrete is

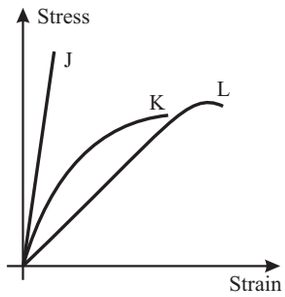
[1 Mark]

(A) 0.5 (B) 1.0
(C) 1.5 (D) 2.0

2014 IIT Kharagpur

11. **Group-I** contains representative stress-strain curves as shown in the figure, while **Group-II** gives the list of materials. Match the stress strain curves with the corresponding materials.

[1 Mark]



Group-I

P. Curve J
Q. Curve K
R. Curve L

Group-II

1. Cement paste
2. Coarse aggregate
3. Concrete

	P	**Q**	**R**
(A)	1	3	2
(B)	2	3	1
(C)	3	1	2
(D)	3	2	1

2015 IIT Kanpur

12. Consider the following statements for air-entrained concrete :

(i) Air-entrainment reduces the water demand for a given level of workability

(ii) Use of air-entrained concrete is required in environments where cyclic freezing and thawing is expected.

Which of the following is **TRUE**?

[1 Mark]

(A) Both (i) and (ii) are True

(B) Both (i) and (ii) are False

(C) (i) is True and (ii) is False

(D) (i) is False and (ii) is True

13. Workability of concrete can be measured using slump, compaction factor and Vee-bee time. Consider the following statements for workability of concrete : **[1 Mark]**

(i) As the slump increases, the Vee-bee time increases

(ii) As the slump increases, the compaction factor increases

Which of the following is **TRUE**?

(A) Both (i) and (ii) are True

(B) Both (i) and (ii) are False

(C) (i) is True and (ii) is False

(D) (i) is False and (ii) is True

2016 IISc Bangalore

14. The compound which is largely responsible for initial setting and early strength gain of Ordinary Portland Cement is **[1 Mark]**

(A) C_3A (B) C_3S

(C) C_2S (D) C_4AF

15. Bull's trench kiln is used in the manufacturing of **[1 Mark]**

(A) Lime (B) Cement

(C) Bricks (D) None of these

16. As per Indian standards for bricks, minimum acceptable compressive strength of any class of burnt clay bricks in dry state is **[1 Mark]**

(A) 10.0 MPa (B) 7.5 MPa

(C) 5.0 MPa (D) 3.5 MPa

2017 IIT Roorkee

17. Group I gives a list of test methods and test apparatus for evaluating some of the properties of Ordinary Portland Cement (OPC) and concrete. Group II gives the list of these properties. **[2 Marks]**

Group I

P. Le Chatelier test

Q. Vee-Bee test

R. Blaine air permeability test

S. The Vicat apparatus

Group II

1. Soundness of OPC
2. Consistency and setting time of OPC
3. Consistency or workability of concrete
4. Fineness of OPC

The correct match of the items in Group I with the items in Group II is

(A) P-1, Q-3, R-4, S-2

(B) P-2, Q-3, R-1, S-4

(C) P-4, Q-2, R-4, S-1

(D) P-1, Q-4, R-2, S-3

18. Consider the following statements : **[2 Marks]**

P. Walls of one brick thick are measured in square meters

Q. Walls of one brick thick are measured in cubic meters.

R. No deduction in the brickwork quantity if made for openings in walls up to $0.1 m^2$ area.

S. For the measurement of excavation from the borrow pit in a fairly uniform ground, deadman are left at suitable intervals.

For the above statements, the correct option is

(A) P-false; Q-True; R-false; S-True

(B) P-false; Q-True; R-false; S-False

(C) P-True; Q-False; R-True; S-False

(D) P-True; Q-False; R-True; S-True

2018 IIT Guwahati

19. The Le chatelier apparatus is used to determine **[1 Mark]**

(A) Compressive strength of cement

(B) Fineness of cement

(C) Setting time of cement

(D) Soundness of cement

20. The setting time of cement is determined using **[1 Mark]**

(A) Le chateller apparatus

(B) Briquette testing apparatus

(C) Vicat apparatus

(D) Casagrande's apparatus

2019 IIT Madras

21. A box measuring $50\,cm \times 50\,cm \times 50\,cm$ is filled to the top with dry coarse aggregate of mass $187.5\,kg$. The water absorption and specific gravity of the aggregate are 0.5 % 2.5, respectively. The maximum quantity of water (in kg. round off to 2 decimal places) required to fill the box completely is_____.

[2 Marks]

2020 IIT Delhi

22. During the process of hydration of cement, due to increase in Dicalcium Silicate (C_2S) content in cement clinker, the heat of hydration **[1 Mark]**

(A) increases

(B) initially decreases and then increases

(C) does not change

(D) decreases

23. The Los Angles test for stone aggregates is used to examine **[1 Mark]**

(A) soundness

(B) abrasion resistance

(C) specific gravity

(D) crushing strength

24. **Group-I** gives a list of test methods for evaluating properties of aggregates. **Group-II** gives the list of properties to be evaluated. **[2 Marks]**

Group - I (Test methods)		Group - II (Properties)	
P.	Soundness test	**1.**	Strength
Q.	Crushing test	**2.**	Resistance to weathering
R.	Los Angeles abrasion test	**3.**	Adhesion
S.	Stripping value test	**4.**	Hardness

The correct match of test methods under **Group-I** to properties **Group-II**, is

(A) P-3, Q-4, R-1, S-2

(B) P-4, Q-1, R-2, S-3

(C) P-2, Q-4, R-3, S-1

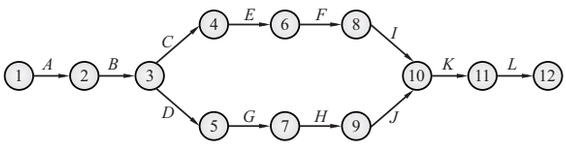
(D) P-2, Q-1, R-4, S-3

Construction Planning & Management

2016 IISc Bangalore

25. A construction project consists of twelve activities. The estimated duration (in days) required to complete each of the activities along with the corresponding network diagram is shown below,

Activity		Duration (Days)
A	Inauguration	1
B	Foundation work	7
C	Structural construction-1	30
D	Structural construction-2	30
E	Brick masonry work	25
F	Plastering	7
G	Flooring	25
H	Electrification	7
I	Plumbing	7
J	Woodwork	7
K	Coloring	3
L	Handing over function	1



Total floats (in days) for the activities 5-7 and 11-12 for the project are, respectively,

[1 Mark]

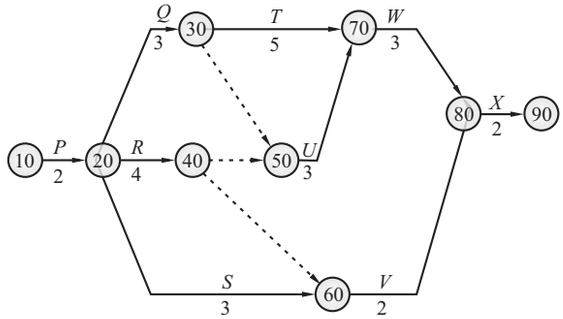
(A) 25 and 1 (B) 1 and 1

(C) 0 and 0 (D) 81 and 0

26. The optimistic time (O), Most likely time (M) and pessimistic Time (P) (in days) of the activities in the critical path are given below in the format O-M-P.

The expected completion time (in days) of the project is _____. **[2 Marks]**

27. The activity on arrow network of activities for a construction project is shown in the figure. The durations (expressed in days) of the activities are mentioned below the arrows.



The critical duration for this construction project is **[2 Marks]**

(A) 13 days (B) 14 days

(C) 15 days (D) 16 days

2017 IIT Roorkee

28. The activity details of a project are given below

Activity	Depends on	Duration (in days)
P	-	6
Q	P	15
R	Q,T	12
S	R	16
T	P	10
U	Q, T	14
V	U	16

The estimated minimum time (in days) for the completion of the project will be _____. **[2 Marks]**

29. Given that the scope of the construction work is well defined with all its drawing, specifications, quantities and estimates, which one of the following types of contract would be most preferred?

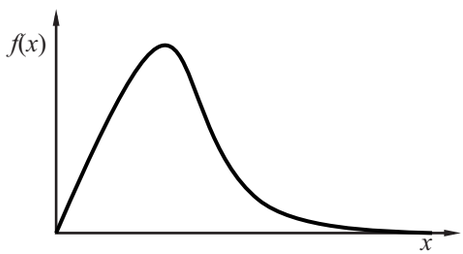
[1 Mark]

(A) EPC contract
(B) Percentage rate contract
(C) Item rate contract
(D) Lump sum contract

30. For a construction project, the mean and standard deviation of the completion time are 200 days and 6.1 days, respectively. Assume normal distribution and use the value of standard normal deviate, $z = 1.64$ for the 95% confidence level. The maximum time required (in days) for the completion of the project would be _____. **[1 Mark]**

2018 IIT Guwahati

31. A probability distribution with right skew is shown in figure **[1 Mark]**

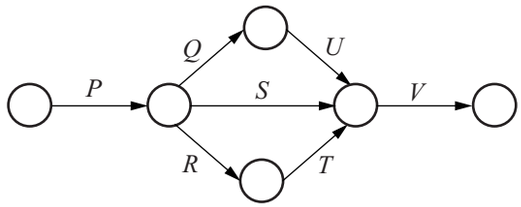


The correct statement for the probability distribution is

(A) Mean is equal to mode
(B) Mean is greater than median but less than mode
(C) Mean is greater than median and mode
(D) Mode is greater than median

2019 IIT Madras

32. The network of a small construction project awarded to a contractors is shown in the following figure. The normal duration, crash duration, normal cost and crash cost of all the activities are shown in the table. The indirect cost incurred by the contractor in INR 5000 per day.



Activity	Normal duration (Days)	Crash duration (Days)	Normal cost (INR)	Crash cost (INR)
P	6	4	15000	25000
Q	5	2	6000	12000
R	5	3	8000	9500
S	6	3	7000	10000
T	3	2	6000	9000
U	2	1	4000	6000
V	4	2	20000	28000

If the project is target for completion in 16 days, the total cost (in INR) to be incurred by the contractor would be _____. **[2 Marks]**

Answers Construction Materials & Management

1.	A	2.	B	3.	C	4.	B	5.	A
6.	A	7.	A	8.	D	9.	A	10.	B
11.	B	12.	A	13.	D	14.	B	15.	C
16.	D	17.	A	18.	D	19.	D	20.	C
21.	50.94	22.	D	23.	B	24.	D	25.	C
26.	37.84	27.	C	28.	51	29.	C	30.	210
31.	C	32.	149500						

Explanations Construction Materials & Management

1. (A)

The tensile strength of cement may be obtained by briquette test.

- A mixture of cement and sand is gauged in the proportion of 1 : 3 by weight.
- The percentage of water to be used is calculated by formula $\left(\frac{P}{5}+2.5\right)\%$

  Where, P = % of water required to produce a paste of standard consistency.
- The mix is filled in the briquette moulds and kept for 24 hours at temperature of $27^0 \pm 2^0 C$.

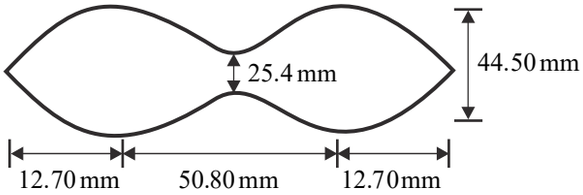


Fig. Standard briquette

- The briquettes $(6.45\,cm^2)$ are kept in clean water and are taken out before testing.
- Six briquettes are tested and average tensile strength is calculated
- Generally tensile strength is 10-25 % of compressive strength.

Hence, the correct option is (A).

Key Point

Various test	Apparatus
Consistency test	Vicat's apparatus
Setting time	Vicat apparatus
Workability	Slump cone test
Tensile strength of concrete	Split cylinder test

2. (B)

Normal consistency test :

- The standard (normal) consistency of a cement paste is defined as that consistency which will permit a vicat plunger having 10 mm diameter and 50 mm length penetrate a depth of 33 to 35 mm from the top (or 5 to 7 mm from the bottom) of the mould.
- Particular percentage of water which allows the plunger to penetrate 5 to 7 mm from bottom is known as the percentage of water required to produce a cement paste of normal consistency. This percentage is generally denoted by P.

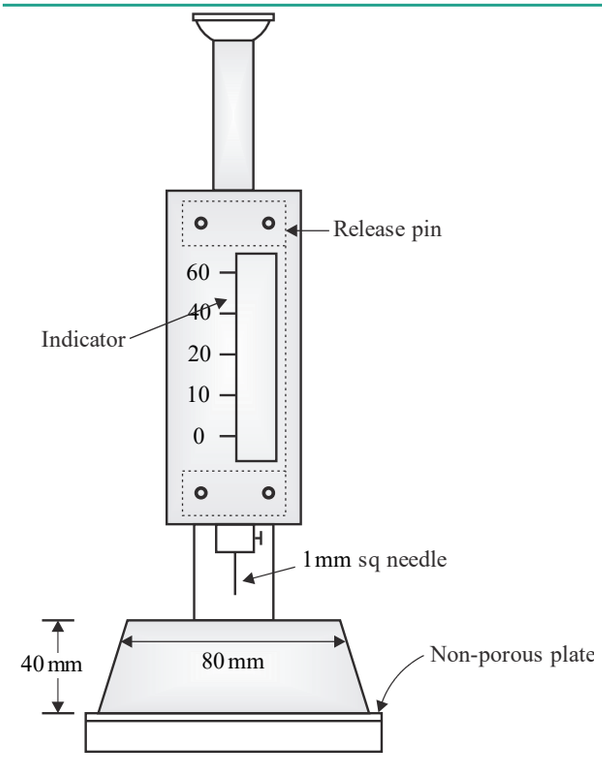


Fig. Vicat apparatus

Initial setting time :

- It is the time elapsed between the moment that the water is added to the cement, to the time that the paste starts loosing its plasticity.
- Take 300 gm of cement and make a cement paste of consistency 0.85 P
- A plunger of 10 mm diameter and 50 mm length which is used for initial setting time.

Final setting Time :

- It is the time elapsed between the moment the water is added to the cement and the time when the paste has completely lost its plasticity and has attained sufficient firmness to resist certain definite pressure.
- A needle with an annular coller of 5 mm diameter which is used for final setting time.

Hence, the correct option is (A).

Key Point

Various test
Slump test - Workability
Le-chatelier's test - Soundness test
Briquette test - Tensile strength of cement

3. (C)

The particle size distribution of an aggregate determine by sieve analysis is termed as grading of concrete. If all particles of an aggregate are of uniform size, the compacted mass will contain more voids, where as an aggregate comprising particles of various sizes will give a mass containing lesser voids. The particle distribution of a mass of aggregate should be such that the smaller particles fill the voids between the larger particles. Hence the basic aim of grading of aggregate in concrete mix is to reduce the voids.

Note : Actually higher density and reduction in voids these two terms are relative to each other. But the answer should be (C).

Hence, the correct option is (C).

4. (B)

Raw materials after heating for 2-3 hours in preheater, it is allowed to fed into "Rotary kiln".

- **Nodule zone :** In this zone calcination of limestone occurs and limestone get disintegrated into two parts.
- **Burning zone :** In this zone the ingredients of calcareous and argil aceous component get united with each other at a very high temperature and this process is called fusion.

Hence, the correct option is (B).

Key Point

Raw materials used in the manufacture of cement are classified into two broad categories.

Argillaceous	Calcareous
Shale and clay	Limestone
Cement rock	Chalk
Blast furnace slag	Marine shells

5. (A)

Workability is the amount of work to produce full compaction.

Following are the factors which affect the workability of concrete

(a) Water content

(b) Mix proportions

(c) Size of aggregate

(d) Shape of aggregate

(e) Surface texture of aggregates

(f) Grading of aggregates

(g) Use of admixtures

- Work ability of concrete is influenced by all given factors but it is most influenced by its water cement ratio.
- The compressive strength of hardened concrete is inversely proportional to the water cement ratio, provided that the mix is of workable consistency, this is so called Abram's law.
- A reduction in the water cement ratio generally results in an increased quality of concrete in terms of density, strength, impermeability reduced shrinkage and creep etc.

Hence, the correct option is (A).

6. (A)

- Workability of concrete –Vee-bee test
- Direct tensile strength of concrete-cylinder splitting test
- Bond between concrete and steel-pull out test
- Fineness of cement- surface area test

Hence, the correct option is (A).

Key Point

Test on cement		
A.	Fineness test	Sieve method
		Air permeability method (Nurse and Blaine method)
		Sedimentation method (Wanger turbidimeter method)
B.	Consistency test	Vicat's apparatus
C.	Setting time	Vicat's apparatus
D.	Soundness test	Le- Chatelier
		Auto clave test
E.	Tensile strength	Briquette test
F.	Heat of hydration	Calorimeter test
G.	Specific gravity test	Le-chatelier's Flask
H.	Compressive strength test	Compressive testing machine (UTM machine)

Test on concrete		
A.	Workability	Slump test
		Compacting factor test
		Vee-Bee consistometor test

B.	Compressive strength test	Compressive testing machine (UTM machine)
		Rebound hammer test
C.	Direct tensile strength of concrete	Cylinder splitting test
D.	Bond between concrete and steel	Pull out test
E.	Dynamic modulus of elasticity	Resonant frequency test

7. (A)

1. With increase in water cement ratio of concrete mix, void ratio will increase. Hence compressive strength of concrete will decrease compressive strength is maximum at an optimum water-cement ratio

2. Main function of water in concrete is hydration of cement for forming hardened material, and workability of concrete mixture.

3. Creep and shrinkage depend upon water cement ratio of concrete mixture.

Hence, the correct option is (A).

8. (D)

1. Modulus of elasticity,

$$E = 5000\sqrt{f_{ck}}$$

$$E \propto \sqrt{f_{ck}}$$

Where f_{ck} = Characteristic strength of concrete modulus of elasticity increases with compressive strength.

2. High strength concrete is more brittle as compared to low strength concrete

3. Shear strength of concrete $\tau_c \propto f_{ck}$

Hence, the correct option is (D).

9. (A)

(A) Resonant frequency test is used to find out dynamic modulus of elasticity whereas destructive test like cube test etc. measures the static modulus of elasticity.

(B) Rebound hammer test is a non-destructive test used to find out compressive strength of concrete.

(C) Split cylinder test is used to find out the direct tensile strength of concrete.

(D) Compaction factor test is used to measure the workability of concrete.

Hence, the correct option is (A).

10. (B)

In the compaction factor test, the degree of workability of concrete is measured in terms of internal energy required to compact the concrete thoroughly.

Compaction factor

$$= \frac{\text{Weight of partially compacted concrete}}{\text{Weight of fully compacted concrete}}$$

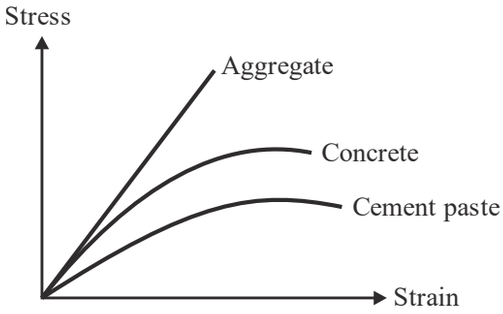
The maximum possible value of compaction factor for fresh concrete can be 1.

Hence, the correct option is (B).

Key Point

Degree of workability	Compacting factor
Very low	0.78
Low	0.85
Medium	0.92
High	0.95

11. (B)



Hence, the correct option is (B).

12. (A)

An air entraining agent introduces air in the form of bubbles that occupy upto 5 % of the volume of concrete distributed uniformly throughout cement paste. Thus for the same amount of water/cement ratio, we get higher workability.

An air entraining agent improves freezing and thowing resistance since cracks in the concrete during freezing and melting of water is avoided because water can take up the air voids present is concrete.

Hence, the correct option is (A).

13. (D)

Slump test : Slumps is the subsidence of the specimen recorded in mm.

The difference in levels between the height of the mould and that of the highest point of the subsided concrete is measured.

The difference in height in mm is taken as slump of concrete. As water content increases, slump increases.

Compaction factor test : In the compaction factor test, the degree of workability of concrete is measured in terms of internal energy required to compact the concrete thoroughly.

$$C.F. = \frac{\text{Weight of patially compacted concrete}}{\text{Weight of fully compacted concrete}}$$

Vee bee test : This test is preferred for finding workability of stiff concrete mix having very low workability. The time required for the shape of concrete fully assumes a cylindrical shape, in second is known as vee-bee degree.

- As the slump increases, the vee-bee time decreases

  Hence first statement false.

- As the slump increases, the compaction factor increases

Hence, the correct option is (D).

14. (B)

1. Tricalcium silicate (C_3S)
   - It is considered as very good strength compound it enables the clinker to grind easily.
   - It hydrates rapidly generates high heat and develop early hardness and strength.
   - The hydration of C_3S is mainly responsible for 7 days strength and hardness.
2. Dicalcium silicate (C_2S)
   - It hydrates and hardens slowly and takes long time to add to the strength. It is responsible for ultimate strength.
   - After 28 days the gain of strength is due to C_2S
3. Tricalcium Aluminate (C_3A)
   - It rapidly reacts with water and is responsible for flash set of finely grounded clinker.
   - It contributes in 24 hours strength after addition of water but it contribute less.

4. Tetra- calcium Aluminate ferrite (C_4AF)
   - It is also responsible for high heat of hydration as compare to C_2S and but less than C_3A.
   - Its contribution in strength is very less.

   The compound which is largely responsible for initial setting and early strength gain of OPC is C_3S.

Hence, the correct option is (B).

Key Point

Key Point

Composition of cement clinker

The principal mineral compounds	Name	Symbol	Percentage
$3CaO.SiO_2$	Alite	C_3S	30-50 %
$2CaO.SiO_2$	Belite	C_2S	20-45 %
$3CaO.A_2O_3$	Celite	C_3A	8-12 %
$4CaO.Al_2O_3.Fe_2O_3$	Felite	C_4AF	6-10 %

15. (C)

Kilns : A kiln is a large oven which is used to burn bricks. The kilns which are used in the manufacture of bricks are of the following two types :

(1) Intermittent kilns :
- (a) Intermittent up-drought kilns
- (b) Intermittent down-drought kilns

(2) Continuous kiln :
- (a) Bull's trench kiln
- (b) Hoffman's kiln
- (c) Tunnel kiln

Hence, the correct option is (C).

16. (D)

As per Indian standards for bricks, minimum acceptable compressive strength of any class of burnt clay bricks in dry state is 3.5 MPa.

Hence, the correct option is (D)

Key Point

Class of brick	Minimum crushing strength
First class	$\not< 10\ N/mm^2$
Second class	$\not< 7.5\ N/mm^2$

17. (A)

Le-Chatelier test	Soundness of OPC
Vee-Bee test	Consistency of workability of concrete
Blaine air permeability test	Fineness of OPC
Vicat apparatus	Consistency and setting time of OPC

Hence, the correct option is (A).

18. (D)

1. Walls of one brick thick are measured in square meters.
2. No deduction in the brickwork quantity is made for openings in walls up to $0.1 m^2$ area.
3. For the measurement of excavation from the borrowpit in a fairly uniform ground, deadman are left at suitable intervals.

Hence, the correct option is (D).

19. (D)

The soundness of cement is determined either by Le-Chatelier's method of by means of autoclave test.

Le-Chatelier's method detects unsoundness due to free lime only.

Hence, the correct option is (D).

20. (C)

The setting time of cement is determined using vicat apparatus.

Hence, the correct option is (C).

Key Point

Apparatus		Determine	
1.	Vicat apparatus	**(i)**	Consistency, Initial and final setting of cement
2.	Briquette testing apparatus	**(ii)**	Tensile strength of cement
3.	Le- chatelier apparatus	**(iii)**	Soundness of cement
4.	Slump cone apparatus	**(iv)**	Workability of concrete
5.	Permeability apparatus with manometer and flow meter	**(v)**	Fineness of cement
6.	Le-Chatelier flask	**(vi)**	Specific gravity of cement

21. 50.94

Volume of the box $= 0.5 \times 0.5 \times 0.5 = 0.125\,m^3$

Mass of aggregate $= 187.5\,kg$

Specific gravity of aggregate $(G_{agg}) = 2.5$

$$\text{Volume of aggregate} = \frac{187.5}{2.5 \times 1000} = 0.075\,m^3$$

$$\text{Volume of empty space} = 0.125 - 0.075 = 0.05\,m^3$$

Water absorption $= 0.5\%$

$$\text{Volume of water absorbed} = \frac{0.5}{100} \times \frac{187.5}{1000} = 9.375 \times 10^{-4}$$

Total volume of water that can be filled

$$= 9.375 \times 10^{-4} + 0.05 = 0.0509\,m^3$$

Mass of water $= 50.94\,kg$

Hence, the maximum quantity of water required to fill the box completely is **50.94 kg**.

22. (D)

Due to increase in C_2S heat of hydration decreases.

Hence, the correct option is (D).

Key Point

(i) Rate of hydration,

$C_4AF > C_3A > C_3S > C_2S$

(ii) Dicalcium silicate,

($2CaO \times SiO_2$ or C_2S)

- Last compound formed during hydration of cement.
- Responsible for progressive later stage strength.
- Structure requires later stage strength proportion of this component increase e.g. hydraulic structure, bridges.

23. (B)

- Abrasion resistance is determined by Los Angles test.
- The soundness of cement is determined either by 'Le Chatelier's method' or by means of a 'Auto clave test'.
- The specific gravity of cement is obtained by using 'Le Chatelier's' flask.

Hence, the correct option is (B).

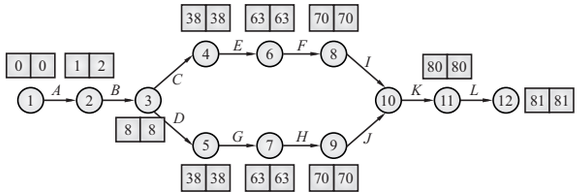
24. (D)

1. Soundness test is done to evaluate resistance against weathering of aggregate.
2. Crushing test is done to evaluate strength of aggregate.
3. Loss angles abrasion test is to evaluate hardness of aggregate.
4. Stripping value test is to evaluate ratio of uncovered area observed to the total area

of aggregate. The test is conducted to determine the effects of moisture upon the adhesion of binding material.

Hence, the correct option is (D).

25. (C)



Now, for activity 5 – 7,

Early Start Time $(EST) = 38$

Late Start Time $(LST) = 38$

Total float (F_T) = Late start time (LST)

–Early start time (EST)

$(F_T) = 38 - 38 = 0$

Similarly, for activity 11 – 12,

Early Start Time $(EST) = 80$

Late Start Time $(LST) = 80$

Total float (F_T) = Late start time (LST)

–Early start time (EST)

$(F_T) = 80 - 80 = 0$

Hence, the correct option is (C).

Key Point

Total float can be determined using following expressions/methods

1. Total float,
   (F_T) = Late Start Time (LST)
   –Early Start Time (EST)
2. Total float
   (F_T) = Late Finish Time (LFT)
   –Early Finish Time (EFT)
3. For any activity, if there is no margin left for the occurrence of events (the slack of all events are zero), the total float will be zero.

26. 37.84

Process	Optimistic time (O)	Most likely time (M)	Pessimistic time (P)
1. $(E-F)$	8	10	14
2. $(F-G)$	6	8	11
3. $(G-H)$	5	7	10
4. $(H-I)$	7	12	18

Now, the expected completion time is given by,

$$T = \frac{O + 4M + P}{6}$$

$$= \frac{8 + 4\times10 + 14}{6} + \frac{6 + 4\times8 + 11}{6} + \frac{5 + 4\times7 + 10}{6} + \frac{7 + 4\times12 + 18}{6}$$

$$= 10.33 + 8.17 + 7.17 + 12.17$$

$$T = 37.84 \text{ days}$$

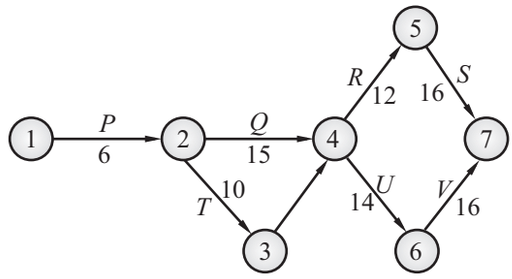
Scan for Video Solution

27. (C)

It can be observed that path *P-Q-T-W-X* consumes maximum amount of time, hence it is the critical path of the project.

Critical time $= 2 + 3 + 5 + 3 + 2 = 15$ days

28. 51



Time taken by the critical path is considered as the shortest possible time to finish the project as the delay critical activity tends to delay in overall project. Here, the time taken by the critical path (*P-Q-U-V*) is

$6 + 15 + 14 + 16 = 51$ days

29. (C)

In case, when the scope of the construction work is well defined with all its drawing, specifications, quantities and estimates, *Item rate contract* also called a *schedule contract/unit price contract* is mostly preferred.

The item rate contract is most commonly used for all types of engineering works financed by public or government bodies. This type of contract, is suitable for works which can be split into various items and quantities under each item can be estimated with accuracy.

Key Point

1. **EPC (Engineering Procurement Construction) contract :** The process of tendering, engineering and construction may have the detailed design of the project, procure all the equipment and materials necessary. The Front End Engineering Design phase (FEED) packages are used as the basis for bidding on when the client offers the EPC work to the market.
2. **Percentage rate contract :** It is similar to the item rate contract in almost all respects as the contract documents are the same as in item rate contract, making payments to the contractor, the engineer is similar. The method of tendering the unit rates are different in this case. While tendering, the contractors do not have to write the rate or estimated cost of each item, but a percentage figure by which the estimate unit rates are to be increased or decreased, the same percentage figure being applicable to all the items.
3. **Lumpsum concract :** In this type of contract, the contractor offers to do the whole work as shown in drawings and described by specifications, for a total stipulated sum of money. There are no individual rates quoted, thus it becomes difficult to make adjustments in the contract value if any changes are to be made in the work later on.

   A lump sum contract is more suitable for works for which contractors have prior construction experience. This experience enables the contractors to submit a more realistic bid. This type of contract is not suitable for difficult foundations, excavations of uncertain character and projects susceptible to unpredictable hazards and variations.

30. 210

Given : Mean deviation of the completion time $(T_M) = 200\,\text{days}$

Standard deviation of the completion time $(\sigma) = 6.1\,\text{days}$

Standard normal deviate $(z) = 1.64$

Let, the maximum time required for the completion of the project be $T_{\max}$

Now, we know that

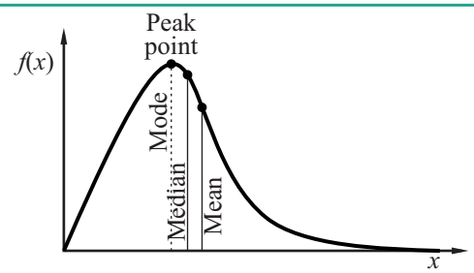
$$z = \frac{T_{\max} - T_M}{\sigma}$$

$$1.64 = \frac{T_{\max} - 200}{6.1}$$

$$T_{\max} = 1.64 \times 6.1 + 200 = 210\,\text{days}$$

31. (C)

The given case is of unsymmetrical distribution (positive skewed or right skewed distribution). In this case, mean is greater than median and mode (Mean > median > mode) as shown in figure.



Hence, the correct option is (C)

Key Point

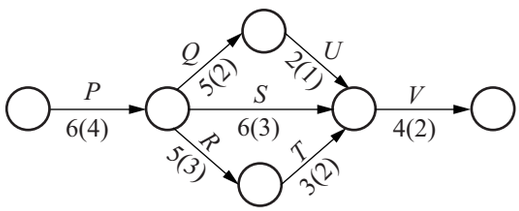
Mode : It is the most frequently occurring data value in a series of numbers. The terms mode, median and mean are used to identify the midpoint in a given skewed distribution data set. For the symmetric distribution, the mode, median and the mean is same i.e., the axis of symmetry.

Median : It is the mid data point in a data series organized in sequence.

Mean : It is the sum of all the data divided by the number of data sets.

32. 149500

Activity	t_n (days)	t_c (days)	C_n (INR)	C_c (INR)	Cost slope $C_s = \frac{C_c - C_n}{t_n - t_c}$
P	6	4	15000	25000	INR 5000/d
Q	5	2	6000	12000	INR 2000/d
R	5	3	8000	9500	INR 750/d
S	6	3	7000	10000	INR 1000/d
T	3	2	6000	9000	INR 3000/d
U	2	1	4000	6000	INR 2000/d
V	4	2	20000	28000	INR 4000/d

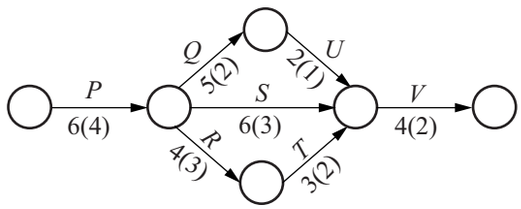


Considering normal durations, the total cost of the project

$$TC = DC + IC$$
$$= \text{INR } 66000 + \text{INR } 5000 \times 18$$
$$= \text{INR } 156000$$

Stage 1 : Crash activity R by 1 day

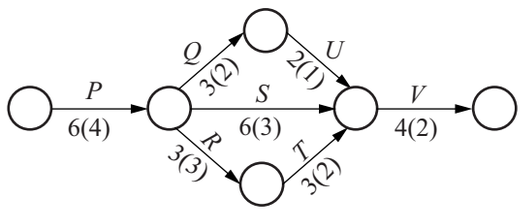
$\therefore$ Project becomes



$$TC = (\text{INR } 6600 + \text{INR } 750) + \text{INR } 5000 \times 17$$
$$= 151750$$

Stage 2 : Crash activity Q and R (or U and R) by 1 day

$\therefore$ Project becomes



$$TC = (\text{INR } 66750 + \text{INR } 2000 + \text{INR } 750) + (\text{INR } 5000 \times 16)$$
$$= 149500$$

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Space for Notes

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1 Fundamental Concepts

1990 IISc Bangalore

1.1 The required slope correction for a length of 60 m, along a gradient of 1 in 20 is **[1 Mark]**

(A) 7.50 cm (B) 0.750 cm

(C) 75.0 cm (D) 5.50 cm

1991 IIT Madras

1.2 The length of a base line measured on ground at an elevation of 300 metres above mean sea level is 2250 meters. The required correction to reduce to sea level length (given the radius of earth is 6370 km) is_____. **[1 Mark]**

1993 IIT Bombay

1.3 A line measured on a uniform slope with a 30 m tape pulled with a force of 5kg. The length of the line was found to be 525.80 m. The temperature at the time of measurement was 30^0C and the tape is correct at 24^0C when there is no pull. Determine the horizontal distance between the end points of the line. The difference of levels of the two ends of the line is 9.14m. The coefficient of thermal expansion of the material of the tape is 0.00001 per ^{0}C. Cross-sectional area of the tape is $2mm^2$ and its modulus of elasticity is $21\times10^5 kg/cm^2$. **[1 Mark]**

2007 IIT Kanpur

1.4 The plan of a map was photo copied to a reduced size such that a line originally 100 mm, measures 90 mm. The original scale of the plan was 1:1000. The revised scale is **[2 Marks]**

(A) 1 : 900 (B) 1 : 1111

(C) 1 : 1121 (D) 1 : 1221

2008 IISc Bangalore

1.5 The type of surveying in which the curvature of the Earth is taken into account is called **[1 Mark]**

(A) Geodetic surveying

(B) Plane surveying

(C) Preliminary surveying

(D) Topographical surveying

1.6 The plan of a survey plotted to a scale of 10 m to 1 cm is reduced in such a way that a line originally 10 cm long now measures 9 cm. The area of the reduced plan is measured as $81\,cm^2$. The actual area (m^2) of the survey is **[2 Marks]**

(A) 10000 (B) 6561

(C) 1000 (D) 656

2014 IIT Kharagpur

1.7 The survey carried out to delineate natural features, such as hills, rivers, forests and man-made features, such as towns, villages, buildings, roads, transmission lines and canals is classified as **[1 Mark]**

(A) Engineering survey

(B) Geological survey

(C) Land survey

(D) Topographic survey

2019 IIT Madras

1.8 A survey line was measured to be 285.5 m with a tape having a nominal length of the type was found to be 0.05 m too short. If the line lay on a slope of 1 in 10. The reduced length (horizontal length) of the line for plotting of survey work would be

(A) 285.6 mm (B) 284.5 mm

(C) 283.6 mm (D) 285.0 mm

Answers Fundamental Concepts

1.1	A	1.2	– 0.106	1.3	525.82	1.4	B	1.5	A
1.6	A	1.7	D	1.8	C				

Explanations Fundamental Concepts

1.1 (A)

Given : Gradiant = 1 in 20 (i.e. if we move 20 horizontal then we also move 1 verticle)

$20H:1V$

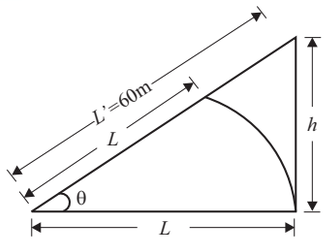
$3\times20H:3\times1V$

$60H:3V$

Measured Length $L' = 60\text{m}$

Correct Length = L m

Method 1



$$L'^2 = L^2 + h^2$$

$$L^2 = (L')^2 - h^2$$

$$L = \sqrt{(L')^2 - h^2}$$

Correction due to slope $C_{slope} = L' - L$

$$C_{slope} = L' - \sqrt{L'^2 - h^2}$$

$h = 3,\ L' = 60\text{m}$

$$C_{slope} = 60 - \sqrt{60^2 - 3^2}$$

$$= 0.0750\text{m or } 7.50\text{cm}$$

Hence, the correct option is (A).

Method 2

Angle of sloping ground, $\theta = \tan^{-1}\left(\frac{1}{20}\right)$

$$\theta = 2.862^0$$

Horizontal distance or correct length

$$= L' \times \cos\theta$$

$$= 60 \times \cos(2.862^0) = 59.925 \text{ m}$$

Correction due to slope,

$$C_{slope} = L' - L$$

$$= 60 - 59.925 = 0.0748 \text{ m}$$

or $C_{slope} = 7.48\text{cm}$

Hence, the correct option is (A).

Method 3

This is approximate method.

Correction due to slope,

$$C_{slope} = \frac{h^2}{2L'} = \frac{3^2}{2\times 60}$$

$$= 0.0750\text{m} = 7.50\text{cm}$$

Note : Slope correction is always negative.

Hence, the correct option is (A).

Scan for Video Solution

1.2 – 0.106

Given : Radius of earth R = 6370 kM

Altitude above mean sea level (MSL) h = 2250m

Measured length L = 300m

Mean sea level correction,

$$C_{MSL} = \frac{-hL}{R} = -\frac{2250\times 300}{6370\times 1000}$$

$$C_{MSL} = -0.106\,\text{m}$$

Note : Mean sea level correction (C_{MSL}) is always negative.

Hence, the required correction to reduce to sea level length is $-\mathbf{0.106\,m}$.

Scan for Video Solution

1.3 525.82

Given : Measured length of line $L' = 525.80$

Temperature at the time of measurement,

$T_m = 30^0\text{C}$ and $T_0 = 24^0\text{C}$

Pull applied at the time of measurement $P_m = 5\text{kg}$

Temperature correction,

$$C_{Temp} = L'\alpha(T_m - T_0)$$

$$= 525.80\times 0.00001\times(30-24)$$

$$= 0.0315 \text{ m}$$

Pull correction,

$$C_{pull} = \frac{(P_m - P_0)L'}{AE}$$

$$= \frac{(5-0)\times 525.80}{2\times 10^{-2}\times 21\times 10^5} = 0.0626\,\text{m}$$

Slope correction,

$$C_{slope} = -\frac{h^2}{2L'}$$

$$= -\frac{(9.14)^2}{2\times 525.80} = -0.0794\,\text{m}$$

Total correction,

$$C_{total} = C_{Temp} + C_{Pull} + (-C_{Slope})$$

$$= 0.0315 + 0.0626 - 0.0794$$

$$= +0.0147$$

Note :

Correction	Nature
C_{Temp}	Positive (due to temperature rise)
C_{Pull}	Positive (due to pulled with a force)
C_{Slope}	Negative (Slope correction is always subtractive)

Corrected length = Measured length ± Correction

$$= L' + C_{Total} = 525.80 + 0.0147$$

$$= 525.8147 \text{ m}$$

Hence, the horizontal distance between the end points of the line is **525.8147 m**.

1.4 (B)

Given : Original sale $= \frac{1}{1000}$

Shrunk length $= 90\,\text{mm}$

Original length $= 100\,\text{mm}$

$$\text{Shrunk factor} = \frac{\text{Shrunk length}}{\text{Original length}}$$

$$S.F. = \frac{90}{100} = 0.9$$

Shrunk scale or revised scale

$$= S.F. \times \text{Original length}$$

$$= 0.9 \times \frac{1}{1000} = \frac{1}{\frac{10,000}{9}} = \frac{1}{1111.11}$$

Hence, the correct option is (B).

Key Point

- Shrinkage ratio or shrunk factor $= \frac{\text{Shrunk length}}{\text{Actual length}}$
- $(\text{Shrinkage ratio})^2 = \frac{\text{Measured area}}{\text{Actual area}}$
- Correct length × Correct scale = (Measured scale) × Measured length

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1.5 (A)

Plane surveying : The type of surveying in which curvature of earth is not considered is called plane surveying.

Preliminary survey : This type of survey in which the soil property is examined by digging the ground upto some a certain depth is called preliminary survey.

Topographical survey : To show Natural features of the country such as rivers hills, Lake, forest and man made features such as town villages, building, road canal etc is called topographical survey.

Geodetic surveying : The type of surveying in which curvature of earth is taken into account.

Hence, the correct option is (A).

Scan for Video Solution

1.6 (A)

Given : Scale $= \frac{1}{1000}$, Reduced area $= 81\,\text{m}^2$

$$\text{Shrunk factor} = \frac{\text{Shrunk length}}{\text{Original length}}$$

$$S.F. = \frac{9\,\text{cm}}{10\,\text{cm}} = 0.9$$

$$\text{Actual area} = \frac{\text{Measured area}}{(S.F.)^2} = \frac{81\,\text{cm}^2}{(0.9)^2}$$

$$= 100\,\text{cm}^2$$

In original map 1 cm $= 10\,\text{m}$.

So, $1\,\text{cm}^2 = 100\,\text{m}^2$

$$100\,\text{cm}^2 = 100 \times 100\,\text{m}^2 = 10,000\,\text{m}^2$$

Hence, the correct option is (A).

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1.7 (D)

Engineering survey : To prepare detail drawing of project involving roads, railways etc. and to determine the quantities on datas for designing of engineering works.

Land survey : It is a survey being done on land. It involves running of survey lines and determining their length.

Geological survey : The type of survey in which, information about both the surface and subsurface is acquired.

Hence, the correct option is (D).

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1.8 (C)

Given :

Length of measured survey line $(L') = 285.5$

Nominal length of of tape = 30 m

Gradient = 1 in 10

Actual tape length = 30 – 0.05 = 29.95 m

Standardization correction :

Actual tape length < Nominal length of tape

$$29.95\text{m} < 30\text{m}$$

Correction per tape length = 29.95 – 30

$$= -0.05\text{m}$$

Standardization correction,

$$C_s = \frac{\text{Measured length}}{\text{Nominal length of tape}} \times \text{Correction}$$

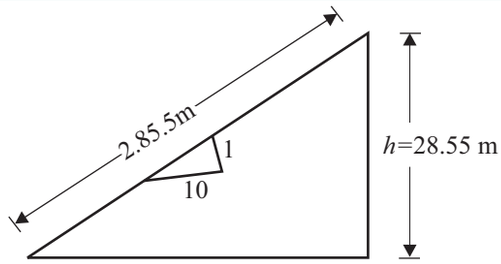
$$= \frac{285.5}{30} \times 0.05 = 0.4758\,\text{m} \quad \text{(negative)}$$

Slope correction :

Slope correction,

$$C_{slope} = -\frac{h^2}{2L'} = \frac{28.55^2}{2 \times 285.5}$$

$$= 1.4275 \quad \text{(negative)}$$



Total correction,

$$C_{Total} = C_S + C_{Slope}$$

$$= -0.4758 - 1.4275$$

$$= -(0.4758 + 1.4275) = -1.9033$$

Correct length L = Measured length – Correction

Correct length $L = 285.5 - 1.9033 = 283.596\,\text{m}$

Hence the correct length (L) of measured survey line is **283.596 m**.

Key Point

Correction due to slope always negative.

Correction for standardization.

- Length of tape too long positive.
- Length of tape too short negative.

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2 Differential Equations

1993 IIT Bombay

2.1 The differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}+\sin y=0 \text{ is} \quad \textbf{[1 Mark]}$$

(A) Linear (B) Non-linear

(C) Homogeneous (D) of degree two

1994 IIT Kharagpur

2.2 The differential equation

$$\frac{d^4y}{dx^4}+P\frac{d^2y}{dx^2}+ky=0 \text{ is} \quad \textbf{[1 Mark]}$$

(A) linear of fourth order.

(B) non-linear of fourth order.

(C) non-homogeneous.

(D) linear and fourth degree.

2.3 The necessary and sufficient condition for the differential equation of the form $M(x,y)\,dx+N(x,y)\,dy=0$ to be exact is **[1 Mark]**

(A) $M=N$ (B) $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y}$

(C) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ (D) $\frac{\partial^2 M}{\partial x^2}=\frac{\partial^2 N}{\partial y^2}$

1995 IIT Kanpur

2.4 The differential equation

$$y''+(x^3+\sin x)^5\,y'+y=\cos x^3 \text{ is}$$

[1 Mark]

(A) homogeneous.

(B) non-linear.

(C) second order linear.

(D) non-homogeneous with constant coefficient.

1997 IIT Madras

2.5 For the differential equation,

$$f(x,y)\frac{dy}{dx}+g(x,y)=0 \text{ to be exact}$$

[1 Mark]

(A) $\frac{\partial f}{\partial y}=\frac{\partial g}{\partial x}$ (B) $\frac{\partial f}{\partial x}=\frac{\partial g}{\partial y}$

(C) $f=g$ (D) $\frac{\partial^2 f}{\partial x^2}=\frac{\partial^2 g}{\partial y^2}$

2.6 The differential equation $\frac{dy}{dx}+Py=Q$, is a linear equation of first order only if **[1 Mark]**

(A) P is a constant but Q is a function of y.

(B) P and Q are functions of y (or) constants.

(C) P is a function of y but Q is a constant.

(D) P and Q are functions of x (or) constants.

1998 IIT Delhi

2.7 The solution of differential equation

$$\frac{d^4y}{dx^4} - y = 15\cos 2x \text{ is}$$

(A) $C_1e^x + C_2e^{-x} + C_3\cos x + C_4\sin x + \cos 2x$

(B) $C_1e^{2x} + C_2e^x + C_3\cos x + C_4\sin x + \cos x$

(C) $C_1e^{2x} + C_2e^{-2x} + C_3\cos x + C_4\sin x + \cos 2x$

(D) $C_1e^{-2x} + C_2e^{-x} + C_3\cos x + C_4\sin x + \cos x$

1999 IIT Bombay

2.8 If c is a constant, then solution of the equation $\frac{dy}{dx} = 1 + y^2$ is **[1 Mark]**

(A) $y = \sin(x+c)$ (B) $y = \cos(x+c)$

(C) $y = \tan(x+c)$ (D) $y = e^x + c$

2000 IIT Kharagpur

2.9 If $f(x,y,z) = (x^2+y^2+z^2)^{\frac{-1}{2}}$,

$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ is equal to

[1 Mark]

(A) Zero

(B) 1

(C) 2

(D) $-3\left(x^2+y^2+z^2\right)^{\frac{5}{2}}$

2001 IIT Kanpur

2.10 The solution for the following differential equation with boundary conditions $y(0) = 2$ and $y'(1) = -3$ is, where $\frac{d^2y}{dx^2} = 3x - 2$. **[2 Marks]**

(A) $y = \frac{x^3}{3} - \frac{x^2}{2} + 3x - 6$

(B) $y = 3x^3 - \frac{x^2}{2} - 5x + 2$

(C) $y = \frac{x^3}{2} - x^2 - \frac{5x}{2} + 2$

(D) $y = x^3 - \frac{x^2}{2} + 5x + \frac{3}{2}$

2.11 The number of boundary conditions required to solve the differential equation $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$ is **[1 Mark]**

(A) 2 (B) 0

(C) 4 (D) 1

2004 IIT Delhi

2.12 Biotransformation of an organic compound having concentration (x) can be modeled using an ordinary differential equation $\frac{dx}{dt} + kx^2 = 0$, where k is the reaction rate constant. If $x = a$ at $t = 0$, the solution of the equation is **[2 Marks]**

(A) $x = ae^{-kt}$ (B) $\frac{1}{x} = \frac{1}{a} + kt$

(C) $x = a(1-e^{-kt})$ (D) $x = a + kt$

2005 IIT Bombay

2.13 Transformation of linear form by substituting $v = y^{1-n}$ of the equation $\frac{dy}{dt} + p(t)y = q(t)\,y^n; n > 0$ will be

[2 Marks]

(A) $\frac{dv}{dt} + (1-n)pv = (1-n)q$

(B) $\frac{dv}{dt} + (1-n)pv = (1+n)q$

(C) $\frac{dv}{dt}+(1+n)pv=(1-n)q$

(D) $\frac{dv}{dt}+(1+n)pv=(1+n)q$

2.14 The solution of $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+17y=0$; $y(0)=1$, $\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}}=0$ in the range $0<x<\frac{\pi}{4}$ is given by **[2 Marks]**

(A) $e^{-x}\left(\cos 4x+\frac{1}{4}\sin 4x\right)$

(B) $e^{x}\left(\cos 4x-\frac{1}{4}\sin 4x\right)$

(C) $e^{-4x}\left(\cos 4x-\frac{1}{4}\sin x\right)$

(D) $e^{-4x}\left(\cos 4x-\frac{1}{4}\sin 4x\right)$

2006 IIT Kharagpur

2.15 The solution of the differential equation $x^2\frac{dy}{dx}+2xy-x+1=0$, given that at $x=1$, $y=0$ is **[2 Marks]**

(A) $\frac{1}{2}-\frac{1}{x}+\frac{1}{2x^2}$ (B) $\frac{1}{2}-\frac{1}{x}-\frac{1}{2x^2}$

(C) $\frac{1}{2}+\frac{1}{x}+\frac{1}{2x^2}$ (D) $-\frac{1}{2}+\frac{1}{x}+\frac{1}{2x^2}$

2007 IIT Kanpur

2.16 The degree of the differential equation $\frac{d^2x}{dt^2}+2x^3=0$ is **[1 Mark]**

(A) 0 (B) 1

(C) 2 (D) 3

2.17 The solution for the differential equation $\frac{dy}{dx}=x^2y$ with condition that $y=1$ at $x=0$ is **[1 Mark]**

(A) $y=e^{\frac{1}{2x}}$ (B) $\ln(y)=\frac{x^3}{3}+4$

(C) $\ln(y)=\frac{x^2}{2}$ (D) $y=e^{\frac{x^3}{3}}$

2008 IISc Bangalore

2.18 Solution of $\frac{dy}{dx}=-\frac{x}{y}$ at $x=1$ and $y=\sqrt{3}$ is **[2 Marks]**

(A) $x-y^2=2$ (B) $x+y^2=4$

(C) $x^2-y^2=-2$ (D) $x^2+y^2=4$

2.19 The equation $k_x\frac{\partial^2 h}{\partial x^2}+k_z\frac{\partial^2 h}{\partial z^2}=0$ can be transformed to $\frac{\partial^2 h}{\partial x_t^2}+\frac{\partial^2 h}{\partial z^2}=0$ by substituting **[2 Marks]**

(A) $x_t=x\frac{k_z}{k_x}$ (B) $x_t=x\frac{k_x}{k_z}$

(C) $x_t=x\sqrt{\frac{k_x}{k_z}}$ (D) $x_t=x\sqrt{\frac{k_z}{k_x}}$

2.20 The general solution of $\frac{d^2y}{dx^2}+y=0$ is **[1 Mark]**

(A) $y=P\cos x+Q\sin x$

(B) $y=P\cos x$

(C) $y=P\sin x$

(D) $y=P\sin 2x$

2009 IIT Roorkee

2.21 Solution of the differential equation $3y\frac{dy}{dx}+2x=0$ represents a family of **[2 Marks]**

(A) ellipses (B) circles
(C) parabolas (D) hyperbolas

2010 IIT Guwahati

2.22 The partial differential equation that can be formed from $z = ax + by + ab$ has the form $\left(\text{with } p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}\right)$.

[2 Marks]

(A) $z = px + qy$ (B) $z = px + pq$
(C) $z = px + qy + pq$ (D) $z = qy + pq$

2.23 The solution to the ordinary differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ is

[2 Marks]

(A) $y = c_1e^{3x} + c_2e^{-2x}$
(B) $y = c_1e^{3x} + c_2e^{2x}$
(C) $y = c_1e^{-3x} + c_2e^{2x}$
(D) $y = c_1e^{-3x} + c_2e^{-2x}$

2.24 The order and degree of the differential equation $\frac{d^3y}{dx^3} + 4\sqrt{\left(\frac{dy}{dx}\right)^3 + y^2} = 0$ are respectively **[1 Mark]**

(A) 3 and 2 (B) 2 and 3
(C) 3 and 3 (D) 3 and 1

2011 IIT Madras

2.25 The solution of the ordinary differential equation $\frac{dy}{dx} + 2y = 0$ for the boundary condition, $y = 5$ at $x = 1$ is **[2 Marks]**

(A) $y = e^{-2x}$ (B) $y = 2e^{-2x}$
(C) $y = 10.95e^{-2x}$ (D) $y = 36.95e^{-2x}$

2.26 The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x$, with the condition that $y = 1$ at $x = 1$, is **[2 Marks]**

(A) $y = \frac{2}{3x^2} + \frac{x}{3}$ (B) $y = \frac{x}{2} + \frac{1}{2x}$
(C) $y = \frac{2}{3} + \frac{x}{3}$ (D) $y = \frac{2}{3x} + \frac{x^2}{3}$

2013 IIT Bombay

2.27 Laplace equation for water flow in soil is given by $\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = 0$.

Head H does not vary in y and z-direction. Boundary condition

$$x = 0,\ H = 5\,\text{m},\ \frac{dH}{dx} = -1.$$

What is value of H (in m) at $x = 1.2$?

2014 IIT Kharagpur

2.28 The integrating factor for the differential equation $\frac{dP}{dt} + k_2P = k_1L_0e^{-k_1t}$ is

[1 Mark]

(A) e^{-k_1t} (B) e^{-k_2t}
(C) e^{k_1t} (D) e^{k_2t}

2.29 Water is flowing at a steady rate through a homogenous and saturated horizontal soil strip of 10 m length. The strip is being subjected to a constant water head (H) of 5 m at the beginning and 1 m at the end. If the governing equations of flow in the soil strip is $\frac{d^2H}{dx^2} = 0$ (where, x is the distance along the soil strip), the value of H (in m) at the middle of the strip is_____.

2015 IIT Kanpur

2.30 Consider the following second order linear differential equation **[2 Marks]**

$$\frac{d^2y}{dx^2} = -12x^2 + 24x - 20$$

The boundary conditions are :

$x = 0, y = 5$ and at $x = 2, y = 21$.

The value of y at $x = 1$ is_____.

2.31 Consider the following difference equation

$$x(y\,dx + x\,dy)\cos\frac{y}{x} = y(x\,dy - y\,dx)\sin\frac{y}{x}$$

Which of the following is the solution of the above equation (c is an arbitrary constant)? **[2 Marks]**

(A) $\frac{x}{y}\cos\frac{y}{x} = C$ (B) $\frac{x}{y}\sin\frac{y}{x} = C$

(C) $xy\cos\frac{y}{x} = C$ (D) $xy\sin\frac{y}{x} = C$

2016 IISc Bangalore

2.32 The type of partial differential equation $\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + 3\frac{\partial^2 P}{\partial x \partial y} + 2\frac{\partial P}{\partial x} - \frac{\partial P}{\partial y} = 0$ is **[1 Mark]**

(A) Elliptic (B) Parabolic

(C) Hyperbolic (D) None of these

2.33 The respective expressions for complementary function and particular integral part of the solution of the differential equation **[2 Marks]**

$\frac{d^4y}{dx^4} + 3\frac{d^2y}{dx^2} = 108x^2$ are

(A) $[c_1 + c_2x + c_3\sin\sqrt{3}x + c_4\cos\sqrt{3}x]$ and $[3x^4 - 12x^2 + c]$

(B) $[c_2x + c_3\sin\sqrt{3}x + c_4\cos\sqrt{3}x]$ and $[5x^4 - 12x^2 + c]$

(C) $[c_1 + c_3\sin\sqrt{3}x + c_4\cos\sqrt{3}x]$ and $[3x^4 - 12x^2 + c]$

(D) $[c_1 + c_2x + c_3\sin\sqrt{3}x + c_4\cos\sqrt{3}x]$ and $[5x^4 - 12x^2 + c]$

2.34 The solution of the partial differential equation $\frac{\partial u}{\partial t} = \alpha\frac{\partial^2 u}{\partial x^2}$ is of the form **[1 Mark]**

(A) $C\cos(kt)\left[C_1e^{(\sqrt{k/\alpha})x} + C_2e^{-(\sqrt{k/\alpha})x}\right]$

(B) $Ce^{kt}\left[C_1e^{(\sqrt{k/\alpha})x} + C_2e^{-(\sqrt{k/\alpha})x}\right]$

(C) $Ce^{kt}\left[C_1\cos(\sqrt{k/\alpha})x + C_2\sin(-\sqrt{k/\alpha})x\right]$

(D) $C\sin(kt)\left[C_1\cos(\sqrt{k/\alpha})x + C_2\sin(-\sqrt{k/\alpha})x\right]$

2017 IIT Roorkee

2.35 Consider the following partial differential equation

$$3\frac{\partial^2\phi}{\partial x^2} + B\frac{\partial^2\phi}{\partial x\partial y} + 3\frac{\partial^2\phi}{\partial y^2} + 4\phi = 0$$

For this equation to be classified as parabolic, the value of B^2 must be _____. **[1 Mark]**

2.36 The solution of the equation $\frac{dQ}{dt} + Q = 1$ with $Q = 0$ at $t = 0$ is **[2 Marks]**

(A) $Q(t) = e^{-t} - 1$ (B) $Q(t) = 1 + e^{-t}$

(C) $Q(t) = 1 - e^{t}$ (D) $Q(t) = 1 - e^{-t}$

2.37 Consider the following second-order differential equation

$$y'' - 4y' + 3y = 2t - 3t^2$$

The particular solution of the differential equation is **[2 Marks]**

(A) $-2 - 2t - t^2$ (B) $-2t - t^2$

(C) $2t - 3t^2$ (D) $-2 - 2t - 3t^2$

2.38 The tangent to the curve represented by $y = x\ln x$ is required to have 45^0

inclination with the x-axis. The coordinates of the tangent point would be **[2 Marks]**

(A) $(1,0)$ (B) $(0,1)$

(C) $(1,1)$ (D) $(\sqrt{2},\sqrt{2})$

2018 IIT Guwahati

2.39 Variation of water depth (y) in a gradually varied open channel flow is given by the first order differential equation

$$\frac{dy}{dx}=\frac{1-e^{-\frac{10}{3}\ln(y)}}{250-45e^{-3\ln(y)}}$$

Given initial conditions : $y(x=0)=0.8$ m. The depth (in m, up to three decimal places) of flow at a downstream section at $x=1$m from one calculation step of Single Step Euler Method is _____.

2.40 The solution (up to three decimal places) at $x=1$ of the differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$ subject to boundary conditions $y(0)=1$ and $\frac{dy}{dx}(0)=-1$ is _____. **[2 Marks]**

2.41 The solution at $x=1, t=1$ of the partial differential equation, $\frac{\partial^2 u}{\partial x^2}=25\frac{\partial^2 u}{dt^2}$ subject to initial condition of $u(0)=3x, \frac{\partial u}{\partial t}(0)=3$ is, **[2 Marks]**

(A) 1 (B) 2

(C) 4 (D) 6

2.42 The solution of the equation $x\frac{dy}{dx}+y=0$ passing through the point (1, 1) is **[1 Mark]**

(A) x (B) x^2

(C) x^{-1} (D) x^{-2}

2019 IIT Madras

2.43 Consider a two dimensional flow through isotropic soil along x-direction and z-direction. If h is the hydraulic head, the Laplace's equation of continuity is expressed as

(A) $\frac{\partial h}{\partial x}+\frac{\partial h}{\partial x}\frac{\partial h}{\partial z}+\frac{\partial h}{\partial z}=0$

(B) $\frac{\partial h}{\partial x}+\frac{\partial h}{\partial z}=0$

(C) $\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial x \partial z}+\frac{\partial^2 h}{\partial z^2}=0$

(D) $\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial z^2}=0$

2.44 Consider the ordinary differential equation $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$. Given the values of $y(1)=0$ and $y(2)=2$, the value of $y(3)$ (round off to 1 decimal place), is _____. **[2 Marks]**

2.45 An ordinary differential equation is given below $\left(\frac{dy}{dx}\right)(x\ln x)=y$. The solution for the above equation is **[2 Marks]**

(**Note :** K denotes a constant in the options)

(A) $y=Kxe^{x}$ (B) $y=Kxe^{-x}$

(C) $y=K\ln x$ (D) $y=Kx\ln x$

2.46 Euclidean norm (length) of the vector $[4 \quad -2 \quad -6]^T$ is **[1 Mark]**

(A) $\sqrt{48}$ (B) $\sqrt{56}$

(C) $\sqrt{24}$ (D) $\sqrt{12}$

2.47 A one-dimensional domain is discretized into N sub-domain of width Δx with node numbers $i = 0,1,2,3,.....,N$. If the time scale is discretized in steps of Δt, the forward-time and centred-space finite difference approximation at i^{th} node and n^{th} time step, for the partial differential equation $\frac{\partial V}{\partial t} = \beta \frac{\partial^2 V}{\partial x^2}$ is **[2 Marks]**

(A) $\frac{V_{i+1}^{(n+1)} - V_i^{(n)}}{\Delta t} = \beta \left[\frac{V_{i+1}^{(n)} - 2V_i^{(n)} + V_{i-1}^{(n)}}{2\Delta x} \right]$

(B) $\frac{V_i^{(n+1)} - V_i^{(n)}}{\Delta t} = \beta \left[\frac{V_{i+1}^{(n)} - 2V_i^{(n)} + V_{i-1}^{(n)}}{(\Delta x)^2} \right]$

(C) $\frac{V_i^{(n)} - V_i^{(n-1)}}{\Delta t} = \beta \left[\frac{V_{i+1}^{(n)} - 2V_i^{(n)} + V_{i-1}^{(n)}}{(\Delta x)^2} \right]$

(D) $\frac{V_i^{(n)} - V_i^{(n-1)}}{2\Delta t} = \beta \left[\frac{V_{i+1}^{(n)} - 2V_i^{(n)} + V_{i-1}^{(n)}}{2\Delta x} \right]$

2020 IIT Delhi

2.48 In the following partial differential equation, θ is a function of t and z, and D and K are functions of θ

$$D(\theta)\frac{\partial^2 \theta}{\partial z^2} + \frac{\partial K(\theta)}{\partial z} - \frac{\partial \theta}{\partial t} = 0$$

The above equation is **[1 Mark]**

(A) a second order non-linear equation

(B) a second degree linear equation

(C) a second degree non-linear equation

(D) a second order linear equation

2.49 For the ordinary differential equation $\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x = 0$, with initial conditions $x(0) = 0$ and $\frac{dx}{dt}(0) = 10$, the solution is **[2 Marks]**

(A) $10e^{2t} + 10e^{3t}$

(B) $5e^{2t} + 6e^{3t}$

(C) $-10e^{2t} + 10e^{3t}$

(D) $-5e^{2t} + 6e^{3t}$

2.50 The following partial differential equation is defined for $u : u(x, y)$

$$\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial x^2};\ y \geq 0;\ x_1 \leq x \leq x_2$$

The set of auxiliary conditions necessary to solve the equation uniquely, is **[1 Mark]**

(A) two initial conditions and one boundary condition

(B) three initial conditions

(C) three boundary conditions

(D) one initial condition and two boundary conditions

2.51 The ordinary differential equation $\frac{d^2u}{dx^2} - 2x^2u + \sin x = 0$ is **[1 Mark]**

(A) linear and homogeneous

(B) linear and non-homogeneous

(C) non-linear and non-homogeneous

(D) non-linear and homogeneous

2.52 An ordinary differential equation is given below

$$6\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$

The general solution of the above equation (with constants C_1 and C_2), is **[2 Marks]**

(A) $y(x) = C_1 e^{\frac{x}{3}} + C_2 e^{-\frac{x}{2}}$

(B) $y(x) = C_1 x e^{-\frac{x}{3}} + C_2 e^{\frac{x}{2}}$

(C) $y(x) = C_1 e^{-\frac{x}{3}} + C_2 e^{\frac{x}{2}}$

(D) $y(x) = C_1 e^{-\frac{x}{3}} + C_2 x e^{\frac{x}{2}}$

Answers Differential Equations

2.1	B & C	2.2	A	2.3	C	2.4	C	2.5	B
2.6	D	2.7	A	2.8	C	2.9	A	2.10	C
2.11	C	2.12	B	2.13	A	2.14	A	2.15	A
2.16	B	2.17	D	2.18	D	2.19	D	2.20	A
2.21	A	2.22	C	2.23	C	2.24	A	2.25	D
2.26	D	2.27	3.8	2.28	D	2.29	3	2.30	18
2.31	C	2.32	C	2.33	A	2.34	B	2.35	36
2.36	D	2.37	A	2.38	A	2.39	0.793	2.40	0.368
2.41	D	2.42	C	2.43	D	2.44	6	2.45	C
2.46	B	2.47	B	2.48	A	2.49	C	2.50	D
2.51	B	2.52	A						

Explanations Differential Equations

2.1 (B) & (C)

Given : $\frac{d^2y}{dx^2}+\frac{dy}{dx}+\sin y=0$

A differential equation is said to be linear if the dependent variable and its derivatives appear in first degree.

Since, expansion of $\sin y$ contains the power of y higher than 1 hence, it is not linear in y.

So, the given equation is a non-linear differential equation.

The given differential equation is also homogeneous equation.

Hence, the correct options are (B) and (C).

2.2 (A)

The general form of a n^{th} order linear differential equation is

$$\frac{d^ny}{dx^n}+P_1\frac{d^{n-1}y}{dx^{n-1}}+P_2\frac{d^{n-2}y}{dx^{n-2}}+....$$
$$+P_{n-1}\frac{dy}{dx}+P_ny=Q(x)$$

The given differential equation is a linear differential equation of fourth order. In the above equation $\frac{d^4y}{dx^4}$ is the highest order derivative.

Order = 4

Hence, the correct option is (A).

2.3 (C)

Given : $M(x,y)\,dx+N(x,y)\,dy=0$

The necessary and sufficient condition for the above differential equation to be exact is,

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

Hence, the correct option is (C).

2.4 (C)

Given : $y''+\left(x^3\sin x^5\right)y'+y=\cos x^3$

It is observed that,

(i) There are no partial derivatives.

(ii) All the derivatives are of first degree.

(iii) The constant terms are non-zero, and not a function of the dependent variable 'y'.

So, this is a non-homogenous second order ordinary linear differential equation.

Hence, the correct option is (C).

2.5 (B)

Given : $f(x,y)\frac{dy}{dx}+g(x,y)=0$

$$f(x,y)dy+g(x,y)dx=0 \quad \ldots(i)$$

Comparing with normal differential equation,

$$Mdx+Ndy=0$$

For exact differential equation,

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

$$\frac{\partial g(x,y)}{\partial y}=\frac{\partial f(x,y)}{\partial x}$$

$$\frac{\partial g}{\partial y}=\frac{\partial f}{\partial x}$$

Hence, the correct option is (B).

2.6 (D)

A differential equation of the form $\frac{dy}{dx}+Py=Q$ is called Leibniz linear differential equation where P and Q are function of x (but not y) or constants.

Hence, the correct option is (D).

2.7 (A)

Given : $\frac{d^4y}{dx^4}-y=15\cos 2x$

$$\left(D^4-1\right)y=15\cos 2x$$

This is in the form of a non-homogenous linear differential equation.

$$[f(D)]y=\phi$$

The auxiliary equation is given by,

$$f(m)=0$$

$$m^4-1=0$$

$$\left(m^2-1\right)\left(m^2+1\right)=0$$

$$m=\pm 1,\ \pm i$$

$$m_1=1,\ m_2=-1,\ m_3=i,\ m_4=-i$$

The roots are real and distinct, and a pair of complex number $(a\pm ib)$.

The complementary function is given by,

$$\text{C.F.}=C_1e^{1x}+C_2e^{-1x}+e^{0x}\left[C_3\cos x+C_4\sin x\right]$$

$$\text{C.F.}=C_1e^{x}+C_2e^{-x}+C_3\cos x+C_4\sin x$$

The particular integral is given by,

$$\text{P.I.}=\frac{1}{f(D)}\times 15\cos 2x$$

$$\text{P.I.}=\left(\frac{1}{D^4-1}\right)15\cos 2x \qquad \left[D^2=-4, D^4=16\right]$$

$$\text{P.I.}=15\times\left(\frac{1}{16-1}\right)\cos 2x=\cos 2x$$

The complete solution is given by,

$$y=\text{C.F.}+\text{P.I.}$$

$$y=C_1e^{x}+C_2e^{-x}+C_3\cos x+C_4\sin x+\cos 2x$$

Hence, the correct option is (A).

2.8 (C)

Given : $\frac{dy}{dx}=1+y^2$

$$\frac{dy}{1+y^2}=dx$$

Integrating both sides,

$$\int\frac{dy}{1+y^2}=\int dx$$

$$\tan^{-1}y=x+c$$

$$y=\tan(x+c)$$

Hence, the correct option is (C).

2.9 (A)

Given : $f(x,y,z)=(x^2+y^2+z^2)^{\frac{-1}{2}}$

$$\frac{\partial f}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{\frac{-3}{2}} \times 2x$$

$$\frac{\partial f}{\partial x} = -x(x^2+y^2+z^2)^{\frac{-3}{2}}$$

and $$\frac{\partial^2 f}{\partial x^2} = \left[-1\times(x^2+y^2+z^2)^{\frac{-3}{2}}\right]$$

$$-\left[x\times\left(\frac{-3}{2}\right)\times(x^2+y^2+z^2)^{\frac{-5}{2}}\times 2x\right]$$

$$\frac{\partial^2 f}{\partial x^2} = -(x^2+y^2+z^2)^{\frac{-5}{2}}(x^2+y^2+z^2-3x^2)$$

$$\frac{\partial^2 f}{\partial x^2} = (x^2+y^2+z^2)^{\frac{-5}{2}}(2x^2-y^2-z^2) \quad \ldots(i)$$

Similarly,

$$\frac{\partial^2 f}{\partial y^2} = (x^2+y^2+z^2)^{\frac{-5}{2}}(-x^2+2y^2-z^2) \quad \ldots(ii)$$

and $$\frac{\partial^2 f}{\partial z^2} = (x^2+y^2+z^2)^{\frac{-5}{2}}(-x^2-y^2+2z^2) \quad \ldots(iii)$$

From equation (i), (ii) and (iii),

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2} = (x^2+y^2+z^2)^{\frac{-5}{2}}\times(0)$$

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2} = 0$$

Hence, the correct option is (A).

2.10 (C)

Given : $\frac{d^2y}{dx^2} = 3x-2;\ y(0)=2,\ y'(1)=-3$

Integrating on both sides,

$$\frac{dy}{dx} = \frac{3x^2}{2}-2x+C$$

Again integrating on both sides,

$$y = \frac{3}{2}\frac{x^3}{3}-\frac{2x^2}{2}+Cx+D$$

$$y = \frac{x^3}{2}-x^2+Cx+D \quad \ldots(i)$$

Using boundary conditions :

(i) When $x=0, y=2$

$$2 = \frac{0}{2}-0+C\times 0+D \Rightarrow D=2$$

and $y' = \frac{3x^2}{2}-2x+C \quad \ldots(ii)$

(ii) When $x=1,\ y'=-3$

$$-3 = \frac{3}{2}-2+C \Rightarrow C=\frac{-5}{2}$$

Put the values of C and D in equation (i)

$$y = \frac{x^3}{2}-x^2-\frac{5}{2}x+2$$

Hence, the correct option is (C).

2.11 (C)

The general solution of Laplace equation contains four arbitrary constant. Therefore we require four boundary conditions.

Hence, the correct option is (C).

2.12 (B)

Given : $\frac{dx}{dt}+kx^2=0$

$$\frac{dx}{dt} = -k\,x^2$$

$$\frac{dx}{x^2} = -k\,dt$$

Integrating on both sides,

$$\int\frac{dx}{x^2} = -k\int dt$$

$$\frac{-1}{x} = -kt+C \quad \ldots(i)$$

Using boundary condition :

When $t=0,\ x=a$

$$\frac{-1}{a} = -k\times 0+C \Rightarrow C=\frac{-1}{a}$$

Put the value of C in equation (i),

$$-\frac{1}{x} = -kt - \frac{1}{a}$$

$$\frac{1}{x} = kt + \frac{1}{a}$$

Hence, the correct option is (B).

2.13 (A)

Given : $\frac{dy}{dt} + p(t)y = q(t)\,y^n ; \quad n > 0$

and $v = y^{1-n}$

Differentiating both sides with respect to t,

$$\frac{dv}{dt} = (1-n)y^{-n}\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{1}{(1-n)y^{-n}}\frac{dv}{dt}$$

Put the above value in given differential equation,

$$\frac{1}{(1-n)y^{-n}}\frac{dv}{dt} + p(t)y = q(t)y^n$$

Multiplying $(1-n)y^{-n}$ on both sides

$$\frac{dv}{dt} + p(t)(1-n)y^{1-n} = q(t)(1-n)$$

Since, $y^{1-n} = v$,

Hence, $\frac{dv}{dt} + (1-n)pv = (1-n)q$

[p is $p(t)$ and q is $q(t)$]

Hence, the correct option is (A).

2.14 (A)

Given : $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 0$

$$\left(D^2 + 2D + 17\right)y = 0$$

This is in the form of a homogenous linear differential equation.

$$[f(D)]y = 0$$

The auxiliary equation is given by,

$$f(m) = 0$$

$$m^2 + 2m + 17 = 0$$

$$m = -1 \pm 4i$$

$$m_1 = -1 + 4i,\ m_2 = -1 - 4i$$

The roots are a pair of complex number $(a \pm ib)$.

The complementary function is given by,

$$\text{C.F.} = e^{-x}\left[C_1 \cos 4x + C_2 \sin 4x\right]$$

Particular integral (P.I.) is 0, because it is a homogenous equation.

Therefore, the complete solution is

$$y = e^{-x}(C_1 \cos 4x + C_2 \sin 4x) \quad \ldots(\text{i})$$

Using boundary conditions :

(i) When $x = 0,\ y = 1$

$$1 = e^{-0}(C_1 \cos 0 + C_2 \sin 0) \quad \Rightarrow C_1 = 1$$

Differentiating equation (i) with respect to x,

$$\frac{dy}{dx} = e^{-x}(-4C_1 \sin 4x + 4C_2 \cos 4x) - e^{-x}[C_1 \cos 4x + C_2 \sin 4x]$$

$$\frac{dy}{dx} = e^{-x}[(-4C_1 - C_2)\sin 4x + (4C_2 - C_1)\cos 4x]$$

(ii) When $x = \frac{\pi}{4},\ \frac{dy}{dx} = 0$

Hence, $0 = -(4C_2 - C_1)e^{-\pi/4}$

$$4C_2 = C_1$$

$$C_2 = \frac{C_1}{4} = \frac{1}{4}$$

Hence, $C_1 = 1$ and $C_2 = \frac{1}{4}$

Put the values of C_1 and C_2 in equation (i),

$$y = e^{-x}(\cos 4x + \frac{1}{4}\sin 4x)$$

Hence, the correct option is (A).

Key Point

For any homogeneous differential equations of the form :

$$[f(D)]y(t)=0$$

The particular integral (P.I.) is always zero.

2.15 (A)

Given : $x^2\frac{dy}{dx}+2xy-x+1=0$

$$x^2\frac{dy}{dx}+2xy=x-1$$

Dividing the given differential equation by x^2,

$$\frac{dy}{dx}+\left(\frac{2}{x}\right)y=\left(\frac{x-1}{x^2}\right)$$

Above differential equation is in the form of Leibniz linear equation.

$$\frac{dy}{dx}+Py=Q$$

where, $P=\frac{2}{x}$, $Q=\frac{x-1}{x^2}$

The integrating factor is given by,

$$\text{I.F}=e^{\int Pdx}=e^{\int\frac{2}{x}dx}=e^{2\log x}=e^{\log_e x^2}=x^2$$

The complete solution is given by,

$$y\times\text{I.F.}=\int Q\times(\text{I.F})\,dx+C$$

$$y\times x^2=\int\left(\frac{x-1}{x^2}\right)\times x^2dx+C$$

$$yx^2=\int(x-1)\times dx+C$$

$$yx^2=\frac{x^2}{2}-x+C \qquad \ldots(\text{i})$$

Using boundary condition :

When $x=1, y=0$

$$0\times1^2=\frac{1^2}{2}-1+C$$

$$0=\frac{1}{2}-1+C \quad\Rightarrow\quad C=\frac{1}{2}$$

Put the value of C in equation (i),

$$yx^2=\frac{x^2}{2}-x+\frac{1}{2}$$

$$y=\frac{1}{2}-\frac{1}{x}+\frac{1}{2x^2}$$

Hence, the correct option is (A).

2.16 (B)

Given : $\frac{d^2x}{dt^2}+2x^3=0$

In above equation, $\frac{d^2x}{dt^2}$ is the highest order derivative. Order = 2.

So, Degree = Power of highest derivative = 1

Hence, the correct option is (B).

2.17 (D)

Given : $\frac{dy}{dx}=x^2y$

$$\frac{dy}{y}=x^2dx$$

Integrating on both sides,

$$\int\frac{dy}{y}=\int x^2dx$$

$$\log_e y=\frac{x^3}{3}+C_1$$

$$y=e^{\frac{x^3}{3}+C_1}=e^{C_1}\times e^{\frac{x^3}{3}}$$

$$y=C\times e^{\frac{x^3}{3}} \quad \left(C=e^{C_1}\right) \qquad \ldots(\text{i})$$

Using boundary condition :

When $x=0$, $y=1$

$$1=C\times e^{\frac{0}{3}}\Rightarrow C=1$$

Put the value of C in equation (i),

Thus, $y=e^{\frac{x^3}{3}}$

Hence, the correct option is (D).

2.18 (D)

Given : $\frac{dy}{dx}=\frac{-x}{y}$

$$y\,dy=-x\,dx$$

Integrating on both sides,

$$\int y\,dy=\int -x dx$$

$$\frac{y^2}{2}=-\frac{x^2}{2}+C \qquad \ldots(i)$$

Using boundary condition :

When $x=1, y=\sqrt{3}$

$$\frac{(\sqrt{3})^2}{2}=-\frac{1}{2}+C$$

$$C=2$$

Put the value of C in equation (i),

$$\frac{y^2}{2}=\frac{-x^2}{2}+2$$

$$x^2+y^2=4$$

Hence, the correct option is (D).

2.19 (D)

Given : $k_x\frac{\partial^2 h}{\partial x^2}+k_z\frac{\partial^2 h}{\partial z^2}=0 \qquad \ldots(i)$

From option (D) :

Put $x_t=x\sqrt{\frac{k_z}{k_x}} \Rightarrow \frac{\partial x_t}{\partial x}=\sqrt{\frac{k_z}{k_x}}$

$$\frac{\partial x}{\partial x_t}=\sqrt{\frac{k_x}{k_z}}$$

$$\frac{\partial^2 h}{\partial x_t^2}=\frac{\partial}{\partial x_t}\left(\frac{\partial h}{\partial x_t}\right)=\frac{\partial}{\partial x_t}\left(\frac{\partial h}{\partial x}\times\frac{\partial x}{\partial x_t}\right)$$

$$\frac{\partial^2 h}{\partial x_t^2}=\frac{\partial}{\partial x_t}\left(\frac{\partial h}{\partial x}\times\sqrt{\frac{k_x}{k_z}}\right)$$

$$\frac{\partial^2 h}{\partial x_t^2}=\sqrt{\frac{k_x}{k_z}}\times\frac{\partial}{\partial x_t}\left(\frac{\partial h}{\partial x}\right)$$

$$\frac{\partial^2 h}{\partial x_t^2}=\sqrt{\frac{k_x}{k_z}}\left(\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right)\times\frac{\partial x}{\partial x_t}\right)$$

$$\frac{\partial^2 h}{\partial x_t^2}=\sqrt{\frac{k_x}{k_z}}\left(\frac{\partial^2 h}{\partial x^2}\times\sqrt{\frac{k_x}{k_z}}\right)=\frac{k_x}{k_z}\times\frac{\partial^2 h}{\partial x^2}$$

$$\frac{\partial^2 h}{\partial x^2}=\frac{k_z}{k_x}\times\frac{\partial^2 h}{\partial x_t^2}$$

Substituting in equation (i),

$$k_x\times\frac{k_z}{k_x}\times\frac{\partial^2 h}{\partial x_t^2}+k_z\frac{\partial^2 h}{\partial z^2}=0$$

$$k_z\frac{\partial^2 h}{\partial x_t^2}+k_z\frac{\partial^2 h}{\partial z^2}=0 \Rightarrow \frac{\partial^2 h}{\partial x_t^2}+\frac{\partial^2 h}{\partial z^2}=0$$

which is the required form.

Hence, $x_t=x\sqrt{\frac{k_z}{k_x}}$ is the correct substitution.

Hence, the correct option is (D).

2.20 (A)

Given : $\frac{d^2y}{dx^2}+y=0$

$$(D^2+1)y=0$$

This is in the form of homogenous linear differential equation.

$$[f(D)]y=0$$

The auxiliary equation is given by,

$$f(m)=0$$

$$m^2+1=0$$

$$m=0\pm i$$

$$m_1=+i,\ m_2=-i$$

The roots are a pair of complex number $(a\pm ib)$.

So, the complementary function is of given by,

$$\text{C.F.}=e^{0x}\left[P\cos x+Q\sin x\right]$$

Particular integral (P.I.) is 0 because it is a homogenous equation.

Therefore, the complete solution is

$$y = P\cos x + Q\sin x$$

where, P and Q are constants.

Hence, the correct option is (A).

2.21 (A)

Given : $3y\frac{dy}{dx} + 2x = 0$

$$\frac{dy}{dx} = \frac{-2x}{3y}$$

$$3ydy = -2xdx$$

Integrating on both sides,

$$\int 3ydy = \int -2xdx$$

$$\frac{3}{2}y^2 = -2\times\frac{x^2}{2} + C$$

$$3y^2 + 2x^2 = C'$$

$$\frac{x^2}{\left(\frac{1}{2}\right)} + \frac{y^2}{\left(\frac{1}{3}\right)} = C'$$

$$\frac{x^2}{\left(\frac{1}{2}C'\right)} + \frac{y^2}{\left(\frac{1}{3}C'\right)} = 1$$

The above equation represents a family of ellipses.

Hence, the correct option is (A).

2.22 (C)

Given : $z = ax + by + ab$ ...(i)

Differentiating equation (i) with respect to x,

$$\frac{\partial z}{\partial x} = a$$

$$p = a \qquad [\text{since, } p = \frac{\partial z}{\partial x}]$$

Differentiating equation (i) with respect to y,

$$\frac{\partial z}{\partial y} = b$$

$$q = b \qquad [\text{since, } q = \frac{\partial z}{\partial y}]$$

Substituting a and b in equation (i) in terms of p and q ,

$$z = px + qy + pq$$

Hence, the correct option is (C).

2.23 (C)

Given : $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

$$\left(D^2 + D - 6\right)y = 0$$

This is in the form of homogenous linear differential equation.

$$[f(D)]y = 0$$

The auxiliary equation is given by,

$$f(m) = 0$$

$$m^2 + m - 6 = 0$$

$$m = -3,\ 2$$

$$m_1 = -3,\ m_2 = 2$$

The roots are real and distinct.

So, the complementary function is given by,

$$\text{C.F.} = c_1e^{-3x} + c_2e^{2x}$$

Particular integral (P.I.) is 0 because it is a homogenous equation.

Therefore, the complete solution is

$$y = c_1e^{-3x} + c_2e^{2x}$$

Hence, the correct option is (C).

2.24 (A)

Given : $y = c_1e^{-3x} + c_2e^{2x}$

$$\frac{d^3y}{dx^3} = -4\sqrt{\left(\frac{dy}{dx}\right)^3 + y^2}$$

Squaring on both sides,

$$\left(\frac{d^3y}{dx^3}\right)^2 = 16\left[\left(\frac{dy}{dx}\right)^3 + y^2\right]$$

In the above equation $\left(\frac{d^3y}{dx^3}\right)$ is the highest order derivative. So, order = 3

Degree = power of highest derivative = 2

Hence, the correct option is (A).

2.25 (D)

Given : $\frac{dy}{dx}+2y=0$

$$\frac{dy}{dx}=-2y$$

Method 1

$$\frac{dy}{y}=-2dx$$

Integrating on both sides,

$$\int \frac{dy}{y}=\int -2dx$$

$$\ln y=-2x+C$$

$$y=e^{-2x+C}=e^{C}e^{-2x}=C_1e^{-2x} \qquad (C_1=e^{C})$$

$$\ldots(i)$$

Using boundary condition :

When $x=1, \quad y=5$

$$5=C_1e^{-2}$$

$$C_1=\frac{5}{e^{-2}}=36.95$$

Put the value of C_1 in equation (i),

$$y=36.95\ e^{-2x}$$

Hence, the correct option is (D).

Method 2

$$\frac{dy}{dx}+2y=0$$

$$(D+2)y=0$$

This is in the form of homogenous linear differential equation.

$$[f(D)]y=0$$

The auxiliary equation is given by,

$$f(m)=0$$

$$m+2=0$$

$$m=-2$$

The complementary function of is given by,

$$\text{C.F.}=C_1e^{mx}=C_1e^{-2x}$$

Particular integral (P.I.) is 0 because it is a homogenous equation.

Therefore, the complete solution is

$$y=C_1e^{-2x}$$

Using boundary condition :

When $x=1, \quad y=5$

$$5=C_1e^{-2} \text{ or } C_1=\frac{5}{e^{-2}}=5e^2$$

Put the value of C_1 in equation (i),

$$y=\frac{5}{e^{-2}}e^{-2x}=5e^2e^{-2x}=36.95e^{-2x}$$

Hence, the correct option is (D).

Scan for Video Solution

2.26 (D)

Given : $\frac{dy}{dx}+\frac{y}{x}=x$

Above differential equation is in the form of Leibniz linear equation,

$$\frac{dy}{dx}+Py=Q$$

where, $P=\frac{1}{x}$ and $Q=x$

The integrating factor is given by,

$$\text{I.F.}=e^{\int pdx}=e^{\int \frac{1}{x}\cdot dx}=e^{\ln x}=x$$

The complete solution is given by,

$$y\times\text{I.F.}=\int Q\times\text{I.F.}+C$$

$$y \times x = \int x \times x \, dx + C$$

$$yx = \frac{x^3}{3} + C$$

$$y = \frac{x^2}{3} + \frac{C}{x} \qquad \ldots(i)$$

Using boundary condition :

When $x = 1, y = 1$

$$1 = \frac{1}{3} + C$$

$$C = \frac{2}{3}$$

Put the value of C in equation (i),

$$y = \frac{x^2}{3} + \frac{2}{3x}$$

Hence, the correct option is (D).

2.27 3.8

Given : $\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = 0$

Since, head H does not vary in y and z-direction.

Therefore, $\frac{\partial^2 H}{\partial x^2} = 0$

Integrating both sides,

$$\frac{\partial H}{\partial x} = C_1$$

$$H = C_1 x + C_2 \qquad \ldots(i)$$

$$H' = C_1$$

Using boundary conditions :

(i) When $x = 0, H = 5$

$$5 = C_1 \times 0 + C_2 \Rightarrow C_2 = 5$$

(ii) When $x = 0, \frac{dH}{dx} = -1$

$$-1 = C_1 \Rightarrow C_1 = -1$$

Put the values of C_1 and C_2 in equation (i),

$$H = -x + 5$$

At $x = 1.2$ m,

$$H = 5 - 1.2 = 3.8 \text{ m}$$

Hence, the value of H is **3.8**.

2.28 (D)

Given : $\frac{dP}{dt} + k_2 P = k_1 L_0 e^{-k_1 t}$

Above differential equation is in the form of Leibniz linear equation.

$$\frac{dP}{dt} + KP = Q$$

where, $K = k_2$, $Q = k_1 L_0 e^{-k_1 t}$

The integrating factor is given by,

$$\text{I.F.} = e^{\int K\,dt} = e^{\int k_2\,dt} = e^{k_2 t}$$

Hence, the correct option is (D).

2.29 3

Given : $\frac{d^2 H}{dx^2} = 0$

Integrating on both sides,

$$\frac{dH}{dx} = C_1 \qquad \ldots(i)$$

$$H = C_1 x + C_2 \qquad \ldots(ii)$$

Using boundary conditions :

(i) When $x = 0, H = 5$

$$5 = C_1 \times 0 + C_2$$

$$C_2 = 5$$

(ii) When $x = 10, H = 1$

$$1 = C_1 \times 10 + 5$$

$$C_1 = \frac{-2}{5}$$

Put the values of C_1 and C_2 in equation (ii),

$$H = -\frac{2}{5}x + 5$$

At the middle of strip i.e. $x = 5$

$$H = -\frac{2}{5}\times 5 + 5 = 3\,\text{m}$$

Hence, the value of H is **3**.

2.30 18

Given : $\dfrac{d^2y}{dx^2} = -12x^2 + 24x - 20$

Integrating both sides with respect to x,

$$\frac{dy}{dx} = -4x^3 + 12x^2 - 20x + c_1$$

$$y = -x^4 + 4x^3 - 10x^2 + c_1x + c_2 \qquad \ldots(\text{i})$$

Using boundary conditions :

(i) When $x = 0,\ y = 5$

$$5 = -0 + 4\times 0 - 10\times 0 + c_1\times 0 + c_2$$

$$c_2 = 5$$

(ii) When $x = 2,\ y = 21$

$$21 = -2^4 + 4\times 2^3 - 10\times 2^2 + c_1\times 2 + c_2$$

$$21 = -16 + 32 - 40 + 2c_1 + 5$$

$$2c_1 = 21 + 16 - 32 + 40 - 5$$

$$2c_1 = 40$$

$$c_1 = 20$$

Put the values of c_1 and c_2 in equation (i),

$$y = -x^4 + 4x^3 - 10x^2 + 20x + 5$$

At $x = 1$,

$$y = -1 + 4 - 10 + 20 + 5$$

$$y = 18$$

Hence, the value of y is **18**.

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2.31 (C)

Given :

$$x(y\,dx + x\,dy)\cos\left(\frac{y}{x}\right) = y(xdy - ydx)\sin\left(\frac{y}{x}\right)$$

$$\frac{x(y\,dx + x\,dy)}{y(xdy - ydx)} = \frac{\sin\left(\frac{y}{x}\right)}{\cos\left(\frac{y}{x}\right)}$$

$$\frac{y\,dx + x\,dy}{xdy - ydx} = \frac{y}{x}\tan\left(\frac{y}{x}\right)$$

Let $y = vx$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = v + \frac{xdv}{dx} \Rightarrow dy = vdx + xdv$$

$$\frac{vxdx + x(vdx + xdv)}{x(vdx + xdv) - vxdx} = \frac{vx}{x}\tan\left(\frac{vx}{x}\right)$$

$$\frac{2vxdx + x^2dv}{x^2dv} = v\tan v$$

$$\frac{2v}{x}\frac{dx}{dv} + 1 = v\tan v$$

$$\frac{2v}{x}\frac{dx}{dv} = v\tan v - 1$$

$$\frac{2dx}{x} = \frac{(v\tan v - 1)}{v}dv$$

$$\frac{2dx}{x} = \left(\tan v - \frac{1}{v}\right)dv$$

Integrating both sides,

$$\int\frac{2dx}{x} = \int\left(\tan v - \frac{1}{v}\right)dv$$

$$2\ln x = \ln(\sec v) - \ln v + \ln C$$

$$\ln x^2 = \ln\left(\frac{\sec v}{v}\times C\right)$$

$$x^2 = C\frac{\sec v}{v}$$

Put, $v = \dfrac{y}{x}$ on above equation,

$$x^2 = C\frac{\sec\left(\frac{y}{x}\right)}{\frac{y}{x}}$$

$$x = \frac{C}{y}\sec\left(\frac{y}{x}\right)$$

$$xy = C\sec\left(\frac{y}{x}\right) = \frac{C}{\cos\left(\frac{y}{x}\right)}$$

$$xy\cos\left(\frac{y}{x}\right) = C$$

Hence, the correct option is (C).

2.32 (C)

Given : $\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + 3\frac{\partial^2 P}{\partial x \partial y} + 2\frac{\partial P}{\partial x} - \frac{\partial P}{\partial y} = 0$

Comparing the above equation with general form of a linear second order partial differential equation,

$$a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^2 u}{\partial x \partial y} + c\frac{\partial^2 u}{\partial y^2} + d\frac{\partial u}{\partial x} + e\frac{\partial u}{\partial y} + fu = g$$

where, $a = 1,\ b = 3,\ c = 1$

$$b^2 - 4ac = 9 - 4 = 5 > 0$$

Hence, Partial differential equation is hyperbolic.

Hence, the correct option is (C).

2.33 (A)

Given : $\frac{d^4 y}{dx^4} + 3\frac{d^2 y}{dx^2} = 108x^2$

This is in the form of a non-homogenous linear differential equation.

$$[f(D)]y = \phi$$

The auxiliary equation is given by,

$$f(m) = 0$$

$$m^4 + 3m^2 = 0$$

$$m_1 = 0,\ m_2 = 0,\ m_3 = +\sqrt{3}\,i,\ m_4 = \sqrt{3}\,i$$

The complementary function is given by,

$$\text{C.F.} = (c_1 + c_2 x)e^{m_1 x} + e^{ax}[c_3 \sin bx + c_4 \cos bx]$$

$$\text{C.F.} = (c_1 + c_2 x)e^{0x} + e^{0x}[c_3 \sin\sqrt{3}x + c_4 \cos\sqrt{3}x]$$

$$\text{C.F.} = c_1 + c_2 x + c_3 \sin\sqrt{3}x + c_4 \cos\sqrt{3}x$$

The particular integral is given by,

$$\text{P.I.} = \frac{1}{f(D)} \times 108x^2$$

$$\text{P.I.} = \frac{1}{(D^4 + 3D^2)} \times 108x^2$$

$$\text{P.I.} = \frac{1}{3D^2\left[1 + \frac{D^4}{3D^2}\right]} \times 108x^2$$

$$\text{P.I.} = \frac{1}{3D^2\left[1 + \frac{D^2}{3}\right]} \times 108x^2$$

$$\text{P.I.} = \frac{1}{3D^2}\left[1 + \frac{D^2}{3}\right]^{-1} \times 108x^2$$

$$\left[\text{By binomial expansion : } (1+x)^{-n} = 1 - nx + \frac{n(n+1)}{2!}x^2 - \ldots.\right]$$

$$\text{P.I.} = \frac{1}{3D^2}\left[1 - \frac{D^2}{3} + \ldots.\right] \times 108x^2$$

$$\text{P.I.} = \frac{1}{3D^2}\left[108x^2 - \frac{D^2}{3}[108x^2]\right]$$

$$\text{P.I.} = \frac{1}{3D^2}\left[108x^2 - \frac{108}{3} \times 2\right]$$

$$\text{P.I.} = \frac{1}{3D^2}\left[108x^2 - 72\right]$$

$$\text{P.I.} = \frac{1}{3}\left[9x^4 - 36x^2\right] + c$$

$$\text{P.I.} = 3x^4 - 12x^2 + c$$

The complete solution is given by,

$$y = \text{C.F.} + \text{P.I.}$$

$$y = c_1 + c_2 x + c_3 \sin\sqrt{3}\,x + c_4 \cos\sqrt{3}\,x + 3x^4 - 12x^2 + c$$

Hence, the correct option is (A).

2.34 (B)

Given : $\frac{\partial u}{\partial t}=\alpha\frac{\partial^2 u}{\partial x^2}$

The above given equation is heat equation.

Therefore, solution to the above equation is,

$$u(x,t)=(A\cos px+B\sin px)\times Ce^{-p^2\alpha t}$$

Put $p=\sqrt{\frac{-K}{\alpha}}=\sqrt{\frac{K}{\alpha}}\times i$,

Now, put the value of p in given equation,

$$u(x,t)=\left[A\cos\sqrt{\frac{K}{\alpha}}ix+B\sin\sqrt{\frac{K}{\alpha}}ix\right]Ce^{Kt}$$

$$u(x,t)=\left[A\cosh\sqrt{\frac{K}{\alpha}}x+iB\sinh\sqrt{\frac{K}{\alpha}}x\right]Ce^{Kt}$$

$$u(x,t)=Ce^{Kt}\left[A\left(\frac{e^{\sqrt{\frac{K}{\alpha}}x}+e^{-\sqrt{\frac{K}{\alpha}}x}}{2}\right)+iB\left(\frac{e^{\sqrt{\frac{K}{\alpha}}x}-e^{-\sqrt{\frac{K}{\alpha}}x}}{2}\right)\right]$$

$$u(x,t)=Ce^{Kt}\left[e^{\sqrt{\frac{K}{\alpha}}x}\left(\frac{A+iB}{2}\right)+e^{-\sqrt{\frac{K}{\alpha}}x}\left(\frac{A-iB}{2}\right)\right]$$

$$u(x,t)=Ce^{Kt}\left[C_1e^{\sqrt{\frac{K}{\alpha}}x}+C_2e^{-\sqrt{\frac{K}{\alpha}}x}\right]$$

Hence, the correct option is (B).

2.35 36

Given : $3\frac{\partial^2\phi}{\partial x^2}+B\frac{\partial^2\phi}{\partial x\partial y}+3\frac{\partial^2\phi}{\partial y^2}+4\phi=0$

Comparing the above equation with general form of a linear second order partial differential equation,

$$a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2 u}{\partial x\partial y}+c\frac{\partial^2 u}{\partial y^2}+d\frac{\partial u}{\partial x}+e\frac{\partial u}{\partial y}+fu=g$$

where, $a=3$, $b=B, c=3$

For the equation to be parabolic,

$$b^2-4ac=0$$

$$B^2-4\times3\times3=0$$

$$B^2=36$$

Hence, the value of B^2 is **36**.

Scan for Video Solution

2.36 (D)

Given : $\frac{dQ}{dt}+Q=1$

$$(D+1)Q=1$$

This is in the form of a non-homogenous linear differential equation.

$$[f(D)]Q=\phi$$

The auxiliary equation is given by,

$$f(m)=0$$

$$m+1=0$$

$$m=-1$$

The complementary function is given by,

$$\text{C.F.}=C_1e^{mt}=C_1e^{-t}$$

Particular integral is given by,

$$\text{P.I.}=\frac{1}{f(D)}\times(1)$$

$$\text{P.I.}=\frac{1}{D+1}=1(1+D)^{-1}$$

$$\text{P.I.}=1(1-D+D^2-D^3.....)$$

[By binomial expansion : $(1+x)^{-n}=1-nx+\frac{n(n+1)}{2!}x^2-....$]

$$\text{P.I.}=1-0+0.....=1$$

The complete solution is given by,

$$Q(t)=\text{C.F.}+\text{P.I.}$$

$$Q(t)=C_1e^{-t}+1 \qquad ...(i)$$

Using boundary condition :

When $t=0,\ Q=0$

$$0=C_1e^{-0}+1$$

$$0=C_1+1$$

$$C_1=-1$$

Put the value of C_1 in equation (i),

$$Q(t)=1-e^{-t}$$

Hence, the correct option is (D).

2.37 (A)

Given : $y''-4y'+3y=2t-3t^2$

$$(D^2-4D+3)\,y=2t-3t^2$$

The particular integral is given by,

$$\text{P.I.}=\frac{1}{f(D)}(2t-3t^2)$$

$$\text{P.I.}=\frac{1}{D^2-4D+3}(2t-3t^2)$$

$$\text{P.I.}=\frac{1}{3\left[1+\frac{D^2-4D}{3}\right]}(2t-3t^2)$$

$$\text{P.I.}=\frac{1}{3}\left[1+\frac{D^2-4D}{3}\right]^{-1}(2t-3t^2)$$

$$\left[\begin{array}{l}\text{By binomial expansion:}\\ (1+x)^{-n}=1-nx+\frac{n(n+1)}{2!}x^2\\ \qquad\qquad -\frac{n(n+1)(n+2)}{3!}x^3+....\end{array}\right]$$

$$\text{P.I.}=\frac{1}{3}\left[1-\left(\frac{D^2-4D}{3}\right)+\left(\frac{D^2-4D}{3}\right)^2\right.$$
$$\left.-\left(\frac{D^2-4D}{3}\right)^3+.....\right](2t-3t^2)$$

$$\text{P.I.}=\frac{1}{3}\left[(2t-3t^2)-\frac{1}{3}\{D^2(2t-3t^2)\right.$$
$$-4D(2t-3t^2)\}+\frac{1}{9}\{D^4(2t-3t^2)$$
$$\left.+16D^2(2t-3t^2)-8D^3(2t-3t^2)\}.....\right]$$

$$\text{P.I.}=\frac{1}{3}\left[(2t-3t^2)-\frac{1}{3}\{-6-4(2-6t)\}\right.$$
$$\left.+\frac{1}{9}\{0+16(-6)-0\}\right]$$

$$\text{P.I.}=\frac{1}{3}\left[2t-3t^2-\frac{1}{3}(-6-8+24t)\right.$$
$$\left.+\frac{1}{9}(-96)\right]$$

$$\text{P.I.}=\frac{1}{3}\left[2t-3t^2-8t+\frac{14}{3}-\frac{96}{9}\right]$$

$$\text{P.I.}=\frac{1}{3}\left[-3t^2-6t-6\right]$$

$$\text{P.I.}=-t^2-2t-2$$

$$\text{P.I.}=(-2-2t-t^2)$$

Hence, the correct option is (A).

2.38 (A)

Given : Tangent to the curve is,

$$y=x\ln x$$

Co-ordinates of the tangent point is given by,

$$\tan\theta=\frac{dy}{dx}=\ln x+1$$

$$\tan 45^0=\ln x+1$$

$$1=\ln x+1$$

$$\ln x=0$$

$$\therefore\quad x=1$$

Putting $x=1$ in the equation of curve, we get $y=0$.

Hence, the correct option is (A).

2.39 0.793

Given : Differential equation,

$$\frac{dy}{dx}=\frac{1-e^{-\frac{10}{3}\ln(y)}}{250-45e^{-3\ln(y)}}$$

According to Euler method,

$$y_1=y_0+h\,f\,(x_0,y_0)$$

At $x_0=0,\ y_0=0.8\,\text{m}$

$$\frac{dy}{dx}=\frac{1-e^{\frac{-10}{3}\ln(0.8)}}{250-45e^{-3\ln(0.8)}}$$

$$\frac{dy}{dx}=-6.8\times10^{-3}=-0.0068=f(x_0,\ y_0)$$

$$y_1=y_0+f\,(x_0,y_0)\times1$$

$$y_1=0.8+(-0.0068)=0.7932\,\text{m}$$

Hence, the depth of the flow is **0.793**.

2.40 0.368

Given : $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$

$$(D^2+2D+1)y=0$$

This is in the form of a homogenous linear differential equation.

$$[f(D)]\,y=0$$

The auxiliary equation is given by,

$$f(m)=0$$

$$m^2+2m+1=0$$

$$m=-1,-1$$

The complementary function is given by,

So, C.F. $=(C_1+C_2x)e^{-x}$

Particular integral (P.I) is 0 because it is homogenous equation.

The complete solution is given by,

$$y=(C_1+C_2x)e^{-x} \qquad \ldots(\text{i})$$

Differentiating above equation with respect to x,

$$\frac{dy}{dx}=-(C_1+C_2x)e^{-x}+C_2e^{-x}$$

Using boundary conditions :

(i) When $x=0,\ y=1$

From equation (i),

$$1=(C_1+C_2\times0)e^0$$

$$1=C_1$$

(ii) When $\frac{dy}{dx}=-1,\ x=0$

$$-1=-(1+C_2\times0)e^{-0}+C_2e^{-0}$$

$$-1=-1\times1+C_2$$

$$C_2=0$$

Put values of C_1 and C_2 in equation (i),

$$y=(1+0\times x)\,e^{-x}$$

$$y=e^{-x}$$

At $x=1$,

$$y=e^{-1}=\frac{1}{e}=\frac{1}{2.718}=0.368$$

Hence, the solution of the differential equation is **0.368**.

2.41 (D)

Given : $\frac{\partial^2u}{\partial x^2}=25\frac{\partial^2u}{\partial t^2}$

D-alembert's formula for the equation,

$$\frac{\partial^2u}{\partial t^2}=c^2\frac{\partial u^2}{\partial x^2},-\infty<x<\infty,t>0$$

and $c>0$

Satisfying the conditions

$$u(x,0)=f(x)\text{ and }\frac{\partial u(x,0)}{\partial t}=g(x),$$

where, $f(x)$ is the initial displacement

and $g(x)$ is the initial velocity.

The general solution is given by,

$$u(x,\,t)=\frac{1}{2}[f(x-ct)+f(x+ct)]+\frac{1}{2c}\int_{x-ct}^{x+ct}g(s)\,ds$$

$$\frac{\partial^2 u}{\partial x^2}=25\frac{\partial^2 u}{\partial t^2}\qquad\text{[Given]}$$

Initial conditions:

$$u(0)=3x=f(x)$$

$$\frac{\partial u}{\partial t}(0)=3=g(x)$$

Comparing the given equation with standard equation,

$$\frac{\partial^2 u}{\partial t^2}=\frac{1}{25}\frac{\partial^2 u}{\partial x^2}\ \text{i.e.}\ c=\frac{1}{5}$$

Therefore, by D-alembert's solution is,

$$u(x,\,t)=\frac{1}{2}\left[f\left(x-\frac{1}{5}t\right)+f\left(x+\frac{1}{5}t\right)\right]+\frac{1}{2\times\frac{1}{5}}\int_{x-\frac{1}{5}t}^{x+\frac{1}{5}t}g(s)\,ds$$

At $x=1,\ t=1$

$$u(1,\,1)=\frac{1}{2}\left[3\left(1-\frac{1}{5}\right)+3\left(1+\frac{1}{5}\right)\right]+\frac{5}{2}\int_{\left(1-\frac{1}{5}\right)}^{\left(1+\frac{1}{5}\right)}3\,ds$$

$$u(1,\,1)=\frac{1}{2}\left[3\times\frac{4}{5}+3\times\frac{6}{5}\right]+\frac{5}{2}\times3[5]_{4/5}^{6/5}$$

$$u(1,\,1)=\frac{1}{2}\left[\frac{12}{5}+\frac{18}{5}\right]+\frac{15}{2}\left[\frac{6}{5}-\frac{4}{5}\right]$$

$$u(1,\,1)=\frac{1}{2}\left[\frac{30}{5}\right]+\frac{15}{2}\times\frac{2}{5}$$

$$u(1,\,1)=3+3=6$$

Hence, the correct option is (D).

2.42 (C)

Given : $x\frac{dy}{dx}+y=0$

$$x\frac{dy}{dx}=-y$$

$$\frac{dy}{y}=-\frac{dx}{x}$$

Integrating on both sides,

$$\int\frac{dy}{y}=-\int\frac{dx}{x}$$

$$\ln(y)=-\ln(x)+\ln C$$

$$\ln y=\ln(x^{-1})+\ln C$$

$$y=x^{-1}\times C\qquad\ldots(i)$$

At $x=1$ and $y=1$,

$$1=1^{-1}\times C$$

$$C=1$$

Put the value of C in equation (i),

$$y=x^{-1}\times1$$

Therefore, $y=x^{-1}$

Hence, the correct option is (C).

2.43 (D)

For function $h=f(x,y)$ the Laplace equation of continuity is

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=0$$

Hence, the correct option is (D).

2.44 6

Given : Cauchy's differential equation :

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$$

Conditions : $y(1)=0$

$$y(2)=2$$

$$y(3)=?$$

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0 \qquad \ldots(i)$$

Putting $x=e^t$, we get $x\frac{dy}{dx}=Dy$

$$x^2\frac{d^2y}{dx^2}=D(D-1)y, \text{ where } D=\frac{d}{dt}$$

$$\therefore \quad D[D-1]y-2Dy+2=0$$

$$[D^2-3D+2]y=0 \qquad \ldots(ii)$$

Equation is homogenous differential equation,

To solve auxiliary equation,

$$D^2-3D+2=0$$

$$\therefore \quad D=1,\ 2$$

Hence solution :

$$y=C_1e^t+C_2e^{2t}$$

Or $$y=C_1x+C_2x^2 \qquad \ldots(iii)$$

Now substituting the given conditions $y(1)=0$, $y(2)=2$ in equation (iii),

$$\therefore \quad y(1)=0\Rightarrow C_1+C_2=0 \qquad \ldots(iv)$$

$$y(2)=2\Rightarrow 2C_1+4C_2=2 \qquad \ldots(v)$$

Solving equations (iv) and (v),

$$C_1=-1$$

$$C_2=1$$

Putting the value of C_1 and C_2 in equation (iii),

$$y=x^2-x$$

$$\therefore \quad y(3)=3^2-3=9-3=6$$

Hence, the value of $y(3)$ is **6.**

2.45 (C)

Given : $x\ln x\left(\frac{dy}{dx}\right)=y$

Rearranging the above equation,

$$\frac{dy}{y}=\frac{dx}{x\ln x}$$

Integrating both sides,

$$\int\frac{dy}{y}=\int\frac{dx}{x\ln x}$$

$$\ln y=\int\frac{dx}{x\ln x} \qquad \ldots(i)$$

Let $\ln x=t$

$$\frac{dx}{x}=dt$$

Substituting the values in equation (i),

$$\ln y=\int\frac{dt}{t}$$

$$\therefore \quad \ln y=\ln t+\ln K$$

$$\ln y=\ln Kt$$

$$y=Kt \qquad \ldots(ii)$$

Putting $t=\ln x$ in equation (ii),

$$y=K\ln x$$

Hence, the correct option is (C).

2.46 (B)

Given :

$$x=[4\quad -2\quad -6]^T$$

$$x=\begin{bmatrix}4\\-2\\-6\end{bmatrix}$$

Euclidean norm length

$$=\sqrt{16+4+36}=\sqrt{56}$$

Hence, the correct option is (B).

2.47 (B)

Given :

$$\frac{\partial V}{\partial t}=\beta\frac{\partial^2V}{\partial x^2} \qquad \ldots(i)$$

Approximate time derivative in equation (i) with forward difference,

$$\frac{\partial V}{\partial t}=\frac{V_i^{n+1}-V_i^n}{\Delta t}+0(\Delta t) \quad \ldots(ii)$$

Note that the right hand side only in value V at $x=x_1$

Use the central difference approximation to $\left(\frac{\partial^2 V}{\partial x^2}\right)_{x_i}$ and evaluate all the terms at time n.

$$\left.\frac{\partial^2 V}{\partial x^2}\right|_{x_i}=\frac{V_{i-1}^n-2V_i^n+V_{i+1}^n}{\Delta x^2}+0(\Delta x^2) \quad \ldots(iii)$$

Putting the values from equation (ii) and (iii) in equation (i), we get

$$\frac{V_i^{(n+1)}-V_i^{(n)}}{\Delta t}=\beta\left[\frac{V_{i+1}^{(n)}-2V_i^{(n)}+V_{i-1}^{(n)}}{(\Delta x)^2}\right]$$

$$+0(\Delta t)+0(\Delta x^2)$$

$$\therefore \quad \frac{V_i^{(n+1)}-V_i^{(n)}}{\Delta t}=\beta\left[\frac{V_{i+1}^{(n)}-2V_i^{(n)}+V_{i-1}^{(n)}}{(\Delta x)^2}\right]$$

Hence, the correct option is (B).

2.48 (A)

Given : $D(\theta)\frac{\partial^2\theta}{\partial z^2}+\frac{\partial K(\theta)}{\partial z}-\frac{\partial\theta}{\partial t}=0$

In first term of given PDE,

$$D=f(\theta)$$

Equation contains product of dependent variable with it's derivative, so it is non-linear equation.

The given PDE is 2nd order derivative, so it is a second order non-linear equation.

Hence, the correct option is (A).

2.49 (C)

Given : $\frac{d^2x}{dt^2}-5\frac{dx}{dt}+6x=0$,

where $x(0)=0$ and $\frac{dx}{dt}(0)=10$

Auxilary equation is $m^2-5m+6=0$

$$m=2,\ 3$$

So, complimentary function is given by,

$$\text{C.F.}=c_1e^{2t}+c_2e^{3t}$$

and $\quad \text{PI}=0$

Solution is $x=CF+PI$

$$=C_1e^{2t}+C_2e^{3t}$$

And $\quad \frac{dx}{dt}=2C_1e^{2t}+3C_2e^{3t}$

Here, $\quad C_1=-10,\ C_2=10$

$$x=-10e^{2t}+10e^{3t}$$

Hence, the correct option is (C).

2.50 (D)

Given : PDE, $\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial x^2}, y\geq 0, x_1\leq x\leq x_2$

As $y\geq 0$, PDE is parabolic which requires two boundary conditions (corresponding to x) and one initial condition (corresponding to y) to solve.

Hence, the correct option is (D).

2.51 (B)

Given : Differential equation

$$\frac{d^2u}{dx^2}-2x^2u+\sin x=0$$

$$\frac{d^2u}{dx^2}-2x^2u=-\sin x$$

The above equation is in the form of linear non-homogenous differential equation of second order.

$$\frac{d^2u}{dx^2}+P(x)u=Q(x)$$

Hence, the correct option is (B).

2.52 (A)

Given : $6\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=0$

Auxiliary equation is given by,

$$6m^2+m-1=0$$

$$m=\frac{1}{3},\ m=-\frac{1}{2}$$

So, general solution is given by,

$$y=C_1e^{\frac{1}{3}x}+C_2e^{\frac{-1}{2}x}$$

Hence, the correct option is (A).

❖❖❖❖

1 Number System & Series

2010 IIT Guwahati

1.1 If 137 + 276 = 435 how much is 731 + 672 ? **[2 Marks]**

(A) 534 (B) 1403

(C) 1623 (D) 1513

1.2 Given digits 2, 2, 3, 3, 3, 4, 4, 4, 4. How many distinct 4 digit numbers greater than 3000 can be formed? **[2 Marks]**

(A) 50 (B) 51

(C) 52 (D) 54

2013 IIT Bombay

1.3 A number is as much greater than 75 as it is smaller than 117. The number is **[1 Mark]**

(A) 91 (B) 93

(C) 89 (D) 96

2014 IIT Kharagpur

1.4 Find the odd one in the following group ALRVX, EPVZB, ITZDF, OYEIK **[2 Marks]**

(A) ALRVX (B) EPVZB

(C) ITZDF (D) OYEIK

2015 IIT Kanpur

1.5 How many four digit numbers can be formed with the 10 digits 0, 1, 2, ……..9 if no number can start with 0 and if repetitions are not allowed? **[2 Marks]**

2016 IISc Bangalore

1.6 The sum of the digits of a two-digit number is 12. If the new number formed by reversing the digits is greater than the original number by 54, find the original number. **[1 Mark]**

(A) 39 (B) 57

(C) 66 (D) 93

2017 IIT Roorkee

1.7 What is the value of x when **[1 Mark]**

$$81\times\left(\frac{16}{25}\right)^{x+2}\div\left(\frac{3}{5}\right)^{2x+4}=144$$

(A) 1

(B) −1

(C) −2

(D) Cannot be determined

1.8 The last digit of $(2171)^7+(2172)^9+(2173)^{11}+(2174)^{13}$ is **[2 Marks]**

(A) 2 (B) 4

(C) 6 (D) 8

2018 IIT Guwahati

1.9 Consider a sequence of numbers a_1, a_2, $a_3, \ldots a_n$ where $a_n=\frac{1}{n}-\frac{1}{n+2}$, for each integer ($n$ > 0). What is the sum of the first 50 terms? **[2 Marks]**

(A) $\left(1+\frac{1}{2}\right)-\frac{1}{50}$

(B) $\left(1+\frac{1}{2}\right)+\frac{1}{50}$

(C) $\left(1+\frac{1}{2}\right)-\left(\frac{1}{51}+\frac{1}{52}\right)$

(D) $1-\left(\frac{1}{51}+\frac{1}{52}\right)$

1.10 For non-negative integers, a, b, c, what would be the value of $a+b+c$ if $\log a+\log b+\log c=0$? **[1 Mark]**

(A) 3 (B) 1

(C) 0 (D) -1

1.11 $\frac{a+a+a+\ldots+a}{n \text{ times}}=a^2b$ and $\frac{b+b+b+\ldots+a}{m \text{ times}}=ab^2$, where a, b, n and m are natural numbers. What is the value of $\left(\frac{m+m+m+\ldots+m}{n \text{ times}}\right)$ $\left(\frac{n+n+n+\ldots+n}{m \text{ times}}\right)$? **[1 Mark]**

(A) $2a^2b^2$ (B) a^4b^4

(C) $ab(a+b)$ (D) a^2+b^2

1.12 A faulty wall clock is known to gain 15 minutes every 24 hours. It is synchronized to the correct time at 9 AM on 11th July. What will be the correct time to the nearest minute when the clock shows 2 PM on 15th July of the same year? **[2 Marks]**

(A) 12:45 PM (B) 12:58 PM

(C) 1:00 PM (D) 2:00 PM

2019 IIT Madras

1.13 On a horizontal ground, the base of a straight ladder is 6 m away from the base of a vertical pole. The ladder makes an angle of 45^0 to the horizontal. If the ladder is resting at a point located at one – fifth of the height of the pole from the bottom, the height of the pole is _____ meters. **[1 Mark]**

(A) 15 (B) 25

(C) 35 (D) 30

2020 IIT Delhi

1.14 Insert seven numbers between 2 and 34, such that the resulting sequence including 2 and 34 is an arithmetic progression. The sum of these inserted seven numbers is _____. **[1 Mark]**

(A) 130 (B) 120

(C) 124 (D) 126

1.15 The sum of two positive numbers is 100. After subtracting 5 from each number, the product of the resulting numbers is 0. One of the original numbers is _____. **[1 Mark]**

(A) 90 (B) 80

(C) 95 (D) 85

1.16 The unit's place in 26591749^{110016} is _____. **[2 Marks]**

(A) 9 (B) 3

(C) 1 (D) 6

1.17 If $f(x)=x^2$ for each $x\in(-\infty,\infty)$, then $\frac{f(f(f(x)))}{f(x)}$ is equal to _____. **[1 Mark]**

(A) $(f(x))^3$ (B) $(f(x))^2$

(C) $(f(x))^4$ (D) $f(x)$

Answers — Number System & Series

1.1	C	1.2	B	1.3	D	1.4	D	1.5	4536
1.6	A	1.7	B	1.8	B	1.9	C	1.10	A
1.11	B	1.12	B	1.13	D	1.14	D	1.15	C
1.16	C	1.17	A						

Explanations — Number System & Series

1.1 (C)

Given : $137 + 276 = 435$

7 and 6 added is becoming 5 means the given two numbers are added on base 8.

$$\begin{array}{r} (137)_8 \\ +\ (276)_8 \\ \hline (435)_8 \end{array}$$

Add another two given set of numbers also on base 8.

$$\begin{array}{r} (731)_8 \\ +\ (672)_8 \\ \hline (1623)_8 \end{array}$$

Thus, the overall problem was based on identifying base which was 8 and adding numbers on base 8.

Hence, the correct option is (C).

1.2 (B)

Given : Digits are 2, 2, 3, 3, 3, 4, 4, 4, 4

As the number is greater than 3000. So thousand's place can be either 3 or 4.

Let us consider the following two cases.

Case 1 : When thousand's place is 3

$\underline{3}\ \underline{a}\ \underline{b}\ \underline{c}$

If there is no restriction on number of two's, three's and four's. Then each of a, b, c can be filled with 2 or 3 or 4 each in 3 ways.

So, $3 \times 3 \times 3 = 27$ numbers are there. Out of which 3222, 3333 are invalid as 2 can be used twice and 3 thrice only.

So, number of such valid numbers beginning with 3 are $27 - 2 = 25$

Case 2 : When thousand's place is 4

$\underline{4}\ \underline{a}\ \underline{b}\ \underline{c}$

Without restriction on number of 2's, 3's and 4's a, b, c can be filled in 27 ways (as explained in case 1)

Out of these 27 numbers, 4222 is only invalid as two have to be used twice only. So valid numbers are $27 - 1 = 26$

Total numbers from case 1 and case 2 is

$25 + 26 = 51$

Hence, the correct option is (B).

1.3 (D)

Let the unknown number be y.

According to question,

$$y = 75 + x \quad \ldots \text{(i)}$$

$$y = 117 - x \quad \ldots \text{(ii)}$$

From equation (i) and (ii),

$$y = 96$$

Hence, the correct option is (D).

1.4 (D)

Given : ALRVX, EPVZB, ITZDF, OYEIK

Method 1

Option (A) : It consists of only one vowel.

Option (B) : It consists of only one vowel.

Option (C) : It consists of only one vowel.

Option (D) : It consists of three vowels.

Hence, the correct option is (D).

Method 2

Option (A) : A – L – R – V – X

A – L : 10 alphabets between A and L.

L – R : 5 alphabets between L and R.

R – V : 3 alphabets between R and V.

V – X : 1 alphabet between V and X.

Similar for option (B) and (C).

Option (D) : OYEIK

O – Y : 9 alphabets between O and Y.

Y – E : 5 alphabets between Y and E.

E – I : 3 alphabets between E and I.

I – K : 1 alphabet between I and K.

Hence, the correct option is (D).

1.5 4536

Given :

(i) No number can start with zero.

(ii) Repetition is not allowed.

In thousandth place, 9 digits excepts 0 can be placed.

In hundredth place, 9 digits can be placed (including 0, excluding the one used in thousandth place).

In tenth place, 8 digit can be placed (excluding the ones used in thousandth and hundredth place)

In ones place, 7 digits can be placed (excluding the ones used in thousandth, hundredth and tenth place)

Total number of combinations,

$= 9 \times 9 \times 8 \times 7 = 4536$

Hence, 4 digit numbers that can be formed are **4536.**

1.6 (A)

Given :

(i) Sum of digits of two digit number is 12

(ii) Reverse of the two-digit number is greater than original number by 54.

Method 1

The new number formed by reversing the digits is greater than the original number is possible in options A and B only because reverse of 93 will be a smaller number and reverse of 66 will be similar to itself.

From option (A) :

Sum of two digits in the number = 3 + 9 = 12

After reversing the two digits number = 93

The difference between the new number formed and original number = 93 – 39 = 54

Hence, the correct option is (A).

Method 2

Let original number be xy.

Then, $(10x + y) + 54 = 10y + x$

$9x + 54 = 9y$

$9x - 9y = -54$

$x - y = -6$ … (i)

Also, $x + y = 12$ … (ii)

From equations (i) and (ii),

$x = 3,\ y = 9$

Hence, the correct option is (A).

Method 3

Let the digit at one's place will be x and ten's place will be $(12 - x)$.

Hence, the original number $= 10(12 - x) + x$

$= 120 - 10x + x$

$= 120 - 9x$ … (i)

The number obtained by interchanging the digits

$= 10x + 12 - x$

$= 9x + 12$

As per given condition,

$$9x+12=54+120-9x$$

$$9x+9x=174-12$$

$$18x=162$$

$$x=9$$

From equation (i),

Original number $=120-9\times9=120-81=39$

Hence, the correct option is (A).

1.7 (B)

Given : $81\times\left(\frac{16}{25}\right)^{x+2}\div\left(\frac{3}{5}\right)^{2x+4}=144$

$$\left[\left(9\times\frac{4}{5}\right)^{2}\right]^{x+2}\div\left(\frac{3}{5}\right)^{2x+4}=144$$

$$\left[\left(\frac{36}{5}\right)^{x+2}\right]^{2}\div\left[\left(\frac{3}{5}\right)^{x+2}\right]^{2}=144$$

$$\left[\frac{36}{5}\times\frac{5}{3}\right]^{x+2}=12$$

$$12^{x+2}=12$$

So, $x+2=1$

$$x=-1$$

Hence, the correct option is (B).

1.8 (B)

Cyclicity of 1 is always 1.

Cyclicity of 2 is,

$$2^1\rightarrow2$$

$$2^2\rightarrow4$$

$$2^3\rightarrow8$$

$$2^4\rightarrow6$$

$$2^5\rightarrow2$$

Cyclicity of 3 is,

$$3^1\rightarrow3$$

$$3^2\rightarrow9$$

$$3^3\rightarrow7$$

$$3^4\rightarrow1$$

Cyclicity of 4 is,

$$4^1\rightarrow4$$

$$4^2\rightarrow6$$

Unit digit of $2171^7=1$

Unit digit of $2172^9=2$

Unit digit of $2173^{11}=7$

Unit digit of $2174^{13}=4$

Therefore, the last digit of $(2171)^7+(2172)^9+(2173)^{11}+(2174)^{13}=1+2+7+4=\mathbf{14}$

Hence, the correct option is (B).

1.9 (C)

Given :

Sequence of numbers $a_1, a_2.....a_n$

Where $a_n=\frac{1}{n}-\frac{1}{n+2}$

So, $a_1=1-\frac{1}{3},\ a_2=\frac{1}{2}-\frac{1}{4},\ a_3=\frac{1}{3}-\frac{1}{5}$

$$\text{Sum}=\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+.....+\frac{1}{50}-\frac{1}{52}$$

$$\text{Sum}=\left(1+\frac{1}{2}+\frac{1}{3}.....\frac{1}{50}\right)-\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}.....\frac{1}{52}\right)$$

$$\text{Sum}=1+\frac{1}{2}-\frac{1}{51}-\frac{1}{52}$$

$$\text{Sum}=1+\frac{1}{2}-\left(\frac{1}{51}+\frac{1}{52}\right)$$

Hence, the correct option is (C).

1.10 (A)

Given : $\log a+\log b+\log c=0$

$$\log abc=0$$

$$abc=1$$

Since, a, b and c are non-negative numbers and their product is 1, their sum can not be negative or 0.

So, option (C) and (D) are incorrect.

Since, a, b and c are greater than equal to zero, their sum can not be 1.

So, if $a+b+c=1+1+1=3$ [option (A)]

Hence, the correct option is (A).

1.11 (B)

Given :

(i) $\underbrace{a+a+\ldots+a}_{\text{n times}}=a^2b$

$$n\times a=a^2b$$

$$n^2a^2=a^4b^2 \quad \ldots \text{(i)}$$

(ii) $\underbrace{b+b+\ldots+b}_{\text{m times}}=ab^2$

$$m\times b=ab^2$$

$$m^2b^2=a^2b^4 \quad \ldots \text{(ii)}$$

$$\underbrace{m+m+\ldots+m}_{\text{n times}}\times\underbrace{n+n+\ldots+n}_{\text{m times}}=m\times n\times n\times m$$

$$\underbrace{m+m+\ldots+m}_{\text{n times}}\times\underbrace{n+n+\ldots+n}_{\text{m times}}=m^2n^2$$

Multiplying equation (i) and (ii),

$$m^2n^2a^2b^2=a^6b^6$$

$$m^2n^2=a^4b^4$$

Hence, the correct option is (B).

1.12 (B)

Given : Faulty clock gains 15 minutes every 24 hours

On 11th July at 9 am, clock is synchronized to the correct time.

From 11th July, 9 am to 15th July 9 am = 4 days

Total minutes gain $=15\times4=60$ min $=1$ hour .

Every hour the clock gains $\frac{15}{24}$ min $=0.625$ min

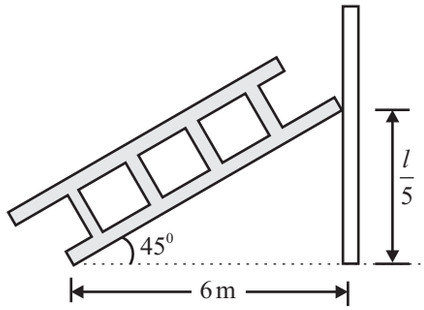
From 15th July 9 am to 15th July 2 pm, total minutes gain $=5\times0.625$ min $=3.125$ min .

So, from 11th July at 9 am to 15th July 2 pm, clock has gain 63.125 minutes, which means the correct time is 12.57 pm or approximately 12:58 pm.

Hence, the correct option is (B).

1.13 (D)

According to given data, the figure can be drawn as shown below,



Let length of pole be l (in meters).

Then, $\tan 45^0=\frac{l/5}{6}$

$$\frac{l}{5}=6\Rightarrow l=30\,\text{m}$$

Hence, the correct option is (D).

1.14 (D)

Let the sequence after inserting 7 terms is given by,

$$2,\ A_1,\ A_2,\ A_3,\ldots\ldots,A_7,\ 34$$

So, the total number of terms of arithmetic progression, $n=9$

$$T_9=34$$

The n^{th} term of the arithmetic progression is given by,

$$a+(n-1)d=34$$

$$2+(9-1)d=34$$

$$8d=32$$

$$d=4$$

Inserted sequence is as shown below,

6, 10, 14, 18, 22, 26, 30

$$\text{Sum} = \left(\frac{6+30}{2}\right)\times 7 = 126$$

Hence, the correct option is (D).

1.15 (C)

Assume, two positive numbers are a and b,

As per question,

$$a+b=100 \quad \ldots \text{(i)}$$

$$(a-5)(b-5)=0 \quad \ldots \text{(ii)}$$

$(a-5)(b-5)$ is zero only when $(a-5)$ or $(b-5)$ is zero

Assuming, $(a-5)=0$

$$a=5$$

$$b=95$$

Hence, the correct option is (C).

1.16 (C)

$(26591749)^{110016} = (26591749)^{2k}$

Unit place $= 9^2 = 9^{even} = 81$

So, the unit digit is 1

Hence, the correct option is (C).

Note : Cyclicity of 9 is (9, 1, 9, 1, ….)

1.17 (A)

Given :

$$f(x)=x^2$$

$$f(f(x))=(x^2)^2=x^4$$

$$f(f(f(x))=(x^4)^2=x^8$$

$$\frac{f(f(f(x)))}{f(x)}=\frac{x^8}{x^2}=x^6$$

$$=(f(x))^3$$

Hence, the correct option is (A).

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