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GATE SYLLABUS

Electrostatics; Maxwell's equations: differential and integral forms and their interpretation, boundary conditions, wave equation, Poynting vector; Plane waves and properties: reflection and refraction, polarization, phase and group velocity, propagation through various media, skin depth; Transmission lines: equations, characteristic impedance, impedance matching, impedance transformation, S-parameters, Smith chart; Waveguides: modes, boundary conditions, cut-off frequencies, dispersion relations; Antennas: antenna types, radiation pattern, gain and directivity, return loss, antenna arrays; Basics of radar; Light propagation in optical fibers.

ESE SYLLABUS

Elements of vector calculus, Maxwell's equations-basic concepts; Gauss', Stokes' theorems; Wave propagation through different media; Transmission Lines-different types, basics, Smith's chart, impedance matching/transformation, S-parameters, pulse excitation, uses; Waveguides-basics, rectangular types, modes, cut-off frequency, dispersion, dielectric types; Antennas-radiation pattern, monopoles/dipoles, gain, arrays-active/passive, theory, uses.

UGC-NET SYLLABUS

Maxwell's equations, Time varying fields, Wave equation and its solution, Rectangular waveguide, Poynting vector, Antenna parameters, Half-wave antenna, Transmission lines, Characteristic of Impedance matching, Smith chart.

CHAPTER 2

Electrostatic

Learning Objectives :

After reading this chapter you should be able to :

- describe Coulomb's law and its application
- explain electric field intensity due to various charge distribution
- describe Gauss's law and concept of Gaussian surface
- describe potential difference and potential gradient
- explain capacitive system and capacitance of different capacitors
- understand electric boundary conditions
- apply method of images

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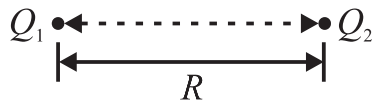
2.1 Coulomb's Law

The Coulomb's law states that force between the two point charges Q_1 and Q_2,

(i) acts along the line joining the two point charges.



(ii) is directly proportional to the product (Q_1Q_2) of the two charges.

(iii) is inversely proportional to the square of the distance between them.

Consider two point charges Q_1 and Q_2 as shown in figure, separated by the distance R. The charge Q_1 exerts a force on Q_2, while Q_2 also exerts a force on Q_1.

(iv) The force acts along the line joining Q_1 and Q_2. The force exerted between them is repulsive if the charges are of same polarity while it is attractive if the charges are of different polarity.

Mathematically the force F between the charges can be expressed as,

$$F \propto \frac{Q_1Q_2}{R^2}$$

where, $Q_1Q_2 =$ Product of the two charges, $R =$ Distance between the two charges

The Coulomb's law also states that this force depends on the medium in which the point charges are located.

$$\therefore \qquad F = k\frac{Q_1Q_2}{R^2} \quad \text{where, } k = \text{Constant of proportionality} = \frac{1}{4\pi\varepsilon}$$

$$\varepsilon = \varepsilon_0\varepsilon_r = \text{Permittivity of the medium in which charges are located.}$$

where, $\varepsilon_0 =$ Permittivity of the free space or vacuum $= \frac{1}{36\pi}\times 10^{-9} = 8.854\times 10^{-12}$ F/m

$\varepsilon_r =$ Relative permittivity or dielectric constant of the medium.

In free space, $k = \frac{1}{4\pi\varepsilon_0} = \frac{1\times 36\pi}{4\pi\times 1\times 10^{-9}} = 9\times 10^9$ F/m

Hence the Coulomb force in free space is given by,

$$F = \frac{9\times 10^9 \times Q_1Q_2}{R^2}$$

Vector form of Coulomb's Law : The force exerted between the two point charge has a fixed direction which is a straight line joining the two charges. Hence the force exerted between the two charges can be expressed in a vector form.

Consider the two point charges Q_1 and Q_2 located at the points having position vectors $\vec{r}_1$ and $\vec{r}_2$ as shown in the figure.

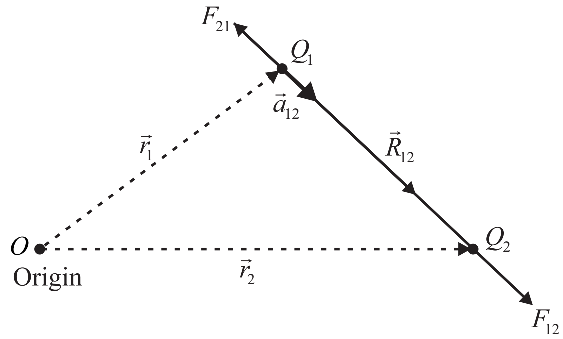


Fig. Vector form of Coulomb's law

Then the force exerted by Q_1 and Q_2 acts along the direction $\vec{R}_{12}$ where $\vec{a}_{12}$ is unit vector along $\vec{R}_{12}$. Hence the force in the vector form can be expressed as,

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\vec{a}_{12}$$

Where $\vec{a}_{12} =$ Unit vector along $\vec{R}_{12} = \frac{\text{Vector}}{\text{Magnitude of vector}}$

$$\therefore \qquad \vec{a}_{12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{R}_{12}|} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

where, $|\vec{R}_{12}| = R_{12} =$ Distance between the two charges.

Applications :

(i) Find the force between a pair of charges.

(ii) Find the potential at a point due to fixed charge.

(iii)Find the electric field at a point due to a fixed charge.

(iv)Find the displacement flux density indirectly.

(v) Find the potential and electric field due to any type of charge distribution.

(vi)Find the charge if the force and the electric field are known.

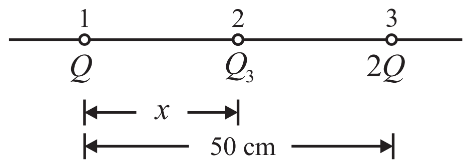
Limitation of Coulomb's Law :

It is difficult to apply the law when charges are of arbitrary shape. Here, the distance, can't be determined accurately as the centers of arbitrarily shaped charge bodies can't be identified accurately.

Solved Example 1

A system of three electric charges lying in a straight line is in equilibrium. Two of the charges are positive with magnitudes Q and $2Q$ and are 50 cm apart. Determine the sign, magnitude and position of the third charge.

Sol. Let Q_3 be the third charge located at a distance x cm from the first charge. Hence, the three charges can be shown as in figure below.



Force on Q due to Q_3 and $2Q$,

$$\frac{kQ_3.Q}{x^2}+\frac{kQ.2Q}{(50)^2}=0 \qquad \ldots(i)$$

Force on Q_3 due to Q and $2Q$,

$$\frac{k\cdot Q_3Q}{x^2}-\frac{kQ_3 2Q}{(50-x)^2}=0 \qquad \ldots(ii)$$

After solving equation (ii),

$$\frac{k\cdot Q_3Q}{x^2}=\frac{kQ_3 2Q}{(50-x)^2}$$

$$(50-x)^2=2x^2 \qquad \ldots(iii)$$

After solving equation (iii),

$$x=20.71$$

Substitute $x=20.71$ in equation (i),

$$Q_3=-\frac{x^2\times 20}{(50)^2}$$

$$Q_3=-3.43Q$$

Ans.

2.2 Electric Field Intensity and Electric Flux Density

The electric field intensity (or electric field strength) $\vec{E}$ is the force per unit charge when placed in an electric field.

$$\vec{E}=\frac{Q}{4\pi\varepsilon_0 R^2}\vec{a}_R$$

Salient Features of Electric Field Intensity :

1. It has unit of Newton/Coulomb or Volts/metre.
2. It is a vector quantity, which means that it is having both direction and magnitude.
3. Its direction is the same as that of Coulomb's force.
4. Its magnitude depends on the magnitude of Coulomb's force and the charge on which the force is acting.
5. It depends on the permittivity of the medium.
6. It depends on the distance of the charge from another charge which produces Coulomb's force.
7. It depends on the location of the charges.
8. It originates from a positive charge and terminates on a negative charge.

9. Consider a charge Q_1 as shown in the figure. The unit positive charge $Q_2 = 1\text{C}$ is placed at a distance R from Q_1. Then the force acting on Q_2 due to Q_1 is along the unit vector $\vec{a}_R$. As the charge Q_2 is unit charge, the force exerted on Q_2 is nothing but electric field intensity $\vec{E}$ of Q_1 at the point where unit charge is placed.

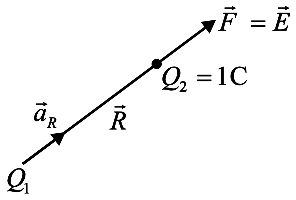


Fig. Concept of electric field intensity

$$\vec{E} = \frac{\vec{F}}{Q_2}$$

$$\vec{E} = \frac{Q_1}{4\pi\varepsilon_0 R^2}\hat{a}_R = \frac{Q\vec{R}}{4\pi\varepsilon_0 R^3}$$

Electric Flux Density :

The flux due to the electric field E can be calculated by using the general definition of flux

$$\vec{D} = \varepsilon_0 \vec{E} \qquad \ldots\text{(i)}$$

Electric flux ψ in terms of $\vec{D}$,

$$\psi = \int_S \vec{D}\cdot d\vec{s} \qquad \ldots\text{(ii)}$$

The vector field $\vec{D}$ is called the electric flux density and is measured in coulombs per square meter. For historical reasons, the electric flux density is also called **electric displacement**.

Properties of Electric flux density are as follows :

(i) It is a vector quantity.

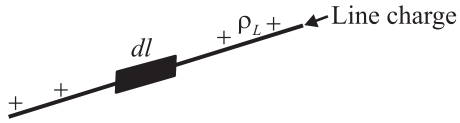
(ii) It is inversely proportional to r^2, where r being the radius of sphere.

(iii)It is independent of medium.

2.3 Continuous Charge Distribution

1. Line charge distribution.
2. Surface charge distribution.
3. Volume charge distribution.

1. Line charge distribution : In case of line charge distribution, charge is distributed along the line therefore line charge density exists along the line. If charge is uniformly distributed along a line then charge density will be constant.



Line charge density is given by,

$$\lambda = \rho_L = \frac{dQ}{dl} \text{ C/m}$$

$$dQ = \rho_L dl$$

$$Q = \int_L \rho_L dl$$

2. **Surface charge distribution :** In case of surface charge distribution charge is distributed along the surface therefore surface charge density exists along the surface.
   If charge is uniformly distributed along the surface, then charge density will be constant.

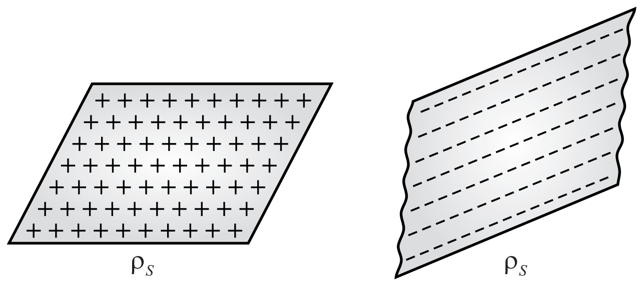


Fig. Surface charge distribution

$$\rho_s = \frac{dQ}{ds} \text{ C/m}^2$$

$$\rho_s ds = dQ$$

$$Q = \int_s \rho_s ds$$

$$Q = \int_{x=0}^{x_0} \int_{y=0}^{y_0} \rho_s dx\, dy$$

3. **Volume charge distribution :** In case of volume charge distribution, charge is distributed along the volume therefore volume charge density exists along the volume.

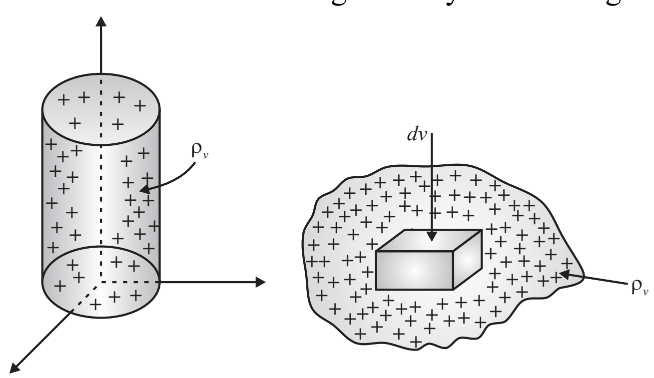


Fig. Volume charge distribution

If charge is distributed uniformly along the volume then charge density will be constant

$$\rho_v = \frac{dQ}{dv} \text{ C/m}^3$$

$$dQ = \rho_v dv$$

$$Q = \int_v \rho_v dv$$

$$Q = \int_{x=0}^{x_0} \int_{y=0}^{y_0} \int_{z=0}^{z_0} \rho_v \, dx \, dy \, dz$$

2.4 Electric Field Due to Various Charge Distribution

2.4.1 Electric Field Due to Infinite Line Charge Distribution

Consider an infinitely long straight line carrying uniform line charge having density ρ_L C/m. Let this line lies along z-axis from $-\infty$ to ∞ and hence called infinite line charge. Let point P is on y-axis at which electric field intensity is to be determined. The distance of point P from the origin is 'r' as shown in the below figure (a).

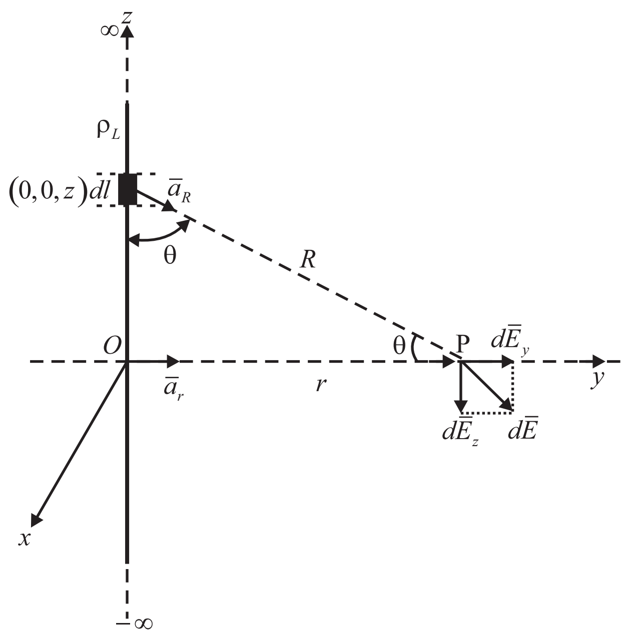


Fig. (a) Field due to infinite line charge

Consider a small differential length dl carrying a charge dQ, along the line as shown in the above figure (a). It is along z-axis hence $dl = dz$.

$$dQ = \rho_L \, dl = \rho_L dz \qquad \ldots\text{(i)}$$

The co-ordinates of dQ are $(0, 0, z)$ while the co-ordinates of point P are $(0, r, 0)$. Hence the distance vector $\vec{R}$ can be written as,

$$\vec{R} = \vec{r}_p - \vec{r}_{dl} = \left[r\vec{a}_y - z\vec{a}_z\right]$$

$$\therefore \qquad \left|\vec{R}\right| = \sqrt{r^2 + z^2}$$

$$\therefore \qquad \vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{r\vec{a}_y - z\vec{a}_z}{\sqrt{r^2 + z^2}} \qquad \ldots\text{(ii)}$$

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\vec{a}_R$$

$$= \frac{\rho_L dz}{4\pi\varepsilon_0\left(\sqrt{r^2+z^2}\right)^2}\left[\frac{r\vec{a}_y - z\vec{a}_z}{\sqrt{r^2+z^2}}\right] \quad \ldots\text{(iii)}$$

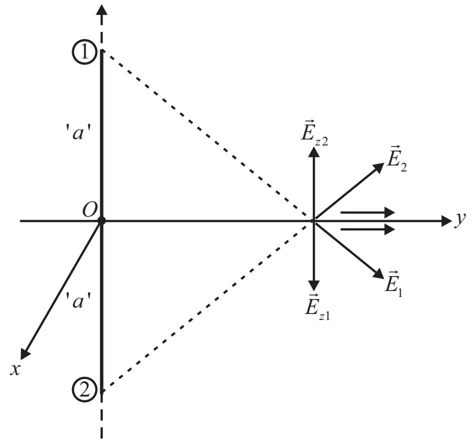


Fig. (b)

Note : For every charge on positive z-axis there is equal charge present on negative z-axis. Hence the z component of electric field intensities produced by such charges at point P will cancel each other. Hence effectively there will not be any z component of $\vec{E}$ at P. This is shown in above figure (b).

Hence the equation of $d\vec{E}$ can be written by eliminating $\vec{a}_z$ component,

$$\therefore \quad d\vec{E} = \frac{\rho_L dz}{4\pi\varepsilon_0\left(\sqrt{r^2+z^2}\right)^2}\frac{r\vec{a}_y}{\sqrt{r^2+z^2}} \quad \ldots\text{(iv)}$$

Now by integrating $d\vec{E}$ over the z-axis from $-\infty$ to ∞ we can obtain total $\vec{E}$ at point P.

$$\therefore \quad \vec{E} = \int_{-\infty}^{\infty}\frac{\rho_L}{4\pi\varepsilon_0\left(r^2+z^2\right)^{3/2}} r dz \vec{a}_y$$

For the such an integration, use the substitution

$$z = r\tan\theta \quad \text{i.e. } r = \frac{z}{\tan\theta}$$

$$\therefore \quad dz = r\sec^2\theta\, d\theta$$

Here r is not the variable of integration.

For $z = -\infty$, $\quad \theta = \tan^{-1}(-\infty) = -\pi/2 = -90^0$

For $z = +\infty$, $\quad \theta = \tan^{-1}(\infty) = +\pi/2 = +90^0$

$$\therefore \quad \vec{E} = \int_{\theta=-\pi/2}^{\pi/2}\frac{\rho_L}{4\pi\varepsilon_0\left[r^2+r^2\tan^2\theta\right]^{3/2}} r \times r\sec^2\theta\, d\theta\, \vec{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\int_{-\pi/2}^{\pi/2}\frac{r^2\sec^2\theta d\theta}{r^3\left[1+\tan^2\theta\right]^{3/2}}\vec{a}_y$$

But $1+\tan^2\theta = \sec^2\theta$

$$\therefore \qquad \vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{-\pi/2}^{\pi/2}\frac{\sec^2\theta\, d\theta}{r\sec^3\theta}\vec{a}_y = \frac{\rho_L}{4\pi\varepsilon_0 r}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta\, \vec{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r}\left[\sin\theta\right]_{-\pi/2}^{\pi/2}\vec{a}_y = \frac{\rho_L}{4\pi\varepsilon_0 r}\left[\sin\frac{\pi}{2} - \sin\left(\frac{-\pi}{2}\right)\right]\vec{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r}\left[1-(-1)\right]\vec{a}_y = \frac{\rho_L}{4\pi\varepsilon_0 r}\times 2\vec{a}_y$$

$$\therefore \qquad \vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\vec{a}_y \text{ V/m} \qquad \ldots(\text{v})$$

The $\vec{a}_y$ is unit vector along the distance r which is perpendicular distance of point P from the line charge. Thus in general $\vec{a}_y = \vec{a}_r$.

Hence the result of $\vec{E}$ can be expressed as,

$$\boldsymbol{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\vec{a}_r \text{ V/m}} \qquad \ldots(\text{vi})$$

where $r =$ Perpendicular distance of point P from the line charge.

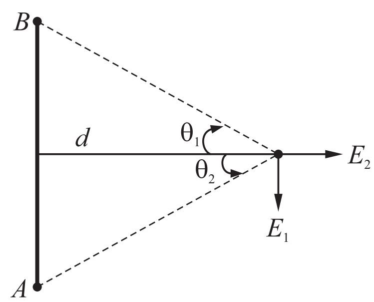
$\vec{a}_r =$ Unit vector in the direction of the perpendicular distance of point P from the line charge.

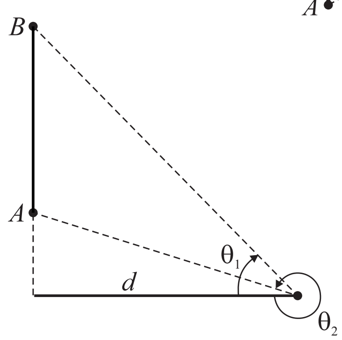
Remember

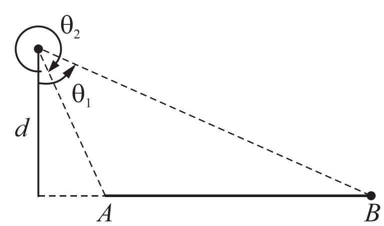
Electric field due to finite length wire :

$$\vec{E}_{||} = \frac{K\rho_l}{d}[\cos\theta_2 + \cos\theta_1]$$

$$\vec{E}_{\perp} = \frac{K\rho_l}{d}[\sin\theta_1 + \sin\theta_2]$$







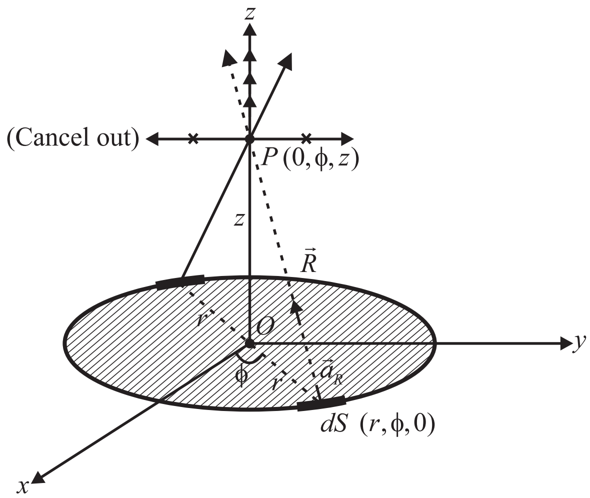
2.4.2 Electric Field Due to Circular Ring

Assumptions :

(i) The charged circular ring is placed in xy plane with center at origin.

(ii) The point of interest is placed on z-axis at (0, 0, z).

(iii) We use cylindrical coordinate system.



The differential charge dQ on the circular ring with radius r is

$$dQ = \rho_l\, dl = \rho_l\,(r d\phi) \quad \ldots\text{(i)}$$

The electric field due to charge dQ is,

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 |\vec{R}|^2}\vec{a}_R \quad \ldots\text{(ii)}$$

Where the vector joining dQ and point P is,

$$\vec{R} = (0-r)\vec{a}_r + (\phi-\phi)\vec{a}_\phi + (z-0)\vec{a}_z = -r\,\vec{a}_r + z\vec{a}_z$$

Then $$|\vec{R}| = \sqrt{r^2+z^2}$$

$$\vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{-r\vec{a}_r + z\vec{a}_z}{\sqrt{r^2+z^2}} \quad \ldots\text{(iii)}$$

Putting equation (i) and (iii) in equation (ii), we get

$$d\vec{E} = \frac{dQ}{4\pi\,\varepsilon_0 |\vec{R}|^2}\vec{a}_r = \frac{\rho_l\, dl}{4\pi\varepsilon_0 (r^2+z^2)} \times \frac{-r\,\vec{a}_r + z\,\vec{a}_z}{\sqrt{r^2+z^2}}$$

$$= \frac{\rho_l\,(r\,d\phi)}{4\pi\,\varepsilon_0 (r^2+z^2)^{3/2}}(-r\,\vec{a}_r + z\vec{a}_z)$$

The total electric field is obtained by integration. But in the process due to symmetry horizontal components of the field $(\vec{a}_r)$ gets cancelled.

$$\vec{E} = \int d\vec{E} = \int_0^{2\pi} \frac{\rho_l\,(r d\phi)}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}(z\,\vec{a}_z)$$

where z is the height of the point from the plane of the ring, r is the radius of the ring.

This is the expression for $\vec{E}$ due to circular charged ring at a point on the axis of the ring.

$$\vec{E} = \frac{\rho_l r z}{2\varepsilon_0 \left(r^2 + z^2\right)^{3/2}} \vec{a}_z \text{ V/m}$$

2.4.3 Electric Field Due to Circular Disk

As the circular charged sheet is a solid cylinder with zero-height and radius a, the solution becomes simple if we tackle the problem in cylindrical coordinate system.

Place the sheet charge over the sheet charge over the xy-plane with the observation point P over the positive z-axis below figure. Let us suppose the radius of the sheet be a and hence the charge extends to a in all directions. i.e. $\rho \to a$.

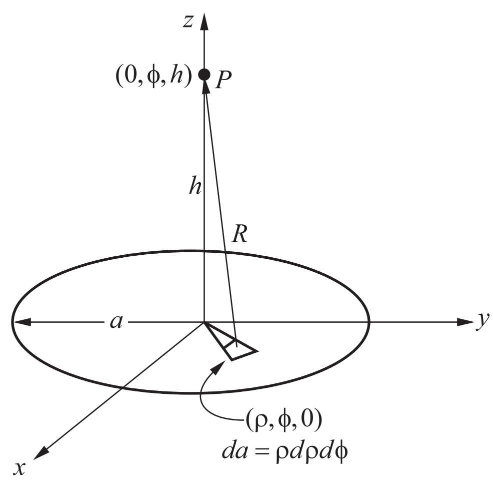


Fig. Electric field intensity due to circular charge disc

The field intensity at $(0, \phi\ h)$ due to differential plane charge at $(\rho, \phi, 0)$ is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{R^2}\vec{R} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\rho d\rho d\phi}{(\rho^2+h^2)}\frac{(h\vec{a}_z - \rho\vec{a}_\rho)}{(\rho^2+h^2)^{1/2}}$$

Since for every dQ over the plane charge, there is another charge dQ on the sheet diametrically opposite to it resulting in the cancellation ρ components. Thus

$$E = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{a}\frac{\sigma h\rho d\rho d\phi}{(\rho^2+h^2)^{3/2}}\vec{a}_z = \frac{\sigma h\vec{a}_z}{2\varepsilon_0}\left[\frac{-1}{\left(\rho^2+h^2\right)^{1/2}}\right]_0^a$$

$$= \frac{\sigma h}{2\varepsilon_0}\left[\frac{-1}{(a^2+h^2)^{1/2}} + \frac{1}{h}\right]\vec{a}_z$$

If the observation is below the sheet then the expression for the field becomes

$$E = \frac{\sigma h}{2\varepsilon_0}\left[\frac{-1}{(a^2+h^2)^{1/2}} + \frac{1}{h}\right](-\vec{a}_z)$$

We can observe that the circular disc becomes an infinite plane sheet if we extend the sheet to infinity $\rho \to a \to \infty$. The field intensity of the disc also becomes that of the infinite plane sheet.

2.4.4 Electric Field Due to Infinite Surface Charge Distribution

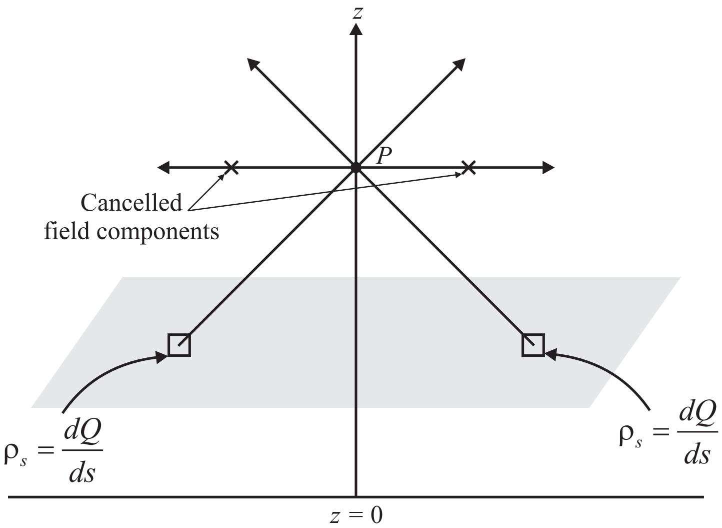
Consider an infinite sheet of charge having uniform charge density ρ_s C/m^2 placed in xy plane as shown in the figure. The point P at which $\vec{E}$ to be calculated is on z-axis by using cylindrical co-ordinates. Consider the differential surface area ds carrying a charge dQ. The normal direction to ds is z direction hence ds normal to z direction is $\rho\, d\rho\, d\phi$.

Now $\quad dQ = \rho_s ds = \rho_s \rho\, d\rho\, d\phi$

Hence, $\quad d\vec{E} = \dfrac{dQ}{4\pi\varepsilon_0 R^2}\vec{a}_R = \dfrac{\rho_s \rho\, d\rho\, d\phi}{4\pi\varepsilon_0 R^2}\vec{a}_R$

The distance vector $\vec{R}$ has two components as shown in the figure (b).

1. The radial component ρ along $-\vec{a}_\rho$, i.e. $-\rho\vec{a}_\rho$.
2. The component z along $\vec{a}_z$, i.e. $z\vec{a}_z$



With these two components $\vec{R}$ can be obtained from the differential area towards point P as,

$$\vec{R} = -\rho\vec{a}_\rho + z\vec{a}_z$$

$$|\vec{R}| = \sqrt{(-\rho)^2 + (z)^2} = \sqrt{\rho^2 + z^2}$$

$$\therefore \quad \vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{-\rho\vec{a}_\rho + z\vec{a}_z}{\sqrt{\rho^2 + z^2}}$$

$$d\vec{E} = \frac{\rho_s\, \rho\, d\rho\, d\phi}{4\pi\varepsilon_0\left(\sqrt{\rho^2 + z^2}\right)^2}\left[\frac{-\rho\vec{a}_\rho + z\vec{a}_z}{\sqrt{\rho^2 + z^2}}\right]$$

For infinite sheet in xy plane, ρ varies from 0 to ∞ while ϕ varies from 0 to 2π.

As there is symmetry about z-axis from all radial direction, all $\vec{a}_\rho$ components of $\vec{E}$ are going to cancel each other and net $\vec{E}$ will not have any radial component.

Hence while integrating $d\vec{E}$ there is no need to consider $\vec{a}_\rho$ component. Though if considered, after integration procedure, it will get mathematically cancelled.

$$\vec{E} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\infty} d\vec{E} = \int_0^{2\pi}\int_0^{\infty} \frac{\rho_s \rho\, d\rho\, d\phi}{4\pi\varepsilon_0(\rho^2+z^2)^{\frac{3}{2}}}(z\vec{a}_z)$$

Put $\rho^2 + z^2 = u^2$ hence $2\rho\, d\rho = 2u\, du$

For $\rho = 0,\ u = z$ and $\rho = \infty,\ u = \infty$

$$\therefore \quad \vec{E} = \int_0^{2\pi}\int_{u=z}^{\infty} \frac{\rho_s}{4\pi\varepsilon_0}\frac{u\,du}{(u^2)^{\frac{3}{2}}} d\phi\, z\,\vec{a}_z = \int_0^{2\pi}\int_{u=z}^{\infty} \frac{\rho_s}{4\pi\varepsilon_0}\frac{du}{(u^2)} d\phi (z\,\vec{a}_z)$$

$$\vec{E} = \int_0^{2\pi} \frac{\rho_s}{4\pi\varepsilon_0} d\phi\, z\,\vec{a}_z \left[-\frac{1}{u}\right]_z^{\infty} \qquad \left\{ \text{as } \int \frac{1}{u^2} = \int u^{-2} = \frac{u^{-1}}{-1} = \frac{-1}{u} \right\}$$

$$\vec{E} = \frac{\rho_s}{4\pi\varepsilon_0}[\phi]_0^{2\pi} z\vec{a}_z \left[-\frac{1}{\infty} - \left(-\frac{1}{z}\right)\right] = \frac{\rho_s}{4\pi\varepsilon_0} 2\pi\vec{a}_z$$

$$\therefore \quad \vec{E} = \frac{\rho_s}{2\varepsilon_0}\vec{a}_z \text{ V/m}$$

This result is for points above xy-plane. Below the xy-plane, the unit vector changes to $-\vec{a}_z$.

The generalized form if $\vec{a}_n$ is direction normal to the surface containing charge, the above result can be expressed as,

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\vec{a}_n \text{ V/m} \qquad \ldots\text{(i)}$$

Where $\vec{a}_n$ = Direction normal to the surface charge

Thus for the point below xy-plane, $\vec{a}_n = -\vec{a}_z$ hence,

$$\therefore \quad \vec{E} = -\frac{\rho_s}{2\varepsilon_0}\vec{a}_z \text{ V/m} \qquad \ldots\text{(ii)}$$

For points below xy-plane.

From equation (i) electric filed intensity at point P, due to surface charge distribution on infinite plane is independent of distance between point P and infinite plane.

Remember

1. $\vec{E}$ due to infinite sheet of charge at a point is not dependent on the distance of that point from the plane containing the charge.
2. The direction of $\vec{E}$ is perpendicular to the infinite charge plane.
3. The magnitude of $\vec{E}$ is constant everywhere and given by $|\vec{E}| = \frac{\rho_s}{2\varepsilon_0}$.

Solved Example 2

A line charge, $\rho_L = 50$ nC/m, is located along the line $x = 2,\ y = 5$, in free space.

(a) Find E at $P(1,\ 3,\ -4)$.

(b) If the surface x = 4 contains a uniform surface charge density, $\rho_S = 18$ nC/m, at what point in the z = 0 plane is $E_{\text{total}} = 0$?

Sol. The line charge density $\rho_L = 50$ nC/m, on line $x = 2,\ y = 5$

(a) Due to line charge, the radial distance between the line charge and point $P(1,\ 3,\ -4)$ is

$$R = \sqrt{(1-2)^2 + (3-5)^2} = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$$

$$\vec{E}_1 = \frac{\rho_L}{2\pi\varepsilon_0 R}\vec{a}_R$$

Where, $$\vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{(1-2)\vec{a}_x + (3-5)\vec{a}_y}{\sqrt{5}} = \frac{-\vec{a}_x - 2\vec{a}_y}{\sqrt{5}}$$

Thus $$\vec{E}_1 = \frac{50\times 10^{-9+12}}{2\pi\times 8.854\times 5}\left(-\vec{a}_x - 2\vec{a}_y\right) = -179.8\vec{a}_x - 359.51\vec{a}_y \text{ V/m}$$

(b) Surface charge density, $\rho_S = 18$ nC/m^2 at plane $x = 4$.

Let at point $(x,\ y,\ 0)$, the $E_{Total} = 0$

At point $(x,\ y,\ 0)$, the electric field intensity due to line charge only is

$$\vec{E}_L = \frac{50\times 10^{-9+12}}{2\pi\times 8.854}\times\frac{(x-2)\vec{a}_x + (y-5)\vec{a}_y}{(x-2)^2 + (y-5)^2} = 899.18\times\frac{(x-2)\vec{a}_x + (y-5)\vec{a}_y}{(x-2)^2 + (y-5)^2}$$

Due to surface charge density the electric field intensity is,

$$\vec{E}_S = \frac{\rho_S}{2\varepsilon_0}\vec{a}_n$$

The unit vector normal to the sheet directed away from it is $\vec{a}_n = -\vec{a}_x$

$$\vec{E}_S = \frac{\rho_S}{2\varepsilon_0}\vec{a}_n = \frac{-18\times 10^{-9+12}}{2\times 8.854}\vec{a}_x = -1016.95\ \vec{a}_x$$

$$E_{total} = 899.18\times\frac{(x-2)\vec{a}_x + (y-5)\vec{a}_y}{(x-2)^2 + (y-5)^2} - 1016.95\vec{a}_x = 0$$

Solving we get $y = 5$ m and $x = 2.88$ m **Ans.**

Solved Example 3

The electric flux density is given as $\vec{D} = x^3\vec{a}_x + x^2 y\vec{a}_z$. Find the charge density inside a cube of side 2 m placed centered at the origin with its sides along the co-ordinate axes.

Sol. The volume charge density is

$$\rho = \nabla.\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\rho = \frac{\partial}{\partial x}\left(x^3\right) + 0 + \frac{\partial}{\partial z}\left(x^2 y\right) \quad \rho = 3x^2$$

The charge enclosed by the cube is

$$Q = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \rho dxdydz = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 3x^2 dxdydz$$

$$Q = 12\int_{-1}^{1} x^2 dx = 12.\frac{2}{3} = 8 \text{ C}$$

Ans.

Solved Example 4

The electric flux density is given by $\vec{D} = \frac{100\cos 2\theta}{r}\vec{a}_\theta$ C/m^2. Find the charge enclosed within the region $1 < r < 2,\ 0 < \theta < \pi/2$ rad.

Sol. The volume charge density is

$$\rho = \nabla.\ \vec{D} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(D_\theta \sin\theta\right)$$

since D_r and D_ϕ are = 0.

$$\rho = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left\{\frac{100\cos 2\theta}{r}\sin\theta\right\}$$

$$\rho = \frac{100}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left\{\frac{\sin 3\theta - \sin\theta}{2}\right\}$$

$$\rho = \frac{50}{r^2\sin\theta}\left\{3\cos 3\theta - \cos\theta\right\}$$

The charge enclosed in the region is

$$Q = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\int_{r=1}^{2} \frac{50}{r^2\sin\theta}\left\{3\cos 3\theta - \cos\theta\right\} r^2 \sin\theta\, dr d\theta d\phi z$$

$$Q = 2\pi(50)\int_1^2 dr \int_0^{\pi/2}\left(3\cos 3\theta - \cos\theta\right) d\theta$$

$$Q = 100\pi\left[\sin 3\theta - \sin\theta\right]_0^{\pi/2} = -200\pi \text{C}$$

Ans.

TEST 1

Q.1 The electrostatic force of repulsion between two α-particles of charges 4.0×10^{-19} C each and separated by a distance of 10^{-10} cm is

Given : $\varepsilon_0 = 8.854\times10^{-12}$ C^2/Nm^2

(A) 57.6×10^{-4} N (B) 28.8×10^{-4} N

(C) 14.4×10^{-4} N (D) 3.6×10^{-4} N

Q.2 Point charges of $Q_1 = 2$ nC and $Q_2 = 3$ nC are located at a distance apart. With regard to this situation, which one of the following statements is not correct ?

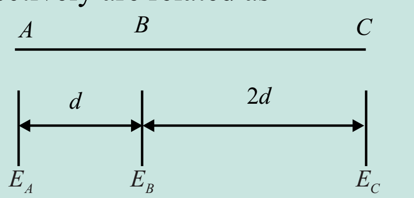
(A) The force on the 3 nC charge is repulsive.

(B) A charge of – 5 nC placed midway between Q_1 and Q_2 will experience no force.

(C) The forces Q_1 and Q_2 are same in magnitude.

(D) The forces on Q_1 and Q_2 will depend on the medium in which they are placed.

Q.3 A plane electromagnetic wave is traveling in a highly dissipative medium in the direction ABC as shown in the figure. The electric field E_A, E_B and E_C at points A, B and C respectively are related as



(A) $E_A^2\ E_C = E_B^2$ (B) $E_A\ E_C^2 = E_B^3$

(C) $E_A\ E_C = E_B^2$ (D) $E_A\ E_B = E_C^2$

Q.4 The plane $x = 3$ has a layer of charge density $2\,\text{nC/m}^2$. A line charge of density 20 nC/m is located at $x = 1$, $z = 4$. The force acting on unit meter length of the line charge is

(A) $\dfrac{2\times10^{-9}\times20\times10^{-9}}{4\pi\varepsilon_0}\hat{a}_x \text{N}$

(B) $\dfrac{2\times10^{-9}\times20\times10^{-9}}{2\pi\varepsilon_0}\hat{a}_z \text{N}$

(C) $\dfrac{2\times10^{-9}\times20\times10^{-9}}{2\varepsilon_0}\hat{a}_y \text{N}$

(D) $\dfrac{2\times10^{-9}\times20\times10^{-9}}{2\varepsilon_0}\hat{a}_x \text{N}$

Q.5 A positive charge of Q coulombs is located at point $A(0, 0, 3)$ and a negative charge of magnitude Q coulombs is located at point $B(0,0, -3)$. The electric field intensity at point $C(4, 0, 0)$ is in the

(A) Negative x-direction

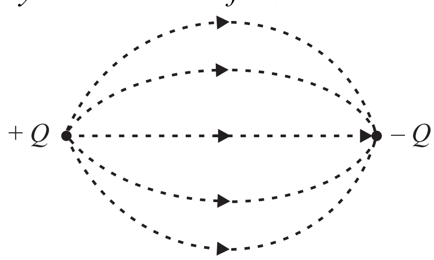
(B) Negative z-direction

(C) Positive x-direction

(D) Positive z-direction

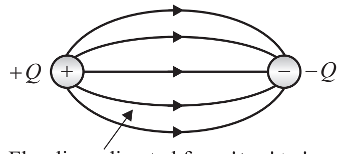
2.5 Gauss's Law

Statement : *Gauss's law states that the total electric flux ψ passing through any closed surface is equal to the total charge enclosed by that closed surface.*



$$Q_{enc.} = \psi$$

Proof :



According to Gauss law the electric flux is charge,

$$\text{Unit}[Q] = \text{coulomb}$$

$$\text{Unit}[\psi] = \text{coulomb}$$

Consider a positive charge Q situated at the center of an imaginary sphere of radius r. Electric flux density is defined as flux per unit surface area,

$$D = \frac{d\psi}{ds}\ \text{C/m}^2$$

$$D = \frac{\psi}{s}\ \text{C/m}^2$$

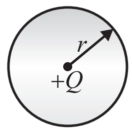


Fig. Spherical shell

For spherical coordinates the differential surface is

$$ds = r^2 \sin\theta\, d\theta\, d\phi\, \vec{a}_r \Big|_{r=c}$$

$$S = 4\pi r^2$$

So, $$D = \frac{\psi}{4\pi r^2}$$

$$\vec{D} = \frac{\psi}{4\pi r^2}\vec{a}_r$$

$\therefore$ $$\vec{D} = \frac{d\psi}{ds}$$

$$d\psi = \vec{D}.d\vec{s}$$

$$\psi = \int_s \vec{D}.d\vec{s}$$

Since a point charge is situated at center, the flux density,

So, $$\vec{D} = \frac{Q_{enc}}{4\pi r^2}\vec{a}_r$$

$$\vec{D}.d\vec{s} = \frac{Q_{enc}}{4\pi r^2}.\vec{a}_r.\left[r^2 \sin\theta\, d\theta\, d\phi\, \vec{a}_r\right]$$

$$\vec{D}.d\vec{s} = \frac{Q_{enc}}{4\pi}\sin\theta\, d\theta\, d\phi$$

$$\psi = \int \vec{D}.d\vec{s} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{Q_{enc}}{4\pi}\sin\theta\, d\theta\, d\phi$$

$$\psi = \frac{Q_{enc}}{4\pi}[-\cos\theta]_0^{\pi}[\phi]_0^{2\pi}$$

$$\psi = \frac{Q_{enc}}{4\pi}[2][2\pi]$$

$$\psi = Q_{enc}$$

Let ρ_v be the volume charge density, then total charge inside the volume

$$Q = \oint_S \vec{D}\cdot d\vec{s} = \int_v \rho_v dv$$

By applying divergence theorem,

$$\oint_S \vec{D}\cdot d\vec{s} = \int_v \nabla\cdot\vec{D}\,dv$$

Comparing the two volume integrals, we get

$$\rho_v = \nabla\cdot\vec{D}$$

This equation is known as **Maxwell's 1st equation** in differential form.

2.6 Applications of Gauss's Law

Gauss's Law finds applications in determining electric field intensity due to commonly known charge distributions. Gauss's law can be applied effectively where the charge distribution has a symmetry.

2.6.1 Electric Field Due to a Point Charge

The field due to point charge Q at the origin is in radial direction i.e. $\vec{a}_r$ (in spherical) direction. The only component of $\vec{D}$ is D_r.

(i) This field is the function of r only.

(ii) Select a surface which satisfies the conditions of Gaussian surface. The Gaussian surface is a spherical surface centred at origin.

Charge enclosed within gaussian surface :

Since charge Q is at the center of the spherical surface, the only charge enclosed by the Gaussian surface is the charge Q i.e.

$$Q_{enclosed} = Q \quad \text{...(i)}$$

Flux through the closed surface : The field $\vec{D}$ has only D_r Component in $\vec{a}_r$ direction and $d\vec{s}$ for sphere is also in same direction $(d\vec{s} = r^2\sin\theta\, d\theta\, d\phi\, \vec{a}_r)$.

$\therefore$ $$\vec{D}\cdot d\vec{s} = D_r\, ds$$

The flux through the closed surface is given by,

$$\psi = \oint \vec{D}\bullet d\vec{s} = \oint D_r\, ds = D_r \oint ds = D_r(4\pi r^2) \quad \text{...(ii)}$$

From equation (i) and (ii), and using Gauss's law, total flux through the closed surface = charge enclosed

i.e. $$D_r(4\pi r^2) = Q \quad \Rightarrow D_r = \frac{Q}{4\pi r^2}$$

$$\vec{D} = D_r\,\vec{a}_r = \frac{Q}{4\pi r^2}\vec{a}_r \text{ C/m}^2$$

The field intensity is given by,

$$\vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{Q}{4\pi\varepsilon r^2}\vec{a}_r \text{ V/m}$$

2.6.2 Electric Field Due to Infinite Line Charge Distribution

(i) Consider an infinite line charge of density ρ_L C/m lying along z-axis from $-\infty$ to $+\infty$.

(ii) Consider the Gaussian surface as the right circular cylinder with z-axis as its axis and radius r as shown in the figure. The length of the cylinder is L.

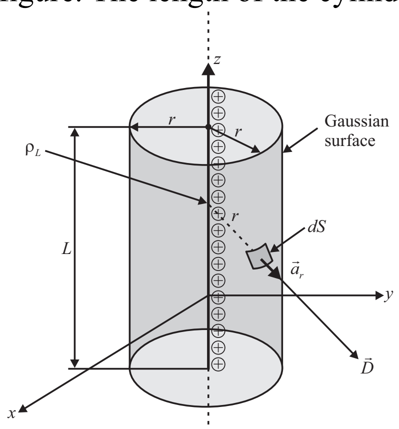


Fig. Right circular cylinder with infinite line charge

(iii)The flux density at any point on the surface is directed radially outwards i.e. in the $\vec{a}_r$ direction according to cylindrical co-ordinate system.

(iv)Consider only differential surface area dS as shown which is a radial distance r from the line charge. The direction normal to dS is $\vec{a}_r$.

(v) As the line charge is along z-axis, there cannot be any component of $\vec{D}$ in z direction. So $\vec{D}$ has only radial component.

From Gauss's law, $Q = \oint_S \vec{D}\cdot d\vec{S}$

$$Q = \oint_{\text{side}} \vec{D}\cdot d\vec{S} + \oint_{\text{top}} \vec{D}\cdot d\vec{S} + \oint_{\text{bottom}} \vec{D}\cdot d\vec{S}$$

$\vec{D} = D_r\vec{a}_r$ has only radial component.

$d\vec{S} = r\,d\phi\,dz\,\vec{a}_r$ normal to $\vec{a}_r$ direction.

$$\vec{D}\cdot d\vec{S} = D_r r\,d\phi\,dz\left(\vec{a}_r\cdot\vec{a}_r\right) = D_r\,r\,d\phi\,dz \qquad [\because\ (\vec{a}_r\cdot\vec{a}_r = 1)]$$

Now D_r is constant over the side surface.

(vi)As $\vec{D}$ has radial component and no component along $\vec{a}_z$ and $-\vec{a}_z$ hence integrations over top and bottom surfaces is zero.

$$\oint_{\text{top}} \vec{D}\cdot d\vec{S} = \oint_{\text{bottom}} \vec{D}\cdot d\vec{S} = 0$$

$$Q = \oint_{side} \vec{D} \cdot d\vec{S} = \oint_{side} D_r \, r \, d\phi \, dz$$

$$Q = \int_{z=0}^{L} \int_{\phi=0}^{2\pi} D_r \, r \, d\phi \, dz = r D_r [z]_0^L [\phi]_0^{2\pi}$$

$$Q = 2\pi r D_r L$$

$$D_r = \frac{Q}{2\pi r L}$$

$$\vec{D} = D_r \vec{a}_r = \frac{Q}{2\pi r L} \vec{a}_r$$

We know that, $\frac{Q}{L} = \rho_L$ C/m

$$\vec{D} = \frac{\rho_L}{2\pi r} \vec{a}_r \text{ C/m}^2$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{\rho_L}{2\pi \varepsilon_0 r} \vec{a}_r \text{ V/m}$$

2.6.3 Electric Field Due to Infinite Sheet of Charge

(i) Consider the infinite sheet of charge of uniform charge density ρ_S C/m^2, lying in the $z = 0$ plane i.e. xy plane as shown in the figure.

(ii) Consider a rectangular box as a Gaussian surface which is cut by the sheet of charge to give $dS = dx\,dy$.

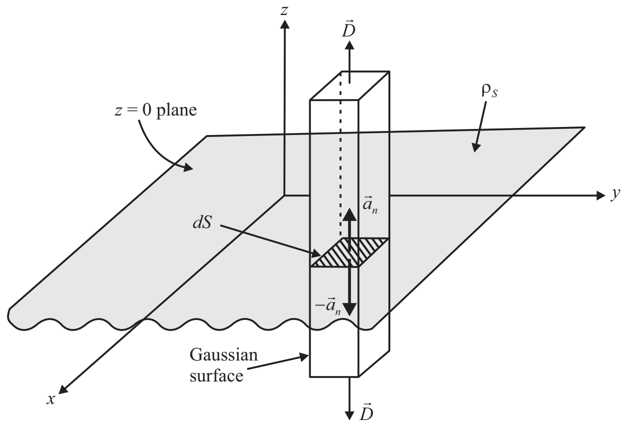


Fig. Infinite sheet of charge

(iii) $\vec{D}$ acts normal to the plane i.e. $\vec{a}_n = \vec{a}_z$ and $-\vec{a}_n = -\vec{a}_z$ direction.

Hence $\vec{D} = 0$ in x and y directions.

(iv) The charge enclosed can be written as,

$$Q = \int_S \vec{D} \cdot d\vec{S} = \int_{sides} \vec{D} \cdot d\vec{S} + \int_{top} \vec{D} \cdot d\vec{S} + \int_{bottom} \vec{D} \cdot d\vec{S}$$

$\int_{sides} \vec{D} \cdot d\vec{S} = 0$ because $\vec{D}$ has no component in x and y directions

$\vec{D} = D_z \vec{a}_z$ for top surface, $d\vec{S} = dx\,dy\,\vec{a}_z$

$$\vec{D}\cdot d\vec{S} = D_z dx\,dy\left(\vec{a}_z\cdot\vec{a}_z\right) = D_z\,dx\,dy$$

$$\vec{D} = D_z\left(-\vec{a}_z\right) \text{ for bottom surface, } d\vec{S} = dx\,dy\left(-\vec{a}_z\right)$$

$$\vec{D}\cdot d\vec{S} = D_z\,dx\,dy\left(\vec{a}_z\cdot\vec{a}_z\right) = D_z\,dx\,dy$$

$$Q = \int_{\text{top}} D_z\,dx\,dy + \int_{\text{bottom}} D_z\,dx\,dy$$

$$\int_{\text{top}} dx\,dy = \int_{\text{bottom}} dx\,dy = A = \text{Area of surface}$$

$$Q = 2D_z A$$

$$Q = \rho_s \times A \text{ as } \rho_s = \text{Surface charge density}$$

$$\rho_S = 2D_z$$

$$D_z = \frac{\rho_S}{2}$$

$$\vec{D} = D_z\vec{a}_z = \frac{\rho_S}{2}\vec{a}_z \text{ C/m}^2$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{\rho_S}{2\varepsilon_0}\vec{a}_z \text{ V/m}$$

2.7 Electric Field Intensity Due to Conducting Sphere

Consider an imaginary spherical shell of radius 'a'. The charge is uniformly distributed over its surface with a density ρ_S C/m^2. We have to find $\vec{E}$ at a point P located at a distance r from the centre such that $r > a$ and $r \le a$, using Gauss's law.

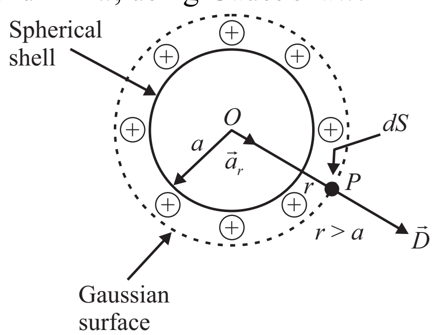


Fig. (a) Spherical shell of charge

Case 1 : Point *P* outside the shell ($r > a$)

Consider a point P at a distance r from the origin such that $r > a$. The Gaussian surface passing through point P is a concentric sphere of radius r. Due to spherical Gaussian surface, the flux lines are directed radially outwards and are normal to the surface. Hence electric flux density $\vec{D}$ is also directed radially outwards at point P and has component only in $\vec{a}_r$ direction. Consider a differential surface area at P normal to $\vec{a}_r$ direction hence $dS = r^2\sin\theta\,d\theta\,d\phi$ in spherical system.

$$d\psi = \vec{D}\cdot d\vec{S} = \left[D_r\vec{a}_r\right]\cdot\left[r^2\sin\theta\,d\theta\,d\phi\,\vec{a}_r\right]$$

$$= D_r r^2\sin\theta\,d\theta\,d\phi$$

$$\psi = \int_S D_r r^2 \sin\theta\, d\theta\, d\phi = D_r r^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta\, d\theta\, d\phi$$

$$\psi = D_r r^2 \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi} = 4\pi r^2 D_r$$

$$\psi = Q \qquad \text{... Gauss's law}$$

$$Q = 4\pi r^2 D_r$$

$$D_r = \frac{Q}{4\pi r^2}$$

$$\vec{D} = D_r \vec{a}_r = \frac{Q}{4\pi r^2} \vec{a}_r \text{ C/m}^2$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2} \vec{a}_r \text{ V/m} \qquad \text{...(i)}$$

Thus, for $r > a$ the field E is inversely proportional to the square of the distance from the origin.

If the surface charge density is $\rho_s \text{C/m}^2$ then

$$Q = \rho_S \times \text{ Surface area of shell}$$

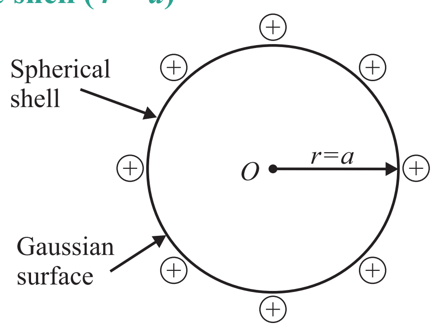
$$\therefore \qquad Q = \rho_S \times 4\pi a^2$$

From equation (i),

$$\vec{E} = \frac{\rho_S 4\pi a^2}{4\pi\varepsilon_0 r^2} \vec{a}_r = \frac{\rho_S a^2}{\varepsilon_0 r^2} \vec{a}_r \text{ V/m}$$

$$\vec{D} = \varepsilon_0 \vec{E} = \frac{\rho_S a^2}{r^2} \vec{a}_r \text{ C/m}^2$$

Case 2 : Point *P* is on the shell ($r = a$)



On the shell, $r = a$

The Gaussian surface is same as the shell itself and $\vec{E}$ can be obtained by putting $r = a$ in the equation (i),

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 a^2} \vec{a}_r \text{ V/m}$$

Case 3 : Point *P* inside the shell ($r < a$)

The Gaussian surface, passing through the point P is again a spherical surface with radius $r < a$. But it can be seen in figure (b) that the entire charge is on the surface and no charge is enclosed by the spherical shell. And when the Gaussian surface is such that no charge is enclosed, irrespective of any charges present outside, the total charge enclosed is zero.

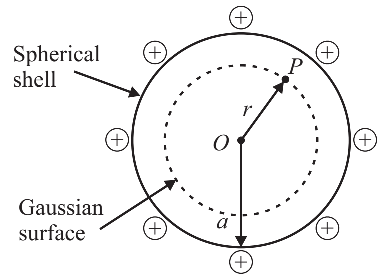


Fig.(b)

$\therefore \quad \psi = Q = \int_S \vec{D} \cdot d\vec{S} = 0$

But, $\quad \int_S d\vec{S} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta \, d\theta \, d\phi = 4\pi r^2$

Thus $\quad \int_S d\vec{S} \neq 0$

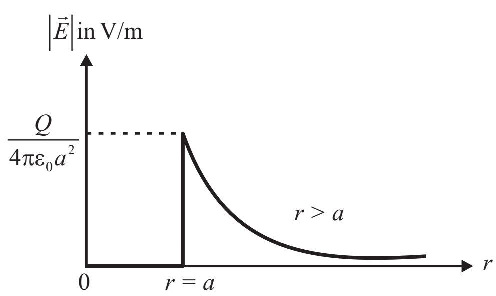
Hence to satisfy that total charge enclosed is zero, inside the spherical shell.

$$\vec{D} = 0 \text{ and } \vec{E} = \frac{\vec{D}}{\varepsilon_0} = 0$$

Note : Electric flux density and electric field at any point inside a spherical shell is zero.

Plot of $\vec{E}$ against r :

The variation of $\vec{E}$ against the radial distance r measured from the origin is shown in the figure(c).



r	$\vec{E}$
$r < a$	$\vec{E} = 0$
$r = a$	$\vec{E} = \frac{Q}{4\pi\varepsilon_0 a^2} \vec{a}_r$
$r > a$	$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \vec{a}_r$

Fig. (c)

2.8 Electric Field Intensity Due to Non-conducting Sphere

Consider a sphere of radius 'a' with a uniform charge density of ρ_v C/m^3. Let us find $\vec{E}$ at a point P located at a radial distance r from centre of the sphere such that $r \leq a$ and $r > a$, using Gauss's law. The sphere is shown in the figure (a).

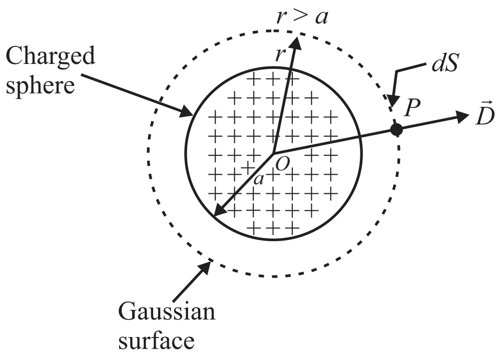


Fig. (a) Uniformly charged sphere

Case 1 : The point *P* is outside the sphere ($r > a$)

The Gaussian surface passing through point P is a spherical surface of radius r. The flux lines and $\vec{D}$ are directed radii outwards along $\vec{a}_r$ direction. The difference area dS is considered at point P which is normal to $\vec{a}_r$ direction.

$$d\vec{S} = r^2 \sin\theta \, d\theta \, d\phi \, \vec{a}_r$$

Differential flux,

$$d\psi = \vec{D} \cdot d\vec{S} = D_r \vec{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \vec{a}_r$$

$$= D_r \, r^2 \sin\theta \, d\theta \, d\phi$$

Net flux, $$\psi = Q = \int_S \vec{D} \cdot d\vec{S} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} D_r r^2 \sin\theta \, d\theta \, d\phi$$

$$= D_r r^2 \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi} = D_r r^2 4\pi$$

Electric Flux density,

$$D_r = \frac{Q}{4\pi r^2}$$

In vector form,

$$\vec{D} = \frac{Q}{4\pi r^2} \vec{a}_r \text{ C/m}^2$$

Electric field intensity,

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2} \vec{a}_r \text{ V/m} \quad \ldots\text{(i)}$$

The total charge enclosed can be obtained as,

$$Q = \int_v \rho_v \, dv = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{a} \rho_v r^2 \sin\theta \, dr \, d\theta \, d\phi = \rho_v \left[\frac{r^3}{3}\right]_0^a \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$= \frac{4}{3} \pi a^3 \rho_v \text{ C}$$

$$\therefore \quad \vec{E} = \frac{\frac{4}{3}\pi a^3 \rho_v}{4\pi\varepsilon_0 r^2} \vec{a}_r = \frac{a^3 \rho_v}{3\varepsilon_0 r^2} \vec{a}_r \quad \ldots\text{(ii)}$$

From equation (i) and (ii), we get

$$\vec{D} = \frac{a^3 \rho_v}{3r^2} \vec{a}_r$$

Case 2 : The point *P* on the sphere (*r* = *a*)

The Gaussian surface is same as the surface of the charged sphere. Hence results can be obtained directly substituting $r = a$ in the equation (ii),

$$\therefore \qquad \vec{E} = \frac{a^3 \rho_v}{3\varepsilon_0 a^2} \vec{a}_r = \frac{\rho_v a}{3\varepsilon_0} \vec{a}_r$$

Electric Flux density,

$$\vec{D} = \varepsilon_0 \vec{E} = \frac{\rho_v a}{3} \vec{a}_r$$

Case 3 : The point *P* is inside the sphere (*r* < *a*)

The Gaussian surface is a spherical surface of radius r where $r < a$. Consider differential surface area dS as shown in the figure (b).

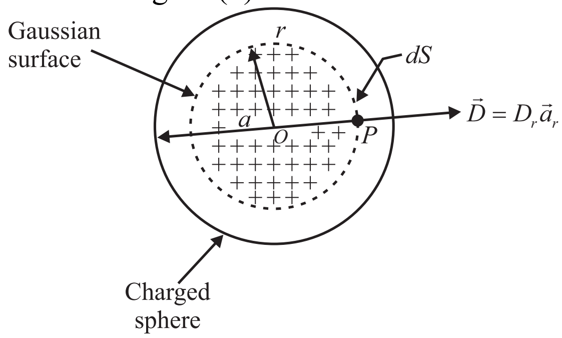


Fig. (b)

$d\vec{S}$ and $\vec{D}$ are directed radially outwards.

$$\therefore \qquad \vec{D} = D_r \vec{a}_r \text{ while } d\vec{S} = r^2 \sin\theta \, d\theta \, d\phi \, \vec{a}_r$$

$$\therefore \qquad d\psi = \vec{D} \cdot d\vec{S} = D_r r^2 \sin\theta \, d\theta \, d\phi \qquad [\because \ \vec{a}_r \cdot \vec{a}_r = 1]$$

Net flux,

$$\psi = Q = \int_S \vec{D} \cdot d\vec{S} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} D_r r^2 \sin\theta \ d\theta \, d\phi = D_r r^2 [-\cos\theta]_0^{\pi} [\phi]_0^{2\pi} = 4\pi r^2 D_r$$

$$D_r = \frac{Q}{4\pi r^2}$$

In vector form,

$$\vec{D} = \frac{Q}{4\pi r^2} \vec{a}_r \text{ C/m}^2 \qquad \text{...(iii)}$$

The charge enclosed is by the sphere of radius r only and not by the entire sphere. The charge outside the Gaussian surface will not affect $\vec{D}$.

$$\therefore \qquad Q = \int_v \rho_v dv = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{r} \rho_v \, r^2 \sin\theta \, dr \, d\theta \, d\phi = \frac{4}{3}\pi r^3 \rho_v \qquad \text{where } r < a$$

From equation (iii), we get

$$\vec{D} = \frac{\frac{4}{3}\pi r^3 \rho_v}{4\pi r^2} \vec{a}_r$$

$\therefore \quad \vec{D} = \frac{r}{3}\rho_v \vec{a}_r \quad ..... \; 0 < r \le a$

$\therefore \quad \vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{r}{3\varepsilon_0}\rho_v \vec{a}_r \quad ...... \; 0 < r \le a$

Variation of $\vec{E}$ against r :

From the equations and it can be seen that for $r > a$, the $\vec{E}$ is inversely proportional to square of the distance while for $r < a$ it is directly proportional to the distance r. At $r = a, \left|\vec{E}\right| = \frac{\rho_v a}{3\varepsilon_0}$ depends on the radius of the charged sphere.

For $r > a$, the graph of $\left|\vec{E}\right|$ against r is parabolic while for $r < a$ it is a straight line as shown in the figure (c).

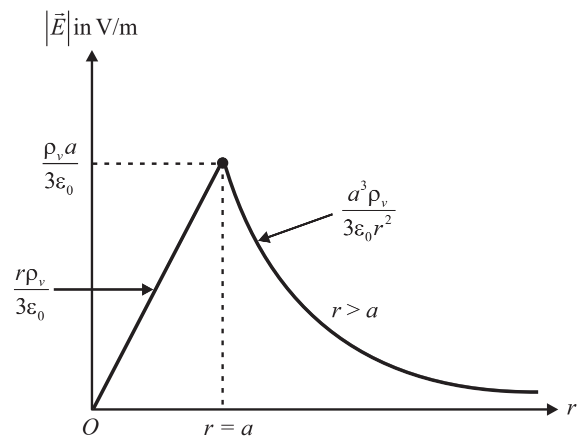


Fig. (c) : The graph of $\left|\vec{D}\right|$ against r is exactly similar in nature as $\left|\vec{E}\right|$ against r

Solved Example 5

Given that $\vec{D} = \left(\frac{10}{3}\right)x^3\,\vec{a}_x\ \text{C/m}^2$, evaluate both sides of the divergence theorem for the volume of a cube 2 m on the edge, centred at the origin and with edges parallel to the axis.

Sol. **Given :** $\vec{D} = \left(\frac{10}{3}\right)x^3\vec{a}_x = D_x\vec{a}_x$

The divergence theorem is given by,

$$\oint_s \vec{D}\cdot d\vec{s} = \int_v \left(\nabla\cdot\vec{D}\right)dv$$

LHS : $\oint_s \vec{D}\cdot d\vec{s}$

Since the closed surface is formed by six surfaces,

$$\oint_s \vec{D}\cdot d\vec{s} = \int_{\text{Front}} \vec{D}\cdot d\vec{s} + \int_{\text{Back}} \vec{D}\cdot d\vec{s} + \int_{\text{Top}} \vec{D}\cdot d\vec{s} + \int_{\text{Bottom}} \vec{D}\cdot d\vec{s} + \int_{\text{Left}} \vec{D}\cdot d\vec{s} + \int_{\text{Right}} \vec{D}\cdot d\vec{s}$$

Because the cube is centered at the origin, the front face is at $x = 1$ and back face is at $x = -1$ (Remember side of the cube is 2m). Similarly, left side is at $y = -1$, right side at $y = 1$. Also top side at $z = 1$ while bottom side at $z = -1$.

For these surfaces,

$d\vec{s}$ for Front face $= dy\,dz\,\vec{a}_x$　　(x = constant = + 1)

$d\vec{s}$ for Back face $= dy\,dz(-\vec{a}_x)$　　(x = constant = – 1)

$d\vec{s}$ for Top face $= dx\,dy(\vec{a}_z)$　　(z = constant = + 1)

$d\vec{s}$ for Bottom face $= dx\,dy(-\vec{a}_z)$　　(z = constant = – 1)

$d\vec{s}$ for Left face $= dx\,dz(-\vec{a}_y)$　　(y = constant = – 1)

$d\vec{s}$ for Right face $= dx\,dz(\vec{a}_y)$　　(y = constant = + 1)

Since $\vec{D}$ is having only D_x term,

$$\int_{Top} \vec{D}\cdot d\vec{s} = \int_{Bottom} \vec{D}\cdot d\vec{s} = \int_{Left} \vec{D}\cdot d\vec{s} = \int_{Right} \vec{D}\cdot d\vec{s} = 0 \quad (\vec{a}_x\cdot\vec{a}_y = \vec{a}_x\cdot\vec{a}_z = 0)$$

$$\therefore \quad \oint_s \vec{D}\cdot d\vec{s} = \int_{Front} \vec{D}\cdot d\vec{s} + \int_{Back} \vec{D}\cdot d\vec{s}$$

Now, solve each integral on the right of above equation separately,

$$\int_{Front} \vec{D}\cdot d\vec{s} = \int_{-1}^{1}\int_{-1}^{1}\left(\frac{10}{3}x^3\,\vec{a}_x\right)\cdot dy\,dz\,\vec{a}_x = \int_{-1}^{1}\int_{-1}^{1}\frac{10}{3}x^3\,dy\,dz \quad (\because\ \vec{a}_x\cdot\vec{a}_x = 1)$$

$$\int_{Front} \vec{D}\cdot d\vec{s} = \frac{10}{3}(1)^3[y]_{-1}^{1}[z]_{-1}^{1} = \frac{40}{3}\text{C} \qquad \text{(for front face x = 1)}$$

$$\int_{Back} \vec{D}\cdot d\vec{s} = \int_{-1}^{1}\int_{-1}^{1}\left(\frac{10}{3}x^3\,\vec{a}_x\right)\cdot dy\,dz(-\vec{a}_x)$$

$$\int_{Back} \vec{D}\cdot d\vec{s} = -\frac{10}{3}(-1)^3[y]_{-1}^{1}[z]_{-1}^{1} \qquad \text{(for back face x = – 1)}$$

$$\int_{Back} \vec{D}\cdot d\vec{s} = \frac{40}{3}\text{C}$$

$$\therefore \quad \oint_s \vec{D}\cdot d\vec{s} = \frac{40}{3} + \frac{40}{3} = \frac{80}{3}\text{C} \qquad \text{i.e. LHS} = \frac{80}{3}\text{C}$$

To find RHS : $\int_v (\nabla\cdot\vec{D})\,dv$

Divergence in Cartesian coordinate system is given by,

$$\nabla\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \frac{\partial D_x}{\partial x} = 10x^2 \qquad (\because\ D_y = D_z = 0)$$

$$\int_v (\nabla\cdot\vec{D})\,dv = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}(10x^2)\,dx\,dy\,dz = 10\left[\frac{x^3}{3}\right]_{-1}^{1}[y]_{-1}^{1}[z]_{-1}^{1} = \frac{80}{3}\text{C}$$

$\therefore$ 　LHS = RHS

Solved Example 6

A vector field is given by $A(\rho,\phi,z)=30e^{-\rho}\,\vec{a}_\rho-2z\,\vec{a}_z$ verify divergence theorem for the volume enclosed by $\rho=2$, $z=0$ and $z=5$.

Sol. Given : $A(\rho,\phi,z)=30e^{-\rho}\,\vec{a}_\rho-2z\,\vec{a}_z$

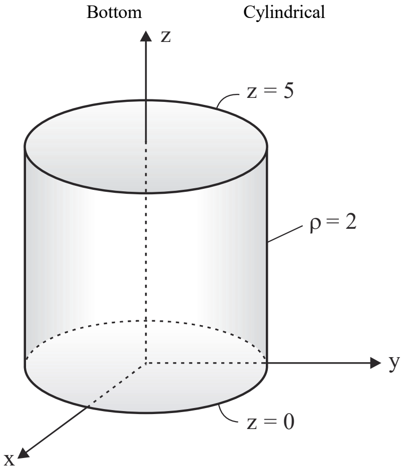
Given volume represents cylinder as shown in the figure (a). The divergence theorem for $\vec{A}$ is,

$$\oint \vec{A}\cdot d\vec{s}=\int_v(\nabla\cdot\vec{A})\,dv$$

To find LHS :

Since the closed surface consists of three surface

$$\oint_s \vec{A}\cdot d\vec{s}=\int_{Top}\vec{A}\cdot d\vec{s}+\int_{Bottom}\vec{A}\cdot d\vec{s}+\int_{Cylindrical}\vec{A}\cdot d\vec{s}$$



For these surface differential areas are

$d\vec{s}$ for top surface $=\rho\,d\rho\,d\phi(\vec{a}_z)$ (For z = 5)

$d\vec{s}$ for bottom surface $=\rho\,d\rho\,d\phi(-\vec{a}_z)$ (For z = 0)

$d\vec{s}$ for cylindrical surface $=\rho\,d\phi\,dz(\vec{a}_\rho)$ (For $\rho=2$)

Solving each integral separately,

$$\int_{Top}\vec{A}\cdot d\vec{s}=\int_0^{2\pi}\int_0^{2}(30e^{-\rho}\,\vec{a}_\rho-2z\,\vec{a}_z)\cdot\rho\,d\rho\,d\phi(\vec{a}_z)=0$$

$$\int_{Top}\vec{A}\cdot d\vec{s}=-2\times5\int_0^{2\pi}\int_0^{2}\rho\,d\rho\,d\phi=-10\left[\frac{\rho^2}{2}\right]_0^2[\phi]_0^{2\pi}=-40\pi=-125.66\text{ C} \quad (\text{Since } z=5)$$

$$\int_{Bottom}\vec{A}\cdot d\vec{s}=\int_0^{2\pi}\int_0^{2}(30e^{-\rho}\vec{a}_\rho-2z\,\vec{a}_z)\cdot\rho\,d\rho\,d\phi(-\vec{a}_z)=0 \quad (\text{Since } z=0)$$

$$\int_{Cylin.}\vec{A}\cdot d\vec{s}=\int_0^{5}\int_0^{2\pi}(30e^{-\rho}\vec{a}_\rho-2z\,\vec{a}_z)\cdot\rho\,d\phi\,dz\,\vec{a}_\rho$$

$$\int_{Cylin.}\vec{A}\cdot d\vec{s}=30e^{-2}\cdot2\int_0^{5}\int_0^{2\pi}d\phi\,dz \quad (\text{Since } r=2)$$

$$\int_{\text{Cylin.}} \vec{A}\cdot d\vec{s} = 60\,e^{-2}\,[\phi]_0^{2\pi}\,[z]_0^5 = 600\,e^{-2}\,\pi = 255.10 \text{ C}$$

$\therefore$ LHS $= -125.66 + 255.10 = 129.44$ C

To find RHS : $\int_v \left(\nabla\cdot\vec{A}\right) dv$

Divergence in cylindrical coordinate system is given by,

$$\nabla\cdot\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

Here, the first term is calculated as,

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\cdot 30\,e^{-\rho}) = \frac{30}{\rho}\left\{\rho(-1)\,e^{-\rho} + e^{-\rho}\cdot 1\right\} = -30\,e^{-\rho} + \frac{30\,e^{-\rho}}{\rho}$$

The second terms is, $\frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} = 0$ (Since $A_\phi = 0$)

The third term is, $\frac{\partial A_z}{\partial z} = \frac{\partial}{\partial z}(-2z) = -2$

$$\therefore \quad \nabla\cdot\vec{A} = -30\,e^{-\rho} + \frac{30\,e^{-\rho}}{\rho} - 2$$

$$\therefore \quad \text{RHS} = \int_v \left(\nabla\cdot\vec{A}\right) dv = \int_0^5\int_0^{2\pi}\int_0^2 (-30\,e^{-\rho} + 30\,e^{-\rho}\rho^{-1} - 2)\,\rho\, d\rho\, d\phi\, dz$$

$$\int_v \left(\nabla\cdot\vec{A}\right) dv = \int_0^5\int_0^{2\pi}\int_0^2 (-30\,\rho\, e^{-\rho} + 30\,e^{-\rho} - 2\rho)\; d\rho\, d\phi\, dz$$

$$\int_v \left(\nabla\cdot\vec{A}\right) dv = [\phi]_0^{2\pi}\,[z]_0^5\left\{\int_0^2 (-30\,\rho\, e^{-\rho} + 30\,e^{-\rho} - 2\rho)\, d\rho\right\}$$

$$\int_v \left(\nabla\cdot\vec{A}\right) dv = 10\pi\left\{-30\left[\rho\cdot\frac{e^{-\rho}}{-1} - \int 1\cdot\frac{e^{-\rho}}{-1}d\rho\right]_0^2 + \left[30\,\frac{e^{-\rho}}{-1}\right]_0^2 - 2\left[\frac{\rho^2}{2}\right]_0^2\right\}$$

$$\int_v \left(\nabla\cdot\vec{A}\right) dv = 10\pi(60e^{-2} - 4) = 129.44 \text{ C}$$

$\therefore$ LHS = RHS

Hence divergence theorem is verified.

Solved Example 7

A volume charge density inside a sphere is $\rho = 10e^{-20r}\,\text{C/m}^3$. Find the total charge enclosed within the sphere. Also find the electric flux density on the surface of the sphere.

Sol. **Given :** $\rho = 10e^{-20r}\,\text{C/m}^3$

The total charge enclosed is given by,

$$Q = \int_v \rho\, dv = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{2} \rho\, r^2 \sin\theta dr\, d\theta\, d\phi = \int_0^{2\pi} d\phi\int_0^{\pi}\sin\theta\, d\theta\int_0^2 10e^{-20r} r^2 dr$$

$$Q = 40\pi\int_0^2 r^2 e^{-20r}\, dr = \frac{40\pi}{4000} = \frac{\pi}{100}$$ **Ans.**

Since the distribution of charge is spherically symmetric, the electric flux density is also uniform on the surface of the sphere. By Gauss's law, the total electric flux from the surface of a sphere is Q.

$$4\pi r^2 D = Q$$

$$D = \frac{Q}{4\pi r^2} = \frac{\pi/100}{4\pi(2)^2} = 6.25\times10^{-4}\,\text{C/m}^2$$ **Ans.**

TEST 2

Q.1 The following point charges are located in air :

$+0.008\,\mu\text{C at }(0,0)\text{ m}$

$+0.05\,\mu\text{C at }(3,0)\text{ m}$

$-0.009\,\mu\text{C at }(0,4)\text{ m}$

The total electric flux over a sphere of 5 m radius with centre (0,0) is

(A) $0.058\,\mu\text{C}$ (B) $0.049\,\mu\text{C}$

(C) $0.029\,\mu\text{C}$ (D) $0.016\,\mu\text{C}$

Q.2 The vector statement of Gauss's law is

(A) $\oint_V \vec{D}\cdot da = \int_S \sigma\, dv$

(B) $\int_V \vec{D}\cdot da = \oint_S \rho\, dv$

(C) $\oint_S D\cdot da = \int_V \rho^2\, dv$

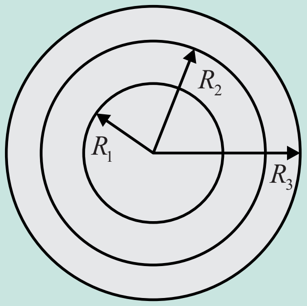
(D) $\oint_S D\cdot da = \int_V \rho\, dv$

Q.3 Given that the electric flux density $\vec{D} = z\rho(\cos^2\phi)\vec{a}_z\ \text{C/m}^2$. The charge density at point $\left(1,\frac{\pi}{4},3\right)$ is

(A) 3 (B) 1

(C) 0.5 (D) $0.5\vec{a}_x$

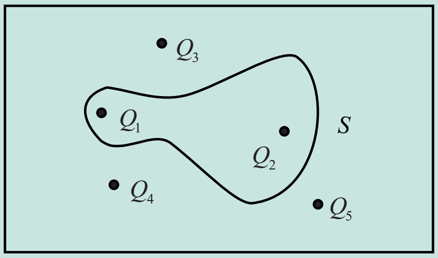
Q.4 Three concentric conducting spherical surfaces of radii R_1, R_2 and R_3 $(R_1 < R_2 < R_3)$ carry charges of $-1, -2$ and 4 coulombs respectively. The charges on the inner and outer surfaces of the outermost sphere will be respectively (in coulombs)



(A) 0, 4 (B) 3, 1

(C) −3, 7 (D) −2, 6

Q.5 The net flux of electric field emanating from the surface 'S' with location of point charges as shown in the given figure is



(A) $\frac{Q_1+Q_2}{\varepsilon_0}$

(B) $\frac{Q_1+Q_2+Q_3+Q_4+Q_5}{\varepsilon_0}$

(C) $\frac{Q_3+Q_4+Q_5}{\varepsilon_0}$

(D) $\varepsilon_0(Q_1+Q_2)$

2.9 Potential Difference and Absolute Potential

Potential difference is defined as amount of work done in moving unit positive charge from one finite distance to another finite distance.

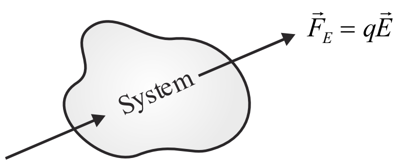
Absolute potential or potential is defined as amount of work done in moving unit positive charge from ∞ to finite distance.

Absolute potential is an absolute value of potential difference where initial or reference point is placed at ∞ and another point is placed at finite distance.

Potential difference is defined as work done per unit charge.

$$dV = \frac{dW}{dq}$$

$$dW = \vec{F}_{\text{applied}} \cdot d\vec{l}$$



Under equilibrium

$$\vec{F}_{\text{applied}} + \vec{F}_E = 0$$

$$\vec{F}_{\text{applied}} = -\vec{F}_E$$

$$\vec{F}_{\text{applied}} = -q\vec{E}$$

$$\vec{F}_{\text{applied}} \cdot d\vec{l} = -q\vec{E} \cdot d\vec{l}$$

$$dW = -q\vec{E} \cdot d\vec{l}$$

$$dV = \frac{dW}{dq} = -\vec{E} \cdot d\vec{l}$$

$$V = -\int \vec{E} \cdot d\vec{l}$$

2.10 Potential Due to Various Charge Distributions

2.10.1 *V* due to Line Charge Distribution

The differential charge on line is $dQ = \rho_l \, dl$

The voltage at point P due to charge dQ is,

$$dV = \frac{dQ}{4\pi\varepsilon_0 R} = \frac{\rho_l dl}{4\pi\varepsilon_0 R}$$

The total voltage due to line charge is, $V_l = \int_l dV$

i.e. $$V_l = \frac{1}{4\pi\varepsilon_0} \int_l \frac{\rho_l dl}{R}$$

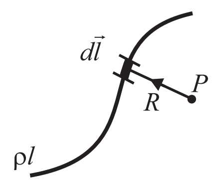


Fig. (a) To find *V* due to line

2.10.2 *V* due to Surface Charge distribution

The differential charge is, $dQ = \rho_s\, ds$

The voltage due to surface change is,

$$dV = \frac{dQ}{4\pi\varepsilon_0 R} = \frac{\rho_s ds}{4\pi\varepsilon_0 R}$$

The total voltage due to surface charge, $V_s = \int_s dV$

i.e. $$V_s = \frac{1}{4\pi\varepsilon_0}\int_s \frac{\rho_s ds}{R}$$

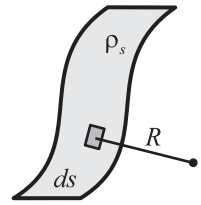


Fig. (b) To find *V* due to surface

2.10.3 *V* due to Volume Charge Distribution

If the volume charge density is ρ_v then the differential charge is,

$$dQ = \rho_v\, dv$$

and $$dV = \frac{dQ}{4\pi\varepsilon_0 R} = \frac{\rho_v dv}{4\pi\varepsilon_0 R}$$

The total voltage due to volume charge,

$$V_v = \int_v dv \text{ i.e. } V_v = \frac{1}{4\pi\varepsilon_0}\int_v \frac{\rho_v dv}{R}$$

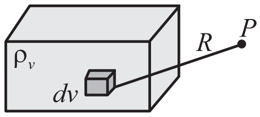


Fig. (c) To find *V* due to volume

2.10.4 *V* due to Discrete Charge Point Charge

In this case find voltages due to each charge separately and the total voltage is sum of all. Let Q_1, Q_2, Q_3 be the point charges at R_1, R_2, R_3 respectively. Then the potential due to them is,

$$V_p = \frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2} + \frac{Q_3}{4\pi\varepsilon_0 R_3} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R_1} + \frac{Q_2}{R_2} + \frac{Q_3}{R_3}\right)$$

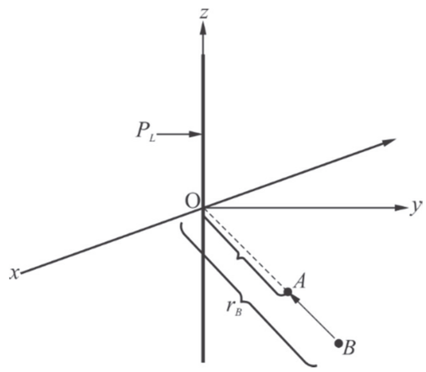
This can be extended to any number of charges. If *N* charges are present,

Then, $$V_p = \frac{1}{4\pi\varepsilon_0}\sum_{n=1}^{N}\frac{Q_n}{R_n}$$

2.11 Potential Difference Due to Infinite Long Line Charge

Consider an infinite line charge along *z*-axis having uniform line charge density ρ_L C/m.

The point B is at a radial distance r_B while point A is at a radial distance r_A from the charge, as shown in the figure shown below



The E due to infinite line charge along z-axis is known and given by

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0}\vec{a}_r$$

While $d\vec{L} = dr\vec{a}_r$

in cylindrical system in radial direction.

$$\therefore \quad V_{AB} = -\int_B^A \vec{E}.d\vec{L} = -\int_{r_B}^{r_A} \frac{\rho L}{2\pi\varepsilon_0 r}\vec{a}_r.dr\vec{a}_r$$

$$= -\int_{r_B}^{r_A} \frac{\rho_L}{2\pi\varepsilon_0}dr = \frac{-\rho_L}{2\pi\varepsilon_0 r}dr = \frac{-\rho L}{2\pi\varepsilon_0}\int_{R_B}^{r_A}\frac{1}{r}dr$$

$$= -\int_{r_B}^{r_A} \frac{\rho_L}{2\pi\varepsilon_0}dr = \frac{-\rho_L}{2\pi\varepsilon_0 r}dr = \frac{-\rho L}{2\pi\varepsilon_0}\int_{R_B}^{r_A}\frac{1}{r}dr$$

$$\therefore \quad V_{AB} = \frac{\rho_L}{2\pi\varepsilon_0}\ln\frac{r_B}{r_A}V$$

Important note : This is a standard result and may be used to find potential difference between the points due to infinite line charge. Remember that r_A and r_B are radial distances in cylindrical co-ordinate system i.e. perpendicular distances from charge, thus do not forget to find perpendicular distances r_A and r_B while using this result. The result can be used for any zero reference as potential difference calculation does not depend on the reference.

Solved Example 8

Two point charges $-4\ \mu C$ and $5\ \mu C$ are located at (2, –1, 3) and (0, 4 –2), respectively. Find the potential at (1, 0, 1), assuming zero potential at infinity.

Sol. Given : $Q_1 = -4\ \mu C, \quad Q_2 = 5\mu C$

$$V(r) = \frac{Q_1}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|} + \frac{Q_2}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|} + C_0$$

If $V(\infty) = 0,\ C_0 = 0$

$$|\vec{r}-\vec{r}_1| = |(1, 0, 1)-(2, -1, 3)| = |(-1, 1, -2)| = \sqrt{6}$$

$$|\vec{r}-\vec{r}_2| = |(1, 0, 1)-(0, 4, -2)| = |(1, -4, 3)| = \sqrt{26}$$

Hence,
$$V(1, 0, 1) = \frac{10^{-6}}{4\pi\times\frac{10^{-9}}{36\pi}}\left[\frac{-4}{\sqrt{6}} + \frac{5}{\sqrt{26}}\right]$$

$$V(1, 0, 1) = 9\times10^3(-1.633+0.9806)$$

$$V(1, 0, 1) = -5.872\ \text{kV}$$

Solved Example 9

Find the potentials at $r_A = 4$ m and at $r_B = 16$ m due to a point charge Q = 900 pC at the origin and zero reference at infinity. Also find the potential at r_A with respect to r_B.

Sol. The potential at r distance from origin having charge Q and zero reference at infinity is given by,

$$V = \frac{1}{4\pi\varepsilon}\frac{Q}{r} \quad \text{where, } \varepsilon = \varepsilon_0 = \text{ free space permittivity } = \left(\frac{1}{36\pi}\right)\times 10^{-9} \text{ F/m}$$

Potential at $r_A = 4$ m

$$V_A = \frac{1}{4\pi\varepsilon}\frac{Q}{4} = 9\times 10^9 \times \frac{900\times 10^{-12}}{4}$$

$$V_A = \frac{8.1}{4} = 2.025 \text{ Volts}$$ **Ans.**

Potential at $r_B = 16$ m,

$$V_B = \frac{1}{4\pi\varepsilon}\frac{Q}{16}$$

$$V_B = 9\times 10^9 \times \frac{900\times 10^{-12}}{16}$$

$$V_B = \frac{8.1}{16} = 0.50625 \text{ Volts}$$ **Ans.**

Potential at r_A with respect to r_B is given by,

$$V_{AB} = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_A} - \frac{1}{r_B}\right] \quad \text{where } r_A = 4\text{m}, \ r_B = 16\text{m}$$

$$V_{AB} = 9\times 10^9 \times 900\times 10^{-12}\left[\frac{1}{4} - \frac{1}{16}\right]$$

$$V_{AB} = 8.1\left[\frac{4-1}{16}\right] = 1.5187 \text{ Volts}$$ **Ans.**

Remember

Alternatively,

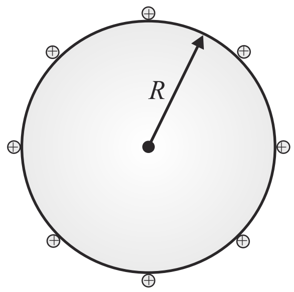
$$V_{AB} = V_A - V_B$$

$$V_{AB} = 2.025 - 0.50625$$

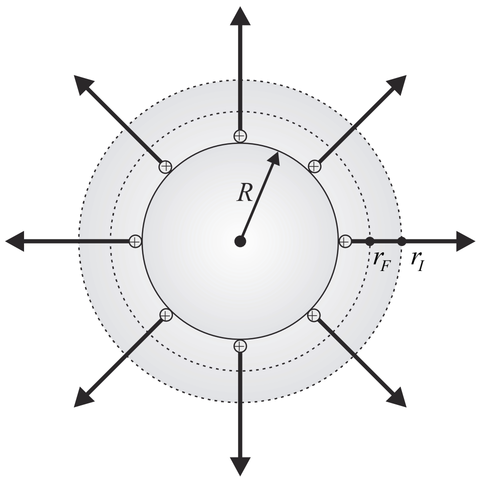
$$V_{AB} = 1.518 \text{ Volts}$$

2.12 Potential Difference and Potential Due to Conducting Sphere of Radius 'R'

Assume that charge is uniformly distributed across the surface of conducting sphere.



Case I : $r > R$



Electric field intensity due a conducting sphere for $r > R$ is given by,

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a}_r$$

Potential difference is given by,

$$V_{FI} = -\int_{r_I}^{r_F} \vec{E}\cdot\vec{dl}$$

$$V_{FI} = -\int_{r_I}^{r_F} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dr = \frac{-Q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_{r_I}^{r_F} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_F}-\frac{1}{r_I}\right]$$

$$V_{FI} = V_F - V_I = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_F}-\frac{1}{r_I}\right]$$

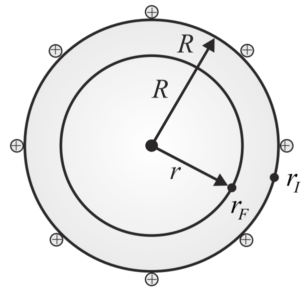
If $r_I = \infty$, $V_I = 0$ minimum potential

Special case :

If $r_I = \infty$, $r_F = R$

$$V_{FI} = V_F = \frac{Q}{4\pi\varepsilon_0 R}$$ maximum potential

Case II : $r < R$



Electric field intensity inside the conducting sphere is zero.

$$\vec{E} = 0$$

$$\therefore \quad V_{FI} = 0$$

Potential difference inside conducting sphere is zero but potential is not zero.

If we take any observation point at inside the sphere of radius 'R' then potential of that point is same as potential of the surface of sphere i.e. $\frac{Q}{4\pi\varepsilon_0 R}$ and potential difference between two points inside the sphere is zero.

$$r_I = R$$

$$V_I = \frac{Q}{4\pi\varepsilon_0 R}$$

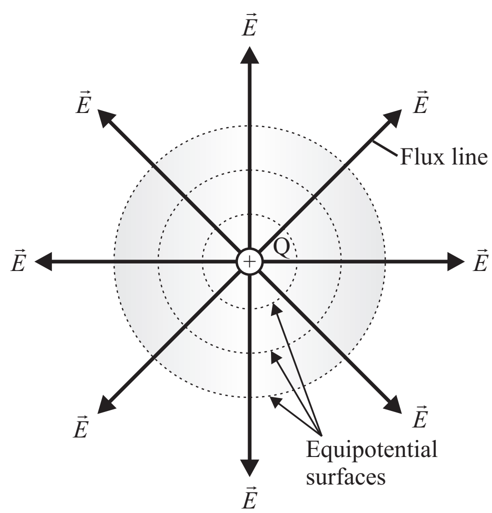
$$V_{FI} = V_F - V_I = 0 \quad \Rightarrow \quad V_F = V_I$$

$$V_F = \frac{Q}{4\pi\varepsilon_0 R} = V_I$$

The above value of potential is constant for all points on the conducting sphere.

2.13 Equipotential Surfaces

Any surface or volume over which the electric potential is constant be called an equipotential. The volume or surface may be that of a material body or simply a surface or volume of space.



In an electric field, there are many points at which the electric potential is same. This is because, the potential is a scalar quantity which depends on the distance between the point at which potential is to be obtained and the location of the charge. If the surface is imagined, joining all such points which are at the same potential, then such a surface is called equipotential surface. Consider a point charge located at the origin of a sphere, then potential at a point which is at a radial distance r from the point charge is given by,

No work is done in moving a charge from one point to another along an equipotential line or surface $\left(V_A - V_B = 0\right)$ and hence

$$\int_L \vec{E}.d\vec{l} = 0$$

Lines of force or flux lines (or the direction of E) are always normal to equipotential surfaces.

2.14 Relationship between Electric Field Intensity E and Electric Potential V

Electric field is defined as,

$$\vec{E} = E_x\vec{a}_x + E_y\vec{a}_y + E_z\vec{a}_z \quad \ldots\text{(i)}$$

Differential potential is defined by,

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz \quad \ldots\text{(ii)}$$

Differential length is given as,

$$d\vec{l} = dx\,\vec{a}_x + dy\,\vec{a}_y + dz\,\vec{a}_z \quad \ldots\text{(iii)}$$

From equation (i) and (iv),

$$E \cdot d\vec{l} = E_x dx + E_y dy + E_z dz$$

We know that,

$$dV = -\vec{E} \cdot d\vec{l}$$

$$-\vec{E} \cdot d\vec{l} = -(E_x dx + E_y dy + E_z dz) \quad \ldots\text{(iv)}$$

Compare equation (iv) and (i),

$$\frac{\partial v}{\partial x} = -E_x$$

$$\frac{\partial v}{\partial y} = -E_y$$

$$\frac{\partial v}{\partial z} = -E_z$$

From equation (i),

$$\vec{E} = -\frac{\partial v}{\partial x}\vec{a}_x - \frac{\partial v}{\partial y}\vec{a}_y - \frac{\partial v}{\partial z}\vec{a}_z$$

$$\vec{E} = -\left[\frac{\partial v}{\partial x}\vec{a}_x + \frac{\partial v}{\partial y}\vec{a}_y + \frac{\partial v}{\partial z}\vec{a}_z\right] \quad \ldots\text{(v)}$$

$$V = f\,(x, y, z)$$

Potential gradient is given as,

$$\nabla V = \frac{\partial V}{\partial x}\vec{a}_x + \frac{\partial V}{\partial y}\vec{a}_y + \frac{\partial V}{\partial z}\vec{a}_z \qquad \ldots(vi)$$

From equation (v) and (vi),

$$\vec{E} = -\nabla V$$

Electric field is defined as negative gradient of potential.

Remember

1. Electric field intensity is perpendicular to the equipotential surface (a surface on which potential is constant) and is directed from higher to lower potential i.e. towards the decreasing value of potential.
2. Maximum rate of change of potential with respect to distance gives the magnitude of electric field intensity.

The potential difference between the two points A and B is independent of the path taken.

Hence, $$V_{BA} = -V_{AB}$$

i.e. $$V_{BA} + V_{AB} = \oint \vec{E}\cdot d\vec{l} = 0$$

$$\oint_L \vec{E}\cdot d\vec{l} = 0 \qquad \ldots(i)$$

This shown that the line integral of $\vec{E}$ along a closed path as shown in figure must be zero. Physically, this implies that no net work is done in moving a charge along a closed path in an electrostatic field.

Equation (i) is referred to as Maxwell's 2nd equation in integral form.

Applying Stokes theorem to equation (i), we get

$$\oint_L \vec{E}\cdot d\vec{l} = \oint_S (\nabla\times\vec{E})\cdot d\vec{s} = 0$$

$$\nabla\times\vec{E} = 0 \qquad \ldots(ii)$$

Any vector field that satisfies equation (i) or (ii) is said to be **conservative, or irrotational**.

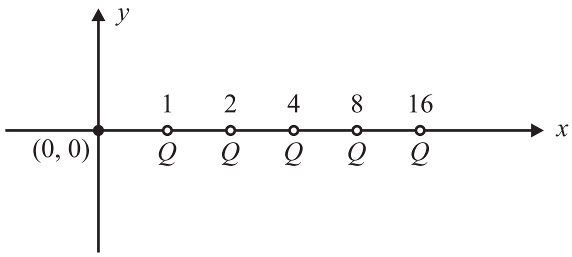
In other words, vectors whose line integral does not depend on the path of integration are called conservative vectors. Thus an electrostatic field is a conservative field.

Equation (ii) is referred to as Maxwell's 2nd equation in differential form.

Solved Example 10

An infinite number of charges each equal to Q are placed along the x-axis at $x = 1$, $x = 2$, $x = 4$, $x = 8$……… and so on. Find the potential and electric field at the point $x = 0$ due to this set of this set of charges.

Sol. Given : An infinite number of charges each equal to Q are placed along the x-axis at $x = 1$, $x = 2$, $x = 4$, $x = 8$……… and so on as shown in figure.



Resultant potential at the origin,

$$V_T = \frac{Q}{4\pi\varepsilon}\sum_{i=1}^{\infty}\frac{1}{r}$$

$$V_T = \frac{Q}{4\pi\varepsilon}\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+..........\right]$$

$$V_T = \frac{Q}{4\pi\varepsilon}\left[\frac{a}{1-r}\right]$$

Here $r=\frac{1}{2},\ a=1$

$$V_T = \frac{Q}{4\pi\varepsilon}\cdot\frac{1}{1-\frac{1}{2}} = \frac{Q}{2\pi\varepsilon}$$ **Ans.**

Resultant electric field intensity at the origin,

$$E_T = \frac{Q}{4\pi\varepsilon}\sum_{i=1}^{\infty}\frac{1}{r_i^2}$$

$$E_T = \frac{Q}{4\pi\varepsilon}\left[1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+.........\right]$$

$$E_T = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{1-\frac{1}{4}}\right] = \frac{Q}{4\pi\varepsilon}\times\frac{4}{3}$$

$$E_T = \frac{Q}{3\pi\varepsilon}$$ **Ans.**

TEST 3

Q.1 If the potential difference between points $A(1,0,0)$ and $B(2,0,0)$ is 10 V, determine d for point $C(d,0,0)$, when V_{BC} is 6 V in a uniform field.

(A) 1 m (B) 2 m

(C) 6 m (D) 5 m

Q.2 Which one of the following is the correct statement?

Equipotential lines and field lines

(A) are parallel

(B) are anti-parallel

(C) are orthogonal

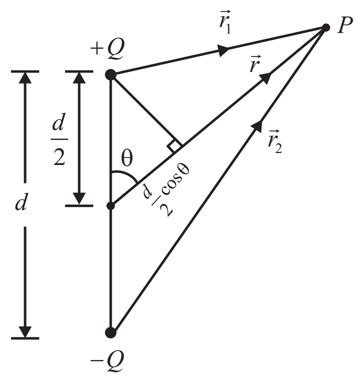
(D) bear no definite relationship

2.15 Electric Field and Electric Potential Due to Dipole

(i) When two equal and opposite charges are separated by finite distance then they contribute dipole.

(ii) Dipole is directed from negative charge to positive charge, while electric field is directed from positive charge to negative.

(iii) A diploe moment is a measurement of the separation of two oppositely charged charges.



Potential due to positive charge is,

$$V^+ = \frac{Q}{4\pi\varepsilon_0 r_1} \quad \ldots(i)$$

Potential due to negative charge is,

$$V^- = \frac{-Q}{4\pi\varepsilon_0 r_2} \quad \ldots(ii)$$

$$r_1 \approx r - \frac{d}{2}\cos\theta \quad \ldots(iii)$$

$$r_2 \approx r + \frac{d}{2}\cos\theta \quad \ldots(iv)$$

Overall potential at observation point is,

$$r = V^+ + V^-$$

$$V = \frac{Q}{4\pi\varepsilon_0 r_1} - \frac{Q}{4\pi\varepsilon_0 r_2}$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left\{\frac{1}{r_1} - \frac{1}{r_2}\right\} = \frac{Q}{4\pi\varepsilon_0}\left\{\frac{1}{r - \frac{d}{2}\cos\theta} - \frac{1}{r + \frac{d}{2}\cos\theta}\right\}$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left\{\frac{r + \frac{d}{2}\cos\theta - r + \frac{d}{2}\cos\theta}{r^2 - \left(\frac{d}{2}\cos\theta\right)^2}\right\} = \frac{Q}{4\pi\varepsilon_0}\left\{\frac{d\cos\theta}{r^2 - \left(\frac{d}{2}\cos\theta\right)^2}\right\}$$

$$r >> \frac{d}{2}\cos\theta$$

$$r^2 >>>> \left(\frac{d}{2}\cos\theta\right)^2$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left\{\frac{d\cos\theta}{r^2}\right\}$$

$$V = \frac{Q\,d\cos\theta}{4\pi\varepsilon_0 r^2}$$

Dipole moment is given by,

$$\vec{p} = Q\vec{d}$$

Dipole moment is a vector quantity.

$$V = \frac{\vec{p}\cdot\vec{a}_r}{4\pi\varepsilon_0 r^2}$$

$$\vec{p}\cdot\vec{a}_r = |\vec{p}||\vec{a}_r|\cos\theta$$

$$\vec{p}\cdot\vec{a}_r = Qd\cos\theta$$

Electric field is given by,

$$\vec{E} = -\nabla V$$

$$\vec{E} = -\left\{\frac{\partial V}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial V}{\partial\theta}\vec{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\vec{a}_\phi\right\}$$

$$\vec{E} = -\left\{\frac{-2Qd\cos\theta}{4\pi\varepsilon_0 r^3}\vec{a}_r - \frac{1}{r}\frac{Qd(\sin\theta)}{4\pi\varepsilon_0 r^2}\vec{a}_\theta\right\}$$

$$\vec{E} = \frac{Qd}{4\pi\varepsilon_0 r^3}\{2\cos\theta\vec{a}_r + \sin\theta\vec{a}_\theta\}$$

Remember

$$V\{\text{point charge}\} = \frac{Q}{4\pi\varepsilon_0 r} \quad\Rightarrow\quad V \propto \frac{1}{r}$$

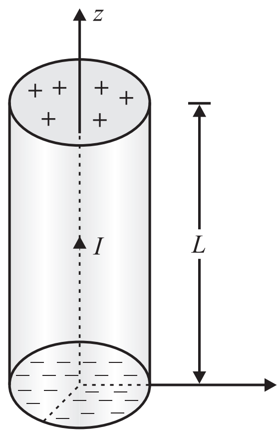
$$V\{\text{Dipole}\} = \frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2} \quad\Rightarrow\quad V \propto \frac{1}{r^2}$$

$$E\{\text{point charge}\} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a}_r \quad\Rightarrow\quad E \propto \frac{1}{r^2}$$

$$E\{\text{Dipole}\} = \frac{Qd}{4\pi\varepsilon_0 r^3}\{2\cos\theta\vec{a}_r + \sin\theta\vec{a}_\theta\} \quad\Rightarrow\quad E \propto \frac{1}{r^3}$$

2.16 Electric Current Density

Definition : *Current density is defined as amount of current in unit area. Current density is a vector quantity whereas current is a scalar quantity. It is directed along direction of flow of current.*



$$\vec{J} = J\,\vec{a}_z$$

$$\vec{J} = \frac{dI}{ds}$$

$$dI = \vec{J}\,d\vec{s}$$

$$I = \oint \vec{J}\cdot d\vec{s} \qquad \ldots\text{(i)}$$

Differential surface is given by,

$$d\vec{s} = d\rho(\rho d\phi)\vec{a}_z + (\rho d\phi\, dz)\vec{a}_\rho + dz\, d\rho\, \vec{a}_\phi$$

$$\vec{J}\cdot d\vec{s} = \vec{J}\cdot\vec{a}_z(d\vec{s}) = J\rho\, d\rho\, d\phi$$

From equation (i),

$$I = J\int_0^a \rho\, d\rho \int_{\phi=0}^{2\pi} d\phi$$

$$I = J\left(\frac{\rho^2}{2}\right)_0^a 2\pi$$

$$I = J\left(\frac{a^2}{2}\times 2\pi\right)$$

$$I = J\pi a^2 = J\times \pi a^2$$

$$J = \frac{I}{\pi a^2} = \frac{I}{A_c} \rightarrow \text{cross sectional area}$$

J **is also referred to as conduction current density according to find theory ohm's law.**

$$\vec{J} = \sigma\vec{E}$$

2.17 Energy Density in Electrostatic Fields

Total electrostatic energy stored in a medium with volumetric charge density ρ_v is given by,

$$W_E = \frac{1}{2}\int \rho_v V\, dv$$

$$W_E = \frac{1}{2}\int_v (\varepsilon_0 E^2)\, dV$$

$$W_E = \frac{1}{2}\int_v (E.D)\, dV = \frac{1}{2}\int_v \varepsilon_0\, E^2\, dV$$

$$W_E = \frac{1}{2}(Q_1 V_1 + Q_2 V_2 + Q_3 V_3)$$

$$W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k \qquad \text{...(i)}$$

From this, we can define electrostatic energy density w_E (in J/m^3) as

$$w_E = \frac{dW_E}{dv} = \frac{1}{2}D.E = \frac{1}{2}\varepsilon_0 E^2 = \frac{D^2}{2\varepsilon_0}$$

So, equation (i) may be written as

$$W_E = \int_v w_E\, dv$$

Solved Example 11

The potential function $V = 3x + 5y$ volts exists in free space. Find the stored energy in a $2\ \text{m}^3$ volume (spherical) centered at the origin.

Sol. **Given :** $V = 3x + 5y$

Electric field in terms of potential function is given by,

$$\vec{E} = -\nabla V$$

$$\vec{E} = -\left[\frac{\partial}{\partial x}(3x+5y)\vec{a}_x + \frac{\partial}{\partial y}(3x+5y)\vec{a}_y\right]$$

$$\vec{E} = -3\vec{a}_x - 5\vec{a}_y$$

$$|E| = \sqrt{9+25} = 5.8$$

Energy stored is given by,

$$W_E = \frac{1}{2}\varepsilon\int_V |E|^2\, dv$$

$$W_E = \frac{1}{2}\varepsilon(5.8)^2 \times 2 = \frac{10^{-9}}{36\pi}\times(5.8)^2 = 2.97\times 10^{-10}\ \text{Joules}$$

Ans.

2.18 Dielectric Material

Dielectric Material : The conductors have large number of free charges while insulators and dielectric materials don't have free charges.

The charges in dielectric are bounded by the finite forces and hence called bounded charges. As they are bound and not free they can't contribute to the conduction process but is subjected to an electric field E they shift their relative positions against the normal atomic forces. This shift in the relative position of bound charges allow the dielectric to store the energy.

The shift in positive charges and negative charge are in opposite direction and under the influence of an applied filed, such charge acts like small electric dipole.

When the dipole results from the displacement of the bound charge the dielectric is said to be polarized.

When the dipole is formed due to polarization their exits an electric dipole moment $\vec{P}$

$$\vec{P} = Q\vec{d}$$

Where $Q =$ magnitude of one of the two charges,

$\vec{d} =$ distance vector from negative to positive charge

Let $n =$ number of dipoles per unit volume

$\Delta v =$ total volume of the dielectric

$N =$ total dipole

$N = n\,\Delta v$

Total dipole moment is given by,

$$\vec{P}_{total} = Q_1\vec{d}_1 + Q_2\vec{d}_2.............$$

$$\vec{P}_{total} = \sum_{i=1}^{n\Delta v} Q_i\, d_i$$

The polarization, $\vec{P}_0$ is defined as the total dipole moment per unit volume.

$$\vec{P}_0 = \frac{\sum_{i=1}^{N} Q_i\, d_i}{\Delta v}$$

Properties of dielectric Materials :

1. The dielectrics do not contain any free charge but contain bound charges.
2. Bound charges are under the atomic force, hence can't contribute to the conduction.
3. Due to external electric field the bound charges shift their relative position and electric dipole gets induced inside the dielectric this is called polarization.
4. Due to polarization, dielectric can store the energy.
5. Due to polarization, the flux density of the dielectric increase by amount equal to the polarization.

6. The induced dipoles produce their own electric field and align in the direction of the applied electric field.
7. When polarization occurs the volume charge density is formed inside the dielectric while surface charge density is formed over the surface of the dielectric

Dielectric Strength :

1. The ideal dielectric is non-conducting but practically no dielectric can be ideal.
2. Under large electric field the dielectric becomes conducting due to presence of large number of free electron this condition of dielectric is called dielectric breakdown.
3. The breakdown depends on the nature of the material, magnitude of applied electric field and atmospheric conditions, such as temperature, humidity etc.

The minimum value of applied electric field at which the dielectric breakdown is called dielectric strength of that dielectric.

The dielectric strength is measured in V/m or kV/cm.

Bounded charge is given by,

$$Q_b = -\oint_s p_0 \cdot d\vec{s} \quad \ldots(i)$$

$p_0 =$ polarization vector

Free charge is given by,

$$Q_f = \oint_s \vec{D} \cdot d\vec{s} \quad \ldots(ii)$$

Total charge is given by,

$$Q_T = \oint_s \varepsilon_0 \vec{E} \cdot d\vec{s} \quad \ldots(iii)$$

Total charge = Bound charge + free charge

$$Q_T = Q_b + Q_f$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{s} = \oint_s \vec{D} \cdot d\vec{s} - \oint p_0 \cdot d\vec{s}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{s} + \oint p_0 \cdot d\vec{s} = \oint_s \vec{D} \cdot d\vec{s}$$

$$\oint (\varepsilon_0 \vec{E} + \vec{p}_0) \cdot d\vec{s} = \oint_s \vec{D} \cdot d\vec{s}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{p}_0$$

$$\varepsilon_0 \varepsilon_r \vec{E} = \varepsilon_0 \vec{E} + \vec{p}_0$$

$$\varepsilon_0 \varepsilon_r \vec{E} - \varepsilon_0 \vec{E} = \vec{p}_0$$

$$\varepsilon_0 (\varepsilon_r - 1) \vec{E} = \vec{p}_0$$

$$\varepsilon_0 \chi_e \vec{E} = \vec{p}_0$$

$\chi_e =$ Electric susceptibility $= \varepsilon_r - 1$

$$\vec{p}_0 = \varepsilon_0 \chi_e \vec{E}$$

(i) If $\chi_e = 0,\ \varepsilon_r = 1$

$$\vec{p}_0 = 0$$

$$Q_b = 0$$

$$Q_T = Q_f$$

(ii) $\chi_e > D$

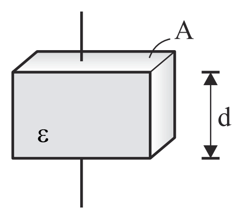
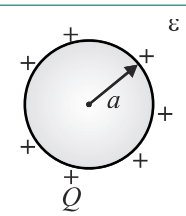
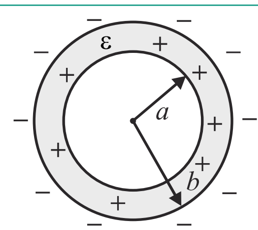
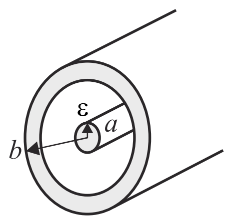
$$\vec{p}_0 \neq 0$$

$$Q_b \neq 0$$

$$Q_T = Q_b + Q_f$$

$\because$ Q_b negative $Q_T < Q_f$

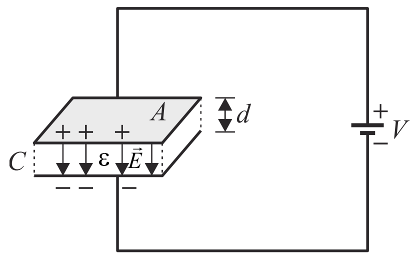
2.19 Capacitances for Different Types of Capacitors

Capacitor	Structure	Capacitance
(i) Parallel plate		$C = \frac{\varepsilon A}{d}$ F
(ii) Isolated sphere		$C = 4\pi\varepsilon a$ F
(iii) Concentric spheres		$C = \frac{4\pi\varepsilon}{\left(\frac{1}{a}\right)-\left(\frac{1}{b}\right)}$ F
(iv) Coaxial cable		$C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)} \frac{\text{F}}{\text{m}}$

2.20 Energy Stored in a Capacitor

It is seen that capacitor can store the electrostatic energy. Consider a parallel plate capacitor C with supplied voltage equal to V.

Let $\vec{a}_n$ be the direction normal to the plates.



$$\therefore \qquad \vec{E} = \frac{V}{d}\vec{a}_n \qquad \ldots(i)$$

The energy stored

$$W_E = \frac{1}{2}\int_{vol} \vec{D}\cdot\vec{E}\,dv$$

$$W_E = \frac{1}{2}\int_{vol} \varepsilon\vec{E}\cdot\vec{E}\,dv \qquad \text{but } \vec{E}\cdot\vec{E} = |\vec{E}|^2$$

$$W_E = \frac{1}{2}\int_{vol} \varepsilon|\vec{E}|^2\,dv \qquad \text{but } |\vec{E}| = \frac{V}{d}$$

$$W_E = \frac{1}{2}\varepsilon\frac{V^2}{d^2}\int_{vol} dv \qquad \text{but } \int_{vol} dv = \text{Volume} = A\times d$$

$$W_E = \frac{1}{2}\varepsilon\frac{V^2Ad}{d^2} = \frac{1}{2}\frac{\varepsilon A}{d}V^2$$

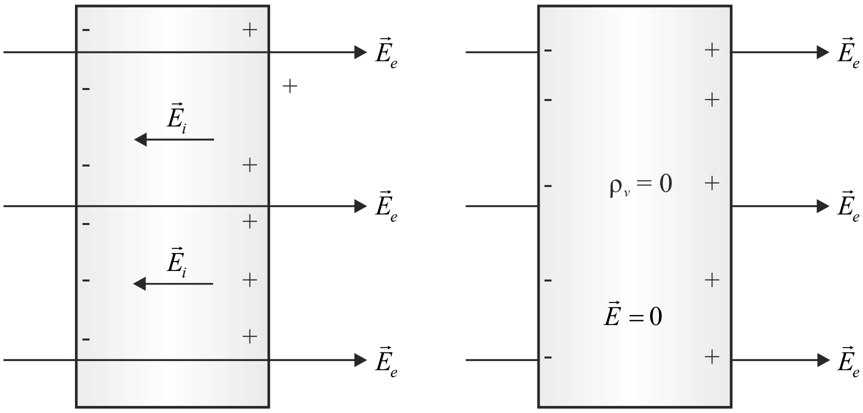
$$W_E = \frac{1}{2}CV^2$$

2.21 Conductors

When an external electric field $\vec{E}_e$ is applied, the positive free charges are pushed along the same direction as the applied field, while the negative free charges move in the opposite direction. This charge migration takes place very quickly. The free charges do two things. First, they accumulate in the surface of the conductor and form an induced surface charge. Second, the induced charges set up an internal induced field $\vec{E}_i$, which cancels the externally applied field $\vec{E}_e$.

A perfect conductor $(\sigma = \infty)$ cannot contain an electrostatic field within it.

$$\vec{E} = 0, \quad \rho_v = 0, \quad V_{ab} = 0 \text{ inside a conductor.}$$



(a) (b)

2.22 Continuity Equation : Conservation of Charge

Statement : *According to continuity equation the time rate of decrease of charge within a given volume must be equal to the net outward current flow through the surface of the volume.*

The continuity equation of the current is based on the **principle of conservation of charge**. The principle states that, the charges can neither be created nor destroyed. The current through the closed surface, is given by,

$$I = \oint_S \vec{J} \cdot d\vec{s}$$

According to law of charge, the outward flow of positive charge must be balanced by a decrease of positive charge within the closed surface.

$$I = \oint_S \vec{J} \cdot d\vec{s} = -\frac{dQ}{dt} \qquad \ldots(i)$$

Now, according to divergence theorem,

$$\oint_S \vec{J} \cdot d\vec{s} = \int_V (\nabla \cdot \vec{J})\, dv \qquad \ldots(ii)$$

Also,
$$-\frac{dQ}{dt} = -\frac{d}{dt}\int dQ = -\frac{d}{dt}\int_{Vol} \rho_v dv$$

$\therefore$
$$-\frac{dQ}{dt} = -\frac{d}{dt}\int_{Vol} \rho_v dv \qquad (\because dQ = \rho_v dv) \qquad \ldots(iii)$$

Substitute the value of $\oint_S \vec{J} \cdot d\vec{s}$ and $-\dfrac{dQ}{dt}$ in equation (i)

$$\oint_S \vec{J} \cdot d\vec{s} = -\frac{dQ}{dt}$$

$$\int_V (\nabla \cdot \vec{J})\, dv = -\frac{d}{dt}\int_V \rho_v dv$$

$$\int_V (\nabla \cdot \vec{J})\, dv = -\int_V \frac{\partial \rho_v}{\partial t} dv$$

Hence, we have

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

This is the differential or point form of continuity equation.

The equation states that the current or the charge per second, diverging from a small volume per unit volume is equal to the time rate of decrease of charge per unit volume at every point.

Now from Ohm's law in point form,

$$\vec{J} = \sigma \vec{E}$$

$$\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t}$$

So, $$\nabla \cdot (\sigma \vec{E}) = \frac{-\partial \rho_v}{\partial t}$$

But $$\vec{D} = \varepsilon \vec{E}$$

$$\nabla \cdot \left(\frac{\sigma \vec{D}}{\varepsilon} \right) = \frac{-\partial \rho_v}{\partial t}$$

$$\frac{\sigma}{\varepsilon} (\nabla \cdot \vec{D}) = \frac{-\partial \rho_v}{\partial t}$$

According to Gauss's law,

$$\nabla \cdot \vec{D} = \rho_v$$

$$\frac{\sigma}{\varepsilon} \rho_v = \frac{-\partial \rho_v}{dt}$$

By using variable separation method, solution of differential equation, we get

$$\log \rho_v = -\frac{\sigma}{\varepsilon} t + K$$

At $t = 0$, $\rho_v = \rho_0$,

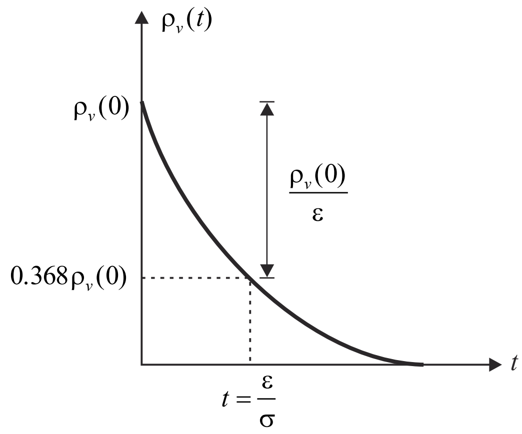
$$\log \rho_0 = K$$

$$\log \rho_v = -\frac{\sigma}{\varepsilon} t + \log \rho_0$$

$$\log \left(\frac{\rho_v}{\rho_0} \right) = -\frac{\sigma}{\varepsilon} t$$

$$\frac{\rho_v}{\rho_0} = e^{\frac{-\sigma}{\varepsilon} t}$$

$$\rho_v = \rho_0 e^{\frac{-\sigma}{\varepsilon} t}$$



Relaxation time : It is defined as the time required for the volume charge density to becomes 36.8% of its original value at time $t = 0$.

$$\tau = \frac{\varepsilon}{\sigma}$$

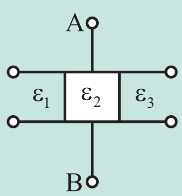
$$\rho_v = \rho_0 e^{\frac{-t}{\tau}}$$

At $t = \tau$, $\rho_v = \rho_0 e^{-1}$

$$\rho_v = 0.368\rho_0$$

TEST 4

Q.1



The space between the plates of a parallel-plate capacitor of capacitance C is filled with three dielectric slabs of identical size as shown in the figure. If dielectric constants are ε_1, ε_2 and ε_3, the new capacitance is

(A) $\frac{C}{3}$

(B) $\frac{(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)C}{3}$

(C) $(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)C$

(D) $\frac{9(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)C}{\varepsilon_1\varepsilon_2\varepsilon_3}$

Q.2 In a capacitor, the electric charge is stored in

(A) Dielectric

(B) Metal plates

(C) Dielectric as well as metal plates

(D) Neither dielectric nor metal plates

Q.3 A parallel plate capacitor of $100\,\text{pf}$ having an air dielectric is charged to 10 kilovolts. It is then electrically isolated. The plates are pulled away from each other until the distance is ten times more than before. Estimate the energy needed to pull the plates.

(A) 0.05 Joules (B) 50 Joules

(C) 500 Joules (D) – 50 Joules

2.23 Electric Boundary Conditions

When an electric field passes from one medium to other medium, it is important to study the conditions at the boundary between the two media. The conditions existing at the boundary of the two media when field passes from one medium to other are called **boundary conditions**. Depending upon the nature of the media, there are two situations of the boundary conditions.

(i) Boundary between conductor and free space,

(ii) Boundary between two dielectrics with different properties.

For free space is nothing but a dielectric hence the boundary conditions, between conductor and a dielectric, the Maxwell's equations for electrostatics are required.

(a) $\oint \vec{E}\cdot d\vec{l} = 0$ (b) $\oint \vec{D}\cdot d\vec{s} = Q$

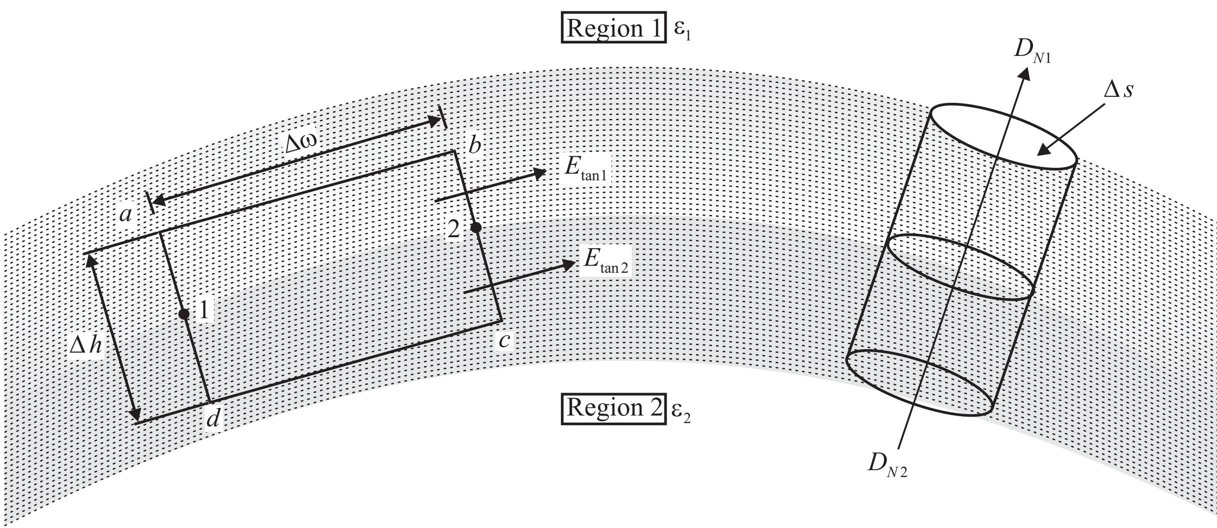
Similarly the field intensity $\vec{E}$ is required to be decomposed into two components namely, tangential to the boundary $(\vec{E}_{\tan})$ and normal to the boundary $(\vec{E}_N)$,

$$\therefore \quad \vec{E} = \vec{E}_{\tan} + \vec{E}_N$$

Boundary Condition Between two Perfect Dielectrics :

Let us consider the boundary between two perfect dielectrics. One dielectric has permittivity ε_1 while the other has permittivity ε_2. The interface is shown in the figure.

The $\vec{E}$ and $\vec{D}$ are to be obtained again by resolving each into two components, tangential to the boundary and normal to the surface.



$$E_{\tan 1} = E_{\tan 2}$$

Thus the tangential components of field intensity at the boundary both the dielectrics remain same i.e., electric field intensity is continuous across the boundary.

The relation between $\vec{D}$ and $\vec{E}$ is known as,

$$\vec{D} = \varepsilon \vec{E}$$

Hence if $D_{\tan 1}$ and $D_{\tan 2}$ are magnitudes of the tangential components of $\vec{D}$ in dielectric 1 and 2 respectively then,

$$D_{\tan 1} = \varepsilon_1 E_{\tan 1}$$

and $$D_{\tan 2} = \varepsilon_2 E_{\tan 2}$$

$$\therefore \quad \frac{D_{\tan 1}}{D_{\tan 2}} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

Thus tangential components of $\vec{D}$ undergoes some change across the interface hence tangential $\vec{D}$ is said to be discontinuous across the boundary.

For normal components :

$$D_{N1} = D_{N2} \qquad (\rho_s = 0)$$

The normal component of flux density $\vec{D}$ is continuous at the boundary between the two perfect dielectrics.

$$D_{N1} = \varepsilon_1 E_{N1}$$

$$D_{N2} = \varepsilon_2 E_{N2}$$

$$\therefore \quad \frac{D_{N1}}{D_{N2}} = \frac{\varepsilon_1}{\varepsilon_2}\frac{E_{N1}}{E_{N2}} = 1 \qquad (\text{as } D_{N1} = D_{N2})$$

$$\therefore \quad \frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$$

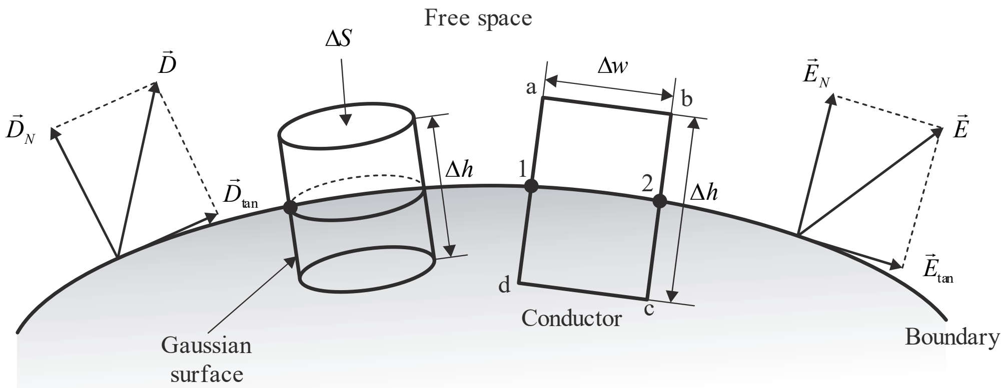
The normal components of the electric field intensity $\vec{E}$ are inversely proportional to the relative permittivities of the two media.

Boundary Condition Between Conductor and Free Space :

Consider a boundary between conductor and free space. The conductor is ideal having infinite conductivity. Such conductors are copper, silver etc. having conductivity of the order of 10^6 S/m and can be treated ideal. For ideal conductors it is known that,

1. The field intensity inside a conductor is zero and the flux density inside a conductor is zero.
2. No charge can exist within a conductor. The charge appears on the surface in the form of surface charge density.
3. The charge density within the conductor is zero.

Thus $\vec{E}$, $\vec{D}$ and ρ_v **within the conductor are zero.** While ρ_s is the surface charge density on the surface of the conductor.



$\vec{E}$ at the boundary :

Let $\vec{E}$ be the electric field intensity, in the direction shown in the figure making some angle with the boundary. This $\vec{E}$ can be resolved into two components :

1. The component tangential to the surface $(\vec{E}_{\tan})$.
2. The component normal to the surface $(\vec{E}_N)$.

$$E_{\tan} = 0$$

Thus the tangential component of the electric field intensity is zero at the boundary between conductor and free space.

Thus the $\vec{E}$ at the boundary between conductor and free space is always in the direction perpendicular to the boundary.

For free space $\vec{D} = \varepsilon_0 \vec{E}$

$\therefore$ $$D_{\tan} = \varepsilon_0 E_{\tan} = 0$$

Thus the tangential component of electric flux density is zero at the boundary between conductor and free space.

Thus electric flux density of $\vec{D}$ is also only in the normal direction at the boundary between the conductor and the free space.

$\vec{D}_N$ at the boundary :

$$D_N = \rho_s$$

Thus the flux leaves the surface normally and the normal component of flux density is equal to the surface charge density.

$\therefore$ $$D_N = \varepsilon_0 E_N = \rho_s$$

$\therefore$ $$E_N = \frac{\rho_s}{\varepsilon_0}$$

Thus that as the tangential component of $\vec{E}$, i.e. $\vec{E}_{\tan} = 0$, the surface of the conductor is an equipotential surface. The potential difference along any path on the surface of the conductor is $-\int \vec{E} \cdot d\vec{l}$ and as $\vec{E} = E_{\tan} = 0$, the potential difference is zero. Thus all points on the conductor surface are at the same potential.

Solved Example 12

Just inside the surface of dielectric slab the electric field is 10 V/m and makes an angle of 60^0 with the surface. If the dielectric constant of the slab is 4, find the electric field in direction just above the surface.

Sol. The tangential and normal components of the field in medium (1) (dielectric) are

$$E_{t_1} = 10\cos 60^0 = 5 \text{ V/m}$$

$$E_{n_1} = 10\sin 60^0 = 5\sqrt{3} \text{ V/m}$$

From the boundary condition Equation

$$E_{t_2} = E_{t_1} = 5 \text{ V/m and } D_{n_1} = D_{n_2} \Rightarrow \varepsilon_0 E_{n_2} = \varepsilon_0 (4) E_{n_1}$$

$$E_{n_2} = 4E_{n_1} = 20\sqrt{3} \text{ V/m}$$

Total electric field above the surface is

$$E_2 = \sqrt{E_{t_2}^2 + E_{n_2}^2}$$

$$E_2 = \sqrt{(20\sqrt{3})^2 + (5\sqrt{3})^2} = 35 \text{ V/m}$$

Angle which the field makes with the surface is

$$\alpha = \tan^{-1}\left(\frac{E_{n_2}}{E_{t_2}}\right) = \tan^{-1}\left(\frac{20\sqrt{3}}{5}\right) = 81.79^0$$ **Ans.**

2.24 Poisson's and Laplace's Equations

Poisson's and Laplace's equation are used for finding the charge and potential at some boundaries of the region.

Practically charge and potential may be known at some boundaries of the region only, for those values it is must to obtain potential and $\vec{E}$ throughout the region. Such "electrostatics problem are called boundary value problems.

Using Poisson's and Laplace's equation it is easy, if the charge distribution or potential throughout the region is known. To solve such problems, Poisson's and Laplace's equation must be known.

From the point form the Gauss's law, Poisson's equation can be derived,

$$\nabla \cdot \vec{D} = \rho_v \qquad (\because \text{Point form of Gauss's law}) \qquad \ldots\text{(i)}$$

Where, $\vec{D}$ = Electric flux density

ρ_v = Volume charge density.

Now, relation between electric flux density and electric field intensity for homogenous, isotropic and linear medium. Thus

$$\vec{D} = \varepsilon \vec{E} \qquad \ldots\text{(ii)}$$

Substitute the value of $\vec{D}$ from equation (ii) and (i), we get,

Now, $\nabla(\varepsilon \vec{E}) = \rho_v$

From the gradient relationship,

$$\vec{E} = -\vec{\nabla} V$$

$$\therefore \quad \nabla \cdot \varepsilon(-\nabla V) = \rho_v$$

$$-\varepsilon \nabla \cdot \nabla V = \rho_v$$

$$\nabla \cdot \nabla V = -\frac{\rho_v}{\varepsilon}$$

Now, $\nabla \cdot \nabla = \nabla^2$

So, $$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \quad \ldots\text{(iii)}$$

The equation (iii) is called Poisson's equation.

For certain region, volume charge density is zero $(\rho_v = 0)$, which is true for dielectric medium then the Poisson's equation,

$$\nabla^2 V = 0 \quad (\because \text{ for charge free region})$$

This is special case of Poisson's and is called Laplace's equation. The ∇^2 operation is called the Laplace's of V.

The expression for $\nabla^2 V$ in Cartesian, cylindrical and spherical coordinates system are given by,

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \quad \text{(in Cartesian system)}$$

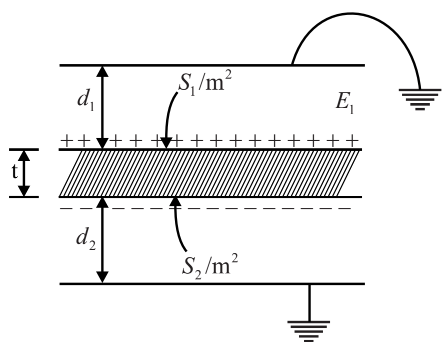
$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \quad \text{(in Cylindrical system)}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

(in Spherical system)

Solved Example 13

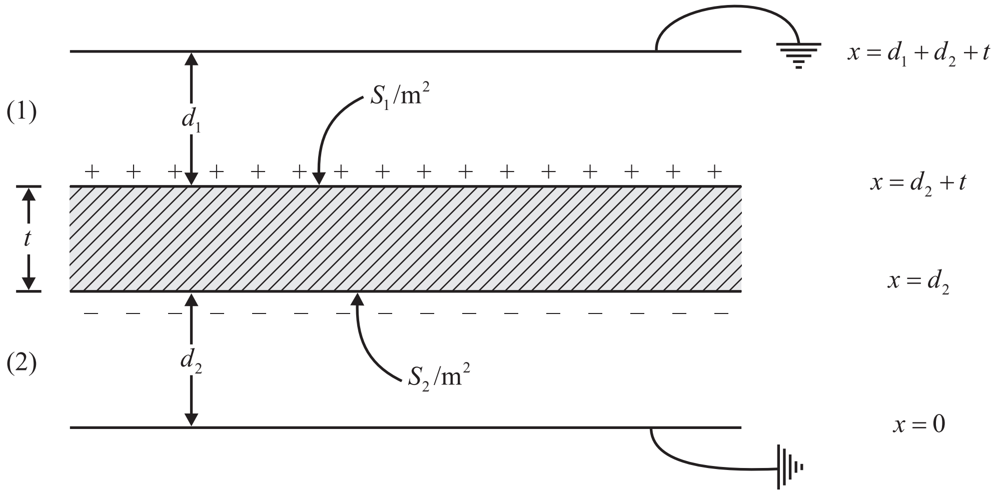
An infinite charged conducting plate is placed between and parallel to two infinite conducting grounded planes as shown. The ratio of charge densities S_1 and S_2 on the two sides of the plate will be



(A) $(d_1 + t)/(d_2 + t)$ (B) $(d_2 + t)/(d_2 + t)$

(C) d_2 / d_1 (D) d_1 / d_2

Sol.



Applying Laplace's equation, because $\rho_v = 0$ (non-conducting medium between plates).

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} = 0$$

Integrating twice, we get for region (2),

$$V_2 = A_2 x + B_2 \quad \ldots\text{(i)}$$

$$V_2(x=0) = 0 \Rightarrow A_2(0) + B_2 = 0 \Rightarrow B_2 = 0 \quad \ldots\text{(ii)}$$

For region (i),

$$V_1 = A_1 x + B_1 \quad \ldots\text{(iii)}$$

$$V_1(x = d_1 + d_2 + t) = 0$$

$$A_1(d_1 + d_2 + t) + B_1 = 0 \Rightarrow t = \frac{-B_1}{A_1} - d_1 - d_2$$

$$V_2(x = d_2) = V_1(x = t + d_2)$$

$$A_2 d_2 + B_2 = A_1(t + d_2) + B_1$$

From (ii), $B_2 = 0$

$$\therefore \quad A_2 d_2 = A_1\left(\frac{-B_1}{A_1} - d_1 - d_2 + d_2\right) + B_1$$

$$A_2 d_2 = -A_1 d_1 - A_1 d_2 + A_1 d_2$$

$$\frac{d_1}{d_2} = \frac{-A_2}{A_1} \quad \ldots\text{(iv)}$$

From boundary conditions between conductor and free space,

$$\because \quad D_{n_1} = S_1, \; \varepsilon_0 E_{n_1} = S_1$$

$$-\varepsilon_0 \frac{dV_1}{dx} = \rho_{s_1} \Rightarrow -\varepsilon_0 A_1 = S_1 \quad \ldots(v)$$

$$D_{n_2} = S_2,\ \varepsilon_0 E_{n_2} = S_2$$

$$-\varepsilon_0 \frac{dV_2}{dx} = S_2 \Rightarrow -\varepsilon_0 A_2 = S_2 \quad \ldots(vi)$$

From (iv), (v) and (vi),

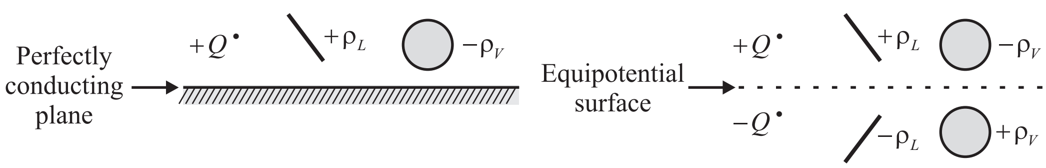
$$\frac{A_1}{A_2} = \frac{S_1}{-S_2} = \frac{d_2}{d_1}$$ **Ans.**

$\therefore$ Ratio is independent of t.

2.25 Method of Images

Consider a dipole field. The plane exists midway between the two charges, is a zero potential infinite plane. Such a plane may be represented by very thin conducting plane which is infinite. The conductor is an equipotential surface at a potential $V = 0$ and $\vec{E}$ is only normal to the surface. Thus if out of dipole, only positive charge is considered above a conducting plane then fields at all points in upper half of plane are same. In other words, if there is a charge above a conducting plane then same fields at the points above the plane can be obtained by replacing conducting plane by equipotential surface and an image of the charge at a symmetrical location below the plane. Such an image is negative of the original charge.

According to method of image if a charge is placed above a perfect conductor then its images are itself created below the equipotential surface.



The conditions to be satisfied to apply the method of images are :

1. The image charges must be located in the conducting region.
2. The image charges must be located such that on the conducting surface the potential is zero or constant.

The first condition is to satisfy the Poisson's equation while the other to satisfy the boundary conditions.

If a point charge is placed between two semi-infinite perfectly grounded conducting sheet placed at an angle θ degree then number of image charges is given by,

$$N = \left(\frac{360^0}{\theta} - 1\right).$$

TEST 5

Q.1 The normal components of electric flux density across a dielectric-dielectric boundary

(A) are discontinuous.

(B) are continuous.

(C) depend on the magnitude of the surface charge density.

(D) depend on electric field intensity.

Q.2 Consider the following statements in connection with boundary relations of electric field

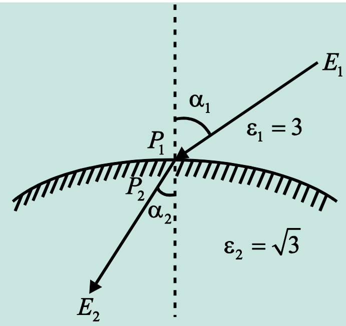
1. In a single medium electric field is continuous.
2. The tangential components are the same on both sides of a boundary between two dielectrics.
3. The tangential electric field at the boundary of a dielectric and a current carrying conductor with finite conductivity is zero.
4. Normal component of the flux density is continuous across the charge-free boundary between two dielectrics.

Which of these statements is/are correct?

(A) 1 only (B) 1, 2 and 3

(C) 1, 2 and 4 (D) 3 and 4 only

Q.3 Two dielectric media with permittivities 3 and $\sqrt{3}$ are separated by a charge-free boundary as shown in figure. The electric field intensity in medium 1 at point P_1 has magnitude E_1 and makes an angle $\alpha_1 = 60^0$ with the normal. The direction of the electric field intensity at point (P_2, α_2) is



(A) $\sin^{-1}\left(\frac{\sqrt{3}E_1}{2}\right)$ (B) 45^0

(C) $\cos^{-1}\left(\frac{\sqrt{3}E_1}{2}\right)$ (D) 30^0

Q.4 In a multilayer dielectric material on either side of a boundary

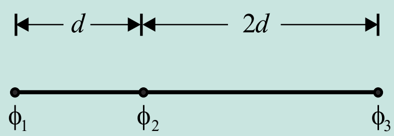
(A) The normal component of electrical displacement is same.

(B) The normal component of electric field is same.

(C) The tangential component of electrical displacement is necessarily zero.

(D) The tangential component of electrical field is necessarily zero.

Q.5 The three values of a one-dimensional potential in the given figure and satisfying Laplace's equation are related as



(A) $\phi_2 = \frac{2\phi_3 + \phi_1}{3}$ (B) $\phi_2 = \frac{2\phi_1 + \phi_3}{3}$

(C) $\phi_2 = \frac{2\phi_1 + 2\phi_3}{3}$ (D) $\phi_2 = \frac{\phi_1 + 3\phi_3}{2}$

Important Formulas

1. Coulomb's law is given by,

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}\hat{a}_R$$

2. Electric field intensity at a point due to presence of charge Q at distance 'r' is given by,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{a}_R$$

3. Electric field due to an infinite line charge is given by,

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{a}_r$$

4. Electric field due to an infinite sheet of surface charge is given by,

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a}_r$$

5. Gauss's Divergence theorem is given by,

$$\nabla \cdot D = \rho_V$$

6. Electric flux density due to uniformly charged sphere is

(i) $\vec{D} = 0$ (for $r < R$)

(ii) $\vec{D} = \dfrac{Q}{4\pi r^2}$ (for $r = R$)

(iii) $\vec{D} = \dfrac{Q}{4\pi R^2}$ (for $r > R$)

7. Electric flux density due to uniformly charged sphere where Q is a distributed on complete volume

(i) $\vec{D} = \dfrac{Q}{4\pi}\left(\dfrac{r}{R^3}\right)\hat{a}_r$ (for $r < R$)

(ii) $\vec{D} = \dfrac{Q}{4\pi R^2}\hat{a}_r$ (for $r = R$)

(iii) $\vec{D} = \dfrac{Q}{4\pi r^2}\hat{a}_r$ (for $r > R$)

8. Scalar potential is related to electric field as

$$V = -\int E dl$$

or $$E = -\nabla V$$

9. Energy density in electrostatic field is given as

$$W_E = \frac{1}{2}\int_V \varepsilon_0 E^2 dv$$

10. Electric Boundary conditions are given as

$$E_{1t} = E_{2t}$$

$$D_{1N} = D_{2N} \quad \text{(for surface charge } \rho_s = 0\text{)}$$

Numerical Answer Type Questions

Q.1 A capacitor is made with a polymeric dielectric having an ε_r of 2.26 and a dielectric breakdown strength of 50 kV/cm. The permittivity of free space is 8.85 pF/m. If the rectangular plate of the capacitor have a width of 20 cm and a length of 40 cm, then the maximum electric charge in (μC) the capacitor is ______.

Q.2 Two point charges $Q_1 = 10\,\mu\text{C}$ and $Q_2 = 20\,\mu\text{C}$ are placed at coordinates (1, 1, 0) and $(-1,-1,0)$ respectively. The total electric flux (in μC) passing through a plane $z = 20$ will be _____.

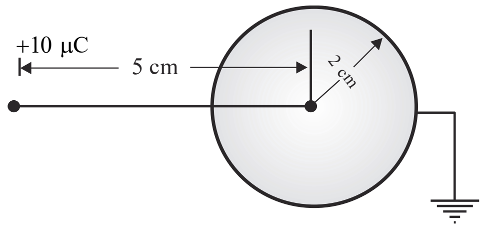
Q.3 An electrostatic potential is given by $\phi = 2x\sqrt{y}$ volts in the rectangular co-ordinate system. The magnitude of the electric field at x = 1 m, y = 1 m is ______V/m.

Q.4 The ratio of the charges stored by two metallic spheres raised to the same potential is 6. The ratio of the surface area of the sphere is _____.

Q.5 In the field of a charge Q at the origin, the potentials at A (2, 0, 0) and B (1 / 2, 0, 0) are $V_A = 15\,\text{volt}$ and $V_B = 30\,\text{volt}$ respectively, the potential at C (1, 0, 0) will be ____.

Q.6 Two small diameter 5 g dielectric balls can slide freely on a vertical non-conducting thread. Each ball carries a negative charge of $2\,\mu\text{C}$. If the lower ball is restrained from moving, then the separation between the two balls will be ______mm.

Q.7 A point charge of $+10\mu C$ placed at a distance of 5 cm from the centre of a conducing grounded sphere of radius 2 cm is shown in the diagram given below:



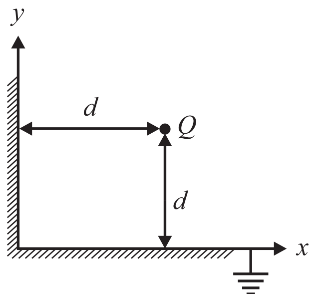
The total induced charge (in μC) on the conducting sphere will be _____.

Q.8 The displacement flux density at a point on the surface of a perfect conductor is $\bar{D} = 2\left(\vec{a}_x - \sqrt{3}\vec{a}_z\right)\text{C/m}^2$ and is pointing away from the surface. The surface charge density at the point (C/m^2) will be ______.

Q.9 A non-conducting ring of radius 0.5 m carries a total charge of $1.11\times10^{-10}\,\text{C}$ distributed non-uniformly on its circumference producing an electric field $\vec{E}$ everywhere in space. The value of the line integral $\int_{l=\infty}^{l=0} -\vec{E}\cdot\vec{dl}$ ($l = 0$ being central of the ring) in volt is ______.

Multiple Choice Questions

Q.1 Two semi-infinite conducting sheets are placed at right angles to each other as shown in the figure. A point charge of $+Q$ is placed at a distance of d from both sheets. The net force on the charge is $\frac{Q^2}{4\pi\varepsilon_0}\frac{K}{d^2}$, where K is given by



(A) 0

(B) $-\frac{1}{4}\hat{i}-\frac{1}{4}\hat{j}$

(C) $-\frac{1}{8}\hat{i}-\frac{1}{8}\hat{j}$

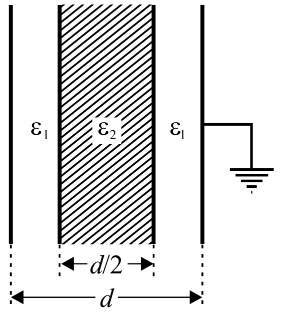
(D) $\frac{1-2\sqrt{2}}{8\sqrt{2}}\hat{i}+\frac{1-2\sqrt{2}}{8\sqrt{2}}\hat{j}$

Q.2 C_0 is the capacitance of a parallel plate capacitor with air as dielectric (as in figure (a)). If, half of the entire gap as shown in figure (b) is filled with a dielectric of permittivity ε_r, the expression for the modified capacitance is,

(A) $\frac{C_0}{2}(1+\varepsilon_r)$	(B) $(C_0+\varepsilon_r)$

(C) $\frac{C_0}{2}\varepsilon_r$	(D) $C_0(1+\varepsilon_r)$

Q.3 A parallel plate capacitor consisting two dielectric materials is shown in the figure. The middle dielectric slab is placed symmetrically with respect to the plates.

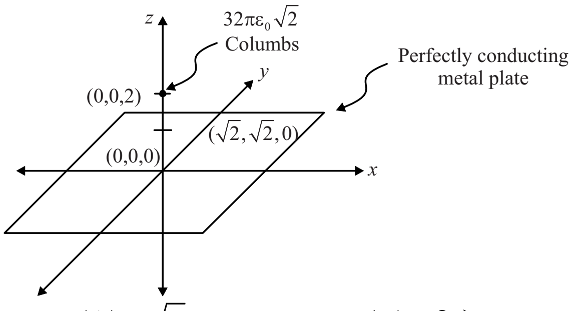


If the potential difference between one of the plates and the nearest surface of dielectric interface is 2 Volts, then the ratio $\varepsilon_1:\varepsilon_2$ is

(A) 1 : 4	(B) 2 : 3

(C) 3 : 2	(D) 4 : 1

Q.4 A perfectly conducting metal plate is placed in x-y plane in a right handed coordinate system. A charge of $+32\pi\varepsilon_0\sqrt{2}$ columbs is placed at coordinate (0,0,2). ε_0 is the permittivity of free space. Assume $\vec{a}_x, \vec{a}_y, \vec{a}_z$ to be unit vectors along x, y and z axis respectively. At the coordinate $(\sqrt{2},\sqrt{2}\,0)$, the electric field vector $\vec{E}$ (Newton's/Columb) will be



(A) $2\sqrt{2}\vec{a}_z$	(B) $-2\vec{a}_z$

(C) $2\vec{a}_z$	(D) $-2\sqrt{2}\vec{a}_z$

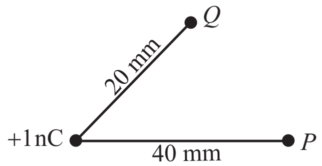
Q.5 If the electric field intensity is given by

$$\vec{E} = (x\vec{u}_x + y\vec{u}_y + z\vec{u}_z) \text{ volt/m}$$

the potential difference between $x(2, 0, 0)$ and $y(1, 2, 3)$ is

(A) + 1 Volt (B) – 1 Volt
(C) + 5 Volt (D) + 6 Volt

Q.6 A point of +1nC is placed in a space with permittivity of 8.85×10^{-12} F/m as shown in figure. The potential difference V_{PQ} between two points P and Q at distance of 40 mm and 20 mm respectively from the point charge is



(A) 0.22 kV (B) –225 V
(C) –2.24 kV (D) 15 kV

Q.7 In a coaxial transmission line $(\varepsilon_r = 1)$, the electric field intensity is given by

$$E = \frac{100}{\rho}\cos(10^9 t - 6z)\vec{a}_\rho \text{ V/m}$$

The displacement current density is

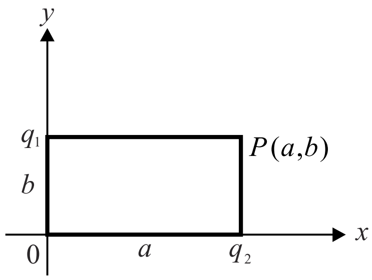
(A) $-\frac{100}{\rho}\sin(10^9 t - 6z)\vec{a}_\rho$ A/m²

(B) $\frac{116}{\rho}\sin(10^9 t - 6z)\vec{a}_\rho$ A/m²

(C) $-\frac{0.9}{\rho}\sin(10^9 t - 6z)\vec{a}_\rho$ A/m²

(D) $-\frac{216}{\rho}\cos(10^9 t - 6z)\vec{a}_\rho$ A/m²

Q.8 Two point charges $q_1 = 2\mu\text{C}$ and $q_2 = 1\mu\text{C}$ are placed at distances b = 1 cm and a = 2 cm from the origin on the y and x axes as shown in figure. The electric field vector at point $P(a, b)$ that will subtend at angle θ with the x axis is



(A) $\tan\theta = 1$ (B) $\tan\theta = 2$
(C) $\tan\theta = 3$ (D) $\tan\theta = 4$

Q.9 A point charge is located at origin. At point (a, a), electric field is $\vec{E}_1$. At point $(-a, a)$ the electric field is $\vec{E}_2$ and at a point $(-a, -a)$ the electric field is $\vec{E}_3$. Which one of the following is correct?

(A) $\vec{E}_1 \cdot \vec{E}_2 = 0$

(B) $|\vec{E}_1 \times \vec{E}_3| = 0$

(C) Both $\vec{E}_1 \cdot \vec{E}_2 = 0$ and $|\vec{E}_1 \times \vec{E}_3| = 0$

(D) Neither $\vec{E}_1 \cdot \vec{E}_2 = 0$ nor $|\vec{E}_1 \times \vec{E}_3| = 0$

Q.10 A charge 'Q' is divided between two point charges. What should be the values of this charge on the objects so that the force between them is maximum ?

(A) $\frac{Q}{3}$ (B) $\frac{Q}{2}$
(C) $(Q-2)$ (D) $2Q$

Q.11 At a point (x, y, z) potential is given by

$$A(x^2 + y^2 + z^2).$$

The potential difference between points P (1, 0, 2) and Q (1, 1, 2) is

(A) 7 V (B) 8A V
(C) 9A V (D) 9 V

Q.12 There are three charges, which are given by $Q_1 = 1\mu\text{C}$, $Q_2 = 2\mu\text{C}$ and $Q_3 = 3\mu\text{C}$. The field due to each charge at a point P in free space is

$(\vec{a}_x + 2\vec{a}_y - \vec{a}_z)$, $(\vec{a}_y + 3\vec{a}_z)$ and $(2\vec{a}_x - \vec{a}_y)$ N/C.

The total field at the point P due to all three charge is given by,

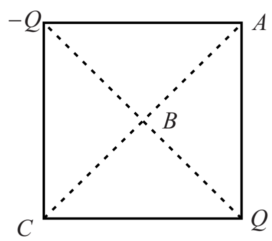
(A) $1.6\vec{a}_x + 2.2\vec{a}_y + 2.5\vec{a}_z$ N/C

(B) $0.3\vec{a}_x + 0.2\vec{a}_y + 0.2\vec{a}_z$ N/C

(C) $3\vec{a}_x + 2\vec{a}_y + 2\vec{a}_z$ N/C

(D) $0.6\vec{a}_x + 0.2\vec{a}_y + 0.5\vec{a}_z$ N/C

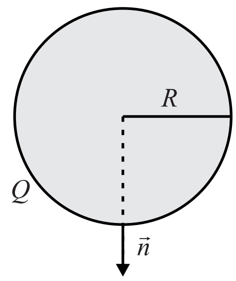
Q.13 Two point charges Q and $-Q$ are located on two opposite corners of a square as shown in the figure. If the potential at the corner A is taken as 1 V, then the potential at B, the centre of the square will be



(A) Zero (B) $\frac{1}{\sqrt{2}}$ V

(C) 1V (D) $\sqrt{2}$V

Q.14 A point charge Q is located on the surface of a sphere of radius R as shown in the figure. The average electric field on the surface of the sphere will be



(A) Infinite

(B) $\frac{Q}{4\pi\varepsilon_0}\frac{1}{R^2}(-\vec{n})$

(C) $\frac{Q}{8\pi\varepsilon_0}\frac{1}{R^2}(-\vec{n})$

(D) Zero

Q.15 Consider the following statements regarding field boundary conditions

1. The tangential component of electric field is continuous across the boundary between two dielectrics.
2. The tangential component of the electric field at a dielectric-conductor boundary is non zero.
3. The discontinuity in the normal component of the flux density at a dielectric-conductor boundary is equal to the surface charge density on the conductor.
4. The normal component of the flux-density is continuous across the charge-free boundary between two dielectrics.

(A) 1, 2 and 3 are correct

(B) 1, 3 and 4 are correct

(C) 1, 2 and 4 are correct

(D) 3 and 4 are correct

Q.16 Consider the following statements : In electrostatics, the Equipotential surface is defined as the surface where

1. Electric field intensity is normal everywhere.
2. Electric field intensity is tangential everywhere.
3. No work is done in moving a charge over it.
4. No charge is present.

Of these statements

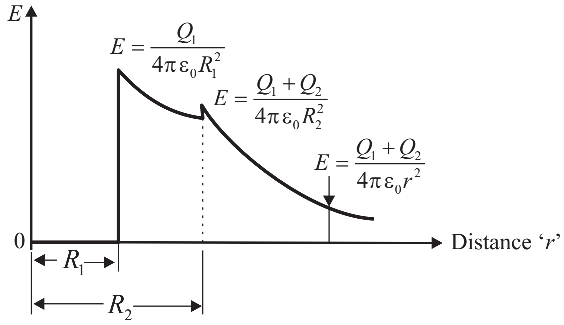
(A) 1 alone is correct

(B) 3 and 4 are correct

(C) 1 and 3 are correct

(D) 2 and 4 are correct

Q.17 The given figure represents the variation of electric field 'E'



(A) Due to a spherical volume charge $Q = Q_1 + Q_2$

(B) Due to two concentric shells of charges Q_1 and Q_2 uniformly distributed over spheres of radii R_1 and R_2.

(C) Due to two point charges Q_1 and Q_2 located at any two points 'r' $(= R_1$ and $R_2)$.

(D) In a single spherical shell of charges Q uniformly distributed, $Q = Q_1 + Q_2$.

Q.18 Charge needed within a unit sphere centred at the origin for producing a potential field,

$$V = -\frac{6r^5}{\varepsilon_0}, \text{ for } r \le 1 \text{ is \_\_\_\_\_\_\_.}$$

(A) 12π C (B) 60π C

(C) -120π C (D) 180π C

Q.19 Plane z = 10 m carries surface charge density $20\,\text{nC/m}^2$. What is the electric field at the origin?

(A) $-10\,\hat{a}_z$ V/m

(B) $-18\,\pi\,\hat{a}_z$ V/m

(C) $72\,\pi\,\hat{a}_z$ V/m

(D) $-360\,\pi\,\hat{a}_z$ V/m

Q.20 Two extensive homogeneous isotropic dielectrics meet on a plane z = 0. For $z \ge 0$, $\varepsilon_{r1} = 4$ and for $z \le 0$, $\varepsilon_{r2} = 2$. A uniform electric field exist at $z \ge 0$ as $\vec{E}_1 = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$ kV/m. What is the value of $\vec{E}_{n_2}$ in the region $z \le 0$?

(A) $3\hat{a}_z$ (B) $5\hat{a}_x - 2\hat{a}_y$

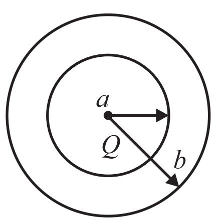
(C) $6\hat{a}_z$ (D) $\hat{a}_x - \hat{a}_y$

Q.21 A positive charge of 1nC is placed at (0, 0, 0.2) where all dimensions are in metres. Consider the x-y plane to be a conducting ground plane. The Z component of the E field at (0, 0, 0.1) is closest to

(A) 899.18 V/m (B) – 899.18 V/m

(C) 999.09 V/m (D) – 999.09 V/m

Q.22 The region between two concentric spheres of radii 'a' and 'b' respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the center. At the center of the spheres is a point charge Q. The value of A such that the electric field in the region between spheres will be constant is :



(A) $\frac{Q}{2\pi a^2}$ (B) $\frac{Q}{2\pi(b^2-a^2)}$

(C) $\frac{2Q}{\pi(a^2-b^2)}$ (D) $\frac{2Q}{\pi a^2}$

Q.23 Three concentric metal shells A, B and C of respectively radii a, b and $c\,(a<b<c)$ have surface charge densities $+\sigma, -\sigma$ and $+\sigma$ respectively. The potential of shell B is :

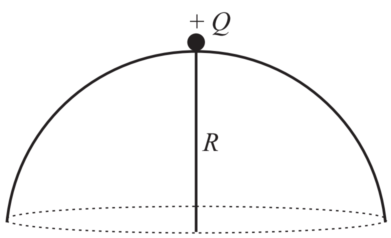
(A) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$

(B) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

(C) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$

(D) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

Q.24 A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



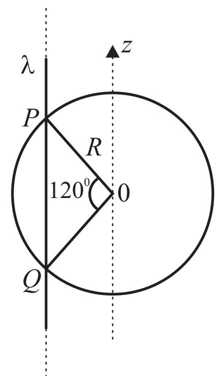
(A) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

(B) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$

(C) The component of the electric field normal to the surface is constant over the surface

(D) The circumference of the surface is an equipotential

Q.25 An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120^0 at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true?



(A) The electric flux through the shell is $\sqrt{3}R\lambda/\varepsilon_0$

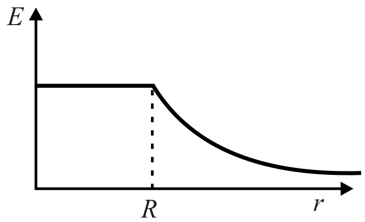
(B) The z-component of the electric filed is zero at all the points on the surface of the shell

(C) The electric flux through the shell is $\sqrt{2}R\lambda/\varepsilon_0$

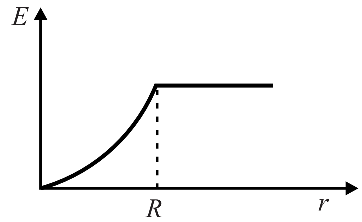
(D) The electric field is normal to the surface of the shell at all points

Q.26 Which of the following graphs shows the variation of electric field E due to a hollow spherical conductor of radius R as a function of distance from the centre of the sphere.

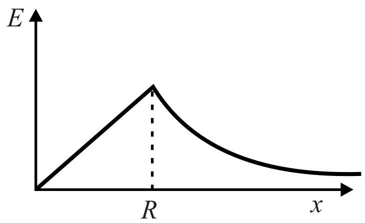
(A)



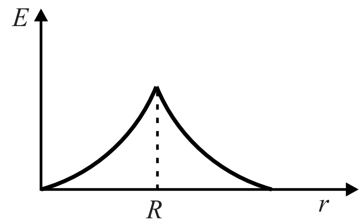
(C)



(B)



(D)



Try Yourself

Q.1 Three electrostatic point charges are located in the *xy*-plane as given below:

$$+Q \text{ at } \left(-\frac{a}{2}, 0\right),\ +Q \text{ at } \left(\frac{a}{2}, 0\right) \text{ and } -2Q \text{ at } \left(0, \frac{a\sqrt{3}}{2}\right)$$

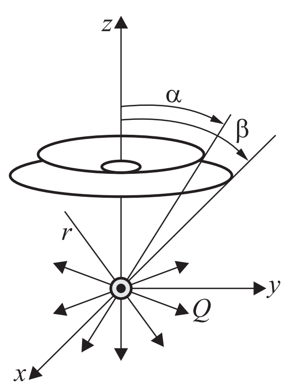
Calculate the coordinate of the point, *P*, on the *y*-axis, where the potential due to these charges is zero. Also, calculate the magnitude of the electric field strength at *P*.

Q.2 Two identical conducting spheres *A* and *B* carry equal charge *Q*. They are separated by a distance much large than their diameters. A third identical conducting sphere *C* carries charge 2*Q*. Sphere *C* is first touched to *A*, then to *B* and finally removed. Find electrostatic force between *A* and *B* which was originally *F*.

Q.3 A parallel plate capacitor has length and width 1 in each and the plate separation is 10 ram, Find the energy stored in capacitor and the force between the plates if air is filled between the plates and p.d of 10 kV applied,

Q.4 An air-insulated parallel-plate capacitor has plate area $76\,\text{cm}^2$ and spacing 1.2 mm. It is charges to 900 V and then disconnected from the charging battery. A plexiglass sheet having $\varepsilon_r = 3.4$ is then inserted to fill the space between the plates. What are (a) the capacitance, (b) the potential difference between the plates, and (c) the stored energy both before and after the plexiglass is inserted?

Q.5 A point charge *Q* is at the origin of a spherical coordinate system. Find the flux which crosses the portion of a spherical shell described by $\alpha \le \theta \le \beta$. What is the result if $\alpha = 0$ and $\beta = \frac{\pi}{2}$.



Q.6 Two homogenous isotropic dielectrics meet on plane $z = 0$. For $z > 0$, $\varepsilon_{r1} = 4$ and for $z < 0$, $\varepsilon_{r2} = 3$. A uniform electric field $\vec{E}_1 = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$ kV/m exists for $z \geq 0$. Find

(i) $\vec{E}_2$ for $z \leq 0$

(ii) The angles E_1 and E_2 make with the interface

(iii)The energy densities (in J/m^3) in both dielectrics

(iv)The energy within a cube of side 2 m centered at (3, 4, –5)

Q.7 A conducting sphere of radius 1 cm is surrounded by a conducting spherical shell of inner radius 3 cm and outer radius 4 cm. If the electric field at $r = 2\,\text{cm}$ is going outwards with magnitude 300 V/cm and at $r = 5\,\text{cm}$ is also going outwards with magnitude 300 V/cm. What is the net charge on conducting spherical shell?

Q.8 Two conducting spheres are far apart. The smaller sphere carries a total charge of *Q*. The large sphere has a radius three times that of the smaller sphere and is neutral. After the two spheres are connected by a conducting wire, find the charges on the smaller and larger spheres.

Answers to Test 1

1.	C	2.	B	3.	B	4.	D	5.	B

Answers to Test 2

1.	B	2.	D	3.	C	4.	B	5.	A

Answers to Test 3

1.	D	2.	C						

Answers to Test 4

1.	B	2.	B	3.	A				

Answers to Test 5

1.	D	2.	C	3.	B	4.	D	5.	B

Answers to Numerical Answer Type Questions

1.	8	2.	15	3.	2.23	4.	36	5.	10
6.	857	7.	4	8.	4	9.	2		

Answers to Multiple Choice Questions

1.	D	2.	A	3.	C	4.	B	5.	C
6.	B	7.	C	8.	B	9.	C	10.	B
11.	A	12.	C	13.	C	14.	C	15.	B
16.	C	17.	D	18.	C	19.	D	20.	C
21.	D	22.	A	23.	D	24.	B	25.	A, B
26.	B								

Scan for Detailed solution of Try yourself

Explanation to Numerical Answer Type Questions

Sol.1

Given : $A = 20 \times 40\,\text{cm}^2 = 8 \times 10^{-2}\,\text{m}^2$

$$\varepsilon = \varepsilon_0 \varepsilon_r = 2.26 \times 8.85 \times 10^{-12}\ \text{F/m}$$

Breakdown strength of the dielectric

$$= 50\,\text{kV/cm} = 5 \times 10^6\ \text{Volts/m}$$

For a distance of separation, d meter ; maximum voltage that can be applied,

$$V_m = 5 \times 10^6 d\ \text{Volts}$$

Maximum charge in the capacitor,

$$Q_m = CV_m, \text{ where } C = \frac{\varepsilon A}{d}$$

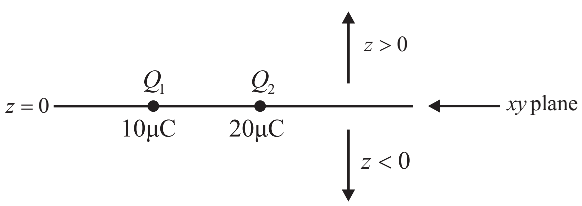
$$Q_m = \frac{\varepsilon A}{d} 5 \times 10^6 d = 5 \times 10^6 \varepsilon A$$

$$Q_m = 5 \times 10^6 \times 2.26 \times 8.85 \times 10^{-12} \times 8 \times 10^{-2}\,\text{C}$$

$$Q_m = 8\,\mu\text{C}$$

Hence, the maximum electric charge in the capacitor is $8\,\mu\text{C}$.

Sol.2



The charges are lying on the xy plane. Hence, half of the flux will pass through above the plane (i.e. $z > 0$) and half will pass through below the plane (i.e. $z < 0$).

Now total flux = Total charge enclosed

$$= 10\,\mu\text{C} + 20\,\mu\text{C} = 30\,\mu\text{C}$$

Total flux passing through infinite plane $z = 20$ will be $\frac{30}{2}\,\mu\text{C} = 15\,\mu\text{C}$

Hence, the total electric flux passing through a plane $z = 20$ is $15\,\mu\text{C}$.

Sol.3

Given : Electrostatic potential,

$$\phi = 2x\sqrt{y}\ \text{Volts.}$$

Electric field intensity is given by,

$$\vec{E} = -\nabla\phi = -\left[\frac{\partial\phi}{\partial x}\vec{a}_x + \frac{\partial\phi}{\partial y}\vec{a}_y\right]$$

$$\vec{E} = -\left[\frac{\partial\left(2x\sqrt{y}\right)}{\partial x}\vec{a}_x + \frac{\partial\left(2x\sqrt{y}\right)}{\partial y}\vec{a}_y\right]$$

$$\vec{E} = -\left[2\sqrt{y}\,\vec{a}_x + \frac{x}{\sqrt{y}}\vec{a}_y\right]$$

At the point $x = 1$ m, $y = 1$ m

$$E_x = 2,\ E_y = 1$$

$$\therefore \quad \vec{E} = -\left[2\vec{a}_x + 1\vec{a}_y + 0\vec{a}_z\right]$$

Magnitude of electric field intensity is given by,

$$\left|\vec{E}\right| = E = \sqrt{2^2 + 1^2} = \sqrt{5}\ \text{V/m}$$

Hence, the correct answer is $\sqrt{5}$.

Sol.4

For a sphere potential is given by,

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

$$\frac{Q_1}{4\pi\varepsilon_0 R_1} = \frac{Q_2}{4\pi\varepsilon_0 R_2}$$

$$\frac{Q_1}{Q_2} = \frac{R_1}{R_2} = 6$$

Surface area of sphere $= 4\pi R^2$

$$\frac{\text{Area}_1}{\text{Area}_2} = \frac{R_1^2}{R_2^2} = 36$$

Hence, the ratio of the surface area of the sphere is 36.

Sol.5

In general, potential due to Point charge is given by,

$$V = \frac{Q}{4\pi\varepsilon_0 R} + C$$

$$V_A = \frac{Q}{4\pi\varepsilon_0 (2)} + C = 15\,(\text{Given})$$

$$V_B = \frac{Q}{4\pi\varepsilon_0 \left(\frac{1}{2}\right)} + C = 30\,(\text{Given})$$

From equation (i) and (ii)

$$V_A - V_B = -15 = \frac{Q}{4\pi\varepsilon_0 (2)} - \frac{2Q}{4\pi\varepsilon_0}$$

$$-15 = Q\left[\frac{1}{8\pi\varepsilon_0} - \frac{2}{4\pi\varepsilon_0}\right]$$

$$Q = \frac{-15 \times 8\pi\varepsilon_0}{(1-4)} = 40\pi\varepsilon_0$$

Put the value Q in equation (i) from we get,

$$V = \frac{40\pi\varepsilon_0}{4\pi\varepsilon \times 2} + C = 15$$

Then $C = 10$

Hence, the potential at C (1, 0, 0) is 10 V.

Sol.6

From Coloumb's law, we have

$$\frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = mg$$

Where mg = Gravitational force on mass m

Where r is the separation between the two balls.

$$\frac{2\times10^{-6} \times 2\times10^{-6} \times 9\times10^{9}}{r^2} = 5\times10^{-3} \times 9.8$$

$$r^2 = 0.7347$$

$$r = 0.857 \text{ m} = 857 \text{ mm}$$

Hence, the separation between the two balls is 857 mm.

Sol.7

Capacitance of the sphere is given by,

$$C = 4\pi\varepsilon_0 a$$

Voltage at the centre of sphere is given by,

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Where a = radius of sphere ...(i)

r = distance between the centre of sphere and point charge ...(ii)

Total induces charge on the conducting sphere.

$$Q_{ind} = CV$$

$$Q_{ind} = 4\pi\varepsilon_0 a \frac{Q}{4\pi\varepsilon_0 r}$$

$$Q_{ind} = \frac{Q \times a}{r} = \frac{10 \times 2}{5} \times \mu C = 4\mu C$$

Hence, the induces charge on the conducting sphere is $4\,\mu\text{C}$.

Sol.8

Given : $\bar{D} = 2\vec{a}_x - 2\sqrt{3}\vec{a}_y$ $\quad \bar{D} = |\bar{D}|\vec{a}_n = \rho_s \vec{a}_n$

$$|\bar{D}| = \sqrt{16} = 4$$

$$\therefore \quad \bar{D} = 4\left\{\frac{2\vec{a}_x - 2\sqrt{3}\vec{a}_y}{4}\right\} = \rho_s \vec{a}_n$$

$$\therefore \quad \rho_s = 4\,\text{C/m}^2$$

Hence the surface charge density is 4.

Sol.9

Given : Total charge $= 1.11\times10^{-10}\,\text{C}$

Radius = 0.5 m

$\int_{-\infty}^{0} -\vec{E}\cdot d\vec{l}$ = Potential at centre of non-conducting ring.

$$V = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r} = \frac{9\times10^{9} \times 1.11\times10^{-10}}{0.5} = 2\text{ volt}$$

Hence, the potential at centre of non-conducting ring is 2 volt.

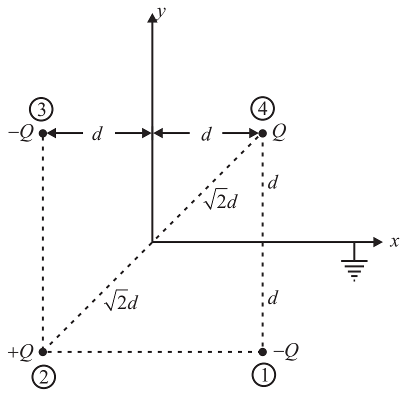
Explanations to Multiple Choice Questions

Sol.1

Given : Net force on the charge is

$$\frac{Q^2}{4\pi\varepsilon_0}\frac{K}{d^2} \qquad \ldots(i)$$

Since there is two semi-infinite conducting sheet and there are connected to ground we can apply method of images. Due to image of charge, the new charges will be created as shown in the figure.



Force on (4) due to (1), (2), (3) will be

$$\vec{F}_{total} = \vec{F}_{41} + \vec{F}_{42} + \vec{F}_{43}$$

Force on (4) due to (1),

$$\vec{F}_{41} = \frac{-Q^2}{4\pi\varepsilon_0}\cdot\frac{1}{(4d^2)}\vec{a}_y$$

Force on (4) due to (2),

$$\vec{F}_{42} = \frac{+Q^2}{4\pi\varepsilon_0}\cdot\frac{1}{8d^2}\left(\frac{\vec{a}_y}{\sqrt{2}} + \frac{\vec{a}_x}{\sqrt{2}}\right)$$

Force on (4) due to (3),

$$\vec{F}_{43} = \frac{-Q^2}{4\pi\varepsilon_0}\cdot\frac{1}{4d^2}\vec{a}_x$$

The net force on charge Q is,

$$F_{total} = \frac{Q^2}{4\pi\varepsilon_0 d^2}\left(\frac{1-2\sqrt{2}}{8\sqrt{2}}\vec{a}_x + \frac{1-2\sqrt{2}}{8\sqrt{2}}\vec{a}_y\right) \qquad \ldots(ii)$$

On comparing equation (i) and (ii), we get

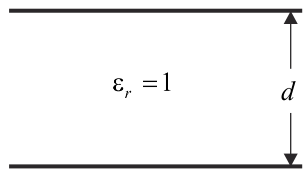
$$K = \frac{1-2\sqrt{2}}{8\sqrt{2}}\vec{a}_x + \frac{1-2\sqrt{2}}{8\sqrt{2}}\vec{a}_y$$

Hence, the correct option is (D).

Sol.2

Let A be the area of the parallel plate capacitor and d be the distance between the plates.

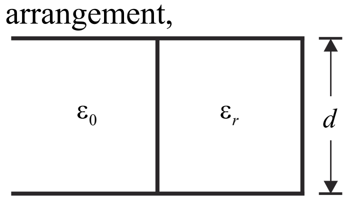
With air dielectric :



Capacitance,

$$C_0 = \frac{\varepsilon_0 A}{d} \qquad \ldots(i)$$

With new arrangement,

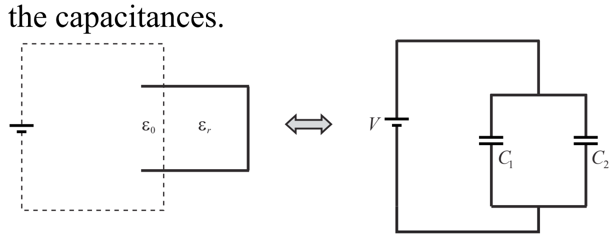


Let C_1 be the capacitance of half portion with air as dielectric medium and C_2 be capacitance with a dielectric of permittivity ε_r.

Then, $C_1 = \dfrac{\varepsilon_0\left(\frac{A}{2}\right)}{d}$ (As area becomes half)

and $C_2 = \dfrac{\varepsilon_0\varepsilon_r\left(\frac{A}{2}\right)}{d}$ (As area becomes half)

Now, these two capacitances will be in parallel if voltage is applied between the plates as same potential difference will be there between both the capacitances.



$\therefore$ Equivalent capacitance is

$$C_{eq.} = C_1 + C_2$$

$$C_{eq.} = \frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_0 \varepsilon_r A}{2d}$$

$$C_{eq} = \frac{\varepsilon_0 A}{2d}(1+\varepsilon_r) = \frac{C_0}{2}(1+\varepsilon_r)$$

Using equation (i),

$$C_{eq} = \frac{C_0}{2}(1+\varepsilon_r)$$

Hence, the correct option is (A).

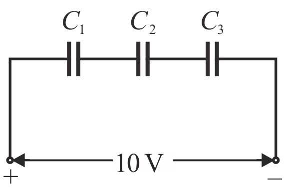
Sol.3

Let A = Area of plates

Let $C_1 = C_3$ be capacitance formed with dielectric having dielectric constant ε_1.

C_{eq} be the equivalent capacitance.

C_2 be the capacitance formed with dielectric having dielectric constant ε_2.



Then $C_1 = C_3 = \dfrac{\varepsilon_0 \varepsilon_1 A}{\frac{d}{4}} = \dfrac{4\varepsilon_0 \varepsilon_1 A}{d}$

And $C_2 = \dfrac{\varepsilon_0 \varepsilon_2 A}{\frac{d}{2}} = \dfrac{2\varepsilon_0 \varepsilon_2 A}{d}$

Also, equivalent capacitance = C_{eq}

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{2}{C_1} + \frac{1}{C_2}$$

($\because$ $C_1 = C_3$, C_1, C_2, C_3 all are in series)

$$\frac{1}{C_{eq}} = \frac{2d}{4\varepsilon_0 \varepsilon_1 A} + \frac{d}{2\varepsilon_0 \varepsilon_2 A}$$

$$C_{eq} = \frac{2\varepsilon_1 \varepsilon_2 \varepsilon_0 A}{d(\varepsilon_1 + \varepsilon_2)}$$

V_{eq} = total voltage = 10 V

$V_1 = V_3 = 2$ V.

We know that, $C \propto \dfrac{1}{V}$

$$\frac{C_{eq}}{C_1} = \frac{V_1}{V_{eq}}$$

$$C_{eq} = \frac{C_1}{5}$$

$$\frac{2\varepsilon_0 \varepsilon_1 \varepsilon_2 A}{d(\varepsilon_1 + \varepsilon_2)} \cdot \frac{d}{4\varepsilon_0 \varepsilon_1 A} = \frac{1}{5}$$

$$\frac{\varepsilon_2}{2(\varepsilon_1 + \varepsilon_2)} = \frac{1}{5}$$

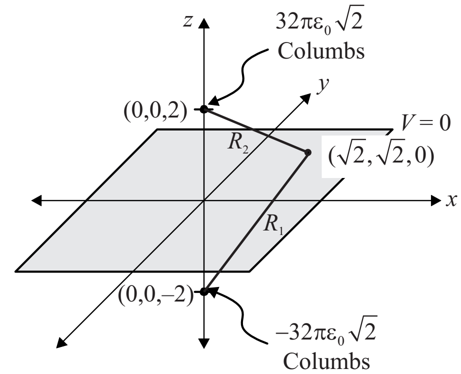
$\Rightarrow$ $5\varepsilon_2 = 2\varepsilon_1 + 2\varepsilon_2 \Rightarrow 2\varepsilon_1 = 3\varepsilon_2$

$$\varepsilon_1 : \varepsilon_2 = 3 : 2$$

Hence, the correct option is (C).

Sol.4

Given : $Q = 32\pi\varepsilon_0\sqrt{2}$ C at coordinate (0,0,2).



Due to charge at (0, 0, 2), and conductor plane there is an image at (0, 0, –2).

Electric field intensity at any point is given by,

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^{3/2}}\vec{R}$$

Electric field intensity due to charge at (0, 0, 2),

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R_1^{3/2}}\vec{R}_1$$

$$\vec{E}_1 = \frac{32\pi\varepsilon_0\sqrt{2}}{4\pi\varepsilon_0(2+2+4)^{\frac{3}{2}}}(\sqrt{2}\,\vec{a}_x + \sqrt{2}\,\vec{a}_y - 2\,\vec{a}_z)$$

Electric field intensity due to charge at (0, 0, –2) ,

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R_2^{3/2}}\vec{R}_2$$

$$\vec{E}_2 = \frac{-32\pi\varepsilon_0\sqrt{2}}{4\pi\varepsilon_0(2+2+4)^{\frac{3}{2}}}(\sqrt{2}\,\vec{a}_x + \sqrt{2}\,\vec{a}_y + 2\,\vec{a}_z)$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 0\,\vec{a}_x + 0\,\vec{a}_y - 2\,\vec{a}_z = -2\,\vec{a}_z$$

Hence, the correct option is (B).

Sol.5

Given : $\vec{E} = \left(x\vec{u}_x + y\vec{u}_y + z\vec{u}_z\right)$

Potential difference is given by,

$$V_{xy} = V_x - V_y = -\int_y^x \vec{E}\cdot d\vec{l}$$

Here, $d\vec{l} = dx\vec{u}_x + dy\vec{u}_y + dz\vec{u}_z$

$$\vec{E}\cdot d\vec{l} = xdx + ydy + zdz$$

$$V_{xy} = -\int_y^x \vec{E}\cdot d\vec{l} = -\int_1^2 xdx - \int_2^0 ydy - \int_3^0 zdz$$

$$= -\left(\left[\frac{x^2}{2}\right]_1^2 - \left[\frac{y^2}{2}\right]_2^0 - \left[\frac{z^2}{2}\right]_3^0\right)$$

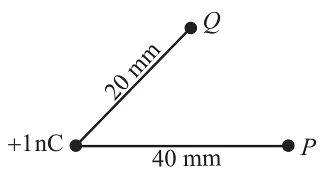
$$V_{xy} = -\left[2-\frac{1}{2}\right] - [-2] - \left[-\frac{9}{2}\right]$$

$$= -\frac{3}{2} + 2 + \frac{9}{2}$$

$$V_{xy} = 3 + 2 = 5\,\text{V}$$

Hence, the correct option is (C).

Sol.6



Potential due to a point charge is given by,

$$V = \frac{q}{4\pi\varepsilon r}$$

Potential difference between P and Q can be written as,

$$V_P - V_Q = \frac{q}{4\pi\varepsilon_0 r_1} - \frac{q}{4\pi\varepsilon_0 r_2}$$

$$V_P - V_Q = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= 10^{-9} \times 9 \times 10^{9} \times \left[\frac{1}{40\times 10^{-3}} - \frac{1}{20\times 10^{-3}}\right]$$

$$V_P - V_Q = \frac{-10^{9}\times 9\times 10^{9}}{40\times 10^{-3}}\text{Volts}$$

$$V_P - V_Q = -225 \text{ Volts}$$

Hence, the correct option is (B).

Sol.7

Given : $\vec{E} = \dfrac{100}{\rho}\cos(10^9 t - 6z)\vec{a}_\rho$ V/m

$$\varepsilon_r = 1$$

The displacement current density is given by,

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0\varepsilon_r\frac{\partial \vec{E}}{\partial t}$$

$$\vec{J}_D = \varepsilon_0\frac{\partial}{\partial t}\left[\frac{100}{\rho}\cos(10^9 t - 6z)\vec{a}_\rho\right]$$

$$\vec{J}_D = -\varepsilon_0 \times 10^9 \times \frac{100}{\rho}\sin(10^9 t - 6z)\vec{a}_\rho$$

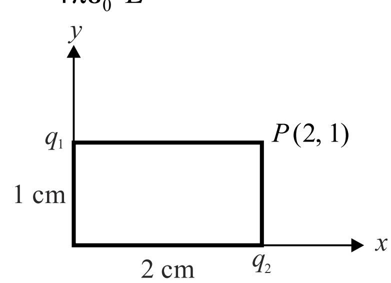
$$\vec{J}_D = \frac{-0.9}{\rho}\sin(10^9 t - 6z)\vec{a}_\rho \text{ A/m}^2$$

Hence, the correct option is (C).

Sol.8

Field intensity is given by,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{E}\vec{a}_R$$



Field intensity at P due to q_1 will be

$$\vec{E}_1 = \frac{2}{4\pi\varepsilon_0}\frac{\vec{a}_x}{(2)^2}$$

Field intensity at P due to q_2 will be

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{\vec{a}_y}{(1)^2}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = \frac{2}{4\pi\varepsilon_0}\frac{\vec{a}_x}{4} + \frac{1}{4\pi\varepsilon_0}\frac{\vec{a}_y}{1}$$

$$\vec{E} = (0.5\vec{a}_x + \vec{a}_y)K$$

The electric field vector point P that will subtend at angle θ with x axis is

$$\tan\theta = \frac{1}{0.5} = 2$$

$$\tan\theta = 2$$

Hence, the correct option is (B).

Sol.9

The given situation is shown in figure below.

Field intensity at any point is given by,

$$\vec{E} = \frac{KQ}{R^2}\vec{a}_R$$

Field intensity at point (a, a) can be written as,

$$\vec{E}_1 = KQ\frac{(a\vec{a}_x + a\vec{a}_y)}{(a^2+a^2)^{\frac{3}{2}}} = \frac{KQ}{2\sqrt{2}a^2}(\vec{a}_x + \vec{a}_y)$$

$$\vec{E}_1 = \frac{KQ}{2\sqrt{2}a^2}(\vec{a}_x + \vec{a}_y)$$

Field intensity at point $(-a, a)$ can be written as,

$$\vec{E}_1 = KQ\frac{(-a\vec{a}_x + a\vec{a}_y)}{(a^2+a^2)^{\frac{3}{2}}} = \frac{KQ}{2\sqrt{2}a^2}(-\vec{a}_x + \vec{a}_y)$$

Field intensity at point $(-a, -a)$ can be written as,

$$\vec{E}_1 = KQ\frac{(-a\vec{a}_x - a\vec{a}_y)}{(a^2+a^2)^{\frac{3}{2}}} = \frac{KQ}{2\sqrt{2}a^2}(-\vec{a}_x - \vec{a}_y)$$

$$\vec{E}_1 \cdot \vec{E}_2 = \left[\frac{KQ}{2\sqrt{2}a^2}(\vec{a}_x + \vec{a}_y)\right]\cdot\left[\frac{KQ}{2\sqrt{2}a^2}(-\vec{a}_x + \vec{a}_y)\right]$$

$$\vec{E}_1 \cdot \vec{E}_2 = \frac{-K^2Q^2}{8a^4} + \frac{K^2Q^2}{8a^4} = 0$$

$$\vec{E}_1 \cdot \vec{E}_3 = \frac{KQ}{2\sqrt{2}a^2}\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 1 & 1 & 0 \\ -1 & -1 & 0 \end{vmatrix}$$

$$\vec{E}_1 \times \vec{E}_3 = \vec{a}_z(-1+1) = 0$$

Hence, the correct option is (C).

Sol.10

Let charge on the object be q and $Q-q$.

Force between them $F = \dfrac{q(Q-q)}{4\pi\varepsilon_0 d^2}$

Where d is the distance between them.

For maximum F, numerator of equation (i) is maximum. Let $q(Q-q) = y$.

$\therefore$ y should be maximum.

Differentiating w.r.t 'y',

$$\frac{dy}{dq} = Q - 2q$$

Equating to zero (to get maxima of y), we get

$$Q - 2q = 0 \Rightarrow q = \frac{Q}{2}$$

Thus, the charge should be equally distributed between the objects.

Hence, the correct option is (B).

Sol.11

Given : $V = x^2 + y^2 + z^2$

Electric field intensity is given by,

$$\vec{E} = -\nabla V$$

$$\vec{E} = -\frac{\partial}{\partial x}(x^2+y^2+z^2)\vec{a}_x - \frac{\partial}{\partial y}(x^2+y^2+z^2)\vec{a}_y - \frac{\partial}{\partial z}(x^2+y^2+z^2)\vec{a}_z$$

$$\vec{E} = -\left(2x\,\vec{a}_x + 2y\,\vec{a}_y + 2z\,\vec{a}_z\right)$$

Potential difference is given by,

$$V_{PQ} = -\int_0^1 \vec{E}\cdot d\vec{l} = \int_0^1 [2x + 2y + 2z]\,dy$$

$$V_{PQ} = \int_0^1 \vec{E}\cdot d\vec{l} = \left[2xy + 2\left(\frac{y^2}{2}\right) + 2zy\right]_0^1$$

$$x = 1,\ z = 2$$

$$V_{PQ} = V = 2 + 1 + 2(2) = 7\,\text{V}$$

Hence, the correct option is (A).

Sol.12

Given : $\vec{E}_1 = \vec{a}_x + 2\vec{a}_y - \vec{a}_z$

$$\vec{E}_2 = \vec{a}_y + 3\vec{a}_z$$

$$\vec{E}_3 = 2\vec{a}_x - \vec{a}_y$$

The total electric field at any point is equal to the summation of the electric fields due to the individual electric charges at that point.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$\vec{E} = 3\vec{a}_x + 2\vec{a}_y + 2\vec{a}_z \ \text{N/C}$$

Hence, the correct option is (C).

Sol.13

Potential difference is given by,

$$V = -\int \vec{E}\cdot d\vec{l}$$

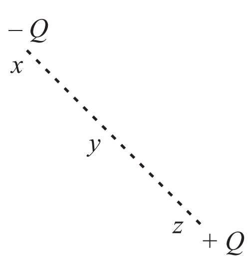
$$|dV| = -|\vec{E}||d\vec{l}|\cos\theta$$

If $\theta = 90^0$

$dV = 0$ i.e., voltage is constant

i.e. , if we move perpendicular to field, voltage is constant.

In given problem $+Q, -Q$ are diagonally opposite.



Plane xyz is perpendicular plane to both $-Q$ and $+Q$ i.e., on xyz plane voltage is constant.

In the problem ABC plane is also perpendicular to $-Q$, $+Q$ charges. Therefore voltage is constant given

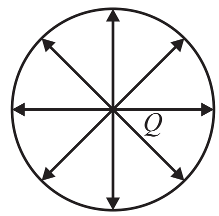
$$V_A = 1$$

$$\therefore \quad V_B = 1\,\text{V}$$

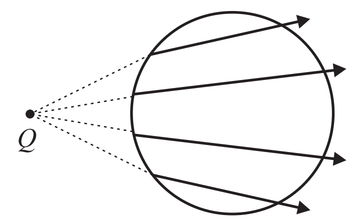
Hence, the correct option is (C).

Sol.14

If a point charge exists inside a closed surface. Then total electric flux coming out of closed surface is charge enclosed.



If a point charge is outside of closed surface flux entering = flux leaving net contribution is zero.

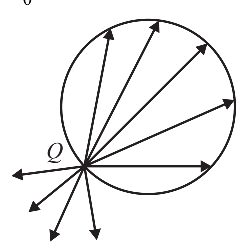


Half of electric flux leaving closed surface, half of flux is not entering into closed surface.

$$\oint \vec{D}\cdot d\vec{s} = \frac{Q}{2}$$

$$D = \frac{Q}{2\times 4\pi r^2}$$

$$E = \frac{Q}{8\pi\varepsilon_0 R^2}(-\vec{n})$$



Hence, the correct option is (C).

Sol.15

Refer section 2.23

Sol.16

Properties of equipotential surface :

(i) It tells the direction of the electric field.

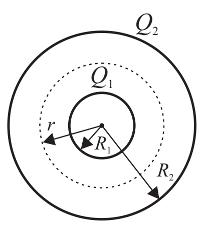
(ii) Electric field is always right angle to the equipotential surface.

(iii) No work is done in moving a test charge over on equipotential surface.

(iv) No two equipotential surface intersect each other.

Hence, the correct option is (C).

Sol.17



$$R_1 \le r \le R_2$$

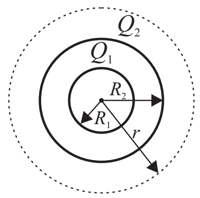
Applying Gauss Law for

$$\oiint_s \vec{E}\cdot d\vec{s} = \frac{Q_1}{\varepsilon_0}$$

$$E\cdot 4\pi r^2 = \frac{Q_1}{\varepsilon_0}$$

$$E = \frac{Q_1}{4\pi\varepsilon_0 r^2}$$

Applying Gauss law for $r \ge R_2$,



$$\oint \vec{E}\cdot d\vec{s} = \frac{Q_1+Q_2}{\varepsilon_0}$$

$$E\cdot 4\pi r^2 = \frac{Q_1+Q_2}{\varepsilon_0}$$

$$E = \frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2}$$

Hence, the correct option is (D).

Sol.18

Given : $V = \dfrac{(6r^5)}{\varepsilon_0}$ for $r \le 1\,\text{m}$

Electric field intensity is given by,

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial r}\vec{a}_r$$

$$\vec{E} = -\frac{30r^4}{\varepsilon_0}\vec{a}_r$$

Electric flux density is given by,

$$\vec{D} = \varepsilon_0\vec{E} = -\frac{30r^4}{\varepsilon_0}\varepsilon_0\vec{a}_r = -30r^4\vec{a}_r$$

By Gauss's law,

$$\psi_{net} = Q_{enc} = \oint_s \vec{D}\cdot\vec{ds} = -30(1)^4\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\sin\theta\, d\theta\, d\phi$$

$$\psi_{net} = -120\pi\,\text{C}$$

Hence, the correct option is (C).

Sol.19

Electric field due to surface change density is given by,

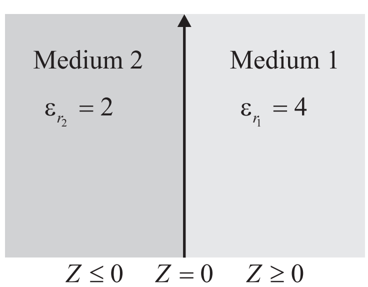
$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\vec{a}_n$$

$$\vec{E} = \frac{-20\times10^{-9}}{2\times\dfrac{10^{-9}}{36\pi}}\vec{a}_z = -360\pi\vec{a}_z$$

Hence, the correct option is (D).

Sol.20

Given : $\vec{E}_1 = 5\vec{a}_x - 2\vec{a}_y + 3\vec{a}_z$,



From above figure

$$\vec{a}_n = \vec{a}_z$$

$$\vec{E}_{n_1} = (E_1\cdot\vec{a}_n)\vec{a}_n$$

$$\vec{E}_{n_1} = \{(5\vec{a}_x - 2\vec{a}_y + 3\vec{a}_z)\vec{a}_z\}\vec{a}_z$$

$$\vec{E}_{n_1} = 3\vec{a}_z$$

From the boundary condition

$$\vec{D}_{n_1} = \vec{D}_{n_2}$$

$$\varepsilon_1\vec{E}_{n_1} = \varepsilon_2\vec{E}_{n_2}$$

$$\vec{E}_{n_2} = \frac{4}{2}\vec{E}_{n_1}$$

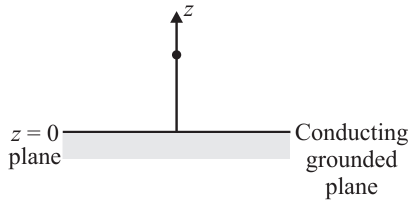
$$\vec{E}_{n_2} = 2\times 3\vec{a}_z$$

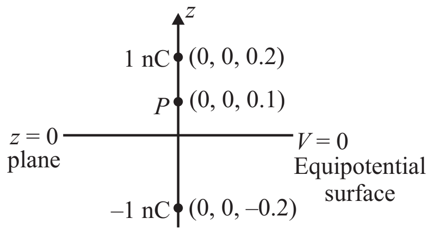
$$\vec{E}_{n_2} = 6\vec{a}_z$$

Hence, the correct option is (C).

Sol.21

Given : Charge = 1 nC at (0, 0, 0.2). Since the charge is placed above conducting grounded plane there will be an image charge below the grounded conducting plane as per method of image concept.





Electric field at any point is given by,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q\bar{R}}{|\bar{R}|^3}$$

$\vec{R} =$ displacement vector between charge to point of interest.

Electric filed at point P due to point charge $+1\,\text{nC}$,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{1\times10^{-9}\left[(0-0)\vec{a}_x + (0-0)\vec{a}_y + (0.1-0.2)\vec{a}_z\right]}{(0.1)^3}$$

$$\vec{E}_1 = -898.755\,\text{V/m}$$

Electric field point P due to pint charge $-1\,\text{nC}$,

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{-1\times10^{-9}\left[(0-0)\vec{a}_x + (0-0)\vec{a}_y + (0.1+0.2)\vec{a}_z\right]}{(0.3)^3}$$

$$\vec{E}_2 = -99.86\,\text{V/m}$$

Total electric field at point P due to both charges,

$$\vec{E}_1 = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = -898.755 - 99.86 = -997.61\,\text{V/m}$$

Hence, the correct option is (D).

Sol.22

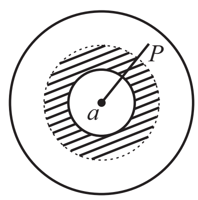
Charge in the shaded region

$$= \int_a^r 4\pi r^2 \frac{A}{r}.dr = 2\pi A\left(r^2 - a^2\right)$$

Total field at

$$P = \frac{1}{4\pi\varepsilon a}\cdot\frac{Q}{r^2} + \frac{1}{4\pi\varepsilon a}\cdot 2\pi A\left(1 - \frac{a^2}{r^2}\right)$$

For field to be independent of $r : Q = 2\pi A a^2$



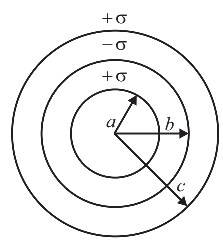
Hence, the correct option is (A).

Sol.23

$$V_B = \frac{k(\sigma\times4\pi a^2)}{b} - \frac{k(\sigma\times4\pi b^2)}{b} + \frac{k(\sigma\times4\pi c)^2}{c}$$

$$V_B = \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{b} - b + c\right]$$

$$V_B = \frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$



Hence, the correct option is (D).

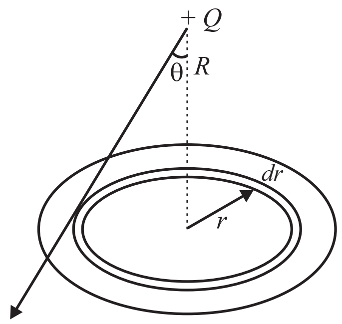
Sol.24

Since the charge lies outside the sphere, net flux passing through the sphere is zero.

$$\phi_{\text{curved surface}} + \phi_{\text{disc}} = 0$$

Option (C) is incorrect

$$\phi_{\text{curved surface}} = -\phi_{\text{disc}}$$



$$\cos\theta = \frac{R}{\sqrt{R^2 + r^2}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(R^2 + r^2\right)}$$

$$\phi_{disc} = \int \vec{E}\cdot d\vec{A}$$

$$= \int\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(R^2 + r^2\right)} \times 2\pi r dr \times \cos\theta\right)$$

$$= \frac{Q\cdot 2\pi}{4\pi\varepsilon_0}\int\frac{Q\cdot 2\pi}{4\pi\varepsilon_0}\int\frac{rdr}{R^1 + r^2} \times \frac{R}{\left(R^2 + r^2\right)^{1/2}}$$

$$\Rightarrow \quad \phi_{\text{curved surface}} = \frac{Q}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{2}}\right]$$

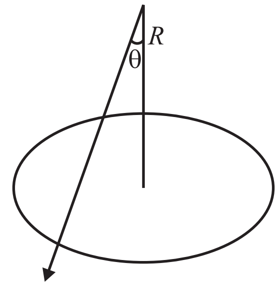
Option (A) is correct

Potential at any point on the circumference of the flat surface is

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2 + R^2}} = \frac{Q}{4\pi\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0\left(\sqrt{2}R\right)}$$

Hence it is equipotential

Option (D) is correct



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(R/\cos\theta)^2} = \frac{\theta}{4\pi\varepsilon_0 R^2}\cos^2\theta$$

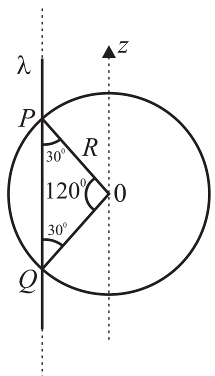
$$E_{normal} = E\cos\theta = \frac{Q}{4\pi\varepsilon_0 R^2}\cos^2\theta$$

Which is not constant

Hence, the correct option is (B).

Sol.25

Given : Uniform line charge density is λ.



Using sine formula,

$$\frac{PQ}{\sin 120^0} = \frac{R}{\sin 30^0}$$

$$\frac{PQ}{\sqrt{3}/2} = \frac{R}{1/2}$$

$$PQ = R\sqrt{3}$$

$$Q_{enc} = P_L \times \text{length}$$

$$Q_{enc} = \lambda\sqrt{3}R$$

Gauss's law is given by,

$$\psi = \frac{Q_{enc}}{\epsilon_0} = \frac{\sqrt{3}\lambda R}{\epsilon_0}$$

Hence, the correct option is (A, B).

Sol.26

Electric field due to a hollow spherical conductor is governed by following equation

$$E = 0\text{, for } r < R$$

and $$E = \frac{Q}{4\pi\varepsilon_0 r^2} \text{ for } r \geq R$$

i.e., inside the conductor field will be zero and outside the conductor will vary according to $E \propto \frac{1}{r^2}$.

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