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GATE - 2022

Electronics & Comm. Engg.

SAMPLE PDF

ELECTRONICS & COMMUNICATION ENGINEERING

VOLUME - 02

This sample PDF of **GATE Previous Years Solution Book** contains randomly selected questions with solutions from some of the chapters of every subject along with part of concept refresher **Synopsis** of those chapters to let the aspirants have an idea about the content, style and appearance of the book.

Previous Years marks distribution analysis is also given in tabular form with index page of every subject, which contains analysis of GATE papers from 2003 onwards as GATE pattern has turned objective since 2003.

GATE ACADEMY PUBLICATIONS®

GATE 2022

Electronics & Communication Engineering

(Volume - II)

TOPIC WISE GATE SOLUTIONS

1987-2021

Umesh Dhande

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steps to success...

I am thankful to

My wife ***Sakshi Dhande****, Son* ***Advait,***

who are my source of encouragement and inspiration

and

my dearest father ***Shri Hariram Dhande,***

mother ***Indu Dhande,***

who are responsible for what I am today.

Last but not the least special thanks to

my brother ***Dr. Rakesh Dhande***

and sisters ***Dr. Preeti Fendar, Punam Dhande***

who always motivate me to give my best.

To My Son Advait

IMPORTANCE of GATE

GATE examination has been emerging as one of the most prestigious competitive exam for engineers. Earlier it was considered to be an exam just for eligibility for pursuing PG courses, but now GATE exam has gained a lot of attention of students as this exam open an ocean of possibilities like :

1. **Admission into IISc, IITs, IIITs, NITs**

   A good GATE score is helpful for getting admission into IISc, IITs, IIITs, NITs and many other renowned institutions for M.Tech./M.E./M.S. An M.Tech graduate has a number of career opportunities in research fields and education industries. Students get ₹ 12,400 per month as stipend during their course.

2. **Selection in various Public Sector Undertakings** (**PSUs**)

   A good GATE score is helpful for getting job in government-owned corporations termed as **Public Sector Undertakings** (**PSUs**) in India like IOCL, BHEL, NTPC, BARC, ONGC, PGCIL, DVC, HPCL, GAIL, SAIL & many more.

3. Direct recruitment to Group A level posts in Central government, i.e., Senior Field Officer (Tele), Senior Research Officer (Crypto) and Senior Research Officer (S&T) in Cabinet Secretariat, Government of India, is now being carried out on the basis of GATE score.

4. **Foreign universities through GATE**

   GATE has crossed the boundaries to become an international level test for entry into postgraduate engineering programmes in abroad. Some institutes in two countries **Singapore** and **Germany** are known to accept GATE score for admission to their PG engineering programmes.

5. **National Institute of Industrial Engg. (NITIE)**
   - NITIE offers **PGDIE / PGDMM / PGDPM** on the basis of GATE scores. The shortlisted candidates are then called for group Discussion and Personal Interview rounds.
   - NITIE offers a Doctoral Level Fellowship Programme recognized by Ministry of HRD (MHRD) as equivalent to PhD of any Indian University.
   - Regular full time candidates those who will qualify for the financial assistance will receive ₹ 25,000 during 1st and 2nd year of the Fellowship programme and ₹ 28,000 during 3rd, 4th and 5th year of the Fellowship programme as per MHRD guidelines.

6. **Ph.D. in IISc/ IITs**
   - IISc and IITs take admissions for Ph.D. on the basis of GATE score.
   - Earn a Ph.D. degree directly after Bachelor’s degree through integrated programme.
   - A fulltime residential researcher (RR) programme.

7. **Fellowship Program in management (FPM)**
   - Enrolment through GATE score card
   - Stipend of ₹ 22,000 – 30,000 per month + HRA
   - It is a fellowship program
   - Application form is generally available in month of sept. and oct.

Note : In near future, hopefully GATE exam will become a mandatory exit test for all engineering students, so take this exam seriously. Best of LUCK !

GATE Exam Pattern

Section	**Question No.**	**No. of Questions**	**Marks Per Question**	**Total Marks**
General Aptitude	1 to 5	5	1	5
	6 to 10	5	2	10
Technical + Engineering Mathematics	1 to 25	25	1	25
	26 to 55	30	2	60
Total Duration : 3 hours		**Total Questions : 65**		**Total Marks : 100**
Note : 40 to 45 marks will be allotted to Numerical Answer Type Questions				

Pattern of Questions :

(i) **Multiple Choice Questions (MCQ)** carrying 1 or 2 marks each in all the papers and sections. These questions are objective in nature, and each will have a choice of four answers, out of which the candidate has to select (mark) the correct answer.

Negative Marking for Wrong Answers : For a wrong answer chosen in a MCQ, there will be negative marking. For **1-mark** MCQ, **1/3 mark** will be deducted for a wrong answer. Likewise for, **2-marks** MCQ, **2/3 mark** will be deducted for a wrong answer.

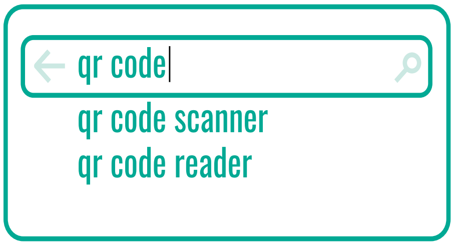
(ii) **Numerical Answer Type (NAT)** Questions carrying **1** or **2** marks each in all the papers and sections. For these questions, the answer is a signed real number, which needs to be entered by the candidate using the virtual numeric keypad on the monitor (Keyboard of the computer will be disabled). No choices will be shown for these type of questions. The answer can be a number such as **10** or **– 10** (an integer only). The answer may be in decimals as well, for example, **10.1** (one decimal) or **10.01** (two decimal) or **–10.001** (three decimal). These questions will be mentioned with, up to which decimal places, the candidates need to make an answer. Also, an appropriate range will be considered while evaluating the numerical answer type questions so that the candidate is not penalized due to the usual round-off errors. Wherever required and possible, it is better to give NAT answer up to a maximum of three decimal places.

Note : There is NO negative marking for a wrong answer in NAT questions.

What is special about this book ?

GATE ACADEMY Team took several years' to come up with the solutions of GATE examination. It is because we strongly believe in quality. We have significantly prepared each and every solution of the questions appeared in GATE, and many individuals from the community have taken time out to proof read and improve the quality of solutions, so that it becomes very lucid for the readers. Some of the key features of this book are as under :

- This book gives complete analysis of questions chapter wise as well as year wise.
- Video Solution of important conceptual questions has been given in the form of QR code and by scanning QR code one can see the video solution of the given question.
- Solutions has been presented in lucid and understandable language for an average student.
- In addition to the GATE syllabus, the book includes the nomenclature of chapters according to text books for easy reference.
- Last but not the least, author's 10 years experience and devotion in preparation of these solutions.
- Steps to Open Video solution through mobile.



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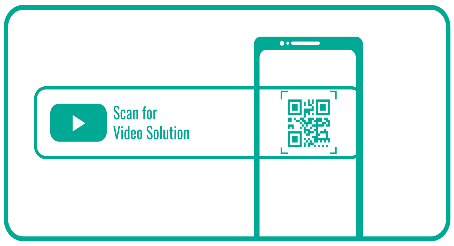
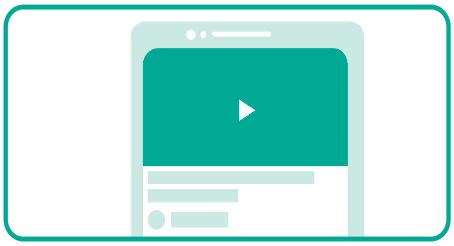
(2) Download & Install any QR Code Scanner App.



(3) Scan the given QR Code for particular question.



(4) Visit the link generated & you'll be redirect to the video solution.

Note : For recent updates regarding minor changes in this book, visit - www.gateacademy.co.in. We are always ready to appreciate and help you.

ACKNOWLEDGEMENT

12 Years of recurring effort went into the volume which is now ready to cover the aptitude of GATE aspirants.

We are glad of this opportunity to acknowledge the views and to express with all the weaknesses of mere words the gratitude that we must always feel for the generosity of them.

We now express our gracious gratitude to the persons who have contributed a lot in order to put forth this into device. They are to be mentioned here and they are Akansha Singh, Durshree Sharma, Gaurav Thakur, K.L. Srinivas, Krishna Kumar, Koushalya Chandrawanshi, Mukesh Kumar Sahu, Neeraj Jagwani, Neelima Patel, Pritesh Patel, Pratima Patel, Reena Sahu, Ravi Yadav, Saurabh Sinha, Shrikant Soni, Shubham Dabir, T. Pushpalata, Vikas Athe, Vivek Kumar, and Vaishali Rathi.

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Lastly, we take this opportunity to acknowledge the service of the total team of publication and everyone who collaborated in producing this work.

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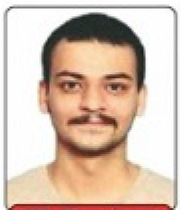
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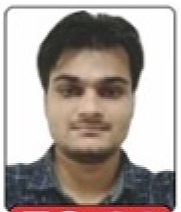
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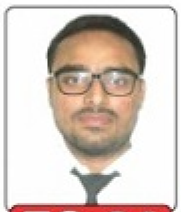
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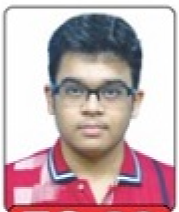
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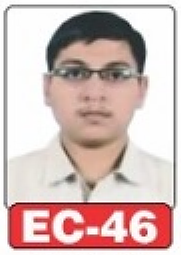
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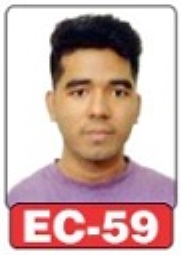
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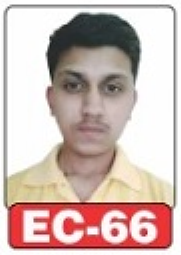
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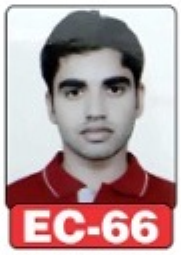
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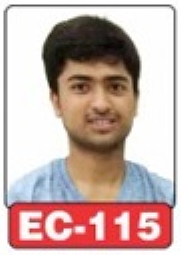
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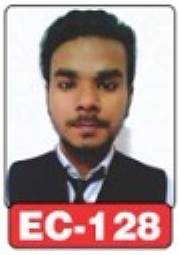
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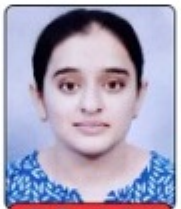
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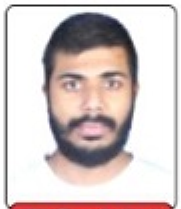
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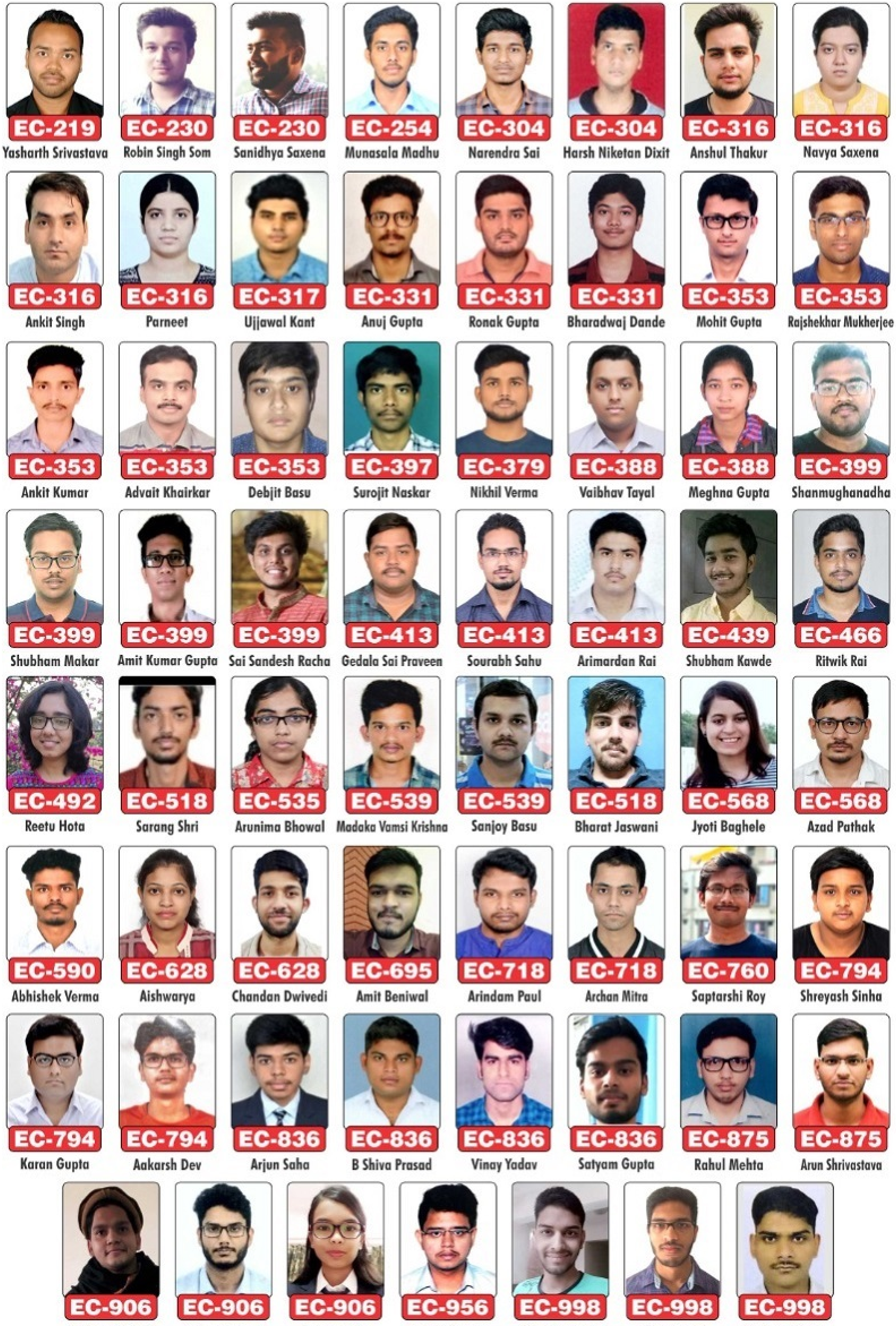

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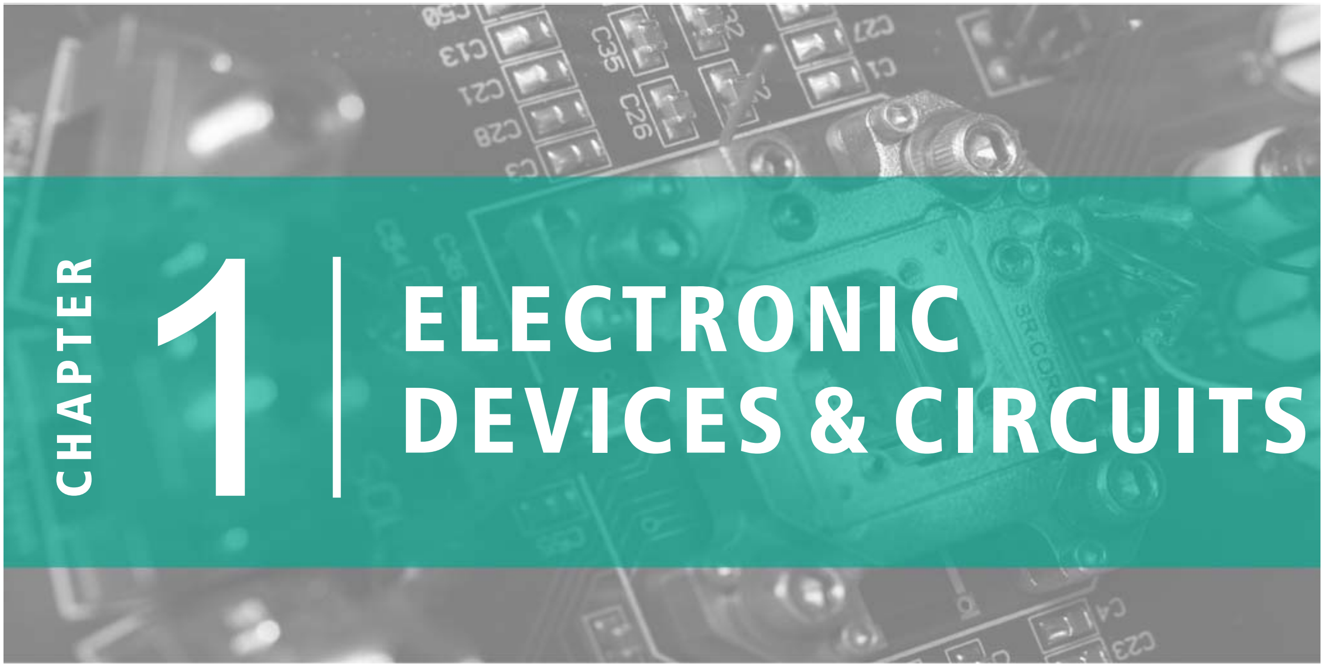


CHAPTER 1

ELECTRONIC DEVICES & CIRCUITS

Marks Distribution of Electronic Devices & Circuits in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	5	5	15
2004	3	7	17
2005	3	3	9
2006	4	4	12
2007	2	6	14
2008	4	4	12
2009	2	3	8
2010	2	4	10
2011	3	3	9
2012	1	3	7
2013	2	–	2
2014 Set-1	2	3	8
2014 Set-2	2	3	8
2014 Set-3	3	3	9

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-4	3	3	9
2015 Set-1	2	4	10
2015 Set-2	2	3	8
2015 Set-3	2	3	8
2016 Set-1	2	4	10
2016 Set-2	1	4	9
2016 Set-3	3	2	7
2017 Set-1	3	3	9
2017 Set-2	2	4	10
2018	3	3	9
2019	3	4	11
2020	2	4	10
2021	2	3	8

Syllabus : Electronic Devices & Circuits

Energy bands in intrinsic and extrinsic semiconductors, equilibrium carrier concentration, direct and indirect band-gap semiconductors.

Carrier transport : diffusion current, drift current, mobility and resistivity, generation and recombination of carriers, Poisson and continuity equations. P-N junction, Zener diode, BJT, MOS capacitor, MOSFET, LED, photo diode and solar cell.

Contents : Electronic Devices & Circuits

1

Basic Semiconductor Physics

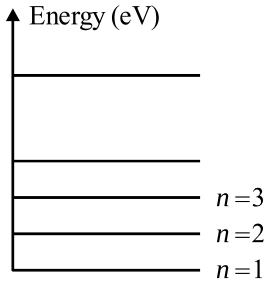
➢ Partial Synopsis

1. Atomic theory :

Energy of an electron in any energy level is given by

$$E_g = -\frac{13.6}{n^2} \qquad \text{where, } n \text{ is quantum no. } (1, 2, 3, \ldots\ldots)$$

$$E_g = hf$$



$1s^2\, 2s^2\, 2p^6 3s^2\, 3p^2$

2. Drift velocity (V_d) :

$$V_d = \frac{-qE\tau}{m^*}$$

Where, $\tau =$ Average relaxation time / Mean time

$m^* =$ Effective mass

3. Mobility (μ) :

$$\mu = \frac{V_d}{E} = \frac{-q\tau}{m^*}$$

$\mu_n > \mu_p$ i.e., mobility of free electrons is always greater than mobility of hole.

Where, m^* is the effective mass of electron or holes.

Effective mass is the mass of electrons or holes in the crystal to fit the Newton's law for them. We know, in crystal various forces act on electrons or holes, so the effective mass of electrons and holes comes from including all these forces.

4. Conductivity (σ) :

From ohm's law, $J = \sigma E$

$$\sigma = \frac{J}{E}$$

Charge density, $\rho = nq$, unit in $\frac{C}{(cm)^3}$

$$\sigma = \sigma_n + \sigma_p$$

$$\sigma = nq\mu_n + pq\mu_p$$

5. Drift current :

$$J_d = \sigma E$$

$$J_d = \sigma_n E + \sigma_p E$$

$$J_d = nq\mu_n E + pq\mu_p E$$

Also, $J = \frac{I}{A}$

$\therefore$ $I = nq\mu_n E A + pq\mu_p EA$

6. Intrinsic semiconductor :

$$\sigma_i = n_i q(\mu_n + \mu_p)$$

$$J = \sigma_i E$$

For Si, $E_g(T) = [1.21 - 3.6\times10^{-4} T]$ eV

For Ge, $E_g(T) = [0.785 - 2.26\times10^{-4} T]$ eV

At 300 K, $n = p = n_i$

(Electron current, I_n) $\neq$ (Hole current, I_p)

Reason : (Mobility of free e^-, μ_n) $\neq$ (Mobility of hole, μ_P)

7. Mass – Action Law :

- Under thermal equilibrium

$$np = n_i^2$$

- $n_i^2 = A_0 T^3 e^{\frac{-E_{g0}}{kT}}$
- $n_i = \sqrt{N_C N_V}\, e^{-E_g/2kT}$
- $N_C = 2\left[\frac{2\pi m_n^* kT}{h^2}\right]^{3/2}$
- $N_V = 2\left[\frac{2\pi m_p^* kT}{h^2}\right]^{3/2}$

8. Charge Densities in a Semi-Conductor:

- By neutrality of s.c. material

$$N_D^+ + p = N_A^- + n$$

Where, N_D^+ = Donor ion concentration

N_A^- = Acceptor ion concentration

$p =$ Hole concentration

$n =$ Free electron concentration

- In thermal equilibrium, minority charge carriers concentration is constant

→ Minority concentration can be increased by increasing the temperature only.

→ Majority concentration depends on "Doping concentration".

9. Diffusion current :

- Hole diffusion current density,

$$J_p = -qD_P \frac{dp(x)}{dx}$$

$-ve$ sign is intentionally given because by convention, we take

(direction of current) = (direction of holes movement)

And by default (natural process) $= \frac{dp(x)}{dx}$ is $-ve$

- Electron diffusion current density,

$$J_n = q\,D_n \frac{dn(x)}{dx}$$

📖 Key Point

In a semiconductor, both diffusion and drift current exist.

i.e. $I_{total} = I_{drift} + I_{diffusion}$

➢ Sample Questions

1987 IIT Bombay

1.1 Consider two energy levels : E_1, E eV above the Fermi level and E_2, E eV below the Fermi level. P_1 and P_2 are respectively the probabilities of E_1 being occupied by an electron and E_2 being empty. Then

(A) $P_1 > P_2$

(B) $P_1 = P_2$

(C) $P_1 < P_2$

(D) P_1 and P_2 depend on number of free electrons.

2003 IIT Madras

1.2 The electron concentration in a sample of uniformly doped n-type silicon at 300^0K varies linearly from $10^{17}/\text{cm}^3$ at $x = 0$ to $6\times10^{16}/\text{cm}^3$ at $x = 2\ \mu\text{m}$.

Assume a situation that electrons are supplied to keep this concentration gradient constant with time. If electronic charge is 1.6×10^{-19} Coulomb and the diffusion constant $D_n = 35\ \text{cm}^2/\text{s}$, the current density in the silicon, if no electric field present is

(A) zero (B) 120 A/cm^2

(C) +1120 A/cm^2 (D) –1120 A/cm^2

2006 IIT Kharagpur

1.3 Under low level injection assumption, the injected minority carrier current for an extrinsic semiconductor is essentially the

(A) diffusion current.

(B) drift current.

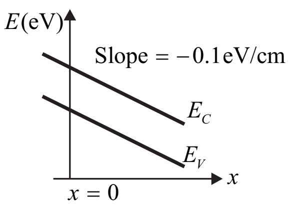
(C) recombination current.

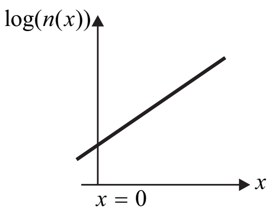
(D) induced current.

2015 IIT Kanpur

1.4 The energy band diagram and the electron density profile $n(x)$ in a semiconductor are shown in the figures.

Assume that $n(x) = 10^{15} e^{\left(\frac{q\alpha x}{kT}\right)}\ \text{cm}^{-3}$, with $\alpha = 0.1$ V/cm and x expressed in cm. Given $\frac{kT}{q} = 0.026$ V, $D_n = 36\ \text{cm}^2\text{s}^{-1}$ and $\frac{D}{\mu} = \frac{kT}{q}$. The electron current density (in A/cm^2) at $x = 0$ is **[Set - 02]**





(A) -4.4×10^{-2} (B) -2.2×10^{-2}

(C) 0 (D) 2.2×10^{-2}

2017 IIT Roorkee

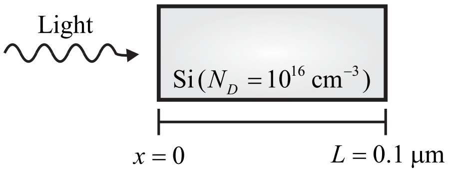
1.5 As shown, a uniformly doped Silicon (Si) bar of length L = 0.1 μm with a donor concentration $N_D = 10^{16}\ \text{cm}^{-3}$ is illuminated at $x = 0$ such that electron and hole pairs are generated at the rate of

$$G_L = G_{L_0}\left(1 - \frac{x}{L}\right),\ 0 \le x \le L,$$

where $G_{L_0} = 10^{17}/\text{cm}^3\ \text{s}$.

Hole life time is 10^{-4} s electronic charge $q = 1.6\times10^{-19}$ C, hole diffusion coefficient $D_p = 100\ \text{cm}^2/\text{s}$ and low level injection condition prevails. Assuming a linearly decaying steady state excess hole concentration that goes to 0 at x = L, the magnitude of the diffusion current density at x = L/2, in A/cm^2, is ________.

[Set - 01]



2021 IIT Bombay

1.6 A bar of silicon is doped with boron concentration of 10^{16} cm^{-3} and assumed to be fully ionized. It is exposed to light such that electron-hole pairs are generated throughout the volume of the bar at the rate of 10^{20} cm^{-3} s^{-1}. If the recombination lifetime is 100 μs and intrinsic carrier concentration of silicon is 10^{10} cm^{-3} and assuming 100% ionization of boron, then the approximate product of steady-state electron and hole concentration due to this light exposure is

(A) $10^{20}\ \text{cm}^{-6}$ (B) $2\times10^{20}\ \text{cm}^{-6}$

(C) $10^{32}\ \text{cm}^{-6}$ (D) $2\times10^{32}\ \text{cm}^{-6}$

Explanations Basic Semiconductor Physics

1.1 (B)

Given :

(i) E_1 is above the fermi level,

$E_1 - E_F = E$ eV

(ii) E_2 is below the fermi level,

$E_2 - E_F = -E$ eV

(iii) P_1 = Probability that E_1 is occupied by electron

(iv) P_2 = Probability that E_2 is empty

Method 1

Fermi dirac probability distribution function is given by,

$$f_n(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

where, $E =$ Given energy level

E_F = Fermi level energy

Probability that E_1 is occupied by electron is given by,

$$P_1 = f_n(E_1) = \frac{1}{1+e^{(E_1-E_F)/kT}}$$

$$P_1 = \frac{1}{1+e^{E/kT}} \qquad \ldots \text{(i)}$$

Probability that E_2 is not occupied by electron is given by,

$$P_2 = 1 - f_n(E_2) = 1 - \frac{1}{1+e^{(E_2-E_F)/kT}}$$

$$P_2 = \frac{e^{-E/kT}}{1+e^{-E/kT}} = \frac{1}{1+e^{E/kT}} \qquad \ldots \text{(ii)}$$

From equations (i) and (ii),

$$P_1 = P_2$$

Hence, the correct option is (B).

Method 2

Fermi dirac probability distribution function is given by,

$$f_n(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

$$\frac{kT}{q} = 26 \text{ mV at } T = 300^0 \text{K}$$

At $T = 0^0$K :

(i) **Case 1 : When** $E > E_F$,

$$f_n(E) = \frac{1}{1+e^{\infty}} = \frac{1}{\infty} = 0$$

When given energy level is more than fermi energy level then probability of finding electron in given energy level at 0^0K will be zero.

(ii) **Case 2 : When** $E < E_F$,

$$f_n(E) = \frac{1}{1+e^{-\infty}} = \frac{1}{1+0} = 1$$

When energy level is less than fermi energy level then probability of finding electron in given energy level at 0^0K will be 100%.

(iii) **Case 3 : When** $E = E_F$,

$$f_n(E) = \frac{1}{1+e^{0}} = \frac{1}{2} = 0.5$$

Since, $E_1 > E_F$, probability of finding electron i.e. P_1 in E_1 will be close to zero. Since, $E_2 < E_F$, probability of finding electron i.e. P_3 in E_2 will be close to one. Hence, probability of not finding electron in E_2 will be,

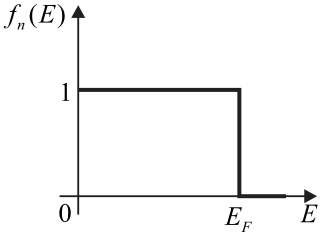
$$P_2 = 1 - P_3 \approx 1 - 1 \approx 0$$

So, $P_1 = P_2$

Hence, the correct option is (B).

📖 Key Point

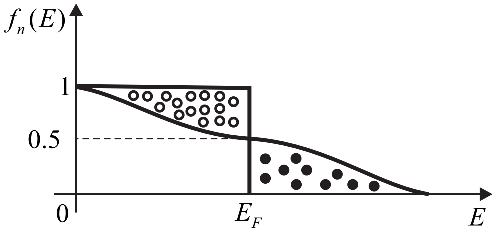
1. Fermi dirac distribution function describes the probability of existence of electron as a function of energy.
2. $f_n(E)$ denotes the ratio of filled quantum state to the total quantum state at any energy level E.
3. Fermi dirac distribution function at 0^0K is shown below,



This rectangular distribution implies that at 0^0K, every available quantum state up to E_f are filled with electron and all the quantum state above E_F are empty.

$$f_n(E) = \begin{cases} 1; & E < E_F \\ 0; & E > E_F \end{cases}$$

4. Fermi dirac distribution function at is shown below,



(i) There is some non-zero probability that some energy state above E_F will be occupied by electron and some energy state below E_F will be empty.

(ii) Fermi energy Level E_F is defined as energy level at which fermi dirac distribution function $f(E)$ will be 50% of its maximum value.

(iii) **In case of metal or conductor,**

$$f_n(E) = 1$$

$$\% f_n(E) = 100\%$$

5. The Fermi level represents the energy state with 50% probability of being filled if no forbidden band exists.

1.2 (D)

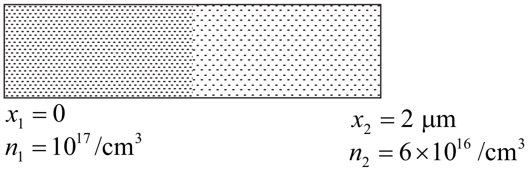
Given :

(i) Uniformly doped n-type Silicon.

(ii) Diffusion constant, $D_n = 35\ \text{cm}^2/\text{s}$

(iii) Electronic charge, $q = 1.6\times10^{-19}\ \text{C}$

(iv) Electric field, $E = 0\ \text{V/cm}$



Concentration gradient is given by,

$$\frac{dn}{dx} = \frac{n_2 - n_1}{x_2 - x_1}$$

$$\frac{dn}{dx} = \frac{6\times10^{16} - 10^{17}}{2\times10^{-4} - 0} = -2\times10^{20}\ /\ \text{cm}^4$$

Current density for n-type semiconductor is given by,

$$J_n = J_{n\,(\text{drift})} + J_{n\,(\text{diffusion})}$$

$$J_n = nq\mu_n E + qD_n\frac{dn}{dx}$$

$$J_n = qD_n\frac{dn}{dx} \qquad [\text{Given: } E = 0]$$

$$J_n = 35\times1.6\times10^{-19}\times(-2)\times10^{20}$$

$$J_n = -1120\ \text{A/cm}^2$$

Hence, the correct option is (D).

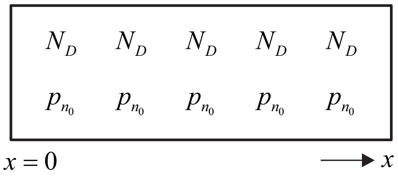
1.3 (A)

Low level injection is a process which injects minority carrier concentration into an extrinsic semiconductor.

Generally, doping is used to increase majority carrier concentration and low level injection is used to increase minority carrier concentration.

Assume uniformly doped *n*-type semiconductor, hence

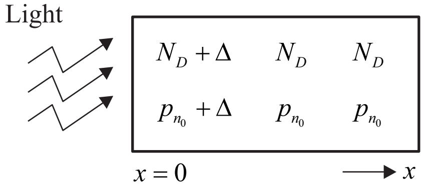
$$n = N_D$$



Let minority carriers be p_n, then

$$p_n\big|_{x=0} = p_{n_0} = \frac{n_i^2}{N_D}$$

When light is illuminated at one side of semiconductor, the covalent bonds are broken. So, the electron-hole $(e^- \text{-} h)$ pair are generated.



If Δ number of e^- and holes are generated, then total number of e^- becomes $N_D + \Delta$, where N_D is very large compared to Δ.

So, $\quad N_D + \Delta \approx N_D$

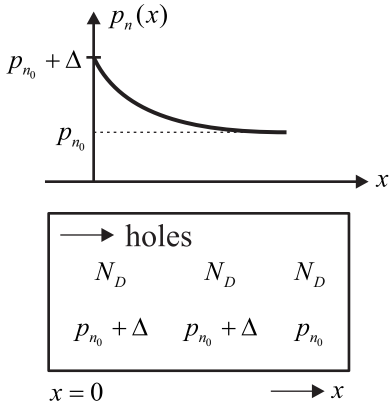
So, the change in majority carrier is negligible.

Compared to minority carrier Δ is very large. Hence, we cannot neglect Δ as compared to minority carrier.

At $x = 0$,

Total number of minority carrier $= p_n + \Delta$.

As x increases, the minority carrier will be reduced to p_{n_0}.



To establish equilibrium condition, the holes will move from higher concentration to the lower concentration, which is called diffusion.

Because of low level injection, large number of minority carrier will be created and they will move from higher concentration to lower concentration due to diffusion process and the current resulting due to diffusion process is called diffusion current.

Hence, the correct option is (A).

1.4 (C)

Given :

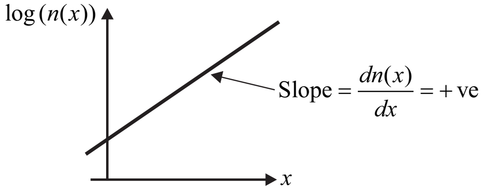
(i) $\quad n(x) = 10^{15} e^{\left(\frac{q\alpha x}{kT}\right)} \text{ cm}^{-3}$

(ii) $\quad \alpha = 0.1 \text{ V/cm}, \ \dfrac{kT}{q} = 0.026 \text{ V}$

(iii) $\quad D_n = 36 \text{ cm}^2\text{s}^{-1}, \ \dfrac{D}{\mu} = \dfrac{kT}{q}$

(iv) $\quad \mu_n = \dfrac{D_n}{kT/q} = \dfrac{D_n}{V_T} = \dfrac{36}{0.026} \text{ cm}^2/\text{V-sec}$

$$n(x) = 10^{15} e^{\left(\frac{q\alpha x}{kT}\right)} \text{ cm}^{-3} \qquad \dots \text{(i)}$$



$$\frac{dn(x)}{dx} = 10^{15} \times \frac{q\alpha}{kT} e^{q\alpha x/kT}$$

$$\left.\frac{dn(x)}{dx}\right|_{x=0} = 10^{15} \times e^{0} \times \frac{0.1}{0.026}$$

$$\left.\frac{dn(x)}{dx}\right|_{x=0} = 10^{15} \times \frac{100}{26} = \frac{10^{17}}{26}$$

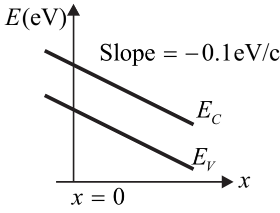
For non-uniform semiconductor, the electron are moving from higher concentration to lower concentration. Hence, electron diffusion current density is given by,

$$J_{n_1}(x)\Big|_{x=0} = qD_n \left.\frac{dn(x)}{dx}\right|_{x=0}$$

$$J_{n_1}(0) = 1.6 \times 10^{-19} \times 36 \times \frac{10^{17}}{26}$$

$$J_{n_1}(0) = \frac{1.6 \times 36}{26} \times 10^{-2}$$

$$J_{n_1}(0) = 2.21 \times 10^{-2} \text{ A/cm}^2 \qquad \dots \text{(ii)}$$



In above figure, slope of energy bands indicates applied electric field. Hence,

Electric field, $E(x) = -0.1$ V/cm

Electron drift current density is given by,

$$J_{n_2}(x)\Big|_{x=0} = n(x) q\mu_n E(x)\Big|_{x=0}$$

From equation (i),

$$n(0) = 10^{15} \text{ /cm}^3$$

Hence,

$$J_{n_2}(0) = 10^{15} \times 1.6 \times 10^{-19} \times \frac{36}{0.026} \times (-0.1)$$

$$J_{n_2}(0) = -2.21 \times 10^{-2} \text{ A/cm}^2 \quad \dots \text{(iii)}$$

From equation (ii) and (iii), total electron current density at $x = 0$ is given by,

$$J_n(0) = J_{n_1}(0) + J_{n_2}(0)$$

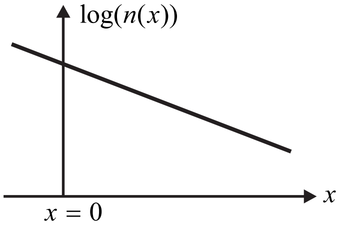
$$J_n(0) = 2.21 \times 10^{-2} + (-2.21 \times 10^{-2})$$

$$J_n(0) = 0 \text{ A/cm}^2$$

Hence, the correct option is (C).

Key Point

If diagram is given in this way,



In that case, $\frac{dn}{dx} = -\text{ve}$

$$J_{n_1}(x) = -\text{ve}$$

$$J_{n_1}(x) = -2.2 \times 10^{-2} \text{ A/cm}^2$$

1.5 **16**

Given :

(i) Electron-hole pairs generation rate,

$$G_L = G_{L_0}\left(1 - \frac{x}{L}\right)$$

(ii) $G_{L_0} = 10^{17} \text{/cm}^3\text{s}$

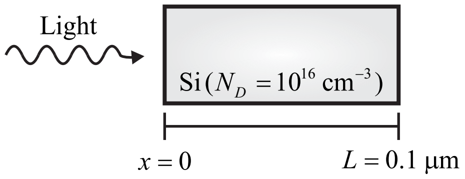
(iii) Length of the bar, $L = 0.1 \times 10^{-4}$ cm

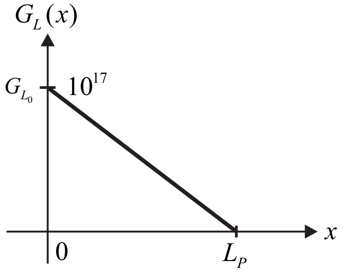
(iv) Hole diffusion coefficient,

$D_p = 100 \text{ cm}^2\text{/sec}$

(v) Hole life time, $\tau_p = 10^{-4}$ sec,

(vi) Electronic charge, $q = 1.6 \times 10^{-19}$ C





Hole diffusion current density is given by,

$$J_p = -qD_p \left.\frac{dp}{dx}\right|_{x=L/2} \qquad \ldots \text{(i)}$$

Concentration of hole in Si bar is given by,

$$p = G_L \times \tau_p$$

$$p = G_{L_0}\left(1-\frac{x}{L}\right)\times 10^{-4} = 10^{17}\left(1-\frac{x}{L}\right)\times 10^{-4}$$

$$p = 10^{13}\left(1-\frac{x}{L}\right) \text{ /cm}^3$$

Hence, $\dfrac{dp}{dx} = 10^{13}\left(0-\dfrac{1}{L}\right) = \dfrac{-10^{13}}{L}$

$$\frac{dp}{dx} = \frac{-10^{13}}{0.1\times 10^{-4}} = -10^{18} \text{ /cm}^4 \quad \ldots \text{(ii)}$$

Since, hole concentration gradient is independent of length of Si bar hence, current density will be independent of the length of the silicon bar.

From equations (i) and (ii),

$$J_p = -1.6\times 10^{-19}\times 100\times(-10^{18})$$

$$J_p = 16 \text{ A/cm}^2$$

Hence, the magnitude of the diffusion current density at $x = L/2$ is 16 A/cm^2.

1.6 (D)

Given :

Boron concentration $N_A = 10^{16}\text{cm}^{-3}$

So, $N_A = p_{p_0} = 10^{16}\text{cm}^{-3}$

Generation Rate $G_p = 10^{20}\text{cm}^{-3}\text{s}^{-1}$

Recombination life time $\tau_p = 100\ \mu\text{sec}$

Intrinsic carrier concentration $n_i = 10^{10}\text{ cm}^{-3}$

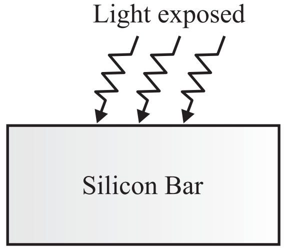
Using mass action law under thermal equilibrium

$$n_{p_0} = \frac{n_i^2}{p_{p_0}} = \frac{n_i^2}{N_A} = \frac{(10^{10})^2}{10^{16}}$$

$$n_{p_0} = 10^4\text{cm}^{-3} \quad \text{(Under no light exposed)}$$

So minority carrier concentration n_{p_0} when light is not exposed is 10^4 cm^{-3}

When light exposed,



So after exposing light, excess carrier concentration is

$$\Delta n = \Delta p$$

So, generation rate G_p is

$$G_p = \frac{\Delta p}{\tau_p}$$

$$\Delta p = G_p \times \tau_p = 10^{20}\times 100\,\mu\text{sec}$$

$$\Delta p = 10^{16}\text{cm}^{-3}$$

$$\Delta n = 10^{16}\text{cm}^{-3}$$

So after exposing light, under equilibrium,

1. Electrons concentration :

$$n_{p_0} + \Delta n = (10^4 + 10^{16})\text{cm}^{-3} \cong 10^{16}\text{cm}^{-3}$$

2. Hole concentration :

$$p_{p_0} + \Delta_p = (10^{16} + 10^{16})\text{cm}^{-3}$$

$$= 2\times 10^{16}\text{cm}^{-3}$$

Thus, product of hole concentration and electron concentration after exposing the light is

$$(n_{p_0} + \Delta_n)(p_{p_0} + \Delta_p) = 10^{16}\times 2\times 10^{16}$$

$$= 2\times 10^{32}\text{cm}^{-6}$$

Hence, the correct option is (D).

❖❖❖❖

2

p-n Junction Diodes

➢ Partial Synopsis

1. Built-In Potential (V_0):

$$V_0 = kT \ln \frac{N_D N_A}{n_i^2}$$, Open-circuited Potential

2. Depletion and Concentration Relation:

(a) $N_D^+ x_{n_0} = N_A^- x_{p_0}$

(b) $x_{p_0} = \frac{WN_D}{N_A + N_D}$

For $N_D >> N_A$

$x_{p_0} \approx W$

Also, $x_{n_0} = \frac{WN_A}{N_A + N_D}$

For $N_D << N_A$

$x_{n_0} \approx W$

3. Biasing:

(a) Junction voltage, $V_j = V_0 - V_D$

Where, $V_D =$ Applied voltage across diode.

4. Law of junction:

(a) Concentration of holes in n-region at the junction.

$$P_n(0) = P_{n_0} \; e^{\frac{V}{V_T}}$$

Where $V =$ Terminal voltages

Also, concentration of electron in p-region at junction.

$$n_p(0) = n_{p_0} \; e^{\frac{V}{V_T}}$$

(b) Diode current,

$$I = I_0\left(e^{\frac{V}{\eta V_T}} - 1\right)$$

Where, $\eta = 1$ (For Ge) and $\eta = 1$ (by default)

$\eta = 2$ (For Si)

$$I_0 = \frac{AqD_p p_{n0}}{L_p} + \frac{AqD_n n_{p0}}{L_n} = \text{Reverse saturation current}$$

p_{n0} = holes in n-side

$n_{p0} = e^-$ in p-side

Key Point

1. $I = -I_0$ ($-ve$ sign represents that in Reverse Bias, the flow of current is opposite to the direction of forward current).

➢ Sample Questions

1987 IIT Bombay

2.1 The diffusion capacitance of a *p-n* junction

(A) decreases with increasing current and increasing temperature.

(B) decreases with decreasing current and increasing temperature.

(C) increases with increasing current and increasing temperature.

(D) does not depend on current and temperature.

1993 IIT Bombay

2.2 The built-in potential (diffusion potential) in a *p-n* junction

(A) is equal to the difference in the Fermi-level of the two sides and expressed in volts.

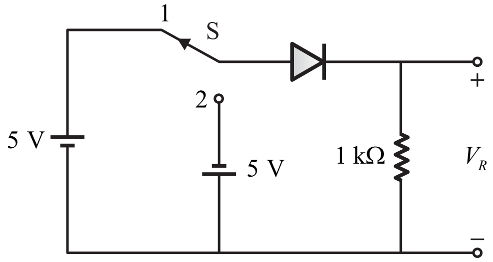
(B) increases with the increase in the doping levels of the two sides.

(C) increases with the increase in temperature.

(D) is equal to the average of the Fermi levels of the two sides.

2006 IIT Kharagpur

2.3 In the circuit shown below, the switch was connected to position 1 at $t < 0$ and at $t = 0$, it is changed to position 2. Assume that the diode has zero voltage drop and a storage time t_s for $0 < t \le t_s$, V_R is given by (all in Volts)



(A) $V_R = -5$ (B) $V_R = +5$

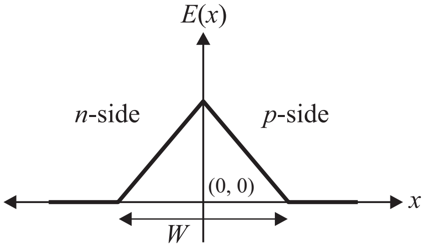
(C) $0 \le V_R < 5$ (D) $-5 \le V_R < 0$

2017 IIT Roorkee

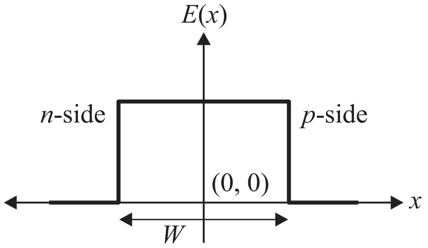
2.4 An abrupt *p-n* junction (located at $x = 0$) is uniformly doped on both *p* and *n* sides. The width of the depletion region is W and the electric field variation in the *x*-direction is $E(x)$.

Which of the following figures represents the electric field profile near the *p-n* junction? **[Set - 02]**

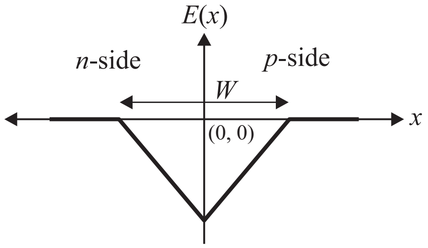
(A)



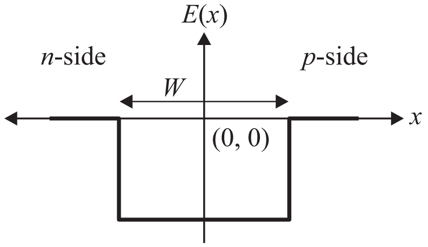
(B)



(C)



(D)



2021 **IIT Bombay**

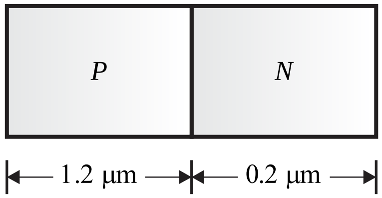
2.5 A silicon P-N junction is shown in the figure. The doping in the P region is $5\times10^{16}\,\text{cm}^{-3}$ and doping in the N region is $10\times10^{16}\,\text{cm}^{-3}$. The parameters given are

Built in voltage $(\phi_{bi})=0.8$ V

Electron charge $(q)=1.6\times10^{-19}$ C

Vacuum permittivity $(\varepsilon_0)=8.85\times10^{-12}$ F/m

Relative permittivity of silicon $(\varepsilon_{si})=12$



The magnitude of reverse bias voltage that would completely deplete one of the two regions (P or N) prior to the other (rounded off to one decimal place) is________ V.

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Explanations ***p-n* Junction Diodes**

2.1 **(B)**

For a *p-n* junction diode, diffusion capacitance is given by,

$$C_D=\tau g=\frac{\tau}{r}=\frac{\tau I_D}{\eta V_T}$$

where, r = AC resistance or dynamic resistance

I_D = Diode forward current

η = Ideality factor

V_T = Voltage temperature coefficient

$$C_D=\frac{\tau I_D}{\eta T}\times11600 \qquad \left[V_T=\frac{T(\text{K})}{11600}\right]$$

From above expression,

$$C_D\propto I_D \text{ and } C_D\propto\frac{1}{T}$$

Hence, diffusion capacitance decreases with decreasing current and increasing temperature.

Hence, the correct option is (B).

2.2 (A) and (B)

Built-in potential of *p*-*n* junction diode is given by,

$$V_{bi}=V_T\ln\left[\frac{N_AN_D}{n_i^2}\right]\text{ V}$$

$$V_{bi}\propto T\text{ and }V_{bi}\propto\frac{1}{n_i^2}$$

where, $n_i^2=A_0T^3e^{-E_{g0}/kT}$

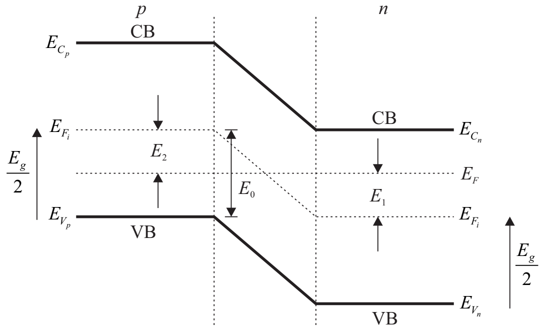
When temperature increase, thermal voltage equivalent (V_T) as well as n_i increases and both compensate each other. Therefore, there is no change in V_{bi} due to temperature.

Hence, it is clear that built-in potential V_{bi} will depend only on doping level. Therefore, V_{bi} will increase if doping level will increase.

Hence, the correct options are (A) and (B).

Key Point

Energy band diagram of *p*-*n* junction diode :



From above figure,

$$E_1=kT\ln\left[\frac{N_D}{n_i}\right],\qquad E_2=kT\ln\left[\frac{N_A}{n_i}\right]$$

$$E_0=E_1+E_2$$

$$E_0=kT\ln\left[\frac{N_A}{n_i}\right]+kT\ln\left[\frac{N_D}{n_i}\right]$$

$$E_0=kT\ln\left[\frac{N_AN_D}{n_i^2}\right]\text{ eV}$$

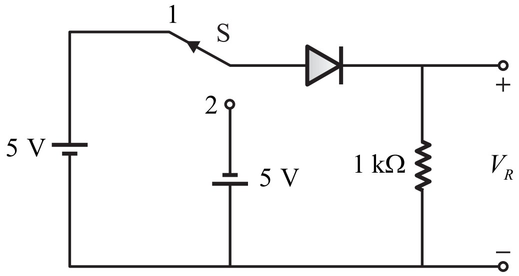
$$V_0=V_T\ln\left[\frac{N_AN_D}{n_i^2}\right]\text{ V}$$

2.3 (A)

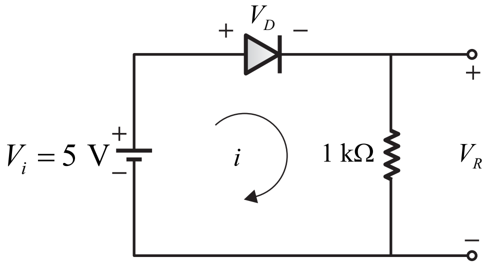
Given :

(i) Diode has zero voltage drop

(ii) Storage time = t_s



Case 1 : When switch is connected to position 1 ;

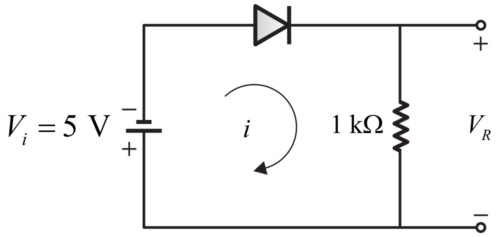


Since, voltage drop across diode is zero volt, therefore $V_D=0$ V. (Diode is ON i.e. short circuit)

$$i=\frac{5}{1}=5\text{ mA}$$

$$V_R=+5\text{ V}$$

Case 2 : When switch is connected to position 2 ;

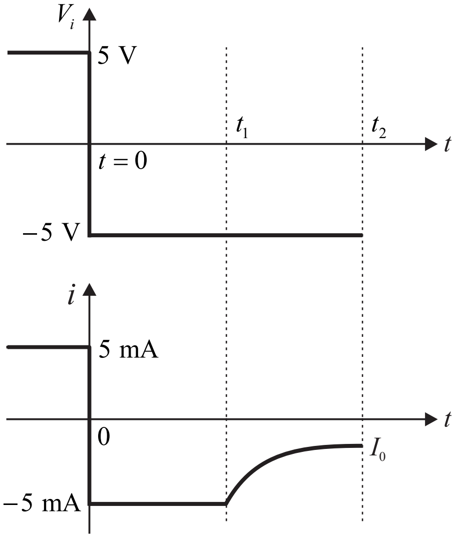


For $0<t\le t_s$,

where, t_s = Storage time

$$i=-5\text{ mA},\quad V_R=-5\text{ V}$$

Explanation :



At $t < 0$, during forward biased, the holes are moving from *p* to *n* and electrons from *n* to *p*, so there are large number of holes in *n*-side (i.e. minority charge carriers) supplied from *p*-side due to forward current. Similarly, for electrons in *p*-side.

In forward biased diode, voltage drop is very small i.e. $V_D = 0$.

So, forward current, $I_D = \dfrac{V_i}{R_L} = 5 \text{ mA}$

At $t = 0$, switch is changing from position 1 to position 2 i.e. $V_i = -5 \text{ V}$

However due to the stored excess minority carriers (holes in *n*-side and electrons in *p*-side), large reverse current $-I_R$ flows. Because of higher concentration of stored minority carriers the diode offers low resistance even in reverse bias during storage period t_s.

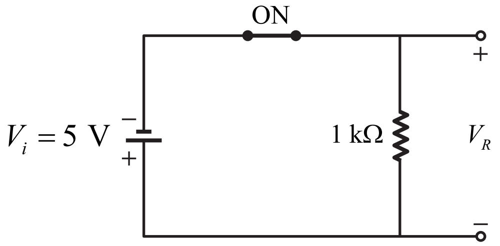
Reverse current $= -I_R \simeq \dfrac{V_R}{R_L}$

After time t_s, when almost all the stored charge carriers would have been removed, the reverse current decreases upto reverse saturation I_0 value during transition time t_t. After t_t, I_0 flows only due to minority charge carriers (corresponding to equilibrium concentration).

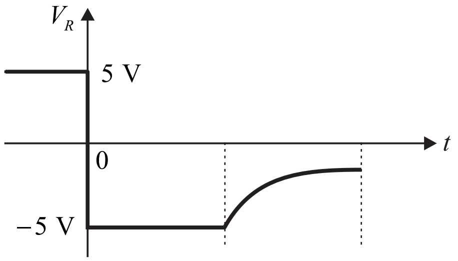
Total time taken by diode to reach the OFF state is t_{rr} (reverse recovery time).

$$t_{rr} = t_s + t_t$$

The voltage across $R_L = V_R = V_i - V_D$



During time t_s, $V_i = -V_R = -5 \text{ V}$



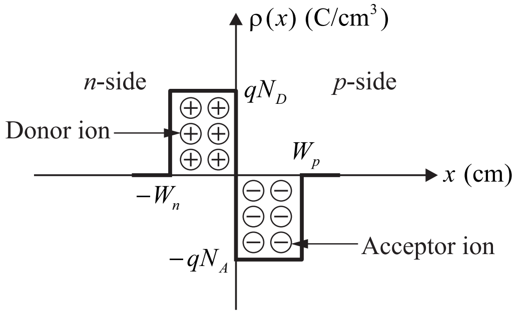
and $\quad V_D = 0 \text{ V}$ [Given]

So $\quad V_R = -5 \text{ V}$

Hence, the correct option is (A).

2.4 (A)

An electric field is created in the depletion region by the separation of positive and negative space charge density.



From Poisson's equation,

$$\frac{d^2}{dx^2}V(x)=\frac{-\rho(x)}{\varepsilon}$$

$$E(x)=\frac{-dV(x)}{dx}$$

$$\frac{dE(x)}{dx}=\frac{\rho(x)}{\varepsilon} \qquad \ldots \text{(i)}$$

where, $V(x)$ = Electric potential,

$E(x)$ = Electric field,

$\rho(x)$ = Volume charge density

From figure, volume charge density is,

$$\rho(x)=\begin{cases} qN_D; & -W_n<x<0 \\ -qN_A; & 0<x<W_p \end{cases}$$

From equation (i),

$$E(x)=\int\frac{\rho(x)}{\varepsilon}dx$$

$$E(x)=\int\frac{qN_D}{\varepsilon}dx=\frac{qN_D}{\varepsilon}x+C_1 \qquad \ldots \text{(ii)}$$

At $x=-W_n,\ E(x)=0$

$$0=\frac{-qN_DW_n}{\varepsilon}+C_1$$

$$C_1=\frac{qN_DW_n}{\varepsilon}$$

From equation (ii), electric field $E(x)$ at *n*-side is given by,

$$E(x)=\frac{qN_D}{\varepsilon}x+\frac{qN_DW_n}{\varepsilon}$$

$$E(x)=\frac{qN_D}{\varepsilon}(x+W_n)\ ;\quad -W_n<x<0 \qquad \ldots \text{(iii)}$$

In the *p*-region, electric field is determined from,

$$E(x)=\int\frac{-qN_A}{\varepsilon}dx=\frac{-qN_A}{\varepsilon}x+C_2 \ldots \text{(iv)}$$

At $x=W_p,\ E(x)=0$

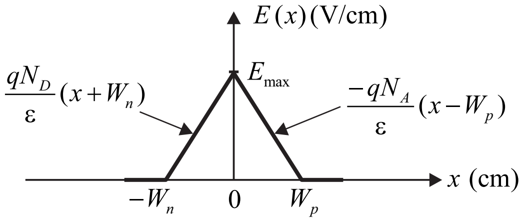
$$0=\frac{-qN_AW_p}{\varepsilon}+C_2$$

$$C_2=\frac{qN_AW_p}{\varepsilon}$$

From equation (iv), electric field $E(x)$ at *p*-side is given by,

$$E(x)=\frac{-qN_A}{\varepsilon}x+\frac{qN_AW_p}{\varepsilon}$$

$$E(x)=\frac{-qN_A}{\varepsilon}(x-W_p)$$



Hence, the correct option is (A).

2.5 **8.24**

Given:

Doping in P-region $N_A=5\times10^{16}\ \text{cm}^{-3}$

Doping in N-region $N_D=10^{17}\ \text{cm}^{-3}$

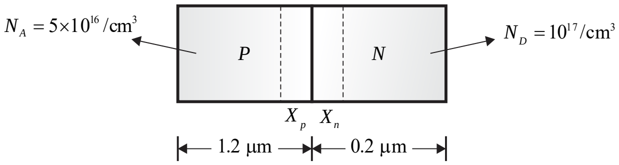
Built in voltage $(\phi_{bi})=0.8\ \text{V}$

Electron charge $(q)=1.6\times10^{-19}\text{C}$

Vacuum permittivity $(\varepsilon_0)=8.85\times10^{-12}$ F/m

Relative permittivity of silicon $(\varepsilon_{si})=12$

Given p-n junction is shown below,



So width of depletion region, $W=X_n+X_p$

From charge equality concept,

$$X_n.N_D=X_p.N_A$$

$$X_n.N_D = (W - X_n) N_A$$

$$X_n(N_D + N_A) = W.N_A$$

$$X_n = \frac{WN_A}{N_A + N_D} = \frac{W \times 5\times10^{16}}{(5\times10^{16}) + (10\times10^{16})}$$

$$X_n = \frac{W\times(5\times10^{16})}{15\times10^{16}} = \frac{W}{3}$$

So, $X_p = W - X_n = W - \frac{W}{3}$

$$X_p = \frac{2}{3}\text{ W}$$

Suppose when N-is depleted completely, during reverse bias so,

$$X_n = 0.2\ \mu\text{m}$$

$\therefore$ $W = 3X_n = 0.6\ \mu\text{m}$

$$X_p = \frac{2}{3}\times W = \frac{1.2}{3} = 0.4\ \mu\text{m}$$

i.e. when N is depleted completely (i.e. $0.2\ \mu\text{m}$) then and only then P side is depleted only by $0.4\ \mu\text{m}$ only. Thus

$$\left.\begin{matrix} X_n = 0.2\ \mu\text{m} \\ W = 0.6\ \mu\text{m}\end{matrix}\right\} \rightarrow (N \text{ region depleted completely})$$

So, total depletion width (W) under reverse bias is,

$$W = \sqrt{\frac{2\varepsilon(\phi_{bi}+V_R)}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}$$

$$W^2 = \frac{2\varepsilon(\phi_{bi}+V_R)}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)$$

$$(0.6\times10^{-6}\times100)^2 = \frac{2\times12\times8.85\times10^{-14}}{1.6\times10^{-19}}$$

$$\left(\frac{1}{5\times10^{16}}+\frac{1}{10\times10^{16}}\right)(\phi_{bi}+V_R)$$

$$(0.6\times10^{-4})^2 = 3.9825\times10^{-10}\ (\phi_{bi}+V_R)$$

$$\phi_{bi}+V_R = 9.0395$$

$$V_R = 9.0395 - \phi_{bi}$$

$$V_R = 9.0395 - 0.8$$

$$V_R = 8.2395\text{ V} \approx 8.24\text{ V}$$

Thus, the magnitude of reverse bias voltage that would completely deplete N region is 8.24 V.

Hence, the correct answer for V_R is 8.24 V.

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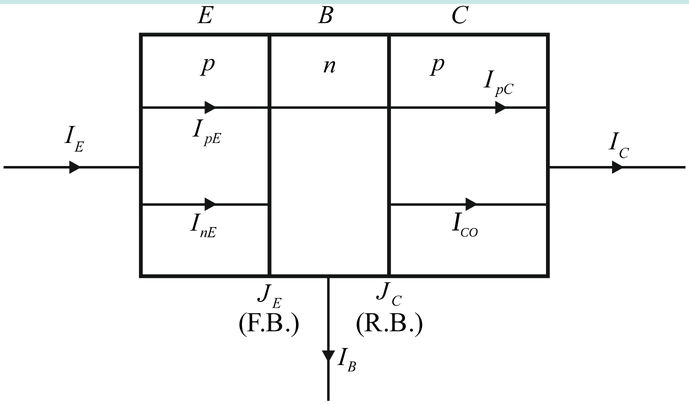
3

Basics of BJT

➢ Partial Synopsis

Active Mode :



Where, I_{pE} = Current due to diffusion of holes from p to n

I_{nE} = Current due to diffusion of electrons from n to p

Results :

(a) $I_C = I_{pC} + I_{CO}$

(b) $I_C = \alpha I_E + I_{CO}$ (Common base Equation).

(c) Emitter injection Efficiency/ Emitter Insertion factor

$$\gamma^* \cong \frac{I_{pE}}{I_{pE} + I_{nE}}$$

(d) Base Transport Factor

$$\beta^* = \alpha_T = \frac{I_{pC}}{I_{pE}}$$

(e) $I_E = I_{pE} + I_{nE}$

(f) $I_E = I_B + I_C$

(g) $\gamma^* \beta^* = \dfrac{I_{pC}}{I_{pE} + I_{nE}} = \alpha =$ Amplification Factor

Known as Amplification factor of Current Gain (CB) configuration

(h) $I_C = \alpha I_E + I_{CBO}$, $I_c = I_S e^{V_{BE}/V_T}$

Where I_{CBO} is current between Collector & Base when emitter is Open Circuited.

(i) $I_C = \alpha I_E - I_{CO}\left(e^{\frac{V_{CE}}{\eta V_T}} - 1\right)$

(j) Base current in BJT can be given by,

$$I_B = \left(\frac{I_S}{\beta}\right)e^{\frac{V_{BE}}{V_T}}$$

(k) Emitter current in BJT is given by,

$$I_E = \left(\frac{I_S}{\alpha}\right)e^{\frac{V_{BE}}{V_T}}$$

Ebers moll Model:

- Two diodes connecting back to back, we can construct a transistor until, a dependent current source is connected in parallel with the diodes.
- The given Model can work in all 3 conditions i.e. (Active, cut-off and saturation)

CB, CE, CC configuration:

(A) CB Configuration

- $\alpha = \dfrac{I_C}{I_E}$
- $I_C = \alpha I_E + I_{CBO}$

  (Where I_{CBO} is the reverse saturation current in CB configuration when emitter terminal is open etc.)
- CB amplifier circuit is used, where the low input impedance is required, like RF amplifier.

(B) CE Configuration

- $\beta = \dfrac{\alpha}{1-\alpha}$ and $\alpha = \dfrac{\beta}{1+\beta}$
- $I_C = \beta I_B + (\beta + 1) I_{CBO}$
- $I_C = \beta I_B + I_{CEO}$
- $I_{CEO} = (\beta + 1) I_{CBO}$
- CE configuration, commonly used in low noise amplifier.

(C) CC Configuration:

- $\gamma = \beta + 1 = \dfrac{1}{1-\alpha}$

- Application in “Super- Heterodyne Receiver”
- Practically, the below configuration is not possible.
- Better “Current Amplifier”
- $I_E = \gamma I_B + \frac{I_{CBO}}{1-\alpha}$
- CC is basically used for “Voltage Follower”.

Early effect /Base-Width Modulation:

- Consequences of Early- Effect.

(i) $I_E \uparrow, I_B \downarrow, \text{I}_C \uparrow$

(ii) $\alpha \uparrow$

(iii) Punch through / Reach Through occurs

Explanation :

Due to any R.B of J_{BC}, the effective base-width may be reduced to zero. This is known as “Punch Through” occurs.

At “ Reach through” condition, the usefulness of the transistor is lost.

Then, the depletion region, resistance increases. Hence more F.B. of the junction J_{EB} is required.

➢ Sample Questions

1988 IIT Kharagpur

3.1 For a *p*-*n* junction match the type of breakdown with phenomenon

1. Avalanche breakdown
2. Zener breakdown
3. Punch through

A. Collision of carriers with crystal ions

B. Early effect

C. Rupture of covalent bond due to strong electric field

(A) 1-B, 2-A, 3-C
(B) 1-C, 2-A, 3-B
(C) 1-A, 2-B, 3-C
(D) 1-A, 2-C, 3-B

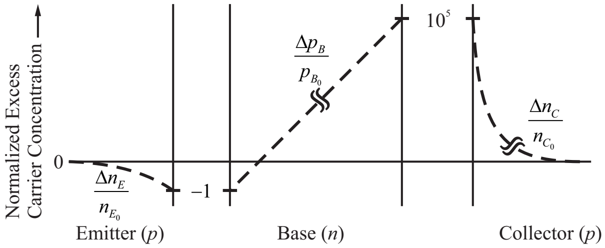
1997 IIT Madras

3.2 In a bipolar transistor at room temperature, if the emitter current is doubled, the voltage across its base-emitter junction

(A) doubles
(B) halves
(C) increases by about 20 mV
(D) decreases by about 20 mV

2017 IIT Roorkee

3.3 For a narrow base *p*-*n*-*p* BJT, the excess minority carrier concentrations (Δn_E for emitter, Δp_B for base, Δn_C for collector) normalized to equilibrium minority carrier concentrations (n_{E_0} for emitter, p_{B_0} for base, n_{C_0} for collector) in the quasi-neutral emitter, base and collector regions are shown below. Which one of the following biasing modes is the transistor operating in? **[Set - 01]**



X and *Y* axes are not to scale

(A) Forward active (B) Saturation

(C) Inverse active (D) Cutoff

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Explanations Basics of BJT

3.1 (D)

1. **Avalanche breakdown :**

(i) It is due to electron multiplication, which leads to multiple collision between electrons and ions in the depletion layer.

(ii) It is due to impact ionization.

(iii) Avalanche breakdown occurs for breakdown voltage greater than 6 V.

2. **Zener breakdown :**

(i) It is due to large electric field intensity, which in turn leads to tearing off or rupturing off of covalent bonds within the depletion layer.

(ii) Zener breakdown occurs for breakdown voltage below 6 V.

3. **Punch through :**

When effective base width reduces to zero due to early effect then this is called **punch through effect**.

Hence, the correct option is (D).

3.2 (C)

For a BJT, emitter current is given by,

$$I_E = I_0\left[e^{\frac{V_{BE}}{V_T}} - 1\right]$$

Let $$I_{E_1} = I_0\left[e^{\frac{V_{BE_1}}{V_T}} - 1\right] \quad \text{… (i)}$$

and $$I_{E_2} = I_0\left[e^{\frac{V_{BE_2}}{V_T}} - 1\right]$$

$$2I_{E_1} = I_0\left[e^{\frac{V_{BE_2}}{V_T}} - 1\right] \quad [\text{Given : } I_{E_2} = 2I_{E_1}]$$

… (ii)

From equation (i) and (ii),

$$\frac{2I_{E_1}}{I_{E_1}} = \frac{I_0\left[e^{\frac{V_{BE_2}}{V_T}} - 1\right]}{I_0\left[e^{\frac{V_{BE_1}}{V_T}} - 1\right]}$$

$$2 = \frac{e^{\frac{V_{BE_2}}{V_T}}}{e^{\frac{V_{BE_1}}{V_T}}} \quad [e^{\frac{V_{BE_2}}{V_T}} >> 1 \text{ and } e^{\frac{V_{BE_1}}{V_T}} >> 1]$$

$$2 = e^{\frac{(V_{BE_2} - V_{BE_1})}{V_T}}$$

$$\ln 2 = \frac{V_{BE_2} - V_{BE_1}}{V_T}$$

$$V_{BE_2} - V_{BE_1} = V_T \times \ln 2$$

Since, $V_T = 0.026$ V

$$V_{BE_2} - V_{BE_1} = 0.026 \times 0.693$$

$$V_{BE_2} - V_{BE_1} = 18 \text{ mV} \approx 20 \text{ mV}$$

$$V_{BE_2} = V_{BE_1} + 20 \text{ mV}$$

Hence, voltage across base-emitter junction will increase by 20 mV.

Hence, the correct option is (C).

3.3 (C)

Given :

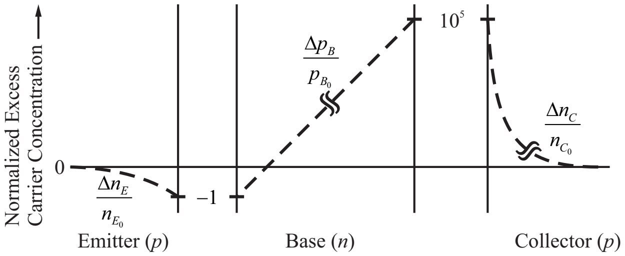


Figure shows the minority carrier distribution in the *p-n-p* transistor for the inverse-active mode. In this case, the *B-E* junction is reverse biased and the *B-C* junction is forward biased. Electrons from the collector are now injected into the base. The gradient in the minority carrier electron concentration in the base is in the opposite direction compared with the forward-active mode, so the emitter and collector currents will change direction.

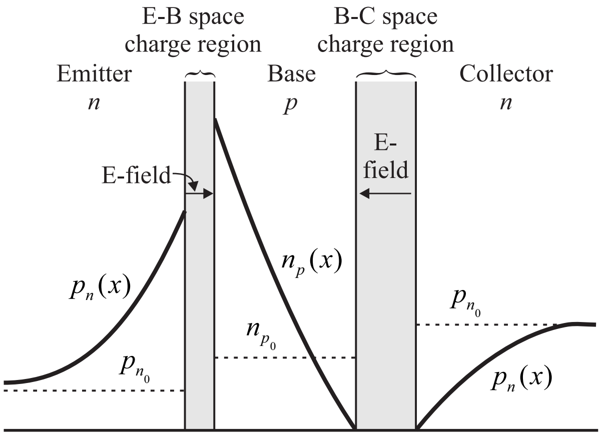
Hence, the correct option is (C).

Key Point

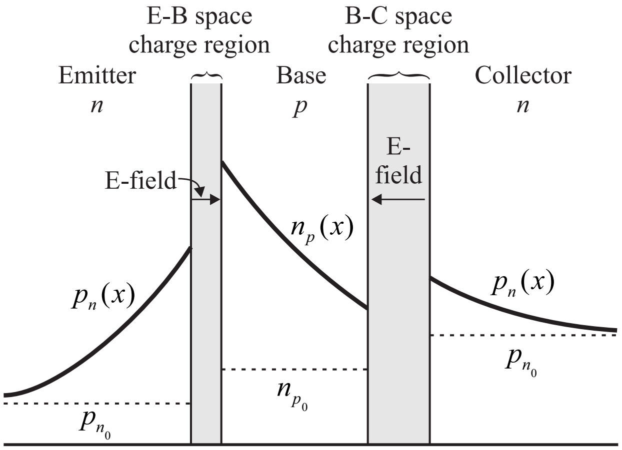
(i) For *n-p-n* transistor :

Variation of minority carrier concentration (in different operating mode) is shown below,

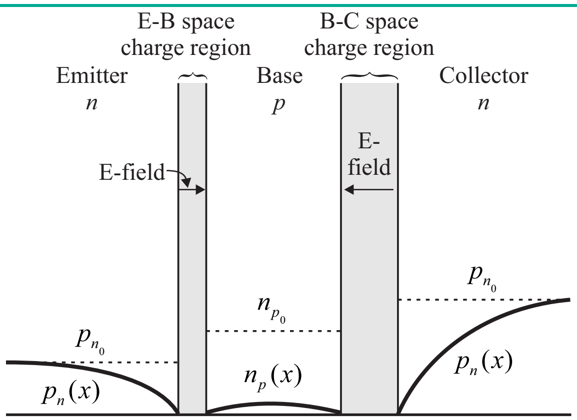
(a) Forward active mode :



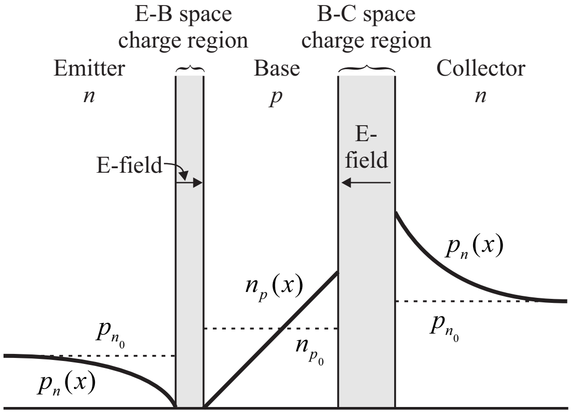
(b) Saturation mode :



(c) Cutoff mode :



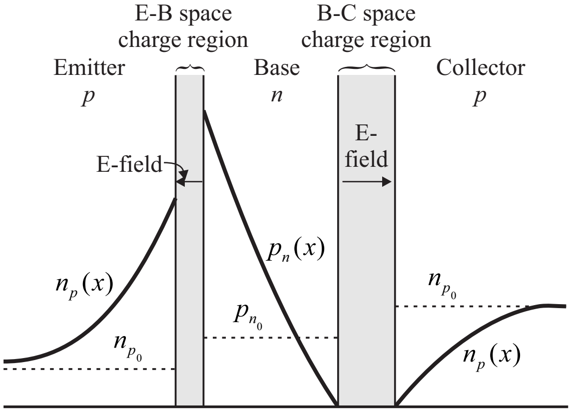
(d) Inverse active mode :



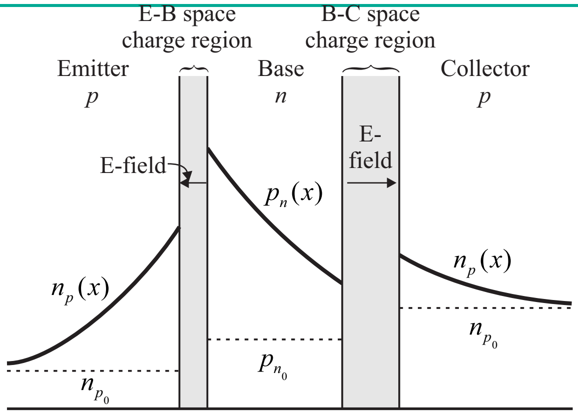
(ii) For *p-n-p* transistor :

Variation of minority carrier concentration (in different operating mode) is shown below,

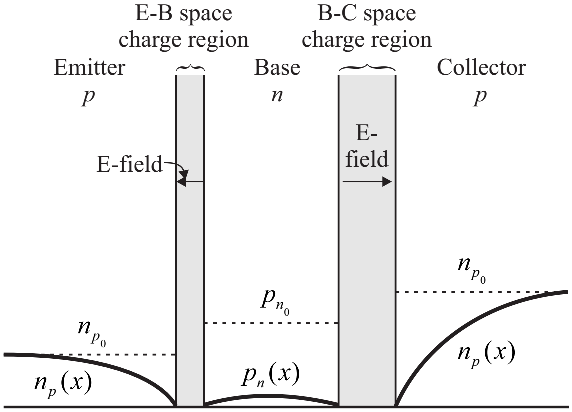
(a) Forward active mode :



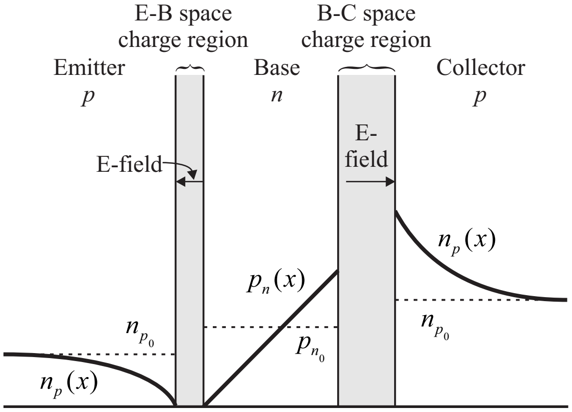
(b) Saturation mode :



(c) Cutoff mode :



(d) Inverse active mode :



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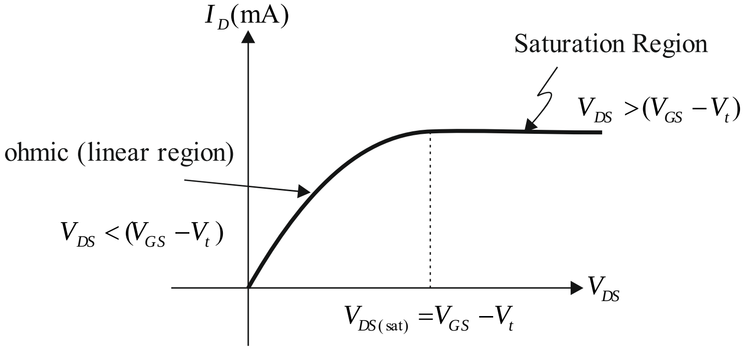
4

JFET & MOSFET

➢ Partial Synopsis

Current Characteristic:



Drain current :

(A) In Triode region,

$$I_D = \mu_n C'_{ox} \frac{W}{L}\left[(V_{GS} - V_t)V_{DS} - \frac{V_{DS}^2}{2}\right]_{NMOS}$$

(B) In saturation region,

$$I_D = \frac{\mu_n C'_{ox}}{2}\frac{W}{L}\left[(V_{GS} - V_t)^2\right]_{NMOS}$$

Transconductance Parameter :

(a) $\mu_n C'_{ox} = K'_n$

(b) $\mu_P C'_{ox} = K'_P$

where, K'_n, K'_p is the process transconductance parameter

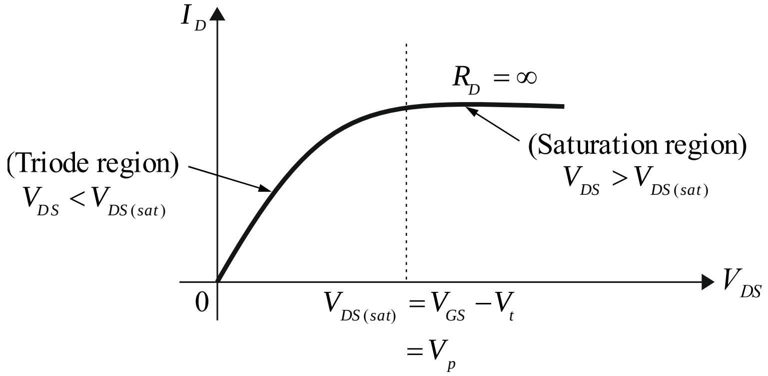
Unit of $\mu_P C'_{ox} = \dfrac{\text{cm}^2}{V\sec} \times \dfrac{F}{\text{cm}^2} = \dfrac{F}{V\sec} = \dfrac{C}{V.\sec} \times \dfrac{1}{V}$

Unit is $\dfrac{A}{V^2}$

Drain Resistance, r_d (valid in Triode Region) :

$$r_d = \frac{1}{K_n{}'\dfrac{W}{L}(V_{GS} - V_t)}$$

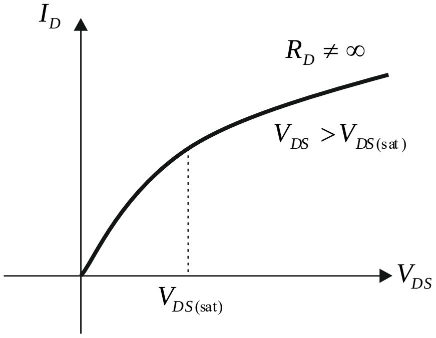
Where, aspect ratio $=\dfrac{W}{L}=\dfrac{\text{Width of channel}}{\text{Length of channel}}$



where, V_p is pinch off voltage.

"Finite Output Resistance" in saturation region :

Under channel length modulation



Here after V_{DS} , there is a finite output Resistance.

➢ Sample Questions

1988 **IIT Kharagpur**

4.1 In MOSFET devices the *n*-channel type is better than the *p*-channel type in the following respects

(A) it has better noise immunity.

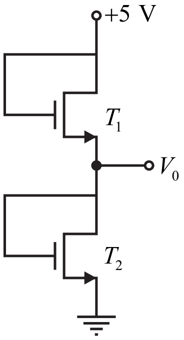
(B) it is faster.

(C) it is TTL compatible.

(D) it has better drive capability.

2005 **IIT Bombay**

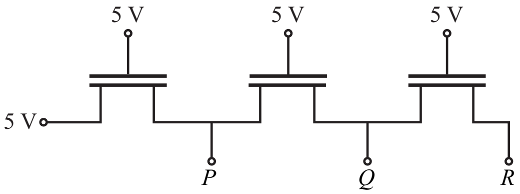
4.2 Both transistors T_1 and T_2 shown in the figure, have a threshold voltage of 1 volt. The device parameters K_1 and K_2 of T_1 and T_2 are, respectively, 36 $\mu A/V^2$ and 9 $\mu A/V^2$. The output voltage V_0 is



(A) 1 V
(B) 2 V
(C) 3 V
(D) 4 V

2014 IIT Kharagpur

4.3 In the following circuit employing pass transistor logic, all NMOS transistors are identical with a threshold voltage of 1 V. Ignoring the body-effect, the output voltages at *P*, *Q* and *R* are, **[Set - 01]**



(A) 4 V, 3 V, 2 V (B) 5 V, 5 V, 5 V
(C) 4 V, 4 V, 4 V (D) 5 V, 4 V, 3 V

2019 IIT Madras

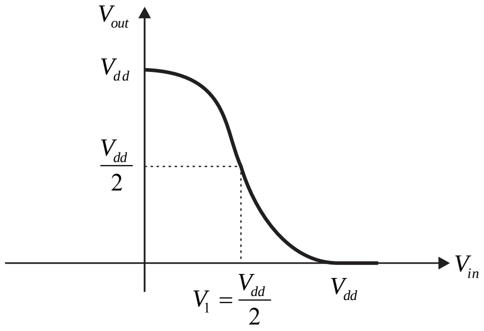
4.4 A CMOS inverter, designed to have a mid-point voltage V_1 equal to half of V_{dd} as shown in figure, has following parameters :

$$V_{dd} = 3\text{V}, \quad \mu_n C_{ox} = 100\ \mu\text{A/V}^2$$

$$V_{tn} = 0.7\text{V for NMOS}$$

$$\mu_p C_{ox} = 40\ \mu\text{A/V}^2$$

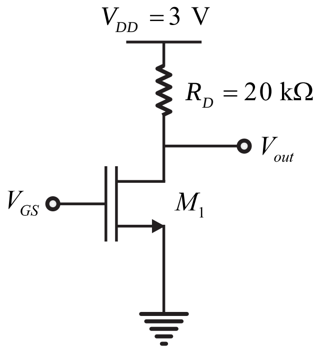
$$V_{tp} = 0.9\text{V for PMOS}$$



The ratio of $\left(\frac{W}{L}\right)_n$ to $\left(\frac{W}{L}\right)_p$ is equal to __________ (rounded off upto 3 decimal places).

2021 IIT Bombay

4.5 For the transistor M_1 in the circuit shown in the figure, $\mu_n C_{ox} = 100\,\mu\text{A/V}^2$ and $(W/L) = 10$ where μ_n is the mobility of electron, C_{ox} is the oxide capacitance per unit area, W is the width and L is the length.



The channel length modulation coefficient is ignored. If the gate-to-source voltage V_{GS} is 1 V to keep the transistor at the edge of saturation, then the threshold voltage of the transistor (rounded off to one decimal place) is __________V.

❖❖❖❖

Concept of CMOS Inverter Part I :

Scan for Video Explanation

Concept of CMOS Inverter Part II :

Scan for Video Explanation

4.1 (B)

Difference between *n*-channel type MOSFET and *p*-channel type MOSFET :

(i) **Source, drain and substrate :** NMOS is built with *n*-type source and drain and a *p*-type substrate, while PMOS is built with *p*-type source and drain and a *n*-type substrate.

(ii) **Carrier :** In NMOS, carrier are electrons, while in PMOS, carrier are holes.

(iii) **Gate voltage :** When a high voltage is applied to the gate, NMOS will conduct, while when a high voltage is applied to the gate, PMOS will not conduct.

When a low voltage is applied to the gate, NMOS will not conduct, while when a low voltage is applied to the gate, PMOS will conduct.

(iv) **Speed :** NMOS are considered to be faster than PMOS, since the carrier in NMOS, which are electron, travel twice as fast as holes, which are carriers in PMOS.

Electron moves faster than hole, because mobility of electron is higher than mobility of hole.

(v) **Noise immunity :** NMOS is less immune to noise, while PMOS is more immune to noise.

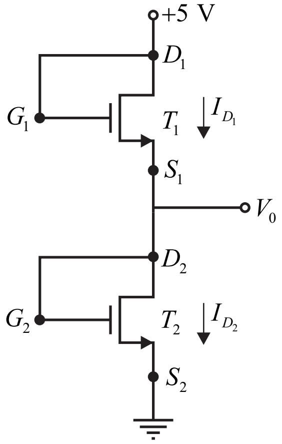
(vi) **Impedance :** For same geometry and operating condition, the NMOS can provide one half of the impedance provided by a PMOS.

(vii) **Required area :** NMOS IC would be smaller than PMOS IC (that gives the same functionality), since NMOS has lesser impedance than PMOS.

Hence, the correct option is (B).

4.2 (C)

Given :



(i) Threshold voltage, $V_{Th} = 1$ Volt

(ii) $K_1 = 36\ \mu\text{A/V}^2,\ K_2 = 9\ \mu\text{A/V}^2$

Drain and gate terminal of both MOSFETs are shorted, therefore both MOSFETs will be in saturation region.

Drain current of T_1 is given by,

$$I_{D_1} = K_1(V_{GS_1} - V_{Th})^2$$

Drain current of T_2 is given by,

$$I_{D_2} = K_2(V_{GS_2} - V_{Th})^2$$

Both MOSFETs are connected in series, hence same drain current will flow through both MOSFET. Hence,

$$I_{D_1} = I_{D_2}$$

$$K_1(V_{GS_1} - V_{Th})^2 = K_2(V_{GS_2} - V_{Th})^2 \qquad \ldots \text{(i)}$$

From the given circuit,

$$V_{GS_1} = V_{G_1} - V_{S_1} = V_{D_1} - V_{S_1} = 5 - V_0$$

$$V_{GS_2} = V_{G_2} - V_{S_2} = V_{D_2} - V_{S_2} = V_0$$

From equation (i),

$$K_1(5 - V_0 - V_{Th})^2 = K_2(V_0 - V_{Th})^2$$

$$36(5 - V_0 - 1)^2 = 9(V_0 - 1)^2$$

$$6(4 - V_0) = \pm 3(V_0 - 1)$$

$$V_0 = 3 \text{ V}, 7 \text{ V}$$

Case 1 : For $V_0 = 7$ V;

$$V_{GS_1} = 5 - 7 = -2 \text{ V}$$

$$V_{GS_2} = V_0 = 7 \text{ V}$$

For $V_{GS_1} < V_{Th}$, T_1 will not work in saturation region.

Hence, $V_0 = 7$ V, is not possible.

Case 2 : For $V_0 = 3$ V ;

$$V_{GS_1} = 5 - V_0 = 5 - 3 = 2 \text{ V}$$

$$V_{GS_2} = V_0 = 3 \text{ V}$$

For, $V_{GS_1} > V_{Th}$ and $V_{GS_2} > V_{Th}$, both T_1 and T_2 will work in saturation region.

Thus, the correct value of V_0 is 3 V.

Hence, the correct option is (C).

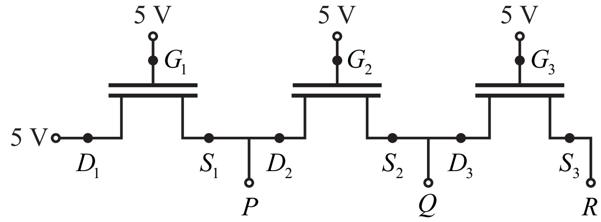
4.3 **(C)**

Method 1

Given : For NMOS transistors

$$V_{Th} = 1 \text{ V}$$

Given circuit is shown below,



In pass transistors logic, V_{GS} is automatically maintained at V_{Th}.

If V_G is fixed, V_S should be adjusted so that $V_G - V_S = V_{Th}$.

$V_G = 5$ V for all NMOS transistors and $V_{Th} = 1$ V for all transistors.

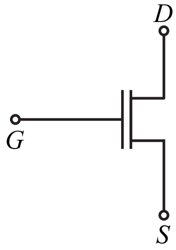
Hence, $V_S = 5 - 1 = 4$ V, For all NMOS transistors.

Method 2

In NMOS transistor, gate voltage (V_G) work as a controlled input. When NMOS work as a Switch, then

If $V_G = 0$ V, then NMOS will be OFF.

And if $V_G = +5$ V, then NMOS will be ON.

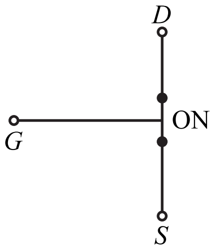


(i) For $V_{DS} < V_{GS} - V_{Th}$

$$V_D - V_S < V_G - V_S - V_{Th}$$

$$V_D < V_G - V_{Th}$$

When NMOS will satisfy above equation, then NMOS will be in linear region and hence it will be ON.



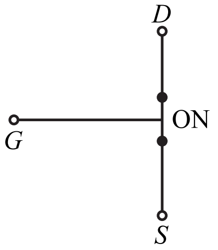
Therefore, $V_D = V_S$

(ii) For $V_{DS} \geq V_{GS} - V_{Th}$

$$V_D - V_S \geq V_G - V_S - V_{Th}$$

$$V_D \geq V_G - V_{Th}$$

When NMOS will satisfy above equation, then NMOS will be in saturation region and hence it will be ON.



Therefore, $V_D = V_S = V_G - V_{Th}$

For first NMOS :

$$V_{D_1} = 5 \ \ \& \ V_{G_1} - V_{Th} = 4\,\text{V}$$

Hence, $V_{D_1} > V_{G_1} - V_{Th}$

Therefore, first NMOS is working in saturation region.

So, $V_{S_1} = V_P = V_{G_1} - V_{Th} = 4\,\text{V}$

Second NMOS :

$$V_P = V_{D_2} = 4\,\text{V and } V_{G_2} - V_{Th} = 4 \text{ V}$$

Hence, $V_{D_2} = V_{G_2} - V_{Th}$

Therefore, second NMOS is working in saturation region.

So, $V_Q = V_{G_2} - V_{Th} = 4\,\text{V}$

Third NMOS :

$$V_Q = V_{D_3} = 4\,\text{V} \ \text{ and } \ V_{G_3} - V_{Th} = 4 \text{ V}$$

Hence, $V_{D_3} = V_{G_3} - V_{Th}$

Therefore, third NMOS is working in saturation region.

So, $V_{S_3} = V_R = V_{G_3} - V_{Th} = 4 \text{ V}$

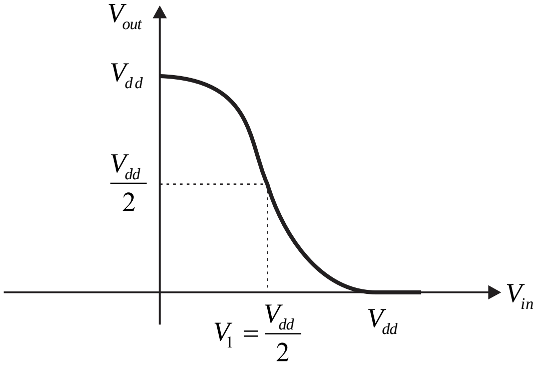
From above, it is clear that NMOS transistor passes zero (0) and PMOS transistor passes one (1).

Hence, the correct option is (C).

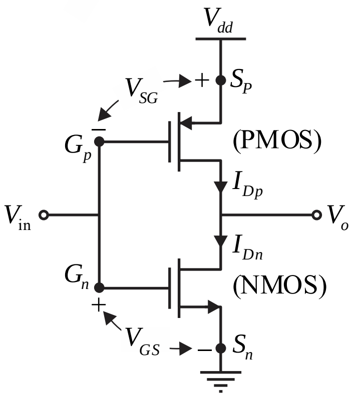
Given : $V_{dd} = 3\,\text{V}$

$\mu_n C_{ox} = 100 \ \mu\text{A/V}^2, \ V_{tn} = \ 0.7\text{V}$ for NMOS

$\mu_p C_{ox} = 40 \ \mu\text{A/V}^2, \ V_{tp} = \ 0.9\text{V}$ for PMOS



A CMOS invertor circuit,



Form figure of transfer characteristics of CMOS inverter, we say that both the transistor in saturation region.

So, $I_{Dn} = I_{Dp}$ … (i)

and $V_{in} = \frac{V_{dd}}{2}$ and $V_{out} = \frac{V_{dd}}{2}$

If $V_{in} = \frac{V_{dd}}{2} = 1.5\,\text{V} = V_{GS}$

From PMOS $V_{dd} = V_{SG} + V_{in}$

$$V_{dd} = V_{SG} + \frac{V_{dd}}{2}$$

$$V_{SG} = \frac{V_{dd}}{2} = 1.5\ \text{V}$$

From equation (i),

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{GS} - V_{Tn})^2$$

$$= \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p \left(V_{SG} - |V_{Tp}|\right)^2$$

$$\frac{\left(\frac{W}{L}\right)_n}{\left(\frac{W}{L}\right)_p} = \frac{\mu_p C_{ox}}{\mu_n C_{ox}} \frac{\left(V_{SG} - |V_{Tp}|\right)^2}{(V_{GS} - V_{Tn})^2}$$

$$\frac{\left(\frac{W}{L}\right)_n}{\left(\frac{W}{L}\right)_p} = \frac{40 \times (1.5 - 0.9)^2}{100 \times (1.5 - 0.7)^2}$$

$$\frac{\left(\frac{W}{L}\right)_n}{\left(\frac{W}{L}\right)_p} = \frac{40 \times (0.6)^2}{100 \times (0.8)^2} = 0.225$$

OR

If we consider

$$I_{Dp} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p \left(V_{GS} - V_{Tp}\right)^2$$

$$(V_{GS})_p = -1.5\,\text{V}$$

$$V_{Tp} = -0.9\,\text{V}$$

From equation (i),

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{GS} - V_{Tn})^2$$

$$= \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p \left(V_{GS} - V_{Tp}\right)^2$$

$$\frac{\left(\frac{W}{L}\right)_n}{\left(\frac{W}{L}\right)_p} = \frac{\mu_p C_{ox}}{\mu_n C_{ox}} \frac{\left(V_{GS} - V_{Tp}\right)^2}{(V_{GS} - V_{Tn})^2}$$

$$\frac{\left(\frac{W}{L}\right)_n}{\left(\frac{W}{L}\right)_p} = \frac{40 \times \left(-1.5 - (-0.9)\right)^2}{100 \times (1.5 - 0.7)^2}$$

$$\frac{\left(\frac{W}{L}\right)_n}{\left(\frac{W}{L}\right)_p} = \frac{40 \times (0.6)^2}{100 \times (0.8)^2} = 0.225$$

Hence, the ratio of $\left(\frac{W}{L}\right)_n$ to $\left(\frac{W}{L}\right)_p$ is $= 0.225$.

4.4 **0.5**

Given :

(i) $\mu_n C_{ox} = 100\,\mu\text{A/V}^2$

(ii) $\frac{W}{L} = 10$

(iii) $V_{GS} = 1\ \text{V}$

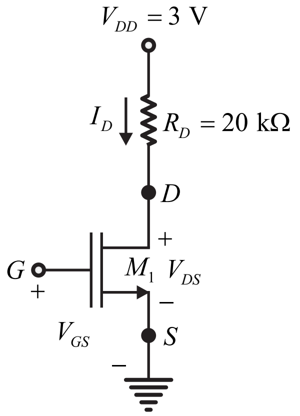
Drain to source current (I_{DS}) when N-MOS is saturation is given by,

$$I_D = I_{DS} = \frac{\mu_n C_{ox}}{2} \times \frac{W}{L}(V_{GS} - V_T)^2$$

(For NMOS in saturation)

$$I_D = I_{DS} = \frac{100 \times 10^{-6}}{2} \times 10(1 - V_T)^2$$

$$I_D = \frac{1}{2}(1 - V_T)^2\ \text{mA}$$



Apply KVL at outer loop of N-MOS,

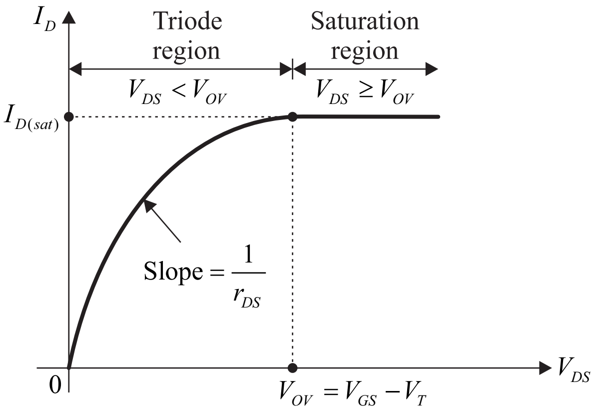
$$V_{DS} = 3 - 20 \times I_{DS}$$

Put the value of I_{DS} in above equation, we get

$$V_{DS} = 3 - \frac{20}{2}(1 - V_T)^2$$

$$V_{DS} = 3 - 10(1 - V_T)^2$$

MOSFET operates in saturation if $V_{DS} \geq V_{OV}$ as shown below,



So, $V_{DS} \geq V_{OV} = V_{GS} - V_T$

So, we take, $V_{DS} = V_{GS} - V_T$

$$V_{GS} - V_T = 3 - 10(1 - V_T)^2$$

$$1 - V_T = 3 - 10(1 - V_T)^2$$

Let, $1 - V_T = x$

$$3 - 10x^2 = x$$

$$10x^2 + \times - 3x = 0$$

We get, $x = -\dfrac{1 \pm \sqrt{1+120}}{20}$

$$x = \frac{1 \pm 11}{20}$$

∴ $x = 0.5$ and -0.6

When $x = 0.5$	**When** $x = -0.6$
$1 - V_T = 0.5$ $V_T = 0.5\text{ V}$	$1 - V_T = -0.6$ $V_T = 1.6\text{ V}$

For MOSFET to be in saturation

If $V_T < V_{GS}$

i.e., $V_T < 1$

∴ $V_T = 0.5\text{ V}$

Hence, the threshold voltage of the transistor to be at the edge of saturation is **0.5 V**.

Key Point

If we choose $V_T = 1.6$ V, then the MOSFET goes in cut-off region, because $V_T > V_{GS}$.

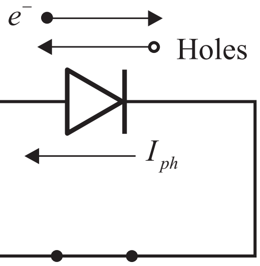
❖❖❖❖

6

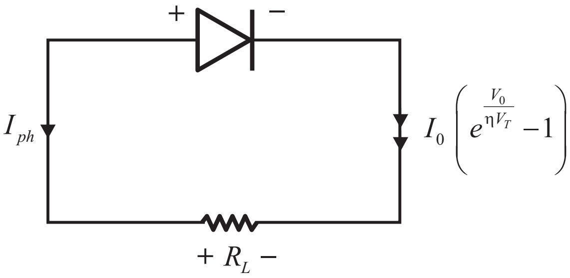
Special Purpose Diodes

➢ Partial Synopsis

Solar cell (R.B) :



Magnitude of reverse photocurrent, I_{ph} is directly proportional to the intensity of light.



Total current $I=I_0\left(e^{\frac{V_0}{\eta V_T}}-1\right)-I_{ph}$

Key Point

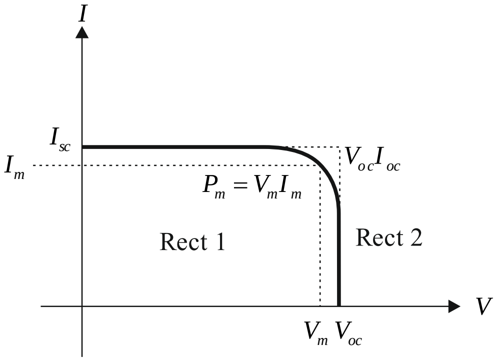
$-ve$ sign is with I_{ph} because I_{ph} is flowing from n to p

- Parameters of solar cell:

  (a) Percentage rise of minority $=\dfrac{(P_{n0}+\Delta P)-(P_{n0})}{P_{n0}}$

  Where, $\Delta P \rightarrow$ excess generated carriers

  (b) Efficiency $\eta=\dfrac{(\text{maxmium power delivered by solar cell})}{(\text{Incident power per unit area from sun})}$



🕮 Key Point

The maximum area of rectangle that can be drawn gives the maximum power of solar cell.

(c) Fill factor $(FF) = \dfrac{\text{Area of Rect1}}{\text{Area of Rect2}} = \dfrac{V_m I_m}{V_{oc} I_{sc}}$

$$\text{Also, } \eta = \frac{\text{Output}}{\text{input}} = \frac{V_m I_m}{(\text{Power/ Area})} = \frac{(V_{oc} I_{sc})(\text{FF})}{\left(\dfrac{\text{Power}}{\text{Area}}\right)(\text{Area})} = \frac{\text{FF} \times V_{oc} \times I_{sc}}{\text{Power}}$$

(d) Calculation of open circuited voltage (V_{oc})

$$I = I_0 e^{\frac{V_{oc}}{\eta V_T}} - I_{ph}(\text{Intensity of light})$$

$$0 = I_0 e^{\frac{V_{oc}}{\eta V_T}} - (I_{ph})(\text{Intensity of light})$$

$$I_{ph} = I_0 e^{\frac{V_{oc}}{\eta V_T}}$$

$$V_{oc} = \eta V_T \ln\left(\frac{I_{ph}}{I_0}\right)$$

🕮 Key Point

Solar cell operates in 4th quadrant of V-I characteristics.

➢ Sample Questions

1987 **IIT Bombay**

6.1 Direct band gap semiconductors

(A) exhibit short carrier life time and they are used for fabricating BJTs.

(B) exhibit long carrier life time and they are used for fabricating BJTs.

(C) exhibit short carrier life time and they are used for fabricating lasers.

(D) exhibit long carrier life time and they are used for fabricating lasers.

2006 **IIT Kharagpur**

6.2 The value of voltage (V_D) across a tunnel-diode corresponding to peak and valley currents are V_p, V_v respectively.

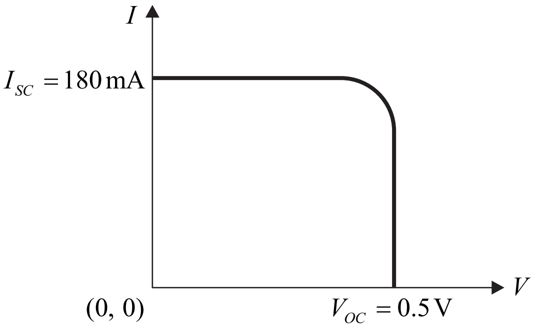
The range of tunnel-diode voltage V_D for which the slope of its $I - V_D$ characteristics is negative would be

(A) $V_D < 0$ (B) $0 \le V_D < V_p$

(C) $V_p \le V_D \le V_v$ (D) $V_D \ge V_v$

2016 IISc Bangalore

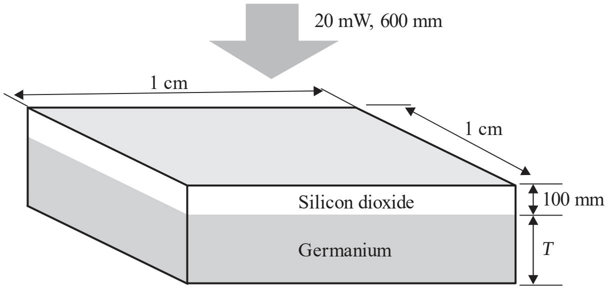
6.3 The figure shows the *I-V* characteristics of a solar cell illuminated uniformly with solar light of power $100\ \text{mW/cm}^2$. The solar cell has an area of $3\ \text{cm}^2$ and a fill factor of 0.7.



The maximum efficiency (in %) of the device is _____. **[Set - 03]**

2019 IIT Madras

6.4 A Germanium sample of dimensions 1 cm × 1 cm is illuminated with a 20 mW, 600 nm laser light source as shown in the figure. The illuminated sample surface has a 100 nm of loss-less Silicon dioxide layer that reflects one-fourth of the incident light. From the remaining light, one-third of the power is reflected from the Silicon dioxide-Germanium interface, one-third is absorbed in the Germanium layer, and one-third is transmitted through the other side of the sample. If the absorption coefficient of Germanium at 600 nm is $3 \times 10^4\ \text{cm}^{-1}$ and the bandgap is 0.66 eV, the thickness of the Germanium layer, rounded off to 3 decimal places, is _______ μm.



Explanations Special Purpose Diodes

6.1 (C)

Comparison between Direct band gap semiconductor & Indirect band gap semiconductor :

S. No.	Direct band gap semiconductor	Indirect band gap semiconductor
1.	During recombination most of the energy dissipates in the form of light.	During recombination most of the energy dissipates in the form of heat.
2.	During recombination, most of the free e^- will be directly falling from conduction band to valence band and hence termed as direct band gap semiconductor.	During recombination most of the free e^- falling from conduction band and will be going to intermediate level & then fall into valence band, hence named indirect band gap semiconductor.
3.	In direct band gap semiconductor some of the free e^- falling from conduction band will be colliding with the crystal and crystal will absorb the energy from the following e^- and release small energy in the form of heat.	In indirect band gap semiconductor some of the free e^- falling from conduction band will escape the collision and they will directly fall into valence band and small energy is dissipated in the form of light.

4.	The direction of falling e^- will remain the same i.e. conduction band to Valence band.	The direction of falling e^- will remain same i.e. conduction band to Valence band.
5.	**Smaller carrier lifetime.**	**Larger carrier lifetime.**
6.	The e^- can release energy without change in momentum i.e. momentum of e^- remains constant.	The e^- can't release energy without a change in momentum i.e. momentum of e^- will change.
7.	**Example :** Gallium arsenide (GaAs), Gallium nitride (GaN), Gallium antimonide (GaSb), Indium arsenide (InAs), Indium antimonide (InSb), Cadmium sulphide (CdS), Cadmium selenide (CdSe)	**Example :** Silicon (Si) Germanium (Ge)

Hence, the correct option is (C).

🕮 Key Point

(i) Si is never used in fabrication of LED because it is an indirect band gap semiconductor.

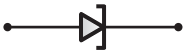
(ii) **Semiconductor LASER is fabricated with direct band gap semiconductor.**

6.2 (C)

Tunnel Diode :

The Tunnel diode is a *p-n* junction that operates in certain region of its $I-V$ characteristics by the quantum mechanical tunneling of electrons through the potential barrier of the junction.

Symbol :



or

Characteristics of Tunnel Diode :

(i) Tunnel diode is based on heavy doping $(1:10^3)$ or $10^{20}\ /\text{cm}^3$. Due to heavy doping depletion region decreases to approximately $100\overset{0}{\text{A}}$.

(ii) In tunnel diode the current is due to drift whereas in *p-n* junction diode the current is due to diffusion.

(iii) Tunnel diode characteristic is also called as "Voltage controlled negative resistance" i.e. current decreases rapidly at some critical voltage.

(iv) Tunnel diode consists negative resistance region i.e. it is not working on ohm's law.

(v) Electrons moving in *p-n* diode are due to diffusion and in tunnel diode are due to drift. Since drift is more faster than diffusion process, so Tunnel diode is considered as high speed switch.

(vi) Ge or GaAs are used in the construction of tunnel diode because Si based tunnel diode gives less value of I_P / I_V. (This ratio is often used as figure of merit of Tunnel diode.)

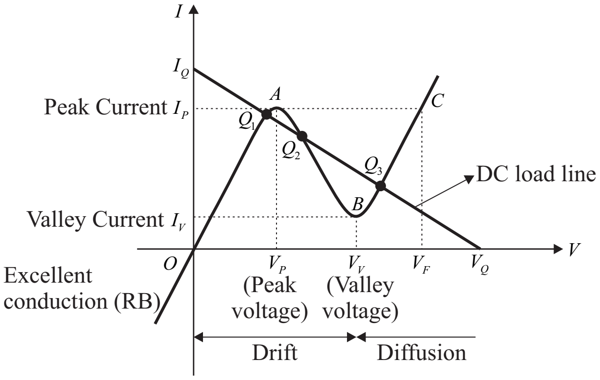
Tunneling :

A charge particle need not to have energy equal to height of barrier to move from one side to other of a diode instead it can penetrate through a barrier. This quantum mechanical effect is called as "Tunneling".

There are two conditions when free electrons tunnel from filled to empty state.

(i) Width of depletion region should be very narrow approximately $100\overset{0}{\text{A}}$.

(ii) At one side of a diode, filled state should exist and at the other side at the same energy level empty state (no electrons) should exist.



Region OA :

This region consists positive resistance i.e. tunneling current increases with increase in forward bias.

Region AB :

This region of characteristics is known as negative resistance region because the tunneling current decreases with the increase in bias. This negative resistance region is useful in oscillators.

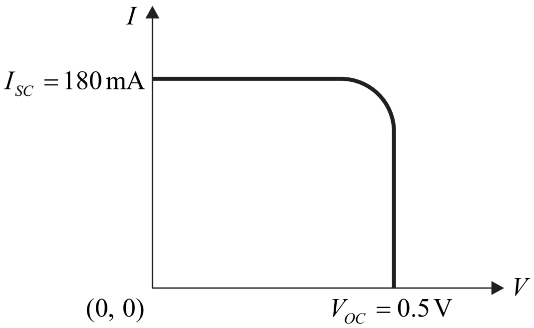
Region BC :

If forward bias is increased beyond the negative resistance region, the current begins to increase again. Once the bands have passed each other the characteristics reassembles that of a conventional diode. The forward current is now dominated by the diffusion current.

Hence, the correct option is (C).

6.3 21

Given :



Fill factor = 0.7, Area (A) = 3 cm^2,

Solar light power = 100 mW/cm^2

Fill factor is given by,

$$\textbf{Fill factor} = \frac{\text{Maximum obtained power}}{V_{OC} I_{SC}}$$

$$0.7 = \frac{\text{Maximum obtained power}}{0.5 \times 180 \times 10^{-3}}$$

Maximum obtained power,

$$P_m = 0.063 \text{ W}$$

Efficiency is given by,

$$\eta = \frac{P_m \times 100}{G \times A}$$

where, G is input solar light power in W/cm^2, and A is area of solar cell.

$$\%\eta = \frac{0.063 \times 100}{100 \times 10^{-3} \times 3}\% = 21\%$$

Hence, the maximum efficiency of the device is **21%.**

6.4 0.231

Given : P_{in} (input optical power) = 20 mW

Out of total incident power $\frac{1}{4}$th of power is reflected by silicon dioxide.

So, the power striking the Germanium surface is

$$P_{Ge} = P_{in}' = \frac{3}{4} P_{in} = 15 \text{ mW}$$

Out of total incident power on Germanium surface $\frac{1}{3}$rd is reflected, so power entering germanium substrate is

$$P(x) = \frac{2}{3} P_{in}'$$

Given that, $\frac{1}{3}$rd of power entering the germanium is absorbed and

Absorption coefficient,

$$(\alpha) = 3\times 10^4 \text{ cm}^{-1} = 3\times 10^6 \,(\text{m}^{-1})$$

Now, incident optical power and absorbed optical power are related as

$$P_{in} e^{-\alpha l} = P_{absorb}$$

where, P_{in} = Power entering the substrate

α = Absorption coefficient

l = Thickness of substrate

P_{abs} = Power absorbed in substrate

$$\frac{2}{3} P_{in}'\, e^{-\alpha l} = \frac{1}{3} P_{in}'$$

$$e^{-\alpha l} = \frac{1}{2}$$

$$-\alpha\, l = -0.693$$

$$l = \frac{0.693}{\alpha} = 0.231\,\mu\text{m}$$

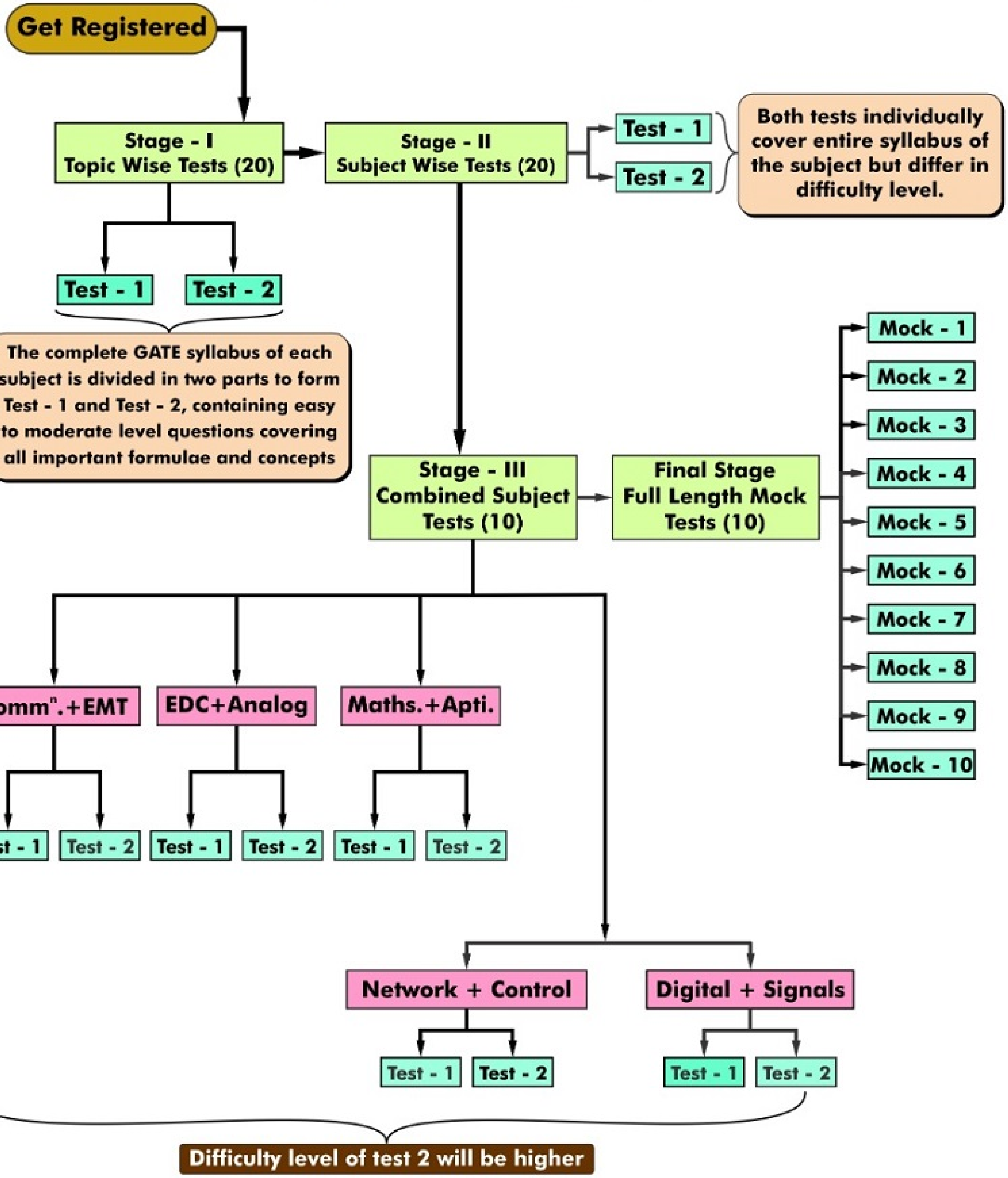
Hence, the thickness of germanium layer is $0.231\,\mu\text{m}$.

❖❖❖❖

GATE ACADEMY

TEST SERIES STRUCTURE (EC)

(One Year Program)



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Stage - I
Topic Wise Tests (20)

Test - 1
Test - 2

The complete GATE syllabus of each subject is divided in two parts to form Test - 1 and Test - 2, containing easy to moderate level questions covering all important formulae and concepts

Stage - II
Subject Wise Tests (20)

Test - 1
Test - 2

Both tests individually cover entire syllabus of the subject but differ in difficulty level.

Stage - III
Combined Subject Tests (10)

Comm^{n}.+EMT
Test - 1
Test - 2

EDC+Analog
Test - 1
Test - 2

Maths.+Apti.
Test - 1
Test - 2

Network + Control
Test - 1
Test - 2

Digital + Signals
Test - 1
Test - 2

Difficulty level of test 2 will be higher

Final Stage
Full Length Mock Tests (10)

Mock - 1
Mock - 2
Mock - 3
Mock - 4
Mock - 5
Mock - 6
Mock - 7
Mock - 8
Mock - 9
Mock - 10

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Rahul Chauhan

EE-53

Pritam Saurav

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Arihant Jain

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B Anurag

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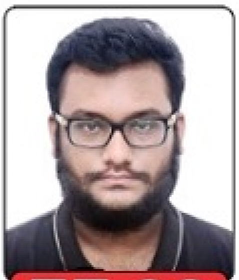
Tushar Chaudhary



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Sanket Tekawade

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Subham Dutta

EE-137

Mona Kumari

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Aman Kumar

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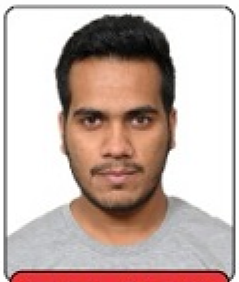
Wasim Ahmed



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Anant Kumar

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Anagha Paul

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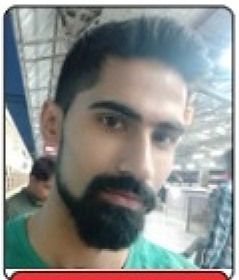

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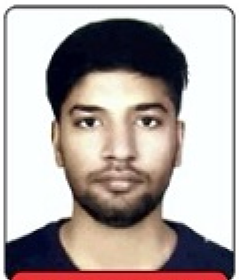

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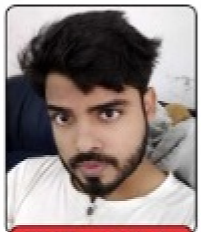
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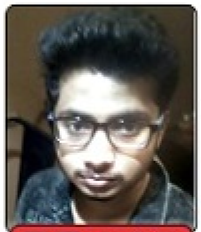
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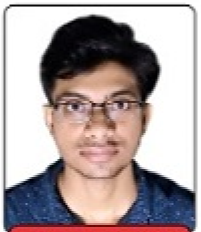
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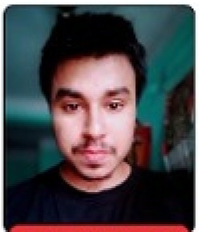
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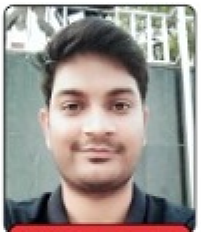
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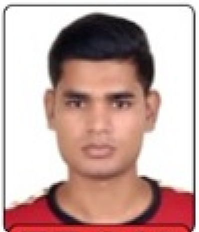
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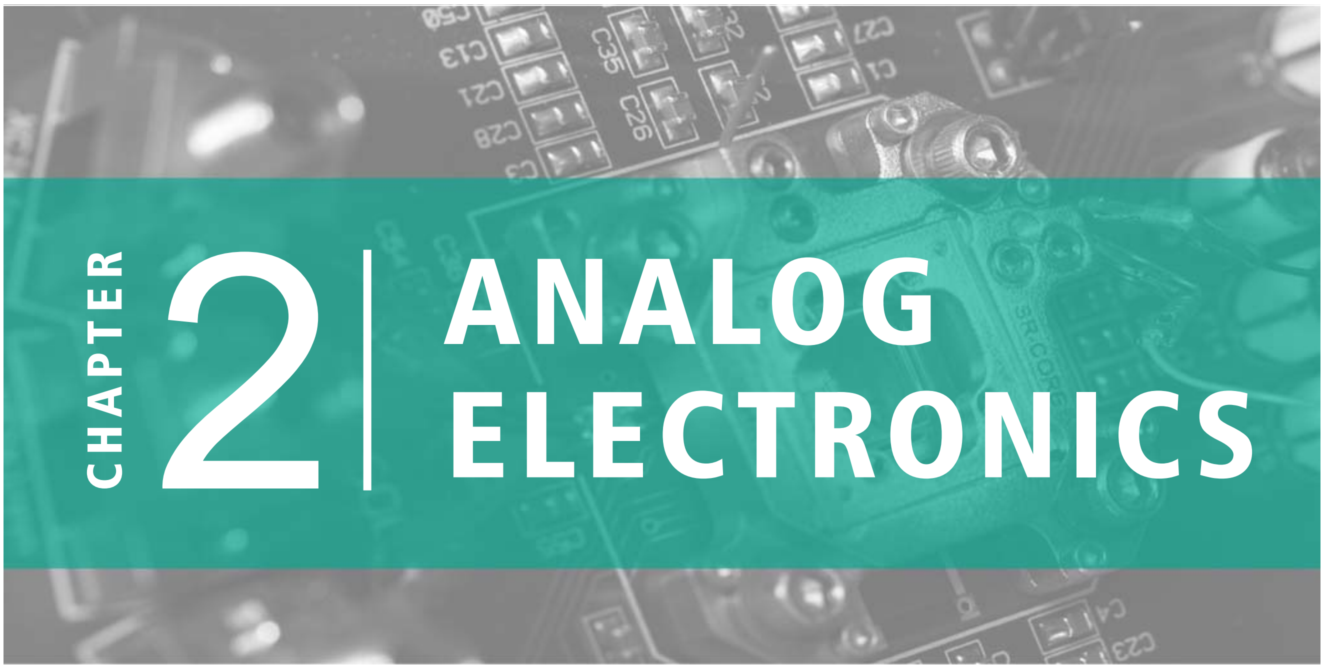


CHAPTER 2

ANALOG ELECTRONICS

Marks Distribution of Analog Electronics in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	5	8	21
2004	4	6	16
2005	3	11	25
2006	2	7	16
2007	2	8	19
2008	1	8	17
2009	–	6	12
2010	3	3	9
2011	2	4	10
2012	2	3	8
2013	3	6	15
2014 Set-1	4	3	10
2014 Set-2	4	3	10
2014 Set-3	3	3	9

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-4	3	3	9
2015 Set-1	2	3	8
2015 Set-2	3	4	11
2015 Set-3	3	3	9
2016 Set-1	4	2	8
2016 Set-2	4	3	10
2016 Set-3	3	4	11
2017 Set-1	3	4	11
2017 Set-2	4	3	10
2018	3	4	11
2019	1	5	11
2020	3	5	13
2021	3	2	7

Syllabus : Analog Electronics

Diode circuits: clipping, clamping and rectifiers. BJT and MOSFET amplifiers: biasing, ac coupling, small signal analysis, frequency response. Current mirrors and differential amplifiers. Op-amp circuits: Amplifiers, summers, differentiators, integrators, active filters, Schmitt triggers and oscillators.

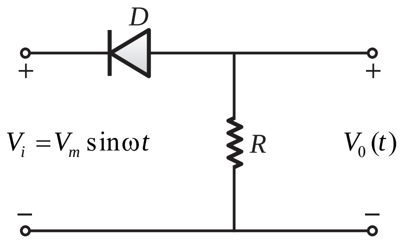
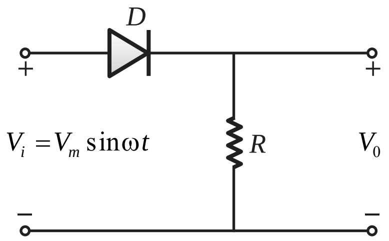
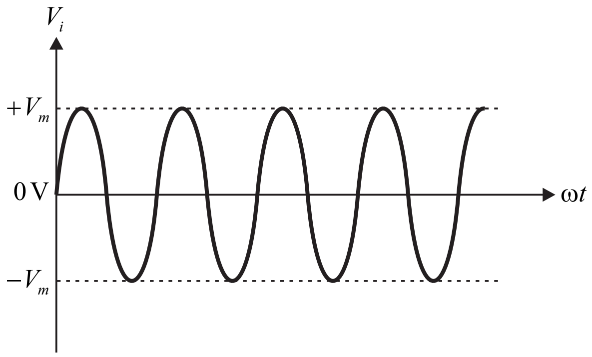
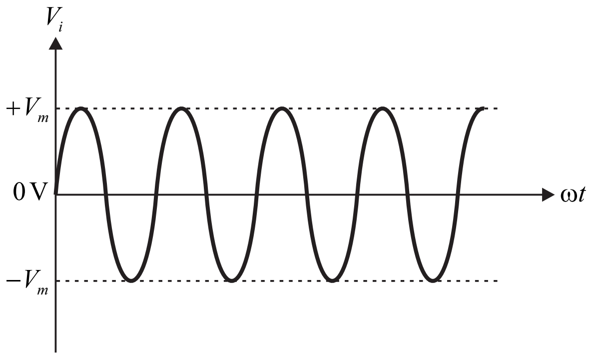
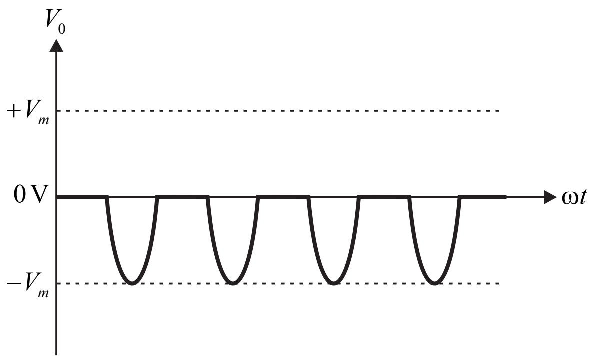
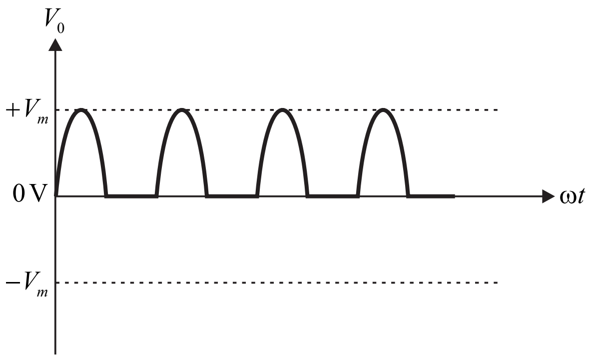
Contents : Analog Electronics

1

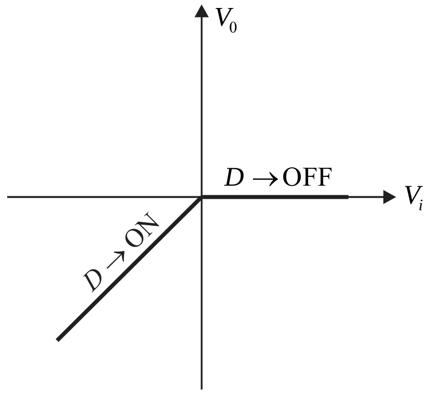
Diode Circuits & Applications

➢ Partial Synopsis

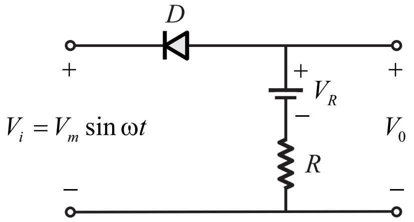
Series Clipper Circuits

Positive series clipper circuit	Negative series clipper circuit
Circuit Diagram : 	**Circuit Diagram :** 
Input waveform : 	**Input waveform :** 
Output waveform : 	**Output waveform :** 

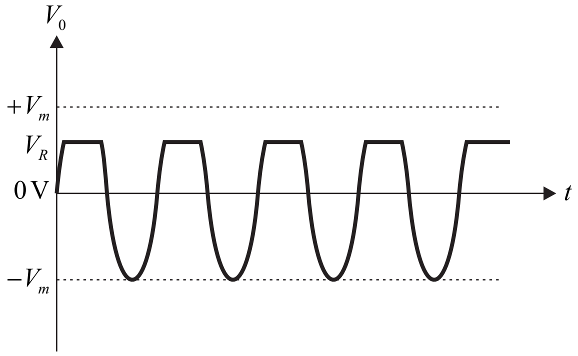
Transfer characteristics :



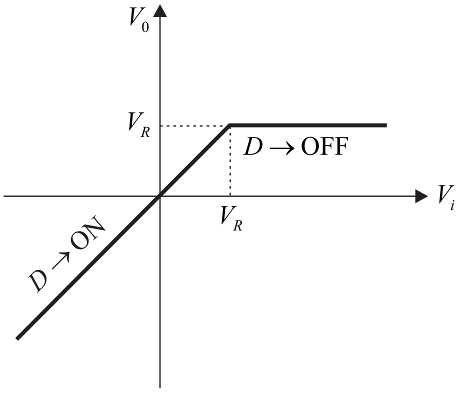
Positive series clipper circuit (with reference voltage, (V_R)) :



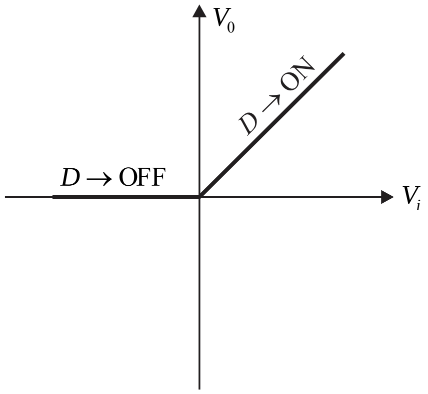
Output waveform :



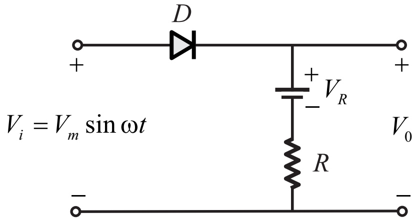
Transfer characteristics :



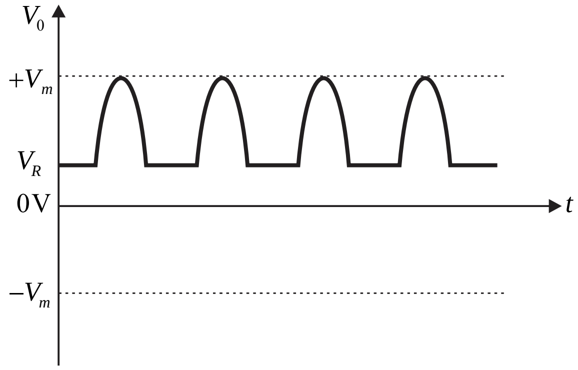
Transfer characteristics :



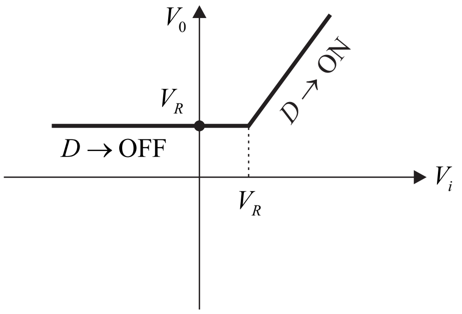
Negative Series Clipper Circuit (with reference voltage) :



Output waveform :



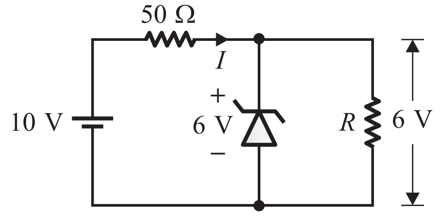
Transfer characteristics :



➢ Sample Questions

1992 IIT Delhi

1.1 The 6 V Zener diode shown below has zero Zener resistance and a knee current of 5 mA. The minimum value of R so that the voltage across R does not fall below 6 V is

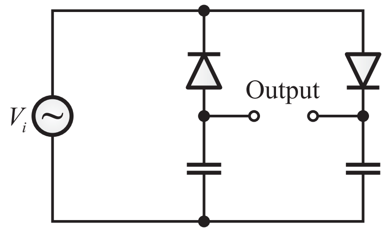


(A) 1.2 kΩ (B) 50 Ω

(C) 80 Ω (D) 0 Ω

2003 IIT Madras

1.2 The circuit shown in the figure is best described as a



(A) bridge rectifier

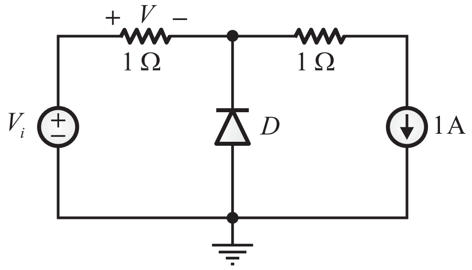
(B) ring modulator

(C) frequency discriminatory

(D) voltage doubler

2009 IIT Roorkee

1.3 In the circuit below, the diode is ideal. The voltage V is given by



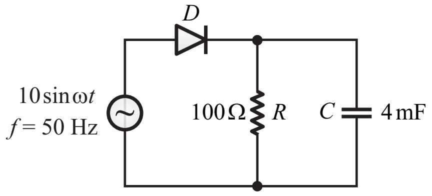
(A) $\min(V_i, 1)$ (B) $\max(V_i, 1)$

(C) $\min(-V_i, 1)$ (D) $\max(-V_i, 1)$

2014 IIT Kharagpur

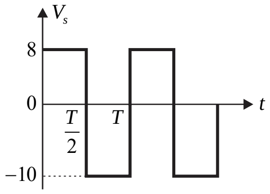
1.4 The figure shows a half-wave rectifier. The diode D is ideal. The average steady state current (in Amperes) through the diode is approximately ________.

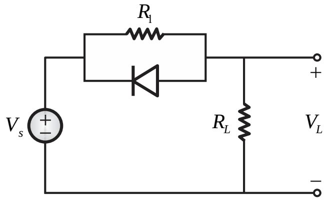
[Set - 03]



2019 IIT Madras

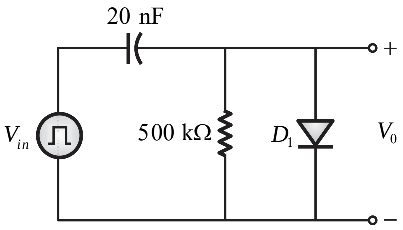
1.5 In the circuit shown, V_s is a square wave of period T with maximum and minimum values of 8 V and – 10 V, respectively. Assume that the diode is ideal and $R_1 = R_2 = 50\ \Omega$. The average value of V_L is_______ volts (rounded off to 1 decimal place).





2021 IIT Bombay

1.6 An asymmetrical periodic pulse train V_{in} of 10 V amplitude with on-time $T_{ON} = 1$ ms and off-time $T_{OFF} = 1\ \mu s$ is applied to the circuit shown in figure. The diode D_1 is ideal.



The difference between the maximum voltage and minimum voltage of the output wave form V_0 (in integer) is _____V.

Explanations | Diode Circuits & Applications

Concept of Clamper & Peak Detector Circuits

Scan for Video Explanation

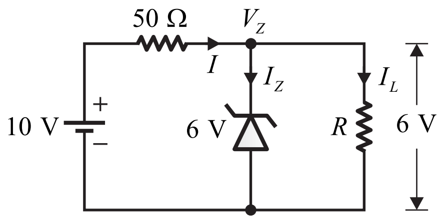
Multiplier Circuit by using Clamper & Peak Detector Circuits

Scan for Video Explanation

1.1 (C)

Given : $V_Z = 6\text{ V}$, $I_{knee} = I_{Z(\text{min})} = 5\text{ mA}$

The regulator circuit is shown below,



Concept of load regulation :

(i) Variable load resistor.

(ii) Fixed input voltage (or fixed input current).

For variable load R, input current I is given by,

$$I = I_{Z(\text{min})} + I_{L(\text{max})} \quad \ldots (i)$$

$$I = I_{Z(\text{max})} + I_{L(\text{min})} \quad \ldots (ii)$$

Current through $50\ \Omega$ resistor is given by,

$$I = \frac{V_{in} - V_z}{R} = \frac{10-6}{50} = 80\text{ mA}$$

From equation (i),

$$80 = 5 + I_{L(\text{max})} \quad \left[\text{Given} : I_{Z(\text{min})} = 5\text{ mA}\right]$$

$$I_{L(\text{max})} = 75\text{ mA}$$

For minimum value of R, current I_L has to be maximum.

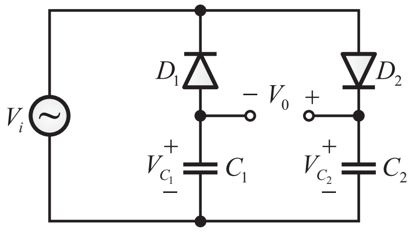
$$R_{\text{min}} = \frac{V_Z}{I_{L(\text{max})}} = \frac{V_R}{I_{L(\text{max})}}$$

$$R_{\text{min}} = \frac{6}{75\times10^{-3}} = 80\ \Omega$$

Hence, the correct option is (C).

1.2 (D)

Given circuit is shown below,



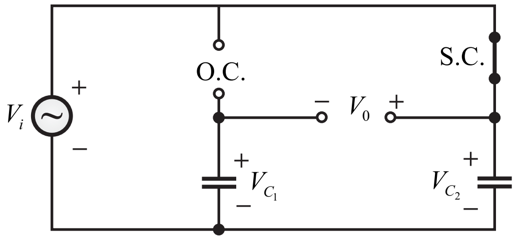
Output voltage V_0 is given by,

$$V_0 = V_{C_2} - V_{C_1}$$

Case 1 : During 1st positive half cycle,

Diode D_1 is in reverse bias, hence OFF.

Diode D_2 is in forward bias, hence ON.



Capacitor C_2 will start charging from zero volt to maximum of input voltage i.e. V_m.

In this case, charging time constant,

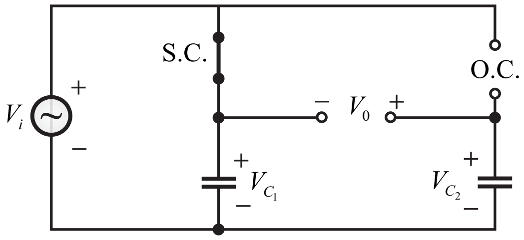
$$\tau_C = 0\text{ sec}$$

Therefore, $V_{C_2} = V_m$

Case 2 : During 1st negative half cycle,

Diode D_1 is in forward bias, hence ON.

Diode D_2 is in reverse bias, hence OFF.



Capacitor C_1 starts charging in negative direction from zero volt to minimum of input voltage i.e. $-V_m$.

In this case, charging time constant,

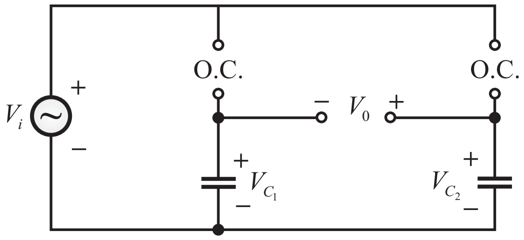
$$\tau_C = 0 \text{ sec}$$

Therefore, $V_{C_1} = -V_m$

Case 3 : During 2nd positive half cycle,

The positive terminal of diode D_1 is at potential $-V_m$. Diode D_1 will be forward biased when the potential of negative terminal of D_1 will be lower or more negative than $-V_m$. Since, the potential of negative terminal of D_1 can not be less than $-V_m$, hence D_1 will be always reverse biased.

The negative terminal of diode D_2 is at potential V_m. Diode D_2 will be forward biased when the potential of positive terminal of D_2 will be higher than V_m. Since, the potential of positive terminal of D_2 can not be higher than V_m, hence D_2 will be always reverse biased.



At steady state, diodes will be OFF and capacitors will not discharge.

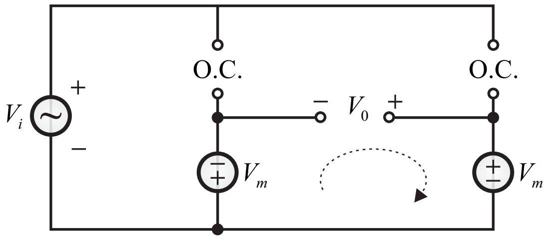
From figure, $V_0 = V_{C_2} - V_{C_1}$

$$V_0 = V_m - (-V_m)$$

$$V_0 = 2V_m$$

OR

At steady state :



Applying KVL in loop shown in figure,

$$+V_m - V_0 + V_m = 0$$

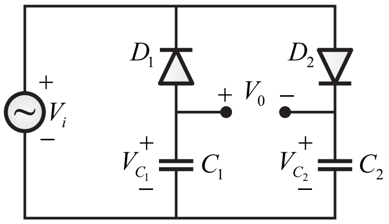
$$V_0 = 2V_m$$

Thus, the circuit behaves like a voltage doubler circuit.

Hence, the correct option is (D).

Key Point

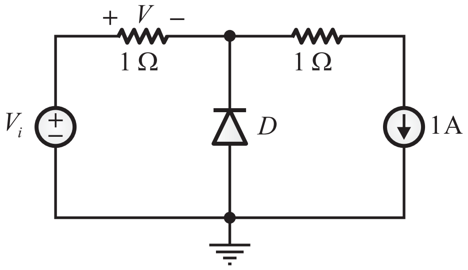
In the above question, polarity of V_0 is not given. So if we assume polarity of V_0 as shown in figure below,



Then, $V_0 = -2V_m$ (Voltage doubler circuit)

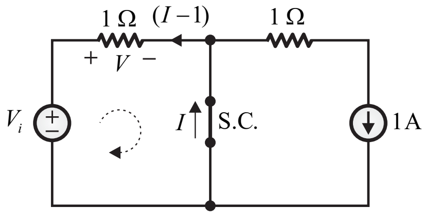
1.3 (A)

Given circuit is shown below,



Case 1 : Assume diode is ON,

Hence, diode can be replaced by short circuit.



Applying KVL in the loop shown,

$$-V_i + V + 0 = 0$$

$$V = V_i \qquad \dots \text{(i)}$$

where, $V = -(I-1) \times 1 = 1 - I$

$$I = 1 - V$$

From equation (i),

$$I = 1 - V_i \qquad \ldots \text{(ii)}$$

For diode to be ON, current I must be positive i.e. $I > 0$.

$$1 - V_i > 0$$

$$V_i < 1\text{V}$$

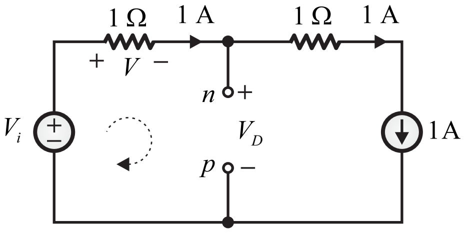
Hence, for diode to be ON, the voltage V_i must be less than 1 V.

If $V_i = 0.5\text{V}$, then $V = 0.5\text{V}$

Option	V	Status
(A) $\min(V_i, 1)$	0.5 V	Correct
(B) $\max(V_i, 1)$	1 V	Wrong
(C) $\min(-V_i, 1)$	–0.5 V	Wrong
(D) $\max(-V_i, 1)$	1 V	Wrong

Case 2 : Assume diode is OFF,

Hence, diode can be replaced by open circuit.



From figure, $V = 1 \times 1 = 1\text{V}$

Applying KVL in the loop shown,

$$-V_i + V + V_D = 0$$

$$V_D = V_i - V$$

$$V_D = V_i - 1$$

For diode to be OFF, V_D must be positive i.e. $V_D > 0\text{V}$.

$$V_i - 1 > 0$$

$$V_i > 1\text{V}$$

Hence, for diode to be OFF, V_i must be greater than 1 V and V must be equal to 1 V.

Let, $V_i = 1.5\text{V}$ and $V = 1$ V

Option	V	Status
(A) $\min(V_i, 1)$	1 V	Correct
(B) $\max(V_i, 1)$	1.5 V	Wrong
(C) $\min(-V_i, 1)$	–1.5 V	Wrong
(D) $\max(-V_i, 1)$	1 V	Correct

Hence, from the Case 1 and Case 2 it is clear that option (A) is common for both cases.

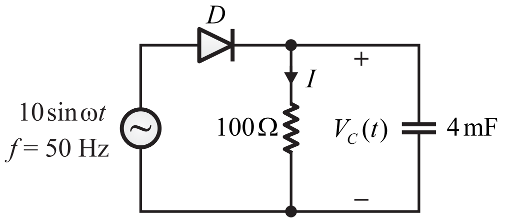
Hence, the correct option is (A).

Scan for
Video Solution

1.4 **0.1**

Given :

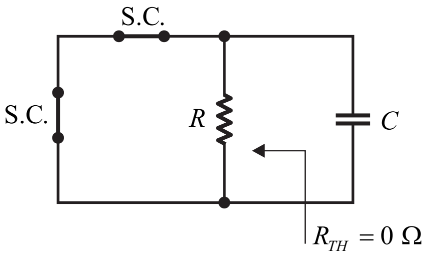
(i)



(ii) This circuit is used in AM receiver to detect the envelope of the amplitude modulated carrier waveform. The name of the circuit is peak detector (Envelope detector).

(iii) In envelope detector circuit, charging time should be small and discharging time should be large.

When diode is ON charging time constant is given by,

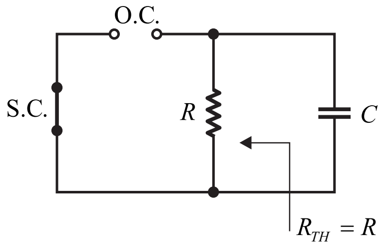


Charging time constant, $\tau_C = R_{TH}C$

$$\tau_C = 0 \text{ sec}$$

Therefore, capacitor will charge immediately.

When diode is OFF, discharging time constant is given by,



$$\tau_D = R_{TH}C = RC$$

$$\tau_D = 100 \times 4 \times 10^{-3} \text{ sec}$$

$$\tau_D = 400 \text{ msec}$$

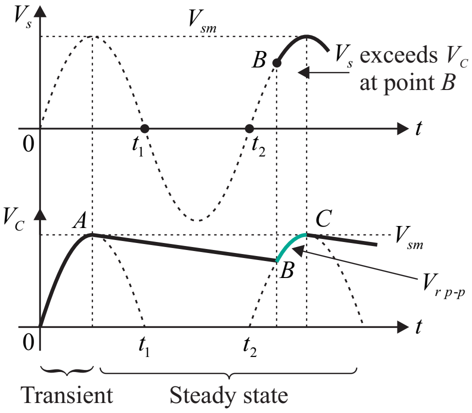
Time period of input signal is given by,

$$T = \frac{1}{f} = \frac{1}{50} = 20 \text{ msec}$$

[Given : $f = 50$ Hz]

Since, $T << \tau_D$

[Condition for envelope detector]



At steady state, capacitor voltage varies between B and C,

$$V_{dc} = V_m - \frac{V_{r\,p-p}}{2}$$

Where, $V_{r\,p-p} = \dfrac{I_{dc}}{2fC}$

$$I_{dc}R_L = V_m - \frac{I_{dc}}{4fC}$$

$$I_{dc}\left[R_L + \frac{1}{4fC}\right] = V_m$$

$$I_{dc} = \frac{V_m}{R_L + \dfrac{1}{4fC}} = \frac{10}{100 + \dfrac{1}{4 \times 50 \times 4 \times 10^{-3}}}$$

$$I_{dc} = 0.09 \approx 0.1 \text{ A}$$

Hence, the average steady state current is **0.1 A.**

1.5 **- 3**

Given circuit and input waveform are shown below,

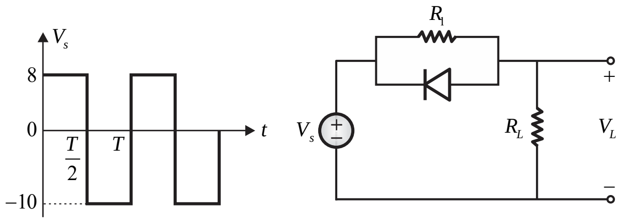


Fig. (a)

Where, $R_1 = R_L = 50\ \Omega$

For $0 < t < T/2$, $V_s = 8\text{V}$

So, diode will be off and can be replaced with open circuit

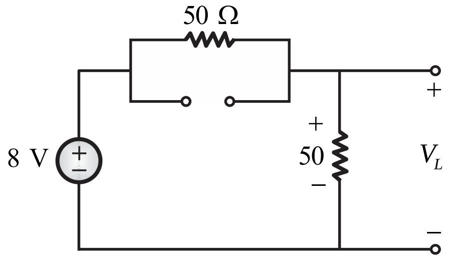


Fig. (b)

$$\therefore \qquad V_L = \left[\frac{50}{50+50}\right] \times 8 = 4 \text{ V}$$

For $T/2 < t < T$, $V_s = -10\text{V}$. So, diode will be ON and hence it will be short circuited and R_1 can be neglected as it comes across short circuit.

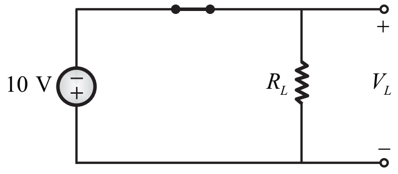


Fig. (c)

$$V_L = -10 \text{ V}$$

Therefore, equivalent output waveform will be

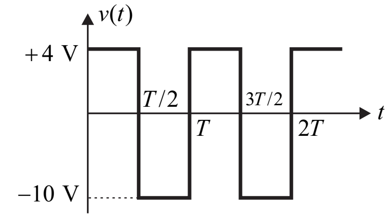


Fig. (d)

Average value of $V_L(t)$ is given as

$$V_{av} = \frac{1}{T}\int_0^T v_L(t)\,st$$

$$V_{L_{av}} = \frac{1}{T}\left[\int_0^{T/2} 4\cdot dt + \int_{T/2}^{T} (-10)\,dt\right]$$

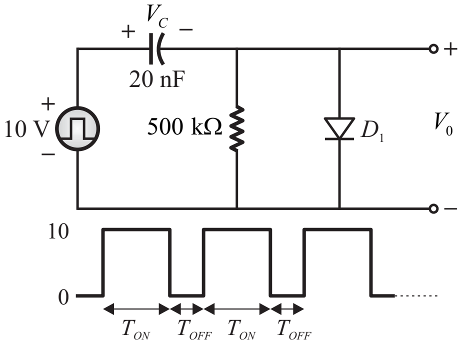
$$V_{L_{av}} = \frac{1}{T}\left[4\times\frac{T}{2} - 10\times\frac{T}{2}\right]$$

$$V_{L_{av}} = \frac{1}{T}\times(-6)\times\frac{T}{2} = -3\text{ V}$$

Hence, the average value of V_L is **– 3 V**

1.6 10

Given circuit and asymmetrical periodic input V_{in} is shown below,



(i) $T_{ON} = 1$ msec

(ii) $T_{OFF} = 1$ μsec

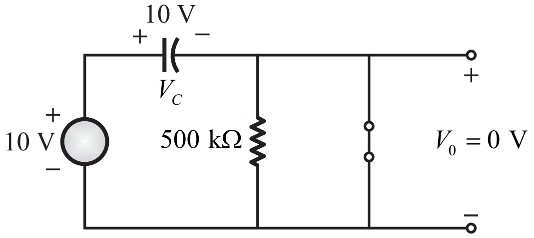
(iii) Initially capacitor C is uncharged

(iv) Diode D_1 is ideal diode.

Method 1

(i) For T_{ON} :

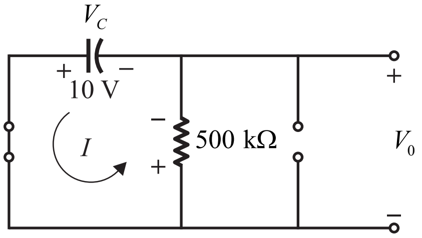
Diode D_1 is ON and will be short circuit, so capacitor will get charged instantly with $V_C = 10$ V as shown below, because $RC >> T_{ON}$.



Thus, voltage $V_0 = 0$ volt.

(ii) For T_{OFF} :

Given input voltage $V_{in} = 0$, diode is OFF and will be open circuit, so circuit becomes as,

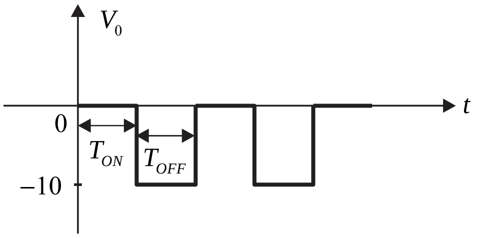


Current, $I = \frac{10}{500} = \frac{1}{50}$ mA

Voltage, $V_0 = -I\times 500$

$$= -\frac{1}{50}\times 500 = -10\text{ V}$$

Thus output waveform under steady state is,



Here, V_0 (maximum) = 0 V, V_0 (minimum) = – 10 V

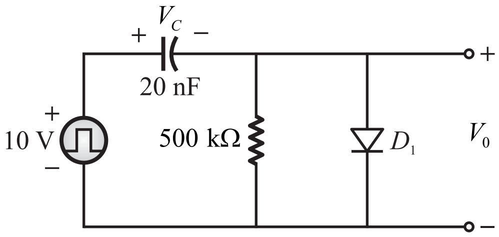
Thus, difference between maximum and minimum output voltages,

$$V_0(\text{maximum}) - V_0(\text{minimum})$$
$$= 0-(-10) = +10\text{ V}$$

Hence, the correct answer is 10 V.

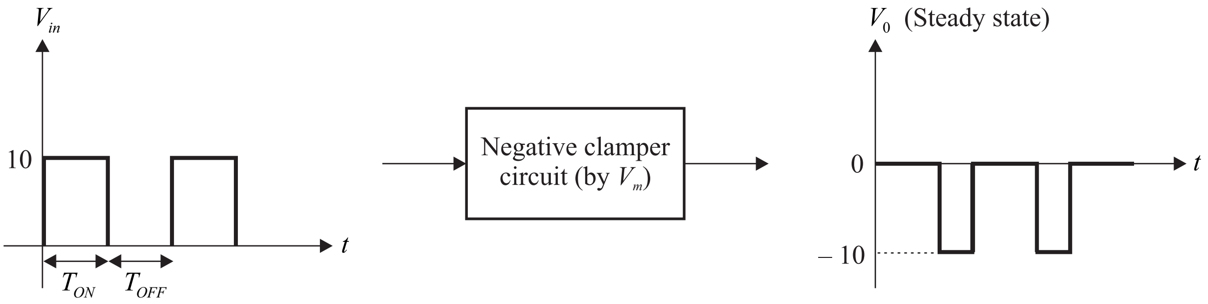
Method 2

The given circuit is shown below



V_{in} is asymmetrical pulse with peak voltage $V_m = 10\,\text{V}$, $T_{ON} = 1\,\text{msec}$ and $T_{OFF} = 1\,\mu\text{sec}$.

As, $RC >> T_{ON}$ and $RC >> T_{OFF}$, the circuit will behave as negative clapper circuit.



So, difference between maximum voltage and minimum voltage of output V_0 is,

$$V_{0(\max)} - V_{0(\min)} = 0 - (-10) = +10\,\text{Volt}$$

Hence, the correct answer is 10.

❖❖❖❖

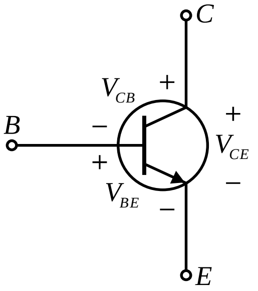
2

BJT Biasing & Region of Operation

➢ Partial Synopsis

To Identify the Region of Operation

Method 1 :



Assume Transistor is in active region ($I_C = \beta I_B$) :

1. Apply KVL to Base-Emitter (BE) circuit and calculate I_B.
2. Replace $I_C = \beta_{dc} I_B$ if required.
3. If emitter resistor R_E exist then replace emitter current $I_E = I_B + I_C$

$$I_E = (1+\beta_{dc})I_B$$

4. Apply KVL in Collector-Emitter (CE) circuit and calculate V_{CE} .
5. Calculate V_{CB} ,

$$V_{CB} = V_{CE} + V_{EB}$$

$$V_{CB} = V_{CE} - V_{BE}$$

Where $V_{BE} = 0.7\text{ V}$

6. If $V_{CB} = +\text{ve}$ for npn transistor and –ve for pnp transistor, then transistor will be in active region otherwise saturation region.

$$\left.\begin{array}{ll} V_{CB} = +\text{ve} & \text{(npn)} \\ V_{CB} = -\text{ve} & \text{(pnp)} \end{array}\right\} \quad \text{Condition for active region}$$

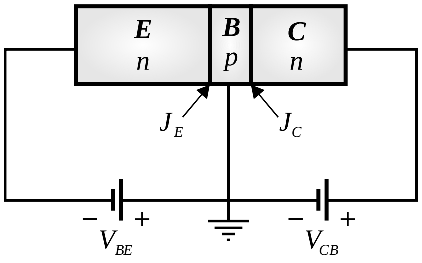


Fig.(a) npn transistor (Active region)

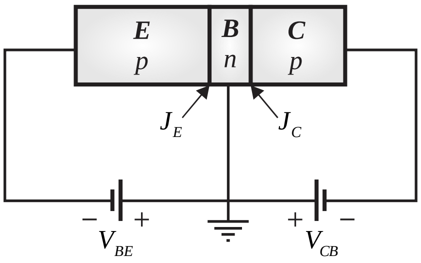


Fig.(b) pnp transistor (Active region)

For npn transistor :

$V_{BE} > 0$, i.e. Emitter base junction J_E = FB,

V_{CB}	J_C	Region
+ ve	RB	Active
− ve	FB	Saturation

Collector base junction J_C = RB,

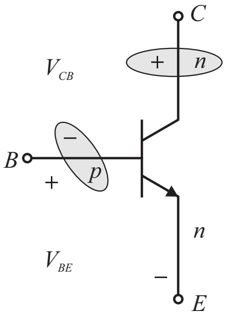


Fig. V_{CB} = + ve (Active)

Collector base junction J_C = FB,

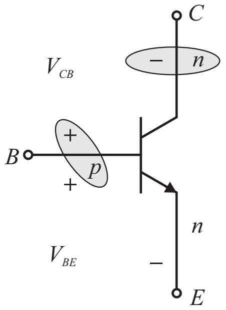


Fig. V_{CB} = – ve (Saturation)

For pnp transistor :

If $V_{BE} < 0$, i.e. J_E = FB,

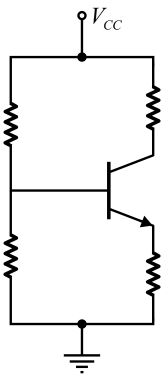
V_{CB}	J_C	Region
− ve	RB	Active
+ ve	FB	Saturation

Sample Questions

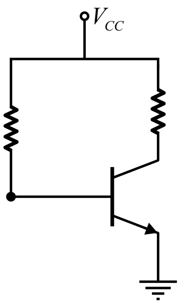
1989 **IIT Kanpur**

2.1 From the four biasing circuits shown in figure for a BJT, indicate the one which can have maximum bias stability

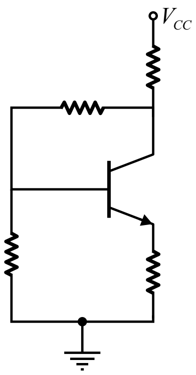
(A)



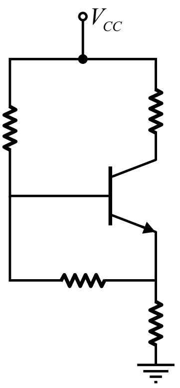
(B)



(C)

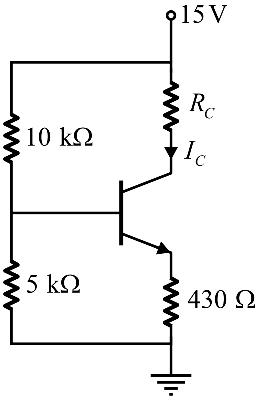


(D)



2000 **IIT Kharagpur**

2.2 In the circuit of the figure, assume that the transistor is in the active region. It has a large β and its base-emitter voltage is 0.7 V. The value of I_C is



(A) Indeterminate since R_C is not given

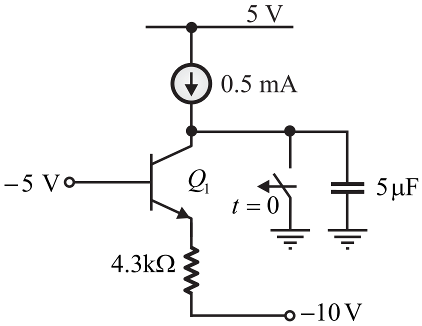
(B) 1 mA

(C) 5 mA

(D) 10 mA

2011 IIT Madras

2.3 For the BJT Q_1 in the circuit shown below, $\beta = \infty, V_{BE,\,ON} = 0.7\,\text{V}$, $V_{CE\,(\text{sat})} = 0.7\,\text{V}$. The switch is initially closed. At time $t = 0$, the switch is opened. The time t at which Q_1 leaves the active region is

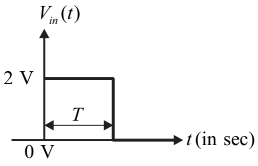


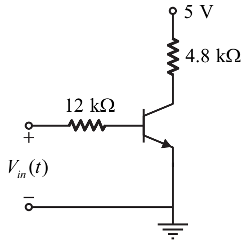
(A) 10 ms (B) 25 ms

(C) 50 ms (D) 100 ms

2017 IIT Roorkee

2.4 In the figure shown, the *n-p-n* transistor acts as a switch.





For the input $V_{in}(t)$ as shown in the figure, the transistor switches between the cutoff and saturation regions of operation, when T is large. Assume collector-to-emitter voltage at saturation $V_{CE\,(sat)} = 0.2\,\text{V}$ and base-to-emitter voltage $V_{BE} = 0.7\,\text{V}$. The minimum value of the common-base current gain (α) of the transistor for the switching should be __________. **[Set - 01]**

❖❖❖❖

Explanations BJT Biasing & Region of Operation

Concept of Current Mirror Circuit

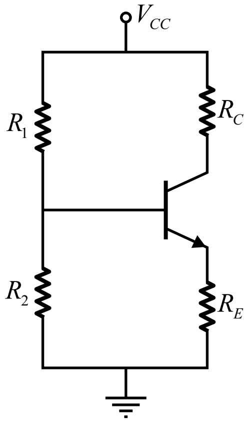
Scan for Video Explanation

2.1 (A)

Option (B) represents a fixed bias circuit without emitter resistance R_E and option (C) represents collector to base bias circuit with emitter resistance R_E.

In BJT, self bias circuit or voltage (potential) divider circuit provides the maximum bias stability because it provides stable I_C, irrespective of variation in temperature and β.

Self bias circuit is shown below,



Hence, the correct option is (A).

2.2 (D)

Given :

(i) Operating region of transistor is active region.

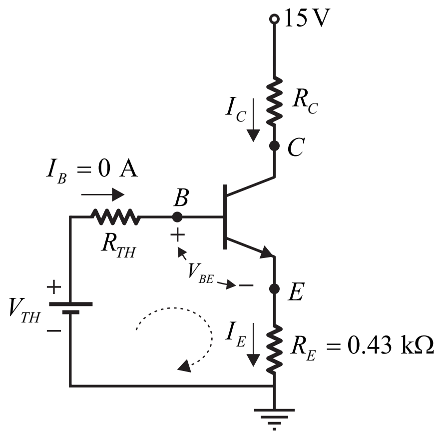
(ii) Base-emitter voltage, $V_{BE} = 0.7$ V

(iii) β is large. Assume $\beta = \infty$, so $I_B = 0$ A and $I_C = I_E$.

(iv) Given circuit is self bias circuit/ voltage divider circuit.

(v) $R_1 = 10\ \text{k}\Omega,\ R_2 = 5\ \text{k}\Omega,\ R_E = 0.43\ \text{k}\Omega$

Thevenin equivalent circuit of self bias circuit :



$$V_{TH} = \frac{R_2 \times 15}{R_1 + R_2} = \frac{5 \times 15}{10+5} = 5\ \text{V} \quad \text{[By VDR]}$$

$$R_{TH} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{10 \times 5}{10+5} = 3.34\ \text{k}\Omega$$

Applying KVL in the input loop,

$$-V_{TH} + (I_B \times R_{TH}) + V_{BE} + (I_E \times R_E) = 0$$

$$-V_{TH} + (0 \times R_{TH}) + V_{BE} + [I_C \times R_E] = 0$$

$$I_C = \frac{V_{TH} - V_{BE}}{R_E} = \frac{5-0.7}{0.430} = 10\ \text{mA}$$

Hence, the correct option is (D).

2.3 (C)

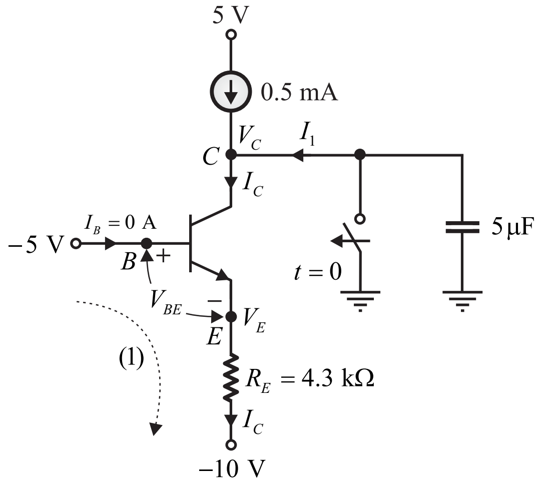
Given :

(i) β is very large, hence $I_B \approx 0$ and therefore $I_C \approx I_E$.

(ii) $V_{BE(\text{ON})} = 0.7$ V

(iii) $V_{CE(\text{sat})} = 0.7$ V

Given circuit is shown below,



From figure, −10 V is more negative as compared to −5 V. Therefore, input junction is in forward bias.

Note : If instead of −10 V, R_E was connected to ground, then emitter base junction will be in reverse bias.

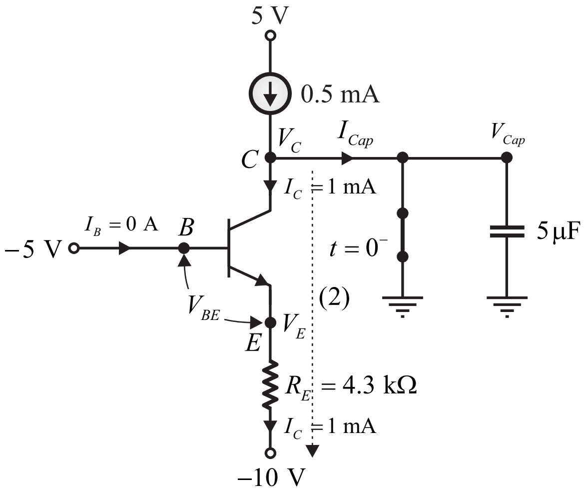
Applying KVL in input loop,

$$5 + 0.7 + 4.3 \times I_C - 10 = 0$$

$$4.3 I_C = 4.3$$

$$I_C = 1\ \text{mA}$$

At $t = 0^-$, switch is closed.



Capacitor voltage $V_C(0^-) = 0$ V

Collector voltage $V_C = 0$ V

Note : $I_C = 1$ mA is fixed because other elements are fixed, which are responsible for current I_C.

Applying KVL in loop (2),

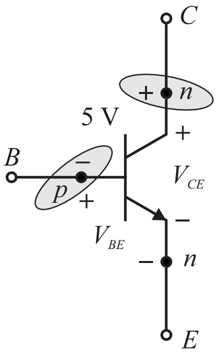
$$-V_C + V_{CE} + 4.3 \times 1 - 10 = 0$$

$$V_C = V_{CE} + 4.3 \times 1 - 10$$

$$0 = V_{CE} + 4.3 \times 1 - 10$$

$$V_{CE} = 5.7 \text{ V}$$

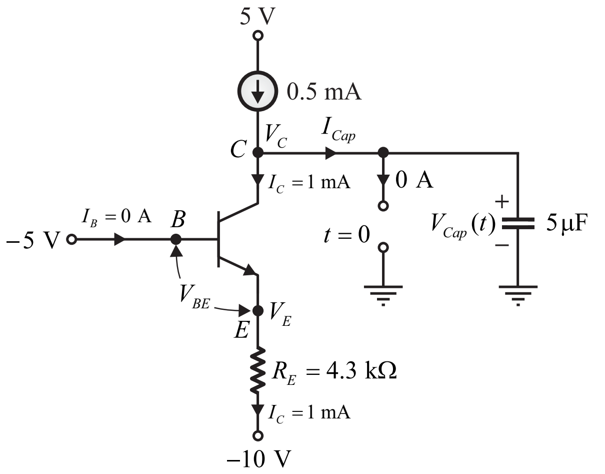
$$V_{CB} = V_{CE} - V_{BE} = 5.7 - 0.7 = 5 \text{ V}$$



From above figure, collector junction is in reverse bias.

At $t = 0^-$ given transistor is in active region.

At $t = 0$, switch is opened.



From above figure, $V_C(t) = V_{Cap}(t)$

Applying KCL at collector terminal,

$$-0.5 + I_{Cap} + 1 = 0$$

$$0.5 = I_{Cap} + 1$$

$$I_{Cap} = -0.5 \text{ mA} \qquad \text{[Constant value]}$$

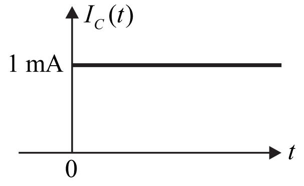
Here, capacitor will linearly charge in opposite direction or linearly discharge.

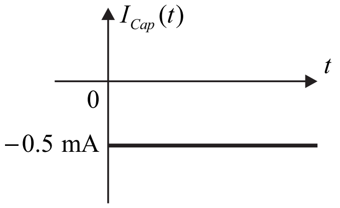
Capacitor voltage is given by,

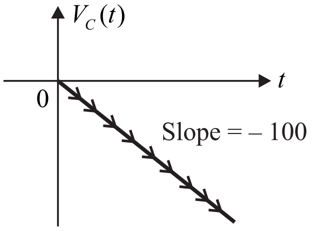
$$V_{Cap}(t) = \frac{1}{C}\int_0^t I_{Cap}(t)\,dt$$

$$V_{Cap}(t) = \frac{1}{5 \times 10^{-6}}\int_0^t -0.5 \times 10^{-3}\,dt$$

$$V_{Cap}(t) = -100t \qquad \text{... (i)}$$







When transistor enters into saturation region,

$$V_{CE(\text{sat})} = 0.7 \text{ V} \qquad \text{[Given]}$$

From figure,

$$V_{Cap}(t) = V_{CE} + I_C R_E - 10$$

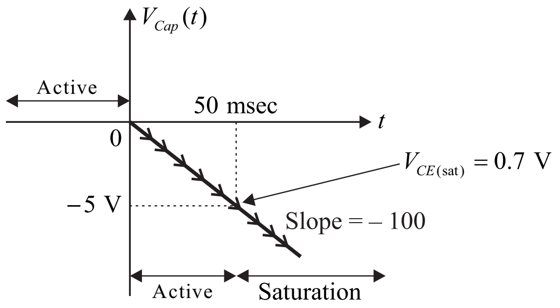
$$V_{Cap}(t) = 0.7 + 1 \times 4.3 - 10$$

$$V_{Cap}(t) = -5 \text{ V} \qquad \ldots \text{(ii)}$$

Equating equation (i) and (ii),

$$-5 = -100 \times t$$

$$t = \frac{5}{100} = 0.05 \text{ sec} = 50 \text{ msec}$$



Upto 50 msec transistor is in active region. After 50 msec transistor leaves the active region and goes to saturation region.

Hence, the correct option is (C).

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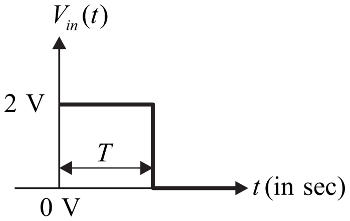
2.4 **0.90**

Given : $V_{CE(\text{sat})} = 0.2 \text{ V}, V_{BE} = 0.7 \text{ V}$

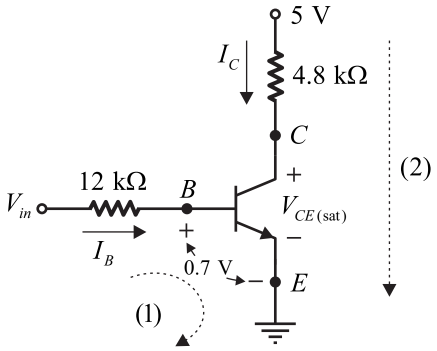


For 0 to T, transistor will in saturation region. Condition for saturation region is given by,

$$I_B \geq I_{B(\min)}$$

$$I_B \geq \frac{I_{C(\text{sat})}}{\beta_{\min}}$$

$$\beta_{\min} \geq \frac{I_{C(\text{sat})}}{I_B}$$



Applying KVL in loop (1),

$$-V_{in} + I_B \times 12 + 0.7 = 0$$

$$I_B = \frac{V_{in} - 0.7}{12} = \frac{2 - 0.7}{12}$$

$$I_B = \frac{1.3}{12} = 0.108 \text{ mA}$$

Applying KVL in loop (2),

$$-5 + 4.8 \times I_C + V_{CE(\text{sat})} = 0$$

$$I_C = \frac{5 - 0.2}{4.8} = 1 \text{ mA}$$

Current gain β is given by,

$$\beta_{\min} = \frac{I_{C(\text{sat})}}{I_B} = \frac{1}{0.108} = 9.25$$

Current gain α in terms of β is given by,

$$\alpha = \frac{\beta}{1+\beta} = \frac{9.25}{10.25} = 0.90$$

After $t > T$, $V_{in} = 0$ V

$V_{in} < 0.7\text{ V}$, input junction is in reverse bias, therefore transistor is in cutoff region.

t	**V_{in}**	**Region**
$t < T$	2 V	Saturation
$t > T$	0	Cutoff

Hence, the minimum value of the common-base current gain is **0.90.**

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3

Low Frequency BJT Amplifier

➢ Partial Synopsis

Common Emitter (CE) Amplifier with R_E

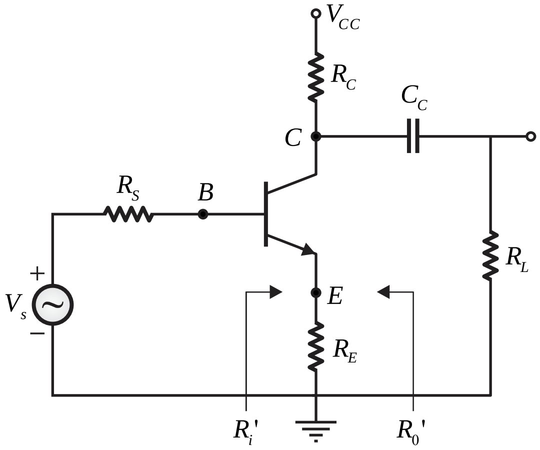


Fig. (a) Common emitter with R_E

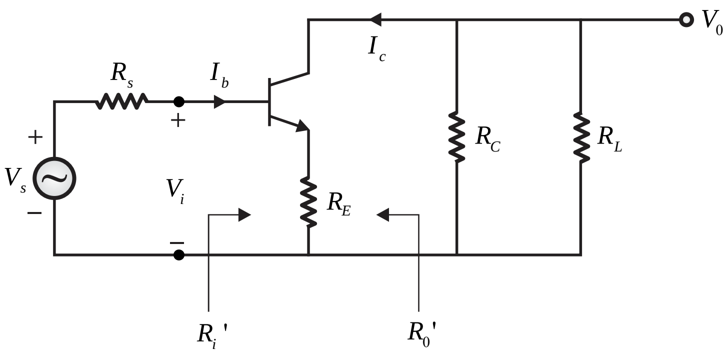


Fig. (b) AC equivalent of Common emitter with R_E

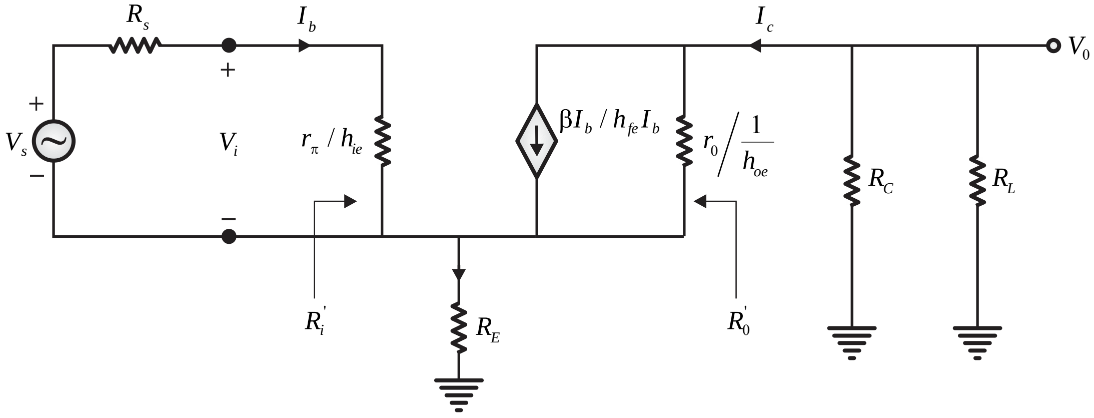


Fig. (c) π-model of common emitter with R_E

Summary of Internal Parameter of CE Amplifier with R_E

Name of Internal parameter	**Internal parameter of common emitter with R_E**	**Approximate hybrid model**	**r_e-Model**
Current gain	$A_I' = \frac{-I_c}{I_b}$	$A_I' = -h_{fe}$	$A_I' = -\beta$
Input resistance	$R_i' = \frac{V_i}{I_b}$	$R_i' = h_{ie} + (1+h_{fe})R_E$	$R_i' = r_\pi + (1+\beta)R_E$
Voltage gain	$A_V' = \frac{V_0}{V_i}$	$A_V' = A_I' \times \frac{R_L'}{R_i'}$ $A_V' \approx \frac{-R_L'}{R_E}$ $R_L' = R_C \parallel R_L$	$A_V' = A_I' \times \frac{R_L'}{R_i'}$ $A_V' = \frac{-\beta R_L'}{r_\pi + (1+\beta)R_E} \approx \frac{-R_L'}{R_E}$ $[(1+\beta)R_E >> r_\pi]$ $R_L' = R_C \parallel R_L$
Output resistance	$R_0' = \left.\frac{V_{dc}}{I_{dc}}\right\vert_{V_s=0}$	$R_0' = \infty$	$R_0' = \infty \quad [V_A = \infty]$

Common Collector (CC) Amplifier

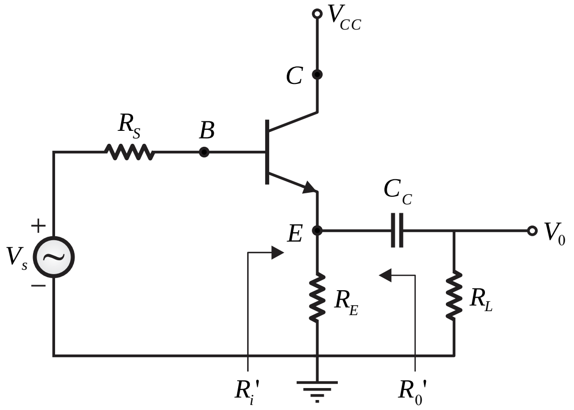


Fig. (a) Common collector amplifier

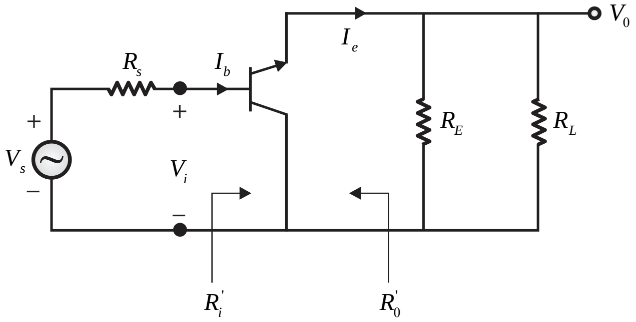


Fig. (b) AC equivalent of Common collector amplifier

Summary of Internal Parameter of CC Amplifier

Name of Internal parameter	**Internal parameter of CC Amplifier**	**Approximate hybrid model**	**r_e-Model**
Current gain	$A_I' = \frac{-I_e}{I_b}$	$A_I' = 1 + h_{fe}$	$A_I' = 1 + \beta$
Input resistance	$R_i' = \frac{V_i}{I_b}$	$R_i' = h_{ie} + (1+h_{fe})R_L'$	$R_i' = r_\pi + (1+\beta)R_L'$

Voltage gain	$A_V' = \frac{V_0}{V_i}$	$A_V' = \frac{A_I' R_L'}{R_i'} \approx 1$ $R_L' = R_E \parallel R_L$	$A_V' = \frac{(1+\beta) R_L'}{r_\pi + (1+\beta) R_L'} \approx 1.0$ $R_L' = R_E \parallel R_L$
Output resistance	$R_0' = \frac{V_{dc}}{I_{dc}}\Big\vert_{V_s=0}$	$R_0' = \frac{R_S' + h_{ie}}{1+h_{fe}}$ $R_S' =$ Effective source impedance	$R_0' = \frac{R_S' + r_\pi}{1+\beta}$ If $r_\pi > R_S'$ $R_0' = \frac{r_\pi}{1+\beta} \approx \frac{r_\pi}{\beta}$ $R_0' = \frac{r_\pi}{\beta} = \frac{1}{g_m}$

➢ Sample Questions

1987　IIT Bombay

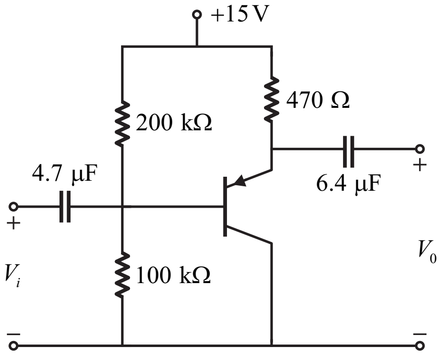
3.1 The configuration of cascode amplifier is

(A) CE-CE　　(B) CE-CB

(C) CC-CB　　(D) CC-CC

1993　IIT Bombay

3.2 For the Amplifier circuit of figure. The transistor has a β of 800. The mid band voltage gain V_0 / V_i of the circuit will be

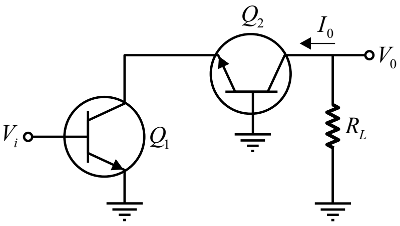


(A) 0　　(B) < 1

(C) ≈ 1　　(D) 800

1999　IIT Bombay

3.3 In the cascode amplifier shown in the figure, if the common-emitter stage (Q_1) has a transconductance g_{m_1}, and the common base stage (Q_2) has a transconductance g_{m_2}, then the overall transconductance $g_m (= I_0 / V_i)$ of the cascode amplifier is

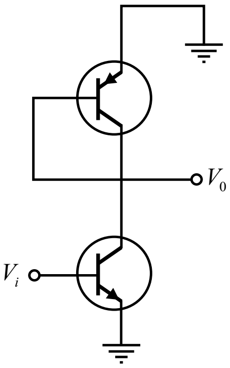


(A) g_{m_1}　　(B) g_{m_2}

(C) $\frac{g_{m_1}}{2}$　　(D) $\frac{g_{m_2}}{2}$

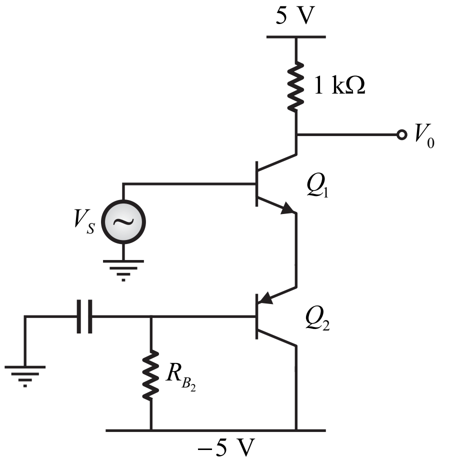
2015　IIT Kanpur

3.4 In the ac equivalent circuit shown, the two BJTs are biased in active region and have identical parameters with $\beta >> 1$. The open circuit small signal voltage gain is approximately ______. **[Set - 02]**



2017 **IIT Roorkee**

3.5 In the circuit shown, transistors Q_1 and Q_2 are biased at a collector current of 2.6 mA. Assuming that transistor current gains are sufficiently large to assume collector current equal to emitter current and thermal voltage of 26 mV, the magnitude of voltage gain $\left(\frac{V_0}{V_S}\right)$ in the midband frequency range is ________ (up to second decimal place). **[Set - 02]**



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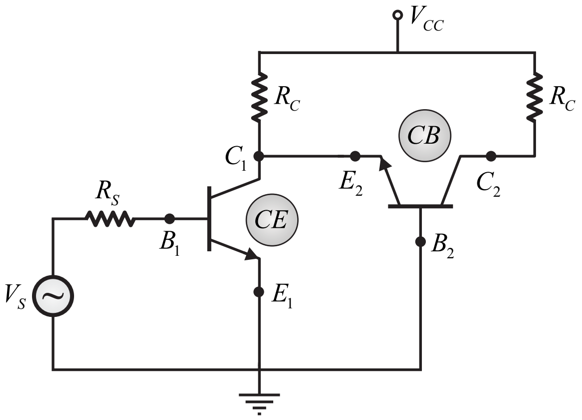
Explanations Low Frequency BJT Amplifier

3.1 **(B)**

Cascode amplifier :



Cascode amplifier is the cascade connection of common emitter and common base configuration where common emitter followed by common base configuration.

Hence, the correct option is (B).

Key Point

(i) Cascode amplifier has high output impedance and large voltage gain.

(ii) In a cascode configuration, the collector of the leading transistor is connected to the emitter of the following transistor.

(iii) Cascode amplifier is also known as direct coupled amplifier because output of CE configuration is directly connected to input of CB configuration without using any passive component.

3.2 **(C)**

Given : $\beta = 800$

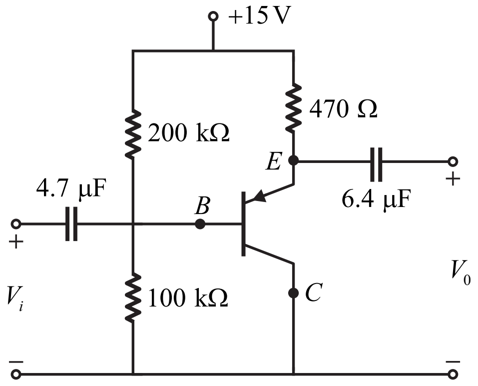
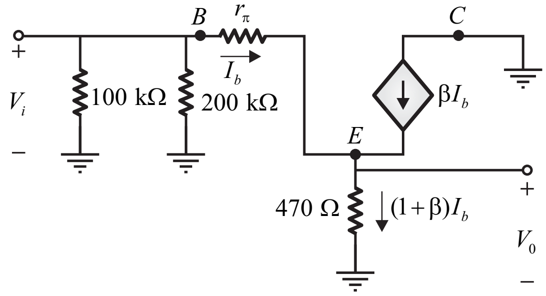


Fig. CC amplifier

Small signal equivalent circuit (r_e-model) is given by,



Output voltage V_0 is given by,

$$V_0 = 470(1+\beta)I_b \qquad \dots \text{(i)}$$

Input voltage is given by

$$V_i = r_\pi I_b + 470\, I_b(1+\beta) \qquad \dots \text{(ii)}$$

From equation (i) and equation (ii),

Voltage gain is given by,

$$A_V = \frac{V_0}{V_i} = \frac{470(1+\beta)I_b}{[r_\pi + (1+\beta)470]\,I_b}$$

Normally, the range of r_π lies between 1 kΩ to 5 kΩ.

Since, $0.47(1+\beta) = 801 \times 0.47 = 376.47$ kΩ

So, $0.47(1+\beta) >> r_\pi$

$$A_V \approx 1$$

Note : There is no need of DC analysis for calculation of r_π because the value of β is very high, so we can neglect r_π compared to $(1+\beta)R_E$ If we calculate r_π using DC analysis then $r_\pi \approx 0.4$ kΩ.

Hence, the correct option is (C).

Key Point

(i) The given circuit is an emitter follower or common collector amplifier. For common collector amplifier voltage gain is close to unity.

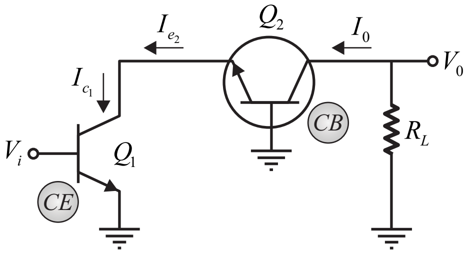
$$A_V \approx 1$$

(ii) We can directly use standard result without drawing r_e-model.

$$A_V = \frac{A_I R_L}{R_i} = \frac{(1+\beta)R_L}{r_\pi + (1+\beta)R_L}$$

3.3 (A)

Given circuit is shown below,



Transconductance of Q_1 is given by,

$$g_{m_1} = \frac{I_{c_1}}{V_i}$$

Overall transconductance is given by,

$$g_m = \frac{I_{c_2}}{V_i} = \frac{I_0}{V_i}$$

$$g_m = \frac{I_0}{I_{e_2}} \times \frac{I_{e_2}}{I_{c_1}} \times \frac{I_{c_1}}{V_i} \qquad \dots \text{(i)}$$

From figure, $I_{e_2} = I_{c_1}$ $\qquad \dots$ (ii)

Current gain in common base configuration (Q_2) is given by,

$$\alpha = \frac{I_0}{I_{e_2}} \qquad \text{[From figure]} \qquad \dots \text{(iii)}$$

From equation (i), (ii) and (iii),

$$g_m = \alpha \times 1 \times g_{m_1}$$

For common base configuration, current gain is almost equal to 1.

Hence, $g_m = g_{m_1}$

Hence, the correct option is (A).

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Key Point

$g_m\,[\text{cascode amplifier}] = \alpha[\text{CB}] \times g_m\,[\text{CE}]$

3.4 **–1**

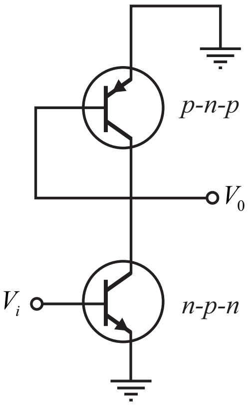
Given :

(i) Identical parameter

Therefore, $I_{b_1} = I_{b_2} = I_b$

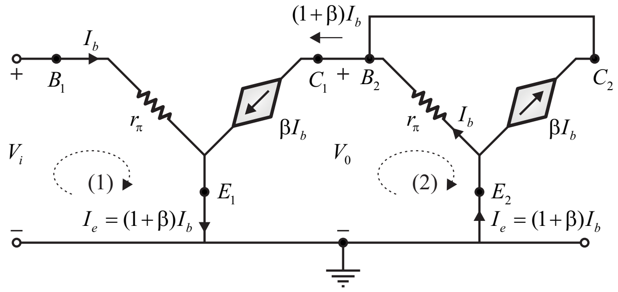
(ii) $\beta >> 1$, $I_e \approx I_c$, $(1+\beta)I_b = \beta I_b$

Circuit is shown below,



Method 1

The equivalent r_e-model of given circuit is shown below,



Applying KVL in loop (1),

$$V_i = r_\pi I_b$$

Applying KVL in loop (2),

$$V_0 = -r_\pi I_b$$

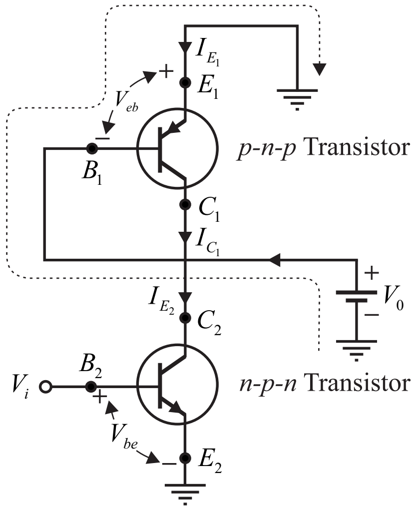
Voltage gain A_V is given by,

$$A_V = \frac{V_0}{V_i} = \frac{-r_\pi I_b}{r_\pi I_b} = -1$$

Hence, the open circuit small signal voltage gain is **–1.**

Method 2

Given circuit is shown below,



Applying KVL in the loop shown in figure,

$$-V_0 - V_{eb(p\text{-}n\text{-}p)} = 0$$

$$V_0 = -V_{eb(p\text{-}n\text{-}p)} \qquad \ldots \text{(i)}$$

From figure,

$$V_i = V_{be\,(n\text{-}p\text{-}n)} \qquad \ldots \text{(ii)}$$

From equation (i) and (ii), voltage gain is given by,

$$A_V = \frac{V_0}{V_i} = \frac{-V_{eb(p\text{-}n\text{-}p)}}{V_{be(n\text{-}p\text{-}n)}} \qquad \ldots \text{(iii)}$$

Since, both BJTs have identical parameters, hence

$$V_{eb(p\text{-}n\text{-}p)} = V_{be(n\text{-}p\text{-}n)} \qquad \ldots \text{(iv)}$$

where, $V_{eb(p\text{-}n\text{-}p)}$ = Internal AC input voltage of *p-n-p* transistor

$V_{be(n\text{-}p\text{-}n)}$ = Internal AC input voltage of *n-p-n* transistor

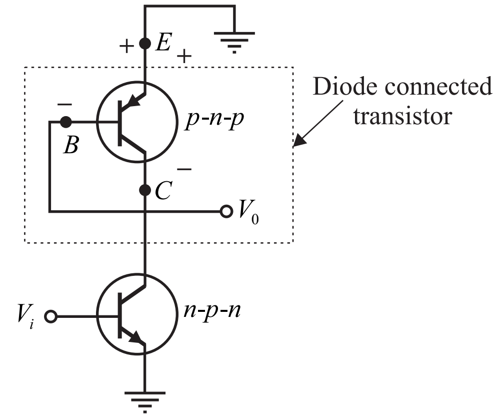
From equation (iii) and (iv),

$$A_V = -1$$

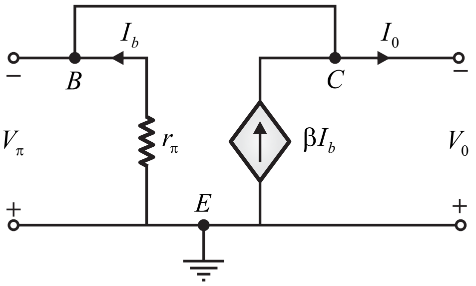
Hence, the open circuit small signal voltage gain is **–1.**

Method 3

Given figure has a diode connected *p-n-p* transistor which is shown below,



Equivalent small signal r_e-model of diode connected *p-n-p* transistor is shown below,



From figure, $V_\pi - V_0 = 0$

$$V_0 = V_\pi \qquad \ldots \text{(i)}$$

Applying KCL at collector terminal,

$$I_0 - I_b - \beta I_b = 0$$

$$I_0 = I_b + \beta I_b = (1+\beta) I_b \qquad \ldots \text{(ii)}$$

$$R_0 = \frac{V_0}{I_0}$$

From equation (i) and (ii),

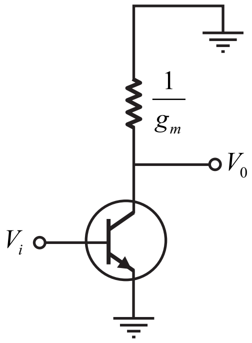
$$R_0 = \frac{V_\pi}{I_b(1+\beta)}$$

Since, $\beta >> 1$,

$$R_0 = \frac{V_\pi}{\beta I_b} = \frac{V_\pi}{g_m V_\pi} \qquad [g_m V_\pi = \beta I_b]$$

$$R_0 = \frac{1}{g_m}$$

Hence, equivalent circuit of the given circuit is shown below,



Internal voltage gain is given by,

$$A_V = \frac{V_0}{V_i} = -g_m R_L \qquad \ldots \text{(iv)}$$

From figure,

$$R_L = \frac{1}{g_m} \qquad \ldots \text{(v)}$$

From equation (iv) and (v),

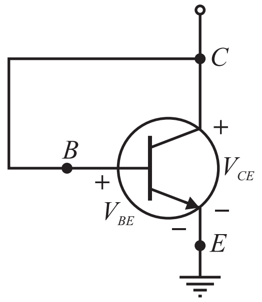
$$A_V = -1$$

Hence, the open circuit small signal voltage gain is **–1.**

Scan for Video Solution

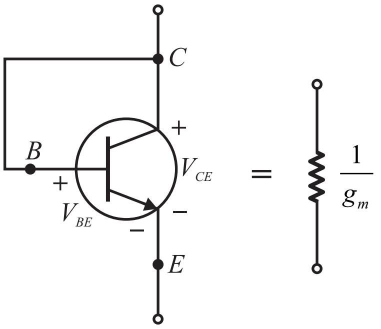
Key Point

(i) BJT will behave as a diode if collector is shorted to base i.e. $V_{CB} = 0\,\text{V}$. This type of BJT is referred as **Diode connected transistor.**

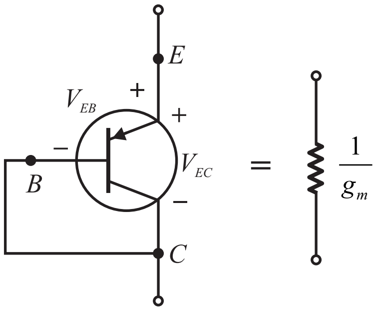


From figure, $V_{CB} = 0\,\text{V}$, $V_{CE} = V_{BE}$

(ii) ***n-p-n* diode connected transistor :**



(iii) ***p-n-p* diode connected transistor :**



(iv) (a) A diode connected transistor always works in active region.

(b) A diode connected MOSFET always works in saturation region.

3.5 **50**

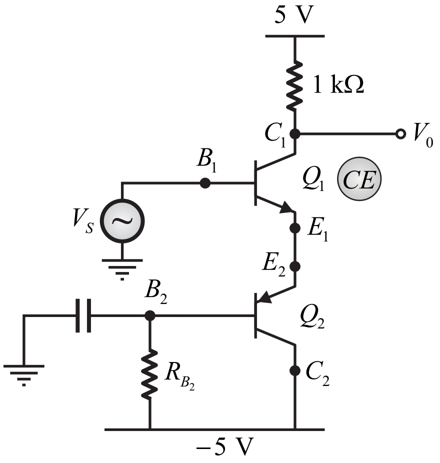
Given :

(i) Collector current = $I_{C_1} = I_{C_2} = 2.6$ mA

(ii) β is large i.e. $I_{B_1} = I_{B_2} = 0$, hence

$I_{C_1} = I_{E_1}$, $I_{C_2} = I_{E_2}$.

(iii) Thermal voltage = $V_T = 26$ mV



Method 1

AC equivalent model :

(i) All capacitors are short circuited.

(ii) All DC voltage sources are replaced by short circuit.

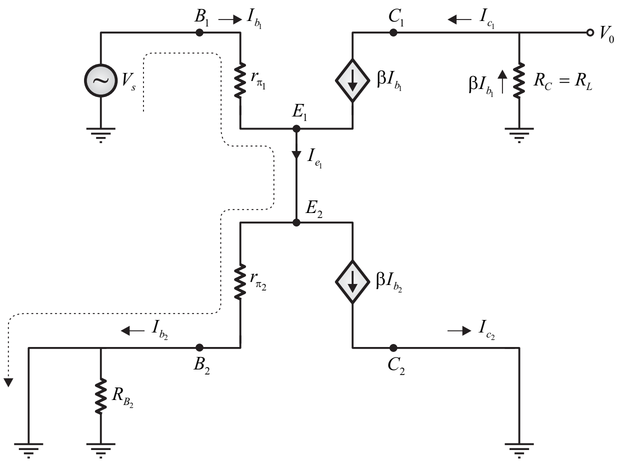


Fig. (a)

Modified figure is shown below,

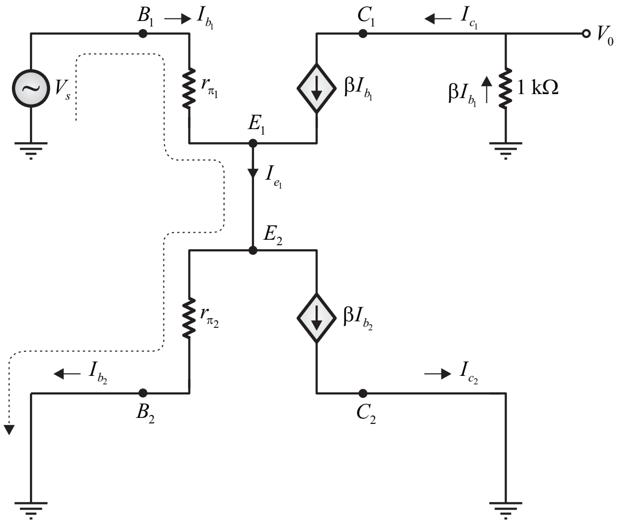


Fig. (b)

From the above circuit, output voltage V_0 is given by,

$$V_0 = -\beta I_{b_1} \times R_L \qquad \ldots \text{(i)}$$

Applying KVL in the loop shown in above figure,

$$V_s = r_{\pi_1} I_{b_1} + r_{\pi_2} I_{b_2} \qquad \ldots \text{(ii)}$$

Applying KCL at node E_1 i.e. E_2,

$$-I_{b_1} - \beta I_{b_1} + I_{b_2} + \beta I_{b_2} = 0$$

$$(1+\beta)I_{b_1} = (1+\beta)I_{b_2}$$

$$I_{b_1} = I_{b_2} \qquad \dots \text{(iii)}$$

From equation (ii) and (iii),

$$V_s = (r_{\pi_1} + r_{\pi_2})I_{b_1}$$

where, $r_{\pi_1} = \dfrac{\beta}{g_{m_1}}$, $r_{\pi_2} = \dfrac{\beta}{g_{m_2}}$

Hence, $$V_s = \left(\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}}\right)\beta I_{b_1} \qquad \dots \text{(iv)}$$

From equation (i) and (iv),

$$V_s = -\left(\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}}\right) \times \frac{V_0}{R_L}$$

$$\frac{V_0}{V_s} = \frac{-R_L}{\dfrac{1}{g_{m_1}} + \dfrac{1}{g_{m_2}}}$$

where, $g_{m_1} = \dfrac{I_{C_1}}{V_T} = \dfrac{2.6}{26} = 0.1$ A/V and

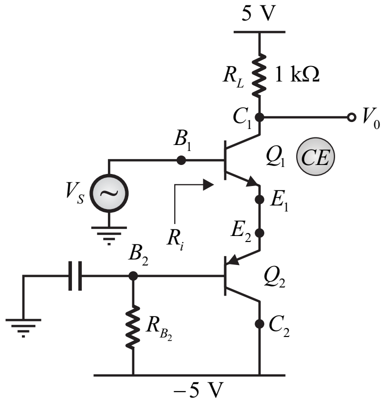
$$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{2.6}{26} = 0.1 \text{ A/V}$$

Hence, $$\frac{V_0}{V_s} = \frac{-10^3}{\dfrac{26}{2.6} + \dfrac{26}{2.6}} = \frac{-10^3}{20} = -50$$

$$\left|\frac{V_0}{V_s}\right| = 50$$

Hence, the magnitude of voltage gain is **50.**

Method 2

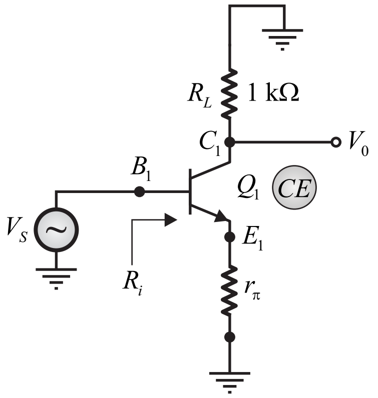


Voltage gain of CE amplifier is given by,

$$A_V = \frac{V_0}{V_s} = \frac{A_I R_L}{R_i}$$

where, $R_L = R_C = 1\text{ k}\Omega$, $A_I = -\beta$ ... (i)

Calculation of R_i :



$R_i = R_{i_1} + r_\pi$ where, $R_{i_1} = r_\pi$ [input resistance of single stage CE amplifier]

$$R_i = r_\pi + r_\pi = 2r_\pi$$

From equation (i),

$$A_V = \frac{V_0}{V_s} = \frac{-\beta R_C}{2r_\pi} = \frac{-g_m R_C}{2}$$

where, $g_m = \dfrac{I_C}{V_T} = \dfrac{2.6}{26} = 0.1 \text{ A/V} = 100 \text{ mA/V}$

$$A_V = \frac{V_0}{V_s} = \frac{-100\times 1}{2} = -50$$

Magnitude of voltage gain, $\left|\dfrac{V_0}{V_S}\right| = 50$.

Hence, the magnitude of voltage gain is **50.**

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4

JFET & MOSFET Amplifier with Biasing

➢ Partial Synopsis

JFET amplifier is an important topic for all the exam but it is not present in the syllabus of GATE exam since 2016.

1. Common source amplifier without R_S :

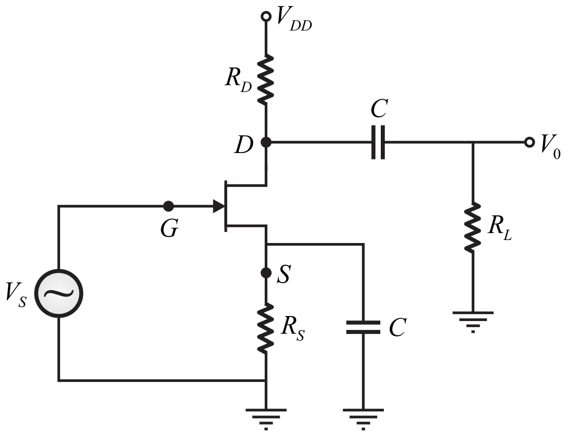


Fig. CS JFET amplifier (*n*-channel)

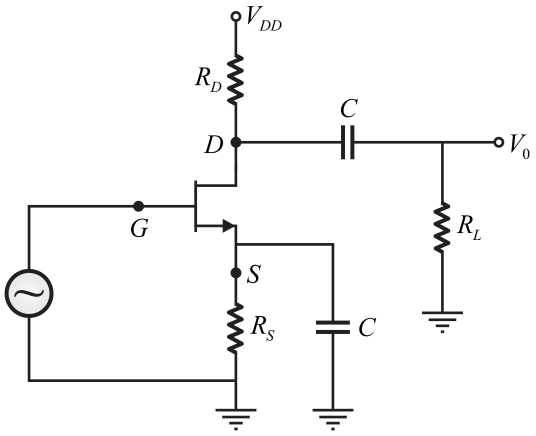


Fig. CS MOSFET amplifier (*n*-channel enhancement)

Sr.	Internal parameter of amplifier	CS without R_S [JFET]	CS without R_S [MOSFET]
1.	R_i	$R_i = \infty$	$R_i = \infty$
2.	R_0	$R_0 = r_d$ (Datasheet parameter)	$R_0 = r_0$ $[\lambda \neq 0]$ $R_0 = \infty$ $[\lambda = 0]$
3.	A_V	$A_V = -g_m R_L''$ $(R_L'' = r_d \parallel R_D \parallel R_L)$	$A_V = -g_m R_L''$ $(R_L'' = r_d \parallel R_D \parallel R_L)$
4.	A_I	Does not exist $\left[I_g = 0 \text{ A}\right]$	Does not exist $\left[I_g = 0 \text{ A}\right]$

2. Common source amplifier with R_S :

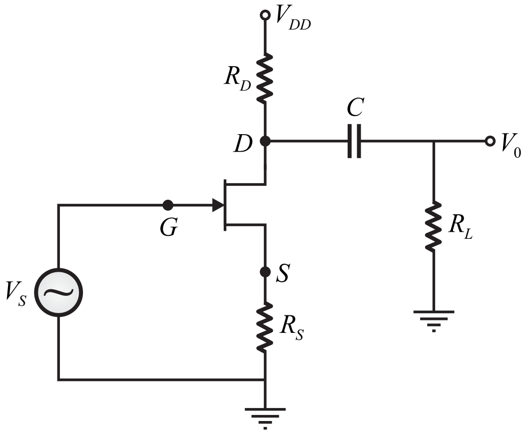


Fig. CS JFET amplifier (*n*-channel)

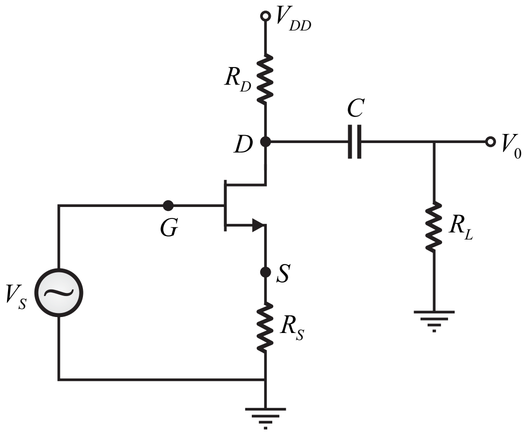


Fig. CS MOSFET amplifier (*n*-channel enhancement)

Sr.	Internal parameter of amplifier	CS with R_S [JFET]	CS with R_S [MOSFET]
1.	R_i	$R_i = \infty$	$R_i = \infty$
2.	R_0	$R_0 = r_d + (1+\mu)R_S$ $(\mu = g_m r_d)$	$R_0 = r_d(1+\mu)R_S$ $R_0 = \infty$ [if $\lambda = 0 \Rightarrow r_d = \infty$]
3.	A_V	$A_V = \dfrac{-\mu R_L'}{R_L' + r_d + R_S(1+\mu)}$ $(R_L' = R_D \parallel R_L)$	$A_V = \dfrac{-\mu R_L'}{R_L' + r_0 + R_S(1+\mu)}$ $(R_L' = R_D \parallel R_L)$
4.	A_I	Does not exist $\left[I_g = 0 \text{ A}\right]$	Does not exist $\left[I_g = 0 \text{ A}\right]$

➤ Sample Questions

1990 IISc Bangalore

4.1 Which of the following can be considered to be the advantage of FET amplifier as compared to BJT amplifiers?

1. Higher input impedance
2. Good bias stability
3. Higher gain-bandwidth product
4. Lower noise figure

Select the correct answer using the codes given below.

Codes :

(A) 1, 2 and 3

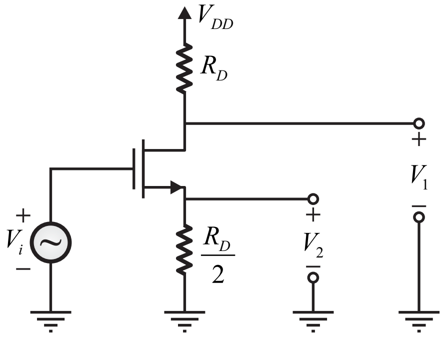
(B) 1, 2 and 4

(C) 2, 3 and 4

(D) 1, 3 and 4

1998 IIT Delhi

4.2 In the MOSFET amplifier of the figure, the signal output V_1 and V_2 obey which of the following relationship.

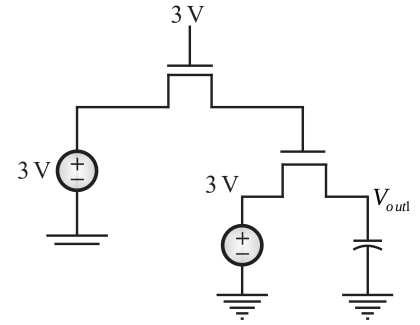


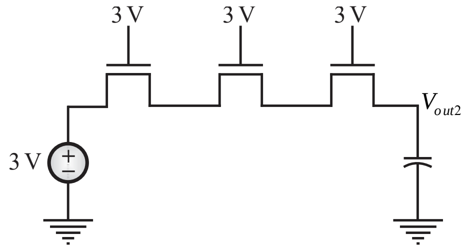
(A) $V_1 = \frac{V_2}{2}$ (B) $V_1 = -\frac{V_2}{2}$

(C) $V_1 = 2V_2$ (D) $V_1 = -2V_2$

2019 IIT Madras

4.3 In the circuits shown, the threshold voltage of each NMOS transistor is 0.6 V. Ignoring the effect of channel length modulation and body bias, the values of V_{out1} and V_{out2}, respectively, in volts, are



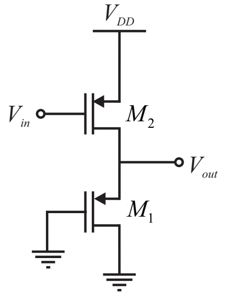


(A) 2.4 and 2.4 (B) 2.4 and 1.2

(C) 1.8 and 1.2 (D) 1.8 and 2.4

2021 IIT Bombay

4.4 In the circuit shown in the figure, the transistors M_1 and M_2 are operating in saturation. The channel length modulation coefficients of both the transistors are non-zero. The transconductance of the MOSFETs M_1 and M_2 are g_{m1} and g_{m2}, respectively, and the internal resistance of the MOSFETs M_1 and M_2 are r_{01} and r_{02}, respectively.



Ignoring the body effect, the ac small signal voltage gain $(\partial V_{out} / \partial V_{in})$ of the circuit is

(A) $-g_{m2}(r_{01} \| r_{02})$

(B) $-g_{m2}\left(\frac{1}{g_{m1}} \| r_{02}\right)$

(C) $-g_{m1}\left(\frac{1}{g_{m2}} \| r_{01} \| r_{02}\right)$

(D) $-g_{m2}\left(\frac{1}{g_{m1}} \| r_{01} \| r_{02}\right)$

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Explanations JFET & MOSFET Amplifier with Biasing

4.1 (B)

FET has following characteristics as given below,

(i) High input impedance

(ii) Low output impedance

(iii) Good bias stability

(iv) Low gain-bandwidth product

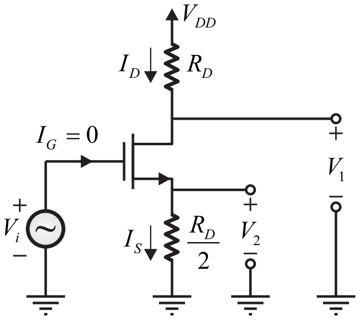
(v) Lower noise figure

(vi) Less noisy

Hence, the correct option is (B).

4.2 (B)

Given circuit is shown below,



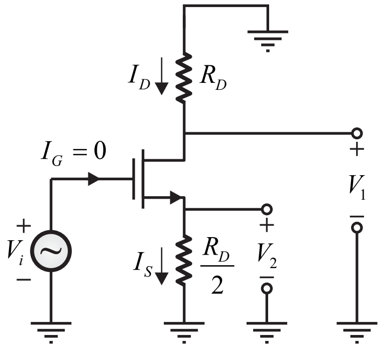
From above figure, $I_G = 0$

So, $I_D = I_S$

AC equivalent circuit :

(i) All capacitors are short circuited.

(ii) All DC voltage sources are replaced by short circuit.



From figure,

$$V_2 = I_S \times \frac{R_D}{2} \qquad \text{[Ohm's law]}$$

$$V_2 = I_D \times \frac{R_D}{2} \qquad [I_D = I_S] \ldots \text{(i)}$$

Applying KVL at drain terminal,

$$I_D R_D + V_1 = 0$$

$$V_1 = -I_D \times R_D \qquad \ldots \text{(ii)}$$

From equation (i) and (ii),

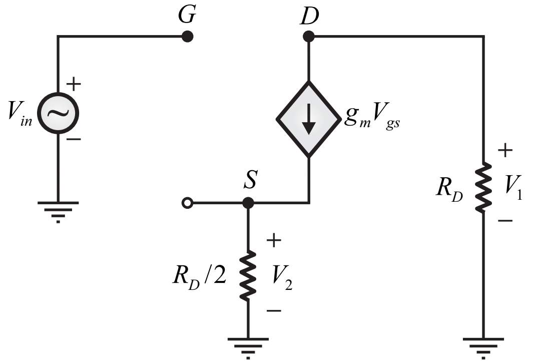
$$V_2 = \frac{-V_1}{2}$$

$$V_1 = -2V_2$$

Hence, the correct option is (D).

OR

Using AC equivalent circuit :



From the AC equivalent circuit shown in below,

$$V_1 = -g_m V_{gs} \cdot R_D \qquad \ldots \text{(i)}$$

$$V_2 = g_m V_{gs} \cdot \frac{R_D}{2} \qquad \ldots \text{(ii)}$$

Dividing equation (i) by equation (ii),

$$\frac{V_2}{V_1} = \frac{-g_m V_{gs} \cdot R_D}{g_m V_{gs} \cdot \dfrac{R_D}{2}} = -2$$

Therefore, $V_1 = -2V_2$

Hence, the correct option is (D).

4.3 (D)

Method 1

Given : Threshold voltage of each NMOS transistor is 0.6 V.

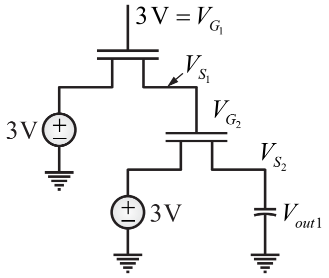


Fig. (a)

In a pass transistor logic, V_{GS} is automatically maintained at V_{TH}.

If V_G is fixed, V_S should be adjusted so that $V_G - V_S = V_{TH}$.

$V_G = 3\text{ V}$ for all NMOS transistors and

$V_{TH} = 0.6\,\text{V}$ for all transistor

So, $V_S = V_G - V_{TH}$

From figure (a),

$$V_{S_1} = V_{G_1} - V_{TH} = 3 - 0.6 = 2.4\ \text{V}$$

$$V_{S_2} = V_{G_2} - V_{TH} = 2.4 - 0.6 = 1.8\,\text{V} = V_{out1}$$

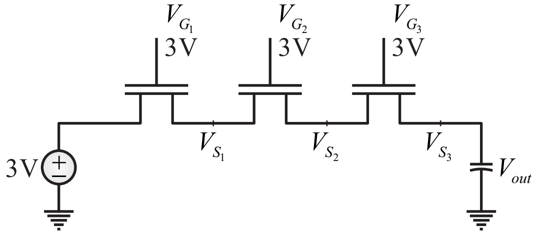


Fig. (b)

Similarly, from figure (b)

$$V_{S_1} = V_{G_1} - V_{TH} = 3 - 0.6 = 2.4\,\text{V}$$

$$V_{S_2} = V_{G_2} - V_{TH} = 3 - 0.6 = 2.4\,\text{V}$$

$$V_{S_3} = V_{G_3} - V_{TH} = 3 - 0.6 = 2.4\,\text{V} = V_{out2}$$

Hence, the correct option is (D).

Method 2

Given circuit (I) is shown in below figure and the n-MOS used have threshold voltage as 0.6 V. For pass transistor logic V_{GS} is maintained as V_{TH}.

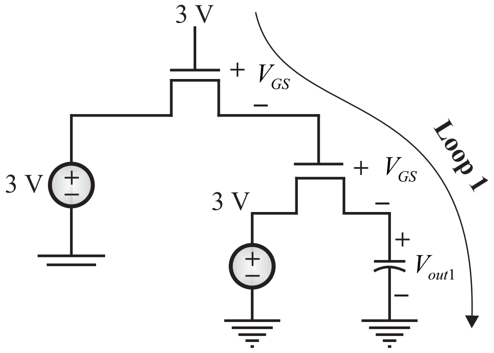


Fig. (c)

Applying KVL in loop (1), we get

$$-3 + V_{GS} + V_{GS} + V_{out_1} = 0$$

$$V_{out_1} = 3 - 2V_{GS} = 3 - 2 \times 0.6$$

$$V_{out_1} = 1.8\ \text{V}$$

Similarly from circuit (II),

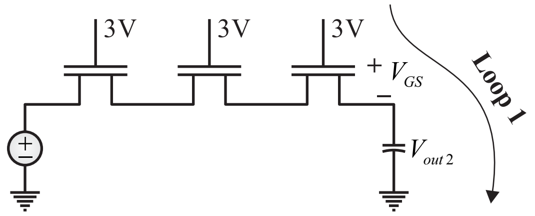


Fig. (d)

Applying kVL in loop (1)

$$-3 + V_{GS} + V_{out_2} = 0$$

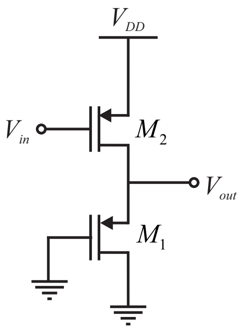
$$V_{out_2} = 3 - V_{GS} = 3 - 0.6$$

$$V_{out_2} = 2.4\ \text{volt}$$

Hence, the correct option is (D).

4.4 (D)

Given circuit is shown below,



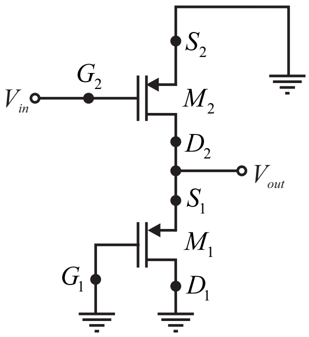
Channel length modulation coefficient $\lambda \neq 0$, it means internal resistance of MOSFET M_1 and M_2 are not zero.

That is $r_{01} \neq 0$, $r_{02} \neq 0$

AC equivalent circuit :

(i) All capacitors are short circuited.

(ii) All DC voltage sources replaced by ground.



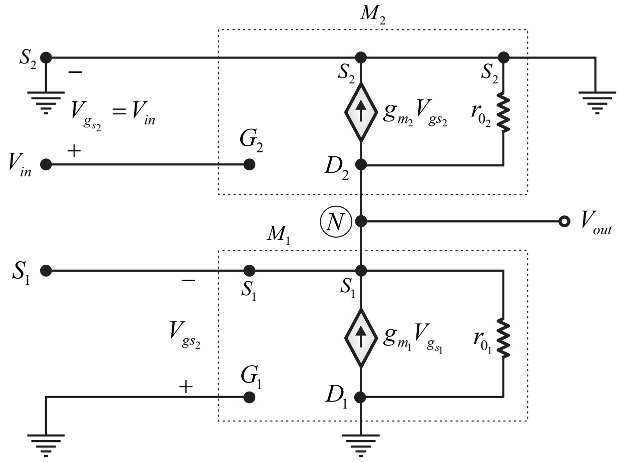
From figure $V_{g_1} = 0\text{ V}$, $V_{s_2} = 0\text{ V}$ and $V_{d_1} = 0\text{ V}$

$$V_{s_1} = V_{d_2} = V_{out},\ V_{g_2} = V_{in}.$$

So, $V_{gs_1} = V_{g_1} - V_{s_1} = 0 - V_{out} = -V_{out}$

$$V_{gs_2} = V_{g_2} - V_{s_2} = V_i - 0 = V_i$$

The AC equivalent model of given circuit is shown below,



Applying KCL at node N,

$$\frac{V_{out}}{r_{01}} - g_{m_1}V_{gs_1} + g_{m_2}V_{gs_2} + \frac{V_{out}}{r_{02}} = 0 \quad \ldots\text{(i)}$$

Here, $V_{gs_1} = -V_{out}$ …(ii)

$V_{gs_2} = V_{in}$ …(iii)

So, put equation (ii) and (iii) into equation (i),

$$V_{out}\left[\frac{1}{r_{01}} + \frac{1}{r_{02}} + g_{m_1}\right] = -g_{m_2}V_i$$

So, $$\frac{V_{out}}{V_i} = -g_{m_2}\left[\frac{1}{g_{m_1}} \,||\, r_{01} \,||\, r_{02}\right]$$

Hence, $$\frac{\partial V_{out}}{\partial V_i} = -g_{m_2}\left[\frac{1}{g_{m_1}} \,||\, r_{01} \,||\, r_{02}\right]$$

Hence, the correct option is (D).

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5 Feedback Amplifiers

➢ Partial Synopsis

Classification of Feedback Topology :

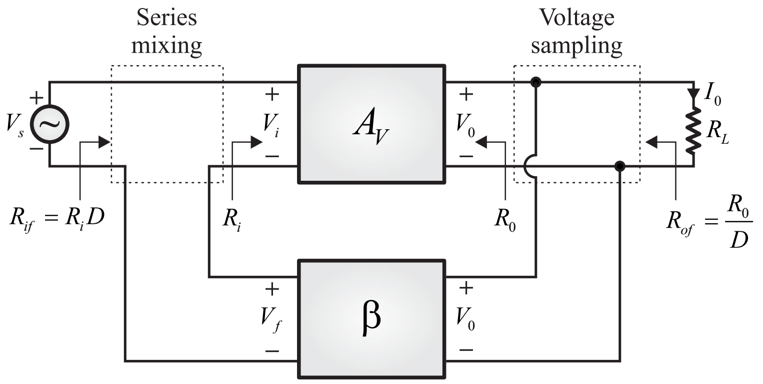
1. Voltage series topology :

Also known as,

(i) Series-shunt topology

(ii) Voltage-voltage topology

(iii) Voltage amplifier



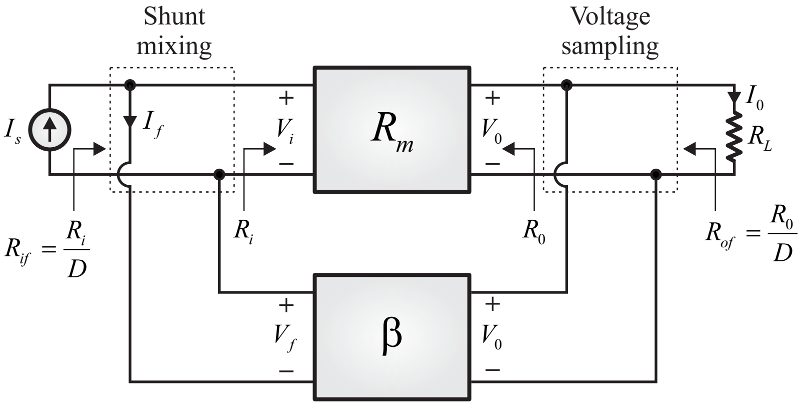
2. Voltage shunt topology :

Also known as,

(i) Shunt-shunt topology

(ii) Current-voltage topology

(iii) Transresistance amplifier



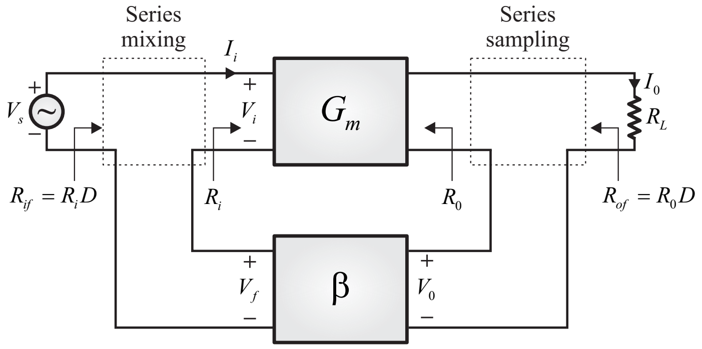
3. Current series topology :

Also known as,

(i) Series-series topology

(ii) Voltage-current topology

(iii) Transconductance amplifier



4. Current shunt topology :

Also known as,

(i) Shunt-series topology

(ii) Current-current topology

(iii) Current amplifier

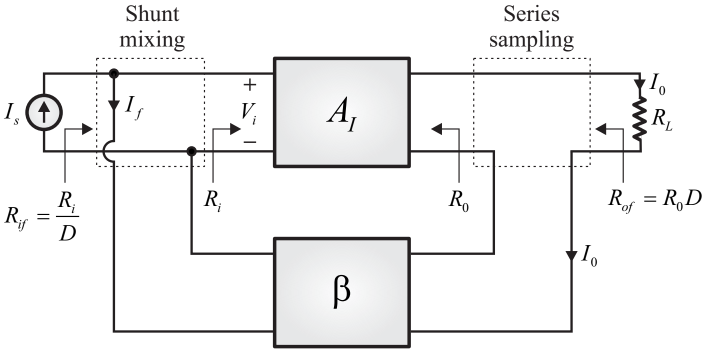


Table : 5.1

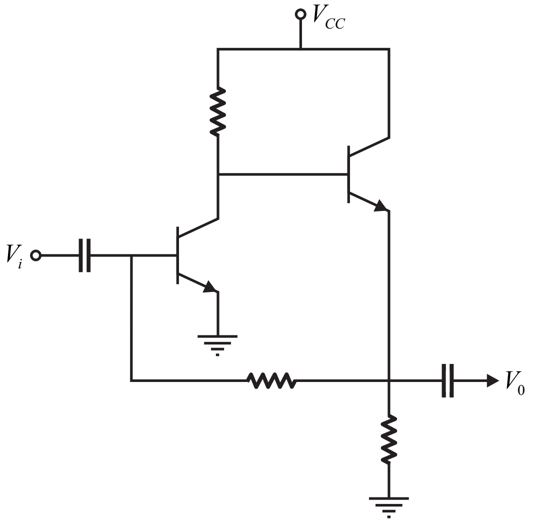
Feedback topology	Feedback gain	R_{if}	R_{of}	De-sensitivity (D)
Voltage-series (A_{Vf}) or Voltage-voltage or Series-shunt	$A_{Vf} = \frac{A_V}{1+A_V\beta}$	R_iD	$\frac{R_0}{D}$	$D = 1 + A_V\beta$
Voltage-shunt (R_{mf}) or Current-voltage or Shunt-shunt	$R_{mf} = \frac{R_m}{1+R_m\beta}$	$\frac{R_i}{D}$	$\frac{R_0}{D}$	$D = 1 + R_m\beta$

Current-series (G_{mf}) or Voltage-current or Series-series	$G_{mf} = \dfrac{G_m}{1+G_m\beta}$	$R_i D$	$R_0 D$	$D = 1 + G_m \beta$
Current-shunt (A_{If}) or Current-current or Shunt- series	$A_{If} = \dfrac{A_I}{1+A_I\beta}$	$\dfrac{R_i}{D}$	$R_0 D$	$D = 1 + A_I \beta$

➢ Sample Questions

1989 IIT Kanpur

5.1 The feedback amplifier shown in figure has



(A) current-series feedback with large input impedance and large output impedance.

(B) voltage-series feedback with large input impedance and low output impedance.

(C) voltage-shunt feedback with low input impedance and low output impedance.

(D) current-shunt feedback with low input impedance and large output impedance.

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5.2 In a negative feedback amplifier using voltage - series (i.e., voltage-sampling, series mixing) feedback

(A) R_i decreases and R_0 decreases.

(B) R_i decreases and R_0 increases.

(C) R_i increases and R_0 decreases.

(D) R_i increases and R_0 increases.

(R_i and R_0 denote the input and output resistances respectively)

2018 IIT Guwahati

5.3 A good trans-impedance amplifier has

(A) low input impedance and high output impedance.

(B) high input impedance and high output impedance.

(C) high input impedance and low output impedance.

(D) low input impedance and low output impedance.

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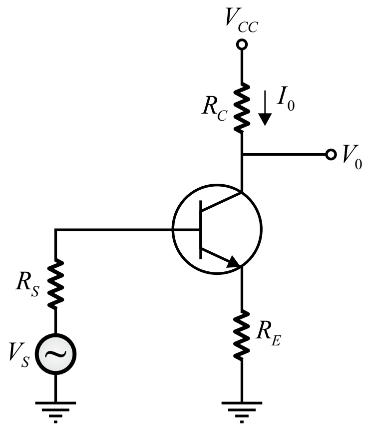
5.4 The feedback topology in the amplifier circuit (the base bias circuit is not shown for simplicity) in the figure is **[Set - 02]**



(A) voltage shunt feedback

(B) current series feedback

(C) current shunt feedback

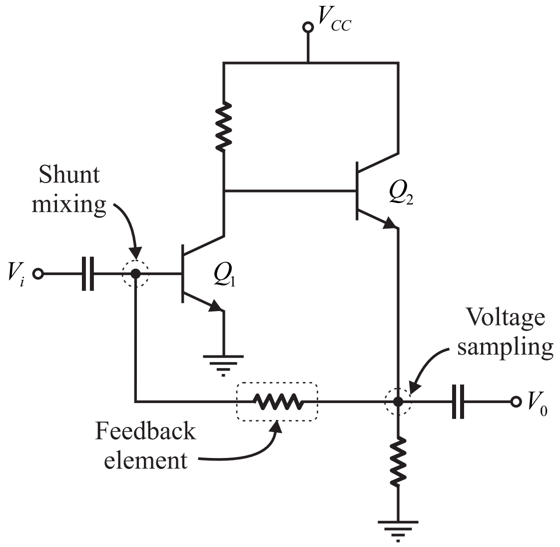
(D) voltage series feedback

❖❖❖❖

Explanations Feedback Amplifiers

5.1 (1)

Feedback amplifier is shown below,



(i)2 Feedback element is directly connected to output voltage, hence it is voltage sampling.

(ii)3 Feedback element is directly connected to the input voltage in shunted form, hence it is shunt mixing.

Therefore, feedback circuit is voltage-shunt feedback.

For voltage shunt feedback,

(i) Input impedance decreases by factor $(1+R_m\beta)$.

$$R_{if} = \frac{R_i}{1+R_m\beta}$$

(ii) Output impedance decreases by factor $(1+R_m\beta)$.

$$R_{of} = \frac{R_0}{1+R_m\beta}$$

where,

$R_m =$ Open loop gain for transresistance amplifier

$\beta =$ Feedback gain

Hence, the correct options is (C).

5.2 (C)

For voltage series feedback (voltage amplifier),

(i) Input impedance increases by factor $(1+A_V\beta)$.

$$R_{if} = R_i D = R_i(1+A_V\beta)$$

(ii) Output impedance decreases by factor $(1+A_V\beta)$.

$$R_{of} = \frac{R_0}{D} = \frac{R_0}{(1+A_V\beta)}$$

Hence, the correct option is (C).

5.3 **(D)**

Transresistance or transimpedance amplifier is normally used as a current to voltage convertor which requires low input and low output impedances for its proper operation.

Requirements of input and output impedances for different amplifiers are listed below :

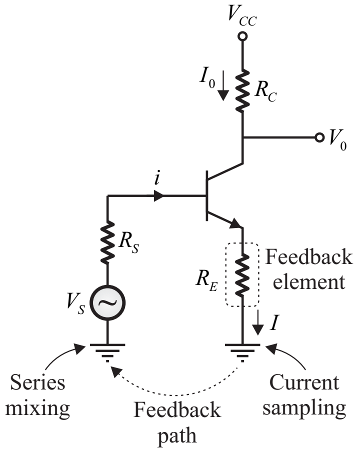
Feedback Topology	Type of amplifier	R_{in}	R_{out}	Conversion
Series-shunt (Voltage series)	Voltage amplifier	High	Low	Voltage to voltage
Shunt-series (Current shunt)	Current amplifier	Low	High	Current to current
Series-series (Current series)	Transconductance	High	High	Voltage to current
Shunt-shunt (Voltage shunt)	Transimpedance amplifier	Low	Low	Current to voltage

Hence, the correct option is (D).

Scan for Video Solution

5.4 **(B)**

The given configuration is a common emitter with emitter resistance is as shown below,



(i) Feedback element R_E is not connected to output voltage directly hence it is current sampling.

(ii) Feedback element is not directly connected to the input in shunted form, hence it is series mixing or voltage mixing.

Hence, it is a current series feedback topology.

Hence, the correct option is (B).

❖❖❖❖

6

Operational Amplifiers

➢ Partial Synopsis

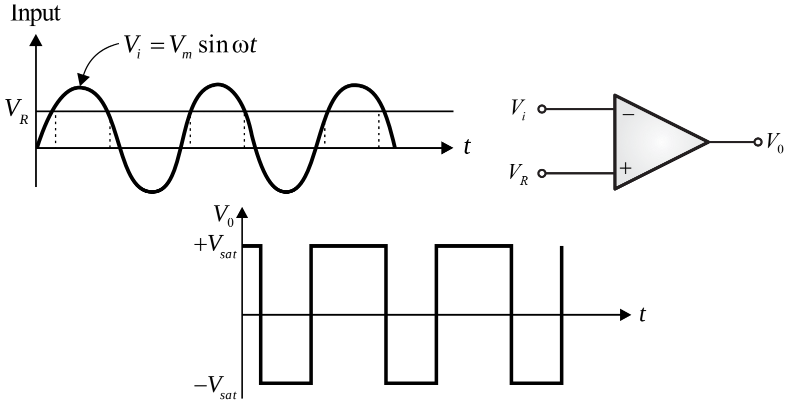
Non-linear Application of Op-Amp

1. Comparators :

- When Op-Amp acts as comparator,

  If $V_{NI} > V_I$ then, $V_0 = +V_{sat}$

  $V_I > V_{NI}$ then, $V_0 = -V_{sat}$



- The output of a comparator is either HIGH $(+V_{sat})$ or LOW $(-V_{sat})$.

2. Schmitt Trigger :

- Schmitt trigger is basically a comparator circuit with positive feedback and hence it is also called as re-generative comparator.

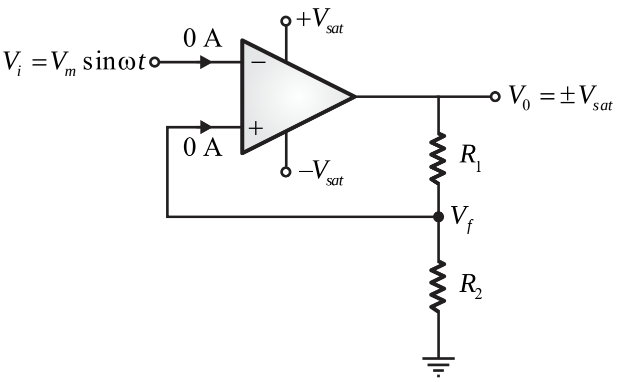
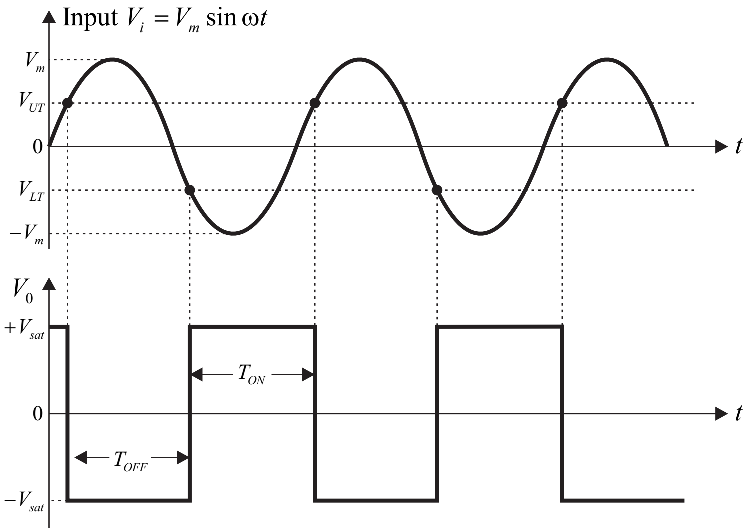


Fig. Schmitt trigger

- The upper and lower threshold voltage,

$$V_{UT} = \frac{V_{sat}R_2}{R_1 + R_2},\ V_{LT} = \frac{-V_{sat}R_2}{R_1 + R_2}$$

- Output waveform (V_0) is shown below,



- From output waveform :

(a) $T_{ON} = T_{OFF}$

(b) % Duty cycle $= 50\,\%$

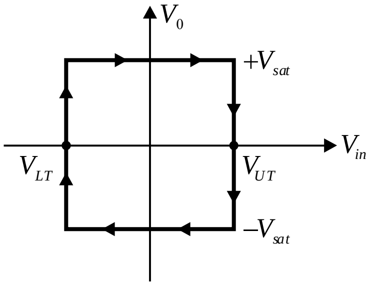
(c) $\text{Avg}(V_0) = 0$

(d) $\text{Area}^+ = \text{Area}^-$

(e) Symmetrical square wave

(f) RMS value of Output $= V_{sat}$

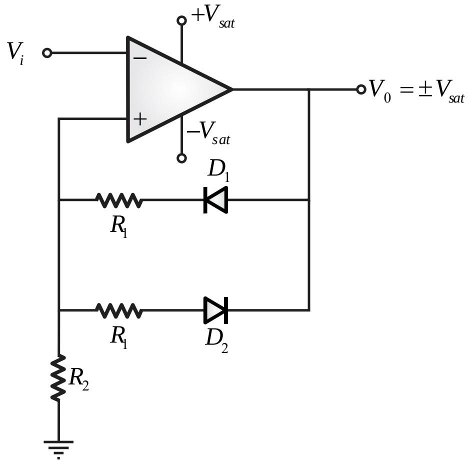
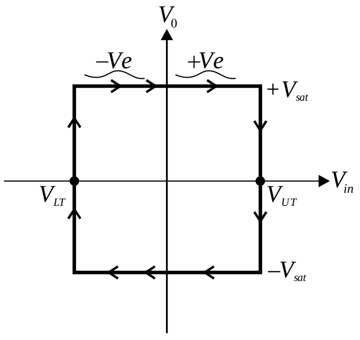
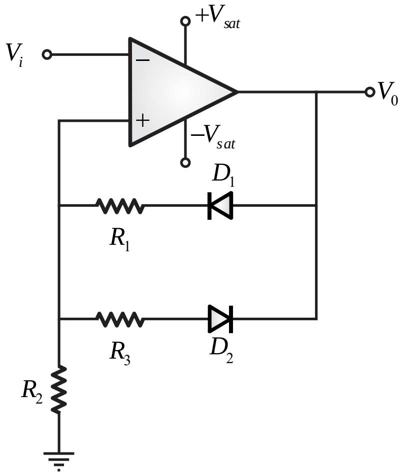
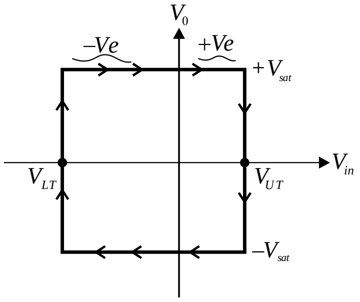
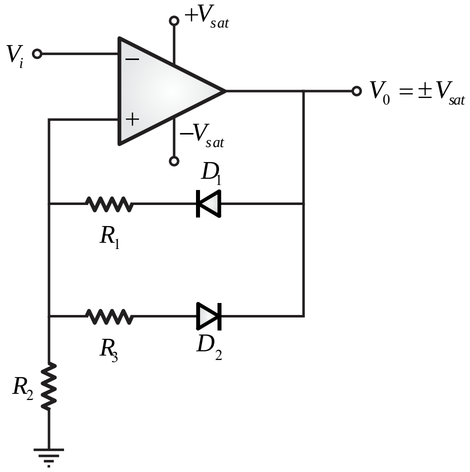
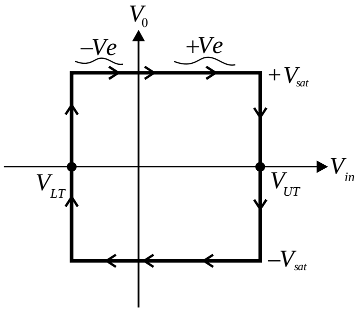
- Hysteresis Curve :



$$V_{hys} = V_{UT} - V_{LT}$$

$$V_{hys} = \frac{2V_{sat}R_2}{R_1 + R_2}$$

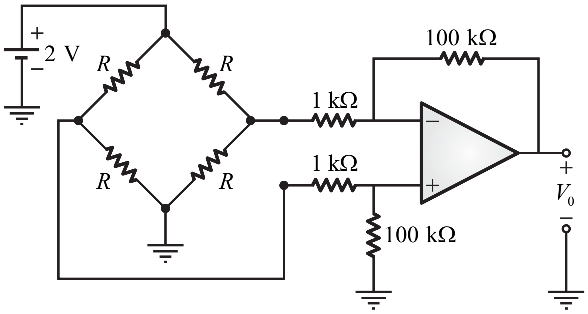
Special Cases of Schmitt Trigger

S. No.	Schmitt Trigger Circuit	Transfer characteristics	Summary
1.	 **Consider $R_1 = R_2$**	 $\lvert V_{UT} \rvert = \lvert V_{LT} \rvert$	Conclusion from transfer characteristics, (a) Area of $(+ve) =$ Area of $(-ve)$ (b) Average value of $(V_0) = 0$ (c) $T_{ON} = T_{OFF}$ (d) % duty cycle $= 50\%$ (e) Output is symmetrical square wave form
2.	 **Consider $R_1 > R_3$**	 $\lvert V_{UT} \rvert < \lvert V_{LT} \rvert$	Conclusion from transfer characteristics, (a) Area of $(+ve) <$ Area of $(-ve)$ (b) Average value of $(V_0) = -ve$ (c) $T_{ON} < T_{OFF}$ (d) % duty cycle $< 50\%$ (e) Output is asymmetrical square wave form
3.	 **Consider $R_3 > R_2$**	 $\lvert V_{UT} \rvert > \lvert V_{LT} \rvert$	Conclusion from transfer characteristics, (a) Area of $(+ve) >$ Area of $(-ve)$ (b) Average value of $(V_0) = +ve$ (c) $T_{ON} > T_{OFF}$ (d) % duty cycle $> 50\%$ (e) Output is asymmetrical square wave form.

➢ Sample Questions

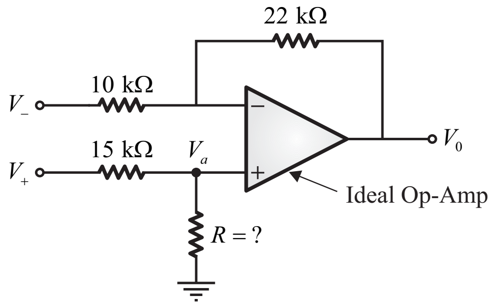
1987 IIT Bombay

6.1 In figure if the CMRR of the operational amplifier is 60 dB, then the magnitude of the output voltage is _________ mV.



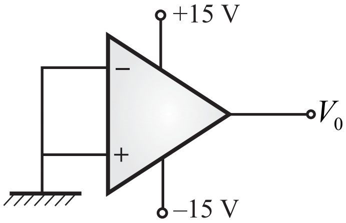
1995 IIT Kanpur

6.2 In the given circuit, if the voltage inputs V_+ and V_- are to be amplified by the same amplification factor, the value of 'R' should be



2000 IIT Kharagpur

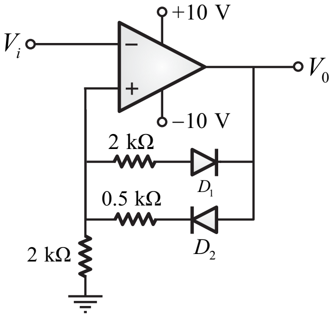
6.3 If the Op-Amp in given figure, has an input offset voltage of 5 mV and an open-loop voltage gain of 10000, then V_0 will be



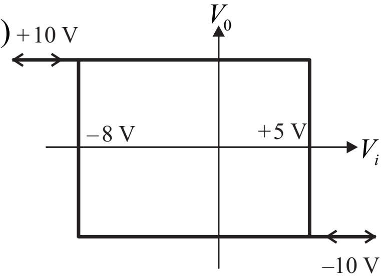
(A) 0 V
(B) 5 mV
(C) +15 V or – 15 V
(D) + 50 V or – 50 V

2005 IIT Bombay

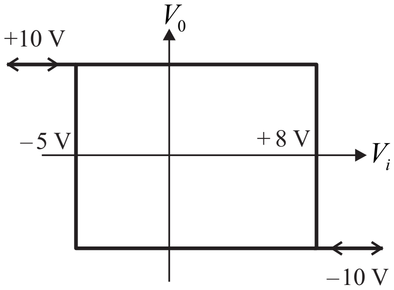
6.4 Given the ideal operational amplifier circuit shown in the figure. Indicate the correct transfer characteristics assuming ideal diodes with zero cut-in voltage.



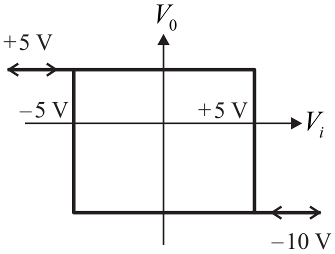
(A)



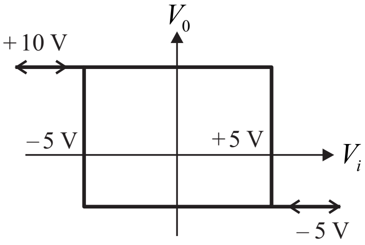
(B)



(C)

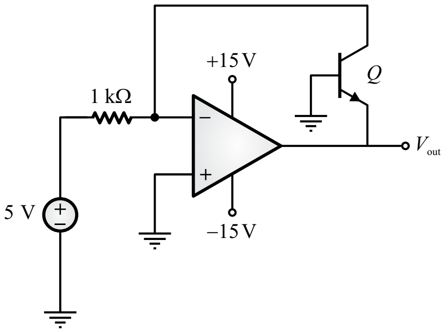


(D)



2013 IIT Bombay

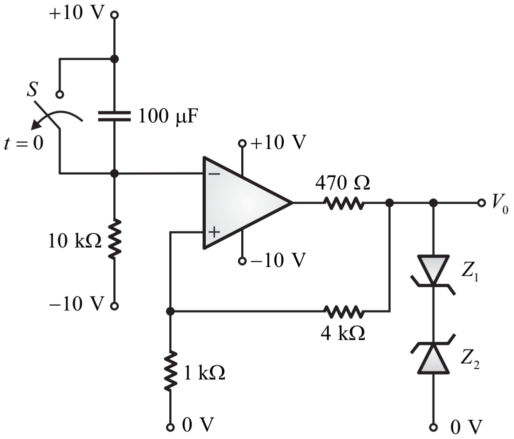
6.5 In the circuit shown below what is the output voltage (V_{out}) if a silicon transistor Q and an ideal Op-Amp are used?



(A) − 15 V
(B) − 0.7 V
(C) + 0.7 V
(D) + 15 V

2016 IISc Bangalore

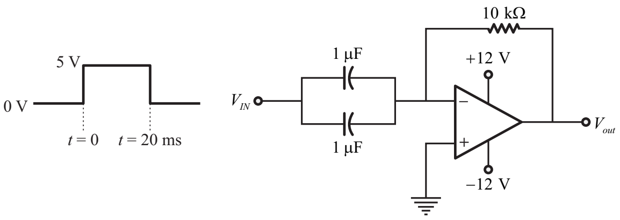
6.6 In the op-amp circuit shown, the Zener diodes Z_1 and Z_2 clamp the output voltage V_0 to $+5$ V or -5 V. The switch S is initially closed and is opened at time $t = 0$.



The time $t = t_1$ (in seconds) at which V_0 changes state is ______. **[Set - 02]**

2021 IIT Bombay

6.7 A circuit with an ideal OPAMP is shown in the figure. A pulse V_{IN} of 20 ms duration is applied to the input. The capacitors are initially uncharged.



The output voltage V_{out} of this circuit at $t = 0^+$ (in integer) is ______ V.

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Explanations Operational Amplifiers

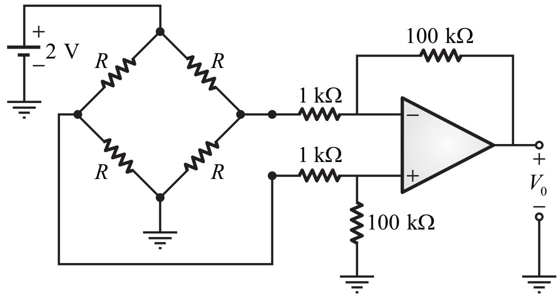
Concept and Validity of Virtual Ground

Scan for Video Explanation

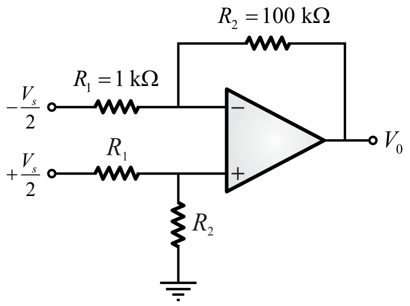
6.1 **100**

Given : CMRR of given Op-Amp is $60\,\text{dB}$.

Given circuit is shown below,



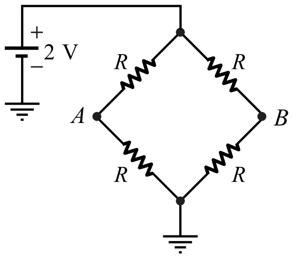
Op-Amp circuit without bridge is shown below,



From above circuit, differential gain is given by,

$$|A_d| = \frac{R_2}{R_1} = 100$$

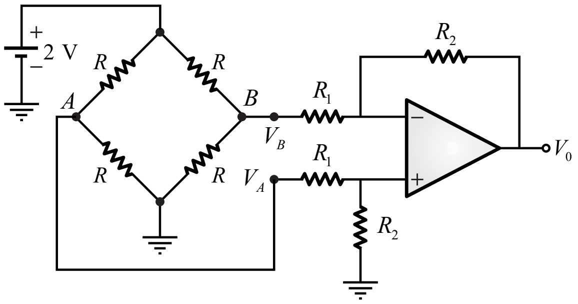
Bridge circuit of given Op-Amp circuit is shown below,



In the above circuit bridge is balance.

$$\text{Hence, } V_A = \frac{R}{R+R} \times 2 = 1\,\text{Volt}$$

$$V_B = \frac{R}{R+R} \times 2 = 1\,\text{Volt}$$



Differential input voltage is given by,

$$V_d = V_A - V_B = 0$$

Output voltage when CMRR $\neq \infty$, is given by,

$$V_0 = A_d V_d + A_c V_c$$

Since, $V_d = 0$

Hence, $V_0 = A_c V_c$ … (i)

Common mode input voltage is given by,

$$V_c = \frac{V_A + V_B}{2} = 1\,\text{Volt}$$

Since, $\text{CMRR}\big|_{dB} = 20\log(\text{CMRR})$

$$60 = 20\log \text{CMRR}$$

$$\text{CMRR} = (10)^{60/20} = 1000$$

$$\text{CMRR} = \left|\frac{A_d}{A_c}\right|$$

$$\text{Hence, } A_c = \frac{A_d}{\text{CMRR}} = \frac{100}{1000} = 0.1$$

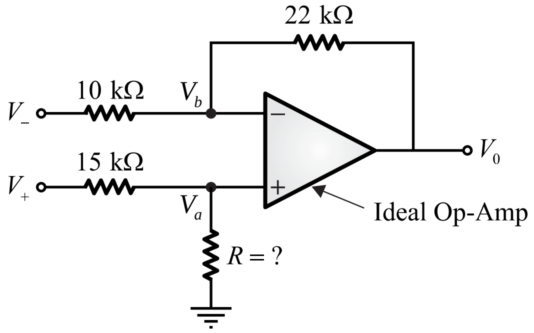
From equation (i),

$$V_0 = 0.1 \times 1 = 0.1\,\text{Volt} = 100\text{ mV}$$

Hence, the magnitude of the output voltage is **100 mV.**

6.2 33

Given : Input voltage V_+ and V_- are to be amplified by the same amplification factor.



Method 1

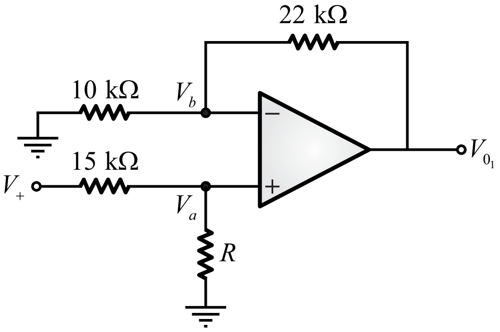
Amplification factor or voltage gain for input voltage V_+ is given by,

$$|A_{V_1}| = \left|\frac{V_{0_1}}{V_+}\right|_{\text{When } V_- = 0}$$

Amplification factor or voltage gain for input voltage V_- is given by,

$$|A_{V_2}| = \left|\frac{V_{0_2}}{V_-}\right|_{\text{When } V_+ = 0}$$

Case 1 : When $V_- = 0$



$$V_a = \frac{R}{R+15}V_+ \qquad \text{[By VDR]}$$

From virtual ground concept,

$$V_b = V_a = \frac{R}{R+15}V_+ \qquad \ldots\text{(i)}$$

Applying KCL at inverting terminal,

$$\frac{V_b - 0}{10} + \frac{V_b - V_{0_1}}{22} = 0$$

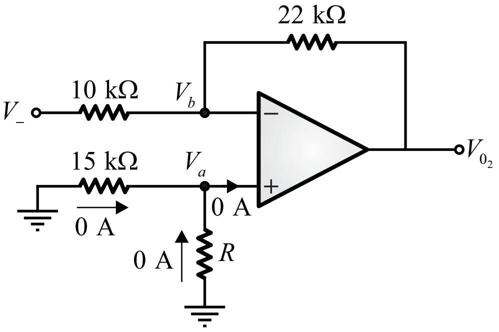
$$V_{0_1} = \left(1 + \frac{22}{10}\right)V_b \qquad \ldots\text{ (ii)}$$

From equation (i) and equation (ii),

$$V_{0_1} = \frac{32}{10} \times \frac{R}{R+15} \times V_+$$

$$\frac{V_{0_1}}{V_+} = A_{V_1} = \frac{32}{10} \times \frac{R}{R+15} \qquad \ldots\text{ (iii)}$$

Case 2 : When $V_+ = 0$



From virtual ground concept,

$$V_b = V_a = 0 \text{ V}$$

Above figure represents an inverting Op-Amp. Gain of inverting Op-Amp is given by,

$$A_{V_2} = \frac{V_{0_2}}{V_-} = \frac{-22}{10} \qquad \ldots\text{ (iv)}$$

Now, since $|A_{V_1}| = |A_{V_2}|$ [Given]

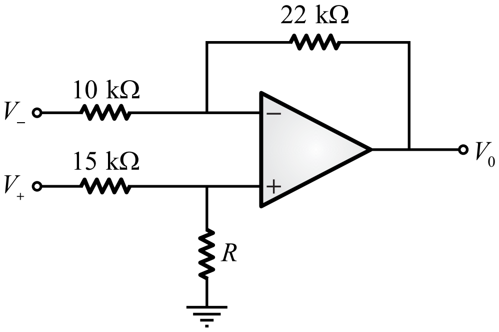
So, $$\frac{32}{10} \times \frac{R}{R+15} = \frac{22}{10}$$

$$32R = 22R + 330$$

$$R = 33 \text{ k}\Omega$$

Hence, the value of R is **33 kΩ .**

Method 2



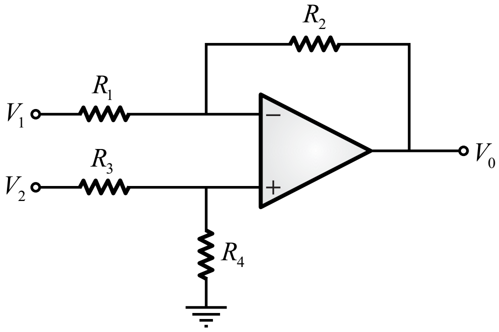
When the input voltages are amplified by the same amplification factor, then

$$\frac{R}{15} = \frac{22}{10}$$

$$R = 33 \text{ k}\Omega$$

Hence, the value of R is **33 kΩ .**

📖 Key Point



From above figure,

$$\left|\frac{V_{0_1}}{V_1}\right|_{\text{when } V_2 = 0} = \frac{R_2}{R_1}$$

and $$\left|\frac{V_{0_2}}{V_2}\right|_{\text{when } V_1=0} = \frac{R_1+R_2}{R_1}\times\frac{R_4}{R_3+R_4}$$

If V_1 and V_2 are amplified by the same amplification factor, then

$$\left|\frac{V_{0_2}}{V_2}\right|_{\text{when } V_1=0} = \left|\frac{V_{0_1}}{V_1}\right|_{\text{when } V_2=0}$$

So, $$\frac{R_1+R_2}{R_1}\times\frac{R_4}{R_3+R_4} = \frac{R_2}{R_1}$$

$$\frac{R_4}{R_3+R_4} = \frac{R_2}{R_1+R_2}$$

$$1+\frac{R_3}{R_4} = 1+\frac{R_1}{R_2}$$

$$\boldsymbol{\frac{R_3}{R_4} = \frac{R_1}{R_2}}$$

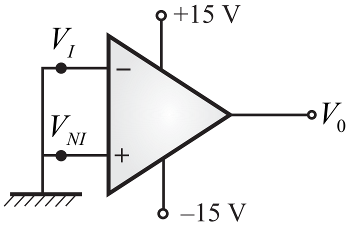
6.3 (C)

Given :

Input offset voltage, $V_{ios} = 5\text{ mV}$

Open loop voltage gain, $A_{OL} = 10000$

Given circuit is shown below,



Method 1

Open loop gain, $A_{OL} = \dfrac{V_0}{V_d}$

$$A_{OL} \neq \infty \quad \text{i.e. } V_d \neq 0$$

$$V_d = V_{NI} - V_I$$

Case 1 :

If $V_{NI} = V_{ios}$, then $V_I = 0$

$$V_d = V_{ios} - V_I = 5\text{ mV} - 0$$

$$V_d = 5\text{ mV} \qquad \dots \text{(i)}$$

Case 2 :

If $V_I = V_{ios}$, then $V_{NI} = 0$

$$V_d = V_{NI} - V_{ios} = 0 - 5\text{ mV}$$

$$V_d = -5\text{ mV} \qquad \dots \text{(ii)}$$

From case 1 and case 2, differential input voltage V_d can be written as given below,

$$V_d = \pm 5\text{ mV}$$

Output voltage is given by,

$$V_0 = A_{OL}V_d$$

$$V_0 = 10000 \times (\pm 5\text{ mV}) = \pm 50\text{ V}$$

But output voltage cannot be greater than saturation voltage.

Therefore, $V_0 = \pm 15\text{ V}$

Hence, the correct option is (C).

Method 2

Output voltage $V_0 = A_{OL} \times V_{ios}$

$$V_0 = 10^4 \times 5 \times 10^{-3}$$

$$V_0 = 50\text{ V}$$

Since, in Open loop Op-Amp, output can not be greater than $\pm V_{sat}$. Therefore output voltage will be $+V_{sat}(=15\text{ V})$ or $-V_{sat}(=-15\text{ V})$.

Hence, the correct option is (C).

Key Point

For ideal Op-Amp, $A_{OL} = \infty$ then $V_d = 0$

$$V_{NI} - V_I = 0$$

$$V_{NI} = V_I$$

$$V_{ios} = V_{NI} = V_I$$

Output voltage,

$$V_0 = A_{OL}V_d$$

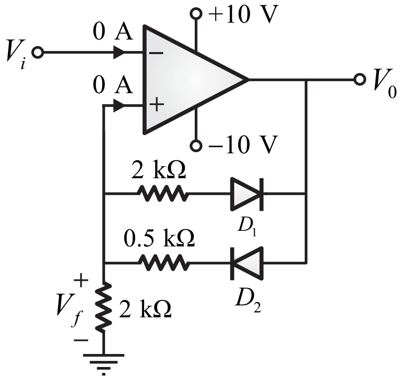
$$V_0 = 0\text{ V}$$

Input offset voltage : This is the input voltage which is used to compensate the output voltage i.e. $V_0 = 0\text{ V}$.

This definition of V_{ios} is valid only for ideal Op-Amp.

6.4 (B)

Given circuit is shown below,

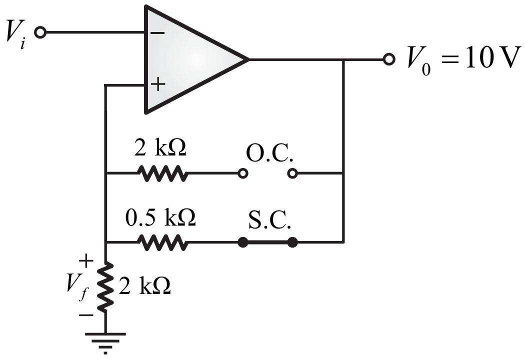


The given Op-Amp circuit has a positive feedback. It is referred as Schmitt trigger circuit.

V_0 can be either $+V_{sat}\,(+10\text{ V})$

or $-V_{sat}\,(-10\text{ V})$

1. When $V_0 = +10\text{ V}$, D_2 is ON (i.e. short circuit) and D_1 is OFF (i.e. open circuit)



$$V_f = \frac{2}{2+0.5} \times V_0 = \frac{10 \times 2}{2+0.5} = 8\text{ V}$$

$$V_f = V_{UTP} = 8\text{ V}$$

where, $V_{UTP} =$ Upper trigger (threshold) point voltage

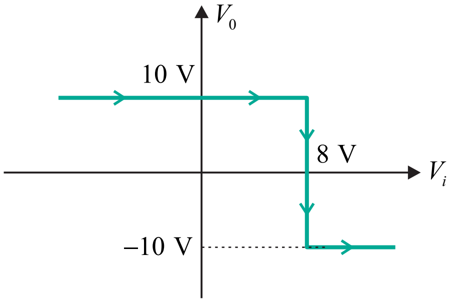
Concept of V_{UTP} :

(i) If $V_i < V_{UTP}$, then inverting terminal will be at lower potential than non-inverting terminal.

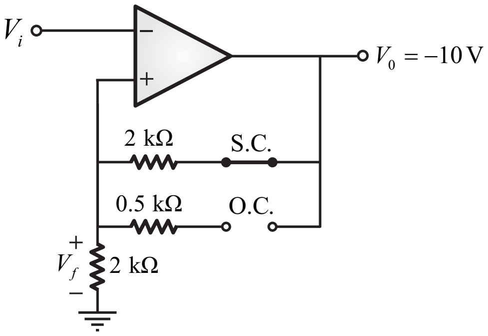
Hence, $V_0 = +V_{sat}$

(ii) If $V_i > V_{UTP}$, then non-inverting terminal will be at lower potential than inverting terminal.

Hence, $V_0 = -V_{sat}$



2. When $V_0 = -10\text{ V}$, D_1 is ON (i.e. short circuit) and D_2 is OFF (i.e. open circuit)



$$V_f = \frac{2}{2+2} \times V_0 = \frac{-10 \times 2}{2+2} = -5\text{ V}$$

$$V_f = V_{LTP} = -5\text{ V}$$

where, $V_{LTP} =$ Lower trigger (threshold) point voltage

Concept of V_{LTP} :

(i) If $V_i < V_{LTP}$, then inverting terminal will be at lower potential than non-inverting terminal.

Hence, $V_0 = +V_{sat}$

(ii) If $V_i > V_{LTP}$, then non-inverting terminal will be at lower potential than inverting terminal.

Hence, $V_0 = -V_{sat}$

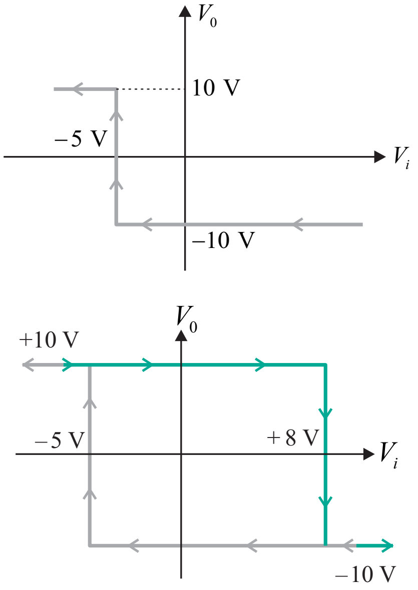


Fig. Transfer characteristics

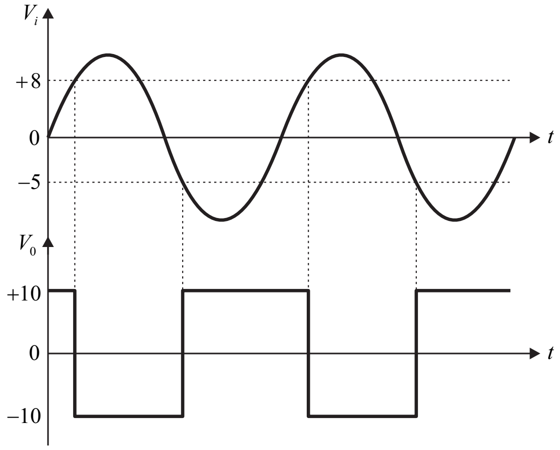
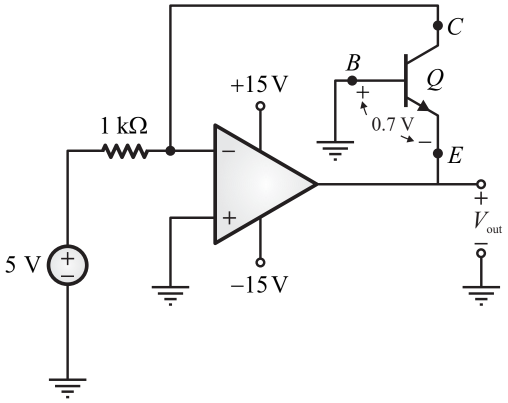


Fig. Output waveform

Hence, the correct option is (B).

6.5 (B)

Given circuit represent the logarithmic amplifier by using transistor is as shown below,



From figure, $V_B = 0,\ V_E = V_{out}$

Transistor is in active region.

Hence, $V_{BE} = 0.7\text{ V}$

So,
$$V_{BE} = V_B - V_E = 0.7\text{ V}$$
$$0 - V_{out} = 0.7\text{ V}$$
$$V_{out} = -0.7\text{ V}$$

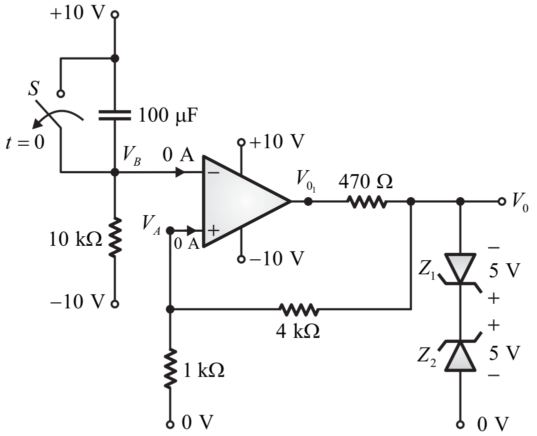
Hence, the correct option is (B).

Key Point

For designing and proper operation of Logarithmic amplifier using transistor, transistor should be in active region.

6.6 0.798

Given circuit is shown below,



Given figure represent astable multivibrator (positive feedback circuit).

Op-Amp output is given by,

$$V_{0_1} = \pm V_{sat} = \pm 10\text{ V}$$

Circuit output, $V_0 = \pm 5\text{ V}$

[Due to back to back connected Zener diodes]

V_A and V_B	V_{0_1}	Z_1	Z_2	V_0
$V_B > V_A$ $(V_- > V_+)$	$-V_{sat} = -10\text{ V}$	Break down (Reverse bias)	ON (Forward bias)	–5 V
$V_B < V_A$ $(V_- < V_+)$	$+V_{sat} = +10\text{ V}$	ON (Forward bias)	Break down (Reverse bias)	+5 V

Initially switch is closed i.e. initial voltage across capacitor is 0 V (uncharged capacitor).

At $t < 0 / t = 0^-$:

$$V_C(0^-) = 0 \text{ V}$$

$$V_B(0^-) = 10 - V_C(0^-) = 10 \text{ V}$$

If $V_0 = 5$ V, i.e. Z_1 is ON, and Z_2 is in breakdown region.

The upper trigger point voltage is given by,

$$V_{UTP} = V_A = \frac{1}{1+4} \times 5 = 1 \text{ V} \quad \text{[By VDR]}$$

If $V_0 = -5$ V, i.e. Z_1 is in breakdown region, and Z_2 is ON.

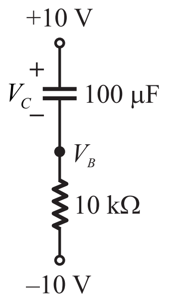
The lower trigger point voltage is given by,

$$V_{LTP} = V_A = \frac{1}{1+4} \times (-5) = -1 \text{ V}$$

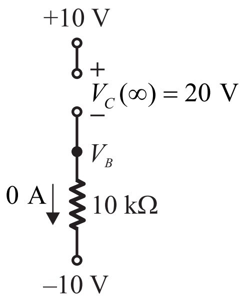
[By VDR]

At $t = 0$, S is opened and at $t = \infty$, capacitor becomes in steady state, hence it will act as open circuit.

At $t = 0$



At $t = \infty$



Capacitor voltage is given by,

$$V_C(t) = V_C(\infty) + \left[V_C(0^+) - V_C(\infty)\right] e^{-t/\tau}$$

(i) $V_C(0^-) = V_C(0^+) = 0$ V

[From property of capacitor]

(ii) $\tau = RC = 10 \times 10^3 \times 100 \times 10^{-6} = 1$ sec

(iii) $V_C(\infty) = 20$ V

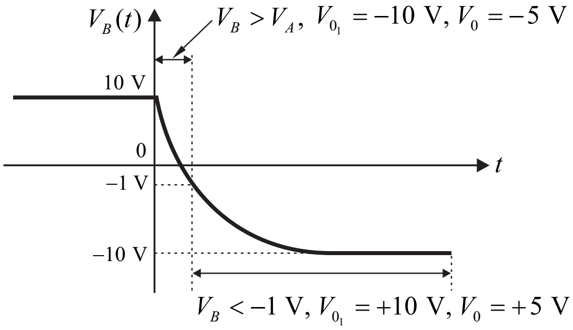
$$V_C(t) = 20(1 - e^{-t}) \qquad \dots \text{(i)}$$

From figure,

$$V_B(t) = 10 - V_C(t) = 10 - (20 - 20e^{-t})$$

$$V_B(t) = -10 + 20e^{-t} \qquad \dots \text{(ii)}$$

t	V_B
0	10 V
∞	– 10 V



At $t = 0$, $V_B = 10$ V, V_A may be –1 V or +1 V.

Therefore, $V_B > V_A$ and hence

$$V_{0_1} = -V_{sat} = -10 \text{ V}$$

Then, $V_0 = -5$ V,

$$V_A = V_{LTP} = -1 \text{ V}$$

$$V_{0_1} = +V_{sat}$$

Hence, $V_0 = +5$ V

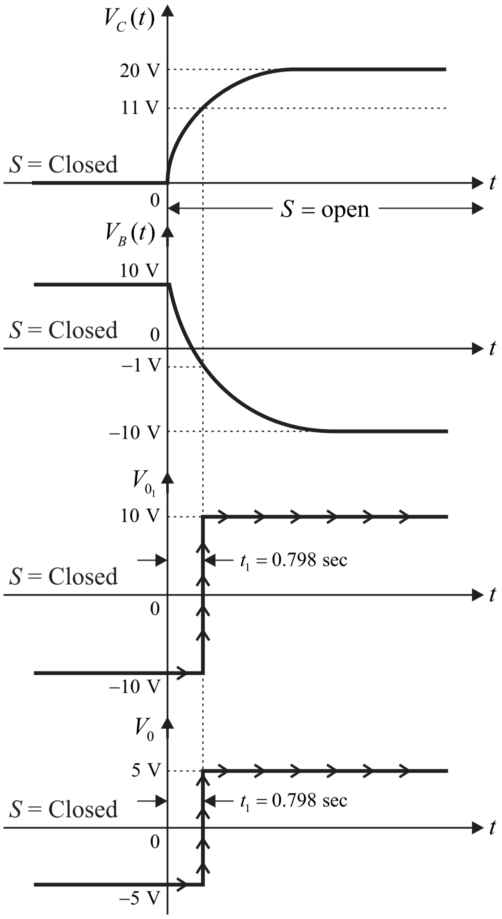
So, V_0 changes from – 5 V to + 5 V at the time instant t_1 when $V_B(t)$ is reduced to – 1 V.

$$-10 + 20e^{-t_1} = -1 \quad \text{[From equation (ii)]}$$

$$e^{-t_1} = \frac{9}{20}$$

$$e^{t_1} = \frac{20}{9}$$

$$t_1 = \ln\left(\frac{20}{9}\right) = 0.798 \text{ sec}$$



OR

Calculation of t_1 from capacitor voltage :

From figure,

$$10 = V_C(t) + V_B(t)$$

$$V_B(t) = 10 - V_C(t)$$

V_0 changes from –5 V to 5 V at $t = t_1$.

When $V_B(t) = -1$ V then $V_C(t) = 11$

$$20 - 20e^{-t_1} = 11 \quad \text{[From equation (ii)]}$$

$$20e^{-t_1} = 9$$

$$e^{-t_1} = \frac{9}{20}$$

$$e^{t_1} = \frac{20}{9}$$

$$t_1 = \ln\left(\frac{20}{9}\right) = 0.798 \text{ sec}$$

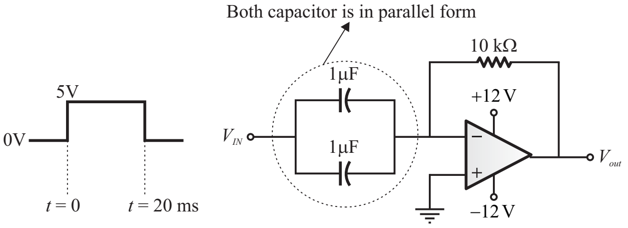
Hence, the time $t = t_1$ at which V_0 changes state is **0.798 sec.**

Key Point

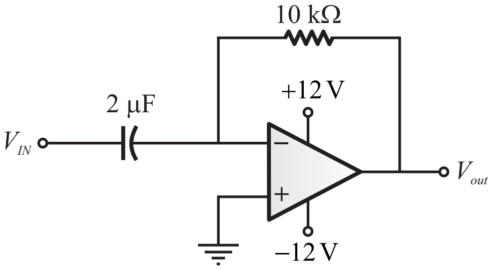
1. Back to back Zener diode used for limiting output voltage.
2. Limiting resistor (470 Ω) is used for maintaining the circuit in proper way. Due to this $V_{0_1} = \pm 10$ V and $V_0 = \pm 5$ V.

6.7 - 12

The given circuit is shown below,



Above figure can be redrawn as

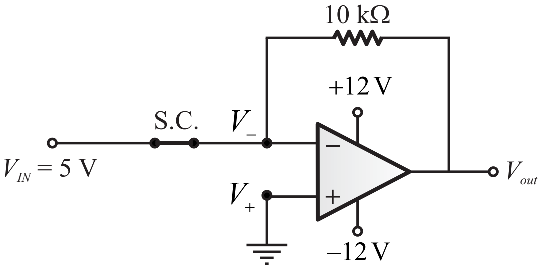


As capacitors are initially uncharged i.e.,

$$V_C(0^-) = 0 \text{ V}.$$

Therefore, at $t = 0^+$, capacitor will be short circuit as $V_C(0^-) = V_C(0^+)$.

So, above circuit can be redrawn as,



From above circuit it is clear that,

$$V_- = V_{IN} = +5 \text{ V}$$

$$V_+ = 0 \text{ V}$$

Here, $V_- \neq V_+$

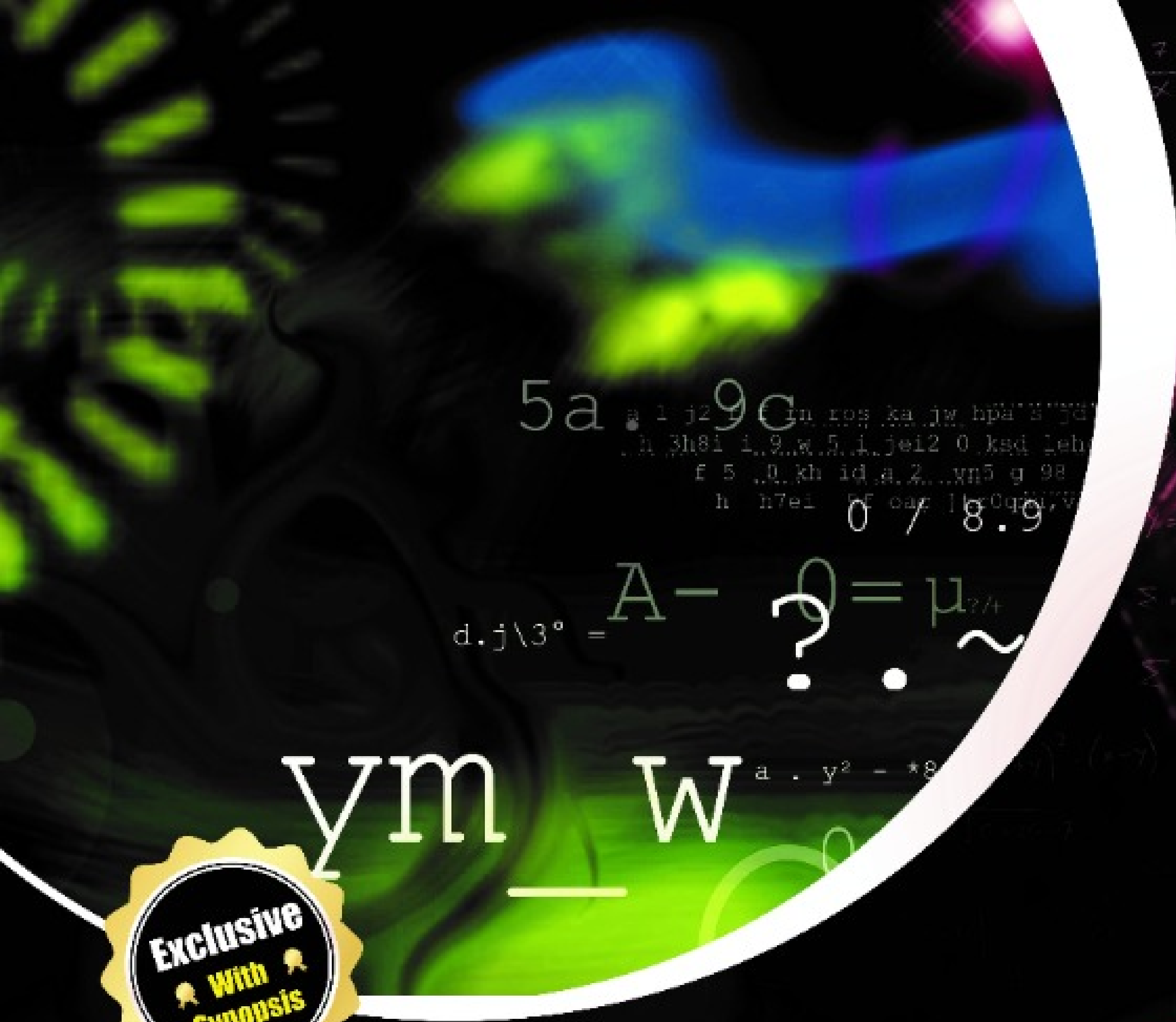
It means, voltage at both terminals of op-amp are unequal and fixed so virtual ground concept is not valid here and op-amp will work as comparator.

Thus, $V_- > V_+$

So, $V_{out} = -V_{sat} = -12\,\text{V}$

Hence, the output voltage V_{out} of this circuit at $t = 0^+$ is $-12\,\text{V}$.

❖❖❖❖



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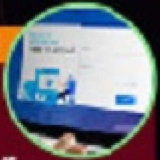

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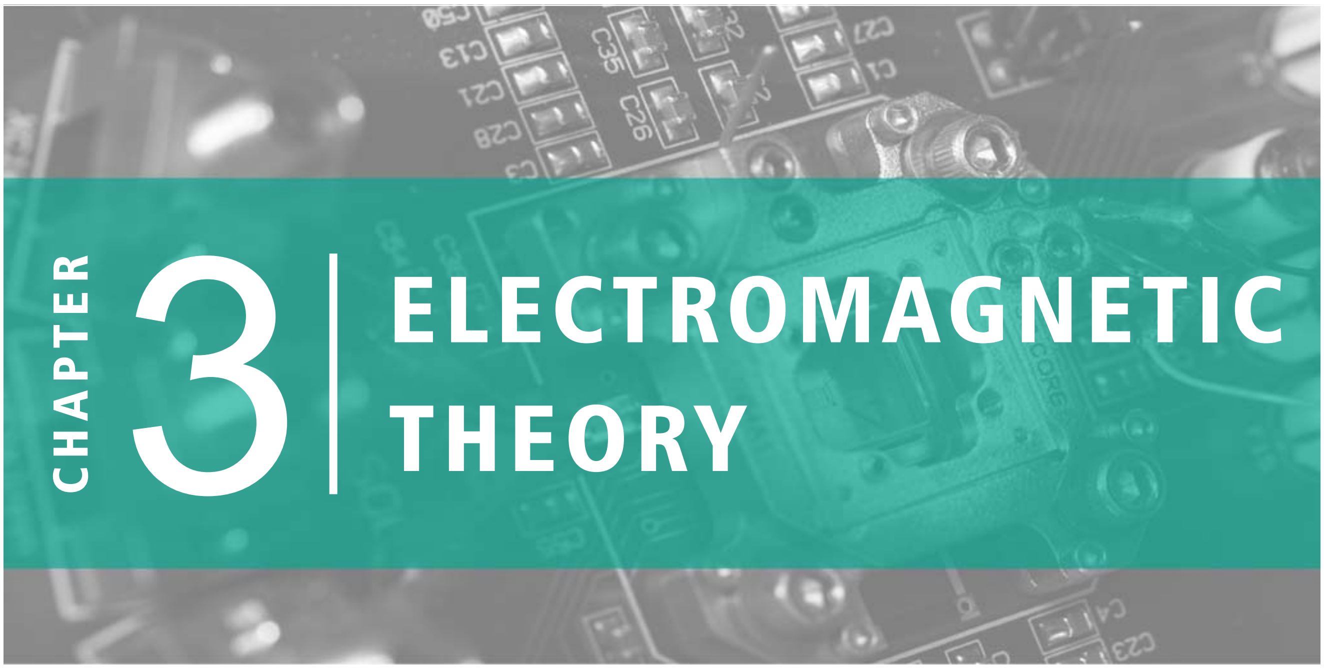


CHAPTER 3

ELECTROMAGNETIC THEORY

Marks Distribution of Electromagnetic Theory in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	2	7	16
2004	2	6	14
2005	2	6	14
2006	2	8	18
2007	2	7	16
2008	2	5	12
2009	2	3	8
2010	3	2	7
2011	3	3	9
2012	4	4	12
2013	1	2	5
2014 Set-1	2	3	8
2014 Set-2	2	3	8
2014 Set-3	2	2	6

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-4	2	2	6
2015 Set-1	2	3	8
2015 Set-2	2	3	8
2015 Set-3	2	4	10
2016 Set-1	2	4	10
2016 Set-2	3	4	11
2016 Set-3	2	4	10
2017 Set-1	2	3	8
2017 Set-2	2	3	8
2018	2	3	8
2019	3	3	9
2020	3	2	7
2021	5	3	11

Syllabus : Electromagnetic Theory

Maxwell's equations: differential and integral forms and their interpretation, boundary conditions, wave equation, Poynting vector. Plane waves and properties: reflection and refraction, polarization, phase and group velocity, propagation through various media, skin depth. Transmission lines: equations, characteristic impedance, impedance matching, impedance transformation, S-parameters, Smith chart. Rectangular and circular waveguides, light propagation in optical fibers, dipole and monopole antennas, linear antenna arrays.

Contents : Electromagnetic Theory

1

Basics of Electromagnetics

➢ Partial Synopsis

Coordinate Systems and their Related Parameters

S.N.	Parameters	Cartesian	Cylindrical	Spherical
1.	Variables	x, y, z	ρ, ϕ, z	r, θ, ϕ
2.	Range of variables	$-\infty < x < \infty$ $-\infty < y < \infty$ $-\infty < z < \infty$	$0 \le \rho < \infty$ $0 \le \phi \le 2\pi$ $-\infty < z < \infty$	$0 \le r < \infty$ $0 \le \theta \le \pi$ $0 \le \phi \le 2\pi$
3.	Representation	$\vec{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z$	$\vec{A} = A_\rho\,\hat{a}_\rho + A_\phi\,\hat{a}_\phi + A_z\,\hat{a}_z$	$\vec{A} = A_r\,\hat{a}_r + A_\theta\,\hat{a}_\theta + A_\phi\,\hat{a}_\phi$
4.	Differential length parameter	$dx =$ differential length in x direction $dy =$ differential length in y direction $dz =$ differential length in z direction	$d\rho =$ differential length in ρ direction $\rho d\phi =$ differential length in ϕ direction $dz =$ differential length in z direction	$dr =$ differential length in r direction $rd\theta =$ differential length in θ direction $r\sin\theta\,d\phi =$ differential length in ϕ direction
5.	Differential displacement	$d\vec{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$	$d\vec{l} = d\rho\,\hat{a}_\rho + \rho d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$	$d\vec{l} = dr\,\hat{a}_r + rd\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$
6.	Differential volume	$dV = dx\,dy\,dz$	$dV = \rho d\rho\; d\phi\; dz$	$dV = r^2\sin\theta\; dr\,d\theta\,d\phi$
7.	Differential surface area (With unit normal)	$d\vec{S}_x = dy\,dz\,\hat{a}_x$ $d\vec{S}_y = dx\,dz\,\hat{a}_y$ $d\vec{S}_z = dx\,dy\,\hat{a}_z$	$d\vec{S}_\rho = \rho\,d\phi\,dz\,\hat{a}_\rho$ $d\vec{S}_\phi = d\rho\,dz\,\hat{a}_\phi$ $d\vec{S}_z = \rho d\rho\,d\phi\,\hat{a}_z$	$d\vec{S}_r = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r$ $d\vec{S}_\theta = r\sin\theta\,d\phi\,dr\,\hat{a}_\theta$ $d\vec{S}_\phi = r\,dr\,d\theta\,\hat{a}_\phi$

Gradient

- Gradient is only defined for scalar point function.
  Consider scalar point function $V = f(x, y, z)$.

$$\text{Grad } V = \nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

- Resultant of gradient is a vector quantity.
- The magnitude of ∇V equals the maximum rate of change in V per unit distance.
- ∇V points in the direction of the maximum rate of change in V.
- The projection (or component) of ∇V in the direction of a unit vector $\hat{a}$ is $\nabla V \cdot \hat{a}$ and is called the directional derivative of V along $\hat{a}$. This is the rate of change of V in the direction of $\hat{a}$.

Divergence

- It is only defined for vector point function.

  Consider vector point function $\vec{V} = V_x\hat{a}_x + V_y\hat{a}_y + V_z\hat{a}_z$

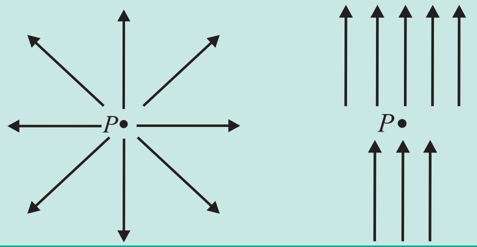
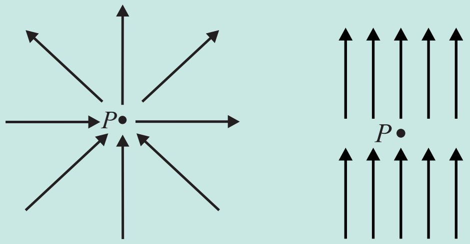
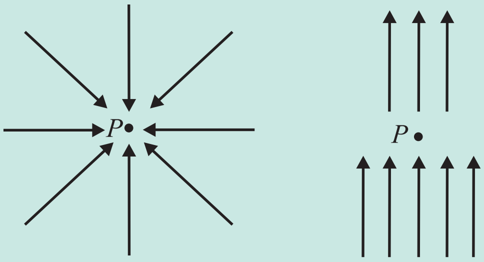
$$\text{Div } \vec{V} = \nabla \cdot \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

- Resultant of a divergence is a scalar quantity.

Remember

If divergence of any vector quantity is zero, then the vector is referred as a **solenoidal** vector or **incompressible** vector or **divergenceless**.

- Divergence of the vector field $\vec{V}$ at a given point is a measure of how much the field diverges from that point.

	Net flow is outward and hence divergence is positive.
	Net flow of vector quantity is zero, hence divergence is zero i.e., solenoidal vector.
	Net flow of vector quantity is inward, hence divergence is negative.

Curl

- It is only defined for a vector quantity.

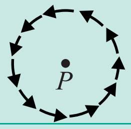
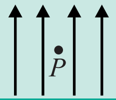
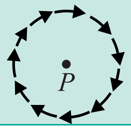
  Consider vector point function $\vec{V} = V_x\hat{a}_x + V_y\hat{a}_y + V_z\hat{a}_z$

$$\text{Curl } \vec{V} = \nabla\times\vec{V} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

- Resultant of curl is also a vector quantity.
- Curl describes the rotation of vector quantity.

Remember

If curl of vector is zero, then vector is referred as **irrotational** or **conservative** vector.

	Curl at point P points out of the page.
	Curl at point P is zero.
	Curl at point P points inside the page.

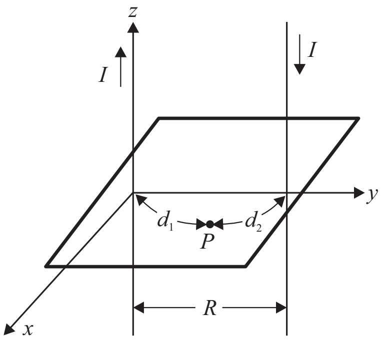
Remember

- Div [Curl $\vec{A}$] = 0 or D.C. = 0
- Curl [Grad ϕ] = 0 or C.G. = 0
- $\nabla\times(\vec{A}+\vec{B}) = \nabla\times\vec{A}+\nabla\times\vec{B}$
- $\nabla\times(V\vec{A}) = V\nabla\times\vec{A}+\nabla V\times\vec{A}$

➢ Sample Questions

1987 **IIT Bombay**

1.1 Two long parallel wires in free space are separated by a distance R and carry currents of equal magnitude by opposite in direction. At any general point, the z-component of



(A) The magnetic vector potential is $\frac{\mu_0 I}{4\pi}\ln\left(\frac{d_2^2}{d_1^2}\right)$.

(B) The magnetic induction is $\frac{\mu_0 I}{2\pi}\left(\frac{d_2}{d_1}\right)$.

(C) The magnetic induction is zero.

(D) The magnetic vector potential is $\frac{\mu_0 I}{4\pi}\left(\frac{d_1^2}{d_2^2}\right)$.

1993 IIT Bombay

1.2 A long solenoid of radius R, and having N turns per unit length carries a time dependent current $I(t) = I_0\cos(\omega t)$. The magnitude of induced electric field at a distance $R/2$ radially from the axis of the solenoid is

(A) $\frac{R}{2}\mu_0 N I_0 \omega\sin(\omega t)$

(B) $\frac{R}{4}\mu_0 N I_0 \omega\cos(\omega t)$

(C) $\frac{R}{4}\mu_0 N I_0 \omega\sin(\omega t)$

(D) $R\mu_0 N I_0 \omega\sin(\omega t)$

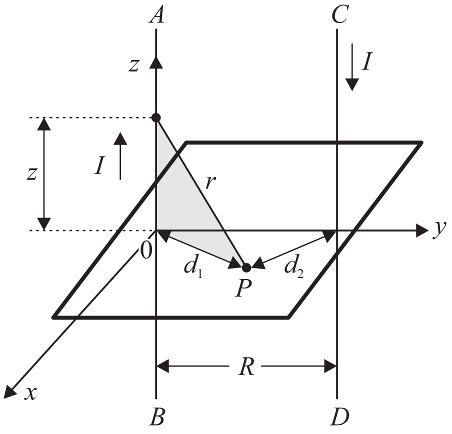
2021 IIT Bombay

1.3 For a vector field $\vec{D} = \rho\cos^2\phi\,\hat{a}_\rho + z^2\sin^2\phi\,\hat{a}_\phi$ in a cylindrical coordinate system (ρ, ϕ, z) with unit vector $\hat{a}_\rho, \hat{a}_\phi$ and $\hat{a}_z$, the net flux of $\vec{D}$ leaving the closed surface of the cylinder $(\rho = 3, 0 \le z \le 2)$ (Round off to 2 decimal places) is ______.

Explanations Basics of Electromagnetics

1.1 (A)

Magnetic vector potential A, at point P in x-y plane is shown below,



Let us take current I of equal magnitude but opposite in direction through the wires AB and CD.

Magnetic vector potential A_1 due to wire AB is given by,

$$A_1 = \frac{\mu_0 I}{4\pi}\int_{-L}^{L}\frac{dz}{r}$$

$$A_1 = \frac{\mu_0 I}{4\pi}\int_{-L}^{L}\frac{1}{\sqrt{z^2+d_1^2}}dz$$

[From figure, $r = \sqrt{z^2+d_1^2}$]

$$A_1 = \frac{\mu_0 I}{2\pi}\int_{0}^{L}\frac{1}{\sqrt{z^2+d_1^2}}dz$$

$$A_1 = \frac{\mu_0 I}{2\pi}\left[\ln(z+\sqrt{z^2+d_1^2})\right]_0^L$$

$$\left[\int\frac{1}{\sqrt{x^2\pm a^2}}dx = \ln\left[x+\sqrt{x^2\pm a^2}\right]\right]$$

$$A_1 = \frac{\mu_0 I}{2\pi}\left[\ln(L+\sqrt{L^2+d_1^2}) - \ln(0+\sqrt{0+d_1^2})\right]$$

$$A_1 = \frac{\mu_0 I}{2\pi}\left[\ln(L+\sqrt{L^2+d_1^2}) - \ln d_1\right]$$

Since $L >> d_1$,

$$A_1 = \frac{\mu_0 I}{2\pi}\left[\ln(L+L) - \ln d_1\right]$$

$$A_1 = \frac{\mu_0 I}{2\pi}\left[\ln(2L) - \ln d_1\right] \quad \ldots \text{(i)}$$

Similarly, magnetic vector potential A_2 due to wire CD is given by,

$$A_2 = -\frac{\mu_0 I}{2\pi}\left[\ln(2L) - \ln d_2\right] \quad \ldots \text{(ii)}$$

[Negative sign is due to direction of current through the wire CD]

Hence, total magnetic vector potential A due to both wires will be,

$$A = A_1 + A_2$$

From equation (i) and (ii),

$$A = \left(\frac{\mu_0 I}{2\pi}\left[\ln(2L) - \ln d_1\right]\right)$$

$$+\left(-\frac{\mu_0 I}{2\pi}\left[\ln(2L) - \ln d_2\right]\right)$$

$$A = \frac{\mu_0 I}{2\pi}\left[\ln d_2 - \ln d_1\right] = \frac{\mu_0 I}{2\pi}\ln\frac{d_2}{d_1}$$

$$A = \frac{\mu_0 I}{4\pi}\ln\frac{d_2^2}{d_1^2} \qquad \left[m\log_e a = \log_e a^m\right]$$

Hence, the correct option is (A).

Key Point

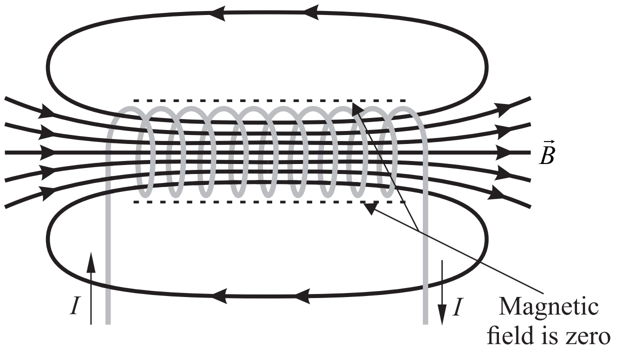
(i) Vector magnetic potential, A exists when J is present.

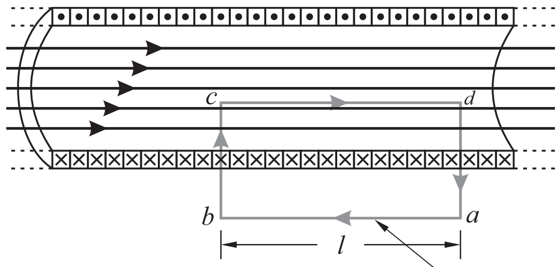
(ii) It has applications to obtain radiation characteristics of antenna, apertures and also to obtain radiation leakage from transmission lines, waveguide and microwave ovens.

1.2 (C)

Given : A long solenoid of radius R, and having N turns per unit length carries a time dependent current $I(t) = I_0\cos(\omega t)$.

A solenoid consists of a long conducting wire made up of many loops packed closely together. For coil that are closely together, the magnetic field is approximately uniform towards the center. If the current flows through the conducting wire then it generates magnetic field and it is uniform.





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From Ampere's law,

$$\oint \vec{H}\cdot d\vec{l} = I_{\text{enc}}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enc}} \qquad \ldots \text{(i)}$$

$$\oint \vec{B}\cdot d\vec{l} = \underbrace{\int_a^b \vec{B}\cdot d\vec{l}}_{\vec{B}=0} + \underbrace{\int_b^c \vec{B}\cdot d\vec{l}}_{\theta=90^0} + \int_c^d \vec{B}\cdot d\vec{l}$$

$$+\underbrace{\int_d^a \vec{B}\cdot d\vec{l}}_{\theta=90^0}$$

$$\oint \vec{B}\cdot d\vec{l} = \int_c^d B\,dl\cos 0^0 = Bl \qquad \ldots \text{(ii)}$$

If I is the current in the wire of the solenoidal then the enclosed I is given by, nI where n is number of loops.

From equation (i) and (ii),

$$Bl = nI\mu_0$$

$$B = \frac{nI\mu_0}{l} = NI\mu_0$$

where, $\frac{n}{l} =$ number of loops per unit length

$$B = \mu_0 NI \qquad [H = NI(t) \text{ and } B = \mu H]$$

$$B = \mu_0(H) = \mu_0 NI(t)$$

$$B = \mu_0 NI_0 \cos(\omega t)$$

According to Faraday's law,

$$\oint E \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$E \cdot 2\pi \frac{R}{2} = \mu_0 NI_0 \omega \sin(\omega t) \frac{\pi R^2}{4}$$

$$E = \frac{R}{4} \mu_0 NI_0 \omega \sin(\omega t)$$

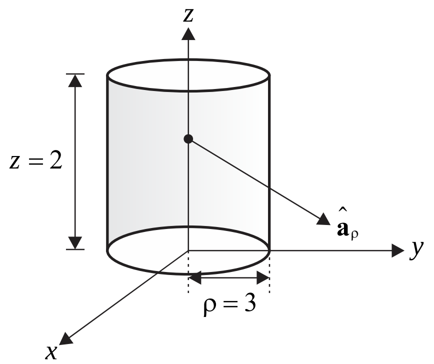
Hence, the correct option is (C).

1.3 **56.548**

Method 1

Given : $\vec{D} = \rho \cos^2 \phi \hat{a}_\rho + z^2 \sin^2 \phi \hat{a}_\phi$

Cylinder $(\rho = 3, 0 \le z \le 2)$ is shown below,



From this given cylinder,

$$\rho = 3\,\text{m},\ z = 0 \text{ to } 2\,\text{m},\ \phi = 0 \text{ to } 2\pi$$

Shaded portion shows closed surface of cylinder and unit vector that is perpendicular this surface is $\hat{a}_\rho$ thus, net flux (ψ) leaving the closed surface of cylinder is

$$\psi = \oiint_s \vec{D}.\vec{ds} = \int_{\phi=0}^{2\pi} \int_{z=2}^{2} (\rho \cos^2 \phi\ \hat{a}_\rho + z^2 \sin^2 \phi\ \hat{a}_\phi) \cdot (\rho\, d\phi\, dz\ \hat{a}_\rho)$$

$$\psi = \int_{\phi=0}^{2\pi} \int_{z=0}^{2} \rho^2 \cos^2 \phi\, d\phi\, dz \Big|_{\rho=3}$$

$$[\because \hat{a}_\rho.\hat{a}_\rho = 1 \text{ and } \hat{a}_\rho.\hat{a}_\phi = 0]$$

$$\psi = \rho^2 \int_{z=0}^{2} dz \int_{\phi=0}^{2\pi} \cos^2 \phi\, d\phi$$

$$\psi = 3^2 [z]_{z=0}^{z=2} \int_{\phi=0}^{2\pi} \left(\frac{1 + \cos 2\phi}{2}\right) d\phi$$

$$\psi = 9[2-0]\left[\frac{\phi}{2} + \frac{\sin 2\phi}{4}\right]_{\phi=0}^{\phi=2\pi}$$

$$\psi = 18(\pi - 0) = 18\pi = 56.548 \text{ C}$$

Hence, the correct answer is 56.548.

Method 2

Given : $\vec{D} = \rho \cos^2 \phi \hat{a}_\rho + z^2 \sin^2 \phi \hat{a}_\phi$

Net flux of $\vec{D}$ leaving the closed surface of cylinder $\psi = \oiint \vec{D} \cdot d\vec{S}$

By Gauss divergence theorem

$$\psi = \oiint \vec{D} \cdot d\vec{S} = \int_\rho \int_\phi \int_z \vec{\nabla} \cdot \vec{D}\, dV$$

$$\rho \to 0 \text{ to } 3$$

$$\phi \to 0 \text{ to } 2\pi$$

$$z \to 0 \text{ to } 2$$

$$\vec{\nabla} \cdot \vec{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho D_\rho + \frac{1}{\rho} \frac{\partial}{\partial \phi} D_\phi + \frac{\partial}{\partial z} D_z$$

$$\vec{\nabla} \cdot \vec{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \cdot \rho \cos^2 \phi + \frac{1}{\rho} \frac{\partial}{\partial \phi} z^2 \sin^2 \phi + \frac{\partial}{\partial z} 0$$

$$\vec{\nabla} \cdot \vec{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho^2 \cos^2 \phi + \frac{1}{\rho} (z^2 \cdot 2 \sin \phi \cos \phi)$$

$$\vec{\nabla} \cdot \vec{D} = \frac{1}{\rho} 2\rho \cos^2 \phi + \frac{1}{\rho} z^2 \sin 2\phi$$

$$\vec{\nabla}\cdot\vec{D} = 2\cos^2\phi + \frac{z^2}{\rho}\sin 2\phi$$

So, $$\psi = \int_{\rho=0}^{3}\int_{\phi=0}^{2\pi}\int_{z=0}^{2} \vec{\nabla}\cdot\vec{D}\,dV$$

where $dV = \rho d\rho d\phi dz$

$$\psi = \int_{\rho=0}^{3}\int_{\phi=0}^{2\pi}\int_{z=0}^{2}\left(2\cos^2\phi + \frac{z}{\rho}\sin^2\phi\right)\rho d\rho d\phi dz$$

$$\psi = \int_{0}^{3}\int_{0}^{2\pi}\int_{0}^{2} 2\rho\cos^2\phi d\rho d\phi dz$$

$$+\int_{0}^{3}\int_{0}^{2\pi}\int_{0}^{2} z^2\sin 2\phi d\rho d\phi dz$$

$$\psi = \left[\frac{2\rho^2}{2}\right]_0^3 [z]_0^2 \int_0^{2\pi}\frac{1+\cos 2\phi}{2}d\phi$$

$$+\left[\frac{z^3}{3}\right]_0^2 [\rho]_0^3 \left[\frac{-\cos 2\phi}{2}\right]_0^{2\pi}$$

$$\psi = 9.2\left[\frac{\phi}{2}+\frac{\sin 2\phi}{4}\right]_0^{2\pi}$$

$$+\frac{8}{3}\cdot 3\left[\frac{-\cos 4\pi + \cos 0}{2}\right]$$

$$\psi = 18\left[\frac{2\pi}{2}+\frac{\sin 4\pi}{4}-0\right]+8\left[\frac{-1+1}{2}\right]$$

$$\psi = 18\pi = 56.54$$

Hence, the correct answer is 56.54.

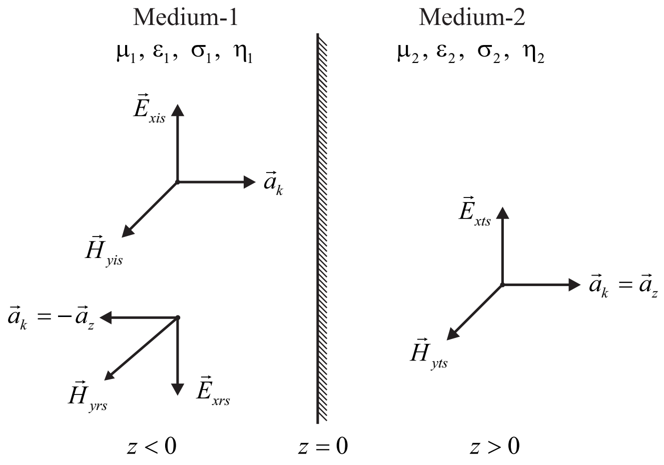
❖❖❖❖

2

Plane Wave Propagation

➢ Partial Synopsis

Reflection Coefficient for Magnetic Field



Reflection Coefficient for magnetic field is given by,

$$\alpha_H = \frac{H_{Yro}}{H_{Yio}} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

Transmission Coefficient for magnetic field is given by,

$$\beta_H = \frac{H_{Yto}}{H_{Yio}} = \frac{2\eta_1}{\eta_1 + \eta_2} = 1 + \alpha_H$$

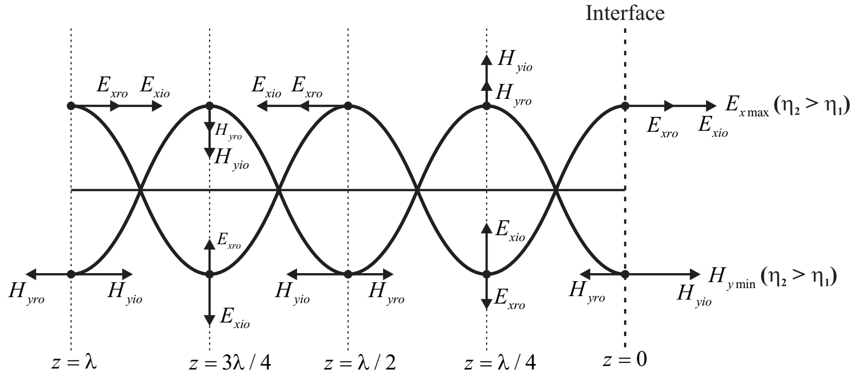
Electric and Magnetic field patterns for two different Media

Case 1 : $(\eta_2 > \eta_1)$

$$\alpha_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \rightarrow \text{positive} \qquad \alpha_H = \frac{\eta_1 - \eta_2}{\eta_2 + \eta_1} \rightarrow \text{negative}$$

z	$\beta z = \frac{2\pi}{\lambda} \cdot z$	$E_{xis}(z) = E_{xio} e^{-j\beta z} + E_{xro} e^{j\beta z}$	$H_{yis}(z) = H_{yio} e^{-j\beta z} - H_{yro} e^{j\beta z}$
0	0	$E_{xis}(0) = E_{xio} + E_{xro}$	$H_{yis}(0) = H_{yio} - H_{yro}$

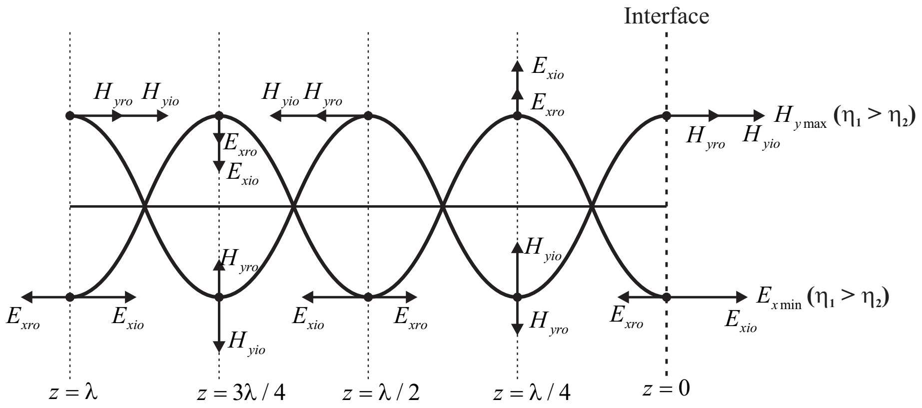
$\frac{\lambda}{4}$	$\frac{\pi}{2}$	$E_{xis}\left(\frac{\lambda}{4}\right)=-j\,E_{xio}+j\,E_{xro}$	$H_{yis}\left(\frac{\lambda}{4}\right)=-j\,H_{yio}-j\,H_{yro}$
$\frac{\lambda}{2}$	π	$E_{xis}\left(\frac{\lambda}{2}\right)=-E_{xio}-E_{xro}$	$H_{yis}\left(\frac{\lambda}{2}\right)=-H_{yio}+H_{yro}$
$\frac{3\lambda}{4}$	$\frac{3\pi}{2}$	$E_{xis}\left(\frac{3\lambda}{4}\right)=jE_{xio}-jE_{xro}$	$H_{yis}\left(\frac{3\lambda}{4}\right)=j\,H_{yio}+j\,H_{yro}$
λ	2π	$E_{xis}(\lambda)=E_{xio}+E_{xro}$	$H_{yis}(\lambda)=H_{yio}-H_{yro}$



Case 2 : $(\eta_1>\eta_2)$

$$\alpha_E=\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\;\rightarrow\;\text{negative}\qquad\qquad \alpha_H=\frac{\eta_1-\eta_2}{\eta_2+\eta_1}\;\rightarrow\text{positive}$$

z	$\beta z=\frac{2\pi}{\lambda}\cdot z$	$H_{yis}(z)=H_{yio}e^{-j\beta z}+H_{yro}e^{j\beta z}$	$E_{xis}(z)=E_{xio}e^{-j\beta z}-E_{xro}e^{j\beta z}$
0	0	$H_{yis}(0)=H_{yio}+H_{yro}$	$E_{xis}(0)=E_{xio}-E_{xro}$
$\frac{\lambda}{4}$	$\frac{\pi}{2}$	$H_{yis}\left(\frac{\lambda}{4}\right)=-j\,H_{yio}+j\,H_{yro}$	$E_{xis}\left(\frac{\lambda}{4}\right)=-j\,E_{xio}-j\,E_{xro}$
$\frac{\lambda}{2}$	π	$H_{yis}\left(\frac{\lambda}{2}\right)=-H_{yio}-H_{yro}$	$E_{xis}\left(\frac{\lambda}{2}\right)=-E_{xio}+E_{xro}$
$\frac{3\lambda}{4}$	$\frac{3\pi}{2}$	$H_{yis}\left(\frac{3\lambda}{4}\right)=jH_{yio}-jH_{yro}$	$E_{xis}\left(\frac{3\lambda}{2}\right)=j\,E_{xio}+j\,E_{xro}$
λ	2π	$H_{yis}(\lambda)=H_{yio}+H_{yro}$	$E_{xis}(\lambda)=E_{xio}-E_{xro}$



Standing wave ratio :

It is defined as the ratio of maximum value of the wave to the minimum value. Standing wave ratio for electric field is ratio of maximum amplitude to the minimum amplitude of the electric field. Similarly, for magnetic field, it is ratio of maximum amplitude to the minimum amplitude of the magnetic field.

$$\text{SWR}_\text{E} = \left|\frac{E_{\max}}{E_{\min}}\right| = \frac{|E_{xio}| + |E_{xro}|}{|E_{xio}| - |E_{xro}|}$$

$$\text{SWR}_\text{E} = \frac{1 + \left|\dfrac{E_{xro}}{E_{xio}}\right|}{1 - \left|\dfrac{E_{xro}}{E_{xio}}\right|} = \frac{1 + |\alpha_E|}{1 - |\alpha_E|}$$

$$\text{SWR}_\text{H} = \left|\frac{H_{\max}}{H_{\min}}\right| = \frac{|H_{yio}| + |H_{yro}|}{|H_{yio}| - |H_{yro}|}$$

$$\text{SWR}_\text{H} = \frac{1 + \left|\dfrac{H_{yro}}{H_{yio}}\right|}{1 - \left|\dfrac{H_{yro}}{H_{yio}}\right|} = \frac{1 + |\alpha_H|}{1 - |\alpha_H|}$$

Standing Wave Ratio (SWR)

$$SWR_E = \left|\frac{E_{\max}}{E_{\min}}\right| = \frac{|E_{Xio}| + |E_{Xro}|}{|E_{Xio}| - |E_{Xro}|} = \frac{1 + |\alpha_E|}{1 - |\alpha_E|}$$

$$SWR_H = \left|\frac{H_{\max}}{H_{\min}}\right| = \frac{|H_{Yio}| + |H_{Yro}|}{|H_{Yio}| - |H_{Yro}|} = \frac{1 + |\alpha_H|}{1 - |\alpha_H|}$$

Poynting Vector and Poynting Theorem

- It is a vector product of electric & magnetic field.
- Poynting vector describes power flow per unit area.
- Direction of poynting vector is same as direction of propogation.

$$\vec{S} = \vec{E} \times \vec{H} \quad \text{Watt/m}^2$$

- Surface integration of poynting vector gives power for a given surface.

$$P = \oint \vec{S} \cdot \overrightarrow{dA} \text{ watt}$$

- We can interpret the RHS as decreasing in the electric & magnetic field power stored in volume & further reduces by the ohmic dissipation.

Poynting Vector in a Medium Media

S. No.	Medium	Poynting Vector (ω / m^2)
1.	Free Space	$S_{avg} = \frac{E^2_{rms}}{\eta_0} \hat{a}_z$
2.	Lossless dielectric	$S_{avg} = \frac{E^2_{rms}}{\eta} \hat{a}_z$
3.	Lossy dielectric	$S_{avg} = \frac{E^2_{rms}}{\eta} e^{-2\alpha z} \cos\theta_\eta \hat{a}_z$
4.	Good conductor	$S_{avg} = \frac{E^2_{rms}}{\eta} e^{-2\alpha z} \cos 45^0 \hat{a}_z$

➢ Sample Questions

2000 IIT Kharagpur

2.1 A *TEM* wave is incident normally upon a perfect conductor. The $\vec{E}$ and $\vec{H}$ fields at the boundary will be, respectively

(A) minimum and minimum.

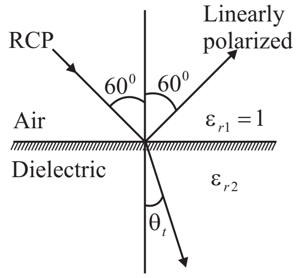
(B) maximum and maximum.

(C) minimum and maximum.

(D) maximum and minimum.

2007 IIT Kanpur

2.2 A right circularly polarized (RCP) plane wave is incident at an angle of 60^0 to the normal, on an air-dielectric interface. If the reflected wave is linearly polarized the relative dielectric constant ε_{r2} is



(A) $\sqrt{2}$ (B) $\sqrt{3}$

(C) 2 (D) 3

2013 IIT Bombay

2.3 Consider a vector field $\vec{A}(r)$. The closed loop line integral $\oint \vec{A} \bullet \vec{dl}$ can be expressed as

(A) $\oiint (\nabla \times \vec{A}) \bullet \vec{dS}$ over the closed surface bounded by the loop.

(B) $\oiiint (\nabla \bullet \vec{A})\, dv$ over the closed volume bounded by the loop.

(C) $\iiint (\nabla \bullet \vec{A})\, dv$ over the open volume bounded by the loop.

(D) $\iint (\nabla \times \vec{A}) \bullet \vec{dS}$ over the open surface bounded by the loop.

2021 IIT Bombay

2.4 Consider a rectangular coordinate system (x, y, z) with unit vector $\mathbf{a_x}, \mathbf{a_y}$ and $\mathbf{a_z}$. A plane wave travelling in the region $z \geq 0$ with electric field vector

$\mathbf{E} = 10\cos(2\times10^8 t + \beta z)\hat{\mathbf{a}}_y$ is incident normally on the plane at $z = 0$, where β is the phase constant. The region $z \geq 0$ is in free space and the region $z < 0$ is filled with a lossless medium (permittivity $\varepsilon = \varepsilon_0$ permeability $\mu = 4\mu_0$, where $\varepsilon_0 = 8.85\times10^{-12}$ F/m and $\mu_0 = 4\pi\times10^{-7}$ H/m). The value of reflection coefficient is

(A) $\frac{1}{3}$

(B) $\frac{3}{5}$

(C) $\frac{2}{5}$

(D) $\frac{2}{3}$

❖❖❖❖

Explanations Plane Wave Propagation

2.1 (C)

Region 2 is a perfect conductor which will behaves as a perfect reflector.

Hence, $E_2 = 0$

$$\eta_2 = \left|\frac{E_2}{H_2}\right| = 0$$

Electric field in region 1,

$$E_{x_{1s}} = E_{x_{io}} e^{-\gamma z} + E_{x_{ro}} e^{\gamma z}$$

$$E_{x_{1s}}(z) = E_{x_{io}} e^{-j\beta z} + E_{x_{ro}} e^{j\beta z} \quad \ldots \text{(i)}$$

Magnetic field in region 1,

$$H_{y_{1s}} = H_{y_{io}} e^{-\gamma z} + H_{y_{ro}} e^{\gamma z}$$

$$H_{y_{1s}}(z) = H_{y_{io}} e^{-j\beta z} + H_{y_{ro}} e^{j\beta z} \quad \ldots \text{(ii)}$$

Reflection coefficient for electric field,

$$\Gamma_E = \frac{E_{x_{ro}}}{E_{x_{io}}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0 - \eta_1}{0 + \eta_1} = -1$$

$$E_{x_{ro}} = \Gamma_E E_{x_{io}} \quad \ldots \text{(iii)}$$

Transmission coefficient for electric field,

$$\tau_E = \frac{E_{x_{to}}}{E_{x_{io}}} = 1 + \Gamma_E = 1 - 1 = 0$$

Transmission coefficient for magnetic field,

$$\Gamma_H = \frac{H_{y_{ro}}}{H_{y_{io}}} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \frac{\eta_1 - 0}{\eta_1 + 0} = 1$$

$$H_{y_{ro}} = \Gamma_H H_{y_{io}} \quad \ldots \text{(iv)}$$

Reflection coefficient for magnetic field,

$$\tau_H = 1 + \Gamma_H = 1 + 1 = 2$$

From equation (i) and (iii),

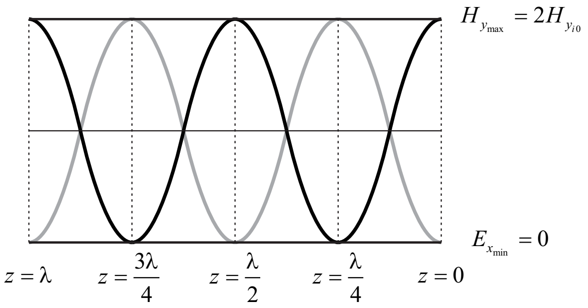
$$E_{x_{1s}}(z) = E_{x_{io}} e^{-j\beta z} + \Gamma_E E_{x_{io}} e^{j\beta z}$$

From equation (ii) and (iv),

$$H_{y_{1s}}(z) = H_{y_{io}} e^{-j\beta z} + \Gamma_H H_{y_{io}} e^{j\beta z}$$

Observing the variations in $E_{x_{1s}}$ and $H_{y_{1s}}$ with respect to z to analyze the maximum and minimum values,

z	βz	$E_{x_{i0}} e^{-j\beta z} + \Gamma_E E_{x_{i0}} e^{j\beta z}$	$H_{y_{io}} e^{-j\beta z} + \Gamma_H H_{y_{io}} e^{j\beta z}$
0	0	$E_{x_{i0}} - E_{x_{i0}} = 0$	$H_{y_{io}} + H_{y_{io}} = 2H_{y_{io}}$
$\frac{\lambda}{4}$	$\frac{\pi}{2}$	$-jE_{x_{i0}} - jE_{x_{i0}} = -j2E_{x_{i0}}$	$-jH_{y_{io}} + jH_{y_{io}} = 0$
$\frac{\lambda}{2}$	π	$-E_{x_{i0}} + E_{x_{i0}} = 0$	$-H_{y_{io}} - H_{y_{io}} = -2H_{y_{io}}$
$\frac{3\lambda}{4}$	$\frac{3\pi}{2}$	$jE_{x_{i0}} + jE_{x_{i0}} = j2E_{x_{i0}}$	$jH_{y_{io}} - jH_{y_{io}} = 0$
λ	2π	$E_{x_{i0}} - E_{x_{i0}} = 0$	$H_{y_{io}} + H_{y_{io}} = 2H_{y_{io}}$



It is a case of short circuit load in transmission line. In this case current is maximum and voltage is minimum.

Current → Magnetic field

Voltage → Electric field

From figure, H is maximum and E is minimum.

Hence, the correct option is (C).

2.2　(D)

Given : Angle of incidence $\theta_i = 60^0$

Method 1

If light strikes on interface so that there is a 90^0 angle between the transmitted and reflected light then reflected light will be linearly polarized. The direction of polarization (the way the electric field vectors point) is parallel to the plane of the interface.

$$\theta_t + \theta_r = 90^0$$

$$\theta_t = 90^0 - \theta_r = 90^0 - 60^0 = 30^0 \ \{\because \theta_i = \theta_r\}$$

From Snell's law of refraction,

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

where,

$$n_1 = \text{refractive index of air} = c\sqrt{\mu_0\varepsilon_0}$$

$$n_2 = \text{refractive index of dielectric } = c\sqrt{\mu_2\varepsilon_2}$$

Hence, $c\sqrt{\mu_0\varepsilon_0}\sin 60^0 = c\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_{r_2}}\sin 30^0$

$$\sqrt{\varepsilon_{r_2}} = \frac{\sin 60^0}{\sin 30^0} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

$$\varepsilon_{r_2} = 3$$

Hence, the correct option is (D).

Method 2

If circularly polarized or elliptically polarized wave incidents at Brewster angle, then only the reflected and transmitted waves are linearly polarized.

When EMW is incident at Brewster angle, the reflected wave is linearly polarized because reflection coefficient for parallel component is zero.

$$\tan\theta_B = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}$$

$$\tan 60^0 = \sqrt{\frac{\varepsilon_{r2}}{1}}$$

$$\sqrt{\varepsilon_{r2}} = \sqrt{3}$$

$$\varepsilon_{r2} = 3$$

Hence, the correct option is (D).

🕮 Key Point

(i) If an arbitrary polarized wave is incident at the Brewster angle, the parallel polarization is completely transmitted, but the perpendicular polarization is only partially transmitted.

The reflected wave has only perpendicular polarization irrespective of the polarization of the incident wave. In other words the reflected wave is linearly polarized (perpendicular polarization) irrespective of the state of polarization of incident wave.

Even a randomly polarized wave incident at Brewster angle produces a linearly polarized wave after reflection.

(ii) Brewster angle is also referred as polarizing angle.

(iii) The reflection coefficient are different for waves parallel and perpendicular to plane of incidence.

2.3　(D)

Stokes theorem states that the circulation of a vector field $\vec{A}$ around closed path is equal to the surface integral of the curl of $\vec{A}$ over the open surface bounded by closed path, provided $\vec{A}$ and $\nabla\times\vec{A}$ are continuous on surface.

$$\oint \vec{A}\cdot d\vec{l} = \iint (\nabla\times\vec{A})\cdot d\vec{S}$$

where $(\nabla\times\vec{A})$ is curl of $\vec{A}$.

Hence, the correct option is (D).

2.4 (A)

Given : Medium 1 $(z \geq 0)$

(i) Medium 1 is free space.

(ii) $\mu_1 = \mu_0,\ \varepsilon_1 = \varepsilon_0$

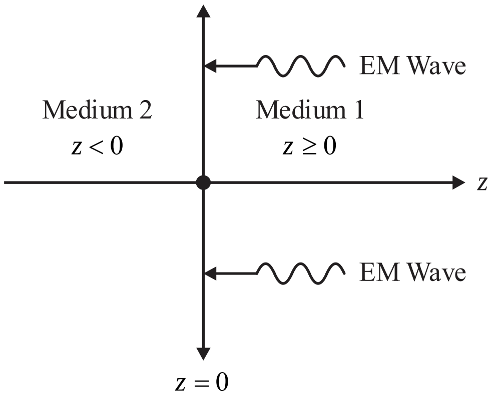
(iii) $E_1 = 10\cos(2\times10^8 t + \beta z)\,\hat{\mathbf{a}}_y$

(iv) $\omega = 2\times10^8$ rad/sec, $E_m = 10$ V/m

Medium 2 $(z < 0)$

(i) Medium 2 is lossless

(ii) $\mu_2 = 4\mu_0,\ \varepsilon_2 = \varepsilon_0$



Reflection coefficient $(\Gamma) = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

$$\Gamma = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}} - \sqrt{\frac{\mu_1}{\varepsilon_1}}}{\sqrt{\frac{\mu_2}{\varepsilon_2}} + \sqrt{\frac{\mu_1}{\varepsilon_1}}}$$

Since, $\varepsilon_2 = \varepsilon_1$

$$\Gamma = \frac{\sqrt{\mu_2} - \sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}}$$

$$\Gamma = \frac{\sqrt{4\mu_0} - \sqrt{\mu_0}}{\sqrt{4\mu_0} + \sqrt{\mu_0}}$$

$$\Gamma = \frac{\sqrt{4} - \sqrt{1}}{\sqrt{4} + \sqrt{1}} = \frac{1}{3}$$

Hence, the correct option is (A).

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3

Polarization

➢ Partial Synopsis

2. Circular / Elliptical Polarization Representation :

$$\vec{E}(z,\,t)=E_{x0}\cos(\omega t-\beta z)\vec{a}_x+e^{j\delta}E_{y0}\cos(\omega t-\beta z)\vec{a}_y$$

Phase difference, $\delta=(2n+1)\dfrac{\pi}{2}$ (odd multiples of 90^0)

where, $n=0,\,1,\,2,\,3,\,..........$

For circular polarization,

$$E_{x0}=E_{y0}=E_0$$

For elliptical polarization,

$$E_{x0}\neq E_{y0}$$

Direction of propagation,

$$\vec{a}_k=\vec{a}_z$$

Case 1 : $n = 0$

$$\delta=\frac{\pi}{2}\quad\Rightarrow\qquad e^{j\pi/2}=\cos 90^0+j\sin 90^0=j$$

$$\vec{E}(z,\,t)=E_0\cos(\omega t-\beta z)\vec{a}_x+jE_0\cos(\omega t-\beta z)\vec{a}_y=E_0(\vec{a}_x+j\vec{a}_y)\cos(\omega t-\beta z)$$

This equation can also be written in following form,

$$\vec{E}(z,\,t)=E_0\cos(\omega t-\beta z)\vec{a}_x+e^{j\pi/2}E_0\cos(\omega t-\beta z)\vec{a}_y$$

$$\vec{E}(z,\,t)=E_0\cos(\omega t-\beta z)\vec{a}_x+E_0\cos(\omega t-\beta z+90^0)\vec{a}_y$$

$$\vec{E}(z,\,t)=E_0\cos(\omega t-\beta z)\vec{a}_x-E_0\sin(\omega t-\beta z)\vec{a}_y$$

$$\boldsymbol{\vec{E}(0,\,t)=E_0\cos(\omega t)\vec{a}_x+E_0\cos(\omega t+90^0)\vec{a}_y}\qquad\text{... (i)}$$

$\boldsymbol{\omega t}$	$\boldsymbol{E_0\cos\omega t}$	$\boldsymbol{E_0\cos(\omega t+90^0)}$	$\boldsymbol{\lvert E\rvert=\sqrt{E_x^2+E_y^2}}$
0^0	E_0	0	E_0
45^0	$\dfrac{E_0}{\sqrt{2}}$	$\dfrac{-E_0}{\sqrt{2}}$	E_0

90^0	0	$-E_0$	E_0
135^0	$\frac{-E_0}{\sqrt{2}}$	$\frac{-E_0}{\sqrt{2}}$	E_0
180^0	$-E_0$	0	E_0
225^0	$\frac{-E_0}{\sqrt{2}}$	$\frac{E_0}{\sqrt{2}}$	E_0
270^0	0	E_0	E_0
315^0	$\frac{E_0}{\sqrt{2}}$	$\frac{E_0}{\sqrt{2}}$	E_0
360^0	E_0	0	E_0

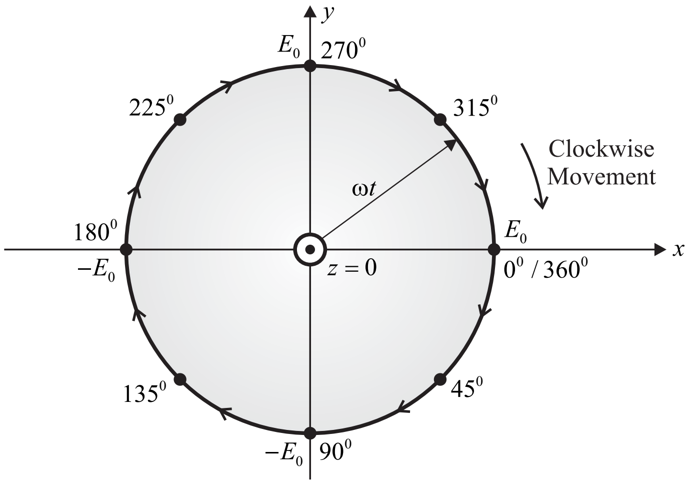
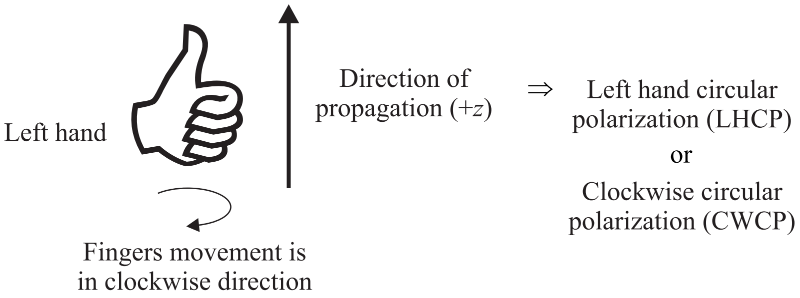


Fig. (a) Left hand circular polarization (LHCP)

Key Point

Thumb represents direction of propagation and fingers represent movement of electric field vector i.e. clockwise or anticlockwise.



For elliptical polarization : $E_{x0} \neq E_{y0}$

Direction of propagation is same as above.

If $E_{x0} < E_{y0}$

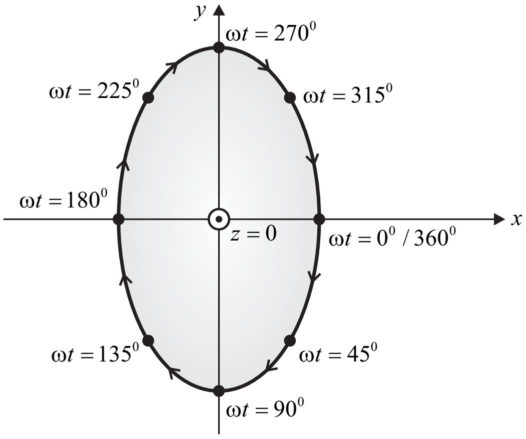


Fig. (a) Left hand elliptical polarization (LHEP)

➢ Sample Questions

2016 **IISc Bangalore**

3.1 If a right-handed circularly polarized wave is incident normally on plane perfect conductor, then the reflected wave will be **[Set - 03]**

(A) right-handed circularly polarized

(B) left-handed circularly polarized.

(C) elliptically polarized with a tilt angle of 45^0.

(D) horizontally polarized.

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Explanations Polarization

Introduction of Polarization

Scan for Video Explanation

Concept of Linear Polarization

Scan for Video Explanation

Concept of Circular & Elliptical Polarization

Scan for Video Explanation

Previous Year GATE Questions & Practice Questions On Polarization (Part 1)

Scan for Video Solution

Previous Year GATE Questions & Practice Questions On Polarization (Part 2)

Scan for Video Solution

Previous Year GATE Questions & Practice Questions On Polarization (Part 3)

Scan for Video Solution

3.1 **(B)**

Consider a right hand circular polarized wave,

$$\vec{E}_i = E_0 \cos(\omega t - \beta z)\vec{a}_x$$

$$+E_0 \cos(\omega t - \beta z - 90^0)\vec{a}_y$$

$$E_{i\|} = E_x = E_0 \cos(\omega t - \beta z)$$

$$E_{i\perp} = E_y = E_0 \cos\left(\omega t - \beta z - 90^0\right)$$

$$E_r = E_{i\|}\Gamma_{E\|} + E_{i\perp}\Gamma_{E\perp} \qquad \ldots \text{(i)}$$

Reflection coefficient of electric field for parallel component is given by,

$$\Gamma_{E\|} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

Since, $\eta_2 = 0$ for perfect conductor

$$\Gamma_{E\|} = -1 = 1\angle 180^0$$

Reflection coefficient of electric field for perpendicular component is given by,

$$\Gamma_{E\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

Since, $\eta_2 = 0$ for perfect conductor

$$\Gamma_{E\perp} = -1 = 1\angle 180^0$$

From equation (i),

$$\vec{E}_r = \left(1\angle 180^0\right) E_0 \cos\left(\omega t + \beta z\right)\vec{a}_x + \left(1\angle 180^0\right) E_0 \cos\left(\omega t + \beta z - 90^0\right)\vec{a}_y$$

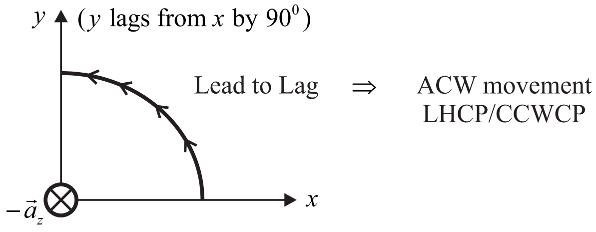
$$\vec{E}_r = E_0 \cos\left(\omega t + \beta z + 180^0\right)\vec{a}_x + E_0 \cos\left(\omega t + \beta z + 90^0\right)\vec{a}_y$$

$$E_{x0} = E_{y0} = E_0$$

Direction of propagation $\vec{a}_k = -\vec{a}_z$

Phase difference $\delta = 90^0$

Refer Case 4



In given wave, E_x leads E_y by angle of 90^0.

Thus, the given wave is left-hand circularly polarized.

Hence, the correct option is (B).

Key Point

General Results :

(i) If right circularly polarized (RCP) wave incidents on perfect conductor then reflected wave is left circularly polarized (LCP).

(ii) If left circularly polarized (LCP) wave incidents on perfect conductor then reflected wave is right circularly polarized (RCP).

(iii) If right circularly polarized (RCP) wave incidents on polystyrene then reflected wave is left elliptically polarized (LEP) and transmitted wave is right elliptically polarized (REP).

(iv) If circularly polarized or elliptically polarized wave incidents at Brewster angle, then the reflected and transmitted waves are linearly polarized.

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4 Transmission Lines

➢ Partial Synopsis

Input impedance of transmission line

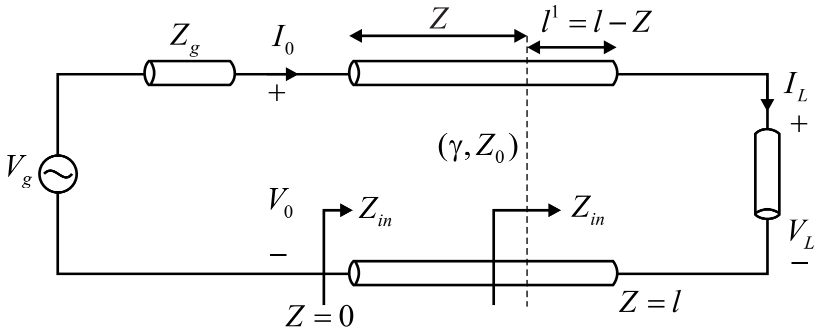


Fig. 3 : Input impedance due to a line terminated by a load.

Incident Voltage is given by,

$$V_0^+ = \frac{1}{2}(V_0 + I_0 Z_0)$$

Reflected Voltage is given by,

$$V_0^- = \frac{1}{2}(V_0 - Z_0 I_0)$$

Input impedance of lossy transmission line is given by :

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right]$$

Input impedance f lossless transmission line :

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L} \right]$$

Input impedance of lossless transmission line for different Load condition

Condition of load	Expression of input impedance
1. Short circuit $(Z_L = 0)$	$Z_{in} = j Z_0 \tan\beta l$
2. Open circuit $(Z_L = \infty)$	$Z_{in} = -j Z_0 \cot\beta l$
3. Matched load $(Z_L = Z_0)$	$Z_{in} = Z_0$ $Z_{in} = Z_L$

Input impedance of shorted transmission line $(Z_L = 0)$

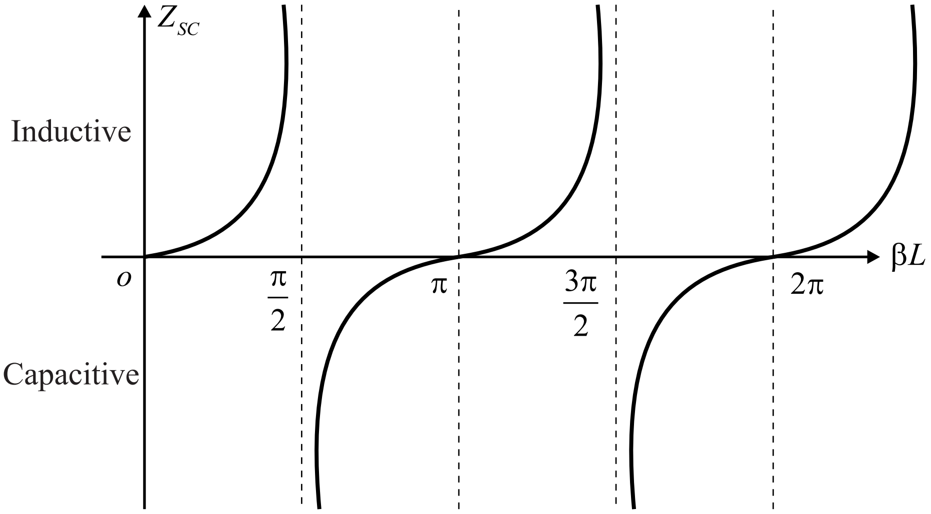


Fig. 4 : Input impedance for shorted T.L.

Input impedance of open-circuited transmission line $(Z_L = \infty)$

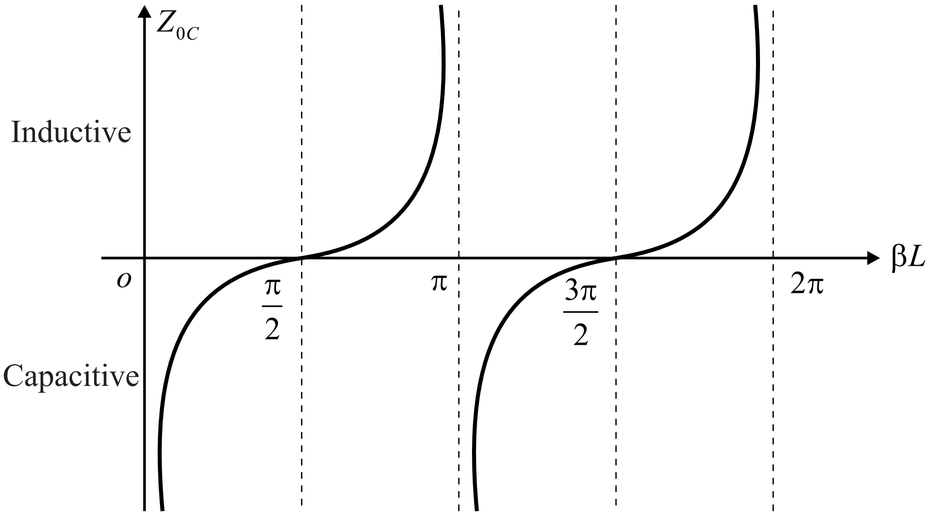


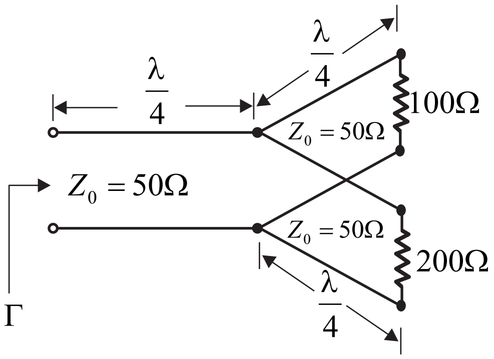
Fig. 5 : Input impedance characteristics for open-circuited T.L.

$$Z_0 = \sqrt{Z_{SC} Z_{OC}}$$

➢ Sample Questions

2007 IIT Kanpur

4.1 The parallel branches of a 2-wire transmission line are terminated in $100\,\Omega$ and $200\,\Omega$ resistors as shown in the figure. The characteristic impedance of the line is $Z_0 = 50\,\Omega$ and each section has a length of $\frac{\lambda}{4}$. The voltage reflection coefficient Γ at the input is



(A) $-j\frac{7}{5}$ (B) $\frac{-5}{7}$

(C) $j\frac{5}{7}$ (D) $\frac{5}{7}$

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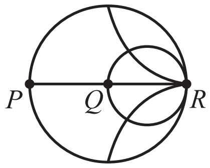
4.2 The points P, Q and R shown on the Smith chart (normalized impedance chart) in the following figure represent



(A) P: Open Circuit, Q: Short Circuit, R: Matched Load

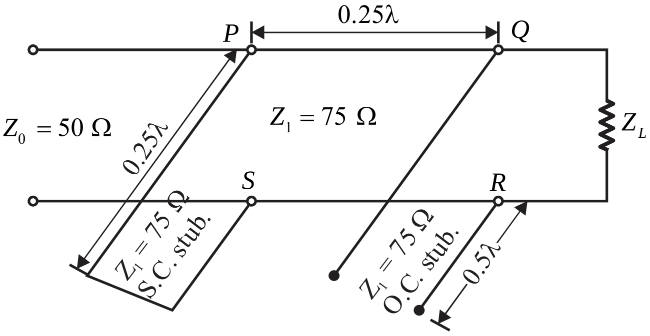
(B) P: Open Circuit, Q: Matched Load, R: Short Circuit

(C) P: Short Circuit, Q: Matched Load, R: Open Circuit

(D) P: Short Circuit, Q: Open Circuit, R: Matched Load

2021 IIT Bombay

4.3 The impedance matching network shown in the figure is to match a lossless line having characteristic impedance $Z_0 = 50\ \Omega$ with a load impedance Z_L. A quarter-wave line having a characteristic impedance $Z_1 = 75\ \Omega$ is connected to Z_L. Two stubs having characteristic impedance of $75\ \Omega$ each are connected to this quarter-wave line. One is a short-circuited (S.C.) stub of length 0.25λ connected across PS and the other one is an open-circuited (O.C.) stub of length 0.5λ connected across QR.



The impedance matching is achieved when the real part of Z_L is

(A) 112.5 Ω (B) 75.0 Ω

(B) 50.0 Ω (D) 33.3 Ω

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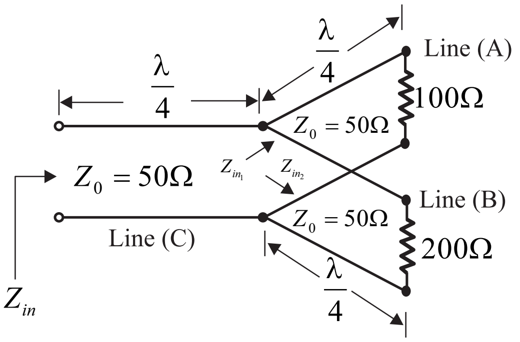
Explanations Transmission Lines

4.1 (D)

Method 1

Given figure is shown below,



Input impedance of $\frac{\lambda}{4}$ length line is given by,

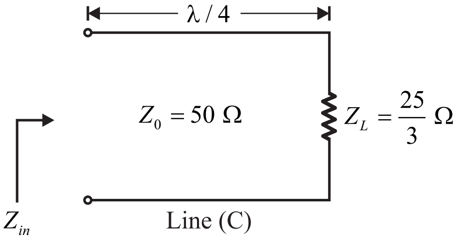
$$Z_{in} = \frac{Z_0^2}{Z_L}$$

For line A, $Z_{in_1} = \frac{(50)^2}{100} = 25\ \Omega$

For line B, $Z_{in_2} = \frac{(50)^2}{200} = \frac{25}{2}\ \Omega$

Effective load impedance at line C,

$$Z_L = \left(\frac{25}{2}\right) || (25) = \frac{\frac{25}{2} \times 25}{\frac{25}{2} + 25} = \frac{25}{3}\ \Omega$$



$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(50)^2}{25/3} = 300\ \Omega$$

Reflection coefficient at input end is given by,

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{300 - 50}{300 + 50} = \frac{5}{7}$$

Hence, the correct option is (D).

Method 2

Reflection coefficient at load will be

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{25}{3} - 50}{\frac{25}{3} + 50} = -\frac{5}{7}$$

Reflection coefficient at any distance 'l' from load is given by,

$$\Gamma_{z=l} = \Gamma_{\text{load}}\, e^{-j2\beta l}$$

$$\Gamma_{z=l} = \frac{-5}{7} \times e^{-j2\times\frac{2\pi}{\lambda}\times\frac{\lambda}{4}} = \frac{-5}{7} e^{-j\pi} = \frac{5}{7}$$

Hence, the correct option is (D).

4.2 (C)

Concept of Smith chart : Smith chart represents polar plot of real part of reflection coefficient Vs imaginary part of reflection coefficient

There are two family of circles on this graph

(i) Constant resistance circle

(ii) Constant reactance circle

Normalized load impedance is given as

$$z_L = \frac{Z_L}{Z_0}$$

If load is complex then $z_L = r + jx$

Reflection coefficient at load is

$$\Gamma_{load} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

$$\Gamma_{load} = |\Gamma| e^{j\phi} \qquad \text{[Polar form]}$$

where $0 \le \phi \le 2\pi$ and $-1 < \Gamma < +1$

Hence $|\Gamma_{load}|_{\max} = 1$

$$z_L = \frac{1 + \Gamma_{load}}{1 - \Gamma_{load}}$$

$$r + jx = \frac{1 + \Gamma_{real} + j\Gamma_{imag}}{1 - \Gamma_{real} - j\Gamma_{imag}}$$

Real part of normalized load impedance that represents resistance is given as

$$r = \frac{1-(\Gamma_{real})^2-(\Gamma_{imag})^2}{(1-\Gamma_{real})^2+(\Gamma_{imag})^2} \qquad \ldots \text{(i)}$$

Imaginary part of normalized load impedance, that represents the reactance is given as

$$x = \frac{2\Gamma_{imag}}{(1-\Gamma_{real})^2+(\Gamma_{imag})^2} \qquad \ldots \text{(ii)}$$

Rearranging equation (i), we get

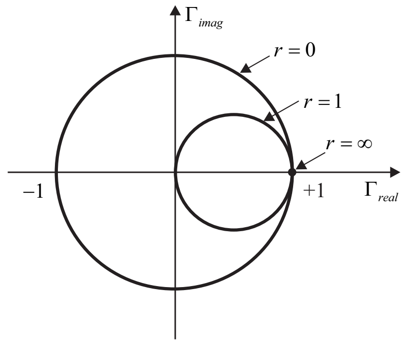
$$\left(\Gamma_{real}-\frac{r}{r+1}\right)^2+(\Gamma_{imag})^2=\left(\frac{1}{r+1}\right)^2 \qquad \ldots \text{(iii)}$$

It represents the equation of **constant resistance circle** having

$$\text{Centre}\left(\frac{r}{r+1},0\right) \text{ and Radius}\left(\frac{1}{r+1}\right)$$

Plot for constant resistance circle :

r	Centre $\left(\frac{r}{r+1},0\right)$	Radius $\left(\frac{1}{r+1}\right)$
0	(0, 0)	1
1	$\left(\frac{1}{2},0\right)$	$\left(\frac{1}{2}\right)$
∞	(1, 0)	0



Rearranging equation (ii), we get

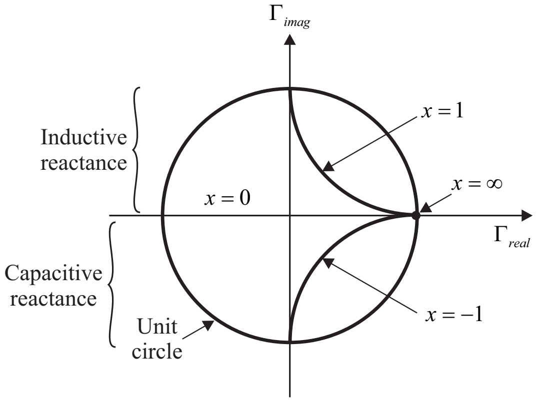
$$(\Gamma_{real}-1)^2+\left(\Gamma_{imag}-\frac{1}{x}\right)^2=\left(\frac{1}{x}\right)^2 \qquad \ldots\text{(iv)}$$

It represents the equation of **constant reactance circle** having

$$\text{Centre}\left(1,\frac{1}{x}\right) \text{ and Radius}\left(\frac{1}{x}\right)$$

Plot for constant reactance circle :

x	Centre $\left(1,\frac{1}{x}\right)$	Radius $\left(\frac{1}{x}\right)$
0	$(1,\infty)$	∞
-1	$(1,-1)$	-1
1	(1, 1)	1
∞	(1, 0)	0



Overlapping of constant resistance and constant reactance circles gives smith chart as shown in figure :

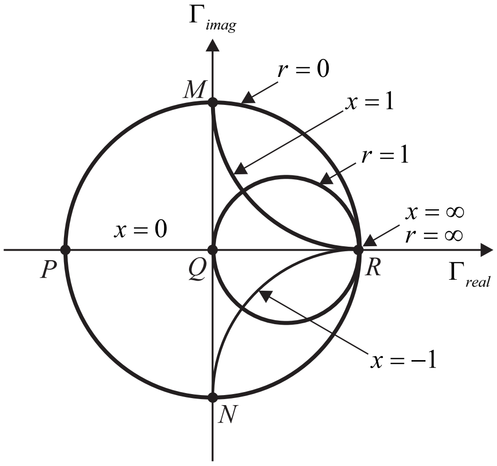
1. **At point *P* :**

$$r=0,\ x=0$$

$$z_L=r+jx=0$$

$$z_L=\frac{Z_L}{Z_0}=0$$

Hence $Z_L=0$



Hence point P represents the condition of short circuited load

2. **At point Q :**

$$r = 1,\ x = 0$$

$$z_L = r + jx = 1$$

$$z_L = \frac{Z_L}{Z_0} = 1$$

$$Z_L = Z_0$$

Hence point Q represents the condition of matched load.

3. **At point R :**

$$r = \infty,\ x = \infty$$

$$z_L = r + jx = \infty$$

$$z_L = \frac{Z_L}{Z_0} = \infty$$

$$Z_L = \infty$$

Hence point R represents the condition of open circuited load.

Similarly points M $(r = 0,\ x = 1)$ and $N(r = 0,$ $x = -1)$ represents purely inductive and purely capacitive load impedance conditions respectively.

Hence, the correct option is (C).

4.3 (A)

Given :

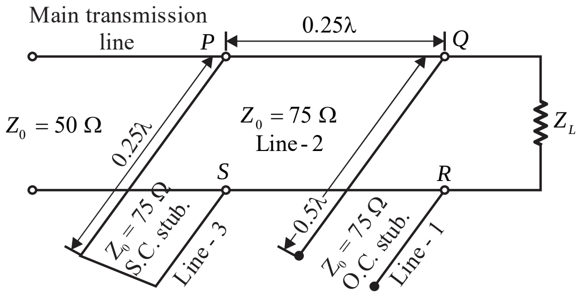
Lossless line characteristic impedance $Z_0 = 50\Omega$

Open circuit stub characteristic impedance, $Z_{oc} = 75\Omega$

Short circuit stub characteristic impedance, $Z_{sc} = 75\Omega$

Method 1

The given arrangement of transmission line is shown below,



Here, $(Z_L)_{Line-1} = (Z_L)_{oc} = \infty$

$$(Z_L)_{Line-3} = (Z_L)_{sc} = 0$$

Input Impedance of $\frac{\lambda}{2}$ long line-1,

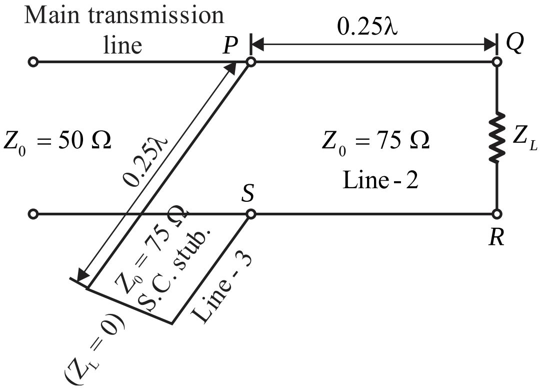
$$Z_{in}\left[l = \frac{\lambda}{2}\right]_{\text{Line-1}} = (Z_L)_{\text{Line-1}} = \infty$$

(Open circuit)

Thus total impedance at terminal QR is,

$$Z_{QR} = Z_L \parallel (Z_{in})_{Line1} = Z_L \parallel \infty = Z_L$$

Now arrangement of Transmission Line becomes as,



Thus Input Impedance of line 2 is,

$$Z_{in}\left[l = \frac{\lambda}{4}\right]_{\text{Line-2}} = \frac{Z_0^{\ 2}}{Z_L} = \frac{75^2}{Z_L}$$

Input Impedance of Line 3 is,

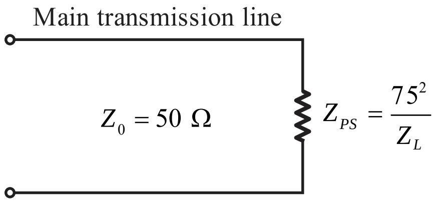
$$Z_{in}\left[l = \frac{\lambda}{4}\right]_{\text{Line-3}} = \frac{Z_0^{\ 2}}{(Z_L)_{Line-3}} = \frac{75^2}{0} = \infty$$

Thus total impedance at terminal PS is,

$$Z_{PS} = (Z_{in})_{Line-2} \parallel (Z_{in})_{Line-3}$$

$$Z_{PS}=\frac{75^2}{Z_L}\parallel\infty=\left(\frac{75^2}{Z_L}\right)$$

Now Transmission Line arrangement becomes as-



Hence Z_{PS} work as load for main Transmission line.

For matching of main Transmission line with load (Z_{PS}),

$$Z_{PS}=Z_o$$

$$\frac{75^2}{Z_L}=50\Rightarrow Z_L=\frac{75^2}{50}=112.5\Omega$$

Hence, the correct option is (A)

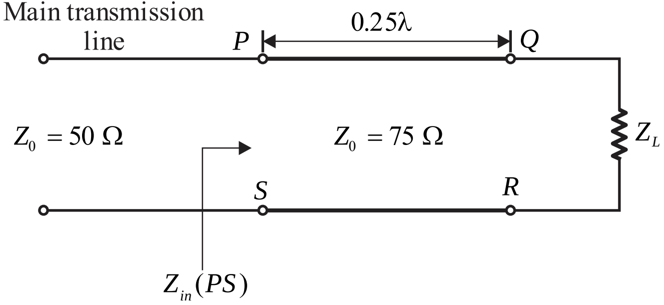
Method 2

From given arrangement, it is clear that,

$$Z_{in}\left[l=\frac{\lambda}{2}\right]_{Line-1}=(Z_L)_{Line-1}=\infty$$

$$Z_{in}\left[l=\frac{\lambda}{4}\right]_{Line-3}=\frac{Z_0^2}{(Z_L)_{Line-3}}=\frac{Z_0^2}{0}=\infty$$

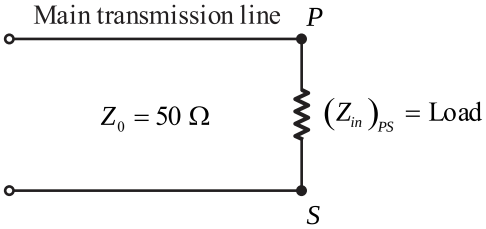
So, input impedance of both line-1 and line-3 are ∞ (i.e. open circuit) so it does not make any effect on main transmission line, so given transmission line configuration becomes as,



Thus Input Impedance at terminal PS is,

$$(Z_{in})_{PS}=\frac{Z_0^{\ 2}}{Z_L}=\frac{75^2}{Z_L}\Omega$$

So $(Z_{in})_{PS}$ work as load for main transmission line so arrangement of transmission line becomes as,



Hence Z_{PS} work as load for main Transmission line, for matching of main Transmission line with load (Z_{PS})

$$Z_{PS}=Z_o$$

$$\frac{75^2}{Z_L}=50$$

$$Z_L=\frac{75^2}{50}=112.5\Omega$$

Hence, the correct option is (A)

❖❖❖❖

6

Waveguides

➢ Partial Synopsis

2. Cut-off frequency :

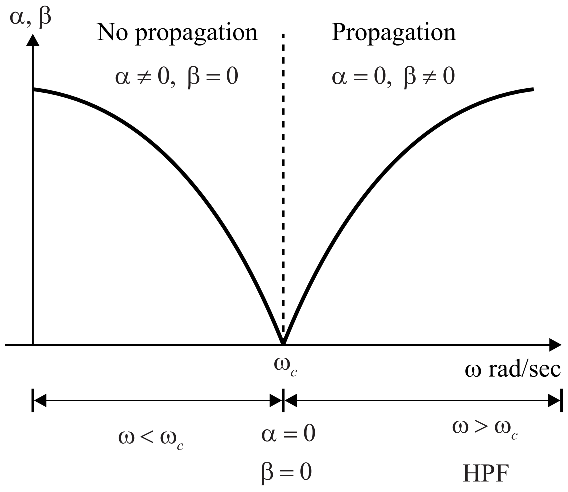
$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2}$$

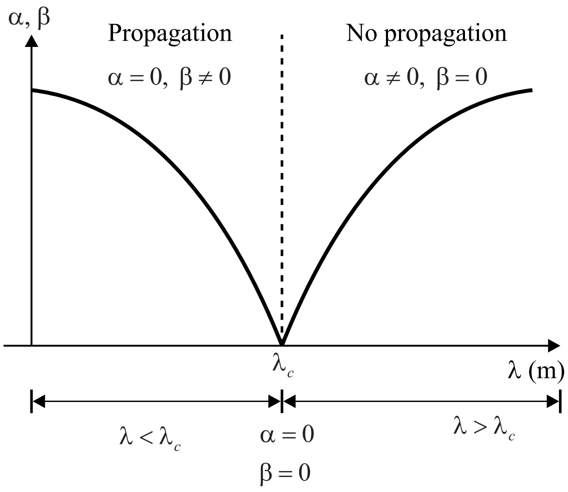
The cut-off frequency is the operating frequency below which attenuations occurs and above which propagation takes place.

3. Cut-off wavelength :

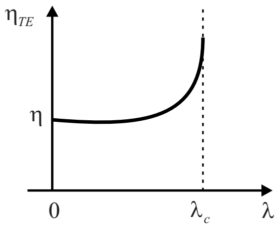
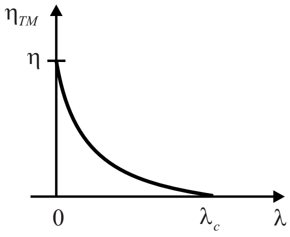
$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Below figure shows the behavior of waveguide as a High pass filter.





4. Summary of Guided Parameters in Rectangular Waveguide

Parameter	TE Modes	TM Modes
Dominant mode	TE_{10}	TM_{11}
E_z, H_z	$E_z = 0,\ H_z \neq 0$ $H_z = H_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$	$E_z \neq 0,\ H_z = 0$ $E_z = E_0 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$
Guided phase velocity	$v_{pz} = \frac{v_p}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$	$v_{pz} = \frac{v_p}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$
Guided group velocity	$v_{gz} = v_p\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$	$v_{gz} = v_p\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$
Guided wavelength	$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$	$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$
Guided phase constant	$\beta_g = \beta\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$	$\beta_g = \beta\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$
Guided intrinsic impedance	$\eta_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$ $\eta_{TE} > \eta$	$\eta_{TM} = \eta\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$ $\eta_{TM} < \eta$
Guided intrinsic impedance vs Wavelength		

(i) TE_{10} has the lowest cut-off frequency (or the longest cut-off wavelength) of all the *TE* modes, therefore it is a dominant *TE* mode.

(ii) Two or more modes having the same cut-off frequency, are *Degenerated modes*. In a rectangular waveguide the corresponding TE_{mn} and TM_{mn} mode are always degenerate.

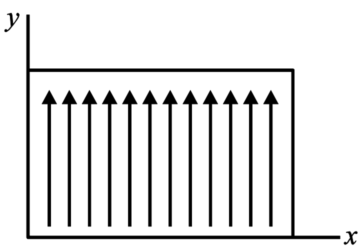
(iii) $\frac{a}{b}$ = aspect ratio

➢ Sample Questions

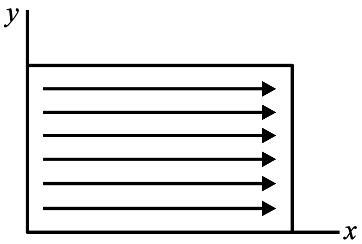
2005 IIT Bombay

6.1 Which one of the following does represent the electric field lines for the TE_{02} mode in the cross-section of a hollow rectangular metallic waveguide ?

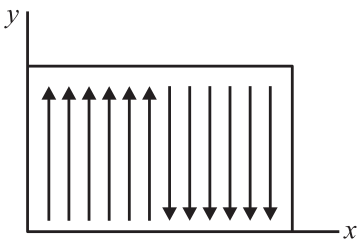
(A)



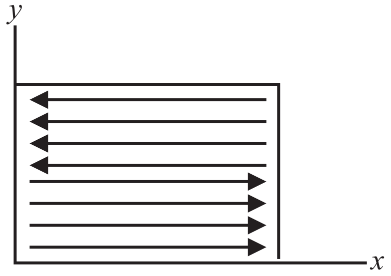
(B)



(C)



(D)



2021 IIT Bombay

6.2 A standard air filled rectangular waveguide with dimensions $a = 8$ cm , $b = 4$ cm operates at 3.4 GHz. For the dominant mode of wave propagation, the phase velocity of the signal is v_p . The value (rounded off to two decimal places) of v_p / c, where c denotes the velocity of light is _______.

❖❖❖❖

Explanations Waveguides

6.1 (D)

Method 1

Field pattern in rectangular waveguide for TE_{mn} mode can be calculated as,

$$E_x = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right)H_0\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$E_y = \frac{-j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right)H_0\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$E_z = 0$$

$$H_x = \frac{j\beta}{h^2}\left(\frac{m\pi}{a}\right)H_0\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$H_y = \frac{j\beta}{h^2}\left(\frac{n\pi}{b}\right)H_0\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$H_z = H_0\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

For TE_{02} mode, $m = 0$, $n = 2$

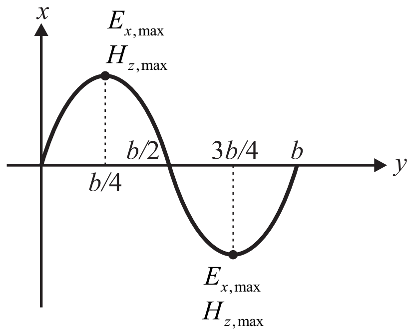
$$E_x = \frac{j\omega\mu}{h^2}\left(\frac{2\pi}{b}\right)\sin\left(\frac{2\pi y}{b}\right)e^{-\gamma z}$$

$E_y = 0$, $E_z = 0$, $H_x = 0$

$$H_y = \frac{j\beta}{h^2}\left(\frac{2\pi}{b}\right)H_0 \sin\left(\frac{2\pi y}{b}\right)e^{-\gamma z}$$

$$H_z = H_0 \cos\left(\frac{2\pi y}{b}\right)e^{-\gamma z}$$

y	E_x	E_y	E_z	H_x
0	0	0	0	0
b/4	$E_{\max}$	0	0	0
b/2	0	0	0	0
3b/4	$-E_{\max}$	0	0	0
b	0	0	0	0



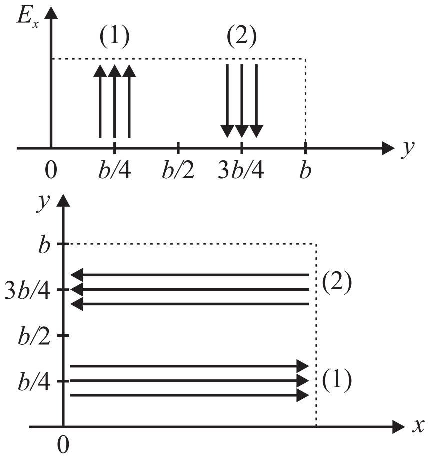


Fig. End view

Hence, the correct option is (D).

Method 2

m = Half cycle variation in x-direction

n = Half cycle variation in y-direction

In the given question

$$TE_{02} \rightarrow m = 0 \text{ and } n = 2$$

which shows two half cycle variation in y-direction.

Hence, the correct option is (D).

6.2 **1.198**

Given : $a = 8$ cm, $b = 4$ cm,

For a standard rectangular waveguide $a > b$

So, cut-off frequency for dominant mode (TE_{10}) is given by,

$$f_C = \frac{c}{2a}$$

$$f_C = \frac{3\times10^8}{2\times8\times10^{-2}} = \frac{30\times10^7}{16\times10^{-2}}$$

$$f_C = 1.875\times10^9 \text{ Hz} = 1.875 \text{ GHz}$$

Given operating frequency is, $f = 3.4$ GHz

So, phase velocity in rectangular waveguide is given by,

$$v_p = \frac{c}{\sqrt{1-\left(\frac{f_C}{f}\right)^2}}$$

$$\frac{v_p}{c} = \frac{1}{\sqrt{1-\left(\frac{1.875}{3.4}\right)^2}} = 1.198$$

Hence, the value of v_p / c is 1.198.

❖❖❖❖

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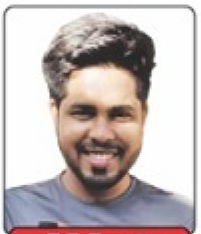
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Divakar Chaudhary


IN-4
Saurabh Jaiswal


IN-6
Raju Sharma

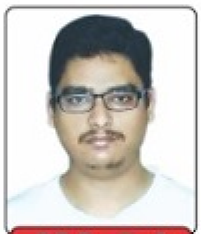

IN-13
Pritam Saurav


IN-20
Vishal Bansal

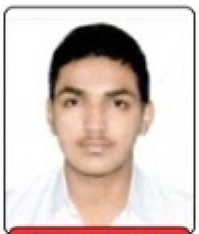

IN-27
Divyansh Trivedi


IN-30
Amirreddy


IN-48
Deepak Maurya


IN-61
Abhishek Singh


IN-73
Vineet Gupta


IN-102
Prashant


IN-115
Aman Jain

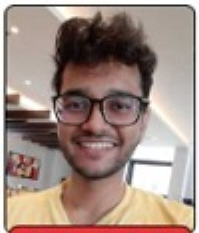

IN-119
Amit Beniwal


IN-119
D Ajay


IN-130
Prashant Kumar


IN-136
Mohit Gupta


IN-165
Akash Dagadugir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Jishnu


IN-199
Aditi Ahuja


IN-248
Prathmesh Bhunje


IN-276
Bharadwaj Dande


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Amit Kumar Gupta


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Nihal Neekra


IN-428
M Prabal Reddy


IN-491
Vishnu Choudhary


IN-510
Archan Mitra


IN-525
Ronak Gupta


IN-579
Aayush Paikra


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Akshyay Khanna


IN-599
Anupam Shukla


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Piyush Songara


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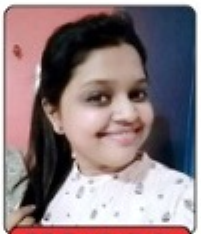
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Akshat Tiwari


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Ankit Meena


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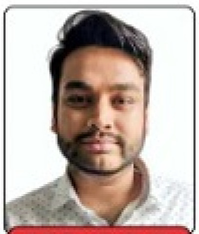
Arpit Chaudhary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Nabarjun Saha


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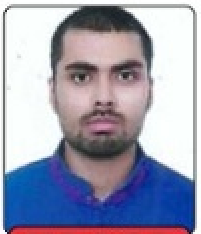
Ghosh Prerna Pradip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CHAPTER 4

COMMUNICATION SYSTEMS

Marks Distribution of Communication Systems in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	3	13	29
2004	5	10	25
2005	2	7	16
2006	–	14	28
2007	3	10	23
2008	1	11	23
2009	3	5	13
2010	3	4	11
2011	2	4	10
2012	4	3	1
2013	2	5	12
2014 Set-1	3	4	11
2014 Set-2	2	4	10
2014 Set-3	3	5	13

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-4	2	4	10
2015 Set-1	2	4	10
2015 Set-2	1	3	7
2015 Set-3	2	3	8
2016 Set-1	4	4	12
2016 Set-2	2	4	10
2016 Set-3	3	3	9
2017 Set-1	3	2	7
2017 Set-2	3	3	9
2018	3	3	9
2019	2	4	10
2020	3	4	11
2021	4	5	14

Syllabus : Communication Systems

Random processes : autocorrelation and power spectral density, properties of white noise, filtering of random signals through LTI systems.

Analog communications : amplitude modulation and demodulation, angle modulation and demodulation, spectra of AM and FM, superheterodyne receivers.

Information theory : entropy, mutual information and channel capacity theorem.

Digital communications : PCM, DPCM, digital modulation schemes (AS5K, PSK, FSK, QAM), bandwidth, inter-symbol interference, MAP, ML detection, matched filter receiver, SNR and BER.

Fundamentals of error correction, Hamming codes, CRC.

Contents : Communication Systems

1

Random Variables & Random Processes

Partial Synopsis

Random Variables

A random variable is a rule or relationship, denoted by X that assigns a real member $X(s)$ to every point in the sample space S.

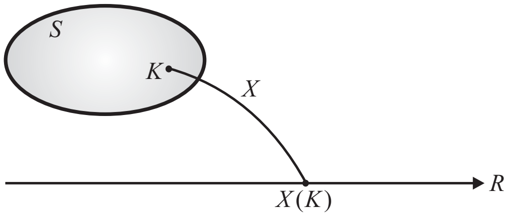


Fig. Random variable *X* as a function

Cumulative Distribuation Function (CDF)

The CDF associated with a random variable is defined as probability that outcome of an experiment will be one of the outcomes for which $X \le x,$ where x is any given number.

$$F_X(x) = P(X \le x) \qquad -\infty < x < \infty$$

The above expression is for continuous time random variable.

Properties of CDF :

1. $0 \le F_X(x) \le 1$
2. $F_X(x_1) \le F_X(x_2)$ if $x_1 < x_2$
3. $\lim\limits_{x\to\infty} F_X(x) = F_X(\infty) = 1$
4. $\lim\limits_{x\to-\infty} F_X(x) = F_X(-\infty) = 0$
5. $P(x_1 < X \le x_2) = P(X \le x_2) - P(X \le x_1) = F_X(x_2) - F_X(x_1)$
6. $P(x_1 \le X \le x_2) = P(X \le x_2) - P(X \le x_1^-) = F_X(x_2) - F_X(x_1^-)$

   For continuous random variable $P(x_1 < X \le x_2) = P(x_1 \le X \le x_2)$
7. $F_X(x) = 1 - F_X(-x)$

For discrete random variable, CDF is given by,

$$F_X(x) = \sum_{i=1}^{N} P(x_i) u(x - x_i)$$

Probability Density Function (PDF)

The probability density function for continuous random variable is defined as,

$$f_X(x) = \frac{d}{dx} F_X(x)$$

For discrete random variable,

$$f_X(x) = \sum_{i=1}^{N} P(x_i)\,\delta(x - x_i)$$

For discrete random variable, PDF also represents the probability mass function

Properties of PDF :

1. $f_X(x) \geq 0$
2. $F_X(x) = \int_{-\infty}^{x} f_X(\varepsilon)\, d\varepsilon$
3. $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$ (Area under PDF is unity)
4. $P(a < x < b) = \int_{a}^{b} f_X(x)\, dx$

Characterstics Parameters of Random variable

Expectation :

The mean (or expected value) of random variable X, denoted by μ_x or $E(X)$, is defined by,

$$\mu_x = E(X) = \overline{X} = \begin{cases} \Sigma\, x_K P(x_K) & X \ ; \text{ discrete} \\ \int_{-\infty}^{\infty} x\, f_X(x)\, dx & X \ ; \text{ continuous} \end{cases}$$

Where, $P(x_k) = P(X = x_k)$

The mean or expected value of a function is given by,

$$E[g(x)] = \begin{cases} \sum_{i} g(x_i)\, P(x_i) & g(X) \ ; \text{ discrete} \\ \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx & g(X) \ ; \text{ continuous} \end{cases}$$

Moment :

- The n^{th} moment of a random variable X is defined by,

$$E(X^n) = \begin{cases} \sum_{K} x_K^n\, P(x_K) & X \ ; \text{ discrete} \\ \int_{-\infty}^{\infty} x^n\, f_X(x)\, dx & X \ ; \text{ continuous} \end{cases}$$

where n represents the n^{th} order of the moment.

- The n^{th} order central moment of the random variable X is defined as,

$$E\left[(X-\bar{X})^n\right]=\begin{cases}\sum\limits_{K}(x_K-\bar{X})^n P(x_K) & X\ ;\ \text{discrete}\\ \int\limits_{-\infty}^{\infty}(x-\bar{X})^n f_X(x)\,dx & X\ ;\ \text{continuous}\end{cases}$$

Variance :

The variance of a random variable X, denoted by σ_X^2 or var (X) is defined by,

$$\sigma_X^2=\text{var}(X)=E\left\{\left[X-E(X)\right]^2\right\}$$

$$\sigma_X^2=\begin{cases}\sum\limits_{K}(x_K-\bar{X})^2 P(x_K)\ X\ ;\ \text{discrete}\\ \int\limits_{-\infty}^{\infty}(x-\bar{X})^2 f_X(x)\,dx X\ ;\ \text{continuous}\end{cases}$$

Properties of mean :

(i) $E[\text{constant}]=\text{constant}$

(ii) $E[CX]=CE[X]=C\bar{X}$

(iii) $E[X+C]=\bar{X}+C$

Properties of variance :

(i) $\text{var}(C)=0$

(ii) $\text{var}(CX)=C^2\,\text{var}(X)$

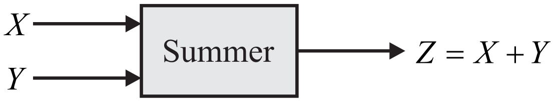
(iii) $\text{var}(X+C)=\text{var}(X)$

Central Limit Theorem

The central limit theorem says that the probability distribution of the sum of a large number of random variables approaches a Gaussian distribution.

OR

The density function of the sum of two statistically independent random variables is the convolution of the individual density functions.



$$f_z(z)=f_X(x)\otimes f_Y(y)$$

➤ Sample Questions

1989 IIT Kharagpur

1.2 Zero mean Gaussian noise of variance N is applied to a half wave rectifier. The mean squared value of the rectifier output will be

(A) Zero (B) $N/2$

(C) $N/\sqrt{2}$ (D) N

2002 IISc Bangalore

1.2 If the variance σ_d^2 of $d(n) = x(n) - x(n-1)$ is one-tenth the variance σ_x^2 of stationary zero-mean discrete-time signal $x(n)$, then the normalized autocorrelation function $\dfrac{R_{XX}(k)}{\sigma_X^2}$ at $k = 1$ is

(A) 0.95 (B) 0.90

(C) 0.10 (D) 0.05

2014 IIT Kharagpur

1.3 The power spectral density of a real stationary random process $X(t)$ is given by

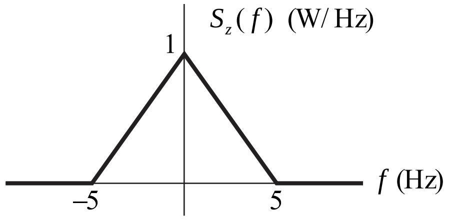
$$S_X(f) = \begin{cases} \dfrac{1}{W}', & |f| \le W \\ 0, & |f| > W \end{cases}$$

The value of the expectation $E\left[\pi X(t) X\left(t - \dfrac{1}{4W}\right)\right]$ is ________.

[Set - 02]

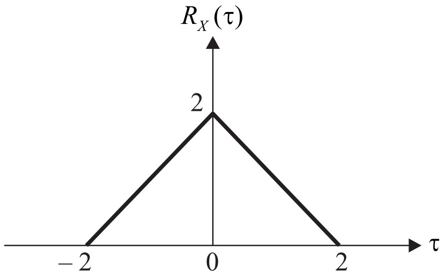
2019 IIT Madras

1.4 Let a random process $Y(t)$ be described as $Y(t) = h(t) * X(t) + Z(t)$, where $X(t)$ is a white noise process with power spectral density $S_X(f) = 5$ W/Hz. The filter $h(t)$ has a magnitude response given by $|H(f)| = 0.5$ for $-5 \le f \le 5$, and zero elsewhere. $Z(t)$ is a stationary random process, uncorrelated with $X(t)$, with power spectral density as shown in the figure. The power in $Y(t)$, in watts, is equal to ______W (rounded off to two decimal places).



2021 IIT Bombay

1.5 The autocorrelation function $R_X(\tau)$ of a wide-sense stationary random process $X(t)$ is shown in the figure,



The average power of $X(t)$ is ______.

Explanations Random Variables & Random Process

1.1 (B)

Half wave rectification is represented as,

$$Y = X \quad \text{for } x \geq 0$$
$$= 0 \quad \text{for } x < 0$$

The PDF of zero mean Gaussian random variable is given by,

$$f(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-\frac{x^2}{2\sigma_X^2}} dx \quad \ldots\text{(i)}$$

where, σ_X^2 = variance of Gaussian random variable X.

The PDF of random variable Y with zero mean and variance N is given by,

$$f(y) = \frac{1}{\sqrt{2\pi N}} e^{-y^2/2N} \quad \ldots\text{(ii)}$$

Mean square value is given by,

$$E(Y^2) = \int_0^\infty y^2 f(y) dy$$

$$E(Y^2) = \int_0^\infty \frac{y^2}{\sqrt{2\pi N}} e^{-y^2/2N} dy$$

Let, $\dfrac{y^2}{2N} = t$

$$2y\,dy = 2N\,dt$$

$$dy = \frac{N\,dt}{y} = \frac{Ndt}{\sqrt{2Nt}}$$

$$dy = \frac{\sqrt{N}}{\sqrt{2}} t^{-1/2} dt$$

$$E(y^2) = \int_0^\infty \frac{2Nt}{\sqrt{2\pi N}} e^{-t} \frac{\sqrt{N}}{\sqrt{2}} t^{-1/2} dt$$

$$E(y^2) = \frac{N}{\sqrt{\pi}} \int_0^\infty t^{1/2} e^{-t} dt \quad \ldots\text{(iii)}$$

Definition of gamma function :

$$\int_0^\infty e^{-t} t^{n-1} dt = \left\lceil n \right. \quad \ldots\text{(iv)}$$

On comparing equation (iii) and (iv),

$$n - 1 = \frac{1}{2}$$

$$n = \frac{3}{2}$$

$$\int_0^\infty t^{1/2} e^{-t} dt = \left\lceil \frac{3}{2} \right. = \left\lceil \frac{1}{2} + 1 \right. = \frac{1}{2}\left\lceil \frac{1}{2} \right. = \frac{\sqrt{\pi}}{2} \quad \ldots\text{(v)}$$

$$\left[\because \left\lceil n+1 \right. = n\left\lceil n \right. \text{ and } \left\lceil \frac{1}{2} \right. = \sqrt{\pi}\right]$$

From equation (iii) and (v),

$$E(Y^2) = \frac{N}{\sqrt{\pi}} \times \frac{\sqrt{\pi}}{2} = \frac{N}{2}$$

Hence, the correct option is (B).

1.2 (A)

Given : $\sigma_d^2 = \dfrac{1}{10}\sigma_X^2$, $E[x(n)] = 0$

$$d(n) = x(n) - x(n-1)$$

Variance of $d(n)$ is given by,

$$\sigma_d^2 = E[d^2(n)] - \{E[d(n)]\}^2 \quad \ldots\text{(i)}$$

$$E[d^2(n)] = E[x^2(n) - 2x(n)x(n-1) + x^2(n-1)]$$

$$E[d^2(n)] = E[x^2(n)] - 2E[x(n)x(n-1)] + E[x^2(n-1)]$$

$$E[d^2(n)] = E[x^2(n)] - 2R_{XX}(1) + E[x^2(n-1)]$$

Autocorrelation function is defined as,

$$R_{XX}(k) = E[x(n)x(n-k)]$$

Since this process is stationary, the mean is independent of time, so

$$E[x(n)] = E[x(n-1)]$$

$$E[d^2(n)] = 2E[x^2(n)] - 2R_{XX}(1) \quad \ldots\text{(ii)}$$

Mean of $d(n)$ can be written as,

$$E[d(n)] = E[x(n) - x(n-1)]$$

$$E[d(n)] = E[x(n)] - E[x(n-1)] = 0 \quad \ldots\text{(iii)}$$

The variance of $x(n)$ can be written as,

$$\sigma_X^2 = E[x^2(n)] - \{E[x(n)]\}^2 \quad \ldots\text{(iv)}$$

On substituting the value of equation (ii) and (iii) in equation (i),

$$\sigma_d^2 = 2E[x^2(n)] - 2R_{XX}(1) \quad \ldots\text{(v)}$$

Given, Mean of $x(n)$ is zero.

$$E[x(n)] = 0$$

From equation (iv),

$$\sigma_X^2 = E[x^2(n)]$$

From equation (v),

$$\sigma_d^2 = 2\sigma_X^2 - 2R_{XX}(1)$$

$$\frac{\sigma_X^2}{10} = 2\sigma_X^2 - 2R_{XX}(1)$$

$$2R_{XX}(1) = 2\sigma_X^2 - \frac{\sigma_X^2}{10}$$

$$\left.\frac{R_{XX}(k)}{\sigma_X^2}\right|_{k=1} = \frac{19}{20} = 0.95$$

Hence, the correct option is (A).

1.3 **4**

Given : The power spectral density

$$S_X(f) = \begin{cases} \dfrac{1}{W} & |f| \le W \\ 0 & |f| > W \end{cases} \quad \ldots\text{(i)}$$

From Wiener-Khinchin relationship, auto-correlation function and power spectral density are Fourier transform pair.

$$R_X(\tau) \xleftrightarrow{F.T.} S_X(f)$$

By taking inverse Fourier transform of equation (i),

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) e^{j2\pi f\tau} df$$

$$R_X(\tau) = \int_{-W}^{W} \frac{1}{W} e^{j2\pi f\tau} df$$

$$R_X(\tau) = \frac{1}{W}\frac{[e^{j2\pi\tau}]_{-W}^{W}}{j2\pi\tau}$$

$$R_X(\tau) = \frac{1}{W}\frac{[e^{j2\pi\tau W} - e^{-j2\pi W\tau}]}{j2\pi\tau}$$

$$R_X(\tau) = 2\frac{\sin(2\pi\tau W)}{(2\pi\tau W)}$$

$$R_X(\tau) = 2\sin c(2\tau W)$$

$$\left\{\because \ \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}\right\} \quad \ldots\text{(ii)}$$

Autocorrelation is given by,

$$R_X(\tau) = E[X(t)X(t-\tau)]$$

The value of expectation $E\left[\pi X(t)X\left(t - \frac{1}{4W}\right)\right]$ can be written as,

$$\pi E\left[X(t)X\left(t - \frac{1}{4W}\right)\right] = \pi R_{XX}\left(\frac{1}{4W}\right) \quad \ldots\text{(iii)}$$

Using equation (ii),

$$R_{XX}\left(\frac{1}{4W}\right) = 2\frac{\sin\left(2\pi\frac{1}{4W}\cdot W\right)}{\left(2\pi\frac{1}{4W}\cdot W\right)}$$

$$R_{XX}\left(\frac{1}{4W}\right) = 2\frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right)} = \frac{4}{\pi}$$

From equation (iii),

$$E\left[\pi X(t)X\left(t - \frac{1}{4W}\right)\right] = \pi \times \frac{4}{\pi} = 4$$

Hence, the value of the expectation is 4.

Short Method

If $A\,\text{rect}\left(\frac{t}{\tau}\right) \xleftrightarrow{F.T.} A\tau\,\text{sinc}(f\tau)$

Where, $A =$ height of pulse

$\tau =$ width of pulse

From duality property,

$$A\tau\,\text{sinc}(t\tau) \xleftrightarrow{F.T.} A\,\text{rect}\left(\frac{-f}{\tau}\right) = A\,\text{rect}\left(\frac{f}{\tau}\right)$$

$$2\,\text{sinc}(2W\tau) \xleftrightarrow{F.T.} \frac{1}{W}\text{rect}\left(\frac{f}{2W}\right)$$

$$E\left[\pi X(t)X\left(t-\frac{1}{4W}\right)\right] = \pi R\left(\frac{1}{4W}\right)$$

$$R_{XX}\left(\frac{1}{4W}\right) = 2\,\text{sinc}(2W\tau)$$

$$R_{XX}\left(\frac{1}{4W}\right) = 2\frac{\sin\left(2\pi\frac{1}{4W}\cdot W\right)}{\left(2\pi\frac{1}{4W}\cdot W\right)}$$

$$R_{XX}\left(\frac{1}{4W}\right) = 2\frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right)} = \frac{4}{\pi}$$

$$E\left[\pi X(t)X\left(t-\frac{1}{4W}\right)\right] = \pi\times\frac{4}{\pi} = 4$$

Hence, the value of the expectation is **4**.

Scan for
Video Solution

1.4 **17.5**

Given : Random process $Y(t)$ as

$$Y(t) = h(t) * X(t) + Z(t)$$

Let, $h(t) * X(t) = X'(t)$

$$Y(t) = X'(t) + Z(t)$$

Given $X(t)$ and $Z(t)$ are uncorrelated. We know that, total area under PSD gives the power of the process.

Autocorrelation of $Y(t)$ is given as

$$R_y(\tau) = E[Y(t)\cdot Y(t+\tau)]$$

$$R_y(\tau) = E[\{X'(t)+Z(t)\}\{X'(t+\tau)+Z(t+\tau)\}]$$

$$R_Y(\tau) = E[\{X'(t)X'(t+\tau)\}] + E[\{X'(t)Z(t+\tau)\}] + E[\{Z(t)X'(t+\tau)\}] + E[\{Z(t)Z(t+\tau)\}]$$

$$R_Y(\tau) = R_{X'}(\tau) + R_{X'Z}(\tau) + R_{ZX'}(\tau) + R_Z(\tau) \quad \text{...(i)}$$

Where, $R_{X'Z}(\tau)$ and $R_{ZX'}(\tau)$ are correlation of $X'(t)$ and $Z(t)$.

From Wiener-Khinchin relationship, autocorrelation function and PSD are Fourier transform pair.

Therefore, taking Fourier transform pair of equation (i),

$$S_Y(f) = S_{X'}(f) + S_{X'Z}(f) + S_{ZX'}(f) + S_Z(f) \quad \text{...(ii)}$$

Given $X(t)$ and $Z(t)$ are uncorrelated and have constant means,

$$S_{X'Z}(\omega) = S_{ZX'}(\omega) = \overline{X'}\,\overline{Z}\,\delta(\omega) \quad \text{...(iii)}$$

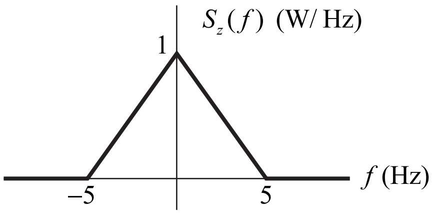
Where, $\overline{X'}$ an $\overline{Z}$ are means of X' and Z

Given $X(t)$ is white noise with PSD $S_X(f) = 5\text{ W/Hz}$

Impulse response $h(t)$ whose system response is

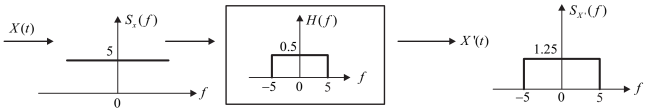
$$H(f) = \begin{cases} 0.5\,; & -5 \le f \le 5 \\ 0\;; & \text{otherwise} \end{cases}$$

PSD of $Z(t)$ is



Now PSD of $S_{X'}(f)$ can be drawn as

$$S_{X'}(f) = |H(f)|^2 \, S_X(f)$$



Now taking Inverse Fourier transform of $S_{X'}(f)$

$$R_{X'}(\tau) = 12.5 \sin c(10\tau)$$

From property of ACF,

$$\lim_{\tau\to\infty} R_{X'}(\tau) = (\overline{X'})^2$$

$$\lim_{\tau\to\infty} R_{X'}(\tau) = \lim_{\tau\to\infty} [12.5 \,\text{sinc}\,(10\tau)]$$

$$(\overline{X'})^2 = 0 \qquad \{\because \lim_{\tau\to\infty} \sin c(\tau) = 0\}$$

$$\overline{X'} = 0$$

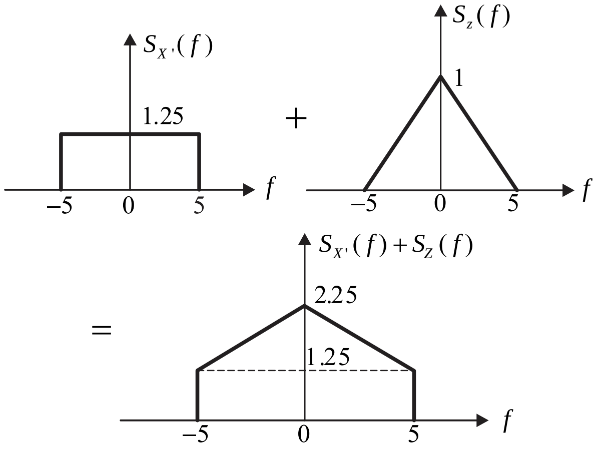
Putting $\overline{X'}$ in equation (iii),

$$S_{X'Z}(f) = S_{ZX'}(f) = 0$$

Now equation (ii) becomes

$$S_Y(f) = S_{X'}(f) + S_Z(f)$$

Power spectral density of is given as $S_{X'}(f) + S_Z(f)$ as shown in figure,



Power in $Y(t)$ is given as area under the power spectral density function.

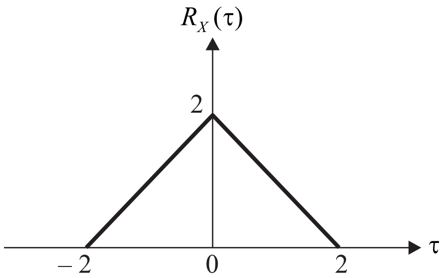
Area under the above figure

$$P_Y = (10 \times 1.25) + \left(\frac{1}{2} \times 1 \times 10\right) = 17.5$$

Hence, the power in $y(t)$ is **17.5 W.**

1.5 **2**

Given : Autocorrelation function $R_X(\tau)$ of wide sense stationary random process $X(t)$ is shown in figure.



Method 1 :

Here, $R_X(0) = 2$

Average power of $X(t)$ is given as mean square value of $X(t)$, i.e.,

$$P_X = E\left[X^2(t)\right] = E[X(t)X(t)] \ldots\text{(i)}$$

Autocorrelation function of $X(t)$ is given by,

$$R_X(\tau) = E\left[X(t) \cdot X(t+\tau)\right]$$

$$R_X(0) = E[X(t)X(t+0)]$$

$$R_X(0) = E[X(t)X(t)] \qquad \ldots\text{(ii)}$$

From equations (i) and (ii),

$$P_X = R_X(0) = 2\,\text{W}$$

Hence, the correct answer is 2 W.

Method 2 :

$R_X(\tau)$ can be represented as,

$$R_X(\tau) = 2tri\left(\frac{\tau}{2}\right)$$

Autocorrelation function of $X(t)$ is related to power spectral density is,

$$R_X(\tau) \xleftrightarrow{\text{F.T.}} S_X(f)$$

From duality property,

$$A tri\left(\frac{\tau}{T}\right) \xleftrightarrow{\text{F.T.}} AT\text{sinc}^2(fT)$$

On comparing $R_X(\tau) = 2tri\left(\frac{\tau}{2}\right)$ with above equation, $A = 2, T = 2$.

$$S_Y(f) = AT\text{sinc}^2(fT)$$

$$S_Y(f) = 2\times 2\times \text{sinc}^2(2f)$$

$$S_Y(f) = 4\text{sinc}^2(2f)$$

The average power of $X(t)$ is given by,

$$P_X = \int_{-\infty}^{\infty} S_X(f)\,df$$

$$P_X = 4\int_{-\infty}^{\infty} \text{sinc}^2(2f)\,df$$

$$P_X = 4[\text{Energy of sinc}^2(2f)]$$

We know that, energy of $\text{sinc}^2(f) = 1$

By using scaling property, energy of $\text{sinc}^2(2f) = \frac{1}{2}$

Therefore, $P_X = 4\times\frac{1}{2} = 2\text{ W}$

Hence, the correct answer is 2 W.

❖❖❖❖

2

Amplitude Modulation

Partial Synopsis

Modulation

Modulation is the process in which characteristics (amplitude, frequency or phase) of one waveform i.e. carrier signal varies in accordance with the characteristics of another waveform i.e. modulating signal.

Amplitude Modulation

The general expression for the amplitude modulated signal is given by,

$$s_{AM}(t) = A_c\left[1 + k_a\, m(t)\right]\cos\omega_c t$$

where,

$A_c =$ Amplitude of the carrier signal,

$f_c =$ Frequency of carrier signal,

$k_a =$ Amplitude sensitivity,

$m(t) =$ Message signal.

Modulation index :

Modulation index is a measure of extent to which modulating signal modulates the carrier signal. It is denoted by m_a.

- $$m_a = k_a\, m(t)\big|_{\max}$$

  For a single tone sinusoidal signal, $m(t) = A_m \cos\omega_m t$

  $$m(t)\big|_{\max} = A_m \quad \Rightarrow \quad m_a = k_a A_m$$

- Modulation index (m_a) is also defined as,

  $$m_a = \frac{A_m}{A_c}$$

- For a single tone sinusoidal signal, envelope of amplitude modulated signal is given by,

  $$E = A_c(1 + m_a \cos\omega_m t)$$

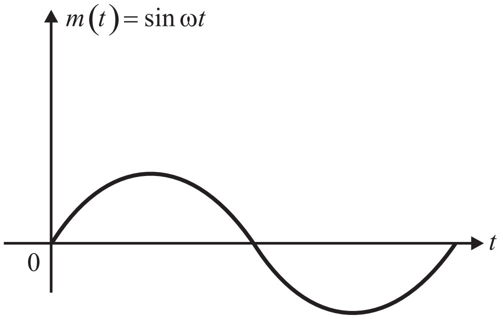


Fig. Modulating signal

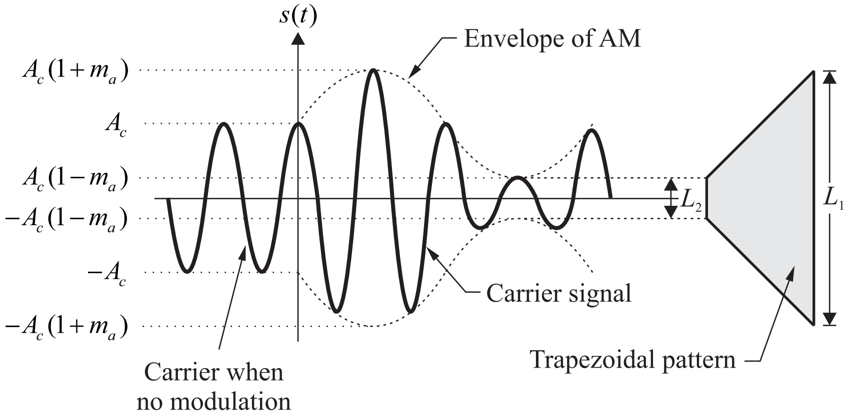


Fig. Amplitude modulated signal

$$E_{\max} = A_c(1+m_a) \quad \text{and} \quad E_{\min} = A_c(1-m_a)$$

$$A_c = \frac{E_{\max}+E_{\min}}{2} \quad \text{and} \quad A_m = \frac{E_{\max}-E_{\min}}{2}$$

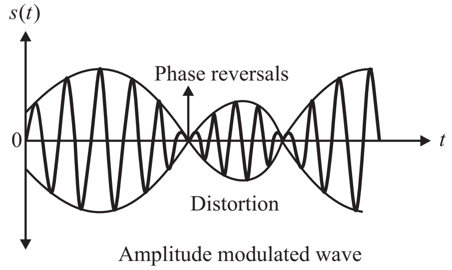
$$m_a = \frac{E_{\max}-E_{\min}}{E_{\max}+E_{\min}}$$

- Trapezoidal pattern for calculation of modulation index.

$$m_a = \frac{L_1 - L_2}{L_1 + L_2}$$

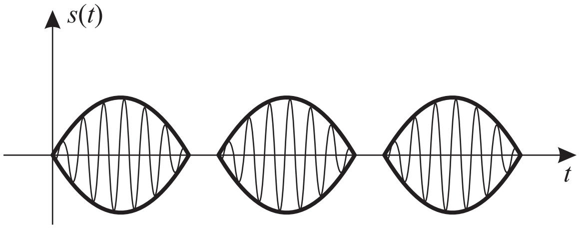
- Over modulation (i.e., $m_a > 1$) cause distortion in the form of

(a) Phase reversal



Amplitude modulated wave
$|K_a m(t)| > 1$

(b) Sideband splatter



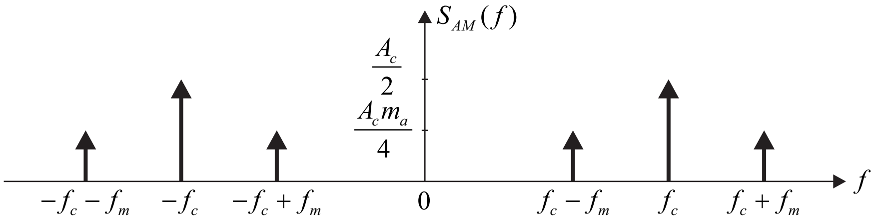
- Sideband splatter increases the bandwidth.

Frequency domain representation of AM signal :

- For single tone sinusoidal modulating signal,

$$s_{AM}(t) = A_c[1 + m_a \cos \omega_m t] \cos \omega_c t$$

$$s_{AM}(t) = A_c \cos \omega_c t + \frac{A_c m_a}{2} \cos(\omega_c + \omega_m)t + \frac{A_c m_a}{2} \cos(\omega_c - \omega_m)t$$



- **Bandwidth of AM signal** $= 2 f_m$

Power calculation in AM :

$$s_{AM}(t) = A_c \cos \omega_c t + A_c K_a\, m(t) \cos \omega_c t$$

where, Carrier power $(P_c) = \frac{A_c^2}{2}$

Sideband power $(P_{SB}) = \frac{A_c^2 k_a^2 \overline{m^2(t)}}{2}$

[where $\overline{m^2(t)}$ is power of the modulating signal]

Total power = carrier power + sideband power

$$P_T = \frac{A_c^2}{2}\left[1 + k_a^2 \overline{m^2(t)}\right] = P_c\left[1 + k_a^2 \overline{m^2(t)}\right]$$

- Power $P_T = P_c\left(1 + \frac{m_a^2}{2}\right)$; for single tone modulation.
- Power $P_T = P_c\left(1 + \frac{m_{aT}^2}{2}\right)$; for multi tone modulation.

where, $m_{aT}^2 = m_{a_1}^2 + m_{a_2}^2 + m_{a_3}^2 + ...m_{a_n}^2$

- Peak envelope power $= \dfrac{A_{\max}^2}{2}$　　$[A_{\max} = A_c(1+m_a)]$

Transmission efficiency :

$$\eta = \frac{\text{Useful power}}{\text{Total power}} = \frac{P_{SB}}{P_T}$$

$$\eta = \frac{k_a^2 \overline{m^2(t)}}{1 + k_a^2 \overline{m^2(t)}}$$

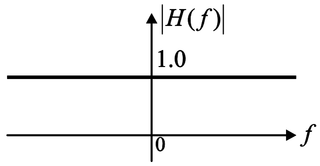
For single tone sinusoidal signal,

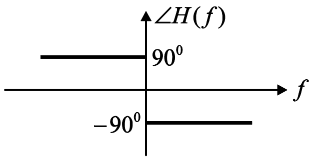
$$\% \eta = \frac{m_a^2}{2+m_a^2} \times 100\%$$

For $m_a = 1,\ \eta = 33.3\%$. This is maximum efficiency that can be achieved using amplitude modulation.

- Frequency response of Hilbert transform is given by,

$$H(f) = -j\,\text{sgn}(f)$$





➢ Sample Questions

1993　IIT Bombay

2.1 Which of the following demodulator (s) can be used for demodulating the signal

$x(t) = 5\ (1 + 2\cos 200\,\pi t)\cos 20000\,\pi t$

(A) Envelope demodulator

(B) Square-law demodulator

(C) Synchronous demodulator

(D) None of the above

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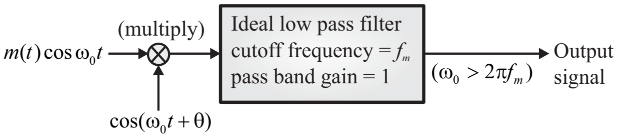
2.2 A message $m(t)$ band-limited to the frequency f_m has a power of P_m. The power of the output signal in given figure is



(A) $\dfrac{P_m \cos\theta}{2}$　　(B) $\dfrac{P_m}{4}$

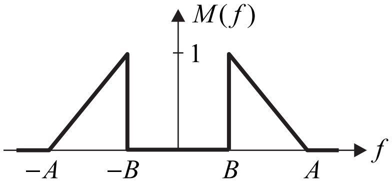
(C) $\dfrac{P_m \sin^2\theta}{4}$　　(D) $\dfrac{P_m \cos^2\theta}{4}$

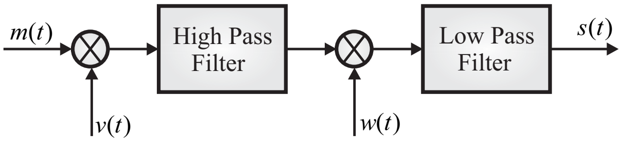
2014　IIT Kharagpur

2.3 In the figure, $M(f)$ is the Fourier transform of the message signal $m(t)$ where $A = 100$ Hz and $B = 40$ Hz. Given $v(t) = \cos(2\pi\, f_c t)$ and

$w(t) = \cos\left(2\pi(f_c + A)t\right)$ where $f_c > A$.

The cut-off frequencies of both the filters are f_c.





The bandwidth of the signal at the output of the modulator (in Hz) is __________. **[Set - 02]**

2021 IIT Bombay

2.4 Consider a carrier signal which is amplitude modulated by a single-tone sinusoidal message signal with a modulation index of 50%. If the carrier and one of the sidebands are suppressed in the modulated signal, the percentage of power saved (rounded off to one decimal place) is _______.

❖❖❖❖

Explanations Amplitude Modulation

2.1 (C)

Given : $x(t) = 5(1 + 2\cos 200\pi t)\cos 20000\pi t$ ...(i)

For single tone modulating signal expression for AM signal is given by,

$$x(t) = A_c(1 + m_a \cos 2\pi f_m t)\cos 2\pi f_c t \quad \text{...(ii)}$$

On comparing equation (i) and (ii),

Modulation index is,

$$m_a = 2$$

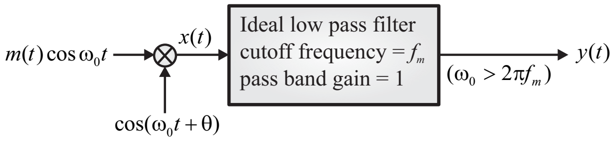
Since the modulation index is more than 1 so, it is over-modulated signal. When $m_a > 1$ then the detection is possible only with synchronous modulator. Such signal cannot be detected by square-law demodulator and envelope detector.

Hence, the correct option is (C).

2.2 (D)

Given : Power = Mean square value

$$= E[m^2(t)] = \overline{m^2(t)} = P_m$$

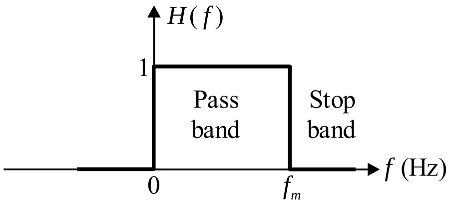


Let $x(t) = m(t)\cos\omega_0 t \cos(\omega_0 t + \theta)$

$$x(t) = \frac{m(t)}{2}\left[\cos(2\omega_0 t + \theta) + \cos\theta\right]$$

$x(t)$ is passed through a low pass filter.

The response of ideal low pass filter with cut-off frequency f_m and passband gain 1 is shown in figure below.



Output of LPF will be,

$$y(t) = \frac{m(t)}{2}\cos\theta$$

Power of the output signal is,

$$\text{Power } = \left(\frac{m(t)}{2}\cos\theta\right)^2 = \frac{\overline{m^2(t)}}{4}\cos^2\theta$$

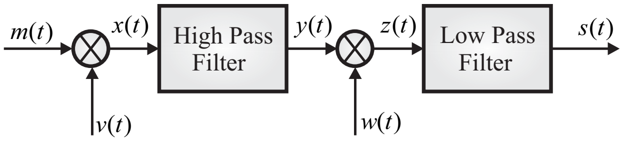
$$\text{Power } = \frac{P_m}{4}\cos^2\theta$$

Hence, the correct option is (D).

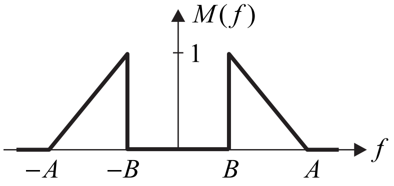
2.3 **60**

Given : $A = 100$ Hz and $B = 40$ Hz,

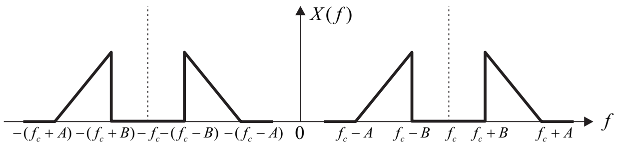
$v(t) = \cos(2\pi f_c t)$ and $w(t) = \cos(2\pi(f_c + A)t)$



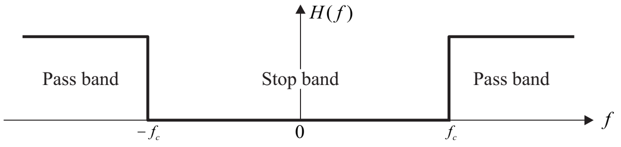
The message spectrum is shown in figure below.



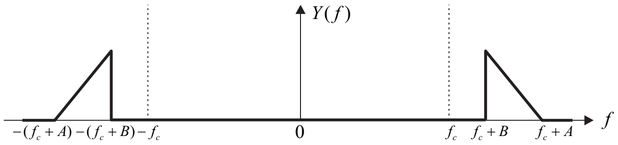
The spectrum of output of 1st multiplier $X(f)$ is shown in figure below.



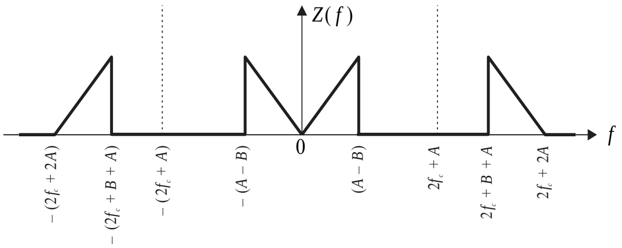
The cut-off frequency of the HPF is ‘ f_c ’.



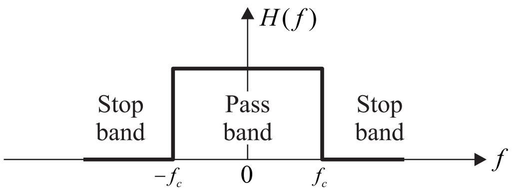
So, the spectrum of output of the HPF $Y(f)$ is shown in figure below.



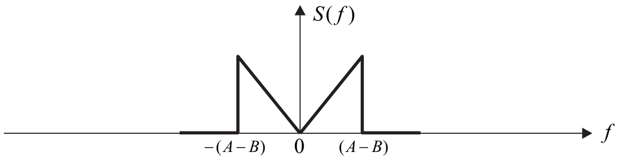
The spectrum of output of second multiplier $Z(f)$ is shown in figure below.



The cut-off frequency of low pass filter is f_c. The spectrum of low pass filter is shown below,



The spectrum of output of LPF $S(f)$ is shown in figure below.



The BW of the above signal can be written as,

$$BW = f_H - f_L = (A - B) - 0$$

$$BW = 60 \text{ Hz}$$

Hence, the bandwidth of the signal is **60 Hz.**

Scan for Video Solution

2.4 **94.44**

Given : Modulation index = 50%

$$m_a = 0.5$$

According to question, carrier and one side band is suppressed, so saved power is given by, Power saved = Carrier Power + One side band Power

$$P_{saved} = P_c + P_c \frac{m_a^2}{4} = P_c \left[1 + \frac{m_a^2}{4}\right]$$

Total power of AM signal $= P_c \left[1 + \frac{m_a^2}{2}\right]$

So, percentage saving in power is given as,

$$\% \text{ Power saving } = \frac{\text{Power saved}}{\text{Total power}} \times 100\%$$

$$\% \text{ Power saving } = \frac{P_c \left[1 + \frac{m_a^2}{4}\right]}{P_c \left[1 + \frac{m_a^2}{2}\right]} \times 100\%$$

$$\% \text{ Power saving} = \frac{1 + \left(\frac{1}{2}\right)^2 \frac{1}{4}}{1 + \left(\frac{1}{2}\right)^2 \frac{1}{2}} \times 100\%$$

$$\% \text{ Power saving } = \frac{1 + \frac{1}{16}}{1 + \frac{1}{8}} \times 100\%$$

$$\% \text{ Power saving} = 94.44\%$$

Hence, the correct answer is 94.44%.

❖❖❖❖

6

Baseband Transmission

➢ Partial Synopsis

Digital Pulse Modulation

In digital pulse modulation technique, the transmitted samples take on only discrete values. Two important types of digital pulse modulation are :

Pulse code modulation (PCM) : PCM is essentially an analog-to-digital conversion of a special type where the information contained in the instantaneous samples of an analog signal is represented by digital words in a serial bit stream. It consists of basic three operations :

Quantization :

- Quantization is a process of approximation or rounding off,

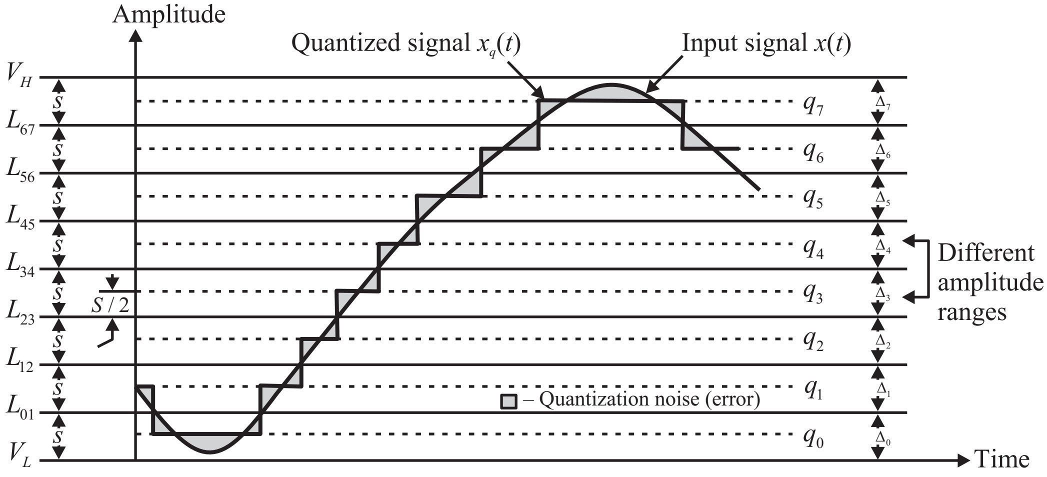


Fig. Process of quantization

- The difference between the quantized value and input value is called. Quantization error or quantization noise,

$$E = x_q(t) - x(t)$$

- The maximum value of quantization error is $\pm S/2$ where S is step size.

- A sampled waveform is quantized into q quantization levels, where q is any integer.
- If the message signal is defined in the range $(-m_p, m_p)$ then the step size of quantizer is

$$S = \frac{2m_p}{q}$$

- For a binary PCM system with n digital codes, the no. of quantization level is defined as,

$$q = 2^n$$

- If the message signal is sampled at the sampling rate f_s and encoded to n number of bits per sample, then the bit rate (bits/sec) of the PCM is defined as,

$$R_b = nf_s$$

and bandwidth is given by,

$$BW = \frac{R_b}{2}$$

- The mean square value of the quantization is given by $\frac{S^2}{12}$.

➢ Sample Questions

1987 IIT Bombay

6.1 Companding in PCM system leads to improved signal-to-quantization noise ratio. This improvement is for

(A) lower frequency components only

(B) higher frequency component only

(C) lower amplitudes only

(D) higher amplitudes only

1999 IIT Bombay

6.2 Four independent messages have bandwidths of 100 Hz, 100 Hz, 200 Hz, and 400 Hz, respectively. Each is sampled at the Nyquist rate, and the samples are time division multiplexed (TDM) and transmitted. The transmitted sample rate (in Hz) is

(A) 1600 (B) 800

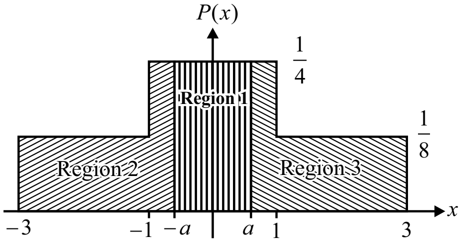
(C) 400 (D) 200

2005 IIT Bombay

Statement for Linked Answer Questions 6.3 & 6.4

A symmetric three-level mid-tread quantizer is to be designed assuming equiprobable occurrence of all quantization levels.

6.3 If the probability density function is divided into three regions as shown in the figure, the value of a in the figure is



(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

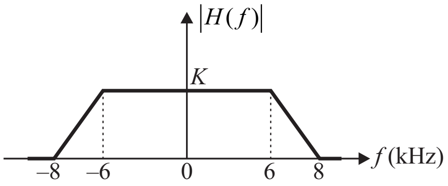
6.4 The quantization noise power for the quantization region between – a and + a in the figure is

(A) $\frac{4}{81}$ (B) $\frac{1}{9}$

(C) $\frac{5}{81}$ (D) $\frac{2}{81}$

2018 **IIT Guwahati**

6.5 A band limited low-pass signal $x(t)$ of bandwidth 5 kHz is sampled at a sampling rate f_s. The signal $x(t)$ is reconstructed using the reconstruction filter $H(f)$ whose magnitude response is shown below :



The minimum sampling rate f_s (in kHz) for perfect reconstruction of $x(t)$ is _______.

2021 **IIT Bombay**

6.6 A speech signal, band limited to 4 kHz, is sampled at 1.25 times the nyquist rate. The speech samples, assumed to be statistically independent and uniformly distributed in the range – 5 V to + 5 V, are subsequently quantized in an 8-bit uniform quantizer and then transmitted over a voice-grade AWGN telephone channel. If the ratio of transmitted signal power to channel noise power is 26 dB, the minimum channel bandwidth required to ensure reliable transmission of the signal with arbitrarily small probability of transmission error (rounded off to two decimal places) is _______ kHz.

❖❖❖❖

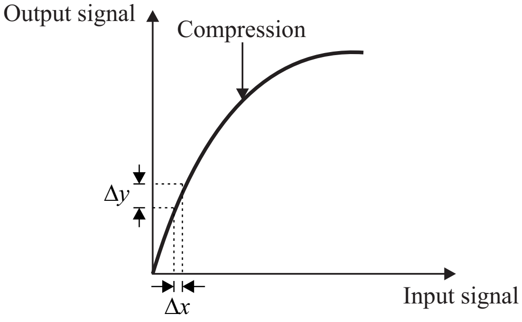
Explanations Baseband Transmission

6.1 **(C)**

Quantization noise power is given by,

$$N_q = \frac{S^2}{12}$$

where, $S =$ Step size



Since, in uniform quantization, step size is constant, hence quantization noise power is uniform, throughout the signal. If the message signal is weak then signal power which is proportional to square of signal voltage will be less. Hence, the signal to noise ratio of the system will be poor. That is why it is important to increase the amplitude of weak signal i.e. low amplitude signal.

Higher amplitudes of signal, will have better SNR than the lower amplitudes. Hence, companding is used for improving SNR at lower amplitudes.

Hence, the correct option is (C).

📖 Key Point

1. With companded PCM systems, the higher-amplitude analog signals are compressed (amplified less than the lower-amplitude signals) prior to transmission and then expanded (amplified more than lower-amplitude signals) in the receiver.
2. In an actual PCM system, the combination of compressor and uniform quantizer is located in PCM transmitter, while the expander is located in the receiver. Companding is an effective means of improving the dynamic range of a PCM-based communication system.

6.2 (A)

Given : $f_{m_1} = 100$ Hz, $f_{m_2} = 100$ Hz,

$f_{m_3} = 200$ Hz and $f_{m_4} = 400$ Hz

Sampling frequency is given by,

$$f_s = 2 f_m$$

In TDM, if the bandwidth of signals are not equal then transmitted sample rate is given by,

$$f_S = f_{S_1} + f_{S_2} + \dots\dots\dots f_{S_M}$$

where, $M =$ number of message signal

Here $M = 4$

$$f_S = f_{S_1} + f_{S_2} + f_{S_3} + f_{S_4}$$

$$f_S = 200 + 200 + 400 + 800$$

$$f_S = 1600 \text{ Hz}$$

Hence, the correct option is (A).

6.3 (B)

Given : All the regions are equiprobable i.e.

$$P(\text{Region1}) = P(\text{Region2}) = P(\text{Region3}) = \frac{1}{3}$$

(Because there are 3 regions)

From property of PDF, area under PDF is equal to probability of that regions.

$$P[x_1 < X \le x_2] = \int_{x_1}^{x_2} P(x)\,dx$$

$$P[-a < X \le a] = \int_{-a}^{a} \frac{1}{4}dx$$

$$\frac{1}{3} = \frac{1}{4}[x]_{-a}^{a}$$

$$\frac{4}{3} = 2a$$

$$a = \frac{2}{3}$$

Hence, the correct option is (B).

6.4 (A)

In PCM with uniform quantization, quantization noise power can be calculated as,

$$N_q = E[X^2] = \int_{-a}^{a} x^2 P(x)dx$$

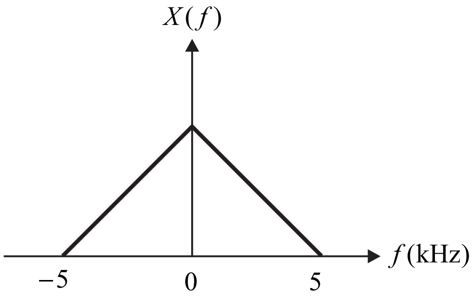
$$N_q = \frac{1}{4}\frac{[x^3]_{-a}^{a}}{3} = \frac{1}{12}[a^3 - (-a)^3]$$

$$N_q = \frac{a^3}{6} = \left(\frac{2}{3}\right)^3 \frac{1}{6} = \frac{4}{81}$$

Hence, the correct option is (A).

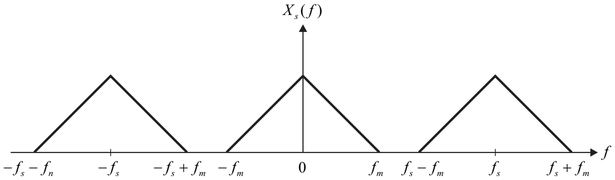
6.5 13

Given : Spectrum of message signal band limited to 5 kHz is shown below,

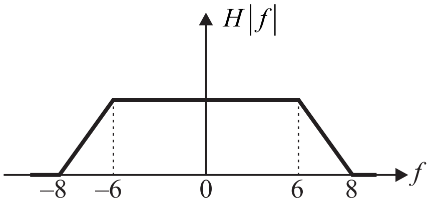


Expression for spectrum of sampled signal is given by,

$$X_s(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(f - nf_s)$$



Magnitude response of reconstruction filter $|H(f)|$ is shown below,



For proper reconstruction of message signal

$$f_s - f_m > 8$$

$$f_s > 8 + f_m$$

$$f_s > 13\,\text{kHz} \qquad [f_m = 5\text{ kHz}]$$

Hence, the minimum sampling rate f_s for perfect reconstruction of $x(t)$ is **13 kHz.**

6.6 **9.26**

Given :

Speech signal/message signal frequency,

$$f_m = 4\text{ kHz}$$

Sampling frequency, $f_s = 1.25\, f_{NR}$

Nyquist rate, $f_{NR} = 2f_m = 2\times 4 = 8\text{ kHz}$

Thus, sampling frequency,

$$f_s = 1.25\, f_{NR} = 1.25\times 8\text{ kHz} = 10\text{ kHz}$$

$\because$ Speech samples are uniformly distributed in the range $-5\,\text{V}$ to 5 V and subsequently quantized in 8-bit uniform quantizer and transmitted over AWGN channel, so number of bits used by quantizer is given by, $n = 8$.

Bit rate is given as,

$$R_b = nf_s = 8\times 10 = 80\text{ kbps}$$

According to channel capacity theorem,

$$C \le R_b$$

$$B\log_2(1+SNR) = R_b \qquad \ldots\text{(i)}$$

Where, $B =$ Channel bandwidth, $SNR =$ Transmitted signal power to channel noise power ratio.

Here, $(SNR)_{dB} = 26\text{ dB}$

$$10\log_{10}(SNR) = 26$$

$$SNR = (10)^{2.6} = 398.1$$

From equation (i),

$$B\log_2(1+398.1) \ge 80\times 10^3$$

$$B\log_2(399.1) \ge 80\times 10^3$$

$$B \ge \frac{80\times 10^3}{\log_2(299.1)} \ge \frac{80\times 10^3}{8.640}$$

$$B \ge 9.59\times 10^3\text{ Hz}$$

$$B_{\min} = 9.259\text{ kHz}$$

Hence, the correct answer is 9.259 kHz.

❖❖❖❖

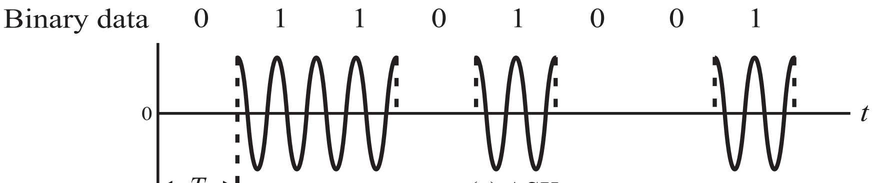
7

Bandpass Transmission

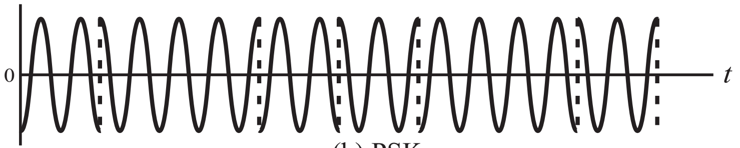
➢ Partial Synopsis

Basic Digital Modulation Schemes

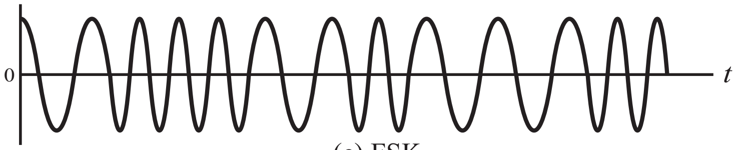
1. **ASK :** In amplitude shift keying or on-off keying, amplitude of carrier signal varies in accordance with the digital data stream.
2. **Frequency shift keying :** In FSK frequency of the carrier signal varies in accordance with the digital data stream.
3. **Phase shift keying :** In PSK, phase of the carrier is varied in accordance with the digital data stream.



(a) ASK



(b) PSK



(c) FSK

Differential Phase Shift Keying (DPSK)

Differential phase shift keying (DPSK) is differentially coherent modulation method. DPSK does not need a synchronous (coherent) carrier at the demodulator. The input sequence of binary bits is modified such that next bit depends upon the previous bit. Therefore in the receiver the previous received bits are used to detect the present bit.

DPSK transmitter :

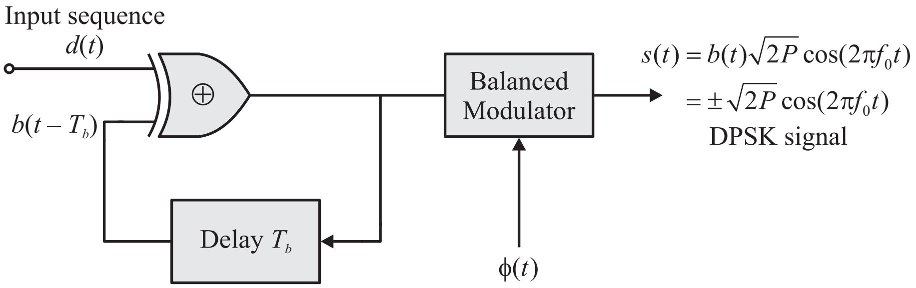


Fig.1 : Block diagram of DPSK generate or transmitter

An arbitrary sequence $d(t)$ is taken. Depending on this sequence $b(t)$ and $b(t-T_b)$ are found. The waveforms are shown below.

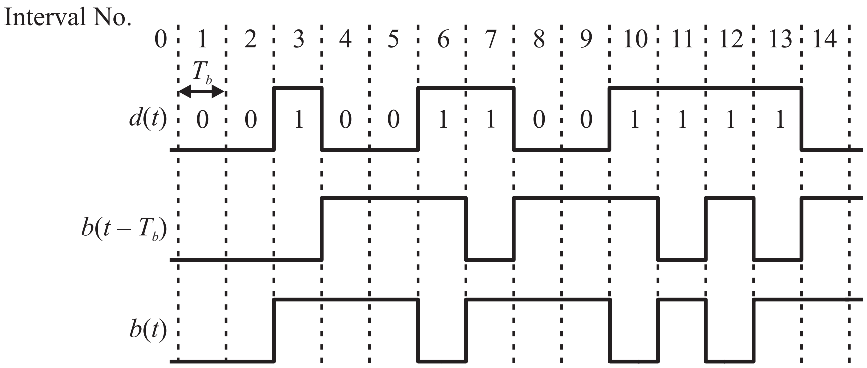


Fig. 2 : DPSK waveforms

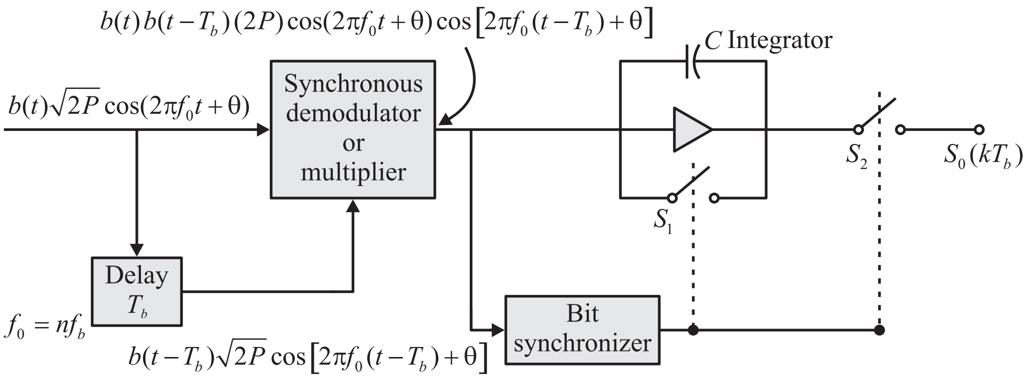
When $d(t)=0,\quad b(t)=b(t-T_b)$

$d(t)=1,\quad b(t)=\overline{b(t-T_b)}$

Modulator output is,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_c t)$$
$$= \pm\sqrt{2P}\cos(2\pi f_0 t)$$

DPSK receiver :



(a)

Fig. 3 : (a) DPSK receiver

Bandwidth of DPSK signal is given by,

$$\text{BW} = f_b$$

M-ary PSK :

Expression for *M*-ary PSK is given by,

$$S_{M\text{-ary PSK}}(t) = A_c \cos\left[\omega_c t + (2m+1)\frac{\pi}{M}\right]$$

where, $m = 0, 1, 2 \ldots\ldots\ldots M-1$

(i) Euclidean distance is given by,

$$d = \sqrt{4E_b N \sin^2\left(\frac{\pi}{M}\right)} = 2E_s \sin\left(\frac{\pi}{M}\right)$$

(ii) Bandwidth is given by,

$$\text{BW} = \frac{2f_b}{N}$$

Remember

(i) Bandwidth reduces as *M* increases.

(ii) As distance reduces for *M*-ary PSK, therefore probability of error increases.

(iii)QPSK is better, as bandwidths is less than BPSK but probability of error remains constant.

(iv)*M*-ary FSK is better than *M*-ary PSK.

Comparison of Digital Modulation Systems

S.N.	Parameter	Binary ASK	Binary FSK	Binary PSK
1.	Variable characteristic	Amplitude	Frequency	Phase
2.	Bandwidth (Hz) (spectral efficiency)	$2f_b$	$4f_b$	$2f_b$
3.	Noise immunity	Low	High	High
4.	Error probability	High	Low	Low
5.	Performance in presence of noise	Bad	Better than ASK	Better than FSK
6.	Complexity	Simple	Moderately complex	Very complex
7.	Bit rate	Suitable upto 100 bits/sec	Suitable upto about 1200 bits/sec	Suitable for high bit rates
8.	Detection method	Envelope	Envelope	Coherent

➢ Sample Questions

1988　IIT Kharagpur

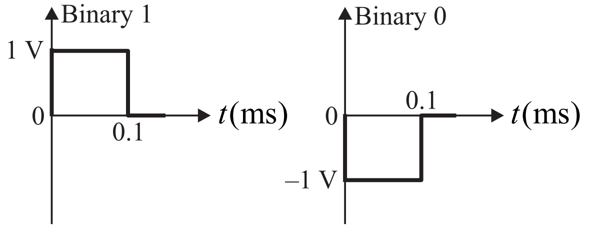
7.1　The message bit sequence input to a DPSK modulator is 1, 1, 0, 0, 1, 1. The carrier phase during the reception of the first two message bits is π, π. The carrier phase for the remaining four message bits is

(A) $\pi, \pi, 0, \pi$　(B) $0, 0, \pi, \pi$

(C) $0, \pi, \pi, \pi$　(D) $\pi, \pi, 0, 0$

2004　IIT Delhi

7.2　A source produces binary data at the rate of 10 kbps. The binary symbols are represented as shown in below figure.



The source output is transmitted using two modulation schemes, namely Binary PSK (BPSK) and Quadrature PSK (QPSK). Let B_1 and B_2 be the bandwidth requirements of BPSK and QPSK respectively. Assuming that the bandwidth of the above rectangular pulses is 10 kHz, B_1 and B_2 are

(A) $B_1 = 20\,\text{kHz}$, $B_2 = 20\,\text{kHz}$

(B) $B_1 = 10\,\text{kHz}$, $B_2 = 10\text{kHz}$

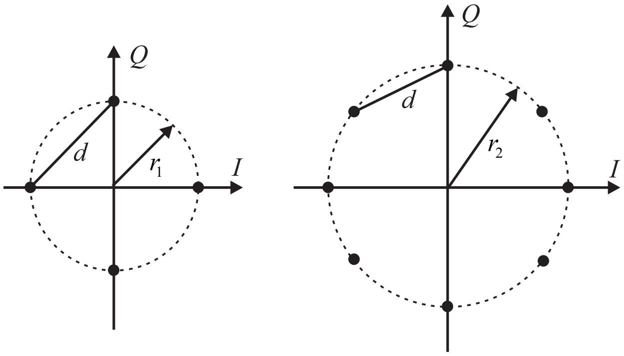
(C) $B_1 = 20\,\text{kHz}$, $B_2 = 10\text{kHz}$

(D) $B_1 = 10\,\text{kHz}$, $B_2 = 20\,\text{kHz}$

2011　IIT Madras

Statement for Linked Answer Questions 7.3 & 7.4

A four-phase and an eight-phase signal constellation are shown in the figure below.



7.3　For the constraint that the minimum distance between pairs of signal points be d for both constellations, the radii r_1 and r_2 of the circles are

(A) $r_1 = 0.707d, r_2 = 2.782d$

(B) $r_1 = 0.707d, r_2 = 1.932d$

(C) $r_1 = 0.707d, r_2 = 1.545d$

(D) $r_1 = 0.707d, r_2 = 1.307d$

7.4　Assuming high SNR and that all signals are equally probable, the additional transmitted signal energy required by the 8-PSK signal to achieve the same error probability as the 4-PSK signal is

(A) 11.90 dB　(B) 8.73 dB

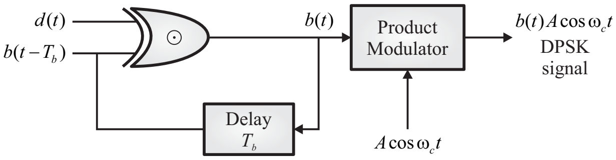
(C) 6.79 dB　(D) 5.33 dB

❖❖❖❖

Explanations Bandpass Transmission

7.1 (C)

For DPSK, transmitter block diagram is shown in figure below.



If it is not given in the questions then take XNOR logic gate for encoding DPSK signal.

Truth table for XNOR gate is given in the table below.

A	B	Z
0	0	1
0	1	0
1	0	0
1	1	1

Assuming reference bit as 1.

$d(t)$		1	1	0	0	1	1
$b(t-T_b)$		1	1	1	0	1	1
$b(t)$	1	1	1	0	1	1	1
Carrier phase		0	0	π	0	0	0

Initial first two message bit is having carrier phase as π, π. But with reference bit 1 we are not getting the correct phase as given in question.

Now assuming reference bit as 0.

$d(t)$		1	1	0	0	1	1
$b(t-T_b)$		0	0	0	1	0	0
$b(t)$	0	0	0	1	0	0	0
Carrier phase		π	π	0	π	π	π

The carrier phase for remaining four message bits are 0, π, π, π.

Hence, the correct option is (C).

7.2 (C)

Given : $f_b = 10$ kHz

Bandwidth of BPSK signal is given by,

$$(BW)_{BPSK} = 2f_b$$

$$B_1 = (BW)_{BPSK} = 2\times 10 = 20 \text{ kHz}$$

Bandwidth of QPSK signal is given by,

$$(BW)_{QPSK} = f_b$$

$$B_2 = (BW)_{QPSK} = 10 \text{ kHz}$$

Hence, the correct option is (C).

7.3 (D)

Method 1

For constellation (a),

$$d^2 = r_1^2 + r_1^2$$

$$d^2 = 2r_1^2$$

$$r_1 = \frac{d}{\sqrt{2}} = 0.707d$$

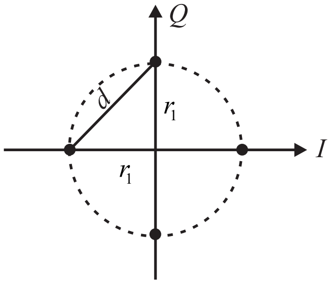


Fig. Constellation (a)

For constellation (b),

$$\text{In } \Delta BDO,\ \cos 45^0 = \frac{OD}{OB}$$

$$OD = \frac{r_2}{\sqrt{2}}$$

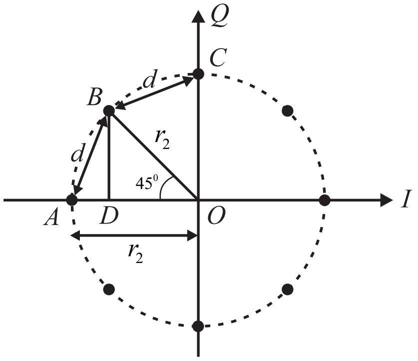


Fig. Constellation (b)

In ΔBDO, $\sin 45^0 = \dfrac{BD}{OB} = \dfrac{BD}{r_2}$

$$BD = r_2 \sin 45^0 = \frac{r_2}{\sqrt{2}}$$

$$DA = OA - OD = r_2 - \frac{r_2}{\sqrt{2}}$$

In ΔBDA, $(AB)^2 = (BD)^2 + (AD)^2$

$$d^2 = \left(\frac{r_2}{\sqrt{2}}\right)^2 + \left(r_2 - \frac{r_2}{\sqrt{2}}\right)^2$$

$$d^2 = r_2^2 (0.2928)^2 + \frac{r_2^2}{2}$$

$$d^2 = 0.585 r_2^2$$

$$r_2 = 1.307d$$

Hence, the correct option is (D).

Method 2

For M-ary, minimum distance is given by,

$$d = 2\sin\left(\frac{\pi}{M}\right)\sqrt{E_s}$$

where, $\sqrt{E_s}$ = distance of any symbol from origin

For 4-ary, $M = 4$

$$d_1 = 2\sin\left(\frac{\pi}{4}\right) r_1 = \sqrt{2}\, r_1$$

$$r_1 = 0.707d$$

For 8-ary, $M = 8$

$$d_2 = 2\sin\left(\frac{\pi}{8}\right) r_2$$

$$r_2 = 1.307d$$

Hence, the correct option is (D).

Scan for
Video Solution

Key point

Probability of error increases as minimum distance between two symbols decreases.

7.4 (D)

Method 1

Average probability of error in M-ary PSK is given by,

$$P_e = \frac{1}{2}\text{erfc}\left[\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{M}\right)\right]$$

For 4-PSK, $M = 4$

$$P_{e1} = \frac{1}{2}\text{erfc}\left[\sqrt{\frac{E_1}{N_0}}\sin\left(\frac{\pi}{4}\right)\right]$$

For 8-PSK, $M = 8$

$$P_{e2} = \frac{1}{2}\text{erfc}\left[\sqrt{\frac{E_2}{N_0}}\sin\left(\frac{\pi}{8}\right)\right]$$

For same probability of error in both PSK system,

$$\frac{1}{2}\text{erfc}\left[\sqrt{\frac{E_1}{N_0}}\sin\left(\frac{\pi}{4}\right)\right] = \frac{1}{2}\text{erfc}\left[\sqrt{\frac{E_2}{N_0}}\sin\left(\frac{\pi}{8}\right)\right]$$

$$\left[\sqrt{\frac{E_1}{N_0}}\sin\left(\frac{\pi}{4}\right)\right]^2 = \left[\sqrt{\frac{E_2}{N_0}}\sin\left(\frac{\pi}{8}\right)\right]^2$$

$$\frac{E_2}{E_1} = \frac{\sin^2\left(\dfrac{\pi}{4}\right)}{\sin^2\left(\dfrac{\pi}{8}\right)} = 3.41$$

In dB, $10\log\left(\frac{E_2}{E_1}\right)=10\log 3.41$

$$10\log E_2-10\log E_1=5.33\text{ dB}$$

$$(E_2)_{dB}-(E_1)_{dB}=5.33\text{ dB}$$

So, additional average transmitted signal energy required by the 8-PSK signal to achieve the same error probability as the 4-PSK signal is 5.33 dB.

Hence, the correct option is (D).

Method 2

Energy of any symbol can be calculated as,

$$E=(\text{distance between origin and symbol})^2$$

$$E[\text{8-PSK}]=(r_2)^2=(1.307d)^2$$

$$E[\text{4-PSK}]=(r_1)^2=(0.707d)^2$$

$$\frac{E[\text{8-PSK}]}{E[\text{4-PSK}]}=\frac{(1.307d)^2}{(0.707d)^2}=3.417$$

In dB, $10\log\left(\frac{E[\text{8-PSK}]}{E[\text{4-PSK}]}\right)=10\log 3.417$

$$10\log E[\text{8-PSK}]$$

$$-10\log E[\text{4-PSK}]=5.33\text{ dB}$$

$$(E_2)_{dB}-(E_1)_{dB}=5.33\text{ dB}$$

Hence, the correct option is (D).

❖❖❖❖

9

Information Theory & Coding

Partial Synopsis

Information

Information contained in any message is inversely proportional to the probability of occurrence of that message.

Measure of information :

Consider the communication system in which allowable message are $m_1, m_2, m_3 .... m_K$ with probability of occurrence $P_1, P_2, P_3 .... P_K$. Then amount of information transmitted through the message m_K with probability P_k is given by,

$$I_K = \log_2\left(\frac{1}{P_K}\right)$$

Unit of information :

When base 2 is used then unit is ‘bit’ when natural logarithmic base is used then unit is ‘Nat’ when base 10 is used then unit is ‘Hartley or decit’.

Properties :

(i) $I_K = 0$ for $P_K = 1$.

(ii) $I_K \geq 0$ for $0 \leq P_K \leq 1$.

(iii) $I_K \geq I_L$ for $P_K < P_L$.

(iv) $I_{K,L} = I_K + I_L$ if event K and L are statistically independent.

Average information or Entropy

Average information content per symbol in a group of symbol is known as entropy and represented by H.

- If we have M symbols whose probability of occurrence is $P_1, P_2 .... P_M$ then entropy H will be

$$H = \sum_{i=1}^{M} P_i \log_2 \frac{1}{P_i} \textbf{ bits/symbol}$$

Properties :

(i) Entropy lies in the range of $M \geq H \geq 0$.

(ii) $H = 0$, if and only if the probability $P_K = 1$ for some K and remaining probabilities in the set are all zero; the lower bound on entropy corresponds to no uncertainty.

(iii) $H = \log_2 K$, if and only if $P_K = \dfrac{1}{K}$ for all K, this upper bound on entropy corresponds to maximum uncertainty.

Entropy of mark-off sources :

Mark-off sources is source which emits the symbols dependently,

$$H = \sum_{i=1}^{n} P_i \sum_{j=1}^{n} P_{ij} \log_2\left(\frac{1}{P_{ij}}\right)$$

where, $P_i =$ Probability that source is in state *i*.

$P_{ij} =$ Probability that it is going state *i* to *j*.

Conditional and Joint Entropy

Let, $P(x_i) =$ Input probability

$P(y_j) =$ Output probability

$P(y_j/x_i) =$ Transition probability

$P(x_i/y_j) =$ Joint probability.

Note : $\sum_{j=1}^{n} P(y_j/x_i) = 1$

$$[P(Y)] = [P(X)]\ P[Y/X]$$

1. $H(X) = \sum_{i=1}^{m} P(x_i)\log_2\left[\dfrac{1}{P(x_i)}\right]$

2. $H(Y) = \sum_{j=1}^{n} P(y_j)\log_2\left[\dfrac{1}{P(y_j)}\right]$

3. $H(X/Y) = \sum_{j=1}^{n}\sum_{i=1}^{m} P(x_i, y_j)\log_2\left[\dfrac{1}{P(x_i/y_j)}\right]$

4. $H(Y/X) = \sum_{j=1}^{n}\sum_{i=1}^{m} P(x_i, y_j)\log_2\left[\dfrac{1}{P(y_j/x_i)}\right]$

5. $H(X,Y) = \sum_{j=1}^{n}\sum_{i=1}^{m} P(x_i, y_j)\log_2\left[\dfrac{1}{P(x_i, y_j)}\right]$

where, $H(X) =$ Average uncertainty of the channel output.

$H(Y) =$ Average uncertainty of the channel output.

$H(X/Y) =$ Measure of average uncertainty remaining about channel input after channel output has been observed.

$H(Y/X) =$ Measure of average uncertainty of channel output given channel input was transmitted.

$H(X,Y) =$ Average uncertainty of communication channel as a whole.

➢ Sample Questions

1988 IIT Kharagpur

9.1 In a digital communication system, transmissions of successive bits through a noisy channel are assumed to be independent events with error probability *p*. The probability of at most one error in the transmission of an 8-bit sequence is

(A) $7\dfrac{(1-p)}{8}\times\dfrac{7(1-p)}{8+\dfrac{p}{8}}$

(B) $(1-p)^8+8p(1-p)^7$

(C) $(1-p)^8+(1-p)^7$

(D) $(1-p)^8+p(1-p)^7$

2007 IIT Kanpur

9.2 During transmission over a certain binary communication channel, bit errors occur independently with probability *p*. The probability of AT MOST one bit in error in a block of *n* bits is given by

(A) p^n

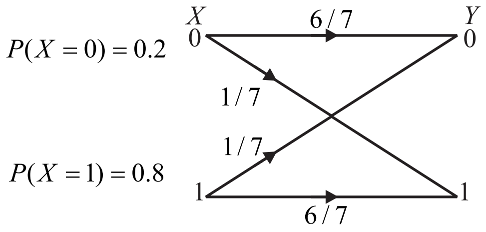
(B) $1-p^n$

(C) $np(1-p)^{n-1}+(1-p)^n$

(D) $1-(1-p)^n$

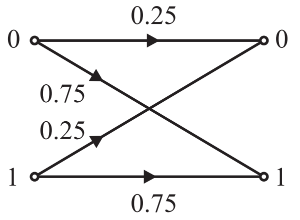
2015 IIT Kanpur

9.3 The input X to Binary Symmetric Channel (BSC) shown in the figure is '1' with probability of 0.8. The cross-over probability is $\dfrac{1}{7}$. If the received bit $Y=0$, the conditional probability that '1' was transmitted is______. **[Set - 01]**



2017 IIT Roorkee

9.4 Consider a binary memoryless channel characterized by the transition probability diagram shown in the figure. **[Set - 02]**



The channel is

(A) Lossless (B) Noiseless

(C) Useless (D) Deterministic

2019 IIT Madras

9.5 A linear hamming code is used to map 4-bit message to 7-bit code word. The encoder mapping is linear. If message 0001 is mapping to the codeword 0000111 and the message is 0011 is mapped to codeword 1100110 then massage 0010 is mapped to

(A) 1100001 (B) 0010011

(C) 1111000 (D) 1111111

2021 IIT Bombay

9.6 A digital transmission system uses a (7, 4) systematic linear Hamming code for transmitting data over a noisy channel. If three of the message code-word pairs in this code $(m_i; c_i)$, where, $\mathbf{c}_i$ is the code-word corresponding to the i^{th} message m_i, are known to be $(1100; 0101100)$, $(1110; 0011110)$, $(0110; 1000110)$, then which of the following is a valid code-word in this code?

(A) 1101001 (B) 1011010

(C) 0001011 (D) 0110100

Explanations Information Theory & Coding

Concept of MAP & ML Decoding

Scan for Video Explanation

Example on MAP & ML Decoding

Scan for Video Explanation

9.1 (B)

$$P(\text{success}) = p,\ P(\text{failure}) = 1 - p$$

According to Poisson's formula for probability,

$$P(x = r) = {}^{n}C_r\, p^r (1-p)^{n-r}$$

$$P(x = \text{at most } 1) = p(x=0) + p(x=1)$$

$$P(x = \text{at most } 1) = 8C_0 (p)^0 \times (1-p)^{8-0}$$

$$+{}^{8}C_1 (p)^1 (1-p)^{8-1}$$

$$P(x = \text{at most } 1) = (1-p)^8 + 8p(1-p)^7$$

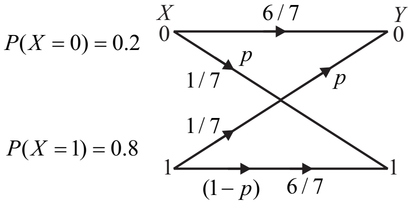
Hence, the correct option is (B).

9.2 (C)

Refer Solution 9.6

9.3 0.4

According to question



$$P(X) = [0.2 \quad 0.8]$$

For BSC channel,

$$P[Y \mid X] = \begin{bmatrix} 1-P & P \\ P & 1-P \end{bmatrix} = \begin{array}{c|cc|l} & Y=0 & Y=1 & \\ \hline & \frac{6}{7} & \frac{1}{7} & \rightarrow X=0 \\ & \frac{1}{7} & \frac{6}{7} & \rightarrow X=1 \end{array}$$

$$P[Y] = P[X]P[Y \mid X]$$

$$P[Y] = [0.2 \quad 0.8]\begin{bmatrix} \frac{6}{7} & \frac{1}{7} \\ \frac{1}{7} & \frac{6}{7} \end{bmatrix}$$

$$P[Y]=\begin{array}{cc} Y=0 & Y=1 \\ \left[\frac{2}{7}\right. & \left.\frac{5}{7}\right] \end{array}$$

According to Baye's theorem,

$$P[X\,|\,Y]=\frac{P[Y\,|\,X]P[X]}{P[Y]}$$

$$P[X=1\,|\,Y=0]=\frac{P[Y=0\,|\,X=1]P[X=1]}{P[Y=0]}$$

$$P[X=1\,|\,Y=0]=\frac{\left(\frac{1}{7}\right)\times 0.8}{0.285}=0.4$$

Hence, if the received bit $Y=0$, the conditional probability that '1' was transmitted is **0.4.**

9.4 **2**

Given : Discrete memoryless source with alphabet $S=\{s_0,\,s_1,\,s_2,\,s_3,\,s_4,....\}$

and $P=\left\{\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16},\frac{1}{32},....\right\}$

Entropy of source X is given by,

$$H=\sum_{i=0}^{N}P_i\log_2\frac{1}{P_i}$$

$$H=\frac{1}{2}\log_2 2+\frac{1}{4}\log_2 4+\frac{1}{8}\log_2 8+\frac{1}{16}\log_2 16+.....$$

$$H=\frac{1}{2}+2\times\left(\frac{1}{2}\right)^2+3\times\left(\frac{1}{2}\right)^3+4\times\left(\frac{1}{2}\right)^4+..... \quad ...(i)$$

Divide both side by 2, So,

$$\frac{H}{2}=\left(\frac{1}{2}\right)^2+2\times\left(\frac{1}{2}\right)^3+3\times\left(\frac{1}{2}\right)^4+..... \quad ...(ii)$$

Subtracting (ii) from (i),

$$\frac{H}{2}=\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+.....$$

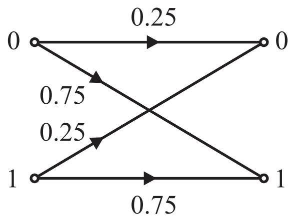
$$\frac{H}{2}=\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)}=1$$

$$H=2\text{ bits/symbol}$$

Hence, the entropy of the source is **2 bits/symbol.**

9.5 **(C)**

Given :



Channel transition matrix is given by,

$$P[Y/X]=\begin{bmatrix}0.25 & 0.75\\0.25 & 0.75\end{bmatrix}$$

Lossless channel : A channel described by a channel matrix with only one nonzero element in each column is called a lossless channel.

Example :

$$P[Y/X]=\begin{bmatrix}\frac{1}{3} & \frac{2}{3} & 0 & 0 & 0\\ 0 & 0 & \frac{3}{4} & \frac{1}{4} & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Thus, given BSC channel is not lossless.

Deterministic channel : A channel described by a channel matrix with only one nonzero element in each row is called a deterministic channel.

Example : $P[Y/X]=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 1 & 0\end{bmatrix}$

Thus, given BSC channel is not deterministic.

Noiseless channel : A channel is called noiseless if it is both lossless and deterministic.

Example : $P[Y/X]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

Thus, given BSC channel is not noiseless.

So, the given channel is useless.

Hence, the correct option is (C).

9.6 (A)

Given :

Data	Codeword
0 0 0 1	0 0 0 0 1 1 1
0 0 1 1	1 1 0 0 1 1 0

Linear Hamming code i.e. mod 2 addition of any two codewords will be codewords of code vector

	Data	Codeword
Mod 2 $\oplus$	0 0 0 1	0 0 0 0 1 1 1
	0 0 1 1	1 1 0 0 1 1 0
	0 0 1 0	1 1 0 0 0 0 1

Hence, the correct option is (A).

9.7 (C)

Given message and corresponding code-words are,

Message					**Code-word**							
m_i	m_1	m_2	m_3	m_4	P_1	P_2	P_3	m_1	m_2	m_3	m_4	C_i
m_{12}	1	1	0	0	0	1	0	1	1	0	0	C_{12}
m_{14}	1	1	1	0	0	0	1	1	1	1	0	C_{14}
m_6	0	1	1	0	1	0	0	0	1	1	0	C_6

From four parity generator we have to select any three equation,

$$P_A=m_1\oplus m_2\oplus m_3$$

$$P_B=m_1\oplus m_2\oplus m_4$$

$$P_C=m_2\oplus m_3\oplus m_4$$

$$P_D=m_1\oplus m_3\oplus m_4$$

From comparing with all three given code-words i.e., C_{12}, C_{14} and C_6

$$P_1=P_B,\ P_2=P_C,\ P_3=P_A$$

So, for message $m_9=1001,\ C_9=0111001$

Option (A) is wrong.

$$m_{10}=1010,\ C_{10}=1101010$$

Option (B) is wrong.

$$m_{11}=1011,\ C_{11}=0001011$$

Option (C) is correct.

$$m_4=0100,\ C_4=1110100$$

Option (D) is wrong.

Hence, the correct option is (C).

❖❖❖❖

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SINCE 2004

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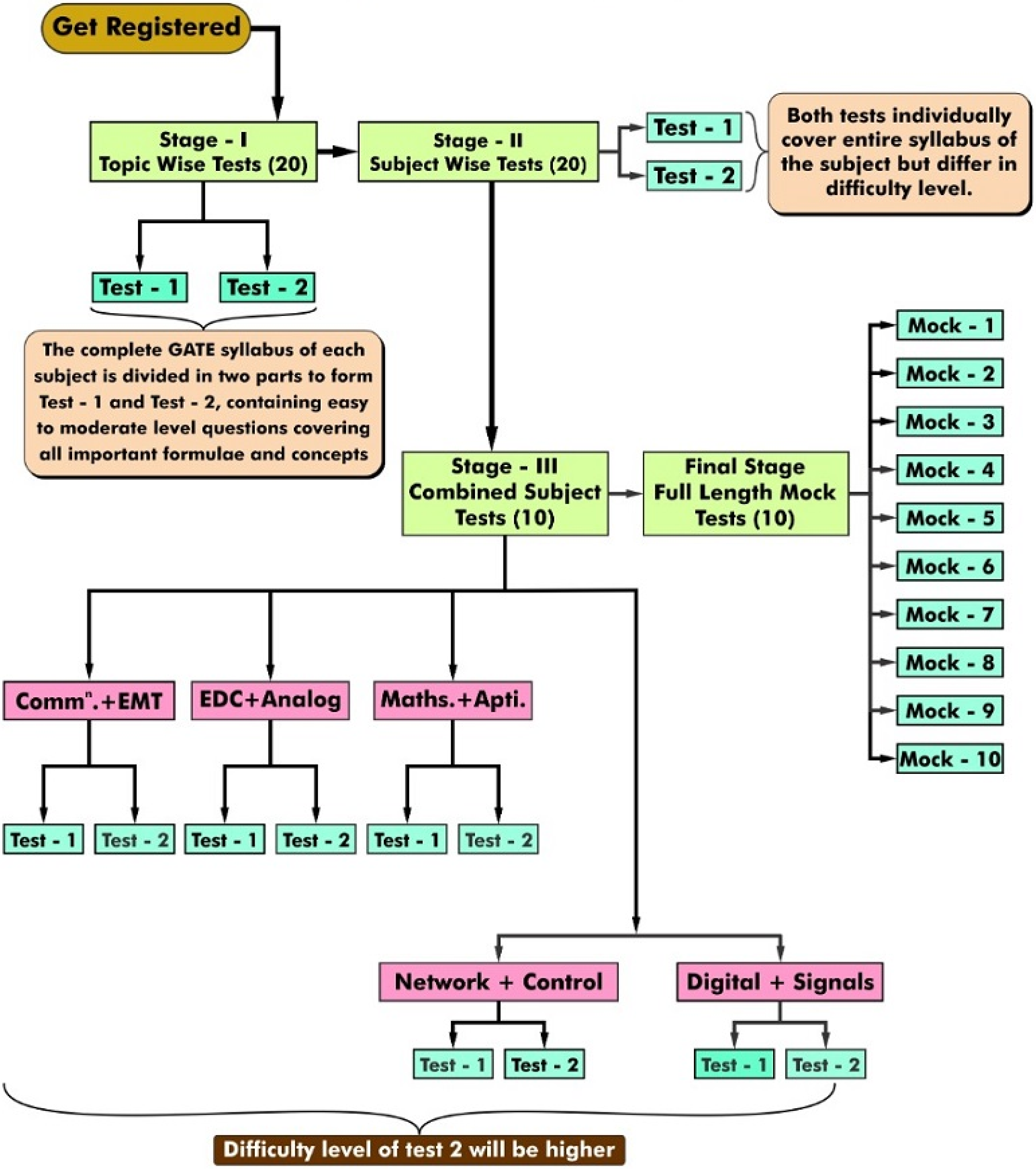
TEST SERIES STRUCTURE (EC)

(One Year Program)



Get Registered

Stage - I
Topic Wise Tests (20)

Test - 1
Test - 2

The complete GATE syllabus of each subject is divided in two parts to form Test - 1 and Test - 2, containing easy to moderate level questions covering all important formulae and concepts

Stage - II
Subject Wise Tests (20)

Test - 1
Test - 2

Both tests individually cover entire syllabus of the subject but differ in difficulty level.

Stage - III
Combined Subject Tests (10)

Comm^{n}.+EMT
Test - 1
Test - 2

EDC+Analog
Test - 1
Test - 2

Maths.+Apti.
Test - 1
Test - 2

Network + Control
Test - 1
Test - 2

Digital + Signals
Test - 1
Test - 2

Difficulty level of test 2 will be higher

Final Stage
Full Length Mock Tests (10)

Mock - 1
Mock - 2
Mock - 3
Mock - 4
Mock - 5
Mock - 6
Mock - 7
Mock - 8
Mock - 9
Mock - 10

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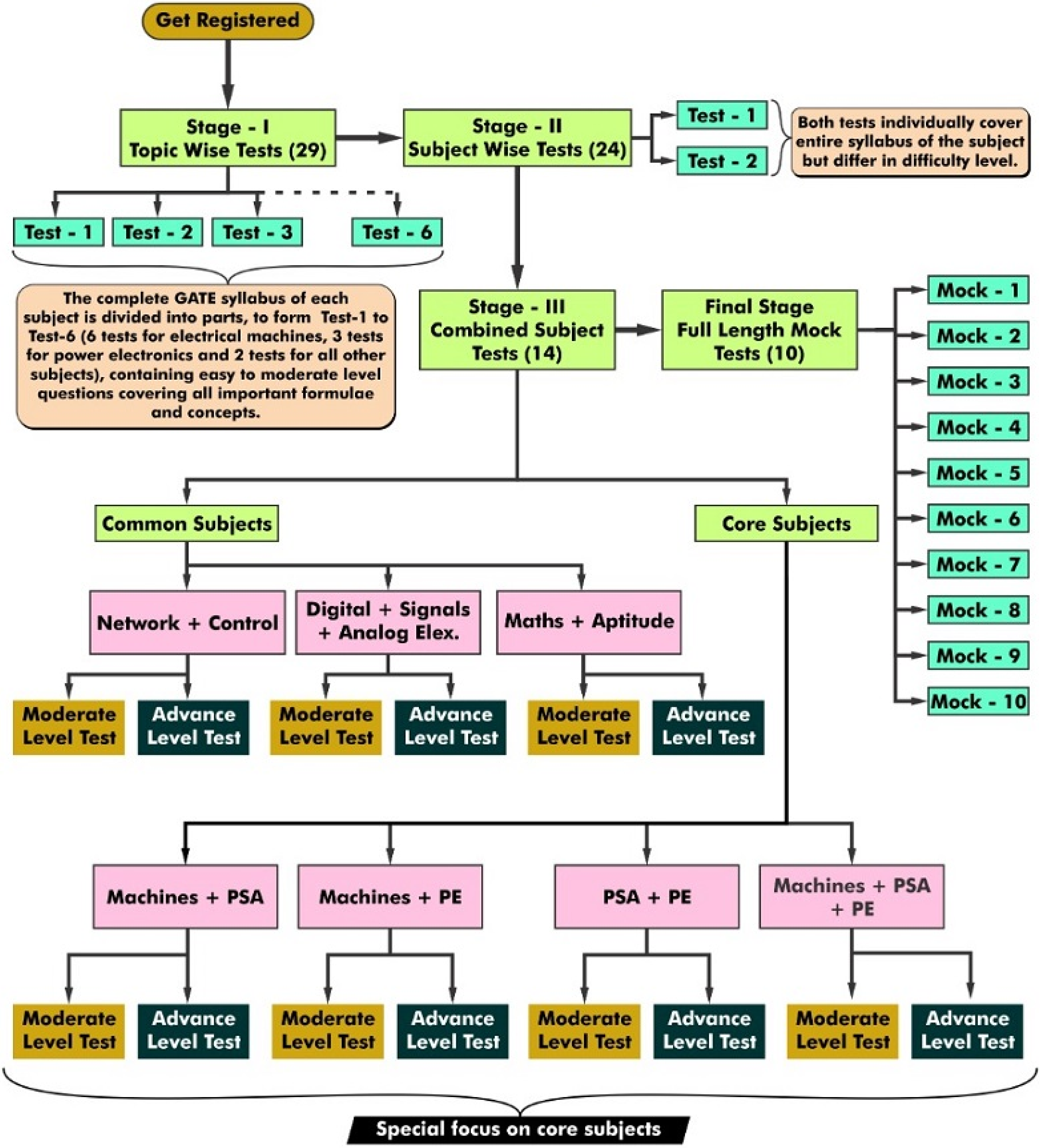
TEST SERIES STRUCTURE (EE)

(One Year Program)



Get Registered

Stage - I
Topic Wise Tests (29)

Test - 1
Test - 2
Test - 3
Test - 6

The complete GATE syllabus of each subject is divided into parts, to form Test-1 to Test-6 (6 tests for electrical machines, 3 tests for power electronics and 2 tests for all other subjects), containing easy to moderate level questions covering all important formulae and concepts.

Stage - II
Subject Wise Tests (24)

Test - 1
Test - 2

Both tests individually cover entire syllabus of the subject but differ in difficulty level.

Stage - III
Combined Subject Tests (14)

Final Stage
Full Length Mock Tests (10)

Mock - 1
Mock - 2
Mock - 3
Mock - 4
Mock - 5
Mock - 6
Mock - 7
Mock - 8
Mock - 9
Mock - 10

Common Subjects

Network + Control
Moderate Level Test
Advance Level Test

Digital + Signals + Analog Elex.
Moderate Level Test
Advance Level Test

Maths + Aptitude
Moderate Level Test
Advance Level Test

Core Subjects

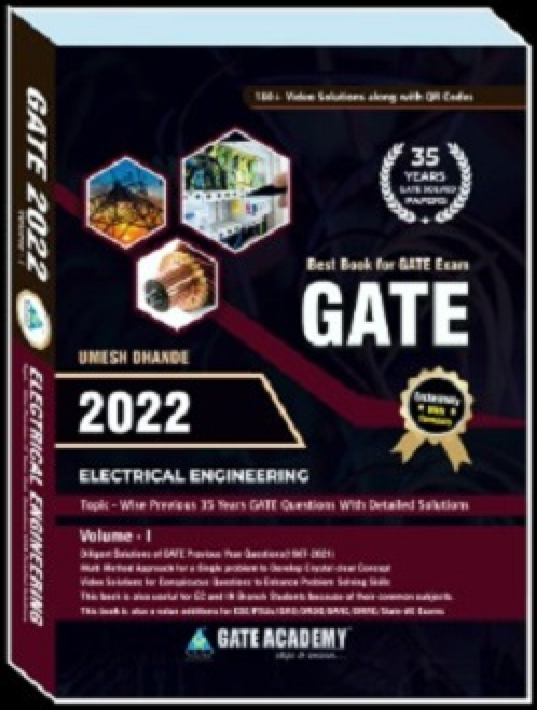
Machines + PSA
Moderate Level Test
Advance Level Test

Machines + PE
Moderate Level Test
Advance Level Test

PSA + PE
Moderate Level Test
Advance Level Test

Machines + PSA + PE
Moderate Level Test
Advance Level Test

Special focus on core subjects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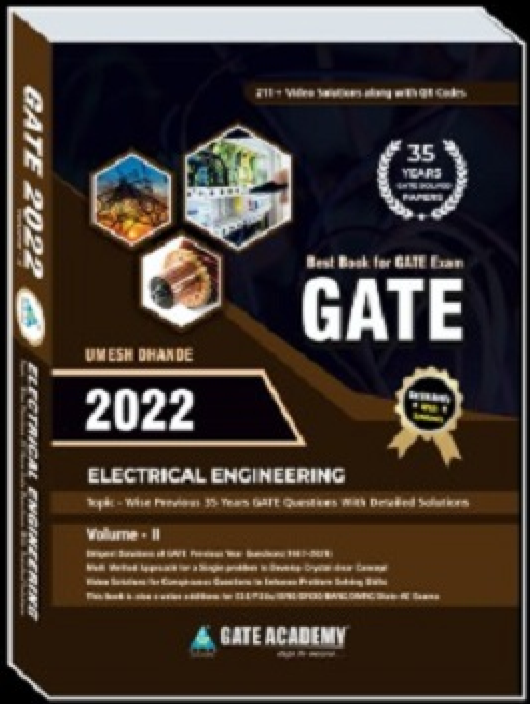
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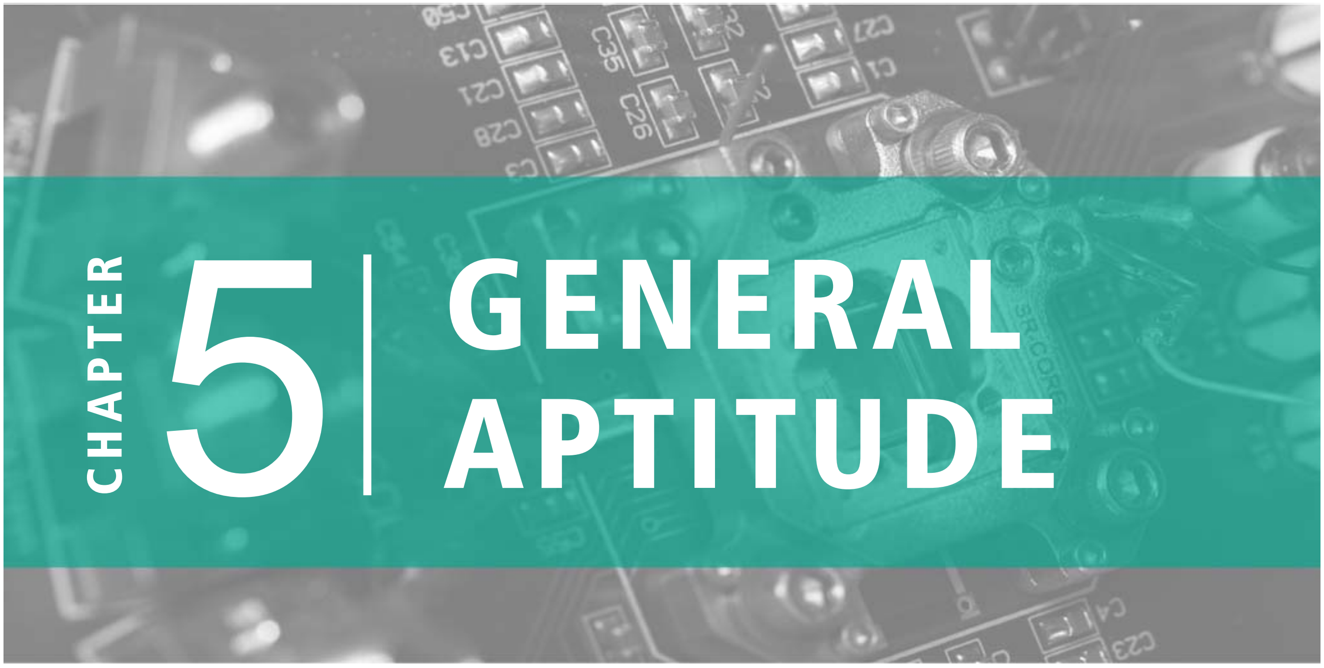


CHAPTER 5 GENERAL APTITUDE

Marks Distribution of General Aptitude in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2010	5	5	15
2011	5	5	15
2012	5	5	15
2013	5	5	15
2014 Set-1	5	5	15
2014 Set-2	5	5	15
2014 Set-4	5	5	15
2015 Set-1	5	5	15
2015 Set-2	5	5	15
2015 Set-3	5	5	15
2016 Set-1	5	5	15

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2016 Set-2	5	5	15
2016 Set-3	5	5	15
2017 Set-1	5	5	15
2017 Set-2	5	5	15
2018	5	5	15
2019	5	5	15
2020	5	5	15
2021	5	5	15

Syllabus : General Aptitude (GA)

Verbal Aptitude : Basic English grammar: tenses, articles, adjectives, prepositions, conjunctions, verb-noun agreement, and other parts of speech Basic vocabulary: words, idioms, and phrases in context Reading and comprehension Narrative sequencing.

Quantitative Aptitude : Data interpretation: data graphs (bar graphs, pie charts, and other graphs representing data), 2- and 3-dimensional plots, maps, and tables Numerical computation and estimation: ratios, percentages, powers, exponents and logarithms, permutations and combinations, and series Mensuration and geometry Elementary statistics and probability.

Analytical Aptitude : Logic: deduction and induction Analogy, Numerical relations and reasoning.

Spatial Aptitude : Transformation of shapes: translation, rotation, scaling, mirroring, assembling, and grouping Paper folding, cutting, and patterns in 2 and 3 dimensions.

Contents : General Aptitude (GA)

1 Verbal Ability

➢ Sample Questions

2012 IIT Delhi

1.1 Choose the most appropriate alternative from the options given below to complete the following sentence :

If the tired soldier wanted to lie down, he _____ the mattress out on the balcony.

(A) should take

(B) shall take

(C) should have taken

(D) will have taken

2015 IIT Kanpur

1.2 Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows :

Statements :

I. All film star are playback singers.

II. All film directors are film stars.

Conclusions :

I. All film directors are playback singers.

II. Some film stars are film directors.

(A) Only conclusion I follows.

(B) Only conclusion I nor II follows.

(C) Neither conclusion I nor II follows.

(D) Both conclusions I and II follow.

2016 IISc Bangalore

1.3 An apple costs Rs. 10. An onion costs Rs. 8.

Select the most suitable sentence with respect to grammar and usage.

(A) The price of an apple is greater than an onion.

(B) The price of an apple is more than onion.

(C) The price of an apple is greater than that of an onion.

(D) Apples are more costlier than onions.

1.4 **M** has a son **Q** and a daughter **R**. He has no other children. **E** is the mother of **P** and daughter-in-law of **M**. How is **P** related to **M**?

(A) **P** is the son-in-law of **M**.

(B) **P** is the grandchild of **M**.

(C) **P** is the daughter-in law of **M**.

(D) **P** is the grandfather of **M**.

2017 IIT Roorkee

1.5 Some tables are shelves. Some shelves are chairs. All chairs are benches. Which of the following conclusions can be deduced from the preceding sentences?

(i) At least one bench is a table.

(ii) At least one shelf is a bench.

(iii) At least one chair is a table.

(iv) All benches are chairs.

(A) Only (i) (B) Only (ii)

(C) Only (ii) and (iii) (D) Only (iv)

1.6 *S*, *T*, *U*, *V*, *W*, *X*, *Y* and *Z* are seated around a circular table. *T*'s neighbors are *Y* and *V*. *Z* is seated third to the left of *T* and second to the right of *S*. *U*'s neighbors are *S* and *Y* ; and *T* and *W* are not seated opposite each other. Who is third to the left of *V* ?

(A) *X* (B) *W*

(C) *U* (D) *T*

1.7 "If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages for though I have spent a lifetime in the country, I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters."

Which of the following statement best reflects the author's opinion?

(A) An intimate association does not allow for the necessary perspective.

(B) Matters are recorded with an impartial perspective.

(C) An intimate association offers an impartial perspective.

(D) Actors are typically associated with the impartial recording of matters.

2018 IIT Guwahati

1.8 A coastal region with unparalleled beauty is home to many species of animals. It is dotted with coral reefs and unspoilt white sandy beaches. It has remained inaccessible to tourists due to poor connectivity and lack of accommodation. A company has spotted the opportunity and is planning to develop a luxury resort with helicopter service to the nearest major city airport. Environmentalists are upset that this would lead to the region becoming crowded and polluted like any other major beach resorts.

Which one of the following statements can be logically inferred from the information given in the above paragraph?

(A) The culture and tradition of the local people will be influenced by the tourists.

(B) The region will become crowded and polluted due to tourism.

(C) The coral reefs are on the decline and could soon vanish.

(D) Helicopter connectivity would lead to an increase in tourists coming to the region.

2021 IIT Bombay

1.9 Given below are two statements and two conclusions.

Statement :

Statement 1 : All purple are green.

Statement 2 : All Black are green.

Conclusion :

Conclusion I : Some black are purple

Conclusion II : No black is purple

Based on the above statements and conclusions, which one of the following options is logically **CORRECT**?

(A) Only conclusion I is correct.

(B) Only conclusion II is correct.

(C) Either conclusion I or II is correct

(D) Both conclusion I & II are correct

❖❖❖❖

Explanations　Verbal Ability

1.1 (C)

There are a number of different ways to use **'should have been' or 'should have done' (i.e. *should have* + *past participle*).**

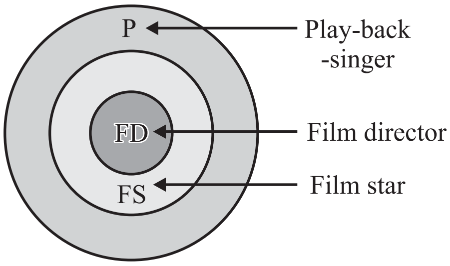
For example :

You may also hear 'should have been/done' used when you expect a confirmation that something had a particular outcome in the past. This usage would be similar to 'must have been/done'

Hence, the correct option is (C).

1.2 (D)

The possible Venn diagram drawn from the given statements is :



From this diagram, conclusions I and II logically follows.

Hence, the correct option is (D).

1.3 (C)

Based on the given sentences option (C) is the correct sentence which is in the comparative degree. Options (A) and (B) convey the wrong comparison and (D) has double comparative and so they are wrong.

Hence, the correct option is (C).

1.4 (B)

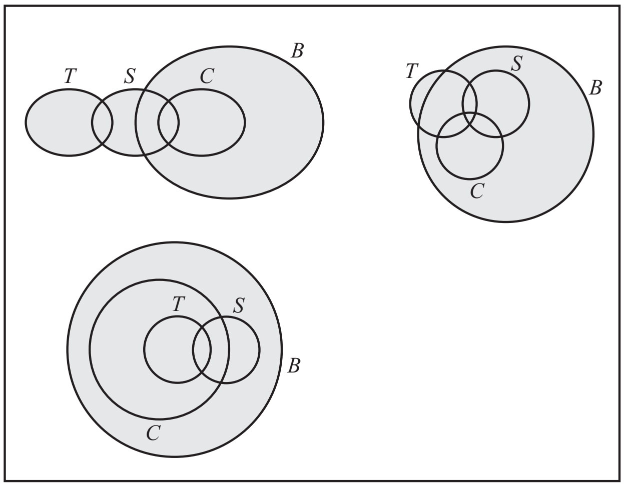
Q and **R** are the son and daughter of **M**, **E** is the mother of **P** and daughter-in-law of **M** means **Q** and **E** are married couples in the family

Therefore, **P** is the grandchild of **M.**

Hence, the correct option is (B).

1.5 (B)

Method 1



Only conclusion (ii) follows.

Hence, the correct option is (B).

Method 2

Some tables are shelves,

$T \in S$　…(i)

Some shelves are chairs,

$S \in C$　…(ii)

All chairs are benches,

$C = B$　…(iii)

Using equation (ii) and (iii),

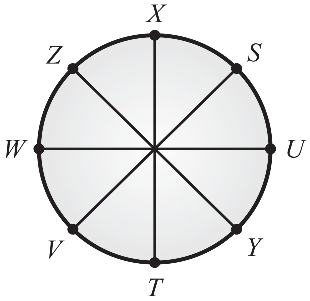
$$S \in B$$

Therefore, at least one shelf is a bench.

Hence, the correct option is (B).

1.6 **(A)**

Following circular seating arrangement can be drawn



Only one such arrangement can be drawn. The person on third to the left of *V* is *X*.

Hence, the correct option is (A).

1.7 **(C)**

It is clear from the last line :

“I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters.”

Hence, the correct option is (C).

1.8 **(B)**

(A) Culture and tradition of local people will be influenced by the tourists : Not exactly meant by passage.

(B) Region will become crowded and polluted by tourism : This concluded most closely to the environment safety.

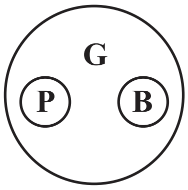
(C) The coral reefs are one decline and could soon vanish : Unless we change the way we live that affect the biodiversity of nature so it is not exact conclusion of the passage.

(D) Helicopter connectivity would lead to increase in tourists coming to region : Inappropriate

Hence the correct option is (B).

1.9 **(C)**

According to the statement and conclusion given in the question we can formed Venn diagram as shown below,



1. can’t say (wrong)
2. can’t say (wrong)

Both the conclusion are wrong by can’t say condition and variable of the conclusion are same. Also one conclusion is positive and another is negative. Hence they full fill the condition of “either or” case.

Hence, the correct option is (C).

◆ ◆ ◆ ◆

2

Numerical Ability

➢ Sample Questions

2013 IIT Bombay

2.1 Find the sum to n terms of the series 10 + 84 + 734 +….

(A) $\frac{9(9^n+1)}{10}+1$ (B) $\frac{9(9^n-1)}{8}+1$

(C) $\frac{9(9^n-1)}{8}+n$ (D) $\frac{9(9^n-1)}{8}+n^2$

2016 IISc Bangalore

2.2 A person moving through a tuberculosis prone zone has a 50% probability of becoming infected. However, only 30% of infected people develop the disease. What percentage of people moving through a tuberculosis prone zone remains infected but does not show symptoms of disease?

(A) 15 (B) 33

(C) 35 (D) 37

2.3 A wire of length 340 mm is to be cut into two parts. One of the parts is to be made into a square and the other into a rectangle where sides are in the ratio of 1 : 2. What is the length of the side of the square (in mm) such that the combined area of the square and the rectangle is a **MINIMUM**?

(A) 30 (B) 40

(C) 120 (D) 180

2017 IIT Roorkee

2.4 Trucks (10 m long) and cars (5 m long) go on a single lane bridge. There must be a gap of at least 20 m after each truck and a gap of at least 15 m after each car. Trucks and cars travel at a speed of 36 km/h. If cars and trucks go alternately, what is the maximum number of vehicles that can use the bridge in one hour?

(A) 1440 (B) 1200

(C) 720 (D) 600

2.5 A rule states that in order to drink beer, one must be over 18 years old. In a bar, there are 4 people P is 16 years old, Q is 25 years old, R is drinking milkshake and S is drinking a beer. What must be checked to ensure that the rule is being followed?

(A) Only P's drink

(B) Only P's drink and S's age

(C) Only S's age

(D) Only P's drink, Q's drink and S's age

2018 IIT Guwahati

2.6 Two alloys A and B contain gold and copper in the ratios of 2:3 and 3:7 by mass, respectively. Equal masses of alloys A and B are melted to make an alloy *C*. The ratio of gold to copper in alloy *C* is,

(A) 5 : 10 (B) 7 : 13

(C) 6 : 11 (D) 9 : 13

2.7 A cab was involved in a hit and run accident at night. You are given the following data about the cabs in the city and the accident.

(i) 85% of cabs in the city are green and the remaining cabs are blue.

(ii) A witness identified the cab involved in the accident as blue.

(iii) It is known that a witness can correctly identify the cab colour only 80% of the time.

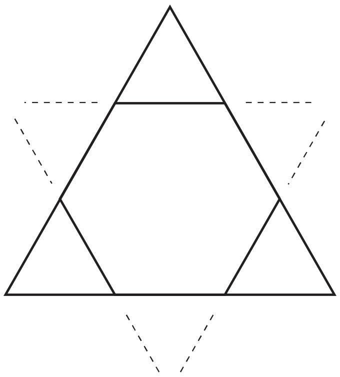
Which of the following options is closest to the probability that the accident was caused by a blue cab?

(A) 12% (B) 15%

(C) 41% (D) 80%

2021 IIT Bombay

2.8



Corners are cut from an equilateral triangle to produce a regular convex hexagon as shown in the figure above. The ratio of the area of the regular convex hexagon to the area of original equilateral triangle is

(A) 2 : 3 (B) 3 : 4

(C) 4 : 5 (D) 5 : 6

❖❖❖❖

Explanations Numerical Ability

2.1 (D)

Given : Series is $10+84+734+.....$

Method 1

Check from the options by substituting

$n = 1, 2\ldots$

For $n = 1$:

From option (A),

$$\frac{9(9^1+1)}{10}+1=10$$

From option (B),

$$\frac{9(9^1-1)}{8}+1=10$$

From option (C),

$$\frac{9(9^1-1)}{8}+1=10$$

From option (D),

$$\frac{9(9^1-1)}{8}+1^2=10$$

For $n = 1$, all options are right.

For $n = 2$, sum must be $10 + 84 = 94$.

Only option (D) is correct for this and we can go through that.

Hence, the correct option is (D).

Method 2

Sum of n terms of series

$$S_n=10+84+734+....$$

$$S_n=(9+1)+(81+3)+(729+5)+....$$

$$S_n = (9+81+729+....)+(1+3+5+....)$$

$$S_n = (9+9^2+9^3+....n \text{ terms})$$
$$+(1+3+5+....n \text{ terms})$$

$$S_n = 9\left[\frac{9^n-1}{9-1}\right]+\frac{n}{2}[2\times1+(n-1)2]$$

$$S_n = \frac{9}{8}(9^n-1)+n^2$$

Hence, the correct option is (D).

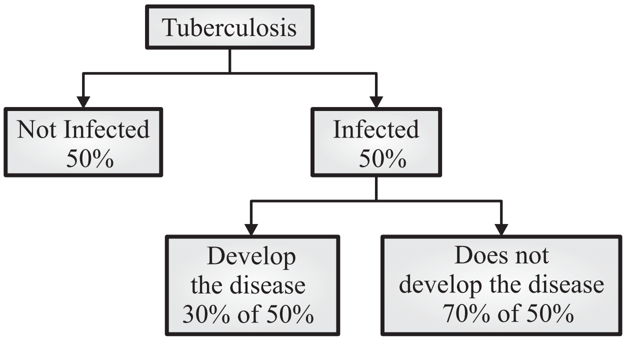
2.2 (C)

Given :

Probability of becoming infected = 50%

Probability of infected people developed the disease = 30%

According to the question, following information can is obtained.



Therefore, percentage of people moving through a tuberculosis prone zone remains infected but does not show symptoms of disease

$$= \frac{70}{100}\times\frac{50}{100} = \frac{35}{100} = 35\%$$

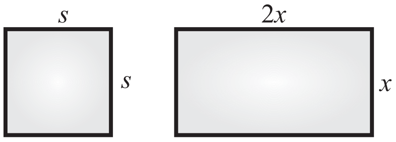
Hence, the correct option is (C).

2.3 (B)

Given : Length of the wire = 340 mm

Also the sides of rectangle are in the ratio 1 : 2.

Method 1



Let length and width of rectangle be $2x$ and x respectively.

Perimeter of rectangle $= 2(2x+x) = 6x$

Perimeter of square $= 340-6x$

Side of square, $s = \dfrac{340-6x}{4}$

Total area

$$= \text{Area of square } + \text{Area of rectangle}$$

$$= \left(\frac{340-6x}{4}\right)^2 + 2x\times x$$

$$= \left(\frac{340-6x}{4}\right)^2 + 2x^2$$

Given condition : Combined area of square + rectangle should be minimum,

Let $f(x) = \left(\dfrac{340-6x}{4}\right)^2 + 2x^2$

For minimum value of $F(x)$

$$f'(x) = 0$$

$$f'(x) = \left[2\times\left(\frac{340-6x}{4}\right)\times\frac{-6}{4}\right]+4x = 0$$

$$-3\left(\frac{340-6x}{4}\right) = -4x$$

$$\frac{340-6x}{4} = \frac{4x}{3}$$

$$\frac{340}{4} = \frac{4x}{3}+\frac{3x}{2}$$

$$\frac{340}{4} = \frac{17x}{6}$$

$$x = \frac{340\times6}{17\times4} = 30 \text{ mm}$$

Side of square,

$$s = \frac{340-6x}{4} = \frac{340-180}{4} = 40 \text{ mm}$$

Hence, the correct option is (B).

Method 2

Checking from the options,

Option (A) :

Side of the square = x = 30 mm

Perimeter of the square = $4\times30 = 120$ mm

Therefore,

Perimeter of the rectangle = 340–120 = 220 mm

$$2x + 2 \times 2x = 220$$

$$x = 37$$

$$2x = 37 \times 2 = 74$$

Area of square = $x^2 = (30)^2 = 900$

Area of rectangle = $x \times 2x = 37 \times 74 = 2738$

Total area $(A) = 900 + 2738 = 3638$ mm^2

Option (B) :

Side of the square = x = 40 mm

Perimeter of the square $= 4 \times 40 = 160$

Perimeter of the square = 340 – 160 = 180 mm

$$2x + 2 \times 2x = 180 \text{ mm}$$

$$6x = 180 \text{ mm}$$

$$x = 30 \text{ mm}$$

Area of the square = 40 × 40 = 1600 mm^2

Area of the rectangle = 30 × 2 × 30 = 1800 mm^2

Total area = 1600 + 1800

Area $(B) = 3400$ mm^2

Option (C) :

Side of the square $= x = 120$

Perimeter of the square $= 4 \times 120 = 480$

Here perimeter of the square > length of wire

So, this option is incorrect.

Option (D) :

Side of the square $= x = 180$

Perimeter of the square $= 4 \times 180 = 720$

Here perimeter of the square > length of wire

So, this option is incorrect.

On comparing the valid areas of options (A) and (B),

$$\text{Area } (B) < \text{Area } (A)$$

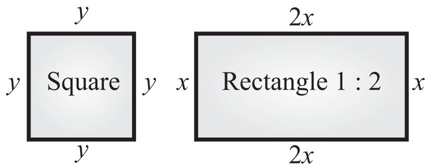
Hence, the correct option is (B).

Method 3

As per data

Wire is divided into two parts –

Let $a + b = 340$



Perimeter of rectangle,

$$b = 2x + x + 2x + x$$

$$b = 6x$$

$$x = b/6$$

and perimeter of square, $a = 4y$

$$y = a/4$$

Sum of area $A =$ Area of square + Area of rectangle

$$A = y^2 + 2x^2$$

$$A = (a/4)^2 + 2(b/6)^2$$

$$A = \frac{a^2}{16} + \frac{b^2}{18}$$

$$A = \frac{a^2}{16} + \frac{(340 - a)^2}{18}$$

For minimum area, $\frac{dA}{da} = 0$

$$\frac{2a}{16} + \frac{2(340 - a)}{18} \times (-1) = 0$$

$$\frac{a}{8} = \frac{(340 - a)}{9}$$

$$9a = 2720 - 8a$$

$$a = 160 \text{ m}$$

Side of square $= \frac{a}{4} = 40$ m

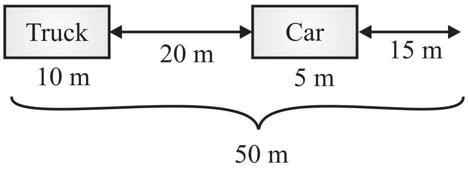
Hence, the correct option is (B).

2.4 (A)

Given :

	Length	**Gap**
Truck →	10 m	20 m
Car →	5 m	15 m

$$\text{Speed} = 36 \text{ km/hr} \times \frac{5}{18} = 10 \text{ m/s}$$



$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{50}{10} = 5 \text{ sec}$$

If 50 m distance is covered in 5 seconds by two vehicles (one truck and one car). Then in one hour maximum number of vehicles passing through the lane

$$= \frac{3600 \times 2}{5} = 1440$$

Hence, the correct option is (A).

2.5 (B)

Given :

P is 16 years old, *Q* is 25 years old, *R* is drinking milkshake and *S* is drinking beer.

Since, age of *P* is less than 18 year, so checking *P*'s drink and since *S* is drinking a beer so it is required to check age of *S* for the rule to be followed.

Also, since *Q* is 25 years old, he may have any drink and since *R* is drinking milk shake so there are no issues.

Therefore, *P*'s and *S*'s age should be checked to ensure that the rule is being followed.

Hence, the correct option is (B).

2.6 (B)

Let the quantity of *A* and *B* each be 10 gm

$$\text{Mass of gold in alloy } A = \frac{2 \times 10}{2+3} = \frac{2 \times 10}{5} = 4 \text{ gm}$$

and Mass of copper in alloy *A*

$$= \frac{3 \times 10}{3+2} = \frac{3 \times 10}{5} = 6 \text{ gm}$$

$$\text{Mass of gold in alloy } B = \frac{3 \times 10}{3+7} = \frac{3 \times 10}{10} = 3 \text{ gm}$$

and Mass of copper in alloy *B*

$$= \frac{7 \times 10}{7+3} = \frac{7 \times 10}{10} = 7 \text{ gm}$$

Total quantity of gold in alloy $C = 4 + 3 = 7$ gm

Total quantity of copper in alloy *C*

$$= 6 + 7 = 13 \text{ gm}$$

The required ratio of alloy $C = 7 : 13$

Hence, the correct option is (B).

Key Point

If two alloys of ratio *A* : *B* and *C* : *D* are mixed together, the new ratio of alloys is given by,

$$\frac{A}{A+B} + \frac{C}{C+D} : \frac{B}{A+B} + \frac{D}{C+D}$$

2.7 (C)

Given : 85% of the cabs are green

15% are blue

Assuming there are 100 cabs. 85 cabs are green and 15 are blue

Witness is correct 80% of the times in identifying the cab is blue

Total number of blue cabs identified correctly

$$= 15 \times 0.8 = 12$$

Witness is incorrect 20% of the times identifying the cab is green.

Total number of green cabs identified incorrectly $= 85 \times 0.2 = 17$

The total cabs identified by the witness

$$= 12 + 17 = 29$$

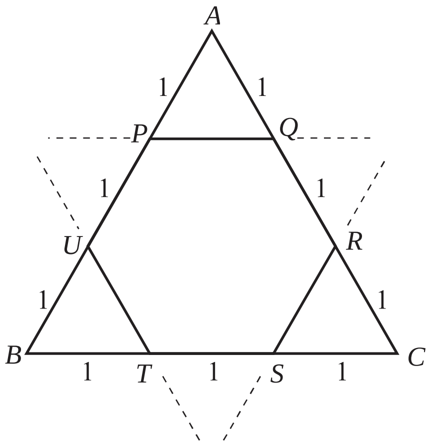
The probability of identifying blue cab correctly

$$= \frac{12}{29} = 41.3\%$$

Hence, the correct option is (C).

2.8 **(A)**

Given : Equilateral triangle



∵ Sides of Hexagon formed by an equilateral triangle to the same equilateral triangle

$=1:3$

∴ The ratio of the area of the regular convex Hexagon (*PQRSTU*) to the area of original equilateral triangle is

$$\Rightarrow \quad \frac{3\sqrt{3}}{2}(1)^2 : \frac{\sqrt{3}}{4}(3)^2$$

$$\Rightarrow \quad \frac{3\sqrt{3}}{2} : \frac{9\sqrt{3}}{4}$$

$\Rightarrow$ 2 : 3

Hence, the correct option is (A).

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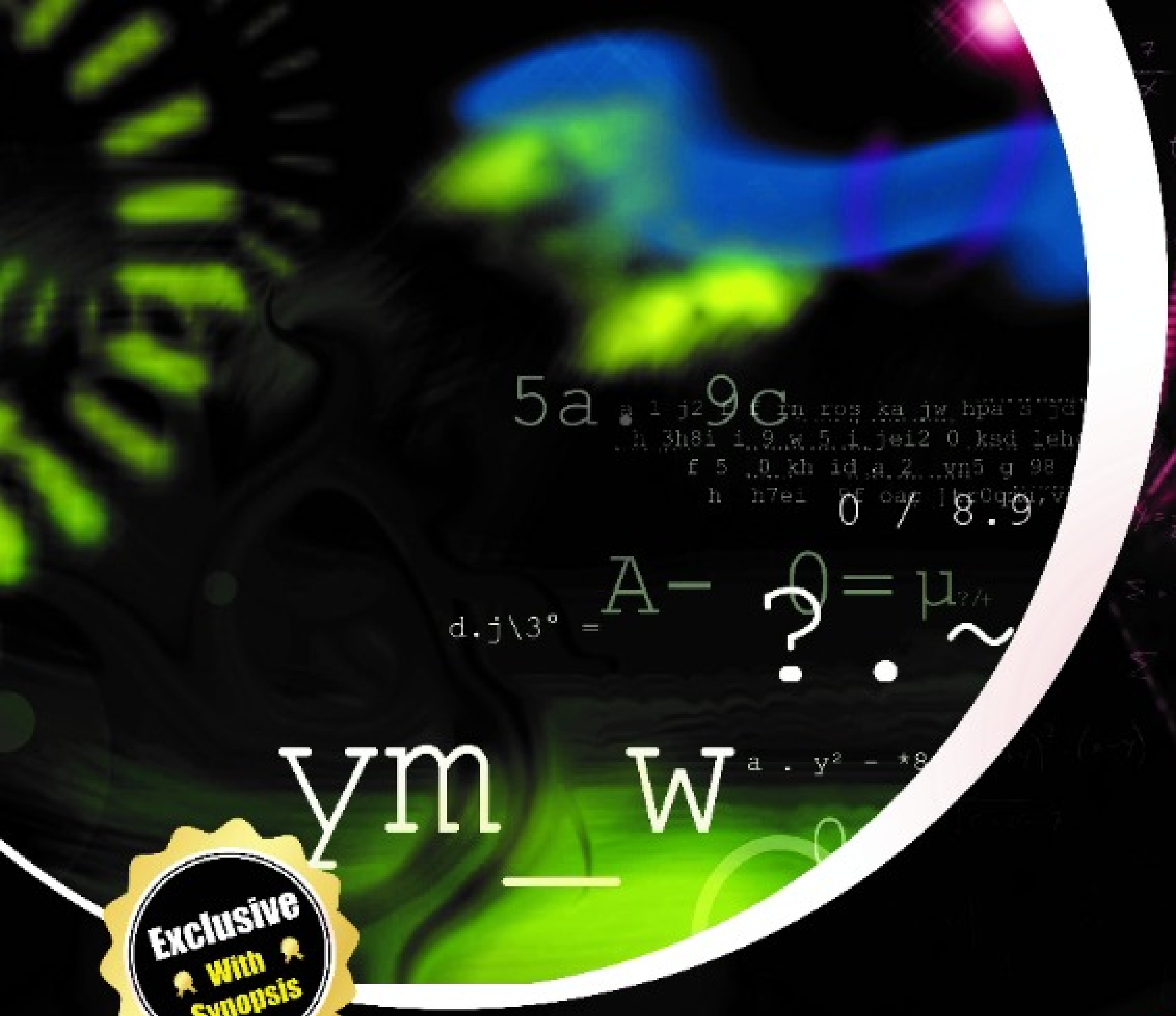

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