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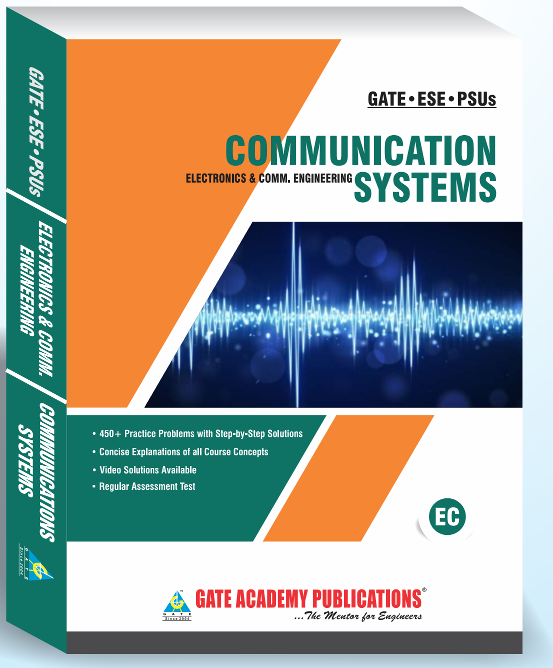

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Electronics & Communication Engineering (EC)

Cover Design : Tarun Tank

Type & Setting : Vikas Deshmukh

First Edition : April 2019
Book Code : GATE-CS-626
ISBN : 978-93-86699-62-6

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CONTENTS

GATE SYLLABUS

Random processes: autocorrelation and power spectral density, properties of white noise, filtering of random signals through LTI systems; Analog communications: amplitude modulation and demodulation, angle modulation and demodulation, spectra of AM and FM, superheterodyne receivers, circuits for analog communications; Information theory: entropy, mutual information and channel capacity theorem; Digital communications: PCM, DPCM, digital modulation schemes, amplitude, phase and frequency shift keying (ASK, PSK, FSK), QAM, MAP and ML decoding, matched filter receiver, calculation of bandwidth, SNR and BER for digital modulation; Fundamentals of error correction, Hamming codes; Timing and frequency synchronization, inter-symbol interference and its mitigation; Basics of TDMA, FDMA and CDMA.

ESE SYLLABUS

Random signals, noise, probability theory, information theory; Analog versus digital communication & applications: Systems- AM, FM, transmitters/receivers, theory / practice / standards, SNR comparison; Digital communication basics: Sampling, quantizing, coding, PCM, DPCM, multiplexing-audio/video; Digital modulation: ASK, FSK, PSK; Multiple access: TDMA, FDMA, CDMA; Optical communication: fibre optics, theory, practice/standards.

UGC-NET SYLLABUS

Basic principles of amplitude, frequency and phase modulation, Demodulation, Intermediate frequency and principle of superheterodyne receiver, Spectral analysis and signal transmission through linear systems, Random signals and noise, Noise temperature and noise figure. Basic concepts of information theory, Digital modulation and Demodulation; PM, PCM, ASK, FSK, PSK, Time-division Multiplexing, Frequency-Division Multiplexing.

CHAPTER 3

Amplitude Modulation

Learning Objectives :

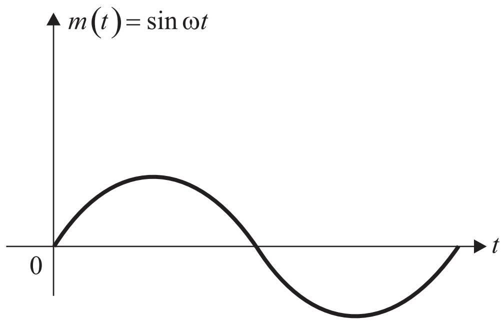
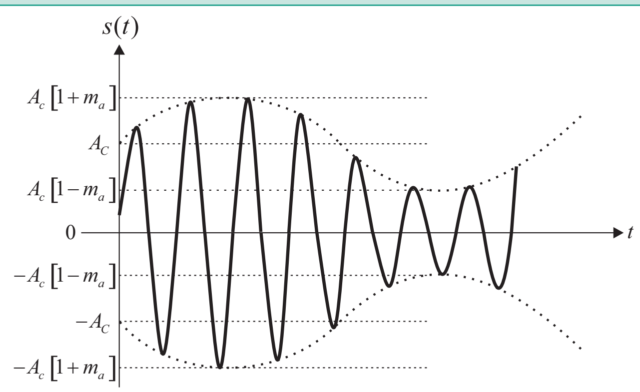
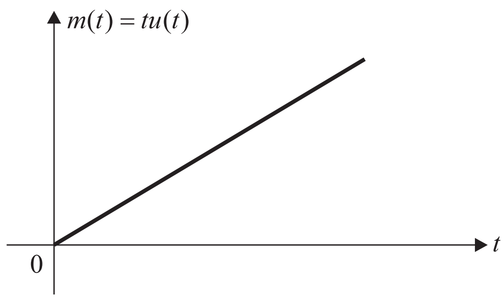
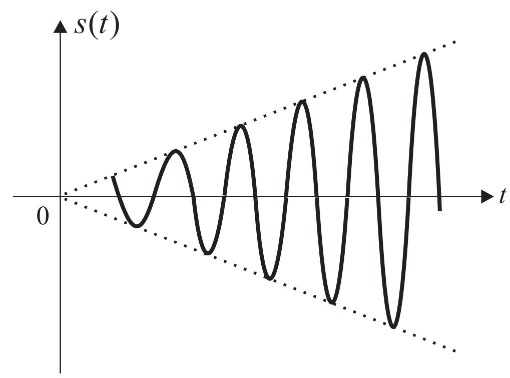
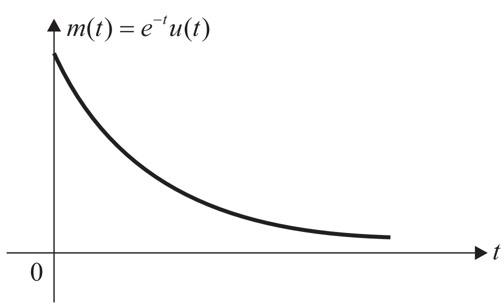
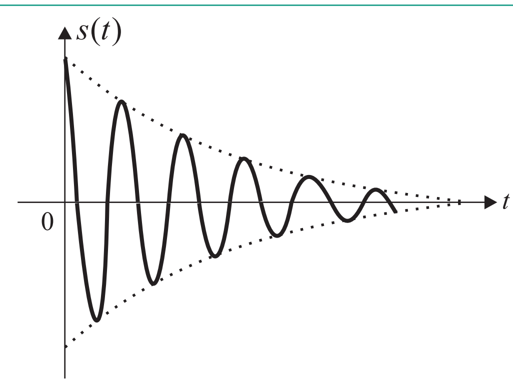
After reading this chapter you should be able to :

- analyze amplitude modulated signals in the time and frequency domain
- describe the AM frequency spectrum and bandwidth
- calculate the sideband power in an AM wave given carrier power and modulation index
- describe double sided suppressed carrier, single sideband modulation techniques
- analyze vestigial sideband modulation technique in time and frequency domains

Table of Contents

3.1 Amplitude Modulation (AM)

In AM, the peak amplitude $A_c \cos(2\pi f_c t + \phi)$ of the carrier is varied linearly with the amplitude of the message signal.

Message	AM signal
	
	
	

3.2 Time Domain Equation of AM

In DSB-FC or AM, the amplitude of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping its (carrier) frequency and phase constant.

Usually, the modulating signal is low-frequency audio signal such as the voice of the human or the music in case of an AM or FM broadcast radio applications. For a single-tone modulating signal,

$$m(t) = A_m \cos(2\pi f_m t) \quad \ldots(i)$$

Instantaneous value of carrier signal,

$$c(t) = A_c \cos(2\pi f_c t) \quad \text{...(ii)}$$

where A_m and A_c are maximum amplitudes of modulating and carrier signal respectively. f_m and f_c are frequency of modulating and carrier signal respectively.

Time domain equation of amplitude modulated (AM) wave is given by,

$$s(t) = A_c \cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t) \quad \text{...(iii)}$$

where k_a is a constant called the **amplitude sensitivity** of the modulator.

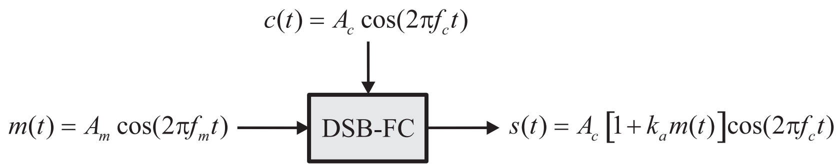
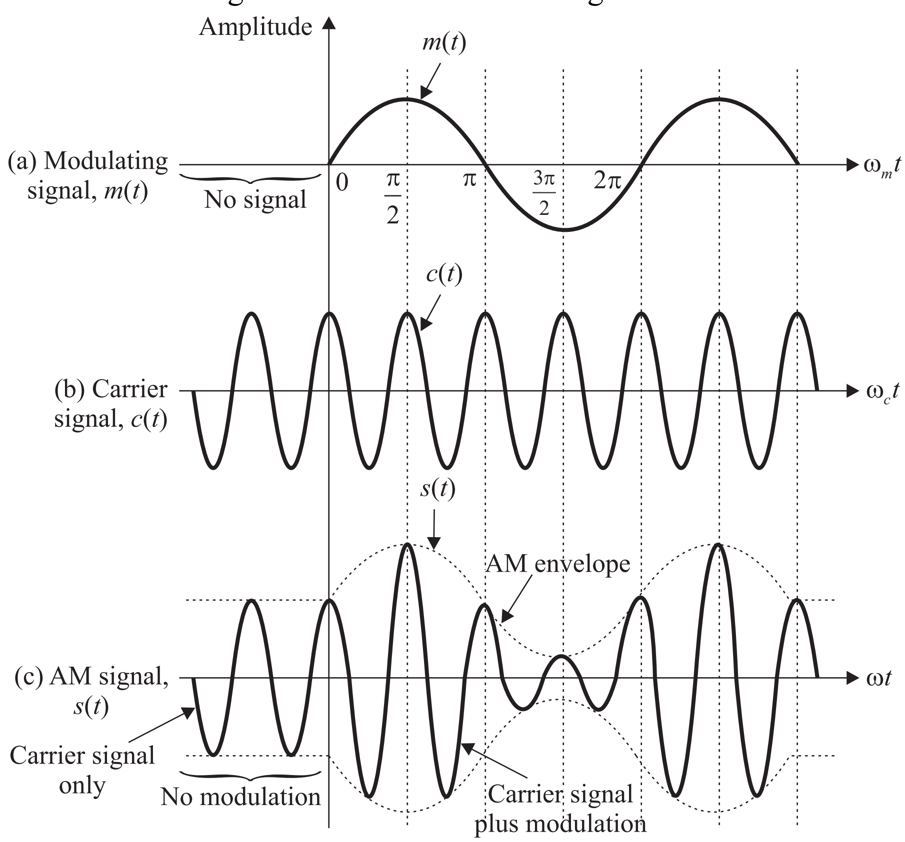


Figure illustrates the relationship among the modulating signal $m(t)$, the carrier signal $c(t)$, and the amplitude modulated signal $s(t)$ for amplitude modulated (AM) signal, as described by equations (i), (ii) and (iii), respectively.

From the amplitude-modulated signal waveform, it is observed that

- When no modulating signal is applied, the modulated AM signal waveform is simply the unmodulated carrier signal.
- When a modulating signal is applied, the amplitude of modulated signal waveform varies in accordance with the amplitude of the modulating signal.
- The frequency of the carrier signal in the amplitude modulated signal waveform remains the same as that of the original unmodulated carrier signal.



3.3 The AM Envelope

Definition : *The time-varying shape of the amplitude-modulated waveform is called the envelope of the AM signal or simply the AM envelope.*

From amplitude-modulated waveform as shown in above figure (c), the following is observed :

- When the peaks of the individual waveforms of the carrier signal in amplitude-modulated wave in positive half and negative half are joined together separately (as shown by dotted lines), the resulting envelopes resemble the original modulating signal.
- The exterior shape of the each half (positive or negative) of the AM envelope is identical to the shape of the modulating signal.
- The modulated AM wave contains the information signal in the amplitude variations of the carrier signal.
- The AM envelope contains all the frequency components (the carrier signal, sum and difference of carrier frequency and modulating frequency) that make up the AM signal.

3.4 Modulation Index

Modulation index is measure of extent to which modulating signal modulates the carrier signal.

Definition 1 :

The ratio of the maximum amplitude of the modulating signal (A_m) and the maximum amplitude of the carrier signal (A_c) is known as the modulation index for AM wave.

That is, $$m_a = \frac{A_m}{A_c}$$

The amplitude modulation index is also known as **depth of modulation, coefficient of modulation, degree of modulation, modulation factor.**

Definition 2 :

Let the general form of AM wave is

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

For this form of AM wave, modulation index is defined as,

$$m_a = \max\left|k_a m(t)\right|$$

When the message contains single frequency or single tone, then the modulation is called single tone modulation.

$$m(t) = A_m \cos(2\pi f_m t)$$

$$m_a = \max\left|k_a A_m \cos(2\pi f_m t)\right| = k_a A_m$$

Definition 3 :

Let the general form of AM wave is,

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

$$s(t) = A_c\left[1 + k_a A_m \cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

$$s(t) = A_c\left[1 + m_a \cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

where $k_a A_m = m_a =$ modulation index or modulation factor.

1. The envelope is $A_c[1+m_a\cos(2\pi f_m t)]$.
2. The maximum value of the positive envelope is $A_c[1+m_a]$.
3. The minimum value of the positive envelope is $A_c[1-m_a]$.
4. To avoid envelope distortion, we require $m_a \le 1$

$$E_{\max} = 1+m_a$$
$$E_{\min} = 1-m_a$$
$$\frac{E_{\max}}{E_{\min}} = \frac{1+m_a}{1-m_a}$$
$$m_a = \frac{E_{\max}-E_{\min}}{E_{\max}+E_{\min}}$$

Definition 4 :

The message signal $m(t)$ will have a non-zero offset such that its maximum $V_{\max}$ and its minimum $V_{\min}$ are not symmetric that is

$$V_{\min} \neq -V_{\max}$$

Modulation index in this case is given by,

$$m_a = \frac{V_{\max}-V_{\min}}{2A_c+V_{\max}+V_{\min}}$$

Percentage modulation index :

The modulation index can be expressed as a percent modulation, m_a. Percent modulation gives the percentage change in the amplitude of the output AM waveform when the carrier signal is amplitude modulated by a modulating signal. It can be calculated by multiplying modulation index by 100. That is,

$$\% m_a = m_a \times 100$$

3.5 Frequency Domain Representation of DSB-FC

The amplitude modulated carrier has new signals at different frequencies, called **side frequencies or sidebands**, occur in the frequency spectrum directly above and below the carrier frequency.

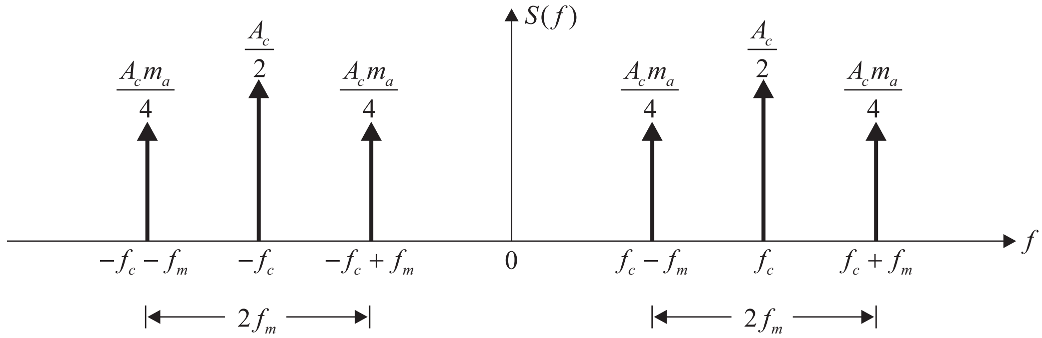
For single tone sinusoidal signal expression for AM is given by,

$$s(t) = A_c[1+m_a\cos(2\pi f_m t)]\cos(2\pi f_c t)$$
$$s(t) = A_c\cos(2\pi f_c t) + m_a A_c\cos(2\pi f_m t)\cos(2\pi f_c t)$$
$$s(t) = A_c\cos(2\pi f_c t) + \frac{m_a A_c}{2}\cos 2\pi(f_c - f_m)t + \frac{m_a A_c}{2}\cos 2\pi(f_c + f_m)t$$

Upper sideband frequency, $f_{USB} = f_c + f_m$

Lower sideband frequency, $f_{LSB} = f_c - f_m$

The spectrum of $s(t)$ is shown in figure below,



Definition of bandwidth : *The bandwidth of the amplitude-modulated signal is equal to the difference between the maximum upper-sideband frequency and the minimum lower-sideband frequency.*

$$BW = f_{USB} - f_{LSB} = (f_c + f_m) - (f_c - f_m) = 2f_m$$

Remember

The bandwidth required for the amplitude modulation is twice the highest frequency of the modulating signal.

3.6 Effect of Modulation Index on AM Envelope

Let $m(t) = A_m \sin \omega_m t$ and $c(t) = A_c \cos \omega_c t$

For single tone sinusoidal signal expression for AM is given by,

$$s(t) = A_c [1 + m_a \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

(i) For $m_a = 0$, $s(t) = A_c \cos \omega_c t$

(ii) For $m_a = 0.3$, $s(t) = A_c[1 + 0.3 \sin \omega_m t]$

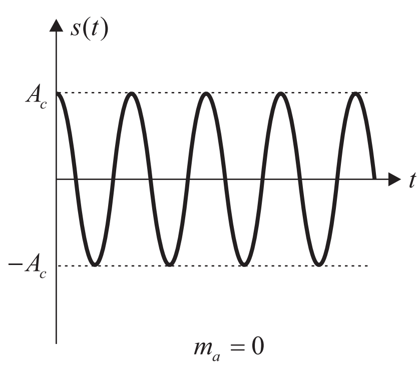
$$s(t)_{\max} = 1.3A_c, \quad s(t)_{\min} = 0.7A_c$$

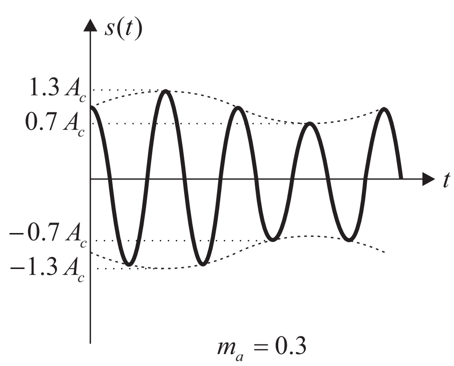
(iii) For $m_a = 0.8$, $s(t) = A_c[1 + 0.8 \sin \omega_m t]$

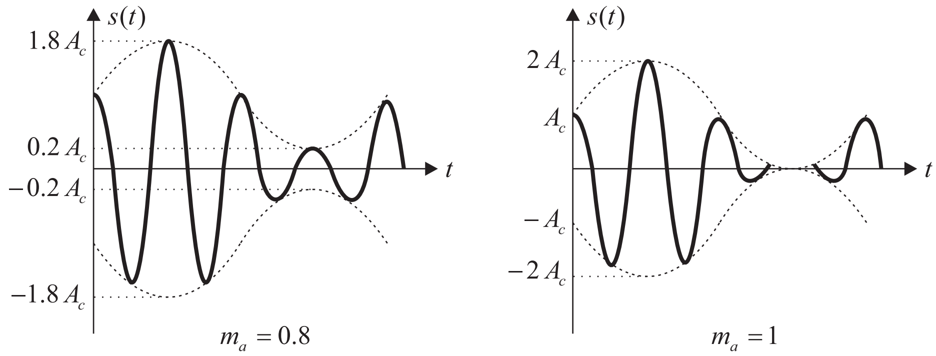
$$s(t)_{\max} = 1.8A_c, \quad s(t)_{\min} = 0.2A_c$$

(iv) For $m_a = 1$, $s(t) = 2A_c[1 + \sin \omega_m t]$

$$s(t)_{\max} = 2A_c, \quad s(t)_{\min} = 0$$







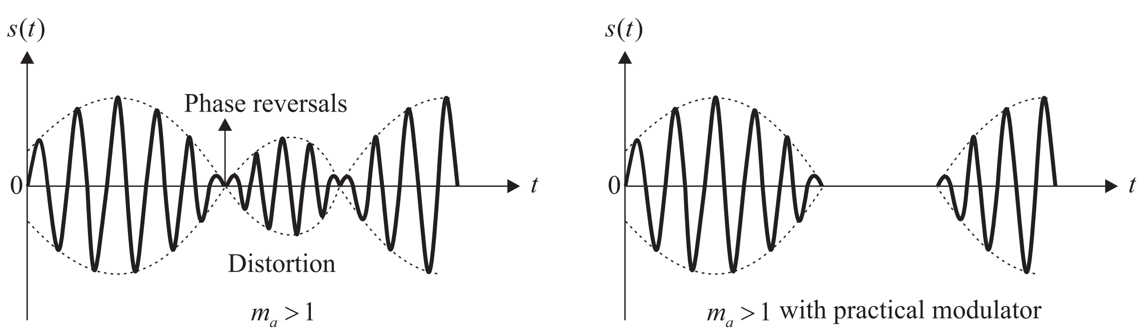
- For $m_a = 1$, the maximum signal voltage of the modulated AM wave varies between zero and twice the maximum amplitude of unmodulated carrier signal.
- For $m_a = 0$, the original unmodulated carrier signal is the resultant waveform because $A_m = 0$ means absence of modulating signal.

3.7 Overmodulation and Splatter

Definition of overmodulation : *When the amplitude modulation index, m_a is greater than one, the AM signal is said to be overmodulated.*

Definition of splatter : *Splatter is a combination of spurious frequencies generated by most practical AM modulator circuits when driven by overmodulation.*

- The bandwidth of amplitude-modulated signal is increased beyond specified limits (10 kHz in case of AM broadcast application) due to overmodulation.
- Splatter are frequency components produced by a transmitter that fall outside its assigned channel.
- The increased bandwidth can cause interference with another AM signal on an adjacent channel.
- The clipping of the modulating wave that occurs at the zero axis changes the envelope wave shape to one that contains higher-order harmonics of the original modulating frequency.



TEST 1

Q.1 Which of the following is the most correct?

(A) A_m should be greater than A_c.

(B) A_c should be greater than A_m.

(C) A_m should be equal to or less than A_c.

(D) A_c must always equal A_m.

Q.2 Which of the following is NOT another name for modulation index?

(A) Modulation reciprocal

(B) Modulation factor

(C) Degree of modulation

(D) Modulation coefficient

Q.3 The degree or depth of modulation expressed as a percentage is computed using the expression

(A) $2A_m$ (B) $\frac{100}{m_a}$

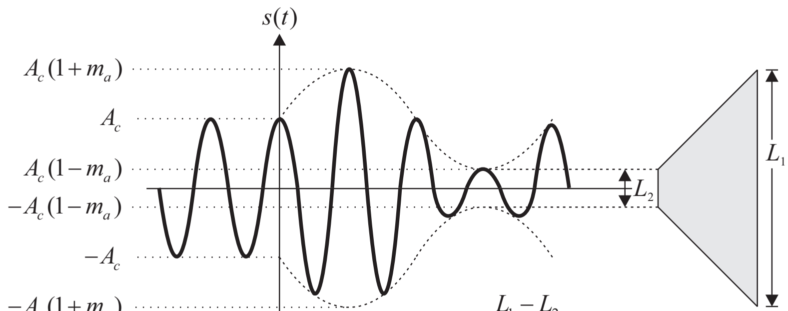
(C) $\frac{m_a}{100}$ (D) $100\% \times m_a$

Q.4 An AM wave displayed on an oscilloscope has values of $E_{max} = 3.8$ and $E_{min} = 1.5$ as read from the graticule. The percentage of modulation is ________%.

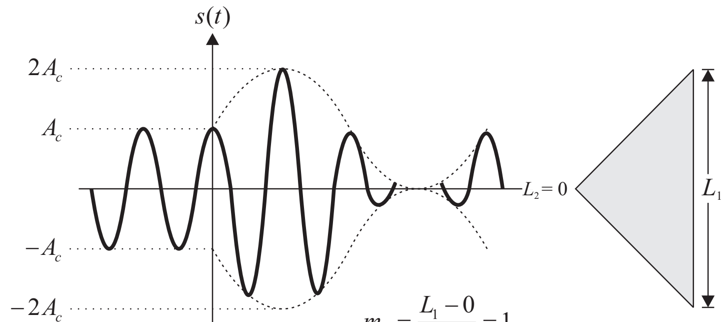
Q.5 To achieve 85% modulation of a carrier of $A_c = 40\text{ V}$, a modulating signal of $A_m =$ ________ is needed.

3.8 Trapezoidal Pattern

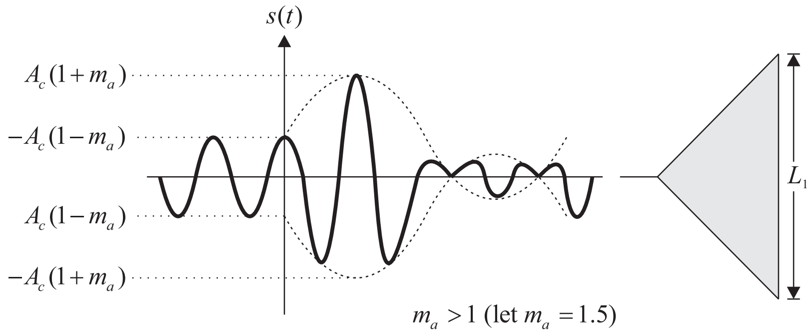
Definition : *Trapezoidal pattern is a display on a standard oscilloscope used for measurement of the modulation characteristics (modulation index, percent modulation, coefficient of modulation, and modulation symmetry) of amplitude-modulated waveform.*



$$m_a = \frac{L_1 - L_2}{L_1 + L_2}$$



$$m_a = \frac{L_1 - 0}{L_1 + 0} = 1$$



Significance of trapezoidal pattern :

Trapezoidal pattern is quite useful when the modulating signal is a non-periodic signal such as speech waveform which contains complex sinusoidal waveforms. AM transmitter modulation characteristics such as modulation symmetry and modulation index can be easily and accurately interpreted by observing trapezoidal patterns of the modulated signal on the standard oscilloscope display.

3.9 Power Calculation of AM

Assuming that the AM signal appear across a load resistance R, so that reactive volt-amperes can be ignored. In general, the power of a signal is given as the ratio of square of RMS value of signal voltage and the load resistance. That is,

$$\text{Signal Power} = \frac{(\text{RMS value of signal voltage})^2}{\text{Load Resistance}}$$

$$P = I_{rms}^2 R = V_{rms}^2 / R$$

For single tone modulating signal expression for AM is given by,

$$s(t) = A_c[1 + m_a \cos 2\pi f_m t]\cos 2\pi f_c t$$

$$s(t) = \underbrace{A_c \cos(2\pi f_c t)}_{\text{Carrier}} + \underbrace{\frac{m_a A_c}{2}\cos 2\pi(f_c - f_m)t}_{\text{Lower sideband}} + \underbrace{\frac{m_a A_c}{2}\cos 2\pi(f_c + f_m)t}_{\text{Upper sideband}}$$

The AM wave has three components : unmodulated carrier, lower sideband and upper sideband. Therefore, the total power of AM wave is the sum of the carrier power P_c and power in the two sidebands i.e. P_{USB} and P_{LSB}.

$$P_T = P_c + P_{USB} + P_{LSB} = P_c + P_{SB}$$

Carrier power :

The peak amplitude of the carrier term in AM signal is the same as that of unmodulated carrier signal. Therefore, the carrier power is given as

$$P_c = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2}{2R}$$

Sideband power :

The average upper sideband power is given by,

$$P_{USB} = \frac{\left(\frac{A_c}{2\sqrt{2}} m_a\right)^2}{R} = \frac{m_a^2 A_c^{\,2}}{8R} = \frac{m_a^2}{4}\frac{A_c^2}{2R} = \frac{m_a^2}{4} P_c$$

The average lower sideband power is given by,

$$P_{LSB} = \frac{\left(\frac{A_c}{2\sqrt{2}} m_a\right)^2}{R} = \frac{m_a^2 A_c^{\,2}}{8R} = \frac{m_a^2}{4}\frac{A_c^2}{2R} = \frac{m_a^2}{4} P_c$$

The average sideband power is given by,

$$P_{USB} = P_{LSB} = \frac{m_a^2 P_c}{4}$$

Total sideband power,

$$P_{SB} = P_{USB} + P_{LSB} = \frac{m_a^2}{4} P_c + \frac{m_a^2}{4} P_c = \frac{m_a^2}{2} P_c$$

Total power :

$$P_T = P_c + P_{SB} = P_c + \frac{m_a^2}{2} P_c$$

Total transmitted power, $P_T = P_c\left[1 + \frac{m_a^2}{2}\right]$

For 100% modulation $m_a = 1$, $P_T = P_c\left[1 + \frac{1^2}{2}\right] = 1.5P_c$

Carrier power, $P_c = \frac{1}{1.5} P_T = 0.666 P_T$

Total sideband power, $P_{SB} = P_{USB} + P_{LSB} = \frac{m_a^2 P_c}{2} = \frac{1^2}{2} P_c = \frac{1}{3} P_T = 0.333 P_T$

Remember

In amplitude modulated wave, the 66.66% of the transmitted power is used by the carrier signal and remaining 33.33% of the power is used by the sidebands $(P_{USB} \text{ and } P_{LSB})$.

3.10 Transmission Efficiency

Definition : *Transmission efficiency is defined as the ratio of the power carried by the sidebands to the total transmitted power.*

Transmission efficiency, $\eta = \frac{P_{USB} + P_{LSB}}{P_T} = \frac{P_{SB}}{P_T}$

Total sideband power, $P_{USB} + P_{LSB} = \frac{m_a^2 A_c^2}{8} + \frac{m_a^2 A_c^2}{8} = \frac{m_a^2 A_c^2}{4} = \frac{m_a^2 P_c}{2}$

$$\eta = \frac{P_{USB} + P_{LSB}}{P_T} = \frac{m_a^2 P_c / 2}{P_c(1 + m_a^2 / 2)} = \frac{m_a^2 / 2}{1 + m_a^2 / 2}$$

$\therefore$ Transmission efficiency,

$$\eta = \frac{P_{USB} + P_{LSB}}{P_T} = \frac{m_a^2}{m_a^2 + 2} \times 100\%$$

From above equation it can be seen that η increases monotonically with m_a.

For $m_a = 1$, $\eta_{\max} = 33.33\%$

In general, modulation efficiency or transmission efficiency or power efficiency is given by,

$$\eta = \frac{k_a^{\,2}\overline{m^2(t)}}{1 + k_a^{\,2}\overline{m^2(t)}}$$

Remember

Smaller value of m_a degrade efficiency. For this reason **Volume Compression** and **Peak Limiting** are commonly used in AM to ensure that full modulation ($m_a = 1$) is maintained most of the time.

3.11 AM Current Distribution

Let I_t be the total current flowing through an antenna resistance, R_a in an AM transmitter.

AM transmit power,

$$P_T = I_t^2 R_a \quad \text{...(i)}$$

If I_c is the carrier current flowing through an antenna resistance, R_a then

Carrier power, $P_c = I_c^2 R_a$ ...(ii)

The ratio $\frac{P_T}{P_c} = \frac{I_t^2}{I_c^2}$ ...(iii)

Total transmit power in AM is given by,

$$P_T = P_c\left(1 + \frac{m_a^2}{2}\right) \quad \text{...(iv)}$$

$$\frac{P_T}{P_c} = \left(1 + \frac{m_a^2}{2}\right) \quad \text{...(v)}$$

Equating (iii) and (v),

$$\frac{I_t^2}{I_c^2} = \left(1 + \frac{m_a^2}{2}\right)$$

$$I_t = I_c\sqrt{\left(1 + \frac{m_a^2}{2}\right)} \quad \text{...(vi)}$$

This is the expression for total current flowing in an antenna in AM transmitter.

Rearranging the terms, we can write

$$\frac{I_t}{I_c} = \sqrt{\left(1+\frac{m_a^2}{2}\right)}$$

$$\frac{I_t^2}{I_c^2} = 1+\frac{m_a^2}{2}$$

$$\frac{m_a^2}{2} = \frac{I_t^2}{I_c^2} - 1$$

$$m_a = \sqrt{2\left(\frac{I_t^2}{I_c^2} - 1\right)} \quad \ldots\text{(vii)}$$

TEST 2

Q.1 The total sideband power is what percentage of the carrier power for 100% modulation?

(A) 25% (B) 50%

(C) 100% (D) 150%

Q.2 An AM signal with a carrier of 1 kW has 100 W in each sideband. The percentage of modulation is _______ percent.

Q.3 An AM transmitter has a carrier power of 200 W. The percentage of modulation is 60 percent. The total signal power is ________ W.

Q.4 The total AM signal power is 2800 W. The carrier power 2000 W. The power in one sideband is _______W. The percentage of modulation is ________.

Q.5 The unmodulated carrier current in an antenna is 1.5 A. When the carrier is modulated by 95 percent, the total antenna current is ________ A.

3.12 Multi-tone Modulation

Time domain equation of amplitude modulated (AM) wave is given by,

$$s(t) = A_c\cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t) = A_c\left[1+k_a m(t)\right]\cos(2\pi f_c t)$$

$$s(t) = A_c\left[1+k_a\left\{m_1(t)+m_2(t)\right\}\right]\cos(2\pi f_c t)$$

$$s(t) = A_c\left[1+k_a\left\{A_{m1}\cos(2\pi f_{m1}t)+A_{m2}\cos(2\pi f_{m2}t)\right\}\right]\cos(2\pi f_c t)$$

$$s(t) = A_c\left[1+m_{a1}\cos(2\pi f_{m1}t)+m_{a2}\cos(2\pi f_{m2}t)\right]\cos(2\pi f_c t) \qquad [\because\ m_a = k_a A_m]$$

$$s(t) = A_c\cos(2\pi f_c t) + m_{a1}A_c\cos(2\pi f_c t).\cos(2\pi f_{m1}t) + m_{a2}A_c\cos(2\pi f_c t).\cos(2\pi f_{m2}t)$$

$$[\because\ 2\cos A\cos B = \cos(A-B)+\cos(A+B)]$$

$$s(t) = A_c\cos(2\pi f_c t) + \frac{m_{a1}A_c}{2}\cos 2\pi(f_c - f_{m1})t + \frac{m_{a1}A_c}{2}\cos 2\pi(f_c + f_{m1})t$$

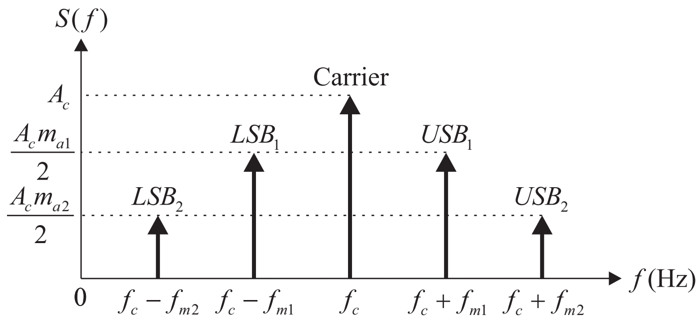
$$+\frac{m_{a2}A_c}{2}\cos 2\pi(f_c - f_{m2})t + \frac{m_{a2}A_c}{2}\cos 2\pi(f_c + f_{m2})t$$

When there are two modulating frequencies, we get two upper sidebands (USB) $f_c + f_{m1}$ and $f_c + f_{m2}$ and two lower sidebands (LSB) $f_c - f_{m1}$ and $f_c - f_{m2}$.

Amplitude of first sideband $= \dfrac{A_c m_{a1}}{2}$

Amplitude of second sideband $= \dfrac{A_c m_{a2}}{2}$

Single sided spectrum :



Total transmitted power :

The total power in the amplitude modulated wave is calculated as follows :

$$P_T = P_c + P_{USB1} + P_{LSB1} + P_{USB2} + P_{LSB2}$$

$$P_T = \frac{A_c^2}{2} + \frac{1}{2}\left(\frac{m_{a1}A_c}{2}\right)^2 + \frac{1}{2}\left(\frac{m_{a1}A_c}{2}\right)^2 + \frac{1}{2}\left(\frac{m_{a2}A_c}{2}\right)^2 + \frac{1}{2}\left(\frac{m_{a2}A_c}{2}\right)^2$$

$$P_T = \frac{A_c^2}{2} + \frac{m_{a1}^2 A_c^2}{4} + \frac{m_{a2}^2 A_c}{4} = \frac{A_c^2}{2}\left[1 + \frac{m_{a1}^2}{2} + \frac{m_{a2}^2}{2}\right]$$

$$P_T = P_c\left[1 + \frac{m_{aT}^2}{2}\right]$$

where $P_c = \dfrac{A_c^2}{2}$ and $m_{aT}^2 = m_{a1}^2 + m_{a2}^2$

When several modulating signals having different peak amplitude levels and frequencies simultaneously amplitude modulate a common carrier signal, total modulation index of resultant complex AM signal is given by the square root of the quadratic sum of the individual modulation indices due to the individual modulating signals.

In general, net modulation index is given by,

$$m_{aT} = \sqrt{m_{a1}^2 + m_{a2}^2 + m_{a3}^2 + \ldots\ldots + m_{an}^2}$$

$$\eta = \frac{\text{Power in sideband}}{\text{Total Power}} = \frac{P_c m_{aT}^2 / 2}{P_c(1 + m_{aT}^2 / 2)} = \frac{m_{aT}^2}{2 + m_{aT}^2}$$

3.13 Peak Envelope Power

For single tone modulating signal expression for AM signal is given by,

$$s(t) = A_c\left[1 + m_a\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

Where, peak envelope = $A_c\left[1 + m_a\right]$

Peak envelope power is given by,

$$PEP = \frac{A_c^2\left[1 + m_a\right]^2}{2}.$$

3.14 Advantages, Disadvantages and Applications of AM

Advantages of AM :

1. AM transmitters are less complex.
2. AM receivers are simple, detection is easy.
3. AM receivers are cost efficient.

Disadvantages of AM :

1. Low transmitter power efficiency.
2. Poor reception quality.
3. Noisy signal reception.
4. Limited operating radio range

Applications of AM :

1. Radio broadcasting.
2. Picture transmission in a TV system.

Remember

In a linear modulation scheme the frequency content of the modulating signal is translated to occupy a different portion of the frequency spectrum.

In AM, low frequency of the modulating signal is translated in accordance with high frequency of carrier signal but no new components are generated. Hence, AM is called a **linear modulation**.

Solved Example 1

The output voltage of a transmitted is given by $v = 500(1 + 0.4\sin 3140t)\sin(6.28\times10^7)t$. This voltage is fed to a load of 600 Ω resistance. Determine (a) f_c (b) f_m (c) P_c (d) P_{av} (e) peak output power.

Sol. **Given :** $m_a = 0.4$ and $v = 500(1 + 0.4\sin 3140t)\sin(6.28\times10^7)t$

(a) The carrier frequency, $f_c = \dfrac{6.28\times10^7}{2\pi} = 10\text{ MHz}$ **Ans.**

(b) The modulating frequency, $f_m = \dfrac{3140}{2\pi} = 499.74\text{ Hz}$ **Ans.**

(c) The carrier voltage, $A_c = 500$ V

The carrier power across resistance $600\ \Omega$ can be calculated as,

$$P_c = \frac{A_c^2}{2R} = \frac{500 \times 500}{2 \times 600} = 208.33 \text{ W}$$ **Ans.**

(d) The total power is given by,

$$P_T = P_c\left(1 + \frac{m_a^2}{2}\right) = \frac{2500}{12}(1 + 0.08) = 225 \text{ watts}$$ **Ans.**

(e) Peak voltage $= A_c(1 + m_a) = 500(1 + 0.4) = 700$ V **Ans.**

$$\text{Peak Power} = \frac{[A_c(1 + m_a)]^2}{2R} = \frac{700^2}{1200} = 408.3 \text{ W}$$ **Ans.**

Solved Example 2

A carrier signal having 10 V peak amplitude is amplitude-modulated by three different modulating frequencies with peak amplitude levels of 2 V, 3 V, and 4 V, respectively. Compute the modulation index of the resultant complex AM signal.

Sol. **Given :** $A_c = 10$ V, $A_{m1} = 2$ V, $A_{m2} = 3$ V, $A_{m3} = 4$ V.

The three respective amplitude modulation indices can be determined as,

$$m_{a1} = \frac{A_{m1}}{A_c} = \frac{2}{10} = 0.2$$

$$m_{a2} = \frac{A_{m2}}{A_c} = \frac{3}{10} = 0.3$$

$$m_{a3} = \frac{A_{m3}}{A_c} = \frac{4}{10} = 0.4$$

The total resultant modulation index is the square root of the quadratic sum of the individual modulation indices due to the individual modulating frequency components. That is,

$$m_{aT} = \sqrt{m_{a1}^2 + m_{a2}^2 + m_{a3}^2} = \sqrt{0.2^2 + 0.3^2 + 0.4^2}$$

$$m_{aT} = 0.538$$ **Ans.**

Solved Example 3

A message signal given by, $m(t) = \left(\frac{1}{2}\right)\cos\omega_1 t - \left(\frac{1}{2}\right)\sin\omega_2 t$ is amplitude-modulated with a carrier of frequency ω_c to generate $s(t) = [1 + m(t)]\cos\omega_c t$. What is the power efficiency achieved by this modulation scheme?

(A) 8.33 % (B) 11.11 % (C) 20 % (D) 25 %

Sol. **Given :** $s(t) = [1 + m(t)]\cos\omega_c t$ ...(i)

where, $$m(t) = \frac{1}{2}\cos\omega_1 t - \frac{1}{2}\sin\omega_2 t$$

Power efficiency is given by,

$$\%\eta = \frac{k_a^2 \overline{m^2(t)}}{1+k_a^2 \overline{m^2(t)}} \times 100 \quad \ldots\text{(ii)}$$

Expression for AM signal is given by,

$$s(t) = A_c\left[1+k_a m(t)\right]\cos\omega_c t \quad \ldots\text{(iii)}$$

On comparing equation (i) and (iii),

$$k_a = 1$$

where, $\overline{m^2(t)} =$ Mean square value = Power.

$$\overline{m^2(t)} = \frac{(1/2)^2}{2} + \frac{(1/2)^2}{2} = \frac{1}{4}$$

From equation (ii),

$$\%\eta = \frac{0.25}{1+0.25} \times 100 = 20\%$$

TEST 3

Q.1 What will be the total modulation index if a wave is amplitude modulated by three sine waves with modulation indices of 25%, 50% and 75%?

(A) $M_t = 1.5$ (B) $M_t = 0.93$

(C) $M_t = 1.22$ (D) $M_t = 1$

Q.2 A carrier is simultaneously modulated by two sine wave with modulation indices of 0.3 and 0.4. If the modulated power is 10 kW, what is the total modulated power?

(A) 11.25 kW (B) 10kW

(C) 10.125 kW (D) 10.5 kW

Q.3 An SSB transmitter has a 150 V supply. Voice peaks produce a current of 2.3 A. The PEP input is _______W.

Q.4 The average output power of an SSB transmitter rated at 12 W PEP is in the ______ to ______ range.

3.15 Generation of AM Signals

3.15.1 Square Law Modulator

1. Square law modulator or non-linear modulator is used for generation of AM wave or DSB-FC wave.
2. An AM wave can be obtained by combining the modulating signal and the carrier through a non- linear device.
3. A non-linear device is the device with a non-linear relation between its current and voltage. The non- linear device could be a diode or a transistor. The transistor has an advantage that it also provides amplification.

4. When the current through the diode is passed through a resistive load, the output voltage $v_0(t)$ is related to the input voltage $v_i(t)$ by the following polynomial approximation

$$v_0(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t).....$$

where, the a_is are constants.

5. For square law device (SLD) the relationship between the output voltage $v_0(t)$ and input voltage $v_i(t)$ is

$$v_0(t) = a_1 v_i(t) + a_2 v_i^2(t)$$

6. The square law modulator circuit consists of the following :
   (i) A carrier signal and a modulating signal
   (ii) A non-linear device
   (iii)A bandpass signal

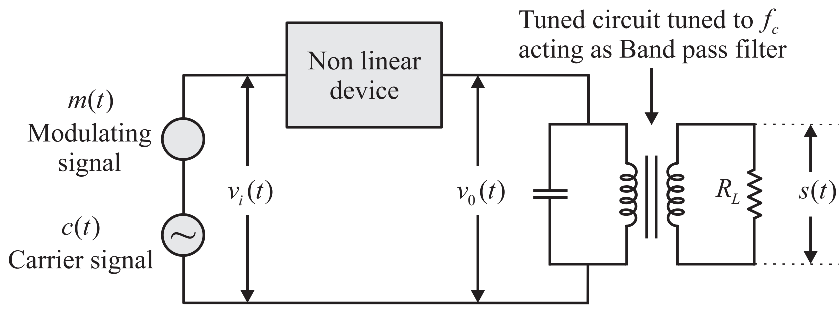
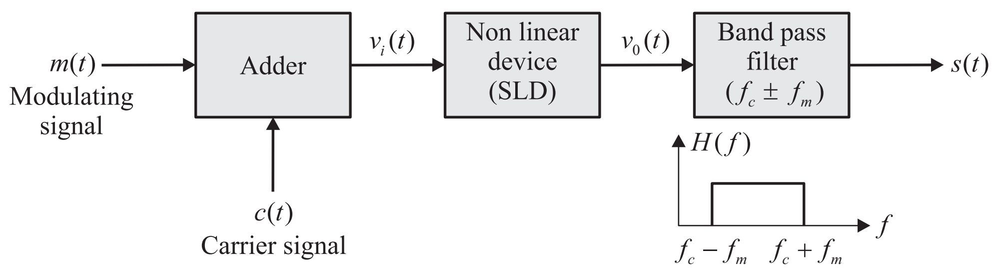


Fig. Square law modulator

7. Block diagram of square law modulator is shown in figure below.



8. Mathematical analysis,

$$v_i(t) = m(t) + c(t) \quad \text{where} \quad c(t) = A_c \cos(2\pi f_c t)$$

$$v_i(t) = m(t) + A_c \cos(2\pi f_c t)$$

Characteristic of square law device can be written as,

$$v_0(t) = a_1 v_i(t) + a_2 v_i^2(t)$$

$$v_0(t) = a_1 \left[m(t) + A_c \cos(2\pi f_c t)\right] + a_2 \left[m(t) + A_c \cos(2\pi f_c t)\right]^2$$

$$v_0(t) = a_1 \left[m(t) + A_c \cos(2\pi f_c t)\right]$$

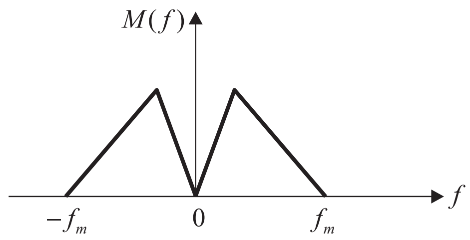
$$+ a_2 \left[m^2(t) + 2m(t) A_c \cos(2\pi f_c t) + A_c^2 \cos^2(2\pi f_c t)\right]$$

$$v_0(t) = a_1 m(t) + a_1 A_c \cos(2\pi f_c t) + a_2 m^2(t)$$
$$+2a_2 m(t) A_c \cos(2\pi f_c t) + a_2 A_c^2 \left[\frac{1+\cos(4\pi f_c t)}{2}\right]$$

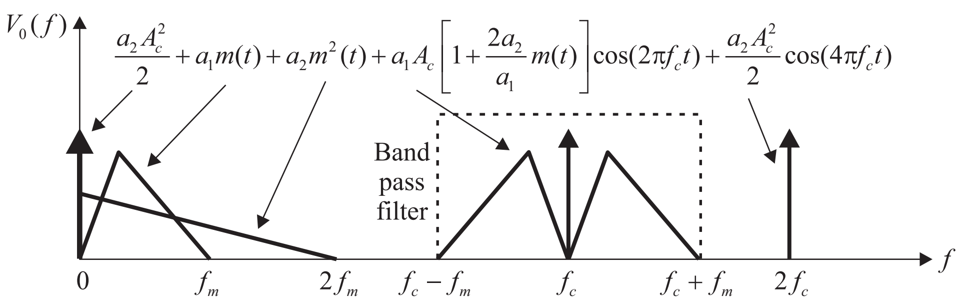
$$v_0(t) = \frac{a_2 A_c^2}{2} + a_1 m(t) + a_2 m^2(t) + a_1 A_c \cos(2\pi f_c t)$$
$$+2a_2 m(t) A_c \cos(2\pi f_c t) + \frac{a_2 A_c^2}{2}\cos(4\pi f_c t)$$

$$v_0(t) = \frac{a_2 A_c^2}{2} + a_1 m(t) + a_2 m^2(t) + a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right]\cos(2\pi f_c t) + \frac{a_2 A_c^2}{2}\cos(4\pi f_c t)$$

The spectrum of $m(t)$ is shown in figure below.



The spectrum of $v_0(t)$ for positive frequencies is shown in figure below,



The desired AM wave is obtained by passing $v_0(t)$ through a band pass filter centred at f_c with a bandwidth corresponding to the AM bandwidth $2f_m$.

$$s(t) = a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right]\cos(2\pi f_c t)$$

Above equation is in the form of standard AM wave,

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t) \qquad \text{where, } k_a = \frac{2a_2}{a_1}$$

BPF produces desire AM wave in frequency range $f_c - f_m$ to $f_c + f_m$ and remove undesired components out of this frequency range.

Remember

The main condition for generation of DSB-FC or AM wave is that the carrier frequency should be atleast three times the maximum frequency of the modulating signal.

$$f_c - f_m \geq 2f_m \quad \Rightarrow \quad f_c \geq 3f_m$$

Solved Example 4

Let $c(t) = A_c \cos(2\pi f_c t)$ and $m(t) = \cos(2\pi f_m t)$. It is given that $f_c >> 5 f_m$. The signal $c(t) + m(t)$ is applied to the input of a non-linear device, whose output $v_0(t)$ is related to the input $v_i(t)$ as $v_0(t) = a v_i(t) + b v_i^2(t)$, where a and b are positive constants. The output of the non-linear device is passed through an ideal band-pass filter with center frequency f_c and bandwidth $3 f_m$, to produce an amplitude modulated (AM) wave. If it is desired to have the sideband power of the AM wave to be half of the carrier power, then a/b is

(A) 0.25 (B) 0.5 (C) 1 (D) 2

Sol. **Given :** $P_{SB} = \frac{1}{2} P_c$

$$v_0(t) = a\left[A_c \cos(2\pi f_c t) + \cos 2\pi f_m t\right] + b\left[A_c \cos 2\pi f_c t + \cos 2\pi f_m t\right]^2$$

$$v_0(t) = aA_c \cos 2\pi f_c t + a\cos 2\pi f_m t + bA_c^2 \cos^2 2\pi f_c t + b\cos^2 2\pi f_m t$$
$$+ 2b\, A_c \cos 2\pi f_c t + \cos 2\pi f_m t$$

$$v_0(t) = aA_c \cos 2\pi f_c t + a\cos 2\pi f_m t + \frac{bA_c^2}{2}(1 + \cos 4\pi f_c t) + \frac{b}{2}(1 + \cos 4\pi f_m)t$$
$$+ bA_c \cos 2\pi (f_c + f_m)t$$

Output the bandpass filter will be,

$$y(t) = aA_c \cos 2\pi f_c t + 2bA_c \cos 2\pi f_c t \cos 2\pi f_m t$$

$$P_c = \frac{(aA_c)^2}{2} = \frac{a^2 A_c^2}{2}$$

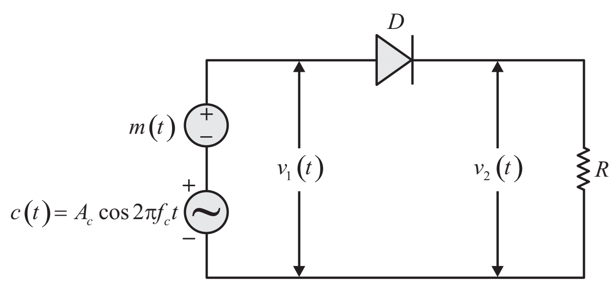
$$P_{SB} = \frac{2(bA_c)^2}{2} = b^2 A_c^2$$

$$b^2 A_c^2 = \frac{1}{2}\frac{a^2 A_c^2}{2}$$

$$\left(\frac{a}{b}\right)^2 = 4 \Rightarrow \frac{a}{b} = 2$$ **Ans.**

3.15.2 Switching Modulator

Figure shows a switching modulator, where the switching action is provided by a single diode. The $m(t) + c(t)$ with $c(t) >> m(t)$, so that switching action of the diode is controlled by $c(t)$. The diode opens and shorts periodically with $\cos \omega_c t$, in effect multiplying signal $v_1(t) \times g_p(t)$. The voltage across R is $v_2(t)$.



$$v_1(t) = m(t) + A_c \cos 2\pi f_c t \quad : \quad A_c >> |m(t)|$$

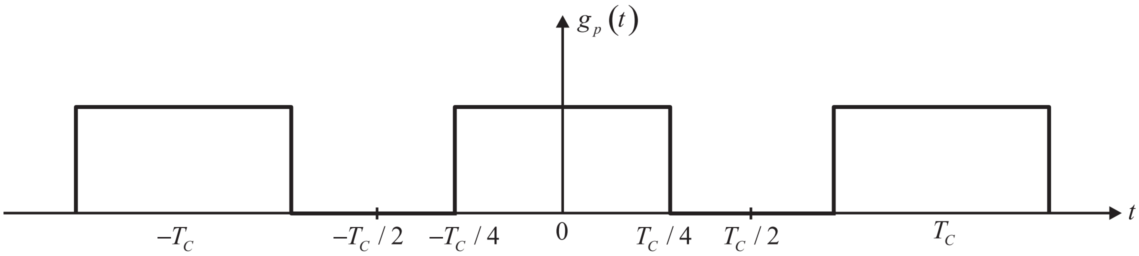
$$v_2(t) = v_1(t) \quad : \quad c(t) > 0$$

$$v_2(t) = 0 \quad : \quad c(t) < 0$$

$$v_1(t) = m(t) + A_c \cos 2\pi f_c t \quad : \quad c(t) > 0$$

$$v_2(t) = 0 \quad : \quad c(t) < 0$$

Analytically $v_2(t) = v_1(t) \times g_p(t)$



Fourier series for $g_p(t)$ with the period $T_c = 1/f_c$.

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi 3 f_c t) + \ldots\ldots..$$

$$\therefore \quad v_2(t) = [m(t) + A_c \cos 2\pi f_c t]\left[\frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi 3 f_c t) + \ldots\ldots..\right]$$

$$v_2(t) = \left[\frac{2}{\pi} m(t)\cos 2\pi f_c t + \frac{A_c}{2}\cos 2\pi f_c t\right] + \text{other terms}$$

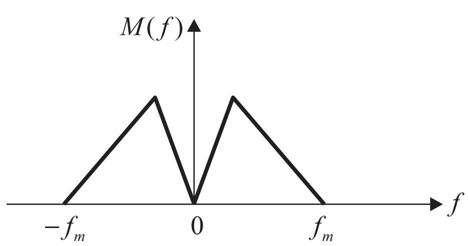
If the above signal is passed through a BPF, it suppresses all the other terms, yielding the desired AM signal at the output.

$$v_2(t) = \frac{A_c}{2}\cos(2\pi f_c t) + \frac{2}{\pi} m(t)\cos(2\pi f_c t)$$

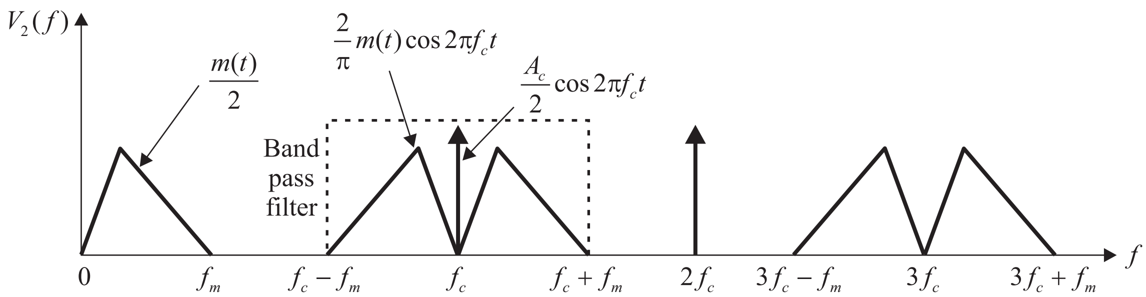
$$v_0(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right]\cos 2\pi f_c t \qquad \text{[AM Signal]}$$

where, $k_a = \dfrac{4}{\pi A_c}$

The spectrum of $m(t)$ is shown in figure below,



The spectrum of $v_0(t)$ for positive frequencies is shown in figure below,



Remember

The main condition for generation of DSB-FC or AM wave is that the carrier frequency should be atleast three times the maximum frequency of the modulating signal.

$$f_c - f_m \geq f_m \quad \Rightarrow \quad f_c \geq 2f_m$$

3.16 Demodulation of AM Signals

3.16.1 Square Law Demodulator (Detector)

1. A square law demodulator is used for the recovery of modulated signal from DSB-FC or AM wave.
2. It uses a non linear square law device like diode which satisfies square law relationship.
3. An AM signal can be demodulated by squaring it and then passing the squared signal through a low pass filter (LPF).

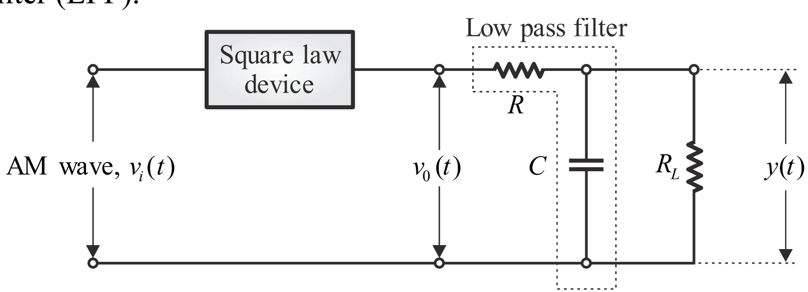
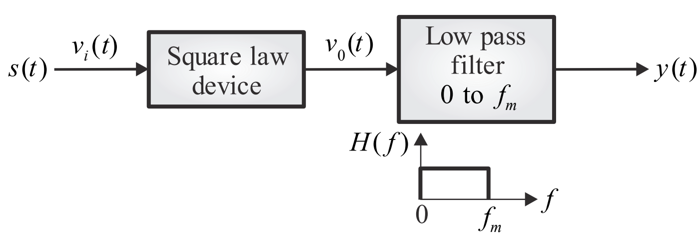


Fig. Square law demodulator

4. Block diagram of square law demodulator is shown in below figure.



5. **Mathematical analysis :**

The characteristics of a non-linear device is given by,

$$v_0(t) = a_1 v_i(t) + a_2 v_i^2(t)$$

where, $v_i(t) =$ Input voltage, $v_0(t) =$ Output voltage, a_1 and a_2 are constants.

The input voltage of the AM wave is given by,

$$v_i(t) = s(t) = A_c\left[1+k_a m(t)\right]\cos(2\pi f_c t)$$

$$\therefore \quad v_0(t) = a_1\left[A_c\left[1+k_a m(t)\right]\cos(2\pi f_c t)\right] + a_2\left[A_c\left[1+k_a m(t)\right]\cos(2\pi f_c t)\right]^2$$

$$v_0(t) = a_1 A_c\left[1+k_a m(t)\right]\cos(2\pi f_c t) + a_2 A_c^2\left[1+k_a m(t)\right]^2\cos^2(2\pi f_c t)$$

$$v_0(t) = a_1 A_c\left[1+k_a m(t)\right]\cos(2\pi f_c t) + a_2 A_c^2\left[1+k_a^2 m^2(t)+2k_a m(t)\right]\left[\frac{1+\cos(4\pi f_c t)}{2}\right]$$

$$v_0(t) = \frac{a_2 A_c^2}{2} + a_2 A_c^2 k_a m(t) + \frac{a_2 A_c^2 k_a^2}{2} m^2(t) + a_1 A_c\left[1+k_a m(t)\right]\cos(2\pi f_c t)$$

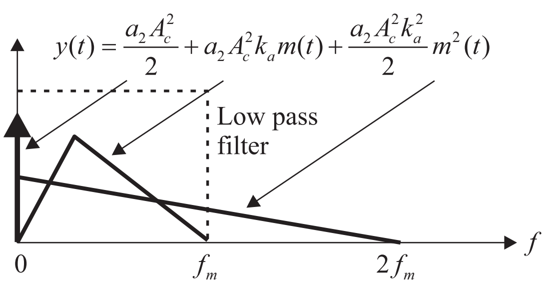
$$+\frac{a_2 A_c^2}{2}\left[1+k_a m(t)\right]^2\cos(4\pi f_c t)$$

The low pass filter passes only the first three terms on the right hand side to the output. Hence the low pass filter output is

$$y(t) = LPF\left[\frac{a_2 A_c^2}{2} + a_2 A_c^2 k_a m(t) + \frac{a_2 A_c^2 k_a^2}{2} m^2(t)\right]$$

$$y(t) = \frac{a_2 A_c^2}{2} + a_2 A_c^2 k_a m(t) + \frac{a_2 A_c^2 k_a^2}{2} LPF\left[m^2(t)\right]$$

$m^2(t)$ has a spectrum extending from dc to $2f_m$. When it is low pass filtered, only the spectrum portion $\left[0 \text{ to } f_m\right]$ gets through and this is indicated in the right hand side of the above equation $LPF\left[m^2(t)\right]$.



For successful demodulation, the output should be proportional to the modulating signal, $m(t)$ alone. However, there is an unwanted signal which gives rise to signal distortion.

Desired component $= a_2 A_c^2 k_a m(t)$

Distortion component $= \dfrac{a_2 A_c^2 k_a^2}{2} \underset{[0,\, f_m]}{LPF}\left[m^2(t)\right]$

The ratio of desired component to the undesired distortion component is,

$$\frac{a_2 A_c^2 k_a m(t)}{\frac{1}{2} a_2 A_c^2 k_a^2 m^2(t)} = \frac{1}{\frac{1}{2} k_a m(t)} = \frac{2}{k_a m(t)}$$

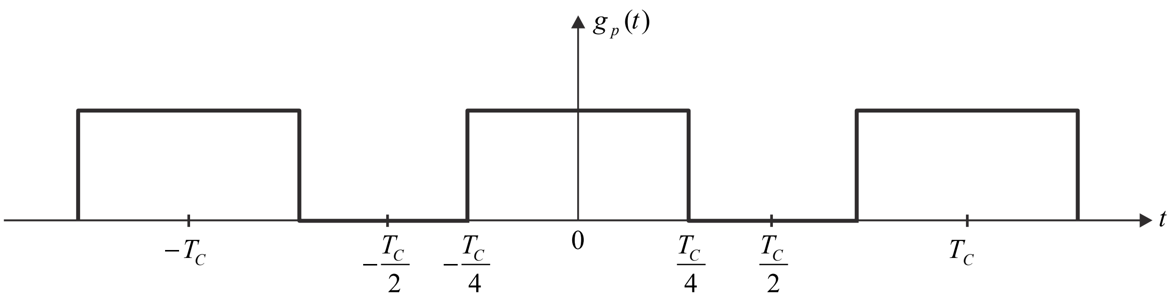
Ideally, this ratio should be infinite; practically, as large as possible; for which $|k_a m(t)| << 2$. This implies that the percentage modulation should be small.

Remember

The square law demodulator allows distortionless recovery of the baseband signal only if the AM wave has a low percentage modulation. This is a major limitation of the square law demodulator since small percentage modulation implies that the transmitted AM wave has low power efficiency and more susceptibility to channel noise.

3.16.2 Rectifier Detector

1. If an AM signal is applied to a diode and a resistor circuit shown in figure, the negative part of the AM wave will be suppressed.
2. The output across the resistor is a half-wave rectified version of the AM signal. In essence, the AM signal is multiplied by $g_p(t)$.



Fourier series for $g_p(t)$ with the period $T_c = 1/f_c$.

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi 3 f_c t) + \text{........}$$

Hence, the rectified output v_R is

$$v_R = g_p(t) A_c [1 + k_a m(t)] \cos\omega_c t$$

$$v_R = A_c [1 + k_a m(t)] \cos\omega_c t \left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \text{.......}\right)\right]$$

$$v_R = [A_c \cos\omega_c t + A_c k_a m(t) \cos\omega_c t]\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \text{.......}\right)\right]$$

$$v_R = \frac{A_c}{\pi} + \frac{A_c k_a m(t)}{\pi} + \text{Other terms}$$

3. When v_R is applied to LPF of cut-off frequency f_m. The output is,

$$v_c = \frac{A_c + A_c k_a m(t)}{\pi} = \frac{A_c}{\pi}[1 + k_a m(t)]$$

4. The dc term $\frac{A_c}{\pi}$ will be blocked by capacitor to give the desired output $\frac{A_c k_a m(t)}{\pi}$.

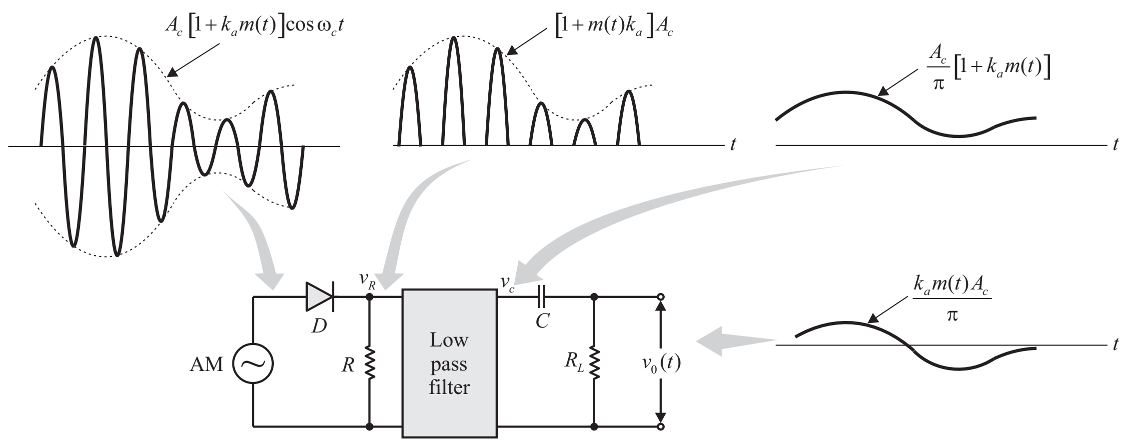


Fig. Rectifier detector for AM

Remember

Rectifier detector is in effect synchronous detection performed without using a local carrier. The high carrier content in AM ensures that its zero crossing are periodic and information about frequency and phase of the carrier at the transmitter is built in to the AM signal itself.

3.16.3 Envelope Detector

1. The envelope detector is a simple and very efficient device which is suitable for the detection of a narrowband AM signal. A narrowband AM wave is the one in which the carrier frequency is much higher as compared to the bandwidth of the modulating signal.
2. Envelope detector is used for recovery of modulating signal from DSB-FC signal.
3. An envelope detector produces output signal that follows the envelope of the input AM signal exactly.
4. Envelope detector is used for recovery of modulating signal for under modulation ($m_a << 1$) system.
5. The circuit diagram shown in figure functions as an envelope detector.

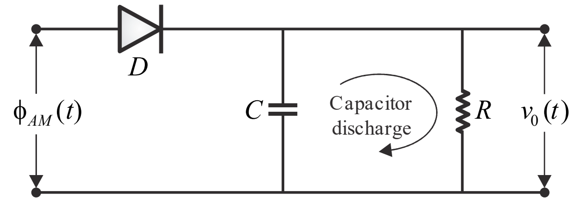
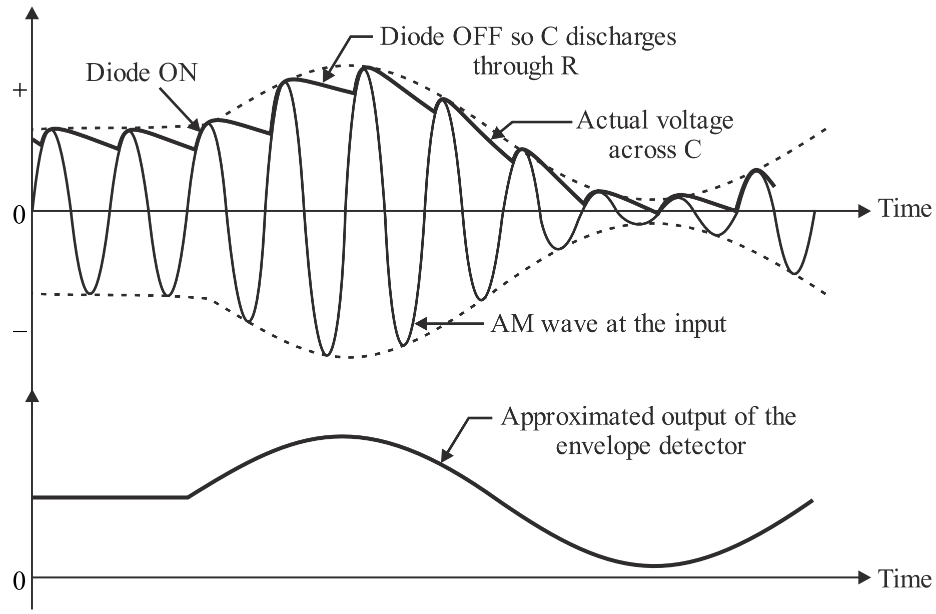


Fig. Envelope Detector

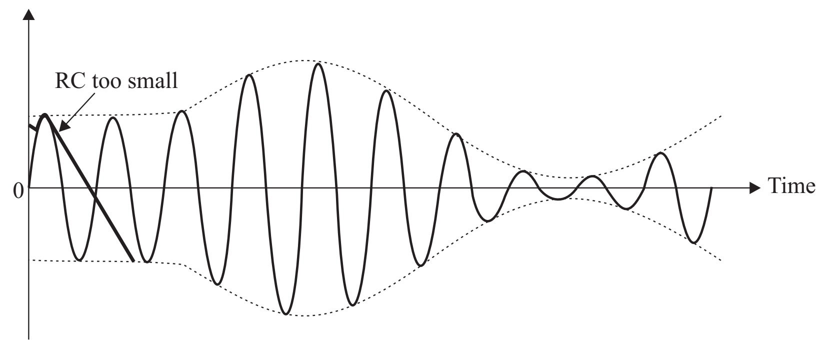
6. In every positive half cycle of the input the detector diode is forward biased. It will charge the filter capacitor C connected across the load resistance R to almost the peak value of the input voltage.
7. As soon as the capacitor charges to the peak value, the diode stops conducting. The capacitor C will discharge through R between the positive peaks as shown in the input output waveform.
8. The discharging process continues until the next positive half cycle. When the input signal becomes greater than the capacitor voltage, the diode conducts again and the process repeats itself.

9. The input output waveform shows the charging discharging of the filter capacitor and the approximate output voltage.
10. It can be seen from these waveforms, that the envelope of an AM wave is being recovered successfully.

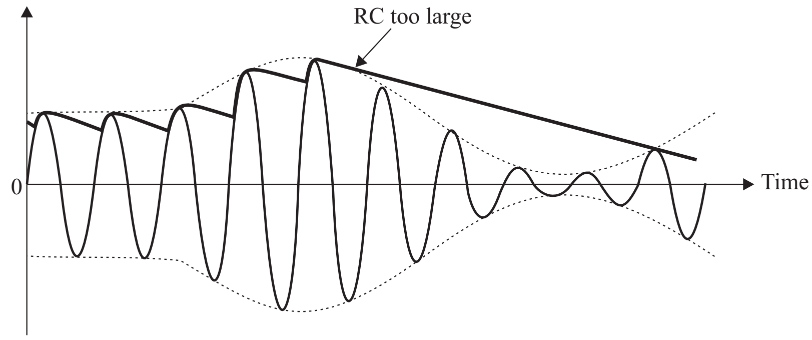


Input output waveforms for an envelope detector

11. If RC is too small, then the output of the filter falls very rapidly after each peak and will not follow the envelope of the modulated signal closely. This corresponding to the case where bandwidth of LPF is too large.



12. If RC is too large, then the discharge of the capacitor is too slow and again the output will not follow the envelope of the modulated signal. This corresponds to the case where the bandwidth of LPF is too small.



13. For good performance of the envelope detection,

(i) $\frac{1}{f_c} << RC << \frac{1}{f_m}$

$T_c << RC << T_m$

(ii) For proper operation of envelope detector, the magnitude of the slope of the capacitor discharge curve $\left|\frac{dv_c(t)}{dt}\right|$ should be larger than the magnitude of the slope of the envelope $\left|\frac{dA(t)}{dt}\right|$.

$$\therefore \quad \left|\frac{dv_c(t)}{dt}\right| \geq \left|\frac{dA(t)}{dt}\right|$$

(iii) $RC \leq \left(\frac{\sqrt{1-m_a^2}}{m_a}\right)\frac{1}{\omega_m}$

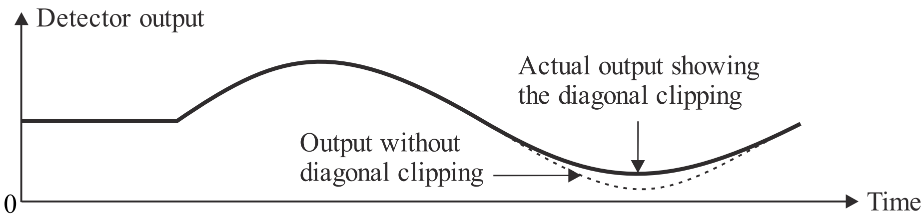
where, f_m is maximum modulating frequency, T_c is the time period of carrier signal and T_m is the time period of modulating signal.

14. **Drawbacks of envelope detector :**

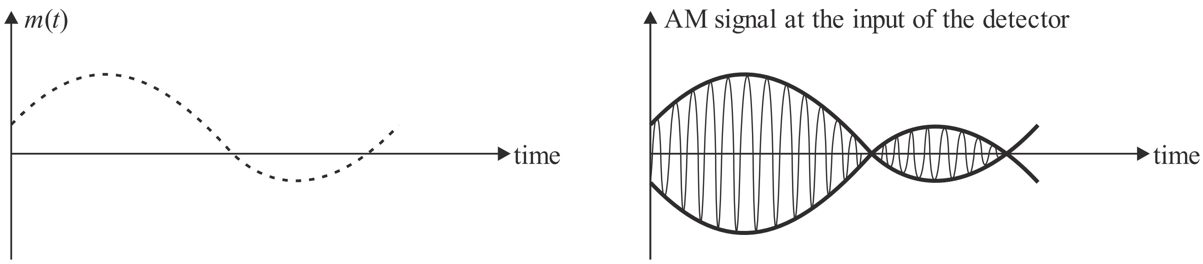
There are two types of distortions which can occur in the detector output. They are :

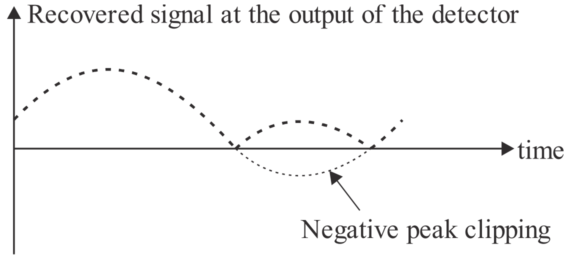
(i) Diagonal clipping : This type of distortions occurs when the *RC* time constant of the load current is too long. Due to this the *RC* circuit cannot follow the fast change in the modulating envelope.

Condition to avoid diagonal clipping, $\frac{1}{f_c} << RC << \frac{1}{f_m}$



(ii) Negative peak clipping : This distortion occurs due to a fact that the modulation index on the output side of the detector is higher than that on its input side. So at higher depths of modulation of the transmitted signal, the over modulation may take place at the output of the detector. The negative peak clipping will take place as a result of this over modulation as shown in figure.





Remember

1. The rectifier detector is basically synchronous detector on the other hand envelope detector is a asynchronous detector.
2. LPF is designed in rectifier detector is to separate $m(t)$ from terms such that $m(t)\cos n\omega_c t$ and it does not depends on m_a. On the other hand the time constant RC of the LPF for the envelope detector does depends on the value of m_a.

Solved Example 5

An AM signal is detected using an envelope detector. The carrier frequency and modulating signal frequency are 1 MHz and 2 kHz respectively. An appropriate value for the time constant of the envelope detector

(A) 500 μsec (B) 20 μsec (C) 0.2 μsec (D) 1 μsec

Sol. **Given :** $f_c = 1\,\text{MHz}$, $f_m = 2\,\text{kHz}$

$$T_m = \frac{1}{f_m} = \frac{1}{2\times 10^3} = 0.5\,\text{msec} = 500\ \mu\text{sec}$$

$$T_c = \frac{1}{f_c} = \frac{1}{1\times 10^6} = 1\,\mu\,\text{sec}$$

For proper operation of envelop detector time constant RC should lie between T_m and T_c.

$$T_c < RC < T_m$$

$$\frac{1}{f_c} < RC < \frac{1}{f_m}$$

$$1\,\mu\text{sec} < RC < 500\,\mu\text{sec}$$

Hence, $RC = 20\,\mu\text{sec}$ because only this lies in the above range.

3.17 Generation of DSB - SC Signals

3.17.1 Balanced modulator

1. Balanced modulator is used for generation of DSB-SC wave. Using two ideal AM generators it is possible to generate a DSB-SC signal.
2. It consists of two amplitude modulators that are interconnected in such a way as to suppress the unwanted carrier in an AM wave and the output consists of upper and lower sidebands only.

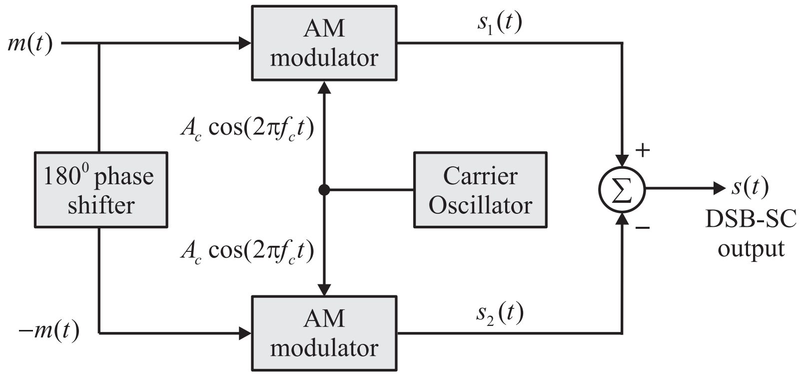


Fig. Balanced Modulator

3. The output of the two AM modulators are as follows :

$$s_1(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

$$s_2(t) = A_c[1 - k_a m(t)]\cos(2\pi f_c t)$$

4. The output of the summer will be

$$s(t) = s_1(t) - s_2(t)$$

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t) - A_c[1 - k_a m(t)]\cos(2\pi f_c t)$$

$$s(t) = A_c\cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t) - A_c\cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t)$$

$$s(t) = 2A_c k_a m(t)\cos(2\pi f_c t)$$

5. The balanced modulator output is equal to the product of the modulating signal $m(t)$ and the carrier signal $c(t)$ except the scaling factor $2k_a$, which is of the form of standard time domain DSB-SC signal.

From modulation theorem, $m(t)\cos(2\pi f_c t) \xleftrightarrow{F.T.} \dfrac{M(f - f_c) + M(f + f_c)}{2}$

Taking Fourier transform on both sides of equation,

$$F[s(t)] = F[2A_c k_a m(t)\cos(2\pi f_c t)] = \frac{2A_c k_a}{2}[M(f - f_c) + M(f + f_c)]$$

$$S(f) = A_c k_a[M(f - f_c) + M(f + f_c)]$$

6. Message spectrum and DSB-SC spectrum is shown in figure below.

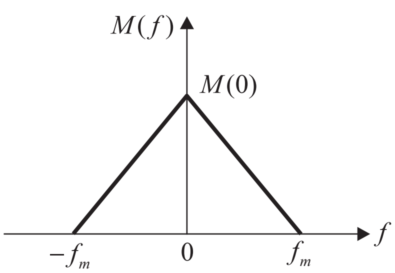


Fig. Message spectrum

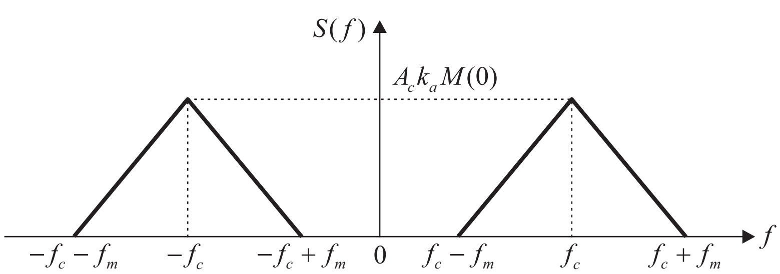


Fig. DSB-SC spectrum

Since the carrier component is eliminated, the output is called DSB-SC signal.

7. Power content in DSB-SC signal is given by,

$$P_T = A_c^2 k_a^2 \overline{m^2(t)}$$

where $\overline{m^2(t)}$ = mean square value

8. Transmission bandwidth required by DSB-SC modulation is given by,

$$BW = 2f_m$$

3.17.2 Ring modulator

1. Ring modulator is another product modulator, which is used to generate DSB-SC signal.
2. In a ring modulator circuit, four diodes are connected in the form of a ring.
3. All the four diodes in ring are controlled by a square wave signal $c(t)$ of frequency f_c applied through a centre tapped transformer.

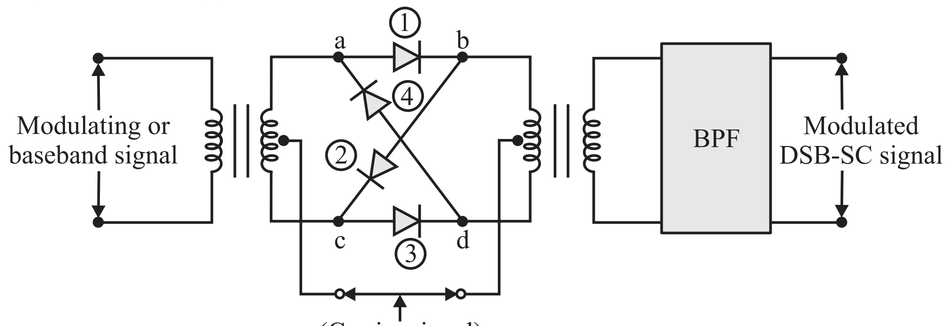


Fig. Circuit diagram of a Ring Modulator

4. In case, when diodes are ideal and transformer are perfectly balanced, the two outer diodes are switched on if the carrier signal is positive whereas the two inner diodes are switched off and thus presenting very high impedance as shown in figure (a). Under this condition, the modulator multiplies the modulating signal $x(t)$ by $+1$.
5. Now, in case when carrier signal is negative, the situation becomes reversed as shown in figure (b). In this case, the modulator multiplies the modulating signal by -1.
6. Figure (a) illustrates the condition when the diodes (1) and (3) are switched ON and diodes (2) and (4) are switched OFF.
Figure (b) illustrates the condition when the diodes (2) and (4) are switched ON and diodes (1) and (3) switched OFF.
Hence, the ring modulator is a product modulator for a square wave carrier and modulating signal.

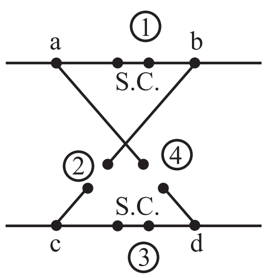


Figure (a)

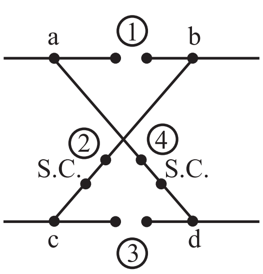
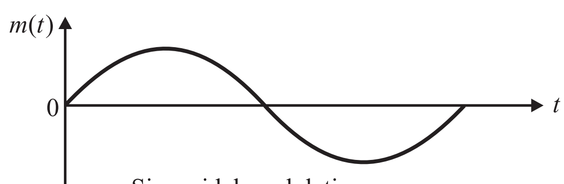
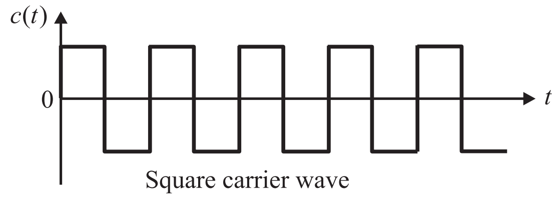


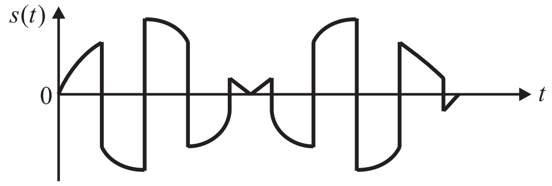
Figure (b)



Sinusoidal modulating wave



Square carrier wave



Modulated wave (DSB-SC) output of the ring modulator

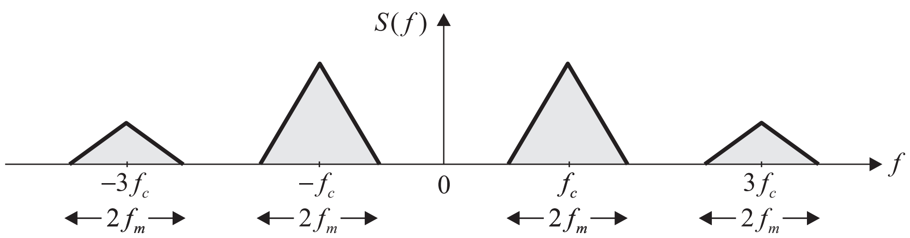
7. Trignometric Fourier series representation of $c(t)$ is given by,

$$c(t)=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)}\left\{\cos\left[2\pi f_c t(2n-1)\right]\right\}$$

$$s(t)=x(t).c(t)=x(t)\times\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)}\left\{\cos\left[2\pi f_c t(2n-1)\right]\right\}$$

   Thus, with the help of above mathematical analysis, it may be verified that the output from ring modulator does not have any components at carrier frequency. Hence the modulated output contains only product terms.

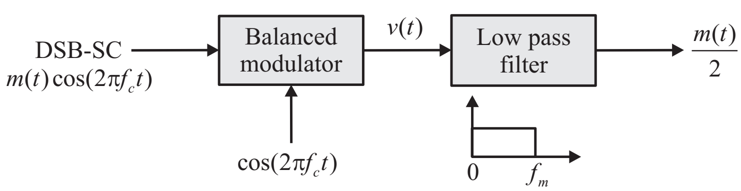
8. A ring modulator is also known as a double balanced modulator since it is balanced with respect to the baseband signal as well as the square wave signal.
9. The frequency spectrum of the ring modulator output contains sidebands around each of the odd harmonics of the square wave carrier signal.



3.18 Detection of DSB-SC

Synchronous detection of DSB-SC :

In synchronous detection method, the received modulated or DSB-SC signal is first multiplied with a locally generated carrier signal $\cos(2\pi f_c t)$ and then passed through a low-pass filer (LPF). At the output of a low-pass filter (LPF), the original modulating signal is recovered.



Synchronous detection of DSB-SC

The balanced modulator (or multiplier or product modulator) output can be given as,

$$v(t) = m(t)\cos(2\pi f_c t).\cos(2\pi f_c t) = m(t)\cos^2(2\pi f_c t) \qquad \left[\because \ \cos^2\theta = \frac{1+\cos 2\theta}{2}\right]$$

$$v(t) = m(t)\left[\frac{1+\cos(2\pi.2 f_c t)}{2}\right] = \frac{m(t)}{2} + \frac{m(t)\cos(2\pi.2 f_c t)}{2}$$

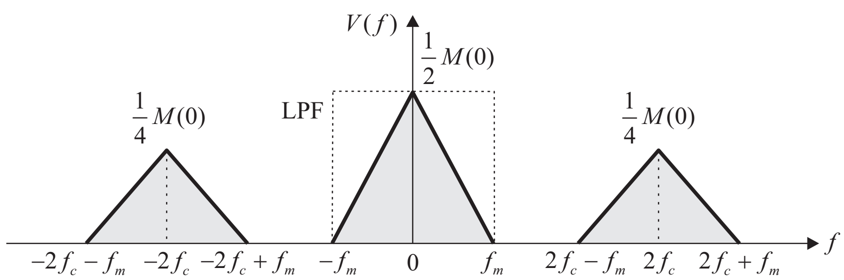
The product waveform is transmitted through a low-pass filter which rejects the double-frequency signal $2f_c$.

Output signal, $s(t) = \dfrac{m(t)}{2}$, is proportional to $m(t)$ and hence the modulating signal is recovered.

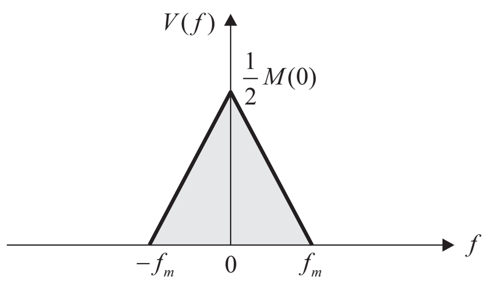
The spectrum of the output of balanced modulator is shown below,

$$v(t) = \frac{m(t)}{2} + \frac{m(t)\cos(2\pi.2 f_c t)}{2} \xleftrightarrow{F.T.} \frac{M(f)}{2} + \frac{1}{2}\left[\frac{M(f-2f_c)+M(f+2f_c)}{2}\right]$$

$$m(t)\cos(2\pi f_c t) \xleftrightarrow{F.T.} \frac{M(f-f_c)+M(f+f_c)}{2}$$

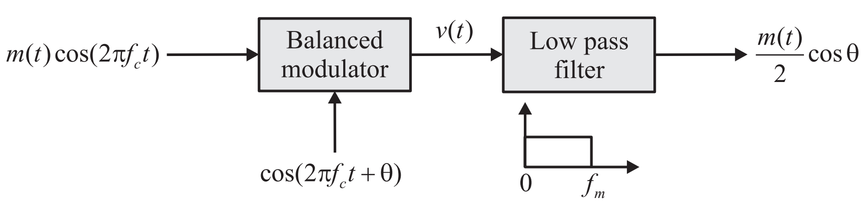


The spectrum of the output of LPF is shown below,



3.19 Effects of Frequency and Phase Error

3.19.1 Phase drift in carrier oscillator



Product waveform, $v(t) = m(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \theta)$

$$[\because 2\cos A\cos B = \cos(A+B) + \cos(A-B)]$$

$\therefore \quad v(t) = \frac{m(t)}{2}\left[\cos(2\pi 2 f_c t + \theta) + \cos\theta\right]$

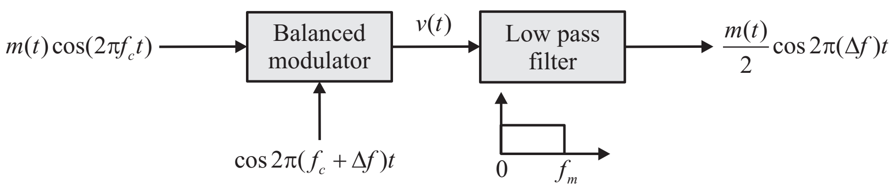
The product waveform is transmitted through a low-pass filter which rejects the double-frequency signal $2f_c$.

Output signal, $s(t) = \frac{m(t)}{2}\cos\theta$

For $\theta = 0$, output signal $s(t) = \frac{m(t)}{2}$ and the modulating signal is recovered.

For $\theta = \frac{\pi}{2} = 90^0$, output signal $s(t) = 0$. This effect is known as ***Quadrature null effect*** where the output of the receiver becomes zero in spite of the fact that there is a transmitted signal.

3.19.2 Frequency drift in carrier oscillator



Product waveform, $v(t) = m(t)\cos 2\pi f_c t \cos 2\pi(f_c + \Delta f)t$

$[\because 2\cos A\cos B = \cos(A+B) + \cos(A-B)]$

$\therefore \quad v(t) = \frac{m(t)}{2}\left[\cos 2\pi(2f_c + \Delta f)t + \cos 2\pi(\Delta f)t\right]$

The product waveform is transmitted through a low-pass filter which rejects the double-frequency signal $2f_c$.

Output signal, $s(t) = \frac{m(t)}{2}\cos 2\pi(\Delta f)t$

The above result is known as ***beat effect.***

Disadvantage : The demerit of the synchronous detection is that it requires an additional system at the receiver to ensure that the locally generated carrier is synchronized with the transmitter carrier to avoid phase and frequency errors making receiver complex and costly.

TEST 4

Q.1 A balanced modulator eliminates which of the following from its output?

(A) Upper sideband

(B) Lower sideband

(C) Carrier

(D) Both sidebands

Q.2 A balanced modulator has a carrier frequency of 1.9 MHz and a modulating sine wave of 2.6 kHz. The output signals are __________ and __________ kHz.

Q.3 The most suitable method for detecting a modulated signal $(2.5 + 5\cos\omega_m t)\cos\omega_c t$ is

(A) envelope detector
(B) synchronous detector
(C) ratio detector
(D) both A and B

Q.4 Time constant when an AM wave $10\left[1+0.6\cos 2\pi\times10^3 t\right]\cos 2\pi\times10^6 t$ is to be detected by a linear diode detector
(A) 0.11 m sec　　(B) 0.21 m sec
(C) 0.31 m sec　　(D) 0.41 m sec

Q.5 A diode detector load consists of 0.01 μF capacitor in parallel with a 5 kΩ resistor. The maximum depth of modulation without diagonal clipping at modulating frequency of 1000 Hz and 10 kHz is
(A) 0.76, 0.24　　(B) 0.95, 0.3
(C) 0.24, 0.76　　(D) 0.3, 0.95

3.20 Advantages, Disadvantages and Applications of DSB-SC

Advantages of DSB-SC :
1. Carrier wave is suppressed.
2. Power saving due to suppression of carrier.
3. The modulation system is simple.
4. Efficiency is more than AM.
5. Linear modulation type is required.
6. It can be used for point to point communication.

Disadvantages of DSB-SC :
1. Design of receiver is complex.
2. Bandwidth required is same as that of AM.

Application of DSB-SC :
Analogue TV systems to transmit color information.

3.21 Hilbert Transform

Hilbert transform is a linear, non-causal and phase shifter transform which gives -90^0 phase shift for positive frequency and 90^0 phase shift for negative frequency.

The Hilbert transform of a function $f(t)$ is defined by,

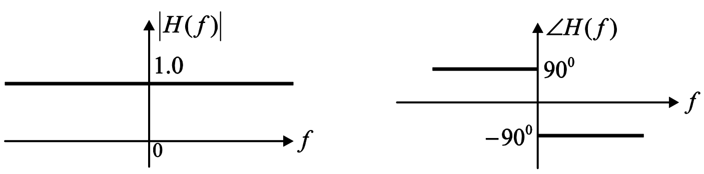
$$H(t)=f(t)\otimes\frac{1}{\pi t}=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(\tau)}{t-\tau}d\tau$$

Frequency response of Hilbert transform is given by,

$$H(f)=-j\,\text{sgn}(f)$$

$$H(f)=|H(f)|\angle H(f)$$

$$|H(f)|=1,-\infty\le f\le\infty \quad\text{and}\quad \angle H(f)=\begin{cases}-90^0, & f>0\\ 0, & f=0\\ 90^0, & f<0\end{cases}$$



3.21.1 Properties of Hilbert Transform

1. The energy content of a signal is equal to the energy content of its Hilbert transform.
2. A signal $x(t)$ and its Hilbert transform $\hat{x}(t)$ have the same autocorrelation function.
3. A signal $x(t)$ and its Hilbert transform $\hat{x}(t)$ are mutually orthogonal.

$$\int_{-\infty}^{\infty} x(t)\hat{x}(t)\,dt = 0$$

4. Applying Hilbert transform operation to a signal twice causes a sign reversal of the signal.

   If $\quad H\left[x(t)\right] = \hat{x}(t)$

   Then $\quad H\left[\hat{x}(t)\right] = -x(t)$

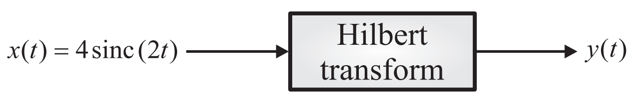
   Here '*H*' denotes the Hilbert transform.
5. The Hilbert transform of an even signal is odd and Hilbert transform of an odd signal is even.

3.21.2 Application of Hilbert Transform

1. For generation of SSB signals.
2. For designing of minimum phase type filters.
3. For representation of band-pass signals.

Solved Example 6

The input $4\,\text{sinc}\,(2t)$ is fed to a Hilbert transform to obtain $y(t)$, as shown in the figure below :



Here $\text{sinc}\,(x) = \dfrac{\sin(\pi x)}{\pi x}$. The value (accurate to two decimal places) of $\int_{-\infty}^{\infty} |y(t)|^2 dt$ is ________.

Sol. Given : $x(t) = 4\,\text{sinc}\,(2t)$

The energy (or power) in a signal and its Hilbert transform are equal

$$\int_{-\infty}^{\infty} |y(t)|^2\, dt = \text{Energy of signal } y(t)$$

So, energy of $y(t)$ will be same as energy of $x(t)$.

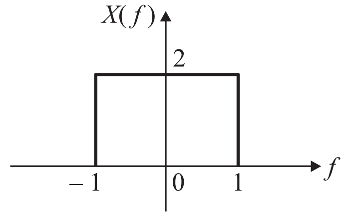
Now, $\qquad A\,\text{rect}\left(\dfrac{t}{\tau}\right) \xleftrightarrow{F.T.} A\tau\,\text{sinc}\,(f\tau)$

Using duality property,

$$A\tau\,\text{sinc}\,(t\tau) \xleftrightarrow{F.T.} A\,\text{rect}\left(\frac{f}{\tau}\right)$$

Here, $A\tau = 4$ and $\tau = 2$. So, $A = 2$

$$4\,\text{sinc}(2t) \xleftrightarrow{F.T.} 2\,\text{rect}\left(\frac{f}{2}\right)$$



Using Parseval's theorem,

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df$$

Energy of $x(t) = \int_{-1}^{1} (2)^2 df = 8$

Hence, energy of $y(t) = 8$

$$\int_{-\infty}^{\infty} |y(t)|^2\, dt = 8$$

Ans.

3.22 Generation of SSB Signals

3.22.1 Filter method (Frequency discrimination method)

Filter method is used for generation of SSB-SC signal. In filter method, a DSB-SC signal is generated first using a balanced modulator or product modulator and then one sideband either upper sideband or lower sideband is removed using a sideband filter. Accordingly, the output is either LSB SSB-SC or USB SSB-SC signal.

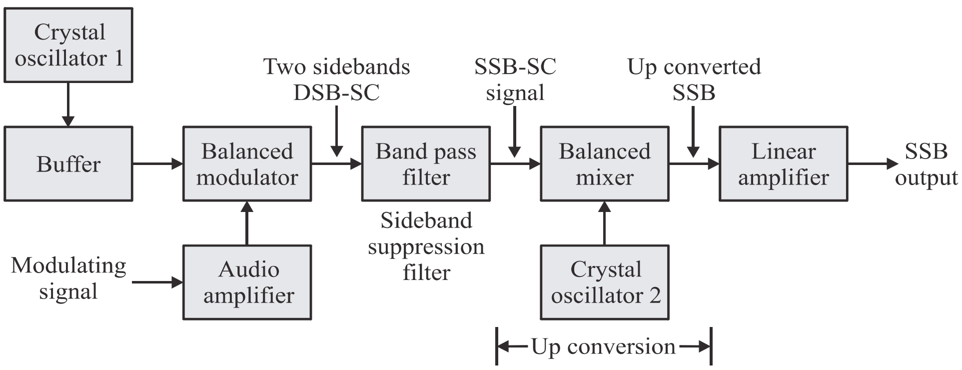


Fig. Filter method for SSB generation

Operation of the filter method :

Filter method can be used for generating the SSB from modulated wave if the message signal does not have any low frequency content.

1. The modulating signal is amplified using an audio amplifier and applied to the balanced modulator. The other input to the balanced modulator is the carrier signal generated by the crystal oscillator 1. This carrier frequency is much less as compared to the carrier which is to be actually transmitted.
2. The balanced modulator will suppress the carrier and will produce upper and lower sidebands at its output which is a DSB-SC signal.
3. Out of the sidebands the unwanted sideband is heavily attenuated by the sideband suppression filter and the other sideband is passed through without much attenuation. In general, the sideband filtering process is not perfect and a small amount of the unwanted sideband gets through. Therefore, the requirements of sideband filter are flat pass band, high attenuation stop band and sharp cut off frequencies.

4. The frequency of the sideband is very low. So the frequency is boosted up to the transmitter frequency by the combination of the balanced mixer and second crystal oscillator. This is called as frequency up conversion.
5. This signal is then amplified using linear amplifiers which are used to avoid any waveform distortion.

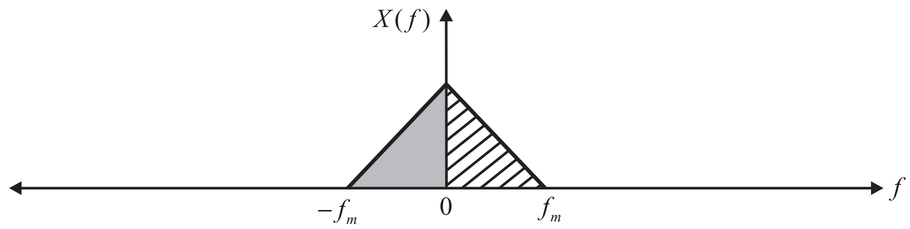
Advantages of filter method :

1. The filter method gives the adequate sideband suppression.
2. The sideband filter also helps to attenuate carrier if present in the output of balanced modulator.
3. The band width is sufficiently flat and wide.
4. This method is simpler as compared to other methods.

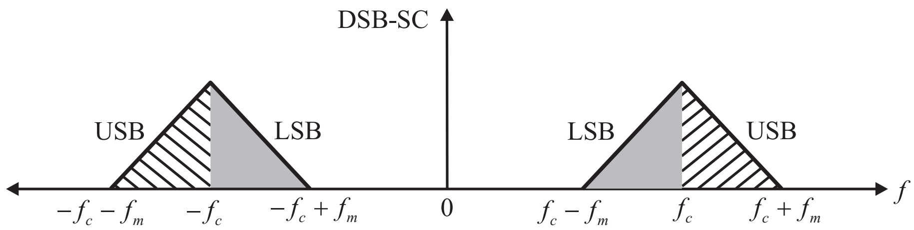
Disadvantages of filter method :

1. Low audio frequencies cannot be used as the filter becomes bulky.
2. Due to the inability of the system to generate SSB at high radio frequencies, the frequency up conversion is necessary.
3. Two expensive filters are to be used one for each sideband.
4. Switching of side band is not possible.
5. We cannot generate SSB-SC at any frequency.

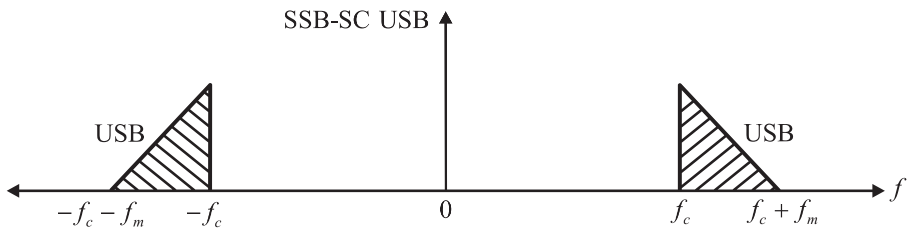
Frequency spectrum of baseband signal $x(t)$:



Frequency spectrum of DSB-SC signal :



Frequency spectrum of SSB-SC signal :



Remember

SSB system reduces the transmission bandwidth by half that of AM or DSB systems. This means that in a given frequency band, twice the number of channels can be accommodated with SSB.

3.22.2 Phase Shift Method

It is the most powerful method for SSB generation. The block diagram for the phase shift method of SSB generation is shown. This system uses two balanced modulators or product modulators 1 and 2 and two -90^0 phase shifting networks.

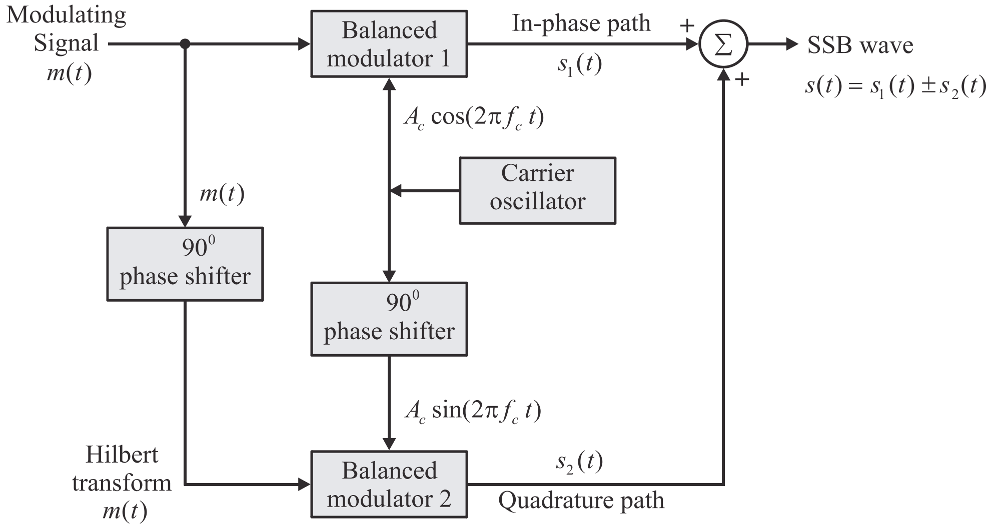


Fig. Phase shift method

Operation of the phase shift method :

1. The SSB modulator uses two balanced modulators 1 and 2.
2. The message signal $m(t)$ and a carrier signal $A_c \cos(2\pi f_c t)$ is directly applied to the balanced modulator 1, producing a DSB-SC wave.
3. The Hilbert transform $\hat{m}(t)$ $(-90^0 \text{ phase shift})$ of $m(t)$ and carrier signal shifted by -90^0 are applied to balanced modulator 2, producing a DSB-SC wave.
4. The output of balanced modulator 1 is $s_1(t) = m(t) A_c \cos(2\pi f_c t)$.
5. The output of balanced modulator 2 is $s_2(t) = \hat{m}(t) A_c \sin(2\pi f_c t)$.
6. These signals $s_1(t)$ and $s_2(t)$ are fed to a summer. The output of the summer is

$$s(t) = s_1(t) \pm s_2(t)$$

$$s(t) = A_c m(t) \cos(2\pi f_c t) \pm A_c \hat{m}(t) \sin(2\pi f_c t)$$

7. The plus sign at the summing junction yields an SSB with only the LSB i.e.

$$s_{LSB}(t) = A_c m(t) \cos(2\pi f_c t) + A_c \hat{m}(t) \sin(2\pi f_c t)$$

Let $m(t) = A_m \cos(2\pi f_m t)$ then its Hilbert transform is

$$\hat{m}(t) = A_m \cos(2\pi f_m t - 90^0) = A_m \sin(2\pi f_m t)$$

$$s_{LSB}(t) = A_c A_m \cos(2\pi f_m t) \cos(2\pi f_c t) + A_c A_m \sin(2\pi f_m t) \sin(2\pi f_c t)$$

$$s_{LSB}(t) = A_c A_m \left[\cos(2\pi f_m t) \cos(2\pi f_c t) + \sin(2\pi f_m t) \sin(2\pi f_c t)\right]$$

$\because$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\therefore$ $s_{LSB}(t) = 2 A_c A_m \cos 2\pi (f_c - f_m) t$

This signal contains only lower sideband frequency component $f_c - f_m$.

8. The minus sign at the summing junction yields an SSB with only the USB i.e.

$$s_{USB}(t) = A_c m(t)\cos(2\pi f_c t) - A_c \hat{m}(t)\sin(2\pi f_c t)$$

$$s_{USB}(t) = A_c A_m \cos(2\pi f_m t)\cos(2\pi f_c t) - A_c A_m \sin(2\pi f_m t)\sin(2\pi f_c t)$$

$$s_{USB}(t) = A_c A_m \left[\cos(2\pi f_m t)\cos(2\pi f_c t) - \sin(2\pi f_m t)\sin(2\pi f_c t)\right]$$

$\because$ $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\therefore$ $s_{USB}(t) = 2A_c A_m \cos 2\pi(f_c + f_m)t$

This signal contains only upper sideband frequency component $f_c + f_m$.

Advantages of phase shift method :

1. Band pass filters are not required.
2. Low audio frequencies may be used for modulation.
3. It can generate SSB at any frequency.
4. Easy switching from one sideband to other sideband is possible.
5. To generate SSB at high radio frequencies, up conversion and hence repetitive mixing is not necessary.

Disadvantages of phase shift method :

1. The output of two balanced modulators must be exactly same, otherwise cancellation will be incomplete.
2. The design of the -90^0 phase shifting networks for the carrier signal and modulating signal is extremely critical. These networks have to provide a correct -90^0 phase shift at all the frequencies which are practically difficult to achieve.

3.22.3 Third method or Weaver's method

Third method of SSB generation is similar to phase-shaft method except that the 90^0 phase shifter is not required for the modulating signal.

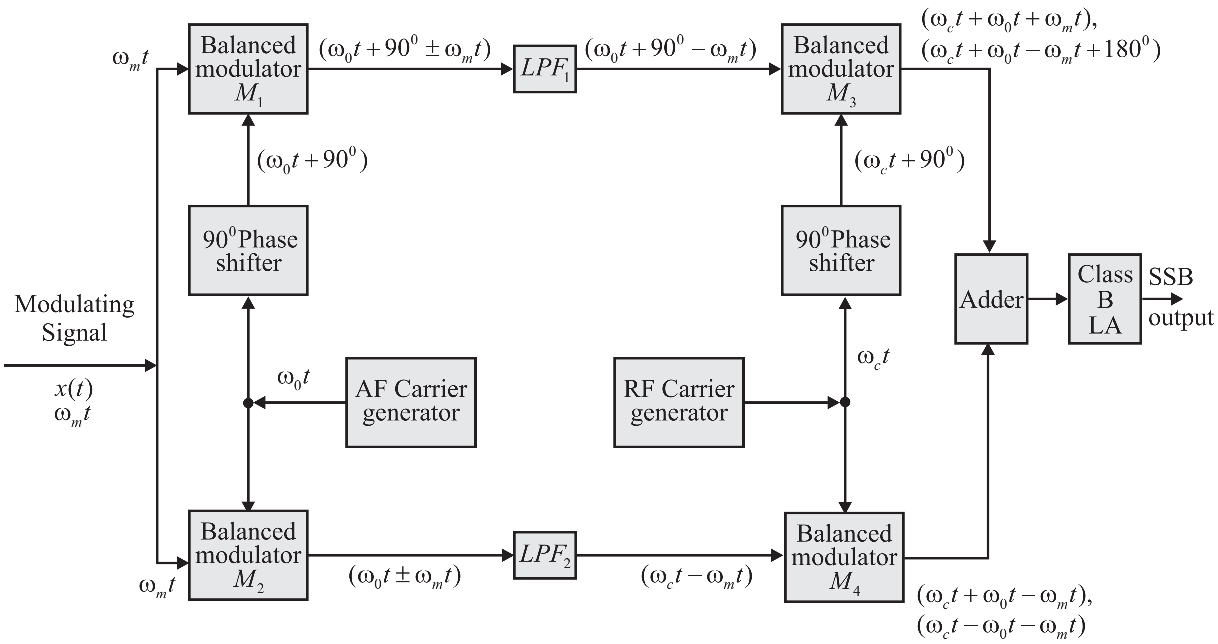


Fig. SSB transmitter using third method

Figure depicts functional block diagram of SSB transmitter employing SSB modulator using third method.

Operation of Weaver's method :

1. This method uses four balanced modulators M_1, M_2, M_3 and M_4.
2. Modulating signal is applied to both M_1 and M_2.
3. The other input signal to balanced modulator M_1 is the AF carrier signal shifted by 90^0.
4. The balanced modulator M_2 receives the other input signal from the AF carrier signal generator.
5. Their outputs are passed through their respective low-pass filters, and then applied to the balanced modulator M_3 and M_4.
6. The other input signal to balanced modulator M_3 is the RF carrier signal shifted by 90^0.
7. The balanced modulator M_4 receives the other input signal from the RF carrier signal generator.
8. Their outputs are then added and applied to class B linear amplifier.

Mathematical Analysis :

The RF carrier generator signal, $v_c(t) = \cos(\omega_c t)$

The AF carrier generator signal, $v_0(t) = \cos(\omega_0 t)$

The modulating signal, $v_m(t) = \cos(\omega_m t)$

The input signals to balanced modulator M_1 are $\cos(\omega_0 t + \pi/2)$ and $\cos(\omega_m t)$.

Therefore, the output of $M_1, v_1 = \cos(\omega_0 t + \pi/2)\cos(\omega_m t)$

$$v_1 = \frac{1}{2}\left[\cos(\omega_0 t + \pi/2 + \omega_m t) + \cos(\omega_0 t + \pi/2 - \omega_m t)\right]$$

$$v_1 = \frac{1}{2}\left[\cos(\omega_0 t + \omega_m t + \pi/2) + \cos(\omega_0 t - \omega_m t + \pi/2)\right]$$

The first term contains higher frequency component and is filtered and is filtered out by low-pass filter LPF_1. Thus,

The output of $LPF_1, v_{11} = \frac{1}{2}\cos\left[(\omega_0 - \omega_m)t + \pi/2\right]$

Now, the input signals to balanced modulator M_2 are $\cos(\omega_0 t)$ and $\cos(\omega_m t)$.

Therefore the output of $M_2, v_2 = \cos(\omega_0 t)\cos(\omega_m t)$

$$v_2 = \frac{1}{2}\left[\cos(\omega_0 t + \omega_m t) + \cos(\omega_0 t - \omega_m t)\right]$$

The first term contains higher frequency component and is filtered out by low-pass filter LPF_2.

The second term is lower frequency component with $\pi/2$ radians lead and is the output of LPF_2. Thus,

The output of $LPF_2, v_{22} = \frac{1}{2}\cos\left[(\omega_0 - \omega_m)t\right]$

Input signals to balanced modulator M_3 are $\cos(\omega_c t + \pi/2)$ and $\frac{1}{2}\cos[(\omega_0 - \omega_m)t + \pi/2]$.

Therefore the output of $M_3, v_3 = \frac{1}{2}\cos(\omega_c t + \pi/2)\cos[(\omega_0 - \omega_m)t + \pi/2]$

$$v_3 = \frac{1}{4}\cos[(\omega_c + \omega_0 - \omega_m)t + \pi/2 + \pi/2] + \frac{1}{4}\cos[(\omega_c - \omega_0 + \omega_m)t + \pi/2 - \pi/2]$$

$$v_3 = \frac{1}{4}[\{-\cos(\omega_c + \omega_0 - \omega_m)t\} + \cos\{(\omega_c - \omega_0 + \omega_m)t\}]$$

The input signals to balanced modulator M_4 are $\cos(\omega_c t)$ and $\cos\{(\omega_0 - \omega_m)t\}$.

Therefore the output of $M_4,\ v_4 = \frac{1}{2}\cos(\omega_c t)\cos\{(\omega_0 - \omega_m)t\}$

$$v_4 = \frac{1}{4}\cos(\omega_c + \omega_0 - \omega_m)t + \frac{1}{4}\cos(\omega_c - \omega_0 + \omega_m)t$$

The output of adder, $v_0 = v_3 + v_4 = \frac{1}{4}\cos[(\omega_c t) - (\omega_0 - \omega_m)t]$

This is SSB with LSB in the output, thus USB is suppressed. Similarly USB can be generated, if needed.

Advantages of Third Method :

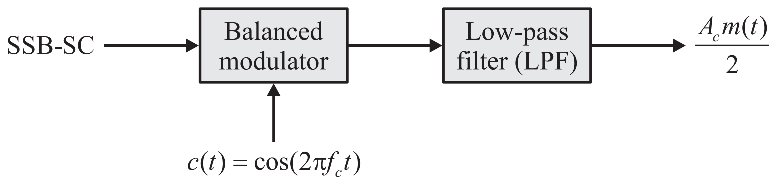
1. Third method of generating SSB signal has same advantages as that of phase-shift method.
2. There is no need of $\pi/2$ phase shifter at low-modulating frequency and whole modulating frequency range.
3. Providing 90^0 phase shift at only one frequency is possible.

Disadvantage of Third Method :

Due to complex design of third Method of SSB generation, it is rarely used.

3.23 Detection of SSB-SC

The SSB-SC Signal can be demodulated by synchronous detector.



The SSB-SC signal can be represented by,

$$s_{LSB}(t) = A_c m(t)\cos(2\pi f_c t) \pm A_c \hat{m}(t)\sin(2\pi f_c t)$$

The output of balanced modulator will be,

$$s_{LSB}(t) = [A_c m(t)\cos(2\pi f_c t) \pm A_c \hat{m}(t)\sin(2\pi f_c t)]\cos 2\pi f_c t$$

$$s_{LSB}(t) = A_c m(t)\cos^2(2\pi f_c t) \pm A_c \hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

$$s_{LSB}(t) = A_c m(t)\frac{[1 + \cos 2(2\pi f_c t)]}{2} \pm A_c \hat{m}(t)\sin 2(2\pi f_c t)$$

The output of lowpass filter will be $\frac{A_c m(t)}{2}$.

3.24 Advantages, Disadvantages and Applications of SSB-SC

Advantages of SSB-SC :

1. Carrier wave and one sideband are suppressed.
2. Power saving due to suppression of carrier and one sideband.
3. SSB-SC signal is a bandwidth efficient system. Less bandwidth requirement, bandwidth $= f_m$, which is half of that required by DSB-FC and DSB-SC signals.

   Carrier component does not contain any information. The information is contained in the sidebands only. But the sidebands are images of each other and hence both of them contain the same information. Thus only one sideband is necessary for transmission of information and if the carrier and one sideband are suppressed at the transmitter, no information is lost. Therefore, channel requires the same bandwidth as the message signal.
4. Reduced interference of noise due to reduced bandwidth.
5. Fading does not occur in SSB transmission. (Fading means that a signal alternately increases and decreases in strength as it is picked up by the receiver. It occur because the carrier and sideband may reach the receiver shifting in time and phase w.r.t. each other).

Disadvantages of SSB-SC :

1. The generation and reception of SSB signal is a complex process.
2. Since carrier is absent, the SSB transmitter and receiver need to have an excellent frequency stability.
3. The SSB modulation is expensive and complex to implement.

Applications of SSB-SC :

1. SSB transmission is used in the application where the power saving is required.
2. SSB is also used in application in which bandwidth requirements are low.

Example : Point to point communication, mobile communications, TV, telemetry, military communications, and radio navigation.

3.25 Vestigial Sideband (VSB)

In VSB modulation instead of rejecting one sideband completely as in SSB modulation scheme, a gradual cut-off of one sideband is allowed. This gradual cut is compensated by a vestige or portion of the other sideband.

In VSB, one sideband and a part of the other sideband called as **vestige** is transmitted. So, the bandwidth required for VSB transmission is somewhat higher than that of SSB modulation.

3.25.1 Generation of VSB

To generate a VSB signal, a DSB-SC signal is generated first and is then passed through a sideband filter as shown in below figure. This filter will pass the wanted sideband as it is along with a part of unwanted sideband.

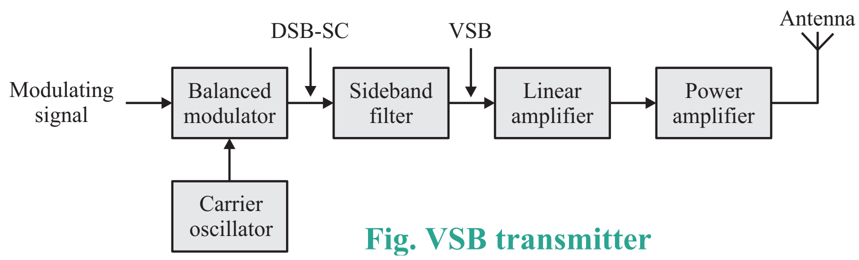
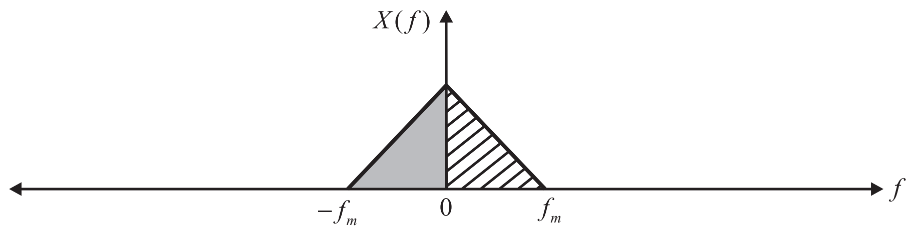


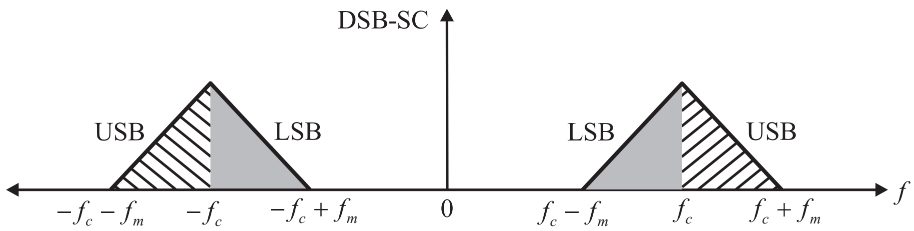
Fig. VSB transmitter

The VSB signal obtained at the output of filter is applied to a chain of linear amplifiers and raised in power by the power amplifiers. The amplified signal is then applied to the transmitting antenna for transmission of signal.

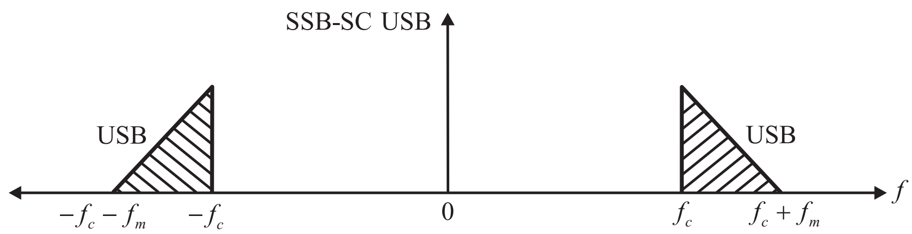
Frequency spectrum of baseband signal *x*(*t*) :



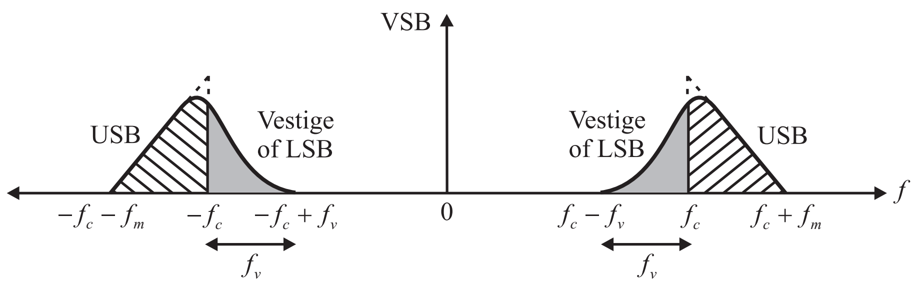
Frequency spectrum of DSB-SC signal :



Frequency spectrum of SSB-SC signal :



Frequency spectrum of VSB signal :



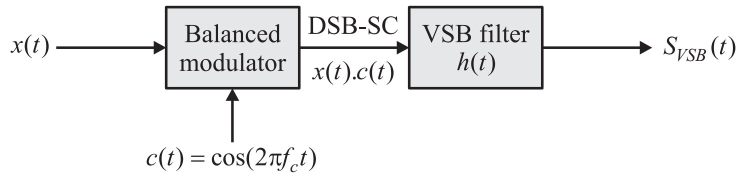
In the frequency spectrum it is assumed that the upper sideband is transmitted as it is and the lower sideband is modified into vestigial sideband.

Transmission bandwidth of VSB is given by, $BW = (f_c + f_m) - (f_c - f_v) = f_m + f_v$

where f_m is the bandwidth of the message signal and f_v is the width of the vestigial sideband.

3.25.2 Modulation of VSB signal

VSB signal can be generated by passing a DSB-SC signal through an appropriate VSB filter having transfer function $H(f)$.



DSB-SC signal $= x(t)c(t) = x(t)\cos(2\pi f_c t)$

VSB-SC signal, $S_{VSB}(t) = x(t)\cos(2\pi f_c t)\otimes h(t)$

The frequency spectrum of VSB signal, $S_{VSB}(f)$ is therefore, given by

$$S_{VSB}(f) = \text{Fourier Transform } S_{VSB}(t)$$

$$x(t)\otimes h(t) \xleftrightarrow{F.T.} X(f)H(f)$$

$$S_{VSB}(f) = FT\left[x(t)\cos(2\pi f_c t)\right]H(f)$$

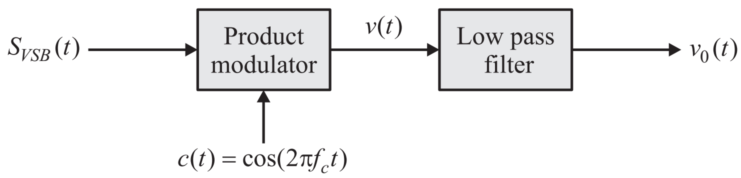
$$x(t)\cos(2\pi f_c t) \xleftrightarrow{F.T.} \frac{1}{2}\left[X(f-f_c)+X(f+f_c)\right]$$

$$S_{VSB}(f) = \frac{1}{2}\left[X(f-f_c)+X(f+f_c)\right]H(f) \quad ...(i)$$

where $X(f)$ is the Fourier transform of modulating signal $x(t)$.

3.25.3 Demodulation of VSB signal

VSB signal can be demodulated by passing a VSB signal through a product modulator along with a carrier signal and then passing through a low pass filter.



The output of the product modulator is given by, $v(t) = S_{VSB}(t)\cos(2\pi f_c t)$

Taking Fourier Transform, $V(f) = \frac{1}{2}\left[S_{VSB}(f-f_c)+S_{VSB}(f+f_c)\right]$ …(ii)

Substituting equation (i) in equation (ii),

$$V(f) = \frac{1}{4}\left[X(f-2f_c)+X(f)\right]H(f-f_c)+\frac{1}{4}\left[X(f)+X(f+2f_c)\right]H(f+f_c)$$

$$V(f) = \frac{1}{4}\left[H(f-f_c)+H(f+f_c)\right]X(f)$$

$$+\frac{1}{4}\left[X(f-2f_c)H(f-f_c)+X(f+2f_c)H(f+f_c)\right]$$

The first term of above equation corresponds to the frequency spectrum of the modulating signal. The second terms of $V(f)$ corresponds to the frequency spectrum of the VSB signal having carrier frequency $2f_c$. The second term can be removed by using low-pass filter (LPF).

The frequency spectrum of the signal $v_0(t)$ at the output of the LPF is given by,

$$V_0(f) = \frac{1}{4}\left[H(f-f_c) + H(f+f_c)\right]X(f)$$

For a distortionless reproduction of the original modulating signal $x(t)$, $V_0(f)$ must be a scaled version of $X(f)$. Hence, for perfect demodulation, the required condition is,

$$H(f-f_c) + H(f+f_c) = \text{constant} \;\; \text{for } |f| \le f_m$$

This is called vestigial symmetry condition.

The response of output low pass equalizer filter is given by,

$$H_0(f) = \frac{1}{H(f-f_c) + H(f+f_c)}$$

3.25.4 Advantages of VSB

1. The main advantage of VSB modulation is the reduction in bandwidth. It is almost as efficient as the SSB.
2. Due to allowance of transmitting a part of lower sideband, the constraint on the filters has been relaxed. So, practically easy to design filters can be used.
3. It possesses good phase characteristics and makes the transmission of low frequency components possible.
4. Practical filters can be used for partial suppression of unwanted sideband.

3.25.5 Disadvantages of VSB

1. During the removal of part of lower sideband, some power is wasted in the filters (VSB filters).
2. Amplitude and phase distortion is introduced at low frequencies.
3. Because of narrow lower sideband used for VSB transmission, critical tuning is required at the receiver.

3.25.6 Applications of VSB

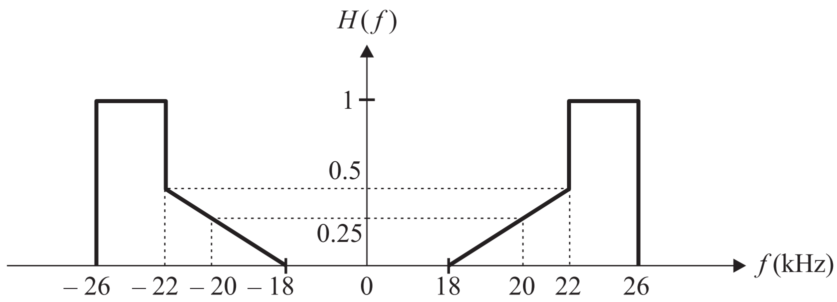
VSB modulation has become standard for the transmission of Television signals, because the video signals need a larger transmission bandwidth if transmitted using DSB-FC or DSB-SC techniques.

Remember

VSB is mainly used as a standard-modulation technique for transmission of video signals in TV signals in commercial television broadcasting because the modulating - video signal has large bandwidth and high-speed data transmission.

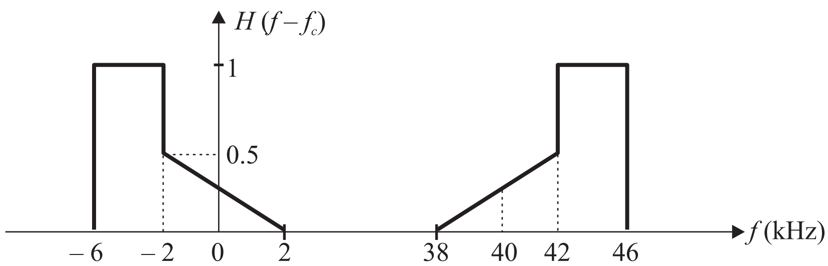
Solved Example 7

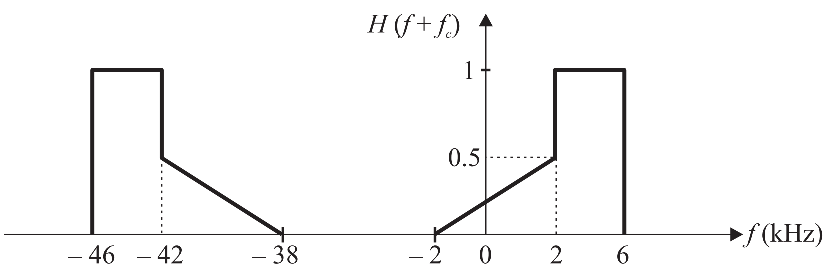
The carrier frequency of a certain VSB is $f_c = 20\,\text{kHz}$ and the baseband signal bandwidth is $6\,\text{kHz}$. The VSB shaping filter of the input which cuts off the LSB gradually over 2 kHz as shown in figure. Draw the response of output filter $H_0(f)$ required for the distortionless reception.

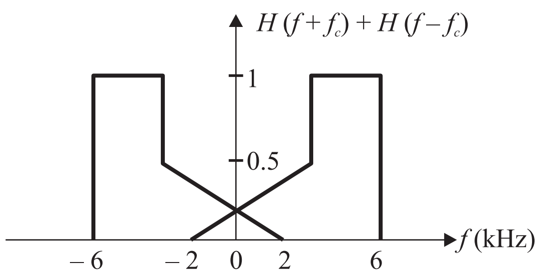


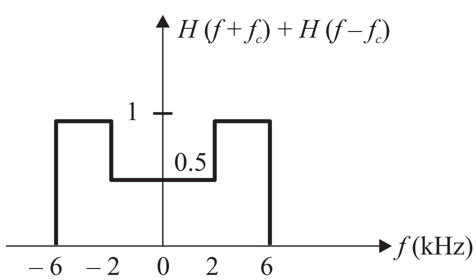
Sol. For distortionless reception, the output filter response is given by,

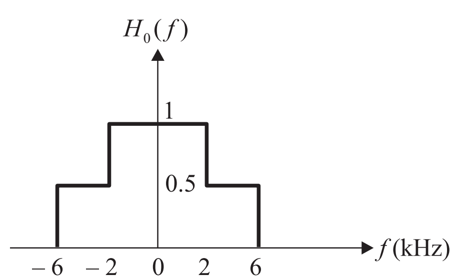
$$H_0(f) = \frac{1}{H(f - f_c) + H(f + f_c)}$$











3.26 Power Saving in AM Systems

Definition : *Power saving is defined as ratio of power saving to the total power.*

$$\text{Power saving} = \frac{\text{Power saved}}{\text{Total power}}$$

For DSB-SC, carrier is suppressed it means we are saving carrier power.

$$\text{Power saving} = \frac{P_c}{P_c\left(1+\frac{m_a^2}{2}\right)} = \frac{2}{2+m_a^2}$$

For SSB-SC, carrier and one of the sideband are suppressed it means we are saving carrier power.

$$\text{Power saving} = \frac{P_c + \frac{P_c m_a^2}{4}}{P_c\left(1+\frac{m_a^2}{2}\right)} = \frac{\left(1+\frac{m_a^2}{4}\right)}{\left(1+\frac{m_a^2}{2}\right)}$$

3.27 Comparison of Various AM Systems

S.	Parameter	DSB-FC Standard AM	DSB-SC	SSB-SC	VSB
1.	Power	High	Medium	Less	Less than DSB-SC but greater than SSB
2.	Bandwidth	$2f_m$	$2f_m$	f_m	$f_m < BW < 2f_m$
3.	Carrier suppression	No	Yes	Yes	No
4.	Sideband suppression	No	No	One sideband suppressed completely	One sideband suppressed partially
5.	Transmission efficiency	Minimum	Moderate	Maximum	Moderate
6.	Receiver complexity	Simple	Complex	Complex	Simple
7.	Modulation type	Non-linear	Linear	Linear	Linear
8.	Applications	Radio communication	Linear Radio communication	Linear point to point mobile communication	Television broadcasting

3.28 Phasor Diagram of AM Signal

For a single tone modulating signal,

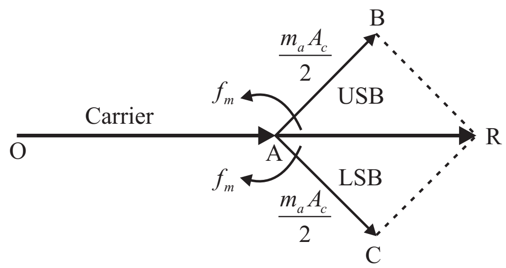
$$s(t) = A_c\left[1+m_a\cos\omega_m t\right]\cos\omega_c t$$

$$s(t) = A_c \cos\omega_c t + \frac{A_c m_a}{2}\cos(\omega_c + \omega_m)t + \frac{A_c m_a}{2}\cos(\omega_c + \omega_m)t$$

$$s(t) = A_c \operatorname{Re}\left[e^{j\omega_c t} + \frac{m_a}{2}e^{j\omega_c t}e^{j\omega_m t} + \frac{m_a}{2}e^{j\omega_c t}e^{-j\omega_m t}\right]$$

Complex envelope is given by,

$$[s(t)]_{CE} = A_c + \frac{A_c m_a}{2}e^{j\omega_m t} + \frac{A_c m_a}{2}e^{-j\omega_m t}$$



The waveform of AM signal can be interpreted as superposition of three waveforms,

(i) Carrier wave $A_c \cos(2\pi f_c t)$ represented by OA.

(ii) Upper sideband $\frac{m_a A_c}{2}\cos 2\pi(f_c + f_m)t$ represented by AB.

(iii)Lower sideband $\frac{m_a A_c}{2}\cos 2\pi(f_c - f_m)t$ represented by AC.

Resultant of sideband is collinear with carrier signal and hence can be detected by envelope detector.

Solved Example 8

Let $A_c = 1,\ m_a = \frac{1}{2}$ and let the upper sideband be attenuated by a factor of 2. Find the expression for the resulting envelope.

Sol. **Given :** $A_c = 1,\ m_a = \frac{1}{2}$

The phase diagram for the case is shown in figure.

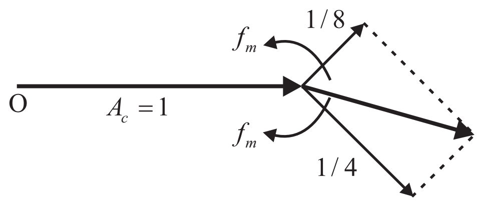


Fig. Phasor diagram for an AM signal with unequal sidebands

As can be seen from the figure, the resultant of the sideband is no longer collinear with the carrier.

$$s(t) = A_c[1 + m_a \cos\omega_m t]\cos\omega_c t$$

$$s(t) = A_c \cos\omega_c t + \frac{A_c m_a}{2}\cos(\omega_c + \omega_m)t + \frac{A_c m_a}{2}\cos(\omega_c + \omega_m)t$$

$$s(t) = A_c \operatorname{Re}\left[e^{j\omega_c t} + \frac{m_a}{2} e^{j\omega_c t} e^{j\omega_m t} + \frac{m_a}{2} e^{j\omega_c t} e^{-j\omega_m t}\right]$$

Complex envelope is given by,

$$[s(t)]_{CE} = A_c + \frac{A_c m_a}{2} e^{j\omega_m t} + \frac{A_c m_a}{2} e^{-j\omega_m t}$$

$$[s(t)]_{CE} = 1 + \frac{1}{8}[\cos(\omega_m t) + j\sin(\omega_m t)] + \frac{1}{4}[\cos(\omega_m t) - j\sin(\omega_m t)]$$

$$[s(t)]_{CE} = 1 + \frac{3}{8}\cos(\omega_m t) - j\frac{1}{8}\sin(\omega_m t)$$

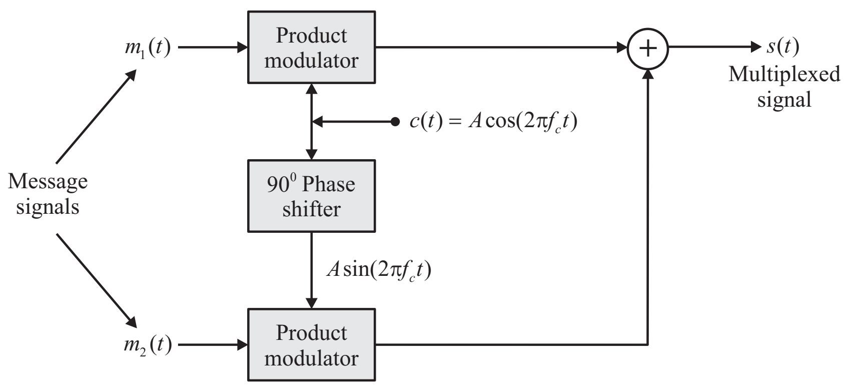
$$A(t) = \left[\left(1 + \frac{3}{8}\cos(\omega_m t)\right)^2 + \left(\frac{1}{8}\sin(\omega_m t)\right)^2\right]^{1/2}$$ **Ans.**

It is not possible for us to recover the message from the above $A(t)$.

3.29 Quadrature Carrier Multiplexing

The transmission bandwidth required by a DSB-SC signal is twice the bandwidth of the message signal. Using Quadrature Carrier Multiplexing or Quadrature Amplitude Modulation (QAM), it is possible to send two DSB-SC signals using carriers of the same frequency but with quadrature phase (i.e. 90^0 out of phase). Both the modulated signals occupy the same frequency band and this, in effect, increases the bandwidth efficiency. In fact, this modulation scheme enables two DSB-SC modulated signals to occupy the same transmission bandwidth and therefore it allows for the separation of the two message signals at the receiver output. It is therefore known as a **bandwidth conservation scheme**.

Block diagram of the QAM transmitter :

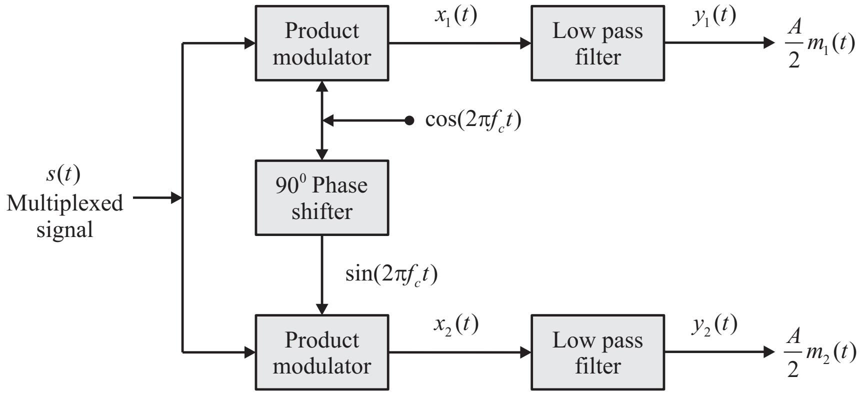


Block diagram of the QAM receiver :

The QAM transmitter consists of two separate product modulators or balanced modulators which are supplied with two carrier waves of the same frequency but differing in phase by 90^0. The output of the two balanced modulators are added in the adder and transmitted. The transmitted signal is thus given by

$$s(t) = m_1(t)A\cos(2\pi f_c t) + m_2(t)A\sin(2\pi f_c t)$$

where, $m_1(t)$ and $m_2(t)$ are the two different message signals applied to the product modulators.



Both $x_1(t)$ and $x_2(t)$ are band-limited in the interval $-f_m \le f \le f_m$, then $s(t)$ will occupy a bandwidth of $2f_m$. This bandwidth $2f_m$ is centred at the carrier frequency f_c, where f_m is the bandwidth of message signal $m_1(t)$ or $m_2(t)$.

Hence, the multiplexed signal consists of the in-phase component $Am_1(t)$ and the quadrature phase component $Am_2(t)$.

The multiplexed signal $s(t)$ from QAM transmitter is applied simultaneously to two separate coherent detectors that are supplied with two local carriers of the same frequency, but differing in phase by 90^0.

Output at receiver before low pass filter,

$$x_1(t) = s(t)\cos(2\pi f_c t) = m_1(t)A\cos^2(2\pi f_c t) + m_2(t)A\cos(2\pi f_c t)\sin(2\pi f_c t)$$

$$x_1(t) = Am_1(t)\left[\frac{1+\cos(4\pi f_c t)}{2}\right] + Am_2(t)\frac{\sin(4\pi f_c t)}{2}$$

$$x_2(t) = s(t).\sin(2\pi f_c t) = m_1(t)A\cos(2\pi f_c t)\sin(2\pi f_c t) + m_2(t)A\sin^2(2\pi f_c t)$$

$$x_2(t) = Am_1(t)\frac{\sin(4\pi f_c t)}{2} + Am_2(t)\left[\frac{1-\cos(4\pi f_c t)}{2}\right]$$

Since, $$\cos^2 A = \frac{1+\cos 2A}{2},\ \sin^2 A = \frac{1-\cos 2A}{2} \text{ and } \sin A\cos A = \frac{\sin 2A}{2}$$

Output at receiver after low pass filter,

$$y_1(t) = \frac{Am_1(t)}{2} \text{ and } y_2(t) = \frac{Am_2(t)}{2}$$

The output of one detector is $(1/2)Am_1(t)$, whereas the output of the second detector is $(1/2)Am_2(t)$. For satisfactory operation of the coherent detector, it is essential to maintain coherent phase and frequency relationship between the oscillators used in the QAM transmitter and receiver parts of the system.

QAM has the problem of co-channel interference i.e. modulated signals having the same carrier frequency interfere with each other. The problem gets worse with frequency error.

QAM finds application in colour television transmission for multiplexing colour information signals.

TEST 5

Q.1 A balanced modulator has a 3 MHz carrier input and a modulating signal input of 1.5 kHz. To pass the lower sideband, a filter must have a center frequency of _______ MHz.

Q.2 In a filter-type SSB generator, a crystal lattice filter is used. The two crystal are 3.0 and 3.0012 MHz. The filter bandwidth is approximately ________ kHz.

3.30 Frequency Division Multiplexing

This technique permits a fixed frequency band to every user in the complete channel bandwidth. Such frequency slot is allotted continuously to that user. As an example consider that the channel bandwidth is 1 MHz. Let there be ten users, each requiring upto 100 kHz bandwidth. Then the complete channel bandwidth of 1 MHz can be divided into ten frequency bands, i.e. each of 100 kHz and every user can be allotted on independent frequency band. This technique is known as **Frequency Division Multiplexing (FDM).**

To transmit a number of several signals over the same channel, the signals must be kept apart so that they do not interfere with each other, and thus they can be separated at the receiving end. This is accomplished by separating the signals either in frequency or in time. The technique of separating the signals in frequency is referred to as ***frequency-division multiplexing***(FDM), whereas the technique of separating the signals in time is called ***time-division multiplexing***(TDM).

Block diagram :

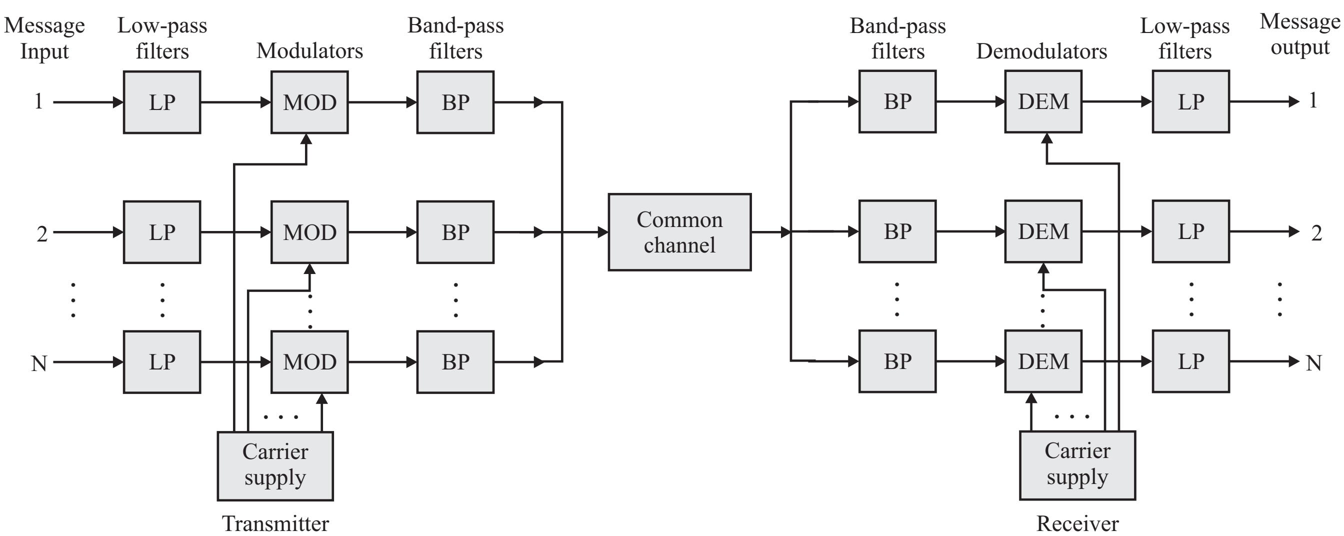


Fig.(a) Block diagram of FDM system

A block diagram of an FDM system is shown in figure (a). The incoming message signals assumed to be the low-pass type are passed through input low-pas filter. This filtering action removes high-frequency components that do not contribute significantly to signal representation but may disturb other message signals that share the common channel. The filtered message signals are then modulated with necessary carrier frequencies with the help of modulators. The most commonly method of modulation in FDM is single modulation, which requires a bandwidth that is approximately equal to that of original message signal. The band pass filters following the modulators are used to restrict the band of each modulated wave to its prescribed range. The outputs of band-pass filters are combined in parallel which form the input to the common channel.

At the receiving end, bandpass filters connected to the common channel separate the message signals on the frequency occupancy basis. Finally, the original message signals are recovered by individual demodulators.

Spectrum of FDM signal :

The figure (b) shows the frequency spectrum of FDM signal. As shown in the figure (b), the adjacent spectrums in the spectrum of an FDM signal do not touch each other. They are separated from each other by guard bands to avoid any interference between them. Wider the guard band, smaller the interference.

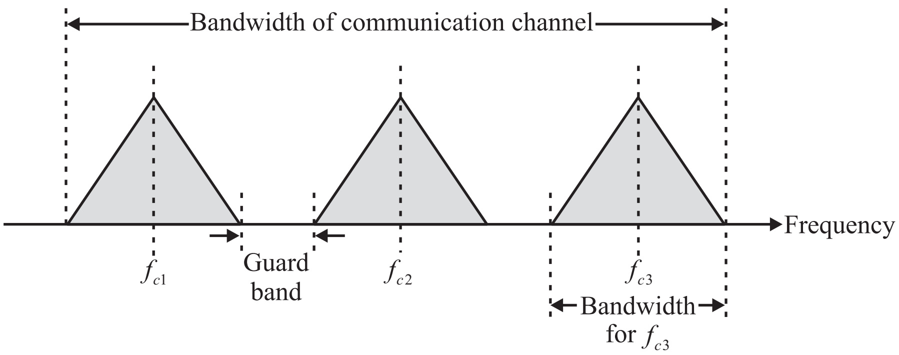


Fig. (b) Spectrum of FDM signal

Advantages of FDM :

1. Number of signals can be transmitted simultaneously.
2. Do not require synchronization between transmitter and receiver.
3. Only a single channel gets affected due to slow narrowband fading.

Disadvantages of FDM :

1. Requires larger bandwidth of communication channel.
2. Suffers from crosstalk problem due to imperfect band-pass filter.
3. Requires complex circuitry at transmitter and receiver.
4. More number of modulators and filters are required.
5. All FDM channels get affected due to wideband fading.

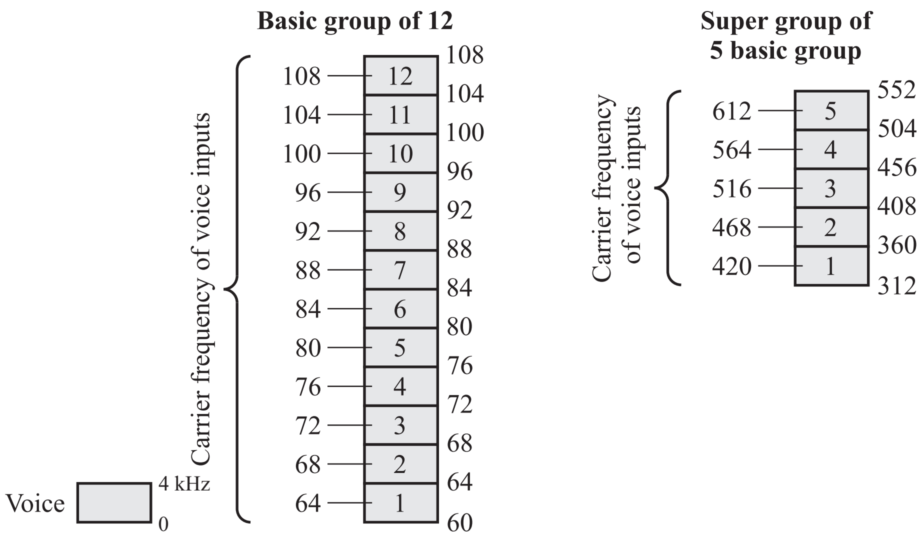
Application of FDM :

1. In radio broadcasting using AM and FM.
2. In TV broadcasting.
3. In telephone systems.
4. In first generation of cellular phones.

The practical implementation of an FDM system usually involves many steps of modulation and demodulation.

1. The first step combines 12 voice inputs into a basic group, which is formed by having the n^{th} input modulate a carrier at frequency

$$f_c = 60 + 4n \text{ kHz} \qquad \text{where } n = 1, 2, \ldots\ldots\ldots\ldots 12$$



2. The lower sidebands are then selected by bandpass filtering and combined to form a group of 12 lower sidebands. Thus, **basic group** occupies the frequency band 60 to 108 kHz.
3. The next step in the FDM hierarchy involves the combination of five basic groups into a **super group**. This accomplished by using the n^{th} group to modulate a frequency

$$f_c = 372 + 48n \text{ kHz} \qquad \text{where } n = 1, 2, 3, 4, 5$$

4. The LSB are selected by filtering and then combined to form a super group occupying the band 312 to 552 kHz. Thus a super group is designed to accommodate 60 independent voice inputs.

3.31 Concept of Pre-envelope and Complex Envelope

Pre-envelope :

The pre-envelope $g_+(t)$ for positive frequencies is defined by,

$$g_+(t) = g(t) + j\,\hat{g}(t)$$

Where, $\hat{g}(t)$ is the Hilbert transform of the signal $g(t)$.

According to this representation, $g(t)$ can be seen as the quadrature function of $g(t)$.

Correspondingly, in the frequency domain we have

$$G_+(f) = \begin{cases} 2G(f), & f > 0 \\ G(0), & f = 0 \\ 0, & f < 0 \end{cases}$$

Complex envelope :

The complex envelope $\tilde{g}(t)$ equals a frequency-shifted version of the pre-envelope $g_+(t)$, as given by,

$$\tilde{g}(t) = g_+(t)e^{-j2\pi f_c t}$$

where, f_c is the carrier frequency of the band-pass signal $g(t)$.

Note : The envelope equals the magnitude of the complex envelope $\tilde{g}(t)$ and also that of the pre-envelope as shown by,

$$a(t) = |\tilde{g}(t)| = |g_+(t)|$$

For a band-pass signal $g(t)$, the pre-envelope $g_+(t)$ is a complex band-pass signal whose value depends on the carrier frequency f_c. On the other hand, the envelope $a(t)$ is always a real low-pass signal and the complex envelope $\tilde{g}(t)$ is a complex low-pass signal.

Important Formulas

1. Modulation index :

$$m_a = \frac{A_m}{A_c}$$

$$m_a = \max |k_a m(t)|$$

$$m_a = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}}$$

2. Bandwidth : The bandwidth of the amplitude-modulated signal is equal to the difference between the maximum upper-sideband frequency and the minimum lower-sideband frequency.

$$BW = f_{USB} - f_{LSB} = 2f_m$$

3. Power :

$$P_c = \frac{A_c^2}{2R}, \quad P_{USB} = \frac{P_c m_a^2}{4}, \quad P_{LSB} = \frac{P_c m_a^2}{4}$$

$$P_T = P_c\left[1 + \frac{m_a^2}{2}\right]$$

4. Transmission efficiency : It is defined as the ratio of the power carried by the sidebands to the total transmitted power.

Transmission efficiency, $\eta = \frac{P_{USB} + P_{LSB}}{P_T} = \frac{P_{SB}}{P_T} == \frac{m_a^2}{m_a^2 + 2}$

5. For multitone net modulation index is given by,

$$m_{aT} = \sqrt{m_{a1}^2 + m_{a2}^2 + m_{a3}^2 + ... + m_{an}^2}$$

Transmission efficiency, $\eta = \frac{\text{Power in sideband}}{\text{Total Power}} = \frac{P_c m_{aT}^2 / 2}{P_c(1 + m_{aT}^2 / 2)} = \frac{m_{aT}^2}{2 + m_{aT}^2}$

6. Peak envelope power is given by,

$$PEP = \frac{A_c^2 [1 + m_a]^2}{2}$$

7. Square law modulator : The main condition for generation of DSB-FC or AM wave is that the carrier frequency should be atleast three times the maximum frequency of the modulating signal.

$$f_c \geq 3f_m$$

8. Switching modulator : The main condition for generation of DSB-FC or AM wave is that the carrier frequency should be atleast two times the maximum frequency of the modulating signal.

$$f_c \geq 2f_m$$

9. Envelope detector :

Range of time constant,

$$\frac{1}{f_c} << RC << \frac{1}{f_m}$$

$$T_c << RC << T_m$$

$$RC \le \left(\frac{\sqrt{1-m_a^2}}{m_a}\right)\frac{1}{\omega_m}$$

10. Hilbert transform :

The Hilbert transform of a function $f(t)$ is defined by,

$$H(t) = f(t) \otimes \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(\tau)}{t-\tau}d\tau$$

Frequency response of Hilbert transform is given by, $H(f) = -j\,\text{sgn}(f)$

11. Power saving :

$$\text{Power saving} = \frac{\text{Power saved}}{\text{Total power}}$$

For DSB-SC, $\text{Power saving} = \dfrac{P_c}{P_c\left(1+\dfrac{m_a^2}{2}\right)} = \dfrac{2}{2+m_a^2}$

For SSB-SC, $\text{Power saving} = \dfrac{P_c + \dfrac{P_c m_a^2}{4}}{P_c\left(1+\dfrac{m_a^2}{2}\right)} = \dfrac{\left(1+\dfrac{m_a^2}{4}\right)}{\left(1+\dfrac{m_a^2}{2}\right)}$

12. Bandwidth of FDM :

$$B_{\text{FDM}} = Nf_m + (N-1)\,\text{Guard band}$$

where, N = Number of channels/users

f_m = Bandwidth of each channel

Numerical Answer Type Questions

Q.1 An AM signal is modulated by a sinusoidal signal so that the modulation index is 0.25. If the carrier is suppressed, the percentage power saving is ______.

Q.2 Consider the message

$$m(t) = 2\cos 2\pi ft + \sin 2\pi ft$$

used to modulate the carrier $\cos 2\pi f_c t$ generate the AM signal

$$s(t) = [A_c + m(t)]\cos 2\pi f_c t.$$

The value of A_c which ensures a modulation index $m_a = 0.5$ is_______.

Q.3 In the trapezoidal pattern displaying modulation, the length of the long vertical side is 5 cm and of the short vertical side 2 cm. The modulation depth is _______.

Q.4 Twelve signals each band-limited to 5 kHz are to be transmitted over a single channel by frequency division multiplexing. If SSB modulation with guard band of 1 kHz is used, then the bandwidth of the multiplexed signal will be ________.

Q.5 In a DSB signal $c(t) = A\cos 2\pi f_c t$ and the message signal is $m(t) = \text{sinc}(t) + \text{sinc}^2(t)$. The bandwidth of the DSB signal is _______.

Q.6 For an AM signal, the bandwidth is 10 kHz and the highest frequency component present is 705 kHz. The carrier frequency used for this AM signal is _______.

Q.7 A given AM broadcast station transmits an average carrier power output of 40 kW and uses a modulation index of 0.707 for sine wave modulation. The maximum (peak) amplitude of the output if the antenna is represented by a $50\,\Omega$ resistive load is _______.

Q.8 A carrier wave of frequency 2.5 GHz amplitude is modulated with two modulating frequencies equal to 1 kHz and 2 kHz. The modulated wave will have the total bandwidth ________.

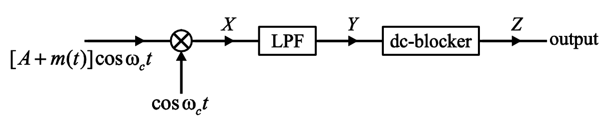
Multiple Choice Questions

Q.1 An AM modulator has output $s(t) = M\cos(2\pi\times 2000t) + N\cos(2\pi\times 1800t) + N\cos(2\pi\times 2200t)$. The carrier power is 50 W and the total sideband power is 3% of the total power. The values of M and N are respectively

(A) 10, 2.4 (B) 10, 1.24

(C) 20, 7 (D) 20, 6

Q.2 Given below figure shown a scheme for coherent demodulation.

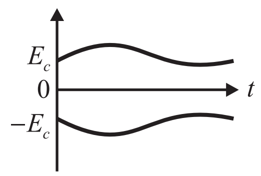


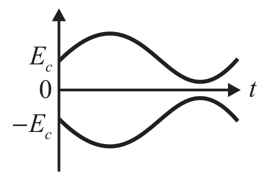
The output is

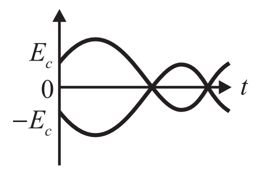
(A) $m(t)$ (B) $m(t)/2$

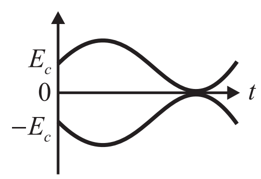
(C) $A + \dfrac{m(t)}{2}$ (D) $A + m(t)$

Q.3 Which envelope represents modulation index is 0.3?

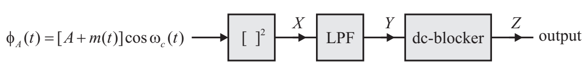
(A)


(B)


(C)


(D)


Q.4 In the early days of radio, AM signals were demodulated by a crystal detector followed by a low-pass filter and a dc blocker as shown in below figure. Assume a crystal detector to be basically a squaring device.



The distortion term in the output $z(t)$ is

(A) $\dfrac{A^2}{2} + Am(t)$ (B) $\dfrac{m^2(t)}{2}$

(C) $Am(t)$ (D) $\dfrac{A^2}{2}$

Q.5 A broadcast AM radio transmitter radiates 125 kW when the modulation percentage is 70. How much is carrier power?

(A) $\approx 25\,\text{kW}$ (B) $\approx 50\,\text{kW}$

(C) $\approx 75\,\text{kW}$ (D) $\approx 100\,\text{kW}$

Q.6 If two signals modulate the same carrier with different modulation depths of 0.3 and 0.9, the resulting modulation signal will

(A) be over-modulated.

(B) have the resultant modulation limited to 1.0.

(C) have the resultant modulation index around 0.82.

(D) have the resultant modulation index around 0.95.

Q.7 In an amplitude modulated (AM) wave with 100% modulation (m_a) the carrier is suppressed. The percentage of power saving will be

(A) 100% (B) 50%

(C) 25% (D) 66.7%

Statement for Linked Answer Questions 8 & 9

An AM signal

$$10(1 + 0.8\cos 2\pi \times 5000t)\cos 2\pi \times 10^6 t$$

is passed through a tuned circuit. The resultant signal is transmitted through a channel. The gain of the tuned circuit at 1 MHz is 0.8 and at 1 MHz + 5 kHz is 0.5.

Q.8 The modulation index of the AM signal at the output of the tuned circuit is

(A) 0.4 (B) 0.5

(C) 0.8 (D) 1.0

Q.9 The transmitted power is

(A) 18 W (B) 9 W

(C) 36 W (D) 66 W

Q.10 The signal $m(t)$ in the DSB-SC signal $v(t) = m(t)\cos\omega_c t$ is to be reconstructed by multiplying $v(t)$ by a signal derived from $v^2(t)$. If $m(t)$ is bandlimited to B

Hz and has a probability density UDF $[-3, 3]$. The expected value of the amplitude at the component of $v^2(t)$ at $2f_c$ is

(A) 1.5 (B) 3.0

(C) 4.5 (D) 6.0

Q.11 In an AM wave represented by $v(t) = 5[1+0.2x(t)]\cos\omega_0 t$, where $x(t)$ is represented by density function UDF $[-2, 2]$. Transmission efficiency of above AM wave is

(A) 3.8 % (B) 5 %

(C) 7.6 % (D) 1.9 %

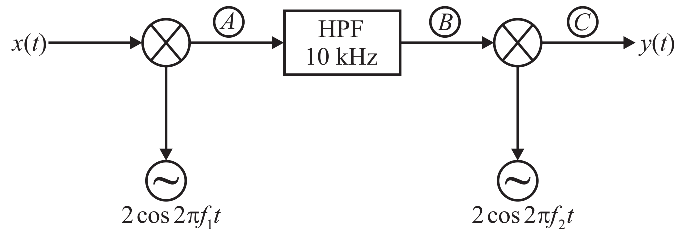
Q.12 A received SSB signal in which the modulation is a signal spectral component has a normalized power of $0.5\,\text{volt}^2$. A carrier is added to the signal and the carrier plus signal are applied to a diode demodulator. The carrier amplitude is to be adjusted so that at the demodulator output 90 percent of the normalized power is in the recovered modulating waveform. Neglect dc components. The carrier amplitude required is

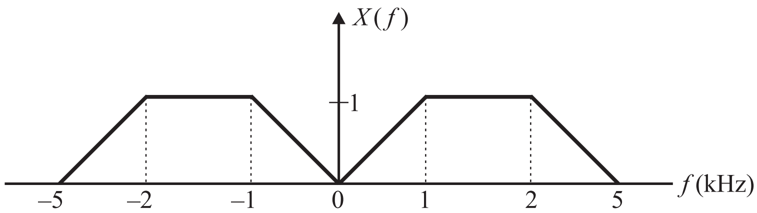
(A) 1 volt (B) 2 volt

(C) 3 volt (D) 4 volt

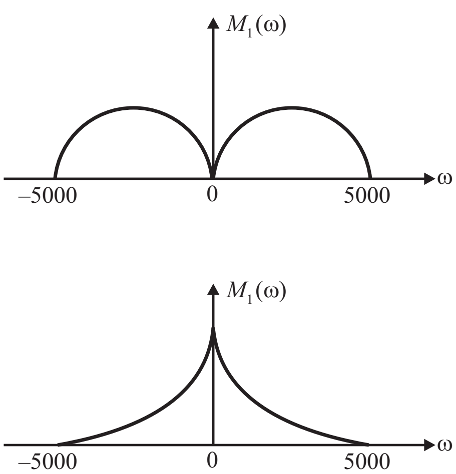
TryYourself

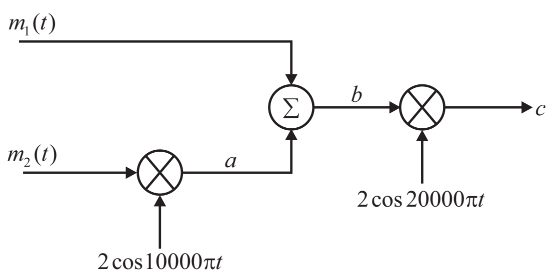
Q.1 Consider the scheme shown in figure. the ideal HPF has the cut off frequency at 10 kHz. Given that $f_1 = 10\,\text{kHz}$ and $f_2 = 15\,\text{kHz}$. Sketch spectrum at point A, B and C.





Q.2 Two signals $m_1(t)$ and $m_2(t)$ both bandlimited to 5000 rad/s, are to be transmitted simultaneously over a channel by the multiplexing scheme shown in figure. The signal at point b is multiplexed signal which now modulates a carries of frequency 20,000 rad/s. The modulated signal at point c is transmitted over a channel.

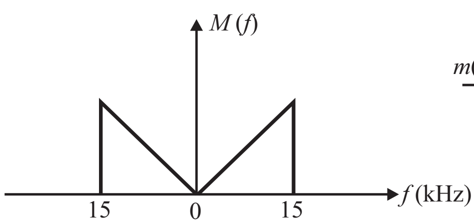


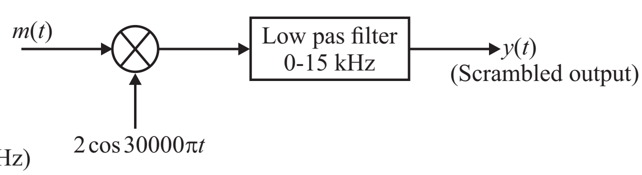


(a) Sketch signal spectra at points a, b and c.

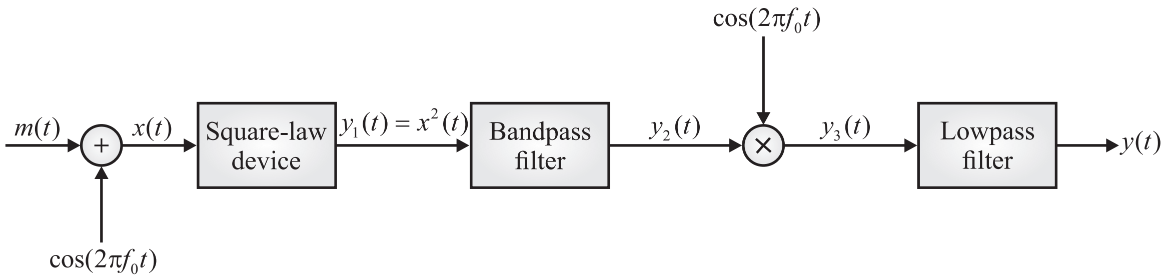
(b) What must be the bandwidth of the channel?

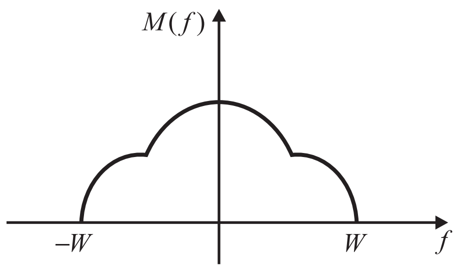
Q.3 System shown in figure is used for scrambling audio signal. The output $y(t)$ is scrambled version of the input $x(t)$. Find the spectrum of scrambled signal $y(t)$ and its bandwidth.





Q.4 The message signal $m(t)$ whose spectrum is shown in the figure is passed through the system shown in same figure.

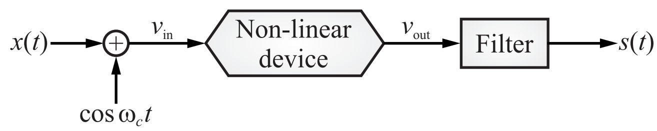




The bandpass filter has a bandwidth of $2W$ centered at f_0 and the lowpass filter has a bandwidth of W. Plot the spectra of the signal $x(t), y_1(t), y_2(t), y_3(t)$ and $y_4(t)$. What are the bandwidths of these signals?

Q.5 The signal $x(t) = 4\sin\left(\dfrac{\pi \times 10^3}{2}t\right)$ is transmitted by DSB what range of carrier frequencies can be used?

Q.6 The signal $x(t) = \dfrac{1}{2}\cos 2\pi 70t + \dfrac{1}{3}\cos 2\pi 120t$ is input to the square-law modulator system with carrier frequency of 10 kHz.



Assume that $v_{out} = a_1 v_{in} + a_2 v_{in}^2$.

(a) Give the center frequency and bandwidth of the filter such that this system will produce a standard AM signal.

(b) Determine values of a_1 and a_2 such that $A_c = 10$ and $k_a = \dfrac{1}{2}$.

Q.7 A modulation system with nonlinear elements produces the signal

$$x_c(t) = aK^2\left[v(t) + A\cos\omega_c t\right]^2 - b\left[v(t) - A\cos\omega_c t\right]^2$$

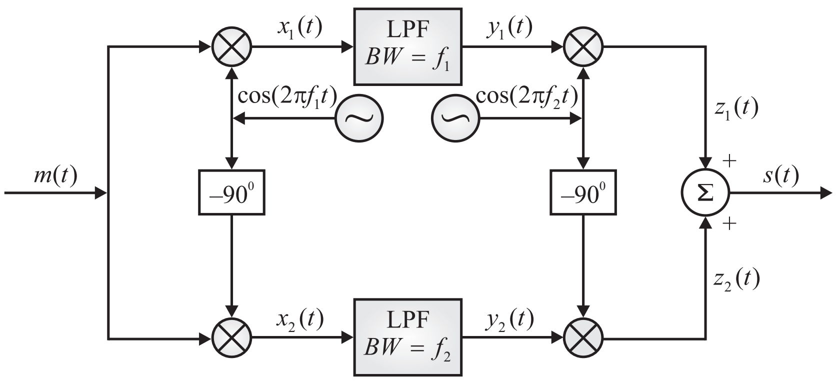
If the carrier has frequency f_c and $v(t) = x(t)$, show that an appropriate choice of K produces DSB modulation without filtering.

Q.8 A DSB-SC signal is given by $m(t)\cos(2\pi 10^6 t)$. The carrier frequency of this signal 1 MHz is to be changed to 400 kHz. The only equipment available is one ring modulator, a BPF centered at the frequency of 400 kHz and one sine wave generator whose frequency can be varied from 150 to 120 kHz. Show how you can obtain the desired signal $c\,m(t)\cos(2\pi\times 400\times 10^3)t$ from $m(t)\cos 2\pi 10^6 t$. Determine the value of c.

Q.9 Find the Hilbert transform of following signals.

(i) $\text{sinc}(t)$ (ii) $\dfrac{2}{1+t^2}$

Q.10 The message signal to the Weaver's SSB modulator shown below is $m(t)=\cos(2\pi f_m t)$.



If $f_1 = 2f_m$, $f_m = 5\,\text{kHz}$ and $s(t)$ is an USSB signal with a carrier frequency of 1 MHz then find the value of the frequency, f_2.

Answers to Test 1

1.	C	2.	A	3.	D	4.	43.4	5.	34

Answers to Test 2

1.	B	2.	63.25	3.	236	4.	400, 89.44	5.	1.8

Answers to Test 3

1.	B	2.	A	3.	345	4.	3 and 4		

Answers to Test 4

1.	C	2.	1897.4, 1902.6	3.	B	4.	B	5.	B

Answers to Test 5

1.	2.9985	2.	1.8						

Answers to NAT Questions

1.	96.96	2.	4.47	3.	0.428	4.	71	5.	2
6.	700	7.	3.414	8.	4				

Answers to Multiple Choice Questions

1.	B	2.	B	3.	A	4.	B	5.	D
6.	D	7.	D	8.	B	9.	C	10.	A
11.	B	12.	C						

Scan for Detailed Solution of Try Yourself

Explanation of NAT Questions

Sol. 1

Given : $m_a = 0.25$

Percentage power saving in DSB-SC is given by,

$$\% P = \frac{\text{Suppressed power}}{\text{Total power}} = \frac{P_c}{P_T} \times 100$$

$$\% P = \frac{P_c}{P_c\left(1+\frac{m_a^2}{2}\right)} = \frac{2}{2+m_a^2} \times 100$$

$$\% P = \frac{2}{2+0.25^2} \times 100 = 96.96\%$$

Hence, the correct option is (D).

Sol. 2

Given : $m(t) = 2\cos 2\pi f t + \sin 2\pi f$

and $m_a = 0.5t$

$$m(t)_{\max} = \sqrt{2^2+1^2} = \sqrt{5}$$

$$\left[\because A\cos\theta + B\sin\theta = \sqrt{A^2+B^2}\cos\left(\theta - \tan^{-1}\frac{B}{A}\right)\right]$$

AM signal, $s(t) = \left[A_c + m(t)\right]\cos 2\pi f_c t$

$$s(t) = A_c\left[1+\frac{m(t)}{A_c}\right]\cos 2\pi f_c t \qquad \ldots\text{(i)}$$

Standard expression of AM signal is given by,

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos 2\pi f_c t \qquad \ldots\text{(ii)}$$

On comparing equation (i) and (ii),

$$k_a = \frac{1}{A_c}$$

Modulation index is given by,

$$m_a = k_a \max[m(t)]$$

$$0.5 = \frac{\sqrt{5}}{A_c}$$

$$A_c = \sqrt{20}$$

Sol. 3

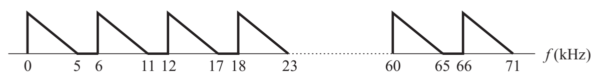
Given : Long vertical side, $L_1 = 5\,\text{cm}$

Short vertical side, $L_2 = 2\,\text{cm}$

Modulation index is given by,

$$m_a = \frac{L_1 - L_2}{L_1 + L_2} = \frac{5-2}{5+2} = 0.428$$

Sol. 4



Bandwidth is given by,

$$BW = f_H - f_L = 71 - 0 = 71\,\text{kHz}$$

Sol. 5

Given : $m(t) = \sin c(t) + \sin c^2(t)$

$f_1 = 0.5\,\text{Hz}$ and $f_2 = 1\,\text{Hz}$

Bandwidth of DSB-SC is given by,

$BW = 2 \times$ Maximum frequency component present in message signal.

$$BW = 2\times 1 = 2\,\text{Hz}$$

Sol.6

Given : Bandwidth of AM,

$2f_m = 10\,\text{kHz} \Rightarrow f_m = 5\,\text{kHz}$

Highest frequency component,

$$f_c + f_m = 705\,\text{kHz}$$

$$f_c = (705-5)10^3 = 700\,\text{kHz}$$

Sol.7

Given : $P_c = 40\,\text{kW}$, $m_a = 0.707$

Carrier power is given by, $P_c = \frac{A_c^2}{2R} = 40\text{ kW}$

$$A_c = 2\,\text{kV}$$

Peak amplitude of AM is given by,

$$E_{\max} = A_c(1+m_a)$$

$$E_{\max} = 2(1+0.707) = 3.414\,\text{kV}$$

Sol.8

Given : $f_c = 2.5\,\text{GHz}$, $f_1 = 10\,\text{kHz}$, $f_2 = 2\,\text{kHz}$

Bandwidth is given by,

$BW = 2\times$ Maximum frequency component present in message signal

$$BW = 2\times 2\times 10^3 = 4\,\text{kHz}$$

Explanation of Multiple Choice Questions

Sol. 1

Given :

$$s(t) = M\cos(2\pi \times 2000t) + N\cos(2\pi \times 1800t) + N\cos(2\pi \times 2200t)$$

$$P_c = 50\,\text{W}, P_{SB} = 0.03\,P_T, P_c = 0.97\,P_T$$

Carrier power is given by,

$$\frac{M^2}{2} = 50$$

$$M = 10$$

$$P_T = \frac{P_c}{0.97} = 51.546\,\text{W}$$

$$P_{SB} = 0.03P_T = 0.03 \times 51.546 = 1.546\,\text{W}$$

Total sideband power can be written as,

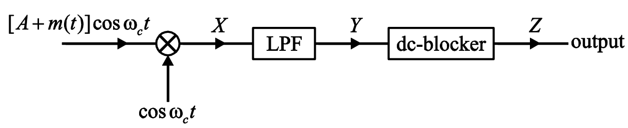
$$\frac{N^2}{2} + \frac{N^2}{2} = 1.546$$

$$N^2 = 1.546 \quad \Rightarrow \quad N = 1.24$$

Hence, the correct option is (B).

Sol. 2

The given figure is shown below.



$$X = [A + m(t)]\cos^2 \omega_c t$$

$$X = \frac{[A + m(t)][1 + \cos 2\omega_c t]}{2}$$

Output of low pass filter will be

$$Y = \frac{A}{2} + \frac{m(t)}{2}$$

Output of the dc blocker will be

$$Z = \frac{m(t)}{2}$$

Hence, the correct option is (B).

Sol. 3

(i) Option (C) represents $m_a > 1$.

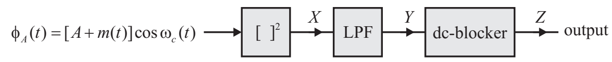
(ii) Option (D) represents $m_a = 1$.

(iii) Option (A) and (B) represent $m_a < 1$ but figure (B) consists higher modulation index.

Hence, the correct option is (A).

Sol.4

The given figure is shown below.



Output of squaring circuit is

$$X = [A + m(t)]^2 \cos^2 \omega_c t$$

$$X = [A^2 + m^2(t) + 2Am(t)]\left[\frac{1 + \cos 2\omega_c t}{2}\right]$$

Output of low pass filter will be

$$Y = \frac{A^2}{2} + Am(t) + \frac{m^2(t)}{2}$$

Output of dc blocker will be,

$$Z = Am(t) + \frac{m^2(t)}{2}$$

$$Am(t) = \text{Desired signal}$$

The distortion term in $z(t)$ will be $\frac{m^2(t)}{2}$.

Hence, the correct option is (B).

Sol.5

Given : $P_T = 125\,\text{kW}$, $m_a = 0.7$

The total power is given by,

$$P_T = P_c\left(1 + \frac{m_a^2}{2}\right)$$

$$125 \times 10^3 = P_c\left[1 + \frac{(0.7)^2}{2}\right]$$

$$P_c = 100.4\,\text{kW}$$

Hence, the correct option is (D).

Sol.6

Given : $m_{a_1} = 0.3$ and $m_{a_2} = 0.9$

Total modulation index is given by,

$$m_{a_T} = \sqrt{m_{a_1}^2 + m_{a_2}^2} = \sqrt{(0.3)^2 + (0.9)^2}$$

$$m_{a_T} = 0.95$$

Hence, the correct option is (D).

Sol.7

Given : $m_a = 1$

With carrier suppressed we get double sideband suppress carrier signal.

In DSB-SC, percentage power saving is given by,

$$\text{Power saving} = \frac{P_c}{P_c\left(1+\frac{m_a^2}{2}\right)} = \frac{2}{2+m_a^2}$$

$$\text{Power saving} = \frac{2}{3}$$

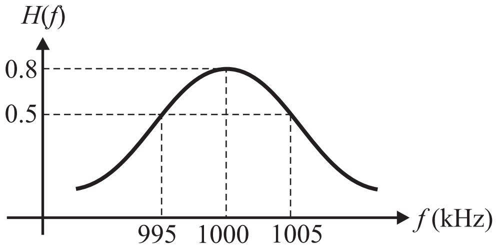
$$\%\text{ power saving} = 66.67\%$$

Hence, the correct option is (D).

Sol.8

Given :

$$s(t) = 10[1+0.8\cos 2\pi\times 5000t]\cos 2\pi\times 10^6 t$$



$$V_c = 10 \text{ at } f = f_c$$

$$V_c m_a = 8 \text{ at } f = f_c \pm f_m$$

At the output of tuned circuit

$$V_c' = 10\times 0.8 = 8 \text{ at } f = f_c$$

$$V_c' m_a' = 8\times 0.5 = 4 \text{ at } f = f_c \pm f_m$$

$$V_c' m_a' = 4$$

$$8m_a' = 4$$

$$m_a' = 0.5$$

Hence, the correct option is (B).

Sol.9

Transmitted power is given by,

$$P_T = P_c\left[1+\frac{m_a^2}{2}\right]$$

$$P_T = \frac{V_c'^2}{2}\left[1+\frac{m_a^2}{2}\right] = \frac{V_c'^2}{2}\left[1+\frac{m'^2_a}{2}\right]$$

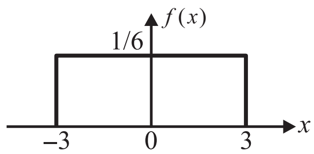
$$P_T = \frac{8^2}{2}\left[1+\frac{0.5^2}{2}\right] = 36\text{ W}$$

Hence, the correct option is (C).

Sol.10

Given :

$m(t)$ is uniformly distributed between (–3, 3).



The area under PDF is always equal to unity.

$$v(t) = m(t)\cos\omega_c t$$

$$v^2(t) = m^2(t)\cos^2\omega_c t$$

$$v^2(t) = \frac{m^2(t)}{2}[1+\cos 2\omega_c t]$$

Amplitude of $v^2(t)$ at $2f_c$ is $\frac{m^2(t)}{2}$.

$$E\left[v^2(t)\right] = \frac{E\left[m^2(t)\right]}{2}$$

$$E\left[v^2(t)\right] = \frac{\text{Mean square value of } m(t)}{2}$$

$$\text{MSV} = \int_{-3}^{3} x^2 f(x)\,dx = \frac{1}{6}\left[\frac{x^3}{3}\right]_{-3}^{3}$$

$$\text{MSV} = \frac{1}{6}\times 18 = 3$$

$$E\left[v^2(t)\right] = \frac{3}{2} = 1.5$$

Hence, the correct option is (A).

Sol.11

Given : $v(t) = 5[1+x(t)]\cos\omega_0 t$

$$v(t) = 5\cos\omega_0 t + x(t)\cos\omega_0 t$$

Total power is given by,

$$P_T = \frac{5^2}{2} + \frac{E\left[x^2(t)\right]}{2}$$

$$\text{Sideband power, } P_{SB} = \frac{E\left[x^2(t)\right]}{2}$$

$E\left[x^2(t)\right]$ represents mean square value.

$$E\left[x^2(t)\right]=\int_{-2}^{2}\frac{1}{4}x^2dx=\frac{1}{4}\left(\frac{x^3}{3}\right)_{-2}^{2}$$

$$E\left[x^2(t)\right]=\frac{1}{4}\times\frac{10}{3}=\frac{4}{3}$$

Transmission efficiency at AM system is given by,

$$\eta\%=\frac{P_{SB}}{P_T}\times 100$$

$$\eta\%=\frac{\frac{E\left[x^2(t)\right]}{2}}{\frac{25}{2}+\frac{E\left[x^2(t)\right]}{2}}\times 100$$

$$\eta\%=\frac{E\left[x^2(t)\right]}{E\left[x^2(t)\right]+25}\times 100$$

$$\eta\%=\frac{4/3}{4/3+25}\times 100=5\%$$

Hence, the correct option is (B).

Sol.12

For a single-tone SSB-SC signal, the waveform after carrier insertion becomes,

$$s'(t)=s(t)+c(t)$$

$$s'(t)=\cos\{(\omega_c+\omega_m)t\}+A\cos(\omega_c t)$$

Using trigonometric relation,

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

$$s'(t)=\cos(\omega_c t)\cos(\omega_m t)-\sin(\omega_c t)\sin(\omega_m t)+A\cos(\omega_c t)$$

$$s'(t)=\left[A+\cos(\omega_m t)\right]\cos(\omega_c t)-\sin(\omega_c t)\sin(\omega_m t)$$

This signal is applied to a diode demodulator. The output of the demodulator is given by,

$$v(t)=\sqrt{(\text{Inphase component})^2+(\text{Quadrature component})^2}$$

$$v(t)=\sqrt{[A+\cos(\omega_m t)]^2+[\sin(\omega_m t)]^2}$$

$$v(t)=\sqrt{A^2+\cos^2(\omega_m t)+2A\cos(\omega_m t)+\sin^2(\omega_m t)}$$

$$v(t)=\sqrt{A^2+1+2A\cos(\omega_m t)}$$

$$v(t)=\left[A^2+1+2A\cos(\omega_m t)\right]^{1/2}$$

$$v(t)=\sqrt{A^2+1}\left[1+\frac{2A}{A^2+1}\cos(\omega_m t)\right]^{1/2}$$

Using binomial expansion,

$$v(t)\cong\sqrt{A^2+1}\left[1+\frac{A}{A^2+1}\cos(\omega_m t)\right]$$

$$v(t)=\sqrt{A^2+1}+\frac{A}{\sqrt{A^2+1}}\cos(\omega_m t)$$

Neglecting the dc component, the normalized power of the detected signal will be

$$P_d=\frac{1}{2}\left(\frac{A}{\sqrt{A^2+1}}\right)^2=\frac{1}{2}\left(\frac{A^2}{A^2+1}\right)$$

For the recovered signal at the demodulator output to be 90% of this normalized power, detector output = 90% of normalized power of SSB modulated waveform

$$P_d=90\%\times P_n$$

Normalized power, $P_n=0.5\ \text{volt}^2$

$$\frac{1}{2}\left(\frac{A^2}{A^2+1}\right)=0.90\times 0.5$$

$$\Rightarrow\quad A^2=0.9(A^2+1)$$

$$A^2-0.9A^2=0.9$$

$$\Rightarrow\quad 0.1A^2=0.9$$

$$\therefore\quad A=3$$

Thus, amplitude of the reinserted carrier must be almost 3 Volt.

Hence, the correct option is (C).

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