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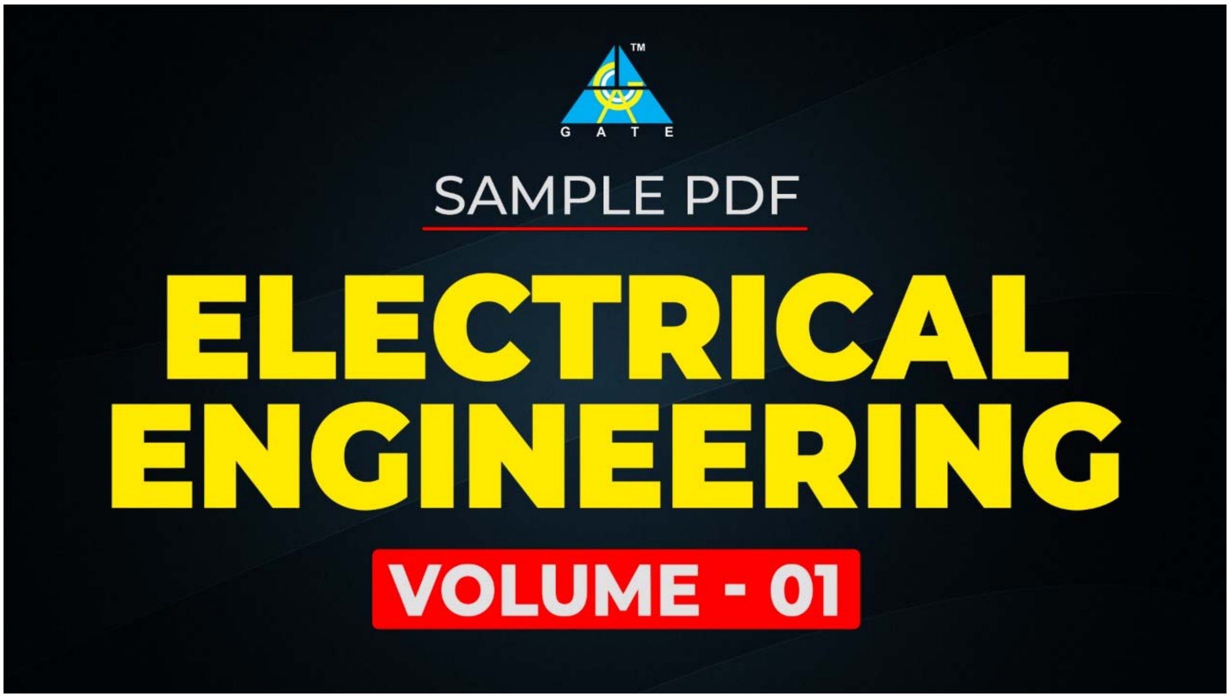

This sample PDF of **GATE Previous Years Solution Book** contains randomly selected questions with solutions from some of the chapters of every subject along with part of concept refresher **Synopsis** of those chapters to let the aspirants have an idea about the content, style and appearance of the book.

Previous Years marks distribution analysis is also given in tabular form with index page of every subject, which contains analysis of GATE papers from 2003 onwards as GATE pattern has turned objective since 2003.

Volume 1 of Electrical Engineering GATE Previous Years Solution Book contains the common subjects of EC, EE and IN and hence it is equally advantageous for GATE aspirants of all these three branches.

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GATE 2022
Electrical Engineering
(Volume - I)

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1987 - 2021

Sakshi Dhande
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IMPORTANCE of GATE

GATE examination has been emerging as one of the most prestigious competitive exam for engineers. Earlier it was considered to be an exam just for eligibility for pursuing PG courses, but now GATE exam has gained a lot of attention of students as this exam open an ocean of possibilities like :

1. **Admission into IISc, IITs, IIITs, NITs**

   A good GATE score is helpful for getting admission into IISc, IITs, IIITs, NITs and many other renowned institutions for M.Tech./M.E./M.S. An M.Tech graduate has a number of career opportunities in research fields and education industries. Students get ₹ 12,400 per month as stipend during their course.

2. **Selection in various Public Sector Undertakings** (**PSUs**)

   A good GATE score is helpful for getting job in government-owned corporations termed as **Public Sector Undertakings** (**PSUs**) in India like IOCL, BHEL, NTPC, BARC, ONGC, PGCIL, DVC, HPCL, GAIL, SAIL & many more.

3. Direct recruitment to Group A level posts in Central government, i.e., Senior Field Officer (Tele), Senior Research Officer (Crypto) and Senior Research Officer (S&T) in Cabinet Secretariat, Government of India, is now being carried out on the basis of GATE score.

4. **Foreign universities through GATE**

   GATE has crossed the boundaries to become an international level test for entry into postgraduate engineering programmes in abroad. Some institutes in two countries **Singapore** and **Germany** are known to accept GATE score for admission to their PG engineering programmes.

5. **National Institute of Industrial Engg. (NITIE)**
   - NITIE offers **PGDIE / PGDMM / PGDPM** on the basis of GATE scores. The shortlisted candidates are then called for group Discussion and Personal Interview rounds.
   - NITIE offers a Doctoral Level Fellowship Programme recognized by Ministry of HRD (MHRD) as equivalent to PhD of any Indian University.
   - Regular full time candidates those who will qualify for the financial assistance will receive ₹ 25,000 during 1st and 2nd year of the Fellowship programme and ₹ 28,000 during 3rd, 4th and 5th year of the Fellowship programme as per MHRD guidelines.

6. **Ph.D. in IISc/ IITs**
   - IISc and IITs take admissions for Ph.D. on the basis of GATE score.
   - Earn a Ph.D. degree directly after Bachelor's degree through integrated programme.
   - A fulltime residential researcher (RR) programme.

7. **Fellowship Program in management (FPM)**
   - Enrolment through GATE score card
   - Stipend of ₹ 22,000 – 30,000 per month + HRA
   - It is a fellowship program
   - Application form is generally available in month of sept. and oct.

Note : In near future, hopefully GATE exam will become a mandatory exit test for all engineering students, so take this exam seriously. Best of LUCK !

GATE Exam Pattern

Section	Question No.	No. of Questions	Marks Per Question	Total Marks
General Aptitude	1 to 5	5	1	5
	6 to 10	5	2	10
Technical + Engineering Mathematics	1 to 25	25	1	25
	26 to 55	30	2	60
Total Duration : 3 hours	**Total Questions : 65**		**Total Marks : 100**	

Note :

(i) 40 to 45 marks will be allotted to Numerical Answer Type Questions.

(ii) MSQ also added from GATE 2021 for which **no negative** marking.

Pattern of Questions :

GATE 2021 would contain questions of THREE different types in all the papers :

(i) **Multiple Choice Questions (MCQ)** carrying 1 or 2 marks each, in all the papers and sections. These questions are objective in nature, and each will have choice of four answers, out of which ONLY ONE choice is correct.

Negative Marking for Wrong Answers : For a wrong answer chosen in a MCQ, there will be negative marking. For 1-mark MCQ, 1/3 mark will be deducted for a wrong answer. Likewise, for 2-mark MCQ, 2/3 mark will be deducted for a wrong answer.

(ii) **Multiple Select Questions (MSQ)** carrying 1 or 2 marks each in all the papers and sections. These questions are objective in nature, and each will have choice of four answers, out of which ONE or MORE than ONE choice(s) are correct.

Note : There is **NO negative** marking for a wrong answer in MSQ questions. However, there is NO partial credit for choosing partially correct combinations of choices or any single wrong choice.

(iii) **Numerical Answer Type (NAT)** Questions carrying 1 or 2 marks each in most of the papers and sections. For these questions, the answer is a signed real number, which needs to be entered by the candidate using the virtual numeric keypad on the monitor (keyboard of the computer will be disabled). No choices will be shown for these types of questions. The answer can be a number such as 10 or -10 (an integer only). The answer may be in decimals as well, for example, 10.1 (one decimal) or 10.01 (two decimals) or -10.001 (three decimals). These questions will be mentioned with, up to which decimal places, the candidates need to present the answer. Also, for some NAT type problems an appropriate range will be considered while evaluating these questions so that the candidate is not unduly penalized due to the usual round-off errors. Candidates are advised to do the rounding off at the end of the calculation (not in between steps). Wherever required and possible, it is better to give NAT answer up to a maximum of three decimal places.

Example : If the wire diameter of a compressive helical spring is increased by 2%, the change in spring stiffness (in %) is _ (correct to two decimal places).

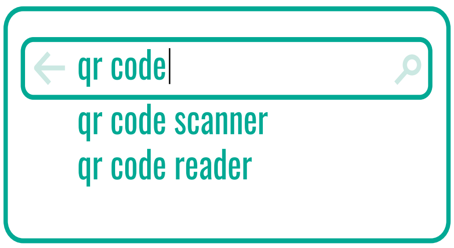
Note : *There is NO negative marking for a wrong answer in NAT questions.*

Also, there is NO partial credit in NAT questions.

What is special about this book ?

GATE ACADEMY Team took several years' to come up with the solutions of GATE examination. It is because we strongly believe in quality. We have significantly prepared each and every solution of the questions appeared in GATE, and many individuals from the community have taken time out to proof read and improve the quality of solutions, so that it becomes very lucid for the readers. Some of the key features of this book are as under :

- This book gives complete analysis of questions chapter wise as well as year wise.
- Video Solution of important conceptual questions has been given in the form of QR code and by scanning QR code one can see the video solution of the given question.
- Solutions has been presented in lucid and understandable language for an average student.
- In addition to the GATE syllabus, the book includes the nomenclature of chapters according to text books for easy reference.
- Last but not the least, author's 10 years experience and devotion in preparation of these solutions.
- Steps to Open Video solution through mobile.



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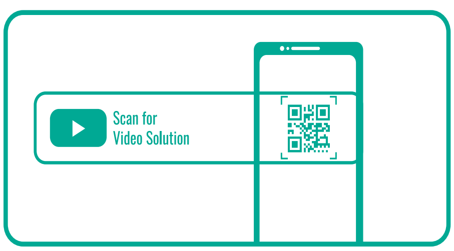

(3) Scan the given QR Code for particular question.

(4) Visit the link generated & you'll be redirect to the video solution.

Note : For recent updates regarding minor changes in this book, visit - www.gateacademy.co.in. We are always ready to appreciate and help you.

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EC-3	EC-4	EC-9	EC-9	EC-9	EC-12	EC-14	EC-18
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EC-46	EC-46	EC-46	EC-59	EC-61	EC-66	EC-66	EC-66
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EC-72	EC-77	EC-77	EC-77	EC-86	EC-86	EC-86	EC-86
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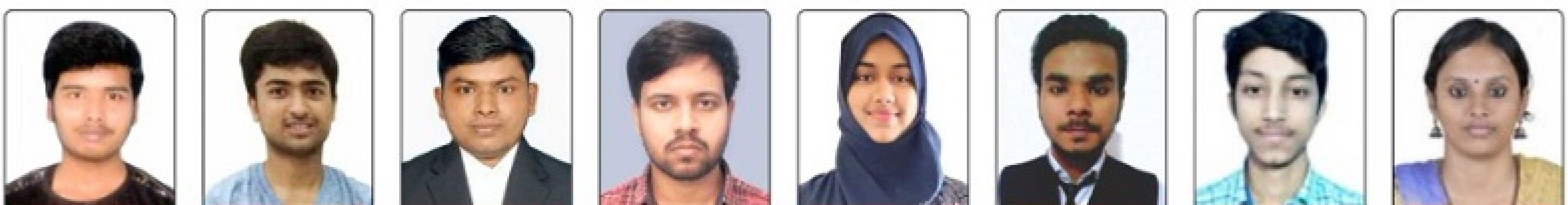

EC-101	EC-115	EC-115	EC-115	EC-128	EC-128	EC-128	EC-139
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EC-139	EC-139	EC-139	EC-152	EC-152	EC-161	EC-161	EC-161
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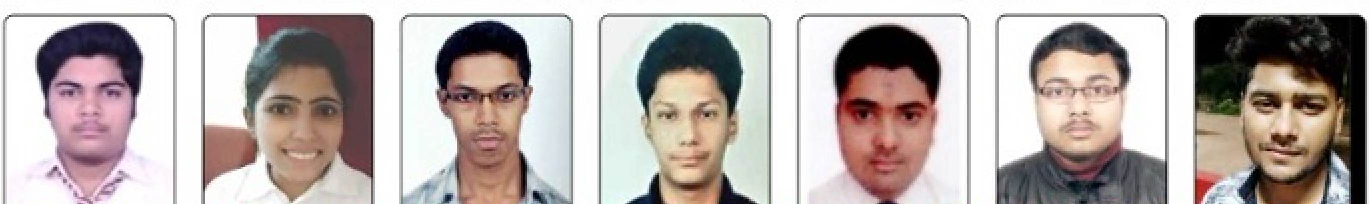

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EC-316	EC-316	EC-317	EC-331	EC-331	EC-331	EC-353	EC-353
Ankit Singh	Parneet	Ujjawal Kant	Anuj Gupta	Ronak Gupta	Bharadwaj Dande	Mohit Gupta	Rajshekhar Mukherjee

EC-353	EC-353	EC-353	EC-397	EC-379	EC-388	EC-388	EC-399
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EC-399	EC-399	EC-399	EC-413	EC-413	EC-413	EC-439	EC-466
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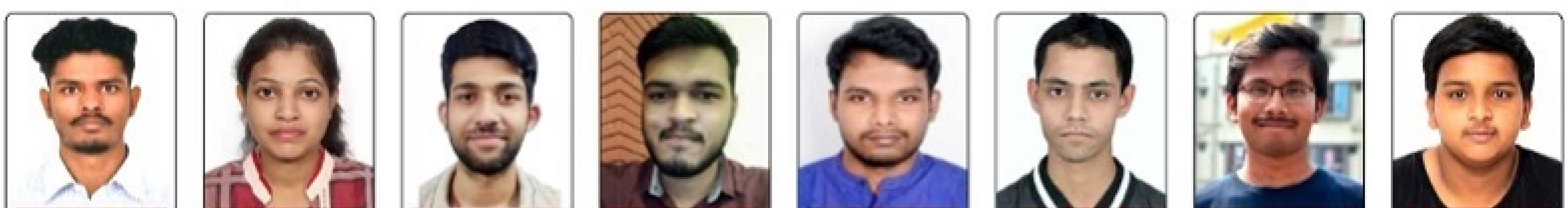

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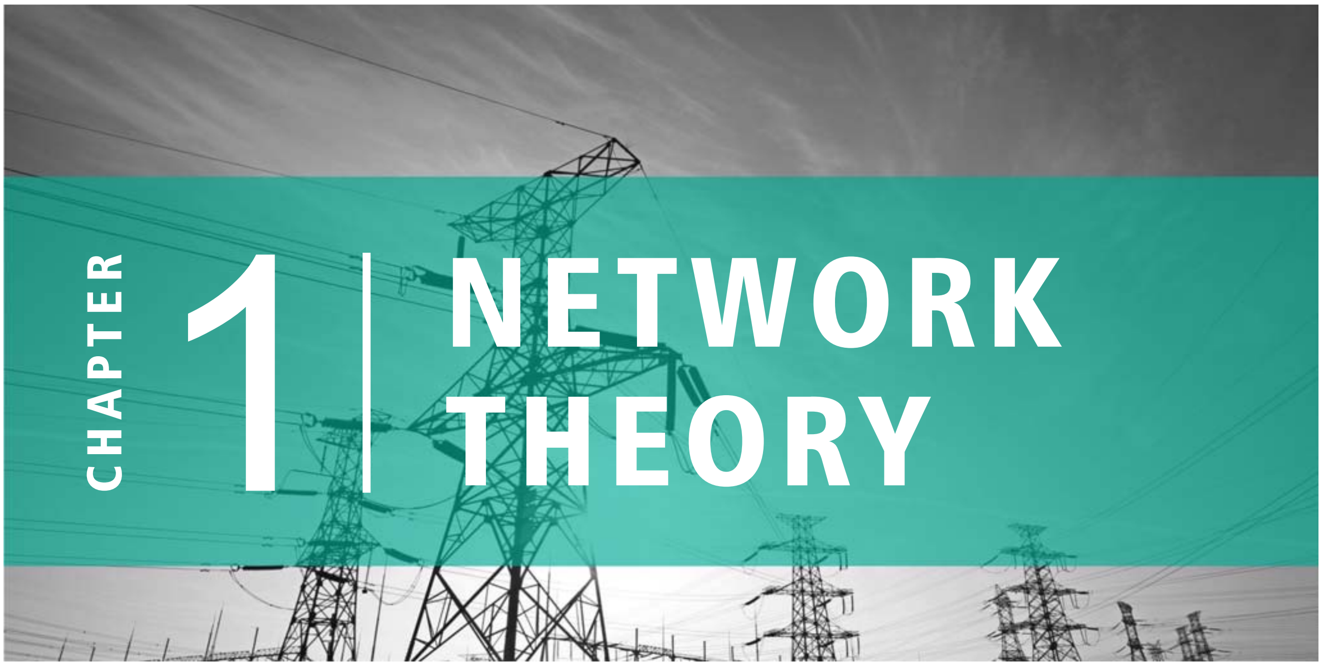


Marks Distribution of Network Theory in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	3	6	15
2004	1	7	15
2005	4	7	18
2006	2	6	14
2007	–	7	14
2008	2	6	14
2009	2	6	14
2010	3	4	11
2011	3	5	13
2012	5	6	17
2013	2	3	8
2014 Set-1	2	2	6
2014 Set-2	3	2	7

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-3	3	3	9
2015 Set-1	4	3	10
2015 Set-2	3	3	9
2016 Set-1	4	5	14
2016 Set-2	5	4	13
2017 Set-1	2	3	8
2017 Set-2	2	2	6
2018	3	4	11
2019	1	3	7
2020	2	2	6
2021	3	5	13

Syllabus : Network Theory

Ideal voltage and current sources, dependent sources, R, L, C, M elements; Network solution methods: KCL, KVL, Node and Mesh analysis; Network Theorems: Thevenin's, Norton's, Superposition and Maximum Power Transfer theorem; Transient response of dc and ac networks, sinusoidal steady-state analysis, resonance, two port networks, balanced three phase circuits, star-delta transformation, complex power and power factor in ac circuits.

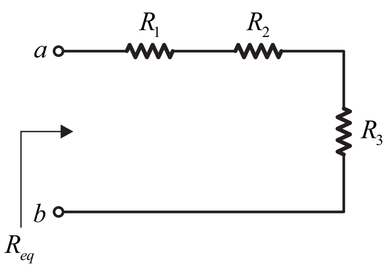
Contents : Network Theory

1 Basic Concept of Networks

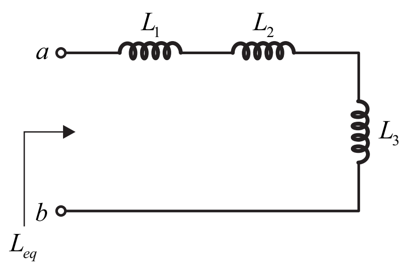
➢ Partial Synopsis

Interconnection of Passive elements :

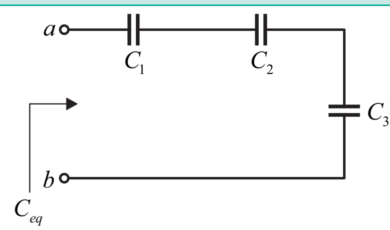
1. Series Connection



$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

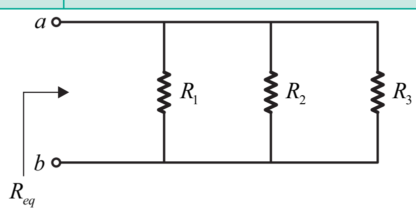


$$\boxed{L_{eq} = L_1 + L_2 + L_3}$$

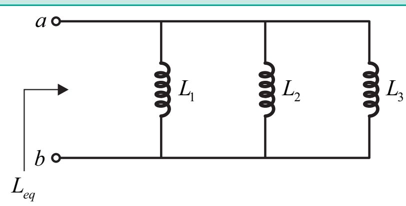


$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

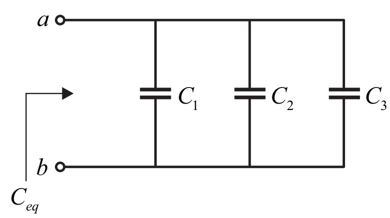
2. Parallel Connection



$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$



$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$



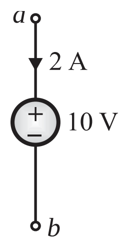
$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

Concept of Absorbed and Delivered Power :

1. If current enters into the positive terminal of voltage source then it is referred as absorbed power (i.e. power is absorbed by voltage source).
2. If current leaves from positive terminal of voltage source then it is referred as delivered power (i.e. power is delivered by voltage source).

Examples :

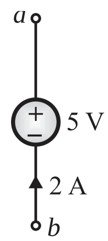
(i)



$P_{absorbed} = 2 \times 10 = 20$ watt

$P_{delivered} = -20$ watt

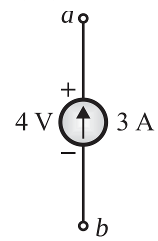
(ii)



$P_{delivered} = 2 \times 5 = 10$ watt

$P_{absorbed} = -10$ watt

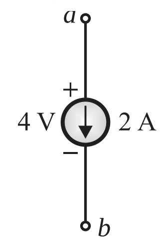
(iii)



$P_{delivered} = 3 \times 4 = 12$ watt

$P_{absorbed} = -12$ watt

(iv)



$P_{absorbed} = 4 \times 2 = 8$ watt

$P_{delivered} = -8$ watt

Note : Same concept is valid for any element at the place of voltage source.

Special cases in application of KCL and KVL :

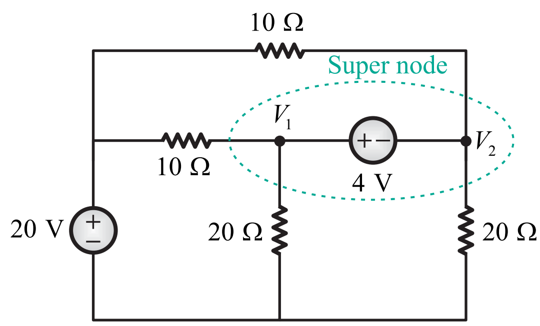
Supernode :

1. If the ideal voltage source (independent or dependent) is connected between two non-reference node then two non-reference node form a generalized node or supernode.
2. A supernode has no voltage of its own.
3. It requires the application of both KVL and KCL.

Step to solve questions on supernode :

1. Identify the super node in the circuit.
2. Apply KCL simultaneously at both the nodes ignoring the branch containing the supernode and write as one equation.
3. Apply KVL to the branch containing supernode.
4. Solve the simultaneous equations to get the node voltage.

Solved example on Supernode :



Applying KCL at node V_1,

$$\frac{V_1 - 20}{10} + \frac{V_1 - 0}{20} + \frac{V_2 - 20}{10} + \frac{V_2 - 0}{20} = 0$$

$$3V_1 + 3V_2 = 80 \qquad \ldots \text{(i)}$$

Applying KVL at the super node,

$$V_1 - 4 - V_2 = 0$$

$$V_1 - V_2 = 4 \qquad \ldots \text{(ii)}$$

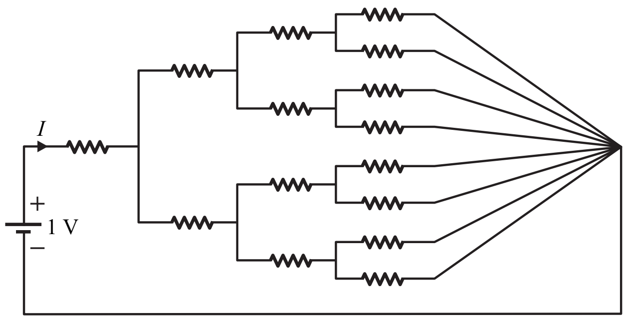
Solving equation (i) and (ii) we get

$$V_1 = 15.33 \text{ V} \qquad V_2 = 11.33 \text{ V}$$

➤ Sample Questions

1992 IIT Delhi

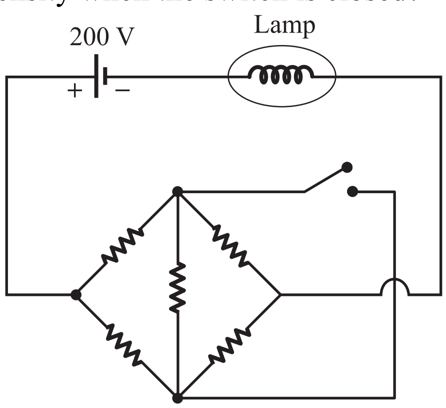
1.1 All the resistances in figure are 1 Ω each. The value of current 'I' is



(A) $\frac{1}{15}$ A (B) $\frac{2}{15}$ A

(C) $\frac{4}{15}$ A (D) $\frac{8}{15}$ A

1.2 All resistances in the circuit in figure are of R ohms each. The switch is initially open. What happens to the lamp's intensity when the switch is closed?



(A) Increases

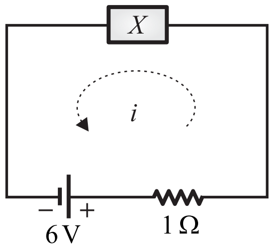
(B) Decreases

(C) Remains same

(D) Answer depends on the value of R

1996 IISc Bangalore

1.3 In the circuit shown in figure, X is an element which always absorbs power. During a particular operation, it sets up a current of 1 amp in the direction shown and absorbs a power P_X. It is possible that X can absorb the same power P_X for another current i. Then the value of this current is



(A) $(3-\sqrt{14})$ amps

(B) $(3+\sqrt{14})$ amps

(C) 5 amps

(D) None of these

2001 IIT Kanpur

1.4 Two incandescent light bulbs of 40 W and 60 W ratings are connected in series across the mains. Then

(A) the bulbs together consume 100 W.

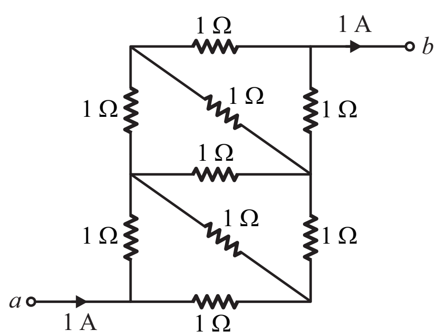
(B) the bulbs together consume 50 W.

(C) the 60 W bulb glows brighter.

(D) the 40 W bulb glows brighter.

2002 IISc Bangalore

1.5 In the resistor network shown in figure, all resistors value 1 Ω. A current of 1 A passes from terminal a to terminal b as shown in figure, voltage between terminal a and b is approximately.

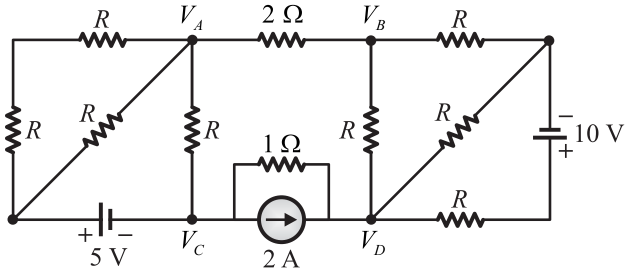


(A) 1.4 V (B) 1.5 V

(C) 0 V (D) 3 V

2012 IIT Delhi

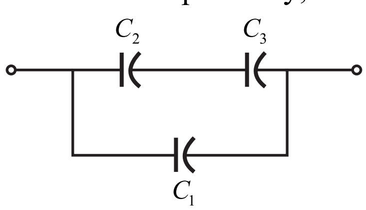
1.6 If $V_A - V_B = 6$ V, then $V_C - V_D$ is



(A) – 5 V (B) 2 V
(C) 3 V (D) 6 V

2013 IIT Bombay

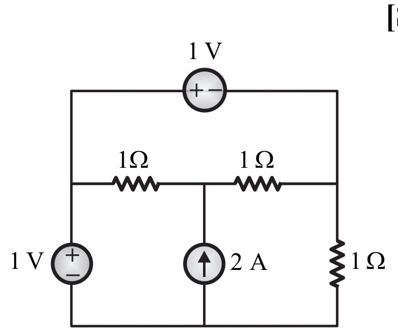
1.7 Three capacitors C_1, C_2 and C_3 whose values are 10 μF, 5 μF and 2 μF respectively, have breakdown voltages of 10 V, 5 V and 2 V respectively. For the interconnection shown below, the maximum safe voltage in volts that can be applied across the combination, and the corresponding total charge in μC stored in the effective capacitance across the terminals are respectively,



(A) 2.8 and 36 (B) 7 and 119
(C) 2.8 and 32 (D) 7 and 80

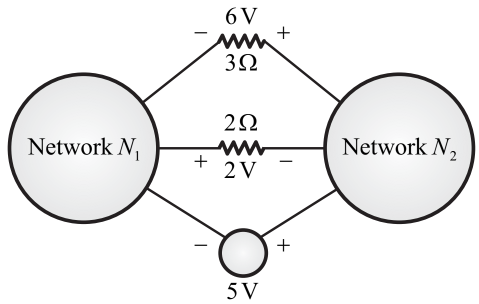
2014 IIT Kharagpur

1.8 The power delivered by the current source, in the figure, is ________ W.
[Set - 03]



2015 IIT Kanpur

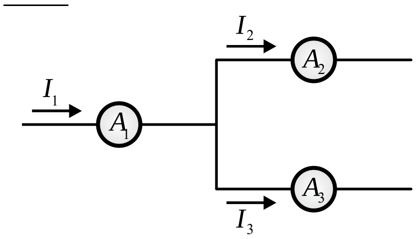
1.9 The voltages developed across the 3 Ω and 2 Ω resistors shown in the figure are 6 V and 2 V respectively, with the polarity as marked. What is the power (in Watt) delivered by the 5 V voltage source? **[Set - 01]**



(A) 5 (B) 7
(C) 10 (D) 14

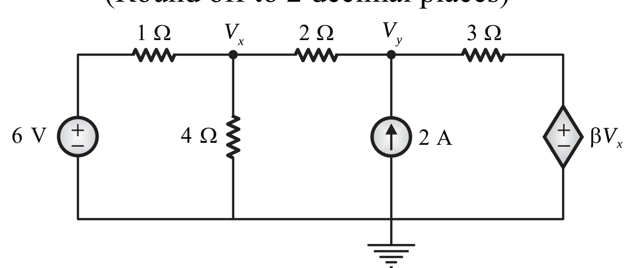
2020 IIT Delhi

1.10 Currents through ammeters A_2 and A_3 in the figure are $1\angle 10^0$ and $1\angle 70^0$ respectively. The reading of the ammeter A_1 (rounded off to 3 decimal places) is ____A.



2021 IIT Bombay

1.11 In the given circuit, for voltage V_y to be zero, the value of β should be _________. (Round off to 2 decimal places)

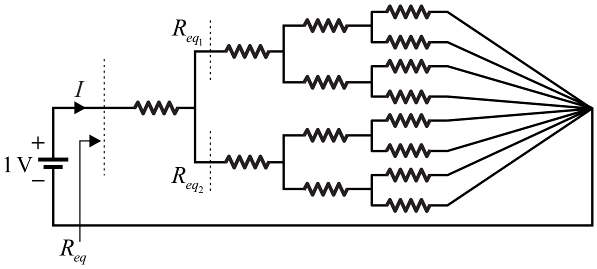


❖❖❖❖

Explanations Basic Concept of Networks

1.1 (D)

Given circuit is shown below,

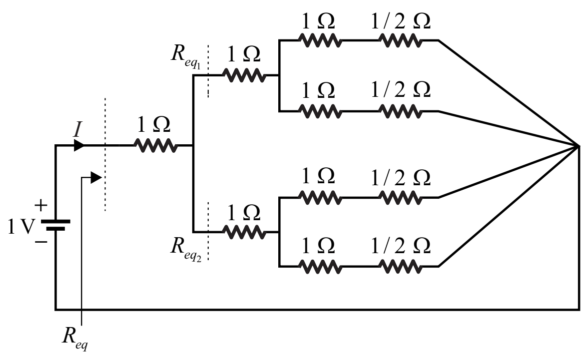


Method 1

Given : All resistances are of 1 Ω.

$$R_{eq_1} = \left\{\left[\left(\frac{1}{2}+1\right) \| \left(\frac{1}{2}+1\right)\right]+1\right\} = \frac{7}{4}\ \Omega$$

$$R_{eq_2} = \left\{\left[\left(\frac{1}{2}+1\right) \| \left(\frac{1}{2}+1\right)\right]+1\right\} = \frac{7}{4}\ \Omega$$



Equivalent resistance is,

$$R_{eq} = \left[R_{eq_1} \| R_{eq_2}\right]+1 = \left[\frac{7}{4} \| \frac{7}{4}\right]+1$$

$$R_{eq} = \frac{15}{8}\ \Omega$$

From above circuit :

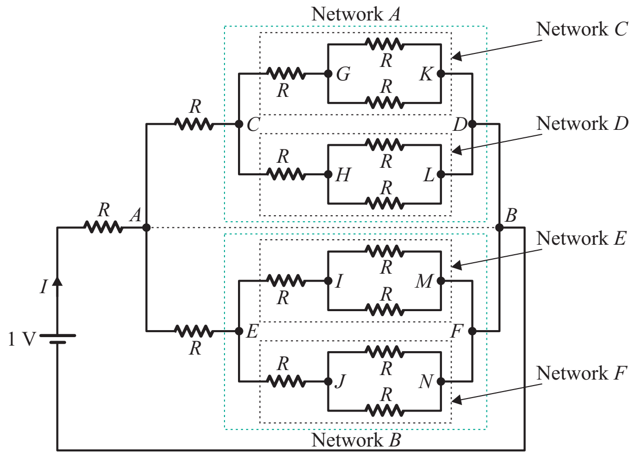
Current is given by,

$$I = \frac{V}{R_{eq}} = \frac{1}{15/8} = \frac{8}{15}\text{ A}$$

Hence, the correct option is (D).

Method 2

Re-arranged given circuit as shown bellows with all resistance $R = 1\ \Omega$.



Here, network A and network B offers horizontal symmetry with respect to dotted line between point A and B it means,

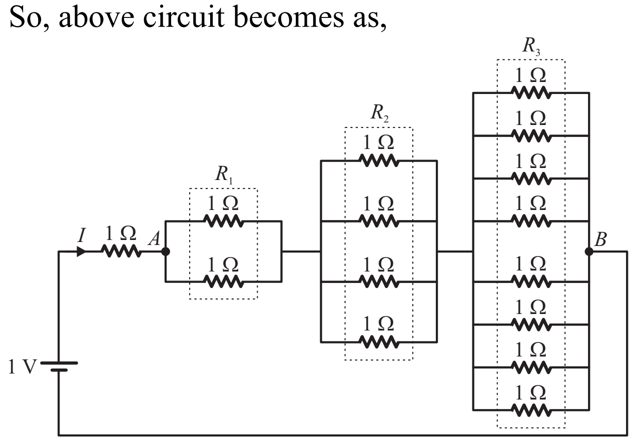
Potential at point C and E are same. So joint point C and E.

Potential at point G, H, I and J are same. So joint point C and E.

Potential at point K, L, M and N are same. So joint point C and E.

Potential at point D and F are same. So joint point C and E.

So, above circuit becomes as,

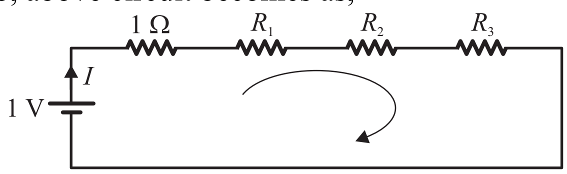


Here, $R_1 = (1 \| 1)\ \Omega = \frac{1}{2}\Omega$

$$R_2 = (1 \| 1 \| 1 \| 1)\ \Omega = \frac{1}{4}\Omega$$

$$R_3 = (1\|1\|1\|1\|1\|1\|1\|1)\ \Omega = \frac{1}{8}\Omega$$

So, above circuit becomes as,



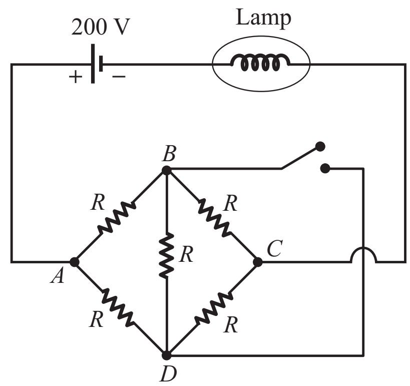
Thus, $I = \dfrac{1}{1 + R_1 + R_2 + R_3} = \dfrac{1}{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}}$

$$I = \frac{1}{15/8} = \frac{8}{15}\ \text{A}$$

Hence, the correct option is (D).

1.2 (C)

Given circuit is shown below,

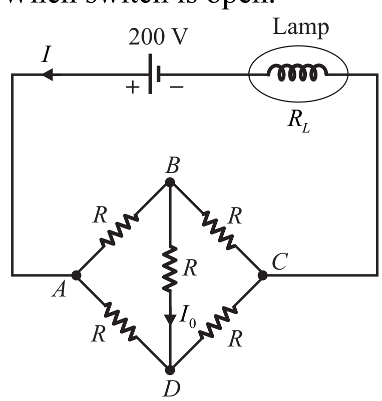


Given : $R_{AB} = R_{BC} = R_{AD} = R_{DC} = R_{BD} = R$

Let us assume the resistance of lamp is R_L.

In the question, there is no discussion of temperature that is why we assume filament resistance does not change with temperature.

Case 1 : When switch is open.



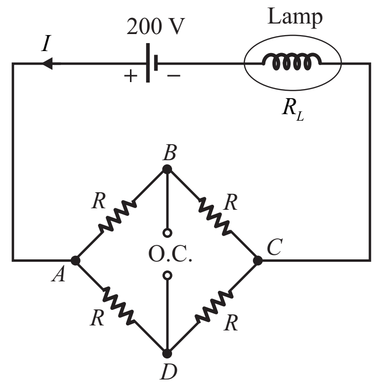
From figure, it is clear that

$$\frac{R_{BC}}{R_{AB}} = \frac{R}{R}, \quad \frac{R_{DC}}{R_{AD}} = \frac{R}{R}$$

Thus, $\dfrac{R_{BC}}{R_{AB}} = \dfrac{R_{DC}}{R_{AD}}$

It satisfy the condition of balanced Wheatstone bridge.

Hence, the Wheat-stone bridge shown in the above network is balanced and therefore no current will flow through R_{BD}. Hence $I_0 = 0$ (It means BD terminal is open circuited).

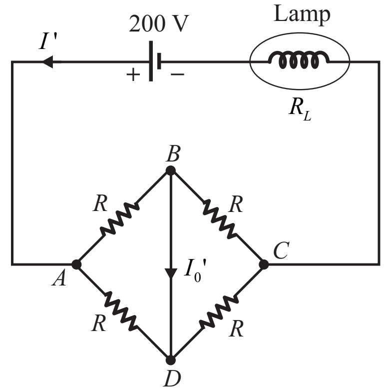


From the above circuit,

$$I = \frac{200}{[(R_{AB} + R_{BC})\|(R_{AD} + R_{DC})] + R_L}$$

$$I = \frac{200}{[2R\|2R] + R_L} = \frac{200}{R + R_L}$$

Case 2 : When switch is closed. BD terminal will be short circuited.



From the above circuit, it is clear that

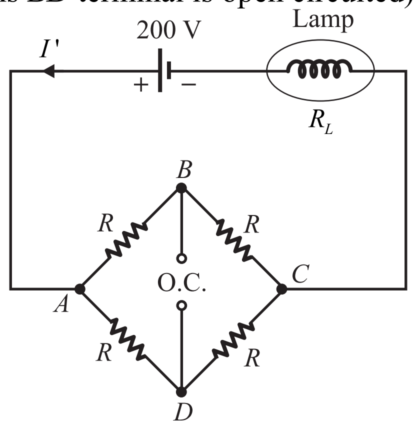
$$\frac{R_{BC}}{R_{AB}} = \frac{R}{R}, \quad \frac{R_{DC}}{R_{AD}} = \frac{R}{R}$$

Thus, $\dfrac{R_{BC}}{R_{AB}} = \dfrac{R_{DC}}{R_{AD}}$

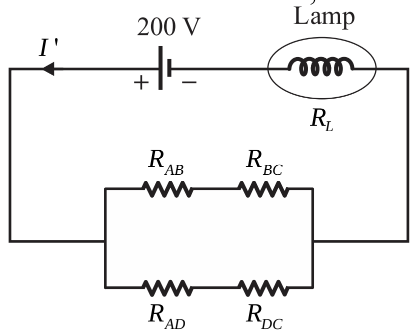
It satisfy the condition of balanced Wheatstone bridge.

Hence, the Wheat-stone bridge shown in the above network is balanced and therefore $I_0' = 0$

(It means *BD* terminal is open circuited).



Above circuit can be reduced as,



From the above circuit,

$$I' = \frac{200}{[(R_{AB} + R_{BC}) \| (R_{AD} + R_{DC})] + R_L}$$

$$I' = \frac{200}{[2R \| 2R] + R_L} = \frac{200}{R + R_L}$$

Since, in both cases, current flowing through lamp is same i.e. $I = I'$, hence in both cases intensity of lamp will remain same.

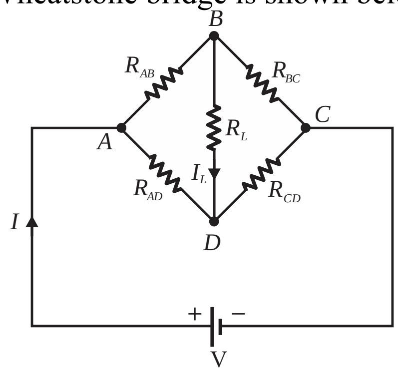
Hence, the correct option is (C).

Note : In general, the current through filament lamp is not directly proportional to the potential difference applied across it because filament gets hot and its temperature increases, so atoms in filament vibrate more, so that filament resistance increases, that is why if we increase potential difference across the filament, the current no longer increases as much.

Key Point

Wheatstone bridge concept :

- Wheatstone bridge is shown below,



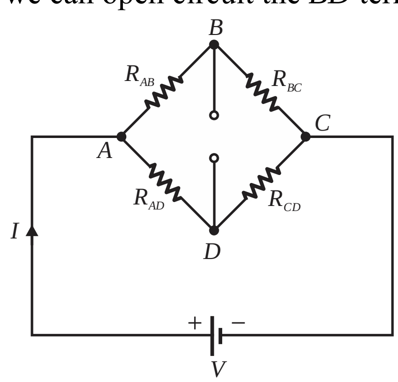
When bridge is balanced then its satisfied the condition of balanced bridge,

$$\frac{R_{AB}}{R_{BC}} = \frac{R_{AD}}{R_{DC}}$$

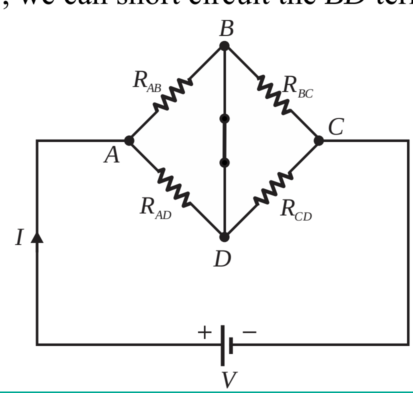
As the condition of bridge balancing is satisfied then current flowing through load (R_L) is zero.

i.e. ($I_L = 0$), then it gives two possiblities,

Possibility-1 : When bridge is balanced, so $I_L = 0$, we can open circuit the *BD* terminal.



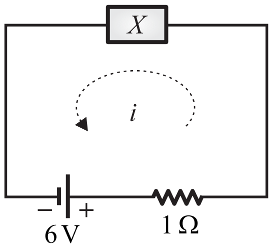
Possibility-2 : When bridge is baalanced, so $V_L = 0$, we can short circuit the *BD* terminal.



- We can use any one of the possibilities of Wheatstone bridge according to the need.

1.3 (C)

Given circuit with unknown element X which always absorb power P_X is shown below,

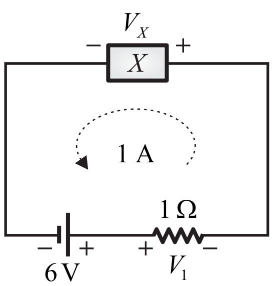


According to Tellegen's theorem, sum of product of voltage and current across each element in any network will be always zero.

i.e. $\sum_{k=1}^{N} V_k i_k = 0$

Now, to solve this question let us consider two different cases as given below.

Case 1 : When current flowing in the circuit is 1 amp and X is power absorbing element that is why current enter at positive terminal of X as shown below,



Applying Tellegen's theorem,

$$\sum_{k=1}^{3} V_k i_k = 0$$

$$1\times(-6)+1\times V_1+1\times V_X = 0 \qquad \ldots\text{(i)}$$

where, $V_1 = 1\times R = 1\times 1 = 1\,\text{V}$

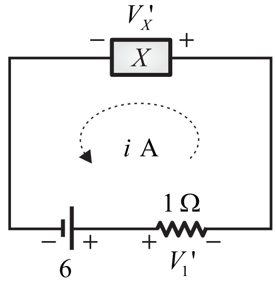
And $V_X = \dfrac{P_X}{1} = P_X$

From equation (i),

$$-6+1+P_X = 0$$

$$P_X = 6-1 = 5\ \text{W}$$

Case 2 : When current flowing in the circuit is i amp.



Applying Tellegen's theorem,

$$\sum_{k=1}^{3} V_k i_k = 0$$

$$i\times(-6)+i\times V_1'+i\times V_X' = 0 \qquad \ldots\text{(ii)}$$

where, $V_1' = i\times R = i\times 1 = i$ Volt

and $V_X' = \dfrac{P_X'}{i}$

Since, power P_X' remains same as in case 1 i.e. $P_X' = P_X$.

Hence, $V_X' = \dfrac{P_X}{i} = \dfrac{5}{i}$

From equation (ii),

$$-6i + i\times i + i\times\frac{5}{i} = 0$$

$$i^2 - 6i + 5 = 0$$

We will get two values of $i = 5, 1$.

Therefore, $i = 5$ A, for which 'X' absorbs same power P_X i.e., 5 W.

Hence, the correct option is (C).

Note : Many of aspirant may think that, why we will get two value of current in the above question for the same absorb power so answer of this query is, if we assume X is as a variable resistor, then it is possible.

1.4 (D)

The incandescent Bulbs B_1 and B_2 are define using rated power at $P_1 = 40\,\text{W}$ and $P_2 = 60\,\text{W}$ at same voltage V.

🕮 Key Point

1. If voltage rating of corresponding bulb is given, then we will use the corresponding voltage rating to calculate the resistance of bulb.

P_1 watt, V_1 volt

P_2 watt, V_2 volt

Then $R_1 = \dfrac{V_1^2}{P_1}$ Then $R_2 = \dfrac{V_2^2}{P_2}$

2. If voltage rating is not given, then we will consider same voltage rating for two bulbs.

P_1 watt

P_2 watt

Then $R_1 = \dfrac{V^2}{P_1}$ Then $R_2 = \dfrac{V^2}{P_2}$

Assuming resistance of 40 W bulb is R_1 and resistance of 60 W bulb is R_2, then

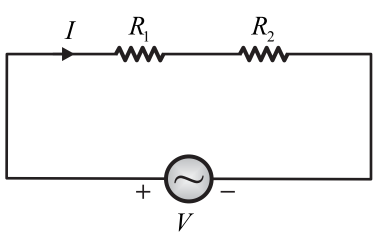
$$P_1 = \frac{V^2}{R_1} \Rightarrow R_1 = \frac{V^2}{P_1} = \frac{V^2}{40}$$

and $$P_2 = \frac{V^2}{R_2} \Rightarrow R_2 = \frac{V^2}{P_2} = \frac{V^2}{60}$$

As $$P_1 < P_2 \Rightarrow R_1 > R_2$$

Therefore, resistance R_1 of 40 W bulb is greater than resistance R_2 of 60 W bulb (Resistance of bulb is calculated by rated power).

According to question, both bulb B_1 and B_2 are connected in series and current will be same.



Current is given by, $I = \dfrac{V}{R_1 + R_2}$

So, to check which bulb glow more we have to find out actual power consumed by bulbs in series combination is given by,

$$P_{S_1} = I^2 R_1 \quad \text{(by bulb } B_1\text{)}$$

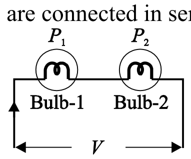
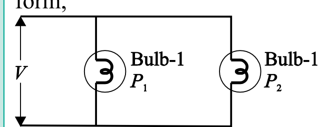
$$P_{S_2} = I^2 R_2 \quad \text{(by bulb } B_2\text{)}$$

It shows that, $R_1 > R_2 \Rightarrow P_{S_1} > P_{S_2}$

Therefore, power consumed by 40 W bulb B_1 will be greater than bulb B_2, which results in brighter glow.

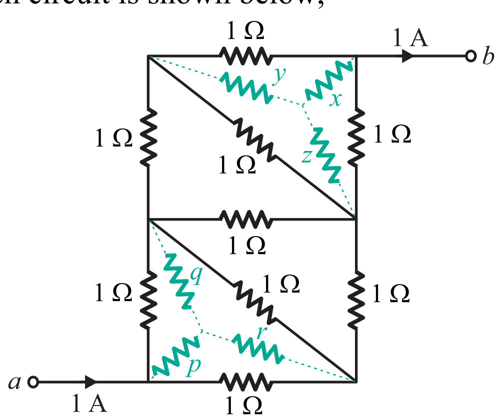
Hence, the correct option is (D).

🕮 Key Point

S. No.	Arrangement	Comment
1.	Bulbs are connected in series  P_1 and P_2 are rated power of bulb 1 and 2 respectively.	**Condition 1 :** If $P_1 > P_2$ then bulb-2 glows more than bulb-1. **Condition 2 :** If $P_1 < P_2$ then bulb-1 glows more than bulb-2.
2.	Bulbs are connected in parallel form,  P_1 and P_2 are rated power of bulb 1 and 2 respectively.	**Condition 1 :** If $P_1 > P_2$, then bulb-1 glows more than bulb-2. **Condition 2 :** If $P_1 < P_2$, then bulb-2 glows more than bulb-1.
3.	Rated power shows maximum power of device that can withstand without damaging itself.	

1.5 (A)

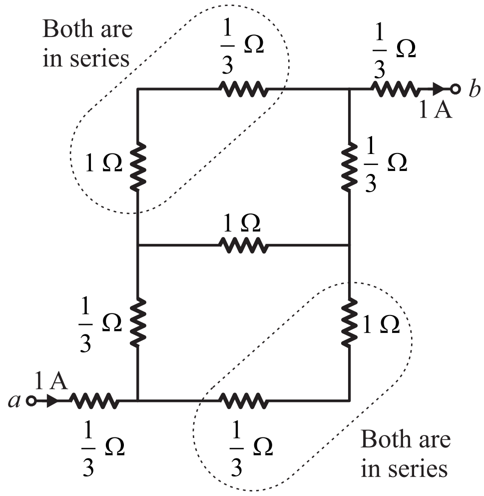
Given circuit is shown below,



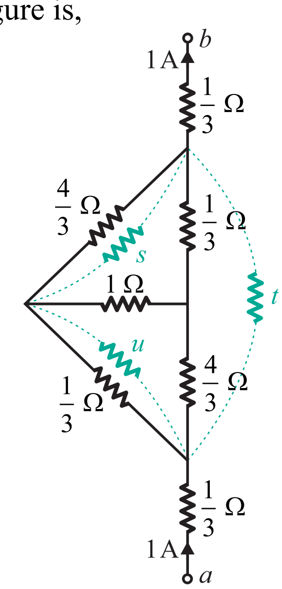
Applying delta to star conversion,

$$x = y = z = \frac{1\times 1}{1+1+1} = \frac{1}{3}\ \Omega$$

$$p = q = r = \frac{1\times 1}{1+1+1} = \frac{1}{3}\ \Omega$$



Modified figure is,

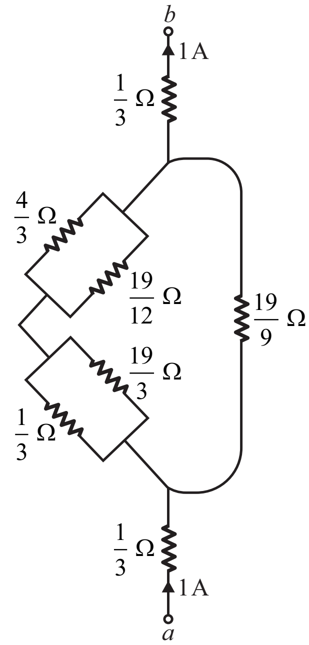


Applying star to delta conversion,

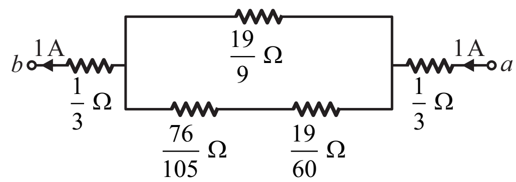
$$t = \frac{\frac{1}{3}\times\frac{4}{3}+1\times\frac{1}{3}+1\times\frac{4}{3}}{1} = \frac{19}{9}\ \Omega$$

$$s = \frac{\frac{1}{3}\times\frac{4}{3}+1\times\frac{1}{3}+1\times\frac{4}{3}}{\frac{4}{3}} = \frac{19}{12}\ \Omega$$

$$u = \frac{\frac{1}{3}\times\frac{4}{3}+1\times\frac{1}{3}+1\times\frac{4}{3}}{\frac{1}{3}} = \frac{19}{3}\ \Omega$$



Simplified figure is,



$$R_{ab} = 1.3636\ \Omega$$

Voltage across between terminal a and b is,

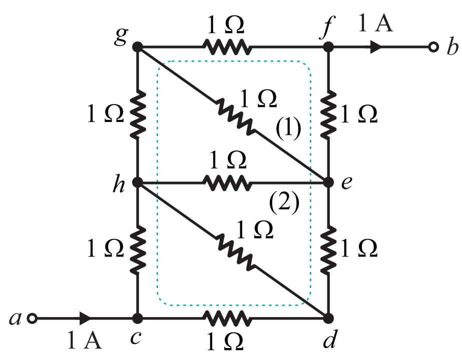
$$V_{ab} = 1.3636\times 1 = 1.3636\ \text{V} \approx 1.4\ \text{V}$$

Hence, the correct option is (A).

☒ Avoid This Mistake

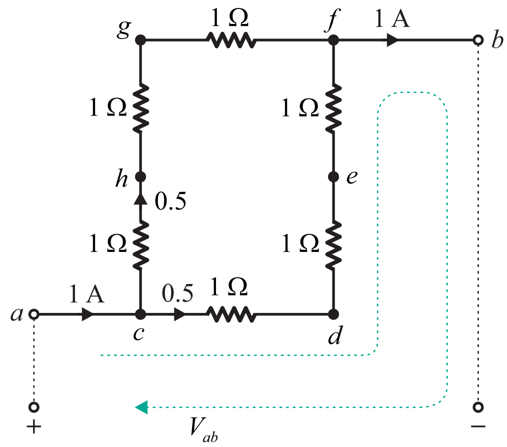
Direct Approach :

[On the basis of balanced Wheat-stone bridge.]



By looking the circuit, the dashed portion can be removed due to symmetry. This is what one can think by just seeing this circuit.

Hence, this circuit will look like as shown below,

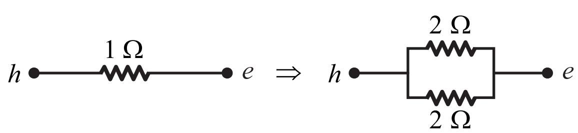


Applying KVL in the loop shown above,

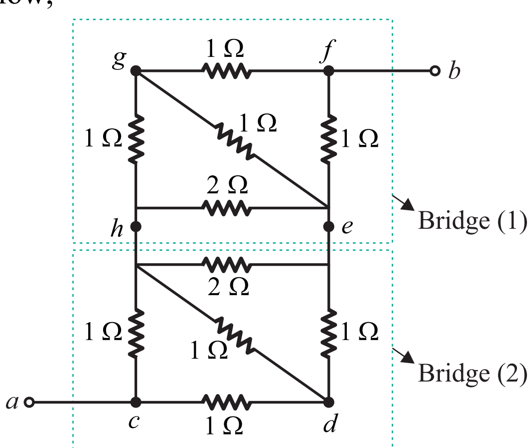
$$V_{ab} = 0.5\times1+0.5\times1+0.5\times1 = 1.5 \text{ V}$$

But this solution is totally wrong. This has been done here to just make you understand what exactly one can do such type of mistake. Here is the explanation i.e. why it's wrong.

The resistance between the terminals is common in two bridges i.e. 1 and 2. Now this has to be break into two resistances whose equivalent is only $1\ \Omega$ to get Wheat-stone bridge condition. Hence,



The given circuit can be drawn as shown below,



From the circuit, it is clear that the bridge 1 and bridge 2 are not balanced.

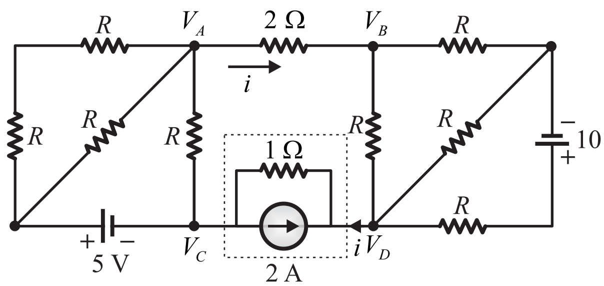
Since, $V_{ge} \neq 0$ V and $V_{hd} \neq 0$ V

So the above answer calculated by balance bridge is not possible. So, follow the star-delta or delta-star to get accurate result.

1.6 (A)

Given : $V_A - V_B = 6$ V

The circuit is shown below,

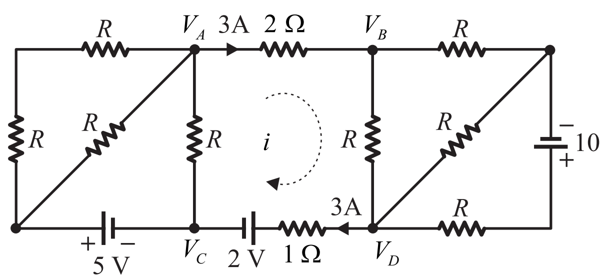


From the circuit, $i = \dfrac{V_A - V_B}{2} = \dfrac{6}{2} = 3$ A

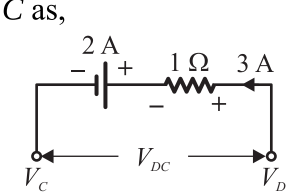
The same current will return and flow through branch between node D and C.

Using source transformation :

Converting 2 A current source into voltage source as shown below,



From the above circuit, apply KVL between node D and C as,

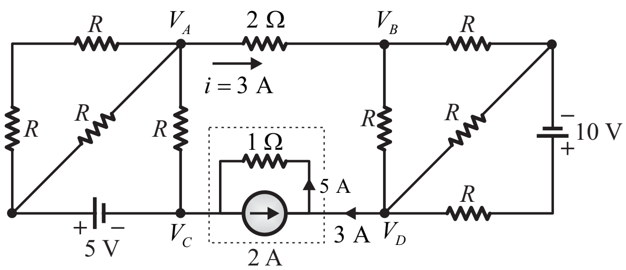


$$V_{DC} = 3\times1+2 = 5 \text{ V}$$

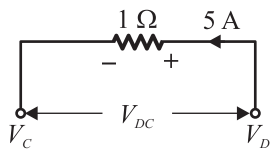
$$V_{CD} = V_C - V_D = -5 \text{ V}$$

Without using source transformation :

We can also solve this question without using source transformation as shown below,



So, voltage across 1 Ω resistor connected between terminal D and C as,



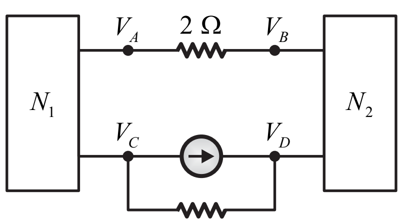
$$V_{DC} = 5 \times 1 = 5\text{ V}$$

$$V_{CD} = -5\text{ V}$$

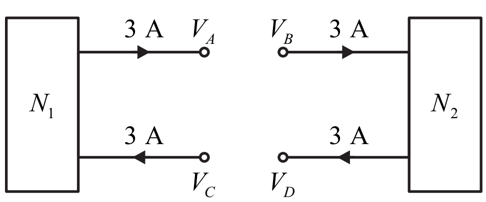
Hence, the correct option is (A).

Scan for Video Solution

Key Point



For a particular one port network,
Incoming current = Outgoing current
Hence,



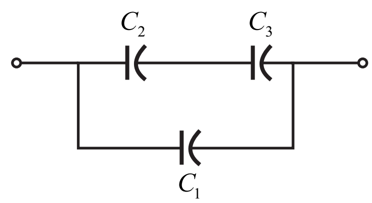
1.7 (C)

Given : $C_1 = 10\ \mu\text{F}, \quad V_1 = 10\text{ V}$

$C_2 = 5\ \mu\text{F}, \quad V_2 = 5\text{ V}$

$C_3 = 2\ \mu\text{F}, \quad V_3 = 2\text{ V}$

Method 1



Charge in a capacitor is given by,

$$Q = CV$$

Capacitor	Breakdown voltage or maximum operating voltage	Maximum charge stored in capacitor
$C_1 = 10\ \mu\text{F}$	10 V	$Q_{1(\max)} = 100\ \mu\text{C}$
$C_2 = 5\ \mu\text{F}$	5 V	$Q_{2(\max)} = 25\ \mu\text{C}$
$C_3 = 2\ \mu\text{F}$	2 V	$Q_{3(\max)} = 4\ \mu\text{C}$

Since, C_2 and C_3 are in series, hence the charge on both capacitor will be same and equal to the min (Q_2, Q_3)

$$Q_{23} = 4\ \mu\text{C}$$

Equivalent capacitance of C_2 and C_3 is,

$$C_{23} = \frac{C_2 C_3}{C_2 + C_3} = \frac{5 \times 2}{5+2} = \frac{10}{7}\ \mu\text{F}$$

Equivalent voltage is,

$$V_{eq} = V_{23} = \frac{Q_{23}}{C_{23}} = \frac{4\ \mu\text{C}}{(10/7)\ \mu\text{F}} = 2.8\text{ V}$$

In parallel, voltage will be same, hence 2.8 V will appear across C_1 also.

Charge stored in C_1 is given by,

$$Q_1 = C_1 V_{eq}$$

$$Q_1 = 10\ \mu\text{F} \times 2.8\text{ V} = 28\ \mu\text{C}$$

In parallel, total charge is given by,

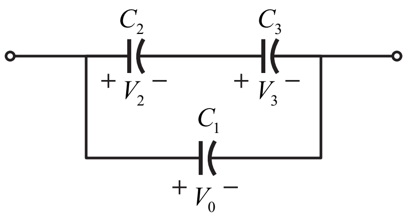
$$Q_T = Q_1 + Q_{23}$$

$$Q_T = (28+4)\,\mu C = 32\,\mu C$$

Hence, the correct option is (C).

Method 2

Capacitor	C_1	C_2	C_3
Value (in μF)	10	5	2
Breakdown Voltage (Volts)	10	5	2



From above figure, $V_0 = V_2 + V_3$

$$V_2 = \frac{C_3}{C_3 + C_2} V_0 = \frac{2}{5+2} V_0 = \frac{2V_0}{7}$$

$$V_3 = \frac{C_2}{C_2 + C_3} V_0 = \frac{5}{2+5} V_0 = \frac{5V_0}{7}$$

$$V_0 \le 10\text{ V},\ V_2 \le 5\text{ V and } V_3 \le 2\text{ V}$$

(i) If $V_0 = 10$ V,

Then $V_2 = \dfrac{2\times 10}{7} = \dfrac{20}{7} = 2.85$ V

So, $V_2 < 5$ V [No Breakdown]

$$V_3 = \frac{5\times 10}{7} = \frac{50}{7} = 7.13\text{ V}$$

So, $V_3 > 2$ V [Breakdown]

Hence, V_0 must be less than 10 V.

(ii) If $V_2 = 5$ V,

$$V_0 = \frac{7}{2}\times 5 = \frac{35}{2}\text{ V}$$

$V_0 > 10$ V [Breakdown]

Hence, V_2 must be less than 5 V.

(iii) If $V_3 = 2$ V,

$$V_0 = \frac{7}{5}\times 2 = 2.8 < 10\text{ V}$$

[No Breakdown]

Also, $V_2 = \dfrac{2V_0}{7} = 0.8\text{ V} < 5\text{ V}$

[No Breakdown]

None of them are in breakdown.

Hence, maximum value of voltage across the combination can be $V_0 = 2.8$ V.

Total charge $Q = C_{eq} V_0$

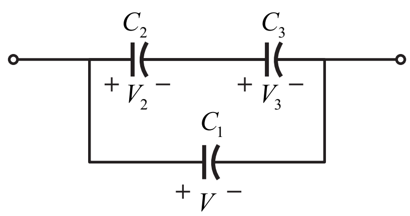
where, $C_{eq} = C_1 + \dfrac{C_2 C_3}{C_2 + C_3}$

$$C_{eq} = 10 + \frac{2\times 5}{2+5} = \frac{80}{7}\ \mu F$$

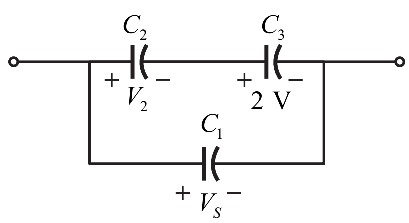
$$Q = \frac{80}{7}\times 2.8 = 32\ \mu C$$

Hence, the correct option is (C).

Method 3



Here, capacitors C_3 has minimum breakdown voltage and hence the breakdown voltage of capacitor C_3 will decide the maximum safe voltage V_S applied across the circuit as shown in figure.



$$V_3 = \frac{C_2}{C_2 + C_3} \times V_S \quad \text{[By VDR]}$$

$$2 = \frac{5}{2+5} \times V_S$$

$$V_S = 2.8 \text{ V}$$

Also, total charge is given by,

$$Q = C_{eq} V_S$$

where, $C_{eq} = C_1 + \dfrac{C_2 C_3}{C_2 + C_3}$

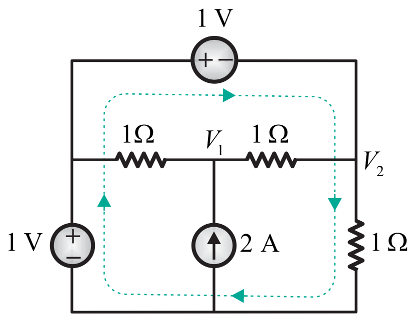
$$C_{eq} = 10 + \frac{2 \times 5}{2+5} = \frac{80}{7} \ \mu\text{F}$$

$$Q = \frac{80}{7} \times 2.8 = 32 \ \mu\text{C}$$

Hence, the correct option is (C).

1.8 3

Given circuit is shown below,



Applying KVL in above loop shown by dotted lines,

$$-1 + 1 + V_2 = 0$$

$$V_2 = 0 \text{ V}$$

Applying KCL at node V_1,

$$\frac{V_1 - 1}{1} + \frac{V_1 - V_2}{1} - 2 = 0$$

$$\frac{V_1 - 1}{1} + \frac{V_1 - 0}{1} - 2 = 0$$

$$2V_1 = 3 \quad \Rightarrow \quad V_1 = \frac{3}{2} \text{ V}$$

Thus, V_1 is the voltage, across the 2 A current source.

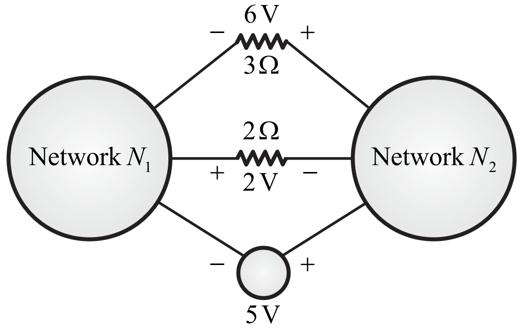
Power delivered by the current source is given by,

$$P_{2\text{ A}} = P_{\text{deliver}} = 2 \times V_1 = 2 \times \frac{3}{2} = 3 \text{ W}$$

Hence, the power delivered by the current source is **3 W.**

1.9 (A)

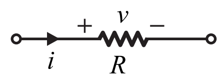
Given circuit is shown below,

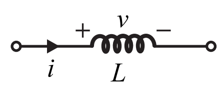


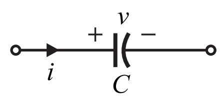
Here, $V_{3\Omega} = 6$ V and $V_{2\Omega} = 2$ V.

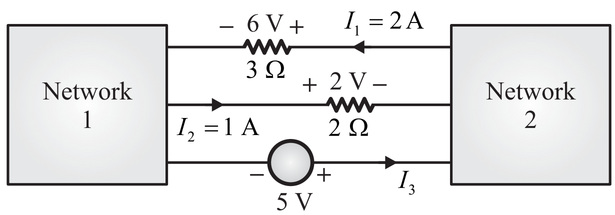
Key Point

Passive sign convention :

1.  $v = iR$

2.  $v = L\dfrac{di}{dt}$

3.  $i = C\dfrac{dv}{dt}$



Taking network 1 as a big node, same can be done by assuming network 2 as a big node.

Applying KCL in the network 1,

$$-I_1 + I_2 + I_3 = 0$$

$$I_3 = I_1 - I_2 = 2 - 1 = 1 \text{ A}$$

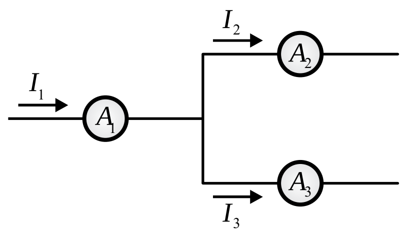
Power delivered by 5 V voltage source is,

$$P_{5\text{ V}} = P_{\text{deliver}} = 5 \times 1 = 5 \text{ Watt}$$

Hence, the correct option is (A).

1.10 **1.732**

Given diagram is shown below



Applying KCL;

$$\vec{I}_1 = \vec{I}_2 + \vec{I}_3$$

$$\vec{I}_1 = 1\angle 10^0 + 1\angle 70^0$$

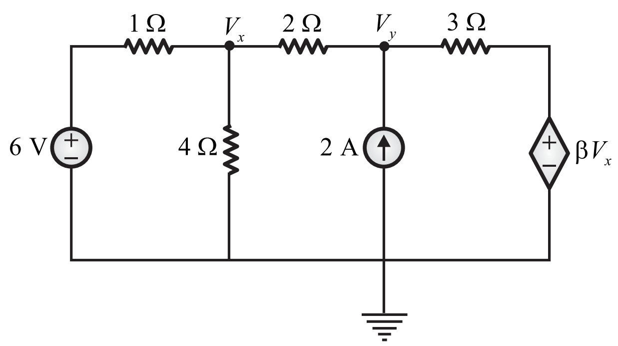
$$\vec{I}_1 = \cos 10^0 + j\sin 10^0$$
$$+\cos 70^0 + j\sin 70^0$$

$$\vec{I}_1 = 1.3268 + j1.113$$

$$\left|\vec{I}_1\right| = 1.732 \text{ A}$$

1.11 **– 3.25**

Given circuit is as shown below,



Applying nodal analysis at node V_x,

$$\frac{V_x}{4} + \frac{V_x - 6}{1} + \frac{V_x - 0}{2} = 0$$

(From the given condition, $V_y = 0$)

$$\frac{3V_x}{4} + V_x = 6$$

$$7V_x = 24$$

$$V_x = \frac{24}{7}$$

Applying nodal analysis at node V_y,

$$2 = \frac{0 - V_x}{2} + \frac{0 - \beta V_x}{3}$$

$$12 = -3V_x - 2\beta V_x$$

$$12 = -(3 + 2\beta)V_x$$

$$12 = -(3 + 2\beta)\frac{24}{7}$$

$$\beta = -3.25$$

Hence, the correct answer is – 3.25.

❖❖❖❖

2 Network Theorems

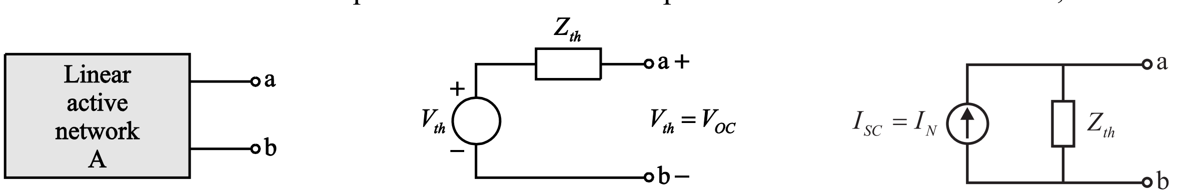
➤ Partial Synopsis

Thevenin's and Norton's Theorem :

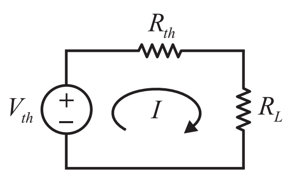
Statement of Thevenin's theorem: A linear network consisting of a number of voltage/current sources and resistances can be replaced by an equivalent network having a single voltage source called Thevenin's voltage (V_{th}) and a single resistance called Thevenin's resistance (R_{th}).

Statement of Norton's theorem : A linear network consisting of a number of voltage/current sources and resistances can be replaced by an equivalent network having a current source called Norton's current (I_N) and a single resistance called Norton's resistance (R_N).

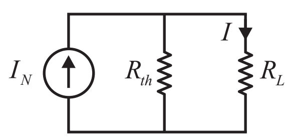
Thevenin's and Norton's equivalent circuits of complex network are shown below,



(a) General network (b) Its Thevenin's equivalent (c) Its Norton's equivalent



$$I = \frac{V_{th}}{R_{th} + R_L}$$



$$I = \frac{R_{th} I_N}{R_{th} + R_L}$$

Steps to find equivalent Thevenin's/Norton's resistance :

📖 Remember :

Case 1 : Circuit containing only independent sources

Voltage sources are replaced by short circuits and current sources are replaced by open circuits. Calculate equivalent resistance seen from open circuited load terminals,

$$R_{th} = R_{eq}$$

Case 2 : Circuit containing independent as well as dependent sources

Replace all independent voltage sources by short circuits and current sources by open circuits but keep dependent sources, then

$$R_{th} = \frac{V_{dc}}{I_{dc}} \qquad \left(\text{Also, } R_{th} = \frac{V_{OC}}{I_{SC}} = \frac{V_{th}}{I_N}\right)$$

Where $V_{dc} = dc$ voltage source applied across load terminals

And, $I_{dc} = dc$ current supplied by V_{dc}

Case 3 : Circuit containing only dependent sources

Keep dependent sources

$$R_{th} = \frac{V_{dc}}{I_{dc}}$$

Where $V_{dc} = dc$ voltage source applied across load terminals

And, $I_{dc} = dc$ current supplied by V_{dc}

In this case, Thevenin's equivalent voltage, $V_{th} = 0$ and $I_N = 0$ since, there is no independent source.

➢ Sample Questions

1991 IIT Madras

2.1 The *V-I* characteristics as seen from terminal pair (A, B) of the network of figure (a) is as shown in figure (b). If a variable resistance R_L is connected across the terminal pair (A, B) the maximum power that can be supplied to R_L would be

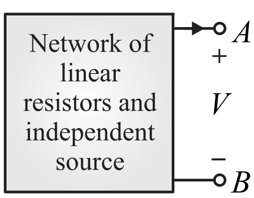


Fig. (a)

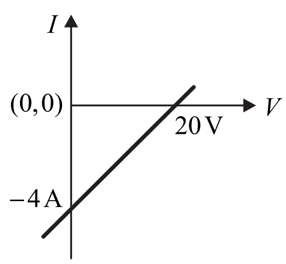
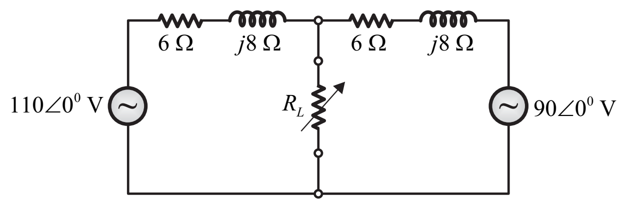


Fig. (b)

(A) 80 W
(B) 40 W
(C) 20 W
(D) Indeterminate unless the actual network is given

2003 IIT Madras

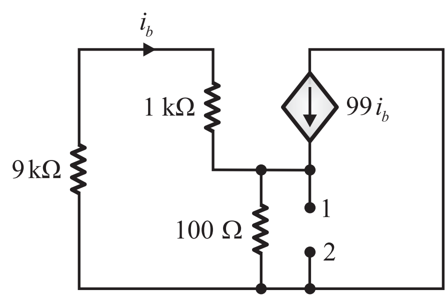
2.2 Two ac sources feed a common variable resistive load as shown in figure. Under the maximum power transfer condition, the power absorbed by the load resistance R_L is



(A) 2200 W   (B) 1250 W
(C) 1000 W   (D) 625 W

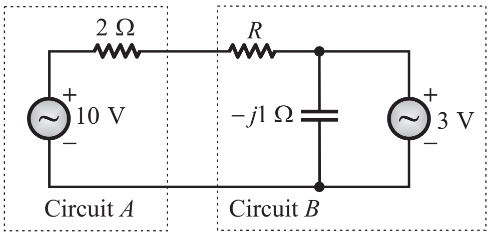
2012 IIT Delhi

2.3 The impedance looking into nodes 1 and 2 in the given circuit is



(A) 50 Ω   (B) 100 Ω
(C) 5 kΩ   (D) 10.1 kΩ

2.4 Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is

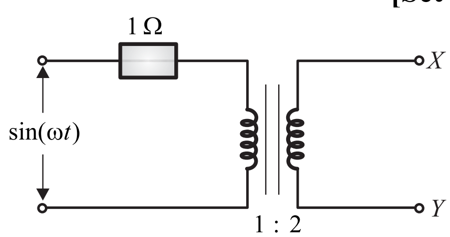


(A) 0.8 Ω (B) 1.4 Ω
(C) 2 Ω (D) 2.8 Ω

2014 IIT Kharagpur

2.5 Assuming an ideal transformer, the Thevenin's equivalent voltage and impedance as seen from the terminals X and Y for the circuit in figure are

[Set - 02]

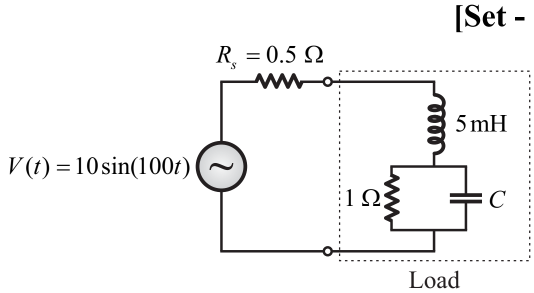


(A) $2\sin(\omega t)$ V, 4 Ω
(B) $1\sin(\omega t)$ V, 1 Ω
(C) $1\sin(\omega t)$ V, 2 Ω
(D) $2\sin(\omega t)$ V, 0.5 Ω

2017 IIT Roorkee

2.6 In the circuit shown below, the value of capacitor C required for maximum power to be transferred to the load is

[Set - 02]



(A) 1 nF (B) 1 μF
(C) 1 mF (D) 10 mF

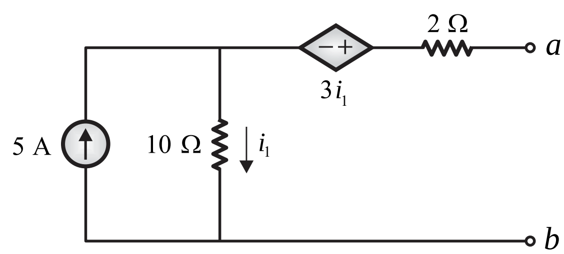
2020 IIT Delhi

2.7 A benchtop dc power supply acts as an ideal 4 A current source as long as its terminal voltage is below 10 V. Beyond this point, it begins to behave as an ideal 10 V voltage source for all load currents going down to 0 A, When connected to an ideal rheostat, find the load resistance value at which maximum power is transferred and the corresponding load voltage and current.

(A) 2.5 Ω, 4 A, 10 V
(B) Short, ∞ A, 10 V
(C) 2.5 Ω, 4 A, 5 V
(D) Open, 4 A, 0 V

2021 IIT Bombay

2.8 For the network shown, the equivalent Thevenin voltage and Thevenin impedance as seen across terminals 'ab' is



(A) 10 V in series with 12 Ω
(B) 65 V in series with 15 Ω
(C) 50 V in series with 2 Ω
(D) 35 V in series with 2 Ω

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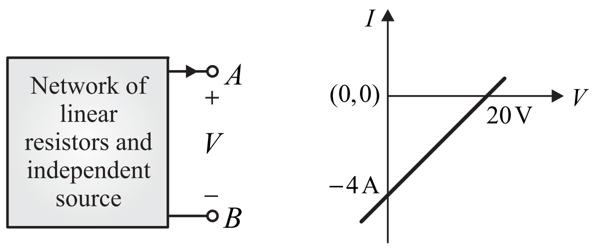
## Explanations		Network Theorems

2.1 (C)

Given network and its I-V characteristics are shown below,



From the given I-V curve, we can conclude that,

Sr.	Parameter	Comment
1.	Open terminal voltage across terminal AB i.e. $V_{OC} = V_{AB} = 20$ V	Open terminal / open circuit means $I = 0$ across the terminal
2.	Short circuit current across terminal AB i.e. $I_{SC} = I_{AB} = -4$ A	Short circuit terminal means $V = 0$ across the terminal

Hence, open terminal voltage across terminal *AB*,

$$V_{OC} = 20 \text{ V}$$

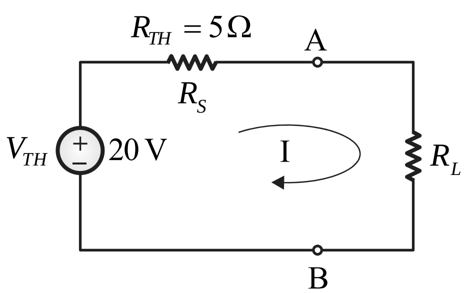
Hence, short circuit current through terminal *AB*,

$$I_{SC} = -4 \text{ A}$$

So the Thevenin equivalent (R_{TH}) resistance across the terminal AB is,

$$R_{TH} = \left|\frac{V_{OC}}{I_{SC}}\right| = \frac{20}{4} = 5\ \Omega$$

Thevenin's equivalent circuit :



In case of resistive network, maximum power transferred to R_L when, load resistance R_L is equal to source resistance R_S,

i.e. $\quad R_L = R_S = R_{TH} = 5\ \Omega$

So the current I in the circuit is given as,

$$I = \frac{20}{5+5} = 2 \text{ A}$$

Now, maximum power that can be supplied to R_L is given by,

$$P_{max} = I^2 R_L = (2)^2 \times 5 = 4 \times 5 = 20 \text{ W}$$

OR

$$P_{max} = \frac{V_{TH}^2}{4R_L} = \frac{(20)^2}{4 \times 5} = 20 \text{ W}$$

Hence, the correct option is (C).

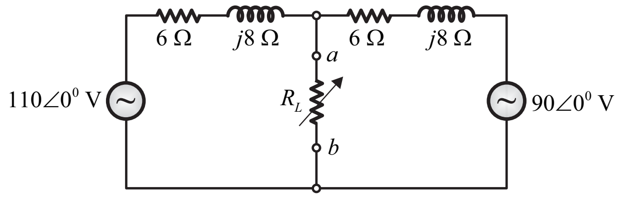
Key Point

1. Open circuit voltage across the given terminal in any network is called as Thevenin voltage (V_{TH}).
2. Short circuit current across the given terminal in any network is called as Norton current (I_N).
3. Thevenin equivalent resistance R_{TH} or Norton equivalent resistance R_N is calculated as,

$$R_{TH} = R_N = \frac{V_{TH}}{I_N}$$

2.2 (D)

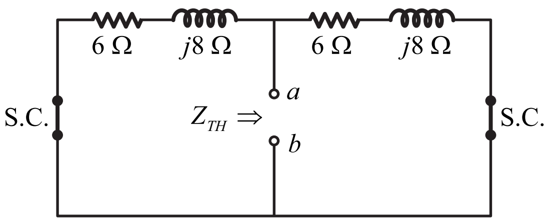
Given circuit with resistive load R_L is shown below,



For check in maximum power absorb by load, we have to convert whole circuit into Thevenin equivalent circuit so apply Thevenin theorem,

(i) Calculation of Z_{TH} :

(when dependent sources are not present) Replace all independent source by their internal resistance, i.e. short circuit all the independent voltage source ($R_{in}=0$) and open circuit all the independent current source $(R_{in}=\infty)$, then circuit becomes,

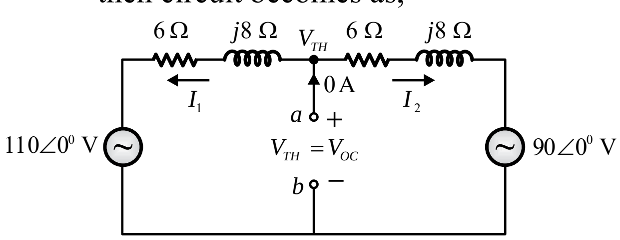


$$Z_{ab}=Z_{TH}=(6+j8)\,||\,(6+j8)$$

$$Z_{TH}=(3+j4)\ \Omega$$

(ii) Calculation of V_{TH} :

First remove R_L across terminal *a* and *b*, then circuit becomes as,



Applying KCL in the above circuit,

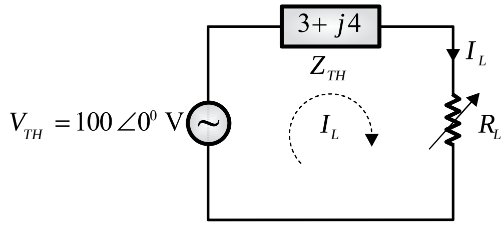
$$I_1+I_2=0$$

$$\frac{V_{TH}-110\angle 0^0}{6+j8}+\frac{V_{TH}-90\angle 0^0}{6+j8}=0$$

$$2V_{TH}=200\angle 0^0\ \text{V}$$

$$V_{TH}=100\angle 0^0\ \text{V}$$

Thevenin's equivalent circuit with load $\mathbf{R_L}$:



In case of complex network, maximum power will be transferred to R_L when,

$$R_L=|Z_{TH}|=\sqrt{3^2+4^2}=5\ \Omega$$

[In this case, load impedance R_L is real at source impedance Z_{TH} is complex so, for the transfer of maximum power to R_L we will take modulus of Z_{TH}]

Load current can be calculated from above circuit as,

$$I_L=\frac{V_{TH}}{3+j4+R_L}=\frac{100}{3+j4+5}$$

$$I_L=11.18\angle -26.56^0\ \text{A}$$

Power absorbed by R_L is given by,

$$P_{R_L}=|I_L|^2 R_L=11.18^2\times 5=625\ \text{W}$$

Hence, the correct option is (D).

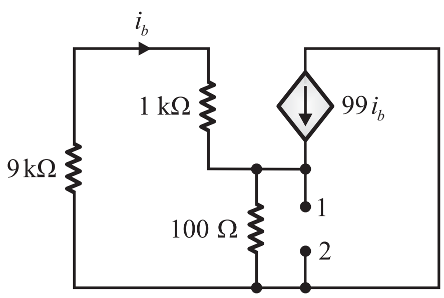
Tip : It is advisable to the aspirants, calculate V_{TH} of this question using the concept of super position theorem (as we did in question 2.5 under method 2) by yourself, it will give a new dimension in your thought process.

Table 2.1 : Maximum Power Transfer Theorem

S. No.	Z_S (Source Impedance)	Z_L (Load Impedance)	Condition for MPT	$P_L(\max)$ [Maximum power delivered to load]	%η (Efficiency)
1.	$R_S + j0$	$R_L + j0$	$R_L = R_S$	$\lvert I^2 \rvert R_L = \dfrac{V_s^2}{4R_s}$	50%
2.	$R_S + jX_S$	$R_L + jX_L$	$R_L = \sqrt{R_S^2 + (X_S + X_L)^2}$	$\lvert I^2 \rvert R_L$	< 50%
3.	$R_S + jX_S$	$R_L + jX_L$	$X_L + X_S = 0$	$\lvert I^2 \rvert R_L = \dfrac{V_s^2 R_L}{(R_s + R_L)^2}$	< 50%
4.	$R_S + jX_S$	$R_L + jX_L$	$Z_L = Z_S^*$	$\lvert I^2 \rvert R_L = \dfrac{V_s^2}{4R_s}$	50%
5.	$R_S + jX_S$	$R_L + j0$	$R_L = \sqrt{R_S^2 + X_S^2}$	$\lvert I^2 \rvert R_L$	< 50%
6.	$R_S + j0$	$R_L + jX_L$	$R_L = \sqrt{R_S^2 + X_L^2}$	$\lvert I^2 \rvert R_L$	< 50%

2.3 **(A)**

Given circuit is shown below,



Here, network contains dependent source only.

Method 1

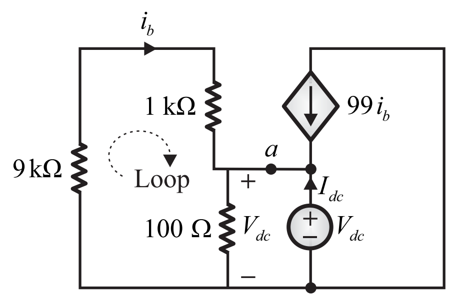
Calculation of R_{TH} :

(when dependent sources are present)

(i) Apply a voltage source 'V_{dc}' between terminal '1' and '2' and assume 'I_{dc}' current flowing to the network.

(ii) Replace all independent source by their internal resistance, i.e. short circuit all the independent voltage source ($R_{in} = 0$) and open circuit all the independent current source ($R_{in} = \infty$).

(iii) Dependent source remains as it is.



The Thevenin equivalent impedance is given by,

$$Z_{TH} = \frac{V_{dc}}{I_{dc}}$$

Applying KCL at node a, in above circuit,

$$-i_b + \frac{V_{dc}}{100} - 99i_b - I_{dc} = 0$$

$$\frac{V_{dc}}{100} - 100 i_b = I_{dc} \quad \ldots(i)$$

Applying KVL in the above shown loop,

$$10 \times 10^3 i_b + V_{dc} = 0$$

$$i_b = \frac{-V_{dc}}{10 \times 10^3} \quad \ldots(ii)$$

Put the value of i_b in equation (i),

$$\frac{V_{dc}}{100} - 100\left[\frac{-V_{dc}}{10^4}\right] = I_{dc}$$

$$\frac{2V_{dc}}{100} = I_{dc}$$

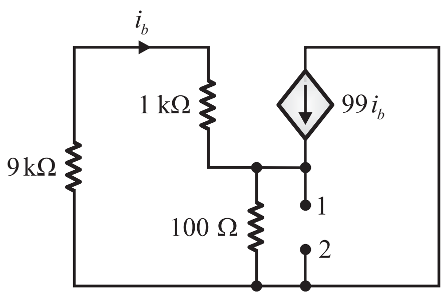
$$\frac{V_{dc}}{I_{dc}} = 50\ \Omega$$

$$Z_{TH} = 50\ \Omega$$

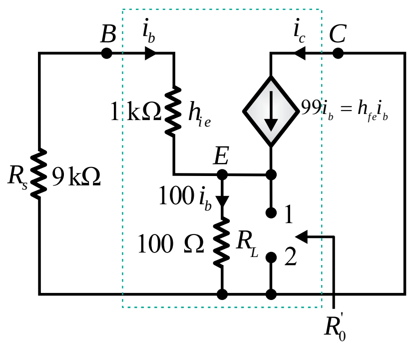
Hence, the correct option is (A).

Method 2

Given circuit is shown below,



Modified circuit is shown below,



Above shown circuit look like a common collector configuration because output is taken from emitter terminals and collector terminals is grounded. So, internal output impedance of common collector configuration is given by,

$$R_0 = \frac{R_s + h_{ie}}{(1+h_{fe})} = \frac{(9+1)10^3}{1+99} = 100\ \Omega$$

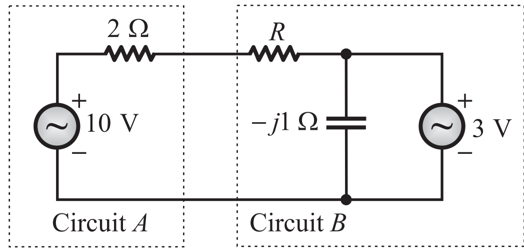
External output impedance is given by,

$$R_0' = R_0 \,||\, R_L = 100\,||\,100 = 50\ \Omega$$

Hence, the correct option is (A).

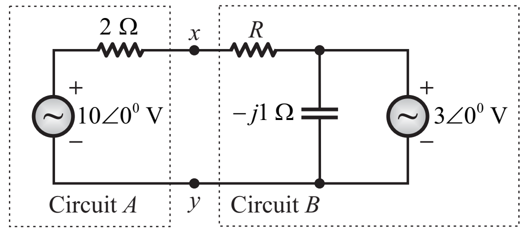
2.4 (A)

Given circuit is shown below,

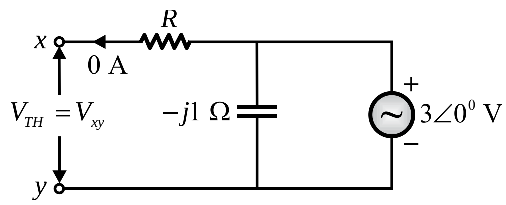


Method 1

Both the voltage sources are in same phase, let $10\angle 0^0$ and $3\angle 0^0$ V.



Circuit B is shown below,



Circuit B can easily converted into Thevenin equivalent as shown below,

Calculation of V_{TH} in circuit B :

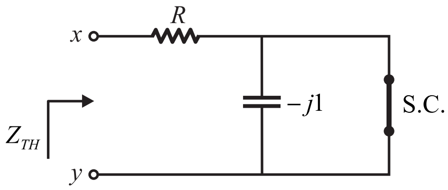
In the circuit B, V_{TH} and $3\angle 0^0$ V are in parallel. So, that Thevenin's or open circuit voltage across terminal xy is given by,

$$V_{TH} = V_{xy} = 3\angle 0^0\ \text{V}$$

Calculation of Z_{TH} in circuit B :

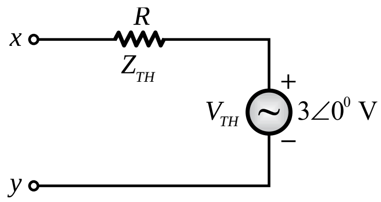
(when dependent sources are not present)

Replace all independent source by their internal resistance, i.e. short circuit all the independent voltage source ($R_{in} = 0$) and open circuit all the independent current source $(R_{in} = \infty)$, then circuit becomes as,

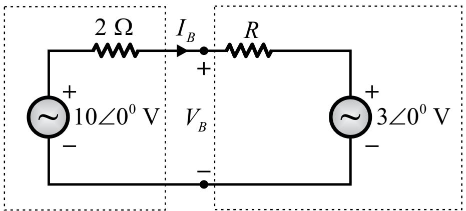


It is clear that, $Z_{TH} = R$

So, Thevenin's equivalent of circuit B is shown below,



So, replace circuit B by Thevenin equivalent circuit in main circuit, then main circuit becomes as,



Circuit A　　Circuit B

Current, $I_B = \dfrac{10-3}{R+2} = \dfrac{7}{(R+2)}\angle 0^0$

The voltage across circuit B is

$$V_B = I_B R + 3\angle 0^0 = \left(\frac{7R}{R+2}+3\right)\angle 0^0$$

Power delivered to circuit B by circuit A,

$$P_B = V_B I_B \cos 0^0 = \frac{7}{(R+2)}\left[\frac{7R}{R+2}+3\right]$$

$$P_B = \frac{7}{(R+2)}\left[\frac{10R+6}{R+2}\right] = 14\frac{(5R+3)}{(R+2)^2}$$

For maximum power $P_B(\text{max})$,

$$\frac{dP_B}{dR} = 0 \qquad \ldots\text{(i)}$$

$$\frac{dP_B}{dR} = \frac{\left[14(R+2)^2\times 5\right]-\left[14(5R+3)\times 2\times(R+2)\right]}{(R+2)^4}$$

From equation (i),

$$\frac{\left[14(R+2)^2\times 5\right]-\left[14(5R+3)\times 2\times(R+2)\right]}{(R+2)^4} = 0$$

$$14(R+2)^2\times 5 = 14(5R+3)\times 2\times(R+2)$$

$$70(R^2+4R+4) = 28(5R^2+13R+6)$$

$$70R^2+84R-112=0$$

$$R = -2,\ 0.8\ \Omega$$

Since, R = Negative [not possible]

So, $R = 0.8\ \Omega$

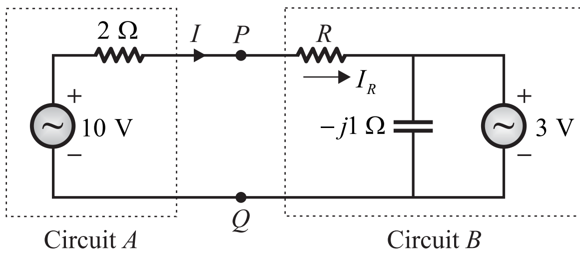
Hence, the correct option is (A).

Key Point

The impedance in parallel with voltage source and impedance in series with current source is **dummy impedance** i.e. they does not affect the equivalent circuit.

Method 2

Maximum power is transferred from circuit A to circuit B, if circuit B offers $2\ \Omega$ to circuit A.



Circuit A　　Circuit B

Fig. 1

So, replace circuit B by $2\,\Omega$ resistance, thus Thevenin equivalent of circuit A is given by,

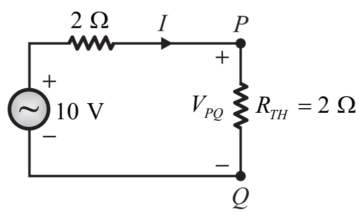


Fig. 2

Then, $I = \dfrac{10}{4} = 2.5$ A

and $V_{PQ} = I\times 2 = 2.5\times 2$

$V_{PQ} = 5$ Volt

From the circuit of figure 1,

$$I_R = \frac{V_{PQ}-3}{R}$$

$$I_R = \frac{5-3}{R} = \frac{2}{R}$$

and $\quad I_R = I \quad \Rightarrow \quad \frac{2}{R} = 2.5$

Therefore, $R = \frac{2}{2.5} = 0.8\ \Omega$

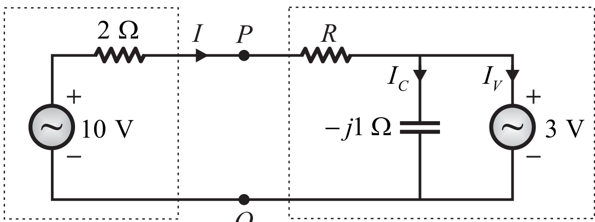
Hence, the correct option is (A).

Key Point

Circuit B is replaced by resistance of $2\ \Omega$ for maximum power transfer to circuit B by circuit A.

Method 3

Objective approach :



Circuit A Circuit B

Applying KVL in the above circuit,

$$I = \frac{10-3}{R+2} = \frac{7}{R+2}$$

Options	R (Ω)	I (A)
(A)	0.8	2.5
(B)	1.4	2.05
(C)	2.0	1.75
(D)	2.8	1.45

Circuit B will consume maximum power when maximum current will flow through the circuit and from the above table it is clear that maximum current is flowing when R is equal to $0.8\ \Omega$.

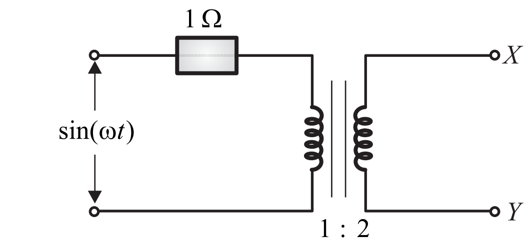
Hence, the correct option is (A).

Tip : It is specially design question and these type of questions requires good concept with clarity, so it is advisable to aspirants, analyze the solution and try to catch up the application of concept, instead of mug-up the concept.

Scan for Video Solution

2.5 **(A)**

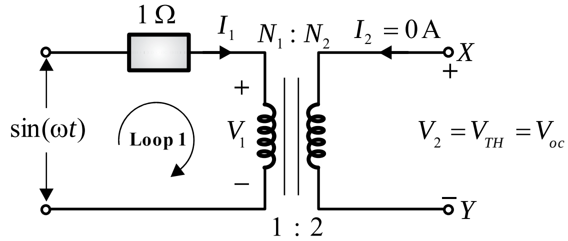
Given circuit is shown below,



Method 1

Apply Thevenin theorem,

(i) Calculation of V_{TH} across terminal XY:



Since, $I_2 = 0$ because of open circuit terminal of XY so, that according to transformer principle,

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$I_1 = \frac{N_2}{N_1} \times I_2$$

$$I_1 = 0\text{ A} \qquad (\because I_2 = 0\text{ A})$$

Apply KVL in loop (1),

$$\sin\omega t = I_1 \times 1 + V_1 \qquad \text{[From figure]}$$

$$V_1 = \sin\omega t$$

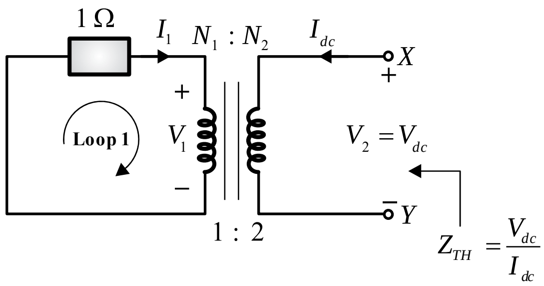
Again apply transformer principle,

$$\frac{V_1}{V_2} = \frac{V_1}{V_{TH}} = \frac{N_1}{N_2} = \frac{1}{2} \qquad \text{[From figure]}$$

$$V_{TH} = 2V_1$$

$$V_{TH} = 2\sin\omega t\text{ V}$$

(ii) Calculation of Z_{TH} across terminal XY:



For transformer principle,

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{1}{2}$$

$$\frac{V_1}{V_{dc}} = \frac{1}{2} \qquad [\because V_{dc} = V_2]$$

$$V_{dc} = 2V_1 \qquad \ldots(i)$$

Again applying transformer principle,

$$\frac{I_2}{I_1} = \frac{I_{dc}}{I_1} = -\frac{N_1}{N_2} = -\frac{1}{2}$$

$$I_{dc} = -\frac{I_1}{2} \qquad \ldots(ii)$$

$$Z_{TH} = \frac{V_{dc}}{I_{dc}} = -\frac{4V_1}{I_1} \qquad \ldots(iii)$$

Applying KVL in loop (1),

$$0 + I_1 \times 1 + V_1 = 0 \qquad \text{[From input side]}$$

$$\frac{V_1}{I_1} = -1$$

From equation (iii),

$$Z_{TH} = -4 \times (-1) = 4\,\Omega$$

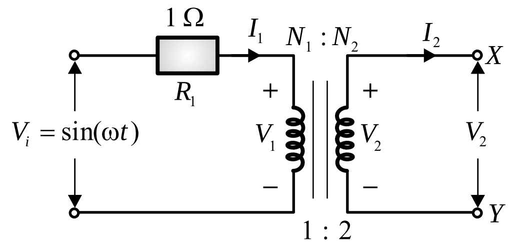
Thus, Thevenin voltage and impedance across terminal X and Y is $2\sin\omega t$ V and $4\,\Omega$ respectively.

Hence, the correct option is (A).

Method 2

Using the concept of transformer referred circuit :

Given circuit is shown below,



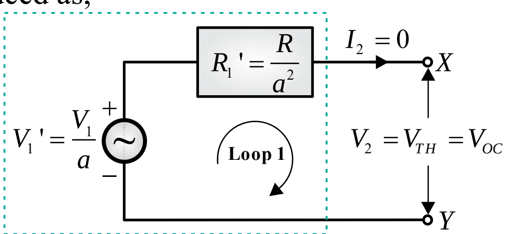
Calculation of V_{TH} across terminal XY :

From basic principle of transformer,

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{1}{2} = 0.5$$

and $$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{2} = 0.5 = a \text{ (Assuming)}$$

Now we can referred, whole primary side to secondary side of transformer and circuit can be reduced as,



(Referred circuit)

Here, $V_1' = \frac{V_1}{a} \Rightarrow$ primary voltage referred to secondary side.

$R_1' = \frac{R_1}{a^2} \Rightarrow$ primary resistance referred to secondary side.

From the circuit it is clear that, XY terminal is open circuited so that $I_2 = 0$ and due to basic transformer, I_1 is also 0, so that,

$$V_1 = V_i \qquad [\because I_1 = 0]$$

Thus, $$V_1' = \frac{V_i}{a} = \frac{V_1}{a} = \frac{\sin\omega t}{0.5} = 2\sin\omega t$$

$$R_1' = \frac{R_1}{a^2} = \frac{1}{0.5^2} = 4\,\Omega$$

Apply KVL in loop (1) of referred circuit,

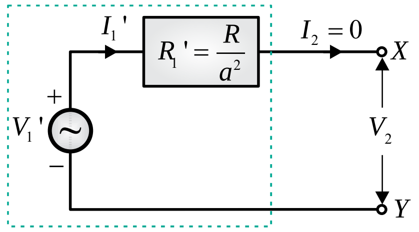
$$V_1' - R_1' I_2 - V_{TH} = 0$$

$$V_1' - 4 \times 0 - V_{TH} = 0$$

$$V_{TH} = V_1' = 2\sin\omega t \text{ V}$$

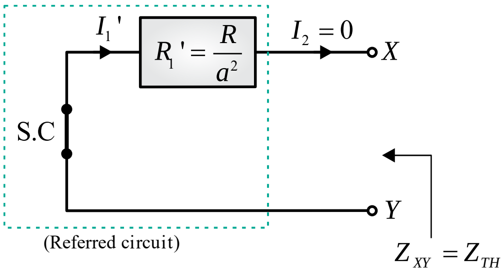
Calculation of Z_{TH} across terminal XY :

From the secondary referred circuit as shown below,



(Referred circuit)

For calculation of Z_{TH} across terminal XY, independent voltage source is short circuited as shown below,



(Referred circuit)

So, $Z_{XY} = Z_{TH} = R_1' = 4\Omega$

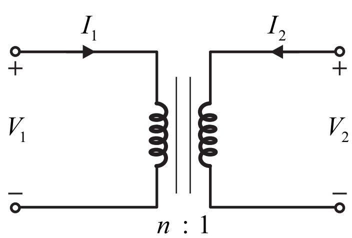
Thus, Thevenin voltage and impedance across terminal X and Y is $2\sin\omega t$ V and $4\,\Omega$ respectively.

Hence, the correct option is (A).

Scan for Video Solution

Key Point

Concept of transformer in two-port network:



(i) $\dfrac{V_1}{V_2} = \dfrac{n}{1} = \dfrac{N_1}{N_2}$

(ii) $\dfrac{I_2}{I_1} = -\dfrac{n}{1} = -\dfrac{N_1}{N_2}$

Note : Negative sign due to output two-port current.

2.6 (D)

Given circuit is shown below,

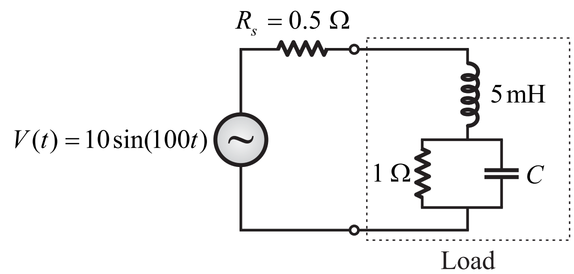


Fig. (a)

Here, $\omega = 100$ rad/sec

$$X_L = j\omega L = j100 \times 5 \times 10^{-3} = j0.5\Omega$$

$$X_C = \frac{1}{j\omega C} = \frac{1}{j100C}\Omega$$

So, figure (a) becomes as,

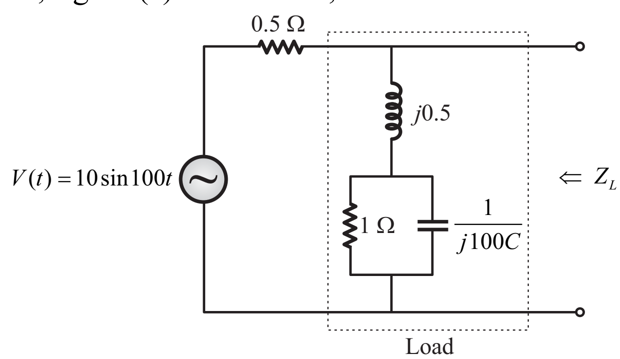


Fig. (b)

Equivalent load impedance,

$$Z_L = j0.5 + \left(1 \| \frac{1}{j100C}\right)$$

$$Z_L = j0.5 + \frac{1}{1 + j100C}$$

$$Z_L = j0.5 + \frac{1 - j100C}{1 + 10^4 C^2}$$

$$Z_L = \underbrace{\frac{1}{1+10^4C^2}}_{\text{Resistive part}} + \underbrace{j\left(0.5 - \frac{100C}{1+10^4C^2}\right)}_{\text{Reactive part}} \quad \ldots\text{(i)}$$

$$Z_L = R_L + jX_L$$

$R_L = f(C)$ and $Z_L = f(C)$

$$Z_L = R_L + jX_L$$

[R_L and X_L both are the function of C, it means C is varied then Z_L is also varied.]

For maximum power transfer,

$$Z_L = Z_s^* = R_s = 0.5\ \Omega$$

So, equation (i) becomes as,

$$0.5 = \frac{1}{1+(100)^2C^2} + j\left[0.5 - \frac{100C}{1+100^2C^2}\right] \quad \ldots\text{(ii)}$$

Case 1 : Comparing real part of equation (ii),

$$0.5 = \frac{1}{1+(100)^2C^2}$$

$$1+(100)^2C^2 = 2$$

$$(100)^2C^2 = 1$$

$$C^2 = \frac{1}{(100)^2}$$

$$C = \frac{1}{100} = 0.01\,\text{F}$$

$$C = 10\,\text{mF}$$

Hence, the correct option is (D).

Case 2 : Comparing imaginary part of equation (ii),

$$0.5 = \frac{100C}{1+(100)^2C^2}$$

$$1+(100)^2C^2 = 200C$$

$$100^2C^2 - 200 + 1 = 0$$

$$C^2 - \frac{200C}{10000} + \frac{1}{10000} = 0$$

$$C^2 - \frac{C}{50} + \frac{1}{10000} = 0$$

$$C = \frac{\frac{1}{50} \pm \sqrt{\frac{1}{2500} - \frac{4}{10000}}}{2}$$

$$C = \frac{1}{100} = 0.01\,\text{F} = 10\,\text{mF}$$

Hence, the correct option is (D).

Case 3 : For maximum power to be delivered to load from source so, load must be purely resistive i.e. reactive part of Z_L must be zero.

Thus, $$j0.5 - \frac{j100C}{1+10^4C^2} = 0$$

$$0.5 = \frac{100C}{1+10^4C^2}$$

$$1+10^4C^2 = 200C$$

$$10^4C^2 - 200C + 1 = 0$$

$$(100C-1)^2 = 0$$

$$C = 10\ \text{mF}$$

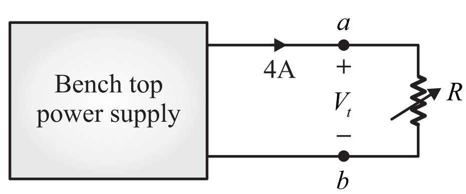
Hence, the correct option is (D).

2.7　(A)

Method 1

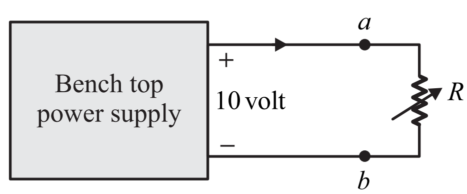
Case I :

Benchtop dc power supply will act as a ideal 4 A current source as long as it a terminal voltage is less than 10 V



Case II :

Benchtop dc power supply will act as a 10 V ideal voltage source for all load currents going down to 0 A



From **Case I** the maximum resistance offered by the ideal 4 A current source is

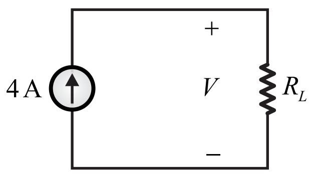
$$R = \frac{V}{I} = \frac{10}{4} = 2.5\,\Omega$$

For maximum power transfer the value of load resistance should be equal to source resistance = 2.5

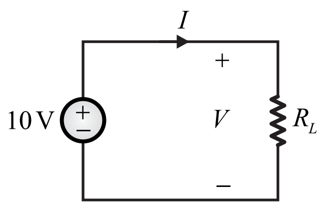
Current and voltage corresponding to load resistance of $2.5\,\Omega$ are 4 A and 10 Volt respectively.

Hence, the correct option is (A).

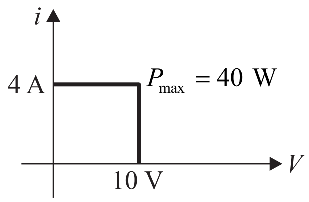
Method 2



$0 < V < 10$ V



$0 < I < 4$



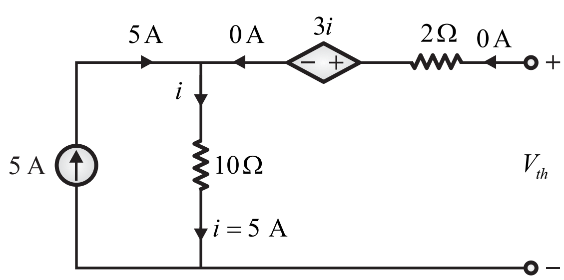
From graph, it is clear that at maximum power $V = 10$ V and $i = 4$ A .

So, $4^2 \times R_L = 40$ $\qquad \dfrac{10^2}{R_L} = 40$

$R_L = 2.5\,\Omega$ $\qquad R_L = 2.5\,\Omega$

2.8 (B)

Calculation of V_{th} :



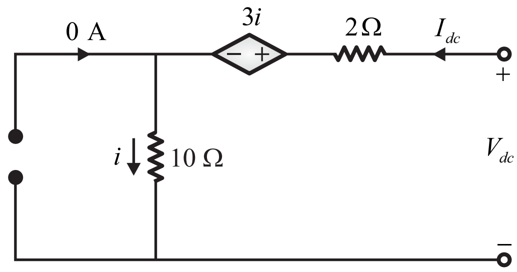
From the circuit diagram,

$$i = 5 \text{ A}$$

$$V_{th} = 3i + 10i = 13i$$

$$V_{th} = 65 \text{ V}$$

Calculation of R_{th} :



From the circuit diagram,

$$i = I_{dc}$$

$$V_{dc} = 2I_{dc} + 3i + 10i$$

$$V_{dc} = 2I_{dc} + 3I_{dc} + 10I_{dc}$$

$$\frac{V_{dc}}{I_{dc}} = 15\ \Omega$$

Hence, the correct option is (B).

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3 Two-Port Networks

➢ Partial Synopsis

Summary :

Sr.	Parameter	Dependent variables	Independent variables	Equation	Matrix Form
1.	Z - Parameter	V_1, V_2	I_1, I_2	$V_1 = Z_{11}I_1 + Z_{12}I_2$ $V_2 = Z_{21}I_1 + Z_{22}I_2$	$[Z]_{2\times2} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$
2.	Y - Parameter	I_1, I_2	V_1, V_2	$I_1 = Y_{11}V_1 + Y_{12}V_2$ $I_2 = Y_{21}V_1 + Y_{22}V_2$	$[Y]_{2\times2} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$
3.	h - parameter	V_1, I_2	I_1, V_2	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$	$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$
4.	g - Parameter	I_1, V_2	V_1, I_2	$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$	$[g] = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}_{2\times2}$
5.	ABCD Parameter	V_1, I_1	$V_2, -I_2$	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$	$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{2\times2}$
6.	ABCD Inverse/ T Inverse Parameter	V_2, I_2	$V_1, -I_1$	$V_2 = A^{-1}V_1 - B^{-1}I_1$ $I_2 = C^{-1}V_1 - D^{-1}I_1$	$[ABCD]^{-1} = [T]^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Symmetricity & Reciprocity Conditions :

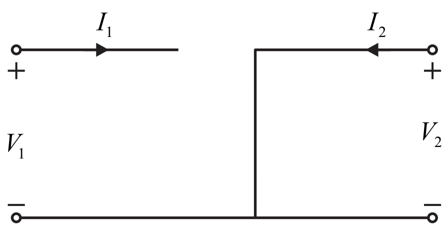
Parameter	Condition for Symmetry	Condition for Reciprocity
Z	$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
Y	$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$

h	$\Delta h = \lvert h \rvert = h_{11}h_{22} - h_{21}h_{12} = 1$	$h_{12} = -h_{21}$
g	$\Delta g = \lvert g \rvert = g_{11}g_{22} - g_{12}g_{21} = 1$	$g_{12} = -g_{21}$
$ABCD$	$A = D$	$\lvert T \rvert = AD - BC = 1$
$A^{-1}B^{-1}C^{-1}D^{-1}$	$A^{-1} = D^{-1}$	$A^{-1}D^{-1} - B^{-1}C^{-1} = 1$

➢ Sample Questions

2006 IIT Kharagpur

3.1 The parameter type and the matrix representation of the relevant two-port parameters that describe the circuit shown are



(A) z parameters, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

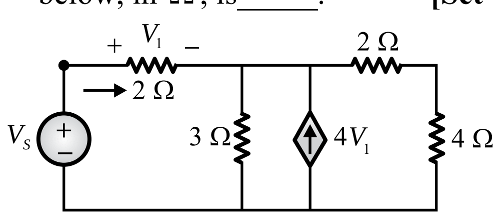
(B) h parameters, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) g parameters, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) z parameters, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

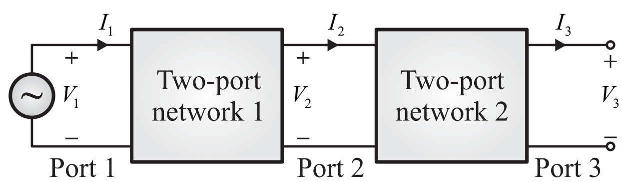
2016 IISc Bangalore

3.2 The driving point input impedance seen from the source V_S of the circuit shown below, in Ω, is______. **[Set - 02]**



2017 IIT Roorkee

3.3 Two passive two-port networks are connected in cascade as shown in figure. A voltage source is connected at port 1. **[Set - 01]**



Given : $V_1 = A_1V_2 + B_1I_2$, $V_2 = A_2V_3 + B_2I_3$

$I_1 = C_1V_2 + D_1I_2$, $I_2 = C_2V_3 + D_2I_3$

A_1, B_1, C_1, D_1, A_2, B_2, C_2 and D_2 are the generalized circuit constants. If the Thevenin equivalent circuit at port 3 consists of a voltage source V_T and an impedance Z_T connected in series, then

(A) $V_T = \dfrac{V_1}{A_1A_2}$, $Z_T = \dfrac{A_1B_2 + B_1D_2}{A_1A_2 + B_1C_2}$

(B) $V_T = \dfrac{V_1}{A_1A_2 + B_1C_2}$, $Z_T = \dfrac{A_1B_2 + B_1D_2}{A_1A_2}$

(C) $V_T = \dfrac{V_1}{A_1 + A_2}$, $Z_T = \dfrac{A_1B_2 + B_1D_2}{A_1 + A_2}$

(D) $V_T = \dfrac{V_1}{A_1A_2 + B_1C_2}$, $Z_T = \dfrac{A_1B_2 + B_1D_2}{A_1A_2 + B_1C_2}$

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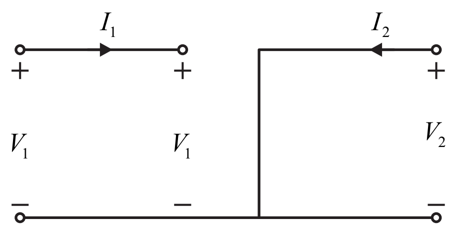
Explanations **Two-Port Networks**

3.1 (C)

Given circuit is shown below,



Port 1 is open circuited, so $I_1 = 0, V_1 \neq 0$

Port 2 is short circuited, so $V_2 = 0, I_2 \neq 0$

Now checking from options one by one

From option (A) :

Standard Z parameter equations,

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Therefore,

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{V_1}{0} = \infty\ \Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{V_1}{I_2}\ \Omega$$

$$Z_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{0}{I_1} = 0\ \Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{0}{I_2} = 0\ \Omega$$

Thus Z parameter are

$$[Z] = \begin{bmatrix} \infty & \dfrac{V_1}{I_2} \\ 0 & 0 \end{bmatrix}$$

Hence, the option A is incorrect.

From option (B) :

Standard h parameter equation,

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Therefore,

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{V_1}{0} = \infty\ \Omega$$

$$h_{12} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{V_2}{0} = \infty$$

$$h_{21} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{I_2}{0} = \infty\ \Omega^{-1}$$

$$h_{22} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{V_1}{0} = \infty$$

Thus h parameter are

$$[h] = \begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$$

Hence, the option B is incorrect.

From option (C) :

Standard g-parameter equation :

$$\boldsymbol{I_1 = g_{11}V_1 + g_{12}I_2}$$

$$\boldsymbol{V_2 = g_{21}V_1 + g_{22}I_2}$$

Therefore,

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \frac{0}{V_1} = 0\ \Omega^{-1}$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = \frac{0}{I_2} = 0$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{0}{V_1} = 0$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{0}{I_2} = 0\ \Omega$$

Therefore g parameters are,

$$[g] = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, the correct option is (C).

From option (D) :

Standard Z parameter equations,

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Therefore,

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{V_1}{0} = \infty\ \Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{V_1}{I_2}\ \Omega$$

$$Z_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{0}{I_1} = 0\ \Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{0}{I_2} = 0\ \Omega$$

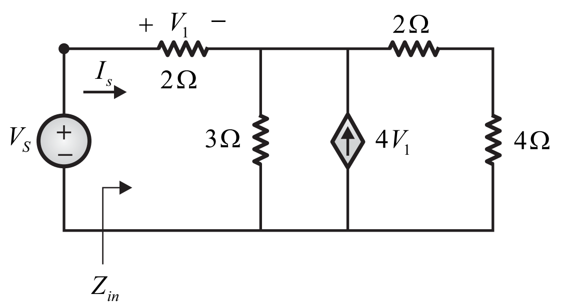
Thus Z parameter are

$$[Z] = \begin{bmatrix} \infty & \dfrac{V_1}{I_2} \\ 0 & 0 \end{bmatrix}$$

Hence, the option D is incorrect.

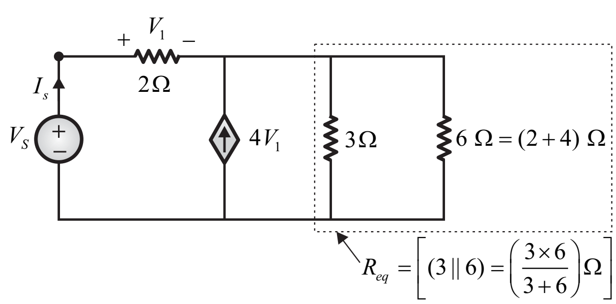
3.2 20

Given circuit is shown below,

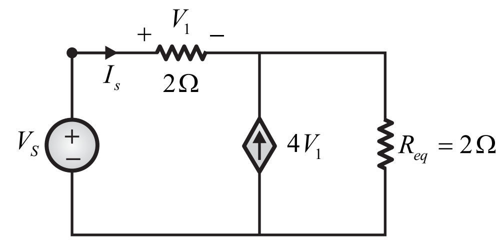


Driving point input impedance is given by,

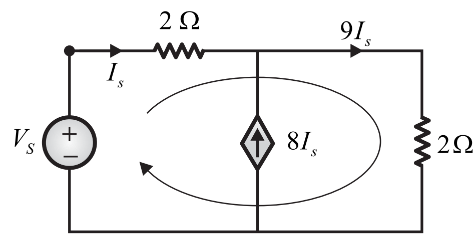
$$Z_{in} = \frac{V_s}{I_s}$$



Above circuit can be reduced as,



From figure, $I_s = \dfrac{V_1}{2}$



Applying KVL in the above super loop,

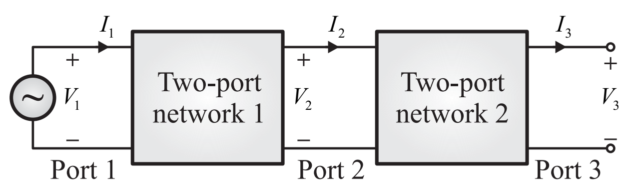
$$-V_s + 2I_s + 9I_s \times 2 = 0$$

$$-V_s + 20I_s = 0$$

$$Z_{in} = \frac{V_s}{I_s} = 20\ \Omega$$

Hence, the driving point input impedance seen from the source V_S is **20 Ω.**

3.3 (D)



Overall *ABCD* parameters of two passive two-port network connected in cascade is given by,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ A_2C_1 + D_1C_2 & C_1B_2 + D_1D_2 \end{bmatrix}$$

Hence, $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ A_2C_1 + D_1C_2 & C_1B_2 + D_1D_2 \end{bmatrix}\begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$

(i) Calculation of V_{TH} :

$$V_{TH} = V_3\big|_{I_3=0}$$

Hence for $I_3 = 0$,

$$V_1 = (A_1A_2 + B_1C_2)V_3$$

Hence, $V_3 = \dfrac{V_1}{A_1A_2 + B_1C_2} = V_{TH}$

(ii) Calculation of Z_{TH} :

$$Z_{TH} = -\frac{V_3}{I_3}\bigg|_{V_1=0\ \text{(i.e. Independent source = 0)}}$$

[Negative sign is due to the direction of current I_3]

Hence for $V_1 = 0$,

$$0 = (A_1A_2 + B_1C_2)V_3 + (A_1B_2 + B_1D_2)I_3$$

$$-\frac{V_3}{I_3} = \frac{A_1B_2 + B_1D_2}{A_1A_2 + B_1C_2}$$

$$Z_{TH} = \frac{A_1B_2 + B_1D_2}{A_1A_2 + B_1C_2}$$

Hence, the correct option is (D).

Key Point

1. When 2-port networks are cascaded then there individual ABCD parameter matrices are multiplied together.
2. When 2-port networks are connected in series form then there individual Z parameter matrices are added together.
3. When 2-port networks are connected in parallel form then there individual Y parameter matrices are added together.
4. When 2-port networks are connected in series-parallel form then there individual *h*-parameter matrices are added together.
5. When 2-port networks are connected in parallel-series form then there individual *g*-parameter matrices are added together.

Scan for Video Solution

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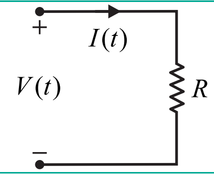
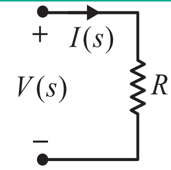
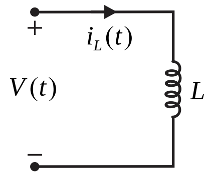
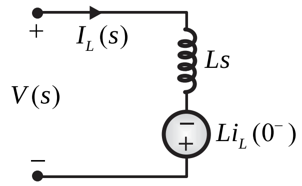
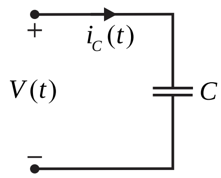
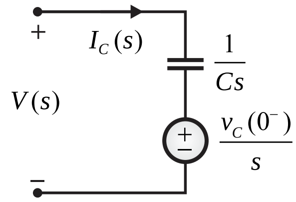
4 Transient Analysis

➢ Partial Synopsis

Transform Networks :

Circuit Elements	Time Domain Equations	Time Domain Circuit	Frequency Domain Equations	Frequency Domain Circuit
Resistance (opposes the flow of current)	$V(t) = R\,I(t)$		$V(s) = R\,I(s)$	
Inductance (opposes the change of current) $I_L(0^-) = I_L(0)$ $= I_L(0^+)$	$V_L(t) = L\dfrac{d}{dt}I_L(t)$ $I_L(t) = \dfrac{1}{L}\int_{-\infty}^{t} V_L(t)\,dt$ $I_L(t) = I_L(0^-) + \dfrac{1}{L}\int_0^t V_L(t)\,dt$		$V_L(s) = LsI_L(s) - LI_L(0^-)$ $I_L(s) = \dfrac{V_L(s)}{Ls} + \dfrac{I_L(0^-)}{s}$	
Capacitance (opposes the change of voltage) $V_C(0^-) = V_C(0)$ $= V_C(0^+)$	$I_C(t) = C\dfrac{d}{dt}V_C(t)$ $V_C(t) = \dfrac{1}{C}\int_{-\infty}^{t} I_C(t)\,dt$ $V_C(t) = V_C(0^-) + \dfrac{1}{C}\int_0^t I_C(t)\,dt$		$V_C(s) = \dfrac{I_C(s)}{Cs} + \dfrac{V_C(0^-)}{s}$ $I_C(s) = CsV_C(s) - CV_C(0^-)$	

📖 Remember :

Following equations can be used to find response of RL and RC circuits after switching :

1. If the switching occurs at $t = 0$.

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right]e^{-t/\tau} \qquad v(t) = v(\infty) + \left[v(0) - v(\infty)\right]e^{-t/\tau}$$

2. If the switching occurs at $t = t_0$.

$$i(t) = i(\infty) + \left[i(t_0) - i(\infty)\right]e^{-(t-t_0)/\tau} \qquad v(t) = v(\infty) + \left[v(t_0) - v(\infty)\right]e^{-(t-t_0)/\tau}$$

3. Both equation can be used only when R, L, C is constant not a function of time for example if $R(t)$ i.e. R is a function of time then we can't use above equation.

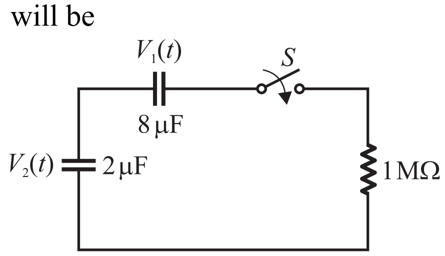
To find different parameters of RL circuit, following facts can be used.

Parameters to find for $t > 0$	$t = 0^+$	$t = \infty$	Requirements
$I_L(t)$	$I_L(0^+) = I_L(0^-)$	$I_L(\infty) =$ Current through inductor under short circuit condition	$I_L(t)$ for $t = \infty$ & $t = 0^-$
$V_L(t)$	$V_L(0^+) \neq V_L(0^-)$	$V_L(\infty) = 0$ volt always	$V_L(t)$ for $t = 0^-$ & $t = 0^+$
$I_R(t)$	$I_R(0^+) \neq I_R(0^-)$	$I_R(\infty) =$ to be calculated under steady state condition with inductor short circuited	$I_R(t)$ for $t = 0^-, 0^+ \& \infty$
$V_R(t)$	$V_R(0^+) \neq V_R(0^-)$	$V_R(\infty) =$ to be calculated under steady state condition with inductor short circuited	$V_R(t)$ for $t = 0^-, 0^+ \& \infty$
$V_C(t)$	$V_C(0^+) = V_C(0^-)$	$V_C(\infty) =$ voltage across capacitor under open circuit condition	$V_C(t)$ for $t = \infty$ & $t = 0^-$
$I_C(t)$	$I_C(0^-) \neq I_C(0^+)$	$I_C(\infty) = 0$ volt always	$I_C(t)$ for $t = 0^-$ & $t = 0^+$
$V_R(t)$	$V_R(0^-) \neq I_R(0^+)$	$V_R(\infty) =$ to be calculated under steady state condition with capacitor open circuited	$V_R(t)$ for $t = 0^-, 0^+ \& \infty$
$I_R(t)$	$I_R(0^-) \neq V_R(0^+)$	$I_R(\infty) =$ to be calculated under steady state condition with inductor short circuited	$I_R(t)$ for $t = 0^-, 0^+ \& \infty$

➢ Sample Questions

1991 IIT Madras

4.1 The switch S in figure is closed at $t = 0$. If $V_2(0) = 10$ V and $V_1(0) = 0$ V respectively, voltage across capacitors in steady state will be



(A) $V_2(\infty) = V_1(\infty) = 0$ V

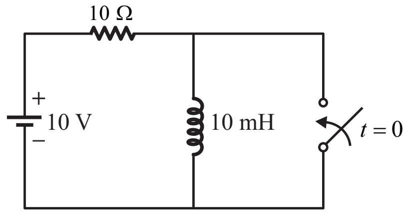
(B) $V_2(\infty) = 2$ V, $V_1(\infty) = 1$ V

(C) $V_2(\infty) = V_1(\infty) = 8$ V

(D) $V_2(\infty) = V_1(\infty) = 2$ V

2005 IIT Bombay

4.2 The circuit shown in the figure is in the steady state, when the switch is closed at t = 0. Assuming that the inductance is ideal, the current through the inductor at $t = 0^+$ will be

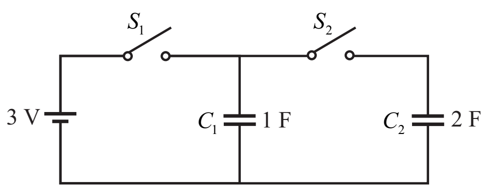


(A) 0 A (B) 0.5 A

(C) 1 A (D) 2 A

2009 IIT Roorkee

4.3 In the figure shown, all elements used are ideal. For time $t<0$, S_1 remained closed and S_2 open. At $t=0$, S_1 is opened and S_2 is closed. If the voltage V_{C_2} across the capacitor C_2 at $t=0$ is zero, the voltage across the capacitor combination at $t=0^+$ will be

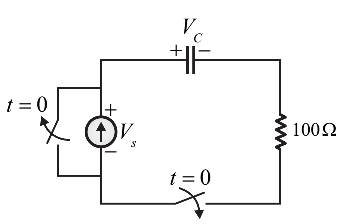


(A) 1 V (B) 2 V

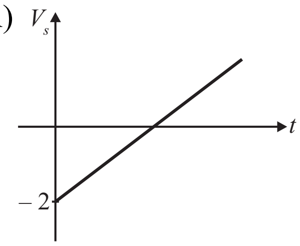
(C) 1.5 V (D) 3 V

2014 IIT Kharagpur

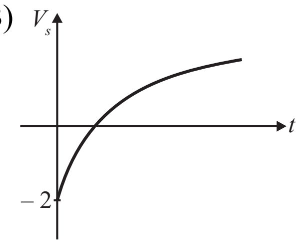
4.4 A combination of 1 μF capacitor with an initial voltage $V_C(0)=-2\text{ V}$ in series with a 100 Ω resistor is connected to a 20 mA ideal dc current source by operating both switches at $t=0$ sec as shown. Which of the following graphs shown in the options approximates the voltage V_s across the current source over the next few seconds? **[Set - 01]**



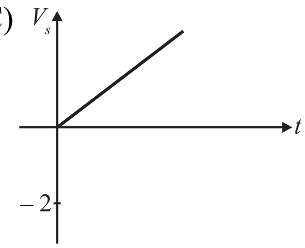
(A)



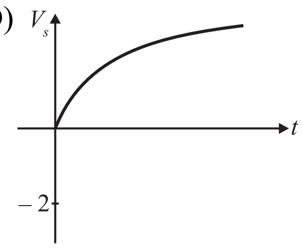
(B)



(C)

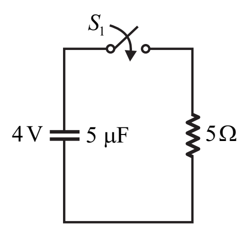


(D)



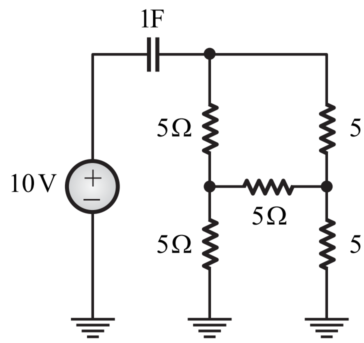
2016 IISc Bangalore

4.5 In the circuit shown below, the initial capacitor voltage is 4 V. Switch S_1 is closed at $t=0$. The charge (in μC) lost by the capacitor from $t=25$ μs to $t=100$ μs is _______. **[Set - 02]**



2017 IIT Roorkee

4.6 The initial charge in the 1 F capacitor present in the circuit shown is zero. The energy in joules transferred from the dc source until steady state condition is reached equals _____. (Give the answer up to one decimal place.) **[Set - 02]**



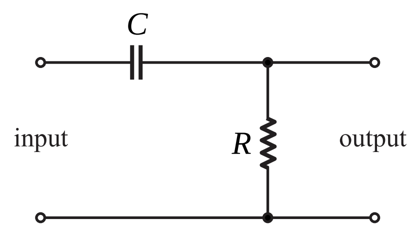
2020 IIT Delhi

4.7 A resistor and a capacitor are connected in series to a 10 V dc supply through a switch. The switch is closed at $t=0$ and the capacitor voltage is found to cross 0 V at $t=0.4\,\tau$, where τ is circuit time constant. The absolute value of percentage change required in the initial capacitor voltage if the zero crossing has to happen at $t=0.2\tau$ is ______ (rounded off to 2 decimal places).

2021 IIT Bombay

4.8 A signal generator having a source resistance of 50 Ω is set to generate a 1 kHz sinewave. Open circuit terminal voltage is 10 V peak-to-peak. Connecting a capacitor across the terminals reduces the voltage to 8 V peak-to-peak. The value of this capacitor is ________ μF . (Round off to 2 decimal places)

4.9 A 100 Hz square wave, switching between 0 V and 5 V, is applied to a CR high-pass filter circuit as shown. The output voltage waveform across the resistor is 6.2 V peak-to-peak. If the resistance R is 820 Ω, then the value C is ______ μF. (Round off to 2 decimal places.)

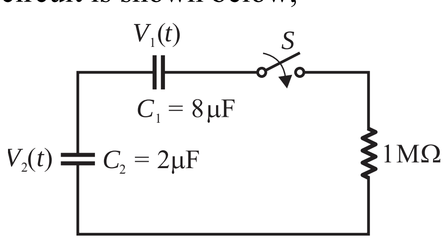


❖❖❖❖

Explanations Transient & Steady State Response

4.1 (D)

Given circuit is shown below,

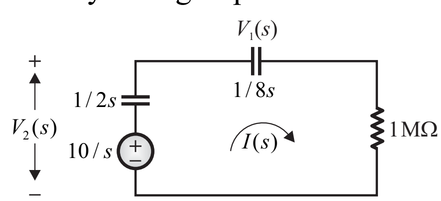


Given : $V_2(0^-)=10\,\text{V},\ V_1(0^-)=0\,\text{V}$

Method 1

Transform domain ($t\geq 0$) :

When switch is closed and convert whole circuit in s-domain by taking Laplace transform.



Applying KVL in above figure,

$$-\frac{10}{s}+I(s)\left[\frac{1}{2s}+\frac{1}{8s}+1\right]=0$$

$$I(s)=\frac{\frac{10}{s}}{\frac{1}{2s}+\frac{1}{8s}+1}=\frac{\frac{10}{s}}{\frac{4+1+8s}{8s}}=\frac{80}{5+8s}$$

From the circuit,

$$V_1(s)=I(s)\times\frac{1}{8s}=\frac{80}{5+8s}\times\frac{1}{8s}$$

At steady state,

$$V_1(\infty)=\lim_{s\to 0} sV_1(s)$$

$$V_1(\infty)=\lim_{s\to 0}\left(s\times\frac{80}{5+8s}\times\frac{1}{8s}\right)$$

$$V_1(\infty)=\frac{80}{5\times 8}=2\text{ V}$$

From the circuit,

$$V_2(s)=\frac{10}{s}-I(s)\times\frac{1}{2s}=\frac{10}{s}-\frac{80}{5+8s}\times\frac{1}{2s}$$

At steady state,

$$V_2(\infty)=\lim_{s\to 0} sV_2(s)$$

$$V_2(\infty)=\lim_{s\to 0}\left[s\times\left(\frac{10}{s}-\frac{80}{5+8s}\times\frac{1}{2s}\right)\right]$$

$$V_2(\infty)=\lim_{s\to 0}\left(10-\frac{40}{5+8s}\right)=10-8=2\text{ V}$$

Hence, the correct option is (D).

Method 2

When two initially charged capacitors are connected in series, then as soon as steady state condition is reached then steady state voltage across both capacitor becomes equal we can use standard formula for calculation of voltage across the capacitors in steady state i.e.,

$$V_1(\infty)=V_2(\infty)=\frac{V_1(0^-)C_1+V_2(0^-)C_2}{C_1+C_2}$$

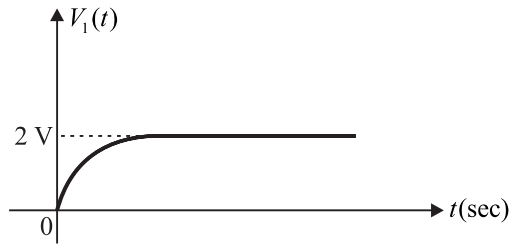
Given : $V_2(0^-)=10\text{ V}$ and $V_1(0^-)=0\text{ V}$

$C_1=8\ \mu\text{F}$ and $C_2=2\ \mu\text{F}$

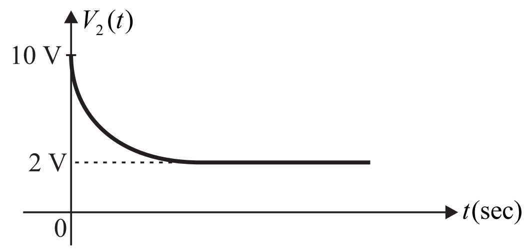
$$V_2(\infty)=V_1(\infty)=\frac{0\times 8\times 10^{-6}+10\times 2\times 10^{-6}}{8\times 10^{-6}+2\times 10^{-6}}$$

$$V_1(\infty)=V_2(\infty)=\frac{20\times 10^{-6}}{10\times 10^{-6}}=2\text{ V}$$

Thus capacitor C_1 voltage follow the charging graph because initial voltage is 0 volt and final is 2 volt as shown below,



Thus capacitor C_2 voltage follow the discharging graph because initial voltage is 10 volt and final is 2 volt as shown below,



Hence, the correct option is (D).

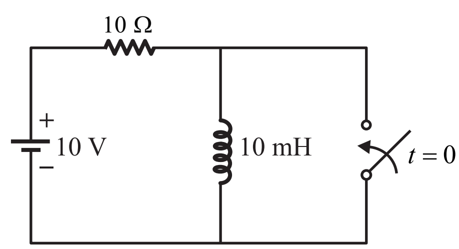
Key Point

According to principle conservation of charge, initial charge in a circuit must be equal to final charge in the circuit.

i.e. $Q_{initial}=Q_{final}$

4.2 (C)

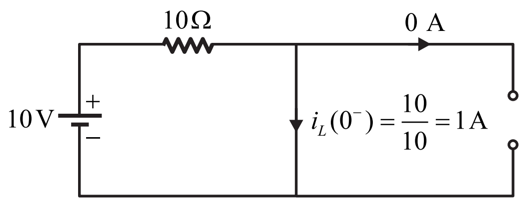
Given circuit is shown below,



At $t=0^-$ / $t<0$ / steady state :

Initially switch was open.

In steady state, inductor behaves as a short circuit.



From figure, $i_L(0^-) = 1\,\text{A}$

From the property of inductor,

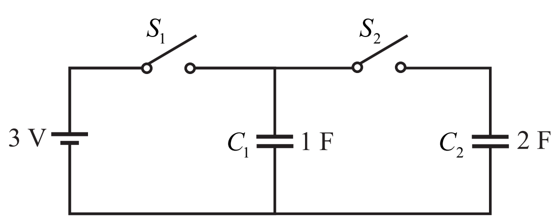
$$i_L(0^-) = i_L(0^+) = 1\,\text{A}$$

Hence, the correct option is (C).

Scan for Video Solution

4.3 (A)

Given circuit is shown below,

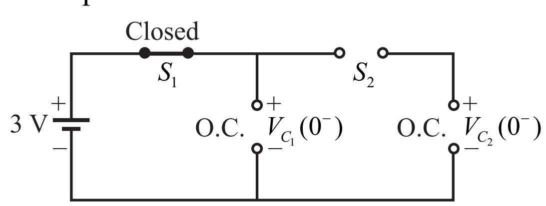


Method 1

(i) At $t = 0^-$ / $t < 0$ / steady state :

Switch S_1 is closed and switch S_2 is open.

In steady state, capacitor behaves as an open circuit.



From figure,

$V_{C_1}(0^-) = 3\,\text{V}$

$V_{C_2}(0^-) = 0\,\text{V}$ (Given)

Charge in capacitor is given by,

$Q = CV$

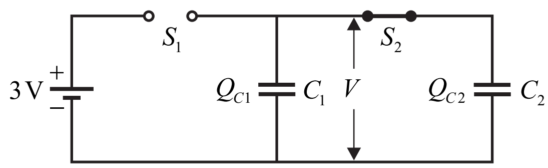
$Q_{C_1}(0^-) = CV_{C_1}(0^-)$

$Q_{C_1}(0^-) = 1 \times 3 = 3\text{ C} = Q_0$

$Q_{C_2}(0^-) = 0 \times 2 = 0\text{ C}$

(ii) At $t = 0^+$:

$S_1 = \text{Open}$, $S_2 = \text{Closed}$



Charge stored (Q_0) initially in C_1 gets redistribute between C_1 and C_2.

Let charge stored in $C_1 = Q_{C_1}$

Charge stored in $C_2 = Q_{C_2}$

According to charge conservation,

$Q_{C_1} + Q_{C_2} = Q_0$

$Q_{C_1} + Q_{C_2} = 3$ ...(i)

From figure,

Voltage across C_1 = Voltage across C_2

$$\frac{Q_{C_1}}{C_1} = \frac{Q_{C_2}}{C_2}$$

$$\frac{Q_{C_1}}{1} = \frac{Q_{C_2}}{2}$$

$Q_{C_2} = 2Q_{C_1}$ ...(ii)

From equation (i) and (ii),

$Q_{C_1} = 1\,\text{C}$, $Q_{C_2} = 2\,\text{C}$

Voltage across capacitor combination is given by,

$$V = \frac{Q_{C_1}}{C_1} = \frac{1}{1} = 1\,\text{V}$$

Or $V = \frac{Q_{C_2}}{C_2} = \frac{2}{2} = 1\,\text{V}$

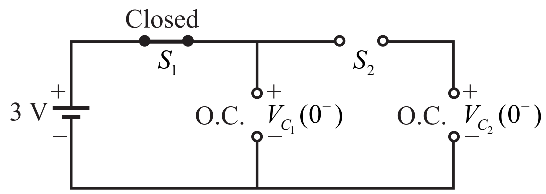
Hence, the correct option is (A).

Method 2

(i) At $t = 0^-$ / $t < 0$ / steady state :

Switch S_1 is closed and switch S_2 is opened.

In steady state, capacitor behaves as an open circuit.



If the ideal voltage is directly connected across capacitor, capacitor voltage will be step signal and capacitor current will impulse signal.

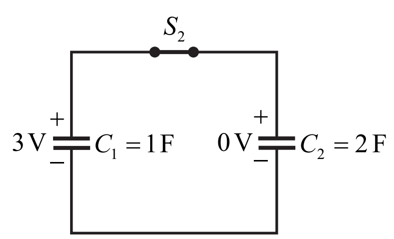
From figure,

$$V_{C_1}(0^-) = 3\,\text{V}$$

$$V_{C_2}(0^-) = 0\,\text{V} \qquad \text{[Given]}$$

(ii) At $t = 0^+$:

$S_1 = \text{Open}$, $S_2 = \text{Closed}$



Charge stored in C_1 is given by,

$$Q_1 = C_1 V_1 = 1 \times 3 = 3\,\text{C}$$

Charge stored in C_2 is given by,

$$Q_2 = C_2 V_2 = 2 \times 0 = 0\,\text{C}$$

Equivalent charge is given by,

$$Q_{eq} = Q_1 + Q_2 = 3 + 0 = 3\,\text{C}$$

Equivalent capacitance is given by,

$$C_{eq} = C_1 + C_2 = 1 + 2 = 3\,\text{F}$$

Voltage across the capacitor combination at $t = 0^+$ is given by,

$$V = \frac{Q_{eq}}{C_{eq}} = \frac{3}{3} = 1\,\text{V}$$

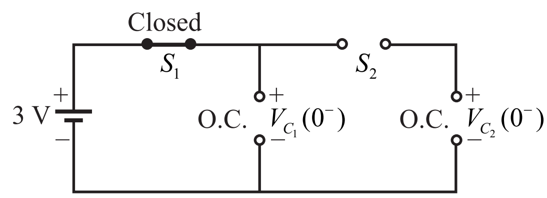
Hence, the correct option is (A).

Method 3 : Using Laplace Transform

(i) At $t = 0^-$ / $t < 0$ / steady state :

Switch S_1 is closed and switch S_2 is open.

In steady state, capacitor behaves as an open circuit.



From figure,

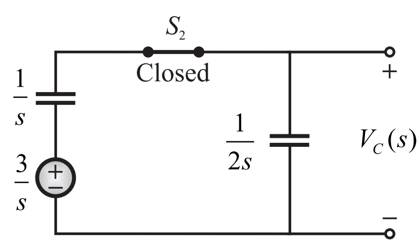
$$V_{C_1}(0^-) = 3\,\text{V}$$

$$V_{C_2}(0^-) = 0\,\text{V} \qquad \text{[Given]}$$

(ii) At $t \geq 0$:

Switch S_1 is opened and S_2 is closed.

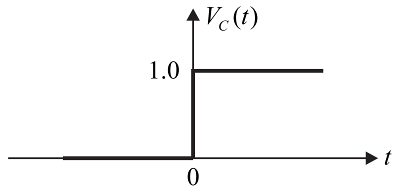
The Laplace transform model is shown below,



$$V_C(s) = \frac{\frac{3}{s} \times \frac{1}{2s}}{\frac{1}{s} + \frac{1}{2s}} = \frac{1}{s} \qquad \text{[From VDR]}$$

Taking inverse Laplace transform of $V_C(s)$,

$V_C(t) = u(t)$ V



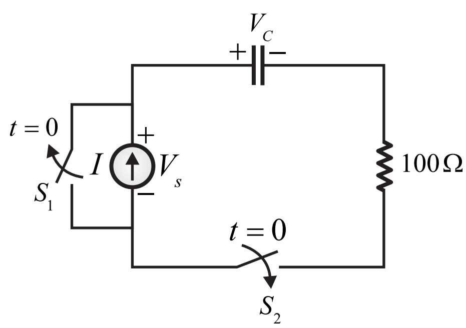
$V_C(t)$ is a unit step function, hence, the voltage at $t = 0^+$ to ∞ will be 1 V.

Hence, the correct option is (A).

Scan for Video Solution

4.4 (C)

Given circuit is shown below,



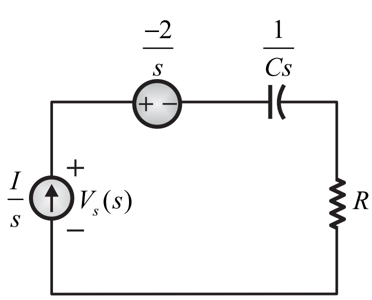
Given : $C = 1\mu\text{F}$, $V_C(0) = -2$ V,

$R = 100\ \Omega$, $I = 20$ mA

From figure,

At $t = 0$, S_1 is open and S_2 is closed.

Transform domain :



Applying KVL in above circuit,

$$-V_s(s) - \frac{2}{s} + \frac{1}{Cs} \times \frac{I}{s} + R \times \frac{I}{s} = 0$$

$$V_s(s) = \frac{-2}{s} + \frac{I}{s}\left(R + \frac{1}{Cs}\right)$$

$$V_s(s) = \frac{-2}{s} + \frac{20 \times 10^{-3}}{s}\left(100 + \frac{10^6}{s}\right)$$

$$V_s(s) = \frac{-2}{s} + \frac{2}{s} + \frac{20 \times 10^3}{s^2}$$

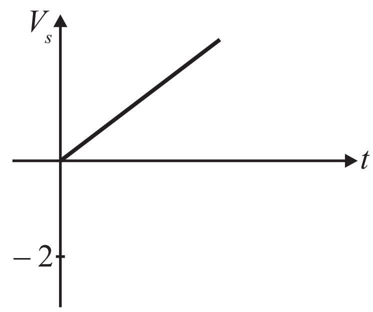
$$V_s(s) = \frac{20 \times 10^3}{s^2}$$

Taking inverse Laplace transform,

$$V_s(t) = 20000t\ u(t)$$

Hence, the above signal is a ramp signal.

The graph for the above analysis is shown below,

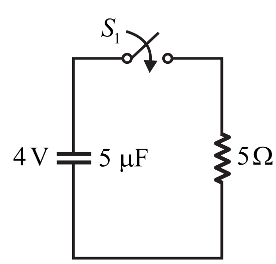


Hence, the correct option is (C).

Scan for Video Solution

4.5 6.9

Given circuit is shown below,



Method 1

Given : $V_C(0^-) = 4$ V

From the property of capacitor,

$$V_C(0^-) = V_C(0^+) = 4\,\text{V}$$

Voltage across capacitor is given by,

$$V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)]e^{\frac{-t}{\tau}} \quad \ldots\text{(i)}$$

(i) At $t \geq 0$ (Transient) :

Switch S_1 is closed.

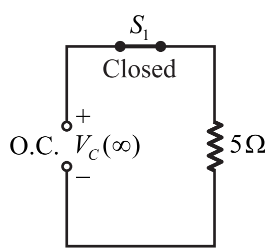
For *R-C* network,
Time constant,

$$\tau = RC = 5 \times 5 \times 10^{-6} = 25\ \mu\text{sec}$$

(ii) At $t = \infty$/steady state :

Switch S_1 is closed.

In steady state, capacitor behaves as an open circuit.



From figure,

$$V_C(\infty) = 0\,\text{V}$$

Put the values of $V_C(0^+)$, $V_C(\infty)$ and τ in equation (i),

$$V_C(t) = 0 + [4 - 0] \times e^{-t/25\times10^{-6}}$$

$$V_C(t) = 4e^{-t/25\times10^{-6}}\ \text{V}$$

Charge in capacitor is given by,

$$q(t) = CV(t)$$

$$q(t) = 5\times10^{-6} \times 4\,\text{e}^{-t/25\times10^{-6}}$$

$$q(t) = 20\,e^{-t/25\times10^{-6}}\ \mu\text{C}$$

At $t = 25\,\mu\text{sec}$,

$$q(t)\big|_{t=25\ \mu\text{sec}} = 20\,e^{-25\times10^{-6}/25\times10^{-6}} = 20\,e^{-1}$$

$$q(t) = 7.357\,\mu\text{C}$$

At $t = 100\ \mu\text{sec}$,

$$q(t)\big|_{t=100\ \mu\text{sec}} = 20\,e^{-100\times10^{-6}/25\times10^{-6}} = 20\,e^{-4}$$

$$q(t) = 0.366\,\mu\text{C}$$

Total charge lost by capacitor from $t = 25\,\mu\text{sec}$ to $t = 100\,\mu\text{sec}$,

$$\Delta q = 7.357 - 0.366 = 6.99\,\mu\text{C}$$

Hence, the charge lost by the capacitor is **6.9 µC.**

Method 2

Given : $V_C(0^-) = 4\,\text{V}$

Current through capacitor is given by,

$$i_C(t) = i_C(\infty) + [i_C(0^+) - i_C(\infty)]e^{-t/\tau} \quad \ldots\text{(i)}$$

(i) At $t \geq 0$ (Transient) :

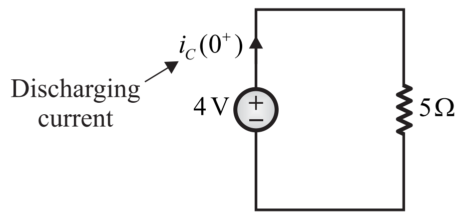
For *R-C* network,
Time constant,

$$\tau = RC = 5 \times 5 \times 10^{-6} = 25\ \mu\text{sec}$$

(ii) At $t = 0^+$:

Capacitor is replaced by a voltage source with initial value i.e.

$$V_C(0^-) = V_C(0^+) = 4\,\text{V}$$



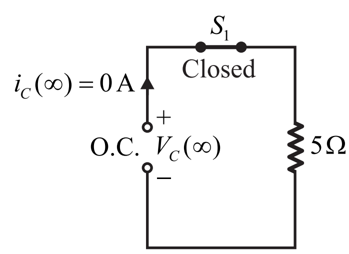
Applying KVL in above figure,

$$-4 + 5i_C(0^+) = 0$$

$$i_C(0^+) = \frac{4}{5} = 0.8\ \text{A}$$

(iii) At $t = \infty$/steady state :

In steady state capacitor behaves as an open circuit.



From circuit, $i_C(\infty) = 0\,\text{A}$

Put the values of $i_C(0^+)$, $i_C(\infty)$ and τ in equation (i),

$$i_C(t) = 0 + (0.8 - 0)e^{-t/25\times10^{-6}}$$

$$i_C(t) = \frac{dq}{dt}$$

$$q(t) = \int_{t_1}^{t_2} i_C(t)\,dt$$

$$q = \int_{25\times10^{-6}}^{100\times10^{-6}} (0.8e^{-t/25\times10^{-6}})\,dt$$

$$q = 0.8\times -25\times10^{-6}\left[e^{-t/25\times10^{-6}}\right]_{25\times10^{-6}}^{100\times10^{-6}}$$

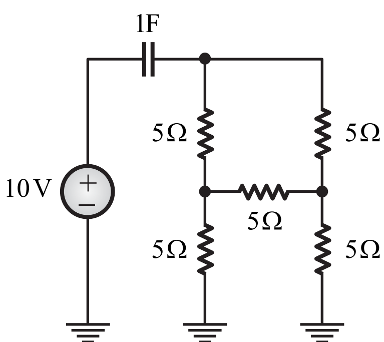
$$q = -20\times10^{-6}[e^{-4} - e^{-1}]$$

$$q = -20\times10^{-6}[0.018 - 0.368] = 6.99\,\mu\text{C}$$

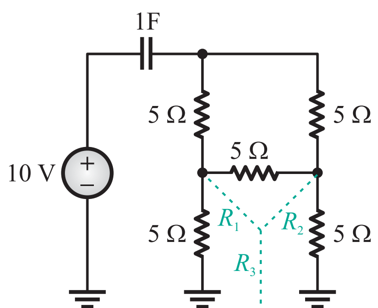
Hence, the charge lost or discharged by the capacitor is **6.9 μC.**

4.6　100

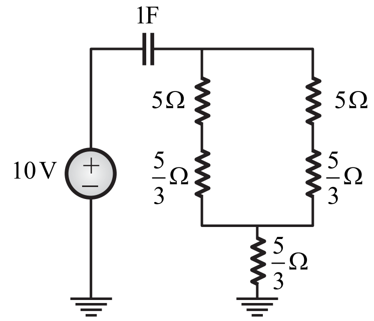
Given circuit is shown below,



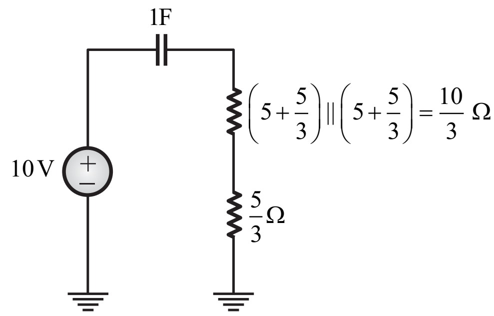
Given : $q(0^-) = 0$ C, $V_C(0^-) = 0$ V



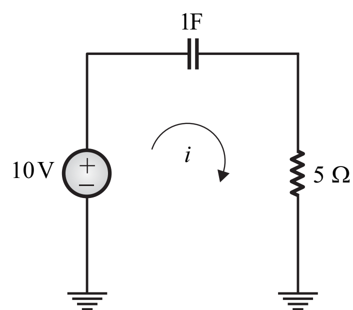
Applying delta to star conversion,



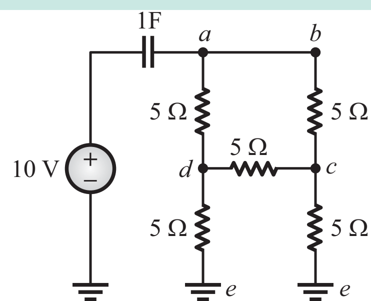
Modified figure is shown below,



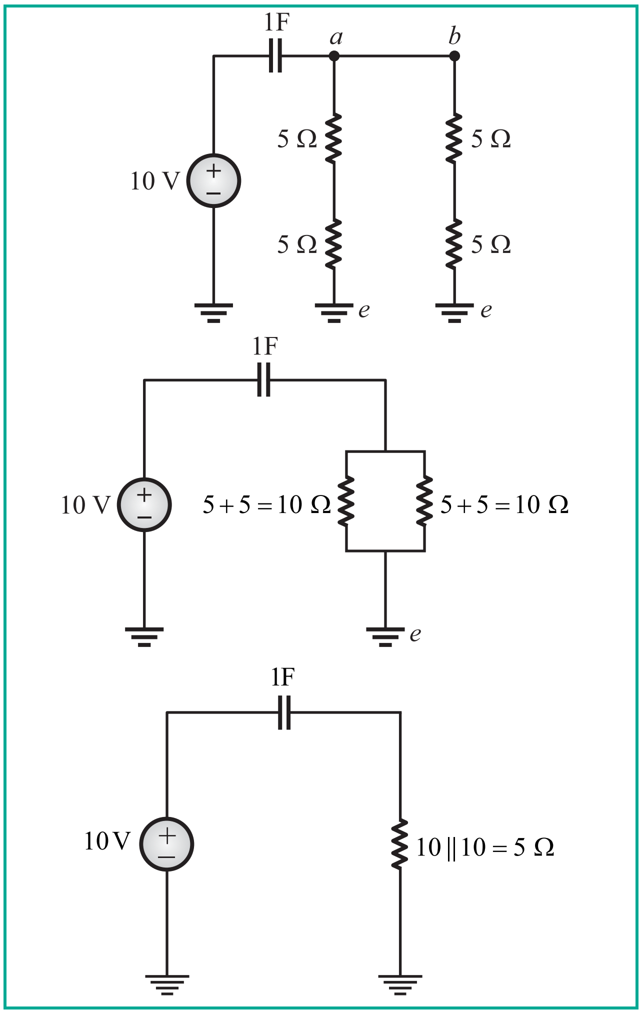
After simplification,



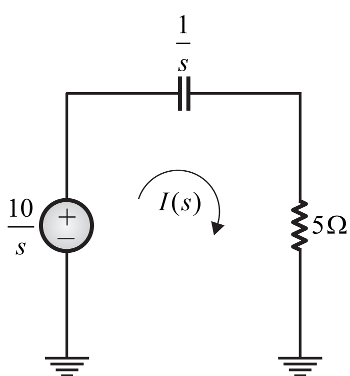
OR



From the above circuit *abcde* is a balance bridge so we can remove 5 Ω resistor connected between *d* and *c*.



Transform domain :



Applying KVL in above figure,

$$-\frac{10}{s}+\frac{I(s)}{s}+5\,I(s)=0$$

$$I(s)=\frac{10}{s\left(5+\frac{1}{s}\right)}=\frac{2}{\left(s+\frac{1}{5}\right)}=\frac{2}{(s+0.2)}$$

Taking inverse Laplace transform, $i(t)=2e^{-0.2t}$.

The energy transferred by the 10 V voltage source is given by,

$$E=\int_0^\infty 10\times 2e^{-0.2t}\,dt$$

$$E=20\left[\frac{e^{-0.2t}}{-0.2}\right]_0^\infty=20\times\frac{-1}{0.2}\left[e^{-\infty}-e^{0}\right]$$

$$E=20\times\frac{-1}{0.2}(0-1)=100\,\text{J}$$

Hence, the energy is **100 J.**

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☒ Avoid This Mistake

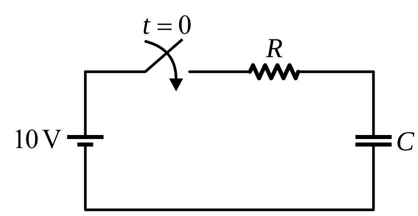
Energy stored by capacitor at steady state :

$$E_C(\infty)=\frac{1}{2}CV_C(\infty)^2=\frac{1}{2}\times 1\times 10^2$$

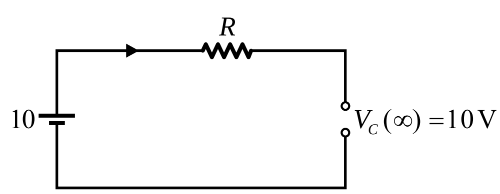
$$E_C(\infty)=50\ \text{J}$$

4.7 **54.98**

Given arrangement in the problem is shown in figure below,

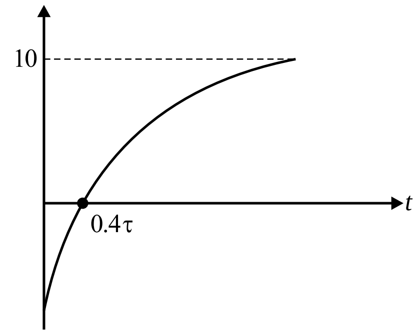


At steady state, capacitor will be open circuited. So the circuit before switching is as shown below,



From given conditions,

Case 1 : $V_c(t)=0$ at $t=0.4\tau$



$$V_c(t) = V_c(\infty) + \{V_c(0^+) - V_c(\infty)\} e^{-t/\tau}$$

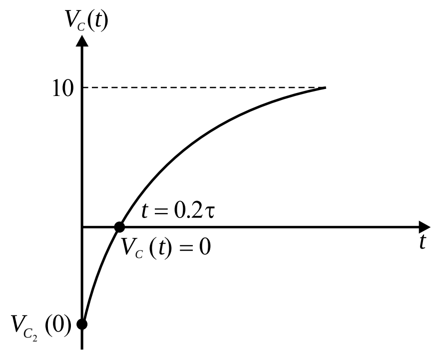
At $t = 0.4\tau$, $V_c(t) = 0$

$$0 = 10 + \{V_{c_1}(0^+) - 10\} e^{-0.4}$$

$$V_{c_1}(0^+) = \frac{10e^{-0.4} - 10}{e^{-0.4}}$$

$$V_{c_1}(t = 0^+) = 10 - 10e^{0.4} = -4.92\text{ V}$$

Case 2 : $V_c(t) = 0$ at $t = 0.2\tau$



$$V_c(t) = V_c(\infty) + \{V_c(0^+) - V_c(\infty)\} e^{-t/\tau}$$

At $t = 0.2\tau$, $V_c(t) = 0$

$$0 = 10 + \{V_{c_2}(0^+) - 10\} e^{-0.2}$$

$$V_{c_2}(0^+) = \frac{10e^{-0.2} - 10}{e^{-0.2}}$$

$$V_{c_2}(t = 0^+) = 10 - 10e^{0.2} = -2.214\text{ V}$$

% change in initial voltage

$$= \frac{V_{c_1}(0^+) - V_{c_2}(0^+)}{V_{c_1}(0^+)} \times 100$$

$$= \frac{-4.92 - (-2.214)}{-4.92} \times 100$$

$$= 0.5499 \times 100 = 54.99\%$$

4.8 **2.38**

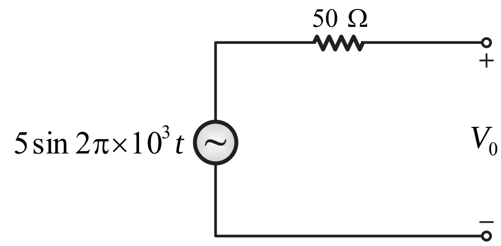
Given :

(i) $V_s = 10$ volts peak-to-peak

(ii) $R_s = 50\ \Omega$

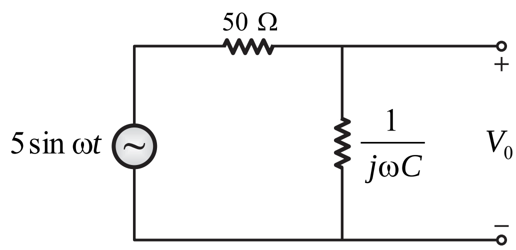
(iii) $f = 1000$ Hz

Case 1 : Under open circuited conduction



$\therefore$ $V_0 = 5\sin(2\pi \times 10^3 t)$

Case 2 : When the capacitor is connected across the terminals, voltage across terminals reduces to 8 volts peak to peak.



Applying voltage divider rule, voltage across capacitor is given by,

$$V_C = \frac{\frac{1}{j\omega C}}{50 + \frac{1}{j\omega C}} \times V_{in} = \frac{V_{in}}{1 + j\omega C \times 50}$$

$$4 = \frac{5}{1 + j\omega C \times 50}$$

$$0.8 = \frac{1}{\sqrt{1 + R^2\omega^2 C^2}}$$

$$0.8 = \frac{1}{\sqrt{1 + R^2 \times (2\pi \times 10^3)^2 C^2}}$$

$$C = 2.38\ \mu\text{F}$$

Hence, the correct answer is 2.38.

4.9 **12.75**

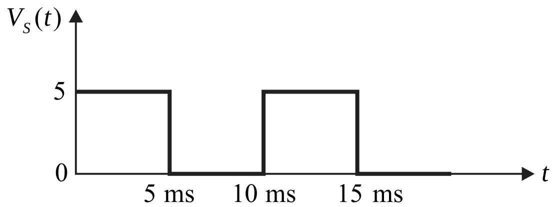
Given :

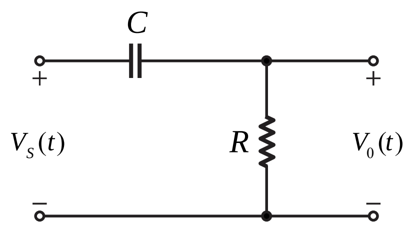
(i) Square waveform of 0 to 5 V amplitude.

(ii) Frequency,

(iii) Resistance, $R = 820\ \Omega$

(iv) Peak-to-peak voltage across resistance, $R = 6.2$ V





From 0 to 5 msec, capacitor charges exponentially

$\therefore \quad V_C(\infty) = 5$ V

$V_C(0) = 0$ V

(As capacitor is initially uncharged)

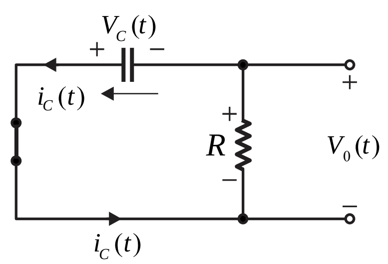
Charging equation of first order RC circuit is given by,

$$\therefore \quad V_C(t) = V_C(\infty) + \left[V_C(0^+) - V_C(\infty)\right] e^{-t/\tau}$$

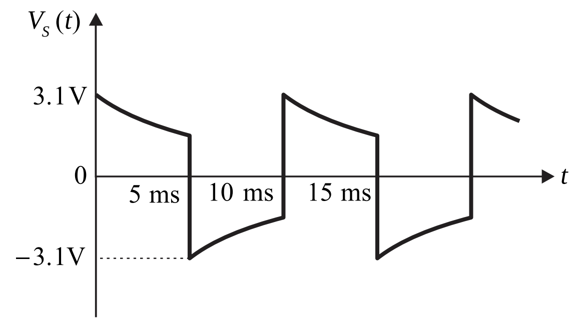
$$V_C(t) = 5 + (0-5)e^{-t/\tau}$$

$$V_C(t) = 5(1 - e^{-t/\tau})$$

From 5 msec to 10 msec, capacitor discharges through resistance.



As the voltage across resistance varies between 3.1 V to –3.1 V (6.2 V peak to peak)



Voltage across resistance is given by,

$$V_R(t) = V_S - V_C(\tau) = 5 - \left[5(1 - e^{-t/\tau})\right] = 5e^{-t/\tau}$$

$$V_R(t) = -3.1 \text{ at } t = 5 \text{ msec}$$

$$-3.1 = 5e^{\frac{-5\times10^{-3}}{820\times C}}$$

$$C = 12.75\ \mu\text{F}$$

Hence, the value of C is **12.75 μF**.

❖❖❖❖

8 Complex Power

➢ Partial Synopsis

Complex power is the product of rms voltage phasor and the complex conjugate of rms current phasor. Complex power is a complex quantity comprises of real and imaginary part. The real part of complex power represents real power and imaginary part represents reactive power.

Mathematically, it is represented by,

$$S = V_{rms} I_{rms}^*$$

where, $V_{rms} = \dfrac{V_m}{\sqrt{2}} \angle \theta_v$ $\qquad$ $I_{rms} = \dfrac{I_m}{\sqrt{2}} \angle \theta_i$

$$S = \left(\frac{V_m}{\sqrt{2}} \angle \theta_v \right) \left(\frac{I_m}{\sqrt{2}} \angle \theta_i \right)^*$$

The expression of complex power in polar form is given by,

$$S = \frac{V_m I_m}{2} \angle \phi \qquad [\phi = \theta_v - \theta_i]$$

It can also be written in exponential form as follows, $S = \dfrac{V_m I_m}{2} e^{j\phi}$

The expression of complex power in rectangular form is given by,

$$S = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + j \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$$

$$S = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$S = P + jQ$$

$$S = \sqrt{P^2 + Q^2} \angle \tan^{-1}\left(\frac{Q}{P} \right)$$

The magnitude and angle of complex power indicates an apparent power and the power factor angle respectively.

Average Power or Real Power :

Average power is the actual power or real power which is actually transferred to the load.

The real part of complex power represents real power or average power.

$$P = \text{Re}(S)$$

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \text{ watts}$$

The real power is measured in watts (W).

Key Point :

(i) If voltage or current having fundamental component or single frequency component then,

Average power or useful power is given by, $P = V_{rms}\, I_{rms} \cos\phi$

where, ϕ is the angle difference between voltage and current.

(ii) If voltage or current or both having more than one frequency components then,

Average power or useful power is given by, $P = V_{1\,rms}\, I_{1\,rms} \cos\phi_1$

where,

$V_{1\,rms}$ is the fundamental RMS voltage, $I_{1\,rms}$ is the fundamental RMS current,

ϕ_1 is the angle difference between fundamental voltage and fundamental current.

Example :

$v(t) = 4\sin\omega t + 2\sin 3\omega t + \sin 5\omega t$ $\quad$ $i(t) = 2\sin(\omega t - 60^0) + \sin(3\omega t - 90^0)$

$$P = \frac{4}{\sqrt{2}} \times \frac{2}{\sqrt{2}} \times \cos[0 - (-60^0)]$$

$$P = 2\text{ W}$$

(iii) Instantaneous power : The amount of power in a circuit at any instant of time is called instantaneous power. It is denoted by $p(t)$.

Therefore, $p(t) = v(t) \times i(t)$

where, $v(t)$ and $i(t)$ are function of time t.

Sample Questions

1996 IISc Bangalore

8.1 A water boiler at home is switched on the ac mains supplying power at 230 V / 50 Hz. The frequency of instantaneous power consumed by the boiler is,

(A) 0 Hz (B) 50 Hz

(C) 100 Hz (D) 150 Hz

2011 IIT Madras

Common Data for Questions 8.2 & 8.3

The input voltage given to a converter is

$$v_i = 100\sqrt{2}\sin(100\pi t)\text{ V}$$

The current drawn by the converter is

$$i_i = \left[10\sqrt{2}\sin(100\pi t - \pi/3) + 5\sqrt{2}\sin(300\pi t + \pi/4) + 2\sqrt{2}\sin(500\pi t - \pi/6)\right]\text{A}$$

8.2 The input power factor of the converter is

(A) 0.31 (B) 0.44

(C) 0.5 (D) 0.71

8.3 The active power drawn by the converter is

(A) 181 W (B) 500 W

(C) 707 W (D) 887 W

2016 IISc Bangalore

8.4 The voltage (V) and current (A) across a load are as follows :

$$v(t) = 100\sin\omega t$$

$$i(t) = 10\sin(\omega t - 60)$$
$$+2\sin(3\omega t) + 5\sin(5\omega t)$$

The average power consumed by the load in W is _____.

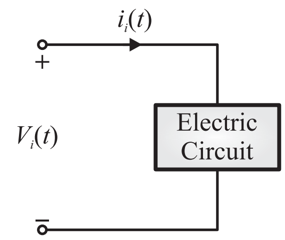
2018 IIT Guwahati

8.5 The voltages across the circuit in the figure, and the current through it are given that the following expressions :

$V_i(t) = 5 - 10\cos(\omega t + 60^0)$ and

$i_i(t) = 5 + X\cos(\omega t)$

Where $\omega = 100\pi$ rad/sec. If the average power delivered to the circuit is zero then the value of X (in Ampere) is ________ (upto two decimal places)



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Explanations Complex Power

8.1 (C)

Given : $V_{rms} = 230\,\text{V}$, $f = 50\,\text{Hz}$

$$V_{peak} = \sqrt{2}V_{rms} = 230\sqrt{2}\ \text{V}$$

So, expression of $v(t)$ is,

$$v(t) = 230\sqrt{2}\cos\omega t$$

where, $\omega = 2\pi\times 50$ rad/sec

Instantaneous power consumed by boiler is given by,

$$p(t) = v(t)\times i(t)$$

where, $i(t) = \dfrac{v(t)}{R}$

$$p(t) = \frac{v^2(t)}{R}$$

where, R is the resistance of boiler circuit coil.

$$p(t) = \frac{(230\sqrt{2}\cos\omega t)^2}{R}$$

$$p(t) = \frac{2(230)^2}{R}[\cos^2\omega t]$$

$$\left[\because\ \cos^2\theta = \frac{\cos 2\theta + 1}{2}\right]$$

$$p(t) = \frac{2(230)^2}{R}\left[\frac{\cos 2\omega t + 1}{2}\right]$$

$$p(t) = \frac{2(230)^2}{R}\left[\frac{\cos 2\times(2\pi\times 50)t + 1}{2}\right]$$

$$[\because\ \omega = 2\pi f]$$

$$p(t) = \frac{2(230)^2}{R}\left[\frac{\cos(2\pi\times 100)t + 1}{2}\right]$$

From the above expression of instantaneous power, the frequency of $p(t)$ is 100 Hz.

Hence, the correct option is (C).

8.2 (B)

Given :

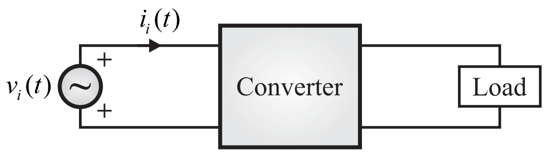
(i) The input current drawn by the converter is,

$$i_1 = \left[10\sqrt{2}\sin\left(100\pi t - \frac{\pi}{3}\right) + 5\sqrt{2}\sin\right.$$

$$\left(300\pi t+\frac{\pi}{4}\right)+2\sqrt{2}\sin\left(500\pi t-\frac{\pi}{6}\right)\Big]\ \text{A}$$

(ii) The input voltage given to the converter is,

$$v_i = 100\sqrt{2}\sin(100\pi t)\ \text{V}$$



Input power factor or supply power factor is given by,

$$\text{IPF} = \frac{V_s I_{s_1}\cos\phi_1}{V_s I_s} = \frac{I_{s_1}}{I_s}\cos\phi_1 \qquad \ldots\text{(i)}$$

where, ϕ_1 = Phase displacement between voltage and fundamental component of current and,

$\cos\phi_1$ = Input fundamental power factor

$$I_{s_1} = 10\ \text{A (rms)}$$

[Component of input current having same frequency as that of input voltage, hence also called fundamental component]

$$I_s = \sqrt{I_{s_1}^2 + I_{s_3}^2 + I_{s_5}^2}$$

$$I_s = \sqrt{\left(\frac{10\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{5\sqrt{2}}{\sqrt{2}}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{2}}\right)^2}$$

$$I_s = \sqrt{10^2+5^2+2^2} = 11.35$$

Substituting the values in equation (i),

$$IPF = \frac{10}{11.35}\cos 60^0 = 0.44$$

Hence, the correct option is (B).

8.3 (B)

Method 1

Active power P is,

$$P = V_s I_{s_1}\cos\phi_1$$

$$P = 100\times 10\times\cos 60^0$$

$$P = 500\ \text{watts}$$

Hence, the correct option is (B).

Method 2

Active power P is, $P = V_s I_s \cos\phi$

where, V_s = supply rms voltage

I_s = supply rms current

$\cos\phi$ = supply input power factor

$$P = 100\times 11.35\times 0.44 = 500\ \text{watts}$$

Hence, the correct option is (B).

8.4 250

Given : $v(t) = 100\sin\omega t$

$$i(t) = 10\sin(\omega t - 60^0) + 2\sin(3\omega t) + 5\sin(5\omega t)$$

Fundamental component of voltage is,

$$v_1 = 100\sin\omega t\ \text{V}$$

$$v_{1\,rms} = \frac{100}{\sqrt{2}}\ \text{V}$$

Fundamental component of current is,

$$i_1 = 10\sin(\omega t - 60^0)\ \text{A}$$

$$i_{1\,rms} = \frac{10}{\sqrt{2}}\ \text{A}$$

Phase difference between these two component is,

$$\phi_1 = 60^0,\ \cos\phi_1 = \cos 60^0 = 0.5$$

Average power or useful power due to fundamental components is given by,

$$P_1 = v_{1\,rms}\, i_{1\,rms}\cos\phi_1$$

$$P_1 = \frac{100}{\sqrt{2}}\times\frac{10}{\sqrt{2}}\times 0.5 = 250\ \text{W}$$

Since, 3^{rd} and 5^{th} harmonics are absent in voltage, there is no average power due to these components.

Average power consumed by the load = Average power due to fundamental components.

$$P_0 = 250\ \text{W}$$

Hence, the average power is **250 W.**

☒ Avoid This Mistake

$$P = V_{rms} I_{rms} \cos\phi_1$$

where, $V_{rms} = \dfrac{100}{\sqrt{2}} = 70.71$ V

$$I_{rms} = \sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2}$$

$$I_{rms} = 8.031 \text{ A}$$

$$\cos\phi_1 = \cos 60^0 = 0.5$$

Hence, $P = 70.71 \times 8.031 \times 0.5 = 283.93$ W

🕮 Key Point

(i) If voltage or current having fundamental component or single frequency component then,

Average power or useful power is given by,

$$P = V_{rms} I_{rms} \cos\phi$$

where, ϕ is the angle between voltage and current.

(ii) If voltage or current or both having more than one frequency components then,

Average power or useful power is given by,

$$P = V_{1\,rms} I_{1\,rms} \cos\phi_1$$

where,

$V_{1\,rms}$ is the fundamental RMS voltage,

$I_{1\,rms}$ is the fundamental RMS current,

ϕ_1 is the angle between fundamental voltage and fundamental current.

8.5 **10**

Given :

(i) $i_i(t) = 5 + X \cos\omega t$

(ii) $V_i(t) = 5 - 10\cos(\omega t + 60)$

$$V_i(t) = 5 + 10\cos(\omega t + 60^0 - 180^0)$$

$$V_i(t) = 5 + 10\cos(\omega t - 120^0)$$

The average power is given by,

$$P_{avg} = V_0 I_0 + \frac{V_m I_m}{2}\cos\phi$$

$$P_{avg} = (5\times 5) + \frac{(10\times X)}{2}\cos 120^0$$

$$0 = 25 + 5X\cos 120^0$$

$$0 = 25 + 5X \times \left(\frac{-1}{2}\right)$$

$$25 = \frac{5X}{2}$$

$$X = 10 \text{ Amp}$$

Hence, the value of X is **10 Amp.**

Scan for
Video Solution

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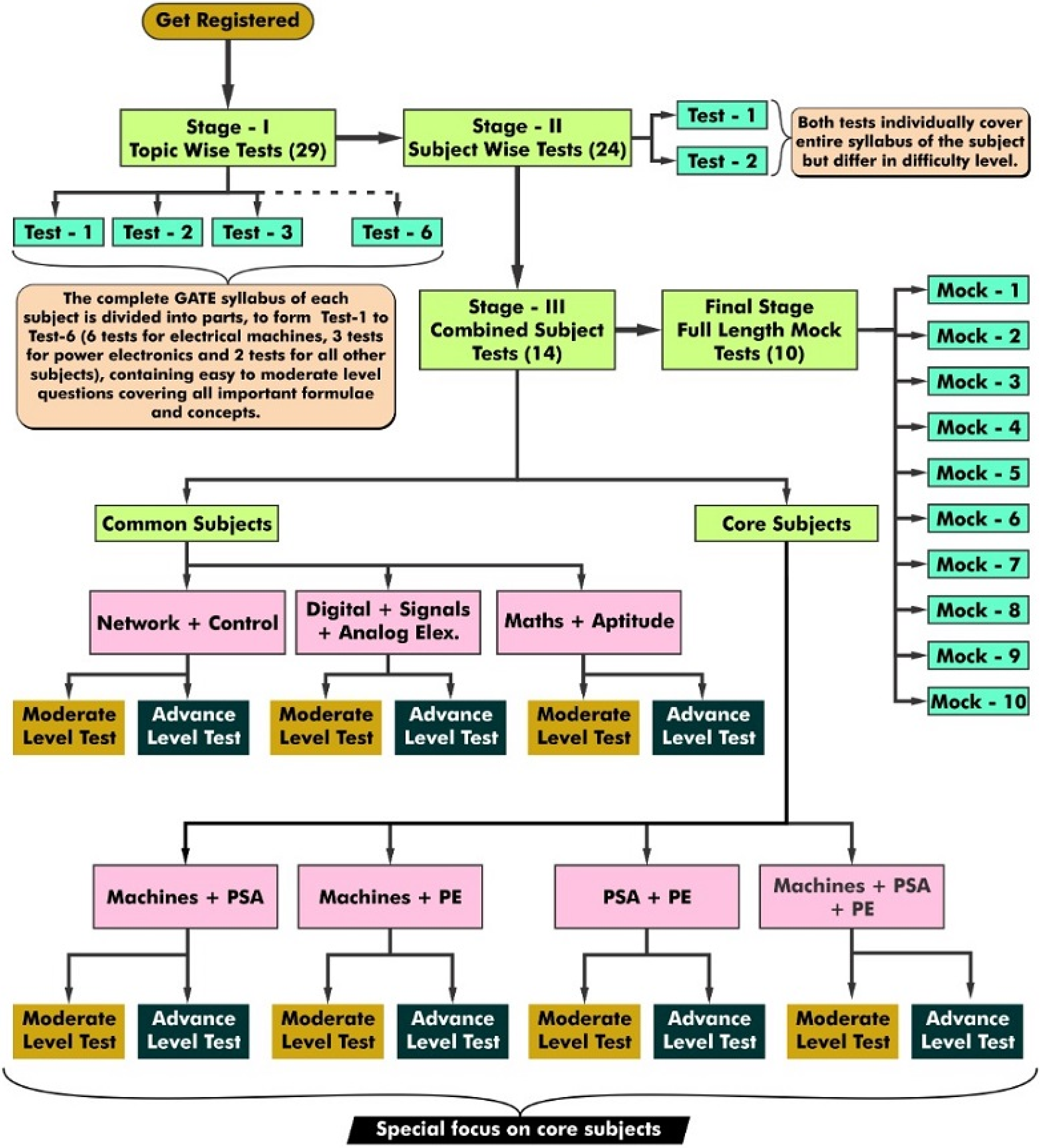

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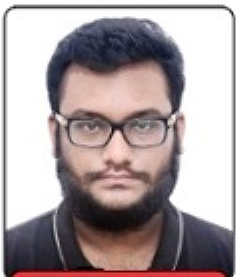
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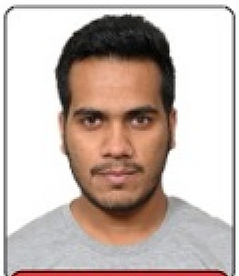
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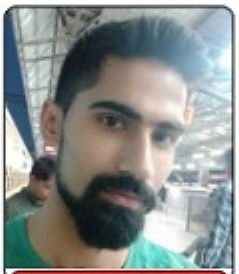
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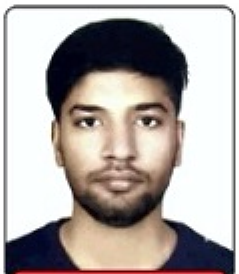
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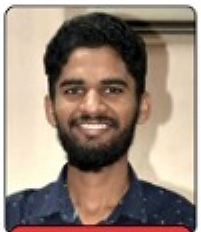
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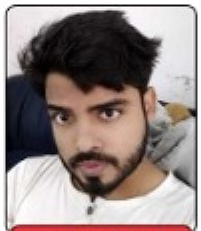
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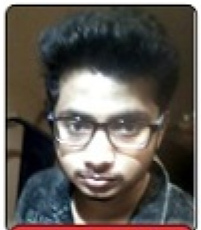
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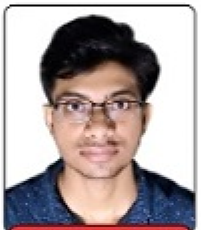
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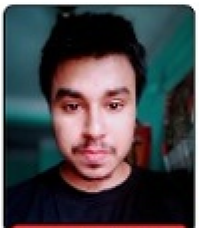
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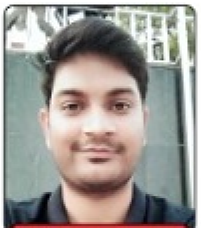
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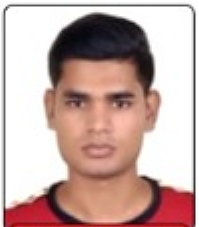
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2003	–	–	–
2004	1	2	5
2005	–	4	8
2006	3	4	11
2007	2	5	12
2008	2	7	16
2009	1	3	7
2010	2	4	10
2011	2	2	6
2012	2	4	10
2013	6	1	8
2014 Set-1	2	3	8
2014 Set-2	2	2	6

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-3	3	3	9
2015 Set-1	2	2	6
2015 Set-2	1	4	9
2016 Set-1	4	3	10
2016 Set-2	5	2	9
2017 Set-1	4	2	8
2017 Set-2	3	2	7
2018	1	3	7
2019	4	1	6
2020	4	3	10
2021	2	3	8

Syllabus : Signals & Systems

Representation of continuous and discrete time signals, shifting and scaling properties, linear time invariant and causal systems, Fourier series representation of continuous and discrete time periodic signals, sampling theorem, Applications of Fourier Transform for continuous and discrete time signals, Laplace Transform and Z transform. R.M.S. value, average value calculation for any general periodic waveform.

Contents : Signals & Systems

1 Basics of Signals

➢ Partial Synopsis

Energy signals : Energy signals are signals which have finite energy. They are mostly defined for non-periodic or finite duration signals.

In continuous domain,

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)\,dt = \int_{-\infty}^{\infty} x^2(t)\,dt \quad \text{where } x(t) \text{ is a real signal}$$

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2\,dt = \int_{-\infty}^{\infty} |x(t)|^2\,dt \quad \text{where } x(t) \text{ is a complex signal}$$

In discrete domain,

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} x^2(n) = \sum_{n=-\infty}^{\infty} x^2(n) \quad \text{where } x(n) \text{ is a real signal}$$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2 = \sum_{n=-\infty}^{\infty} |x(n)|^2 \quad \text{where } x(n) \text{ is a complex signal}$$

Power signals : Power signals are signals which have finite power. They are mostly defined for periodic signals. All periodic signals are power signals but all power signals are not periodic.

In continuous domain,

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt \quad \text{where } x(t) \text{ is a real signal}$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\,dt \quad \text{where } x(t) \text{ is a complex signal}$$

In discrete domain,

$$P = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=-N}^{N} x^2(n) \quad \text{where } x(n) \text{ is a real signal}$$

$$P = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=-N}^{N} |x(n)|^2 \quad \text{where } x(n) \text{ is a complex signal}$$

Relation between energy and power signals :

In continuous domain, $P = \lim\limits_{T\to\infty} \dfrac{E}{T}$ In discrete domain, $P = \lim\limits_{N\to\infty} \dfrac{E}{2N+1}$

$$\text{RMS value} = \sqrt{\text{Power}} = \frac{\text{Energy in one period}}{\text{Time period}}$$

Properties of energy signals :

1. Energy signals always consists finite energy, $0 < E < \infty$
2. Power of energy signal is zero, $P = 0$
3. If $x(t)$ is an even / odd function then energy in left and right halves of $x(t)$ will be same. i.e.

   Energy (L.H.S.) = Energy (R.H.S.), Total energy = 2Energy (L.H.S.) = 2Energy (R.H.S.)
4. $x(t) = x_e(t) + x_0(t)$ $\quad E[x(t)] = E[x_e(t)] + E[x_0(t)]$

Properties of power signals :

1. Power signals always consists finite power, $0 < P < \infty$
2. Energy of power signal is infinite, $E = \infty$
3. For any periodic signal, $P[\text{L.H.S.}] = P[\text{R.H.S.}]$

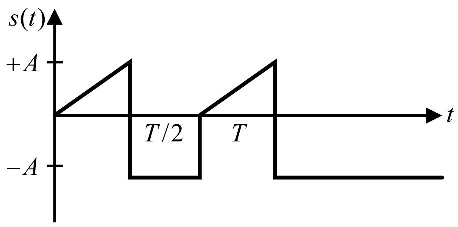
Energy and Power of Some Important Signals

Signals	Energy	Power	Area	Nature
$e^{-at}u(t),\ e^{at}u(-t)$	$E = \frac{1}{2a}$	$P = 0$	$A = \frac{1}{a}$	Energy signal
$e^{-a\|t\|}$	$E = \frac{1}{a}$	$P = 0$	$A = \frac{2}{a}$	Energy signal
$u(t),\ u(n)$	$E = \infty$	$P = \frac{1}{2}$	$A = \infty$	Power signal
$A\text{rect}\left(\frac{t}{\tau}\right)$	$E = A^2\tau$	$P = 0$	$A = A\tau$	Energy signal
$A\text{tri}\left(\frac{t}{\tau}\right)$	$E = \frac{2}{3}A^2\tau$	$P = 0$	$A = A\tau$	Energy signal
$\text{sinc}(t)$	$E = 1$	$P = 0$	$A = 1$	Energy signal
$Sa(t)$	$E = \pi$	$P = 0$	$A = \pi$	Energy signal
$A\cos\omega t,\ A\sin\omega t,$ $A\cos(\omega t+\phi),\ A\sin(\omega t+\phi)$	$E = \infty$	$P = \frac{A^2}{2}$	$A = 0$	Power signal
$Ae^{j\omega t},\ Ae^{j(\omega t+\phi)}$	$E = \infty$	$P = A^2$	$A = 0$	Power signal
$e^{-\pi t^2}$	$E = \frac{1}{\sqrt{2}}$	$P = 0$	$A = 1$	Energy signal
$\delta(t)$	$E = \infty$	$P = \text{Undefined}$	$A = 1$	Neither Energy nor Power signal

➢ Sample Questions

1995 IIT Kanpur

1.1 The RMS value of the waveform $s(t)$ shown in fig is



(A) $\sqrt{\frac{3}{2}}A$ (B) $\sqrt{\frac{2}{3}}A$

(C) $\sqrt{\frac{1}{3}}A$ (D) $\sqrt{2}\,A$

2018 IIT Guwahati

1.2 Consider the two continuous-time signals defined below :

$$x_1(t)=\begin{cases}|t|, & -1\le t\le 1\\ 0, & \text{otherwise}\end{cases}$$

$$x_2(t)=\begin{cases}1-|t|, & -1\le t\le 1\\ 0, & \text{otherwise}\end{cases}$$

These signals are sampled with a sampling period of T = 0.25 seconds to obtain discrete-time signals $x_1[n]$ and $x_2[n]$, respectively. Which one of the following statements is true?

(A) The energy of $x_1[n]$ is greater than the energy of $x_2[n]$

(B) The energy of $x_2[n]$ is greater than the energy of $x_1[n]$.

(C) $x_1[n]$ and $x_2[n]$ have equal energies.

(D) Neither $x_1[n]$ nor $x_2[n]$ is a finite-energy signal.

2020 IIT Delhi

1.3 x_R and x_A are, respectively, the rms and average values of $x(t)=x(t-T)$, and similarly, y_R and y_A are respectively, the rms and average values of $y(t)=kx(t)$. k, T are independent of t. Which of the following is true ?

(A) $y_A = kx_A$; $y_R = kx_R$

(B) $y_A \ne kx_A$; $y_R \ne kx_R$

(C) $y_A \ne kx_A$; $y_R = kx_R$

(D) $y_A = kx_A$; $y_R \ne kx_R$

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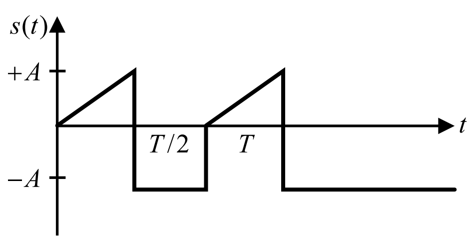
Explanations Basics of Signals

1.1 (B)

Given :



Method 1

From the above figure,

$$s(t)=\begin{cases}\dfrac{2A}{T}t\ ; & 0\le t\le \dfrac{T}{2}\\[2mm] -A\ ; & \dfrac{T}{2}\le t\le T\end{cases}$$

Time period of $s(t)$ is T.

RMS value of $s(t)$ is given by,

$$s_{rms}(t)=\sqrt{\frac{1}{T}\int_0^T s^2(t)dt}$$

$$s_{rms}(t)=\sqrt{\left[\frac{1}{T}\left\{\int_0^{\frac{T}{2}}\left(\frac{2At}{T}\right)^2 dt+\int_{\frac{T}{2}}^{T}A^2dt\right\}\right]}$$

$$s_{rms}(t)=\sqrt{\left[\frac{1}{T}A^2\left[\frac{4}{3T^2}(t^3)_0^{T/2}+(t)_{T/2}^{T}\right]\right]}$$

$$s_{rms}(t)=\sqrt{\frac{A^2}{T}\left[\frac{4}{3T^2}\left(\frac{T^3}{8}-0\right)+\left(T-\frac{T}{2}\right)\right]}$$

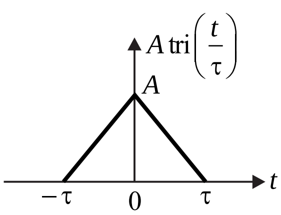
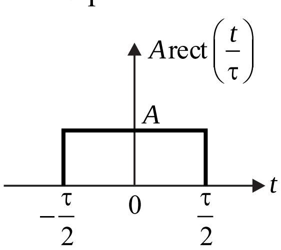
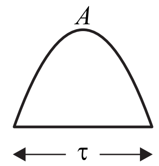
$$s_{rms}(t)=\sqrt{\frac{A^2}{T}\left[\frac{T}{6}+\frac{T}{2}\right]}=\sqrt{\frac{2}{3}}A$$

Hence, the correct option is (B).

Method 2

RMS value of a signal $x(t)$ is given by,

$$x_{rms}=\sqrt{\text{Power}}=\sqrt{\frac{\text{Energy in one time period}}{\text{Time period}}} \quad \ldots(i)$$

Signal	Energy
Triangular wave 	$E=\frac{2A^2\tau}{3}$
Square wave 	$E=A^2\tau$
Sine wave	$E=\frac{A^2\tau}{2}$
	

For duration 0 to $T/2$ the given waveform is a half triangular wave and from $T/2$ to T it is rectangular wave. As there is no effect of shifting on energy of a signal, so total energy in non-overlapping triangular and rectangular function during one time period of given signal is given as

$$E_1=\frac{A^2}{3}\left(\frac{T}{2}\right)+A^2\left(\frac{T}{2}\right)$$

From equation (i),

$$x_{rms}=\sqrt{\left[\frac{\frac{A^2}{3}\left(\frac{T}{2}\right)+A^2\left(\frac{T}{2}\right)}{T}\right]}$$

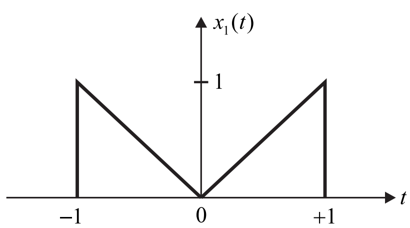
$$x_{rms}=\sqrt{\frac{A^2}{6}+\frac{A^2}{2}}=\sqrt{\frac{2}{3}}A$$

Hence, the correct option is (B).

1.2 (A)

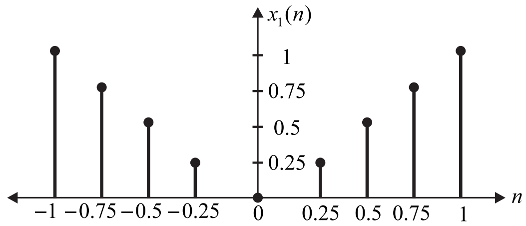
Given :

(a) $x_1(t)=\begin{cases}|t|, & -1\le t\le 1\\ 0, & \text{otherwise}\end{cases}$



Sampling $\downarrow$ $T_s=0.25\text{Sec}$

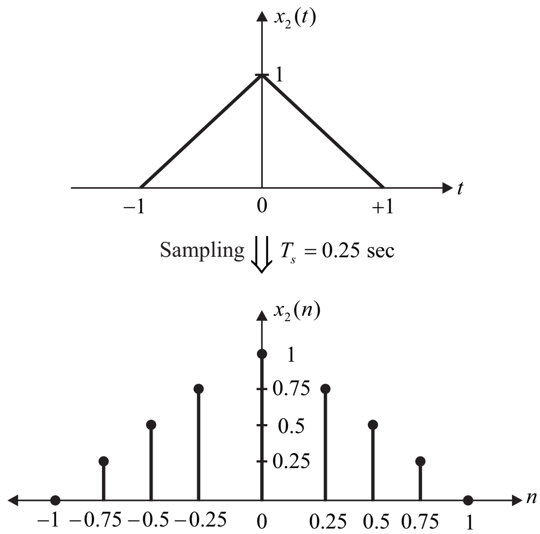
Sampled signal $x_1[n]=x_1(t)\big|_{t=nT_s}$ is shown below,



(b) $x_2(t)=\begin{cases}1-|t|, & -1\le t\le 1\\ 0, & \text{otherwise}\end{cases}$

$x_2(t)$ and sampled signal

$x_2[n]=x_2(t)\big|_{t=nT_s}$ are shown below,



$$E_1=\sum_{n=-\infty}^{\infty}|x_1(n)|^2$$

$$E_1=0^2+2\left[1^2+0.75^2+0.5^2+0.25^2\right]$$

$$E_1=3.75 \qquad \ldots\text{(i)}$$

$$E_2=\sum_{n=-\infty}^{\infty}|x_2(n)|^2$$

$$E_2=1^2+2\left[0.75^2+0.5^2+0.25^2+0^2\right]$$

$$E_2=2.75 \qquad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$E_1>E_2$$

Hence, the correct option is (A).

Key Point

It may seem that energy of continuous time signal should be greater than its discrete counterpart as continuous time signal is present for all instants of time but it is not true. Energy of discrete time signal depends on sampling interval, the less sampling interval will result in more number of samples and hence higher energy in discrete time signal.

1.3 (D)

Given : $x(t-T)=x(t)$, $x(t)$ is periodic with period T

$\therefore$ Average value of $x(t)$

$$x_A=\frac{1}{T}\int_0^T x(t)dt \qquad \ldots\text{(i)}$$

RMS value of $x(t)$

$$x_R=\sqrt{\frac{1}{T}\int_0^T\left|x(t)^2\right|dt} \qquad \ldots\text{(ii)}$$

Given $y(t)=k\cdot x(t)$, period of $y(t)=$ Period of $x(t)=T$

$\therefore$ Average value of $y(t)$

$$y_A=\frac{1}{T}\int_0^T k\cdot x(t)dt$$

$$y_A=k\cdot\left[\frac{1}{T}\int_0^T x(t)dt\right]=k\cdot x_A$$

RMS value of $y(t)$

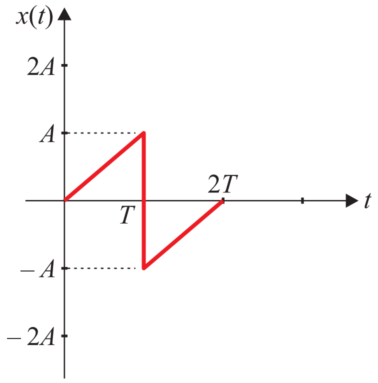
$$y_R=\sqrt{\frac{1}{T}\int_0^T|kx(t)|^2\,dt}$$

$$y_R=|k|\sqrt{\frac{1}{T}\int_0^T\left|x(t)^2\right|dt}=|k|\cdot x_R$$

As rms value can never be negative, so irrespective of the sign of k, y_R will always be positive. So if k is a negative constant then $y_R=k\cdot x_R$ is not true.

Hence, the correct option is (D).

It can be verified by a simple example, as explained below,



Consider a continuous time periodic signal $x(t)$ for which one period is shown in figure,

$$RMS = \sqrt{\text{Power}}$$

$$\text{Power} = \frac{\text{Energy in 1 period}}{\text{Time period}}$$

Energy of $x(t)$ in 1 period,

$$E_x = \frac{A^2T}{3} + \frac{A^2T}{3} = \frac{2A^2T}{3}$$

$\therefore$ Power of $x(t)$,

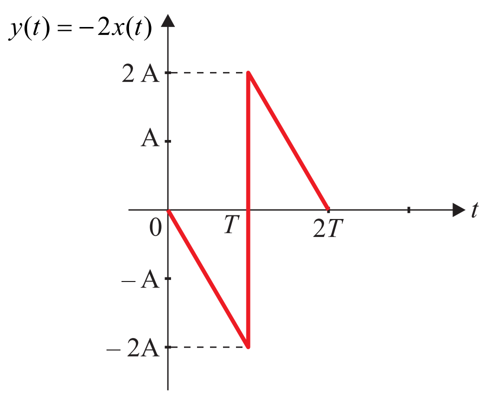
$$P_x = \frac{2A^2T}{3\times 2T} = \frac{A^2}{3}$$

$$\therefore \quad x(t)_{rms} = x_R = \sqrt{\frac{A^2}{3}} = \frac{A}{\sqrt{3}} \quad \ldots\text{(i)}$$

Given : $y(t) = kx(t)$

For $k = -2$, $y(t) = -2x(t)$

$y(t)$ is shown in figure,



$$\text{Energy of } y(t) = \frac{4A^2T}{3} + \frac{4A^2T}{3} = \frac{8A^2T}{3}$$

$$\therefore \quad \text{Power of } y(t) = \frac{8A^2T}{3} \times \frac{1}{2T} = \frac{4A^2}{3}$$

$$\therefore \quad y(t)_{rms} = y_R = \sqrt{\frac{4A^2}{3}} = 2 \times \frac{A}{\sqrt{3}} \quad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$y_R \neq kx_R \text{ as } k = -2,\ k \neq 2$$

Hence, option (A) is not true for any negative value of k.

Given answer in IIT answer key : Option (A).

IIT should have given its correct option as option (D), but they have given option (A) only in their final answer key, which suggests that they have not considered the given relations for **negative values of k**.

In general, option (D) is correct.

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2 Classification of Systems

Partial Synopsis

3. Causal LTI System

An LTI system is said to be causal if the output at any instant depends only on the present and past values of the input.

(a) Consider a continuous-time LTI system whose output $y(t)$ can be obtained using convolution integral given by, $y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$

For $t = 0$, $y(0) = \int_{-\infty}^{\infty} h(\tau)\, x(-\tau)\, d\tau$

If $\tau \geq 0$, the output depends on present and past values of input and the system is causal. But if $\tau < 0$, then output depends on future values of input.

An LTI CT system will be causal if and only if its impulse response $h(t) = 0$ for $t < 0$.

(b) Consider a discrete-time LTI system whose output $y[n]$ can be obtained using convolution sum given by,

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] \qquad \text{For } n = 0, \quad y[0] = \sum_{k=-\infty}^{\infty} h[k]\, x[-k]$$

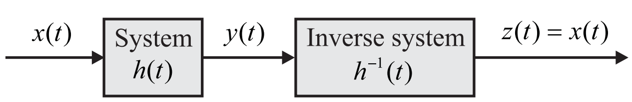
If $k \geq 0$, the output depends on present and past values of input and the system is causal. But if $k < 0$, then output depends on future values of input.

An LTI DT system will be causal if and only if its impulse response $h(n) = 0$ for $n < 0$.

4. Invertible LTI System

An LTI system is said to be invertible if the input of the system can be recovered from the output. If the inverse system is connected in cascade with the original system, then final output will be same as the input.

(a) Consider a CT LTI inverse system as shown in below figure.



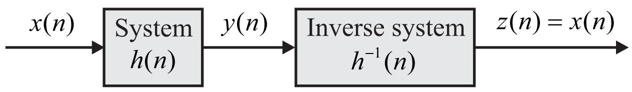
Let $h^{-1}(t)$ represents the impulse response of the inverse system, then in terms of the convolution integral,

$$z(t) = \left\{h^{-1}(t) \otimes h(t)\right\} \otimes x(t) = x(t) \qquad \because \quad x(t) \otimes \delta(t) = x(t)$$

$$\therefore \qquad h^{-1}(t) \otimes h(t) = \delta(t)$$

A CT LTI system is invertible if its impulse response satisfies $h^{-1}(t) \otimes h(t) = \delta(t)$

(b) Consider a DT LTI inverse system as shown in below figure.



Let $h^{-1}(n)$ represents the impulse response of the inverse system, then in terms of the convolution sum,

$$z(n) = \left\{h^{-1}(n) \otimes h(n)\right\} \otimes x(n) = x(n) \qquad \because \quad x(n) \otimes \delta(n) = x(n)$$

$$\therefore \qquad h^{-1}(n) \otimes h(n) = \delta(n)$$

A DT LTI system is invertible if its impulse response satisfies $\boldsymbol{h^{-1}(n) \otimes h(n) = \delta(n)}$

5. Stable LTI System

The necessary and sufficient condition on the continuous time impulse response for stability is that **the impulse response should be absolutely integrable**. Mathematically, $\boldsymbol{\int_{-\infty}^{\infty} |h(t)|dt < \infty}$

Consider a continuous-time LTI system convolution integral whose output is $y(t)$,

$$|y(t)| = \left|\int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau\right| \le \int_{-\infty}^{\infty} |h(\tau)||x(t-\tau)|\, d\tau \le B_x \int_{-\infty}^{\infty} |h(\tau)|\, d\tau$$

The output $|y(t)|$ is bounded when $\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$ or $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

The necessary and sufficient condition on the discrete time impulse response for stability is that **the impulse response should be absolutely summable**. Mathematically, $\boldsymbol{\sum_{n=-\infty}^{\infty} |h(n)| < \infty}$

Consider a discrete-time LTI system convolution sum whose output is $y(n)$,

$$|y[n]| = \left|\sum_{k=-\infty}^{\infty} h[k]\, x[n-k]\right| \le \sum_{k=-\infty}^{\infty} |h[k]||x[n-k]| \le B_x \sum_{k=-\infty}^{\infty} |h[k]|$$

The output $|y(n)|$ is bounded when $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$ or $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

1. **Relation between step and impulse responses of continuous and discrete time LTI systems.**

   For continuous time LTI systems : $s(t)=\int_{-\infty}^{t}h(\tau)\,d\tau$ and $h(t)=\frac{d}{dt}s(t)$

   Where $h(t)$ and $s(t)$ represents impulse and step response of LTI system respectively.

   For discrete time LTI systems : $s[n]=\sum_{k=-\infty}^{n}h[k]$ and $h[n]=s[n]-s[n-1]$

   Where $h[n]$ and $s[n]$ represents impulse and step response of LTI system respectively.

2. Inter connection of two linear / Causal / Time invariant / stable / static systems is always linear / Causal / Time invariant / stable / static but inter connection of two non-linear / non-Causal / Time variant / unstable / dynamic systems may be linear / Causal / Time invariant / stable / static.

Sample Questions

2012 IIT Delhi

2.1 The input $x(t)$ and output $y(t)$ of a system are related as

$$y(t)=\int_{-\infty}^{t}x(\tau)\cos(3\tau)\,d\tau.$$

The system is

(A) time-invariant and stable.
(B) stable and not time-invariant.
(C) time-invariant and not stable.
(D) not time-invariant and not stable.

2017 IIT Roorkee

2.2 Consider the system with following input-output relation

$$y[n]=\left[1+(-1)^n\right]x[n]$$

where, $x[n]$ is the input and $y[n]$ is the output. The system is **[Set - 01]**

(A) invertible and time invariant
(B) invertible and time varying
(C) non-invertible and time invariant
(D) non-invertible and time varying

2021 IIT Bombay

2.3 If the input $x(t)$ and output $y(t)$ of a system are related as $y(t)=\max[0,\,x(t)]$, then the system is

(A) Linear and time-variant
(B) Linear and time-invariant
(C) Non-linear and time-variant
(D) Non-linear and time-invariant

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Explanations　Classification of Systems

2.1 (D)

Given : $y(t)=\int_{-\infty}^{t}x(\tau)\cos(3\tau)\,d\tau$

(i) Time invariancy :

Let input delayed by t_0, then response of the system for delayed input is

$$y(t,t_0)=\int_{\tau=-\infty}^{t}x(\tau-t_0)\cos(3\tau)\,d\tau$$

Let, $\tau - t_0 = \tau_1$

$$y(t,t_0) = \int_{\tau_1=-\infty}^{t-t_0} x(\tau_1)\cos[3(t_0+\tau_1)]\,d\tau_1 \quad \ldots\text{(i)}$$

Replacing t by $t-t_0$ in the given input output relation, delayed response is given as

$$y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\cos(3\tau)\,d\tau \quad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$y(t,t_0) \neq y(t-t_0)$$

Hence, the given system is time variant.

(ii) Stability :

For a system to be BIBO stable, response of the system for every bounded input must be bounded.

Considering a bounded input

$x(\tau) = \cos(3\tau)$.

Then, $y(t) = \int_{-\infty}^{t} \cos^2(3\tau)\,d\tau$

$$y(t) = \int_{-\infty}^{t} \left[\frac{1+\cos 6\tau}{2}\right] d\tau$$

$$y(t) = \frac{1}{2}\left[\tau + \frac{\sin 6\tau}{6}\right]_{-\infty}^{t} = \infty$$

Since, for bounded input $x(t)$, output $y(t)$ is not bounded, thus, the system is not stable.

Hence, the correct option is (D).

Scan for Video Solution

2.2 (D)

Given : $y[n] = \left[1+(-1)^n\right]x[n]$

(i) Time invariancy :

Let for input delayed by k, then response of the system for delayed input is

$$y(n,k) = \left[1+(-1)^n\right]x_1(n-k) \quad \ldots\text{(i)}$$

When n is replaced by $n-k$ in the given input output relation, delayed response is given as

$$y_1(n-k) = \left[1+(-1)^{n-k}\right]x_1(n-k) \quad \ldots\text{(ii)}$$

From equations (i) and (ii),

$$y(n,k) \neq y(n-k)$$

Hence, it is time variant system.

(ii) Invertibility :

Let $x(n) = \delta(n-1)$

$$y(n) = \left[1+(-1)^n\right]\delta(n-1)$$

$$\left[\begin{array}{l}\text{By property of impulse response,}\\ f(n)\delta(n-a) = f(a)\delta(n-a)\end{array}\right]$$

$$y(n) = \left[1+(-1)^1\right]\delta(n-1) = 0$$

Let $x(n) = \delta(n-3)$

$$y(n) = \left[1+(-1)^n\right]\delta(n-3)$$

$$y(n) = \left[1+(-1)^3\right]\delta(n-3) = 0$$

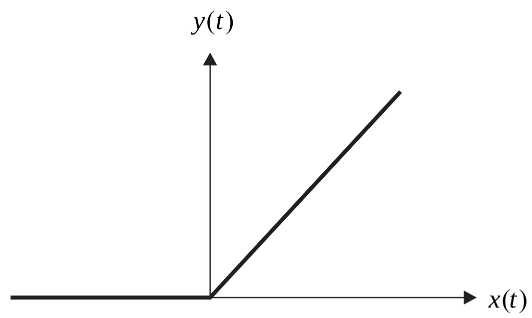
Since, two different inputs give same output, thus, the system is non-invertible.

Hence, the correct option is (D).

Scan for Video Solution

2.3 (D)

Given input output relationship is



$$y(t) = \text{Max}[0, x(t)]$$

$$y(t)=\begin{cases}0, & x(t)\le 0\\ x(t), & x(t)>0\end{cases}$$

As output is splitted not in time but for values of $x(t)$, hence the system will not follow superposition, so it is not a linear system. It can be seen from above figure that the graph between the input and output is not throughout a straight line passing through origin, which is the condition for the system to be linear.

As there is no time scaling or any coefficient multiplied that is function of time and no extra terms other than $x(t)$ and $y(t)$ are present, so the system is time invariant.

Hence, the correct option is (D).

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3 Laplace Transform

➢ Partial Synopsis

9. Time Convolution : Convolution of two signals in time domain is equivalent to their multiplication in s-domain.

For both Bilateral and Unilateral Laplace transform :

$$x_1(t) \overset{LT}{\longleftrightarrow} X_1(s) \qquad \text{ROC}: R_1$$
$$x_2(t) \overset{LT}{\longleftrightarrow} X_2(s) \qquad \text{ROC}: R_2$$
$$x_1(t) \otimes x_2(t) \overset{LT}{\longleftrightarrow} X_1(s).X_2(s) \qquad \text{ROC}: R_1 \cap R_2$$

10. S-domain Convolution : Multiplication of two signals in time domain is equivalent to $1/2\pi j$ times their convolution in s-domain.

For both Bilateral and Unilateral Laplace transform :

$$x_1(t) \overset{LT}{\longleftrightarrow} X_1(s) \qquad \text{ROC}: R_1$$
$$x_2(t) \overset{LT}{\longleftrightarrow} X_2(s) \qquad \text{ROC}: R_2$$
$$x_1(t).x_2(t) \overset{LT}{\longleftrightarrow} \frac{1}{2\pi j}[X_1(s) \otimes X_2(s)] \qquad \text{ROC}: R_1 \cap R_2$$

11. Conjugate :

$$x(t) \overset{LT}{\longleftrightarrow} X(s) \qquad \text{ROC}: R$$
$$x^*(t) \overset{LT}{\longleftrightarrow} X^*(s^*) \qquad \text{ROC}: R$$

Basic Laplace Transform Pairs

S.	$x(t)$	$X(s)$	ROC
1.	$\delta(t)$	1	Entire s-plane
2.	$u(t)$	$\frac{1}{s}$	$\text{Re}(s) > 0$
3.	$-u(-t)$	$\frac{1}{s}$	$\text{Re}(s) < 0$
4.	$e^{-at}u(t)$	$\frac{1}{s+a}$	$\text{Re}(s) > -a$

5.	$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\text{Re}(s) < -a$
6.	$e^{at}u(t)$	$\frac{1}{s-a}$	$\text{Re}(s) > a$
7.	$-e^{at}u(-t)$	$\frac{1}{s-a}$	$\text{Re}(s) < a$
8.	$r(t) = t.u(t)$	$\frac{1}{s^2}$	$\text{Re}(s) > 0$
9.	$t^n.u(t)$	$\frac{n!}{s^{n+1}}$	$\text{Re}(s) > 0$
10.	$t.e^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\text{Re}(s) > -a$
11.	$t^n.e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$	$\text{Re}(s) > -a$
12.	$e^{-at}u(t-b)$	$\frac{e^{-(s+a)b}}{s+a}$	$\text{Re}(s) > -a$
13.	$e^{-a(t-b)}u(t-b)$	$\frac{e^{-sb}}{s+a}$	$\text{Re}(s) > -a$
14.	$\cos\omega_0 t.u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\text{Re}(s) > 0$
15.	$-\cos\omega_0 t.u(-t)$	$\frac{s}{s^2+\omega_0^2}$	$\text{Re}(s) < 0$
16.	$\sin\omega_0 t.u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\text{Re}(s) > 0$
17.	$-\sin\omega_0 t.u(-t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\text{Re}(s) < 0$
18.	$e^{-at}\cos\omega_0 t.u(t)$	$\frac{(s+a)}{(s+a)^2+\omega_0^2}$	$\text{Re}(s) > -a$
19.	$-e^{-at}\cos\omega_0 t.u(-t)$	$\frac{(s+a)}{(s+a)^2+\omega_0^2}$	$\text{Re}(s) < -a$
20.	$e^{-at}\sin\omega_0 t.u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\text{Re}(s) > -a$
21.	$-e^{-at}\sin\omega_0 t.u(-t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\text{Re}(s) < -a$
22.	1	Bilateral LT does not exist	No common ROC
23.	$\text{sgn}(t)$	Bilateral LT does not exist	No common ROC

➢ Sample Questions

1998 IIT Delhi

3.1 The Laplace transform of $(t^2-2t)u(t-1)$ is

(A) $\frac{2}{s^3}e^{-s}-\frac{2}{s^2}e^{-s}$

(B) $\frac{2}{s^3}e^{-2s}-\frac{2}{s^2}e^{-s}$

(C) $\frac{2}{s^3}e^{-s}-\frac{1}{s}e^{-s}$

(D) None of these

2012 IIT Delhi

3.2 Consider the differential equation

$$\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+y(t)=\delta(t)$$

with $y(t)\big|_{t=0^-}=-2$ and $\frac{dy}{dt}\Big|_{t=0^-}=0$.

The numerical value of $\frac{dy}{dt}\Big|_{t=0^+}$ is

(A) -2 (B) -1
(C) 0 (D) 1

2015 IIT Kanpur

3.3 The Laplace transform of $f(t)=2\sqrt{\frac{t}{\pi}}$ is $s^{\frac{-3}{2}}$. The Laplace transform of $g(t)=\sqrt{\frac{1}{\pi t}}$ is **[Set - 02]**

(A) $\frac{3s^{\frac{-5}{2}}}{2}$ (B) $s^{\frac{-1}{2}}$

(C) $s^{\frac{1}{2}}$ (D) $s^{\frac{3}{2}}$

2020 IIT Delhi

3.4 Which of the following statements is true about the two sided Laplace transform?

(A) It exists for every signal that may or may not have a Fourier Transform.

(B) It has no poles for any bounded signal that is non-zero only inside a finite time interval.

(C) If a signal can be expressed as a weighted sum of shifted one sided exponentials, then its Laplace transform will have no poles.

(D) The number of finite poles and finite zeroes must be equal.

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Explanations **Laplace Transform**

3.1 (C)

Method 1

Given : $f(t)=(t^2-2t)u(t-1)$

$$f(t)=(t^2-2t+1-1)u(t-1)$$

$$f(t)=(t^2-2t+1)u(t-1)-u(t-1)$$

$$f(t)=(t-1)^2u(t-1)-u(t-1)$$

Taking Laplace transform of $f(t)$,

$$F(s)=L[f(t)]$$

$$F(s)=\frac{2e^{-s}}{s^3}-\frac{e^{-s}}{s}$$

Hence, the correct option is (C).

Method 2

Given : $x(t) = (t^2 - 2t)u(t-1)$

$$x(t) = t^2 \cdot u(t-1) - 2t \cdot u(t-1)$$

$$= x_1(t) - 2x_2(t)$$

$$X(s) = X_1(s) - 2X_2(s) \qquad \ldots\text{(i)}$$

From basic transform pair,

$$u(t) \longleftrightarrow \frac{1}{s}$$

$$u(t-1) \longleftrightarrow \frac{e^{-s}}{s}$$

[Using time shifting property]

$$t \cdot u(t-1) \longleftrightarrow -\frac{d}{ds}\left(\frac{e^{-s}}{s}\right)$$

[Using multiplication by 't' property]

$$x_2(t) \longleftrightarrow -\left[\frac{s(-e^{-s}) - e^{-s} \cdot 1}{s^2}\right]$$

$$X_2(s) = \frac{e^{-s}}{s} + \frac{e^{-s}}{s^2} \qquad \ldots\text{(ii)}$$

Also, $x_1(t) = t^2 \cdot u(t-t) = t \cdot x_2(t)$

$$X_1(s) = -\frac{d}{ds} X_2(s)$$

$$X_1(s) = -\frac{d}{ds}\left[\frac{e^{-s}}{s} + \frac{e^{-s}}{s^2}\right]$$

$$X_1(s) = -\frac{d}{ds}\left[\frac{e^{-s}}{s}\right] - \frac{d}{ds}\left[\frac{e^{-s}}{s^2}\right]$$

$$X_1(s) = \frac{e^{-s}}{s} + \frac{e^{-s}}{s^2} - \left[\frac{s^2(-e^{-s}) - 2se^{-s}}{s^4}\right]$$

$$X_1(s) = \frac{e^{-s}}{s} + \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s^2} + \frac{2e^{-s}}{s^3}$$

$$X_1(s) = \frac{e^{-s}}{s} + \frac{2e^{-s}}{s^2} + \frac{2e^{-s}}{s^3} \qquad \ldots\text{(iii)}$$

Substituting equation (ii) and (iii) in equation (i),

$$X(s) = X_1(s) - 2X_2(s)$$

$$X(s) = \frac{e^{-s}}{s} + \frac{2e^{-s}}{s^2} + \frac{2e^{-s}}{s^3} - \frac{2e^{-s}}{s} - \frac{2e^{-s}}{s^2}$$

$$X(s) = \frac{2e^{-s}}{s^3} - \frac{e^{-s}}{s}$$

Hence, the correct option is (C).

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3.2 (D)

Given :

Differential equation is,

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t) \qquad \ldots\text{(i)}$$

$$y(t)\Big|_{t=0^-} = -2 \text{ and } \frac{dy}{dt}\Big|_{t=0^-} = 0$$

Taking Laplace transform of equation (i),

$$[s^2Y(s) - sy(0) - y'(0)] + 2[sY(s) - y(0)] + Y(s) = 1$$

$$s^2Y(s) + 2s - 0 + 2[sY(s) + 2] + Y(s) = 1$$

$$(s^2 + 2s + 1)Y(s) + 2s + 4 = 1$$

$$Y(s) = \frac{-2s-3}{s^2+2s+1}$$

$$Y(s) = \frac{-2s-3}{(s+1)^2} \qquad \ldots\text{(ii)}$$

Using partial fraction,

$$Y(s) = \frac{A}{(s+1)} + \frac{B}{(s+1)^2}$$

$$A = -2,\ B = -1$$

From equation (ii),

$$Y(s) = \frac{-2}{s+1} - \frac{1}{(s+1)^2}$$

Taking inverse Laplace transform of $Y(s)$,

$$y(t) = -2e^{-t}u(t) - te^{-t}u(t)$$

Differentiating with respect to t,

$$\frac{dy(t)}{dt} = -[2e^{-t} + te^{-t}]\delta(t) - u(t)\left[-2e^{-t} - te^{-t} + e^{-t}\right]$$

Put $t = 0^+$,

$$\frac{dy(0^+)}{dt} = -[2e^{-0^+} + 0e^{-0^+}]\delta(0^+)$$

$$-u(0^+)\left[-2e^{-0^+} - 0^+ e^{-0^+} + e^{-0^+}\right]$$

$$\frac{dy(0^+)}{dt} = -0-(-2+1) = 1 \quad \left[\delta(0^+) = 0\right]$$

Hence, the correct option is (D).

3.3 (B)

Method 1

Given : $f(t) = 2\sqrt{\frac{t}{\pi}} \xleftrightarrow{\text{LT}} F(s) = s^{\frac{-3}{2}}$

$$g(t) = \sqrt{\frac{1}{\pi t}} = 2\sqrt{\frac{t}{\pi}} \times \frac{1}{2t} = \frac{f(t)}{2t}$$

Taking Laplace transform of $g(t)$,

$$\left[\text{Divide by 't' property, } \left(\frac{x(t)}{t} \xleftrightarrow{L.T} \int_s^\infty X(s)ds\right)\right]$$

$$G(s) = \frac{1}{2}\int_s^\infty F(s)\,ds = \frac{1}{2}\int_s^\infty s^{\frac{-3}{2}}\,ds$$

$$G(s) = \frac{1}{2}\left[\frac{s^{\frac{-3}{2}+1}}{\frac{-3}{2}+1}\right]_s^\infty = \frac{1}{2}\times -2\left[s^{\frac{-1}{2}}\right]_s^\infty$$

$$G(s) = -[\infty^{\frac{-1}{2}} - s^{\frac{-1}{2}}] = s^{\frac{-1}{2}}$$

Hence, the correct option is (B).

Method 2

Given : $f(t) = 2\sqrt{\frac{t}{\pi}} = \frac{2}{\sqrt{\pi}}\cdot t^{1/2}$

$$g(t) = \frac{1}{\sqrt{\pi t}} = \frac{1}{\sqrt{\pi}}\cdot t^{-1/2}$$

As $\quad \frac{d}{dt}f(t) = \frac{d}{dt}\left\{\frac{2}{\sqrt{\pi}}\cdot t^{1/2}\right\}$

$$= \frac{2}{\sqrt{\pi}}\cdot\frac{1}{2}t^{-1/2} = \frac{1}{\sqrt{\pi}}\cdot t^{-1/2}$$

So, $\quad g(t) = \frac{d}{dt}f(t)$

Taking Laplace transforms both sides,

$$G(s) = s\cdot F(s)$$

$$G(s) = s\cdot s^{-3/2} \quad [\text{Given, } F(s) = s^{-3/2}]$$

$$G(s) = s^{-1/2}$$

Hence, the correct option is (B).

Scan for Video Solution

3.4 (B)

From the properties of ROC of Laplace transform :

1. ROC does not contain any pole
2. ROC of transform of a bounded finite duration signal is entire S-plane

It can be said that, if a signal is bounded and exists only for finite duration, then ROC is entire s-plane, so it can not have any pole as ROC does not contain any pole.

Hence, the correct option is (B).

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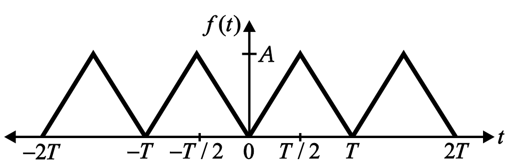
5 Continuous Time Fourier Series

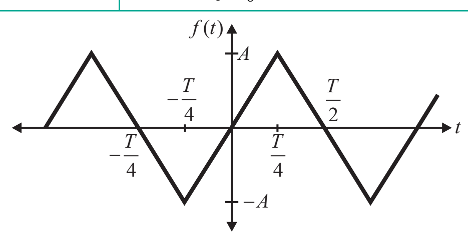
➢ Partial Synopsis

Trigonometric F.S. Coefficients for signals having symmetry

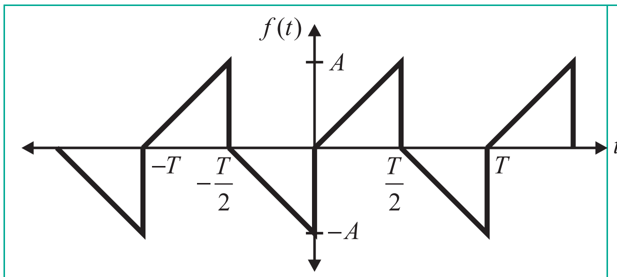
Symmetry	Coefficients			F.S. Representation
Even Symmetry $\boldsymbol{x(t) = x(-t)}$ The trigonometric Fourier series representation of even signals contains cosine terms only. The constant a_0 may or may not be zero.	$\boldsymbol{a_0 \neq 0}$ Or $\boldsymbol{a_0 = 0}$	$\boldsymbol{a_n \neq 0}$	$\boldsymbol{b_n = 0}$	$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t$ $a_0 = \frac{2}{T_0}\int_0^{T_0/2} x(t)dt$ $a_n = \frac{4}{T_0}\int_0^{T_0/2} x(t)\cos n\omega_0 t\, dt$
Odd Symmetry $x(t) = -x(-t)$ The trigonometric Fourier series representation of odd signals contains sine terms only. It has a zero average value, $a_0 = 0$.	$\boldsymbol{a_0 = 0}$	$\boldsymbol{a_n = 0}$	$\boldsymbol{b_n \neq 0}$	$x(t) = \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$ $b_n = \frac{4}{T_0}\int_0^{T_0/2} x(t)\sin n\omega_0 t\, dt$
Half-wave Symmetry $\boldsymbol{x(t) = -x\left(t \pm \frac{T_0}{2}\right)}$ For a signal having half-wave symmetry $a_0 = 0$ and a_n and b_n exists for odd values of n.	$\boldsymbol{a_0 = 0}$	$\boldsymbol{a_{2n} = 0,}$ $\boldsymbol{a_{2n+1} \neq 0}$	$\boldsymbol{b_{2n} = 0,}$ $\boldsymbol{b_{2n+1} \neq 0}$	$x(t) = \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$ $a_n = \frac{4}{T_0}\int_0^{T_0/2} x(t)\cos n\omega_0 t\, dt$, n odd $b_n = \frac{4}{T_0}\int_0^{T_0/2} x(t)\sin n\omega_0 t\, dt$, n odd



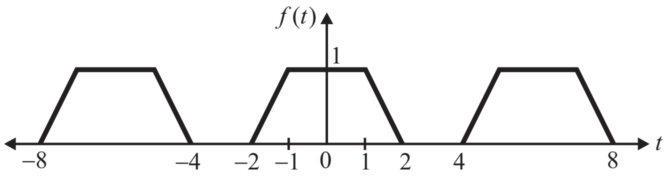
Waveform with even symmetry



Waveform with odd symmetry



Waveform with half-wave symmetry



Waveform with hidden half-wave symmetry

Some waveform show half wave symmetry (hidden) after subtraction of the dc component (a_0), such waveform is shown in figure. The trigonometric Fourier series representation of half-symmetric signals contains only odd harmonics of sine and cosine terms.

Note : RMS value of a periodic waveform $= \sqrt{\frac{1}{T}\int_0^T f^2(t)dt}$

Polar Fourier Series

The polar form or cosine form of Fourier series is expressed as follows

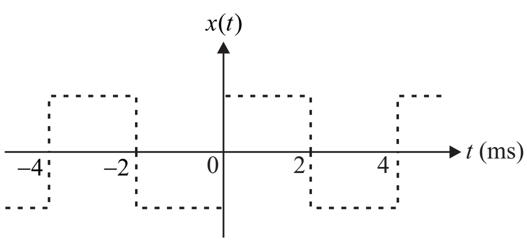
$$\boldsymbol{x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t - \theta_n)}$$

Where, $\boldsymbol{A_0 = a_0}$ $\quad \boldsymbol{A_n = \sqrt{a_n^2 + b_n^2}}$ $\quad \boldsymbol{\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)}$

➢ Sample Questions

1996 IISc Bangalore

5.1 A periodic rectangular signal $x(t)$ has the waveform shown in figure.



Frequency of the fifth harmonic of its spectrum is

(A) 40 Hz (B) 200 Hz

(C) 250 Hz (D) 1250 Hz

2007 IIT Kanpur

5.2 A signal $x(t)$ is given by

$$x(t) = \begin{cases} 1, & \frac{-T}{4} < t \le \frac{3T}{4} \\ -1, & \frac{3T}{4} < t \le \frac{7T}{4} \\ -x(t+T), & \text{Otherwise} \end{cases}$$

Which among the following gives the fundamental Fourier term of $x(t)$?

(A) $\frac{4}{\pi}\cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$

(B) $\frac{\pi}{4}\cos\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

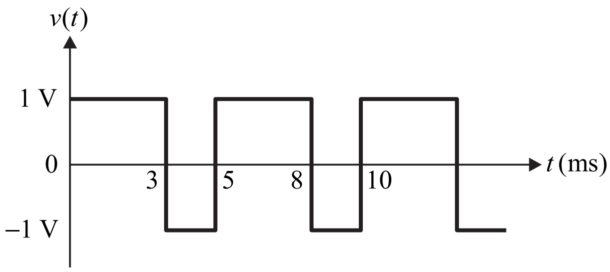
(C) $\frac{4}{\pi}\sin\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$

(D) $\frac{\pi}{4}\sin\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

2014 IIT Kharagpur

Common Data for Questions 5.3 & 5.4

Consider the voltage waveform $v(t)$ as shown in figure. **[Set - 01]**



5.3 The DC component of $v(t)$ is

(A) 0.4 (B) 0.2
(C) 0.8 (D) 0.1

5.4 The amplitude of fundamental component of $v(t)$ is

(A) 1.20 V (B) 2.40 V
(C) 2 V (D) 1 V

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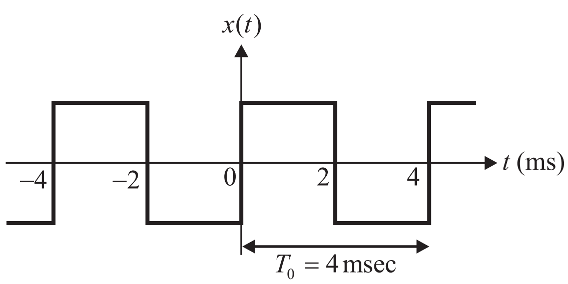
Explanations Continuous Time Fourier Series

5.1 (D)

Given :



Time period of $x(t)$ is 4 msec.

Fundamental frequency is given by,

$$f_0 = \frac{1}{\text{Time period}(T_0)} = \frac{1}{4\,\text{msec}}$$

$$f_0 = \frac{1}{4\times 10^{-3}}\,\text{Hz}$$

Frequency of fifth harmonic

$$= 5f_0 = 5\times\frac{1}{4\times 10^{-3}} = 1250\ \text{Hz}$$

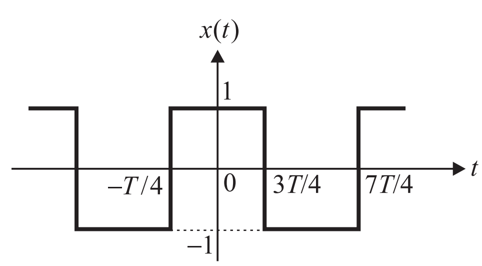
Hence, the correct option is (D).

5.2 (A)

Given :

$$x(t) = \begin{cases} 1\ ; & \dfrac{-T}{4} < t \le \dfrac{3T}{4} \\ -1\ ; & \dfrac{3T}{4} < t \le \dfrac{7T}{4} \\ -x(t+T); & \text{Otherwise} \end{cases}$$

Fourier series exists only for periodic signals. Assume $x(t)$ is a periodic signal. Thus, the waveform of $x(t)$ is given by,



The time period of $x(t)$ is given by,

$$T_0 = \frac{7T}{4} - \left(\frac{-T}{4}\right) = 2T$$

$$x(t) = -x(t+T)$$

$x(t)$ exhibits half wave symmetricity.

Fundamental angular frequency is,

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{\pi}{T}$$

Exponential Fourier series coefficient is given by,

$$C_n = \frac{1}{T_0}\int_{<T_0>} x(t)e^{-j\omega_0 nt}\,dt$$

To get fundamental Fourier term $n = 1$,

$$C_1 = \frac{1}{T_0}\int_{<T_0>} x(t)e^{-j\omega_0 t}\,dt$$

$$C_1 = \frac{1}{2T}\left[\int_{-T/4}^{3T/4} e^{-j\omega_0 t}\,dt + \int_{3T/4}^{7T/4} (-1)\,e^{-j\omega_0 t}\,dt\right]$$

$$C_1 = \frac{1}{2T}\left[\left\{\frac{e^{-j\omega_0 t}}{-j\omega_0}\right\}_{-T/4}^{3T/4} + \left\{\frac{-e^{-j\omega_0 t}}{-j\omega_0}\right\}_{3T/4}^{7T/4}\right]$$

$$C_1 = \frac{1}{j2\pi}\left[\left\{e^{j\frac{\pi}{T}\frac{T}{4}} - e^{-j\frac{\pi}{T}\frac{3T}{4}}\right\}\right.$$

$$\left. + \left\{e^{-j\frac{\pi}{T}\frac{7T}{4}} - e^{-j\frac{\pi}{T}\frac{3T}{4}}\right\}\right]$$

$$C_1 = \frac{1}{j2\pi}[(e^{j\frac{\pi}{4}} - e^{-j\frac{3\pi}{4}}) + (e^{-j\frac{7\pi}{4}} - e^{-j\frac{3\pi}{4}})]$$

$$C_1 = \frac{1}{j2\pi}\left[\cos\frac{\pi}{4} + j\sin\frac{\pi}{4} - \cos\frac{3\pi}{4}\right.$$

$$+ j\sin\frac{3\pi}{4} + \cos\frac{7\pi}{4} - j\sin\frac{7\pi}{4}$$

$$\left. - \cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right]$$

$$C_1 = \frac{1}{j2\pi}\left[\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right.$$

$$\left. + \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right]$$

$$C_1 = \frac{1}{j2\pi}\left[\frac{4}{\sqrt{2}} + j\frac{4}{\sqrt{2}}\right]$$

$$C_1 = \frac{2}{\pi}\left[\frac{1}{j\sqrt{2}} + \frac{1}{\sqrt{2}}\right] = \frac{2}{\pi}\left[\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right]$$

Comparing above equation with

$$C_1 = \frac{a_1}{2} + \frac{jb_1}{2},$$

Then, $a_1 = \frac{4}{\sqrt{2}\pi},\ b_1 = \frac{-4}{\sqrt{2}\pi}$ ...(i)

Fundamental Fourier term $= |A_1|\cos(\omega_0 t + \angle A_1)$ ...(ii)

where, $|A_1| = \sqrt{a_1^2 + b_1^2}$

$$|A_1| = \sqrt{\left(\frac{4}{\sqrt{2}\pi}\right)^2 + \left(\frac{-4}{\sqrt{2}\pi}\right)^2} = \frac{4}{\pi}$$

and $\angle A_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right)$

$$\angle A_1 = \tan^{-1}\left(\frac{-4/\sqrt{2}\pi}{4/\sqrt{2}\pi}\right) = \frac{-\pi}{4}$$

From equation (ii),

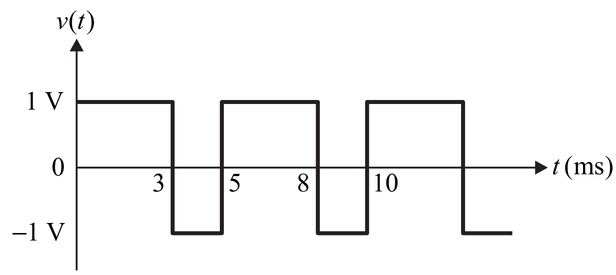
$$\text{Fundamental term} = \frac{4}{\pi}\cos\left(\omega_0 t - \frac{\pi}{4}\right)$$

$$= \frac{4}{\pi}\cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$$

Hence, the correct option is (A).

5.3 (B)

Given :



From the above figure,

$$v(t)=\begin{cases}1\text{V}; & 0<t\le 3\\ -1\text{V}; & 3<t\le 5\end{cases}$$

Time period of $v(t)$ is 5.

$$\omega=\frac{2\pi}{T}=\frac{2\pi}{5}$$

Method 1

The average value of the signal is given by,

$$V_{avg}=\frac{1}{T}\int_0^T V(t)\,dt$$

$$V_{avg}=\frac{1}{5}\left[\int_0^3 (1)dt+\int_3^5(-1)dt\right]$$

$$V_{avg}=\frac{1}{5}[3-0-5+3]=\frac{1}{5}=0.2\text{ V}$$

Hence, the correct option is (B).

Method 2

$$\text{Dc component}=\frac{\text{Area of signal in one period}}{\text{Time period}}$$

$$=\frac{3-2}{5}=\frac{1}{5}=0.2$$

Hence, the correct option is (B).

5.4 (A)

From the waveform,

$$v(t)=\begin{cases}1\text{V}; & 0<t\le 3\\ -1\text{V}; & 3<t\le 5\end{cases}$$

Time period of $v(t)$ is 5.

$$\omega=\frac{2\pi}{T}=\frac{2\pi}{5}$$

Method 1

Amplitude of fundamental component,

$$A_1=\sqrt{a_1^2+b_1^2} \qquad \ldots(\text{i})$$

$$a_1=\frac{2}{T}\int_0^T v(t)\cos\omega t\,dt$$

$$a_1=\frac{2}{5}\left[\int_0^3 1\cdot\cos\left(\frac{2\pi}{5}\right)t\,dt+\int_3^5(-1)\cos\left(\frac{2\pi}{5}\right)t\,dt\right]$$

$$a_1=\frac{2}{5}\left[\left\{\frac{\sin\left(\frac{2\pi}{5}t\right)}{\left(\frac{2\pi}{5}\right)}\right\}_0^3-\left\{\frac{\sin\left(\frac{2\pi}{5}\right)t}{\left(\frac{2\pi}{5}\right)}\right\}_3^5\right]$$

$$a_1=\frac{2}{5}\left[\frac{5}{2\pi}\left\{\sin\left(\frac{6\pi}{5}\right)-0\right\}-\frac{5}{2\pi}\left\{\sin 2\pi-\sin\frac{6\pi}{5}\right\}\right]$$

$$a_1=\frac{2}{5}\times\frac{5}{2\pi}\times\left\{\sin\frac{6\pi}{5}+\sin\frac{6\pi}{5}\right\}$$

$$a_1=\frac{2}{\pi}\sin\left(\frac{6\pi}{5}\right) \qquad \ldots(\text{ii})$$

Similarly, $b_1=\frac{2}{T}\int_0^T v(t)\sin\omega t\,dt$

$$b_1=\frac{2}{5}\left[\int_0^3 1\cdot\sin\left(\frac{2\pi}{5}\right)t\,dt+\int_3^5(-1)\cdot\sin\left(\frac{2\pi}{5}\right)t\,dt\right]$$

$$b_1=\frac{2}{5}\left[\left\{\frac{-\cos\left(\frac{2\pi}{5}\right)t}{\left(\frac{2\pi}{5}\right)}\right\}_0^3-\left\{\frac{-\cos\left(\frac{2\pi}{5}\right)t}{\left(\frac{2\pi}{5}\right)}\right\}_3^5\right]$$

$$b_1=\frac{2}{5}\left[\frac{5}{2\pi}\left\{1-\cos\left(\frac{6\pi}{5}\right)\right\}-\frac{5}{2\pi}\left\{-\cos\frac{10\pi}{5}+\cos\frac{6\pi}{5}\right\}\right]$$

$$b_1=\frac{2}{5}\left[\frac{5}{2\pi}\left\{1-\cos\left(\frac{6\pi}{5}\right)\right\}-\frac{5}{2\pi}\left\{\cos\frac{6\pi}{5}-1\right\}\right]$$

$$b_1=\frac{2}{5}\times\frac{5}{2\pi}\left[2\left\{1-\cos\frac{6\pi}{5}\right\}\right]$$

$$b_1=\frac{2}{\pi}\left\{1-\cos\left(\frac{6\pi}{5}\right)\right\} \qquad \ldots(\text{iii})$$

From equation (i),

Amplitude of fundamental component,

$$A_1=\sqrt{a_1^2+b_1^2}$$

$$A_1=\sqrt{\left[\frac{2}{\pi}\sin\frac{6\pi}{5}\right]^2+\left[\frac{2}{\pi}\left(1-\cos\frac{6\pi}{5}\right)\right]^2}$$

$$A_1 = \sqrt{\frac{4}{\pi^2}\left\{\sin^2\frac{6\pi}{5} + 1 - 2\cos\frac{6\pi}{5} + \cos^2\frac{6\pi}{5}\right\}}$$

$$A_1 = \frac{2}{\pi}\sqrt{2\left(1-\cos\frac{6\pi}{5}\right)}$$

$$A_1 = 1.21\text{ V} \approx 1.20\text{ V}$$

Hence, the correct option is (A).

Method 2

From complex exponential form of Fourier series, the complex coefficient, C_n is,

$$C_n = \frac{1}{T}\int_0^T f(t)\, e^{\frac{-j2\pi nt}{T}}\, dt$$

$$C_1 = \frac{1}{5}\int_0^5 v(t)\cdot e^{-j\frac{2\pi t}{5}}\, dt$$

$$C_1 = \frac{1}{5}\left[\int_0^3 e^{-j\frac{2\pi t}{5}}\, dt - \int_3^5 e^{-j\frac{2\pi t}{5}}\, dt\right]$$

$$C_1 = \frac{1}{5}\left[\left\{\frac{e^{-j\frac{2\pi t}{5}}}{-j\frac{2\pi}{5}}\right\}_0^3 - \left\{\frac{e^{-j\frac{2\pi t}{5}}}{-j\frac{2\pi}{5}}\right\}_3^5\right]$$

$$C_1 = \frac{1}{-j2\pi}\left[\left\{e^{-j\frac{2\pi 3}{5}} - 1\right\} - \left\{e^{-j\frac{2\pi 5}{5}} - e^{-j\frac{2\pi 3}{5}}\right\}\right]$$

$$C_1 = \frac{1}{-j2\pi}\left[e^{-j\frac{2\pi 3}{5}} - 1 - 1 + e^{-j\frac{2\pi 3}{5}}\right]$$

$$C_1 = \frac{1}{j\pi}\left[1 - e^{-j\frac{2\pi 3}{5}}\right]$$

$$C_1 = \frac{1}{j\pi}\left[e^{j\frac{\pi 3}{5}} - e^{-j\frac{\pi 3}{5}}\right] e^{-j\frac{\pi 3}{5}}$$

$$C_1 = \frac{2}{\pi}\sin\frac{3\pi}{5}\, e^{-j\frac{3\pi}{5}}$$

$$|C_1| = \frac{2}{\pi}\left|\sin\frac{3\pi}{5}\right| = 0.6$$

In complex exponential form of Fourier series, half of the power is divided in negative frequencies.

Hence, $A_1 = \sqrt{2\left[|C_1|^2 + |C_{-1}|^2\right]} = 2|C_1|$

$$|C_{-1}| = |C_1| = 0.6$$

$$A_1 = 2\times 0.6\text{ V} = 1.20\text{ V}$$

Hence, the correct option is (A).

❖❖❖❖

6 Continuous Time Fourier Transform

➢ Partial Synopsis

(8) Frequency Differentiation or Multiplication by t : Frequency differentiation leads to multiplication by t in time domain.

$$-j2\pi t.x(t) \xleftrightarrow{FT} \frac{d}{df}X(f) \qquad t.x(t) \xleftrightarrow{FT} \frac{j}{2\pi}\frac{d}{df}X(f)$$

$$-jt.x(t) \xleftrightarrow{FT} \frac{d}{d\omega}X(\omega) \qquad t.x(t) \xleftrightarrow{FT} j\frac{d}{d\omega}X(\omega)$$

(9) Duality : Duality allows to obtain both the dual transform pairs from one equation.

$$x(t) \xleftrightarrow{FT} X(f) \quad \Rightarrow \quad X(t) \xleftrightarrow{FT} x(-f)$$

$$x(t) \xleftrightarrow{FT} X(\omega) \quad \Rightarrow \quad X(t) \xleftrightarrow{FT} 2\pi.x(-\omega)$$

(10) Integration in Time domain :

$$x(t) \xleftrightarrow{FT} X(f) \quad \Rightarrow \quad \int_{-\infty}^{t} x(\tau)d\tau \xleftrightarrow{FT} \frac{X(f)}{j2\pi f} + \frac{1}{2}X(0)\delta(f)$$

$$x(t) \xleftrightarrow{FT} X(\omega) \quad \Rightarrow \quad \int_{-\infty}^{t} x(\tau)d\tau \xleftrightarrow{FT} \frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$$

(11) Conjugation and Conjugate symmetry :

Conjugation $\Rightarrow$ $x^*(t) \xleftrightarrow{FT} X^*(-j\omega)$

Conjugate symmetry $\Rightarrow$ If $x(t)$ is real, then $X(-j\omega) = X^*(j\omega)$

Basic Fourier Transforms

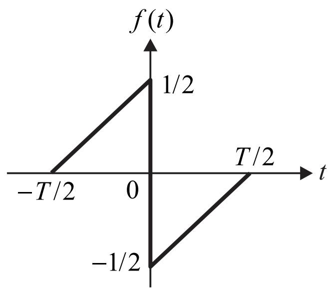
S.	$x(t)$	$X(f)$	$X(\omega)$
1.	$\delta(t)$	1	1
2.	$e^{-at}u(t),\ a>0$	$\frac{1}{a+j2\pi f}$	$\frac{1}{a+j\omega}$
3.	$e^{at}u(-t),\ a>0$	$\frac{1}{a-j2\pi f}$	$\frac{1}{a-j\omega}$

4.	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+(2\pi f)^2}$	$\dfrac{2a}{a^2+\omega^2}$
5.	$A\,\text{rect}\left(\dfrac{t}{\tau}\right)$	$A\tau \sin c(f\tau)$	$A\tau\,\text{Sa}\left(\dfrac{\omega\tau}{2}\right)$
6.	$A\,\text{tri}\left(\dfrac{t}{\tau}\right)=A\left[1-\dfrac{\|t\|}{\tau}\right]$	$A\tau \sin c^2(f\tau)$	$A\tau\,\text{Sa}^2\left(\dfrac{\omega\tau}{2}\right)$
7.	$\text{sgn}(t)$	$\dfrac{1}{j\pi f}$	$\dfrac{2}{j\omega}$
8.	$e^{-a\|t\|}\,\text{sgn}(t)$	$\dfrac{-j4\pi f}{a^2+(2\pi f)^2}$	$\dfrac{-j2\omega}{a^2+\omega^2}$
9.	$t.e^{-at}u(t)$	$\dfrac{1}{(a+j2\pi f)^2}$	$\dfrac{1}{(a+j\omega)^2}$
10.	$t.e^{at}u(-t)$	$\dfrac{-1}{(a-j2\pi f)^2}$	$\dfrac{-1}{(a-j\omega)^2}$
11.	1	$\delta(f)$	$2\pi.\delta(\omega)$
12.	$u(t)$	$\dfrac{\delta(f)}{2}+\dfrac{1}{j2\pi f}$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$
13.	$\cos 2\pi f_0 t$	$\dfrac{\delta(f-f_0)+\delta(f+f_0)}{2}$	$\pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$
14.	$\sin 2\pi f_0 t$	$\dfrac{\delta(f-f_0)-\delta(f+f_0)}{2j}$	$\dfrac{\pi}{j}\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$
15.	$e^{-at}\cos(2\pi f_0 t).u(t)$	$\dfrac{a+j2\pi f}{(a+j2\pi f)^2+(2\pi f_0)^2}$	$\dfrac{a+j\omega}{(a+j\omega)^2+(\omega_0)^2}$
16.	$e^{-at}\sin(2\pi f_0 t).u(t)$	$\dfrac{2\pi f_0}{(a+j2\pi f)^2+(2\pi f_0)^2}$	$\dfrac{\omega_0}{(a+j\omega)^2+(\omega_0)^2}$
17.	$te^{-a\|t\|}$	$\dfrac{-j8\pi fa}{\left[a^2+(2\pi f)^2\right]^2}$	$\dfrac{-j4a\omega}{\left[a^2+\omega^2\right]^2}$
18.	e^{jt}	$\delta\left(f-\dfrac{1}{2\pi}\right)$	$2\pi\delta(\omega-1)$

➢ Sample Questions

2014 IIT Kharagpur

6.1 A function $f(t)$ is shown in the figure.



The Fourier transform $F(\omega)$ of $f(t)$ is **[Set - 03]**

(A) real and even function of ω.
(B) real and odd function of ω.
(C) imaginary and odd function of ω.
(D) imaginary and even function of ω.

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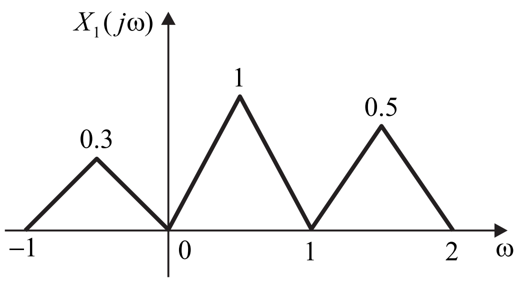
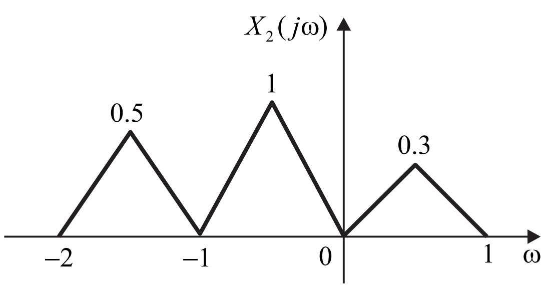
6.2 Suppose $x_1(t)$ and $x_2(t)$ have the Fourier transforms as shown below.





Which one of the following statements is TRUE? **[Set - 01]**

(A) $x_1(t)$ and $x_2(t)$ are complex and $x_1(t).x_2(t)$ is also complex with nonzero imaginary part.
(B) $x_1(t)$ and $x_2(t)$ are real and $x_1(t).x_2(t)$ is also real.
(C) $x_1(t)$ and $x_2(t)$ are complex but $x_1(t).x_2(t)$ is real.
(D) $x_1(t)$ and $x_2(t)$ are imaginary but $x_1(t).x_2(t)$ is real.

6.3 The output of a continuous-time, linear time-invariant system is denoted by $T\{x(t)\}$ where $x(t)$ is the input signal. A signal $z(t)$ is called Eigen-signal of the system T, when $T\{z(t)\} = \gamma\, z(t)$, where γ is a complex number in general and is called an eigenvalue of T. Suppose the impulse response of the system T is real and even. Which of the following statements is TRUE? **[Set - 01]**

(A) $\cos(t)$ is an eigen-signal but $\sin(t)$ is not.
(B) $\cos(t)$ and $\sin(t)$ are both eigen-signals but with different eigenvalues.
(C) $\sin(t)$ is an eigen-signal but $\cos(t)$ is not.
(D) $\cos(t)$ and $\sin(t)$ are both eigen-signals with identical eigenvalues.

2021 IIT Bombay

6.4 Let $f(t)$ be an even function, i.e., $f(-t) = f(t)$ for all t. Let the Fourier transform of $f(t)$ be defined as

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt .$$

Suppose $\dfrac{dF(\omega)}{d\omega} = -\omega F(\omega)$ for all ω

and $F(0) = 1$. Then

(A) $f(0) < 1$ (B) $f(0) > 1$

(C) $f(0) = 1$ (D) $f(0) = 0$

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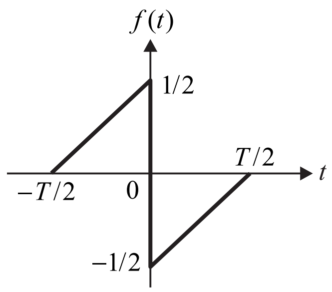
Explanations — Continuous Time Fourier Transform

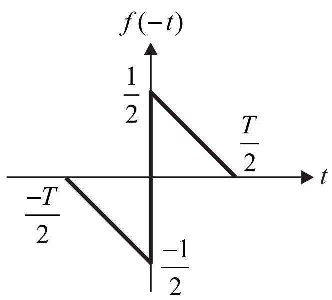
6.1 (C)

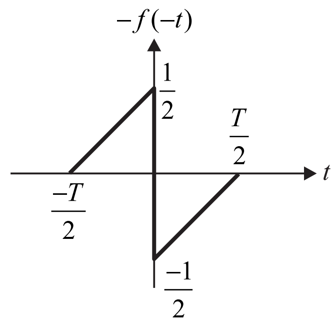
Given :

A function $f(t)$ is shown below,



The waveform of $f(-t)$ is shown below,





Here, $f(t) = -f(-t)$

Signal $f(t)$ is an odd real signal. Therefore, $F(\omega)$ (Fourier transform of $f(t)$) is imaginary and odd function of ω.

Hence, the correct option is (C).

Table 6.1 : Symmetry Conditions of Fourier Transform

$x(t)$	$X(f)$	Example
Even	Even	$\text{rect}(t) \longleftrightarrow \text{sinc}(f)$
Odd	Odd	$\text{sgn}(t) \longleftrightarrow \dfrac{1}{j\pi f}$
Real and even	Real and even	$\text{rect}(t) \longleftrightarrow \text{sinc}(f)$
Imaginary and even	Imaginary and even	$j\,\text{rect}(t) \longleftrightarrow j\,\text{sinc}(f)$

Real and odd	Imaginary and odd	$\text{sgn}(t) \longleftrightarrow \frac{1}{j\pi f}$
Imaginary and odd	Real and odd	$j\,\text{sgn}(t) \longleftrightarrow \frac{1}{\pi f}$
Complex and even	Complex and even	$(1+j)\text{rect}(t) \longleftrightarrow (1+j)\text{sinc}(f)$
Complex and odd	Complex and odd	$(1+j)\text{sgn}(t) \longleftrightarrow \frac{1-j}{\pi f}$
Real even and imaginary odd (Conjugate symmetric)	Real	$\text{rect}(t)+j\,\text{sgn}(t) \longleftrightarrow \text{sinc}(f)+\frac{1}{\pi f}$
Real odd and imaginary even (Conjugate anti-symmetric)	Imaginary	$\text{sgn}(t)+j\,\text{rect}(t) \longleftrightarrow \frac{1}{j\pi f}+j\,\text{sinc}(f)$
Real	Real even and imaginary odd (Conjugate symmetric)	$\text{rect}(t) \longleftrightarrow \text{sinc}(f)$
Imaginary	Real odd and imaginary even (Conjugate anti-symmetric)	$j\,\text{rect}(t) \longleftrightarrow j\,\text{sinc}(f)$

Table 6.2 : Concept of Real and Imaginary / Conjugate symmetric and Conjugate anti symmetric in t-domain :

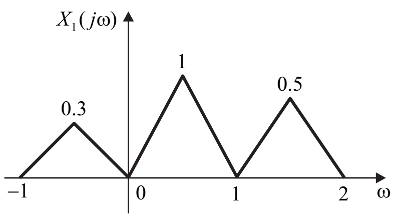
Real	Imaginary	Conjugate symmetric	Conjugate anti symmetric
$x(t)=x^*(t)$	$x(t)=-x^*(t)$	$x(t)=x^*(-t)$	$x(t)=-x^*(-t)$
$x_R(t)=\frac{x(t)+x^*(t)}{2}$	$x_I(t)=\frac{x(t)-x^*(t)}{2}$	$x_{C.S}(t)=\frac{x(t)+x^*(-t)}{2}$	$x_{C.A.S}(t)=\frac{x(t)-x^*(-t)}{2}$

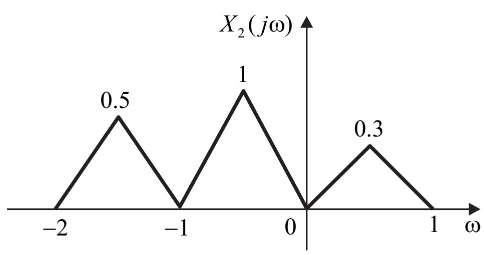
Table 6.3 : Concept of Real and Imaginary / Conjugate symmetric and Conjugate anti symmetric in f-domain :

Real	Imaginary	Conjugate symmetric	Conjugate anti symmetric
$X(f)=X^*(f)$	$X(f)=-X^*(f)$	$X(f)=X^*(-f)$	$X(f)=-X^*(-f)$
$X_R(f)=\frac{X(f)+X^*(f)}{2}$	$X_I(f)=\frac{X(f)-X^*(f)}{2}$	$X_{C.S}(f)=\frac{X(f)+X^*(-f)}{2}$	$X_{C.A.S}(f)=\frac{X(f)-X^*(-f)}{2}$

6.2 (C)

Given :





From the given plots of $X_1(j\omega) \approx X_1(\omega)$ and $X_2(j\omega) \approx X_2(\omega)$, following points can be noted:

1. Both are real, i.e. $X_1^*(\omega) = X_1(\omega)$ and $X_2^*(\omega) = X_2(\omega)$
2. Both are neither even nor odd, i.e.
   $X_1(-\omega) \neq X_1(\omega)$, $X_2(-\omega) \neq X_2(\omega)$
   $X_1(-\omega) \neq -X_1(\omega)$,
   $X_2(-\omega) \neq -X_2(\omega)$
3. $X_1(\omega)$ and $X_2(\omega)$ are related to each other with frequency reversal operation, i.e.,
   $X_2(-\omega) = X_1(\omega)$ or $X_1(-\omega) = X_2(\omega)$

Extending point (1), $X_1^*(\omega) = X_1(\omega)$

Replacing 'ω' by '$-\omega$' both sides,

$$X_1^*(-\omega) = X_1(-\omega)$$

Taking inverse Fourier transform both sides,

$$x_1^*(t) = x_1(-t) \qquad \ldots\text{(i)}$$

[Similar condition is valid for $x_2(t)$ also, i.e. $x_2^*(t) = x_2(-t)$]

As $X_1(\omega)$ is neither even nor odd, i.e. $X_1(-\omega) \neq X_1(\omega)$ and $X_1(-\omega) \neq -X_1(\omega)$.

Taking inverse Fourier transform on both sides of above equations,

$$x_1(-t) \neq x_1(t) \text{ and } x_1(-t) \neq -x_1(t)$$

Substituting $x_1(-t) = x_1^*(t)$ from equation (i), above results can be rewritten as,

$$x_1^*(t) \neq x_1(t) \text{ and } x_1^*(t) \neq -x_1(t)$$

Hence, $x_1(t)$ is not real and $x_1(t)$ is not purely imaginary.

So, we came to conclude that $x_1(t)$ must be complex in nature. Similarly we can prove that $x_2(t)$ is also complex in nature.

Hence, options (B) and (D) can be eliminated.

Now, as $X_2(\omega) = X_1(-\omega)$

$$x_2(t) = x_1(-t) \qquad \ldots\text{(ii)}$$

As $X_1(\omega)$ and $X_2(\omega)$ are real, so following the symmetry condition, their inverse $x_1(t)$ and $x_2(t)$ must be conjugate symmetric which can be seen from equation (i),

Substituting $x_1(-t) = x_1^*(t)$ from equation (i) in equation (ii), we have

$$x_2(t) = x_1^*(t)$$

Therefore, the required product is,

$$x_1(t)x_2(t) = x_1(t)x_1^*(t)$$

$$x_1(t)x_2(t) = |x_1(t)|^2$$

(From property of complex number)

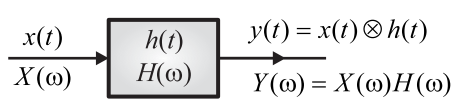
$$x_1(t)x_2(t) = \text{Real}$$

So, finally we can state that $x_1(t)$ and $x_2(t)$ are complex but their product $x_1(t)x_2(t)$ must be real.

Hence, the correct option is (C).

6.3 (D)

Given that the impulse response of the LTI system is real and even and we have to check whether $\cos t$ and $\sin t$ are Eigen functions for the system or not and if they are Eigen functions, then we have to compare Eigen values corresponding to cost and $\sin t$.



As mentioned in the problem, an input $x(t)$ is said to be Eigen function for LTI system, if response of the system for this input is

$$y(t) = \lambda x(t) \qquad \ldots\text{(i)}$$

Where, $\lambda = \text{Constant}$ [Eigen value corresponding to $x(t)$]

As $h(t)$ is real and even,

$$h(-t)=h(t), \qquad h^*(t)=h(t)$$

Taking Fourier transform both sides of above two equations.

$$H(-\omega)=H(\omega) \qquad \ldots\text{(ii)}$$

$$H^*(-\omega)=H(\omega) \qquad \ldots\text{(iii)}$$

To check whether $\cos t$ is Eigen function for the given system or not,

$$x(t)=\cos t$$

$$X(\omega)=\pi\{\delta(\omega-1)+\delta(\omega+1)\}$$

$$Y(\omega)=X(\omega)H(\omega)$$

$$Y(\omega)=H(\omega)\times\pi\{\delta(\omega-1)+\delta(\omega+1)\}$$

$$Y(\omega)=\pi[H(\omega)\delta(\omega-1)+H(\omega)\delta(\omega+1)]$$

Using product property of impulse function,

$$P(\omega)\delta(\omega-\omega_0)=P(\omega_0)\delta(\omega-\omega_0)$$

$$Y(\omega)=\pi[H(1)\delta(\omega-1)+H(-1)\delta(\omega+1)]$$

From equation (ii),

$$H(-1)=H(1)$$

$$Y(\omega)=H(1)\pi[\delta(\omega-1)+\delta(\omega+1)]$$

Taking inverse Fourier transform both sides,

$$y(t)=H(1)\cos t$$

Comparing with equation (i), it can be said that $\cos t$ is an Eigen function with Eigen value $\lambda=H(1)$.

To check whether $\sin t$ is Eigen function for the given system or not,

$$x(t)=\sin t$$

$$X(\omega)=\frac{\pi}{j}\{\delta(\omega-1)-\delta(\omega+1)\}$$

$$Y(\omega)=X(\omega)H(\omega)$$

$$Y(\omega)=H(\omega)\times\frac{\pi}{j}\{\delta(\omega-1)-\delta(\omega+1)\}$$

$$Y(\omega)=\frac{\pi}{j}[H(\omega)\delta(\omega-1)-H(\omega)\delta(\omega+1)]$$

$$Y(\omega)=\frac{\pi}{j}[H(1)\delta(\omega-1)-H(-1)\delta(\omega+1)]$$

From equation (ii),

$$H(-1)=H(1)$$

$$Y(\omega)=H(1)\frac{\pi}{j}[\delta(\omega-1)-\delta(\omega+1)]$$

Taking inverse Fourier transform both sides,

$$y(t)=H(1)\sin t$$

Comparing with equation (i), it can be seen that $'\sin t'$ is also an Eigen function for the given system having same Eigen value $\lambda=H(1)$.

So, $\cos t$ and $\sin t$ both are Eigen functions for the given system with identical Eigen values $\lambda=H(1)$.

Hence, the correct option is (D).

Key Point

Eigen function and Eigen value for continuous time signals : If the output signal is a scalar multiple of input then the signal is referred as an Eigen function (or characteristic function) and the multiplier is referred as an Eigen value (or characteristic value).

Scan for Video Solution

6.4 (A)

Given :

(i) $f(t)$ is even i.e. $f(-t)=f(t)$

(ii) $\dfrac{dF(\omega)}{d\omega}=-\omega F(\omega)$ for all ω

(iii) $F(0)=1$

The only even function, whose differentiation contains itself is a Gaussian function represented as $e^{-a\omega^2}$.

Selecting a Gaussian function which satisfies all mentioned condition,

$$F(\omega)=e^{-\frac{\omega^2}{2}}$$

$$\frac{d}{d\omega}F(\omega) = -\frac{1}{2}\times 2\omega\times e^{-\frac{\omega^2}{2}} = -\omega F(\omega)$$

$$F(0) = e^{-\frac{\omega^2}{2}}\Big|_{\omega=0} = 1$$

So, the required Fourier transform is,

$$f(t) \longleftrightarrow e^{-\frac{\omega^2}{2}} = e^{-\frac{4\pi^2 f^2}{2}} = e^{-2\pi^2 f^2}$$

$$f(t) \longleftrightarrow e^{-\pi(\sqrt{2\pi} f)^2} \qquad \ldots(\text{i})$$

From standard result, Fourier transform only Gaussian function results in Gaussian, i.e.

$$e^{-\pi t^2} \longleftrightarrow e^{-\pi f^2}$$

To obtain a transform as given in equation (i), applying time scaling property, replacing 't' by $\frac{t}{\sqrt{2\pi}}$, we get

$$e^{-\pi\left(\frac{t}{\sqrt{2\pi}}\right)^2} \longleftrightarrow \frac{1}{1/\sqrt{2\pi}} e^{-\pi\left(\frac{f}{1/\sqrt{2\pi}}\right)^2}$$

$$e^{-\frac{t^2}{2}} \longleftrightarrow \sqrt{2\pi}\, e^{-2\pi^2 f^2}$$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \longleftrightarrow e^{-2\pi^2 f^2}$$

Comparing with equation (i),

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$$

$\therefore$ $$f(0) = \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}}\Big|_{t=0} = \frac{1}{\sqrt{2\pi}}$$

$\therefore$ $$f(0) < 1$$

Hence, the correct option is (A).

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7 Z - Transform

Partial Synopsis

Properties of Z-Transform :

(1) Linearity property :

If $x_1[n] \xleftrightarrow{ZT} X_1(z)$	ROC : R_1
$x_2[n] \xleftrightarrow{ZT} X_2(z)$	ROC : R_2
Then $ax_1[n]+bx_2[n] \xleftrightarrow{ZT} aX_1(z)+bX_2(z)$	ROC : Atleast $R_1 \cap R_2$

(2) Time shifting property :

If $x[n] \xleftrightarrow{ZT} X(z)$ ROC : R

Then $x[n \pm k] \xleftrightarrow{ZT} z^{\pm k}\, X(z)$ ROC : R, except possible addition or deletion of $z = 0/\infty$

(3) Scaling property :

If $x[n] \xleftrightarrow{ZT} X(z)$	ROC : R
Then $a^n x[n] \xleftrightarrow{ZT} X\left(\frac{z}{a}\right)$	ROC : $\lvert a \rvert R$

(4) Time reversible property :

If $x[n] \xleftrightarrow{ZT} X(z)$	ROC : R
Then $x[-n] \xleftrightarrow{ZT} X(z^{-1})$	ROC : $\frac{1}{R}$

(5) Convolution property :

If $x_1[n] \xleftrightarrow{ZT} X_1(z)$	ROC : R_1
$x_2[n] \xleftrightarrow{ZT} X_2(z)$	ROC : R_2
Then $x_1[n] \otimes x_2[n] \xleftrightarrow{ZT} X_1(z)\; X_2(z)$	ROC : $R_1 \cap R_2$

(6) Multiplication by n :

If $x[n] \xleftrightarrow{ZT} X(z)$	ROC : R
Then $n\,x[n] \xleftrightarrow{ZT} -z\dfrac{d}{dz}X(z)$	ROC : R

(7) Time differencing :

If $x[n] \xleftrightarrow{ZT} X(z)$	ROC : R
Then $x[n]-x[n-1] \xleftrightarrow{ZT} X(z)(1-z^{-1})$	ROC : $R \cap (\lvert z\rvert > 0)$

Sample Questions

2006 IIT Kharagpur

7.1 The discrete-time signal

$$x[n] \leftrightarrow X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n}$$

Where $\leftrightarrow$ denotes a transform-pair relationship, is orthogonal to the signal

(A) $y_1[n] \leftrightarrow Y_1(z) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$

(B) $y_2[n] \leftrightarrow Y_2(z) = \sum_{n=0}^{\infty} \left(5^n - n\right) z^{-(2n+1)}$

(C) $y_3[n] \leftrightarrow Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|} z^{-n}$

(D) $y_4[n] \leftrightarrow Y_4(z) = 2z^{-4} + 3z^{-2} + 1$

2008 IISc Bangalore

7.2 $H(z)$ is a transfer function of a real system. When a signal $x[n] = (1+j)^n$ is the input to such a system, the output is zero. Further, the region of convergence (ROC) of $\left(1-\frac{1}{2}z^{-1}\right)H(z)$ is the entire z-plane (except $z = 0$). It can then be inferred that $H(z)$ can have a minimum of

(A) one pole and one zero.

(B) one pole and two zeros.

(C) two poles and one zero.

(D) two poles and two zeros.

2021 IIT Bombay

7.3 The causal signal with z-transform $z^2(z-a)^{-2}$ is ($u[n]$ is the unit step signal)

(A) $a^{2n}u[n]$

(B) $(n+1)a^n u[n]$

(C) $n^{-1}a^n u[n]$

(D) $n^2 a^n u[n]$

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Explanations **Z-Transform**

7.1 (B)

Given : $x[n] \leftrightarrow X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n}$

Two discrete time signals $x[n]$ and $y[n]$ are said to be orthogonal, if

$$\sum_{n=0}^{\infty} x[n]y[n] = 0 \text{, for real } x[n] \text{ and } y[n]. \quad \ldots(i)$$

For given $x[n]$,

$$X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n} = \frac{1}{2} + z^2 + \frac{9}{4} z^4 + \ldots$$

Taking inverse Z-transform of $X(z)$,

$$x[n] = \frac{1}{2}\delta(n) + \delta(n+2) + \ldots \quad \ldots(ii)$$

Checking from the option :

(i) From option (A) :

$$Y_1(z) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$$

$$Y_1(z) = 1 + \left(\frac{2}{3}\right)^1 z^{-1} + \left(\frac{2}{3}\right)^2 z^{-2} + \ldots$$

Taking inverse Z-transform of $Y_1(z)$,

$$y_1(n) = \delta(n) + \frac{2}{3}\delta(n-1) + \frac{4}{9}\delta(n-2) + \ldots \quad \ldots(iii)$$

From equation (ii) and (iii),

$$x(n)y_1(n) = \left[\frac{1}{2}\delta(n) + \delta(n+2) + \ldots\right] \times \left[\delta(n) + \frac{2}{3}\delta(n-1) + \frac{4}{9}\delta(n-2) + \ldots\right]$$

$$x(n)\,y_1(n) = \frac{1}{2}\delta(n)\cdot\delta(n) + \frac{1}{3}\delta(n)\cdot\delta(n-1) + \ldots + \delta(n+2)\cdot\delta(n) + \frac{2}{3}\delta(n+2)\cdot\delta(n-1) + \ldots$$

$$\left[\begin{array}{l}\text{Property of impulse response,}\\ \delta(n-a)x(n) = x(a)\cdot\delta(n-a)\end{array}\right]$$

$$x(n)\cdot y_1(n) = \frac{1}{2}\delta(n)\cdot\delta(0) + \frac{1}{3}\delta(n)\cdot\delta(-1) + \ldots$$

$$x(n)\cdot y_1(n) = \frac{1}{2}\delta(n) \neq 0$$

Thus, option (A) does not satisfy orthogonality.

(ii) From option (B) :

$$Y_2(z) = \sum_{n=0}^{\infty} (5^n - n) z^{-(2n+1)}$$

$$Y_2(z) = z^{-1} + 4z^{-3} + 23z^{-5} + \ldots.$$

Taking inverse Z-transform of $Y_2(z)$,

$$y_2(n) = \delta(n-1) + 4\delta(n-3) + 23\delta(n-5) + \ldots$$

$$x(n)\cdot y_2(n) = \left[\frac{1}{2}\delta(n) + \delta(n+2) + \ldots\right] \times \left[\delta(n-1) + 4\delta(n-3) + 23\delta(n-5) + \ldots\right]$$

$$x(n)\,y_2(n) = \frac{1}{2}\delta(n)\,\delta(n-1) + \frac{4}{2}\delta(n)\delta(n-3) + \ldots + \delta(n+2)\,\delta(n-1) + \ldots$$

$$x(n)y_2(n) = \frac{1}{2}\delta(n)\delta(-1) + 2\delta(n)\cdot\delta(-3)\ldots.$$

$$x(n)y_2(n) = 0$$

Thus, option (B) satisfies orthogonality.

(iii) From option (C) :

$$Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|} z^{-n}$$

$$Y_3(z) = \ldots.. + 2^{-2}z^2 + 2^{-1}z^1 + 1 + 2^{-1}z^{-1} + 2^{-2}z^{-2} + \ldots.$$

Taking inverse Z-transform of $Y_3(z)$,

$y_3(n) = ... + 2^{-2}\delta(n+2) + 2^{-1}\delta(n+1) +$

$\delta(n) + 2^{-1}\delta(n-1) + 4^{-1}\delta(n-2) + ...$

$$x(n)\cdot y_3(n) = \left[2^{-1}\delta(n) + \delta(n+2) + ...\right]$$

$$\times\left[.. + 2^{-1}\delta(n+1) + \delta(n) + 2^{-1}\delta(n-1) + ..\right]$$

$$x(n)\, y_3(n) = ... + 2^{-1}\delta(n)\delta(n)$$

$$+\, 4^{-1}\delta(n)\delta(n-1) + ...$$

$$+ 2^{-1}\delta(n+2)\delta(n+1)$$

$$+ \delta(n+2)\delta(n) + ...$$

$$= 2^{-1}\delta(n)\cdot\delta(0) + 4^{-1}\delta(n)\cdot\delta(-1)$$

$$+ 2^{-1}\delta(n+2)\delta(-1) + \delta(n)\cdot\delta(2)$$

$$x(n)\, y_3(n) = 2^{-1}\delta(n) \neq 0$$

Thus, option (C) does not satisfy orthogonality.

(iv) From option (D) :

$$Y_4(z) = 2z^{-4} + 3z^{-2} + 1$$

Taking inverse Z-transform of $Y_4(z)$,

$$y_4(n) = 2\delta(n-4) + 3\delta(n-2) + \delta(n)$$

$$x(n)\cdot y_4(n) = \left[2^{-1}\delta(n) + \delta(n+2) + ...\right]$$

$$\times\left[2\delta(n-4) + 3\delta(n-2) + \delta(n)\right]$$

$$x(n)\, y_4(n) = \delta(n)\delta(n-4)$$

$$+ 3\times 2^{-1}\delta(n)\delta(n-2) + 2^{-1}\delta(n)\delta(n)$$

$$+ 2\delta(n+2)\delta(n-4) + ...$$

$$= \delta(n)\delta(-4) + 3$$

$$\times 2^{-1}\delta(n)\delta(-2) + ...$$

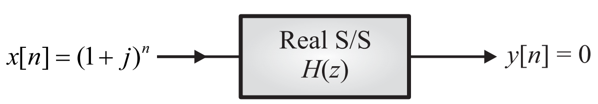
$$x(n)\, y_4(n) = 2^{-1}\delta(n) \neq 0$$

Thus, option (D) does not satisfy orthogonality.

Hence, the correct option is (B).

7.2 (D)

Given : Transfer function of the real system $= H(z)$



For input $x[n] = (1+j)^n$, output $y[n] = 0$

$$x[n] = (1+j)^n = (\sqrt{2}e^{j\frac{\pi}{4}})^n$$

$$x[n] = (z_0)^n \quad \text{where, } z_0 = \sqrt{2}e^{j\frac{\pi}{4}}$$

From the concept of Eigen function,

Inputs of the form $x[n] = (z_0)^n$ are said to be Eigen functions for the LTI systems with transfer function $H(z)$, as the response of LTI systems for such inputs can directly be obtained just by multiplying the input with a constant 'λ', where λ is called the Eigen value corresponding to $x[n] = (z_0)^n$, provided z_0 is a complex number.

i.e., $y[n] = \lambda x[n]$ for $x[n] = (z_0)^n$ and

$$\lambda = H(z)\big|_{z=z_0}$$

Hence, for $x[n] = (\sqrt{2}e^{j\frac{\pi}{4}})^n = (z_0)^n$

$$y[n] = H(z)\big|_{z=z_0=\sqrt{2}e^{j\frac{\pi}{4}}}(z_0)^n$$

$$y[n] = H(\sqrt{2}e^{j\frac{\pi}{4}})(\sqrt{2}e^{j\frac{\pi}{4}})^n$$

But from the given condition $y[n]$ for $x[n] = (\sqrt{2}e^{j\frac{\pi}{4}})^n$ is zero.

$$y[n] = H(\sqrt{2}e^{j\frac{\pi}{4}})(\sqrt{2}e^{j\frac{\pi}{4}})^n = 0$$

$$H(\sqrt{2}e^{j\frac{\pi}{4}}) = 0$$

It implies that the transfer function $H(z)$ has a zero at the complex location $z = \sqrt{2}e^{j\frac{\pi}{4}}$ in the z-plane.

As the given system is real, so complex poles and zeros must be occurring in conjugate pairs.

Hence, if there is a zero at $z = z_0 = \sqrt{2}e^{j\frac{\pi}{4}}$, at least one more zero must be occurring at $z = z_0^* = \sqrt{2}e^{-j\frac{\pi}{4}}$

So, the system will have at least two zeros.

Again from the given condition, ROC of $\left(1-\frac{1}{2}z^{-1}\right)H(z)$ is entire z-plane except $z=0$.

Let $W(z)=\left(1-\frac{1}{2}z^{-1}\right)H(z)$ ...(i)

From convolution property of z-transform.

If $x_1[n]\otimes x_2[n]=x_3[n]$

$X_1[z]\cdot X_2[z]=X_3[z]$

Taking inverse z-transform in equation (i) using the convolution property.

$$w[n]=\left\{\delta[n]-\frac{1}{2}\delta[n-1]\right\}\otimes h[n]$$

$$w[n]=h[n]-\frac{1}{2}h[n-1] \quad \text{...(ii)}$$

As mentioned in the problem, if ROC of $W(z)$ is entire z-plane except $z=0$, i.e. if the ROC is including $z=\infty$ point then it indicates that $w[n]$ must be a causal sequence.

For $w[n]$ to be a causal sequence,

$w[n]=0,\ n<0$

$w[n]=h[n]-\frac{1}{2}h[n-1]=0$ for $n<0$ is possible only if

$h[n]=0$ for $n<0$

It indicates that $h[n]$ is causal.

For $h[n]$ to be causal, the degree of denominator of its transform $H(z)$ must be greater than or equal to the degree of numerator of $H(z)$, i.e. if $H(z)=\frac{N(z)}{D(z)}$, then for $h[n]$ to be causal.

Degree of $D(z)\geq$ Degree of $N(z)$

As we already have obtained the result that there are at least two zeros, so minimum degree of $N(z)$ is 2.

Hence, to satisfy the causality condition,

Degree of $D(z)\geq 2$

Number of poles ≥ 2

Minimum number of poles = 2.

So, according to all given conditions, the system must have at least 2-poles and 2-zeros.

Hence, the correct option is (D).

7.3 (B)

Given : Z-transform of a causal signal is,

$$X(z)=z^2(z-a)^{-2}=\frac{z^2}{(z-a)^2} \quad \text{...(i)}$$

The Z transform pair for $a^n u[n]$ signal is given by

$$a^n u[n]\longleftrightarrow\frac{z}{z-a}$$

Using differentiation in z-domain property,

$$na^n u[n]\longleftrightarrow -z\frac{d}{dz}\left(\frac{z}{z-a}\right)$$

$$\Rightarrow \quad -z\left[\frac{(z-a)\times 1-z\times 1}{(z-a)^2}\right]$$

$$na^n u[n]\longleftrightarrow\frac{az}{(z-a)^2}$$

Using time shifting property,

$$(n+1)a^{n+1}u[n+1]\longleftrightarrow\frac{az}{(z-a)^2}z$$

$$(n+1)a^n u[n+1]\longleftrightarrow\frac{z^2}{(z-a)^2} \quad \text{...(ii)}$$

Comparing equations (i) and (ii), required inverse of given transform is,

$x[n]=(n+1)a^n u[n+1]$

Sequence $u[n+1]$ exist for $-1\leq n<\infty$, but the factor $(n+1)$ is zero for $n=-1$, so $x[n]$ may be expressed as a causal sequence.

$x[n]=(n+1)a^n u[n]$

Hence, the correct option is (B).

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9 Sampling

➤ Sample Questions

2014 IIT Kharagpur

9.1 An input signal $x(t) = 2 + 5\sin(100\pi t)$ is sampled with a sampling frequency of 400 Hz and applied to the system whose transfer function is represented by

$$\frac{Y(z)}{X(z)} = \frac{1}{N}\left(\frac{1-z^{-N}}{1-z^{-1}}\right)$$

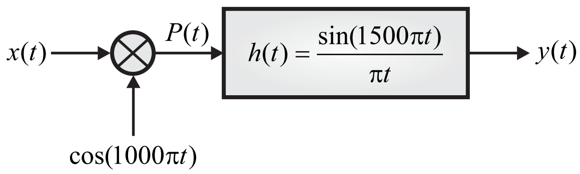
Where, N represents the number of samples per cycle. The output $y(n)$ of the system under steady state is **[Set - 02]**

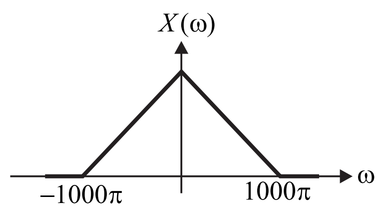
(A) 0 (B) 1

(C) 2 (D) 5

2017 IIT Roorkee

9.2 The output $y(t)$ of the following system is to be sampled, so as to reconstruct it from its samples uniquely. The required minimum sampling rate is **[Set - 02]**





(A) 1000 samples/s

(B) 1500 samples/s

(C) 2000 samples/s

(D) 3000 samples/s

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Explanations **Sampling**

9.1 (C)

Given : $x(t) = 2 + 5\sin 100\pi t$

Sampling frequency $f_s = 400$ Hz

Method 1

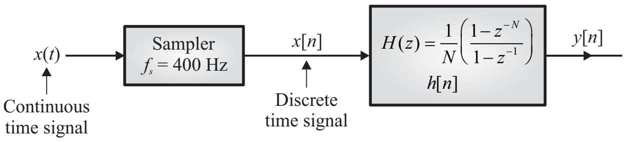
Transfer function of discrete time system,

$$H(z) = \frac{1}{N}\left[\frac{1-z^{-N}}{1-z^{-1}}\right]$$

Where, $N =$ Number of samples taken during every cycle of input $x(t)$

i.e. $N = \dfrac{f_s}{f}$, $f =$ Frequency of $x(t)$

All given conditions can be shown as a block diagram given below,



Output of discrete time system can be obtained as,

$$y[n] = x[n] \otimes h[n] \qquad \ldots\text{(i)}$$

Given, $x(t) = 2 + 5\sin 100\pi t$

As combination of a DC and a periodic signal is always periodic with frequency same as that of periodic part, so frequency of $x(t) =$ frequency of $5\sin 100\pi t$.

Frequency of $x(t)$, $\omega = 100\pi$ rad/sec

$$f = \frac{\omega}{2\pi} = 50 \text{ Hz}$$

From the given condition,

$$N = \frac{f_s}{f} = \frac{400}{50} = 8$$

Key Point

If the sampling frequency is 'K' times the frequency of continuous time periodic signal, then 'K' number of samples will be taken from each period of continuous time signal.

So, transfer function of discrete time system,

$$H(z) = \frac{1}{8}\left[\frac{1-z^{-8}}{1-z^{-1}}\right]$$

$$H(z) = \frac{1}{8}\left[\frac{(1)^2 - (z^{-4})^2}{1-z^{-1}}\right] = \frac{1}{8}\left[\frac{(1-z^{-4})(1+z^{-4})}{(1-z^{-1})}\right]$$

$$H(z) = \frac{1}{8}\left[\frac{(1-z^{-2})(1+z^{-2})(1+z^{-4})}{(1-z^{-1})}\right]$$

$$H(z) = \frac{1}{8}\left[\frac{(1-z^{-1})(1+z^{-1})(1+z^{-2})(1+z^{-4})}{(1-z^{-1})}\right]$$

$$H(z) = \frac{1}{8}\left[(1+z^{-1})(1+z^{-2})(1+z^{-4})\right]$$

$$H(z) = \frac{1}{8}[1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-7}]$$

Taking inverse z-transform on both sides, impulse response of the discrete time system can be obtained as,

$$h[n] = \frac{1}{8}\begin{Bmatrix}\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] \\ +\delta[n-4] + \delta[n-5] + \delta[n-6] + \delta[n-7]\end{Bmatrix}$$

From equation (i),

$$y[n] = x[n] \otimes h[n]$$

$$y[n] = \frac{1}{8}\begin{Bmatrix}x[n] + x[n-1] + x[n-2] + x[n-3] \\ +x[n-4] + x[n-5] + x[n-6] + x[n-7]\end{Bmatrix}$$

$$\{\text{As, } x[n] \otimes \delta[n \pm n_0] = x[n \pm n_0]\}$$

$$y[n] = \frac{1}{8}\sum_{K=0}^{7} x[n-K] \qquad \ldots\text{(ii)}$$

Where, $x[n] = x(t)\big|_{t=nT_s}$

Given, $f_s = 400$ Hz, $T_s = \frac{1}{400}$ sec

$$x[n] = [2 + 5\sin 100\pi t]_{t=\frac{n}{400}}$$

$$x[n] = 2 + 5\sin\frac{\pi}{4}n$$

Hence, from equation (ii), output $y[n]$

$$y[n] = \frac{1}{8}\sum_{K=0}^{7} x[n-K]$$

$$y[n] = \frac{1}{8}\sum_{K=0}^{7}\left\{2 + 5\sin\frac{\pi}{4}(n-K)\right\}$$

$$y[n] = \frac{1}{8}\left[\sum_{K=0}^{7} 2 + 5\sum_{K=0}^{7}\sin\frac{\pi}{4}(n-K)\right]$$

$$y[n] = \frac{1}{8}[s_1 + 5s_2] \qquad \ldots\text{(iii)}$$

Where, $s_1 = \sum_{K=0}^{7} 2 = 2\sum_{K=0}^{7}(1)^K = 2(7+1) = 16$

and $\quad s_2 = \sum_{K=0}^{7}\sin\frac{\pi}{4}(n-K)$

$$s_2 = \sum_{K=0}^{7}\left(\sin\frac{\pi}{4}n\cos\frac{\pi}{4}K - \cos\frac{\pi}{4}n\sin\frac{\pi}{4}K\right)$$

$$s_2 = \sin\frac{\pi}{4}n\sum_{K=0}^{7}\cos\frac{\pi}{4}K - \cos\frac{\pi}{4}n\sum_{K=0}^{7}\sin\frac{\pi}{4}K \qquad \ldots\text{(iv)}$$

$$\sum_{K=0}^{7}\cos\frac{\pi}{4}K = 1+\frac{1}{\sqrt{2}}+0-\frac{1}{\sqrt{2}}-1-\frac{1}{\sqrt{2}}+0+\frac{1}{\sqrt{2}}=0$$

$$\sum_{K=0}^{7}\sin\frac{\pi}{4}K = 0+\frac{1}{\sqrt{2}}+1+\frac{1}{\sqrt{2}}+0-\frac{1}{\sqrt{2}}-1-\frac{1}{\sqrt{2}}=0$$

So, from equation (iv),

$$s_2 = \left(\sin\frac{\pi}{4}n\right)\times 0 - \left(\cos\frac{\pi}{4}n\right)\times 0 = 0$$

Substituting values of s_1 and s_2 in equation (iii),

$$y[n] = \frac{1}{8}[16+5\times 0] = \frac{16}{8} = 2$$

Hence, the steady state value of output $y[n]$ is 2.

Hence, the correct option is (C).

Method 2

Results obtained in method 1.

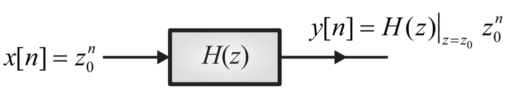
$$H(z) = \frac{1}{8}[(1+z^{-1})(1+z^{-2})(1+z^{-4})]$$

$$x[n] = 2+5\sin\frac{\pi}{4}n$$

$$x[n] = 2e^{j0n} + \frac{5}{2j}e^{j\frac{\pi}{4}n} - \frac{5}{2j}e^{-j\frac{\pi}{4}n}$$

$$x[n] = x_1[n]+x_2[n]-x_3[n] \qquad \ldots\text{(i)}$$

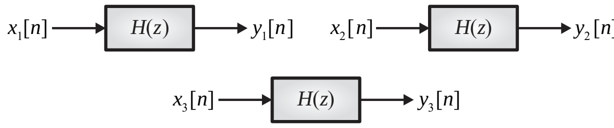
Using the concept of Eigen functions for a discrete time system, output of LTI system in steady state for complex exponential is given as shown below,



Applying property of linearity for the given, LTI system, overall output of system for

$$x[n] = x_1[n]+x_2[n]-x_3[n] \text{ is}$$

$$y[n] = y_1[n]+y_2[n]-y_3[n] \qquad \ldots\text{(ii)}$$



Following equation, $y[n] = H(z)|_{z=z_0}\ z_0^n$

$$x_1[n] = 2e^{j0n} = 2(e^{j0})^n$$

$$z_0 = e^{j0} = 1$$

$$y_1[n] = 2\left[H(z)|_{z=e^{j0}=1}(e^{j0})^n\right]$$

$$H(z)|_{z=1} = \frac{1}{8}\left[(1+z^{-1})(1+z^{-2})(1+z^{-4})\right]_{z=1} = 1$$

$$\therefore \qquad y_1[n] = 2[1\times 1^n] = 2$$

Similarly, $x_2[n] = \frac{5}{2j}e^{j\frac{\pi}{4}n} = \frac{5}{2j}(e^{j\frac{\pi}{4}})^n$

$$z_0 = e^{j\frac{\pi}{4}}$$

$$H(z)|_{z=z_0=e^{j\frac{\pi}{4}}} = \frac{1}{8}\left[(1+e^{-j\frac{\pi}{4}})(1+e^{-j\frac{\pi}{2}})(1+e^{-j\pi})\right]$$

$$H(z_0) = \frac{1}{8}\left[\left(1+\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)(1+0-j)(1-1-0)\right]=0$$

$$\therefore \qquad y_2[n] = \frac{5}{2j}\left[H(e^{j\frac{\pi}{4}})(e^{j\frac{\pi}{4}})^n\right] = 0$$

Similarly for $x_3[n] = \frac{5}{2j}e^{-j\frac{\pi}{4}n} = \frac{5}{2j}(e^{-j\frac{\pi}{4}})^n$

$$z_0 = e^{-j\frac{\pi}{4}}$$

$$H(z_0) = H(e^{-j\frac{\pi}{4}})$$

$$H(z_0) = \frac{1}{8}\left[(1+e^{j\frac{\pi}{4}})(1+e^{j\frac{\pi}{2}})(1+e^{j\pi})\right]$$

$$H(z_0) = \frac{1}{8}\left[\left(1+\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)(1+0+j)(1-1+0)\right]=0$$

$$\therefore \qquad y_3[n] = \frac{5}{2j}\left[H(e^{-j\frac{\pi}{4}})(e^{-j\frac{\pi}{4}})^n\right] = 0$$

Substituting values of $y_1[n]$, $y_2[n]$ and $y_3[n]$ in equation (ii), steady state output of LTI system is given as,

$$y[n] = y_1[n] + y_2[n] - y_3[n]$$

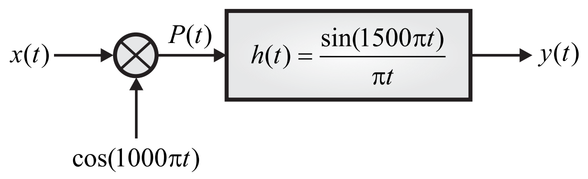
$$y[n] = 2 + 0 - 0 = 2$$

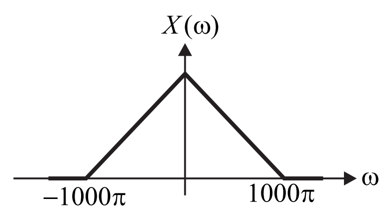
Hence, the steady state output of the given system is 2.

Hence, the correct option is (C).

9.2 (B)

Given :



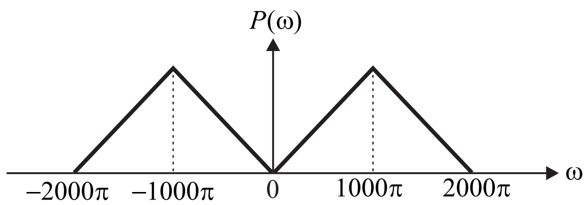


From figure,

$$P(t) = x(t) \cdot \cos(1000\pi t)$$

The spectrum of $P(t)$ is shown in below figure,

$$P(\omega) = \frac{1}{2}[X(\omega - 1000\pi) + X(\omega + 1000\pi)]$$



$$h(t) = \sin\frac{(1500\pi t)}{1500\pi t} \times 1500 \qquad \ldots\text{(i)}$$

$$h(t) = 1500 \sin c(1500t)$$

By duality property,

$$A\text{rect}\left(\frac{t}{\tau}\right) \xleftrightarrow{\text{F.T}} A\tau\text{sinc}(f\tau) = A\tau\text{sinc}\left(\frac{\omega\tau}{2\pi}\right)$$

$$A\tau\text{sinc}\left(\frac{t\tau}{2\pi}\right) \xleftrightarrow{\text{F.T}} 2\pi A\text{rect}\left(\frac{\omega}{\tau}\right) \qquad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$\frac{\tau}{2\pi} = 1500 \quad \Rightarrow \quad \tau = 3000\pi$$

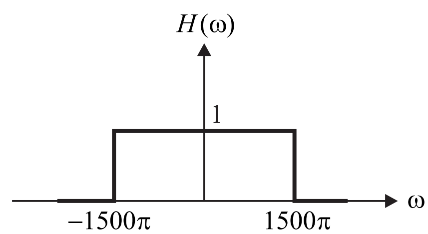
$$A\tau = 1500 \quad \Rightarrow \quad A = \frac{1}{2\pi}$$

From equation (ii),

$$1500\frac{\sin(1500\pi t)}{1500\pi t} \xleftrightarrow{\text{F.T}} \frac{1}{2\pi} 2\pi\,\text{rect}\left(\frac{\omega}{3000\pi}\right)$$

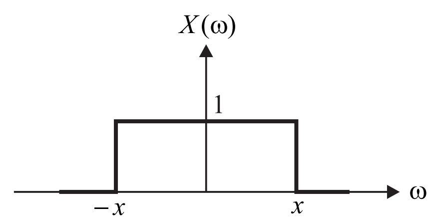
$$= \text{rect}\left(\frac{\omega}{3000\pi}\right)$$

Thus, the waveform of $H(\omega)$ is shown below,

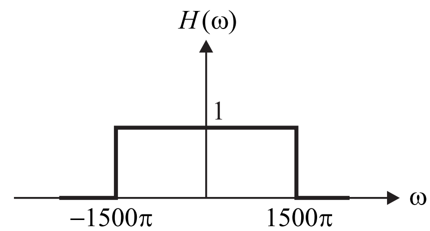


Note :

$$\frac{\sin xt}{\pi t} \xleftrightarrow{\text{F.T.}} \begin{cases} 1; & |\omega| < x \\ 0; & |\omega| > x \end{cases}$$

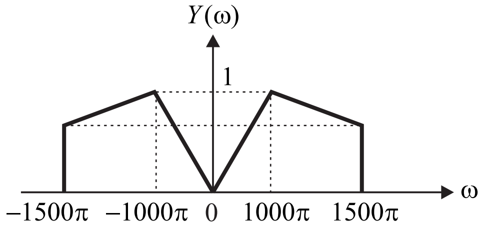


Comparing $\frac{\sin(1500\pi t)}{\pi t}$ with $\frac{\sin xt}{\pi t}$ we can draw its spectrum as,



The spectrum of output $y(t)$ is shown below,

$$Y(\omega) = P(\omega)H(\omega)$$



The maximum frequency of signal $y(t)$ can be written as,

$$\omega_m = 1500\pi \text{ rad/sec}$$

$$f_m = 750\,\text{Hz}$$

The sampling rate is given by,

$$f_s = 2 f_m = 2\times 750 = 1500 \text{ Hz}$$

Hence, the correct option is (B).

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M Prabal Reddy
IN-491
Vishnu Choudhary
IN-510
Archan Mitra
IN-525
Ronak Gupta
IN-579
Aayush Paikra
IN-579
Akshyay Khanna
IN-599
Anupam Shukla
IN-599
Piyush Songara
IN-620
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Marks Distribution of Digital Electronics in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	2	5	12
2004	2	4	10
2005	2	4	10
2006	–	5	10
2007	1	1	3
2008	–	3	6
2009	2	1	4
2010	–	4	8
2011	1	2	5
2012	2	1	4
2013	1	2	5
2014 Set-1	1	2	5

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-2	1	3	7
2014 Set-3	2	3	8
2015 Set-1	1	3	7
2015 Set-2	1	2	5
2016 Set-1	2	2	6
2016 Set-2	1	1	3
2017 Set-1	1	2	5
2017 Set-2	–	1	2
2018	1	2	5
2019	–	2	4
2020	1	1	3
2021	1	1	3

Syllabus : Digital Electronics

Combinational and Sequential logic circuits, Multiplexer, Demultiplexer, Schmitt trigger, Sample and hold circuits, A/D and D/A converters.

Contents : Digital Electronics

1 Boolean Algebra & Minimization

➢ Partial Synopsis

Laws of Boolean Algebra

Basic Laws		
1. $0 \cdot 0 = 0$	2. $0 \cdot 1 = 0$	3. $1 \cdot 0 = 0$
4. $1 \cdot 1 = 1$	5. $0 + 0 = 0$	6. $0 + 1 = 1$
7. $1 + 0 = 1$	8. $1 + 1 = 1$	9. $\overline{1} = 0$
10. $\overline{0} = 1$		
Complementation Laws		
1. $\overline{0} = 1$	2. $\overline{1} = 0$	3. If $A = 0$ then $\overline{A} = 1$
4. If $A = 1$ then $\overline{A} = 0$	5. $\overline{\overline{A}} = A$	
AND Laws		
1. $A \cdot 0 = 0$	2. $A \cdot 1 = A$	3. $A \cdot A = A$
4. $A \cdot \overline{A} = 0$		
OR Laws		
1. $A + 0 = A$	2. $A + 1 = 1$	3. $A + A = A$
4. $A + \overline{A} = 1$		
Commutative Laws		
1. $A + B = B + A$ Can be extended to any number of variables $A + B + C = B + C + A = C + A + B = B + A + C$		
2. $A \cdot B = B \cdot A$ Can be extended to any number of variables $A \cdot B \cdot C = B \cdot C \cdot A = C \cdot A \cdot B = B \cdot A \cdot C$		
Associative Laws		
1. $(A + B) + C = A + (B + C)$ Can be extended to any number of variables		

$$A+(B+C+D)=(A+B+C)+D=(A+B)+(C+D)$$

2. $(A\cdot B)C = A(B\cdot C)$

   Can be extended to any number of variables

$$A(BCD)=(ABC)D=(AB)(CD)$$

Note : Universal gate does not follow associative law $(\overline{\overline{A+B}+C} \neq \overline{A+\overline{B+C}})$.

Distributive Laws

1. $A(B+C)=AB+AC$ 2. $A+BC=(A+B)(A+C)$
3. $A+\bar{A}B=A+B$

Absorption Laws

1. $A+A.B=A$ 2. $A(A+B)=A$

Consensus Theorem

1. $AB+\bar{A}C+BC=AB+\bar{A}C$
2. $\bar{A}B+\bar{A}\bar{C}+\bar{B}\bar{C}=\bar{A}B+\bar{B}\bar{C}$
3. $(A+B)(A+\bar{C})(B+C)=(A+\bar{C})(B+C)$
4. $(\bar{A}+B)(\bar{B}+\bar{C})(\bar{A}+\bar{C})=(\bar{A}+B)(\bar{B}+\bar{C})$

Transposition Theorem

1. $AB+\bar{A}C=(A+C)(\bar{A}+B)$

Demorgan's Theorem

1. $\overline{A+B}=\bar{A}\cdot\bar{B}$

   Can be extended to any number of variables

$$\overline{A+B+C+D+\ldots\ldots\ldots\ldots}=\bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}\ldots\ldots\ldots\ldots$$

2. $\overline{AB}=\bar{A}+\bar{B}$

   Can be extended to any number of variables

$$\overline{A\cdot B\cdot C\cdot D\cdot\ldots\ldots\ldots\ldots}=\bar{A}+\bar{B}+\bar{C}+\bar{D}+\ldots\ldots\ldots\ldots$$

Principle of Duality

If any given Boolean equation is valid, then it's dual will also be valid.

Procedure of finding Dual

- Change all '+' signs to '.' signs and all '.' signs to '+' signs.
- Change all 0s to 1s and all 1s to 0s.
- Do not complement the variables.

Example of Dual functions

Boolean Law/Equation,	Dual of Boolean Law/Equation,
$A\cdot 1=A$	$A+0=A$
$0+1=1$	$1\cdot 0=0$
$A(B+C)=AB+AC$	$A+BC=(A+B)(A+C)$
$A+\bar{A}B=A+B$	$A(\bar{A}+B)=A\cdot B$

Note :

- If $F^D = F$ then function is called Self Dual Function.
- Number of unique function for n variable $= 2^{2^n}$.
- Number of Self Dual Function for n variable $= 2^{2^{n-1}}$.

➢ Sample Questions

2010 IIT Guwahati

Statement for Linked Answer Questions 1.1 & 1.2

The following Karnaugh map represents a functions F

F: $X \backslash YZ$	00	01	11	10
0	1	1	1	0
1	0	0	1	0

1.1 A minimized form of the function F is

(A) $F = \bar{X}Y + YZ$

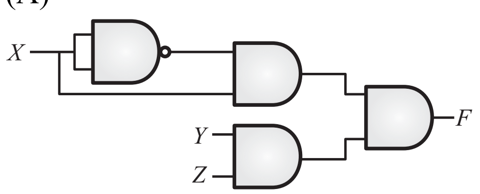
(B) $F = \bar{X}\bar{Y} + YZ$

(C) $F = \overline{XY} + Y\bar{Z}$

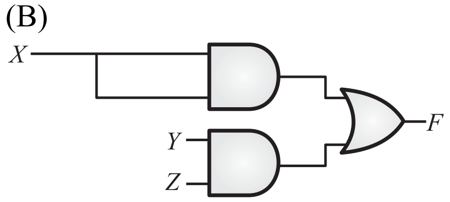
(D) $F = \overline{XY} + \bar{Y}Z$

1.2 Which of the following circuits is a realization of the above functions F?

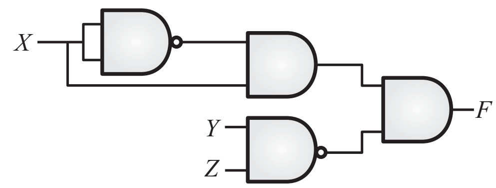
(A)



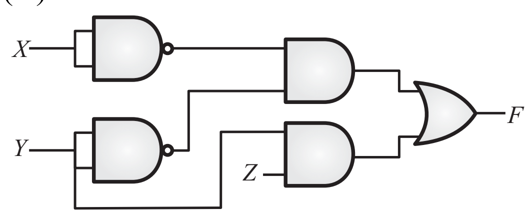
(B)



(C)



(D)



2012 IIT Delhi

1.3 In the sum of products function

$$f(X,Y,Z) = \sum m(2,3,4,5)$$

The prime implicants are

(A) $\bar{X}Y,\ X\bar{Y}$

(B) $\bar{X}Y,\ X\bar{Y}\bar{Z},\ X\bar{Y}Z$

(C) $\bar{X}Y\bar{Z},\ \bar{X}YZ,\ X\bar{Y}$

(D) $\bar{X}Y\bar{Z},\ \bar{X}YZ,\ X\bar{Y}\bar{Z},\ X\bar{Y}Z$

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1.4 $f(A,B,C,D) = \Pi M\,(0, 1, 3, 4, 5, 7, 9, 11, 12, 13, 14, 15)$ is a maxterm representation of Boolean function $f\,(A, B, C, D)$ where A is the MSB and D is the LSB. The equivalent minimized representation of this function is **[Set - 01]**

(A) $(A+\bar{C}+D)(\bar{A}+B+D)$

(B) $A\bar{C}D + \bar{A}BD$

(C) $\bar{A}C\bar{D}+A\bar{B}C\bar{D}+A\bar{B}\bar{C}\bar{D}$

(D) $(B+\bar{C}+D)(A+\bar{B}+\bar{C}+D)$
$(\bar{A}+B+C+D)$

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1.5 The Boolean expression $AB+A\bar{C}+BC$ simplifies to **[Set - 01]**

(A) $BC+A\bar{C}$ (B) $AB+A\bar{C}+B$

(C) $AB+A\bar{C}$ (D) $AB+BC$

2018 | IIT Guwahati

1.6 Digital input signals A, B, C with A as the MSB and C as the LSB are used to realize the Boolean function $F=m_0+m_2+m_3+m_5+m_7$, where m_i denotes the i^{th} minterm. In addition. F has a don't care for m_1. The simplified expression for F is given by

(A) $\bar{A}\bar{C}+\bar{B}C+AC$

(B) $\bar{A}+C$

(C) $\bar{C}+A$

(D) $\bar{A}C+BC+A\bar{C}$

❖❖❖❖

Explanations | Boolean Algebra & Minimization

Concept of Essential Prime Implicants

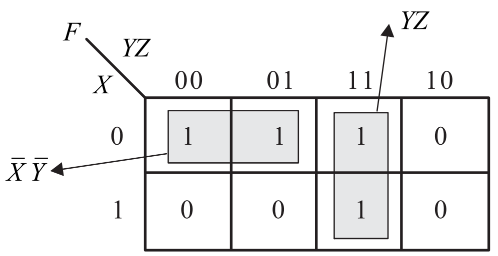
Scan for Video Explanation

Concept of Karnaugh Map (K-Map) SOP & POS

Scan for Video Explanation

1.1 (B)

Given K-map is shown below,



The minimized form of the function F is,

$$F=\bar{X}\bar{Y}+YZ$$

Hence, the correct option is (B).

Key Point

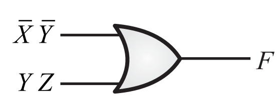
1. To find the minimized expression in POS form, 0's are grouped.
2. To find the minimized expression in SOP form, 1's are grouped.

1.2 (D)

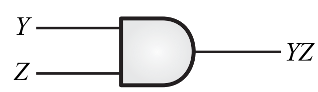
Method 1

$$F=\bar{X}\bar{Y}+YZ$$

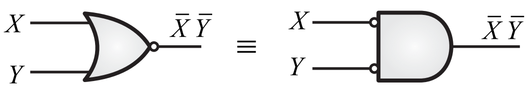
To realize F, an OR gate is needed to SUM UP the two term i.e. $\bar{X}\bar{Y}$ and YZ.



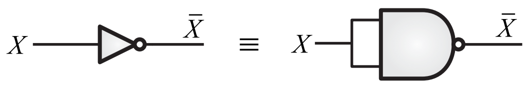
Product term YZ can be generated by an AND gate.



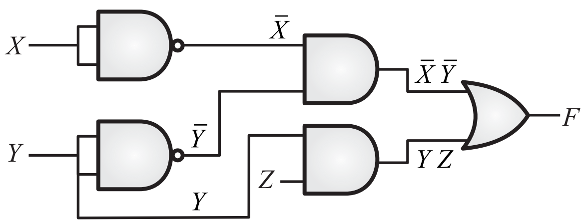
Product term $\bar{X}\,\bar{Y} = \overline{X+Y}$ can be generated by a NOR gate and NOR gate is equivalent to bubbled AND gate.



Where bubble denote the NOT gate and any NOT gate can be obtained from NAND gate by joining its two input together.



Hence, from above explanation F can be realized as shown below.



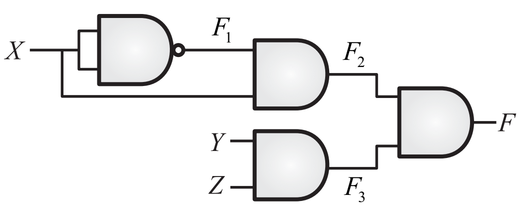
Hence, the correct option is (D).

Method 2

$$F = \bar{X}\,\bar{Y} + Y\,Z$$

Checking from options :

(i) From option (A) :



$$F_1 = \overline{X \cdot X} = \bar{X}$$

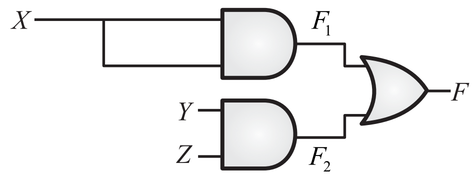
$$F_2 = F_1 \cdot X = \bar{X} \cdot X = 0$$

$$F_3 = YZ$$

$$F = F_2 F_3 = 0 \cdot F_3 = 0$$

Thus, option (A) is not correct.

(ii) From option (B) :



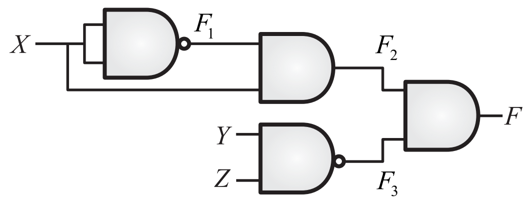
$$F_1 = X \cdot X = X$$

$$F_2 = Y \cdot Z$$

$$F = F_1 + F_2 = X + YZ$$

Thus, option (B) is not correct.

(iii) From option (C) :



$$F_1 = \overline{X \cdot X} = \bar{X}$$

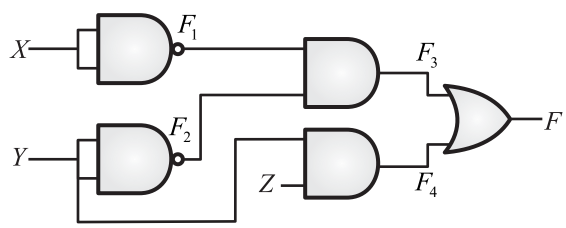
$$F_2 = F_1 \cdot X = \bar{X} \cdot X = 0$$

$$F_3 = \overline{Y \cdot Z}$$

$$F = F_2 F_3 = 0 \cdot \overline{YZ} = 0$$

Thus, option (C) is not correct.

(iv) From option (D) :



$$F_1 = \overline{X \cdot X} = \bar{X}$$

$$F_2 = \overline{Y \cdot Y} = \bar{Y}$$

$$F_3 = F_1 \cdot F_2 = \bar{X} \cdot \bar{Y}$$

$$F_4 = Y \cdot Z$$

$$F = F_3 + F_4 = \bar{X}\bar{Y} + YZ$$

Thus, option (D) is correct.

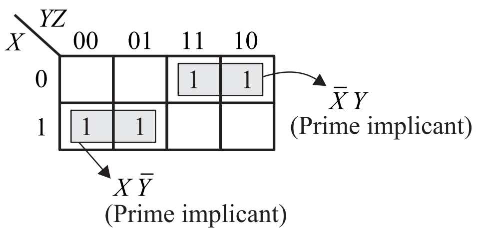
Hence, the correct option is (D).

1.3 (A)

Given : $f(X,Y,Z)=\Sigma m(2,3,4,5)$

$$f(X,Y,Z)=\bar{X}Y\bar{Z}+\bar{X}YZ+X\bar{Y}\bar{Z}+X\bar{Y}Z$$

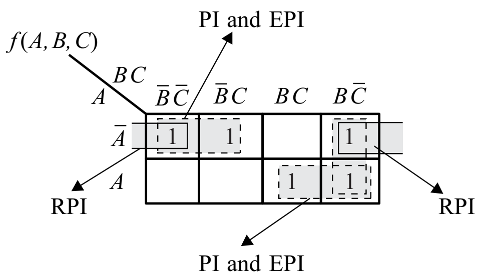
K-map for function $f(X,Y,Z)$ is shown below,



Hence, the correct option is (A).

Key Point

1. **Implicant :** Each individual min-term in canonical SOP form is called implicant.
2. **Prime Implicant (PI) :** Prime implicant is a min-term, which are obtained by combining maximum possible adjacent cells in the K-map.
3. **Essential Prime Implicant (EPI) :** It is a prime implicant which contains atleast one min-terms which is not covered by other prime implicant.
4. **Redundant Prime Implicant (RPI) :** The prime implicant whose each 1 is covered atleast by one EPI is called a redundant prime implicant.
5. **Example :**



Here total number of 1's = 5,

Hence, number of implicant = 5

Implicant $=(\bar{A}\bar{B}\bar{C}),(\bar{A}\bar{B}C),(ABC),$

$(\bar{A}B\bar{C}),(AB\bar{C})$

Number of prime implicant = 4

Prime implicant $=\bar{A}\bar{B},\ \bar{A}\bar{C},\ AB,\ B\bar{C}$

Number of essential prime implicant = 2

Essential prime implicant $=\bar{A}\bar{B},\ AB$

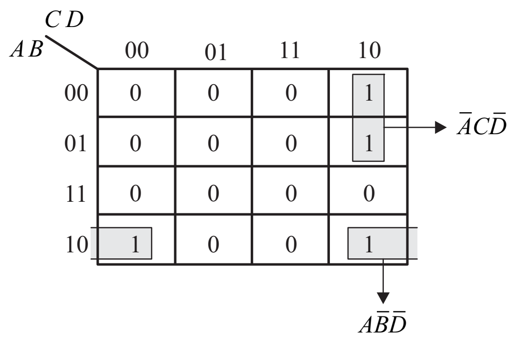
1.4 (C)

Given : The POS function,

$$f(A,B,C,D)=\Pi M(0,1,3,4,5,7,9,11,12,13,14,15)$$

Method 1

K-map of function f in SOP form is,



Thus, Equivalent minimized representation is,

$$f=A\bar{B}\bar{D}+\bar{A}C\bar{D}$$

$$f=A\bar{B}(C+\bar{C})\bar{D}+\bar{A}C\bar{D}$$

$$f=A\bar{B}C\bar{D}+A\bar{B}\bar{C}\bar{D}+\bar{A}C\bar{D}$$

$$f=\bar{A}C\bar{D}+A\bar{B}C\bar{D}+A\bar{B}\bar{C}\bar{D}$$

Hence, the correct option is (C).

Method 2

The POS function,

$$f(A,B,C,D)=\Pi M(0,1,3,4,5,7,9,11,12,13,14,15)$$

The min term of given function is shown below,

$$f(A,B,C,D)=\Sigma m(2,6,8,10)$$

SOP expression of function f is,

$$f(A,B,C,D)=\bar{A}\bar{B}C\bar{D}+\bar{A}BC\bar{D}+A\bar{B}\bar{C}\bar{D}+A\bar{B}C\bar{D}$$

$$f(A,B,C,D)=\bar{A}C\bar{D}(B+\bar{B})+A\bar{B}\bar{C}\bar{D}+A\bar{B}C\bar{D}$$

$$[\because\ B+\bar{B}=1]$$

$$f(A,B,C,D)=\bar{A}C\bar{D}+A\bar{B}C\bar{D}+A\bar{B}\bar{C}\bar{D}$$

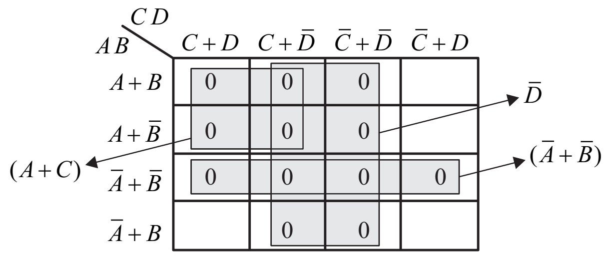
Hence, the correct options is (C).

Method 3

Given : The POS function,

$$f(A,B,C,D)=\Pi M(0,1,3,4,5,7,9,11,12,13,14,15)$$

K-map of function f in POS form is,



The minimize POS form of f :

$$f = \bar{D}(\bar{A}+\bar{B})(A+C)$$

Option (A) and (D) are in POS form but does not matched with f.

So, now we just simplified the function f,

$$f = \bar{D}(\bar{A}\cdot A + A\bar{B} + \bar{A}C + \bar{B}C)$$

$$[\because \bar{A}\cdot A = 0]$$

$$f = \bar{D}(A\bar{B} + \bar{A}C + \bar{B}C)$$

(by consensus law, $A\bar{B} + \bar{A}C + \bar{B}C = A\bar{B} + \bar{A}C$)

$$f = \bar{D}(A\bar{B} + \bar{A}C)$$

$$f = A\bar{B}\bar{D} + \bar{A}C\bar{D} \qquad \text{(SOP form)}$$

Again no option is matching with this minimized SOP form, but only term $\bar{A}C\bar{D}$ is matched with option (B) and (C) respectively.

So, we will try to convert term $A\bar{B}\bar{D}$ into canonical stand form as,

$$f = A\bar{B}\bar{D}(C+\bar{C}) + \bar{A}C\bar{D}$$

$$f = A\bar{B}\,\bar{C}\,\bar{D} + A\bar{B}\,C\,\bar{D} + \bar{A}C\bar{D}$$

Thus, option (C) is matching with above function (f).

Hence, the correct option is (C).

Note : Here options are not in minimized form, so while solving question, keep in mind that, options are in minimized form or not.

1.5 (A)

Method 1

Given : $F(A,\ B,\ C) = AB + A\bar{C} + BC$

$$F(A,B,C) = AB(C+\bar{C}) + A\bar{C} + BC$$

$$[C+\bar{C}=1]$$

$$F(A,B,C) = ABC + AB\bar{C} + A\bar{C} + BC$$

$$F(A,B,C) = (A+1)BC + (B+1)A\bar{C}$$

$$[A+1=1 \text{ and } B+1=1]$$

$$F(A,B,C) = BC + A\bar{C}$$

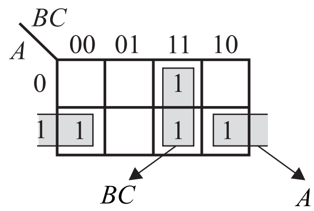
Hence, the correct option is (A).

Method 2

Given Boolean function is,

$$F(A,\ B,\ C) = \underset{(6,7)}{AB} + \underset{(6,4)}{A\bar{C}} + \underset{(7,3)}{BC}$$

K-map,



So, the minimized SOP expression of the above K-map is,

$$F(A,\ B,\ C) = A\bar{C} + BC$$

Hence, the correct option is (A).

Method 3

Given Boolean function is,

$$F(A,\ B,\ C) = AB + A\bar{C} + BC$$

Apply consensus law, we will get

$$F(A,\ B,\ C) = A\bar{C} + BC$$

[AB is redundant term]

Hence, the correct option is (A).

Scan for Video Solution

Key Point

Consensus Law :

(i) In Sum of Product (SOP) form,

$$AB + \bar{A}C + BC = AB + \bar{A}C$$

$$AC + A\bar{B} + BC = A\bar{B} + BC$$

$$\bar{A}\,\bar{B} + A\bar{C} + \bar{B}\,\bar{C} = A\bar{C} + \bar{A}\,\bar{B}$$

(ii) In Product of Sum (POS) form,

$$(A+B)(\bar{A}+C)(B+C)=(A+B)(\bar{A}+C)$$

$$(A+C)(A+\bar{B})(B+C)=(A+\bar{B})(B+C)$$

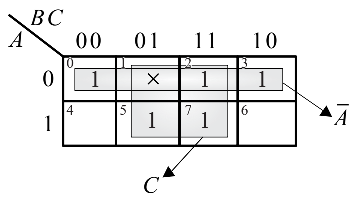
$$(\bar{A}+\bar{C})(\bar{A}+\bar{B})(\bar{B}+C)=(\bar{B}+C)(\bar{A}+\bar{C})$$

1.6 (B)

Given : $F=m_0+m_2+m_3+m_5+m_7$

Also, F has don't care at m_1 ,

Hence, $F(A,B,C)=\Sigma m(0,2,3,5,7)+\Sigma\phi(1)$



$$F=\bar{A}+C$$

Min-term	SOP representation	Binary representation	Decimal representation
m_0	$\bar{A}\bar{B}\bar{C}$	000	0
m_1	$\bar{A}\bar{B}C$	001	1
m_2	$\bar{A}B\bar{C}$	010	2
m_3	$\bar{A}BC$	011	3
m_5	$A\bar{B}C$	101	5
m_7	ABC	111	7

Don't care term

Hence, the correct option is (B).

🕮 Key Point

Don't care conditions :

(i) In n-bit digital system there are 2^n possible input combination, but out of 2^n input combination, some input combination are not occur during the operation of digital system, (it means, certain input combination never occurs and corresponding output never appears) therefore, output for these input combination are indicated by "X" or "d" or "ϕ" in the truth table are called don't care output or don't care condition.

(ii) SOP function with don't care conditions, treated don't care term as 1 only to selected don't care terms in groups.

(iii) POS function with don't care conditions, treated don't care term as 0 only to selected don't care terms in groups.

❖❖❖❖

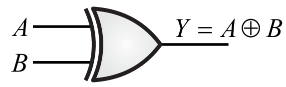
2 Logic Gates

➢ Partial Synopsis

Special Purpose Gates

1. Exclusive OR Gate (EX-OR or XOR) :

- **Symbol of 2-input X-OR gate :**



- **Truth table of 2-input X-OR gate :**

Inputs		Output
A	B	$Y = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Boolean function of 2 input EX-OR gate is given as,

$$Y = A \oplus B = \bar{A}B + A\bar{B} \rightarrow \text{SOP form.}$$

$$Y = A \oplus B = (\bar{A} + \bar{B})(A + B) \rightarrow \text{POS form.}$$

Note :

3 or more input EXOR gate does not exists practically but can be theoretically implemented.

- **Anti-coincidence or inequality detector**

Since an EXOR gate produces an output 1 when the input are not equal it is called anti-coincidence gate or inequality detector.

- **In EXOR operation :**

For implementation of BUFFER circuit, any one of the inputs will be at logic 0,

$$A \oplus 0 = A$$

For implementation of INVERSION circuit, any one of the inputs will be at logic 1,

$$A \oplus 1 = \overline{A}$$

- It acts as an "odd" number of 1's detector
- **Properties of EXOR gate**

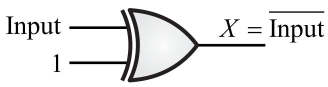
(a) $A \oplus 1 = \overline{A}$ (b) $A \oplus 0 = A$

(c) $A \oplus A = 0$ (d) $A \oplus \overline{A} = 1$

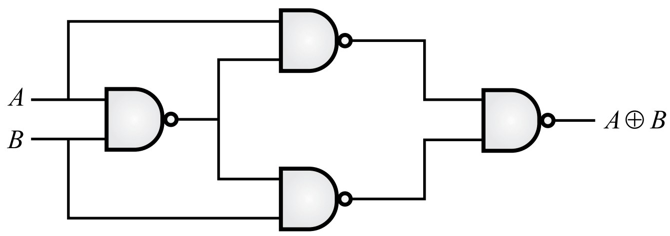
(e) $AB \oplus AC = A(B \oplus C)$ (f) If $A \oplus B = C$ and $A \oplus C = B$, $B \oplus C = A$ then $A \oplus B \oplus C = 0$

- **EXOR gate as an inverter**

An EXOR gate can be used as an inverter by connecting one of the two input terminals to logic 1 and applying the input signal to the other terminal.



- **2-input EXOR gate using NAND gate**



Note :

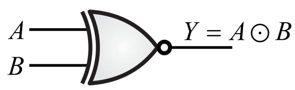
(i) It is also called Stair case switch.

(ii) It is mostly used in Parity generation and detection.

(iii) It is also used in comparator circuit.

2. EX-NOR Gate :

- **Symbol of 2-input X-NOR gate :**



- **Truth Table of 2-input X-NOR gate :**

Inputs		Output
A	B	$Y = \overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

Boolean function of 2 input EX-NOR operation is given as :

$$Y = A \odot B = \overline{A \oplus B} = AB + \overline{A}\overline{B} \rightarrow \text{ SOP form.}$$

$$Y = A \odot B = \overline{A \oplus B} = (\overline{A} + B)(\overline{B} + A) \rightarrow \text{ PSO form.}$$

Note :
3 or more input EX-NOR gate does not exists practically.

- EX-NOR gate is a logic gate whose output is logic high when both the inputs are equal. Hence it is called equality detector.
- **In EX-NOR operation**

  For implementation of buffer circuit, any one of the inputs will be at logic 1,

  $A \odot 1 = A$

  For implementation of inversion circuit, any one of the inputs will be at logic 0,

  $A \odot 0 = \overline{A}$
- EX-NOR gate with even number of 1's detector for even numbers of input.
- EX-NOR gate with odd number of 1's detector for odd numbers of input.

  i.e. for odd inputs, $A \oplus B \oplus C = A \odot B \odot C$.

Note :
For number of Even input $\Rightarrow$ EXOR = $\overline{\text{EXNOR}}$
For number of Odd input $\Rightarrow$ EXOR = EXNOR

- 2-input EXOR and EX-NOR are dual as well as complimented to each other.
- **Some important results :**

  (a) $\overline{A} \oplus B = A \oplus \overline{B} = A \odot B$

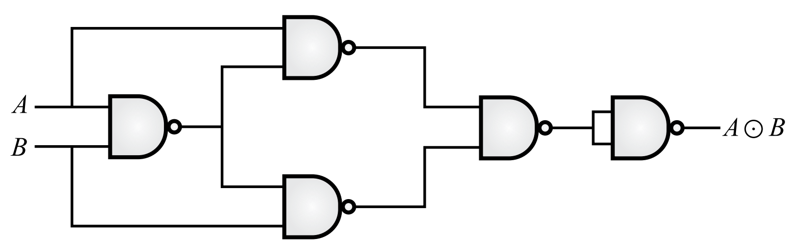
  (b) $\overline{A} \odot B = A \odot \overline{B} = A \oplus B$

  (c) $\overline{A} \oplus B = \overline{A \oplus B} = A \odot B$

  (d) $\overline{A} \odot \overline{B} = A \odot B$

  (e) If $A \odot B = C$, Then

  $A \odot B \odot C = 1$
- **2-input EXNOR gate using NAND gate**



➢ Sample Questions

2001 IIT Kanpur

2.1 The output of a logic gate is "1" when all its inputs are at logic "0". The gate is either

(A) A NAND or an EX-OR gate.
(B) A NOR or an EX-OR gate.
(C) An AND or an EX-NOR gate.
(D) A NOR or an EX-NOR gate.

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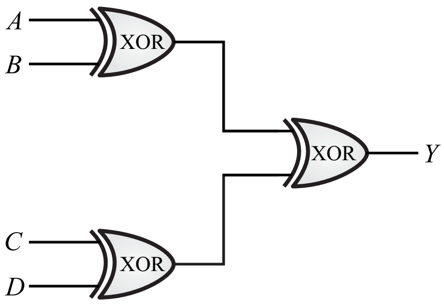
2.2 *A*, *B*, *C* and *D* are input bits and *Y* is the output bit in the XOR gate circuit of the figure below. Which of the following statements about the sum *S* of *A*, *B*, *C*, *D* and *Y* is correct?



(A) *S* is always either zero or odd.
(B) *S* is always either zero or even.
(C) *S* = 1 only if the sum of *A*, *B*, *C* and *D* is even.
(D) *S* = 1 only if the sum of *A*, *B*, *C* and *D* is odd.

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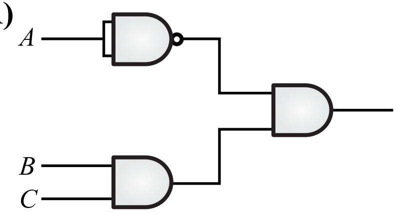
2.3 Which of the following logic circuits is a realization of the function *F* whose Karnaugh map is shown in figure

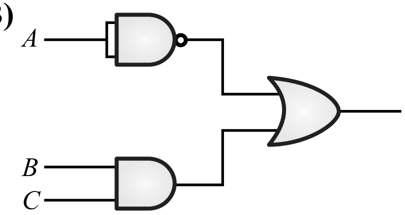
[Set - 01]

C \ AB	00	01	11	10
0	1	1		
1		1	1	

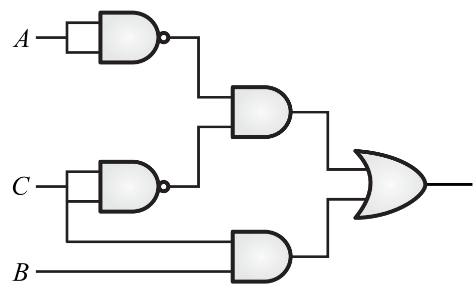
(A)



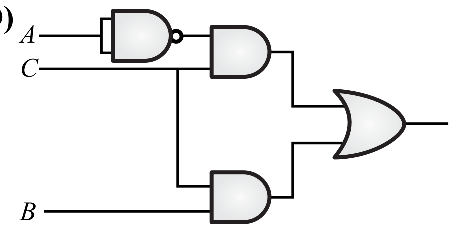
(B)



(C)



(D)



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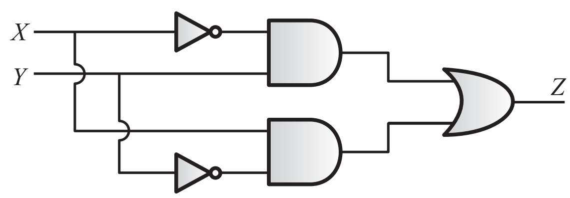
2.4 In the circuit below, *X* and *Y* are digital inputs, and *Z* is a digital output. The equivalent circuit is a



(A) XOR gate (B) XNOR gate
(C) NAND gate (D) NOR gate

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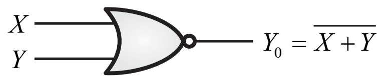
Explanations Logic Gates

2.1 (D)

Given : Output of a logic gate is "1", when all its inputs are at logic "0".

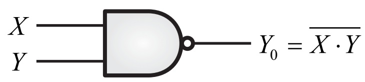
There are only three gates i.e. NOR, NAND and EX-NOR which will produce logic 1 at its output, when all of its input is at logic 0.

NOR gate :



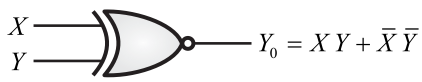
$$Y_0 = 1, \quad \text{when} \quad X = Y = 0$$

NAND gate :



$$Y_0 = 1 \quad \text{when} \quad X = Y = 0$$

EX-NOR gate :

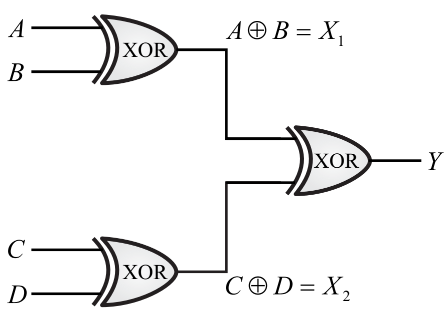


$$Y_0 = 1 \quad \text{when} \quad X = Y = 0$$

Hence, the correct option is (D).

2.2 (B)

Given logic circuit is shown below,



Method 1

From figure,

$$Y = X_1 \oplus X_2$$

$$Y = A \oplus B \oplus C \oplus D$$

Sum is given by,

$$S = A + B + C + D + Y$$

Case 1 : If $A = B = C = D = 0$, $Y = 0$

Then, $S = 0$

Case 2 : If A, B, C, D inputs has odd number of 1's, $Y = 1$ then $S =$ even

Case 3 : If A, B, C, D inputs has even number of 1's, $Y = 0$ then $S =$ even

Therefore, S is always either zero or even.

Hence, the correct option is (B).

Method 2

$$Y = (A \oplus B) \oplus (C \oplus D)$$

$$Y = X_1 \oplus X_2$$

The sum S is given by,

$$S = A + B + C + D + Y$$

The truth table is shown below,

A	B	C	D	X_1	X_2	Y	S
0	0	0	0	0	0	0	0
0	0	0	1	0	1	1	2
0	0	1	0	0	1	1	2
0	0	1	1	0	0	0	2
0	1	0	0	1	0	1	2
0	1	0	1	1	1	0	2
0	1	1	0	1	1	0	2
0	1	1	1	1	0	1	4
1	0	0	0	1	0	1	2
1	0	0	1	1	1	0	2
1	0	1	0	1	1	0	2
1	0	1	1	1	0	1	4
1	1	0	0	0	0	0	2
1	1	0	1	0	1	1	4
1	1	1	0	0	1	1	4
1	1	1	1	0	0	0	4

From the above table, S is always either zero or even.

Hence, the correct option is (B).

Note : We have considered sum "S" as decimal addition. Since there is no information of type of addition.

If we considered sum "S" as binary addition then for all combinations of A, B, C, D and Y we get $S = 0$ which is not present in any option.

Method 3

Sum of two inputs A and B,

$$S = A \oplus B$$

Sum of three inputs A, B and C,

$$S = A \oplus B \oplus C$$

Sum of four inputs A, B, C and D,

$$S = A \oplus B \oplus C \oplus D$$

Sum of S and Y is,

$$S = A \oplus B \oplus C \oplus D \oplus Y$$

$$[Y = (A \oplus B) \oplus (C \oplus D) = A \oplus B \oplus C \oplus D]$$

Thus, $\quad S = Y \oplus Y$

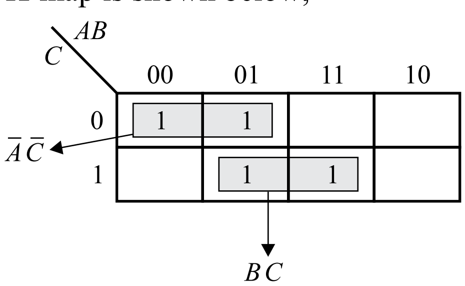
Hence, if the addition of A, B, C, D is in binary then the sum "S" will be zero. If simple addition of A, B, C, D is in decimal then sum "S" will be even.

Hence, the correct option is (B).

2.3 (C)

Method 1

Given K-map is shown below,

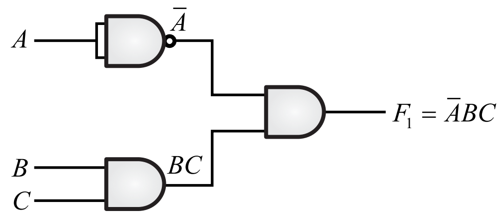


From above K-map,

$$F = BC + \bar{A}\bar{C}$$

Checking from options :

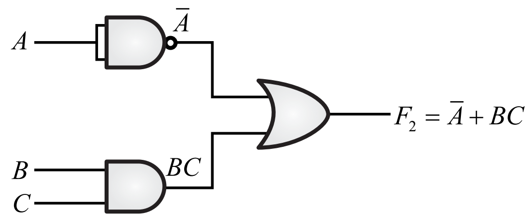
From option (A) :



From above figure, $F_1 = \bar{A}BC$

Hence, option (A) is not correct.

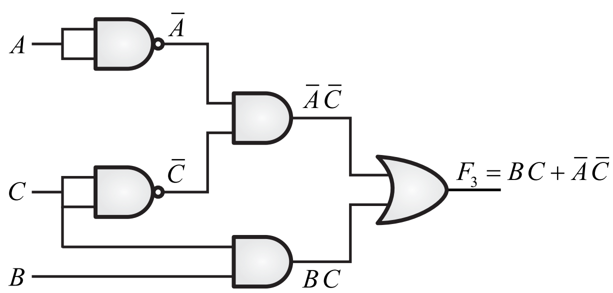
From option (B) :



From above figure, $F_2 = \bar{A} + BC$

Hence, option (B) is not correct.

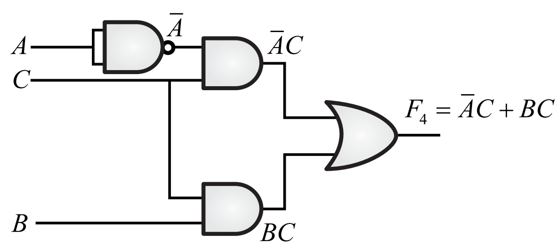
From option (C) :



From above figure, $F_3 = BC + \bar{A}\bar{C}$

Hence, option (C) is correct.

From option (D) :



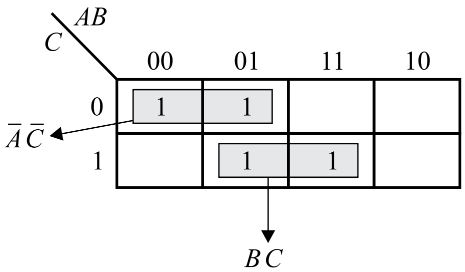
From above figure, $F_4 = \bar{A}C + BC$

Hence, option (D) is not correct.

Hence, the correct option is (C).

Method 2

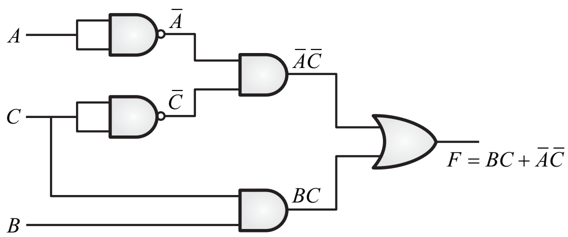
Given K-map is shown below,



From above K-map,

$$F = BC + \bar{A}\bar{C}$$

The above function can realized as,

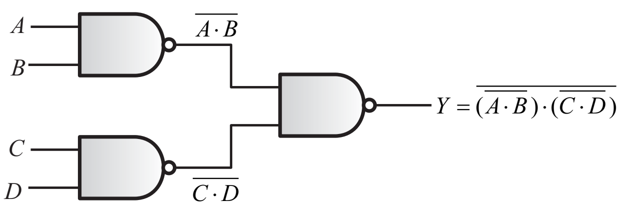


Hence, the correct option is (C).

2.4 (D)

Method 1

Given logic circuit is shown below,



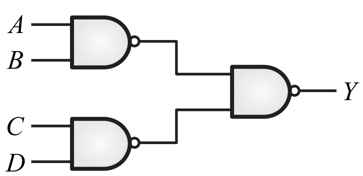
$$f = \overline{\overline{AB}\cdot\overline{CD}} = \overline{\overline{AB}} + \overline{\overline{CD}} = AB + CD$$

[By De-Morgan law $\overline{A\cdot B} = \bar{A} + \bar{B}$ and $\overline{A+B} = \bar{A}\cdot\bar{B}$]

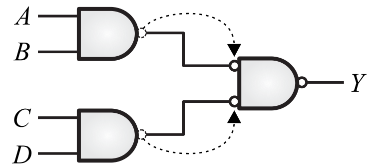
Hence, the correct option is (D).

Method 2

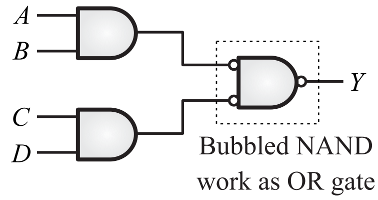
Given logic circuit is shown below,



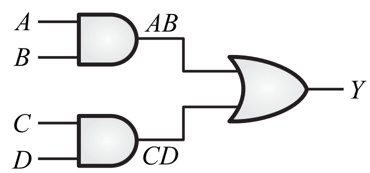
Shifting the bubbles towards the last NAND gate,



The above figure can be reduced as,



Thus, above circuit becomes,



So, $y = AB + CD$

Hence, the correct option is (D).

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3 Combinational Circuits

➢ Partial Synopsis

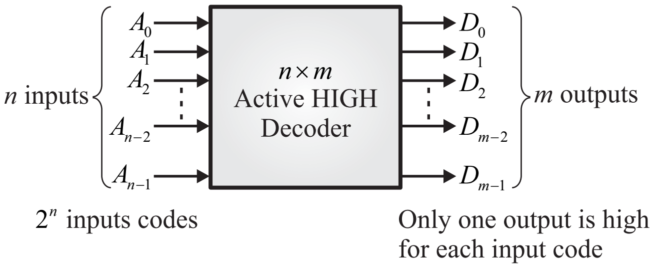
Decoder

- A decoder is a logic circuit that convert an *n*-bit binary input codes into m output lines such that only one output line is activated for each one of the possible combination of inputs.
- For *n*-input bits the maximum number of output lines m will be 2^n i.e. $m \le 2^n$.
- If the *n*-bit decoded information has unused or don't care combinations, the decoder output will have less than 2^n outputs.
- Decoder is used to convert binary data into other codes like binary to octal (3 : 8 decoder), binary to hexadecimal (4 : 16 decoder).
- **Types of decoder :**

  There are two types of decoder as given below :

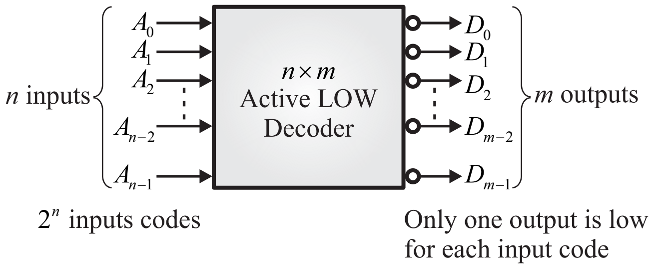
  (i) **Active high decoder :**

  For each input combination only one of the m outputs will be active (HIGH), all the other outputs will remain inactive (LOW).



  (ii) **Active low decoder :**

  For each input combination only one of the m output line will be active (LOW logic level), while all other outputs will be at HIGH logic level.



- **Example of 2x4 decoder :**

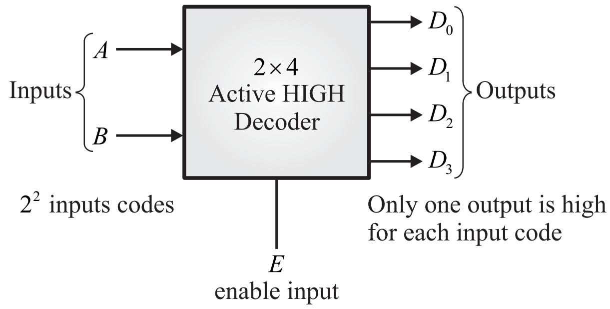
(i) Number of input lines = 2

Number of output lines = 4

Number of input codes $= 2^2 = 4$

(ii) Active high 2×4 decoder

(a) **Block diagram :**

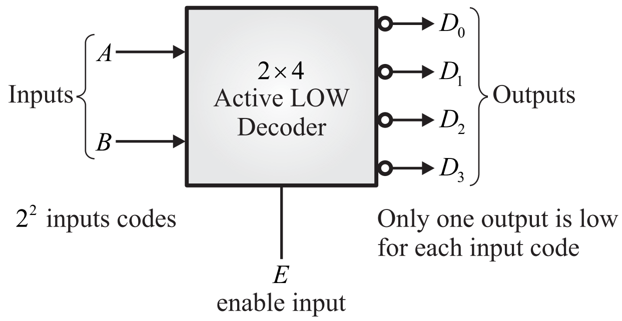


(b) **Truth table :**

Enable	Inputs		Outputs			
E	A	B	D_3	D_2	D_1	D_0
0	×	×	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0

(iii) Active low 2×4 decoder

(a) **Block diagram :**



(b) **Truth table :**

Enable	Inputs		Outputs			
E	A	B	D_3	D_2	D_1	D_0
0	×	×	1	1	1	1
1	0	0	1	1	1	0
1	0	1	1	1	0	1
1	1	0	1	0	1	1
1	1	1	0	1	1	1

Note :

- **2×4 decoder may act like 1×4 demultiplexer and vice-versa.**
- **Decoder and demultiplexer circuits are almost same.**
- **Decoder contains AND gates or NAND gates.**
- **A half adder or half subtractor circuit can be implemented by the 2×4 decoder and an OR gate.**

Comparator

- A comparator is a logic circuit used to compare the magnitudes of two binary numbers.
- Depending on the design, it may either simply provide an output that is active (goes HIGH for example) when the two numbers are equal, or additionally provide outputs that signify which of the number is greater when equality does not hold.
- The X-NOR gate is a basic comparator because its output is a 1 only if its two input bits are equal i.e. the output is a 1 if and only if the input bit coincide.
- **1-bit comparator :**

  Let the 1-bit numbers be $A = A_0$ and $B = B_0$.

  (i) **Truth table :**

A_0	B_0	$L(A<B)$	$E(A=B)$	$G(A>B)$
0	0	0	1	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	0

  L, E, G stands for lower, equal, greater respectively.

  (ii) **SOP expression :**

  $L = \overline{A}_0 B_0$ when $A < B$

  $E = A_0 \odot B_0$ when $A = B$

  $G = A_0 \overline{B}_0$ when $A > B$

- **2-bit comparator :**

  Let the 2-bit numbers are $A = A_1A_0$ and $B = B_1B_0$

  (i) **Truth table :**

Inputs				Outputs		
A_1	A_0	B_1	B_0	$L(A<B)$	$E(A=B)$	$G(A>B)$
0	0	0	0	0	1	0
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0

0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	1	0
1	0	1	1	1	0	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	1	0

(ii) **SOP expression :**

$L = \bar{A}_1 \bar{A}_0 B_0 + \bar{A}_0 B_1 B_0 + \bar{A}_1 B_1$ when $A < B$

$E = (A_0 \odot B_0)(A_1 \odot B_1)$ when $A = B$

$G = A_0 \bar{B}_1 \bar{B}_0 + A_1 \bar{B}_1 + A_1 A_0 \bar{B}_0$ when $A > B$

Note :

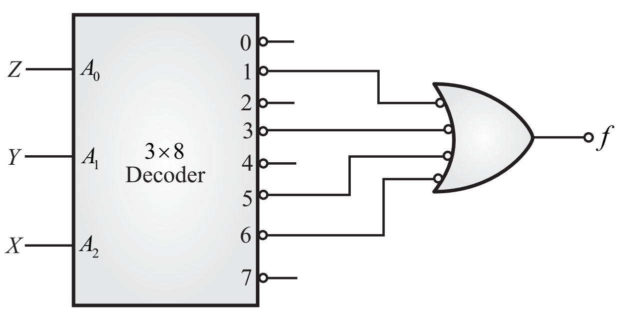
For "*n*-bit" comparator

- **Number of combination which shows equal expression i.e. $A = B$ is 2^n.**
- **Number of combination which shows, greater expression i.e. $A > B$ or lower expression i.e. $A < B$ is $\dfrac{2^{2n} - 2^n}{2}$.**

Sample Questions

2008 IISc Bangalore

3.1 A 3 line to 8 line decoder, with active low outputs, is used to implement a 3 variable Boolean functions as shown in the figure.



The simplified form of Boolean function $F(X, Y, Z)$ implemented in 'Product of Sum' form will be

(A) $(X + Z).(\bar{X} + \bar{Y} + \bar{Z}).(Y + Z)$

(B) $(\bar{X} + \bar{Z}).(X + Y + Z).(\bar{Y} + \bar{Z})$

(C) $(\bar{X} + \bar{Y} + Z).(\bar{X} + Y + Z).$
$(X + \bar{Y} + Z).(X + Y + \bar{Z})$

(D) $(\bar{X} + \bar{Y} + Z).(\bar{X} + Y + \bar{Z}).$
$(X + \bar{Y} + Z).(X + \bar{Y} + \bar{Z})$

2012 IIT Delhi

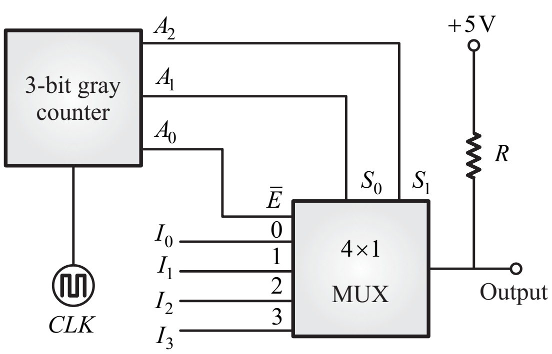
3.2 The output Y of a 2 bit comparator is logic 1 whenever the 2 bit input A is greater than the 2 bit input B. The number of combinations for which the output is logic 1, is

(A) 4 (B) 6

(C) 8 (D) 10

2014 IIT Kharagpur

3.3 A 3-bit gray counter is used to control the output of the multiplexer as shown in the figure. The initial state of the counter is 000_2. The output is pulled high. The output of the circuit follows the sequence. **[Set - 03]**



(A) $I_0, 1, 1, I_1, I_3, 1, 1, I_2$

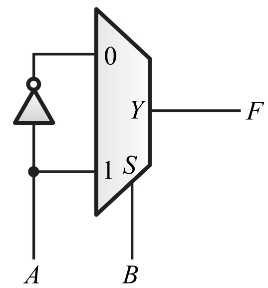
(B) $I_0, 1, I_1, 1, I_2, 1, I_2, 1$

(C) $1, I_0, 1, I_1, 1, I_2, 1, I_3, 1$

(D) $I_0, I_1, I_2, I_3, I_0, I_1, I_2, I_3$

2016 IISc Bangalore

3.4 Consider the following circuit which uses a 2-to-1 multiplexer as shown in the figure below. The Boolean expression for output F in terms of A and B is **[Set - 01]**



(A) $A \oplus B$ (B) $\overline{A+B}$

(C) $A+B$ (D) $\overline{A \oplus B}$

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Explanations Combinational Circuits

Introduction to Multiplexer
Scan for Video Explanation

Designing of Multiplexer
Scan for Video Explanation

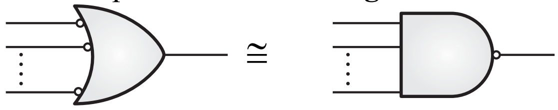
Analysis of Multiplexer
Scan for Video Explanation

Designing of Higher Order MUX using Lower order Multiplexer
Scan for Video Explanation

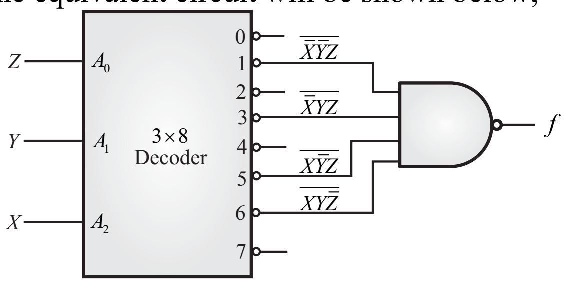
3.1 (A)

Method 1

The **bubbled (negative) OR gate** in the given circuit is equivalent to **NAND gate**.



The equivalent circuit will be shown below,



$$f = \overline{\overline{\bar{X}\,\bar{Y}\,Z}\cdot\overline{\bar{X}\,Y\,Z}\cdot\overline{X\,\bar{Y}\,Z}\cdot\overline{X\,Y\,\bar{Z}}}$$

$$f = \bar{X}\,\bar{Y}\,Z + \bar{X}\,Y\,Z + X\,\bar{Y}\,Z + X\,Y\,\bar{Z}$$

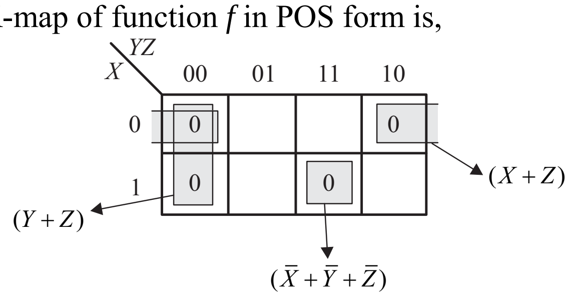
The min-terms expression is given by,

$$f(X,Y,Z) = \sum m(1,3,5,6)$$

Therefore, the max-terms expression will be,

$$f(X,Y,Z) = \Pi M(0,2,4,7)$$

K-map of function f in POS form is,



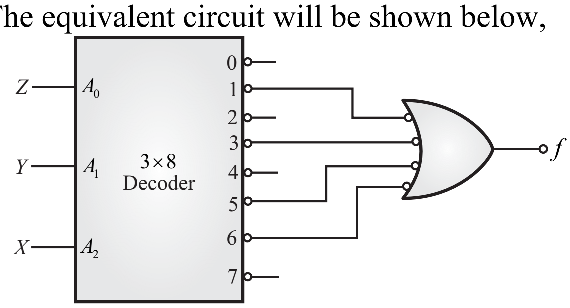
From K-map, function $f(X,Y,Z)$ is given by,

$$f(X,Y,Z) = (X+Z)(\bar{X}+\bar{Y}+\bar{Z})(Y+Z)$$

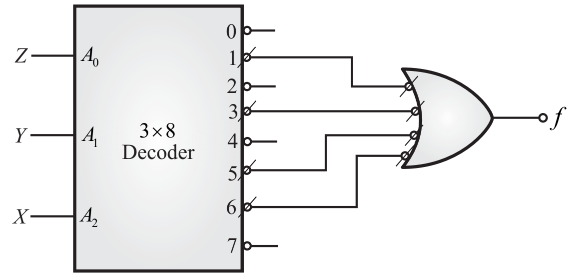
Hence, the correct option is (A).

Method 2

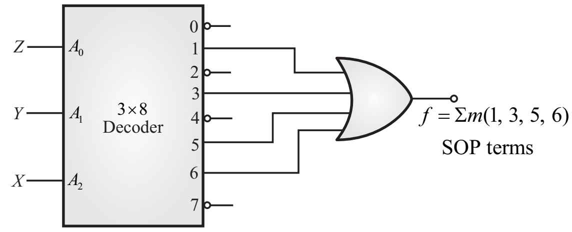
The equivalent circuit will be shown below,



We can cancel out two simultaneous bubbles present in single line,



Now, above circuit becomes,



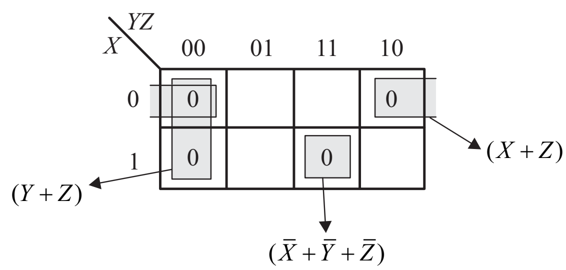
The output of above circuit is,

$$f(X, Y, Z) = \Sigma m(1, 3, 5, 6) \rightarrow \text{SOP terms}$$

Options are provided in POS form so we have,

$$f(X, Y, Z) = \pi M(0, 2, 4, 7) \rightarrow \text{POS terms}$$

K-map of function (f) in POS form,



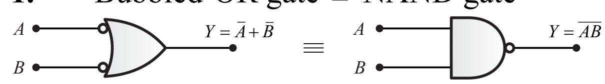
From K-map, function $f(X,Y,Z)$ is given by,

$$f(X,Y,Z) = (X+Z)(\bar{X}+\bar{Y}+\bar{Z})(Y+Z)$$

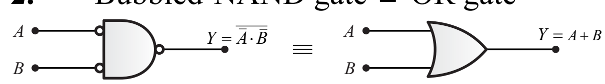
Hence, the correct option is (A).

Key Point

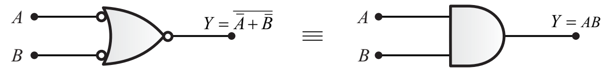
1. Bubbled-OR gate ≡ NAND gate



2. Bubbled-NAND gate ≡ OR gate



3. Bubbled-NOR gate ≡ AND gate



4. Bubbled-AND gate ≡ NOR gate

3.2 (B)

Let 2-bit input $A = A_1A_0$

and 2-bit input $B = B_1B_0$

For a 2-bit comparator, the output is logic 1 whenever the 2-bit input $A = A_1A_0$ is greater than 2-bit input $B = B_1B_0$.

A_1	A_0	B_1	B_0	$Y = A > B$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1 (1)
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1 (2)
1	0	0	1	1 (3)
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1 (4)
1	1	0	1	1 (5)
1	1	1	0	1 (6)
1	1	1	1	0

Thus, the number of combinations for which the output is logic "1" = 6.

Hence, the correct option is (B).

Scan for Video Solution

Key Point

For "*n*-bit" comparator :

1. Total number of combination $= 2^{2n}$
2. Number of combination which shows equal expression i.e. $A = B$ is 2^n.
3. Number of combination which shows, greater expression i.e. $A > B$ or lower expression i.e. $A < B$ is $\dfrac{2^{2n} - 2^n}{2}$.

(n = number of bits)

3.3 (A)

Given circuit is shown below,

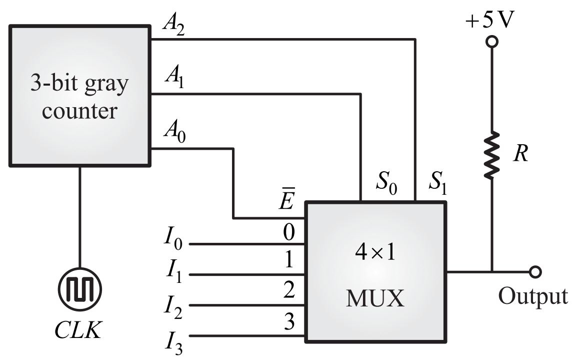


Fig. (a)

The 4×1 MUX in the given circuit is active low enable, since $\overline{E}$ is present.

(i) For $A_0 = 0$, the 4×1 MUX will be enabled.

(ii) For $A_0 = 1$, the 4×1 MUX will be disabled.

The initial state of the counter (i.e. 3 bit gray counter) is 000.

Truth table for gray code counter is shown below,

A_2	A_1	A_0
0	0	0
0	0	1
0	1	1
0	1	0
1	1	0
1	1	1
1	0	1
1	0	0

When $E = 1(\overline{E} = 0)$, MUX is not enable. Therefore, output of MUX is NOT connected to any input of MUX (i.e. I_0, I_1, I_2 and I_3). Hence, final Output of circuit is connected to +5 V i.e. logic high.

A_2	A_1	A_0	$\overline{E} = A_0$	$S_1 = A_2$	$S_0 = A_1$	F
0	0	0	0	0	0	I_0
0	0	1	1	0	0	1
0	1	1	1	0	1	1
0	1	0	0	0	1	I_1
1	1	0	0	1	1	I_3
1	1	1	1	1	1	1
1	0	1	1	1	0	1
1	0	0	0	1	0	I_2

Hence, the correct option is (A).

Key Point

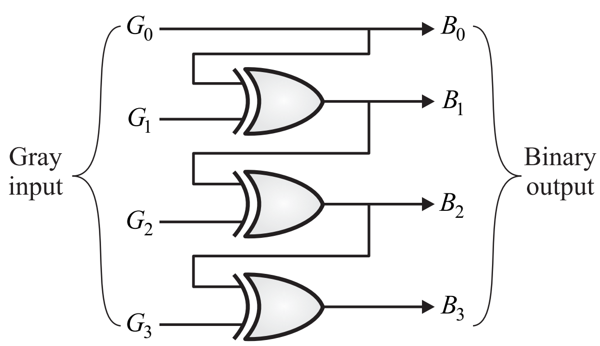
Gray code :

(i) It is unweighted code i.e. there are no specific weights are design to each bit position, due to this reason it is not fall under the category of arthematic code.

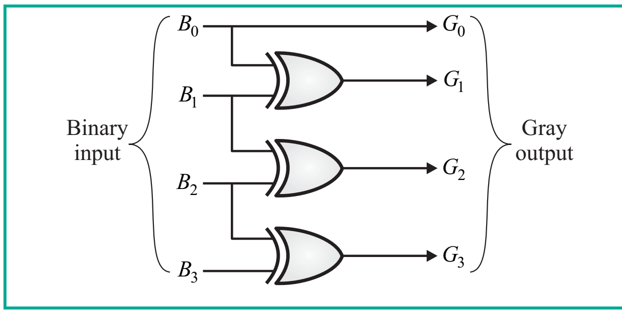
(ii) It is also called as 1-bit change code because each successive code word differ only by 1-bit change.

(iii) It is also called as unit distance code/reflective code.

(iv) 4-bit gray code to binary code converter,

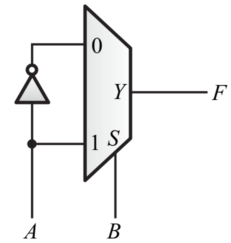


(v) 4-bit binary to gray code converter,



3.4 (D)

Given 2×1 MUX is shown below,



From above 2×1 MUX,

$$F = \overline{S}I_0 + SI_1$$

Here, $S = B$, $I_0 = \overline{A}, I_1 = A$

Now, $F = \overline{B}\,\overline{A} + BA$

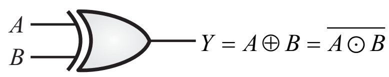
$$F = A \odot B$$

$$F = \overline{A \oplus B}$$

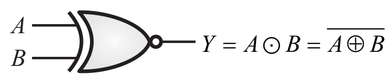
Hence, the correct option is (D).

Key Point

1. For a 2-input EXOR gate,



2. For a 2-input EXNOR gate,



3. 2-input EXOR and EXNOR logic gates are complement to each other.

4. 2-input EXOR and EXNOR logic gates are also dual to each other

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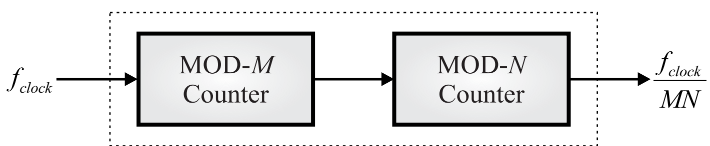
4 Sequential Circuits

➢ Partial Synopsis

Counters

- **Depending upon clock pulse, counter is of two types :**
  (i) **Asynchronous counter :** In this type of counter flip-flops are connected in such a way that the output of 1st flip-flop drives the clock for the 2nd flip-flop, the output of 2nd flip-flop drives the clock for the 3rd flip-flop and so on.
  (ii) **Synchronous counter :** In this type of counter there is no connection between the output of 1st flip-flop and clock input of next flip-flop and so on. In this type of counter all the flip-flops are connected to the same clock.
- **MOD number**
  (i) Number of states present in a counter is known as modulus count or MOD number.
  (ii) The MOD number indicates frequency division obtained from the last flip-flops. For MOD-N counter, the frequency of the output of last flip-flop is $\frac{f_{clock}}{N}$.

  (iii) If two counters are cascaded, with MOD number M and MOD number N, then
    (a) Overall modulus number of cascaded combinations of counter $= M \times N = MN$



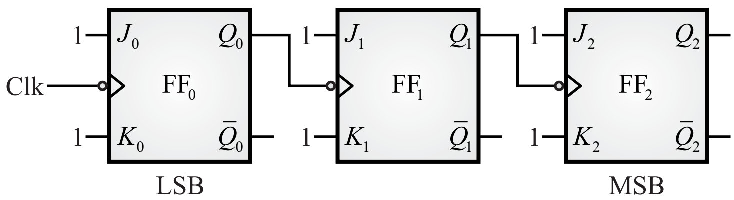
- Maximum decimal count of any counter $= N - 1$

Asynchronous Counter

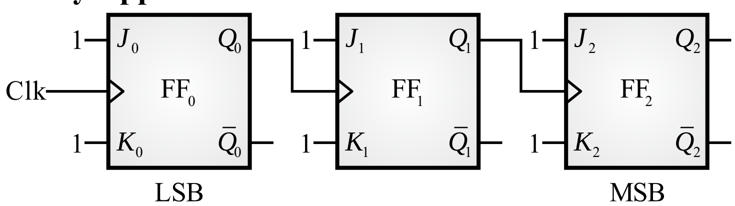
- It is also called ripple counter.
- In asynchronous counter, different clock pulse is applied to different flip-flops.
- In asynchronous counter, all flip-flops are operating in toggle mode.

- Example of asynchronous counter

  (i) 3-bit binary ripple up counter :



  (ii) 3-bit binary ripple down counter :

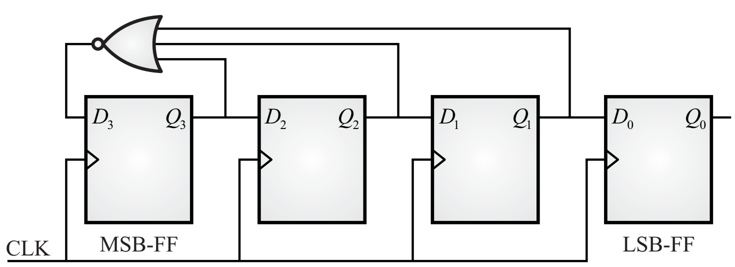


Synchronous Counter

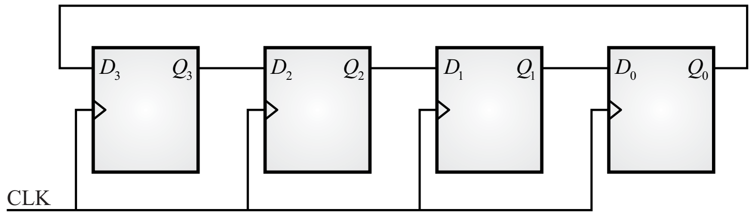
- The synchronous counters are classified as :
  - (i) Shift register counters
    - (a) Ring counter
    - (b) Twisted ring counter/Johnson counter
  - (ii) Series carry counter
  - (iii) Parallel carry counter

Ring Counter

- It is a **synchronous counter.** It is also called **serial-in serial-out (SISO) shift register.**
- In a ring counter, if feedback is used the number of states are reduced.
- An n-bit ring counter has,
  - (i) Number of flip-flop $=$ Mode of counter $= n$
  - (ii) Frequency of output of any flip-flop $= \dfrac{f_{CLK}}{n}$ where, f_{CLK} is frequency of clock signal.
  - (iii) Number of distinct states (or used states) $= n$
  - (iv) Number of unused states $= 2^n - n$
- For decoding ring counter no logic gates are required.
- **4-bit self starting ring counter :**



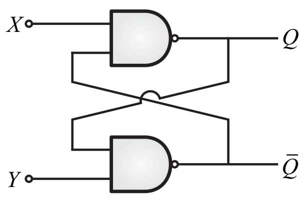
- **4-bit not self starting ring counter :**



➢ Sample Questions

1999 IIT Bombay

4.1 For a flip-flop formed using two NAND gates as shown in figure. The unstable state corresponds to

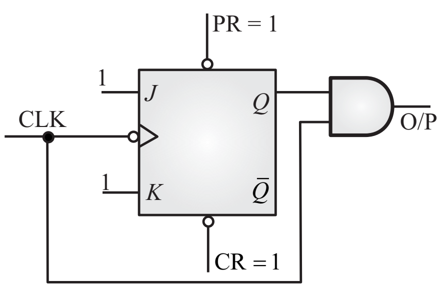


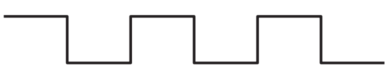
(A) $X = 0, Y = 0$ (B) $X = 0, Y = 1$

(C) $X = 1, Y = 0$ (D) $X = 1, Y = 1$

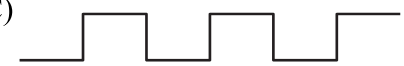
2004 IIT Delhi

4.2 The digital circuit shown in figure generates a modified clock pulse at the output. Choose the correct output waveform from the options given below.



(A) 

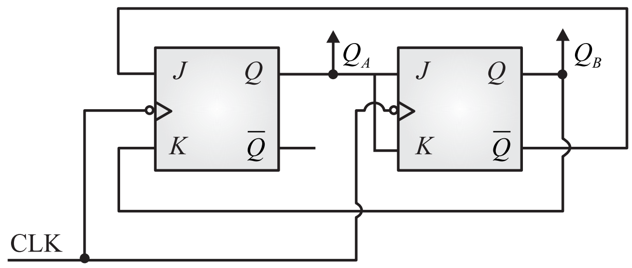
(B) 

(C) 

(D) 

2001 IIT Madras

4.3 A 2-bit counter circuit is shown below,



If the state $Q_A Q_B$ of the counter at the clock time t_n is "10" then the state $Q_A Q_B$ of the counter at $t_n + 3$ (after three cycles) will be

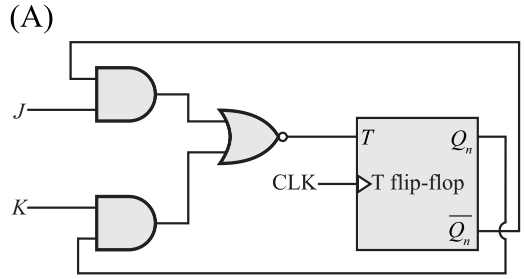
(A) 00 (B) 01

(C) 10 (D) 11

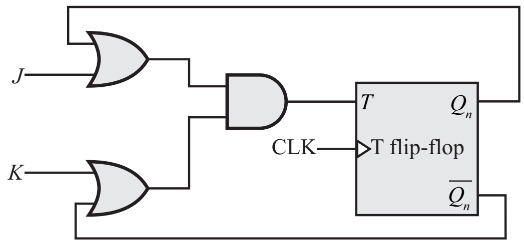
2014 IIT Kharagpur

4.4 A *J*-*K* flip-flop can be implemented by *T* flip-flops. Identify the correct implementation. **[Set - 02]**

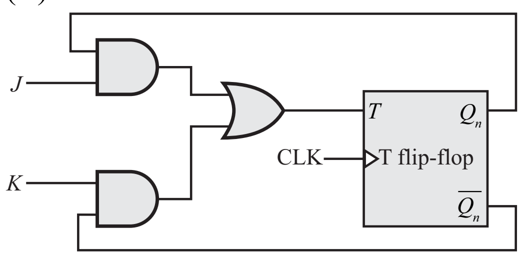
(A)



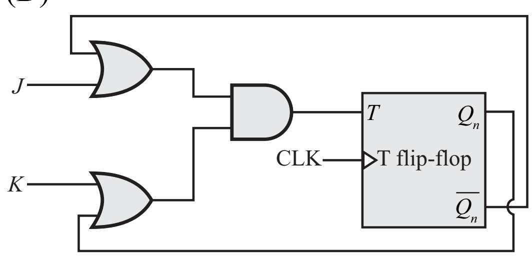
(B)



(C)

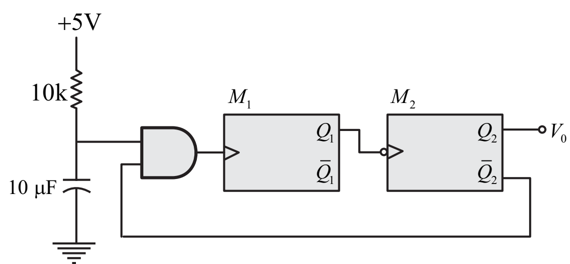


(D)



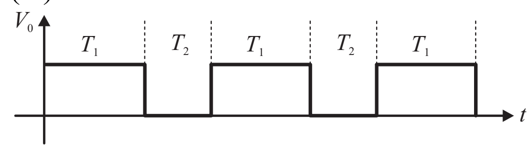
4.5 Two monoshot multivibrators, one positive edge triggered (M_1) and another negative edge triggered (M_2) are connected as shown in figure.

[Set - 03]

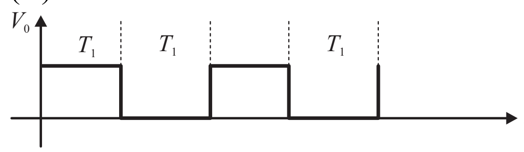


The monoshots M_1 and M_2 when triggered produce pulses of width T_1 and T_2 respectively, where $T_1 > T_2$. The steady state output voltage V_0 of the circuit is

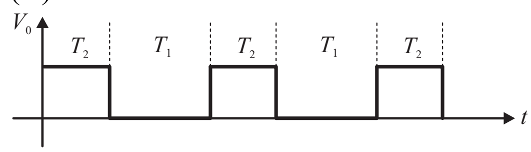
(A)



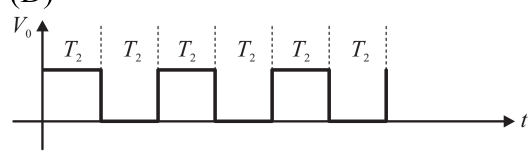
(B)



(C)

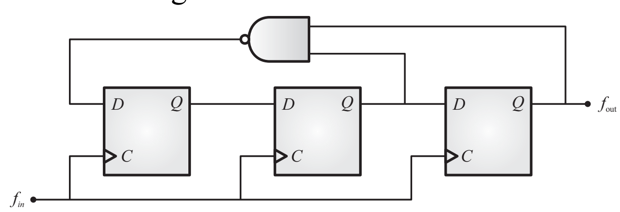


(D)



2018 IIT Guwahati

4.6 Which one of the following statements is true about the digital circuit shown in the figure.



(A) It can be used for dividing the input frequency by 3.

(B) It can be used for dividing the input frequency by 5.

(C) It can be used for dividing the input frequency by 7.

(D) It cannot be reliably used as a frequency divider due to disjoint internal cycles.

2021 IIT Bombay

4.7 A 16-bit synchronous binary up-counter is clocked with a frequency f_{clk}. The two most significant bits are ORed together to form an output Y. Measurements shows that Y is periodic and the duration for which Y the remains high in each period is 24 ms. The clock frequency f_{clk} is _______ MHz. (Round off to 2 decimal places)

❖❖❖❖

Explanations Sequential Circuits

Conclusion of Flip-Flops

Scan for Video Explanation

Concept of Asynchronous Counter

Scan for Video Explanation

Concept of Asynchronous Clear & Preset input

Scan for Video Explanation

Concept of Mod N Asynchronous Counter (Up & Down)

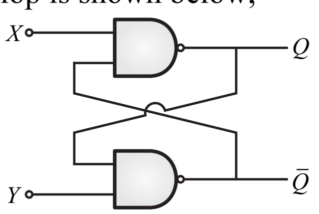
Scan for Video Explanation

Concept of Synchronous Clear & Preset Input

Scan for Video Explanation

4.1 (A)

Given flip-flop is shown below,



The above figure represents *S-R* NAND latch.

Truth table for *S-R* NAND latch is shown below,

X	Y	Q_n	Q_{n+1}	State
0	0	0	X	unstable (invalid)
0	0	1	X	
0	1	0	1	Set
0	1	1	1	
1	0	0	0	Reset
1	0	1	0	
1	1	0	0	No change or Hold
1	1	1	1	

Hence, the correct option is (A).

Key Point

(i) Latches are asynchronous sequential circuit whereas flip-flops are synchronous sequential circuit.

(ii) Latches are level triggered.

(iii) Latch is 1-bit storing element.

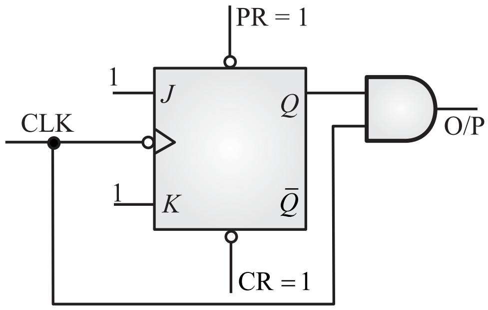
(iv) Latches made up from logic gates connected in cross coupled feedback manner.

(v) S-R NAND latch always lead unstable/invalid states at output when both the input are (0, 0).

(vi) S-R NOR latch always lead unstable/invalid states at output when both the input are (1, 1).

4.2 (B)

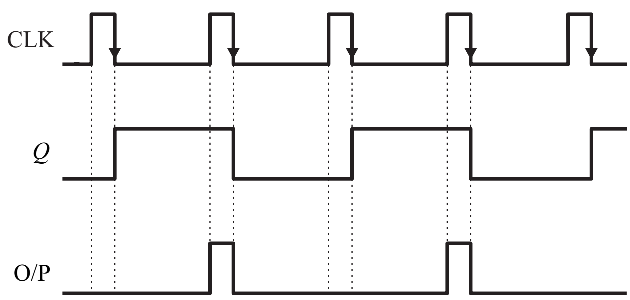
Given *J-K* flip-flop is shown below,



J-K flip flop is in toggle mode, therefore its output frequency is half of the clock frequency. There is no information of *Q* output of *J-K* flip-flop. So by default, assume zero state i.e. initially $Q = 0$.

Flip-flop output *Q* will change for negative edge of clock but output of AND gate will change at positive edge.

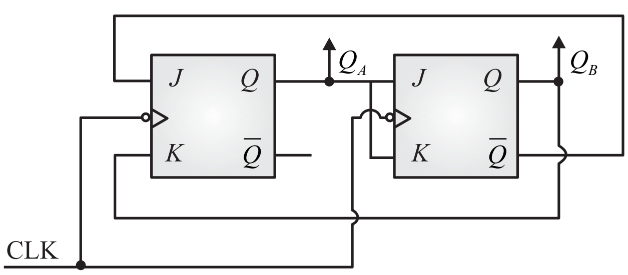
Since, $J = K = 1$; *Q* will toggle with every clock. Output is high (logic 1) only when both '*Q*' and 'CLK' are high.



Hence, the correct option is (B).

4.3 (C)

Given 2 bit counter is shown below,



Initially $Q_A Q_B = 10$,

$$J_{B,\,n} = K_{B,\,n} = Q_{A,\,n}$$

And $J_{A,\,n} = \bar{Q}_{B,\,n},\ K_{A,\,n} = Q_{B,\,n}$

Truth table for the given circuit is shown below,

CLK	Present state		Flip-flop inputs				Next state	
	Q_A	Q_B	$J_A = \bar{Q}_B$	$K_A = Q_B$	$J_B = Q_A$	$K_B = Q_A$	Q_A^+	Q_B^+
1	1	0	1	0	1	1	1	1
2	1	1	0	1	1	1	0	0
3	0	0	1	0	0	0	1	0

From the above table it is clear that output after 3 clock pulse is $Q_A Q_B = 10$.

Hence, the correct option is (C).

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Video Solution

4.4 (B)

Method 1

Key Point

To obtain *J-K* flip flop from *T* flip flop, following steps needs to be taken :

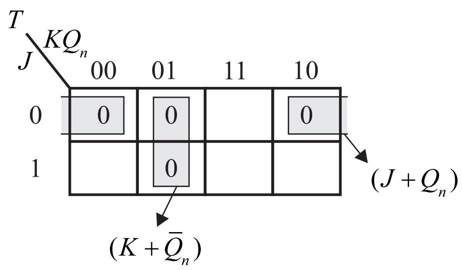
1. Writing characteristic table of *J-K* flip flop.
2. Obtaining excitation table for *T* flip flop.

Excitation table of *T*-Flip flop

J	*K*	Q_n	Q_{n+1}	*T*
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	1

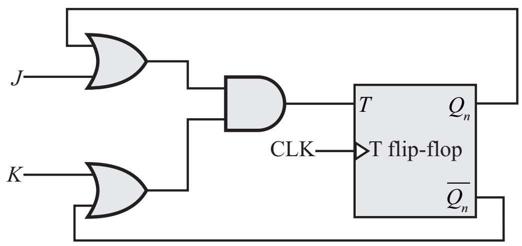
Characteristic table of *J-K* flip flop

K-map of function T in POS form is,



Thus, $T = (J + Q_n).(K + \bar{Q}_n)$

Therefore from the above equation the J-K flip flop can be implemented by T flip flop as shown below,



Hence, the correct option is (B).

Method 2

Here, T flip-flop is given for the implementation of JK flip-flop.

Characteristic equation of T flip-flop,

$$Q_n^+ = T \oplus Q_n = T\bar{Q}_n + \bar{T}Q_n \qquad \ldots\text{(i)}$$

To make T flip-flop work as JK flip-flop, so equation (i) must be in the form of characteristic equation of JK flip-flop

i.e. $$Q_n^+ = J\bar{Q}_n + \bar{K}Q_n \qquad \ldots\text{(ii)}$$

Take the help from option :

From option (A) :

$$T = \overline{J\bar{Q}_n + KQ_n} = (\bar{J} + Q_n)(\bar{K} + \bar{Q}_n)$$

$$\bar{T} = J\bar{Q}_n + KQ_n$$

Put the value of T and $\bar{T}$ in equation (i),

$$Q_n^+ = (\bar{J} + Q_n)(\bar{K} + \bar{Q}_n)\bar{Q}_n + (J\bar{Q}_n + KQ_n)Q_n$$

$$Q_n^+ = \bar{J}\bar{Q}_n(\bar{K} + \bar{Q}_n) + KQ_n \qquad [\because\ Q_n \cdot \bar{Q}_n = 0]$$

$$Q_{n+1} = \bar{J}\,\bar{Q}_n\bar{K} + \bar{J}\,\bar{Q}_n\bar{Q}_n + KQ_n$$

$$Q_{n+1} = \bar{J}\,\bar{Q}_n\bar{K} + \bar{J}\,\bar{Q}_n + KQ_n$$

$$Q_{n+1} = \bar{J}\,\bar{Q}_n(\bar{K} + 1) + KQ_n \qquad [\because\ 1 + \bar{K} = 1]$$

$$Q_n^+ = \bar{J}\bar{Q}_n + KQ_n$$

The above expression does not showing the characteristic equation of JK flip-flop.

Hence, option (A) is incorrect.

From option (B) :

$$T = (J + Q_n)(K + \bar{Q}_n)$$

and $\bar{T} = \overline{(J + Q_n)(K + \bar{Q}_n)} = (\bar{J}\,\bar{Q}_n + \bar{K}Q_n)$

Put the value of T and $\bar{T}$ in equation (i),

$$Q_n^+ = (J + Q_n)(K + \bar{Q}_n)\bar{Q}_n + (\bar{J}\,\bar{Q}_n + \bar{K}Q_n)Q_n$$

$$Q_n^+ = (J + Q_n)\bar{Q}_n + \bar{K}Q_n \qquad [\because\ Q_n.\bar{Q}_n = 0]$$

$$Q_n^+ = J\bar{Q}_n + \bar{K}Q_n$$

The above expression showing the characteristic equation of JK flip-flop.

Hence, option (B) is correct.

From option (C) :

$$T = JQ_n + K\bar{Q}_n$$

$$\bar{T} = (\bar{J} + \bar{Q}_n).(\bar{K} + Q_n)$$

Put the value of T and $\bar{T}$ in equation (i),

$$Q_n^+ = (JQ_n + K\bar{Q}_n)\bar{Q}_n + (\bar{J} + \bar{Q}_n)(\bar{K} + Q_n)Q_n$$

$$Q_n^+ = K\bar{Q}_n + (\bar{J}Q_n)(\bar{K} + Q_n) \qquad [\because\ Q_n.\bar{Q}_n = 0]$$

$$Q_n^+ = \bar{J}Q_n + K\bar{Q}_n$$

The above expression does not showing the characteristic equation of JK flip-flop.

Hence, option (C) is incorrect.

From option (D) :

$$T = (J + \bar{Q}_n).(K + Q_n)$$

$$\bar{T} = (\bar{J}.Q_n) + (\bar{K}.\bar{Q}_n)$$

Put the value of T and $\bar{T}$ in equation (i),

$$Q_n^+ = (J + \bar{Q}_n).(K + Q_n)\bar{Q}_n + (\bar{J}Q_n + \bar{K}\,\bar{Q}_n)Q_n$$

$$Q_n^+ = (J + \bar{Q}_n).(K\bar{Q}_n) + (\bar{J}Q_n) \quad [\because\ Q_n.\bar{Q}_n = 0]$$

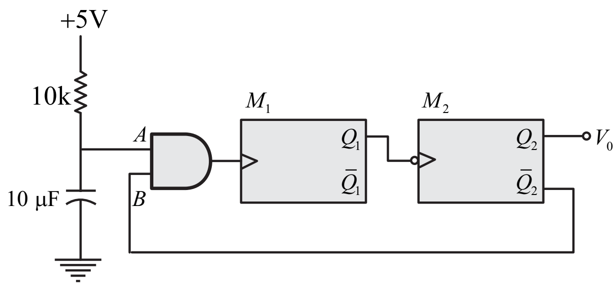
$$Q_n^+ = \bar{J}Q_n + K\bar{Q}_n$$

The above expression does not showing the characteristic equation of JK flip-flop.

Hence, option (D) is incorrect.

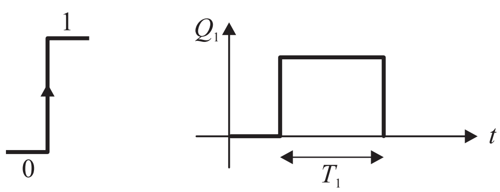
4.5 (C)

Given logic circuit is shown below,

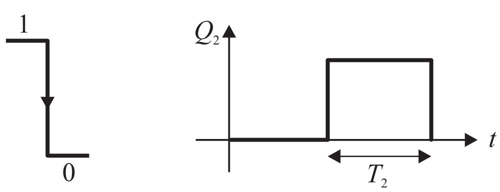


Given :

(i) M_1 is positive edge triggered monoshot multi-vibrator which generates a pulse of width T_1.

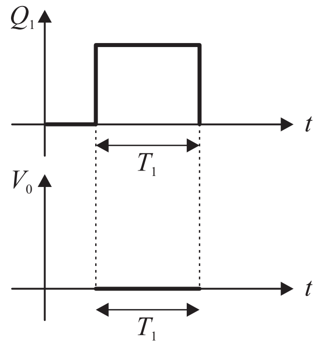


(ii) M_2 is negative edge triggered monoshot multi-vibrator which generates a pulse of width T_2.



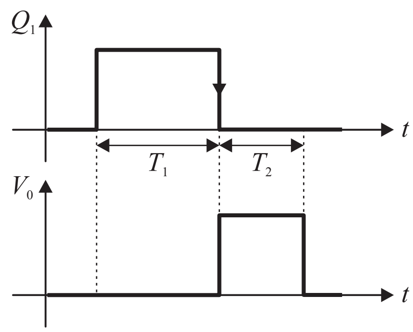
(a) Input A of AND gate is always fixed at 1 because capacitor is charged to '+5 V' i.e. logic high.

(b) Let initially $V_0 = 0 = Q_2$ then $\bar{Q}_2 = 1$, which is fed to input B of AND gate. Now, both input of AND gate is 1 hence output will be high and M_1 is activated and will generate a pulse of duration T_1.

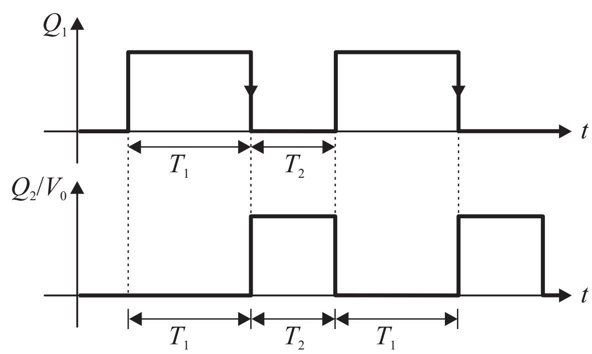


For duration T_1, V_0 will be zero since M_2 is negative edge triggered.

(c) When output of Q_1 is falling from 1 to 0 i.e. at the negative edge, M_2 will be activated and will generated a pulse of duration T_2.



(d) After duration T_2, when $V_0 = Q_2 = 0$ then $\bar{Q}_2$ is again 1 and the whole process is repeated again.

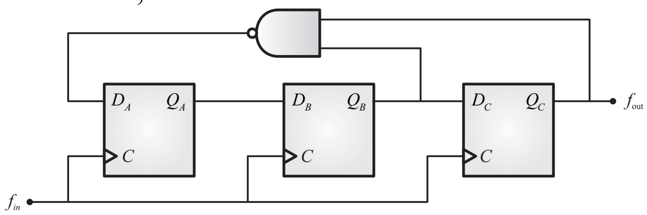


The above figure matches with option (C).

Hence, the correct option is (C).

4.6 (B)

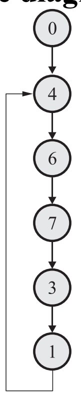
Given logical circuit is shown below,



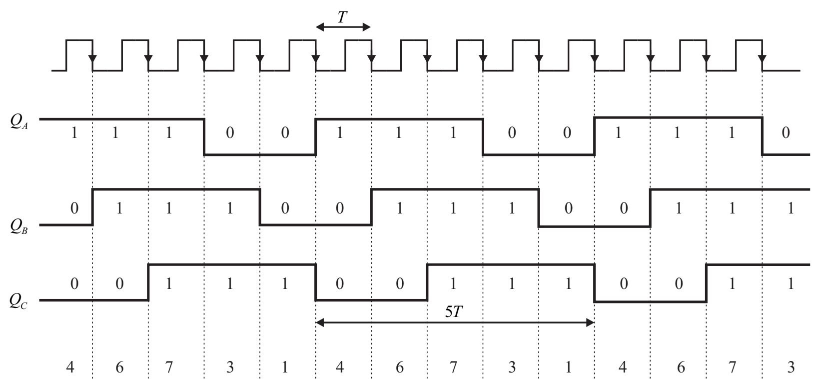
From the above sequential circuit,

Present state			FF inputs			Next state		
Q_A	Q_B	Q_C	$D_A = \overline{Q_B \cdot Q_C}$	$D_B = Q_A$	$D_C = Q_B$	Q_A^+	Q_B^+	Q_C^+
0	0	0	1	0	0	1	0	0
1	0	0	1	1	0	1	1	0
1	1	0	1	1	1	1	1	1
1	1	1	0	1	1	0	1	1
0	1	1	0	0	1	0	0	1
0	0	1	1	0	0	1	0	0

State diagram :



⇒ 5 unique states (4, 6, 7, 3, 1)
⇒ MOD 5 counter
⇒ divide by 5 counter
⇒ It can be used for dividing the input frequency by 5
⇒ State 0 is transient state and it occurs only once at very starting state of counter



Hence, the correct option is (B).

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4.7 2.05

Given counter = 16 bit

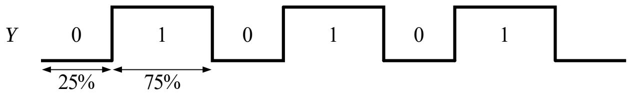
$$2^{16}\text{combination}\begin{cases} \begin{matrix} Q_{15} & Q_{14} & Q_{13} & & Q_1 & Q_0 \\ 0 & 0 & 0 & \ldots\ldots\ldots. & 0 & 0 \\ & & & & & \\ 1 & 1 & 1 & \ldots\ldots\ldots. & 1 & 1 \end{matrix} \end{cases}$$

If we fix $Q_{15}\,Q_{14}$ bit then $(Q_{13}$ to $Q_0)$ will take 2^{14} combination

$Y = Q_{15} + Q_{14}$	Q_{15}	Q_{14}	$Q_{13}\ldots\ldots\ldots Q_1\,Q_0$
0	0 0	0 0	2^{14} combination
1	0 0	1 1	2^{14} combination
1	1 1	0 0	2^{14} combination
1	1 1	1 1	2^{14} combination

So, $Y = 0$ for 2^{14} times. [25 %]

$Y = 1$ for 3×2^{14} times. [75 %]



Given that, $Y = 1$ for 24 ms

So, $\quad Y = 0$ for $\dfrac{24 \times 25}{75} = 8$ ms

So, total time period $= 8 + 24 = 32$ ms

Time period of clock, $T_{clk} = \dfrac{32\,\text{ms}}{2^{16}}$

$$f_{clk} = \frac{2^{16}}{32 \times 10^{-3}}$$

$$f_{clk} = 2.048\,\text{MHz} \approx 2.05\,\text{MHz}$$

Hence, the correct answer is 2.05.

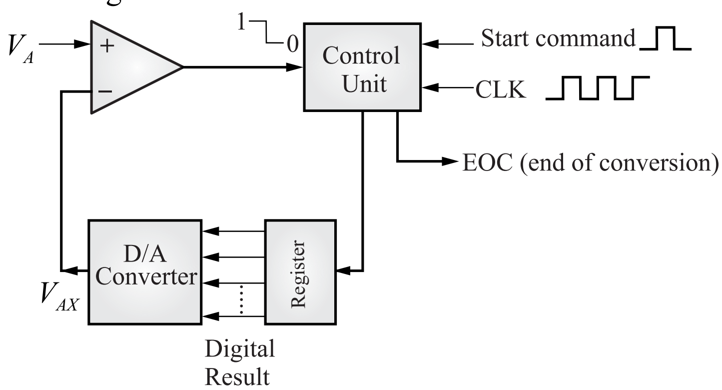
❖❖❖❖

6 ADC & DAC

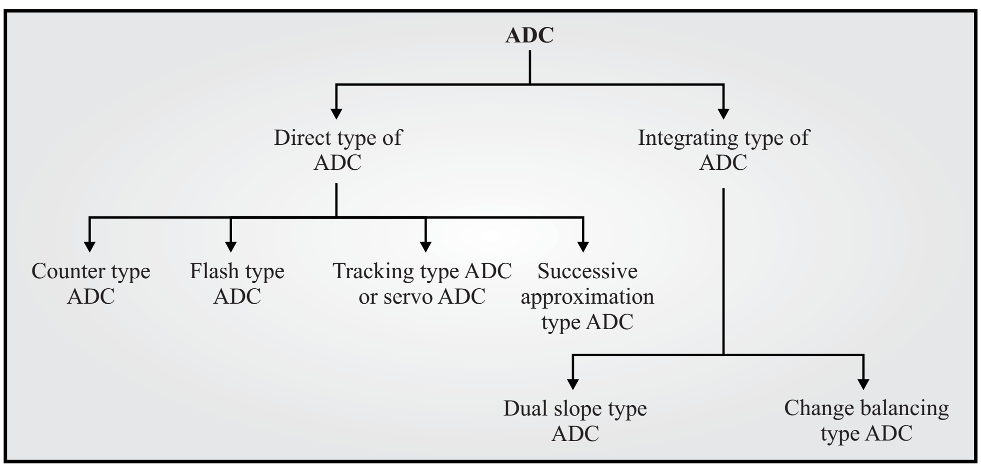
➢ Partial Synopsis

Analog to Digital Converters (ADC)

- It is often required that data taken in a physical system be converted into digital form. Thus, an analog to digital converter produces a digital output that is proportional to the value of input analog signal.
- ADCs are generally more complex and time consuming to design than DACs.
  An ADC takes an analog input voltage and after a certain amount of time produces a digital output code which represents the digital equivalent of the analog input. A general diagram of ADCs is shown in figure below.



Classification of ADC



Flash type/parallel comparator type ADC

- n-bit parallel ADC also called as flash type ADC/simultaneous type ADC.
- For *n*-bit flash type ADC requires,
  1. $2^n - 1$ number of comparator.
  2. 2^n number of resistors.
  3. $2^n \times n$ priority encoder.
  4. Number of priority encoder = 1
  5. Resolution $= \dfrac{V_{\max} - V_{\min}}{2^N - 1}$
- It is fastest type of ADC among all ADC's.
- One drawback with this ADC is, it requires more hardware if number of bits (*n*) increases more.

Maximum conversion time (T_c) for various n-bit ADC's

- Counter type ADC, $T_c = 2^n T_{clock}$
- Successive approximation type ADC, $T_c = nT_{clock}$
- Flash type ADC, $T_c = 1$ clock
- Dual slope integrating type ADC, $T_c = 2^{n+1} T_{clock}$

Sample Questions

1994 IIT Kharagpur

6.1 The number of comparisons carried out in a 4-bit flash type A/D converter is

(A) 16 (B) 15

(C) 4 (D) 3

2001 IIT Kanpur

6.2 Among the following four, the slowest ADC (analog-to-digital converter) is

(A) Parallel-comparator (i.e. flash) type

(B) Successive approximation type

(C) Integrating type

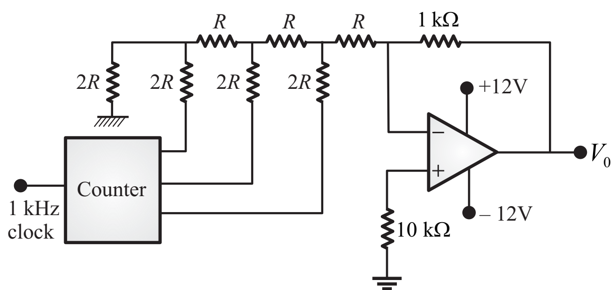
(D) Counting type

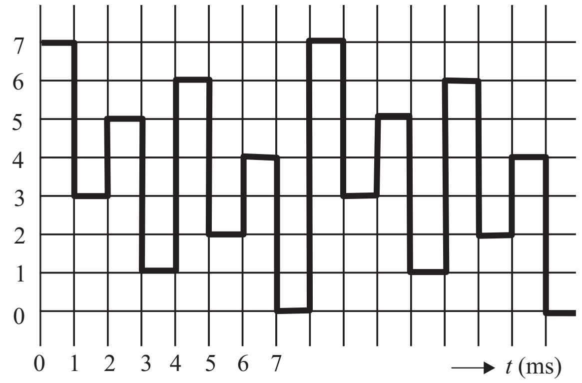
2005 IIT Bombay

6.3 A digital-to-analog converter with a full-scale output voltage of 3.5 V has a resolution close to 14 mV. Its bit size is ________.

2006 IIT Kharagpur

6.4 A student has made a 3-bit binary down counter and connected to the *R*-2*R* ladder type DAC [Gain $= (-1\text{k}\Omega/2R)$] as shown in figure to generate a staircase waveform. The output achieved is different as shown in figure. What could be the possible cause of this error?

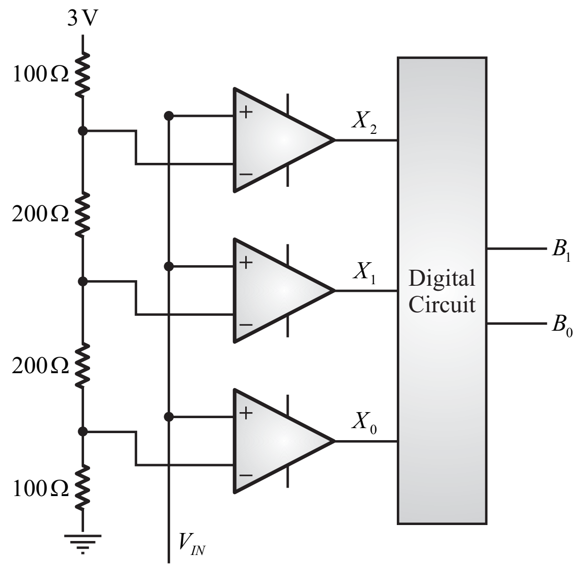




(A) The resistance values are incorrect.
(B) The counter is not working properly.
(C) The connection from the counter to DAC is not proper.
(D) The R and $2R$ resistance are interchanged.

2016 IISc Bangalore

6.5 A 2-bit flash Analog to Digital Converter (ADC) is given below. The input is $0 \le V_{IN} \le 3$ Volts. The expression for the LSB of the output B_0 as a Boolean function of X_2, X_1, and X_0 is **[Set - 01]**



(A) $X_0[\overline{X_2 \oplus X_1}]$ (B) $\overline{X}_0[\overline{X_2 \oplus X_1}]$

(C) $X_0[X_2 \oplus X_1]$ (D) $\overline{X}_0[X_2 \oplus X_1]$

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Explanations ADC & DAC

Concept of DAC and Weighted Resistor DAC
Scan for Video Explanation

Concept of Counter type ADC
Scan for Video Explanation

Concept of ADC and Flash ADC
Scan for Video Explanation

Concept of Resolution, Step size, Full scale output through question
Scan for Video Explanation

Concept of Successive approximation ADC
Scan for Video Explanation

Concept of Dual Slope ADC
Scan for Video Explanation

6.1 (B)

Given : Number of bits, $n = 4$

Number of comparisons

$= $ Number of comparators required

For n-bit flash type A/D converter,

Number of comparators is given by,

$$C = 2^n - 1 = 2^4 - 1$$

$$C = 16 - 1 = 15$$

Hence, the correct option is (B).

Scan for Video Solution

6.2 (C)

Integrating type ADC (or dual slope type ADC) is slowest, since it requires charging and discharging of capacitor.

Hence, the correct option is (C).

Scan for Video Solution

Key Point

Analog to digital converter (ADC) in increasing order of speed :

Dual slope (integrating) type < Counting ramp type < Successive approximation type < Parallel comparator (flash) type.

6.3 8

Given : Full scale output voltage of

DAC = 3.5 V

Resolution $\simeq 14$ mV

Resolution is given by,

$$R = \frac{V_{fullscale}}{2^n - 1}$$

$$2^n - 1 = \frac{3.5}{14 \times 10^{-3}} = 250$$

$$n \ln(2) = \ln(251)$$

$$n = 7.97 \simeq 8 \quad (n \text{ always integer value})$$

Hence, bit size is **8**.

Key Point

(i) Generally resolution refers to the finest minimum change in the signal which is accepted for conversion and it is decided with respect to number of bits.

(ii) Resolution of DAC is smallest change that can occur in the analog output as a result of change in digital input.

(iii) Resolution always equal to weight of LSB in DAC and also known as step size.

(iv) General expression of resolution for n-bit DAC,

$$\text{Resolution} = \frac{\text{Full-scale value}}{2^n - 1}$$

$$\%\text{Resolution} = \left(\frac{1}{2^n - 1}\right) \times 100$$

(v) Resolution for R-2R ladder type n-bit DAC,

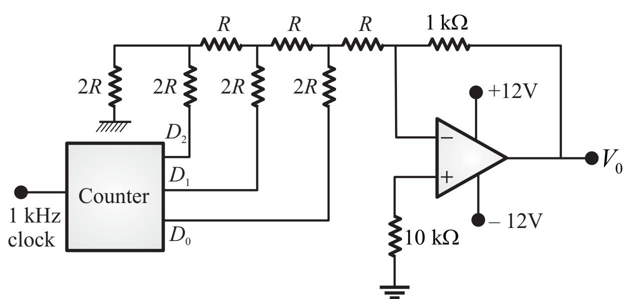
$$\text{Resolution} = \frac{\text{Full-scale value}}{2^n}$$

$$\%\text{Resolution} = \frac{1}{2^n} \times 100$$

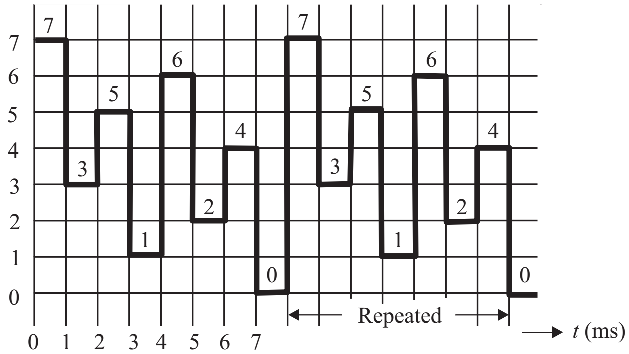
(vi) Resolution always a function of number of bits (n) used in converter if n is increases, then resolution is finer.

6.4 (C)

Given logic circuit is shown below,



Since, it is a down counter it will count from 7 to 0.



Initial stage of the counter $= (111)_2 = (7)_{10}$

So output will be equal to 7 V.

Next state of counter $= (110)_2 = (6)_{10}$

Thus, output should be equal to 6 V.

But output is 3 V that means LSB of counter is connected to MSB of DAC and MSB of counter is connected to LSB of DAC.

Similarly, next state of counter

$$= (101)_2 = (5)_{10}$$

Input to DAC $= (101)_2 = (5)_{10}$

So output = 5 V

When counter goes to $(100)_2$ then input to DAC $= (001)_2 = (1)_{10}$

Thus, output = 1 V

The interconnection from the counter to DAC is not proper. The lines corresponding to bits D_2 to D_1 are interchanged.

Counter output	ADC output	Correct sequence
111	7	7
011	3	6
101	5	5
001	1	4
110	6	3
010	2	2
100	4	1
000	0	0

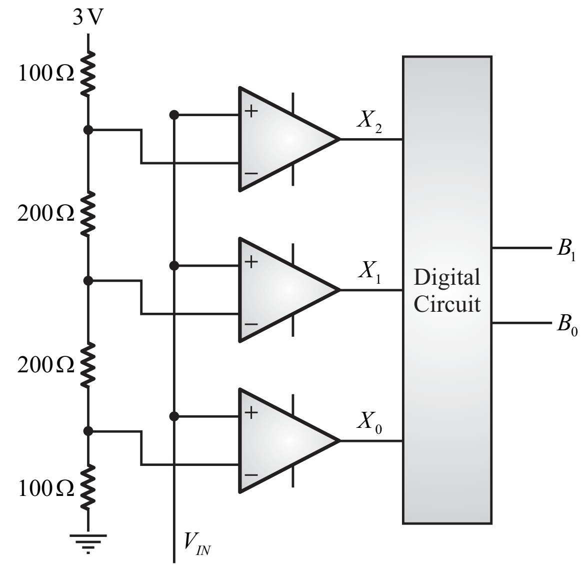
Hence, connections are not proper.

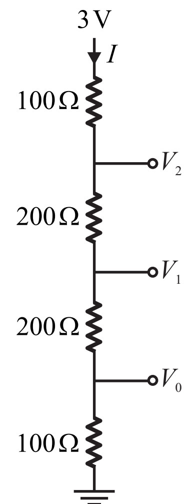
Hence, the correct option is (C).

Scan for Video Solution

6.11 (A)

Given 2-bit flash analog to digital converter is shown below,





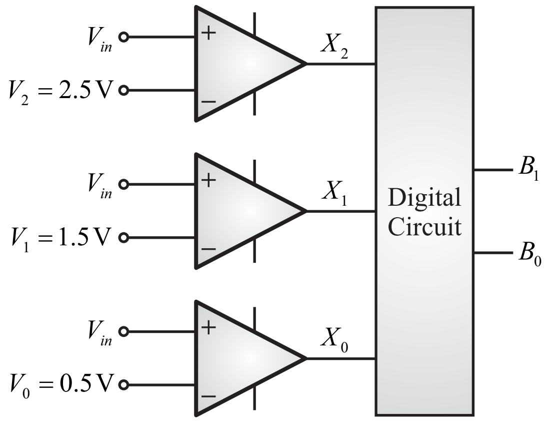
From above figure,

$$I = \frac{3}{100 + 200 + 200 + 100} = 5\,\text{mA}$$

$$V_0 = 100 \times 5 \times 10^{-3} = 0.5\,\text{V}$$

$$V_1 = 300 \times 5 \times 10^{-3} = 1.5\,\text{V}$$

$$V_2 = 500 \times 5 \times 10^{-3} = 2.5\,\text{V}$$



Given : The input voltage range is, $0 \le V_{IN} \le 3$

V_{in} (Volt)	X_2	X_1	X_0	B_1	B_0
$0 \le V_{in} \le 0.5$	0	0	0	0	0
$0.5 < V_{in} \le 1.5$	0	0	1	0	1
$1.5 < V_{in} \le 2.5$	0	1	1	1	0
$2.5 < V_{in} \le 3$	1	1	1	1	1

The expression of B_0 in terms of X_2, X_1 and X_0 is,

$$B_0 = \bar{X}_2\bar{X}_1X_0 + X_2X_1X_0$$
$$= (\bar{X}_2\bar{X}_1 + X_2X_1)X_0$$
$$= (X_2 \odot X_1)X_0$$
$$B_0 = X_0\left[\overline{X_2 \oplus X_1}\right]$$

We can also calculate, expression of B_1 in terms of X_2, X_1 and X_0 is,

$$B_1 = \bar{X}_2X_1X_0 + X_2X_1X_0$$
$$B_1 = X_1X_0(\bar{X}_2 + X_2) \quad [\because\ \bar{X}_2 + X_2 = 1]$$
$$B_1 = X_1X_0$$

Hence, the correct option is (A).

Scan for Video Solution

Key Note

- The above given solution, is not in the minimized form, this above solution is solved according to options.
- The proper solution of this questions as follows, here we used only 4-input combination of X_2, X_1, X_0 as 000, 001, 011 and 111 and corresponding output (B_1, B_0) are 00, 01, 10 and 11 respectively. The remaining input combination are 010, 100, 101, 110 work as don't care terms for their output. Thus the proper truth table given as,

X_2	X_1	X_0	B_1	B_0
0	0	0	0	0
0	0	1	0	1
0	1	0	X	X
0	1	1	1	0
1	0	0	X	X
1	0	1	X	X
1	1	0	X	X
1	1	1	1	1

From the above truth table, we can represent B_1 and B_0 in SOP form as,

$$B_1 = \Sigma m(3,\ 7) + d\Sigma m(2,\ 4,\ 5,\ 6)$$
$$B_0 = \Sigma m(1,\ 7) + d\Sigma m(2,\ 4,\ 5,\ 6)$$

K-map for B_1 and B_0 as,

B_1: X_2 \ X_1X_0	00	01	11	10
0			1	X
1	X	X	X	X

Hence, $B_1 = X_1$

B_0: X_2 \ X_1X_0	00	01	11	10
0		1		X
1	X	X	1	X

Hence, $B_0 = X_2 + \bar{X}_1X_0$

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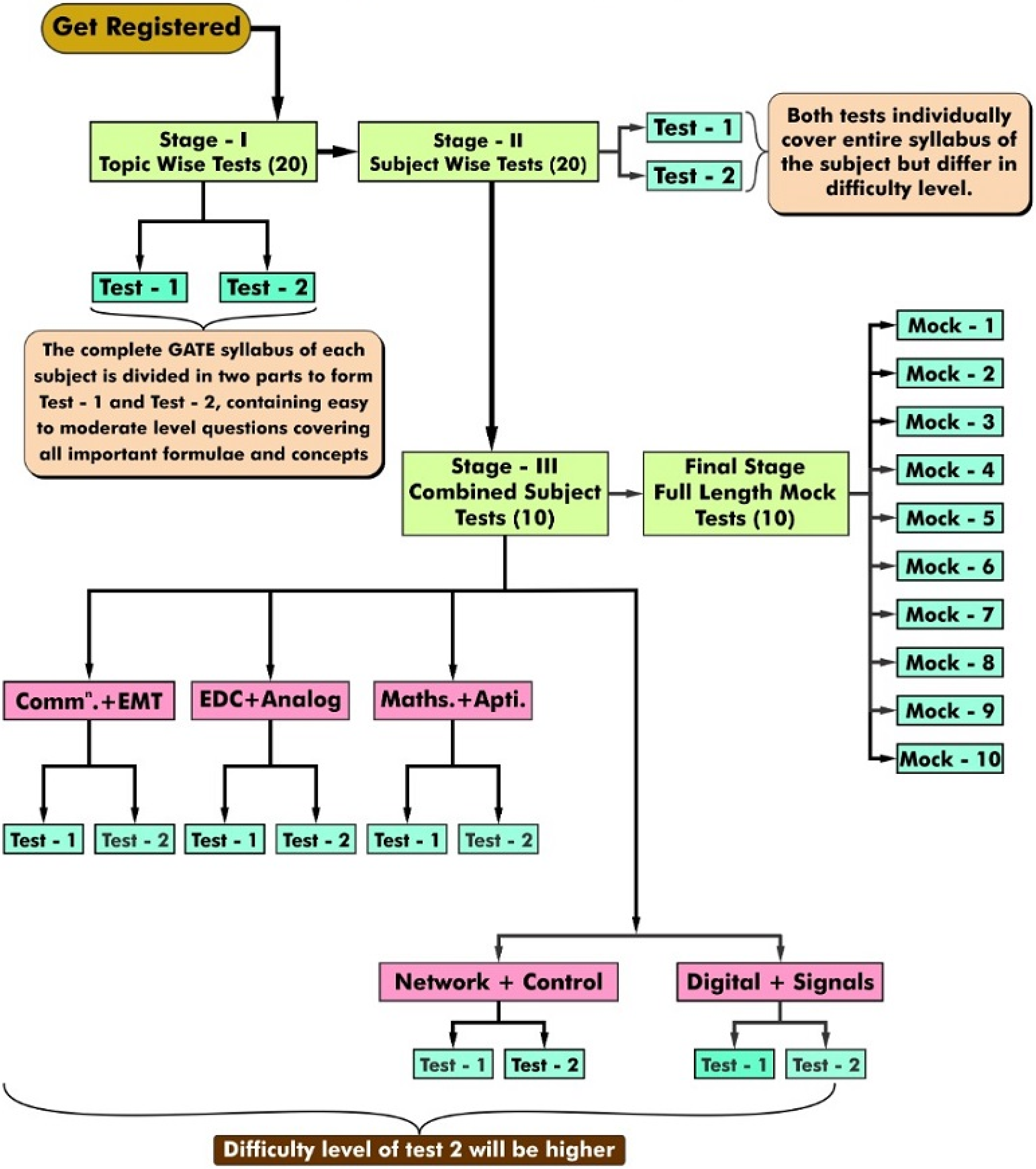

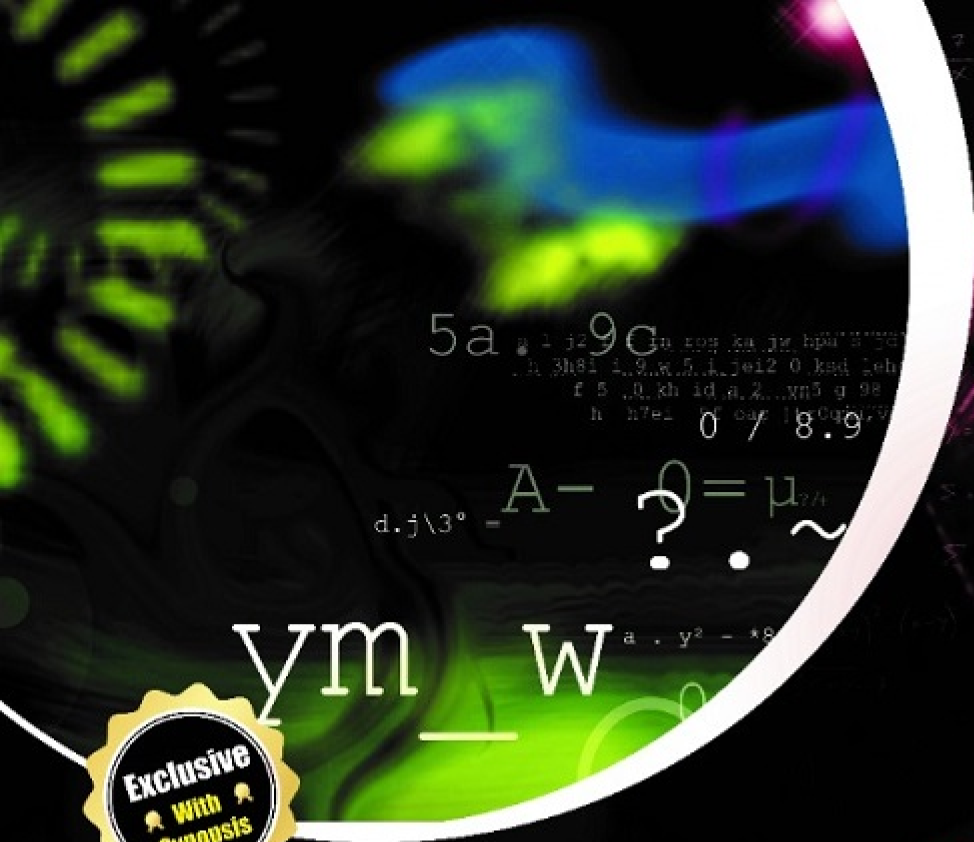

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CHAPTER 4 ANALOG ELECTRONICS

Marks Distribution of Analog Electronics in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	3	4	11
2004	3	6	15
2005	2	6	14
2006	2	3	8
2007	3	3	9
2008	1	6	13
2009	2	3	8
2010	3	1	5
2011	2	2	6
2012	2	2	6
2013	2	3	8
2014 Set-1	2	2	6
2014 Set-2	2	2	6

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-3	1	1	3
2015 Set-1	3	1	5
2015 Set-2	4	1	6
2016 Set-1	1	–	1
2016 Set-2	1	1	3
2017 Set-1	0	2	4
2017 Set-2	0	2	4
2018	1	1	3
2019	2	2	6
2020	2	2	6
2021	2	3	8

Syllabus : Analog Electronics

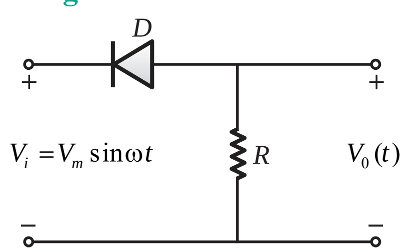
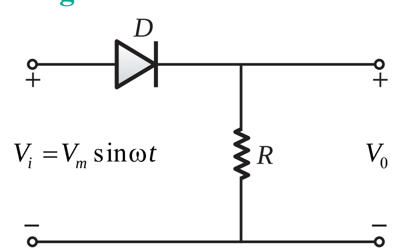
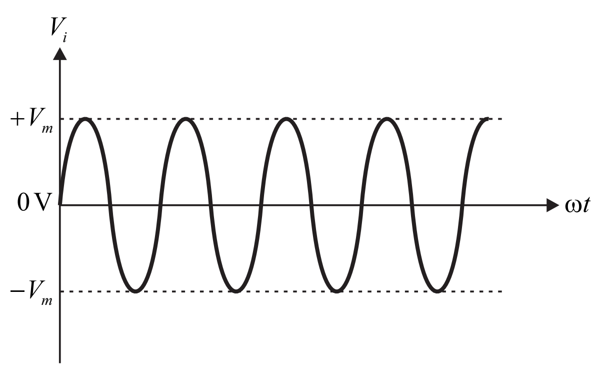
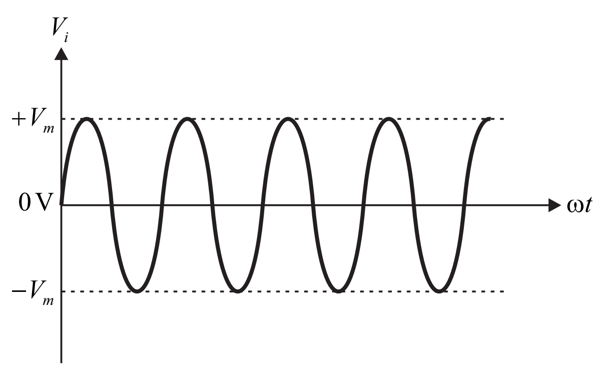
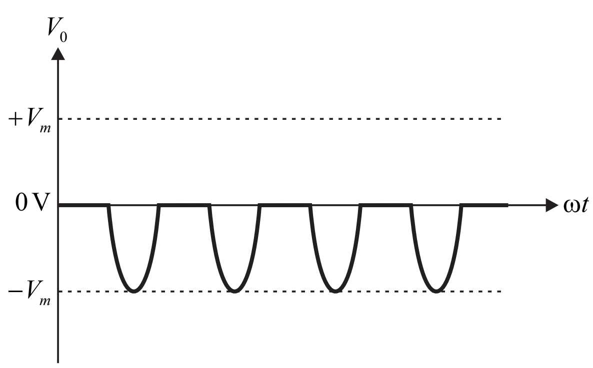
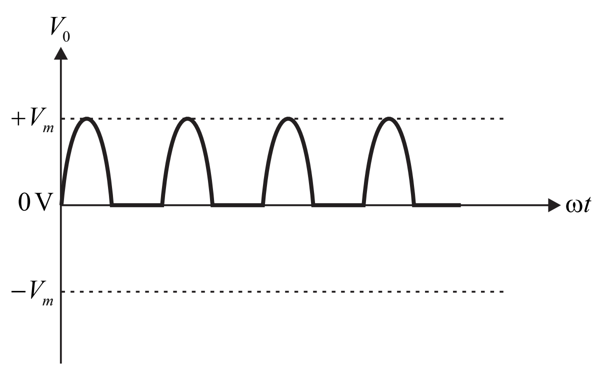
Simple diode circuits: clipping, clamping, rectifiers; Amplifiers: biasing, equivalent circuit and frequency response; oscillators and feedback amplifiers; operational amplifiers: characteristics and applications; single stage active filters, Active Filters: Sallen Key, Butterwoth, VCOs and timers.

Contents : Analog Electronics

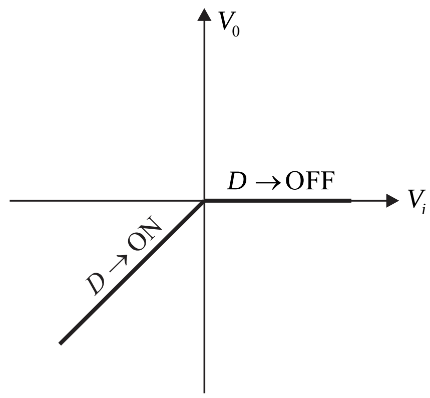
1 Diode Circuits & Applications

➤ Partial Synopsis

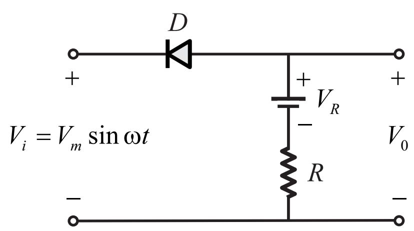
Series Clipper Circuits

Positive series clipper circuit	Negative series clipper circuit
Circuit Diagram : 	**Circuit Diagram :** 
Input waveform : 	**Input waveform :** 
Output waveform : 	**Output waveform :** 

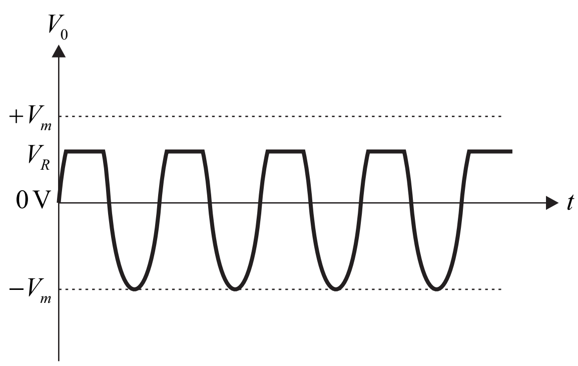
Transfer characteristics :



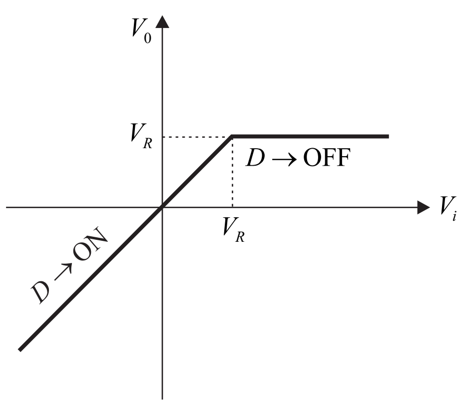
Positive series clipper circuit (with reference voltage, (V_R)) :



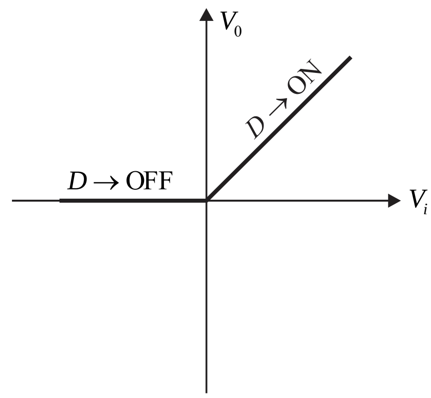
Output waveform :



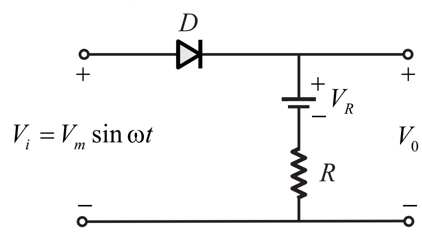
Transfer characteristics :



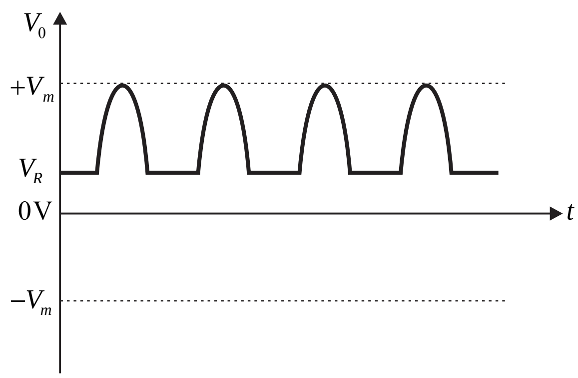
Transfer characteristics :



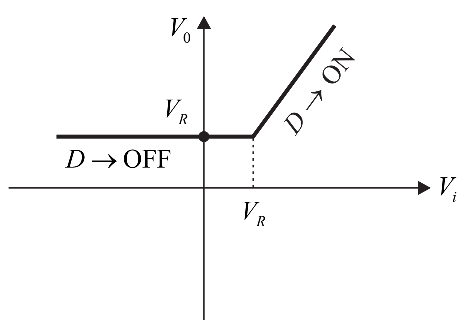
Negative Series Clipper Circuit (with reference voltage) :



Output waveform :



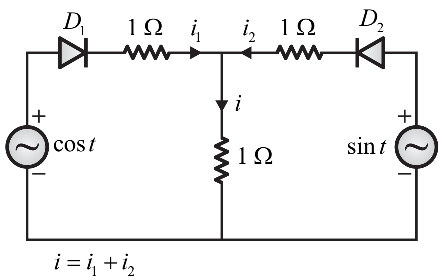
Transfer characteristics :

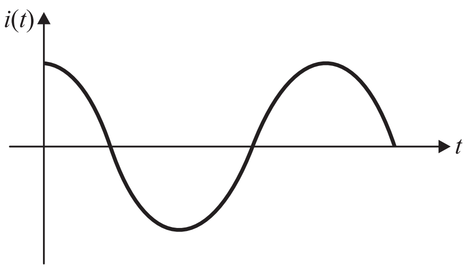


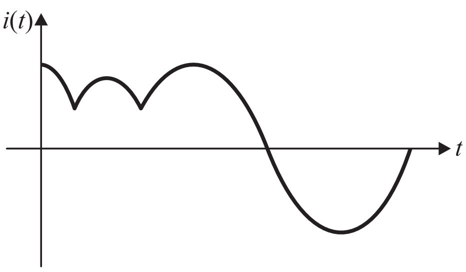
Sample Questions

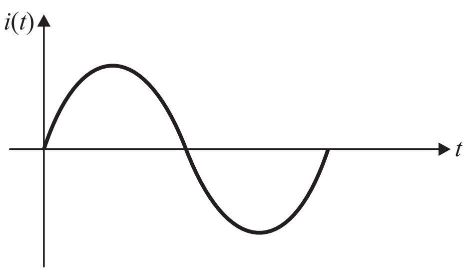
1992 IIT Delhi

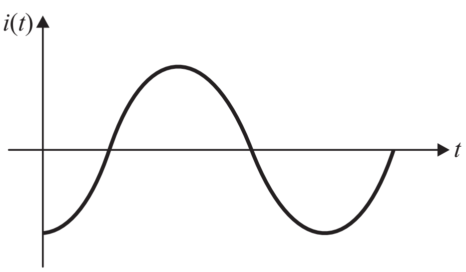
1.1 In the circuit shown in figure the wave form of the current 'i' over one period of the input voltages is (Assume the diode to be ideal).



(A)


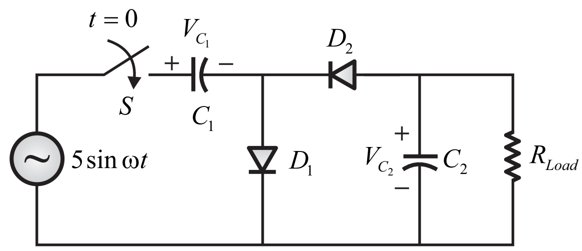
(B)


(C)


(D)


2008 IISc Bangalore

1.2 In the voltage double circuit shown in the figure, the switch 'S' is closed at $t = 0$. Assuming diodes D_1 and D_2 to be ideal, load resistance to be infinite and initial capacitor voltages to be zero, the steady state voltage across capacitors C_1 and C_2 will be



(A) $V_{C_1} = 10$ V, $V_{C_2} = 5$ V

(B) $V_{C_1} = 10$ V, $V_{C_2} = -5$ V

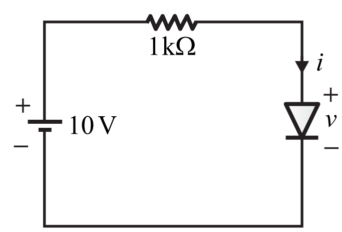
(C) $V_{C_1} = 5$ V, $V_{C_2} = 10$ V

(D) $V_{C_1} = 5$ V, $V_{C_2} = -10$ V

2012 IIT Delhi

1.3 The i-v characteristics of the diode in the circuit given below are

$$i = \begin{cases} 0 & v < 0.7 \text{ V} \\ \dfrac{v - 0.7}{500}\text{ A} & v \geq 0.7 \text{ V} \end{cases}$$



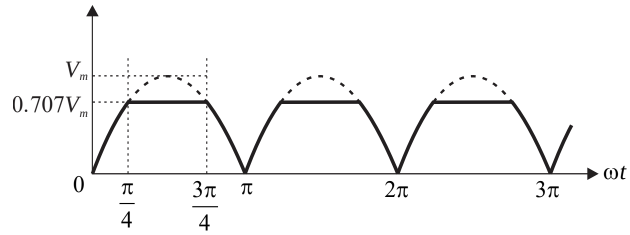
The current in the circuit is

(A) 10 mA (B) 9.3 mA

(B) 6.67 mA (D) 6.2 mA

2021 IIT Bombay

1.4 The waveform shown in solid line is obtained by clipping a full-wave rectified sinusoid (shown dashed). The ratio of the rms value of the full-wave rectified waveform to the rms value of the clipped waveform is ________. (Round off to 2 decimal places,)



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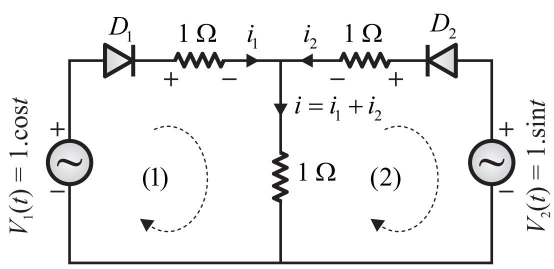
Explanations — Diode Circuits & Applications

1.1 (B)

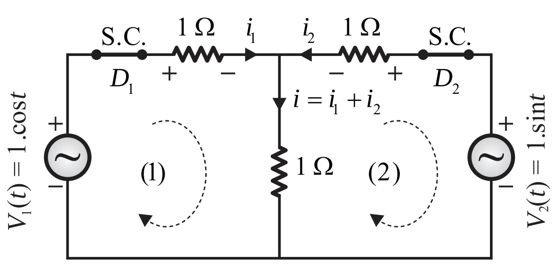
Given circuit with two voltage sources having same frequency is shown below,



Frequency of $V_1(t)$ and $V_2(t)$ is $\omega = 1$ rad/sec

i.e. Time period $T = \frac{2\pi}{\omega} = 6.28$ sec

Assuming both diodes D_1 and D_2 are ON and diode D_1 having current i_1 and diode D_2 having current i_2 as shown below,



Applying KVL in loop (1),

$$-V_1(t) + 1 \times i_1 + 1 \times (i_1 + i_2) = 0$$

$$2i_1 + i_2 = V_1(t) \qquad \dots \text{(i)}$$

Applying KVL in loop (2),

$$V_2(t) - 1 \times i_2 - 1 \times (i_1 + i_2) = 0$$

$$i_1 + 2i_2 = V_2(t) \qquad \dots \text{(ii)}$$

By solving equation (i) and (ii),

$$i_1 = \frac{2V_1(t) - V_2(t)}{3} \text{ and } i_2 = \frac{2V_2(t) - V_1(t)}{3}$$

So, $i_1 = \dfrac{2\cos t - \sin t}{3}$, $i_2 = \dfrac{2\sin t - \cos t}{3}$

(i) For diode D_1 to be ON, i_1 should be greater than zero.

$$i_1 = \frac{2\cos t - \sin t}{3} > 0$$

$$\tan t < 2$$

$$t < 1.11 \text{ sec}$$

For $0 < t < 1.11$ sec, D_1 is ON.

(ii) For diode D_2 to be ON, i_2 should be greater than zero.

$$i_2 = \frac{2\sin t - \cos t}{3} > 0$$

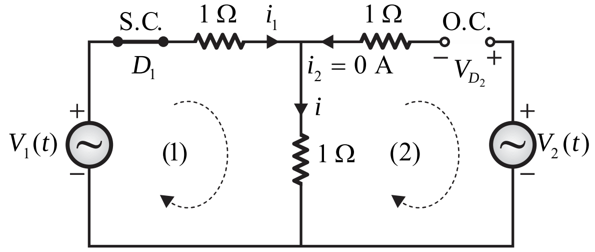
$$\tan t > \frac{1}{2}$$

$$t > 0.46 \text{ sec}$$

For $t > 0.46$ sec, D_2 is ON.

For $0 < t < 0.46$ sec :

D_1 is ON and D_2 is OFF. Hence, modified circuit is shown below,

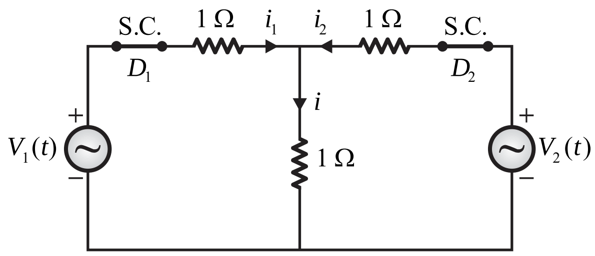


Applying KVL in loop (1),

$$i = i_1 = \frac{V_1(t)}{2} = \frac{\cos t}{2}$$

For 0.46 sec $< t < 1.11$ sec :

Both diodes D_1 and D_2 are ON.



From figure,

$$i = i_1 + i_2$$

$$i_1 = \frac{2V_1(t) - V_2(t)}{3} > 0\,,\ i_2 = \frac{2V_2(t) - V_1(t)}{3} > 0$$

$$i = i_1 + i_2 = \frac{V_1(t) + V_2(t)}{3}$$

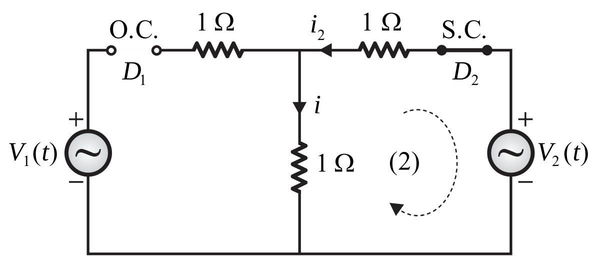
$$i = \frac{\sin t + \cos t}{3} = \frac{1 \times \sin t + 1 \times \cos t}{3}$$

$$i = \frac{\sqrt{2}}{3}\left[\sin t \cos 45^0 + \cos t \sin 45^0\right]$$

$$i = \frac{\sqrt{2}}{3}\sin(t + 45^0) = \frac{\sqrt{2}}{3}\sin\left(t + \frac{\pi}{4}\right)$$

For $1.11 < t < 6.28$ sec :

For this range, only D_2 is ON.



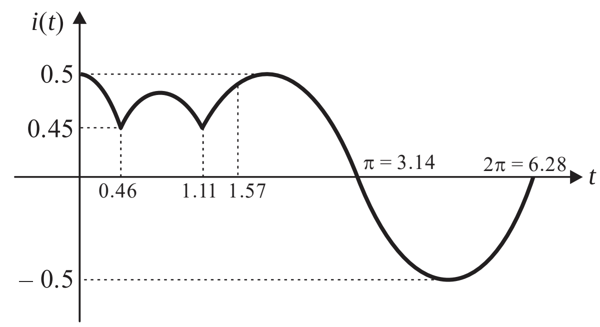
Applying KVL in loop (2),

$$i = \frac{V_2(t)}{1+1} = \frac{\sin t}{2}$$

For one cycle, $0 < t < T$ $(T = 6.28 \text{ sec})$

$$i = \begin{cases} \frac{1}{2}\cos t & ;\ 0 < t < 0.46 \text{ sec} \\ \frac{\sqrt{2}}{3}\sin\left(t + \frac{\pi}{4}\right) & ;\ 0.46 < t < 1.11 \text{sec} \\ \frac{1}{2}\sin t & ;\ 1.11 < t < 6.28 \text{ sec} \end{cases}$$

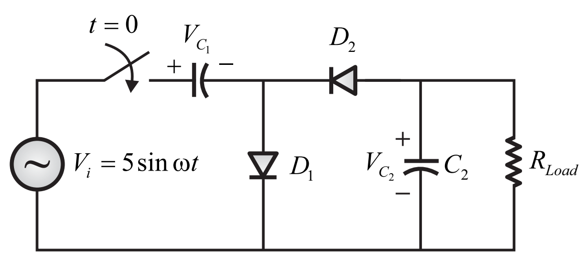
The waveform of $i(t)$ is shown below,



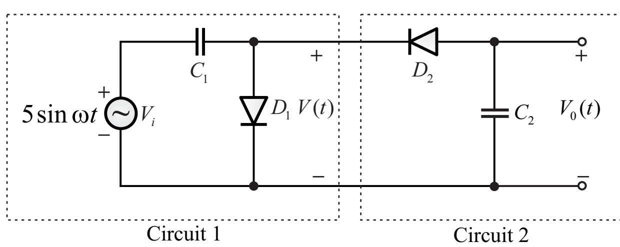
Hence, the correct option is (B).

1.2 **(D)**

Given circuit with diode and capacitor is shown below,



The given circuit can be redrawn as given below,



In above figure, circuit (1) represents **ideal negative clamper circuit** and circuit (2) represents **ideal negative peak detector circuit**. Hence, $V(t)$ is the output of negative clamper circuit and therefore, it will be shifted form of input in downward direction.

$$V(t) = V_i - V_m = 5\sin\omega t - 5$$

In negative clamper circuit, capacitor is charged upto maximum value of input.

Hence, $V_{C_1} = V_m = 5$ V

Output of negative peak detector circuit will be maximum negative value of $V(t)$.

$$V_0(t) = V(t)\big|_{\max} = -5-5 = -10 \text{ V}$$

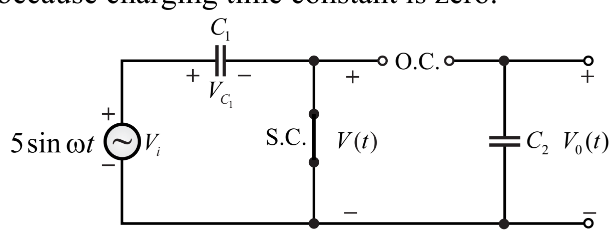
Hence, the correct option is (D).

Details analysis :

Case 1 : During 1st positive half cycle,

Diode D_1 will be forward biased and diode D_2 will be reverse biased.

Hence, charging of capacitor C_1 will start and it will charge upto maximum value of input because charging time constant is zero.

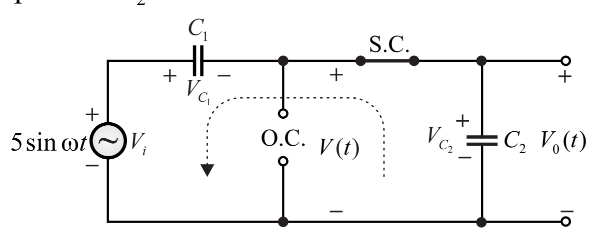


Hence, $V_{C_1} = |5\sin\omega t|_{\max} = 5$ V

Case 2 : During 1st negative half cycle,

Diode D_1 will be reverse biased and diode D_2 will be forward biased.

Hence, charging of capacitor C_2 will start and capacitor C_2 will charge upto maximum negative value of input, which appears across capacitor C_2.



Applying KVL in the loop shown,

$$V_i + 0 - V_{C_2} - V_{C_1} = 0$$

$$V_{C_2} = V_i - V_{C_1}$$

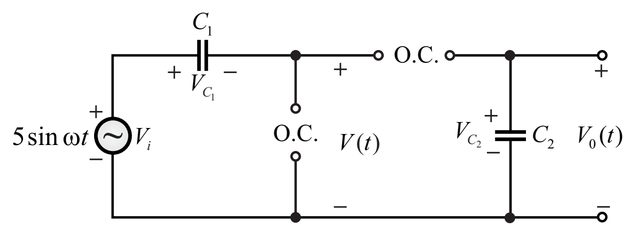
$$V_{C_2} = |5\sin\omega t|_{\min} - 5$$

$$V_{C_2} = -5-5 = -10 \text{ V}$$

Case 3 : During 2nd positive half cycle,

Positive terminal of D_1 is at -5 V and positive terminal of D_2 is at -10 V. Now, diode D_1 and D_2 will be forward biased only when voltage at their negative terminal will be less than that of the positive terminal, which is never possible beyond first negative half cycle and first positive half cycle.

Hence, D_1 and D_2 will always be reverse biased.



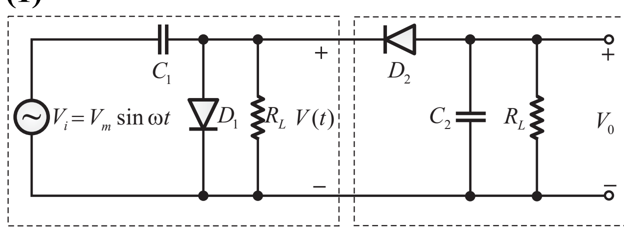
From above circuit,

$$V_0(t) = V_{C_2} = -10 \text{ V}$$

Hence, the correct option is (D).

Key Point

(1)



Circuit 1 **Circuit 2**

Circuit 1 : It is a ideal negative clamper circuit. Output of this circuit will be shifted form of input in downward direction. Hence, output $V(t)$ of this circuit is given by,

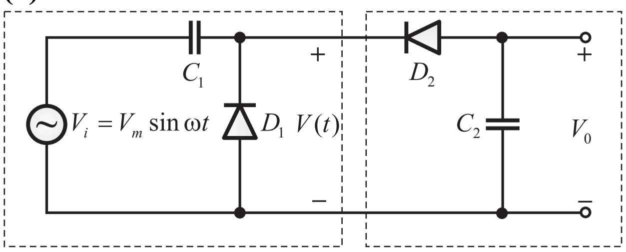
$$V(t) = V_i - V_m = V_m\sin\omega t - V_m$$

Circuit 2 : It is a ideal negative peak detector circuit.

Output of this circuit will be maximum negative value of $V(t)$. Hence, output $V_0(t)$ of this circuit is given by,

$$V_0(t) = |V_m \sin \omega t|_{\min} - V_m = -V_m - V_m = -2V_m$$

(2)



Circuit 1 **Circuit 2**

Circuit 1 : It is a ideal positive clamper circuit.

Output of this circuit will be shifted form of input in upward direction. Hence, output $V(t)$ of this circuit is given by,

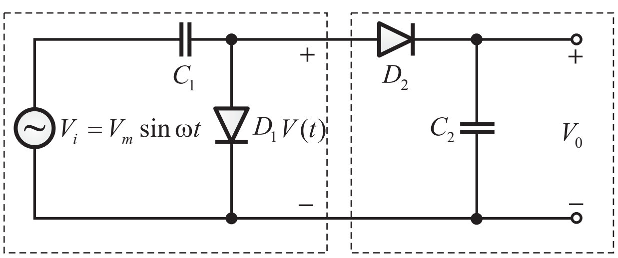
$$V(t) = V_i + V_m = V_m \sin \omega t + V_m$$

Circuit 2 : It is a ideal negative peak detector circuit.

Output of this circuit will be maximum negative value of $V(t)$. Hence, output $V_0(t)$ of this circuit is given by,

$$V_0(t) = |V_m \sin \omega t|_{\min} + V_m = -V_m + V_m = 0\,\text{V}$$

(3)



Circuit 1 **Circuit 2**

Circuit 1 : It is a ideal negative clamper circuit.

Output of this circuit will be shifted form of input in downward direction. Hence, output $V(t)$ of this circuit is given by,

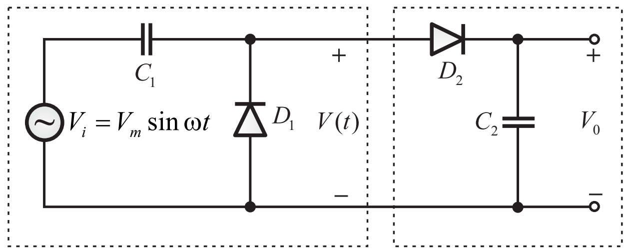
$$V(t) = V_i - V_m = V_m \sin \omega t - V_m$$

Circuit 2 : It is a ideal positive peak detector circuit.

Output of this circuit will be maximum positive value of $V(t)$. Hence, output $V_0(t)$ of this circuit is given by,

$$V_0(t) = |V_m \sin \omega t|_{\max} - V_m = V_m - V_m = 0\,\text{V}$$

(4)



Circuit 1 **Circuit 2**

Circuit 1 : It is a ideal positive clamper circuit.

Output of this circuit will be shifted form of input in upward direction. Hence, output $V(t)$ of this circuit is given by,

$$V(t) = V_i + V_m = V_m \sin \omega t + V_m$$

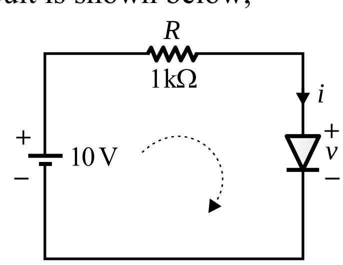
Circuit 2 : It is a ideal positive peak detector circuit.

Output of this circuit will be maximum positive value of $V(t)$. Hence, output $V_0(t)$ of this circuit is given by,

$$V_0(t) = |V_m \sin \omega t|_{\max} + V_m = V_m + V_m = 2V_m$$

1.3 **(D)**

Given circuit is shown below,



The *i*-*v* characteristics is given as,

$$i = \begin{cases} \dfrac{v - 0.7}{500}\,\text{A}; & v > 0.7\,\text{V} \\ 0; & v < 0.7\,\text{V} \end{cases}$$

Method 1

Assuming diode is ON, (i.e. $v > 0.7\text{ V}$)

Applying KVL in the loop shown,

$$-10 + iR + v = 0 \qquad \ldots\text{(i)}$$

Put, $i = \dfrac{v-0.7}{500}, R = 1\text{k}\Omega$ [Given]

So, equation is becomes as,

$$10 = \frac{(v-0.7)}{500} \times 1000 + v$$

$$10 = 2v - 1.4 + v$$

$$3v = 11.4$$

$$v = 3.8\text{ V}$$

Here, we get $v = 3.8\text{ V}$ which is greater than 0.7V. It means our assumption is correct. So, current i,

$$i = \frac{v-0.7}{500}\text{A}$$

$$i = \frac{3.8-0.7}{500}\text{A} \qquad [\text{Put } v = 3.8\text{ V}]$$

$$i = 0.0062\text{ A}$$

$$i = 6.2\text{ mA}$$

Hence, the correct option is (D).

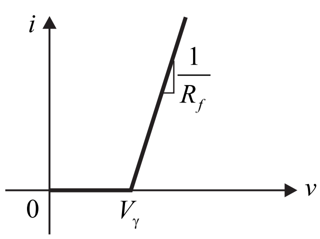
Method 2

From given i-v characteristics,

$$i = \frac{v-0.7}{500} \text{ for } v > 0.7\text{ V} \qquad \ldots\text{(i)}$$

i-v characteristics of forward bias practical diode is given by,

$$i = \frac{v - V_\gamma}{R_f} \qquad \ldots\text{(ii)}$$



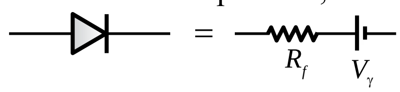
R_f = forward resistance of diode

V_γ = cut-in voltage (offset voltage)

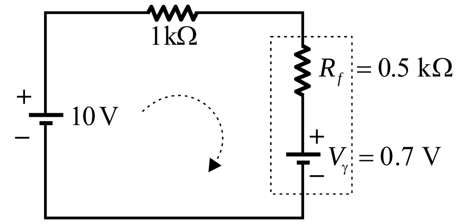
From equation (i) and (ii), we get

$$R_f = 500\ \Omega = 0.5\text{ k}\Omega,\ V_\gamma = 0.7\text{ V}$$

So, given diode can be replace as,



The given circuit is replaced by its equivalent as shown below,



Appling KVL in the above loop,

$$-10 + 1 \times i + 0.5 \times i + 0.7 = 0$$

$$1.5 \times i = 9.3$$

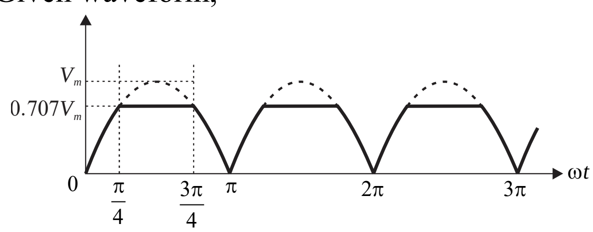
$$i = \frac{9.3}{1.5} = 6.2\text{ mA}$$

Hence, the correct option is (D).

Scan for Video Solution

1.4 1.21

Given waveform,



RMS value of above signal given by,

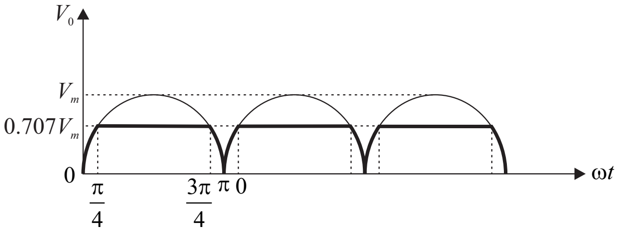
$$V_{rms} = \left[\frac{1}{\pi}\int_0^\pi (\sin\omega t)^2\, d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{\pi}\int_0^\pi \frac{(1-\cos 2\omega t)}{2} d\omega t\right]^{\frac{1}{2}}$$

$$= V_{rms}\left[\frac{1}{2\pi}\left((\pi - 0) - \frac{\sin 2\omega t}{2}\Big|_0^\pi\right)\right]^{\frac{1}{2}}$$

$$= \frac{V_m}{\sqrt{2}} = 0.707V_m$$

RMS value of clipped signal is given by,



$$=\left[\frac{1}{\pi}\int_0^{\frac{\pi}{4}}(V_m\sin\omega t)^2\,d\omega t+\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}(0.707V_m)^2\,d\omega t+\int_{\frac{3\pi}{4}}^{\pi}(V_m\sin\omega t)^2 d\omega t\right]^{\frac{1}{2}}$$

$$=\left\{\frac{1}{\pi}\left[V_m^2\int_0^{\frac{\pi}{4}}\left(\frac{1-\cos 2\omega t}{2}\right)+0.707^2V_m^2\times\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)\right]+V_m^2\int_{\frac{3\pi}{4}}^{\pi}\left(\frac{1-\cos 2\pi\omega t}{2}\right)\right\}^{\frac{1}{2}}$$

$$=\frac{V_m^2}{2\pi}\left[\frac{\pi}{4}-\left(\frac{\sin 2\times\frac{\pi}{4}-\sin 2\times 0}{2}\right)+\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)+\left(\pi-\frac{3\pi}{4}\right)-\sin 2\pi-\sin 2\times\frac{3\pi}{4}\right]^{\frac{1}{2}}$$

$$=\frac{V_m^2}{2\pi}\left[\frac{\pi}{4}-\frac{1}{2}+\frac{\pi}{2}+\frac{\pi}{4}-\frac{1}{2}\right]^{\frac{1}{2}}=V_m\left(\frac{\pi-1}{2\pi}\right)^{\frac{1}{2}}$$

$$=0.5839V_m$$

$$\frac{V_{(rms)}\text{ sinewave}}{V_{(rms)}\text{ clipped wave}}=\frac{0.707V_m}{0.5839V_m}=1.21$$

Hence, the correct answer is 1.21.

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3 BJT & MOSFET Biasing

➢ Partial Synopsis

Procedure to find Region of Operation

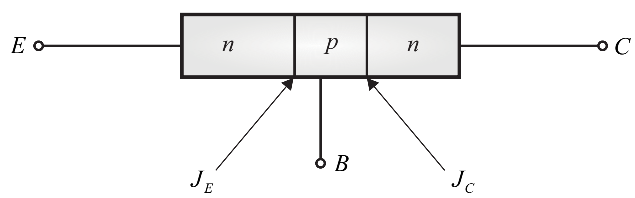


Fig. (a) npn transistor

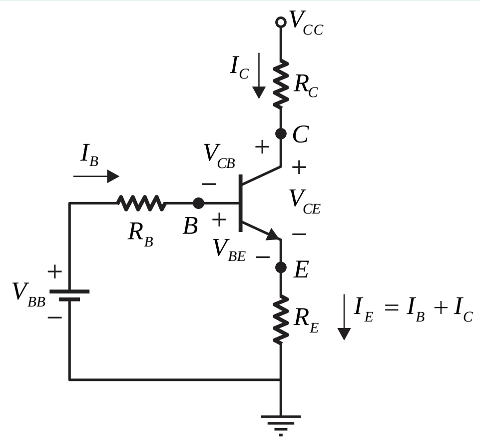


Fig. (b)

1. If J_E and J_C are reverse biased then transistor will be in cut-off region.
2. If J_E is forward biased then transistor can be either in active region or saturation region.
3. Mode of operation and application of transistor based on biasing of the input and output junction as given below,

B-E Junction (J_E)	C-B Junction (J_C)	Region of Operation	Application
FB	RB	Normal Active	Amplifier
FB	FB	Saturation	Switch (ON)
RB	RB	Cut-off	Switch (OFF)
RB	FB	Inverse Active	Attenuator

To Identify the Region of Operation

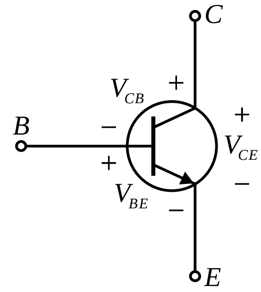
Method 1 :

Assume Transistor is in active region ($I_C = \beta I_B$) :

1. Apply KVL to Base-Emitter (BE) circuit and calculate I_B.
2. Replace $I_C = \beta_{dc} I_B$ if required.

3. If emitter resistor R_E exist then replace emitter current $I_E = I_B + I_C$

$$I_E = (1+\beta_{dc})I_B$$



4. Apply KVL in Collector-Emitter (CE) circuit and calculate V_{CE}.

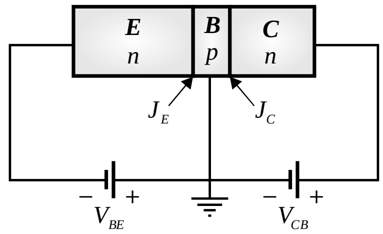
5. Calculate V_{CB},

$$V_{CB} = V_{CE} + V_{EB}$$

$$V_{CB} = V_{CE} - V_{BE}$$

Where $V_{BE} = 0.7$ V

6. If $V_{CB} = +\text{ve}$ for npn transistor and –ve for pnp transistor, then transistor will be in active region otherwise saturation region.



$$\left.\begin{array}{ll} V_{CB} = +\text{ve} & (\text{npn}) \\ V_{CB} = -\text{ve} & (\text{pnp}) \end{array}\right\} \text{Condition for active region}$$

For npn transistor :

$V_{BE} > 0$, i.e. $J_E = \text{FB}$,

V_{CB}	J_C	Region
$+\text{ve}$	RB	Active
$-\text{ve}$	FB	Saturation

$J_C = \text{RB}$,

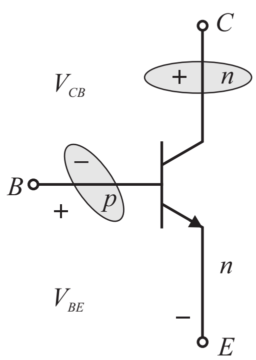


Fig. $V_{CB} = +\text{ve}$ (Active)

$J_C = \text{FB}$,

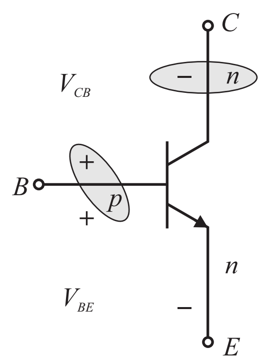


Fig. $V_{CB} = -\text{ve}$ (Saturation)

For pnp transistor :

$V_{BE} < 0$, i.e. $J_E = \text{FB}$,

V_{CB}	J_C	Region
$+\text{ve}$	FB	Saturation
$-\text{ve}$	RB	Active

➢ Sample Questions

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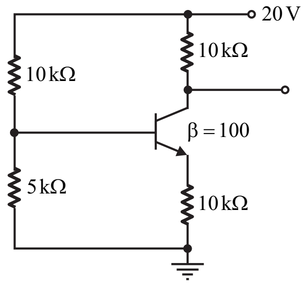
3.1 Figure shown below, shows a common emitter amplifier. The quiescent collector voltage of the circuit is approximately



(A) 20/3 V (B) 10 V

(C) 14 V (D) 20 V

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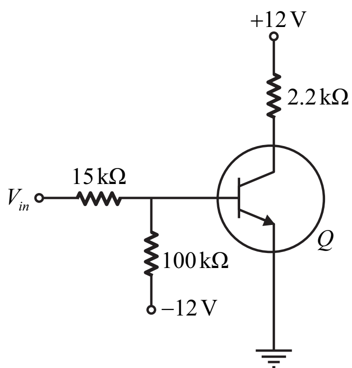
3.2 Consider the circuit shown in figure. If the β of the transistor is 30 and I_{CBO} is 20 nA and the input voltage is + 5 V, the transistor would be operating in



(A) Saturation region

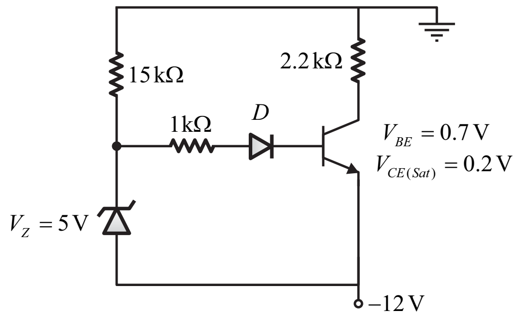
(B) Active region

(C) Breakdown region

(D) Cut-off region

2011 IIT Madras

3.3 The transistor used in the circuit shown below, has a $\beta = 30$ and I_{CBO} is negligible.



If the forward voltage drop of diode is 0.7 V, then the current through collector will be

(A) 168 mA (B) 108 mA

(C) 20.54 mA (D) 5.36 mA

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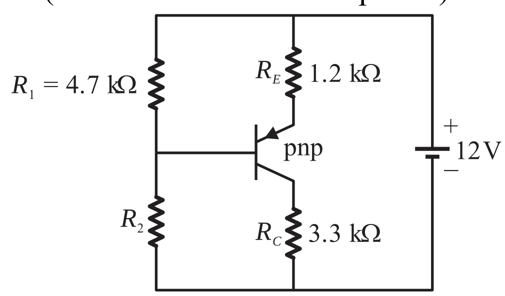
3.4 In the BJT circuit shown, beta of the PNP transistor is 100. Assume $V_{BE} = -0.7\text{ V}$. The voltage across R_C will be 5 V when R_2 is ______ kΩ. (Round off to 2 decimal places)



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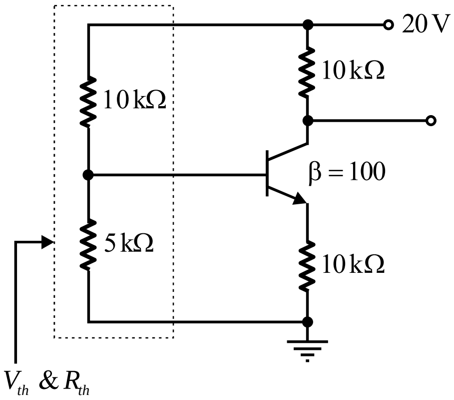
Explanations BJT & MOSFET Biasing

3.1 (C)

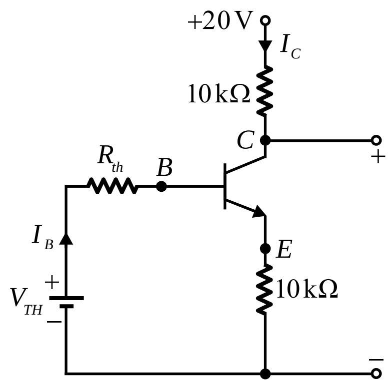
Given :

(i) Common emitter amplifier

(ii) $\beta = 100$



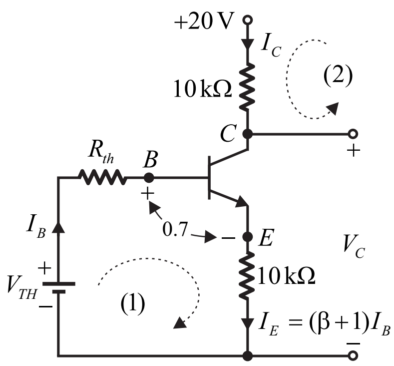
Thevenin equivalent circuit of given self-bias circuit : Left side the above circuit (shown by dotted line) is converted into Thevenin equivalent circuit as shown below,



$$R_{TH} = (5 \| 10) = \frac{5\times 10}{5+10} = \frac{10}{3}\text{ k}\Omega$$

$$V_{TH} = 20\times\frac{5}{5+10} = \frac{20}{3}\text{ V} \quad \text{[By VDR]}$$

So, given self-bias circuit becomes as,



Applying KVL in loop (1),

$$-V_{TH} + R_{TH}\times I_B + 0.7 + 10 I_E = 0$$

$$-\frac{20}{3} + \frac{10}{3}\times I_B + 0.7 + (\beta+1)I_B\times 10 = 0$$

$$I_B = \frac{\frac{20}{3}-0.7}{\frac{10}{3}+(\beta+1)\times 10} = \frac{\frac{20}{3}-0.7}{\frac{10}{3}+101\times 10}$$

$$I_B = 5.84\times 10^{-3}\text{ mA}$$

So, base current I_B is 5.84×10^{-3} mA

Collector current I_C is given by,

$$I_C = \beta I_B$$

$$I_C = 100\times 5.84\times 10^{-3} = 0.584\text{ mA}$$

Applying KVL in loop (2), for calculating collector voltage V_C,

$$-20 + 10 I_C + V_C = 0$$

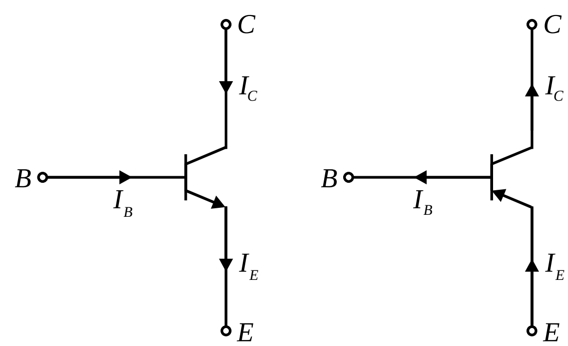
$$V_C = 20 - I_C\times 10 = 20 - 0.584\times 10$$

$$V_C = 14.1\text{ V} \approx 14\text{ V}$$

Hence, the correct option is (C).

Key Point

(i) Symbol of BJT,



(N-P-N) type BJT (P-N-P) type BJT

(ii) For BJT,

$$I_E = I_B + I_C$$

$$I_C = \beta I_B$$

$$I_E = (\beta+1) I_B$$

$$\beta = \frac{\alpha}{1-\alpha}$$

$$\alpha = \frac{\beta}{\beta+1}$$

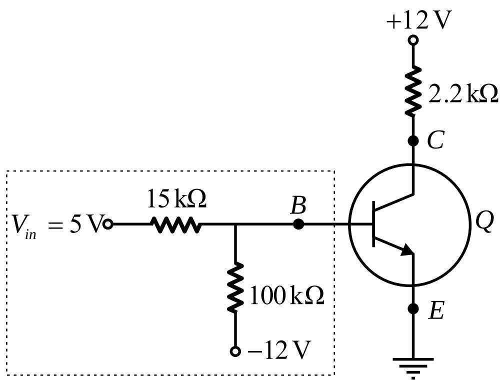
3.2 (B)

Given :

(i) DC current gain, $\beta = 30$

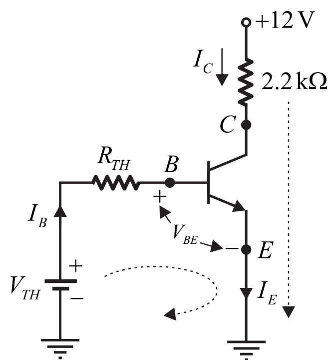
(ii) Input voltage, $V_{in} = 5\text{V}$

(iii) Collector to base reverse saturation current, $I_{CBO} = 20\text{ nA}$



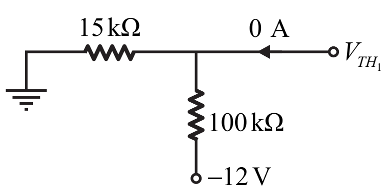
Replace dotted box shown above by Thevenin equivalent circuit as shown below

Thevenin equivalent circuit :



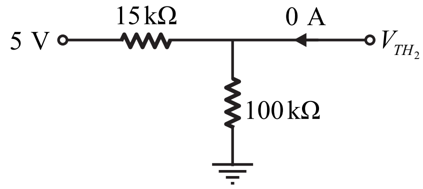
Calculation of V_{TH} using superposition theorem :

(i) Consider – 12 V only,



$$V_{TH_1} = \frac{-12 \times 15}{15 + 100} \text{ V}$$

(ii) Consider 5 V only,



$$V_{TH_2} = \frac{5 \times 100}{100 + 15} \text{ V}$$

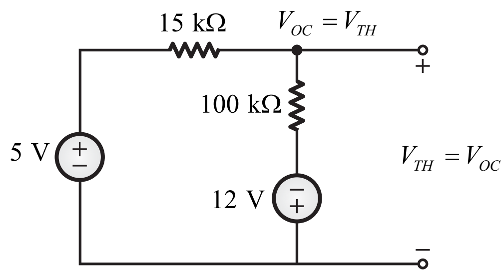
Hence, $V_{TH} = V_{TH_1} + V_{TH_2}$

$$V_{TH} = \frac{5 \times 100}{100 + 15} - \frac{12 \times 15}{100 + 15} = 2.78 \text{ V}$$

and $R_{TH} = 15 \| 100 = 13.04 \text{ k}\Omega$

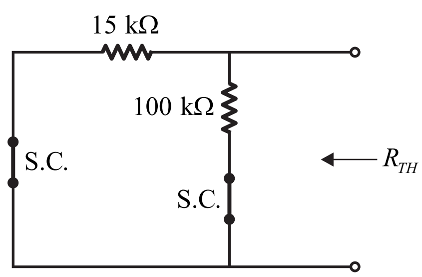
OR

We can also find V_{TH} and R_{TH} as follows



$$\frac{V_{TH} - 5}{15} + \frac{V_{TH} + 12}{100} = 0$$

$$V_{TH} = 2.782 \text{ V}$$



$$R_{TH} = 15 \| 100 = 13.04 \text{ k}\Omega$$

Since, input circuit is forward biased due to V_{TH} ($V_{TH} > 0.7 \text{ V}$), hence BJT will be either in active region or in saturation region.

Method 1

Assuming transistor Q is in active region.

$$V_{BE(\text{active})} = 0.7 \text{ V}$$

Applying KVL in input loop,

$$-V_{TH} + R_{TH} I_B + V_{BE(\text{active})} = 0$$

$$-2.78+13.04\times I_B+0.7=0$$

$$I_B=0.159 \text{ mA}$$

Collector current is given by,

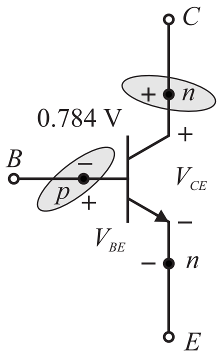
$$I_C=\beta I_B=30\times 0.159=4.78 \text{ mA}$$

Applying KVL in output loop,

$$-12+I_C\times 2.2+V_{CE}=0$$

$$V_{CE}=1.484 \text{ V}$$

$$V_{CB}=V_{CE}-V_{BE}=1.484-0.7=0.784 \text{ V}$$



From above figure, it is clear that, collector side of *n-p-n* transistor is connected to +ve voltage and base side of *n-p-n* transistor is –ve biased, therefore collector to base junction is in reverse bias.

Therefore, the transistor is in active region.

Hence, the correct option is (B).

Method 2

Assuming transistor Q is in saturation region.

$$V_{BE(\text{sat})}=0.8 \text{ V},\ V_{CE(\text{sat})}=0.2 \text{ V}$$

For transistor to be in saturation region,

$$I_{B(\min)}\le I_B \Rightarrow \frac{I_{C(\text{sat})}}{\beta}\le I_B \qquad \dots \text{(i)}$$

Applying KVL in input loop,

$$-V_{TH}+R_{TH}I_B+V_{BE(\text{sat})}=0$$

$$-2.78+13.04\times I_B+0.8=0$$

$$I_B=0.151 \text{ mA} \qquad \dots \text{(ii)}$$

Applying KVL in output loop,

$$-12+I_{C(\text{sat})}\times 2.2+V_{CE(\text{sat})}=0$$

$$I_{C(\text{sat})}=\frac{12-0.2}{2.2}=5.36 \text{ mA}$$

$$I_{B(\min)}=\frac{I_{C(\text{sat})}}{\beta}=\frac{5.36}{30}=0.178 \text{ mA}$$

$$\dots \text{(iii)}$$

From equation (ii) and (iii),

$$I_{B(\min)}>I_B \qquad \dots \text{(iv)}$$

From equation (i) and (iv), the transistor does not satisfy the condition of saturation. Hence, our assumption is wrong.

Therefore, the transistor is in active region.

Hence, the correct option is (B).

Scan for Video Solution

Key Point

1. If J_E = Forward bias,

(i) *n-p-n* transistor

V_{CB}	J_C	Region
+ ve	R.B.	Active
− ve	F.B.	Saturation

(ii) *p-n-p* transistor

V_{CB}	J_C	Region
+ ve	F.B.	Saturation
− ve	R.B.	Active

2. Standard values for BJT :

n-p-n transistor :

Type	$V_{CE(\text{sat})}$	$V_{BE(\text{sat})}$	$V_{BE(\text{active})}$	$V_{BE(\text{cutoff})}$
Si	0.2 V	0.8 V	0.7 V	0.0 V
Ge	0.1 V	0.3 V	0.2 V	−0.1 V

Note : (i) For *p-n-p* transistor, all polarities are reversed.

(ii) If there is no information of type of transistor then we will assume silicon transistor i.e.

$$V_{BE(\text{active})}=0.7 \text{ V}$$

3.3 (D)

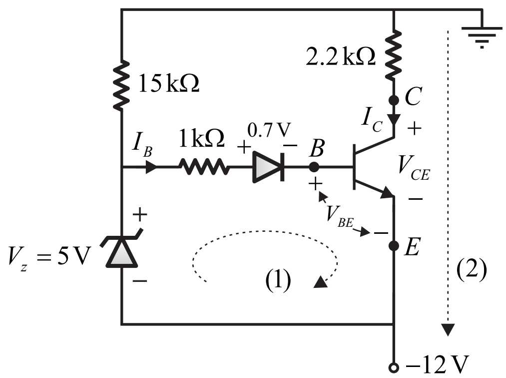
Given :

(i) DC current gain, $\beta = 30$

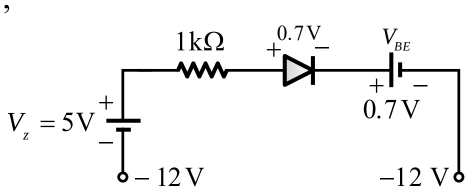
(ii) Collector to base reverse saturation current, $I_{CBO} \approx 0$ A

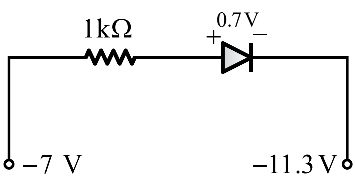
(iii) Diode forward voltage, $V_\gamma = 0.7$ V

(iv) $V_{BE} = 0.7\text{ V}, V_{CE(sat)} = 0.2\text{ V}$



From figure, Zener diode is in reverse breakdown region due to –12 V voltage source. So, Zener diode work as 5 V battery as shown below,





Here, *P*-terminal of diode is at higher potential then N-terminal of diode. Therefore diode and emitter junction J_E ``will be forward biased due to –12 V.

Hence, BJT will be either in active region or in saturation region.

Method 1

Assuming transistor is in active region.

$$V_{BE(\text{active})} = 0.7 \text{ V}$$

Applying KVL in loop (1),

$$-5 + I_B \times 1 + 0.7 + 0.7 = 0$$

$$I_B = 5 - 1.4 = 3.6\text{mA}$$

Collector current is given by,

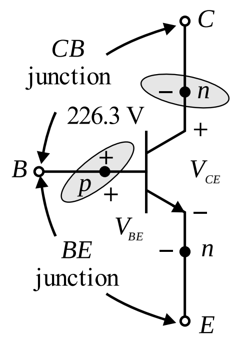
$$I_C = \beta I_B = 30 \times 3.6 = 108 \text{ mA}$$

Applying KVL in loop (2)

$$0 + 2.2\,I_C + V_{CE} - 12 = 0$$

$$V_{CE} = 12 - 2.2 \times 108 = -225.6\text{ V}$$

$$V_{CB} = V_{CE} - V_{BE} = -225.6 - 0.7 = -226.3\text{ V}$$



From above figure, it is clear that collector of given npn BJT is negatively biased and base of given npn BJT is positively biased therefore collector to base junction is in forward bias. But in active result collector to base junction must be in reverse biased. Hence, our assumption is wrong.

Therefore, the transistor is in saturation region.

Applying KVL in loop (2),

$$2.2 I_{C(\text{sat})} + V_{CE(\text{sat})} - 12 = 0$$

Assume $V_{CE(\text{sat})} = 0.2\text{ V}$,

$$2.2\, I_{C(\text{sat})} + 0.2 - 12 = 0$$

$$I_{C(\text{sat})} = 5.36 \text{ mA}$$

Hence, the correct option is (D).

Method 2

Assuming transistor is in saturation region.

$$V_{BE(\text{sat})} = 0.7 \text{ V}, V_{CE(\text{sat})} = 0.2 \text{ V}$$

[Here we are taking $V_{BE(\text{sat})} = 0.7$ V instead of 0.8 as the value for $V_{BE(\text{sat})}$ is not mentioned in the problem, only $V_{BE} = 0.7$ V is given.]

For transistor to be in saturation region,

$$I_{B(\min)} \le I_B \Rightarrow \frac{I_{C(\text{sat})}}{\beta} \le I_B \quad \dots \text{(i)}$$

Applying KVL in loop (1),

$$-5 + I_B \times 1 + 0.7 + 0.7 = 0$$

$$I_B = 5 - 1.4 = 3.6\,\text{mA} \quad \dots \text{(ii)}$$

Applying KVL in loop (2),

$$0 + 2.2\, I_{C(\text{sat})} + V_{CE(\text{sat})} - 12 = 0$$

$$I_{C(\text{sat})} = \frac{12 - V_{CE(\text{sat})}}{2.2} = \frac{12 - 0.2}{2.2} = 5.36 \text{ mA}$$

Hence, $I_{B(\min)} = \frac{I_{C(\text{sat})}}{\beta} = \frac{5.36 \times 10^{-3}}{30} = 1.78 \text{ mA}$

$$\dots \text{(iii)}$$

From equation (ii) and (iii),

$$I_{B(\min)} < I_B \quad \dots \text{(iv)}$$

From equation (i) and (iv), the transistor satisfies condition of saturation. Hence our assumption is correct and transistor is working in saturation region.

Therefore, current through collector is given by,

$$I_{C(\text{sat})} = 5.36 \text{ mA}$$

Hence, the correct option is (D).

Scan for Video Solution

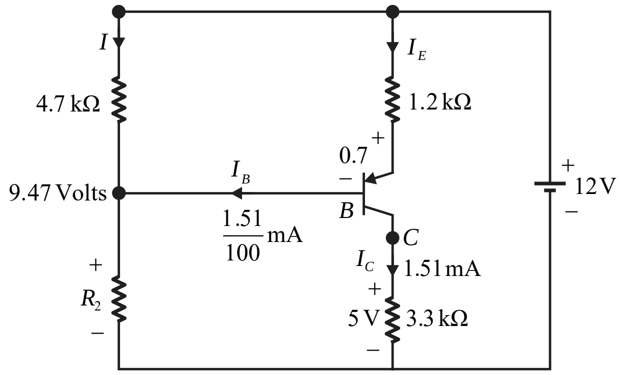
3.4	18.099

Given :

(i) $\beta = 100$

(ii) $V_{BE} = -0.7\,\text{V}$

(iii) $V_s = 12\,\text{V}$



$$I_C = \beta I_B$$

$$1.51 = 100\, I_B$$

$$I_B = 1.51 \times 10^{-2} \text{ mA}$$

$$I_E = (1+\beta) I_B$$

$$I_E = 1.52\,\text{mA}$$

$$12 - 1.2 \times 1.5251 - 0.7 - V_{R_2} = 0$$

$$V_{R_2} = 9.46988$$

$$I = \frac{12 - 9.46988}{4.7} = 0.53\,\text{mA}$$

$$I_{R_2} = 0.53 - \frac{1.51}{100} = 0.52322\,\text{mA}$$

$$R_2 = \frac{V_{R_2}}{I_{R_2}} = \frac{9.46988}{0.52322} = 18.099\,\text{k}\Omega$$

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4 Low Frequency BJT & MOSFET Amplifier

➢ Partial Synopsis

Common Emitter (CE) Amplifier with R_E

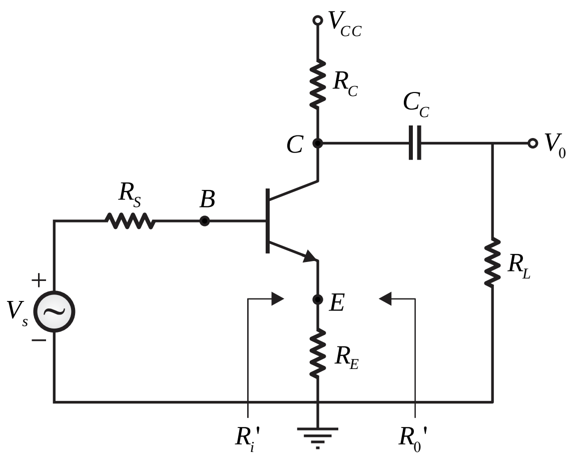

Fig. (a) Common emitter with R_E

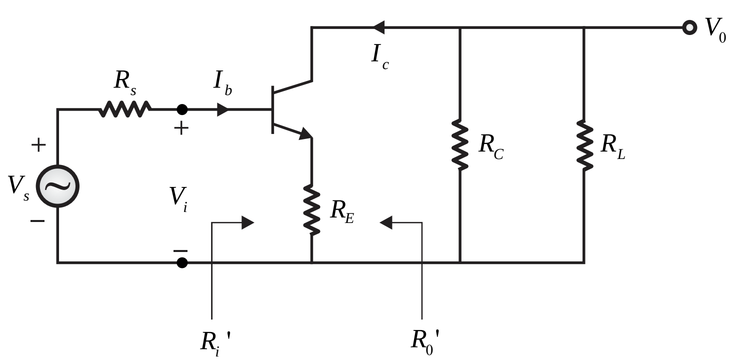

Fig. (b) AC equivalent of Common emitter with R_E

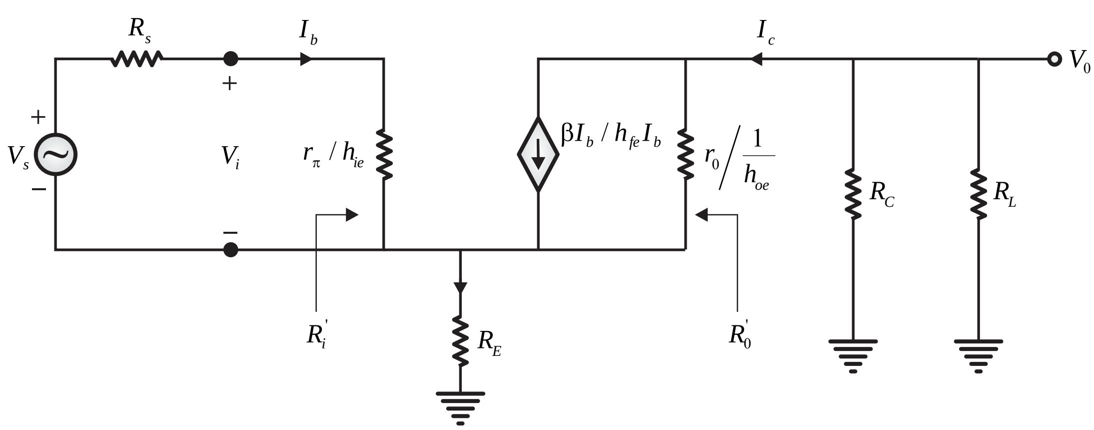

Fig. (c) π-model of common emitter with R_E

Summary of Internal Parameter of CE Amplifier with R_E

Name of Internal parameter	Internal parameter of common emitter with R_E	Approximate hybrid model	r_e-Model
Current gain	$A_I' = \frac{-I_c}{I_b}$	$A_I' = -h_{fe}$	$A_I' = -\beta$
Input resistance	$R_i' = \frac{V_i}{I_b}$	$R_i' = h_{ie} + (1+h_{fe})R_E$	$R_i' = r_\pi + (1+\beta)R_E$
Voltage gain	$A_V' = \frac{V_0}{V_i}$	$A_V' = A_I' \times \frac{R_L'}{R_i'}$ $A_V' \approx \frac{-R_L'}{R_E}$ $R_L' = R_C \parallel R_L$	$A_V' = A_I' \times \frac{R_L'}{R_i'}$ $A_V' = \frac{-\beta R_L'}{r_\pi + (1+\beta)R_E} \approx \frac{-R_L'}{R_E}$ $[(1+\beta)R_E >> r_\pi]$ $R_L' = R_C \parallel R_L$
Output resistance	$R_0' = \left.\frac{V_{dc}}{I_{dc}}\right\|_{V_s=0}$	$R_0' = \infty$	$R_0' = \infty \quad [V_A = \infty]$

Common Collector (CC) Amplifier

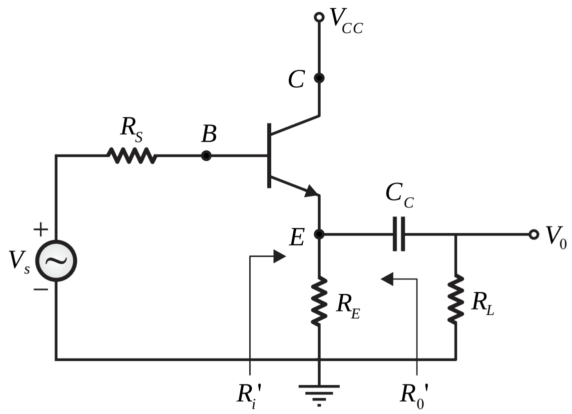

Fig. (a) Common collector amplifier

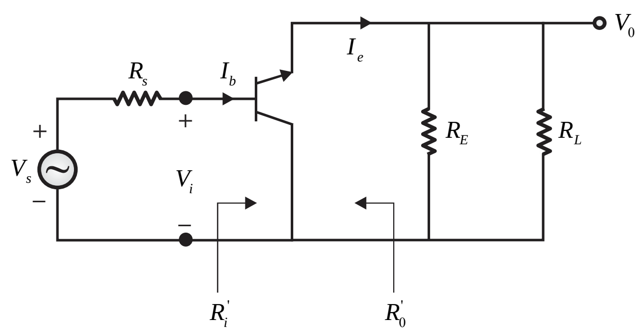

Fig. (b) AC equivalent of Common collector amplifier

Summary of Internal Parameter of CC Amplifier

Name of Internal parameter	Internal parameter of CC Amplifier	Approximate hybrid model	r_e-Model
Current gain	$A_I' = \frac{-I_e}{I_b}$	$A_I' = 1 + h_{fe}$	$A_I' = 1 + \beta$

Input resistance	$R_i' = \frac{V_i}{I_b}$	$R_i' = h_{ie} + (1+h_{fe})R_L'$	$R_i' = r_\pi + (1+\beta)R_L'$
Voltage gain	$A_V' = \frac{V_0}{V_i}$	$A_V' = \frac{A_I' R_L'}{R_i'} \approx 1$ $R_L' = R_E \parallel R_L$	$A_V' = \frac{(1+\beta)R_L'}{r_\pi + (1+\beta)R_L'} \approx 1.0$ $R_L' = R_E \parallel R_L$
Output resistance	$R_0' = \left.\frac{V_{dc}}{I_{dc}}\right\vert_{V_s=0}$	$R_0' = \frac{R_S' + h_{ie}}{1+h_{fe}}$ R_S' = Effective source impedance	$R_0' = \frac{R_S' + r_\pi}{1+\beta}$ If $r_\pi > R_S'$ $R_0' = \frac{r_\pi}{1+\beta} \approx \frac{r_\pi}{\beta}$ $R_0' = \frac{r_\pi}{\beta} = \frac{1}{g_m}$

Sample Questions

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4.1 In an *RC*-Coupled common emitter amplifier, which of the following is true?

(A) Coupling capacitance affects the high frequency (HF) response by pass capacitance affects low frequency (LF) response.

(B) Both coupling and bypass capacitances affect the high frequency (HF) response only.

(C) Both coupling and bypass capacitances affect the low frequency (LF) response only.

(D) Coupling capacitance affects the low frequency (LF) response by pass capacitance affects high frequency (HF) response.

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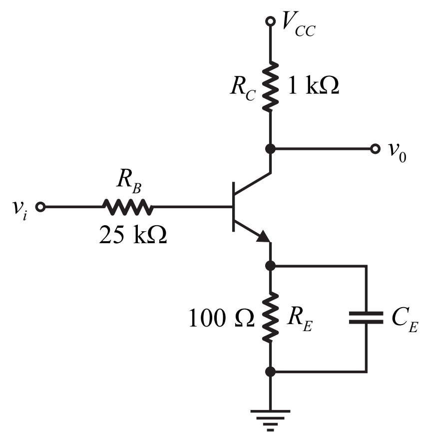
Common Data for Questions 4.2 & 4.3

The transistor in the amplifier circuit shown in figure is biased at $I_C = 1$ mA . Use

$$V_T = \frac{kT}{q} = 26 \text{ mV}, \beta_0 = 200, r_b = 0 \text{ and } r_0 \to \infty$$



4.2 Small-signal mid-band voltage gain v_0 / v_i is

(A) – 8 (B) 38.46

(C) – 6.62 (D) –1

4.3 What is the required value of C_E for the circuit to have a lower cutoff frequency of 10 Hz?

(A) 0.159 mF (B) 1.59 mF

(C) 5 μF (D) 10 μF

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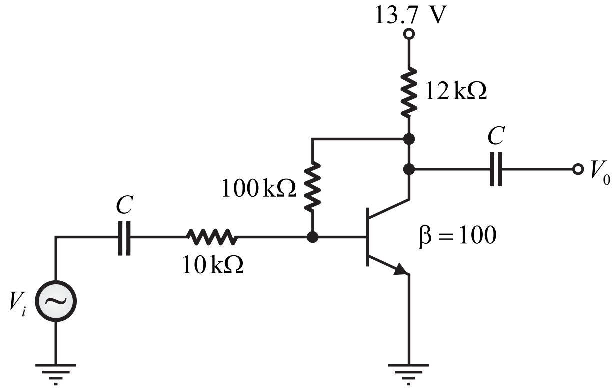
4.4 The voltage gain A_V of the circuit shown below is



(A) $|A_V| \approx 200$ (B) $|A_V| \approx 100$
(C) $|A_V| \approx 20$ (D) $|A_V| \approx 10$

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4.5 A common-source amplifier with a drain resistance, $R_D = 4.7\text{k}\Omega$, is powered using a 10 V power supply. Assuming that the trans-conductance g_m, is $520\,\mu\text{A/V}$, the voltage gain of the amplifier is closest to

(A) 2.44 (B) – 2.44
(C) 1.22 (D) – 1.22

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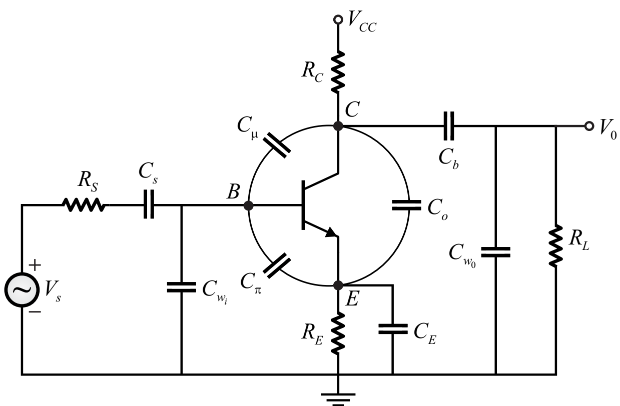
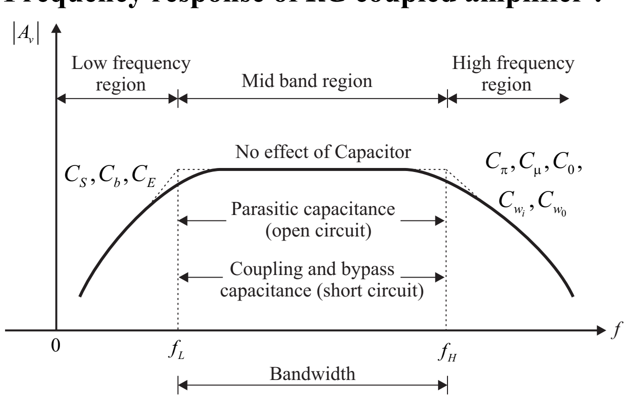
Explanations Low Frequency BJT & MOSFET Amplifier

4.1 (D)

RC coupled amplifier is shown below,



Frequency response of *RC* coupled amplifier :



In Multi-stage *RC*-coupled Amplifier, coupling capacitors are of very high values. Hence, their effect on circuit is considered at low frequency i.e. coupling capacitors are short circuited at low frequency.

Bypass capacitors are of small values. Therefore, their effect on circuit is considered high frequency i.e. bypass capacitors are short circuited at high frequency.

Hence, the correct option is (A).

Key Point

(i) External capacitance limits the low frequency response.

(ii) Internal capacitance limits the high frequency response.

(iii) Parasitic capacitance/stray capacitance $(C_\pi, C_\mu, C_0, C_{w_i}, C_{w_0})$ are **internal capacitance.**

(iv) Coupling capacitance (C_s, C_b) and bypass capacitance (C_E) are **external capacitance.**

4.2 (C)

Given :

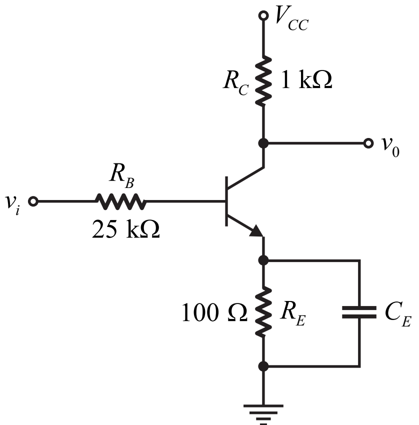
(i) $I_C = 1\text{ mA}$

(ii) $V_T = \dfrac{kT}{q} = 26\text{ mV},\ \beta_0 = 200$

(iii) Base spreading resistance, $r_b = 0$

(iv) Internal output resistance of BJT, $r_0 \to \infty$

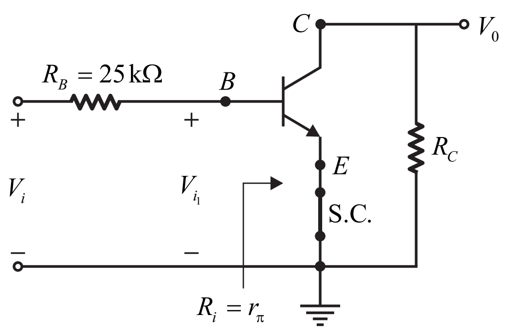
Circuit is shown below,



AC equivalent circuit :

(i) All capacitors are short circuited.

(ii) All DC voltage sources are replaced by short circuit.



Internal input resistance is given by,

$$R_i = r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C}$$

$$R_i = \frac{200 \times 26 \times 10^{-3}}{1 \times 10^{-3}} = 5.2\text{ k}\Omega$$

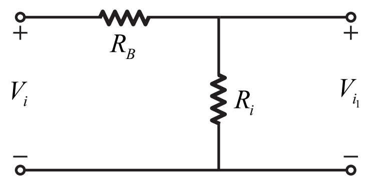
Internal voltage gain is given by,

$$A_{V_1} = \frac{V_0}{V_{i_1}} = -g_m R_L' = \frac{-I_C}{V_T} \times R_C$$

$$A_{V_1} = \frac{-1}{26} \times 1 \times 10^3 = \frac{-1}{26} \times 10^3$$

Overall voltage gain is given by,

$$A_V = \frac{V_0}{V_i} = \frac{V_0}{V_{i_1}} \times \frac{V_{i_1}}{V_i} = A_{V_1} \times \frac{V_{i_1}}{V_i} \quad \ldots \text{(i)}$$



From figure,

$$V_{i_1} = \frac{R_i}{R_i + R_B} \times V_i \qquad \text{[By VDR]}$$

$$\frac{V_{i_1}}{V_i} = \frac{R_i}{R_i + R_b} \quad \ldots \text{(ii)}$$

From equation (i) and (ii),

$$A_V = \frac{-1}{26} \times 10^3 \times \frac{5.2}{5.2 + 25} = -6.62$$

Hence, the correct option is (C).

4.3 (A)

Lower cutoff frequency due to bypass capacitance C_E is given by,

$$f_{L_1} = \frac{1}{2\pi R_E C_E} \text{ and } f_{L_2} = \frac{R_S + r_\pi + (1+\beta) R_E}{2\pi (R_S + r_\pi) R_E C_E}$$

(i) $$C_{E_1} = \frac{1}{2\pi R_E f_{L_1}} = \frac{1}{2\pi \times 100 \times 10}$$

$$C_{E_1} = 1.59 \times 10^{-4}\text{ F} = 0.159\text{ mF}$$

(ii) $$C_{E_2} = \frac{R_S + r_\pi + (1+\beta) R_E}{2\pi (R_S + r_\pi) R_E f_{L_2}}$$

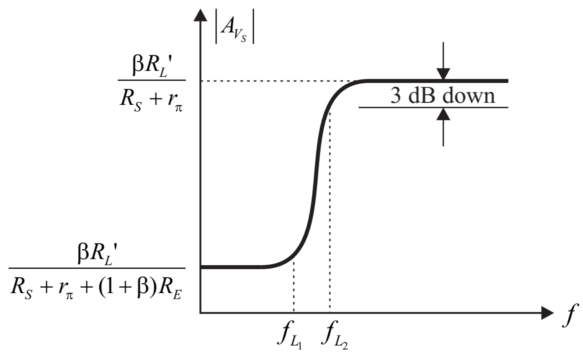
$$C_{E_2} = \frac{25 + 5.2 + (1+200) \times 0.1}{2\pi (25 + 5.2) \times 10 \times 0.1} = 0.265\text{ mF}$$

Only C_{E_1} matches with the options.

Hence, the correct option is (A).

📖 Key Point

Frequency response of common emitter amplifier with bypassed R_E is given by,



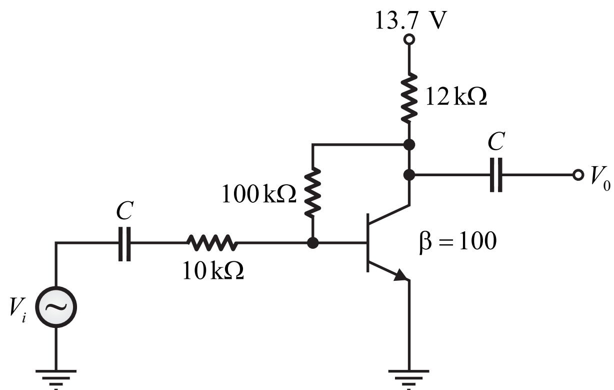
Lower cut-off frequency $f_L = \max(f_{L_1}, f_{L_2})$

When there are more than one f_L exists then higher value of f_L will be dominating.

If both C_{E_1} and C_{E_2} were given in option then we will select C_{E_2} because f_{L_2} is dominating.

4.4　(D)

Given circuit is shown below,

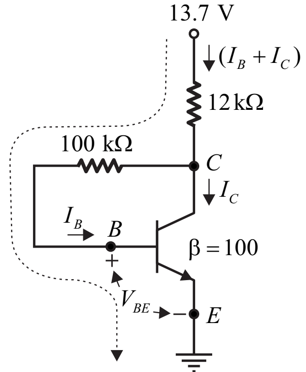


Method 1

DC equivalent circuit (Calculation of g_m) :

(i) All capacitors are open circuited.

(ii) AC voltage sources are replaced by short circuit.



Applying KVL in above shown loop,

$$-13.7 + (I_B + I_C)\times 12 + I_B \times 100 + V_{BE} = 0$$

$$112 I_B + 12 I_C = 13.7 - 0.7$$

$$112 I_B + 12 I_C = 13 \qquad \text{... (i)}$$

Collector current is given by,

$$I_C = \beta I_B = 100\, I_B$$

Put the value of I_C in equation (i),

$$112 I_B + 1200 I_B = 13$$

$$1312 I_B = 13$$

$$I_B = \frac{13}{1312} \approx 0.01 \text{ mA}$$

Hence, $I_C = \beta I_B \approx 1\text{mA}$

Transconductance is given by,

$$g_m = \frac{I_C}{V_T} = \frac{1}{26} = 38.46 \text{ mA/V}$$

AC equivalent circuit (Calculation of R_i' and A_V) :

(i) All capacitors are short circuited.

(ii) All DC voltage sources are replaced by short circuit.

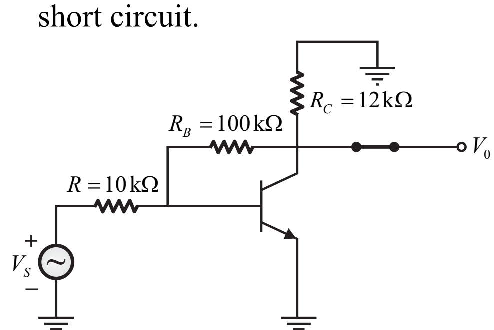
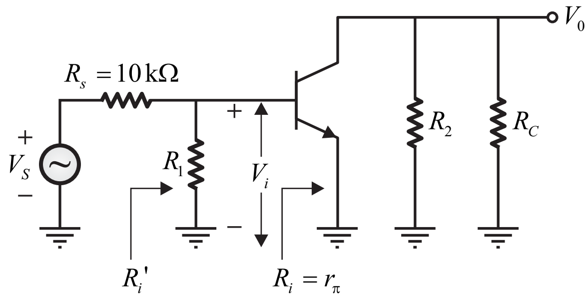


Fig. CE amplifier without R_E

R_B is feedback resistor which can be simplified using **Miller's theorem** as shown below,



where, $R_1 = \dfrac{R_B}{1-A_V}$

and $R_2 = \dfrac{R_B}{1-\dfrac{1}{A_V}}$

Since, for common emitter amplifier,

$$|A_V| >> 1$$

Hence, $1-\dfrac{1}{A_V} \approx 1$

and $R_2 = R_B = 100\text{ k}\Omega$

Hence, total load impedance $R_L = R_2 \| R_C$

$$R_L = 100 \| 12 = 10.7\text{ k}\Omega$$

Voltage gain A_V of a common emitter amplifier is given by,

$$A_V = \frac{A_I R_L}{R_i} = \frac{-\beta R_L}{r_\pi} \quad \dots \text{(i)}$$

Calculation of r_π :

$$\beta = 100,\ g_m = 38.46\text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{38.46\times 10^{-3}} = 2.6\text{ k}\Omega$$

From equation (i),

$$A_V = \frac{-100\times 10.7}{2.6} = -411.53$$

Hence, $R_1 = \dfrac{R_B}{1-A_V} = \dfrac{100}{1+411.53} = 0.2424\text{ k}\Omega$

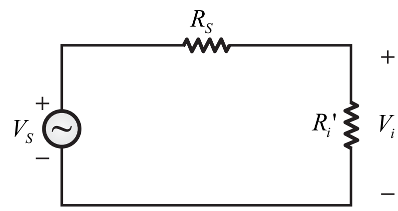
Overall input impedance is given by,

$$R_i' = R_1 \| R_i = R_1 \| r_\pi$$

$$R_i' = 0.2424 \| 2.6 = \frac{0.2424\times 2.6}{0.2424+2.6}$$

$$R_i' = 0.221\text{k}\Omega$$

Input voltage V_i is given by,



$$V_i = \frac{R_i'}{R_i' + R_S} V_S$$

$$\frac{V_i}{V_s} = \frac{0.211}{0.211+10} = 0.0216$$

Overall voltage gain A_{V_S} is given by

$$A_{V_S} = \frac{V_0}{V_S} = \frac{V_0}{V_i}\times\frac{V_i}{V_S} = A_V \times \frac{V_i}{V_S}$$

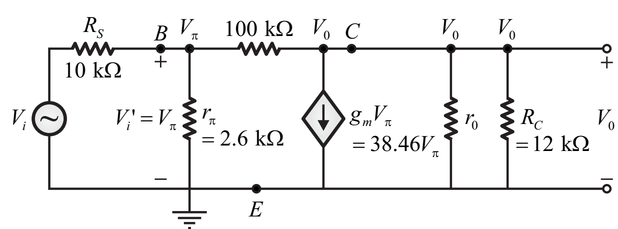
$$A_{V_S} = -411.53\times 0.0216 = -8.89$$

$$|A_{V_S}| \approx 10$$

Hence, the correct option is (D).

Method 2

The r_e-model of given circuit is shown below,



> **Note :** If there is no information of early voltage then assume $V_A = \infty$.

$$r_0 = \frac{V_A}{I_C} = \frac{\infty}{I_C} = \infty$$

$$r_\pi = \frac{\beta}{g_m}$$

$$g_m = \frac{|I_C|}{V_T}$$

$I_C = 1\text{ mA}$ [From DC analysis]

$$g_m = \frac{1}{26} = 38.46 \text{ mA/Volt}$$

$$r_\pi = \frac{100}{38.46} = 2.6 \text{ k}\Omega$$

Applying KCL at node V_0,

$$\frac{V_0}{12} + 38.46V_\pi + \frac{V_0 - V_\pi}{100} = 0 \qquad \ldots \text{(i)}$$

Applying KCL at node V_π,

$$\frac{V_\pi - V_i}{10} + \frac{V_\pi}{2.6} + \frac{V_\pi - V_0}{100} = 0 \qquad \ldots \text{(ii)}$$

From equation (i),

$$100V_0 + 46152V_\pi + 12V_0 - 12V_\pi = 0$$

$$(100+12)V_0 = (12 - 46152)V_\pi$$

$$\frac{V_0}{V_\pi} = -411.96$$

$$V_\pi = -2.42\times10^{-3}V_0 = 0.0024V_0$$

From equation (ii),

$$\frac{-0.0024V_0 - V_i}{10} - \frac{0.0024V_0}{2.6} + \frac{-0.0024V_0 - V_0}{100} = 0$$

$$-0.1V_i = 0.0112V_0$$

$$\frac{V_0}{V_i} = -8.93$$

$$\left|\frac{V_0}{V_i}\right| = 8.93 \approx 10$$

Hence, the correct option is (D).

Method 3

The given circuit is a voltage shunt feedback amplifier and voltage gain of this voltage shunt feedback amplifier is given by,

$$A_V = A_{V_f} = \frac{V_0}{V_i} = \frac{R_{M_f}}{R_S} \qquad \ldots \text{(i)}$$

where, R_S = Source resistance

$$R_{M_f} \approx \frac{1}{\beta}$$

$$\beta = \text{ feedback factor } = \frac{I_f}{V_0} = \frac{-1}{R_B}$$

$$R_{M_f} \approx -R_B$$

From equation (i),

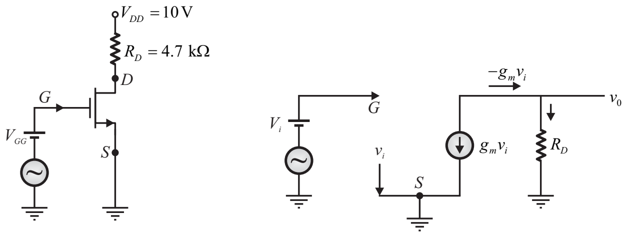
$$A_{V_f} \approx \frac{-R_B}{R_S} \approx \frac{-100}{10} \approx -10$$

Hence, $|A_V| = |A_{V_f}| = 10$

Hence, the correct option is (D).

4.5 (B)

Given for a common source amplifier,



$$R_D = 4.7 \text{ k}\Omega$$

$$V_{in} = 10 \text{ V}$$

$$g_m = 520\ \mu\text{A/V}$$

Voltage gain of common source amplifier is gives as

$$A_V = \frac{V_0}{V_i} = -g_m R_D$$

$$A_V = -520\times10^{-6}\times4.7\times10^3$$

$$A_V = -2.44$$

Hence, the correct option is (B).

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6 Operational Amplifier

➢ Partial Synopsis

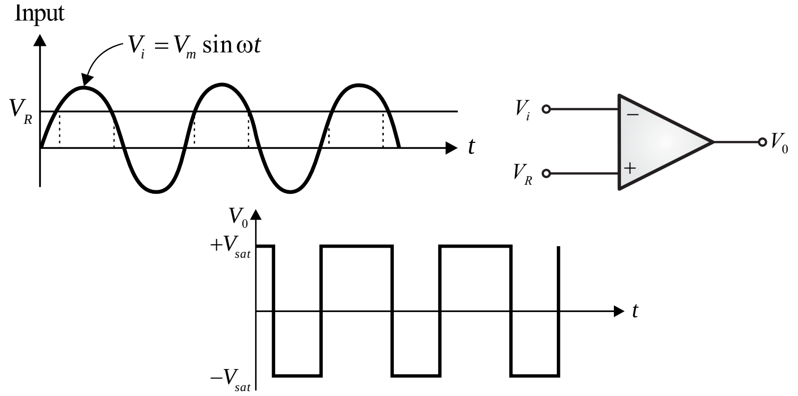
Non-linear Application of Op-Amp

1. Comparators :

- When Op-Amp acts as comparator,

  If $V_{NI} > V_I$ then, $V_0 = +V_{sat}$

  $V_I > V_{NI}$ then, $V_0 = -V_{sat}$



- The output of a comparator is either HIGH $(+V_{sat})$ or LOW $(-V_{sat})$.

2. Schmitt Trigger :

- Schmitt trigger is basically a comparator circuit with positive feedback and hence it is also called as re-generative comparator.

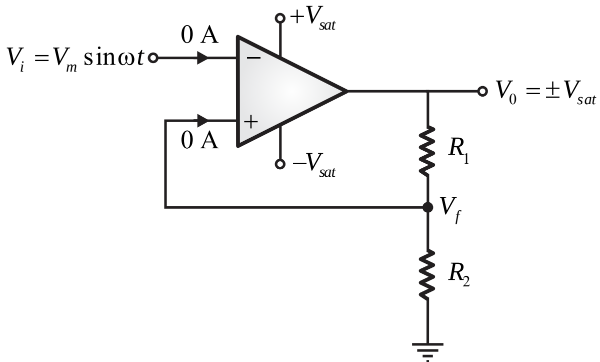
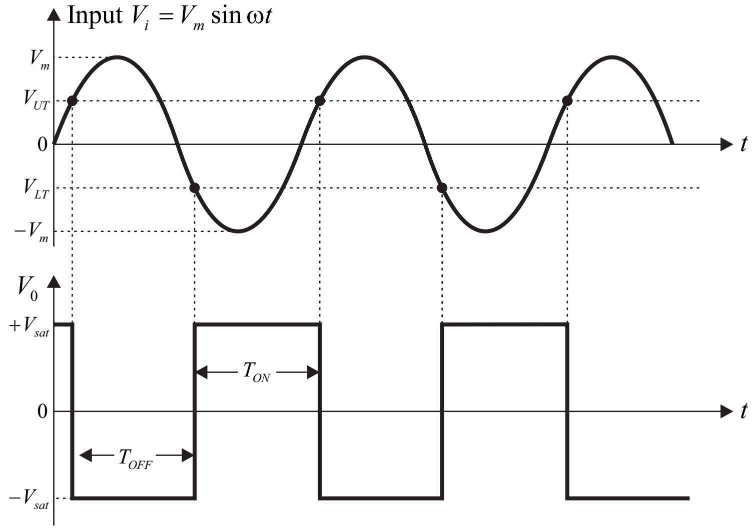


Fig. Schmitt trigger

- The upper and lower threshold voltage,

$$V_{UT} = \frac{V_{sat}R_2}{R_1 + R_2},\ V_{LT} = \frac{-V_{sat}R_2}{R_1 + R_2}$$

- Output waveform (V_0) is shown below,



- From output waveform :

  (a) $T_{ON} = T_{OFF}$

  (b) % Duty cycle $= 50\%$

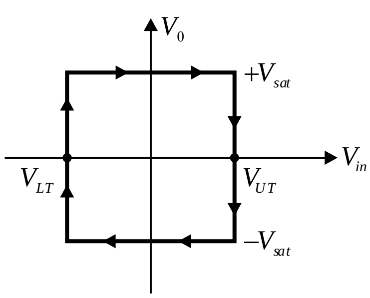
  (c) $\text{Avg}(V_0) = 0$

  (d) $\text{Area}^+ = \text{Area}^-$

  (e) Symmetrical square wave

  (f) RMS value of Output $= V_{sat}$

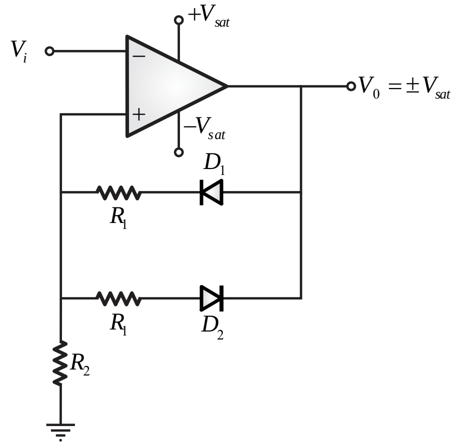
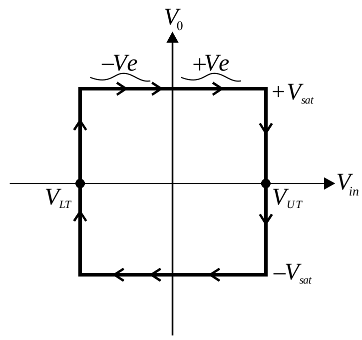
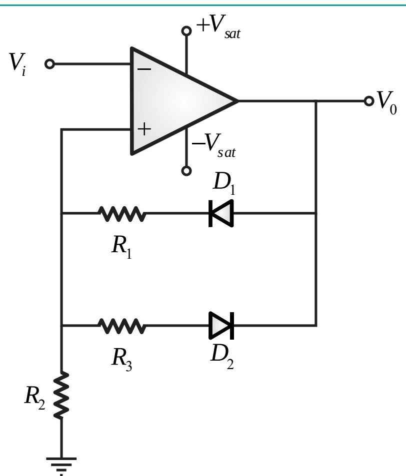
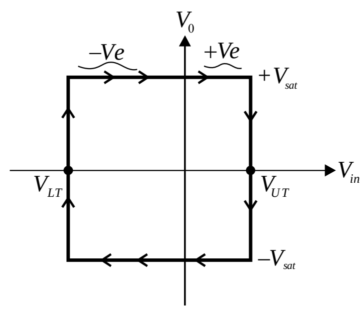
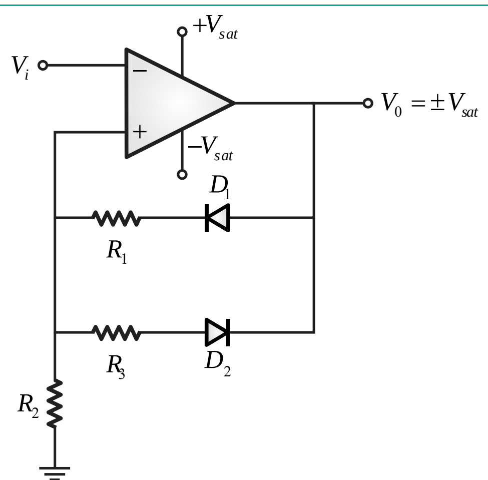
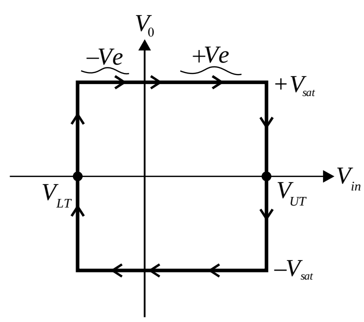
- Hysteresis Curve :



$$V_{hys} = V_{UT} - V_{LT}$$

$$V_{hys} = \frac{2V_{sat}R_2}{R_1 + R_2}$$

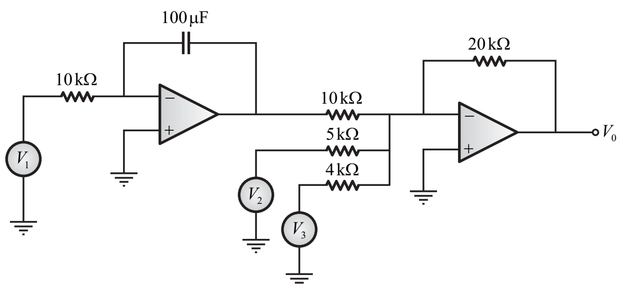
Special Cases of Schmitt Trigger

S. No.	Schmitt Trigger Circuit	Transfer characteristics	Summary
1.	 **Consider $R_1 = R_2$**	 $\lvert V_{UT} \rvert = \lvert V_{LT} \rvert$	Conclusion from transfer characteristics, (a) Area of $(+ve) =$ Area of $(-ve)$ (b) Average value of $(V_0) = 0$ (c) $T_{ON} = T_{OFF}$ (d) % duty cycle $= 50\%$ (e) Output is symmetrical square wave form
2.	 **Consider $R_1 > R_3$**	 $\lvert V_{UT} \rvert < \lvert V_{LT} \rvert$	Conclusion from transfer characteristics, (a) Area of $(+ve) <$ Area of $(-ve)$ (b) Average value of $(V_0) = -ve$ (c) $T_{ON} < T_{OFF}$ (d) % duty cycle $< 50\%$ (e) Output is asymmetrical square wave form
3.	 **Consider $R_3 > R_2$**	 $\lvert V_{UT} \rvert > \lvert V_{LT} \rvert$	Conclusion from transfer characteristics, (a) Area of $(+ve) >$ Area of $(-ve)$ (b) Average value of $(V_0) = +ve$ (c) $T_{ON} > T_{OFF}$ (d) % duty cycle $> 50\%$ (e) Output is asymmetrical square wave form.

➢ Sample Questions

1991 IIT Kanpur

6.1 With the ideal operational amplifiers, the circuit shown in figure, simulates the equation



(A) $V_0 = 2\int V_1\,dt - 4V_2 - 5V_3$

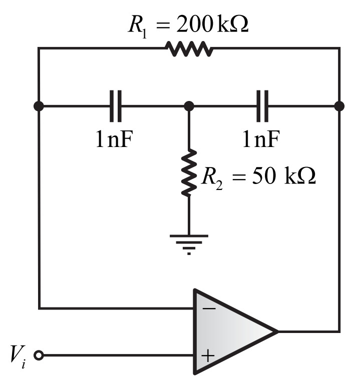
(B) $V_0 = 2\int V_1\,dt + 4V_2 + 5V_3$

(C) $V_0 = -2\int V_1\,dt + 4V_2 + 5V_3$

(D) $V_0 = \int V_1\,dt - 4V_2 - 5V_3$

2004 IIT Delhi

6.2 In the active filter circuit shown in figure, if $Q = 1$, a pair of poles will be realized with ω_0 equal to

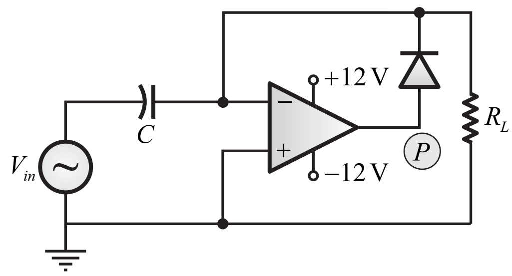


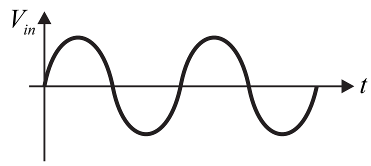
(A) 10000 rad/s (B) 100 rad/s

(C) 10 rad/s (D) 1 rad/s

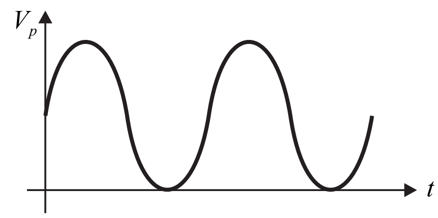
2006 IIT Kharagpur

6.3 For a given sinusoidal input voltage, the voltage waveform at point P of the clamper circuit shown in figure will be

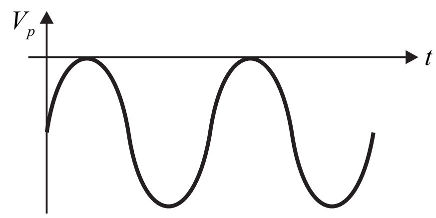




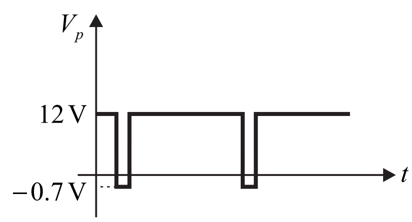
(A)



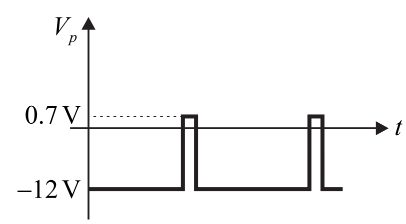
(B)



(C)



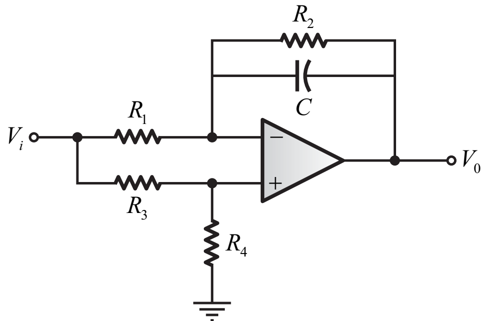
(D)



2008 IISc Bangalore

Statement for Linked Answer Questions 6.4 & 6.5

A general filter circuit is shown in the figure.



6.4 If $R_1 = R_2 = R_A$ and $R_3 = R_4 = R_B$, the circuit acts as a

(A) All pass filter

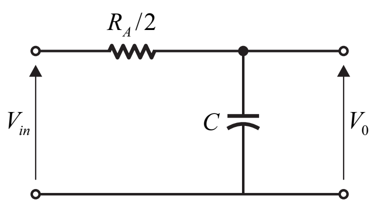
(B) Band pass filter

(C) High pass filter

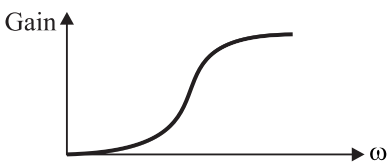
(D) Low pass filter

6.5 The output of the filter in **6.4** is given to the circuit shown in figure.

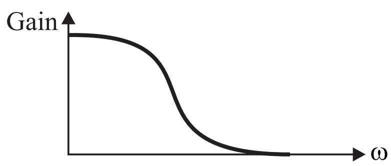
The gain vs frequency characteristic of the output (V_0) will be



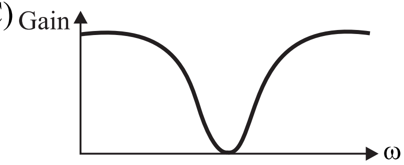
(A)



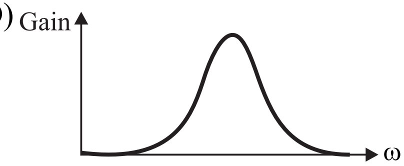
(B)



(C)

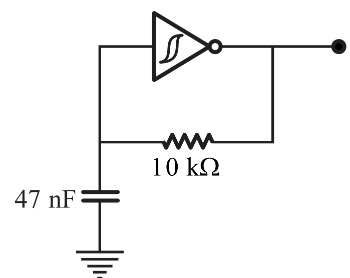


(D)



2021 IIT Bombay

6.6 A CMOS Schmitt-trigger inverter has a low output level of 0 V and a high output level of 5 V. It has input thresholds of 1.6 V and 2.4 V. The input capacitance and output resistance of the Schmitt trigger are negligible. The frequency of the oscillator shown in the figure is _______ Hz. (Round off to 2 decimal places)



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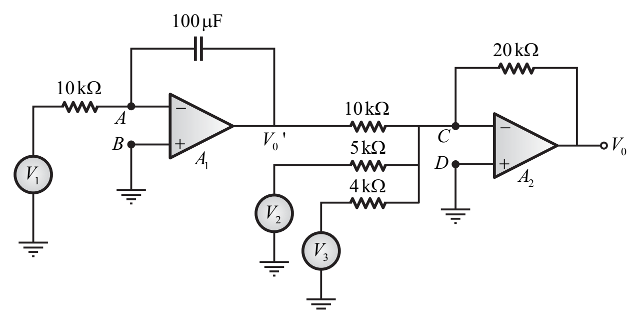
Explanations — Operational Amplifier

Concept and Validity of Virtual Ground

Scan for Video Explanation

6.1 (A)

Given circuit is shown below,



Method 1

From virtual ground concept,

$$V_A = V_B = 0,\ V_C = V_D = 0$$

Applying KCL at node A,

$$\frac{V_1 - V_A}{10\times 10^3} - \frac{V_A - V_0'}{\frac{1}{sC}} = 0$$

$$\frac{V_1}{10^4} + \frac{V_0'}{\frac{1}{sC}} = 0$$

$$V_0' = -\frac{V_1}{10^4 Cs} = -\frac{V_1}{10^4 \times 10^{-4} s}$$

$$V_0' = -\frac{V_1}{s}$$

Taking inverse Laplace transform,

$$V_0' = -\int V_1\, dt \qquad \dots \text{(i)}$$

Applying KVL at node C,

$$\frac{V_0' - V_C}{10\times 10^3} + \frac{V_2 - V_C}{5\times 10^3} + \frac{V_3 - V_C}{4\times 10^3} - \frac{V_C - V_0}{20\times 10^3} = 0$$

$$2V_0' + 4V_2 + 5V_3 + V_0 = 0$$

$$V_0 = -2V_0' - 4V_2 - 5V_3 \qquad \dots \text{(ii)}$$

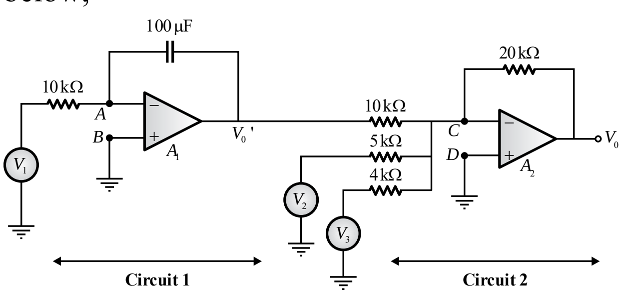
From equation (i) and (ii),

$$V_0 = 2\int V_1\, dt - 4V_2 - 5V_3$$

Hence, the correct option is (A).

Method 2

Given circuit is a combination of ideal integrator and ideal summer/adder as shown below,



Here circuit 1 is ideal integrator, so output of ideal integrator is,

$$V_0' = -\frac{1}{RC}\int V_1\, dt \qquad \dots \text{(i)}$$

$$\because \qquad RC = 10^4 \times 10^{-4} = 1$$

Thus, $\quad V_0' = -\int V_1\, dt \qquad \dots$ (ii)

Now, circuit 2 is ideal summer/adder in inverting configuration so, output V_0 is,

$$V_0 = -\frac{20}{10}V_0' - \frac{20}{5}V_2 - \frac{20}{4}V_3$$

$$V_0 = -2V_0' - 4V_2 - 5V_3$$

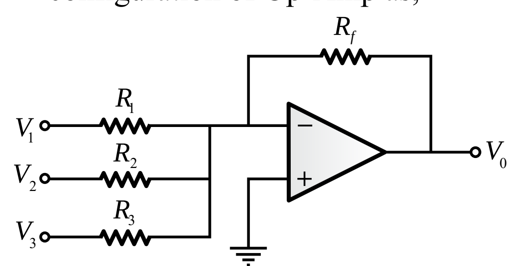
Put the value of V_0' from equation (ii) as,

$$V_0 = +2\int V_1\, dt - 4V_2 - 5V_3$$

Hence, the correct option is (A).

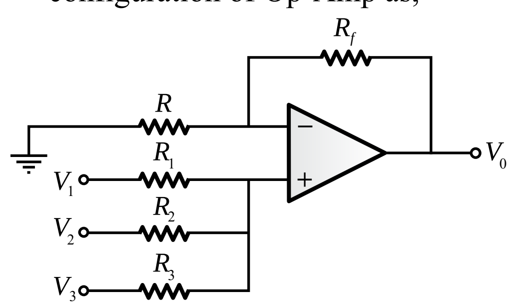
Key Point

(i) 3-input summer using inverting configuration of Op-Amp as,



$$V_0 = -\frac{R_f}{R_1}V_1 - \frac{R_f}{R_2}V_2 - \frac{R_f}{R_3}V_3$$

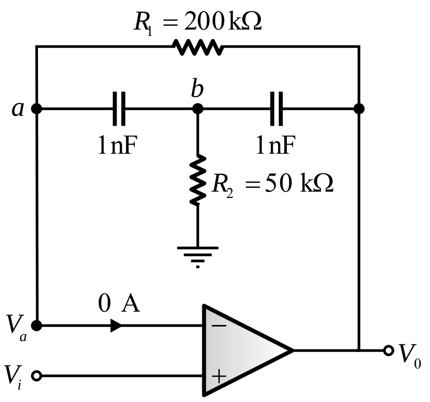
(ii) 3-input summer using non-inverting configuration of Op-Amp as,



$$V_0 = \left[1 + \frac{R_f}{R}\right]\left[\frac{R_1}{R_1 + (R_2 \parallel R_3)}V_1 + \frac{R_2}{R_2 + (R_1 \parallel R_3)}V_2 + \frac{R_3}{R_3 + (R_1 \parallel R_2)}V_3\right]$$

6.2 (A)

Given circuit with ideal Op-Amp is shown below,



Here, quality factor $(Q) = 1$.

Given circuit is 2nd order active filter so we can calculate its transfer function so, from virtual ground concept,

$$V_a = V_i$$

Applying KCL at node *a*,

$$\frac{V_a - V_0}{R_1} + \frac{V_a - V_b}{1/Cs} = 0$$

$$\frac{V_i - V_0}{R_1} + \frac{V_i - V_b}{1/Cs} = 0$$

$$\frac{V_i - V_0}{sR_1C} + V_i - V_b = 0$$

$$V_b = \left(1 + \frac{1}{sR_1C}\right)V_i - \frac{V_0}{sR_1C} \qquad \ldots \text{(i)}$$

Applying KCL at node *b*,

$$\frac{V_b - V_a}{1/Cs} + \frac{V_b - V_0}{1/Cs} + \frac{V_b}{R_2} = 0$$

$$\frac{V_b - V_i}{1/Cs} + \frac{V_b - V_0}{1/Cs} + \frac{V_b}{R_2} = 0$$

$$V_b - V_i + V_b - V_0 + \frac{V_b}{sR_2C} = 0$$

$$V_0 + V_i = \left(2 + \frac{1}{sR_2C}\right)V_b \qquad \ldots \text{(ii)}$$

From equation (i) and (ii),

$$V_0 + V_i = \left(2 + \frac{1}{sR_2C}\right)\left(1 + \frac{1}{sR_1C}\right)V_i - \left(2 + \frac{1}{sR_2C}\right) \times \frac{V_0}{sR_1C}$$

$$V_0\left(1 + \left(2 + \frac{1}{sR_2C}\right)\frac{1}{sR_1C}\right) = \left[\frac{1 + 2R_2Cs}{sR_2C} \times \frac{1 + sR_1C}{sR_1C} - 1\right]V_i$$

$$\frac{V_0}{V_i} = \left[\frac{2s^2R_1R_2C^2 + (2R_2C + R_1C)s - R_1R_2C^2s^2}{R_1R_2C^2\left(s^2 + \frac{2s}{R_1C} + \frac{1}{R_1R_2C^2}\right)}\right]$$

Characteristics equation is given by,

$$s^2 + \frac{2}{R_1C}s + \frac{1}{R_1R_2C^2} = 0 \qquad \ldots \text{(iii)}$$

Characteristics equation of standard second order system is given by,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

where, Quality factor, $Q = \dfrac{1}{2\xi}$

Natural frequency, $\omega_n = \omega_0$

Hence, characteristics of standard second order system is given by,

$$s^2 + \frac{\omega_0}{Q}s + \omega_0^2 = 0 \qquad \ldots \text{(iv)}$$

From equation (iii) and (iv),

$$\frac{\omega_0}{Q} = \frac{2}{R_1C}$$

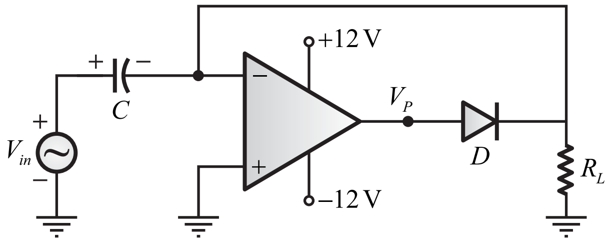
$$\frac{\omega_0}{1} = \frac{2}{10^3 \times 200 \times 10^{-9}} = 10000 \text{ rad/sec}$$

$$\omega_0 = 10000 \text{ rad/sec}$$

Hence, the correct option is (A).

6.3 (D)

Given clamper circuit using Op-Amp is shown below,



Given : $V_{NI} = 0$ V

V_I = Inverting terminal voltage

Case 1 :

V_I = Positive – Voltage

$V_I > V_{NI}$

V_P towards – 12 V

$D = \text{OFF}$

$V_P = -12$ V

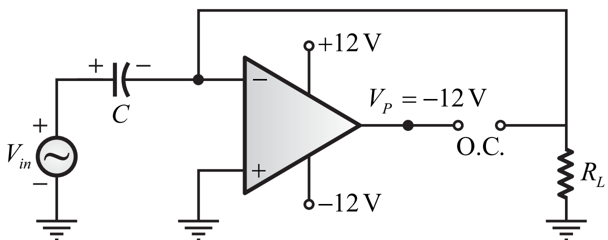


Fig. Open loop system (comparator)

Case 2 :

V_I = Negative – Voltage

$V_{NI} > V_I$

V_P towards + 12 V

$D = \text{ON}$

$V_P = +12$ V

Replaced by 0.7 volts.

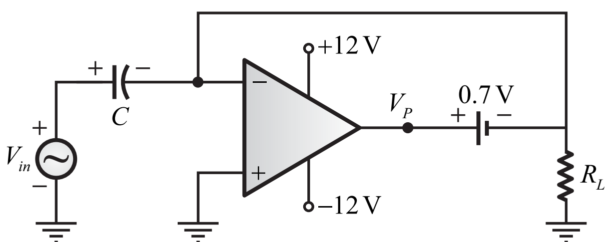
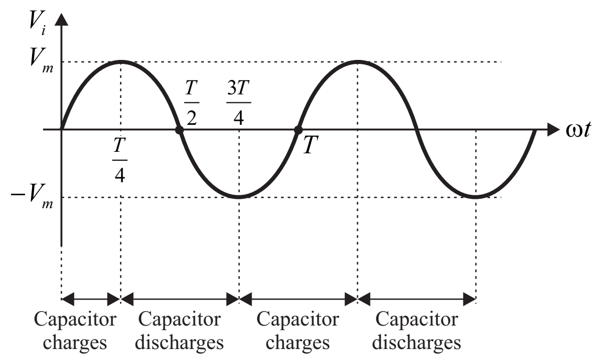


Fig. Closed loop system (negative feedback)

Input signal is given by,



At time, $t = 0$

Assume uncharged capacitor $V_C(0^-) = V_C(0^+) = 0$ V. At $t > 0$, capacitor will start charging but slowly as charging time constant $R_L C > T_i$ (input time period). Now, charging equation of capacitor is given by,

$$V_C(t) = V_m(1 - e^{-t/R_L C}) \quad \text{where } R_L C >> T_i$$

At $t = 0^+$

$V_i(t) > 0$

$V_C(t) > 0$

From figure, $V_I(t) = V_i(t) - V_C(t)$

$V_I(t) > 0$ V

If $V_I > V_{NI}$

$V_P = -12$ V

$D = \text{OFF}$

Given figure represents open loop system and virtual ground concept is not valid.

At $t = T/4$

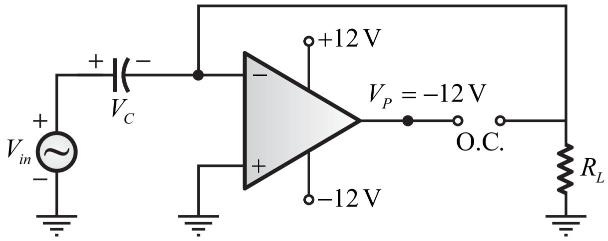
$$V_i\left(t = \frac{T}{4}\right) = V_m$$

$$V_C\left(t = \frac{T}{4}\right) = V_x$$

(some positive finite value such that $V_x << V_m$)

$$V_I\left(t = \frac{T}{4}\right) = V_m - V_x \qquad [V_m - V_x >> 0]$$

$V_I > V_{NI}$



Now $t > T/4$

Capacitor will start discharging,

$$V_C(t) = \text{decreasing } (\downarrow)$$

$$V_i(t) = \text{decreasing } \left[\frac{T}{4} < t < \frac{3T}{4}\right]$$

$$V_I = V_i(t) - V_C(t)$$

[Both $V_C(t)$ and $V_i(t)$ are decreasing]

$$V_I\left(t = \frac{T}{2}\right) = 0 - V_C(t)$$

$$V_I\left(t = \frac{T}{2}\right) = -V_C\left(t = \frac{T}{2}\right)$$

[Which is very small number]

So some where around $t = \frac{T}{2}$, $V_I(t)$ will cross time axis and will become negative

Now, for $t = \frac{T^+}{2}$, $V_{NI} > V_I$

$$V_P = +12 \text{ V}$$

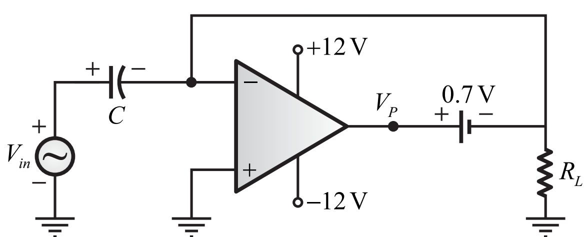


Fig. Closed loop system

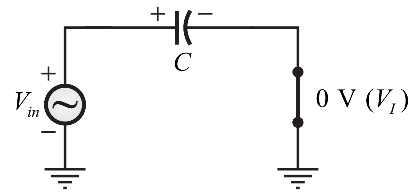
$$V_P = 0.7 \text{ V}$$

Now virtual ground concept is applicable,

i.e. $V_I = V_{NI} = 0$ V

i.e. $V_I = 0$ V [fixed]

that mean,



$$-V_i(t) + V_C(t) = 0$$

So, $V_C(t) = V_i(t)$

Capacitor will be charging exactly as per V_i

Now, $t > T/2$, during negative half cycle of input,

$$V_C\left(t = \frac{3T}{4}\right) = -V_m$$

$$V_i\left(t = \frac{3T}{4}\right) = -V_m$$

But $V_i\left(t = \frac{3T}{4}\right) \neq V_i\left(t = \frac{3T^+}{4}\right)$

and $V_C\left(t = \frac{3T}{4}\right) = V_C\left(t = \frac{3T^+}{4}\right) = V_m$

So, $V_I = V_i(t) - V_C(t)$

$V_i\left(\frac{3T^+}{4}\right)$ is slightly greater than $-V_m$ and

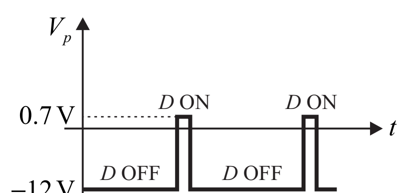
$$V_C\left(\frac{3T^+}{4}\right) = -V_m$$

Hence, V_I = Positive

Now, $V_I > V_{NI}$ $\left(\text{At } t = \frac{3T^+}{4}\right)$

$$V_P = -12 \text{ V}$$

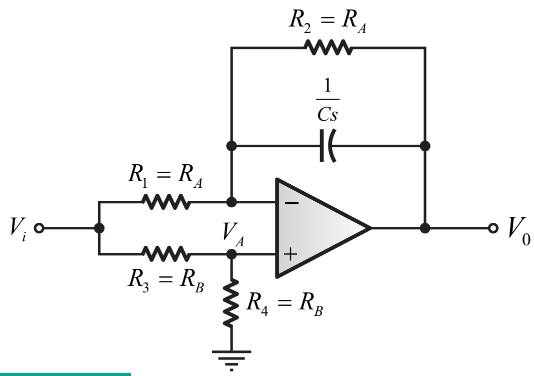
Now virtual ground concept again not valid as $D = \text{OFF}$ and system is open loop so waveform look like.



Hence, the correct option is (D).

6.4 (C)

Given circuit is shown below,



Method 1

From virtual ground concept,

$$V_- = V_+ = V_A$$

Applying KCL at non-inverting terminals,

$$\frac{V_A - V_i}{R_B} + \frac{V_A}{R_B} = 0$$

$$\frac{2V_A}{R_B} = \frac{V_i}{R_B}$$

$$V_A = \frac{V_i}{2} \quad \dots \text{(i)}$$

Applying KCL at inverting terminal,

$$\frac{V_A - V_i}{R_A} + \frac{V_A - V_0}{R_A} + \frac{V_A - V_0}{\frac{1}{Cs}} = 0$$

$$V_A\left(\frac{1}{R_A} + \frac{1}{R_A} + Cs\right) = \frac{V_0}{R_A} + CsV_0 + \frac{V_i}{R_A}$$

$$V_A\left(\frac{2 + R_A Cs}{R_A}\right) = V_0\left(\frac{1 + R_A Cs}{R_A}\right) + \frac{V_i}{R_A} \quad \dots \text{(ii)}$$

From equation (i) and (ii),

$$\frac{V_i}{2}\left(\frac{2 + R_A Cs}{R_A}\right) - \frac{V_i}{R_A} = V_0 \frac{(1 + R_A Cs)}{R_A}$$

$$V_i\left[\frac{2 + R_A Cs}{2R_A} - \frac{1}{R_A}\right] = \frac{V_0 (1 + R_A Cs)}{R_A}$$

$$V_i\left[\frac{2 + R_A Cs - 2}{2R_A}\right] = \frac{V_0 (1 + R_A Cs)}{R_A}$$

$$\frac{V_0}{V_i} = \frac{R_A Cs}{2(1 + R_A Cs)} \quad \dots \text{(iii)}$$

$$\left.\frac{V_0}{V_i}\right|_{s=0} = 0 \text{ and } \left.\frac{V_0}{V_i}\right|_{s=\infty} = \frac{1}{2}$$

Gain of given circuit, is zero at low frequency and $\frac{1}{2}$ at high frequency. Hence, given circuit will act as high pass filter.

Put $s = j\omega$ in above equation (iii),

$$\frac{V_0(j\omega)}{V_i(j\omega)} = \frac{jR_A C\omega}{2(1 + jR_A C\omega)}$$

$$\frac{V_0(j\omega)}{V_i(j\omega)} = \frac{1}{2\left(1 - \frac{j}{R_A C\omega}\right)}$$

Comparing above equation with the standard HPF equation,

$$H(j\omega) = \frac{A_0}{\left(1 - \frac{j\omega_L}{\omega}\right)}$$

$$A_0 = \frac{1}{2} \text{ and } \omega_L = \frac{1}{R_A C} \text{ rad/sec}$$

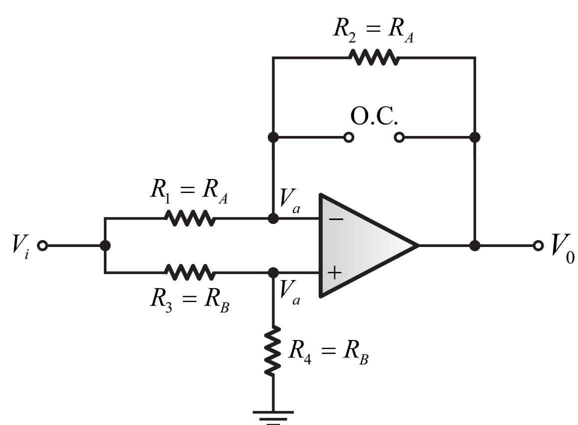
Hence, the correct option is (C).

Method 2

(i) At low frequency $(\omega \to 0)$;

$$X_C = \frac{1}{\omega C} \to \infty \qquad \text{[O.C.]}$$

Hence, modified circuit is shown below,



From figure, apply voltage divider rule,

$$V_a = \frac{V_i \times R_B}{R_B + R_B} = \frac{V_i}{2} \qquad \dots \text{(i)}$$

Applying KCL at inverting terminal,

$$\frac{V_a - V_i}{R_A} + \frac{V_a - V_0}{R_A} = 0$$

$$V_0 = 2V_a - V_i \qquad \dots \text{(ii)}$$

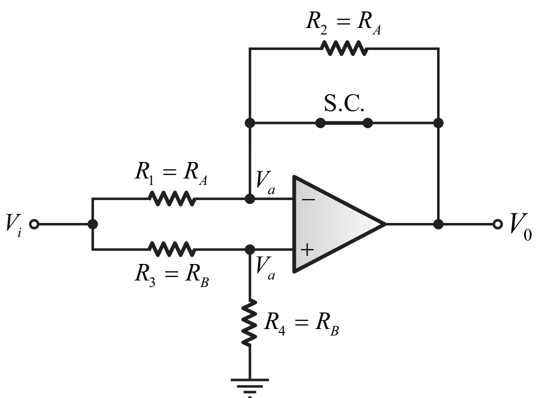
From equation (i) and (ii),

$$V_0 = \left(2 \times \frac{V_i}{2}\right) - V_i = 0$$

(ii) At high frequency $(\omega \to \infty)$;

$$X_C = \frac{1}{\omega C} \to 0 \qquad \text{[S.C.]}$$

Hence, modified circuit is shown below,



From figure, apply voltage divider rule,

$$V_a = V_i \times \frac{R_B}{R_B + R_B} = \frac{V_i}{2} \qquad \dots \text{(iii)}$$

From virtual ground concept,

$$V_0 = V_- = V_a \qquad \dots \text{(iv)}$$

From equation (iii) and (iv),

$$V_0 = \frac{V_i}{2}$$

Hence, $\dfrac{V_0}{V_i} = \dfrac{1}{2}$

From case 1 and case 2, expression of gain can be written as given below,

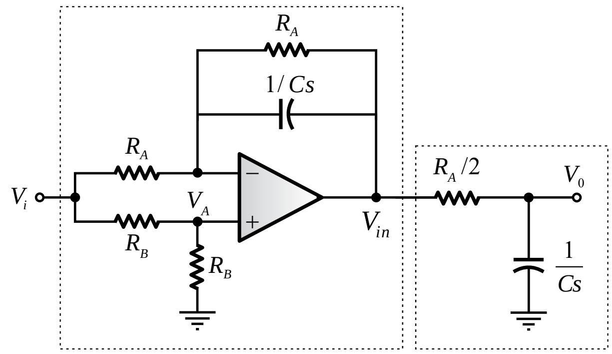
$$\frac{V_0}{V_i} = \begin{cases} 0 \ ; & \text{At low frequency} \\ \dfrac{1}{2} \ ; & \text{At high frequency} \end{cases}$$

Gain of given circuit is zero at low frequency and $\frac{1}{2}$ at high frequency. Hence given circuit will act as high pass filter.

Hence, the correct option is (C).

6.5 (D)

Given circuit is shown below,



Circuit 1 : Active HPF **Circuit 2 : Passive LPF**

(i) Circuit 1 : Active HPF

As explained in previous question, this circuit will act as high pass filter with transfer function as given below,

$$\frac{V_{in}}{V_i} = \frac{R_A Cs}{2(1 + R_A Cs)}, \ \omega_L = \frac{1}{R_A C}$$

$$V_{in} = \frac{R_A Cs}{2(1 + R_A Cs)} V_i \qquad \dots \text{(i)}$$

(ii) Circuit 2 : Passive LPF

Transfer function of this circuit is given by,

$$\frac{V_0}{V_{in}} = \frac{1}{1 + \dfrac{sR_A C}{2}} \qquad \dots \text{(ii)}$$

$$\left.\frac{V_0}{V_{in}}\right|_{s=0} = 1 \text{ and } \left.\frac{V_0}{V_{in}}\right|_{s=\infty} = 0$$

Gain of this circuit is one at low frequency and zero at high frequency. Hence, this circuit will act as low pass filter.

On comparing above equation (ii) with the standard low pass gain equation,

$$H(j\omega) = \frac{A_0'}{1 + \dfrac{j\omega}{\omega_H}}, \ \omega_H = \frac{2}{R_A C} \text{ and } A_0' = 1$$

From equation (i) and (ii) gain of overall circuit is given by,

$$\frac{V_0}{V_i} = \frac{R_A Cs}{(1+R_A Cs)(2+R_A Cs)}$$

From above equation,

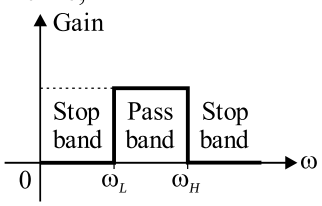
$$\left.\frac{V_0}{V_i}\right|_{s=0} = 0, \text{ Gain is 0 at low frequency}$$

$$\left.\frac{V_0}{V_i}\right|_{s=\infty} = 0, \text{ Gain is 0 at high frequency}$$

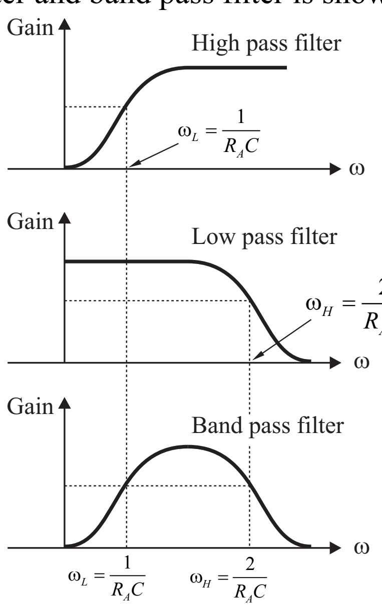
$$\left.\frac{V_0}{V_i}\right|_{s=\text{intermediate frequency}} \neq 0$$

Gain is finite at intermediate frequency

Gain of overall circuit is zero at low frequency and high frequency and gain is not zero at intermediated frequency. Hence, given circuit will act as band pass filter. So ideal response of band pass filter is,



Transfer characteristics of low pass filter, high pass filter and band pass filter is shown below,



Hence, the correct option is (D).

6.6　3158.12

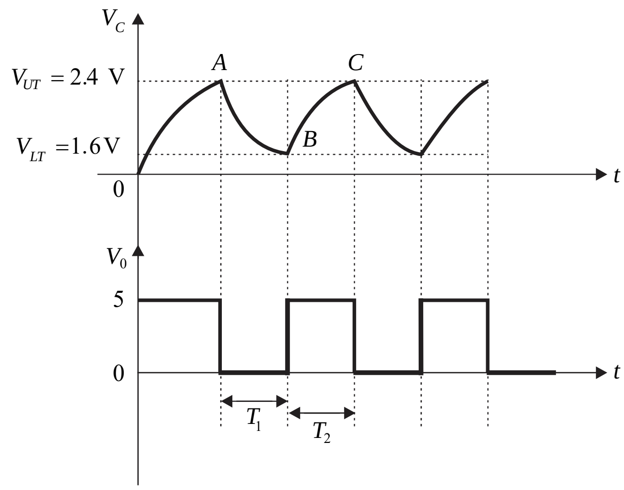
Given :

(i) $V_{LT} = 1.6$ V

(ii) $V_{UT} = 2.4$ V

(iii) $+V_{sat} = 5$ V

(iv) $-V_{sat} = 0$ V



Total time period is given by,

$$T = T_1 + T_2$$

Where, T_1 is OFF time

T_2 is ON time

$$f = \frac{1}{T}$$

Calculation of T_1 :

$$V_c(0^-) = 2.4 \text{ V}$$

$$V_c(\infty) = 0 \text{ V}$$

$$V_c(t) = 0 + (2.4 - 0)e^{-t/RC}$$

At $t = T_1$; $V_c = 1.6$ V

$$1.6 = 2.4e^{-T_1/RC}$$

$$e^{-T_1/RC} = \frac{1.6}{2.4}$$

$$e^{T_1/RC} = \frac{24}{16}$$

$$T_1 = RC \ln(1.5)$$

Calculation of T_2 :

$$V_c(0^-) = 1.6 \text{ V}$$

$$V_c(\infty) = 5 \text{ V}$$

The charging equation of the first order RC circuit is given by

$$V_c(t) = 5 + (16.5)e^{-t/RC}$$

At $t = T_2$

$$V_c = 2.4 \text{ V}$$

$$2.4 = 5 + (16.5)e^{-T_2/RC}$$

$$e^{-T_2/RC} = \frac{34}{26}$$

$$T_2 = RC\log(1.307)$$

Calculation of T :

$$T = T_1 + T_2$$

$$T = RC\ln(1.5 \times 1.307)$$

$$T = 10 \times 10^3 \times 47 \times 10^{-9} \times 0.673$$

$$T = 316.644 \ \mu\text{sec}$$

$$f = \frac{1}{T}$$

$$f = 3158.12 \text{ Hz}$$

Hence, the correct answer is 3158.12.

❖❖❖❖

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CHAPTER 5 CONTROL SYSTEMS

Marks Distribution of Control Systems in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	3	7	17
2004	2	9	20
2005	3	6	15
2006	1	3	7
2007	1	7	15
2008	1	8	17
2009	4	5	14
2010	2	3	8
2011	3	3	9
2012	1	4	9
2013	2	4	10
2014 Set-1	2	2	6
2014 Set-2	2	3	8

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-3	2	3	8
2015 Set-1	2	4	10
2015 Set-2	2	3	8
2016 Set-1	1	4	9
2016 Set-2	1	4	9
2017 Set-1	3	5	13
2017 Set-2	2	3	8
2018	2	2	6
2019	2	3	8
2020	2	4	10
2021	3	3	9

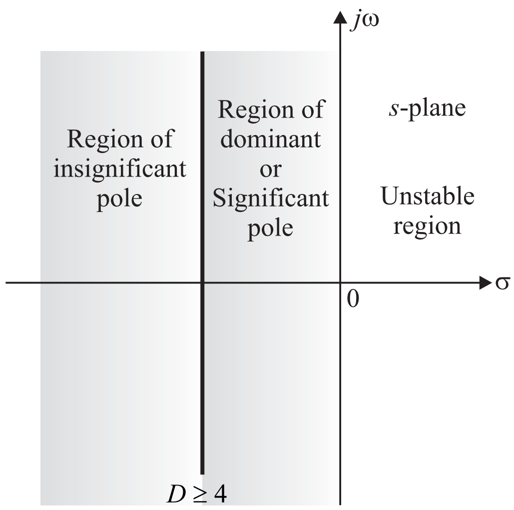
Syllabus : Control Systems

Mathematical modeling and representation of systems, Feedback principle, transfer function, Block diagrams and Signal flow graphs, Transient and Steady-state analysis of linear time invariant systems, Stability analysis using Routh-Hurwitz and Nyquist criteria, Bode plots, Root loci, Lag, Lead and Lead-Lag compensators; P, PI and PID controllers; State space model, Solution of state equations of LTI systems.

Contents : Control Systems

1 Basics of Control System

➢ Partial Synopsis



$$\text{where, } D = \left|\frac{\text{Insignificant pole}}{\text{Significant pole}}\right|$$

Fig. Regions of significant and insignificant poles in the *s*-plane

Remember

To find open loop transfer function from close loop transfer function,

$$\text{C.L.T.F.} = \frac{\text{Numerator}}{\text{Denominator}}\ ;\ \text{O.L.T.F.} = \frac{\text{Numerator}}{\text{Denominator} - \text{Numerator}}$$

To find closed loop transfer function from open loop transfer function,

$$\text{O.L.T.F.} = \frac{\text{Numerator}}{\text{Denominator}}\ ;\ \text{C.L.T.F.} = \frac{\text{Numerator}}{\text{Denominator} + \text{Numerator}}$$

Important Formulas

1. Transfer function :

$$T(s) = \frac{\text{Laplace transform of output}}{\text{Laplace transform of input}} \qquad [\text{Initial value} = 0]$$

2. Time constant :

$$\tau = \frac{1}{|\text{Negative real root}|} \qquad [\text{For first order system}]$$

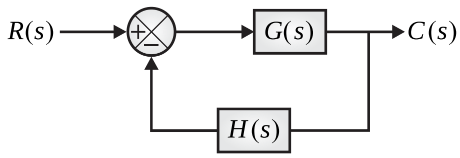
3. Bandwidth :

$$\text{BW} = \frac{1}{\tau} \qquad \text{[For first order system]}$$

4. Relation between impulse response $h(t)$ and step response $s(t)$:

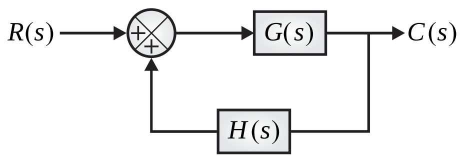
$$h(t) = \frac{d}{dt}s(t) \text{ or } s(t) = \int_{-\infty}^{t} h(t)\,dt$$

5. Negative feedback :



Closed loop transfer function is, $T(s) = \dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$

6. Positive feedback :



Closed loop transfer function is, $T(s) = \dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1-G(s)H(s)}$

7. Sensitivity :

$$S_B^A = \frac{\partial A/A \times 100\%}{\partial B/B \times 100\%} = \frac{\%\text{ change in } A}{\%\text{ change in } B}$$

(i) Open loop system, $S_G^T = 1$, where $T = 1$

(ii) Closed loop system, $S_G^T = \dfrac{1}{1+GH}$, where $T = \dfrac{G}{1+GH}$

$$S_H^T = \frac{-GH}{1+GH} \approx -1 \qquad \text{[Since, } GH >> 1 \text{ very high open loop transfer function]}$$

8. Initial value theorem :

$$x(0^+) = \lim_{t\to 0} x(t) = \lim_{s\to\infty} sX(s)$$

Valid for both unstable and stable system.

Exception :

(i) Initial value theorem is not valid for improper transfer function (number of zeros ≥ number of poles).

➢ Sample Questions

1995 IIT Kanpur

1.1 The closed-loop transfer function of a control system is given by,

$$\frac{C(s)}{R(s)} = \frac{2(s-1)}{(s+2)(s+1)}$$

For a unit step input the output is

(A) $-3e^{-2t} + 4e^{-t} - 1$

(B) $-3e^{-2t} - 4e^{-t} + 1$

(C) zero

(D) infinity

1996 IISc Bangalore

1.2 The unit-impulse response of a unity feedback control system is given by $c(t) = -te^{-t} + 2e^{-t}$, $(t \geq 0)$.

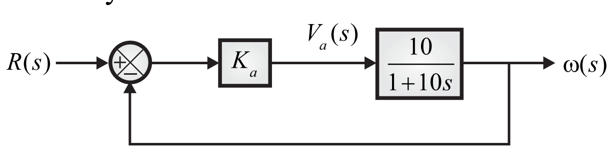
The open loop transfer function is equal to

(A) $\frac{2s+1}{(s+1)^2}$ (B) $\frac{2s+1}{s^2}$

(C) $\frac{s+1}{(s+2)^2}$ (D) $\frac{s+1}{s^2}$

2013 IIT Bombay

1.3 The open-loop transfer function of a dc motor is given as $\frac{\omega(s)}{V_a(s)} = \frac{10}{1+10s}$. When connected in feedback as shown below, the approximate value of K_a that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open-loop system is



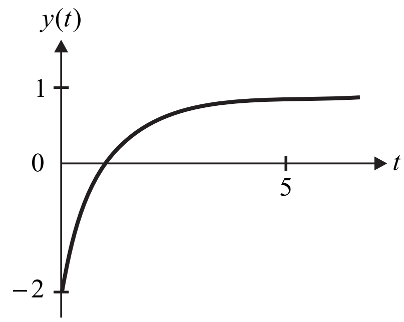
(A) 1 (B) 5

(C) 10 (D) 100

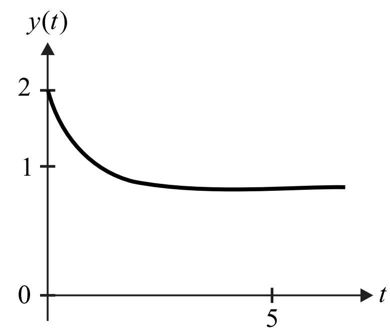
2015 IIT Kanpur

1.4 The unit step response of a system with the transfer function $G(s) = \frac{1-2s}{1+s}$ is given by which one of the following waveforms? **[Set - 01]**

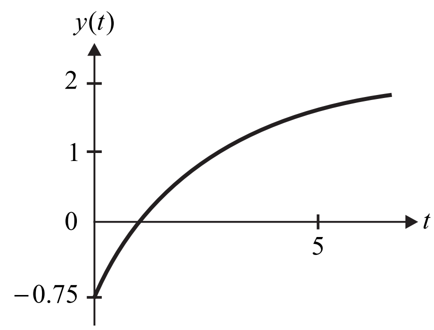
(A)



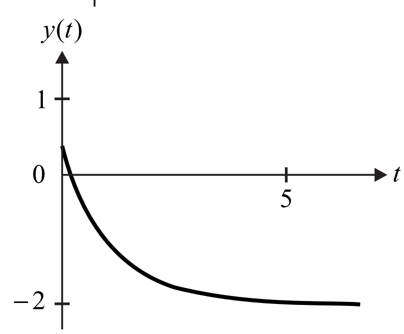
(B)



(C)



(D)



❖❖❖❖

Explanations Basics of Control System

1.1 (A)

Given : Closed loop transfer function is,

$$\frac{C(s)}{R(s)} = \frac{2(s-1)}{(s+2)(s+1)}$$

For step input, $r(t) = u(t)$

Taking Laplace transform of $r(t)$,

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{2(s-1)}{s(s+2)(s+1)}$$

Applying partial fraction,

$$\frac{2(s-1)}{s(s+2)(s+1)} = \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+1)}$$

$$A = sC(s)\big|_{s=0} = -1$$

$$B = (s+2)C(s)\big|_{s=-2} = -3$$

$$C = (s+1)C(s)\big|_{s=-1} = 4$$

Then, $$C(s) = \frac{-1}{s} + \frac{-3}{s+2} + \frac{4}{s+1}$$

Taking inverse Laplace transform of $C(s)$,

$$c(t) = \left[-1 - 3e^{-2t} + 4e^{-t}\right]u(t)$$

Hence, the correct option is (A).

1.2 (B)

Given : Unit-impulse response of unity feedback system is,

$$c(t) = -te^{-t} + 2e^{-t},\ (t \geq 0)$$

Input, $r(t) = \delta(t)$

Taking Laplace transform of $r(t)$ and $c(t)$,

$$R(s) = 1$$

$$C(s) = -\frac{1}{(s+1)^2} + \frac{2}{(s+1)}$$

$$C(s) = \frac{2s+1}{(s+1)^2}$$

Transfer function is given by,

$$T(s) = \frac{C(s)}{R(s)} = \frac{2s+1}{(s+1)^2} = \frac{2s+1}{s^2+2s+1}$$

Closed loop transfer function of a unity negative feedback system is given by,

$$T(s) = \frac{G(s)}{1+G(s)}$$

where, $G(s) =$ open loop transfer function

$$G(s) = \frac{T(s)}{1-T(s)} = \frac{\dfrac{2s+1}{s^2+2s+1}}{1-\dfrac{2s+1}{s^2+2s+1}}$$

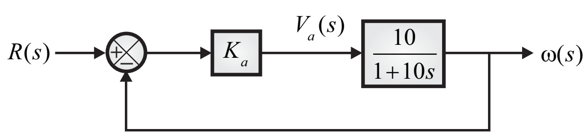
$$G(s) = \frac{2s+1}{s^2}$$

Hence, the correct option is (B).

1.3 (C)

Given : $\tau_{\text{closed loop}} = \frac{1}{100}\tau_{\text{open loop}}$

where, τ represents time constant.

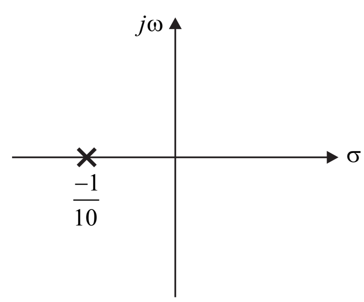


From figure,

Open loop transfer function of DC motor

$$= \frac{\omega(s)}{V_a(s)} = \frac{10}{1+10s}$$

Location of pole of open loop transfer function is shown below,



Time constant is defined as reciprocal of magnitude of negative real root.

Hence, $\tau_{\text{open loop}} = 10$ sec

$$\tau_{\text{closed loop}} = \frac{1}{100}\tau_{\text{open loop}} = 0.1 \text{ sec} \quad \ldots\text{(i)}$$

Closed-loop transfer function for negative unity feedback is given by,

$$T(s) = \frac{G(s)}{1+G(s)}$$

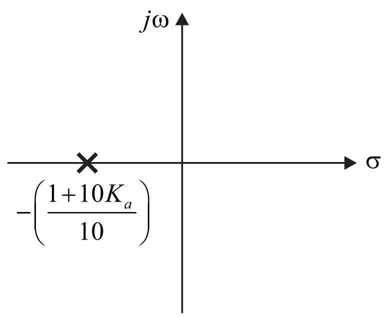
Here, $G(s) = K_a\left(\frac{10}{1+10s}\right)$

$$T(s) = \frac{\omega(s)}{R(s)} = \frac{K_a\left(\frac{10}{1+10s}\right)}{1+K_a\left(\frac{10}{1+10s}\right)}$$

$$T(s) = \frac{10K_a}{1+10s+10K_a}$$

$$= \frac{10K_a}{10s+(10K_a+1)}$$

Location of pole of closed loop transfer function is shown below,



From above figure,

$$\tau_{\text{closed loop}} = \frac{10}{10K_a+1} \quad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$\frac{10}{10K_a+1} = 0.1 \text{ sec}$$

$$\frac{10}{10K_a+1} = \frac{1}{10}$$

$$10K_a + 1 = 100$$

$$10K_a = 99$$

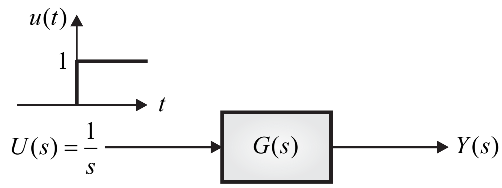
$$K_a = 9.9 \simeq 10$$

Hence, the correct option is (C).

Scan for Video Solution

1.4 (A)

Given : $G(s) = \frac{1-2s}{1+s}$



$$Y(s) = G(s)U(s)$$

$$Y(s) = \frac{(1-2s)}{(1+s)} \times \frac{1}{s}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s+1}$$

$$A = sY(s)\big|_{s=0}$$

$$A = \frac{s(1-2s)}{s(1+s)}\bigg|_{s=0} = 1$$

$$B = (s+1)Y(s)\big|_{s=-1}$$

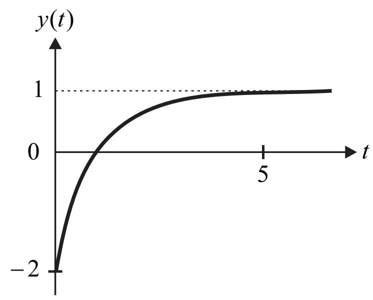
$$B = \frac{1-2s}{3}\bigg|_{s=-1} = -3$$

$$Y(s) = \frac{1}{s} + \frac{-3}{s+1}$$

Taking inverse Laplace transform,

$$y(t) = u(t) - 3e^{-t}u(t)$$

$$y(t) = (1-3e^{-t})u(t)$$



Hence, the correct option is (A).

❖❖❖❖

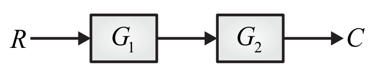
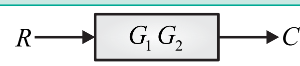
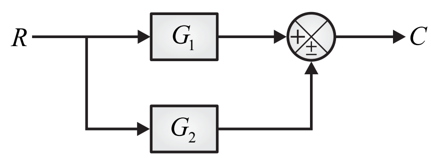
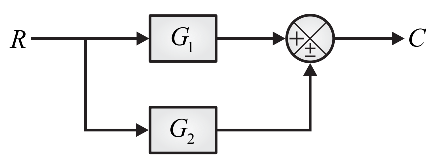
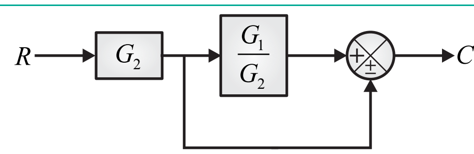
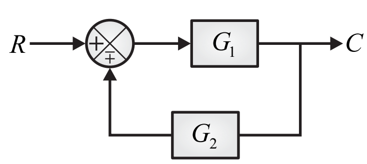
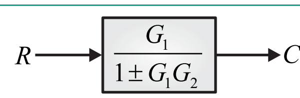
2 Block Diagram & Signal Flow Graph

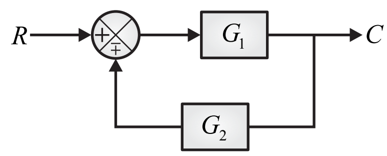
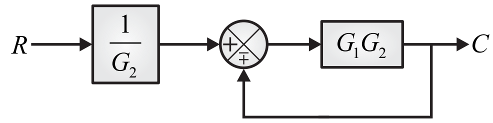
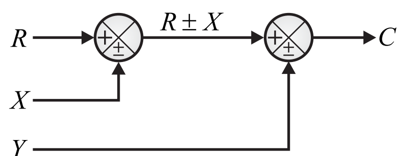
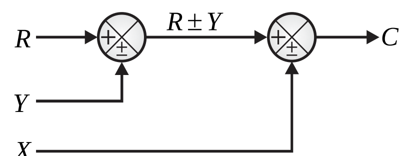
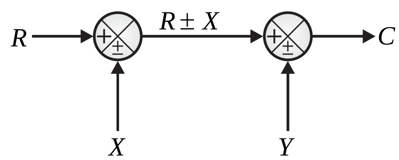
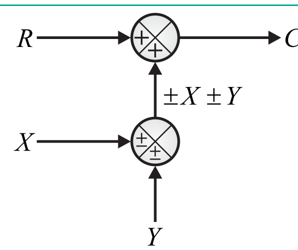
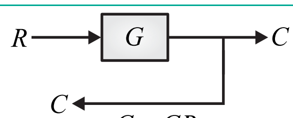
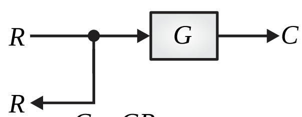
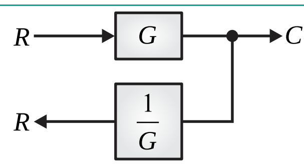
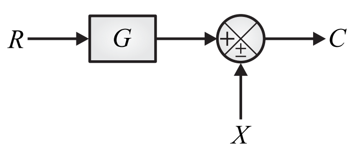
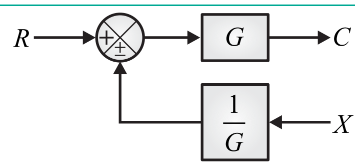
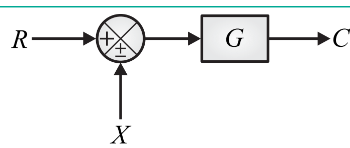
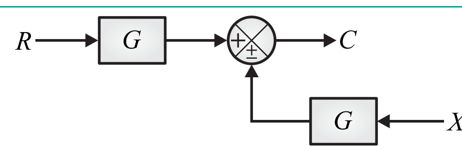
➢ Partial Synopsis

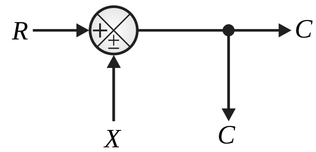
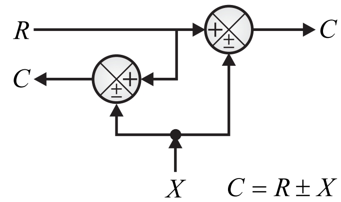
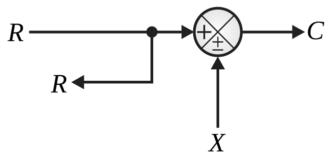
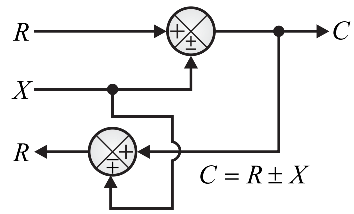
Block Diagram Reduction Rules

Block diagram reduction is used for simplifying (reducing) the block diagram, which is having many blocks, summing points and take off points and to obtain the overall transfer function.

Table : Block Diagram Reduction Rules

Transformation	Original Block Diagram	Equivalent Block Diagram
1. Combining blocks in cascade		 $C = (G_1G_2)R$
2. Combining blocks in parallel or eliminating a forward loop		 $C = (G_1 \pm G_2)R$
3. Removing a block from a forward path	 $C = (G_1 \pm G_2)R$	 $C = G_2\left(\frac{G_1}{G_2} \pm 1\right)R$ $C = (G_1 \pm G_2)R$
4. Eliminating a feedback loop		 $C = \left(\frac{G_1}{1 \pm G_1G_2}\right)R$

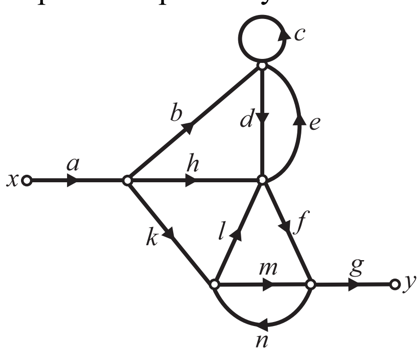
5. Removing a block from a feedback loop	 $C = \frac{G_1}{1 \pm G_1 G_2} R$	 $C = \left(\frac{G_1 G_2}{1 \pm G_1 G_2}\right)\frac{1}{G_2} \times R$ $C = \frac{G_1}{1 \pm G_1 G_2} R$
6. Rearranging summing points	 $C = R \pm X \pm Y$	 $C = R \pm X \pm Y$
7. Rearranging summing points	 $C = R + (\pm X \pm Y)$	 $C = R + (\pm X \pm Y)$
8. Moving a take-off ahead of a block	 $C = GR$	 $C = GR$
9. Moving a take-off point beyond a block	 $C = GR$	 $C = GR$
10. Moving a summing point ahead of a block	 $C = RG \pm X$	 $C = G\left(R \pm \frac{1}{G} X\right) = RG \pm X$
11. Moving a summing point beyond a block	 $C = RG \pm XG = G(R \pm X)$	 $C = RG \pm GX = G(R \pm X)$

12. Moving a take-off point ahead of a summing point	 $C = R \pm X$	 $C = R \pm X$
13. Moving a take-off point beyond of a summing point	 $C = R \pm X$	 $C = R \pm X$

➢ Sample Questions

1991 IIT Madras

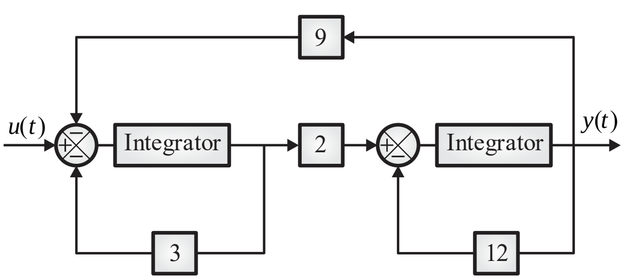
2.1 The signal flow graph of figure shown below has forward paths and feedback loops are respectively



(A) 4, 3 (B) 4, 4
(C) 3, 4 (D) 3, 3

2003 IIT Madras

2.2 The block diagram of a control system is shown in figure. The transfer function $G(s) = \dfrac{Y(s)}{U(s)}$ of the system is



(A) $\dfrac{1}{18\left(1+\dfrac{s}{12}\right)\left(1+\dfrac{s}{3}\right)}$

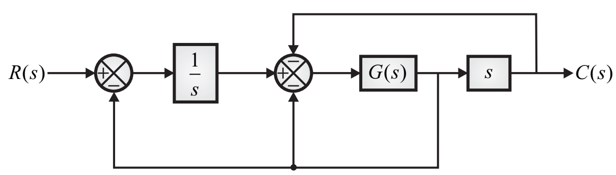
(B) $\dfrac{1}{27\left(1+\dfrac{s}{6}\right)\left(1+\dfrac{s}{9}\right)}$

(C) $\dfrac{1}{27\left(1+\dfrac{s}{12}\right)\left(1+\dfrac{s}{9}\right)}$

(D) $\dfrac{1}{27\left(1+\dfrac{s}{9}\right)\left(1+\dfrac{s}{3}\right)}$

2014 IIT Kharagpur

2.3 The block diagram of a system is shown in the figure.



If the desired transfer function of the system is $\dfrac{C(s)}{R(s)} = \dfrac{s}{s^2+s+2}$ then $G(s)$ is

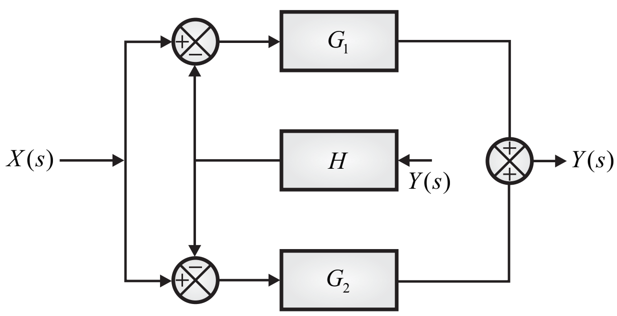
[Set - 03]

(A) 1 (B) s

(C) $1/s$ (D) $\dfrac{-s}{s^3+s^2-s-2}$

2015 IIT Kanpur

2.4 Find the transfer function $\dfrac{Y(s)}{X(s)}$ of the system given below. **[Set - 01]**



(A) $\dfrac{G_1}{1-HG_1}+\dfrac{G_2}{1-HG_2}$

(B) $\dfrac{G_1}{1+HG_1}+\dfrac{G_2}{1+HG_2}$

(C) $\dfrac{G_1+G_2}{1+H(G_1+G_2)}$

(D) $\dfrac{G_1+G_2}{1-H(G_1+G_2)}$

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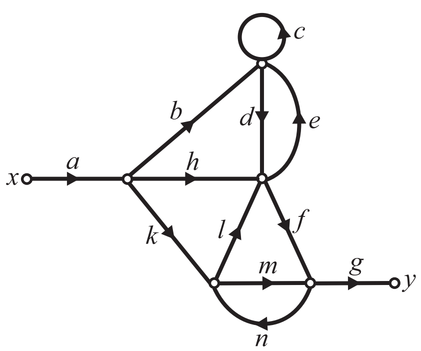
Explanations Block Diagram & Signal Flow Graph

2.1 (B)

The given signal flow graph is shown below,



Forward Path gain :

$$P_1 = abdfg,\ P_2 = ahfg,\ P_3 = aklfg,\ P_4 = akmg$$

Individual loop gain :

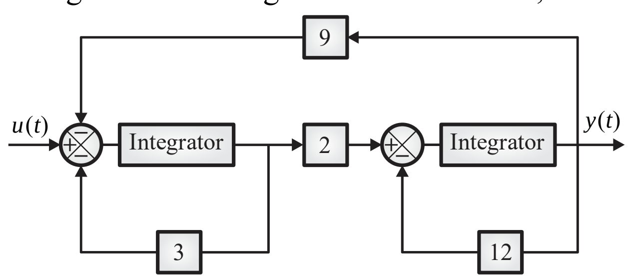
$$L_1 = c,\ L_2 = de,\ L_3 = lfn,\ L_4 = mn$$

Hence, the correct option is (B).

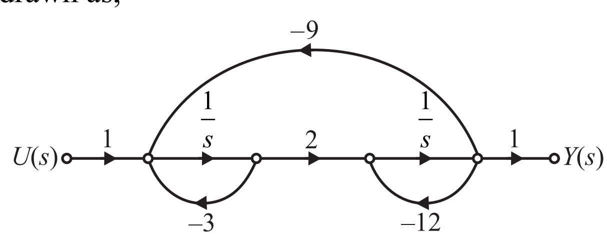
2.2 (B)

Method 1

The given block diagram is shown below,



Signal flow graph of the block diagram can be drawn as,



Forward Path gain : $P_1 = \dfrac{1}{s}\times 2\times\dfrac{1}{s} = \dfrac{2}{s^2}$

Individual loop gain :

$$L_1 = \frac{1}{s}\times(-3) = \frac{-3}{s},\ L_2 = \frac{1}{s}\times(-12) = \frac{-12}{s},$$

$$L_3 = \frac{-1}{s} \times 2 \times \frac{1}{s} \times 9 = \frac{-18}{s^2}$$

Number of two Non-touching loops :

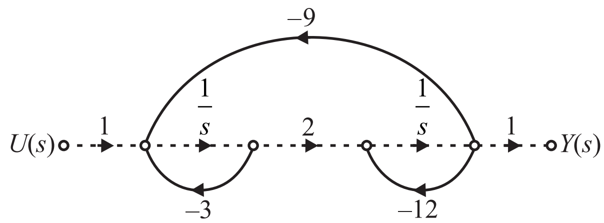
$$L_1 L_2 = \frac{36}{s^2}$$

Determinant :

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2)$$

$$= L_1 = 1 - \left(\frac{-3}{s} + \frac{-12}{s} + \frac{-18}{s^2}\right) + \frac{36}{s^2}$$

Path factor :



All the loops touch forward path.

$$\Delta_1 = 1 - (\text{Isolated loop gain})$$

$$\Delta_1 = 1 - 0 = 1$$

Using Mason's gain formula, transfer function is given by,

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

So, the transfer function is,

$$\frac{Y(s)}{U(s)} = \frac{P_1 \Delta_1}{\Delta}$$

$$\frac{Y(s)}{U(s)} = \frac{\frac{2}{s^2}}{1 + \frac{3}{s} + \frac{12}{s} + \frac{18}{s^2} + \frac{36}{s^2}}$$

$$\frac{Y(s)}{U(s)} = \frac{2}{s^2 + 15s + 54}$$

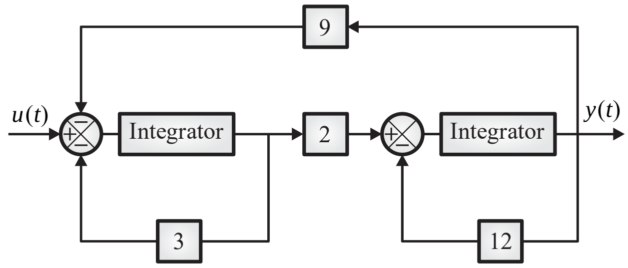
$$\frac{Y(s)}{U(s)} = \frac{2}{(s+6)(s+9)}$$

$$\frac{Y(s)}{U(s)} = \frac{1}{27\left(1 + \frac{s}{6}\right)\left(1 + \frac{s}{9}\right)}$$

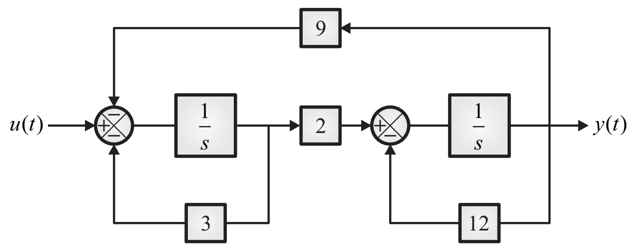
Hence, the correct option is (B).

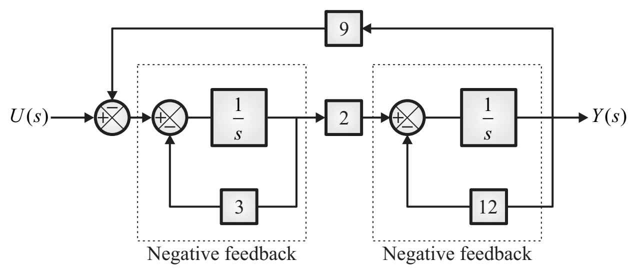
Method 2

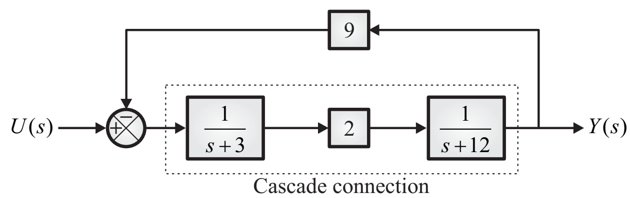
The given block diagram is shown below.

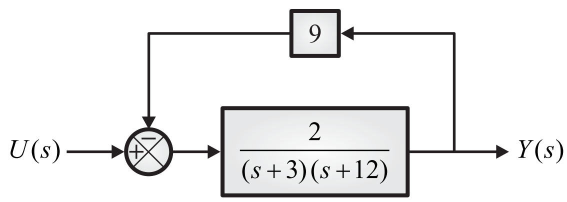


Integrator can be replaced by block $\frac{1}{s}$.









The transfer function of the system is

$$\frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$\frac{Y(s)}{U(s)} = \frac{\frac{2}{(s+3)(s+12)}}{1 + \frac{2 \times 9}{(s+3)(s+12)}}$$

$$\frac{Y(s)}{U(s)} = \frac{2}{s^2 + 15s + 54}$$

$$\frac{Y(s)}{U(s)} = \frac{2}{(s+6)(s+9)}$$

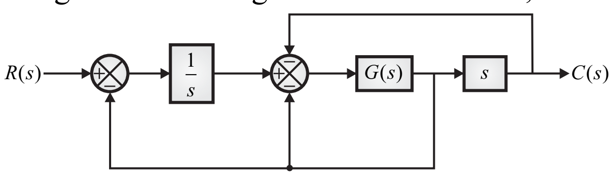
$$\frac{Y(s)}{U(s)} = \frac{1}{27\left(1+\frac{s}{6}\right)\left(1+\frac{s}{9}\right)}$$

Hence, the correct option is (B).

2.3 (B)

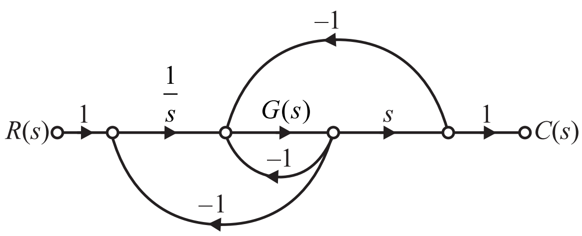
Given : $\frac{C(s)}{R(s)} = \frac{s}{s^2 + s + 2}$

The given block diagram is shown below,



Method 1

Signal flow graph representation of given block diagram is,



Forward path gain : $P_1 = \frac{1}{s} \times G(s) \times s = G(s)$

Individual loop gain :

$$L_1 = -G(s),\ L_2 = -sG(s), L_3 = \frac{-G(s)}{s}$$

Number of two non-touching loops : 0

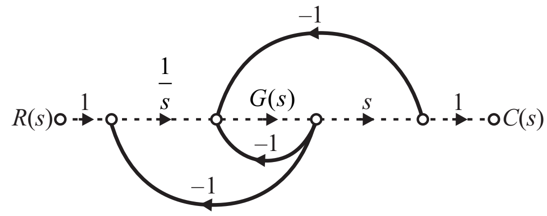
Determinant :

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$\Delta = 1 - \left(-G(s) - sG(s) - \frac{G(s)}{s}\right)$$

$$\Delta = 1 + G(s) + sG(s) + \frac{G(s)}{s}$$

Path factor :



All the loops touch forward path.

$$\Delta_1 = 1 - (\text{Isolated loop gain})$$

$$\Delta_1 = 1 - 0 = 1$$

Using Mason's gain formula, transfer function is given by,

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

So, the transfer function is

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) + sG(s) + \frac{G(s)}{s}}$$

For option (A) : $G(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{1}{1 + 1 + s + \frac{1}{s}} = \frac{s}{s^2 + 2s + 1}$$

It is not matching with given transfer function.

For option (B) : $G(s) = s$

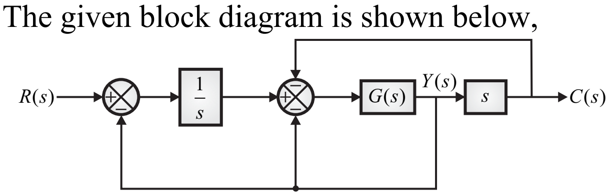
$$\frac{C(s)}{R(s)} = \frac{s}{1 + s + s^2 + 1} = \frac{s}{s^2 + s + 2}$$

This is the desired transfer function of the given system.

Hence, the correct option is (B).

Method 2

The given block diagram is shown below,



From figure,

$$\left\{\left[(R(s) - Y(s))\frac{1}{s}\right] - Y(s) - C(s)\right\}G(s) = Y(s)$$

...(i)

$$Y(s)=\frac{C(s)}{s} \quad \ldots(ii)$$

From equation (i),

$$[R(s)-Y(s)]\frac{G(s)}{s}-Y(s)G(s)-C(s)G(s)=Y(s)$$

$$\frac{R(s)G(s)}{s}-C(s)G(s)=Y(s)+\frac{Y(s)G(s)}{s}+Y(s)G(s)$$

$$\frac{R(s)G(s)}{s}-C(s)G(s)=Y(s)\left[1+G(s)+\frac{G(s)}{s}\right]$$

$$\frac{R(s)G(s)}{s}-C(s)G(s)=\frac{C(s)}{s}\left[1+G(s)+\frac{G(s)}{s}\right]$$

$$R(s)G(s)-sC(s)G(s)=C(s)\left[1+G(s)+\frac{G(s)}{s}\right]$$

$$R(s)G(s)=C(s)\left[1+sG(s)+G(s)+\frac{G(s)}{s}\right]$$

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)+sG(s)+\frac{G(s)}{s}}$$

For option (A) : $G(s)=1$

$$\frac{C(s)}{R(s)}=\frac{1}{1+1+s+\frac{1}{s}}=\frac{s}{s^2+2s+1}$$

It is not matching with given transfer function.

For option (B) : $G(s)=s$

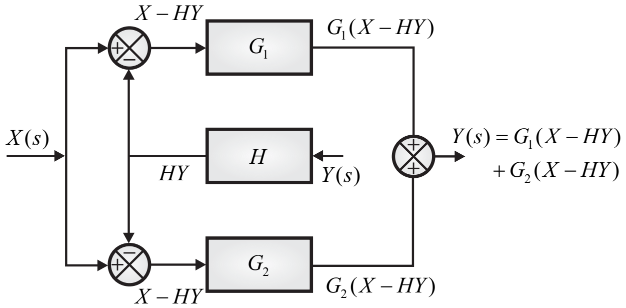
$$\frac{C(s)}{R(s)}=\frac{s}{1+s+s^2+1}=\frac{s}{s^2+s+2}$$

This is the desired transfer function of the given system.

Hence, the correct option is (B).

2.4 (C)

Given block diagram is shown below.



From the block diagram

$$Y=G_1(X-HY)+G_2(X-HY)$$

$$Y=X(G_1+G_2)-HY(G_1+G_2)$$

$$Y[1+H(G_1+G_2)]=X(G_1+G_2)$$

$$\frac{Y}{X}=\frac{G_1+G_2}{1+H(G_1+G_2)}$$

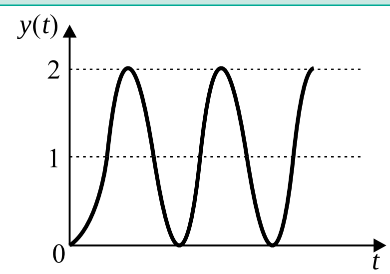
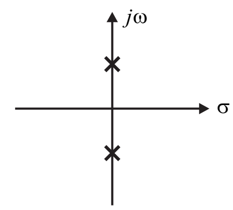
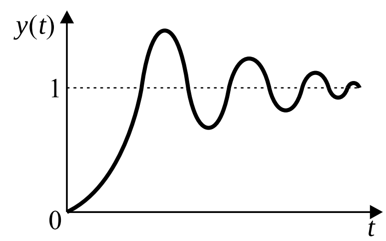
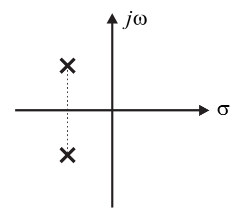
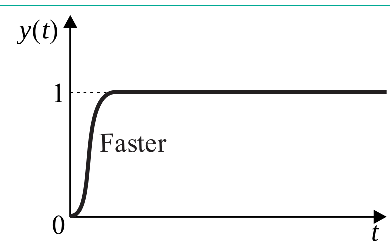
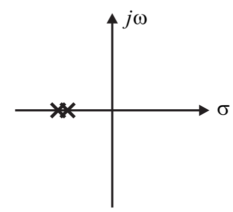
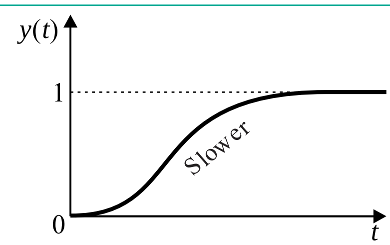
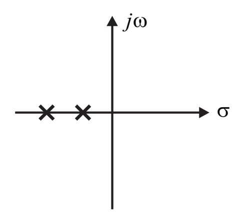
Hence, the correct option is (C).

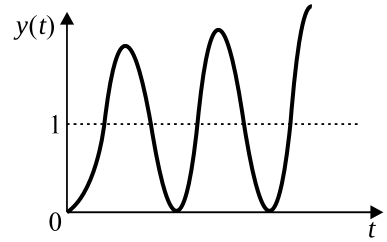
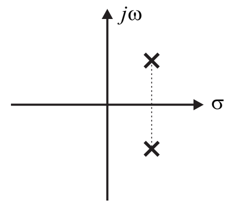
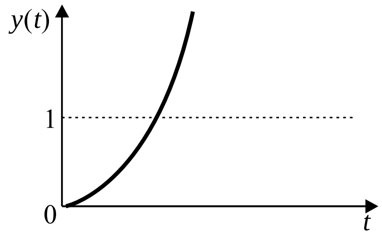
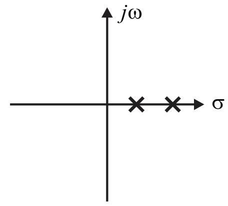
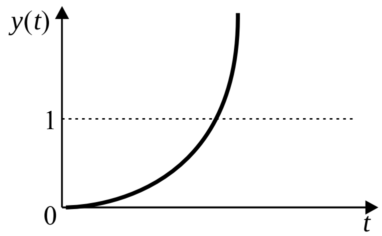
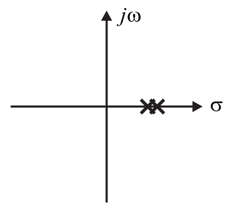
❖❖❖❖

3 Time Response Analysis

➢ Partial Synopsis

Table : Step-response comparison for various characteristics-equation and root locations in the *s*-plane

Damping factor (ξ)	System behavior	Step Response	Stability	Characteristics Roots
$\xi = 0$	Un-damped		Marginally stable	Imaginary 
$0 < \xi < 1$	Under-damped		Stable	Complex Conjugate with negative real part 
$\xi = 1$	Critically damped		Stable	Real, Equal, Negative 
$\xi > 1$	Over-damped		Stable	Real, Unequal, Negative 

$-1<\xi<0$	Negative Under-damped		Unstable	Complex Conjugate with positive real part 
$\xi<-1$	Negative Over-damped		Unstable	Real, Unequal, Positive 
$\xi=-1$	Negative critical damped		Unstable	Real, Equal, Positive 

Damping Ratio for Series and Parallel RLC Circuit

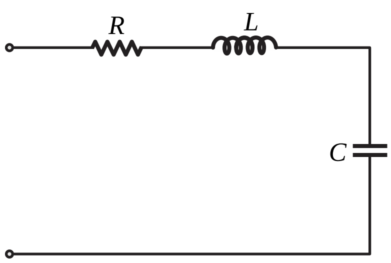


Fig. Series RLC circuit

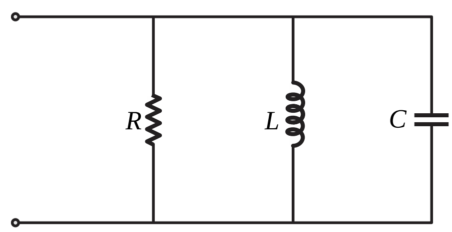


Fig. Parallel RLC circuit

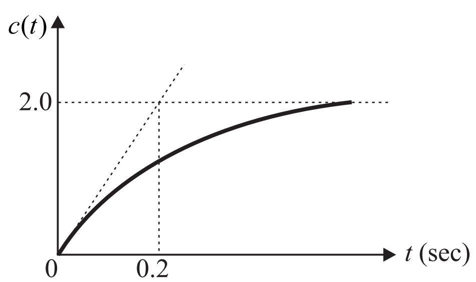
Damping	ξ	Series	Parallel
Un-damped	$\xi=0$	$\frac{R}{2}\sqrt{\frac{C}{L}}=0$ $R=0$	$\frac{1}{2R}\sqrt{\frac{L}{C}}=0$ $R=\infty$
Under-damped	$0<\xi<1$	$0<\frac{R}{2}\sqrt{\frac{C}{L}}<1$ $0<R<2\sqrt{\frac{L}{C}}$	$0<\frac{1}{2R}\sqrt{\frac{L}{C}}<1$ $0<\frac{1}{R}<2\sqrt{\frac{C}{L}}$ $\frac{1}{2}\sqrt{\frac{L}{C}}<R<\infty$

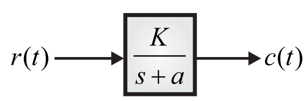
Critical-damped	$\xi = 1$	$\frac{R}{2}\sqrt{\frac{C}{L}} = 1$ $R = 2\sqrt{\frac{L}{C}}$	$\frac{1}{2R}\sqrt{\frac{L}{C}} = 1$ $R = \frac{1}{2}\sqrt{\frac{L}{C}}$
Over-damped	$\xi > 1$	$\frac{R}{2}\sqrt{\frac{C}{L}} > 1$ $R > 2\sqrt{\frac{L}{C}}$	$\frac{1}{2R}\sqrt{\frac{L}{C}} > 1$ $R < \frac{1}{2}\sqrt{\frac{L}{C}}$

➢ Sample Questions

1991 IIT Madras

3.1 A first order system and its response to a unit step input are shown in figure below, the system parameters a and K are respectively



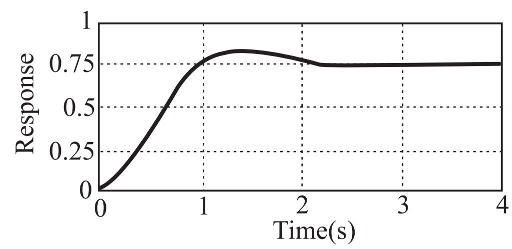


(A) 5, 10 (B) 10, 5

(C) 2, 10 (D) 10, 2

2009 IIT Roorkee

3.2 The unit-step response of a unity feedback system with open loop transfer function $G(s) = \frac{K}{(s+1)(s+2)}$ is shown in the figure. The value of K is



(A) 0.5 (B) 2

(C) 4 (D) 6

2010 IIT Guwahati

3.3 For the system $\frac{2}{(s+1)}$, the approximate time taken for a step response to reach 98% of its final value is

(A) 1 sec (B) 2 sec

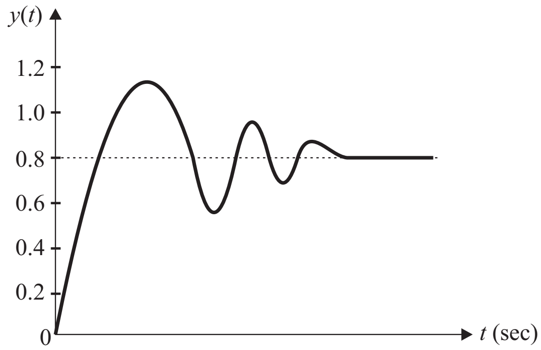
(C) 4 sec (D) 8 sec

2014 IIT Kharagpur

3.4 The closed loop transfer function of a system is $T(s) = \frac{4}{(s^2 + 0.4s + 4)}$. The steady state error due to unit step input is ____________. **[Set - 02]**

2018 IIT Guwahati

3.5 The unit step response $y(t)$ of a unity feedback system with open loop transfer function $G(s)H(s) = \frac{K}{(s+1)^2(s+2)}$ is shown in the figure. The value of K is __________ (up to 2 decimal places).

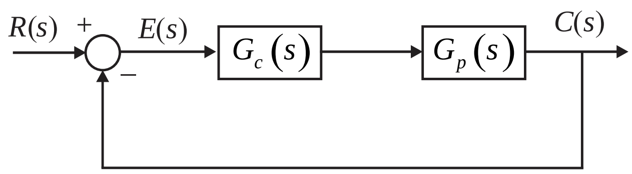


2021 IIT Bombay

3.6 Consider a closed loop system as shown.

$$G_p(s) = \frac{14.4}{s(1+0.1s)}$$

is the plant transfer function and $G_c(s) = 1$ is the compensator. For a unit-step input, the output response has damped oscillations. The damped natural frequency is____ rad/s. (Round off to 2 decimal places).



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Explanations — Time Response Analysis

Analysis of Steady State Error for Non-Unity Feedback System

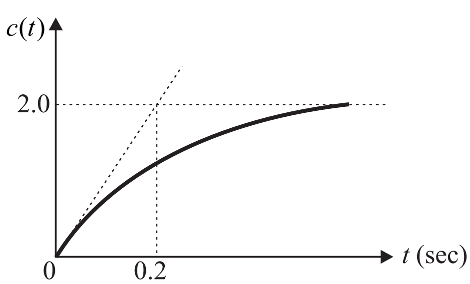
Scan for Video Explanation

Analysis of Steady State Error for Unity Feedback System

Scan for Video Explanation

3.1 (A)

Given :



From figure,

$$c(t) = 2(1-e^{-t/\tau}) \quad \ldots(i)$$

$$\frac{d}{dt}c(t) = \frac{2}{\tau}e^{-t/\tau}$$

$$\left.\frac{d}{dt}c(t)\right|_{t=0} = \frac{2}{\tau} \quad \ldots(ii)$$

From figure,

$$\left.\frac{d}{dt}c(t)\right|_{t=0} = \frac{2-0}{0.2-0} = 10 \quad \ldots(iii)$$

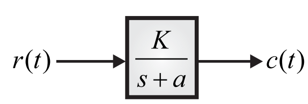
From equation (ii) and (iii),

$$10 = \frac{2}{\tau}$$

$$\tau = 0.2$$

From equation (i),

$$c(t) = 2(1-e^{-t/0.2}) = 2(1-e^{-5t}) \quad \ldots(iv)$$



$$\frac{C(s)}{R(s)} = \frac{K}{s+a}$$

For unit step input $R(s) = \dfrac{1}{s}$

$$C(s)=\frac{K}{s(s+a)}$$

$$C(s)=\frac{K}{a}\left(\frac{1}{s}-\frac{1}{s+a}\right)$$

Taking inverse Laplace transform,

$$c(t)=\frac{K}{a}(1-e^{-at}) \quad \ldots(v)$$

From equation (iv) and (v),

$$a=\frac{1}{\tau}=5$$

$$\frac{K}{a}=2 \Rightarrow \quad K=2a=2\times 5=10$$

Hence, the correct option is (A).

3.2 (D)

Given : $G(s)=\dfrac{K}{(s+1)(s+2)}$

Unit step input, $R(s)=\dfrac{1}{s}$

From figure,

$$\lim_{t\to\infty} y(t)=\lim_{s\to 0} sY(s)=0.75 \quad \ldots(i)$$

Closed-loop transfer function for unity negative feedback is given by,

$$\frac{Y(s)}{R(s)}=\frac{G(s)}{1+G(s)}$$

$$\frac{Y(s)}{R(s)}=\frac{\dfrac{K}{(s+1)(s+2)}}{1+\dfrac{K}{(s+1)(s+2)}}=\frac{K}{s^2+3s+2+K}$$

$$Y(s)=\frac{1}{s}\frac{K}{(s^2+3s+2+K)}$$

Steady state value is given by,

$$\lim_{s\to 0} sY(s)=\lim_{s\to 0} s\frac{1}{s}\frac{K}{(s^2+3s+2+K)}=0.75$$

$$\frac{K}{(2+K)}=0.75 \Rightarrow K=6$$

Hence, the correct option is (D).

3.3 (C)

Method 1

Given : $G(s)=\dfrac{2}{1+s}$ …(i)

Standard first order system is given by,

$$G(s)=\frac{K}{1+s\tau} \quad \ldots(ii)$$

where, $K=$ DC gain

$\tau=$ Time constant

From equations (i) and (ii),

$K=2,\ \tau=1$ sec

The time taken by step response to reach 98% of its final value i.e. settling time.

$t_s\approx 4\tau=4$ sec

Method 2

$$G(s)=\frac{C(s)}{R(s)}=\frac{2}{1+s}$$

For unit step input $R(s)=\dfrac{1}{s}$

Response, $C(s)=\dfrac{2}{s(1+s)}=\dfrac{2}{s}-\dfrac{2}{1+s}$

Taking inverse Laplace transform,

$$c(t)=2\left[1-e^{-t}\right]$$

Final or steady state response is given by,

$$c_{ss}=\lim_{t\to\infty} c(t)=2$$

At settling time t_s, output is given by,

$$c(t_s)=2[1-e^{-t_s}]$$

$$98\%\text{ of } 2=2[1-e^{-t_s}]$$

$$1.96=2\left[1-e^{-t_s}\right]$$

$$e^{-t_s}=0.02$$

$$t_s=-\ln(0.02)=3.91\text{ sec}$$

$$t_s\approx 4\text{ sec}$$

Hence, the correct option is (C).

3.4 **0**

Given : The closed loop transfer function of a system is

$$T(s)=\frac{4}{(s^2+0.4s+4)}$$

$$T(s)=\frac{\dfrac{4}{s^2+0.4s}}{\left(1+\dfrac{4}{s^2+0.4s}\right)}$$

$$T(s)=\frac{\dfrac{4}{s(s+0.4)}}{\left(1+\dfrac{4}{s(s+0.4)}\right)} \quad \text{...(i)}$$

Closed-loop transfer function for negative unity feedback system is given by,

$$T(s)=\frac{G(s)}{1+G(s)} \quad \text{...(ii)}$$

On comparing equation (i) and equation (ii),
Open loop transfer function,

$$G(s)=\frac{4}{s(s+0.4)} \qquad \text{[Type 1 system]}$$

For unit step input (position input) steady state error is given by,

$$e_{ss}=\frac{1}{1+K_p}$$

where, K_p is the position error coefficient.

The position error coefficient is given by,

$$K_p=\lim_{s\to 0} G(s)$$

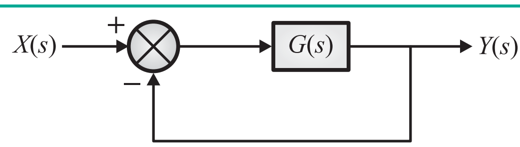
$$K_p=\lim_{s\to 0}\frac{K}{s(s+0.4)}=\infty$$

$$e_{ss}=\frac{1}{1+\infty}=0 \qquad \textbf{[Refer Table 3.1]}$$

Hence, the steady state error is **0.**

3.5 **8**

Given : $G(s)H(s)=\dfrac{K}{(s+1)^2(s+2)}$



CLTF is given by,

$$\frac{Y(s)}{X(s)}=\frac{G(s)}{1+G(s)H(s)}=\frac{G(s)}{1+G(s)}$$

[Since, $H(s)=1$]

$$\frac{Y(s)}{X(s)}=\frac{\dfrac{K}{(s+1)^2(s+2)}}{1+\dfrac{K}{(s+1)^2(s+2)}}$$

$$\frac{Y(s)}{X(s)}=\frac{K}{(s+1)^2(s+2)+K}$$

$$Y(s)=\frac{1}{s}\times\frac{K}{(s+1)^2(s+2)+K}$$

From time response shown in the figure steady state value in time domain is 0.8 i.e. $y(\infty)=0.8$

Final value theorem is given by,

$$y(\infty)=\lim_{s\to 0} sY(s)$$

$$0.8=\frac{K}{2+K}$$

$$K=1.6+0.8K$$

$$K=8$$

Hence, the value of K is **8.**

Scan for Video Solution

3.6 **10.90**

Given : $G_p(s)=\dfrac{14.4}{s(1+0.1s)}$

$$G_p(s)=\frac{14.4}{0.1s(s+10)}$$

$$G_p(s)=\frac{144}{s(s+10)}$$

and $\quad G_c(s)=1$

$$\therefore \qquad G_p(s)_{CLTF} = \frac{144}{s^2+10s+144} \qquad \ldots(i)$$

Now, comparing with the standard equation,

$$\Rightarrow \qquad \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \qquad \ldots(ii)$$

From equation (i) and (ii),

$$\omega_n = 12$$

$$2\xi\omega_n = 10$$

$$\xi = \frac{10}{2\times 12} = \frac{5}{12}$$

Now, $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$\omega_d = 12\sqrt{1-\left(\frac{5}{12}\right)^2} = 12\sqrt{\frac{144-25}{144}}$$

$$\omega_d = 12\sqrt{\frac{119}{144}} = 10.90$$

Hence, the correct answer is 10.90.

❖❖❖❖

5 Root Locus

➢ Partial Synopsis

Rules of Sketching Root Locus

1. Root locus is always symmetrical about the real axis (i.e. x-axis/ σ- axis).
2. Root locus always start from open loop pole $(K=0)$ and terminate either at finite open loop zero or at infinity that means virtual zero $(K=\infty)$.
3. **Existence of any point on root locus :**
   (i) The entire real axis of s-plane is occupied by the root locus for all values of K (i.e. $-\infty \le K \le \infty$).
   (ii) Root locus for $K \ge 0$ are found in the section if the sum of the number of open loop poles and zeros to the right of the section is odd.
   (iii)The remaining section of the real axis are occupied by the root locus for $K \le 0$ (i.e. complementary root locus).
   (iv)Open loop pole and zero are considered as the part of root locus, do not check even and odd concept at this point.
4. **Existence of root locus on real axis :**
   Root locus will exist only on that section of real axis, to the right of which sum of all poles and zeros is an odd number.
5. **Existence of root locus in complex plane/real axis :**
   (i) A point of s-plane will lie on root locus if the angle of $G(s)H(s)$ evaluated at that point is an odd integer multiple of $\pm 180^0$.
   (ii) Substitute the given complex location into the characteristic equation and then calculate the value of K.
   (iii)If K is real and positive for a given complex location then closed loop pole will exist at that location.
   (iv)If K is either negative or imaginary or complex for a given complex location then the location will be invalid and close loop pole will not exist at that location.
6. **Number of root locus branches :**

(i) Number of root locus branches $= P(\text{if}, P > Z)$

(ii) Number of root locus branches $= Z(\text{if}, Z > P)$

(iii) Number of root locus branches $= P = Z(\text{if}, P = Z)$

Note: Number of root locus branch $= \max(P, Z)$

7. **Break points/Saddle points :**

(i) The point at which root locus branches meet or diverge is known as break point or saddle point.

(ii) There are two types of break point :

(a) Break away point (BAP)

(b) Break in point (BIP)

To find break point ;

Step 1 : At first we have to find the characteristics equation, i.e., $1+G(s)H(s)=0$.

Step 2 : Then we have to find the value of K in terms of s, i.e. $K = f(s)$.

Step 3 : Find $\dfrac{dK}{ds}=0$

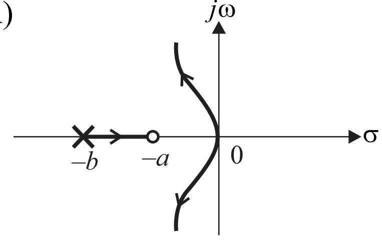
Step 4 : Valid roots of $\dfrac{dK}{ds}=0$ will be the valid break points.

➢ Sample Questions

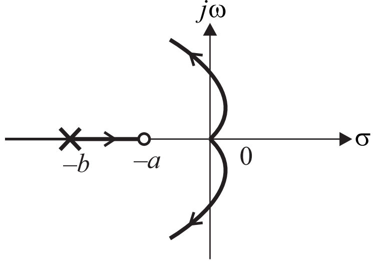
1991 IIT Madras

5.1 A unity feedback system has an open loop transfer function of the form $G(s)=\dfrac{K(s+a)}{s^2(s+b)}; b > a$ which of the loci shown in figure can be valid root-loci for the system?

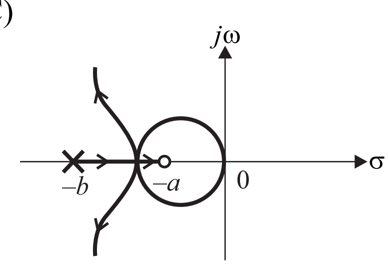
(A)



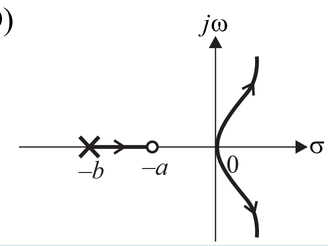
(B)



(C)



(D)

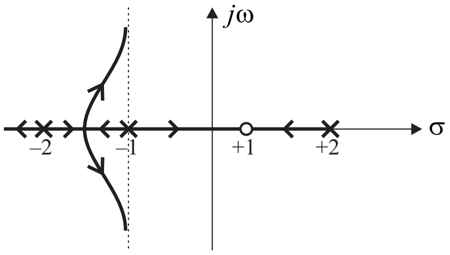


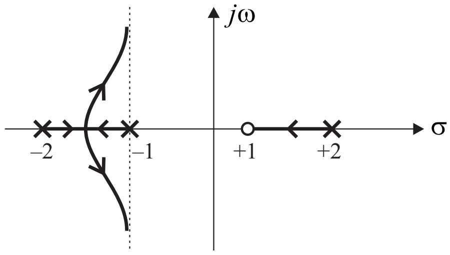
2006 IIT Kharagpur

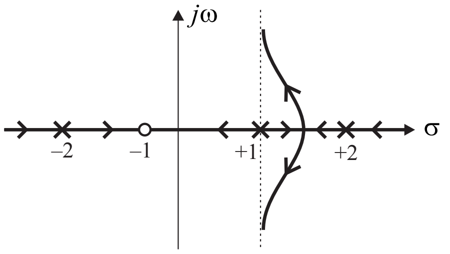
5.2 A closed-loop system has the characteristic function

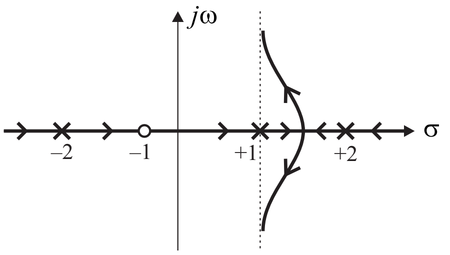
$$(s^2-4)\ (s+1)+K(s-1)=0\ .$$

Its root locus plot against K is

(A)


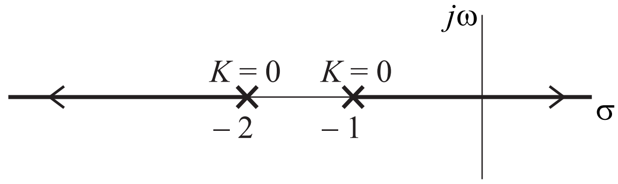
(B)


(C)


(D)


2014 IIT Kharagpur

5.3 The root locus of a unity feedback system is shown in the figure.



The closed loop transfer function of the system is **[Set - 01]**

(A) $\dfrac{C(s)}{R(s)} = \dfrac{K}{(s+1)(s+2)}$

(B) $\dfrac{C(s)}{R(s)} = \dfrac{-K}{(s+1)(s+2)+K}$

(C) $\dfrac{C(s)}{R(s)} = \dfrac{K}{(s+1)(s+2)-K}$

(D) $\dfrac{C(s)}{R(s)} = \dfrac{K}{(s+1)(s+2)+K}$

2016 IISc Bangalore

5.4 The gain at the breakaway point of the root locus of a unity feedback system with open loop transfer function

$G(s) = \dfrac{Ks}{(s-1)(s-4)}$ is **[Set - 02]**

(A) 1 (B) 2

(C) 5 (D) 9

❖❖❖❖

Explanations Root Locus

(A, C)

Given : Open loop transfer function is given by,

$$\text{OLTF} = \frac{K(s+a)}{s^2(s+b)};\ b > a$$

Let us consider two cases as explained below,

Case 1 : $\boldsymbol{a = \dfrac{4}{3}}$ **and** $\boldsymbol{b = 12}$

(i) Number of poles and zeros :

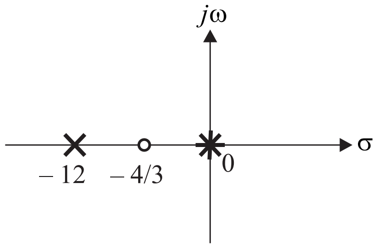
Number of zero $= 1$

Number of poles $= 3$

(ii) Location are poles and zeros :

Location of zero, $s = -\dfrac{4}{3}$

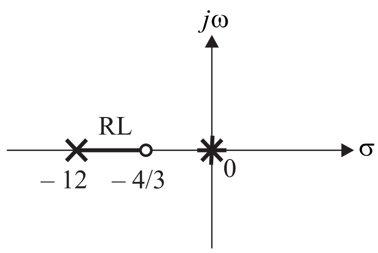
Location of poles, $s = 0, 0, -12$



(iii) Root locus branch on real axis :

Any section on real axis will be part of root locus branch, if the sum of number of open loop poles and open loop zeros on real axis to right of this section will be odd.

Hence, root locus branch on real axis will lie between $-12 < \sigma < -\frac{4}{3}$



(iv) Number of branches (B) :

$B = P = 3 \qquad (P > Z)$

(v) Number of asymptotes (A) :

$A = P - Z = 3 - 1 = 2 \quad (P > Z)$

(vi) Angle of asymptotes $(\angle A)$:

Angle of asymptotes is given by,

$$\angle A = \frac{(2\alpha + 1) \times 180^0}{P - Z}$$

Where, $\alpha = 0, 1, 2, ... \quad (P - Z - 1)$

$\alpha = 0, 1$

$\angle A = 90^0, 270^0$

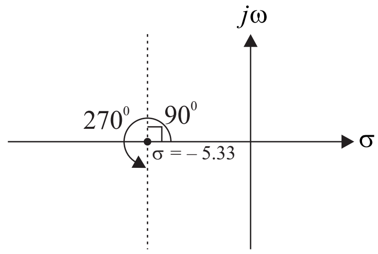
(vii) Centroid (σ) :

The intersection point of asymptotes on the real axis is called centroid.

Centroid is given by,

$$\sigma = \frac{\sum \text{Re(Poles)} - \sum \text{Re(Zeros)}}{P - Z}$$

$$\sigma = \frac{-12 + \frac{4}{3}}{3 - 1} = -5.33$$



(viii) Break-away/break-in point :

Characteristics equation is given by,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K\left(s + \frac{4}{3}\right)}{s^2(s + 12)} = 0$$

$$K = \frac{-s^2(s + 12)}{\left(s + \frac{4}{3}\right)} = \frac{-(s^3 + 12s^2)}{\left(s + \frac{4}{3}\right)}$$

$$\frac{dK}{ds} = 0$$

$$\frac{d}{ds}\left[\frac{-(s^3 + 12s^2)}{\left(s + \frac{4}{3}\right)}\right] = 0$$

$$-\frac{\left(s + \frac{4}{3}\right)(3s^2 + 24s) - (s^3 + 12s^2)}{\left(s + \frac{4}{3}\right)^2} = 0$$

$$3s^3 + 24s^2 + 4s^2 + 32s - s^3 - 12s^2 = 0$$

$$2s^3 + 16s^2 + 32s = 0$$

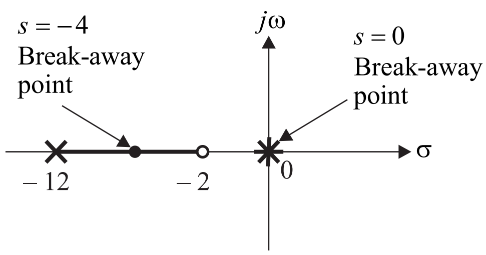
$$s(s^2 + 8s + 16) = 0$$

$$s(s + 4)^2 = 0$$

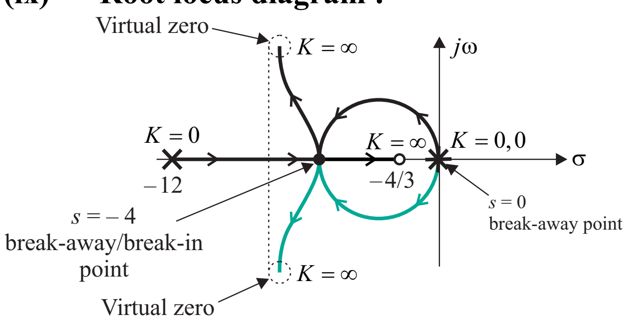
$$s = 0, -4, -4$$

Since, point $s=-4$ is lying on root locus branch of real axis, hence it is a valid **break-away point**.

Since, point $s=0$ is a multiple pole, hence it is a valid **break away point**.



(ix) Root locus diagram :



Case 2 : $a=2$ and $b=4$

(i) Number of poles and zeros :

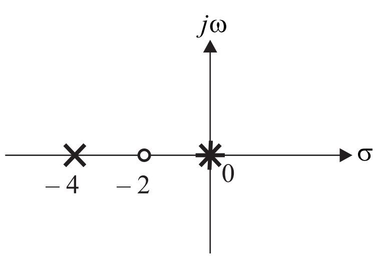
Number of zero $=1$

Number of poles $=3$

(ii) Location are poles and zeros :

Location of zero, $s=-2$

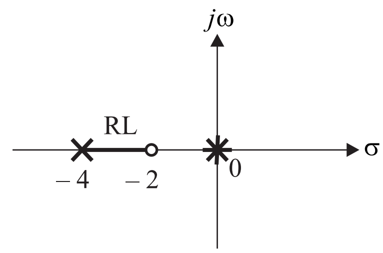
Location of poles, $s=0,\ 0,\ -4$



(iii) Root locus branch on real axis :

Any section on real axis will be part of root locus branch, if the sum of number of open loop poles and open loop zeros on real axis to right of this section will be odd.

Hence, root locus branch on real axis will lie between $-4<\sigma<-2$



(iv) Number of branches (B) :

$B=P=3 \qquad (P>Z)$

(v) Number of asymptotes (A) :

$A=P-Z=3-1=2 \quad (P>Z)$

(vi) Angle of asymptotes $(\angle A)$:

Angle of asymptotes is given by,

$$\angle A=\frac{(2\alpha+1)\times 180^0}{P-Z}$$

Where, $\alpha=0,\ 1,\ 2,\ .....\quad (P-Z-1)$

$\alpha=0,\ 1$

$\angle A=90^0,\ 270^0$

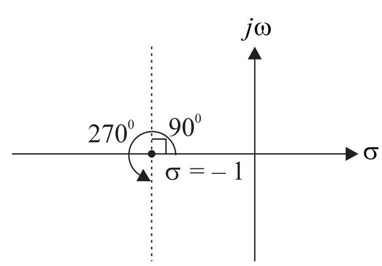
(vii) Centroid (σ) :

The intersection point of asymptotes on the real axis is called centroid.

Centroid is given by,

$$\sigma=\frac{\sum \text{Re(Poles)}-\sum \text{Re(Zeros)}}{P-Z}$$

$$\sigma=\frac{-4+2}{2}=-1$$



(viii) Break-away/break-in point :

Characteristics equation is given by,

$1+G(s)H(s)=0$

$$K = \frac{-s^2(s+4)}{(s+2)} = \frac{-(s^3+4s^2)}{(s+2)}$$

$$\frac{d}{ds}\left[\frac{-(s^3+4s^2)}{(s+2)}\right] = 0$$

$$-\frac{(s+2)(3s^2+8s)-(s^3+4s^2)}{(s+2)^2} = 0$$

$$3s^3+8s^2+6s^2+16s-s^3-4s^2 = 0$$

$$2s^3+10s^2+16s = 0$$

$$s(2s^2+10s+16) = 0$$

$$s = 0,\ s = -2.5 \pm j1.322$$

Since, point $s=0$ is a multiple pole, hence it is a **valid break away point**.

At point $s = -2.5 + j1.332$,

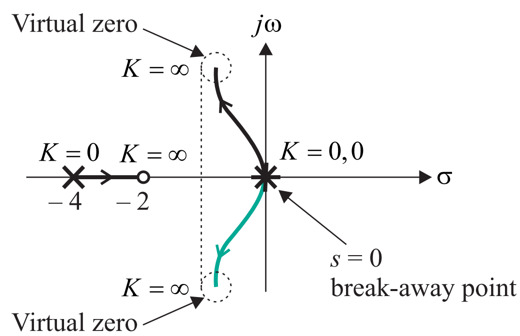
$$K = \frac{-s^2(s+4)}{(s+2)}$$

$$K = \frac{-(-2.5+j1.322)^2(-2.5+j1.322+4)}{(-2.5+j1.322+2)}$$

$$K = 6.5 + j9.26$$

Since, point $s = -2.5 + j1.332$ gives imaginary value of K, hence, it is **invalid break point.**

(ix) Root locus diagram :



Hence, the correct options are (A) and (C).

5.2 (B) 5.1

Given : $(s^2-4)(s+1)+K(s-1)=0$

$$1 + \frac{K(s-1)}{(s^2-4)(s+1)} = 0 \qquad \ldots\text{(i)}$$

Characteristic equation is given by,

$$1 + G(s)H(s) = 0 \qquad \ldots\text{(ii)}$$

On comparing equation (i) and (ii),

$$G(s)H(s) = \frac{K(s-1)}{(s^2-4)(s+1)}$$

Method 1 : Procedure Based

(i) Number of poles and zeros :

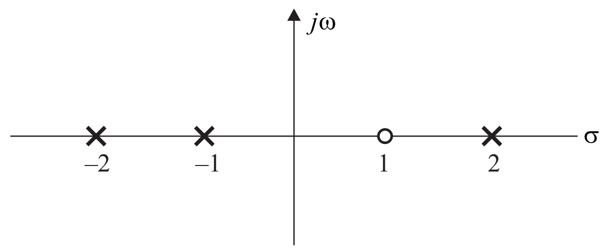
Number of zero = 1

Number of poles = 3

(ii) Location of poles and zeros :

Location of zero, $s = 1$

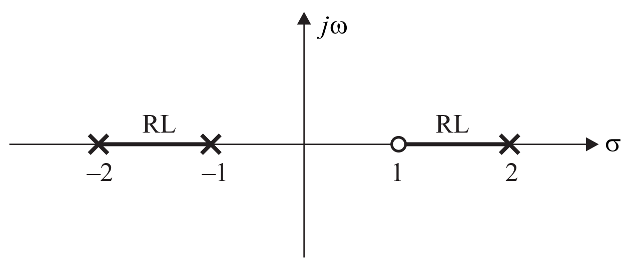
Location of poles, $s = -1, -2, 2$



(iii) Root locus branch on real axis :

Any section on real axis will be part of root locus branch, if the sum of the number of open loop poles and open loop zeros on real axis to right of this section will be odd.

Hence, root locus branch on real axis will lie between $1 < \sigma < 2$ and $-2 < \sigma < -1$.



(iv) Number of branches (B) :

$B = P = 3 \qquad (P > Z)$

(v) Number of asymptotes (A) :

$A = P - Z = 3 - 1 = 2$

(vi) Angle of asymptotes ($\angle A$) :

$$\angle A = \frac{(2\alpha+1)\times 180^0}{P-Z}$$

Where, $\alpha = 0, 1, 2, ...(P - Z - 1)$

$\alpha = 0, 1$

$\angle A = 90^0, 270^0$

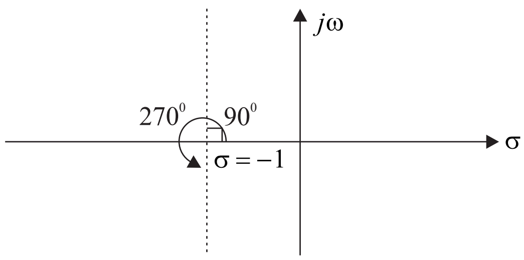
(vii) Centroid (σ) :

The intersection point of asymptotes on the real axis is called centroid.

Centroid is given by,

$$\sigma = \frac{\sum \text{Re(Poles)} - \sum \text{Re(Zeros)}}{P - Z}$$

$$\sigma = \frac{-1-1}{3-1} = -1$$



(viii) Break-away/break-in point :

Characteristic equation is given by,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s-1)}{(s^2-4)(s+1)} = 0$$

$$K = -\frac{(s^2-4)(s+1)}{s-1}$$

$$\frac{dK}{ds} = 0$$

$$\frac{d}{ds}\left[-\frac{(s^2-4)(s+1)}{s-1}\right] = 0$$

$$\frac{d}{ds}\left[\frac{s^3+s^2-4s-4}{s-1}\right] = 0$$

$$(3s^2+2s-4)(s-1) - (s^3+s^2-4s-4) = 0$$

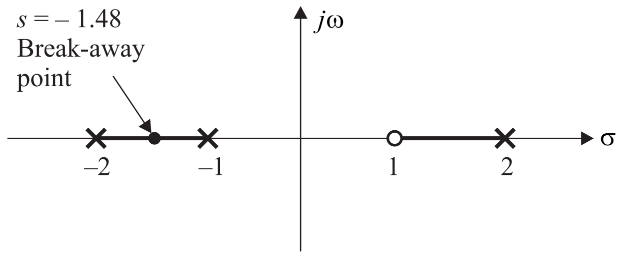
$$3s^3 + 2s^2 - 4s - 3s^2 - 2s$$
$$+4 - s^3 - s^2 + 4s + 4 = 0$$

$$2s^3 - 2s^2 - 2s + 8 = 0$$

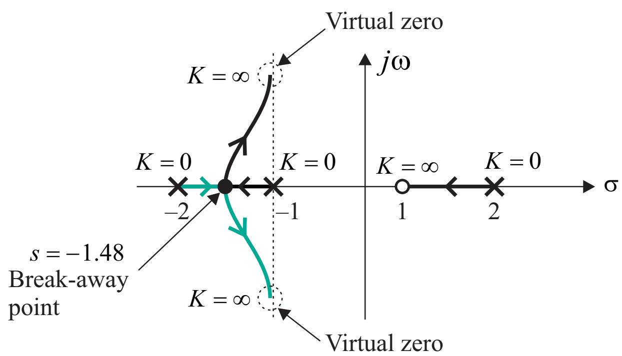
$$s = -1.48, 1.24 \pm 1.07j$$

Since, point $s = -1.48$ is lying between two adjacent poles on root locus branch of real axis, hence it will be **valid break-away point**.

Since, point $s = 1.24 \pm 1.07j$ is not lie in root locus branch. Hence, it is invalid break point.



(ix) Root locus diagram :



Hence, the correct option is (B).

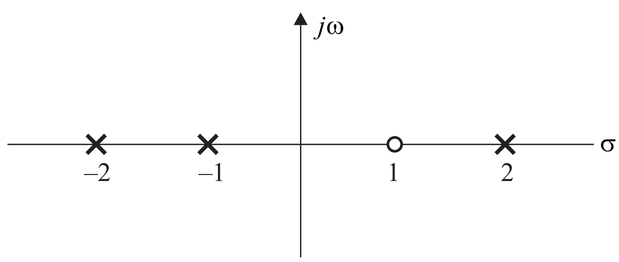
Method 2 : Concept Based

(i) Location of poles and zeros :

Location of zero, $s = 1$

Location of poles, $s = -2, -1, 2$

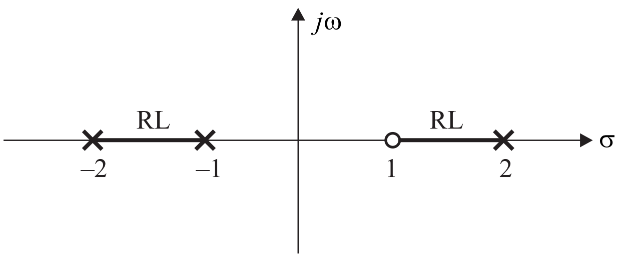
Hence, either option (A) or option (B) is correct.



(ii) Root locus branch on real axis :

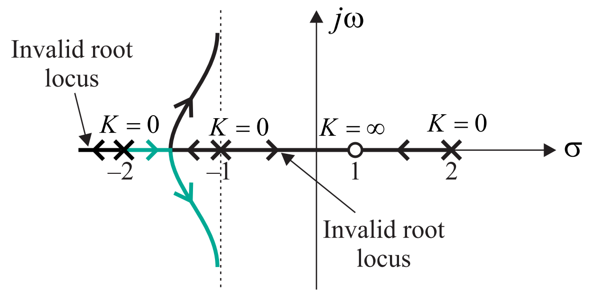
Any section on real axis will be part of root locus branch, if the sum of the number of open loop poles and open loop zeros on real axis to the right of this section will be odd.

Hence, root locus branch on real axis will lie between $-2<\sigma<-1$ and $1<\sigma<2$



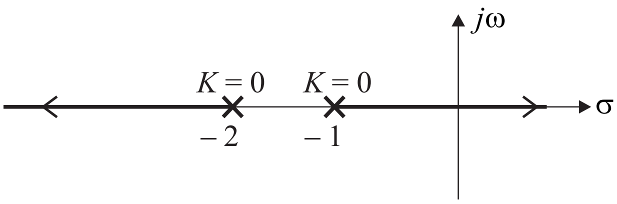
Hence, the correct option is (B).

Note : For option (A)



5.3 (C)

The given root locus diagram is shown below,



From above figure, root loci branches on real axis is lying between $-1<\sigma<\infty$ and $-\infty<\sigma<-2$, total number of open loop poles and zeros to the right of $-1<\sigma<\infty$ and $-\infty<\sigma<-2$ is even, hence given root locus is **complimentary or inverse root locus (CRL).** This indicates presence of positive feedback.

From above figure,

Number of poles = 2

Location of poles, $s=-1,-2$

Hence, based on location of poles and zeros OLTF is given by,

$$G(s)=\frac{K}{(s+1)(s+2)}$$

CLTF for unity positive feedback system with complementary root locus (CRL) is given by,

$$T(s)=\frac{C(s)}{R(s)}=\frac{G(s)}{1-G(s)}$$

$$\text{Hence, } T(s)=\frac{C(s)}{R(s)}=\frac{\frac{K}{(s+1)(s+2)}}{1-\frac{K}{(s+1)(s+2)}}$$

$$T(s)=\frac{C(s)}{R(s)}=\frac{K}{(s+1)(s+2)-K}$$

Hence, the correct option is (C).

5.4 (A)

Given : OLTF $= G(s)=\dfrac{Ks}{(s-1)(s-4)}$

(i) Number of poles and zeros :

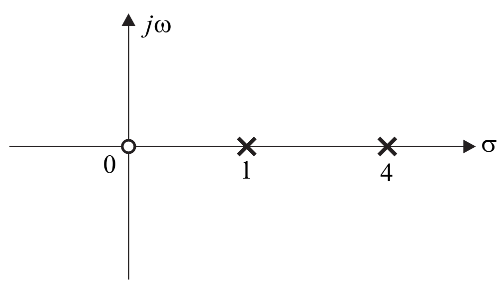
Number of zeros = 1

Number of poles = 2

(ii) Location of poles and zeros :

Location of zero, $s=0$

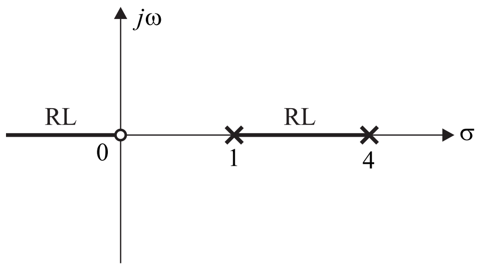
Location of poles, $s=1,4$



(iii) Root locus branch on real axis :

Any section on real axis will be part of root locus branch, if the sum of the number of open loop poles and open loop zeros on real axis to the right of this section will be odd.

Hence, root locus branch on real axis will lie between $1<\sigma<4$ and $\sigma<0$



(iv) Break-away/break-in point :

Characteristic equation is given by,

$$1+G(s)=0$$

$$1+\frac{Ks}{(s-1)(s-4)}=0$$

$$K=-\frac{(s-1)(s-4)}{s} \qquad \ldots\text{(i)}$$

$$\frac{dK}{ds}=0$$

$$\frac{d}{ds}\left[-\frac{s^2-5s+4}{s}\right]=0$$

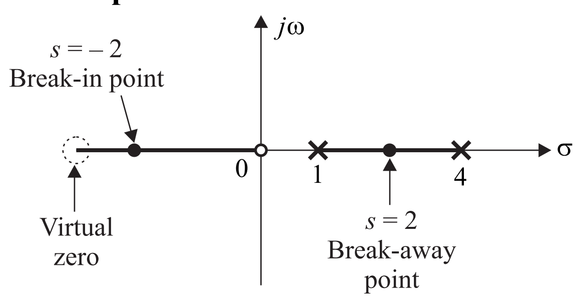
$$(2s-5)s-s^2+5s-4=0$$

$$2s^2-5s-s^2+5s-4=0$$

$$s^2-4=0 \quad \Rightarrow \quad s=-2,\ 2$$

Since, point $s=2$ is lying between two adjacent poles on root locus branch of real axis, hence it will be **break-away point**.

Since, point $s=-2$ is lying between two adjacent zeros (one real and one virtual zero) on root locus branch of real axis, hence it will be **break-in point**.



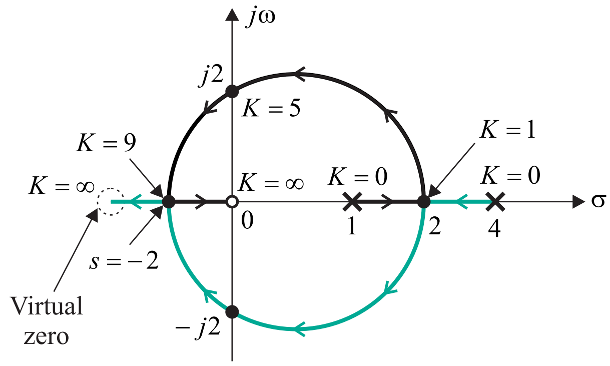
From equation (i), gain at the break away point $s=2$ is given by,

$$K|_{s=2}=-\frac{(2-1)(2-4)}{2}=1$$

Note : From equation (i), gain at the break-in point $s=2$ is given by,

$$K|_{s=-2}=-\frac{(-2-1)(-2-4)}{-2}=9$$

(v) Root locus diagram :



Hence, the correct option is (A).

Note : You can directly find break-in point from step (viii). We are providing all steps to draw root locus diagram.

Key Point

(i) Type of damping with the variation of K in given root locus diagram is given below,

Range of K	Damping	Location of closed loop poles	Stability
$0<K<1$	– ve over damping	Real and unequal	Unstable
$K=1$	– ve critical Damping	Real and equal	Unstable
$1<K<5$	– ve under damping	Complex conjugate	Unstable
$K=5$	Undamped	Imaginary	Marginal stable
$5<K<9$	Under damping	Complex conjugate	Stable
$K=9$	Critical damping	Real and equal	Stable
$9<K<\infty$	Over damping	Real and unequal	Stable

(ii) Root locus represent the path or locus of closed loop poles for different values of K.

❖❖❖❖

8 Bode Plot

➢ Partial Synopsis

Remember

1. If $|K| > 1 \Rightarrow 20\log|K|$ is positive.
2. If $|K| < 1 \Rightarrow 20\log|K|$ is negative.

Case 2 : Poles at origin [i.e. integral factors]

$$G(s) = \frac{1}{s^N}$$

Put $s = j\omega$, $G(j\omega) = \dfrac{1}{(j\omega)^N}$

In dB, magnitude can be written as,

$$M = 20\log_{10}|G(j\omega)| = 20\log_{10}\left|\frac{1}{(j\omega)^N}\right| = 20\log_{10}|(j\omega)^{-N}|$$

$$M = 20\log_{10}|(j\omega)^{-N}| = -20N\log_{10}(\omega)$$

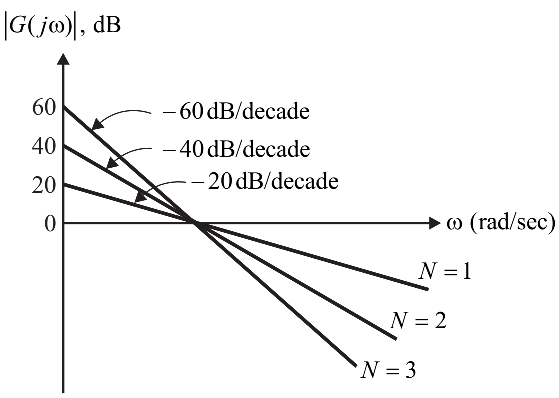
Phase angle can be written as,

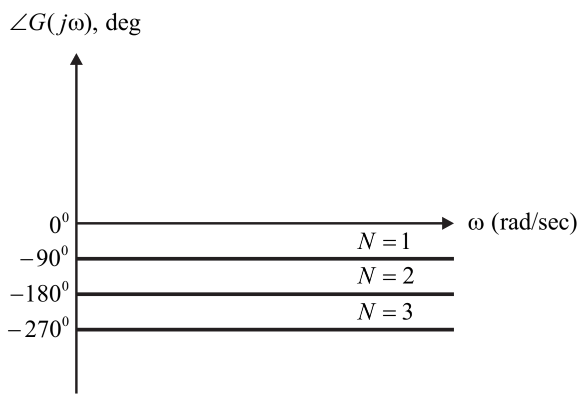
$$\phi = \angle G(j\omega)^{-N} = -90^0 N \qquad \text{where } N = 1, 2, 3, ...$$

The plot $|G(j\omega)|_{\text{dB}}$ versus ω is a straight line.

For $N = 1$, the line has a slope of -20 dB/decade and angle -90^0.

For $N = 2$, the slope of the line will be -40 dB/decade and angle will be -180^0 and so on.





Remember

1. $\frac{\omega_2}{\omega_1} = 10$ represents decade frequency

2. $\frac{\omega_2}{\omega_1} = 2$ represents octave frequency

The relation between octave and decade can be obtained as

$$\frac{\text{Octave}}{\text{Decade}} = \frac{\log_{10} 2}{\log_{10} 10} = 0.3010$$

1 octave = 0.3010 decade

6 dB/octave = 20 dB/decade

12 dB/octave = 40 dB/decade

18 dB/octave = 60 dB/decade

$n \times 6$ dB/octave $= n \times 20$ dB/decade

Number of Open loop poles at origin (type of the system N)	Initial slope 0 dB axis	$\angle G(j\omega)$	Intersection with 0 dB axis
0	0 dB/decade	0^0	Parallel to 0 dB axis
1	-20 dB/decade	-90^0	K
2	-40 dB/decade	-180^0	$\sqrt{K}$
3	-60 dB/decade	-270^0	$K^{1/3}$
⋮	⋮	⋮	⋮
N	$-20N$ dB/decade	$-90^0 N$	$K^{1/N}$

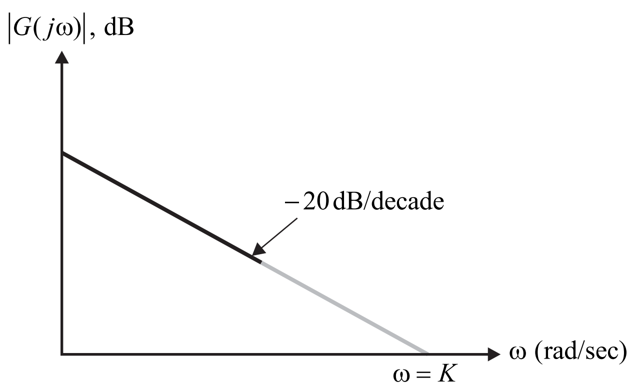


Fig. (a) Type - 1 system

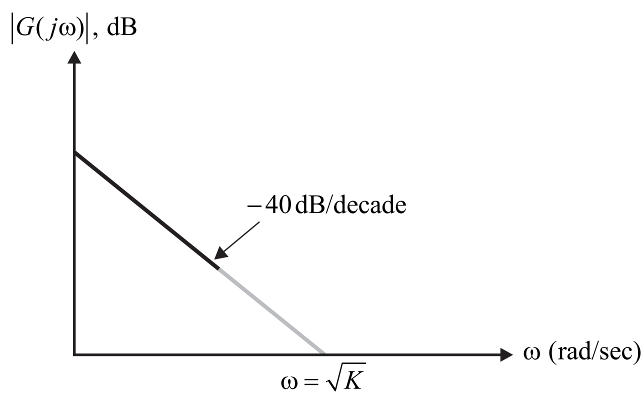
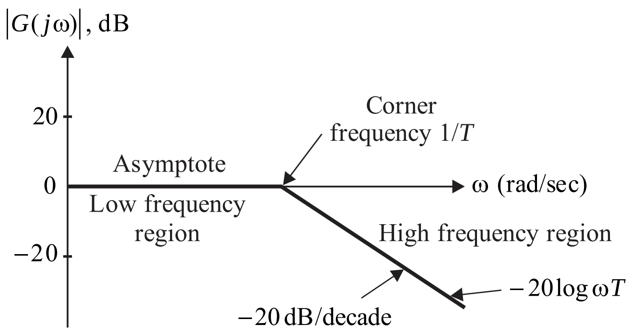


Fig. (b) Type - 2 system

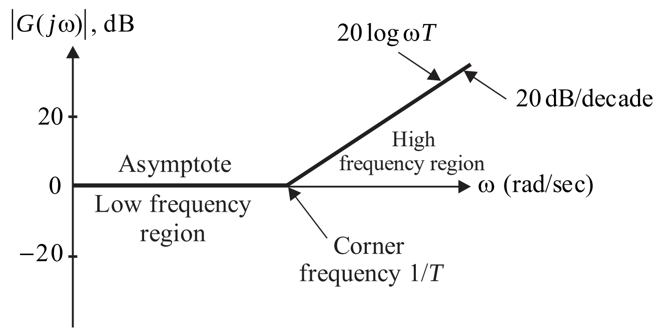
Case 3 : Zeros at origin [i.e. derivative factors]

System type	Initial slope	$\angle G(j\omega)$	Intersection with 0 dB axis
System with one zero at $s = 0$	+20 dB/decade	$+90^0$	$\frac{1}{K}$
System with two zero at $s = 0$	+40 dB/decade	$+180^0$	$\frac{1}{\sqrt{K}}$
System with three zero at $s = 0$	+60 dB/decade	$+270^0$	$\frac{1}{K^{1/3}}$
⋮	⋮	⋮	⋮
System with N zero at $s = 0$	$+20N$ dB/decade	$+90^0 N$	$\frac{1}{K^{1/N}}$

Case 4 : First order pole $\left(\frac{1}{1+sT}\right)$



Case 5 : First order zero $(1+sT)$



Steady State Error Coefficient from Bode Plot

In case of type '0' system, $K = K_p$

In case of type '1' system, $K = K_v$

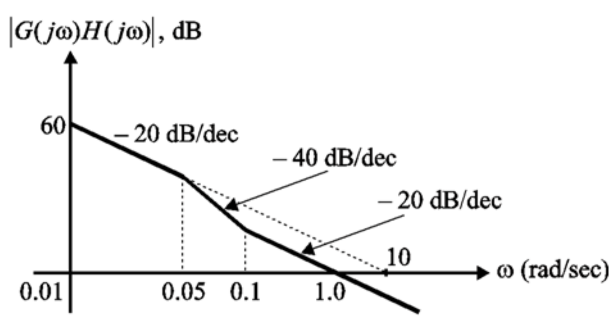
In case of type '2' system, $K = K_a$

where, K is DC gain of bode plot.

➤ Sample Questions

1991　IIT Madras

8.1 The system having the Bode magnitude plot shown in figure below has the transfer function



(A) $\dfrac{60(s+0.01)(s+0.1)}{s^2(s+0.05)^2}$

(B) $\dfrac{10(1+10s)}{s(1+20s)}$

(C) $\dfrac{3(s+0.05)}{s(s+0.1)(s+1)}$

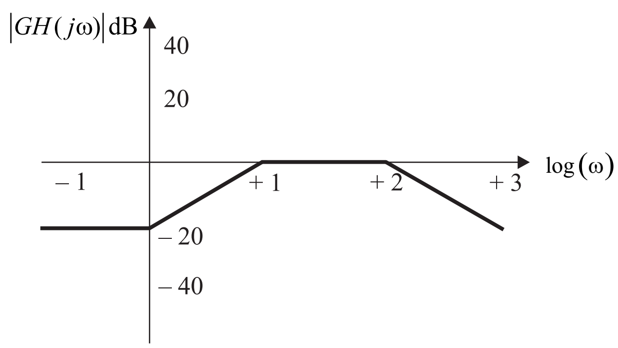
(D) $\dfrac{5(s+0.1)}{s(s+0.05)}$

2006 IIT Kharagpur

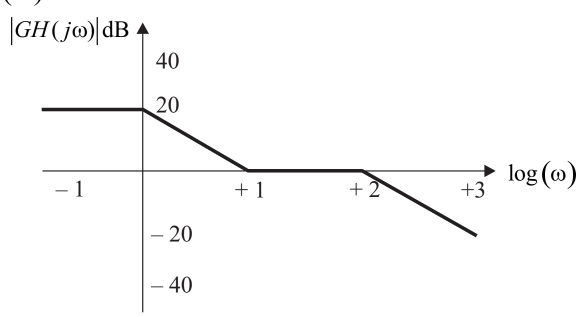
8.2 The Bode magnitude plot of

$$GH(j\omega)=\frac{10^4(1+j\omega)}{(10+j\omega)\,(100+j\omega)^2} \text{ is}$$

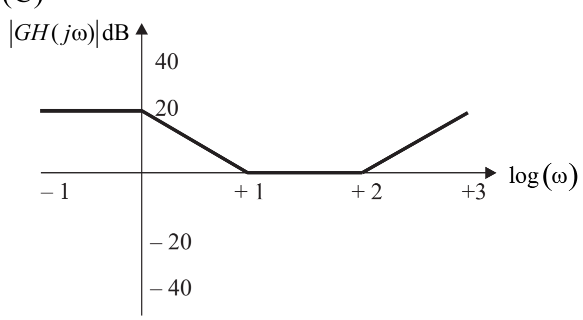
(A)



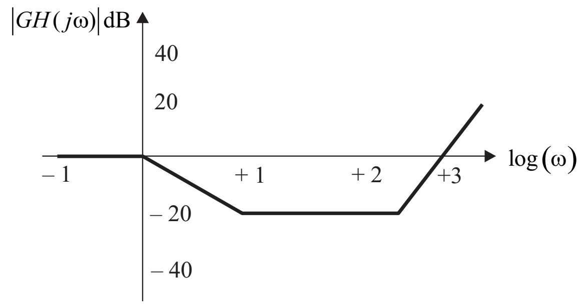
(B)



(C)



(D)

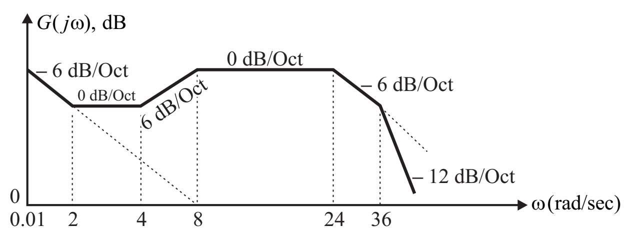


2014 IIT Kharagpur

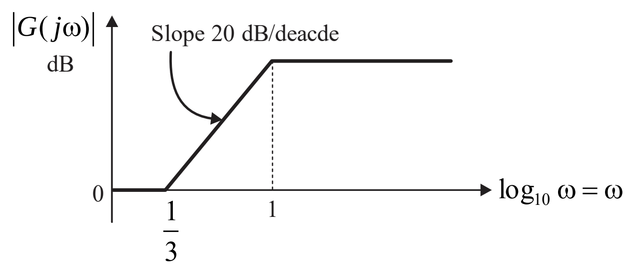
8.3 The Bode magnitude plot of the transfer function

$$G(s)=\frac{K(1+0.5s)(1+as)}{s\left(1+\frac{s}{8}\right)(1+bs)\left(1+\frac{s}{36}\right)}$$

is shown below. Note that – 6 dB/octave $=-20$ dB/decade. The value of a/bK is ____________. **[Set - 01]**



8.4 The magnitude Bode plot of a network is shown in the figure



The maximum phase angle ϕ_m and the corresponding gain G_m respectively are

[Set - 03]

(A) -30^0 and 1.73 dB

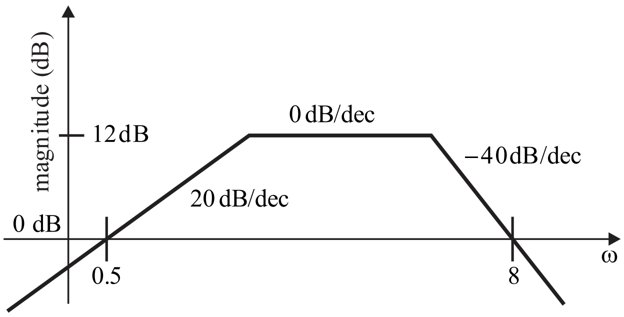
(B) -30^0 and 4.77 dB

(C) 30^0 and 4.77 dB

(D) 30^0 and 1.73 dB

2016 IISc Bangalore

8.5 Consider the following asymptotic Bode magnitude plot (ω is in rad/s).



Which one of the following transfer functions is best represented by the above Bode magnitude plot?

(A) $\dfrac{2s}{(1+0.5s)(1+0.25s)^2}$

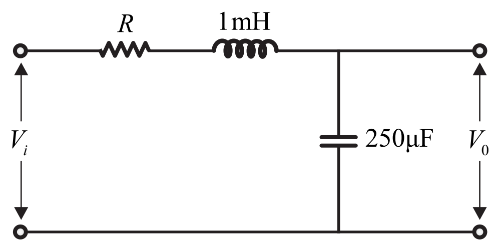
(B) $\dfrac{4(1+0.5s)}{s(1+0.25s)}$

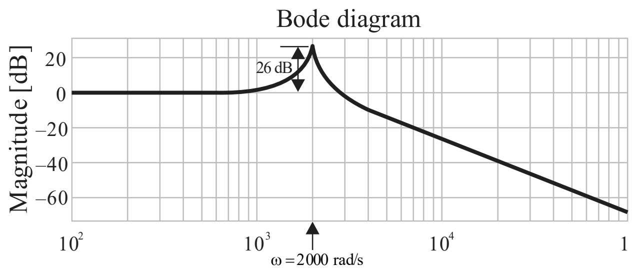
(C) $\dfrac{2s}{(1+2s)(1+4s)}$

(D) $\dfrac{4s}{(1+2s)(1+4s)^2}$

2021 IIT Bombay

8.6 The Bode magnitude plot for the transfer function $\dfrac{V_0(s)}{V_i(s)}$ of the circuit is as shown. The value of R is ________ Ω. (Round off to 2 decimal places)





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Explanations — Bode Plot

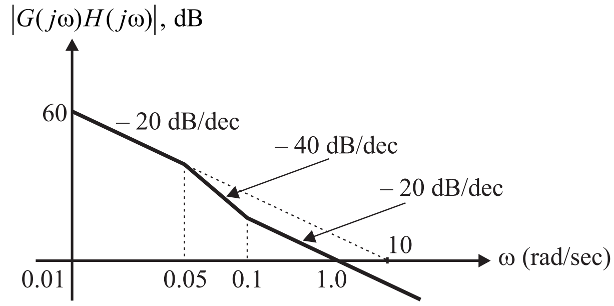
Concept of Asymptotic Bode Phase Plot

Scan for Video Explanation

8.1 (B) and (D)

The given Bode magnitude plot is shown below,

$|G(j\omega)H(j\omega)|$, dB



For the given Bode magnitude plot, there are two corner frequencies : $\omega_1 = 0.05$ and $\omega_2 = 0.1$ rad/sec.

The initial slope is -20 dB/dec and this corresponds to a factor s in the denominator of the transfer function **(one pole at origin).**

At $\omega_1 = 0.05$, the slope changes by -20 dB/dec so resultant slope will be – 40 dB/dec and this is due to the factor $\left(1+\dfrac{s}{0.05}\right)$ in the denominator of the transfer function.

At $\omega_2 = 0.1$, the slope changes by $+20$ dB/dec so resultant slope will be – 20 dB/dec and this is due to the factor $\left(1+\frac{s}{0.1}\right)$ in the numerator of the transfer function.

Calculation of *K* :

$$60 = 20\log K - 20\log(0.01)$$

$$K = 10$$

From figure, $\omega' = K = 10$

[$\omega' =$ Frequency at which Bode plot interest 0 dB line]

Transfer function in Bode plot is given by,

$$G(s) = \frac{K\left(1+\frac{s}{\omega_2}\right)}{s\left(1+\frac{s}{\omega_1}\right)} = \frac{10\left(1+\frac{s}{0.1}\right)}{s\left(1+\frac{s}{0.05}\right)}$$

$$G(s) = \frac{10(1+10s)}{s(1+20s)} = \frac{5(s+0.1)}{s(s+0.05)}$$

Hence, the correct options are (B) and (D).

Key Point

Calculation of error coefficient using bode plot :

Error coefficient method is used to determine the steady state error of CLTF by using OLTF.

(i) For type-1 system,

Initial slope for type-1 system is -20 dB/dec.

This slope will intersect 0 dB line at,

$$\omega' = K = K_v$$

(ii) For type-2 system,

Initial slope for type-2 system is -40 dB/dec.

This slope will intersect 0 dB line at,

$$\omega' = \sqrt{K} \Rightarrow K = K_a = (\omega')^2$$

(iii) For type-n system,

Initial slope for type-n system is $-20n$ dB/dec.

This slope will intersect 0 dB line at,

$$\omega' = K^{1/n} \Rightarrow K = (\omega')^n$$

(iv) When one zero present at origin.

Initial slope for one zero is 20 dB/dec.

This slope will intersect 0 dB line at,

$$\omega' = K^{-1} \Rightarrow K = \frac{1}{\omega'}$$

(v) When *n* number of zeros present at origin.

Initial slope for n number of zeros is $20n$ dB/dec.

This slope will intersect 0 dB line at,

$$\omega' = K^{-1/n} \Rightarrow K = \left(\frac{1}{\omega'}\right)^{1/n}$$

8.2 (A)

The given transfer function is shown below,

$$GH(j\omega) = \frac{10^4(1+j\omega)}{(10+j\omega)\,(100+j\omega)^2}$$

$$GH(j\omega) = \frac{10^4(1+j\omega)}{10\left(1+\frac{j\omega}{10}\right)100^2\left(1+\frac{j\omega}{100}\right)^2}$$

$$GH(s) = \frac{0.1(1+s)}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)^2}$$

where, $K = 0.1$

Gain = 20 log K = 20 log 0.1 = – 20 dB

There are three corner frequencies :

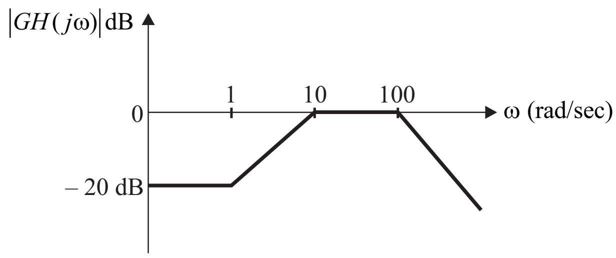
$$\omega_1 = 1,\ \omega_2 = 10 \text{ and } \omega_3 = 100 \text{ rad/sec}.$$

As there are no pole or zero at origin in the transfer function, hence the initial slope will be 0 dB/dec.

(i) At $\omega_1 = 1$: The slope changes by $+20$ dB/dec and resultant slope will be + 20 dB/dec, this is due to the factor $\left(1+\frac{s}{1}\right)$ in the numerator of the transfer function.

(ii) **At $\omega_2 = 10$:** The slope changes by -20 dB/dec and resultant slope will be 0 dB/dec, this is due to the factor $\left(1+\frac{s}{10}\right)$ in the denominator of the transfer function.

(iii) **At $\omega_3 = 100$:** The slope changes by -40 dB/dec and resultant slope will be -40 dB/dec, this is due to the factor $\left(1+\frac{s}{100}\right)^2$ in the denominator of the transfer function.

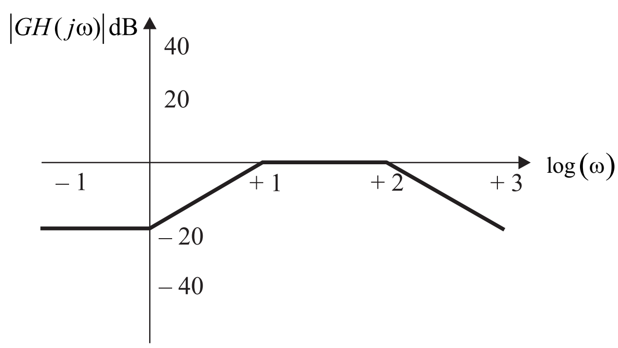


Since, $\log_{10}\omega_1 = \log_{10} 1 = 0$

$\log_{10}\omega_2 = \log_{10} 10 = 1$

$\log_{10}\omega_3 = \log_{10} 100 = 2$

Modified magnitude bode plot is plotted against $\log\omega$ as shown below,



Hence, the correct option is (A).

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Video Solution

☒ Avoid This Mistake

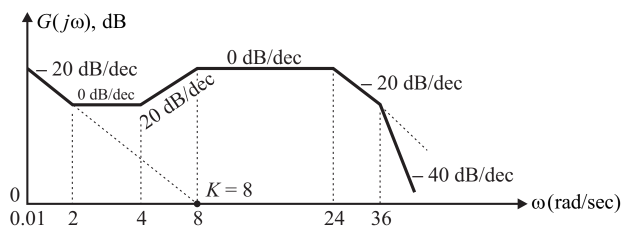
Do not try to compare Bode plot of $|GH(j\omega)|_{dB}$ vs ω with bode plot of $|GH(j\omega)|_{dB}$ vs $\log_{10}\omega$, first convert the ω-axis into $\log_{10}\omega$-axis then compare.

Make sure to check the horizontal axis of bode plot whether it is ω-axis or $\log_{10}\omega$-axis and proceed accordingly.

8.3 **0.75**

Given : $G(s) = \dfrac{K(1+0.5s)(1+as)}{s\left(1+\frac{s}{8}\right)(1+bs)\left(1+\frac{s}{36}\right)}$

The Bode magnitude plot of the given transfer function is shown below,



For the given Bode magnitude plot, there are five corner frequencies :

$\omega_1 = 2$, $\omega_2 = 4$, $\omega_3 = 8$, $\omega_4 = 24$, and $\omega_5 = 36$.

The initial slope is -20 dB/dec and this corresponds to a factor s in the denominator of the transfer function.

At $\omega_1 = 2$, the slope changes by $+20$ dB/dec and resultant slope will be 0 dB/dec, this is due to the factor $\left(1+\frac{s}{2}\right)$ in the numerator of the transfer function.

At $\omega_2 = 4$, the slope changes by $+20$ dB/dec and resultant slope will be $+20$ dB/dec , this is due to the factor $\left(1+\frac{s}{4}\right)$ in the numerator of the transfer function.

At $\omega_3 = 8$, the slope changes by -20 dB/dec and resultant slope will be 0 dB/dec, this is due to the factor $\left(1+\frac{s}{8}\right)$ in the denominator of the transfer function.

At $\omega_4 = 24$, the slope changes by -20 dB/dec and resultant slope will be -20 dB/dec, this is due to the factor $\left(1+\frac{s}{24}\right)$ in the denominator of the transfer function.

At $\omega_5 = 36$, the slope changes by -20 dB/dec and resultant slope will be -40 dB/dec, this is due to the factor $\left(1+\frac{s}{36}\right)$ in the denominator of the transfer function.

For initial slope having – 20 dB/dec,

$$K = \omega$$

where, ω corresponds to 0 dB axis if initial line further extended.

From the figure, for type 1 system,

$$\omega' = K$$

$$K = 8$$

The overall transfer function can be written as,

$$G(s) = \frac{8\left(1+\frac{s}{2}\right)\left(1+\frac{s}{4}\right)}{s\left(1+\frac{s}{8}\right)\left(1+\frac{s}{24}\right)\left(1+\frac{s}{36}\right)}$$

On comparing with given transfer function,

$$a = \frac{1}{4},\ b = \frac{1}{24}$$

$$\frac{a}{bK} = \frac{24}{4\times 8} = 0.75$$

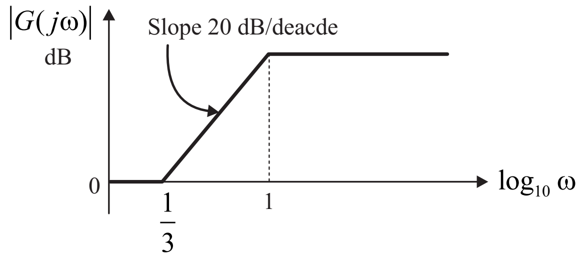
Hence, the value of $\frac{a}{bK}$ is **0.75.**

[Refer Key Point of Solution 8.1]

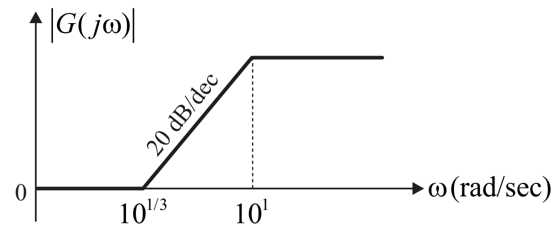
Scan for Video Solution

8.4 (C)

Given : The magnitude Bode plot of a network is shown below,



In ω (rad/sec) axis bode plot is given by,



The transfer function of bode plot is given by,

$$G(s) = \frac{K\left(1+\frac{s}{10^{1/3}}\right)}{\left(1+\frac{s}{10}\right)} = \frac{K(1+0.464s)}{(1+0.1s)} \quad \ldots\text{(i)}$$

Given bode plot represents lead compensator. The transfer function of lead compensator is given by,

$$G_C(s) = \alpha\left(\frac{1+\tau s}{1+\alpha\tau s}\right);\ (\alpha<1) \quad \ldots\text{(ii)}$$

Comparing equation (i) and (ii),

$$\tau = 0.464,\ \alpha\tau = 0.1$$

$$\alpha = 0.215$$

The frequency at which maximum phase occur is given by,

$$\omega_m = \frac{1}{\tau\sqrt{\alpha}}$$

$$\omega_m = 4.647 \text{ rad/sec}$$

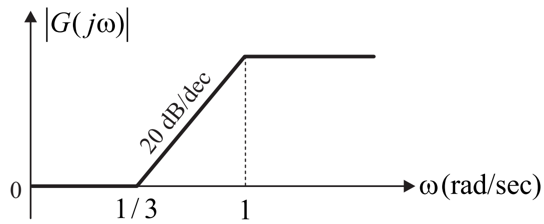
Maximum phase is given by

$$\theta_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

$$\theta_m = \sin^{-1}\left(\frac{1-0.215}{1+0.215}\right) = 40.24^0$$

Since, none of the option is satisfy for given question. Therefore, if we change the *x*-axis from $\log\omega$ to ω then we can get the correct option.

Modified magnitude bode plot is plotted against ω as shown below,



For the given bode magnitude plot, there are two corner frequency $\omega_1 = 1/3$ and $\omega_2 = 1$ rad/sec.

The initial slope is zero.

At $\omega_1 = 1/3$, the slope changes by + 20 dB/dec and resultant slope will be 20 dB/dec and this is due to the factor $\left(1+\frac{s}{1/3}\right)$ in the numerator of the transfer function.

At $\omega_2 = 1$, the slope changes by –20 dB/dec and this is due to the factor $(1+s)$ in the denominator of the transfer function.

Calculation of K :

$$0 = 20\log_{10} K$$

$$K = 1$$

Transfer function in Bode plot is given by,

$$G(s) = \frac{K\left(1+\frac{s}{1/3}\right)}{(1+s)} = \frac{(1+3s)}{(1+s)}$$

Method 1

Put $s = j\omega$ in above equation,

$$G(j\omega) = \frac{1+3j\omega}{1+j\omega} \quad \ldots\text{(iii)}$$

Phase angle is given by,

$$\theta = \angle G(j\omega) = \tan^{-1}(3\omega) - \tan^{-1}(\omega) \quad \ldots\text{(iv)}$$

For maximum phase angle to occur

$$\left.\frac{d\theta}{d\omega}\right|_{\omega=\omega_m} = 0$$

$$0 = \frac{1}{1+(3\omega_m)^2}\times 3 - \frac{1}{(1+\omega_m^2)}$$

$$\frac{3}{1+(3\omega_m)^2} = \frac{1}{(1+\omega_m^2)}$$

$$3(1+\omega_m^2) = 1+9\omega_m^2$$

$$3+3\omega_m^2 = 1+9\omega_m^2$$

$$2 = 6\omega_m^2$$

$$\omega_m^2 = \frac{1}{3}$$

$$\omega_m = \frac{1}{\sqrt{3}} \text{ rad/sec}$$

Therefore, maximum phase from equation (iv) is

$$\theta_m = \tan^{-1}(3\omega_m) - \tan^{-1}(\omega_m)$$

$$\theta_m = \tan^{-1}\left(\frac{3}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\theta_m = \tan^{-1}(\sqrt{3}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\theta_m = 60^0 - 30^0 = 30^0$$

From equation (iii), gain at maximum angular frequency is

$$G_m = \frac{\sqrt{1+9\omega^2}}{\sqrt{1+\omega^2}} = \frac{\sqrt{1+9\times\frac{1}{3}}}{\sqrt{1+\frac{1}{3}}}$$

$$G_m = \frac{\sqrt{4}}{\sqrt{4/3}} = \sqrt{3}$$

Maximum gain in dB,

$$(G_m)_{\text{dB}} = 20\log_{10}\sqrt{3} = 4.77 \text{ dB}$$

Hence, the correct option is (C).

Method 2

Transfer function of the bode plot is given by,

$$G(s)=\frac{(1+3s)}{(1+s)} \quad \ldots(v)$$

Bode plot represent lead compensator. The transfer function of lead compensator is given by,

$$G_C(s)=\alpha\left(\frac{1+\tau s}{1+\alpha\tau s}\right);(\alpha<1) \quad \ldots(vi)$$

Comparing equation (v) and (vi), $\tau=3$,

$$\alpha\tau=1$$

$$\alpha=\frac{1}{3}$$

The frequency at which maximum phase occur is given by,

$$\omega_m=\frac{1}{\tau\sqrt{\alpha}}$$

$$\omega_m=\frac{1}{\sqrt{3}}\text{ rad/sec}$$

Maximum phase is given by,

$$\theta_m=\sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)=\sin^{-1}\left(\frac{1-\frac{1}{3}}{1+\frac{1}{3}}\right)$$

$$\theta_m=\sin^{-1}\left(\frac{1}{2}\right)=30^0$$

Gain at maximum angular frequency is,

$$G_m=\frac{\sqrt{1+9\omega^2}}{\sqrt{1+\omega^2}}=\frac{\sqrt{1+9\times\frac{1}{3}}}{\sqrt{1+\frac{1}{3}}}$$

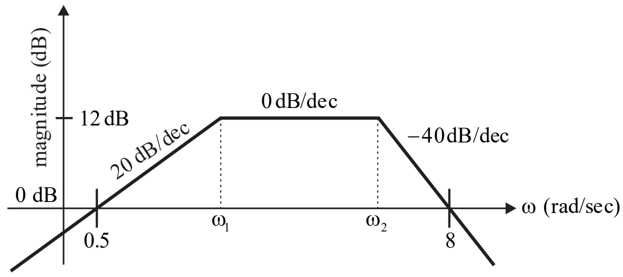
$$G_m=\frac{\sqrt{4}}{\sqrt{4/3}}=\sqrt{3}$$

Maximum gain in dB,

$$(G_m)_{\text{dB}}=20\log_{10}\sqrt{3}=4.77\text{ dB}$$

Hence, the correct option is (C).

8.5 (A)



From given Bode plot, corner frequencies are ω_1 rad/sec and ω_2 rad/sec.

The initial slope is 20 dB/dec and this corresponds to a factor s in the numerator of the transfer function.

At ω_1 : Slope change to 0 dB/dec, signifies a factor of $\left(1+\frac{s}{\omega_1}\right)$ in the denominator of the transfer function.

At ω_2 : Slope changes to − 40 dB/dec that signifies a factor $\left(1+\frac{s}{\omega_2}\right)^2$ in the denominator of the transfer function.

$$\text{Thus, } T(s)=\frac{Ks}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)^2} \quad \ldots(i)$$

Method 1

Calculation of K :

From figure

$$0=20\log K+20\log 0.5$$

$$K=2 \quad \ldots(ii)$$

Calculation of ω_1 and ω_2 :

From figure,

$$20=\frac{12-0}{\log\omega_1-\log 0.5}$$

$$\log\left(\frac{\omega_1}{0.5}\right)=\frac{12}{20}$$

$$\omega_1=2\text{ rad/sec}$$

From figure,

$$-40 = \frac{0-12}{\log 8 - \log \omega_2}$$

$$\log\left(\frac{8}{\omega_2}\right) = \frac{12}{40}$$

$$\omega_2 = 4 \text{ rad/sec}$$

From equation (i),

$$T(s) = \frac{2s}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{4}\right)^2}$$

$$T(s) = \frac{2s}{(1+0.5s)(1+0.25s)^2}$$

Hence, the correct option is (A).

Method 2

From the given bode plot, transfer function has one zero at origin and three poles.

Hence, option (B) and (C) are incorrect.

From figure

$$0 = 20\log K + 20\log 0.5$$

$$K = 2$$

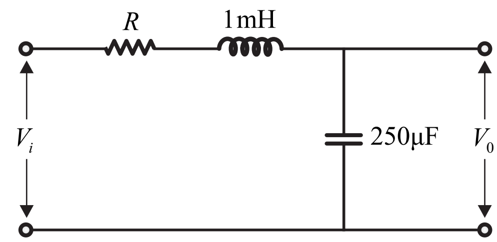
Option (D) has gain $K = 4$ therefore, this option is also incorrect.

Hence, the correct option is (A).

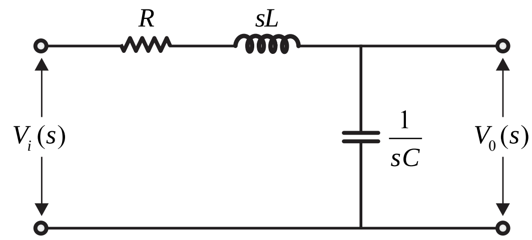
Scan for
Video Solution

8.6 **0.1**

Given circuit is as shown below,



Transforming the above circuit in Laplace domain



Here, $L = 1$ mH, $C = 250$ μF

The transfer function $T(s)$ for above circuit is given by,

$$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}}$$

$$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{\frac{1}{sC}}{\frac{s^2LC + sCR + 1}{sC}}$$

$$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

So, characteristic equation of above transfer function is,

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \qquad \dots \text{(i)}$$

Comparing equation (i) with standard characteristics equation, $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$ then

$\therefore \quad \omega_n = \dfrac{1}{\sqrt{LC}}$

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}} \qquad \text{...(ii)}$$

Now from the Bode plot, resonant peak M_r is 26 dB at $\omega_r = 2000 \text{ rad/sec}$,

$$(M_r)_{\text{dB}} = 26 \text{ dB} = 20\log M_r$$

$$M_r = 19.95 \approx 20$$

So, $\quad M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}} = 20$

Solving above equation, we get $\xi = 0.025$

Now from equation (ii),

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$0.025 = \frac{R}{2}\sqrt{\frac{250\times10^{-6}}{1\times10^{-3}}}$$

$$0.000625 = \frac{R^2}{4}\times 0.25$$

$$R^2 = 0.01\ \Omega$$

$$R = 0.1\ \Omega$$

Hence, the correct answer is 0.1.

❖❖❖❖

10 State Space Analysis

➢ Partial Synopsis

Comparison between Transfer Function Approach and State Variable Approach

S. No.	Transfer Function Approach	State Variable Approach
1.	It is based on the input-output relationship or transfer function.	It is based on the description of the system equations in terms of first order differential equation, which may be combined into vector-matrix.
2.	It is applicable only to LTI systems and it is generally limited to SISO.	It is applicable to linear as well as non-linear, time-invariant as well as variant, SISO as well as MIMO.
3.	In this initial conditions are neglected.	In this initial conditions are considered.
4.	The transfer function of a system is unique.	The state model of a system is not unique.

Solution of State Model Equation

Solution of homogenous state equation

The state equation is given by,

$$\dot{X} = AX(t)$$

The solution of this equation for initial condition $X(0)$ is given by,

$$\left[X(s)\right]_{n\times 1} = [sI - A]^{-1}_{n\times n}[X(0)]_{n\times 1}$$

$$x(t) = \underbrace{e^{At}X(0)}_{\text{Zero input response}}$$

where, $\phi(t) = L^{-1}[sI - A]^{-1}_{n\times n} = e^{At}$.

This is the solution of homogeneous state equation, where e^{At} is termed as **state transition matrix (STM)** and denoted as $\boldsymbol{\phi(t)}$.

Solution of non-homogenous state equation

The state equation is given by,

$$\dot{X} = AX + BU$$

The solution of this equation for initial condition $X(0)$ is given by,

$$X(t) = \underbrace{e^{At}X(0)}_{ZIR} + \underbrace{\int_0^t e^{A(t-\tau)}BU(\tau)\,d\tau}_{ZSR}$$

Where, the forced response or zero state response (ZSR) or the response due to input only is

$$ZSR = L^{-1}\left[\phi(s)BU(s)\right]$$

This is the solution of non-homogeneous state equation, where e^{At} is termed as **state transition matrix (STM)** and denoted as $\boldsymbol{\phi(t)}$.

Properties of State Transition Matrix

$$\phi(t) = L^{-1}[sI - A]^{-1}$$

1. $\phi(0) = I$
2. $\phi^{-1}(t) = \phi(-t)$
3. $\phi(t_2 - t_1)\phi(t_1 - t_0) = \phi(t_2 - t_0)$
4. $\left[\phi(t)\right]^K = \phi(Kt)$; where, K = Positive integer
5. $\phi'(0) = A$
6. $\phi(t_1 + t_2) = \phi(t_1)\phi(t_2)$

Transfer Function from State Model

The state model is given by,

$$\dot{x}(t) = Ax(t) + Bu(t) \quad \ldots\text{(i)}$$

$$y(t) = Cx(t) + Du(t) \quad \ldots\text{(ii)}$$

$$\boldsymbol{T(s) = \frac{Y(s)}{U(s)} = C\left[sI - A\right]^{-1}B + D}$$

The characteristic equation is given by,

$$|sI - A| = 0$$

Sample Questions

1988 IIT Kharagpur

10.1 Given the following state-space description of a system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-2 & 0\\ 0 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

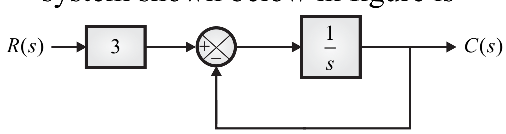
The state-transition matrix will be

(A) $\begin{bmatrix}e^{2t} & 0\\ 0 & e^{4t}\end{bmatrix}$ (B) $\begin{bmatrix}e^{-2t} & 0\\ 0 & e^{4t}\end{bmatrix}$

(C) $\begin{bmatrix}e^{-2t} & 0\\ 0 & e^{-4t}\end{bmatrix}$ (D) $\begin{bmatrix}e^{2t} & 0\\ 0 & e^{-4t}\end{bmatrix}$

1994 IIT Kharagpur

10.2 The matrix of any state-space equations for the transfer function $\dfrac{C(s)}{R(s)}$ of the system shown below in figure is



(A) $\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$ (B) $\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix}$

(C) $[-1]$ (D) $[3]$

2002 IISc Bangalore

10.3 For the system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}2 & 0\\ 0 & 4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 1\end{bmatrix}[u],\ y=\begin{bmatrix}4 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

where $u=\delta(t)$, the initial conditions are zero. The output $y(t)$ is

(A) $y(t)=2e^{2t}$ (B) $y(t)=4e^{2t}$

(C) $y(t)=2e^{4t}$ (D) $y(t)=4e^{4t}$

2003 IIT Madras

10.4 The following equation defines a separately excited dc motor in the form of a differential equation

$$\frac{d^2\omega}{dt^2}+\frac{B}{J}\frac{d\omega}{dt}+\frac{K^2}{LJ}\omega=\frac{K}{LJ}V_a$$

The above equation may be organized in the state-space form as follows :

$$\begin{bmatrix}\dfrac{d^2\omega}{dt^2}\\ \dfrac{d\omega}{dt}\end{bmatrix}=P\begin{bmatrix}\dfrac{d\omega}{dt}\\ \omega\end{bmatrix}+QV_a$$

Where the P matrix is given by

(A) $\begin{bmatrix}-\dfrac{B}{J} & -\dfrac{K^2}{LJ}\\ 1 & 0\end{bmatrix}$ (B) $\begin{bmatrix}-\dfrac{K^2}{LJ} & -\dfrac{B}{J}\\ 0 & 1\end{bmatrix}$

(C) $\begin{bmatrix}0 & 1\\ -\dfrac{K^2}{LJ} & -\dfrac{B}{J}\end{bmatrix}$ (D) $\begin{bmatrix}1 & 0\\ -\dfrac{B}{J} & -\dfrac{K^2}{LJ}\end{bmatrix}$

2008 IISc Bangalore

Statement for Linked Answer Questions 10.5 & 10.6

The state space equation of a system is described by

$$\dot{x}=Ax+Bu$$
$$y=Cx$$

where x is state vector, u is input, y is output

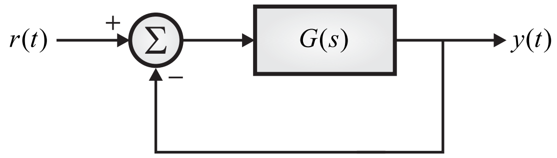
and $A=\begin{bmatrix}0 & 1\\ 0 & -2\end{bmatrix}$, $B=\begin{bmatrix}0\\ 1\end{bmatrix}$, $C=\begin{bmatrix}1 & 0\end{bmatrix}$.

10.5 The transfer function $G(s)$ of this system will be

(A) $\dfrac{s}{(s+2)}$ (B) $\dfrac{s+1}{s(s-2)}$

(C) $\dfrac{s}{(s-2)}$ (D) $\dfrac{1}{s(s+2)}$

10.6 A unity feedback is provided to the above system $G(s)$ to make it a closed loop system as shown in figure.



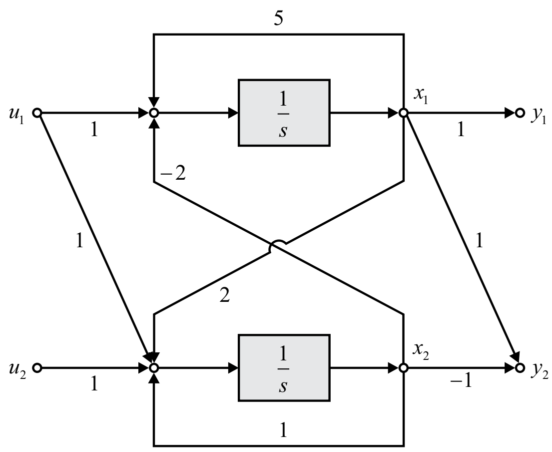
For a unit step input $r(t)$, the steady state error in the output will be

(A) 0 (B) 1

(C) 2 (D) ∞

2015 IIT Kanpur

10.7 In the signal flow diagram given in the figure, u_1 and u_2 are possible inputs whereas y_1 and y_2 are possible outputs. When would the SISO system derived from this diagram be controllable and observable? **[Set - 01]**



(A) When u_1 is the only input and y_1 is the only output.

(B) When u_2 is the only input and y_1 is the only output.

(C) When u_1 is the only input and y_2 is the only output.

(D) When u_2 is the only input and y_2 is the only output.

2018 IIT Guwahati

10.8 Consider a system governed by the following equations

$$\frac{dx_1(t)}{dt} = x_2(t) - x_1(t)$$

$$\frac{dx_2(t)}{dt} = x_1(t) - x_2(t)$$

The initial conditions are such that $x_1(0) < x_2(0) < \infty$. Let $x_{1f} = \lim_{t\to\infty} x_1(t)$ and $x_{2f} = \lim_{t\to\infty} x_2(t)$. Which one of the following is true?

(A) $x_{1f} < x_{2f} < \infty$ (B) $x_{2f} < x_{1f} < \infty$

(C) $x_{1f} = x_{2f} < \infty$ (D) $x_{1f} = x_{2f} = \infty$

2019 IIT Madras

10.9 Consider a state-variable model of a system $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\alpha & -2\beta\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \alpha\end{bmatrix} r$,

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

where y is the output and r is the input. The damping ratio ξ and the undamped natural frequency ω_n (rad/sec) of the system are given by

(A) $\xi = \frac{\sqrt{\alpha}}{\beta}; \omega_n = \sqrt{\beta}$

(B) $\xi = \sqrt{\beta}; \omega_n = \sqrt{\alpha}$

(C) $\xi = \sqrt{\alpha}; \omega_n = \frac{\beta}{\sqrt{\alpha}}$

(D) $\xi = \frac{\beta}{\sqrt{\alpha}}; \omega_n = \sqrt{\alpha}$

2021 IIT Bombay

10.10 The state space representation of a first-order system is given as

$$\dot{x} = -x + u$$
$$y = x$$

Where, x is the state variable, u is the control input and y is the controlled output. Let $u=-kx$ be the control law, where K is the controller gain. To place a closed loop pole at – 2, the value of k is ______.

❖❖❖❖

Explanations **State Space Analysis**

Introduction of State Space Analysis

Scan for Video Explanation

10.1 (C)

Given : $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-2 & 0\\ 0 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$ ...(i)

$$y=[1\quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad \text{...(ii)}$$

State equation is given by,

$$\dot{x}=Ax+Bu \qquad \text{...(iii)}$$

$$y=Cx+Du \qquad \text{...(iv)}$$

Method 1

On comparing equation (i), (ii) with (iii) and (iv),

$$A=\begin{bmatrix}-2 & 0\\ 0 & -4\end{bmatrix},\ B=\begin{bmatrix}0\\ 1\end{bmatrix}, C=[1\quad 0]$$

$$[sI-A]=\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}-\begin{bmatrix}-2 & 0\\ 0 & -4\end{bmatrix}$$

$$[sI-A]=\begin{bmatrix}s+2 & 0\\ 0 & s+4\end{bmatrix}$$

$$[sI-A]^{-1}=\frac{\text{Adj}[sI-A]}{|sI-A|}$$

$$\text{Adj}[sI-A]=\begin{bmatrix}s+4 & 0\\ 0 & s+2\end{bmatrix}$$

$$|sI-A|=(s+2)(s+4)$$

$$[sI-A]^{-1}=\begin{bmatrix}\dfrac{s+4}{(s+2)(s+4)} & 0\\ 0 & \dfrac{s+2}{(s+2)(s+4)}\end{bmatrix}$$

$$[sI-A]^{-1}=\begin{bmatrix}\dfrac{1}{s+2} & 0\\ 0 & \dfrac{1}{s+4}\end{bmatrix} \qquad \text{...(v)}$$

State transition matrix is given by,

$$\phi(t)=e^{At}=L^{-1}[sI-A]^{-1}$$

Taking inverse Laplace transform of equation (v),

$$\phi(t)=e^{At}=\begin{bmatrix}e^{-2t} & 0\\ 0 & e^{-4t}\end{bmatrix}$$

Hence, the correct option is (C).

Method 2

Check by options :

(i) From property of state transition matrix,

$$\phi(0)=I$$

For option (A),

$$\phi(t)=\begin{bmatrix}e^{2t} & 0\\ 0 & e^{4t}\end{bmatrix}$$

$$\phi(0)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

This satisfies the property of STM.

For option (B),

$$\phi(t)=\begin{bmatrix}e^{-2t} & 0\\ 0 & e^{4t}\end{bmatrix}$$

$$\phi(0)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

This satisfies the property of STM.

For option (C),

$$\phi(t)=\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-4t} \end{bmatrix}$$

$$\phi(0)=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=I$$

This satisfies the property of STM.

For option (D),

$$\phi(t)=\begin{bmatrix} e^{2t} & 0 \\ 0 & e^{-4t} \end{bmatrix}$$

$$\phi(0)=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=I$$

This satisfies the property of STM.

All the options are satisfying property of STM.

So we have to check another property of STM.

(ii) From property of state transition matrix,

$$\phi'(0)=A$$

For option (A),

$$\phi'(t)=\begin{bmatrix} 2e^{2t} & 0 \\ 0 & 4e^{4t} \end{bmatrix}$$

$$\phi'(0)=\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\neq A$$

This does not satisfy the property of STM.

For option (B),

$$\phi'(t)=\begin{bmatrix} -2e^{-2t} & 0 \\ 0 & 4e^{4t} \end{bmatrix}$$

$$\phi'(0)=\begin{bmatrix} -2 & 0 \\ 0 & 4 \end{bmatrix}\neq A$$

This does not satisfy the property of STM.

For option (C),

$$\phi'(t)=\begin{bmatrix} -2e^{-2t} & 0 \\ 0 & -4e^{-4t} \end{bmatrix}$$

$$\phi'(0)=\begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}=A$$

This satisfy the property of STM.

For option (D),

$$\phi'(t)=\begin{bmatrix} 2e^{2t} & 0 \\ 0 & -4e^{-4t} \end{bmatrix}$$

$$\phi'(0)=\begin{bmatrix} 2 & 0 \\ 0 & -4 \end{bmatrix}\neq A$$

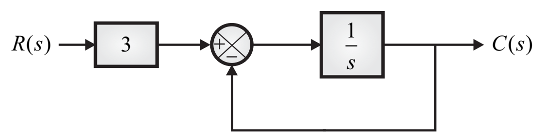
This does not satisfy the property of STM.

Only option (C) is satisfying both the property of STM.

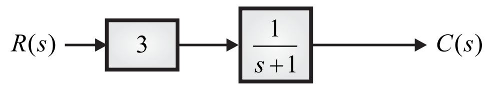
Hence, the correct option is (C).

10.2 (C)

Given :



Method 1



Transfer function of given system is,

$$T(s)=\frac{3}{s+1}$$

$$\frac{C(s)}{R(s)}=\frac{3}{s+1}$$

$$sC(s)+C(s)=3R(s)$$

Taking inverse Laplace transform,

$$\frac{d}{dt}c(t)+c(t)=3r(t)$$

Taking $x_1=c(t)$

So, $\dot{x}_1=\frac{d}{dt}c(t)$

$$\dot{x}_1=3r(t)-x_1$$

$$[\dot{x}_1]=[-1]x_1+[3]r(t) \quad \ldots\text{(i)}$$

The state equation of system is given by,

$$\dot{x}=Ax+Bu \quad \ldots\text{(ii)}$$

On comparing equation (i) and (ii),

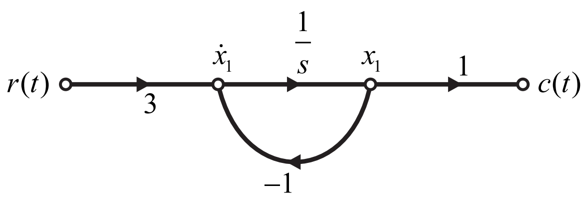
$$A = [-1]$$

Therefore, the system matrix is [–1].

Hence, the correct option is (C).

Method 2

Signal flow graph for given block diagram is shown below.



$$c(t) = x_1$$

$$\dot{x}_1 = 3r(t) - x_1$$

$$[\dot{x}_1] = [-1]x_1 + [3]r(t) \qquad \ldots(i)$$

The state equation of system is given by,

$$\dot{x} = Ax + Bu \qquad \ldots(ii)$$

On comparing equation (i) and (ii),

$$A = [-1]$$

Hence, the correct option is (C).

10.3 (B)

Given : $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} [u]$,

$$y = [4 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \ldots(i)$$

Input, $u = \delta(t)$

State equation is given by,

$$\dot{x} = Ax + Bu \qquad \ldots(ii)$$

Output state equation is given by,

$$y = Cx + Du \qquad \ldots(iii)$$

From equation (i), (ii) and (iii),

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } C = [4 \quad 0]$$

$$D = 0$$

Taking Laplace transform of input,

$$U(s) = 1$$

As the initial conditions are zero. Output is only due to input i.e. ZSR.

Output of the system is given by,

$$T(s) = C[sI - A]^{-1}B + D \qquad \ldots(iv)$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s-2 & 0 \\ 0 & s-4 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\text{Adj}[sI - A]}{|sI - A|}$$

$$\text{Adj}[sI - A] = \begin{bmatrix} s-4 & 0 \\ 0 & s-2 \end{bmatrix}$$

$$|sI - A| = (s-4)(s-2)$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{s-4}{(s-2)(s-4)} & 0 \\ 0 & \frac{s-2}{(s-2)(s-4)} \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{1}{s-2} & 0 \\ 0 & \frac{1}{s-4} \end{bmatrix}$$

From equation (iv),

$$T(s) = [4 \quad 0] \begin{bmatrix} \frac{1}{s-2} & 0 \\ 0 & \frac{1}{s-4} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$T(s) = [4 \quad 0] \begin{bmatrix} \frac{1}{s-2} \\ \frac{1}{s-4} \end{bmatrix}$$

$$T(s) = \frac{4}{s-2} = \frac{Y(s)}{U(s)}$$

$$Y(s) = \frac{4}{s-2}; \text{ where } U(s) = 1$$

Taking inverse Laplace transform,

$$y(t) = 4e^{2t}$$

Hence, the correct option is (B).

10.4 (A)

Given : $\frac{d^2\omega}{dt^2}+\frac{B}{J}\frac{d\omega}{dt}+\frac{K^2}{LJ}\omega=\frac{K}{LJ}V_a$ ...(i)

$$\begin{bmatrix}\frac{d^2\omega}{dt^2}\\ \frac{d\omega}{dt}\end{bmatrix}=P\begin{bmatrix}\frac{d\omega}{dt}\\ \omega\end{bmatrix}+QV_a \quad ...(ii)$$

Let $\omega_1=\omega$,

$$\dot{\omega}_1=\frac{d\omega}{dt}=\omega_2 \quad ...(iii)$$

$$\dot{\omega}_2=\frac{d^2\omega}{dt^2}$$

From equation (i),

$$\dot{\omega}_2=\frac{K}{LJ}V_a-\frac{B}{J}\frac{d\omega}{dt}-\frac{K^2}{LJ}\omega$$

$$\dot{\omega}_2=-\frac{K^2}{LJ}\omega_1-\frac{B}{J}\omega_2+\frac{K}{LJ}V_a \quad ...(iv)$$

From equation (iii) and (iv),

$$\begin{bmatrix}\dot{\omega}_2\\ \dot{\omega}_1\end{bmatrix}=\begin{bmatrix}-\frac{B}{J} & \frac{-K^2}{LJ}\\ 1 & 0\end{bmatrix}\begin{bmatrix}\omega_2\\ \omega_1\end{bmatrix}+\begin{bmatrix}\frac{K}{LJ}\\ 0\end{bmatrix}[V_a] \quad ...(v)$$

From equation (ii) and (v),

$$[P]_{2\times 2}=\begin{bmatrix}-\frac{B}{J} & \frac{-K^2}{LJ}\\ 1 & 0\end{bmatrix}_{2\times 2}$$

$$[Q]_{2\times 1}=\begin{bmatrix}\frac{K}{LJ}\\ 0\end{bmatrix}_{2\times 1}$$

Hence, the correct option is (A).

10.5 (D)

Given : $\dot{x}=Ax+Bu$ and $y=Cx$

where

$$A=\begin{bmatrix}0 & 1\\ 0 & -2\end{bmatrix},\ B=\begin{bmatrix}0\\ 1\end{bmatrix},\ C=\begin{bmatrix}1 & 0\end{bmatrix},$$

$$D=[0]$$

$$[sI-A]=\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}-\begin{bmatrix}0 & 1\\ 0 & -2\end{bmatrix}=\begin{bmatrix}s & -1\\ 0 & s+2\end{bmatrix}$$

$$\text{Adj}[sI-A]=\begin{bmatrix}s+2 & 1\\ 0 & s\end{bmatrix}$$

$$|sI-A|=s(s+2)$$

$$[sI-A]^{-1}=\frac{\text{Adj}[sI-A]}{|sI-A|}$$

$$[sI-A]^{-1}=\frac{1}{s(s+2)}\begin{bmatrix}s+2 & 1\\ 0 & s\end{bmatrix}$$

$$[sI-A]^{-1}=\begin{bmatrix}\frac{1}{s} & \frac{1}{s(s+2)}\\ 0 & \frac{1}{(s+2)}\end{bmatrix}$$

The transfer function is given by,

$$G(s)=C[sI-A]^{-1}B+D$$

$$G(s)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{s} & \frac{1}{s(s+2)}\\ 0 & \frac{1}{(s+2)}\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$G(s)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{s(s+2)}\\ \frac{1}{(s+2)}\end{bmatrix}=\left[\frac{1}{s(s+2)}\right]_{1\times 1}$$

Hence, the correct option is (D).

Scan for Video Solution

10.6 (A)

Given : $G(s)=\frac{1}{s(s+2)}$ and $H(s)=1$

$$r(t)=u(t)$$

Taking Laplace transform of input signal,

$$R(s)=\frac{1}{s}$$

For step input steady state error is given by,

$$e_{ss}=\frac{1}{1+K_p}$$

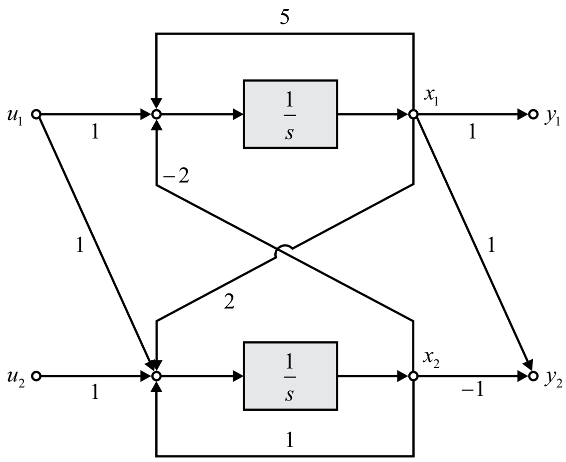
where $K_p=\lim\limits_{s\to 0}\dfrac{1}{s(s+2)}=\infty$

$$e_{ss}=\frac{1}{1+\infty}=0$$

Hence, the correct option is (A).

10.7 (B)

Given :



From given signal flow graph,

$$\dot{x}_1=5x_1-2x_2+u_1$$

$$\dot{x}_2=2x_1+x_2+u_1+u_2$$

$$y_1=x_1$$

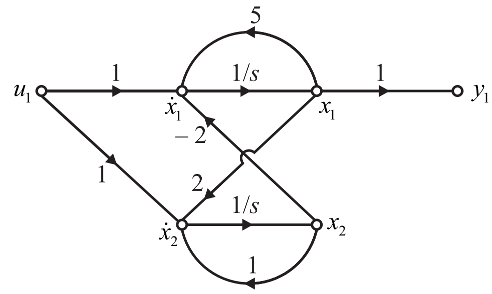
$$y_2=x_1-x_2$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}5 & -2\\ 2 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

For option (A) :

$u_2=0$ and y_1 is the only output.



$$y_1=x_1$$

$$\dot{x}_1=5x_1-2x_2+u_1$$

$$\dot{x}_2=2x_1+x_2+u_1$$

From the above state equation,

$$A=\begin{bmatrix}5 & -2\\ 2 & 1\end{bmatrix}\qquad B=\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 0\end{bmatrix}\qquad C^T=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$AB=\begin{bmatrix}5 & -2\\ 2 & 1\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}=\begin{bmatrix}3\\ 3\end{bmatrix}$$

The controllability matrix is defined as,

$$Q_C=\left[B\ :\ AB\ :\ A^2B\ ...\ :\ A^{n-1}B\right]$$

where, $n=$ number of state variable

If $|Q_C|\neq 0$, then the system is completely state controllable.

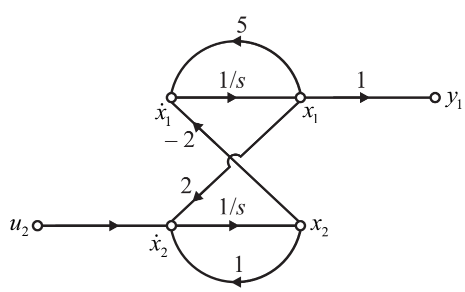
$$Q_C=\left[B\ :\ AB\right]=\begin{bmatrix}1 & 3\\ 1 & 3\end{bmatrix}$$

$$|Q_C|=0$$

Thus, the system is uncontrollable.

For option (B) :

$u_1=0$ and y_1 is the only output.



$$y_1 = x_1$$

$$\dot{x}_1 = 5x_1 - 2x_2$$

$$\dot{x}_2 = 2x_1 + x_2 + u_2$$

From the above state equation,

$$A = \begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \qquad C^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

For controllability,

$$Q_C = [B \ : \ AB]$$

$$Q_C = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$|Q_C| = 2$$

$$|Q_C| \neq 0$$

Thus, the system is controllable.
The observability matrix is defined as,

$$Q_O = \left[C^T \ : \ A^T C^T \ : \ .... \ : \ (A^{n-1})^T C^T \right]$$

where, $n =$ number of state variable

If $|Q_O| \neq 0$, then the system is completely state observable.

$$A^T C^T = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$Q_O = \left[C^T \ : \ A^T C^T \right]$$

$$Q_O = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$$

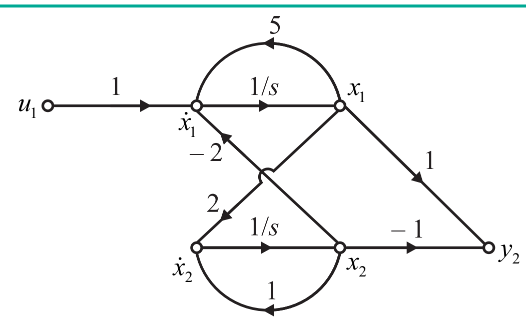
$$|Q_O| = -2$$

$$|Q_O| \neq 0$$

Thus, the system is observable.

For option (C) :

$u_2 = 0$ and y_2 is the only output.



$$y_2 = x_1 - x_2$$

$$\dot{x}_1 = 5x_1 - 2x_2 + u_1$$

$$\dot{x}_2 = 2x_1 + x_2$$

From the above state equation,

$$A = \begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

For controllability,

$$Q_C = [B \ : \ AB]$$

$$Q_C = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}$$

$$|Q_C| = 0$$

Thus, the system is uncontrollable.

For observability,

$$A^T C^T = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$Q_O = \left[C^T \ : \ A^T C^T \right]$$

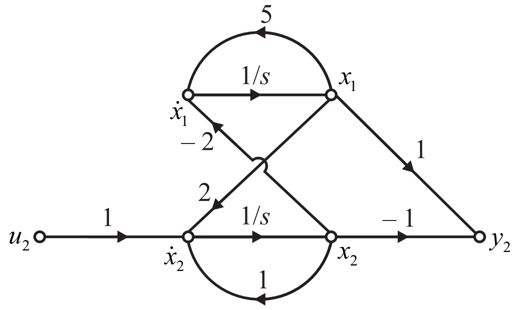
$$Q_O = \begin{bmatrix} 1 & 3 \\ -1 & -3 \end{bmatrix}$$

$$|Q_O| = 0$$

Thus, the system is unobservable.

For option (D) :

$u_1 = 0$ and y_2 is the only output.



$$y_2 = x_1 - x_2$$

$$\dot{x}_1 = 5x_1 - 2x_2$$

$$\dot{x}_2 = 2x_1 + x_2 + u_2$$

From the above state equation,

$$A = \begin{bmatrix} 5 & -1 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

For controllability,

$$Q_C = [B \ : \ AB] = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$|Q_C| = 2$$

$$|Q_C| \neq 0$$

Thus, the system is controllable.

For observability,

$$A^T C^T = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$Q_O = \left[C^T \ : \ A^T C^T\right] = \begin{bmatrix} 1 & 3 \\ 1 & -3 \end{bmatrix}$$

$$|Q_O| = 0$$

Thus, the system is unobservable.

Hence, the correct option is (B).

☒ Avoid This Mistake

From concept of parallel connection

If $u_2 = 0$ then controllable

If $y_1 = 0$ then observable

But given question is not based on parallel decomposition because there is no feedback form.

10.8 (C)

Given : $\dfrac{dx_1(t)}{dt} = x_2(t) - x_1(t)$

$$\frac{dx_2(t)}{dt} = x_1(t) - x_2(t)$$

State space model of above equation is shown below,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s+1 & -1 \\ -1 & s+1 \end{bmatrix}$$

$$\phi(s) = [sI - A]^{-1} = \frac{\text{Adj}[sI - A]}{|sI - A|}$$

$$\text{Adj}[sI - A] = \begin{bmatrix} s+1 & 1 \\ 1 & s+1 \end{bmatrix}$$

$$|sI - A| = (s+1)^2 - 1 = s^2 + 2s$$

$$[sI - A]^{-1} = \frac{1}{s^2 + 2s}\begin{bmatrix} s+1 & 1 \\ 1 & s+1 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \dfrac{s+1}{s(s+2)} & \dfrac{1}{s(s+2)} \\ \dfrac{1}{s(s+2)} & \dfrac{s+1}{s(s+2)} \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \dfrac{1}{2s} + \dfrac{1}{2(s+2)} & \dfrac{1}{2s} - \dfrac{1}{2(s+2)} \\ \dfrac{1}{2s} - \dfrac{1}{2(s+2)} & \dfrac{1}{2s} + \dfrac{1}{2(s+2)} \end{bmatrix}$$

Taking inverse Laplace transform,

$$\phi(t) = \frac{1}{2}\begin{bmatrix} 1 + e^{-2t} & 1 - e^{-2t} \\ 1 - e^{-2t} & 1 + e^{-2t} \end{bmatrix}$$

$$x(t) = \phi(t)x(0) = \frac{1}{2}\begin{bmatrix} 1+e^{-2t} & 1-e^{-2t} \\ 1-e^{-2t} & 1+e^{-2t} \end{bmatrix}\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$$

$$x(t) = \frac{1}{2}\begin{bmatrix} x_1(0)+x_2(0)+e^{-2t}[x_1(0)-x_2(0)] \\ x_1(0)+x_2(0)+e^{-2t}[x_2(0)-x_1(0)] \end{bmatrix}$$

$$x_f = \lim_{t\to\infty} x(t) = \frac{1}{2}\begin{bmatrix} x_1(0)+x_2(0) \\ x_1(0)+x_2(0) \end{bmatrix}$$

Therefore, $x_{1f} = x_{2f} < \infty$

Hence, the correct option is (C).

10.9 (D)

Method 1

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\alpha & -2\beta \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} r$$

$$y = [10]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

System matrix $A = \begin{bmatrix} 0 & 1 \\ -\alpha & -2\beta \end{bmatrix}$

Eigen values of system matrix gives the roots of characteristic equation.

As $[A]$ is a 2×2 matrix, let the Eigen values are λ_1 and λ_2.

From properties of Eigen values

Sum of Eigen values = Trace of matrix

$$\lambda_1 + \lambda_2 = 0 + (-2\beta) = -2\beta \qquad \ldots\text{(i)}$$

Product of Eigen values $= |A|$

$$\lambda_1 \cdot \lambda_2 = 0 - (-\alpha) = \alpha$$

$$\Rightarrow \quad \lambda_2 = \frac{\alpha}{\lambda_1}$$

Substituting value of λ_2 in equation (i),

$$\lambda_1 + \frac{\alpha}{\lambda_1} = -2\beta$$

$$\Rightarrow \quad \lambda_1^2 + \alpha = -2\beta\lambda_1$$

$$\Rightarrow \quad \lambda_1^2 + 2\beta\lambda_1 + \alpha = 0$$

(Characteristic equation)

Comparing with standard equation

$$\omega_n^2 = \alpha$$

$$\Rightarrow \quad \omega_n = \sqrt{\alpha}$$

Also $\quad 2\xi\omega_n = 2\beta$

$$\therefore \quad \xi = \frac{2\beta}{2\omega_n} = \frac{\beta}{\sqrt{\alpha}}$$

Method 2

System matrix $A = \begin{bmatrix} 0 & 1 \\ -\alpha & -2\beta \end{bmatrix}$

Characteristic equation $= |sI - A| = 0$

$$\Rightarrow \quad \begin{vmatrix} s & 0 \\ 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -\alpha & -2\beta \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} s & -1 \\ \alpha & s+2\beta \end{vmatrix} = 0$$

$$\Rightarrow \quad s(s+2\beta) + \alpha = 0$$

$$\Rightarrow \quad s^2 + 2\beta s + \alpha = 0$$

Comparing with standard equation

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

(i) $\quad \omega_n^2 = \alpha$

$$\Rightarrow \quad \omega_n = \sqrt{\alpha}$$

(ii) $\quad 2\xi\omega_n = 2\beta$

$$\xi \times \sqrt{\alpha} = \beta$$

$$\xi = \frac{\beta}{\sqrt{\alpha}}$$

Hence, the correct option is (D).

10.10 1

Given : $\dot{x} = -x + u$

$$y = x$$

$\dot{x} = -x + u$

Let $u = -Kx$

$\dot{x} = -x - Kx$

Method 1

$\dot{x} = x(-1-K)$

$|sI - A| = 0$

$|sI - (-1-K)| = 0$

$s + 1 + K = 0$

$\because$ $s = -2$

$\because$ $-2 + 1 + K = 0$

$K = 1$

Hence, the correct answer is $K = 1$.

Method 2

$y = x$

Let $u = -Kx$

$sx(s) = -x(s) - kx(s)$

$y(s) = x(s)$

$sx(s) + x(s) + Kx(s) = 1$

$x(s)[s + 1 + K] = 1$

Characteristic equation $= [s + 1 + K]$ to place close loop pole at – 2

$1 + K = 2$

$K = 1$

Hence, the correct answer is 1.

❖❖❖❖

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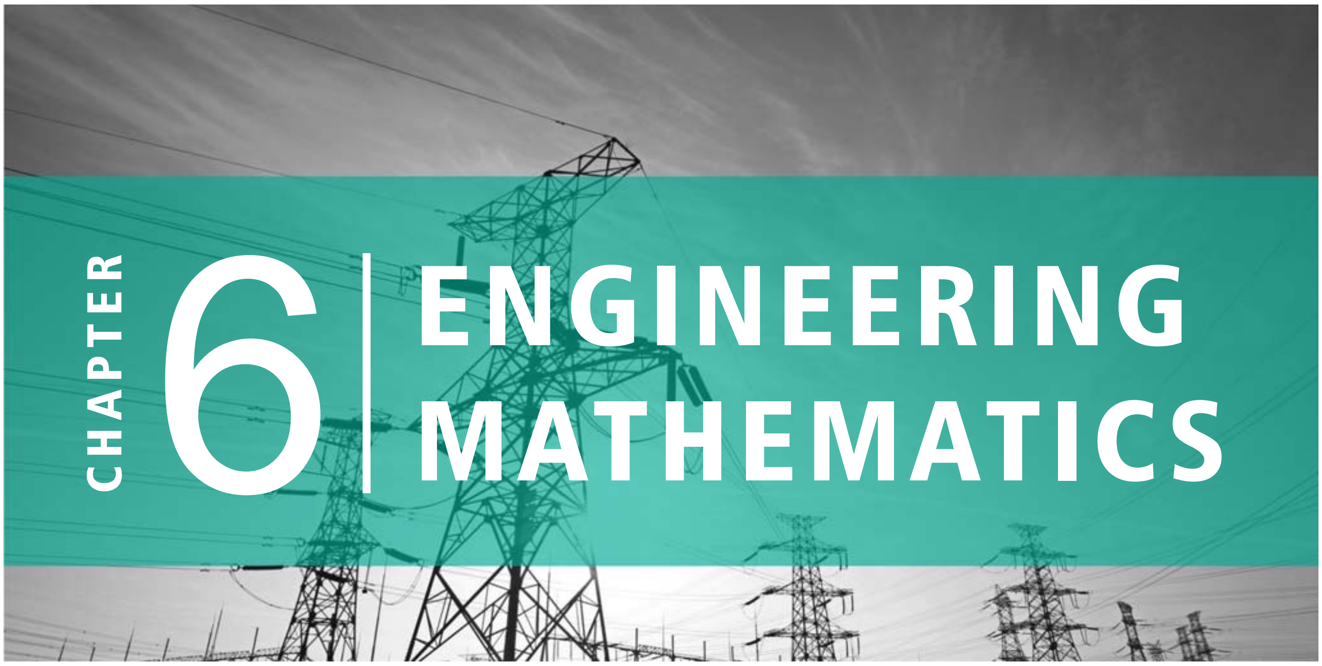


CHAPTER 6 ENGINEERING MATHEMATICS

Marks Distribution of Engineering Mathematics in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	–	–	–
2004	–	–	–
2005	4	5	14
2006	–	5	10
2007	1	8	17
2008	3	5	13
2009	1	5	11
2010	2	4	10
2011	3	4	11
2012	4	3	10
2013	4	4	12
2014 Set-1	5	4	13

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-2	5	3	11
2014 Set-3	3	3	9
2015 Set-1	3	3	9
2015 Set-2	2	3	8
2016 Set-1	5	3	11
2016 Set-2	5	5	15
2017 Set-1	2	4	10
2017 Set-2	5	3	11
2018	5	5	15
2019	6	3	12
2020	4	4	12
2021	4	4	12

Syllabus : Engineering Mathematics

Linear Algebra: Matrix Algebra, Systems of linear equations, Eigenvalues, Eigenvectors.

Calculus: Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, Partial Derivatives, Maxima and minima, Multiple integrals, Fourier series, Vector identities, Directional derivatives, Line integral, Surface integral, Volume integral, Stokes's theorem, Gauss's theorem, Divergence theorem, Green's theorem.

Differential equations: First order equations (linear and nonlinear), Higher order linear differential equations with constant coefficients, Method of variation of parameters, Cauchy's equation, Euler's equation, Initial and boundary value problems, Partial Differential Equations, Method of separation of variables.

Complex variables: Analytic functions, Cauchy's integral theorem, Cauchy's integral formula, Taylor series, Laurent series, Residue theorem, Solution integrals.

Probability and Statistics: Sampling theorems, Conditional probability, Mean, Median, Mode, Standard Deviation, Random variables, Discrete and Continuous distributions, Poisson distribution, Normal distribution, Binomial distribution, Correlation analysis, Regression analysis.

Contents : Engineering Mathematics

1 Linear Algebra

➢ Sample Questions

1994 IIT Kharagpur

1.1 A 5×7 matrix has all its entries equal to -1. The rank of the matrix is

(A) 7 (B) 5

(C) 1 (D) zero

2013 IIT Bombay

1.2 A matrix has Eigen values –1 and –2. The corresponding Eigen vectors are $\begin{bmatrix}1\\-1\end{bmatrix}$ and $\begin{bmatrix}1\\-2\end{bmatrix}$ respectively.

The matrix is

(A) $\begin{bmatrix}1 & 1\\-1 & -2\end{bmatrix}$ (B) $\begin{bmatrix}1 & 2\\-2 & -4\end{bmatrix}$

(C) $\begin{bmatrix}-1 & 0\\0 & -2\end{bmatrix}$ (D) $\begin{bmatrix}0 & 1\\-2 & -3\end{bmatrix}$

2015 IIT Kanpur

1.3 The maximum value of 'a' such that the matrix $\begin{bmatrix}-3 & 0 & -2\\1 & -1 & 0\\0 & a & -2\end{bmatrix}$ has three linearly independent real eigenvectors is **[Set - 01]**

(A) $\frac{2}{3\sqrt{3}}$ (B) $\frac{1}{3\sqrt{3}}$

(C) $\frac{1+2\sqrt{3}}{3\sqrt{3}}$ (D) $\frac{1+\sqrt{3}}{3\sqrt{3}}$

2021 IIT Bombay

1.4 Let A be a 10×10 matrix such that A^5 is null matrix, and let I be the 10×10 identity matrix. The determinant of $A+I$ is _____.

❖❖❖❖

Explanations **Linear Algebra**

1.1 (C)

Given :

A 5×7 matrix has all its entries equal to -1,

$$A=\begin{bmatrix}-1 & -1 & -1 & -1 & -1 & -1 & -1\\-1 & -1 & -1 & -1 & -1 & -1 & -1\\-1 & -1 & -1 & -1 & -1 & -1 & -1\\-1 & -1 & -1 & -1 & -1 & -1 & -1\\-1 & -1 & -1 & -1 & -1 & -1 & -1\end{bmatrix}_{5\times7}$$

Method 1

By elementary transformations,

$$R_2 \rightarrow R_2 - R_1,\ R_3 \rightarrow R_3 - R_1,$$

$$R_4 \rightarrow R_4 - R_1 \text{ and } R_5 \rightarrow R_5 - R_1$$

$$A = \begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{5\times 7}$$

There exists only one non-zero row.

So, $\rho(A) = 1$

Hence, the correct option is (C).

Method 2

The rank of a matrix is defined as the number of linearly independent rows/columns, whichever is minimum in the matrix.

In the given matrix, there exist linear relationships as given below,

$$R_2 = R_1,\ R_3 = R_1,$$

$$R_4 = R_1 \text{ and } R_5 = R_1$$

Thus, only row R_1 is an independent row.

So, $\rho(A) = 1$

Hence, the correct option is (C).

Key Point

These elementary transformations are done in order to convert matrix to its **Echelon form.**

The number of non-zero rows remaining after elementary transformations gives the rank of that matrix.

A matrix is in its Echelon form if :

(i) Leading non-zero elements of a row are behind the leading non-zero elements in its previous row.

(ii) All the zero rows should be below all the non-zero rows.

This method is also known as **Gauss elimination method**.

1.2 (D)

Given :

(i) Eigen values are –1, –2.

(ii) Eigen vector are $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

Let the matrix A be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Method 1

For any Eigen vector $[X]$, of a matrix $[A]$ corresponding to Eigen value λ, the following equation satisfies,

$$[A - \lambda I][X] = 0$$

$$AX = \lambda X$$

For $\lambda = -1$ and $X = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = (-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$a - b = -1 \quad \ldots\text{(i)}$$

$$c - d = 1 \quad \ldots\text{(ii)}$$

For $\lambda = -2$ and $X = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = (-2)\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$a - 2b = -2 \quad \ldots\text{(iii)}$$

$$c - 2d = 4 \quad \ldots\text{(iv)}$$

From equation (i) and (iii),

$$a = 0 \text{ and } b = 1$$

From equation (ii) and (iv),

$$c = -2 \text{ and } d = -3$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Hence, the correct option is (D).

Method 2

Modal matrix can be formed as,

$$[M]=[v_1 \quad v_2]$$

[where, v_1, v_2 are Eigen vectors]

$$[M]=\begin{bmatrix}1 & 1\\-1 & -2\end{bmatrix}$$

The matrix A can be formed as,

$$[A]=[M]\begin{bmatrix}\lambda_1 & 0\\0 & \lambda_2\end{bmatrix}[M]^{-1}$$

$$[A]=\begin{bmatrix}1 & 1\\-1 & -2\end{bmatrix}\begin{bmatrix}-1 & 0\\0 & -2\end{bmatrix}\left(\frac{-1}{1}\begin{bmatrix}-2 & -1\\1 & 1\end{bmatrix}\right)$$

$$[A]=\begin{bmatrix}-1 & -2\\1 & 4\end{bmatrix}\begin{bmatrix}2 & 1\\-1 & -1\end{bmatrix}=\begin{bmatrix}0 & 1\\-2 & -3\end{bmatrix}$$

Hence, the correct option is (D).

Key Point

Concept of Diagonalization :

Any square matrix whose eigen values are distinct, can be represented as :

$$A=[M]\,[D]\,[M^{-1}]$$

where, D is a diagonal matrix whose diagonal elements are Eigen values of A.

M is a non-singular matrix whose columns are respective Eigen vectors of A.

Note : M is also referred to as **Modal matrix**.

Method 3

For any Eigen vector $[X]$ for a matrix $[A]$ corresponding to Eigen value λ, the following equation satisfies,

$$[A-\lambda I][X]=0$$

By the property of square matrix,

Sum of eigen values of a matrix $(\lambda_1+\lambda_2)$

= Trace of the matrix

Product of eigen values of a matrix $(\lambda_1\lambda_2)$

= Determinant of the matrix $(|A|)$

By these properties, options (A) and (B) are not possible.

Option (C) :

For $\lambda=-1$;

$$[A-\lambda I][X]=\begin{bmatrix}-1-(-1) & 0\\0 & -2-(-1)\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$[A-\lambda I][X]=\begin{bmatrix}0 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$[A-\lambda I][X]=\begin{bmatrix}0\\1\end{bmatrix}\neq\begin{bmatrix}0\\0\end{bmatrix}$$

Thus, option (C) is incorrect.

Option (D) :

For $\lambda=-1$;

$$[A-\lambda I][X]=\begin{bmatrix}0-(-1) & 1\\-2 & -3-(-1)\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$[A-\lambda I][X]=\begin{bmatrix}0\\0\end{bmatrix}$$

For $\lambda=-2$;

$$[A-\lambda I][X]=\begin{bmatrix}0-(-2) & 1\\-2 & -3-(-2)\end{bmatrix}\begin{bmatrix}1\\-2\end{bmatrix}$$

$$[A-\lambda I][X]=\begin{bmatrix}0\\0\end{bmatrix}$$

Hence, the correct option is (D).

1.3 (B)

Given : $A=\begin{bmatrix}-3 & 0 & -2\\1 & -1 & 0\\0 & a & -2\end{bmatrix}$

Method 1

The characteristic equation is given by,

$$|A-\lambda I|=0$$

$$\begin{vmatrix}-3-\lambda & 0 & -2\\1 & -1-\lambda & 0\\0 & a & -2-\lambda\end{vmatrix}=0$$

$$(-3-\lambda)\begin{vmatrix}-1-\lambda & 0 \\ a & -2-\lambda\end{vmatrix} - 2\begin{vmatrix}1 & -1-\lambda \\ 0 & a\end{vmatrix} = 0$$

$$-(\lambda+3)\begin{vmatrix}\lambda+1 & 0 \\ -a & \lambda+2\end{vmatrix} - 2\begin{vmatrix}1 & -(1+\lambda) \\ 0 & a\end{vmatrix} = 0$$

$$-(\lambda+3)\left[(\lambda+1)(\lambda+2)\right] - 2[a] = 0$$

$$(\lambda+1)(\lambda+2)(\lambda+3) + 2a = 0$$

$$\lambda^3 + 6\lambda^2 + 11\lambda + (2a+6) = 0$$

$$a = -\frac{1}{2}\left[\lambda^3 + 6\lambda^2 + 11\lambda + 6\right]$$

For extreme value, the stationary points are given by,

$$\frac{da}{d\lambda} = 0$$

$$\frac{da}{d\lambda} = -\frac{1}{2}\left[3\lambda^2 + 12\lambda + 11\right] = 0$$

$$3\lambda^2 + 12\lambda + 11 = 0$$

$$\lambda = \frac{-12 \pm \sqrt{12^2 - 4\times3\times11}}{6} = \frac{-12 \pm \sqrt{12}}{6}$$

$$\lambda = \frac{-12 \pm 2\sqrt{3}}{6} = \frac{-2 \pm \sqrt{3}}{3}$$

$$\lambda_1 = -\left(\frac{2+\sqrt{3}}{3}\right),\ \lambda_2 = -\left(\frac{2-\sqrt{3}}{3}\right)$$

Taking second derivative of a,

$$\frac{d^2a}{d\lambda^2} = -\frac{1}{2}(6\lambda+12) = -3(\lambda+2)$$

For $\lambda_1 = -\left(\frac{2+\sqrt{3}}{3}\right), \frac{d^2a}{d\lambda^2} =$ Negative **[maxima]**

For $\lambda_2 = -\left(\frac{2-\sqrt{3}}{3}\right), \frac{d^2a}{d\lambda^2} =$ Negative **[maxima]**

So, for both λ_1 and λ_2 there is maxima of a

For $\quad \lambda_1 = -\left(\frac{2+\sqrt{3}}{3}\right) = -1.24$

$$a_1 = -\frac{1}{2}\left[\lambda_1^3 + 6\lambda_1^2 + 11\lambda_1 + 6\right]$$

$$a_1 = -\frac{1}{2}\left[(-1.24)^3 + 6(-1.24)^2 + 11(-1.24) + 6\right]$$

$$a_1 = 0.16$$

For $\quad \lambda_2 = -\left(\frac{2-\sqrt{3}}{3}\right) = -0.09$

$$a_2 = -\frac{1}{2}\left[\lambda_2^3 + 6\lambda_2^2 + 11\lambda_2 + 6\right]$$

$$a_2 = -\frac{1}{2}\left[(-0.09)^3 + 6(-0.09)^2 + 11(-0.09) + 6\right]$$

$$a_2 = -2.53$$

So, maximum value of $a = 0.16$.

Checking from the options,

Option (A) $= \frac{2}{3\sqrt{3}} = 0.385$

Option (B) $= \frac{1}{3\sqrt{3}} = 0.19$

Option (C) $= \frac{1+2\sqrt{3}}{3\sqrt{3}} = 0.859$

Option (D) $= \frac{1+\sqrt{3}}{3\sqrt{3}} = 0.525$

Therefore, option (B) gives the closest value of a.

Hence, the correct option (B).

Method 2

Let the given matrix be $A = \begin{bmatrix}-3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2\end{bmatrix}$.

The characteristics equation of A is $|A - \lambda I| = 0$

$$\begin{vmatrix}-3-\lambda & 0 & -2 \\ 1 & -1-\lambda & 0 \\ 0 & a & -2-\lambda\end{vmatrix} = 0$$

$$(-3-\lambda)(-1-\lambda)(-2-\lambda) - 2a = 0$$

$$(\lambda+1)(\lambda+2)(\lambda+3) + 2a = 0 \quad \ldots\text{(i)}$$

If A has three distinct Eigen values then A has three linearly independent eigen vectors.

Let $f(\lambda) = (\lambda+1)(\lambda+2)(\lambda+3)$

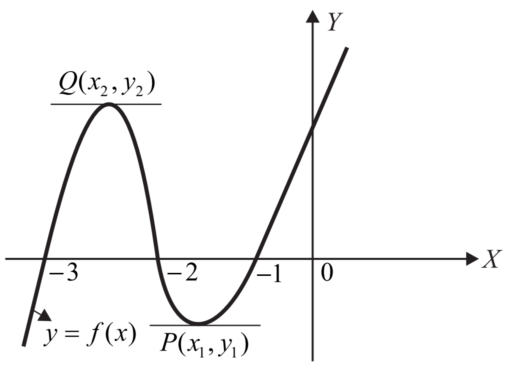
From equation (i),

$$f(\lambda) + 2a = 0$$

$$f(\lambda) = -2a \qquad \ldots\text{(ii)}$$

Consider $f(x) = (x+1)(x+2)(x+3)$

The graph of $f(x)$ is as shown in the figure below,



The number of distinct real roots of an equation $F(x) = k$, (k is real) is same as that of the number of points of intersection of the curve $y = F(x)$ and the line $y = k$.

The curve $y = f(x)$ intersects at three points with a line $y = y_0$ only when $y_1 \le y_0 \le y_2$ i.e. for $f(x) + 2a = 0 \Rightarrow f(x) = -2a$, three distinct real roots exist for

$$y_1 \le -2a \le y_2 \qquad \ldots\text{(iii)}$$

i.e. $$y_1 \le f(x) \le y_2 \qquad \ldots\text{(iv)}$$

[From equation (ii)]

Now we will find y_1 and y_2 [i.e., the minimum and maximum values of $f(x)$]

$$f(x) = (x+1)(x+2)(x+3)$$

$$f(x) = x^3 + 6x^2 + 11x + 6$$

$$f'(x) = 3x^3 + 12x + 11 = 0$$

$$f'(x) = 0 \Rightarrow \quad 3x^3 + 12x + 11 = 0$$

$$x = \frac{-6 \pm \sqrt{3}}{3}$$

and $$f''(x) = 6x + 12$$

At $x = \dfrac{-6+\sqrt{3}}{3}$; $f''(x) = 2\sqrt{3} > 0$ and

At $x = \dfrac{-6-\sqrt{3}}{3}$; $f''(x) = -2\sqrt{3} < 0$

Since, $f(x)$ has a maximum at $x = \dfrac{-6-\sqrt{3}}{3}$

and a minimum at $x = \dfrac{-6+\sqrt{3}}{3}$

The maximum value of $f(x) = y_2 = f(x) = \dfrac{2}{3\sqrt{3}}$

at $$x = \frac{-6-\sqrt{3}}{3}$$

The minimum value of $f(x) = y_1 = f(x)$

at $$x = \frac{-6-\sqrt{3}}{3} = \frac{-2}{3\sqrt{3}}$$

From equation (iv),

$$\frac{-2}{3\sqrt{3}} \le f(x) \le \frac{2}{3\sqrt{3}}$$

$$\frac{-2}{3\sqrt{3}} \le -2a \le \frac{2}{3\sqrt{3}} \qquad \text{[From equation (iii)]}$$

$$\frac{1}{3\sqrt{3}} \ge a \ge \frac{-1}{3\sqrt{3}} \Rightarrow \frac{-1}{3\sqrt{3}} \le a \le \frac{1}{3\sqrt{3}}$$

Therefore, the maximum value of 'a' such that the matrix A has three real linearly independent eigen vectors is $\dfrac{1}{3\sqrt{3}}$.

Hence, the correct option (B).

Key Point

A continuous function $f(x)$ at an stationary point (given by the root of $f'(x) = 0$) is :

(i) Maximum if $f''(x) =$ Negative

(ii) Minimum if $f''(x) =$ Positive

1.4 1

Given : A^5 is a null matrix

$$A^5 = [0]_{10\times 10}$$

Multiplying both side with A^{-1}

$\therefore \quad A^{-1}A^5 = A^{-1}[0]_{10\times 10}$

$IA^4 = 0 \ (\because A^{-1}A = I)$

$A^4 = 0 \qquad \ldots(i)$

Again multiplying equation (i) with A^{-1}

$A^{-1}A^4 = A^{-1}0$

$A^3 = 0$

And so on, finally we will get $A = 0$

We have to find determinant of matrix

$[A+I]$ can be written as,

$[A+I]_{10\times 10} = [I]_{10\times 10}$

Determinant of matrix $[A+I]_{10\times 10}$ is given by,

$|A+I| = |0+I| = |I|$

$|A+I| = 1$

Hence, the correct answer is 1.

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2 Differential Equations

➢ Sample Questions

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2.1 For the differential equation

$$x''(t)+3x'(t)+2x(t)=5$$

the solution $x(t)$ approaches which of the following values as $t\to\infty$?

(A) 0 (B) $\frac{5}{2}$

(C) 5 (D) 10

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2.2 The solution of the differential equation, for $t>0$,

$$y''(t)+2y'(t)+y(t)=0$$

with initial condition $y(0)=0$ and $y'(0)=1$, is ($u(t)$ denotes the unit step function), **[Set - 02]**

(A) $te^{-t}u(t)$

(B) $(e^{-t}-te^{-t})u(t)$

(C) $(e^{-t}+te^{-t})u(t)$

(D) $e^{-t}u(t)$

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2.3 Suppose the circles $x^2+y^2=1$ and $(x-1)^2+(y-1)^2=r^2$ intersect each other orthogonally at the point (u,v). Then $u+v=$ _______.

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Explanations **Differential Equations**

2.1 (B)

Given : $x''(t)+3x'(t)+2x(t)=5$

$$(D^2+3D+2)x(t)=5$$

Method 1

This is in form of a non-homogeneous linear differential equation.

$$[f(D)]x(t)=\phi(t)$$

The auxiliary equation is given by,

$$f(m)=0$$

$$m^2+3m+2=0\Rightarrow(m+2)(m+1)$$

$$m_1=-2,\ m_2=-1$$

The roots are real and distinct. So, the complementary function is given by,

$$\text{C.F.} = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$

$$\text{C.F.} = C_1 e^{-2t} + C_2 e^{-t}$$

and particular integral is given by,

$$\text{P.I.} = \frac{1}{f(D)} \times 5 = \frac{5e^0}{D^2 + 3D + 2}$$

$$\text{P.I.} = \frac{5}{0+0+2} = \frac{5}{2}$$

The complete solution is,

$$x(t) = \text{C.F.} + \text{P.I.}$$

$$x(t) = C_1 e^{-2t} + C_2 e^{-t} + \frac{5}{2}$$

At $t \to \infty$,

$$x(\infty) = C_1 e^{-\infty} + C_2 e^{-\infty} + \frac{5}{2} = \frac{5}{2}$$

Hence, the correct option is (B).

Method 2

Taking Laplace transform of given equation,

$$s^2 X(s) + 3sX(s) + 2X(s) = \frac{5}{s}$$

$$X(s) = \frac{5}{s(s^2 + 3s + 2)}$$

According to final value theorem,

$$x(\infty) = \lim_{s \to 0} sX(s)$$

$$x(\infty) = \lim_{s \to 0} s \times \frac{5}{s(s^2 + 3s + 2)} = \frac{5}{2}$$

Hence, the correct option is (B).

Scan for Video Solution

Key point

When the roots of auxiliary equation are :

(i) Real and distinct (a, b),

$$\text{C.F.} = C_1 e^{at} + C_2 e^{bt}$$

(ii) Real and equal (a, a),

$$\text{C.F.} = (C_1 + C_2 t) e^{at}$$

(iii) Pair of surds $(a \pm \sqrt{b})$,

$$\text{C.F.} = e^{at}(C_1 \cosh \sqrt{b}t + C_2 \sinh \sqrt{b}t)$$

(iv) Pair of complex roots $(a \pm ib)$,

$$\text{C.F.} = e^{at}(C_1 \cos bt + C_2 \sin bt)$$

2.2 (A)

Given :

(i) $y''(t) + 2y'(t) + y(t) = 0$

(ii) Initial conditions :

$y(0) = 0$ and $y'(0) = 1$

Method 1

Taking Laplace transform of given equation,

$$\left[s^2Y(s) - sy(0) - y'(0)\right] + 2\left[sY(s) - y(0)\right] + Y(s) = 0$$

$$(s^2 + 2s + 1)Y(s) - (s+2)y(0) - y'(0) = 0$$

$$(s^2 + 2s + 1)Y(s) - (s+2) \times 0 - 1 = 0$$

$$Y(s) = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2}$$

$$Y(s) = \frac{1}{(s+1)^2}$$

Taking inverse Laplace transform,

$$y(t) = te^{-t}u(t)$$

Hence, the correct option is (A).

Method 2

This is in form of a homogeneous linear differential equation.

$$\left[f(D)\right] y(t) = 0$$

The auxiliary equation is given by,

$$f(m) = 0$$

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0$$

$$m_1 = m_2 = -1$$

The roots are real and equal. So, the complementary function is given by,

$$\text{C.F.} = C_1 + C_2(t)e^{-t}$$

Particular integral (P.I.) is 0, because it is a homogeneous differential equation.

The complete solution is,

$$y(t) = \text{C.F.} + \text{P.I.}$$

$$y = C_1 + C_2(t)e^{-t} \quad \ldots\text{(i)}$$

Using boundary conditions,

$$y(0) = 0$$

$$0 = C_1 + C_2 \times 0$$

$$C_1 = 0$$

Also, $y'(0) = 1$

$$y'(t) = -C_1 e^{-t} + C_2(-te^{-t} + e^{-t})$$

$$1 = C_2(0+1) - C_1$$

$$C_2 = 1$$

From equation (i),

$$y = te^{-t}u(t)$$

Hence, the correct option is (A).

2.3 **1**

Given : $x^2 + y^2 = 1 \quad \ldots\text{(i)}$

$$(x-1)^2 + (y-1)^2 = r^2 \quad \ldots\text{(ii)}$$

Differentiating equation (i) with respect to x,

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

Differentiating equation (ii) with respect to x,

$$2(x-1) + 2(y-1)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(x-1)}{(y-1)}$$

Let m_1 be the slope of equation (i)

$$m_1 = \frac{dy}{dx} = \frac{-x}{y}$$

Let m_2 be the slope of equation (ii)

$$m_2 = \frac{dy}{dx} = \frac{-(x-1)}{(y-1)}$$

Since, equation (i) and (ii) intersect orthogonally each other at the point (u, v)

Therefore, slope of equation (i) and (ii) must be satisfied at the point (u, v)

Thus, $m_1 = \frac{-u}{v}$

$$m_2 = \frac{-(u-1)}{(v-1)} = \frac{1-u}{v-1}$$

From the concept of straight line

$$m_1 \times m_2 = -1$$

$$\left(\frac{-u}{v}\right)\left(\frac{1-u}{v-1}\right) = -1$$

$$\frac{u^2 - u}{v^2 - v} = -1$$

$$u - u^2 = v^2 - v$$

$$u + v = v^2 + u^2 \quad \ldots\text{(iii)}$$

It is given that

$$u^2 + v^2 = 1$$

(Since the point (u, v) is the point of intersection and hence will satisfy the both equation of circles)

Therefore, from equation (iii),

$$u + v = 1$$

Hence, the correct answer is 1.

Key Point

From the concept of straight line, if two line intersect each other orthogonally them product of their slope will be $-\tan\theta \times \cot\theta = m_1 \times m_2 = -1$.

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