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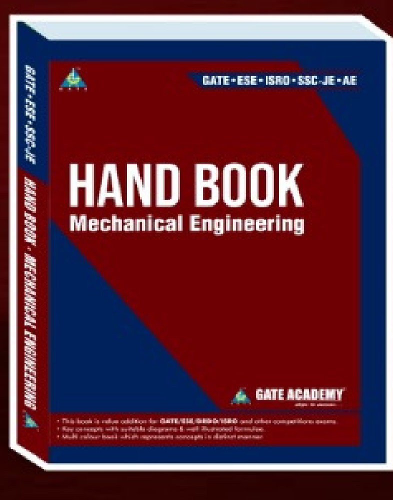

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PREFACE

It is our pleasure, that we insist on presenting **"Hand Book"** authored for **Mechanical Engineering** to all of the aspirants and career seekers. The prime objective of this book is to respond to tremendous amount of ever growing demand for error free, flawless and succinct but conceptually empowered key theory concepts for students.

This book serves to the best supplement for GATE/ESE/PSUs but shall be useful to a larger extent for other discipline as well.

This book contains quite sufficient details on every topic in a very systematic and organized manner. All the chapter are arranged topic wise as given in syllabus and well-illustrated diagrams are also included.

The authors do not sense any deficit in believing that this title will in many aspects, be different from the similar titles within the search of student.

We would like to express our sincere appreciation to **Mrs. Sakshi Dhande Mam** (Co-Director, GATE ACADEMY Learning Pvt. Ltd.) for her constant support and constructive suggestions and comments in reviewing the script.

In particular, we wish to thank **GATE ACADEMY** expert team members for their hard work and consistency while designing the script.

The final manuscript has been prepared with utmost care. However, going a line that, there is always room for improvement in anything done, we would welcome and greatly appreciate the suggestions and corrections for further improvement.

Umesh Dhande
(Director, GATE ACADEMY Learning Pvt. Ltd.)

ACKNOWLEDGEMENT

We are glad of this opportunity to acknowledge the views and to express with all the weaknesses of mere words the gratitude that we must always feel for the generosity of them.

We now express our gracious gratitude to the persons who have contributed a lot in order to put forth this into device. They are to be mentioned here and they are Sujeet Pandey, Prashant Pandey, Naresh Sonkar, Khomesh Sahu, Amit Kumar Singh, Mrinal Banerjee and Mohammad Asif Khan.

We would also like to express our gracious gratitude to the faculty members of Gate Academy who have contributed a lot in order to put forth this into device. They are to be mentioned here and they are **Mr. Deepraj Chandrakar, Mr. Yogesh Tyagi** & **Mr. Venugopal Sharma**.

Special thanks to **Mr. Akshay Mishra,** who has been involved in this project from the beginning and has given his best effort on his part. This book was not possible without his unconditional effort.

Lastly, we take this opportunity to acknowledge the service of the total team of publication and everyone who collaborated in producing this work.

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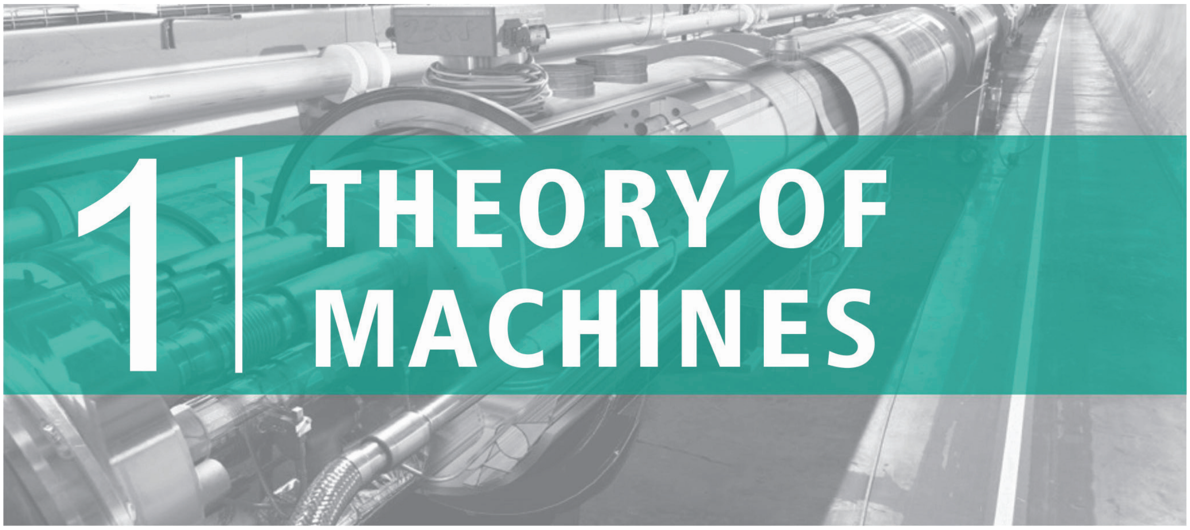
CONTENTS



1 THEORY OF MACHINES

1.1 Mechanism & Machines

Mechanism :

- If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a *mechanism*.
- A mechanism transmits and modifies a motion.

Example : Lever, gears, belt and chain drive.

Machine :

A machine is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work.

Example : Automobiles and refrigerator.

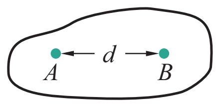
Kinematic Link/Element/Link :

Every part of machine which is having relative motion with respect to other parts is known as *kinematic links*.

Key Point

- It is not necessary links should be rigid, it should be *resistant body* which are able to transfer the motion/power.
- Semi-rigid bodies which are normally flexible, but under certain conditions act as rigid bodies for the limited purpose are called *resistant bodies*.

  Example : A belt is rigid when subjected to tensile forces.



- Under the action of forces, if a body does not suffer any distortion or the distance between any two points is constant, the body is said to be *rigid body*.

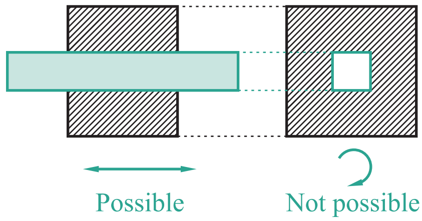
Classification of Link :

Singular link | Binary link | Ternary Link | Quaternary Link

Types of Contrained Motion :

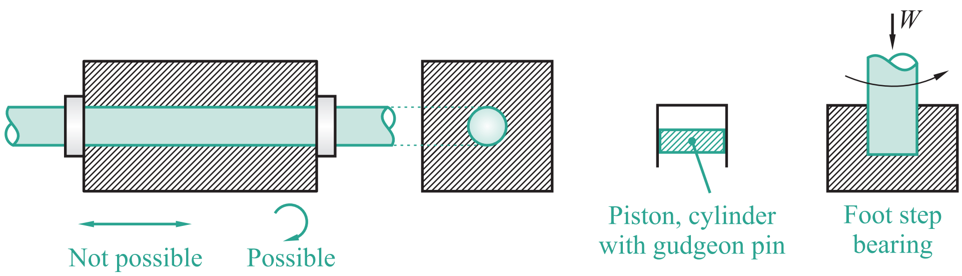
It is also known as desirable motion obtained.

1. Completely constrained motion :



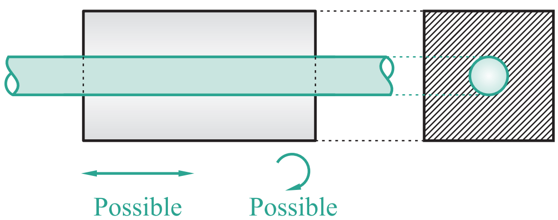
When desired motion is obtained by construction itself, it is called *completely constrained motion.*

2. Successfully constrained motion :



When desired motion is obtained by external means, it is known as *successfully constrained motion.*

3. Incompletely constrained motion :



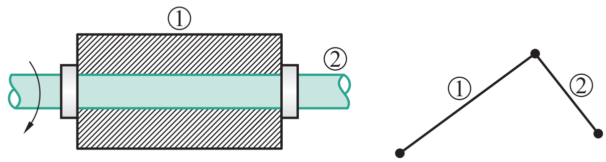
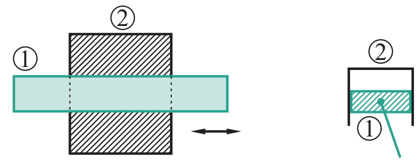
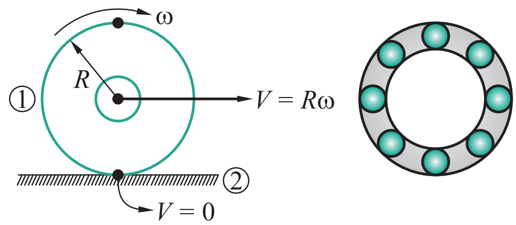
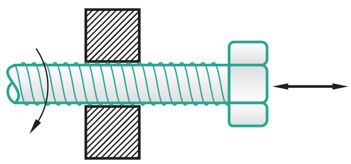
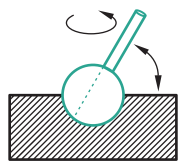
When desired motion is not possible, it is called *incompletely constrained motion.*

Kinematic Pair :

The connection between two links is always known as a pair or joint, but it will be kinematic pair, if motion between link is constrained (completely or successfully).

Classification of Kinematic Pair :

1. According to the type of relative motion :

Pair	Diagram
Turning pair : Also known as *revolute pair* or *pin joint.*	
Sliding pair : Also known as *pure sliding, prismatic pair.*	
Rolling pair : Ball and roller bearing are examples of this.	
Screw (Helical) pair : • Turning + Sliding • Achieved by cutting *threads.*	
Spherical pair : When sphere turns inside a fixed link. **Example :** Ball socket, 3D motion (rotation).	

2. According to the type of contact :

Pair	Examples
Lower pair : • Elements of pair having surface to surface (or area) contact. • Both have similar surface.	• Turning pair. • Sliding pair. • Universal joint. • Screw pair.
Higher pair : • Point or line contact. • Surface are dissimilar	• Rolling pair. • Spherical joint.
Wrapping pair : One link wraps over other links.	• Belt drive. • Chain drive.

3. According to the type of closure :

(i) Permanent closed pair :

- When links are mechanically held together.
- Also known as *self-closed pair*.

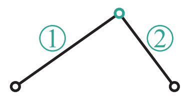
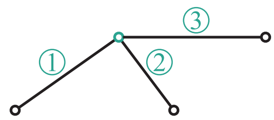
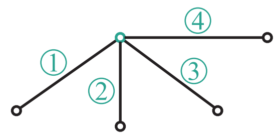
Example : Piston and cylinder.

(ii) Unclosed pair :

- Pairs are connected by action of *external force* (force of gravity or spring action).
- Links are not held mechanically in this.
- Also known as *force-closed pair*.

Example : Cam and follower pair.

Types of Joints :

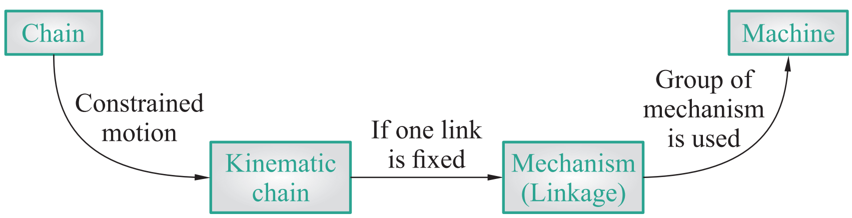
Types of joints	Diagram
Binary joint (B)	 (1, 2)
Ternary joint (T)	 (1, 2) (1, 3) $1T = 2B$
Quaternary joint (Q)	 (1, 2) (1, 3) (1, 4) $1Q = 3B$

Key Point

If 'n' number of links are connected with a single pin then number of binary joint will be 'n-1'.

Kinematic Chain :

When links are connected in such a way that 1st link is connected with the last link it will form a *chain*, but if the motion transfer by the chain is constrained motion, then the chain is known as *kinematic chain*.



Types of kinematic chains :

1. **Open kinematic chain :** When 1st link is not connected with last link it will form an open kinematic chain. **Example :** Robot arm.

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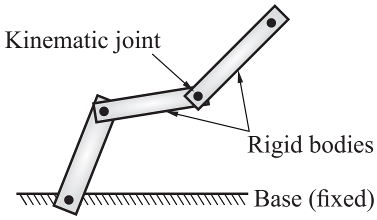
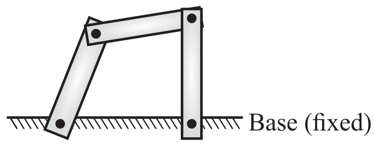


2. **Closed kinematic chain :** When 1st link is connected to the last link it will form a closed kinematic chain.



The following relationship holds for a kinematic chain having lower pair only,

$$l = 2P - 4$$

$$j = \frac{3}{2}l - 2$$

Where, $l =$ Number of binary links, $P =$ Number of lower pair and
$j =$ Number of binary joints.

Key Point

Condition for kinematic chain :

For higher pairs, $\underbrace{j + \frac{h}{2}}_{\text{LHS}} = \underbrace{\frac{3}{2}l - 2}_{\text{RHS}} \rightarrow$ Kinematic chain

($\text{LHS} - \text{RHS} = d$, if $d > 0.5$ then it will be superstructure).
Where, $h =$ Number of higher pair.

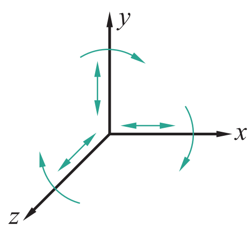
$\Rightarrow \quad j + \frac{h}{2} > \frac{3}{2}l - 2 \rightarrow$ Frame/structure

(If one link of a redundant chain is fixed, it is known as structure or locked system).

$\Rightarrow \quad j + \frac{h}{2} < \frac{3}{2}l - 2 \rightarrow$ Unconstrained

Degrees of Freedom (Mobility of Linkage) :

The number of independent variables required to define motion or position of a system is known as *degree of freedom* of the system.

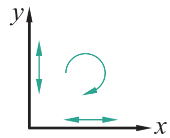


$$DOF = 6 - \text{Restraints (Arrested)}$$

- The motion which is not possible is known as *restraints.*

> **Key Point**
> - Lower pair = 1 *DOF*.
> - Higher pair = 2 *DOF*.
> - Spherical pair = 3 *DOF*.
> - Number of restraints can never be *zero* (joint is disconnected) or *six* (joint becomes solid).

For 2D-planner mechanism :



- Max motion = 3
- If number of link = l, for mechanism $l-1$.

  Grubbler equation for *DOF*, $F=3(l-1)-2j-h$

  Kutzback equation for *DOF*, $F=3(l-1)-2j-h-F_r$

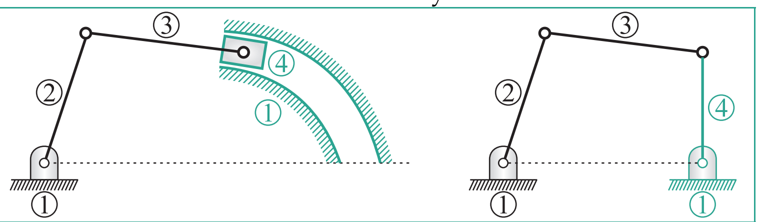
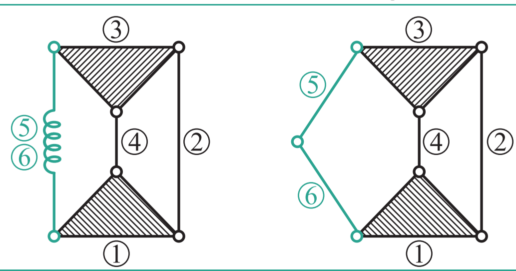
  Where, F_r = Motion which is not the part of mechanism (redundant motion).

> **Key Point**
> - *DOF* of a structure or locked system is zero.
> - If *DOF* is negative, it is known as *superstructure.*

Simple Mechanism :

- A mechanism with four links is known as *simple mechanism,* and the mechanism with more than four links known as *compound mechanism.*
- When all the links lie in the same plane it is known as *planar mechanism* and when all the links of mechanism lie in different planes it is known as *spatial mechanism.*

> **Key Point**
>
> **Equivalent mechanisms :**
> - It is possible to replace turning pairs of plane mechanisms by other types of pairs having one or two degrees of freedom.
> - The new mechanism have same *DOF* and are kinematically similar.

Sliding pairs in place of turning pairs.	
Spring replaced by turning pairs.	

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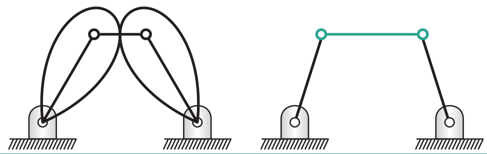
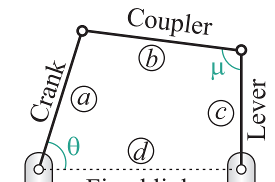
Cam pair in place of turning pairs (higher pair replaced by lower pair).	

Four Bar Chain :



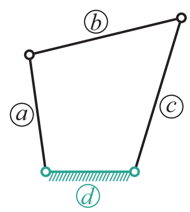
Where, $\theta =$ Input angle and $\mu =$ Transmission angle.

🕮 Key Point

In case of double-crank or crank-rocker mechanism,

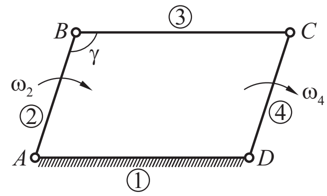
- μ_{max}, when $\theta = 180^0$, $(a+d)^2 = b^2 + c^2 - 2bc\cos(\mu_{max})$.
- μ_{min}, when $\theta = 0^0$, $(d-a)^2 = b^2 + c^2 - 2bc\cos(\mu_{min})$.

Grashof's Law :

Class 1 : Four bar linkage • $d + c < a + b$ (d: Shortest link, c: Longest link)	• When (d) is fixed  *Crank-crank* or *double-crank* or *drag-crank mechanism* or *rotary-rotary converter.* • When (a) or (c) is fixed, the mechanism is *crank-rocker* or *crank lever mechanism* or *rotary-oscillating converter.* • When (b) is fixed, the mechanism is *rocker-rocker* or *double-rocker* or *double-lever mechanism* or *oscilating-oscilating converter.*
Class 2 : Four bar linkage $d + c > a + b$	Fixing any one of the link always results in *rocker-rocker* or *double-rocker mechanism.*
Class 3 : Four bar linkage • $d + c = a + b$ • Similar to class I mechanisms. • Special cases may arise.	**Special cases :** • *Parallel-crank four bar linkage,* in this opposite links are parallel and equal in length. $a = c, b = d$ (Parallelogram) • *Deltoid linkage,* in this equal links ($a = d$, $b = c$) are adjacent to each other, and fixing the shorter link giver *double-crank mechanism.*

Mechanical Advantage :

$$\text{Mechanical advantage} = \frac{\text{Output force/Torque}}{\text{Input force/Torque}}$$



Power input = Power output

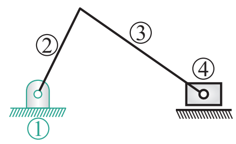
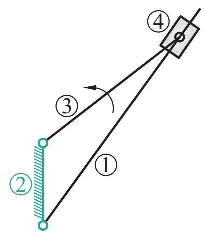
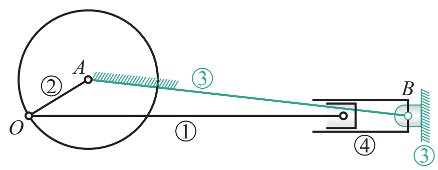
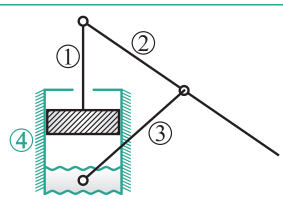
$$T_2\omega_2 = T_4\omega_4$$

$$\text{M.A.} = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}$$

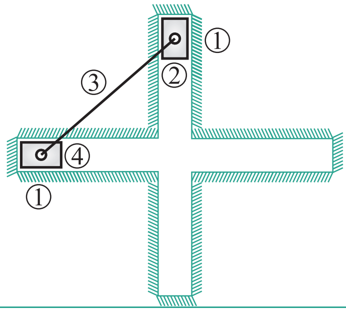
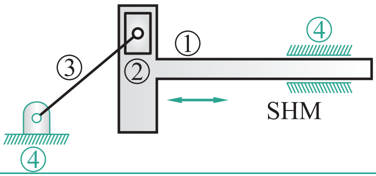
🕮 Key Point

- If γ is equal to 0^0 or 180^0, ω_4 become zero thus mechanical advantage will be *infinity*.
- Extreme position of linkage is known as *toggle postion*.

Slider-Crank Chain :

First inversion : • When link 1 is fixed (cylinder fixed).	 • Reciprocating engine. • Reciprocating compression.
Second inversion : • When link 2 is fixed (crank fixed).	 • Whitworth Quick-return mechanism. • Rotary engine.
Third inversion : When link 3 is fixed (connecting rod fixed).	 • Oscillating cylinder engine. • Crank and slotted lever mechanism.
Fourth inversion : When link 4 is fixed (slider fixed).	 **Hand pump**

Double Slider-Crank Chain :

First inversion : • Elliptical tramnel. • When link 1 is fixed.	
Second inversion : • When any slider (link 2 or link 4) is fixed. • Scotch-yoke mechanism.	
Third inversion : • When link 3 is fixed. • Oldham's coupling. • To connect two shaft having small lateral misalignment.	Maximum velocity of sliding of each tongue is given by $= w \times d$. Where, $d =$ Distance between two parallel shaft.

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2 THERMODYNAMICS

2.5 Second Law of Thermodynamics

Introduction :

- It states that all kinds of energy are not of the same *quality*.
- Electricity or work is a superior form of energy (*high grade energy*), while heat is an inferior form (*low grade energy*).
- 2nd Law relates the direction of flow of heat, *dictates limits on the conversion* of heat into work.
- It also tells us whether a *particular change is feasible or not.*
- Introduces the concept of *entropy.*
- Exergy (or availability) is a measure of the *quality index of energy*.

Thermal Energy Reservoir :

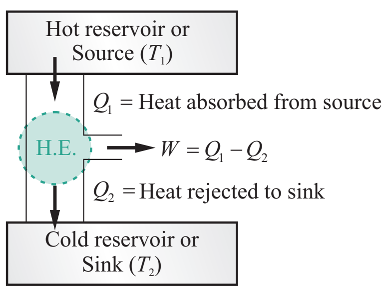
A thermal energy reserviour is defined as a large body of infinite heat capacity, which is capable of absorbing or rejecting an unlimited quantity of heat without suffering appreciable changes in its thermodynamic coordinates.

Heat reserviors are of two types :

1. Heat source
2. Heat sink

Heat Engine :

- Heat engine is a device by which a system is made to undergo cyclic process that result in conversion of heat to work.
- A heat engine cycle is thermodynamics cycle in which there is a net heat transfer to the system and a net-work transfer from the system. The system which executes a heat engine cycle is called a *heat engine*.



Thermal efficiency :

$$\eta_{th} = \frac{\text{Net work output of the cycle}}{\text{Total heat input to the cycle}}$$

$$\eta_{th} = \frac{W_{net}}{Q_1} = \frac{Q_{net}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Key Point

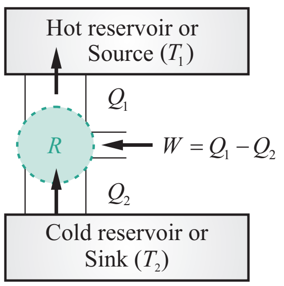
The efficiency of a reversible heat engine in which heat is received solely at T_1 is,

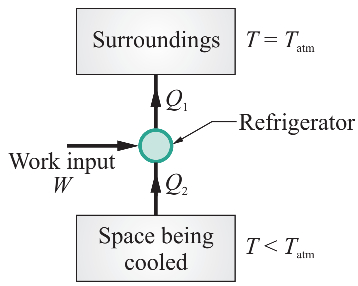
$$\eta_{rev} = \eta_{max} = 1 - \left(\frac{Q_2}{Q_1}\right)_{rev} = 1 - \frac{T_2}{T_1}$$

$$\eta_{rev} = \frac{T_1 - T_2}{T_1}$$

Refrigerator :

- A refrigerator or heat pump is basically a heat engine which runs in reverse directions.
- A *refrigerator* is a device which, operating in a cycle, maintains a body at a temperature lower than the temperature of the surroundings.





$$COP = \frac{\text{Desired effect}}{\text{Work input}} = \frac{Q_2}{W}$$

$$(COP)_{Ref} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

Key Point

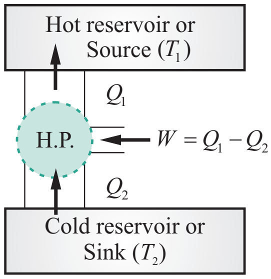
For reversible refrigerator using, $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

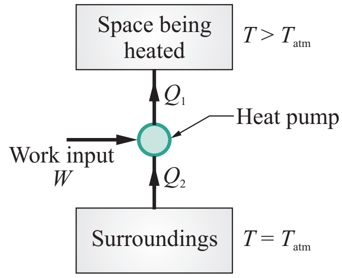
$\Rightarrow$ $$(COP_{Ref})_{rev} = \frac{T_2}{T_1 - T_2}$$

Heat Pump :

- The basic difference between a heat pump and refrigerator is that, the refrigerator gives cooling effect (Q_2) while a heat pump give a heating effect (Q_1).

- A *heat pump* is a device which operating in a cycle, maintains a body, at a temperature higher than the surrounding.





$$COP = \frac{Q_1}{W}$$

$$(COP)_{H.P.} = \frac{Q_1}{Q_1 - Q_2}$$

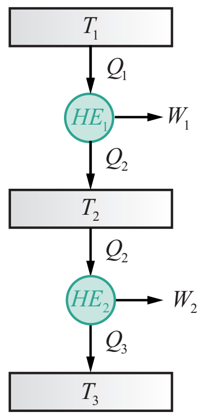
It is found that, $(COP)_{H.P.} = (COP)_{Ref} + 1$

Key Point

For a reversible heat pump, $(COP_{H.P.})_{rev} = \dfrac{T_1}{T_1 - T_2}$

Note :

1. **Efficiency of reversible heat engine :** Changing the sink temperature has a great improvement in efficiency or compare to changing the source temperature, i.e., $T_2 \downarrow$ (Lowered) and $T_1 \uparrow$ (Increased)



$$\Rightarrow \quad \left(\frac{d\eta}{dT_2}\right)_{T_1 = C} > \left(\frac{d\eta}{dT_1}\right)_{T_2 = C}$$

2. **COP of reversible refrigerator :** In refrigerator changing the sink temperature gives a great improvement as compared to changing the source temperature.

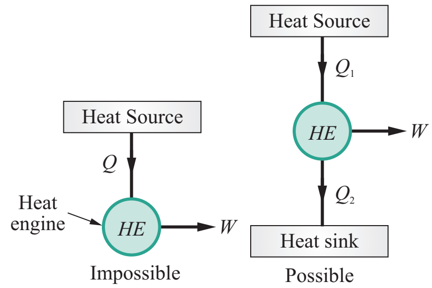
$$\left(\frac{dCOP}{dT_2}\right)_{T_1 = C} > \left(\frac{dCOP}{dT_1}\right)_{T_2 = C}$$

3. **Reversible engines in series :**

- For equal work outputs, $T_2 = \dfrac{T_1 + T_3}{2}$
- For equal efficiencies, $T_2 = \sqrt{T_1 T_3}$
- Overall efficiency, $\eta = \eta_1 + \eta_2 - \eta_1\eta_2$.

Kelvin-Plancks Statement of Second Law :

- *It is impossible to construct an engine that operates in a cycle and produces no effect other than work output and exchange of heat with a single heat reservoir.*
- Kelvin-Planck statement is applied to heat engines.



📖 Key Point

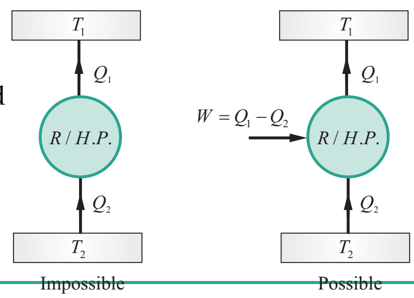
- The Kelvin-Planck statement does not rule out the possibility of a system developing a net amount of work from a heat transfer drawn from a single reservoir (eg. Isothermal process). It only denies this possibility if the system undergoes a thermodynamics cycle.
- If $Q_2 = 0$ (i.e., $W_{net} = Q_1$ or $\eta = 1.00$), the heat engine will produce net-work in a complete cycle by exchanging heat with only reservoir, thus violating the Kelvin-Planck statement. Such a heat engine is called *perpetual motion machine of the second kind* (PMM2). A PMM2 is impossible.

Clausius Statement of Second Law :

- *It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a system at low temperature to another system at high temperature.*
- Clausius statement concerns heat pump and refrigerators.

📖 Key Point

- Clausius statement tells that *COP* of a heat pump/refrigerator cannot be equal to infinity.
- It tells that this spontaneous process cannot proceed in the reverse directions.



Equivalence of Both Statements :

- It is impossible to have a device satisfying one statement and violating the other.
- Any device that violates Clausius statement leads to violation of Kelvin-Planks statement and vice-versa.

Carnot's Theorem :

It states that of all heat engines operating between a given constant temperature source and a given constant temperature sink, none has a higher efficiency than a reversible engine.

$$\eta_B \geq \eta_A$$

Where, E_B = Any reversible heat engine and E_A = Any heat engine.

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Corollaries of Carnot's Theorem :

- The efficiency of all reversible heat engine operating between the same temperature level is the same.
- The efficiency of a reversible engine is independent of the nature or amount of the working substance undergoing the cycle.

Thermodynamic Temperature Scale :

- A temperature scale which is independent of the property of thermometric substance is defined as thermodynamics temperature scale.
- Carnot engine operating between thermal reservoir at temperatures T and T_t (Triple point of water),

$$T = 273.16 \times \frac{Q}{Q_t} = T_t \times \frac{Q}{Q_t}$$

- In the Kelvin scale, the amount of heat supplied plays the role of thermodynamics property.

Key Point

The Kelvin temperature is numerically equal to the ideal gas temperature and *may be measured by means of gas thermometer.*

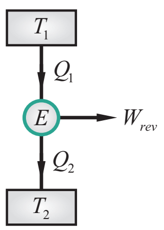
Third Law of Thermodynamics :

It is impossible by one procedure, no matter how idealized, to reduce any system to the absolute zero of temperature in a finite number of operations.

2.6 Entropy

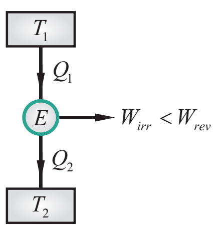
Clausius Inequality :

Reversible heat engine :



$\sum \frac{Q}{T} = 0$ for a reversible engine.

Irreversible heat engine :



$\sum \frac{Q}{T} < 0$ for an irreversible engine.

Clausius theorem : *The algebraic sum of the ratio* $\frac{\delta Q}{T}$ *i.e., the heat interaction to the absolute temperature for a reversible heat engine is equal to zero.*

$$\sum \frac{Q_i}{T_i} = 0 \text{ or } \oint_R \frac{\delta Q}{T} = 0$$

Clausius inequality : The algebraic sum of the ratio $\frac{\delta Q}{T}$ i.e., the heat interaction to the absolute temperature for an irreversible heat engine is less than zero.

$$\sum \frac{Q_i}{T_i} < 0 \text{ or } \oint_I \frac{\delta Q}{T} < 0$$

🕮 Key Point

- In general form, for any engine, $\oint \frac{\delta Q}{T} \le 0$
- If I indicates the amount by which the given cycle is irreversible.

  $\therefore \qquad \oint \frac{\delta Q}{T} + I = 0$
- $\oint \frac{\delta Q}{T} > 0$, violates the second law and is impossible.

Entropy :

The cyclic integral of the quantity $\delta Q/T$ for a reversible process is zero. This suggests that the quantity $\delta Q/T$ is a *point function* and hence a *property* of the system called *entropy*.

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_{rev} = \int_1^2 dS = (S_2 - S_1)\text{ kJ/K}$$

It is an *extensive property*. The units of specific entropy is kJ/kg K.

🕮 Key Point

- Entropy remains constant during a *reversible adiabatic process* and is called *isentropic process*.
- Entropy change in irreversible process is given by, $dS > \frac{\delta Q}{T}$ or $S_2 - S_1 \ge \int_1^2 \frac{\delta Q}{T}$.

Entropy Principle :

Entropy of an isolated system either increase or in the limit remains constant.

$$(dS)_{isolated} \ge 0 \text{ or } (dS)_{universe} \ge 0$$

$$(dS)_{system} + (dS)_{surrounding} \ge 0$$

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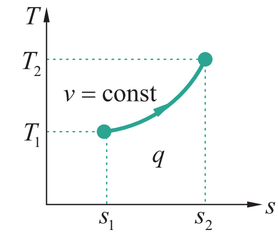
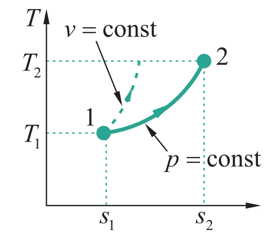
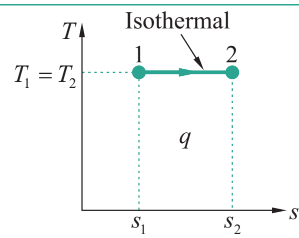
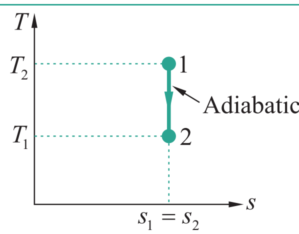
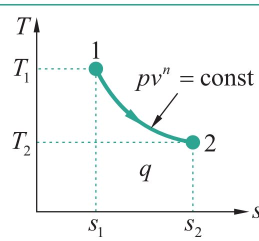
Entropy Change During a Process :

The twin equation of entropy is given by,

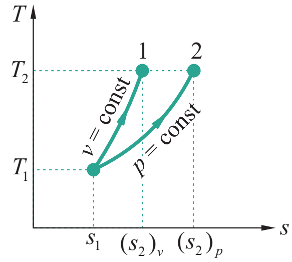
$$Tds = du + pdv \qquad \ldots(i)$$

$$Tds = dh - vdp \qquad \ldots(ii)$$

Equation (i) and (ii) are used for any process (reversible/irreversible) and for any system (open/closed).

Process	*T-s* diagram	Change in entropy
Constant volume		$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} = c_v \log_e \frac{p_2}{p_1}$
Constant pressure		$s_2 - s_1 = c_p \log_e \frac{T_2}{T_1} = c_p \log_e \frac{v_2}{v_1}$
Isothermal process		$s_2 - s_1 = R \log_e \frac{v_2}{v_1} = R \log_e \frac{p_1}{p_2}$
Reversible adiabatic (isentropic)		$dS = 0$ **Note :** Two reversible adiabatic path cannot intersect each other.
Polytropic		$s_2 - s_1 = c_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1}$ $s_2 - s_1 = c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{v_2}{v_1}$ $s_2 - s_1 = c_p \log_e \frac{T_2}{T_1} + R \log_e \frac{p_1}{p_2}$

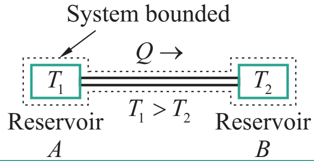
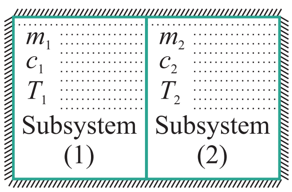
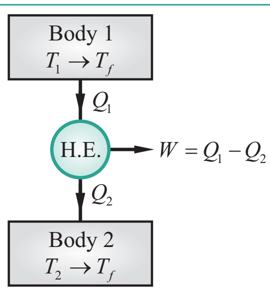
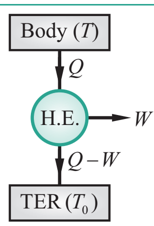
📖 Key Point



Slope of constant volume line, $\frac{dT}{ds} = \frac{T}{c_v}$ and slope of constant pressure line, $\frac{dT}{ds} = \frac{T}{c_p}$.

The constant volume line is steeper than constant pressure lines.

Applications of Entropy Principle :

Condition	Diagram	Equations
Transfer of heat through a finite temperature difference.		$\Delta s_{univ} = \Delta s_A + \Delta s_B$ $\Delta s_{univ} = -\frac{Q}{T_1} + \frac{Q}{T_2} = Q\frac{T_1 - T_2}{T_1 T_2}$
Mixing of two fluids		Final temperature, $T_f = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}$ $(\Delta s)_{univ} = m_1 c_1 \ln\frac{T_f}{T_1} + m_2 c_2 \ln\frac{T_f}{T_2}$ $(\Delta s)_{univ}$ is $(+ve)$ because mixing is irreversible process.
Two finite bodies at temperature T_1 and T_2		$T_f = \frac{T_1 + T_2}{2}$ (for no work output) $T_f = \sqrt{T_1 T_2}$ (for max. work) $W_{max} = c_p\left(T_1 + T_2 - 2\sqrt{T_1 T_2}\right)$ $W_{max} = c_p\left(\sqrt{T_1} - \sqrt{T_2}\right)^2$
Maximum work obtainable from a finite body and TER		$(\Delta s)_{univ} = c_p \ln\frac{T_0}{T} + \frac{Q - W}{T_0}$ $W_{max} = c_p\left[(T - T_0) - T_0 \ln\frac{T}{T_0}\right]$

Key Point

- Every irreversible process is accompanied by entropy increase of the universe.
- This entropy increase quantifies the extent of *irreversibility* of the process.

Entropy Generation :

The entropy of any closed system can increase in two ways :

1. By heat interaction in which there is entropy transfer.
2. By internal irreversibilities or dissipative effects in which work (or K.E.) is dissipated into internal energy increase.

$$\underbrace{s_2 - s_1}_{\text{Entropy change}} - \underbrace{\int_1^2 \frac{\delta Q}{T}}_{\text{Entropy transfer}} = \underbrace{S_{generation}}_{\text{Entropy production}} \quad (S_{gen} \geq 0)$$

The entropy generation depends on the path the system follows. It is an inexact differential.

Key Point

Entropy generation in an open system, $\left(\frac{ds}{dt}\right)_{C.V.} = \dot{s}_{in} - \dot{s}_{out} + \dot{s}_{gen}$.

Steady flow process, $\sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum \frac{\dot{Q}}{T} + \dot{s}_{gen}$.

Entropy Change During Phase Change :

$$\Delta S = \frac{mL}{T} \text{ or } s_{fg} = \frac{h_{fg}}{T_{sat}} = \frac{h_g - h_f}{T_{sat}}$$

Where, L = Latent heat of phase change and T = Saturation temperature.

❖❖❖❖

7 | FLUID MECHANICS

7.4 Fluid Kinematics

Method of Describing Fluid Motion :

1. **Lagrangian method :** In this method individual fluid particles is taken into consideration and for analysis, the observer travels with the fluid particles.
2. **Eulerian method :** In this method a finite volume called control volume is taken into consideration through which fluid flow in and out and observer remains stationary and see what happen in a control volume. This method is commonly used in fluid mechanics.

Types of Fluid Flow :

Steady and unsteady flow :

Steady flow	Unsteady flow
Fluid properties do not change with time.	Change with time.
$\frac{\partial V}{dt} = 0, \frac{\partial p}{\partial t} = 0, \frac{\partial \rho}{\partial t} = 0$	$\frac{\partial V}{dt} \neq 0, \frac{\partial p}{\partial t} \neq 0, \frac{\partial \rho}{\partial t} \neq 0$

This is applicable for all properties.

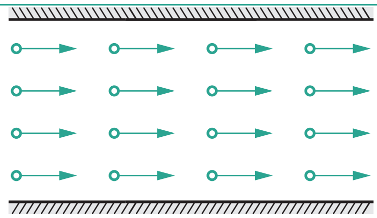
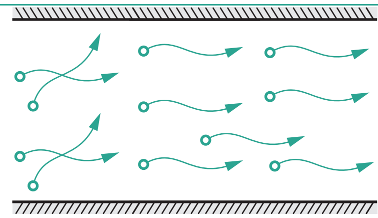
Uniform and non-uniform flow :

Uniform flow	Non-uniform flow
Velocity at any given time does not change with respect to space.	Velocity at any given time changes with respect to space.
$\left(\frac{\partial V}{\partial S}\right)_{t=c} = 0$	$\left(\frac{\partial V}{\partial S}\right)_{t=c} \neq 0$

It is applicable for velocity only.

$\partial S =$ Length of flow in the direction S and $\partial V =$ Change of velocity.

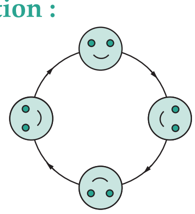
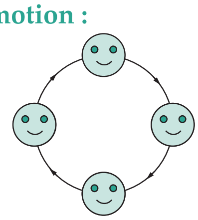
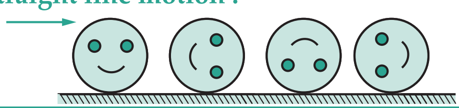
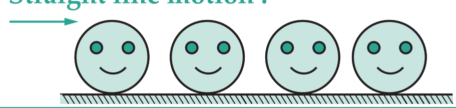
Laminar and turbulent flow :

Laminar flow	Turbulent flow
Fluid particle move along well defined path. The adjacent layer do not cross each other.	Fluid particle move in random order or zig-zag way.
	
Reynold number < 2000 (For pipe flow) Reynold number $< 5\times10^5$ (For flow over flat plate)	Reynold number > 4000 (For pipe flow) Reynold number $= 5\times10^5$ (For flow over flat plate)

Compressible and incompressible flow :

Compressible flow	Incompressible flow
Density of fluid varies point to point.	Density remain constant.
$\rho \neq c$	$\rho = c$

Rotational and irrotational flow :

Rotational flow	Irrotational flow
Fluid particles rotate about its mass centre in both the circular as well as straight line motion	Fluid particles do not rotate about its mass centre in both the circular as well as straight line motion
Circular motion : 	**Circular motion :** 
Straight line motion : 	**Straight line motion :** 

1-D, 2-D, 3-D flow :

$$\left.\begin{array}{ll} u = f(x) & \text{(Steady)} \\ u = f(x, t) & \text{(Unsteady)} \end{array}\right\} \rightarrow \text{1-D}$$

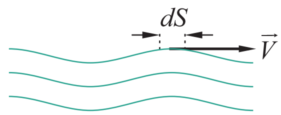
$$\left.\begin{array}{ll} u = f(x, y) & \text{(Steady)} \\ u = f(x, y, t) & \text{(Unsteady)} \end{array}\right\} \rightarrow \text{2-D}$$

$$\left.\begin{array}{ll} u = f(x, y, z) & \text{(Steady)} \\ u = f(x, y, z, t) & \text{(Unsteady)} \end{array}\right\} \rightarrow \text{3-D}$$

Types of Flow Lines :

1. **Stream line :** Stream line is an imaginary line or curve in a flow field such that the tangent drawn at particular point gives the direction of instantaneous velocity at that particular point.

   Consider a small displacement vector (dS) on a flow field.



$$d\vec{S} = dx\hat{i} + dy\hat{j} + dz\hat{k}\ ,\ \vec{V} = u\hat{i} + v\hat{j} + w\hat{k}\ ,\ \vec{V}\times d\vec{S} = 0$$

($\because$ Flow never takes place perpendicular to stream line)

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ u & v & w \\ dx & dy & dz \end{bmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\hat{i}(vdz - wdy) - \hat{j}(udz - wdx) + \hat{k}(udy - vdx) = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

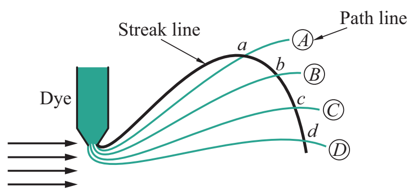
   Comparing both side unit vector,

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \qquad \text{(Stream line equation in 3D flow)}$$

$$\frac{dx}{u} = \frac{dy}{v} \qquad \text{(Stream line equation in 2D flow)}$$

   Slope of stream line in x-y plane, $\frac{dy}{dx} = \frac{v}{u}$.

2. **Streak line :** At any instant of time it is locus of all particles i.e. passes through a fixed point in a flow fluid.



3. **Path line :** Path travelled by a single individual particle over some time interval is called path line.

   Equations for path line, $u = \frac{dx}{dt},\ v = \frac{dy}{dt},\ \omega = \frac{dz}{dt}$.

🕮 Key Point

- In case of steady flow two stream lines can never intersect each other.
- A path line is outcome of lagrangian approach.
- A path line can intersect it self or other path line.
- In case of steady flow, path line, stream line and streak line are identical.

Velocity & Acceleration of Fluid :

Velocity vector, $\overline{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$$|\overline{V}| = \sqrt{u^2 + v^2 + w^2}$$

Where, u, v, w are the components of velocity in x, y, z - direction.

Let a_x, a_y, a_z are component of acceleration in x, y, z - direction respectively.

Local or temporal acceleration

Convective or advective acceleration

$$a_x = \boxed{\frac{\partial u}{\partial t}} + \boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}$$

$$a_y = \boxed{\frac{\partial v}{\partial t}} + \boxed{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}}$$

$$a_z = \boxed{\frac{\partial w}{\partial t}} + \boxed{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}}$$

Local or temporal acceleration : It is defined as the rate of increase of velocity with respect to time at a given point in a flow field. For steady flow, local acceleration does not exist (equal to zero).

Convective or advective acceleration : It is defined as the rate of change of velocity due to the change of position of fluid particles in a fluid flow. For the uniform flow, convective acceleration does not exists (equal to zero).

Key Point

S. No.	Type of flow	Local or temporal acceleration	Convective or advective acceleration	Total or maximum acceleration
1.	Steady + Uniform flow	0	0	0
2.	Unsteady + uniform flow	✓	0	Local
3.	Unsteady and non-uniform flow	✓	✓	Local + Convective
4.	Steady and non-uniform flow	0	✓	Convective

Continuity Equation :

Conservation of mass (m) : Mass flow rate is constant.

For compressible flow, $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

For incompressible flow : Density ρ will be constant, $A_1 V_1 = A_2 V_2$.

AV is known as discharge Q.

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General continuity equation :

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Special case : Steady flow i.e. $\frac{\partial \rho}{\partial t} = 0$.

Then, continuous equation will be, $\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$

Incompressible flow i.e. $\rho =$ constant.

Then, continuity equation will be,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\Delta \bar{V} = 0$$

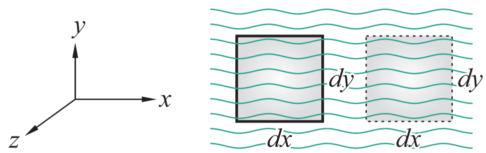
$$\text{div}\bar{V} = 0$$

Key Point

Fluid flow is possible where mass is conserved and continuity equation is satisfied.

Types of Fluid Motion :

1. **Pure translation :** $u =$ Constant, $v =$ Constant .



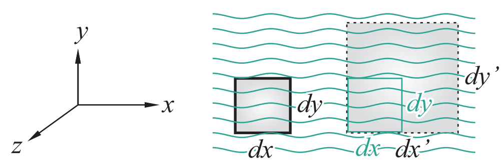
2. **Linear deformation :** $u = f(x), v = f(y)$.

Rate of linear deformation in x -direction,

$$\dot{\varepsilon}_x = \frac{\partial u}{\partial x}$$

Similarly, Rate of linear deformation in y and z - direction,

$$\dot{\varepsilon}_y = \frac{\partial v}{\partial y}, \quad \dot{\varepsilon}_z = \frac{\partial w}{\partial z}$$



Rate of volumetric deformation,

$$\dot{\varepsilon}_V = \dot{\varepsilon}_x + \dot{\varepsilon}_y + \dot{\varepsilon}_z$$

$$\frac{dV}{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

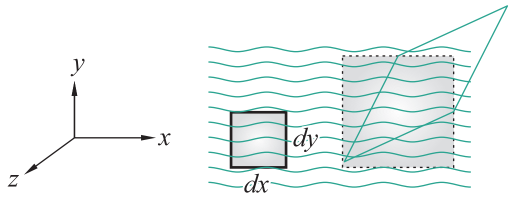
If fluid is incompressible $dV = 0$, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

3. Shear deformation : $u = f(x, y)$, $v = f(x, y)$

Rate of shear deformation in *xy*, *yz* and *zx* - plane are,

$$\dot{\gamma}_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \; \dot{\gamma}_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$$

and $\dot{\gamma}_{zx} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$



4. Rotational flow :

Rotational angular velocity is given by,

$$\overline{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k} \text{ or } \overline{\omega} = \frac{1}{2}(\text{curl}\,\overline{V})$$

$$\therefore \quad \omega_x \hat{i} + \omega_y \hat{j} + \omega_Z \hat{k} = \frac{1}{2}(\nabla \times \overline{V}) \qquad (\because \; \overline{V} = u\hat{i} + v\hat{j} + w\hat{k})$$

$$\omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & \omega \end{vmatrix}$$

Where, u, v, w are component of velocity in x, y and z direction respectively.

Rotational angular velocity component $\omega_x, \omega_y, \omega_z$ in x, y and z direction respectively are given by,

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right), \; \omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \text{ and } \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

So, $\nabla \times \overline{V} = 0$

$\text{Curl}\,\overline{V} = 0$

If curl $\overline{V} = 0$ then flow is irrotational.

Key Point

- For irrotational flow $\omega_x = \omega_y = \omega_z = 0$.
- For $2-D$ flow in xy plane, ω_x and ω_y will always zero and only ω_z will exist. If ω_z is also zero, then flow will be irrotational.
- If any of component is other than zero then flow will be rotational.

Vorticity (Ω) :

Vorticity, $\Omega = 2\overline{\omega} = 2 \times \frac{1}{2}(\nabla \times \overline{V})$

$$\Omega_x \hat{i} + \Omega_y \hat{j} + \Omega_z \hat{k} = \nabla \times \overline{V}$$

So, $$\Omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \; \Omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \text{ and } \Omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.$$

For irrotational flow $\omega_x = \omega_y = \omega_z = 0$, hence component of vorticity i.e. $\Omega_x, \Omega_y, \Omega_z$ will also be zero.

Vortex line equation :

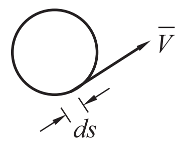
$$\frac{dx}{\Omega_x} = \frac{dy}{\Omega_y} = \frac{dz}{\Omega_z}$$

Key Point

- If flow posses vorticity then the flow is called rotational flow.
- If vorticity is zero then flow is irrotational flow.

Circulation : It is the integral of tangential velocity along the closed path.

$$\Gamma = \oint_{Closed\ path} \bar{V} \cdot d\bar{S}$$



Where, $\bar{V} = u\hat{i} + v\hat{j} + w\hat{k}$, $d\bar{S} = dx\hat{i} + dy\hat{j} + dz\hat{k}$, $\bar{V} \cdot d\bar{S} = udx + vdy + wdz$

$\therefore$ $$\Gamma = \oint_{Closed\ path} udx + vdy + wdz$$

Relation between circulation and vorticity,

Circulation = Vorticity $\times$ Enclosed area

Velocity Potential Function (ϕ) :

$$\phi = f(x, y, z, t) \qquad \text{(For unsteady flow)}$$

$$\phi = f(x, y, z) \qquad \text{(For steady flow)}$$

$$-\frac{\partial \phi}{\partial x} = u, -\frac{\partial \phi}{\partial y} = v, -\frac{\partial \phi}{\partial z} = w$$

If an incompressible fluid flow is possible that means, $\text{div}\bar{V} = 0$

i.e., $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$\therefore$ $$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \qquad \text{(Laplace equation)}$$

Equipotential line : A line along which velocity potential function ϕ is constant is called equipotential line. i.e. $\phi = \text{Constant}, u = v = w = 0$.

Slope of equipotential line, $m_1 = -\frac{u}{v}$

Key Point

- If ϕ exist for an incompressible fluid flow then it satisfies the Laplace equation.
- If ϕ exist for a fluid means that fluid flow is irrotational fluid flow. Therefore, irrotational fluid flow is called potential flow.

Stream Fnction (ψ) :

$$\psi = f(x, y, t) \qquad \text{(For unsteady flow)}$$

$$\psi = f(x, y) \qquad \text{(For steady flow)}$$

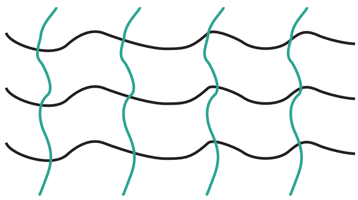
$$\frac{\partial \psi}{\partial x} = v, \frac{\partial \psi}{\partial y} = -u$$

Stream line : A line along which ψ is constant.

Slope of stream line, $m_2 = \frac{v}{u}$

Key Point

- Both the equipotential line and stream line cut to each other orthogonally except stagnation point.



$\therefore \qquad m_1 . m_2 = -1$

- If ϕ exists then flow should be irrotational therefore, irrotational fluid flow is called potential flow. But if ψ exists flow may be rotational or irrotational.
- Difference of two stream functions for two stream lines equal to the flow rate between them i.e. $Q = |\psi_1 - \psi_2|$.

7.5 Fluid Dynamics

Introduction :

It is the study of fluid motion by considering the cause of motion. Motion of any matter is governed by newton's second law of motion which state that "Rate of change of momentum of any body is equal to force on that body".

Mathematically, $\sum \overline{F} = m\overline{a}$

$$\sum \overline{F} = \overline{F}_g + \overline{F}_p + \overline{F}_v + \overline{F}_c + \overline{F}_T + \overline{F}_{\text{minior forces}}$$

Where, $\overline{F}_g$ = Gravitional forces, $\overline{F}_p$ = Pressure force, $\overline{F}_v$ = Viscous force,

$\overline{F}_c$ = Force due to compressibility and $\overline{F}_T$ = Force due to turbulence,

$F_{\text{minior forces}}$ = Forces due to surface tension (generally neglected).

Case 1 : If all these forces are taken into consideration then equation of motion becomes *Newton's equation of motion*.

Case 2 : $F_{\text{minor force}} = 0, \overline{F}_c = 0$.

The equation of motion are called *Reynold's equation of motion.*

Case 3 : $F_{\text{minor force}} = 0, \overline{F}_c = 0, \overline{F}_T = 0$. The equation of motion are known as *Navier-stokes equation of motion*. Which is equation of motion for *Real fluid.*

Case 4 : $F_{\text{minor force}} = 0,\ \overline{F}_c = 0, \overline{F}_T = 0, \overline{F}_v = 0$.

The equation of motion are known as *Euler's equation of motion.* Which is equation of motion for Ideal fluid.

Euler's Equation :

Assumption :

1. Ideal fluid.
2. Flow is in direction of stream line.
3. Steady flow.

$$\frac{dp}{\rho} + VdV + gdz = 0$$

Bernoulli's Equation :

Assumption :

1. Fluid is ideal i.e., viscosity is zero.
2. Steady flow.
3. The flow is incompressible.
4. The flow is irrotational.
5. The flow is along the stream line.

Bernoulli's equation is obtained by integrating the Euler's equation,

$$\int \frac{dp}{\rho} + \int VdV + \int gdz = \text{constant} \qquad \text{(Energy equation)}$$

If flow is incompressible $\rho = c$

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = c$$

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = c$$

Pressure head ($\frac{p}{\rho g}$), K.E. head or velocity head ($\frac{V^2}{2g}$), Potential head (z)

and, $$\frac{P}{\rho g} + z = \text{Piezometric head}$$

Key Point

- If the flow is steady but rotational then Bernoulli's can only be applied between those points which comes in same stream line because constant of integration will be different for different stream line.
- If the flow is steady then Bernoulli's equation can be applied for any instant
- The Bernoulli's equation derived here is applicable only for incompressible fluid. We can obtain Bernoulli's equation for compressible fluid but problem of compressible fluid is easier to solve through steady flow energy equation than Bernoulli's equation.

- Bernoulli's equation for real fluid, $\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f$.

  Where, h_f = Head loss between given points.

Application of Bernoulli's Equation :

1. Venturimeter (To measure the rate of discharge or flow rate)
2. Orificemeter (To measure the rate of discharge or flow rate)
3. Pitot tube (To measure the fluid velocity)

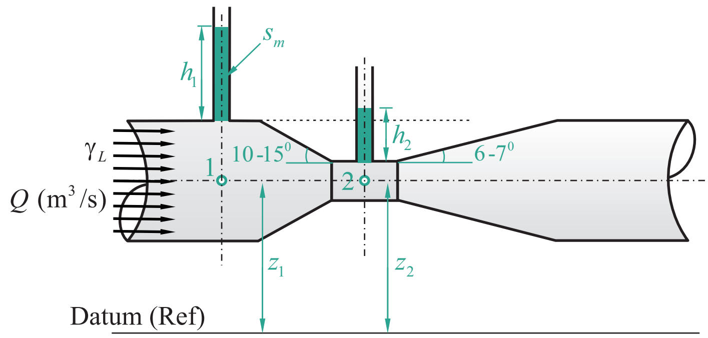
1. Venturimeter :

It consists of three parts :

(i) Sudden converging part (ii) Constant area called *throat*

(iii) Gradual diverging part

It is based on the principle of Bernoulli's equation,



Let d_1 = Diameter at inlet or at section (1), p_1 = Pressure at section (1)

V_1 = Velocity of fluid at section (1), A_1 = Area at section (1) $=\frac{\pi}{4}d_1^2$

and d_2, p_2, V_2, A_2 are corresponding values at section (2).

Applying Bernoulli's equation at section 1 and 2,

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2$$

$$\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=\frac{V_2^2-V_1^2}{2g}$$

Piezometric head

$$h_1-h_2=h=\frac{V_2^2-V_1^2}{2g}$$

Difference in piezometric head

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$$h = x\left(\frac{s_m}{s} - 1\right), \qquad (s_m > s)$$

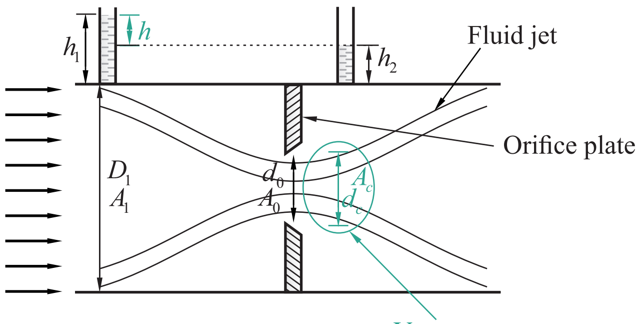
$$h = x\left(1 - \frac{s_m}{s}\right), \qquad (s > s_m)$$

- $V_1 = \dfrac{A_2\sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$
- $Q_{theoritical} = A_1V_1 = \dfrac{A_1A_2\sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$
- Coefficient of discharge, $C_d = 0.97$ to 0.99 for venturimeter (Always less than 1).
- $Q_{actual} = C_d \times Q_{theoritical}$
- It is highly accurate device and costly.

Key Point

- Discharge will remain same if venturimeter is inclined at any angle.
- Deflection of the manometric fluid liquid remain same whether venturimeter is horizontal, vertical or inclined.

2. **Orifice meter :** It works on same principle as that of venturimeter.



Applying Bernoulli's equation at section 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{V_2^2 - V_1^2}{2g}$$

$$h = \frac{V_2^2 - V_1^2}{2g}$$

- $Q_{th} = A_1V_1 = \dfrac{A_1\sqrt{2gh} \times C_cA_0}{\sqrt{A_1^2 - (C_cA_0)^2}}$ $\qquad (A_c = C_cA_0)$

- $Q_{actual} = C_d \times Q_{th} = \dfrac{C_d \times A_1 A_0 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$
- $C_d = 0.65$ to 0.70 for orifice meter.

Key Point

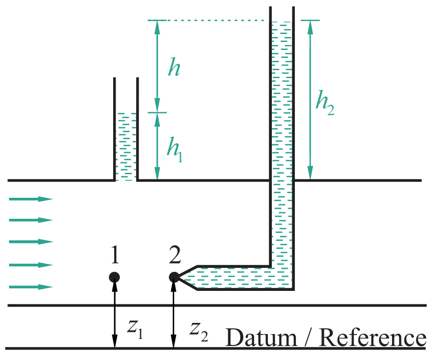
- The coefficient of discharge for orifice meter is much smaller than that for a venturimeter.
- Orifice meter is less accurate as compare to venturimeter and is cheaper device as compare to venturimeter.
- For orifice, $C_c = \dfrac{C_d}{C_v}$.

Coefficient of discharge $(C_d) = \dfrac{\text{Actual discharge } (Q_{ac})}{\text{Theoritical discharge } (Q_{th})}$

Coefficient of velocity $(C_v) = \dfrac{\text{Actual velocity } (V_{ac})}{\text{Theoritical velocity } (V_{th})}$

Coefficient of contraction $(C_c) = \dfrac{\text{Area of jet at vena contracta}}{\text{Area of orifice}}$

3. **Pitot tube :** Pitot tube is used for measuring the velocity of flow at any point in a pipe or a channel.



Bernoulli's equation at point (1) and (2),

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$\frac{V_2^2}{2g} = 0$ (2nd point is stagnation point)

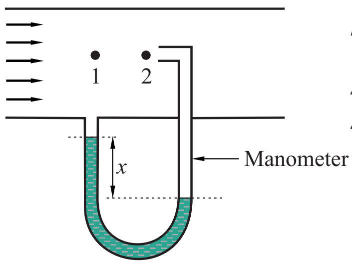
$$h_1 + \frac{V_1^2}{2g} = h_2$$

$$\frac{V_1^2}{2g} = h_2 - h_1 = h \qquad (h = \text{Differences in piezometric head})$$

$$(V_1)_{th} = \sqrt{2gh}$$

$$(V_1)_{actual} = C_v \times (V_1)_{th} \qquad (C_v = 0.97 \text{ to } 0.99)$$

Key Point



$$h = \left| x\left[\frac{s_m}{s_f} - 1\right]\right|$$

s_m = Relative density of manometric fluid

s_f = Relative density of flowing fluid

Instruments & Their Measuring Parameter :

S.No	Instrument	Measuring parameter
1	Venturimeter	Discharge or flow rate
2.	Orificemeter	Discharge or flow rate
3.	Flow nozzle	Discharge or flow rate
4.	Elbow meter	Discharge in vertical segment or flow rate
5.	Nozzel meter	Discharge or flow rate
6.	Pitot tube	Velocity of a fluid
7.	Prandtl tube (Boundary layer theory)	Velocity
8.	Current meter	Velocity in open channel
9.	Weirs	Discharge in open channel
10	Rotameter	Flow rate or discharge in vertical segment
11.	Hot-wire Anemometer	Used for measuring the gas or air velocity
12.	LDA (Lazer doppler Anemometer)	Velocity with high accuracy
13.	Pyrometer	Used for measuring the high temperature
14.	Hydrometer	Specific gravity
15.	Hygrometer	Humidity

Momentum Equation & Its Application :

Newton's 2nd law state that :

1. Rate of change of linear momentum of any matter is equal to force on that matter. Mathematically,

$$\sum F = m\vec{a} = m.\frac{dV}{dt} = \dot{m}dV = \dot{m}[\overline{V_2} - \overline{V_1}]$$

2. Rate of change of angular momentum is equal to torque,

$$T = I_m \alpha$$

Where, I_m = Mass moment of inertia and α = Angular acceleration.

So, $T = I_m \dfrac{d\omega}{dt}$

$$T = \dot{I}_m(\omega_2 - \omega_1)$$

Calculations of Forces in Diffuser, Nozzle, Pipe Bends etc. Under Consideration of Pressure :

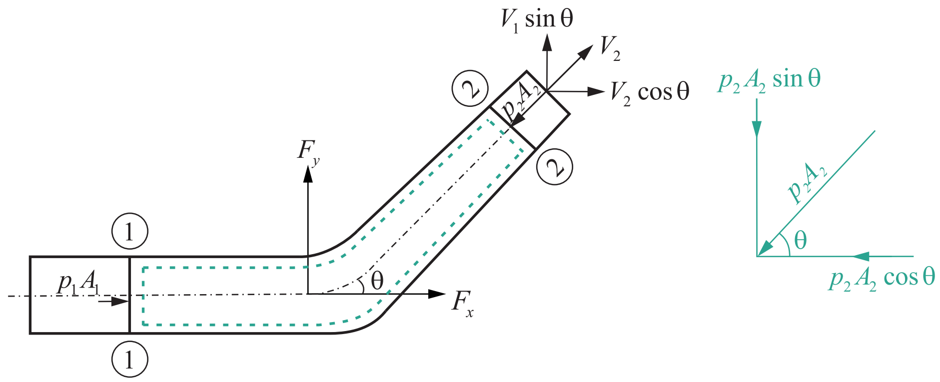


Fig. Force on bend

Net force acting on fluid in the direction of $x =$ Rate of change of momentum in x-direction

$\therefore \quad p_1A_1 - p_2A_2\cos\theta + F_x =$ (Mass per sec) (Change of velocity)

$= \rho Q$ (Final velocity in the direction of x – Initial velocity in the direction of x)

$$= \rho Q(V_2\cos\theta - V_1)$$

$$\therefore \quad F_x = \rho Q(V_1 - V_2\cos\theta) + p_1A_1 - p_2A_2\cos\theta$$

Similarly the momentum equation in y - direction gives,

$$0 - p_2A_2\sin\theta + F_y = \rho Q(V_2\sin\theta - 0)$$

$$\therefore \quad F_y = \rho Q(V_2\sin\theta) + p_2A_2\sin\theta$$

Now the resultant force (F_R) acting on the bend $= \sqrt{F_x^2 + F_y^2}$.

And the angle made by the resultant force with horizontal direction is given by $\tan\alpha = \dfrac{F_y}{F_x}$.

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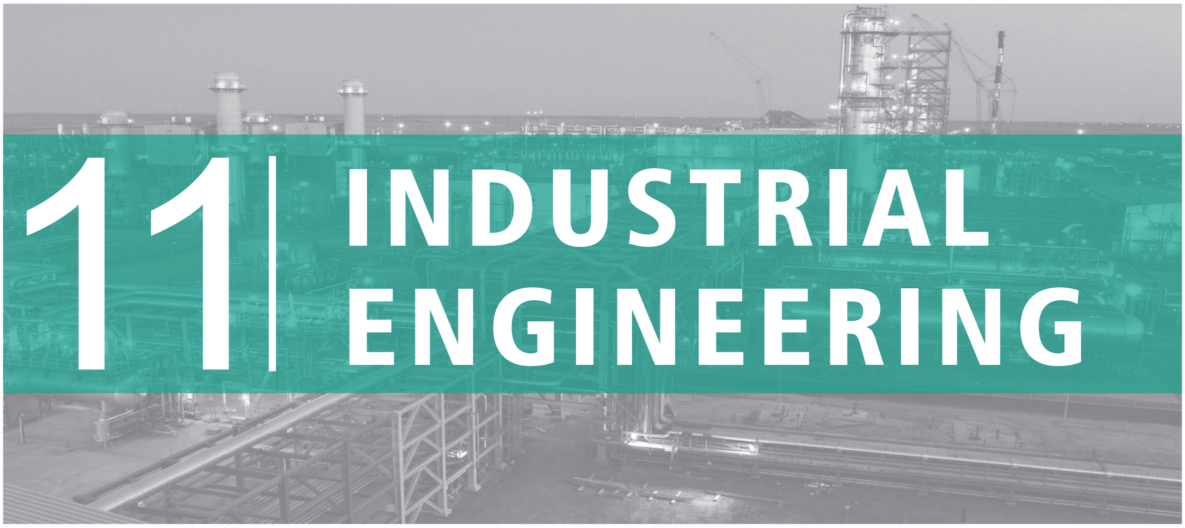


11 INDUSTRIAL ENGINEERING

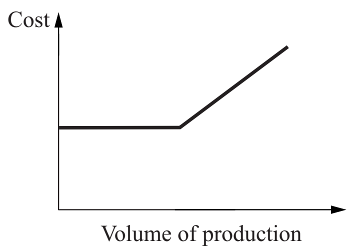
11.1 Break Even Analysis

Introduction :

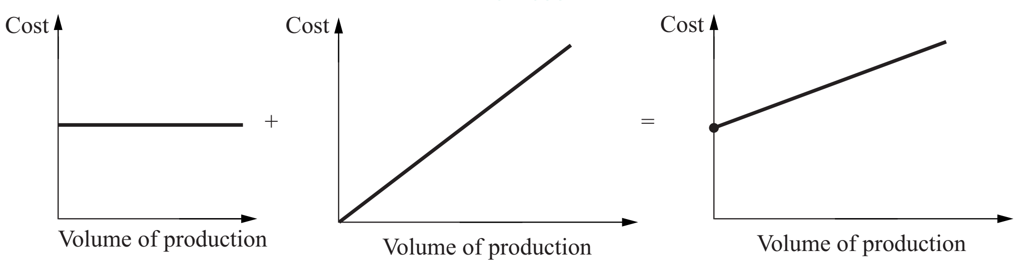
Break even analysis is a preliminary analysis used by organization, to study relation between total cost, selling cost and volume of production.

The total cost is divided into different categories :

1. **Fixed cost :** The cost which do not vary with amount of produced units, they are constant upto specific volume or range of volume e.g. Rent.
2. **Variable cost :** The cost which is directly associated with volume of production e.g. Electricity bill.
3. **Mixed cost :** This cost is a combination of semi-variable and semi-fixed cost.



Mixed cost



Fixed cost **Variable cost** **Total cost**

Assumptions in Break Even Analysis :

1. Fixed cost remains constant.
2. Variable cost varies in direct proportion.
3. Selling price remains constant.
4. No other factor except quantity will affect the cost.
5. Production and sales quantity are equal.

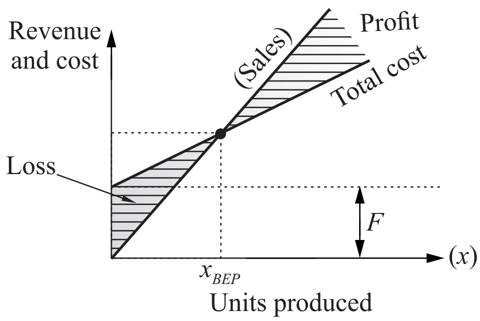
Break Even Point Calculation :

Let, $F =$ Fixed cost, $V =$ Variable cost,

$S =$ Sales price, $P =$ Profit

$x =$ Quantity produced and sold

and $x_{BEP} =$ Break even quantity.



At x_{BEP} units, Revenue generated = Total cost

$$S = TC + \text{Profit}$$

$$S = F + V + \text{Profit} \quad (\because TC = F + V)$$

At Breakeven point, profit = 0

$$S = F + V$$

$$s \times x_{BEP} = F + \nu \times x_{BEP}$$

$\begin{bmatrix} s = \text{Sales price per unit,} \\ \nu = \text{Variable price per unit} \end{bmatrix}$

$$x_{BEP} = \frac{F}{s - \nu}$$

Contribution Margin :

Contribution : It is the measure which tells how much sale of one unit of product will contribute to (Fixed cost and profit), when variable cost has been deducted.

$$\text{Contribution} = F + \text{Profit}$$

$$\text{Contribution} = \text{Total revenue generated} - \text{Variable cost}$$

Illustration 1 :

ABC company invested, $F = ₹2400$, $\nu = ₹12$/piece

Company produced 100 piece at $SP = ₹50$ piece

Contribution $= F + P$

Total revenue $= 50 \times 100 = ₹5000$

Variable cost $= 100 \times 12 = ₹1200$

Fixed cost = ₹2400

Total expense = ₹3600

Profit = ₹1400

Contribution = Profit + Fixed cost

$\Rightarrow \quad 1400 + 2400 = ₹3800$

Costing of product to ₹50 will lead to contribution of covering the fixed cost of ₹2400 and gaining the profit of ₹1400.

Margin of Safety :

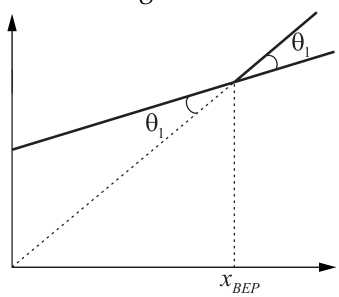
It is termed as excess of actual sales over break even sales,

$$MOS = \text{Actual sales} - \text{Break even sales}$$

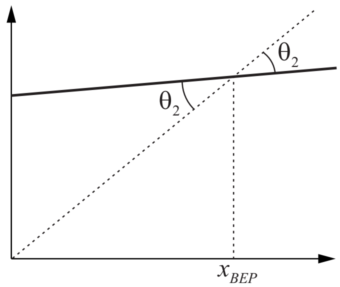
$$MOS \text{ ratio} = \frac{\text{Actual sales} - \text{Break even sales}}{\text{Actual sales}}$$

Angle of incidence : It is angle between lines of total cost and total revenue. It is the indicator of 'Rate of profit acquired'.

When $\theta(\uparrow) \rightarrow$ Profit is at a higher rate.



Production of *A*



Production of *B*

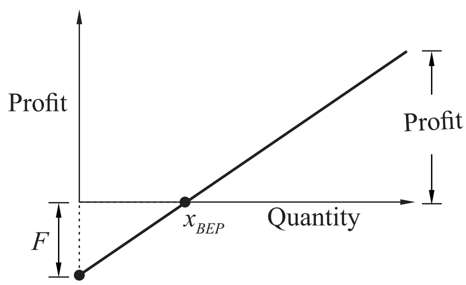
$\because \quad \theta_2 > \theta_1$

$\therefore$ B will earn at a higher profit rate as compared to A.

Profit/volume ratio : It is the relationship between volume of quantity and profit.

$$(P/V)_{ratio} = \frac{\text{Contribution margin}}{\text{Sales}} = \left(\frac{S - V}{S}\right)$$

$$(MOS)_{sales} = \frac{\text{Profit}}{(P/V)_{ratio}}$$



$\Rightarrow$ Higher the profit volume ratio greater be the angle of incidence.

Disadvantage of BEA :

1. It is only valid for short runs and not used for detailed analysis.
2. Actual total cost does not vary in direct proportion unit produced.

Trivia :

Break even analysis is also used for comparison between distinctive production modes.

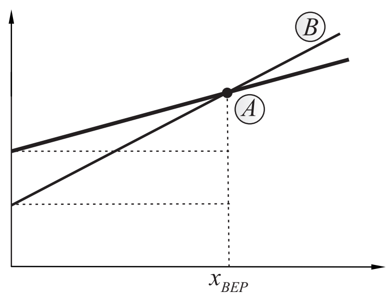
Example : Selection of operation procedure between machine A and machine B.

	Machine A	Machine B
Setup time	30 min	2 hours
Machining time per piece	22 min	5 min
Machine rate	₹ 200/hr	₹ 800/hr

$$TC = \text{Setup time} + \text{Machine cost}$$

$$(TC)_A = \left(\frac{30}{60} + x \times \frac{22}{60}\right) \times 200 \quad \ldots\text{(i)}$$

$$(TC)_B = \left(2 + x \times \frac{5}{60}\right) \times 800 \quad \ldots\text{(ii)}$$



For analysing x_{BEP} equate (i) and (ii),

$$x_{BEP} = 225 \text{ units}$$

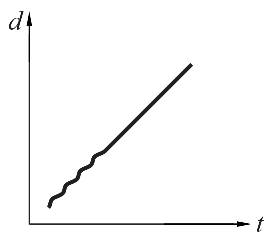
Hence machine B will be profitable for $(x > 225 \text{ units})$ otherwise A is profitable.

11.2 Forecasting

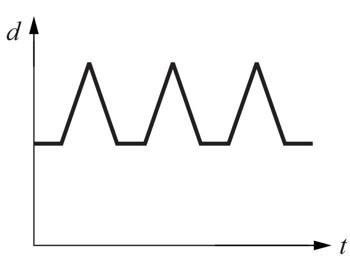
Introduction :

Prediction of future sales or demand of a particular product by careful study of past data and present scenario. It is considered as the first major activity in planning.

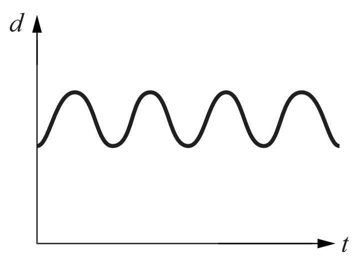
Types of Demand Variation :



Long term upward/downward variation of product



Seasonal variation



Cyclic variation

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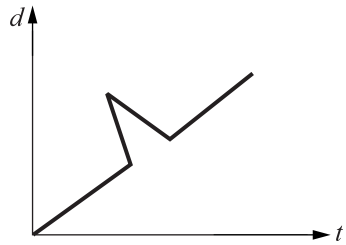
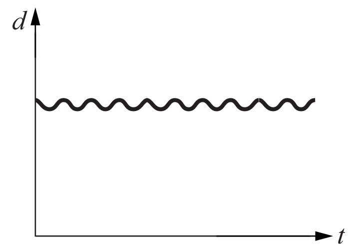


Irregular variation



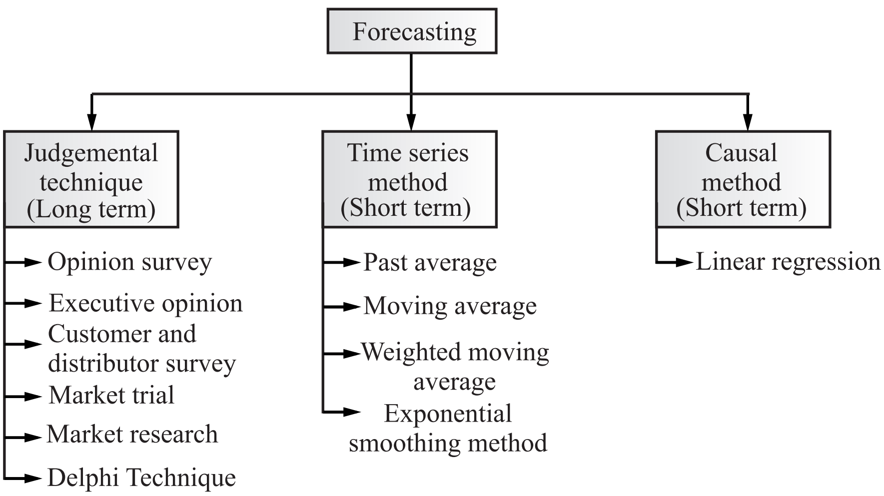
Constant variation

Where, d denotes demand and t denotes time.

Time Horizons in Forecasting :

1. Short term forecasting $\rightarrow$ 1-3 month.
2. Intermediate terms forecasting $\rightarrow$ 3-12 month.
3. Long term forecasting $\rightarrow$ More than 1 year.

Classification of Forecasting Methods :



Trivia :

Judgemental techniques are qualitative, whereas time series and causal analysis are quantitative in nature.

Time Series Method :

1. **Past average :** In this method forecast is equal to average sales of previous period.
2. **Moving average method :** This method also uses past data and calculate a rolling average for a constant period. It gives equal weightage to all periods considered.

$$\text{Moving average} = \frac{\text{Sum of demand for given period}}{\text{Chosen number of period}}$$

3. **Weighted moving average :** This method is also similar to moving average method, but more weightage is given to recent demand and vice versa.

Illustration 2 :

For 4 periods $n = 4$ (1, 2, 3, 4)

$$\Sigma n = \frac{n(n+1)}{2} = 10$$

Weight per period = $\frac{n}{\text{Sum}}, \frac{n-1}{\text{Sum}}, \frac{n-2}{\text{Sum}} \ldots\ldots \frac{1}{\text{Sum}} = \frac{4}{10}, \frac{3}{10}, \frac{2}{10}, \frac{1}{10}$

4. **Exponential smoothing method :** The other methods require a huge database for past data, but this method does not require large calculations and is used to smooth out random fluctuation due to irregular component of time series.

   The most recent observation is given highest weightage,

$$F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1})$$

   Where, F_t = Forecast for next period, F_{t-1} = Forecast of previous period

   D_{t-1} = Demand for previous period and α = Smoothing constant.

   General form, $F_t = \alpha D_{t-1} + \alpha(1-\alpha) D_{t-2} + \alpha(1-\alpha)^2 D_{t-3} \ldots\ldots \quad \left(\alpha = \frac{2}{n+1}\right)$

Trivia :

Exponential smoothing method also accounts for forecasting error, $e = D_{t-1} - F_{t-1}$.

Responsiveness & Stability :

Responsiveness indicates that forecast, as calculated, have a fluctuating and swinging pattern whereas stability means that the forecast shows a flat character.

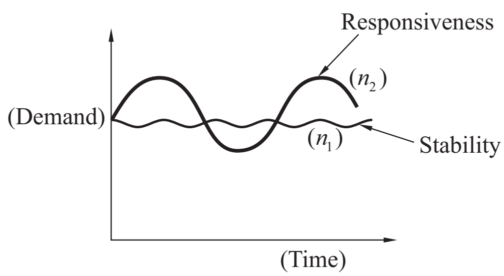
As the value of n increases, the forecast become the stable and lower value of n resulting a forecast being more responsive.

In this example given in the figure,

$$n_1 > n_2, \ \alpha_1 < \alpha_2$$

Where, $n \rightarrow$ Number of observation,

$\alpha \rightarrow$ Smoothing index.



If $n \rightarrow \infty$, for $(\alpha = 0)$,

$$F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1})$$

$$F_t = F_{t-1} \rightarrow \text{Limit of stability}$$

If $n \rightarrow 1$, for $(\alpha = 1)$,

$$(F_t = D_{t-1}) = \text{Limit of responsiveness}$$

$\Rightarrow$ Value of α lies between (0 to 1).

Types of Error :

Error : It is the numerical difference between forecasted demand and actual demand. It is also defined as the effectiveness of firm to calculate and judge variation in demand.

1. **Mean absolute deviation :** It is termed as average of absolute values of difference between actual and forecasted values. Error in terms of magnitude is only considered

$$MAD = \frac{\sum_{t=1}^{n} |D_t - F_t|}{n}$$

2. **Mean forecast error or BIAS :** This error shows magnitude of forecast along with direction.

Negative = Over forecast and Positive = Under forecast.

$$MFE = \frac{\sum_{t=1}^{n} (D_t - F_t)}{n} \qquad \left(\sum_{t=1}^{n} (D_t - F_t) \Rightarrow \text{Run sum forecast error}\right)$$

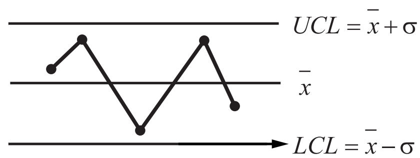
3. **Mean square error (MSE) :**

Average of square of all errors in forecast, $MSE = \frac{\sum_{t=1}^{n} |D_t - F_t|^2}{n}$

Trivia :

Mean sequence error is also used to define control limits in production processes,

$$\sigma = \sqrt{MSE}$$



Where, $UCL \rightarrow$ Upper critical limit, $LCL \rightarrow$ Lower critical limit.

4. **Mean absolute percentage error (MAPE) :**

It gives variation with respect to demand, $MAPE = \frac{\sum_{t=1}^{n} \left|\frac{D_t - F_t}{D_t}\right| \times 100}{n}$

5. **Tracking signal :** It shows how accurate the prediction is going on, in terms of demand and forecasted values by the company. TS is desired to be '0' ideally.

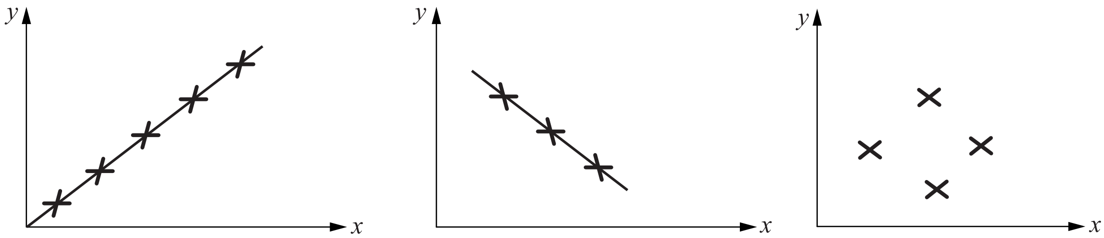
$$TS = \frac{\text{Run sum forecast error}}{MAD} = \frac{\sum_{t=1}^{n} (D_t - F_t)}{MAD}$$

$TS \rightarrow (0) \rightarrow$ Desirable and ± 4 or $\pm 5 \rightarrow$ Acceptable.

Causal Method :

1. **Co-relation method :** In this method the co-relation or dependence between two variable is checked.

$$r = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{\sqrt{\Sigma(x - \bar{x})^2 \Sigma(y - \bar{y})^2}}$$



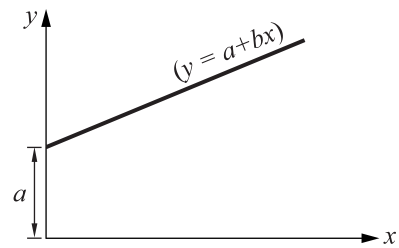
$r = +1$ (Positive co-relation) $r = -1$ (Negative co-relation) $r = 0$ (No co-relation)

2. **Linear regression :**

$y =$ Dependent variable and $x =$ Independent variable.

$$y = a + bx \qquad \text{(Straight line equation)}$$

$$a = \frac{\Sigma y - b\Sigma x}{n},\ b = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$



> **Note :** Least square method is other type of regression analysis in which $(\Sigma x \approx 0)$
>
> $$a = \frac{\Sigma y}{n} \text{ and } b = \frac{\Sigma xy}{\Sigma x^2}.$$

11.3 Material Requirement Planning

Introduction :

Material requirement planning (MRP) is a computational technique for determining the quantity and timing for acquisition of dependent items in order to satisfy master production schedule requirement. It is used for effective manufacturing control.

Purpose of MRP :

1. Inventory reduction.
2. Reduction in manufacturing time.
3. Reduction in delivery lead time.
4. Realistic delivery commitments.

MRP Terminologies :

1. **Lot size :** Quantity of items required to order.
2. **Time phasing :** Scheduling to receive an appropriate lot of material at desired time.
3. **Gross requirement :** Overall quantity of product required to meet planned output level.
4. **Net requirement :** Net quantity of items required for planned output level.

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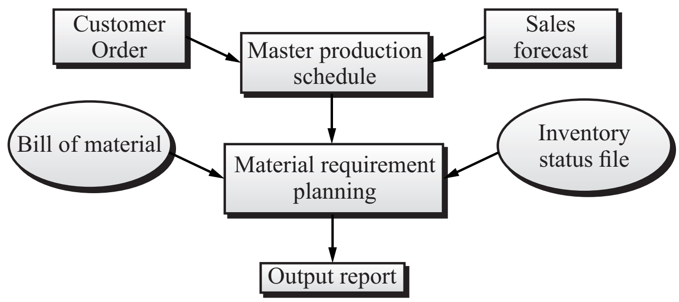
5. **Scheduled receipts :** The awaited amount of items that is to be received from the suppliers.

Note : Net requirement = Gross requirement – Inventory on hand – Scheduled receipt.

6. **Planned order receipts :** The quantity of order that is planned to be ordered for next production cycle.
7. **Planned order release :** This is the quantity of products needed, and order is initiated to suppliers.
8. **Lead time offset :** The supply time between releasing the order and receiving the material in inventory, in short, it is the procurement time.

Block diagram :



Trivia :

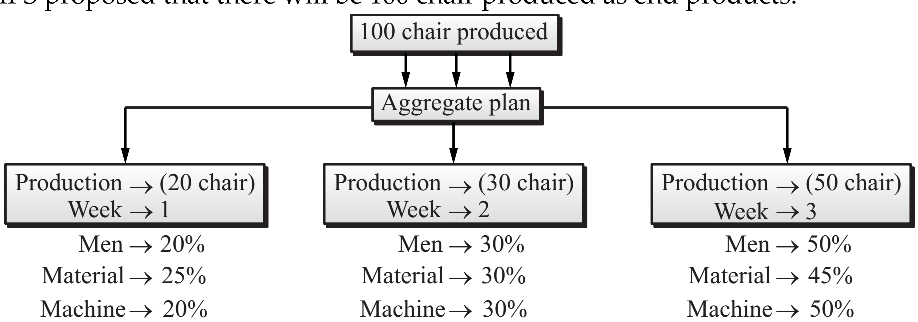
1. **Bill of material file (BOM) :** Engineering changes associated with the product are stored.
2. **Inventory status file :** The file that provides details of availability of items.
3. **Master production schedule (MPS) :** Basic schedule that relates amount of production along with customer orders and sales forecast.
4. **Capacity requirement planning (CRP) :** It is a technique that determines the amount of personnel and equipment capacities to meet production objectives.

Aggregate Planning :

The aggregate planning divides the master production schedule into various segments and checks the requirement of each segment.

Illustration 3 :

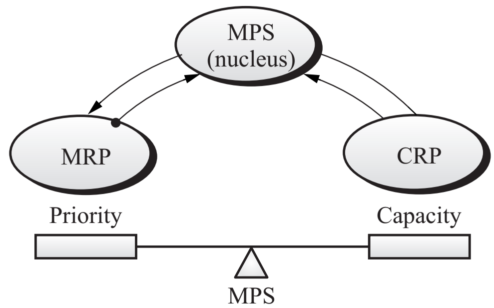
The MPS proposed that there will be 100 chair produced as end products.



In aggregate planning, only fixed amount of assets are assigned. If assets are more than required, then they will be either transferred or taken out of work.

Trivia :

MPS acts like a nucleus between MRP and CRP.



Plant Layout :

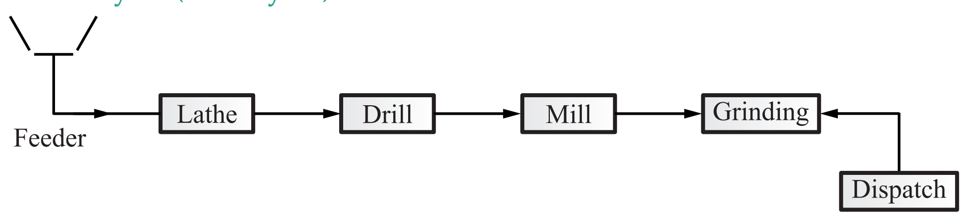
Assignment of men, machine, material as well as material handling equipment at a proper strategic location (for getting optimum product in minimum time) is called plant layout. Plant layout is the floor plan of the production facilities used in production.

Types of Layout :

1. **Process layout (Functional layout) :** Similar machines or similar operations are at one location.

⇒ The material in a process layout can be processed in a order.

2. **Product layout (Line layout) :**

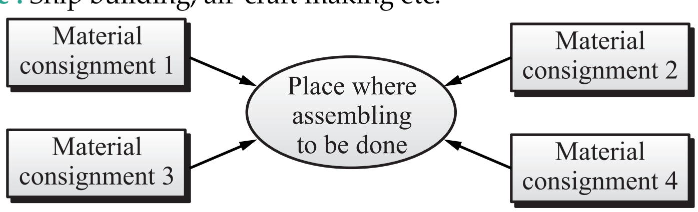


⇒ In this type of layout the operations are linked in assembly lines.

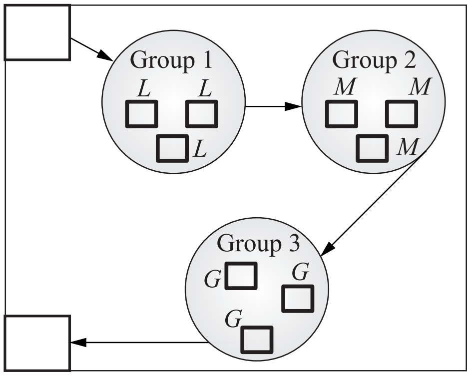
Example : Fabrication of automotives etc.

3. **Fixed position layout :** Materials required for production are assembled at a particular fixed space.

Example : Ship building, air-craft making etc.



4. **Group layout :** It is combination of product layout and process layout.



L= Lathe
M= Mill
G= Grind

⇒ Group layout has arrangement similar to process layout, and the flow of material is analogous to line layout.

Comparison between various layouts :

Factors	Fixed	Product	Process	Group
Operations	Ship building large scale project	Continuous and repetitive **E.g.** Automobile assembly lines	Job or small batch	Small to medium batch **E.g.** Refrigerator
Cost	Moderate to low	Moderate to low	Moderate to low	Moderate to high
Material travel	Variable path	Fixed path	Variable path	Fixed path
Operating facility	General	Special	General	Special
Employee skill	Unskilled/skilled	Unskilled	Skilled	Multi-skilled
Production	Few	Large production with less variability	Small production with huge variability	Moderate production

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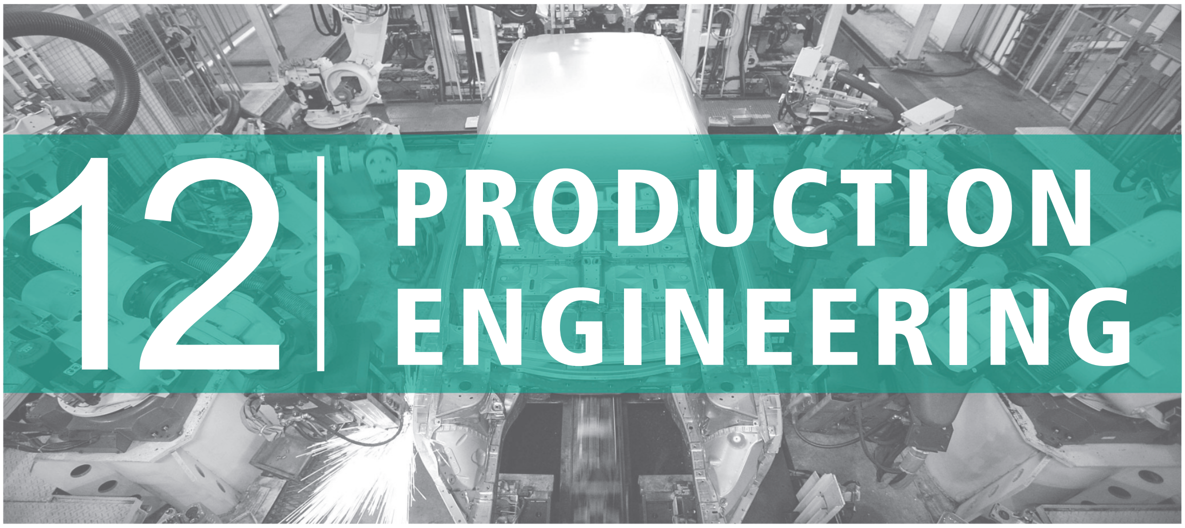


12 PRODUCTION ENGINEERING

12.2 Casting Process

Principle of Casting :

Casting process is one of metal forming process to make several equipment and components. It consists of introducing the molten metal into a cavity or mould of the desired shape and allowing it to solidify. The solidified object is called casting.

Terminology of Casting System :

Schematic diagram of casting process :

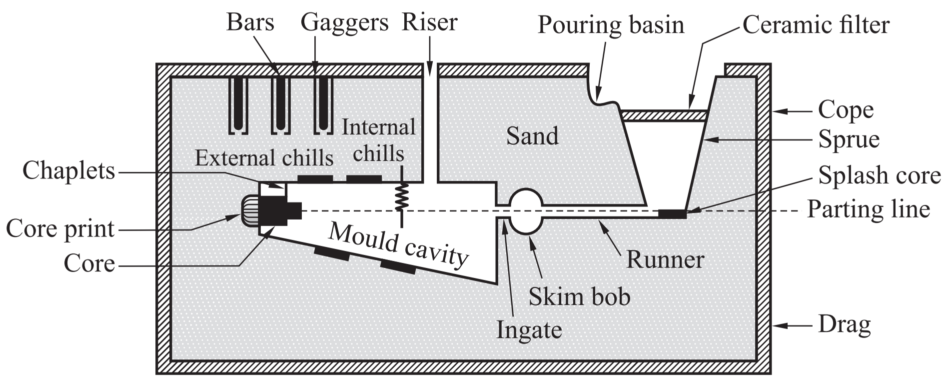


Fig. Casting process

Pattern : It is the replica of the final object to be made.

Flask : A moulding flask is one which holds the sand mould intact. *Drag* is lower moulding flask, *Cope* is upper moulding flask and *Cheek* is intermediate moulding flask used in three-piece moulding.

Parting line : This is the dividing line between the two moulding flasks.

Pouring basin : A small funnel shaped cavity at the top of the mould into which the molten metal is poured.

Sprue : The passage through which the molten metal from the pouring basin reaches the mould cavity.

Runner : The passageways in the parting plane through which molten metal flow is regulated before they reach the mould cavity.

Gate : The actual entry point through which molten metal enters mould cavity.

Riser : The risers are the reservoir of molten metal that provide a continuous flow of molten metal to compensate shrinkage which occurs during the solidification of casting.

Chill : Chills are metallic objects of high heat capacity and high thermal conductivity which are placed in mould or mould cavity to promotes directional solidification.

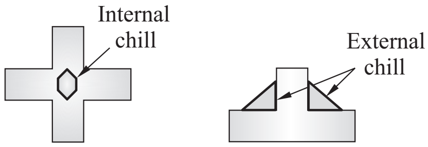


Fig. Casting chills

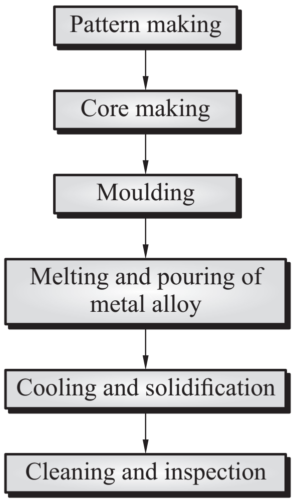
Core : It is used for making hollow cavities in castings.

Core chaplet : Chaplets are used to support cores inside the mould cavity to take care of its own weight and overcome the metallostatic forces. (It can also work as chill).

Core prints : These are a sort of projections at the ends of core to support the core in position in the mould. Core or any core piece can fixed in mould desired location only with help of core print.

Steps of Casting :

Steps of molding and casting process :



Key Point

Casting yield : It is the ratio of the actual casting mass to the mass of metal poured into the mould. It is about 60%.

Pros & Cons of Casting :

Advantages and disadvantages of casting :

S. No.	Advantages	Disadvantages
1.	Molten metal can flow in any small section.	Poor surface finish.
2.	Any intricate shape can be made.	Dimensional inaccuracy.
3.	Possible to cast any material (ferrous and nonferrous).	Moisture content in casting produces defects.

Core Making :

Cores are used to produce the internal features of part such as hole or any opening. Cores are placed in the cavity produced by pattern.

Pattern Making :

The main objective of using a pattern is to reproduce the external shapes of a casting.

Pattern materials are :

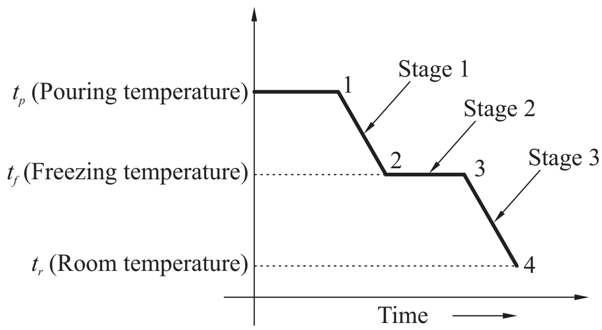
1. Wood
2. Plastic
3. Plaster of paris
4. Wax
5. Metallic
6. Thermocol

Types of patterns :

1. **Single piece pattern :** It is used for very simple casting. Atleast one surface must be flat surface.
2. **Split pattern/Two piece pattern :** It is being used when complexity of part is more and the withdrawal of pattern is difficult.
3. **Gated pattern :** It is used when small components in mass production is required.
4. **Cope and drag pattern :** It is used for production of big size castings.
5. **Match plate pattern :** It is used for production of small size precision casting in mass production casting e.g. piston ring.
6. **Loose piece patterns :** It is used when parts with internal webs are produced.
7. **Sweep pattern :** It is used when 2-D pattern used to produce symmetrical 3-D casting e.g. cone, bells of temples.
8. **Skeleton pattern :** It is used to prepare shells and drums by loam sand (Loam sand = 50% clay + 50% silica used).
9. **Follow board pattern :** It is used when thin or overhanging sections in casting is required.

Types of Pattern Allowances :

1. **Shrinkage or contraction allowance :** The various metals used for casting contract after solidification in the mould. Since the contraction is different for different materials, therefore it will also differ with the form or type of metal.



   1-2 : Liquid shrinkage.

   2-3 : Solidification shrinkage (Phase transformation shrinkage).

   3-4 : Solid shrinkage.

Key Point

- Shrinkage is independent of pouring temperature.
- Grey cast-iron has negative shrinkage allowance. Because on solidification its volume increases. For grey cast iron the pattern used is smaller than the casting.
- Stage 1 and stage 2 shrinkage are compensated by riser.
- Stage 3 are compensated by giving shrinkage allowance i.e., oversize pattern.

2. **Draft allowance :** It is a taper which is given to all the vertical walls of the pattern for easy and clean withdraw of the pattern from the sand without damaging the mould cavity. Inner surfaces of the pattern require higher draft than outer surfaces.
3. **Finish or machining allowance :** The allowance is provided on the pattern if the casting is to be machined. This allowance is given in addition to shrinkage allowance, which depends upon the type of the casting metal, size and the shape of the casting.

Key Point

The ferrous metals require more machining allowance than nonferrous metals.

4. **Distortion or camber allowance :** This allowance is provided on patterns used for casting of such design in which the contraction is not uniform throughout. e.g U-shape and T-shape casting.
5. **Rapping or shaking allowance :** This allowance is provided in the pattern to compensate for the rapping of mould because the pattern is to be rapped before removing it from the mould. It depends on the skill of operator and is negative in nature.

Moulding Sand :

Sand conditioning : It is the process of making the sand suitable for moulding. It consists of distributing the binder uniformly, controlling the moisture content, eliminating foreign particles, aerating and at proper temperature.

Composition of moulding sand :

Silica → 75 - 85%, Clay → 10 - 20%, Water → 2 - 8% and additives → 1 - 6%.

Naturally available clay :

1. Bentonite, 2. Kaolinite.

Types of moulding sand :

S. No.	Types	Characteristics
1.	Green sand	• Mixture of silica sand with 18 to 30% clay and 2 to 6% water. • Coal dust is mixed in green sand to prevent defects in casting.
2.	Dry sand	The moisture available in the moulding sand evaporates because of high temperature of molten metal then the sand is called as dry sand.
3.	Loam sand	• Contains 50% clay and dries hard. • It contains fire clay also. • Moisture content 18 to 20%. • It is used for casting like chemical pans, drums, etc.
4.	Facing sand	The sand which is used near the mould cavity with more clay and fine silica sand is called facing sand.
5.	Backing sand	• The sand which is used near the mould flask away from the cavity with less moulding sand properties to support the mould. • It is weak in bonding strength due to sharp edge of grain size.
6.	Parting sand	• It is used to separate the moulding boxes to each other by spreading fine dry sand. • It is clean, clay and water free silica sand.
7.	Core sand	• It is used for making cores. • It is silica sand mixed with core oil, linseed oil, resin, light mineral oil and other binders.

Key Point

Moulding methods : Hand hammering, jolt machine, squeezing, slinging and blowing.

Sand moulding methods : Bench, floor, plate, pit and machine moulding.

Additive used in moulding sand :

- **Wood flour/saw dust :** To improve green strength and collapsibility of moulding sand.
- **Starch and dextrin :** These are organic binders used to improve resistance to deformation of the mould and improves the skin hardness of the mould.

- **Iron oxide and aluminium oxide :** Used to improve hard strength of green sand.
- **Coal dust, sea coal, silica flour :** These are carbonous materials which are used in the moulding sand to improve surface finish and resistance to metal penetration.
- **Linseed oil molasses :** Improve strength and hardness of moulding sand.

Properties of Moulding Material :

Refractoriness : It is the ability of material to withstand the high temperature of motion metals. It should be high.

Green strength : The moulding sand that contains moisture is termed as green strength. The green sand should have enough strength so that the constructed mould retains its shape. *Universal Testing Machine* is used to test green strength.

Green compressive strength $\rightarrow$ 130 – 160 kPa.

Green shear strength $\rightarrow$ 10 – 40 kPa.

Dry strength : When molten metal is poured into a mould, the sand around the mould cavity is quickly converted into dry sand as the moisture in the sand immediately evaporates due to the heat in the molten metal. At this stage, it should retain the mould cavity and at the same time withstand the metallostatic forces.

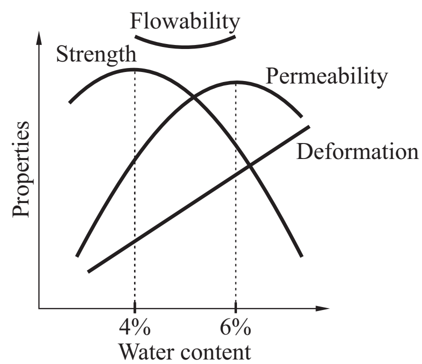
Dry compressive strength $\rightarrow$ 120 - 140 kPa

Dry shear strength $\rightarrow$ 30 - 80 kPa.

Hot strength : It is the strength of the sand that is required to hold the shape of the mould cavity, after all the moisture is eliminated.

Permeability : It is ability of moulding sand which enables trapped gases to escape from mould to produce sound casting. This will be expressed by permeability number (p_n).

$$p_n = \frac{Vh}{pAt}$$



Here, V = Volume of air passing through the specimen ($2000\ \text{cm}^3$),

h = Height of standard specimen (2 inch = 5.08 cm),

p = Pressure of the air passing through the specimen,

A = Cross sectional area of cylindrical specimen (dia = 2 inch),

t = Time required to pass through the specimen.

Grain fineness numbers : GFN will indicate the average grain size distribution of a given moulding. Greater the GFN lower the grain size.

Flowability : The ability of the sand to flow over and around the pattern when the mould is rammed.

Fluidity : The ability of liquid metal to flow into the cavity.

Properties	Fluidity
Pouring temperature ↑	↑
Viscosity ↑	↓
Density ↑	↓
Percentage of water in sand ↑	↓
Surface finish ↑	↓

Adhesiveness : The bond formation between two different material i.e., moulding sand and mould flask, between moulding sand and pattern.

Cohesiveness : Bond formation between 2 similar materials i.e., between 2 sand grains.

Toughness : Ability to resist impact and shock loads by the moulding sands. *Shatter index test* is done for toughness testing, shock observed when molten metal is poured.

Collapsibility : The properties of the moulding sand which creates no resistance to metal contraction is called collapsibility. During the solid contraction of the casting part if the mould creates resistance cracks will appear over the casting. High collapsibility is preferred to improve collapsibility. Saw dust or wood powder is added to moulding sand.

Since when molten metal is poured wood powder burns to ash due to heat and hence shrinks in size causing the mould near casting to easily collapse and provide resistance less shrinkage.

Sprue Design :

Pressure anywhere in liquid metal stream does not fall below atmosphere, otherwise porous casting results. This is called *aspiration effect*. This phenomenon is shown in figure (a) below.

Here pressure at point 2 is given by, $\frac{p_2}{\rho} + gh_2 = 0$ or $p_2 = -\rho g h_2$.

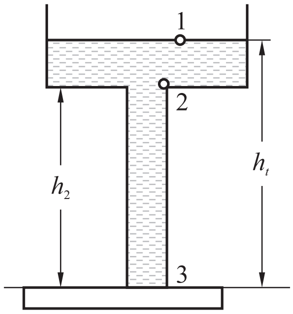


Fig. (a) Straight sprue

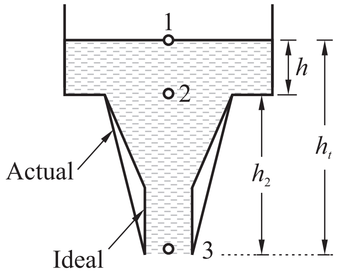


Fig. (b) Taper sprue

It means that at point 2 in the sprue, there will be vaccum and will produce aspiration effect. Hence, straight design is not acceptable.

If the shape of sprue is somewhat tapered, then pressure throughout the stream would be just atmospheric. The exact tapering can be obtained by the equation of continuity.

Denoting the top and choke sections of the sprue by subscript 't' 'c' respectively

$$A_t V_t = A_c V_c$$

$$A_t = A_c \times \frac{V_c}{V_t}$$

The velocities are proportional to the square root of the potential heads.

$$A_c = A_t \sqrt{\frac{h}{h_t}}$$

Where, $h = (h_t - h_2)$ and $V_c = \sqrt{2gh_t}$, $V_t = \sqrt{2gh}$.

Profile of the sprue should be parabolic, thus it is too inconvenient in practice that is why tapered sprue are used.

Key Point

- When there is sudden change in flow direction, in that situation also aspiration effect comes into picture.
- Straight tapered sprue is able to effectively reduce the air aspiration as well as increase the flow rate compared to a cylindrical straight sprue.

Riser Design :

Purposes of riser :

1. Compensates liquid and solidification shrinkage.
2. Promotes directional solidification.
3. Indicates whether casting is full or not.

Solidification time is the basis for riser designing. Solidification time for both casting and riser is given by Chvorinov's Rule.

Chvorinov's Rule :

Total solidification time $(t_s) = K\left(\frac{V}{A}\right)^n$

Where, $n = 1.5$ to 2.0, V = Volume of mould (represent the amount of heat content),

A = Surface area responsible for heat transfer (represent the amount of heat transfer)

In general, $t_s = K\left(\frac{V}{A}\right)^2$

Where, K = Solidification factor/mould constant (sec/m^2).

(V/A) ratio for various shapes :

1.	Cube	$\frac{V}{A} = \frac{a}{6}$
2.	Sphere	$\frac{V}{A} = \frac{D}{6}$

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3.	Cylindrical ($h = D$) • For side riser $h = D$ • For top riser $h = \frac{D}{2}$	$\frac{V}{A} = \frac{D}{6}$
4.	Slab	$\frac{V}{A} = \frac{lbh}{2(lb+bh+hl)}$

Condition for directional solidification, $\frac{\left(\frac{V}{A}\right)_r^2}{\left(\frac{V}{A}\right)_c^2} = \frac{t_r}{t_c} > 1$.

There are three methods which employed for riser design, are as follows :

1. Caine's method :

Freezing ratio, $X = \frac{\text{Cooling rate of casting}}{\text{Cooling rate of riser}} = \frac{(A/V)_{casting}}{(A/V)_{riser}}$

Riser should solidify last, so $X > 1$

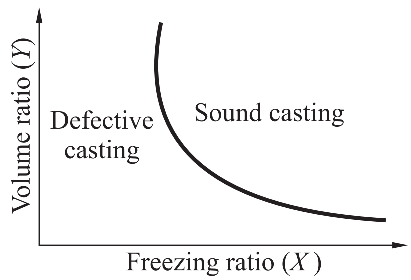
Volume ratio, $Y = \frac{V_r}{V_c}$

Caine's equation, $X = \frac{a}{Y-b} + c$

Where, $a =$ Freezing characteristic constant,

$b =$ Liquid solid solidification contraction

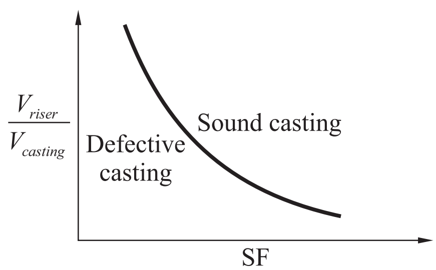
and $c =$ Relative freezing rate of riser and casting.



2. Shape Factor Method/Naval Research Laboratory Method :

Shape factor of casting, SF $= \frac{L+W}{T}$

Where, $L =$ Length, $W =$ Width and $T =$ Average thickness of casting.



Geometry	Cube	Sphere	Cylinder ($H = D$)
SF (Shape factor)	2	2	2

3. Modulus method :

Modulus, $M = \frac{\text{Volume } (V)}{\text{Area responsible for heat transfer } (A)} = \frac{V}{A}$

$$M_{Riser} = 1.2\, M_{casting}$$

Optimum size of riser :

Shape	Top riser	Side riser
Cylindrical ($h =$ Height, $R =$ Radius)	$h = R$	$h = 2R$
Square parallelepiped ($h =$ Height, $a =$ Side of square)	$h = \frac{a}{2}$	$h = a$

Gating System :

We need to design gating system elements such that molten metal can enter into the mould cavity without increase in velocity and turbulence of the molten metal within a specified time.

Types of gates : Top, bottom, parting and step gate. Parting gate is commonly used.

Component of gating system :

1. Pouring basin
2. Sprue
3. Runner
4. Ingate
5. Venting (vent)

Accessories in gating system :

S. No.	Accessories	Description
1.	Strainer	• It acts as a filter for separating the impurities present in molten metal. • It is provided in sprue. • It's made by ceramic material with high porosity.
2.	Splash core	• At the bottom of sprue. • Splash core is mainly used to prevent the sand erosion at bottom of the sprue, during the pouring of molten metal. • Made by ceramic material with low porosity.
3.	Skim bob	• The semicircular cut given on the top and bottom of the runner is called skim bob. • Mainly used for separating the impurities present in the molten metal.

Key Point

Vent : Small opening in the mould to facilitate escape of air and gases.

Gating ratio : Gating ratio reveals, that the system is pressurized and unpressurised.

$A_s : A_r : A_g =$ Gating ratio.

$A_s =$ Cross sectional area of the bottom of the sprue.

$A_r =$ Cross sectional area of runner.

$A_g =$ Cross sectional area of ingate.

1. **Pressurised gating system :** If the total cross-section area decreases towards the mould cavity. This provides a choke effect which pressurizes the liquid metal in the system. Useful for ferrous element (steel, gray CI).

   Gating ratio = 1 : 2 : 1, 1 : 0.75 : 0.5, 4 : 2 : 1.

2. **Non-pressurised gating system :** If the total cross-sectional area increases towards the mould cavity, so that the passages remain incompletely filled. It is an unpressurised system. Useful for non-ferrous elements (Al, Mg alloys).

   Gating ratio = 1 : 4 : 4, 1 : 2 : 2, 1 : 2 : 3, 1 : 2 : 4.

Key Point

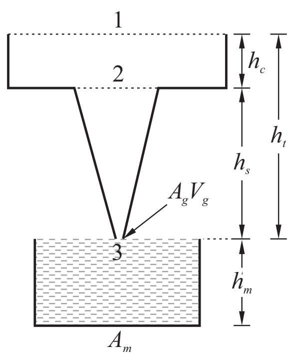
Choke area (CA) : Choke area is the minimum cross sectional area among all gating elements.

$$CA = \frac{m}{\rho t_f C_d \sqrt{2gh_t}}$$

Where, t_f = Filling time, C_d = Coefficient of discharge, m = Mass flow,

h_t = Molten metal head set by pouring basin and ρ = Density of molten metal.

Design of top gate :



Velocity at gate, $V_g = V_3 = \sqrt{2gh_t}$

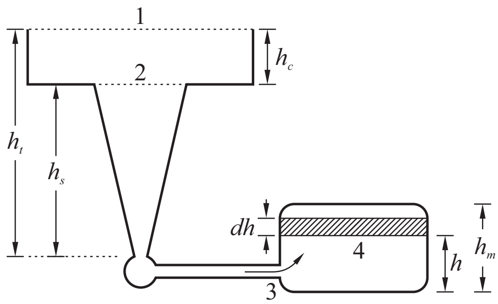
Filling time, $t_f = \frac{V}{A_g V_g} = \frac{A_m h_m}{A_g \sqrt{2gh_t}}$

Where, t_f = Filling time , V = Volume of mould,

A_g = Area of ingate, A_m = Area of mould,

h_m = Height of mould and h_t = Total height of sprue.

Design of bottom gate :



Velocity at gate, $V_g = V_3 = \sqrt{2g(h_t - h)}$

Filling time (t_f) is given by integrating following,

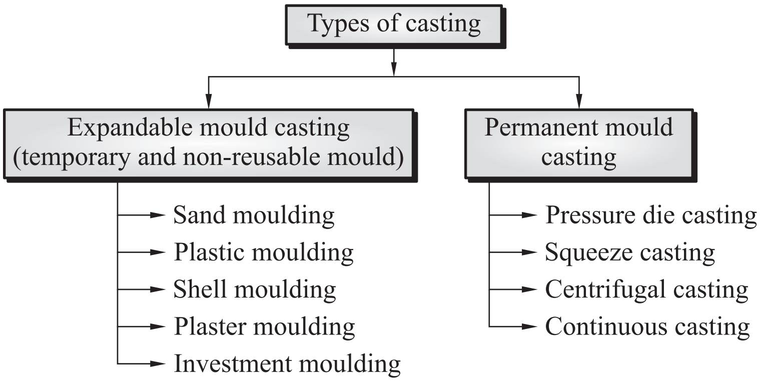
$$A_m dh = A_g V_g dt = A_g \sqrt{2g(h_t - h)}\, dt$$

or $$dt = \left[\frac{A_m}{A_g} \times \frac{1}{\sqrt{2g}} \times \frac{dh}{\sqrt{h_t - h}}\right]$$

On integration, $$t_f = \frac{A_m}{A_g} \times \frac{1}{\sqrt{2g}} \times 2\left[\sqrt{h_t} - \sqrt{h_t - h_m}\right]$$

Types of Casting :

Types of casting process :



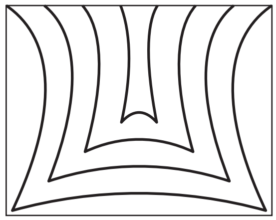
Comparison of different casting processes :

S. No.	Method of casting	Material choice	Surface smoothness	Surface details	Remarks
1.	Sand casting	Wide	Poor	Poor	Usually require machining before use
2.	Shell mould casting	Wide	Good	Good	Best for low casting methods
3.	Permanent mould casting	Al, brass, bronze some gray iron	Good	Good	Economical for mass production only
4.	Plaster mould casting	Narrow : Brass bronze Al	Good	Good	Little finishing required
5.	Investment casting	Wide : Includes materials hard to forge and machine	Excellent	Excellent	Best for too complicated parts
6.	Die casting	Narrow : Non-ferrous	Good	Good	Most economical
7.	Centrifugal casting	Wide	Good	Good	Economical for mass production.

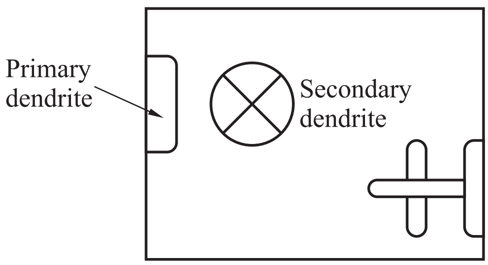
Solidification Mechanism in Casting :

There are two types of solidification mechanisms in casting as follows :

1. **Skin forming :** It take place either in pure material or the alloys having eutectic composition. It move towards centre and solidification appear layer by layer.



2. **Dendritic growth :** It take place when mushy zone appear during solidification of alloy. In this solidification is not uni-directional. To provide unidirectional solidification chills are used.



Defects in Casting :

1. **Porosity :**
   - Poring temperature should be maintained properly to reduce porosity.
   - Small holes, uniformly dispersed through casting.
2. **Blow holes :**
   - It is fairly large, well rounded cavity produced by gases which displace the molten metal at the cope surface of a casting.
   - A controlled content of moisture and volatile constituents in the sand may also helps in avoid the blow holes.
3. **Gas holes :** When the evolved gases or dissolved gases in the molten metal are not able to leave the mass of molten metal as it solidifies and get trapped within the casting.
4. **Scar :** Shallow blow on the flat casting surface.
5. **Blister :** Scar covered by the thin layer of metal.
6. **Pin holes :** When the hot metal comes into the contact of moisture, H_2O is disintegrates into H_2 and O_2. As H_2 comes out slowly it produces large number of fine holes over the casted part.

7. **Drop :** An irregularly shaped projection on the cope surface of a casting is called drop. This is caused by dropping of sand from the cope or other overhang projections in the mould.

8. **Inclusion :** It is non-metallic particle in the metal matrix.

9. **Dross :** These are lighter impurities appearing on the top surface of the casting.

10. **Scab :** Due to hydrostatic pressure of liquid metal in the riser if the impression is appears in the roof of casted part it is called scab.

11. **Swell :** Found on vertical surfaces of a casting due to hydrostatic pressure.

12. **Penetration :** If the mould surface is too soft and porous, the liquid metal may flow between the sand particles upto a distance, into the mould. This cause rough, porous projection and this defect is called penetration.

13. **Misrun :** Misrun due to insufficient heat, metal start freezing before reaching the farthest point of the mould cavity. The defect that thus called misrun.

14. **Cold shut :** For a casting with gates at its two sides, the misrun may show up at the centre of the casting. This defect is called cold shut.

15. **Hot tear :** A crack that develops in a casting due to high residual stress.

16. **Shift :** Misalignment between two halves of a mold or of a core.

Defects in casting due to improper sand condition :

Sand condition	Defects condition
High moisture	Blow holes, fine holes, gas holes, scabs, rough surface, hot tears.
Low moisture	Drops, sand wash
High permeability	Blow holes, pin holes, gas holes, scabs, dirt.
Low green strength	Drops and dirt.
High green strength	Scabs, rough surface and blow holes.
High dry strength	Hot tears.
Low dry strength	Dirt, rough surface.

❖❖❖❖

6 | HEAT TRANSFER

6.1 Introduction to Heat Transfer

It is that discipline of thermal engineering that concerns with the generation, use, conversion and exchange of thermal energy (heat) between physical systems. Heat transfer is classified into various mechanism such as,

1. Conduction or diffusion
2. Convection
3. Radiation
4. Transfer of energy by phase change.

Fundamental modes of heat transfer are as follows :

1. **Conduction :** The flow of internal energy from a region of higher temperature to one of lower temperature by the interaction of the adjacent particles (atoms, molecules, ions, electrons, etc.) in the intervening space.

   Example : A cold cast iron skillet is placed onto a stovetop.

2. **Convection :** Mode of heat transfer between a surface and a fluid moving over it.

   Example : Steaming cup of hot tea-the steam is showing heat being transferred into the air.

3. **Radiation :** It is a form of electromagnetic energy, which do not require any medium for its propagation.

   Example : Infrared light, radio wave, ionization of food.

State of Heat Transfer :

Steady state :

The temperature of the system varies with position but not with respect to time, i.e.,

$$\frac{dT}{dt} = 0 \text{ and } \frac{dT}{dx} \neq 0$$

Where, $dT =$ Change in temperature, $dt =$ Change in time and $dx =$ Change in thickness.

Unsteady state :

The temperature of the body varies with position and time, i.e.,

$$\frac{dT}{dt} \neq 0 \text{ and } \frac{dT}{dx} \neq 0$$

6.2 Conduction

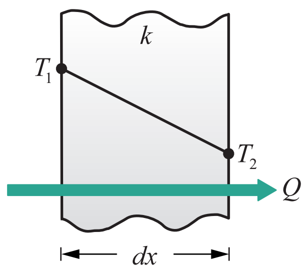
Conduction is a phenomena of heat transfer from one part of matter to another part of the same matter or from one substance to another when it is placed in physical contact without actual displacement of the molecules or particles.

Fourier's Law of Heat Conduction :

It states that the rate of heat flow by conduction through the body is directly proportional to the surface area (A) perpendicular to the direction of heat flow and the temperature gradient (dT/dx) through the body.

Assumptions :

1. Conduction of heat takes place under steady state condition.
2. Flow of heat is unidirectional.
3. The temperature gradient is constant and the temperature profile is linear.
4. No internal heat generation.
5. The bounding surfaces are isothermal in character.
6. Material should be homogeneous and isotropic i.e., value of thermal conductivity is constant in all direction.
7. Heat is transferred such that no phase change is taking place.



Heat flow through a body per unit time (in W), $Q = -kA\frac{dT}{dx}$

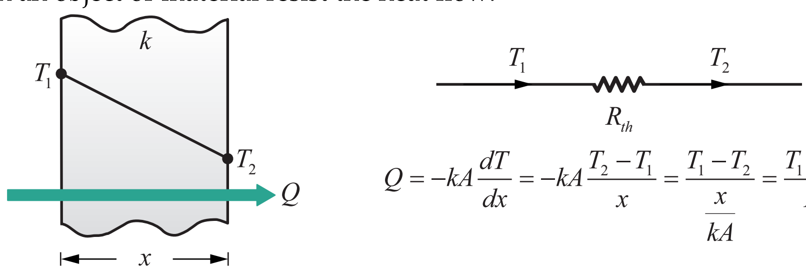
$$q = \frac{Q}{A} = -k\frac{dT}{dx}$$

Where, $k =$ Thermal conductivity (W/mK) and

$q =$ Heat flux (W/m^2).

Thermal Resistance :

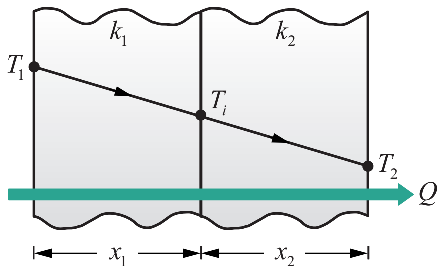
Thermal resistance is a heat property and a measurement of a temperature difference by which an object or material resist the heat flow.



$$Q = -kA\frac{dT}{dx} = -kA\frac{T_2 - T_1}{x} = \frac{T_1 - T_2}{\frac{x}{kA}} = \frac{T_1 - T_2}{R_{th}}$$

where, $R_{th} = \frac{x}{kA}$

Case 1 : Thermal resistance in series,



$$R_{th} = R_1 + R_2 = \frac{x_1}{k_1 A} + \frac{x_2}{k_2 A}$$

and, $$Q = \frac{T_1 - T_2}{R_{th}} = \frac{T_1 - T_2}{\frac{x_1}{k_1 A} + \frac{x_2}{k_2 A}}$$

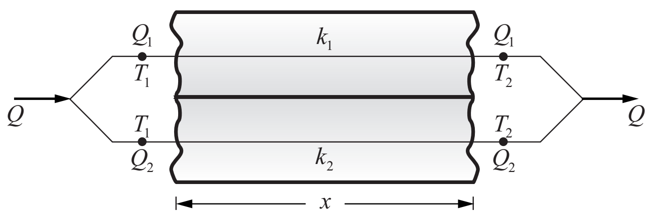
Again, $$T_1 - T_i = QR_1 = \frac{Qx_1}{k_1 A}$$

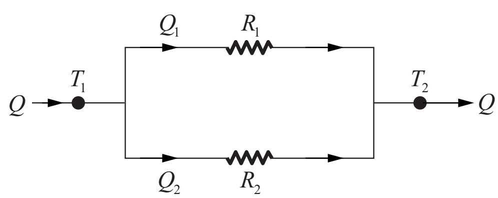
Where, $T_i =$ Intermediate temperature.

For many thermal resistance in series the general equation, we have

$$R_{th} = \Sigma(R_{th})_i = \sum_{i=1}^{n} \frac{x_i}{k_i A_i} \text{ and } Q = \frac{T_1 - T_{n+1}}{\sum_{i=1}^{n} \frac{x_i}{k_i A_i}}$$

Case 2 : Thermal resistance in parallel,





For two thermal resistance in parallel we have,

$$\frac{1}{R_{th}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \qquad \left(\because R_1 = \frac{x}{k_1 A_1} \text{ and } R_2 = \frac{x}{k_2 A_2}\right)$$

Now, $$R_{th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{x\left(\frac{1}{k_1 A_1} \cdot \frac{1}{k_2 A_2}\right)}{x\left(\frac{1}{k_1 A_1} + \frac{1}{k_2 A_2}\right)} = \frac{\left(\frac{1}{k_1 A_1} \cdot \frac{1}{k_2 A_2}\right)}{\left(\frac{1}{k_1 A_1} + \frac{1}{k_2 A_2}\right)}$$

and $$Q = \frac{T_1 - T_2}{R_{th}}$$

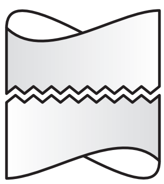
The general equation for many thermal resistances in parallel, we have

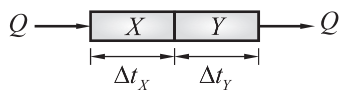
$$\frac{1}{R_{th}} = \sum_{i=1}^{n} \frac{1}{(R_{th})i} = \frac{1}{\sum_{i=1}^{n} \frac{x_i}{k_i A_i}}$$

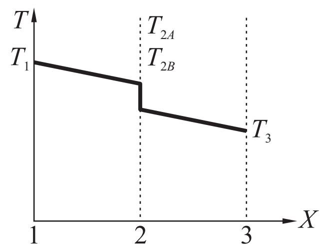
and $$Q = \frac{T_1 - T_{n+1}}{\sum_{i=1}^{n} (R_{th})_i}$$

Key Point

Thermal contact resistance : Whenever a contact is made between two similar or dissimilar metallic or non-metallic solid surfaces, the contact will not be complete because of surface roughness as well as pressure. From the microscopic view point, some areas on the surfaces shall come in contact but remaining areas cannot approach together leaving a void which is generally filled by a gaseous material like air. The resistance offered by the material trapped in the (microscopic) non-contact regions is quantified into thermal contact resistance.







$$R_{th(contact)} = \frac{\text{Temperature drop at interface}}{(Q/A)} = \left[\frac{T_{2A} - T_{2B}}{Q/A}\right]$$

Thermal Conductivity (k) :

Thermal conductivity refers to the intrinsic ability of material to transfer or conduct heat. It is defined as the rate at which heat is transferred by conduction through a unit cross section at area of a material when a temperature gradient exists perpendicular to area.

Materials	Thermal conductivity (k) (W/mK)
Diamond	2300
Aluminium	205
Brass	107
Copper	385
Steel	20-45
Brick (masonary)	0.65
Brick (first clay)	0.75-1.75

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Glass (window)	0.75
Saw dust	0.07
Wood	0.17
Air	0.024
Ice	2.25
Water	0.55-0.7
Freon	0.0083

Key Point

Trends of thermal conductivity :

- $k_{solid} > k_{liquid} > k_{gas}$ (due to shape & size of matter or orientation of atoms in molecules)
-

S.No.	Phase	Temperature	Effect	Cause
1.	Solid	Increases	Conductivity decreases	The crystal configuration of solids is affected and they start to occupy random positions.
2.	Liquid	Increases	Conductivity decreases	The liquids expand and try to attain gaseous configuration.
3.	Gas	Increases	Conductivity increases	The gas molecules collide at higher rate and momentum transfer due to collision increases.

This is the general case, anomalies are not considered.

Thermal Diffusivity :

Thermal diffusivity is a material-specific property for characterizing unsteady heat conduction. This value describes how quickly a material reacts to a change in temperature.

$$\alpha = \frac{\text{Heat conducted}}{\text{Heat capacity}} = \frac{k}{\rho c_p} \text{m}^2/\text{s}$$

- The larger the thermal diffusivity, the faster the propagation of heat into the medium.
- k represents how well a material conducts heat and heat capacity (ρc_p) represents how much energy a material stores per unit volume.
- When thermal conductivity (k) increases then thermal diffusivity (α) increases and $k \to \infty$ and $\alpha \to \infty$ (in case of super conductor).

General Heat Conduction Equation :

(A) Rectangular coordinate :

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_g}{k}=\frac{\rho c_p}{k}.\frac{\partial T}{\partial t}=\frac{1}{\alpha}.\frac{\partial T}{\partial \tau}$$ (Fourier - Biot equation)

Where, c_p = Specific heat of conducting material, α = Thermal diffusivity and τ = Time.

Special cases :

1. When there is no internal heat generation, equation becomes

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}.\frac{\partial T}{\partial \tau}$$

[Unsteady state $\left(\frac{\partial T}{\partial \tau}\neq 0\right)$ heat flow with no internal heat generation]

or, $$\nabla^2 T=\frac{1}{\alpha}\times\frac{\partial T}{\partial \tau}$$ (Diffusion equation)

Where, ∇^2 = Laplacian operator.

2. When temperature is independent of time, then the conduction takes place in the steady state i.e., $\left(\frac{\partial T}{\partial \tau}=0\right)$ and the equation will be,

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_g}{k}=0$$

or $$\nabla^2 T+\frac{q_g}{k}=0$$ (Poisson's equation)

3. If there is no internal heat generation along with steady state condition,

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0$$

or, $$\nabla^2 T=0$$ (Laplace equation)

4. One dimensional steady state heat transfer with internal heat generation,

$$\frac{\partial^2 T}{\partial x^2}+\frac{q_g}{k}=0$$

5. One dimensional unsteady state without internal heat generation,

$$\frac{\partial^2 T}{\partial x^2}=\frac{1}{\alpha}.\frac{\partial T}{\partial \tau}$$

(B) Cylindrical coordinate :

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\left(\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\left(\frac{\partial^2 T}{\partial \phi^2}\right)+\frac{\partial^2 T}{\partial z^2}+\frac{q_g}{k}=\frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)$$

Case 1 : $T = f(r, \tau)$

Then, $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)$$

Case 2 : $T = f(r)$,

Then, $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

Case 3 : For steady state unidirectional radial heat flow, with internal heat generation.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{q_g}{k} = 0$$

(C) Spherical coordinate :

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\psi}\frac{\partial}{\partial \psi}\left(\sin\psi\frac{\partial T}{\partial \psi}\right) + \frac{1}{r^2\sin^2\psi}\left(\frac{\partial^2 T}{\partial \phi^2}\right) + \frac{q_g}{k} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial t}\right)$$

If temperature varies only in radial direction,

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2\partial T}{\partial r}\right) + \frac{\dot{q}}{k} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)$$

or $$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\left(\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{k} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)$$

Heat Conduction Through Different Geometrical Shape :

(A) Plane wall :

Assumption :

1. The contact should be perfect between the adjacent layers
2. There is no fall of temperature at the interface
3. The temperature is continuous at the interface, although there is discontinuity in temperature gradient.

Uniform thermal conductivity : Consider a plane wall of homogenous material through which heat is flowing only in x-direction,

Let, L = Thickness of the plane wall, k = Thermal conductivity of the wall material.

T_1 = Temperatures maintained at face 1, T_2 = Temperatures maintained at face 2,

A = Cross sectional area of the wall (perpendicular to x- direction).

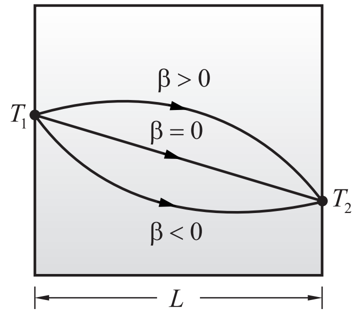
$\therefore$ $$Q = \frac{-kA(T_2 - T_1)}{L} = \frac{T_1 - T_2}{(R_{th})_{cond.}}$$

Here, $(R_{th})_{cond.}$ = Thermal resistance to heat conduction.

Key Point

Conduction of heat takes place in the direction of decreasing temperatures and when temperature decreases with increasing x, the temperature gradient becomes negative. The negative sign ensure that transfer of heat in the positive x-direction is a positive quantity.

Variation of thermal conductivity with temperature :



$$\dot{Q} = k_m A\left[\frac{T_1 - T_2}{L}\right] \text{ and } k_m = k_0(1+\beta T)$$

Where, k_0 = Thermal conductivity at zero temperature,

k_m = Mean thermal conductivity and β = Slope of temperature variation line.

Key Point

- In case of plane wall, $k_m = \left[1+\beta\left(\frac{T_1+T_2}{2}\right)\right]k_0$
- $\beta = +ve$ for nonmetals, $\beta = -ve$ for metals except aluminium and uranium.

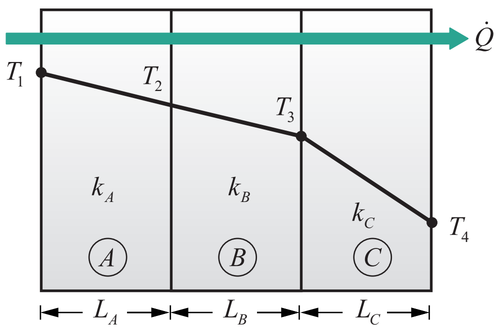
(B) Composite wall :

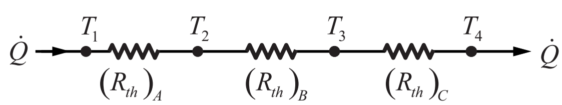
Let, L_A, L_B, L_C = Thickness of slabs *A*, *B* and *C*,

k_A, k_B, k_C = Thermal conductivities of the slabs *A*, *B* and *C*,

T_1 = Temperature at the wall surface 1, T_2 = Temperature at the interface 2,

T_3 = Temperature at the interface 3, T_4 = Temperature at the wall surface 4.





$$(R_{th})_A = \frac{L_A}{k_A.A}, \; (R_{th})_B = \frac{L_B}{k_B.A} \text{ and } (R_{th})_C = \frac{L_C}{k_C.A}$$

Since, the quantity of heat transmitted per unit of time through each slab is same,

$$\dot{Q} = \frac{k_A.A(T_1 - T_2)}{L_A} = \frac{k_B.A(T_2 - T_3)}{L_B} = \frac{k_C.A(T_3 - T_4)}{L_C}$$

$$\dot{Q} = \frac{(T_1 - T_4)}{\left[\frac{L_A}{k_A.A} + \frac{L_B}{k_B.A} + \frac{L_C}{k_C.A}\right]} = \frac{(T_1 - T_4)}{[(R_{th})_A + (R_{th})_B + (R_{th})_C]}$$

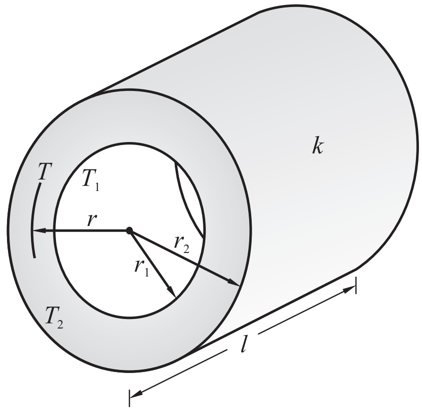
(C) Hollow cylinder : Consider a hollow cylinder made of material having constant thermal conductivity and insulated at both ends.

Let, r_1 = Inner radius, r_2 = Outer radius

T_1 = Temperature of inner surfaces,

T_2 = Temperature of outer surfaces

k = Constant thermal conductivity with the given temperature range.



Temperature distribution : $\dfrac{T-T_1}{T_2-T_1} = \dfrac{\ln\left(\dfrac{r}{r_1}\right)}{\ln\left(\dfrac{r_2}{r_1}\right)}$.

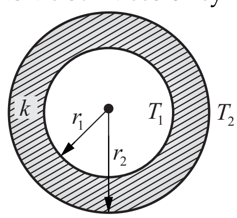
Heat generation : $\dot{Q} = \dfrac{T_1 - T_2}{\dfrac{\ln(r_2/r_1)}{2\pi kL}}$.

Key Point

- The temperature distribution is *logarithmic*.
- Temperature at any point in the cylinder can be expressed as a function of *radius only*.
- **Resistance offered by cylinder :** $R_{cylinder} = \dfrac{\ln\left(\dfrac{r_2}{r_1}\right)}{2\pi kL}$
- **Logarithmic mean area (A_m) for the hollow cylinder :** $A_m = \dfrac{A_2 - A_1}{\ln\left(\dfrac{A_2}{A_1}\right)}$

  Here, A_1 = Area of inside surface of cylinder, A_2 = Area of outside surfaces of cylinder.
- **Logarithmic mean radius (r_m) :**

  $$r_m = \frac{r_2 - r_1}{\ln\left(\dfrac{r_2}{r_1}\right)}$$



  Here, r_1 = Inside radius and r_2 = Outside radius.

(D) Composite cylinder :

Consider flow of heat through a composite cylinder as shown in figure.

Let, T_1, T_2, T_3 = Temperatures at the points 1, 2 and 3,

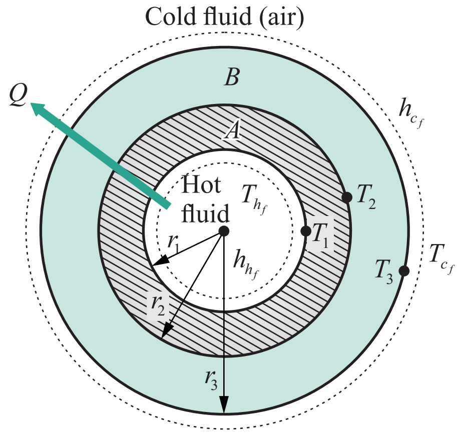
L = Length of the composite cylinder,

h_{hf}, h_{cf} = Inside and outside heat transfer coefficient of hot fluid and cold fluid,

k_A, k_B = Thermal conductivities of the inside layer (A) and outside layer (B),

T_{h_f} = Temperature of the hot fluid flowing inside the cylinder,

T_{c_f} = Temperature of the cold fluid (atmospheric air).

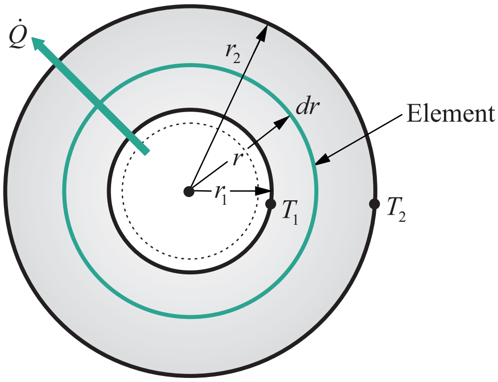


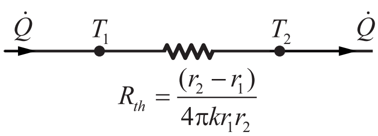
$$\dot{Q} = \frac{2\pi L(T_{hf} - T_{cf})}{\frac{1}{h_{hf}.r_1} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{1}{h_{cf}.r_3}}$$

Key Point

Better insulator should be placed just next to the pipe for lesser heat transfer.

(E) Hollow sphere : Consider a hollow sphere made of material having constant thermal to conductivity.





Let, r_1 = Inner radius, r_2 = Outer radius, T_1 = Temperature of inner surfaces,

T_2 = Temperature of outer surfaces, k = Constant thermal conductivity with the given temperature range.

Temperature distribution, $\frac{T - T_1}{T_2 - T_1} = \frac{(1/r) - (1/r_1)}{(1/r_2) - (1/r_1)}$

Where, T = Temperature at the given radius r.

Heat generation, $\dot{Q} = \frac{T_1 - T_2}{\left[\frac{(r_2 - r_1)}{4\pi k r_1 r_2}\right]}$ $[T_1 > T_2]$

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Key Point

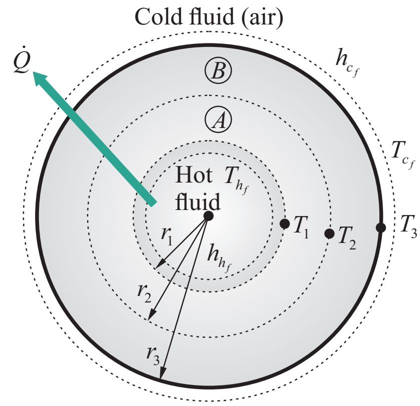
- The temperature distribution follows a hyperbolic variation.
- Logarithmic mean area for the hollow sphere, $A_m = \sqrt{A_1 A_2}$.
  Where, A_1 = Area of inner surface and A_2 = Area of outer surface.
- Mean radius of sphere, $r_m = \sqrt{r_1 r_2}$.

(F) Composite sphere :

$$\dot{Q} = \frac{4\pi(T_{h_f} - T_{c_f})}{\frac{1}{h_{h_f} r_1^2} + \frac{r_2 - r_1}{k_A r_1 r_2} + \frac{r_3 - r_2}{k_B r_2 r_3} + \frac{1}{h_{c_f} r_3^2}}$$

If there are n concentric spheres then the above equations can be written as,

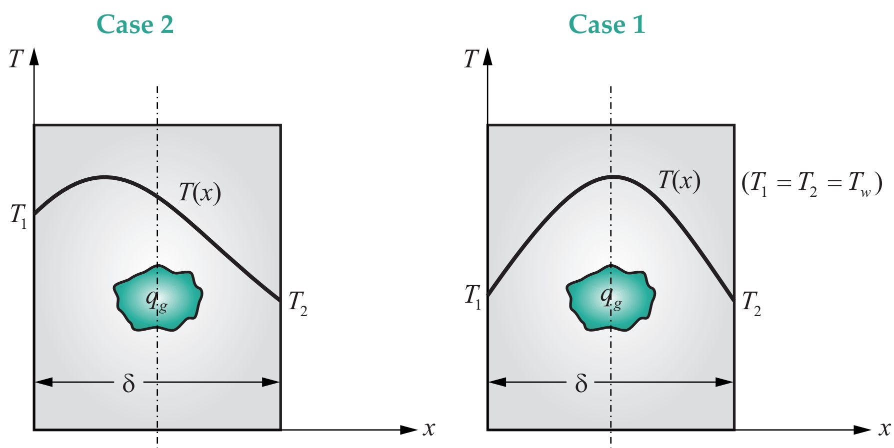
$$\dot{Q} = \frac{4\pi(T_{h_f} - T_{c_f})}{\left[\frac{1}{h_{h_f}.r_1^2} + \sum_{n=1}^{n=n}\left(\frac{r_{(n+1)} - r_n}{k_n.r_n.r_{(n+1)}}\right) + \frac{1}{h_{c_f}.r_{(n+1)}^2}\right]}$$



Plane Wall with Uniform Heat Generation :

Assumptions :

1. Steady state conditions.
2. 1D-heat flow.
3. Constant k.
4. Uniform volumetric heat generation (q_g per unit volume) within the wall.



Appropriate heat flow equation, $\frac{d^2T}{dx^2} + \frac{q_g}{k} = 0$

Integrating twice we get, $T = \frac{-q_g}{2k}x^2 + c_1 x + c_2$ (General solution)

Case 1 : Both surface maintained at same temperature :

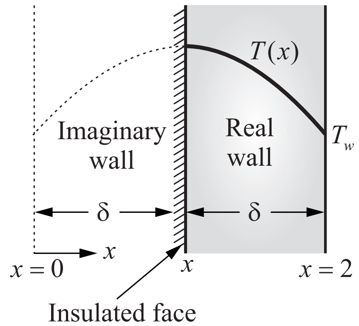
Temperature distribution or profile, $T = \frac{q_g}{2k}(\delta - x)x + T_w$

The temperature distribution is parabolic and symmetrical about the mid plane. Maximum temperature at $x = \delta/2$,

$$T_{max} = \frac{q_g}{8k}\delta^2 + T_w$$

Key Point

Maximum temperature at mid-plane (insulated end),



$$T_{max} = \frac{q_g}{2k}\delta^2 + T_w$$

Case 2 : Temperature of both surface are different

Maximum value of temperature, $\frac{T_{max} - T_2}{T_1 - T_2} = \frac{B+1}{2B} \times \frac{B+1}{2} = \frac{(B+1)^2}{4B}$

Where, B is parameter and constant having value, $B = \frac{q_g}{2k}\frac{\delta^2}{(T_1 - T_2)}$

Key Point

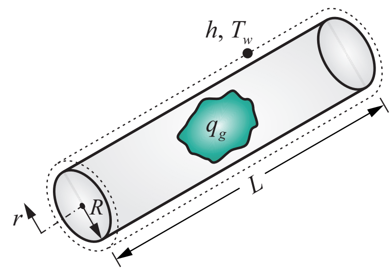
- Maximum temperature is at the left hand face i.e., T_1 is maximum.
- If there is no internal heat generation $(q_g = 0)$ the above case reduce to normal conduction condition.

Cylinder with Uniform Heat Generation :

Assumption :

1. Steady state condition.
2. 1D radial conduction.
3. Constant k.
4. Uniform volumetric heat generation.

Temperatures distribution is given by,



$$T = -\frac{q_g}{4k}r^2 + c_1 \log_e(r) + c_2 \qquad \text{(General solution)}$$

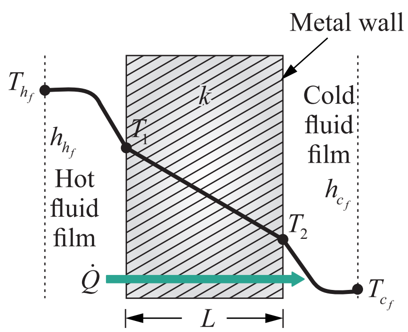
Solid cylinder :

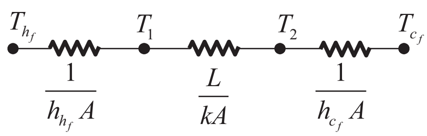
Temperature distribution, $T = T_w + \frac{q_g}{4k}(R^2 - r^2)$

Temperature distribution is parabolic and T_{max} occur at center $(r = 0)$,

$$T_{max} = T_w + \frac{q_g}{4k}R^2$$

Overall Heat Transfer Coefficient (U) :





$$\dot{Q} = UA\,\Delta T = \frac{\Delta T}{R_{total}}$$

$$\Rightarrow \qquad U = \frac{1}{A(R_{total})} \quad \Rightarrow \quad \dot{Q} = \frac{\Delta T}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

Heat Transfer from Extended Surface (Fins) :

Condition of fin	Heat transfer
Infinitely long	$Q_{fin} = \sqrt{hPkA_c}\,\theta_0$
Tip is insulated	$Q_{fin} = \sqrt{hPkA_c}\,\theta_0 \tanh(mL_c)$
Losing heat at tip	$Q_{fin} = \sqrt{hPKA_c}\,\theta_0 \left[\frac{\tanh(mL) + \frac{h}{km}}{1 + \frac{h}{km}\tanh(mL)}\right]$

$$m = \sqrt{\frac{hP}{kA_c}} \quad \text{(unit of } m \text{ is 1/m)}$$

Where, A_c = Cross sectional area, L_c = Corrective length and P = Perimeter.

Fin Performance :

Condition of fin	Efficiency (η)	Effectiveness (ε)
For adiabatic tip.	$\eta_{fin} = \dfrac{\tanh(mL)}{mL}$	$\varepsilon_{fin} = \dfrac{\tanh(mL)}{\sqrt{\dfrac{hA_c}{kP}}}$
Infinitely long.	$\eta_{fin} = \dfrac{1}{mL}$	$\varepsilon_{fin} = \sqrt{\dfrac{kP}{hA_c}}$
A straight rectangular fin of thickness δ and width b.		$\varepsilon_{fin} = \sqrt{\dfrac{2k}{h\delta}}$
When convective heat transfer occurs at tip.	$\eta_{conv.\,fin} = \dfrac{\text{Tanh}\times mL_c}{mL_c}$	$\varepsilon_{conv.\,fin} = \sqrt{\dfrac{kP}{hA_c}} \times \text{Tanh}\times mL_c$
Insulated fin.	$\eta_{ins.\,fin} = \dfrac{\text{Tanh}\times mL}{mL}$	$\varepsilon_{ins.\,fin} = \sqrt{\dfrac{kP}{hA_c}} \times \text{Tanh}\times mL$

🕮 Key Point

Corrected length (L_c)

$$L_c = L + \frac{t}{2},$$ (For rectangular fin of thickness 't')

$$L_c = L + \frac{D}{4},$$ (For circular fin of diameter 'D')

- Efficiency of fin decreases with an increase in length
- Efficiency of fin increases for small values of mL.
- An increases in the fin effectiveness can be obtained by extending the length of fin but that rapidly becomes a losing proposition in terms of efficiency.
- Fins should be closely placed just to increase surface area but not that close that it blocks the fluid flow or trap air.

Time Constant :

A characteristics time that governs the approach of an exponential function to a steady-state value. Mathematically,

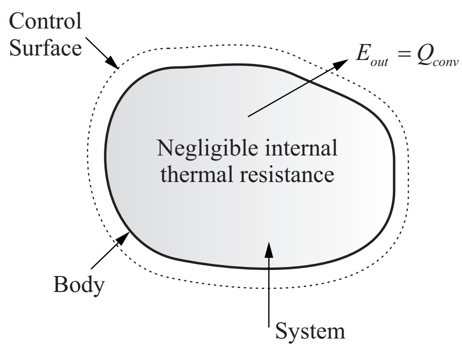
$$\tau = \frac{\rho VC}{hA}$$

Slow system response when large time constant occurs and fast system response obtained in case of low time constant. It may achieved by,

1. Increasing the heat transfer coefficient.
2. Use of metals with low specific heat as well as low density.

Lumped Parameter Analysis :

The process in which the internal resistance is assumed negligible in comparison with its surface resistance is called the Newtonian heating or cooling process. The temperature in this process is considered to be uniform at a given time. This type of analysis is called lumped parameter analysis because the whole solid, whose energy at any time is a function of its temperature and total heat capacity is assumed as one lump.



Let us consider a body whose initial temperature (T_i) throughout and which is placed suddenly in ambient air or any liquid at constant temperature (T_a). The transient response of body can be determined by relating its rate of change of internal energy with convective heat exchange at the surface.

Boundary conditions :

$\tau = 0$ when $T = T_i$

$\tau > 0$ when $T = f(\tau)$

$$Q_{conv} = -\rho VC \frac{dT}{d\tau} = hA_s(T - T_a) \quad \ldots(i)$$

Integrating above equation after rearranging, we get

$$\int \frac{dT}{(T - T_a)} = \frac{hA_s}{\rho VC} \int d\tau$$

$$\ln(T - T_a) = \frac{hA_s}{\rho VC}\tau + c_1 \quad \ldots(ii)$$

Applying boundary condition,

At $\tau = 0$, $\quad T = T_i$

$$C_1 = \ln(T_i - T_a)$$

Hence, after putting value in equation (ii),

$$\ln(T_i - T_a) = \frac{-hA_s}{\rho VC} + \ln(T_i - T_a)$$

$$\frac{T - T_a}{T_i - T_a} = \frac{\theta}{\theta_i} = e^{\left[\frac{-hA_s}{\rho VC}\tau\right]} \quad \ldots(iii)$$

The power on exponential i.e. $\frac{hA_s}{\rho VC}\tau$ can be arranged in dimensionless form.

$$\frac{hA_s}{\rho VC}\tau = \left(\frac{hV}{kA_s}\right)\left(\frac{A_s{}^2 k}{\rho V^2 C}\tau\right) = \left(\frac{hL_c}{k}\right)\left(\frac{\alpha\tau}{L_c^2}\right)$$

Where, $\alpha = \frac{k}{\rho C}$ = Thermal diffusivity.

Key Point

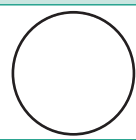
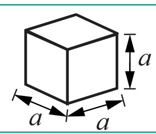
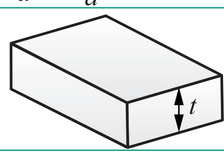
- The non-dimensional factor $\frac{hL_c}{k}$ is called Biot number (Bi). It gives an indication of the ratio of internal (conduction) resistance to surface (convection) resistance. If $\text{Bi} < 0.1$, the lumped heat capacity approach is used.
- The non-dimensional factor $\frac{\alpha\tau}{L_c^2}$ is called Fourier number (F_0). If signifies the degree of penetration of heating or cooling effect through a solid.

$$F_0 = \frac{\alpha\tau}{L_c^{\ 2}}$$

- By using both dimensionless factor equation (iii) will becomes,

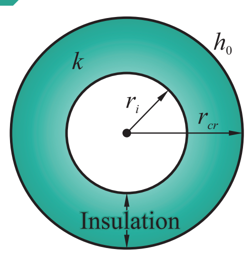
$$\frac{\theta}{\theta_i} = \frac{T - T_a}{T_i - T_a} = e^{-BiF_o}$$

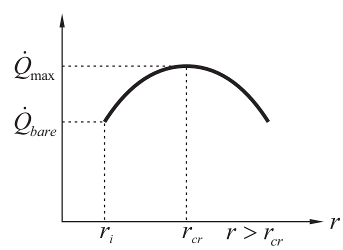
Characteristics Length :

S. No.	Geometry	Shape	Characteristics length $L_c = \frac{\text{Volume of body}}{\text{Surface area of content}}$
1.	Sphere		$L_c = \frac{4/3\pi R^3}{4\pi R^2} = \frac{R}{3}$
2.	Cylinder		$L_c = \frac{\pi R^2 L}{2\pi RL} = R/2$
3.	Cube		$L_c = \frac{a^3}{6a^2} = \frac{a}{6}$
4.	Plate		$L_c = t$

Critical Thickness of Insulation :

- The rate of heat transfer from the cylinder increases with the addition of insulation for $(r_2 < r_{cr})$ reaches a maximum when $r_2 = r_{cr}$ and starts to decrease for $r_2 > r_{cr}$.



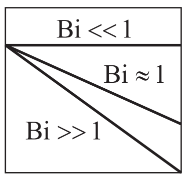


- The radius of electric wires may be smaller than the critical radius.
- For cylinder, $r_{cr} = \dfrac{k}{h_0}$
- For sphere, $r_{cr} = \dfrac{2k}{h_0}$
- Upto critical radius convection dominates as increasing radius (r) convection resistance decreases thus heat transfer increases.

$$R_{convection} = \left[\frac{1}{h(\text{Area})}\right] = \left[\frac{1}{hA}\right]$$

Heisler Charts :

Heisler Charts are extensively used to determine the temperature distribution and heat flow rate when both conduction and convection resistance are almost of equal importance i.e. (Biot number) $\text{Bi} = 1$.



$0 < \text{Bi} < 100$

$\text{Bi} << 1$	$R_{conduction} << R_{convection}$
$\text{Bi} \approx 1$	$R_{convection} = R_{conduction}$
$\text{Bi} >> 1$	$R_{conduction} >> R_{convection}$

Analysis is done for finite solids.

Governing heat equation, $\dfrac{\partial^2 T}{\partial x^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial \tau}$ where, $T = f(\tau, x)$.

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