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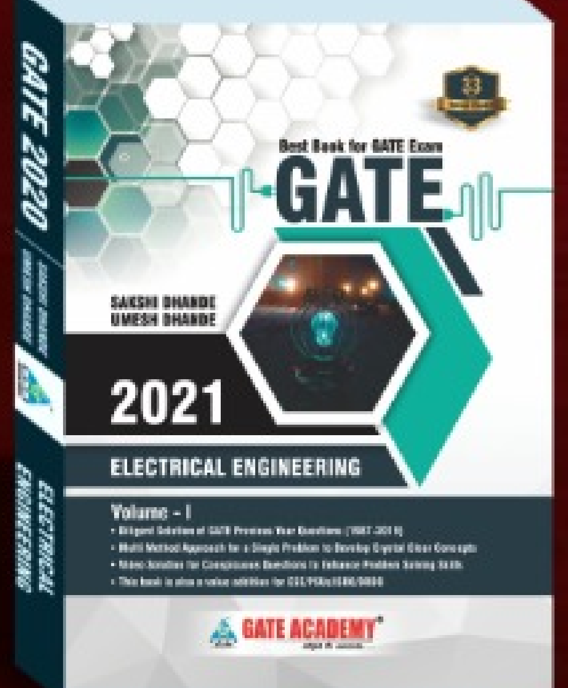

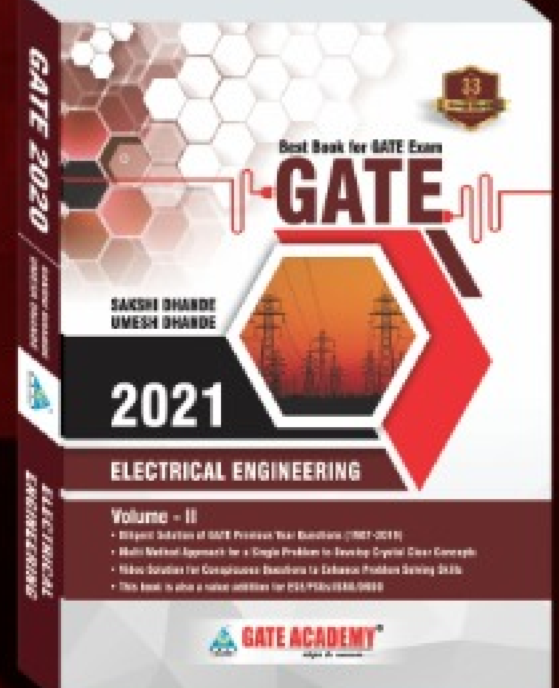

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GATE

SAKSHI DHANDE
UMESH DHANDE

2021

ELECTRICAL ENGINEERING

Volume - I

- Diligent Solution of GATE Previous Year Questions (1987-2020)
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- Video Solution for Conspicuous Questions to Enhance Problem Solving Skills
- This book is also a value addition for ESE/PSUs/ISRO/DRDO

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GATE 2021
Electrical Engineering
(Volume - I)

TOPIC WISE GATE SOLUTIONS
1987 - 2020

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IMPORTANCE of GATE

GATE examination has been emerging as one of the most prestigious competitive exam for engineers. Earlier it was considered to be an exam just for eligibility for pursuing PG courses, but now GATE exam has gained a lot of attention of students as this exam open an ocean of possibilities like :

1. **Admission into IISc, IITs, IIITs, NITs**

   A good GATE score is helpful for getting admission into IISc, IITs, IIITs, NITs and many other renowned institutions for M.Tech./M.E./M.S. An M.Tech graduate has a number of career opportunities in research fields and education industries. Students get ₹ 12,400 per month as stipend during their course.

2. **Selection in various Public Sector Undertakings** (**PSUs**)

   A good GATE score is helpful for getting job in government-owned corporations termed as **Public Sector Undertakings** (**PSUs**) in India like IOCL, BHEL, NTPC, BARC, ONGC, PGCIL, DVC, HPCL, GAIL, SAIL & many more.

3. Direct recruitment to Group A level posts in Central government, i.e., Senior Field Officer (Tele), Senior Research Officer (Crypto) and Senior Research Officer (S&T) in Cabinet Secretariat, Government of India, is now being carried out on the basis of GATE score.

4. **Foreign universities through GATE**

   GATE has crossed the boundaries to become an international level test for entry into postgraduate engineering programmes in abroad. Some institutes in two countries **Singapore** and **Germany** are known to accept GATE score for admission to their PG engineering programmes.

5. **National Institute of Industrial Engg. (NITIE)**
   - NITIE offers **PGDIE / PGDMM / PGDPM** on the basis of GATE scores. The shortlisted candidates are then called for group Discussion and Personal Interview rounds.
   - NITIE offers a Doctoral Level Fellowship Programme recognized by Ministry of HRD (MHRD) as equivalent to PhD of any Indian University.
   - Regular full time candidates those who will qualify for the financial assistance will receive ₹ 25,000 during 1st and 2nd year of the Fellowship programme and ₹ 28,000 during 3rd, 4th and 5th year of the Fellowship programme as per MHRD guidelines.

6. **Ph.D. in IISc/ IITs**
   - IISc and IITs take admissions for Ph.D. on the basis of GATE score.
   - Earn a Ph.D. degree directly after Bachelor’s degree through integrated programme.
   - A fulltime residential researcher (RR) programme.

7. **Fellowship Program in management (FPM)**
   - Enrolment through GATE score card
   - Stipend of ₹ 22,000 – 30,000 per month + HRA
   - It is a fellowship program
   - Application form is generally available in month of sept. and oct.

Note : **In near future, hopefully GATE exam will become a mandatory exit test for all engineering students, so take this exam seriously. Best of LUCK !**

GATE Exam Pattern

Section	Question No.	No. of Questions	Marks Per Question	Total Marks
General Aptitude	1 to 5	5	1	5
	6 to 10	5	2	10
Technical + Engineering Mathematics	1 to 25	25	1	25
	26 to 55	30	2	60
Total Duration : 3 hours	**Total Questions : 65**		**Total Marks : 100**	
Note : 40 to 45 marks will be allotted to Numerical Answer Type Questions				

Pattern of Questions :

(i) **Multiple Choice Questions (MCQ)** carrying 1 or 2 marks each in all the papers and sections. These questions are objective in nature, and each will have a choice of four answers, out of which the candidate has to select (mark) the correct answer.

Negative Marking for Wrong Answers : For a wrong answer chosen in a MCQ, there will be negative marking. For **1-mark** MCQ, **1/3 mark** will be deducted for a wrong answer. Likewise for, **2-marks** MCQ, **2/3 mark** will be deducted for a wrong answer.

(ii) **Numerical Answer Type (NAT)** Questions carrying **1** or **2** marks each in all the papers and sections. For these questions, the answer is a signed real number, which needs to be entered by the candidate using the virtual numeric keypad on the monitor (Keyboard of the computer will be disabled). No choices will be shown for these type of questions. The answer can be a number such as **10** or **– 10** (an integer only). The answer may be in decimals as well, for example, **10.1** (one decimal) or **10.01** (two decimal) or **–10.001** (three decimal). These questions will be mentioned with, up to which decimal places, the candidates need to make an answer. Also, an appropriate range will be considered while evaluating the numerical answer type questions so that the candidate is not penalized due to the usual round-off errors. Wherever required and possible, it is better to give NAT answer up to a maximum of three decimal places.

Note : There is NO negative marking for a wrong answer in NAT questions.

What is special about this book ?

GATE ACADEMY Team took several years' to come up with the solutions of GATE examination. It is because we strongly believe in quality. We have significantly prepared each and every solution of the questions appeared in GATE, and many individuals from the community have taken time out to proof read and improve the quality of solutions, so that it becomes very lucid for the readers. Some of the key features of this book are as under :

☞ This book gives complete analysis of questions chapter wise as well as year wise.

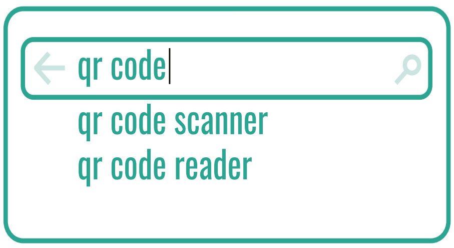
☞ Video Solution of important conceptual questions has been given in the form of QR code and by scanning QR code one can see the video solution of the given question.

☞ Solutions has been presented in lucid and understandable language for an average student.

☞ In addition to the GATE syllabus, the book includes the nomenclature of chapters according to text books for easy reference.

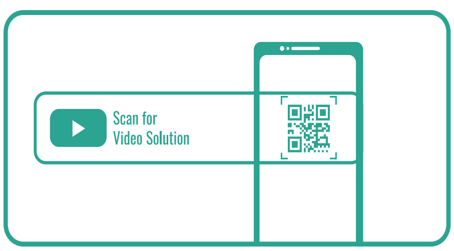
☞ Last but not the least, author's 10 years experience and devotion in preparation of these solutions.

☞ Steps to Open Video solution through mobile.



(1) Search for QR Code scanner in Google Play / App Store.

(2) Download & Install any QR Code Scanner App.



(3) Scan the given QR Code for particular question.

(4) Visit the link generated & you'll be redirect to the video solution.

Note : For recent updates regarding minor changes in this book, visit - www.gateacademy.co.in. We are always ready to appreciate and help you.

PREFACE

It is our pleasure, that we insist on presenting **"GATE 2021 Electrical Engineering (Volume - I)"** authored for Electrical Engineering to all of the aspirants and career seekers. The prime objective of this book is to respond to tremendous amount of ever growing demand for error free, flawless and succinct but conceptually empowered solutions to all the question over the period 1987 - 2020.

This book serves to the best supplement the texts for Electronics & Communication Engineering but shall be useful to a larger extent for **Electronics & Communication Engineering and Instrumentation Engineering** as well. Simultaneously having its salient feature the book comprises :

- Step by step solution to all questions.
- Complete analysis of questions chapter wise as well as year wise.
- Detailed explanation of all the questions.
- Solutions are presented in simple and easily understandable language.
- Video solutions for good questions.
- It covers all GATE questions from 1987 to 2020 (34 years).

The authors do not sense any deficit in believing that this title will in many aspects, be different from the similar titles within the search of student.

In particular, we wish to thank GATE ACADEMY expert team members for their hard work and consistency while designing the script.

The final manuscript has been prepared with utmost care. However, going a line that, there is always room for improvement in anything done, we would welcome and greatly appreciate suggestion and correction for further improvement.

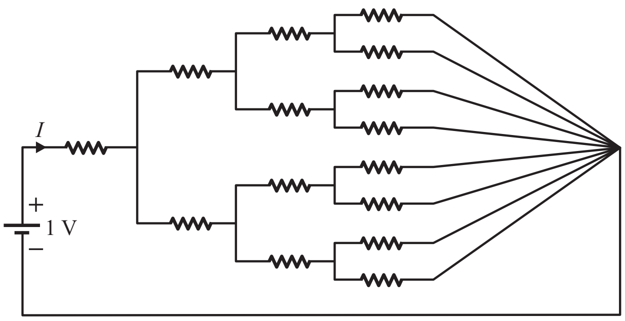
Sakshi Dhande
(Co-Director, GATE ACADEMY Learning Pvt. Ltd.)
Umesh Dhande
(Director, GATE ACADEMY Learning Pvt. Ltd.)

CONTENTS

1 Basic Concepts of Networks

1992 IIT Delhi

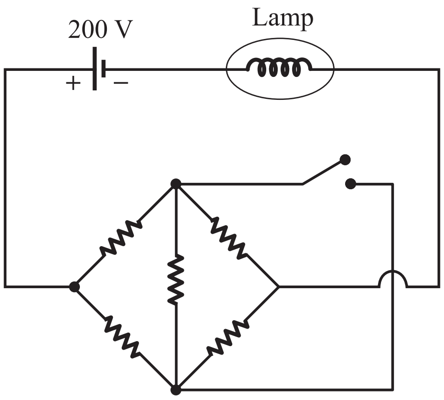
1.1 All the resistances in figure are 1 Ω each. The value of current '*I*' is



(A) $\frac{1}{15}$ A (B) $\frac{2}{15}$ A

(C) $\frac{4}{15}$ A (D) $\frac{8}{15}$ A

1.2 All resistances in the circuit in figure are of *R* ohms each. The switch is initially open. What happens to the lamp's intensity when the switch is closed?



(A) Increases

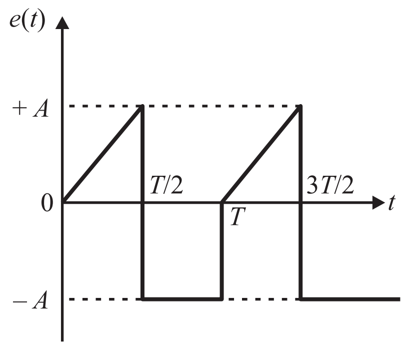
(B) Decreases

(C) Remains same

(D) Answer depends on the value of *R*

1995 IIT Kanpur

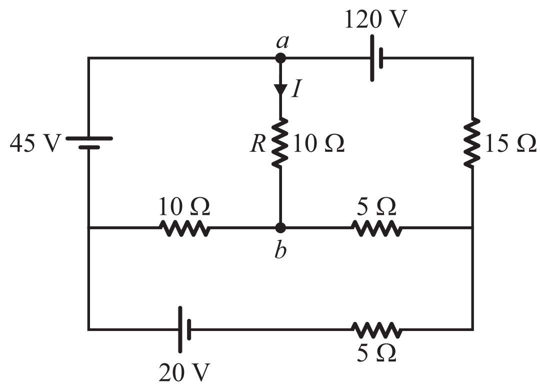
1.3 The rms value of the periodic waveform $e(t)$, shown in the figure is



(A) $\sqrt{\frac{3}{2}}A$ (B) $\sqrt{\frac{2}{3}}A$

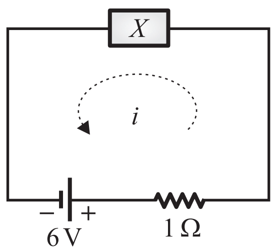
(C) $\sqrt{\frac{1}{3}}A$ (D) $\sqrt{2}A$

1.4 For the circuit shown in figure, the current through the resistance *R* connected between points *a* and *b* is ______ A.



1996 IISc Bangalore

1.5 In the circuit shown in figure, X is an element which always absorbs power. During a particular operation, it sets up a current of 1 amp in the direction shown and absorbs a power P_X. It is possible that X can absorb the same power P_X for another current i. Then the value of this current is



(A) $(3-\sqrt{14})$ amps

(B) $(3+\sqrt{14})$ amps

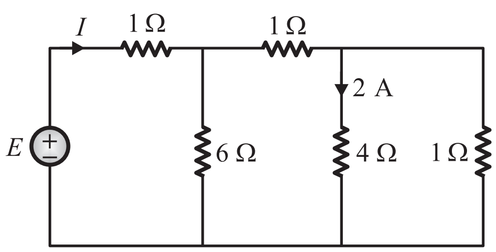
(C) 5 amps

(D) None of these

1997 IIT Madras

1.6 A 10 V battery with an internal resistance of 1 Ω is connected across a non-linear load whose V-I characteristic is given by $7I = V^2 + 2V$. The current delivered by the battery is _______ A.

1.7 The value of E and I for the circuit shown in figure, are _______ V and _______ A.



1.8 A practical current source is usually represented by

(A) a resistance in series with an ideal current source.

(B) a resistance in parallel with an ideal current source.

(C) a resistance in parallel with an ideal voltage source.

(D) none of these

1.9 Energy stored in capacitor over a cycle, when excited by an a.c. source is

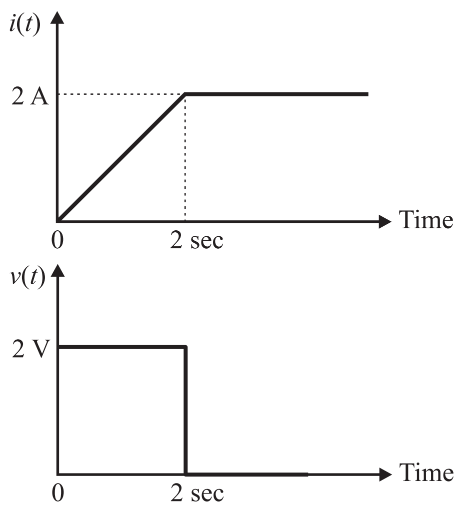
(A) the same as that due to a d.c. source of equivalent magnitude.

(B) half of that due to a d.c. source of equivalent magnitude.

(C) zero.

(D) none of the above

1.10 The voltage and current waveforms for an element are shown in figure. The circuit element is _______ and its value is _______.



1999 IIT Bombay

1.11 The RMS value of a half wave rectified symmetrical square wave current of 2 A is

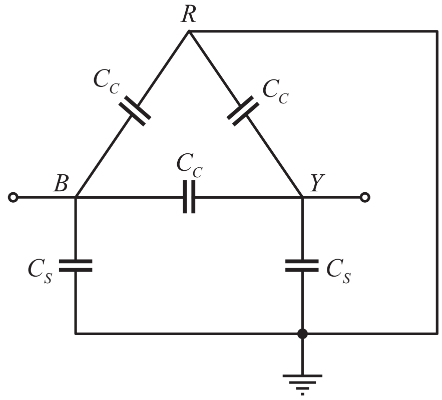
(A) $\sqrt{2}$ A

(B) 1 A

(C) $\frac{1}{\sqrt{2}}$ A

(D) $\sqrt{3}$ A

1.12 For the circuit shown in figure, the capacitance measured between terminals B and Y will be



(A) $C_C + (C_S/2)$　　(B) $C_S + (C_C/2)$

(C) $(C_S + 3C_C)/2$　　(D) $3C_C + 2C_S$

1.13 When a resistance R is connected to a current source, it consumes a power of 18 W. When the same R is connected to a voltage source having the same magnitude as the current source, the power absorbed by R is 4.5 W. The magnitude of the current source and the value of R are

(A) $\sqrt{18}$ A and 1 Ω

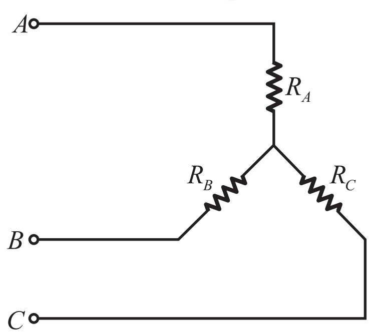
(B) 3 A and 2 Ω

(C) 1 A and 18 Ω

(D) 6 A and 0.5 Ω

2001 IIT Kanpur

1.14 Consider the star network shown in figure. The resistance between terminals A and B with terminal C open is 6 Ω, between terminals B and C with terminal A open is 11 Ω and between terminals C and A with terminal B open is 9 Ω. Then



(A) $R_A = 4\Omega,\ R_B = 2\Omega,\ R_C = 5\Omega$

(B) $R_A = 2\Omega,\ R_B = 4\Omega,\ R_C = 7\Omega$

(C) $R_A = 3\Omega,\ R_B = 3\Omega,\ R_C = 4\Omega$

(D) $R_A = 5\Omega,\ R_B = 1\Omega,\ R_C = 10\Omega$

1.15 Two incandescent light bulbs of 40 W and 60 W ratings are connected in series across the mains. Then

(A) the bulbs together consume 100 W.

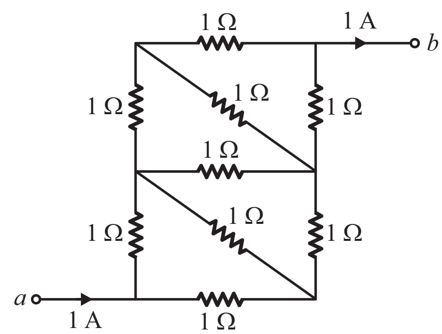
(B) the bulbs together consume 50 W.

(C) the 60 W bulb glows brighter.

(D) the 40 W bulb glows brighter.

2002 IISc Bangalore

1.16 In the resistor network shown in figure, all resistors value 1 Ω. A current of 1 A passes from terminal a to terminal b as shown in figure, voltage between terminal a and b is approximately.

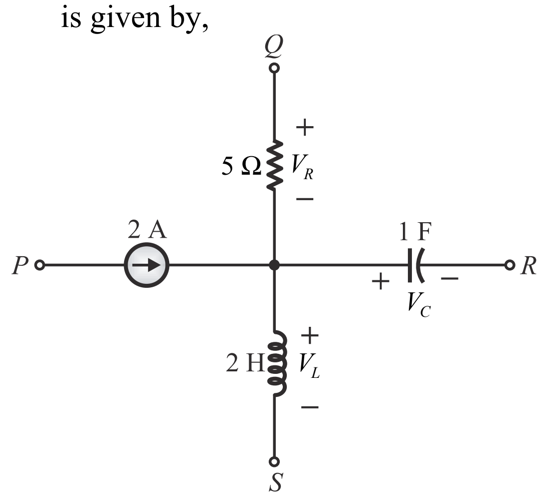


(A) 1.4 V　　(B) 1.5 V

(C) 0 V　　(D) 3 V

2003 IIT Madras

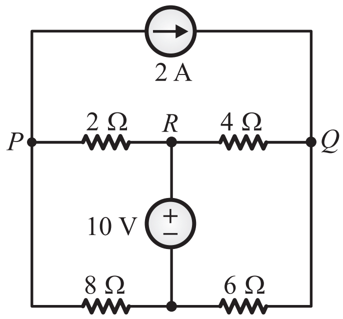
1.17 A segment of a circuit is shown in figure $V_R = 5$ V, $V_C = 4\sin 2t$ V. The voltage V_L is given by,



(A) $3 - 8\cos 2t$ V (B) $32 \sin 2t$ V

(C) $16 \sin 2t$ V (D) $16 \cos 2t$ V

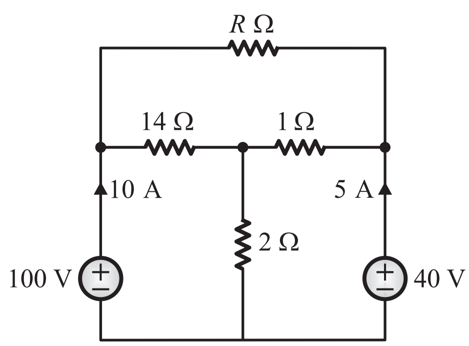
1.18 In figure, the potential difference between points P and Q is



(A) + 12 V (B) 10 V

(C) – 6 V (D) 8 V

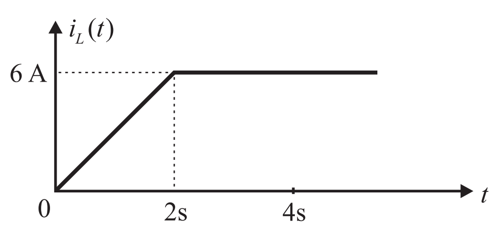
1.19 In figure, the value of R is



(A) 10 Ω (B) 18 Ω

(C) 24 Ω (D) 12 Ω

1.20 Figure shows the waveform of the current passing through an inductor of resistance 1 Ω and inductance 2 H. The energy absorbed by the inductor in the first four seconds is

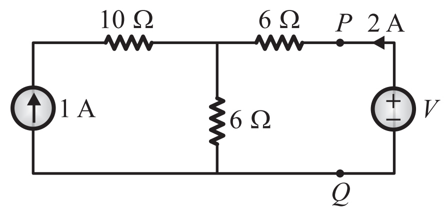


(A) 144 J (B) 98 J

(C) 132 J (D) 168 J

2004 IIT Delhi

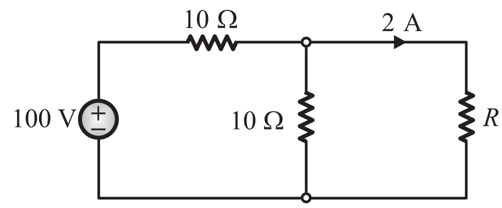
1.21 In the figure the value of the source voltage V is



(A) 12 V (B) 24 V

(C) 30 V (D) 44 V

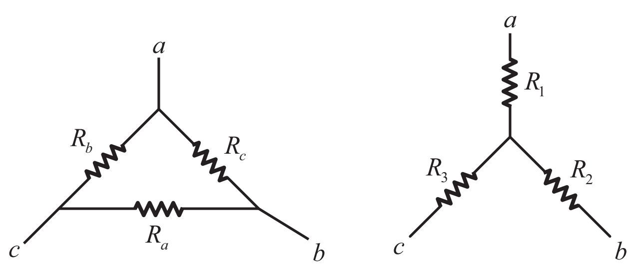
1.22 In figure the value of resistance R in Ω is



(A) 10 (B) 20

(C) 30 (D) 40

1.23 In figure R_a, R_b and R_c are 20 Ω, 10 Ω and 10 Ω respectively. The resistances R_1, R_2 and R_3 in Ω of an equivalent star-connection are



(A) 2.5, 5, 5 (B) 5, 2.5, 5

(C) 5, 5, 2.5 (D) 2.5, 5, 2.5

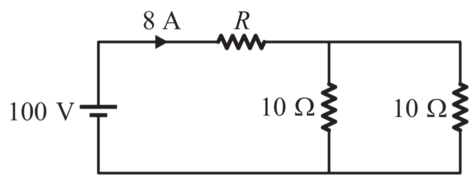
1.24 The rms value of the current in a wire which carries a d.c. current of 10 A and a sinusoidal alternating current of peak value 20 A is

(A) 10 A (B) 14.14 A

(C) 15 A (D) 17.32 A

2005 IIT Bombay

1.25 In the figure given below, the value of R is



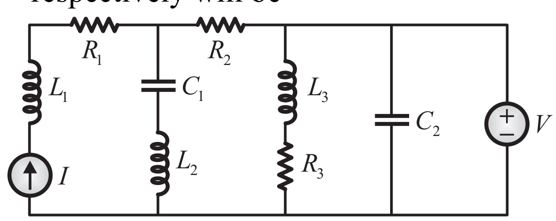
(A) 2.5 Ω (B) 5.0 Ω
(C) 7.5 Ω (D) 10.0 Ω

1.26 The RMS value of the voltage $u(t) = 3 + 4\cos(3t)$ is

(A) $\sqrt{17}$ V (B) 5 V
(C) 7 V (D) $(3 + 2\sqrt{2})$ V

2006 IIT Kharagpur

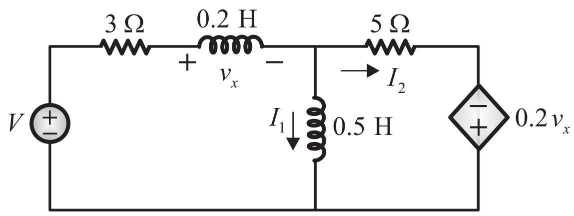
1.27 In the circuit shown in the figure, the current source $I = 1$ A, the voltage source $V = 5$ V, $R_1 = R_2 = R_3 = 1\,\Omega$, $L_1 = L_2 = L_3 = 1$ H, $C_1 = C_2 = 1$ F. The currents (in A) through R_3 and the voltage source V respectively will be



(A) 1, 4 (B) 5, 1
(C) 5, 2 (D) 5, 4

2007 IIT Kanpur

1.28 The state equation for the current I_1 in the network shown below in terms of the voltage v_x and the independent source V, is given by



(A) $\frac{dI_1}{dt} = -1.4v_x - 3.75I_1 + \frac{5}{4}V$

(B) $\frac{dI_1}{dt} = -1.4v_x - 3.75I_1 - \frac{5}{4}V$

(C) $\frac{dI_1}{dt} = -1.4v_x + 3.75I_1 + \frac{5}{4}V$

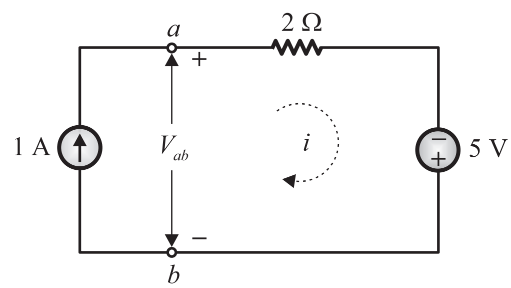
(D) $\frac{dI_1}{dt} = -1.4v_x + 3.75I_1 - \frac{5}{4}V$

1.29 A 3 V dc supply with an internal resistance of 2 Ω supplies a passive non-linear resistance characterized by the relation $V_{NL} = I_{NL}^2$. The power dissipated in the non-linear resistance is

(A) 1.0 W (B) 1.5 W
(C) 2.5 W (D) 3.0 W

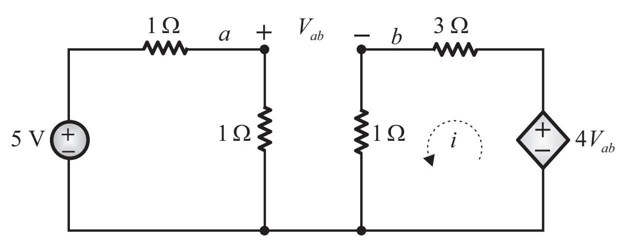
2008 IISc Bangalore

1.30 Assuming ideal elements in the circuit shown below, the voltage V_{ab} will be



(A) – 3 V (B) 0 V
(C) 3 V (D) 5 V

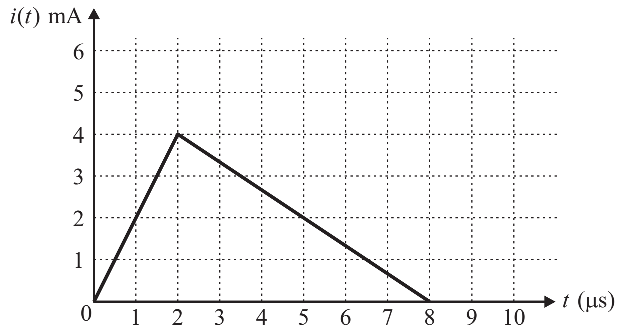
1.31 In the circuit shown in the figure, the value of the current i will be given by



(A) 0.31 A (B) 1.25 A
(C) 1.75 A (D) 2.5 A

Statement for Linked Answer Questions 1.32 & 1.33

The current $i(t)$ sketched in the figure flows through an initially uncharged 0.3 nF capacitor.



1.32 The charge stored in the capacitor upto $t = 5\ \mu s$, will be

(A) 8 nC (B) 10 nC

(C) 13 nC (D) 16 nC

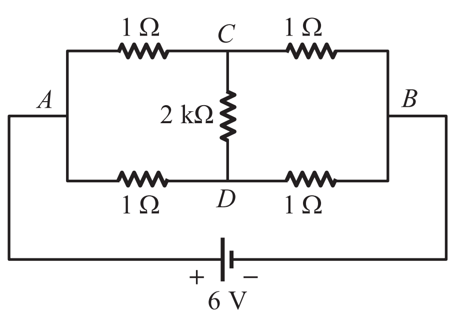
1.33 The capacitor charged upto $5\ \mu s$, as per the current profile given in the figure, is connected across an inductor of 0.6 mH. Then the value of voltage across the capacitor after $1\ \mu s$ will approximately be

(A) 18.8 V (B) 23.5 V

(C) – 23.5 V (D) – 30.6 V

2009 IIT Roorkee

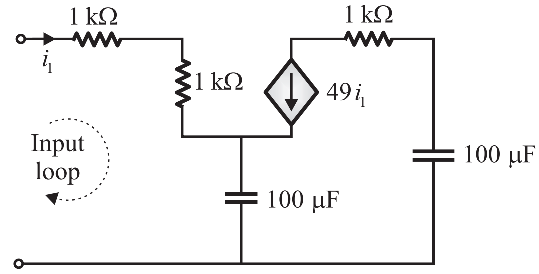
1.34 The current through the $2\ k\Omega$ resistance in the circuit shown is



(A) 0 mA (B) 1 mA

(C) 2 mA (D) 6 mA

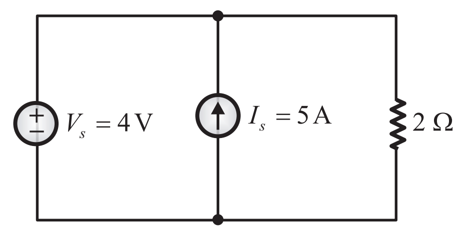
1.35 The equivalent capacitance of the input loop of the circuit shown is



(A) 2 μF (B) 100 μF

(C) 200 μF (D) 4 μF

1.36 For the circuit shown, find out the current flowing through the $2\ \Omega$ resistance. Also identify the changes to be made to double the current through the $2\ \Omega$ resistance.



(A) (5 A; Put $V_s = 20$ V)

(B) (2 A; Put $V_s = 8$ V)

(C) (5 A; Put $I_s = 10$ A)

(D) (7 A; Put $I_s = 12$ A)

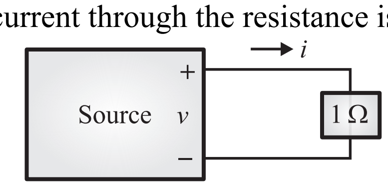
1.37 How many 200 W / 220 V incandescent lamps connected in series would consume the same total power as a single 100 W / 220 V incandescent lamp?

(A) not possible (B) 4

(C) 3 (D) 2

2010 IIT Guwahati

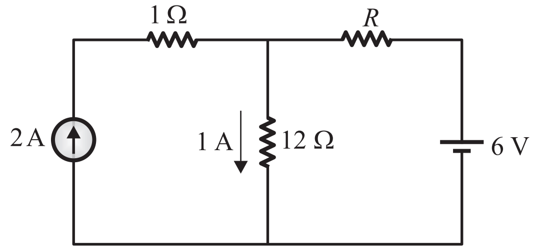
1.38 As shown in the below figure, a $1\ \Omega$ resistance is connected across a source that has a load line $v + i = 100$. The current through the resistance is



(A) 25 A (B) 50 A

(C) 100 A (D) 200 A

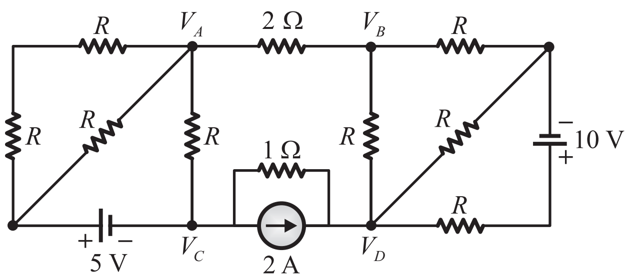
1.39 If the 12 Ω resistor draws a current of 1 A as shown in the figure, the value of resistance R is



(A) 4 Ω (B) 6 Ω
(C) 8 Ω (D) 18 Ω

2012 IIT Delhi

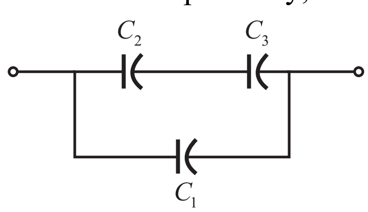
1.40 If $V_A - V_B = 6$ V, then $V_C - V_D$ is



(A) – 5 V (B) 2 V
(C) 3 V (D) 6 V

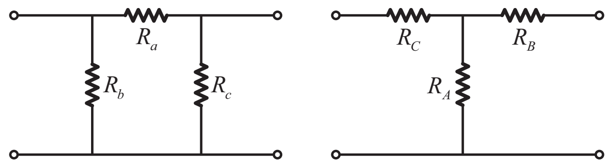
2013 IIT Bombay

1.41 Three capacitors C_1, C_2 and C_3 whose values are 10 μF, 5 μF and 2 μF respectively, have breakdown voltages of 10 V, 5 V and 2 V respectively. For the interconnection shown below, the maximum safe voltage in volts that can be applied across the combination, and the corresponding total charge in μC stored in the effective capacitance across the terminals are respectively,



(A) 2.8 and 36 (B) 7 and 119
(C) 2.8 and 32 (D) 7 and 80

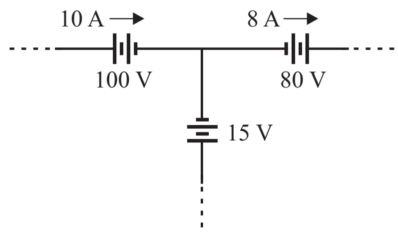
1.42 Consider a delta connection of resistors and its equivalent star connection is shown below. If all elements of the delta connection are scaled by a factor k, $k > 0$, the elements of the corresponding star equivalent will be scaled by a factor of



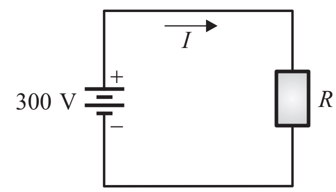
(A) k^2 (B) k
(C) $1/k$ (D) $\sqrt{k}$

2014 IIT Kharagpur

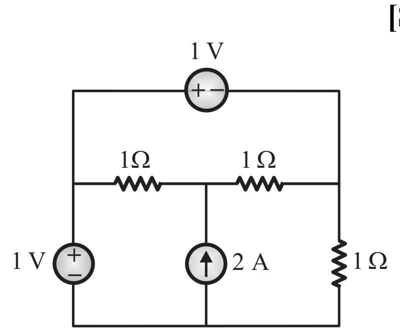
1.43 The three circuit elements shown in the figure are part of an electric circuit. The total power absorbed by the three circuit elements in watts is _______. **[Set - 01]**



1.44 In the figure, the value of resistor R is $\left(25+\frac{I}{2}\right)$ ohms, where I is the current in amperes. The current I is _______.
[Set - 01]

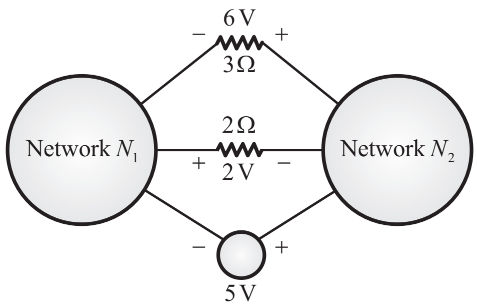


1.45 The power delivered by the current source, in the figure, is ________ W.
[Set - 03]



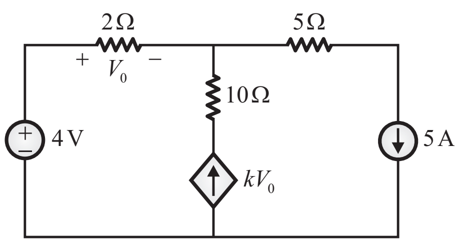
2015 IIT Kanpur

1.46 The voltages developed across the 3 Ω and 2 Ω resistors shown in the figure are 6 V and 2 V respectively, with the polarity as marked. What is the power (in Watt) delivered by the 5 V voltage source? **[Set - 01]**

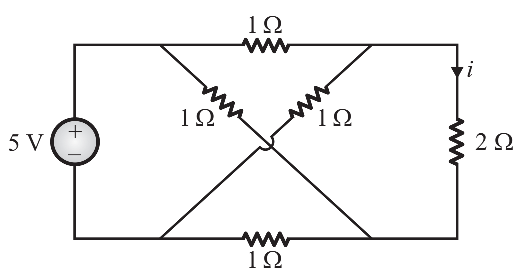


(A) 5 (B) 7
(C) 10 (D) 14

1.47 In the given circuit, the parameter k is positive, and the power dissipated in the 2 Ω resistor is 12.5 W. The value of k is _____. **[Set - 01]**



1.48 The current i (in Ampere) in the 2 Ω resistor of the given network is _____. **[Set - 02]**



2016 IISc Bangalore

1.49 R_A and R_B are the input resistances of circuits as shown below. The circuits extend infinitely in the direction shown. Which one of the following statements is TRUE? **[Set - 01]**

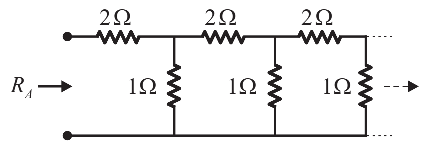


Fig. (i)

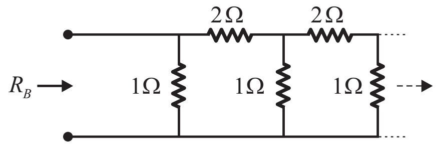
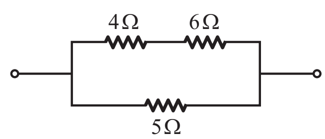


Fig. (ii)

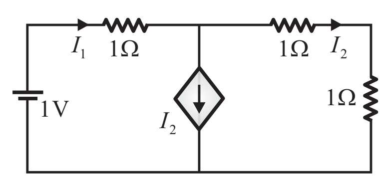
(A) $R_A = R_B$ (B) $R_A = R_B = 0$

(C) $R_A < R_B$ (D) $R_B = \dfrac{R_A}{(1+R_A)}$

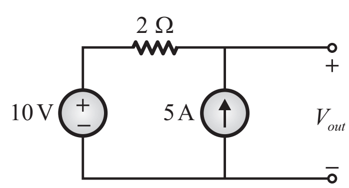
1.50 In the portion of a circuit shown, if the heat generated in 5 Ω resistance is 10 calories per second, then heat generated by the 4 Ω resistance, in calories per second, is _______. **[Set - 01]**



1.51 In the given circuit, the current supplied by the battery, in ampere, is _______. **[Set - 01]**

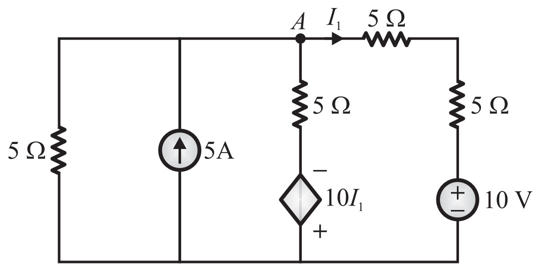


1.52 In the circuit shown below, the voltage and current sources are ideal. The voltage (V_{out}) across the current source, in volts, is **[Set - 02]**



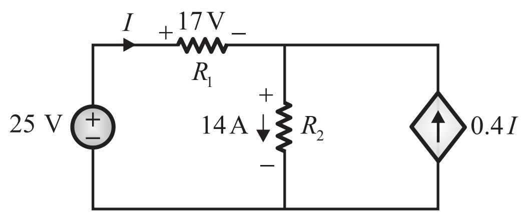
(A) 0 (B) 5
(C) 10 (D) 20

1.53 In the circuit shown below, the node voltage V_A is ________V.

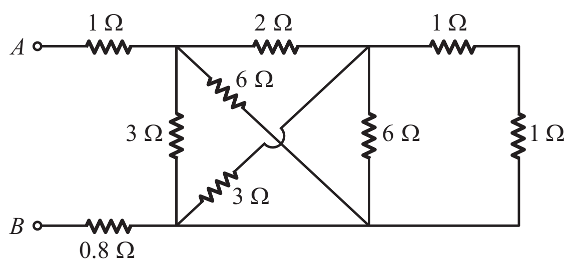


2017 IIT Roorkee

1.54 The power supplied by the 25 V source in the figure shown below is ________W.
[Set - 01]

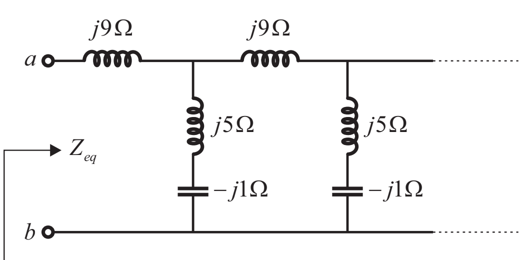


1.55 The equivalent resistance between the terminals A and B is _______Ω. **[Set - 01]**



2018 IIT Guwahati

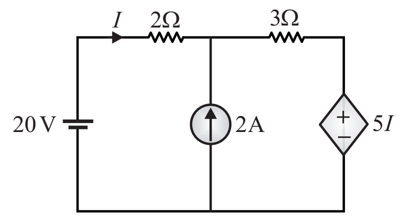
1.56 The equivalent impedance Z_{eq} for the infinite ladder circuit shown in the figure is



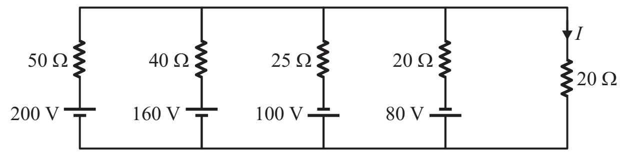
(A) $j12\ \Omega$ (B) $-j12\ \Omega$
(C) $j13\ \Omega$ (D) $13\ \Omega$

2019 IIT Madras

1.57 The current I flowing in the circuit shown below in amperes (round off to one decimal place) is ________.

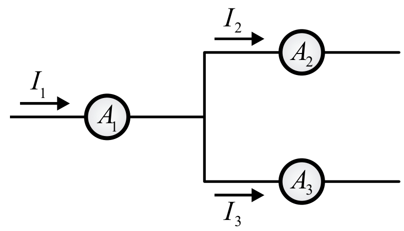


1.58 The current I flowing in the circuit shown below in amperes is ________.



2020 IIT Delhi

1.59 Currents through ammeters A_2 and A_3 in the figure are $1\angle 10^0$ and $1\angle 70^0$ respectively. The reading of the ammeter A_1 (rounded off to 3 decimal places) is _____A.



1.60 A benchtop dc power supply acts as an ideal 4 A current source as long as its terminal voltage is below 10 V. Beyond this point, it begins to behave as an ideal 10 V voltage source for all load currents going down to 0 A, When connected to an ideal rheostat, find the load resistance value at which maximum power is transferred and the corresponding load voltage and current.
(A) 2.5 Ω, 4 A, 10 V
(B) Short, ∞ A, 10 V
(C) 2.5 Ω, 4 A, 5 V
(D) Open, 4 A, 0 V

❖❖❖❖

Answers Basic Concepts of Networks

1.1	D	1.2	C	1.3	B	1.4	4.29	1.5	C
1.6	5	1.7	31, 13	1.8	B	1.9	C	1.10	Inductor, 2
1.11	A	1.12	C	1.13	B	1.14	B	1.15	D
1.16	A	1.17	B	1.18	C	1.19	D	1.20	C
1.21	C	1.22	B	1.23	A	1.24	D	1.25	C
1.26	A	1.27	D	1.28	A	1.29	A	1.30	A
1.31	B	1.32	C	1.33	D	1.34	A	1.35	A
1.36	B	1.37	D	1.38	B	1.39	B	1.40	A
1.41	C	1.42	B	1.43	330	1.44	10	1.45	3
1.46	A	1.47	0.5	1.48	0	1.49	D	1.50	2
1.51	0.5	1.52	D	1.53	11.428	1.54	250	1.55	3
1.56	A	1.57	1.4	1.58	0	1.59	1.732	1.60	A

Explanations Basic Concepts of Networks

Concepts of Absorbed and Delivered Power

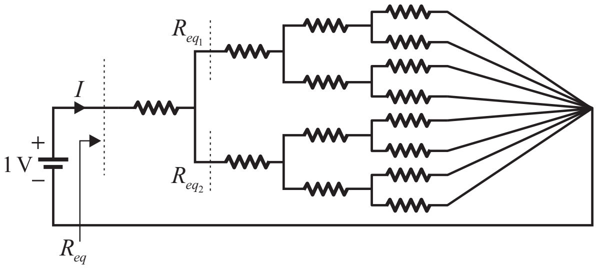
Scan for Video Explanation

Concepts of Voltmeter and Ammeter

Scan for Video Explanation

1.1 (D)

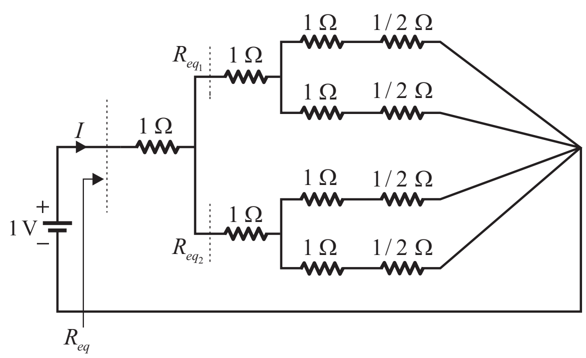
Given circuit is shown below,



Given : All resistances are of 1 Ω.

$$R_{eq_1} = \left\{\left[\left(\frac{1}{2}+1\right) \| \left(\frac{1}{2}+1\right)\right]+1\right\} = \frac{7}{4}\ \Omega$$

$$R_{eq_2} = \left\{\left[\left(\frac{1}{2}+1\right) \| \left(\frac{1}{2}+1\right)\right]+1\right\} = \frac{7}{4}\ \Omega$$



Equivalent resistance is,

$$R_{eq} = \left[R_{eq_1} \| R_{eq_2}\right]+1 = \left[\frac{7}{4} \| \frac{7}{4}\right]+1$$

$$R_{eq} = \frac{15}{8}\ \Omega$$

From above circuit :

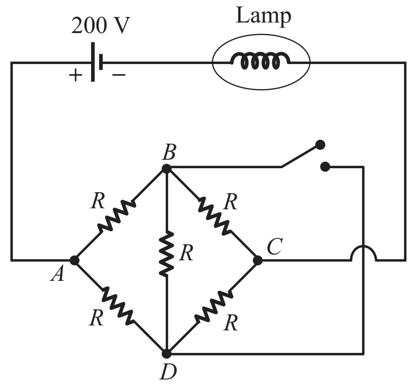
Current is given by,

$$I = \frac{V}{R_{eq}} = \frac{1}{15/8} = \frac{8}{15}\,\text{A}$$

Hence, the correct option is (D).

1.2　(C)

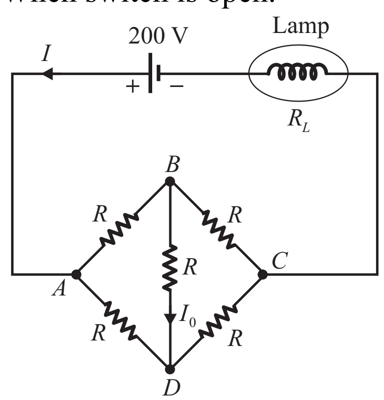
Given circuit is shown below,



Given : $R_{AB} = R_{BC} = R_{AD} = R_{DC} = R_{BD} = R$

Let us assume the resistance of lamp is R_L.

Case 1 : When switch is open.

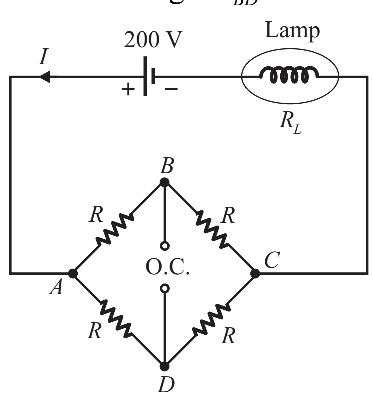


From figure,

$$\frac{R_{BC}}{R_{AB}} = \frac{R}{R}, \quad \frac{R_{DC}}{R_{AD}} = \frac{R}{R}$$

Thus, $\dfrac{R_{BC}}{R_{AB}} = \dfrac{R_{DC}}{R_{AD}}$

Hence, the Wheat-stone bridge shown in the above network is balanced and therefore no current will flow through R_{BD}. Hence $I_0 = 0$.

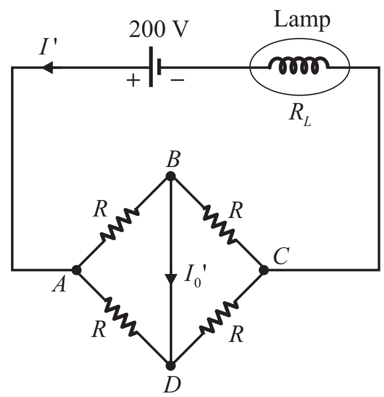


From the above circuit,

$$I = \frac{200}{[(R_{AB} + R_{BC}) \| (R_{AD} + R_{DC})] + R_L}$$

$$I = \frac{200}{[2R \| 2R] + R_L} = \frac{200}{R + R_L}$$

Case 2 : When switch is closed. *BD* terminal will be short circuited.

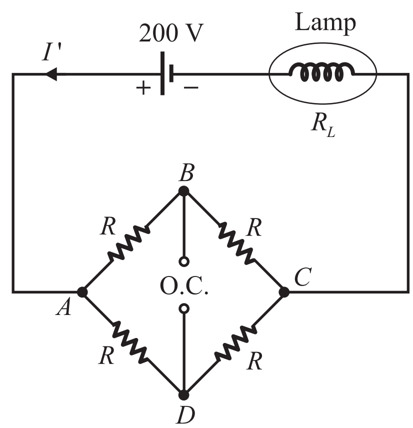


From the above circuit,

$$\frac{R_{BC}}{R_{AB}} = \frac{R}{R}, \quad \frac{R_{DC}}{R_{AD}} = \frac{R}{R}$$

Thus, $\dfrac{R_{BC}}{R_{AB}} = \dfrac{R_{DC}}{R_{AD}}$

Hence, the Wheat-stone bridge shown in the above network is balanced and therefore $I_0' = 0$.



From the above circuit,

$$I' = \frac{200}{[(R_{AB} + R_{BC}) \| (R_{AD} + R_{DC})] + R_L}$$

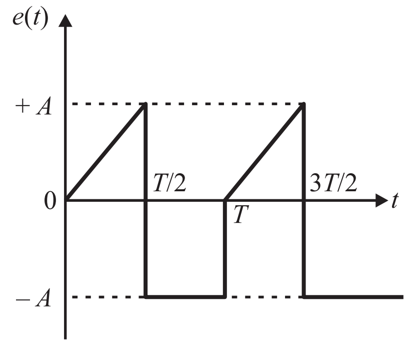
$$I' = \frac{200}{[2R \| 2R] + R_L} = \frac{200}{R + R_L}$$

Since, in both cases, current flowing through lamp is same i.e. $I = I'$, hence in both cases intensity of lamp will remain same.

Hence, the correct option is (C).

1.3 (B)

Given waveform is shown below,



For the given signal,

$$e(t) = \begin{cases} \dfrac{2A}{T}t \;; & 0 < t < \dfrac{T}{2} \\ -A \;; & \dfrac{T}{2} < t < T \end{cases}$$

RMS value of periodic waveform is given by,

$$e(t)_{rms} = \sqrt{\frac{1}{T}\int_0^T [e(t)]^2 dt}$$

$$e(t)_{rms} = \sqrt{\frac{1}{T}\left[\int_0^{T/2}\left(\frac{2A}{T}t\right)^2 dt + \int_{T/2}^{T}(-A)^2 dt\right]}$$

$$e(t)_{rms} = \sqrt{\frac{1}{T}\left\{\frac{4A^2}{T^2}\left[\frac{t^3}{3}\right]_0^{T/2} + \left[A^2 t\right]_{T/2}^{T}\right\}}$$

$$e(t)_{rms} = \sqrt{\frac{1}{T}\left[\frac{4A^2}{T^2}\times\frac{1}{3}\left(\frac{T}{2}\right)^3 + A^2\left(T-\frac{T}{2}\right)\right]}$$

$$e(t)_{rms} = \sqrt{\frac{1}{T}\left[\frac{4A^2}{T^2}\times\frac{1}{3}\times\frac{T^3}{8} + A^2\times\frac{T}{2}\right]}$$

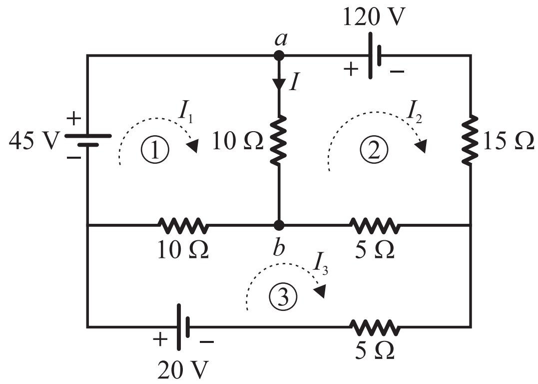
$$e(t)_{rms} = \sqrt{\frac{1}{T}\left[\frac{A^2T}{6} + \frac{A^2T}{2}\right]}$$

$$e(t)_{rms} = A\sqrt{\frac{1}{T}\left[\frac{T}{6} + \frac{T}{2}\right]} = \sqrt{\frac{2}{3}}A$$

Hence, the correct option is (B).

1.4 4.29

Given circuit is shown below,



Applying KVL in loop (1),

$$-45 + 10(I_1 - I_2) + 10(I_1 - I_3) = 0$$

$$20I_1 - 10I_2 - 10I_3 = 45 \qquad \ldots\text{(i)}$$

Applying KVL in loop (2),

$$120 + 15I_2 + 5(I_2 - I_3) + 10(I_2 - I_1) = 0$$

$$-10I_1 + 30I_2 - 5I_3 = -120 \qquad \ldots\text{(ii)}$$

Applying KVL in loop (3),

$$10(I_3 - I_1) + 5(I_3 - I_2) + 5I_3 - 20 = 0$$

$$-10I_1 - 5I_2 + 20I_3 = 20 \qquad \ldots\text{(iii)}$$

From equation (i),

$$20I_1 - 10I_2 - 45 = 10I_3$$

$$I_3 = 2I_1 - I_2 - 4.5 \qquad \ldots\text{(iv)}$$

Put the value of I_3 in equation (ii),

$$-10I_1 + 30I_2 - 5(2I_1 - I_2 - 4.5) = -120$$

$$-20I_1 + 35I_2 + 22.5 = -120$$

$$-20I_1 + 35I_2 = -142.5 \qquad \ldots\text{(v)}$$

Put the value of I_3 in equation (iii),

$$-10I_1 - 5I_2 + 20(2I_1 - I_2 - 4.5) = 20$$

$$30I_1 - 25I_2 = 20 + 90$$

$$30I_1 - 25I_2 = 110 \qquad \ldots\text{(vi)}$$

Solving equations (v) and (vi),

$$I_1 = 0.5227\text{ A}$$

$$I_2 = -3.772\text{ A}$$

Current through resistance R is,

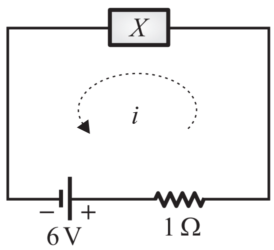
$$I = I_1 - I_2$$

$$I = 0.5227 - (-3.772) = 4.294\text{ A}$$

Hence, current through the resistance R is **4.29 A.**

1.5 (C)

Given circuit is shown below,

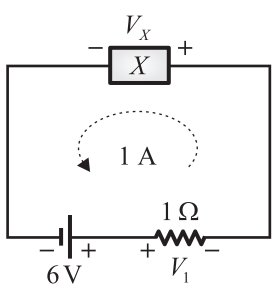


According to Tellegen's theorem, sum of product of voltage and current across each element in any network will be always zero.

i.e. $$\sum_{k=1}^{N} V_k i_k = 0$$

Now, to solve this question let us consider two different cases as given below.

Case 1 : When current flowing in the circuit is 1 amp.



Applying Tellegen's theorem,

$$\sum_{k=1}^{3} V_k i_k = 0$$

$$1\times(-6) + 1\times V_1 + 1\times V_X = 0 \qquad \ldots\text{(i)}$$

where, $V_1 = 1\times R = 1\times 1 = 1\text{V}$

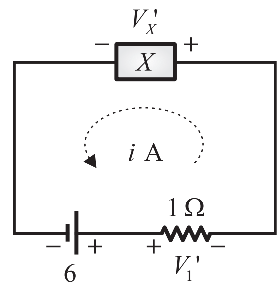
And $$V_X = \frac{P_X}{1} = P_X$$

From equation (i),

$$-6 + 1 + P_X = 0$$

$$P_X = 6 - 1 = 5\ \text{ W}$$

Case 2 : When current flowing in the circuit is i amp.



Applying Tellegen's theorem,

$$\sum_{k=1}^{3} V_k i_k = 0$$

$$i\times(-6) + i\times V_1' + i\times V_X' = 0 \qquad \ldots\text{(ii)}$$

where, $V_1' = i\times R = i\times 1 = i$ Volt

and $$V_X' = \frac{P_X'}{i}$$

Since, power P_X' remains same as in case 1 i.e. $P_X' = P_X$.

Hence, $$V_X' = \frac{P_X}{i} = \frac{5}{i}$$

From equation (ii),

$$-6i + i\times i + i\times\frac{5}{i} = 0$$

$$i^2 - 6i + 5 = 0$$

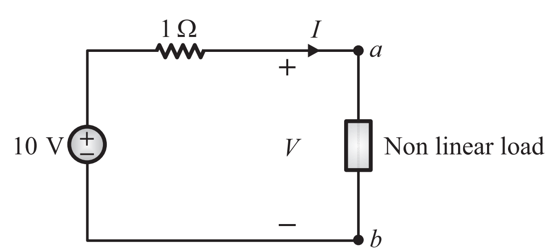
$$i = 5, 1$$

Therefore, $i = 5$ A, for which 'X' absorbs same power P_X i.e., 5 W.

Hence, the correct option is (C).

1.6 5

Given : $7I = V^2 + 2V \qquad \ldots\text{(i)}$



Applying KVL in the above circuit,

$$-10 + I \times 1 + V = 0$$

$$V + I = 10, \quad I = (10 - V) \quad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$V^2 + 2V - 7(10 - V) = 0$$

$$V^2 + 2V - 70 + 7V = 0$$

$$V^2 + 9V - 70 = 0$$

$$V = 5, -14$$

For $V = 5$ V,

$$I = \frac{V^2 + 2V}{7} = \frac{35}{7} = 5\text{ A}$$

For $V = -14$ V,

$$I = \frac{V^2 + 2V}{7} = \frac{168}{7} = 24\text{ A}$$

Note : In this question battery delivers the power while 1 Ω resistor and non-linear load will absorb the power. Hence, current must enter at positive terminal at load terminal *a*. Hence, *V* and *I* must be positive.

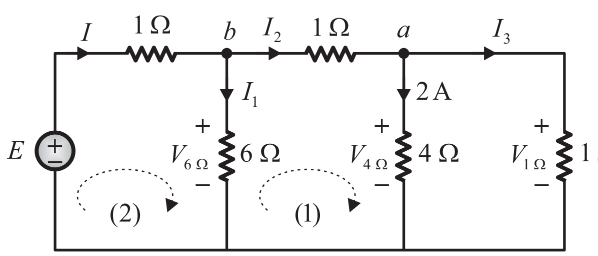
Therefore, for $V = 5$ V, $I = 5$ A.

$$P_{load} = VI = 5 \times 5 = 25\text{ W}$$

Hence, the current delivered by the battery will be **5 A** only.

1.7 **31, 13**

Given circuit is shown below,



Voltage across 4 Ω resistor is given by,

$$V_{4\,\Omega} = V_{1\,\Omega} = 4 \times 2 = 8\text{ V}$$

Current through 1 Ω resistor is given by,

$$I_3 = \frac{8}{1} = 8\text{ A}$$

Applying KCL at node *a*,

$$I_2 = I_3 + 2 = 8 + 2 = 10\text{ A}$$

Applying KVL in loop (1),

$$V_{6\,\Omega} = 8 + 10 \times 1 = 18\text{ V}$$

Current through 6 Ω resistor is given by,

$$I_1 = \frac{18}{6} = 3\text{ A}$$

Applying KCL at node *b*,

$$I = I_1 + I_2 = 10 + 3 = 13\text{ A}$$

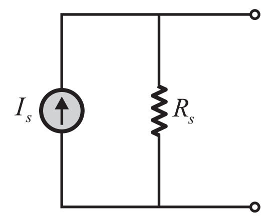
Applying KVL in loop (2),

$$E = 18 + 13 \times 1 = 18 + 13 = 31\text{ V}$$

Hence, the value of *E* and *I* are **31 V** and **13 A.**

1.8 **(B)**

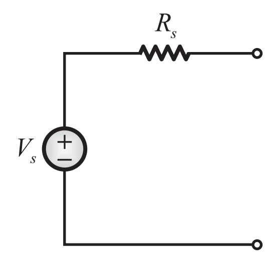
Practical current source is usually represented by a resistance in parallel with an ideal current source as shown below,



Hence, the correct option is (B).

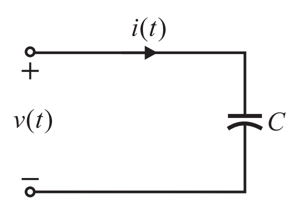
Key Point

A practical voltage source is represented by a resistor in series with an ideal voltage source.



1.9 **(C)**

Let, $v(t) = V_m \sin \omega t$



Current through capacitor is given by,

$$i(t) = C\frac{dv(t)}{dt} = C\frac{d(V_m \sin \omega t)}{dt}$$

$$i(t) = V_m C \cos \omega t \times \omega = V_m \omega C \cos \omega t$$

Instantaneous power is given by,

$$p(t) = v(t) i(t)$$

$$p(t) = V_m \sin \omega t \times V_m \omega C \cos \omega t$$

$$p(t) = V_m^2 \omega C \sin \omega t \cos \omega t$$

$$p(t) = \frac{V_m^2 \omega C}{2} \sin 2\omega t$$

Energy in the capacitor over one cycle is given by,

$$E = \int_0^{\frac{2\pi}{\omega}} p(t)\, dt = \frac{V_m^2 \omega C}{2} \int_0^{\frac{2\pi}{\omega}} \sin(2\omega t)\, dt$$

$$E = -\frac{V_m^2 \omega C}{2}\left[\frac{\cos 2\omega t}{2\omega}\right]_0^{\frac{2\pi}{\omega}}$$

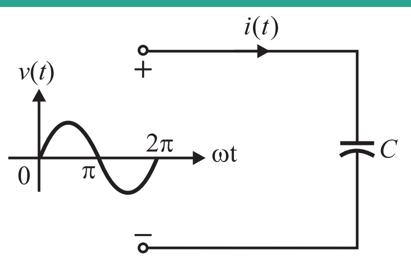
$$E = -\frac{V_m^2 \omega C}{2} \times \frac{1}{2\omega}\left[\cos\left(2\omega \times \frac{2\pi}{\omega}\right) - \cos 0\right]$$

$$E = -\frac{V_m^2 C}{4}[1-1] = 0$$

Therefore, energy stored in capacitor over a cycle, when excited by an a.c. source is zero.

Hence, the correct option is (C).

Key Point



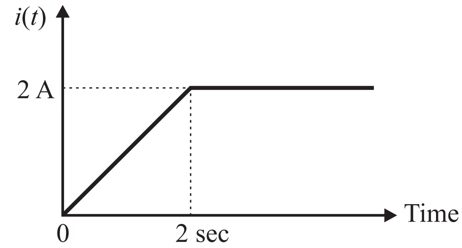
For positive half cycle, capacitor stores energy.

For negative half cycle, capacitor delivers that stored energy to the source.

Therefore, energy stored in capacitor over a cycle when excited by an ac source is zero.

1.10 Inductor, 2

Given waveform is shown below,



$$i(t) = \begin{cases} t; & 0 < t < 2 \\ 2; & t > 2 \end{cases}$$

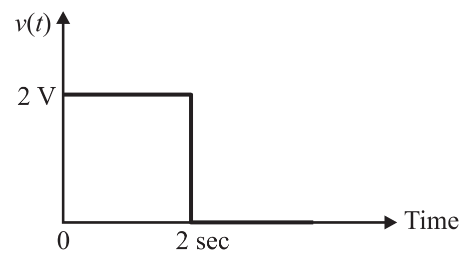
Voltage across inductor is given by,

$$V_L(t) = L\frac{d}{dt}i(t)$$

$$\frac{d}{dt}i(t) = \begin{cases} 1\,; & 0 < t < 2 \\ 0\,; & t > 2 \end{cases}$$

$$V_L(t) = 2\frac{d}{dt}i(t) = \begin{cases} 2; & 0 \le t \le 2 \\ 0; & t \ge 2 \end{cases}$$

Since, this is the given $v(t)$ as shown below,



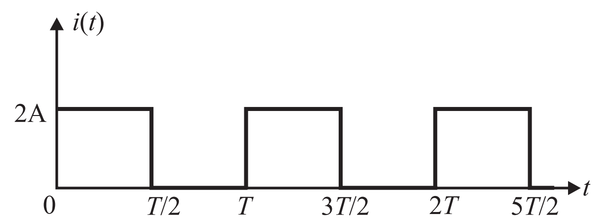
Therefore, on comparing

$$V_L(t) = v(t) = 2\frac{d}{dt}i(t) \Rightarrow L = 2\text{H}$$

Hence, the circuit element is **inductor** and its value is **2 H.**

1.11 (A)

A half wave rectified symmetrical square wave current is shown below,



RMS value of periodic waveform is given by,

$$i(t)_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

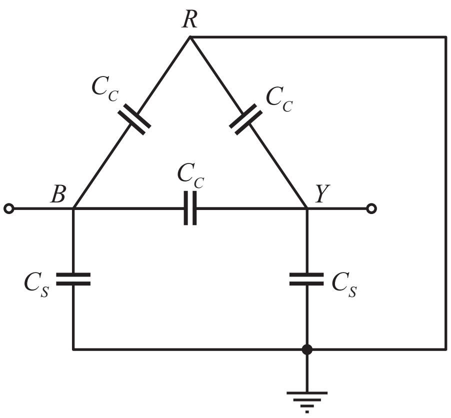
$$i(t)_{rms} = \sqrt{\frac{1}{T}\left[\int_0^{T/2}(2)^2\,dt + \int_{T/2}^{T}(0)^2\,dt\right]}$$

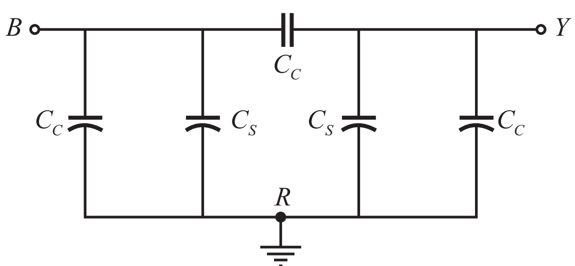
$$i(t)_{rms} = \sqrt{\frac{1}{T}\times 4\times\frac{T}{2}} = \sqrt{2}\text{ A}$$

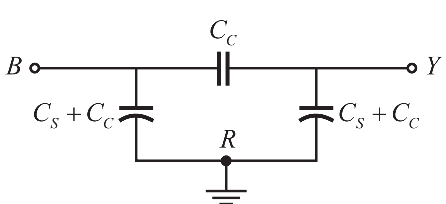
Hence, the correct option is (A).

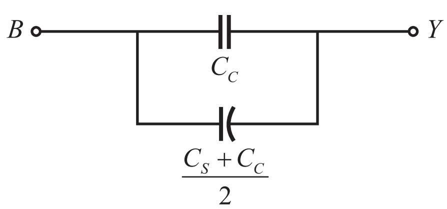
1.12 (C)

Given circuit is shown below,









Equivalent capacitance across BY is,

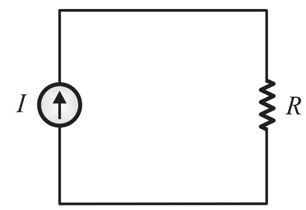
$$C_{BY} = \frac{C_S + C_C}{2} + C_C = \frac{C_S + 3C_C}{2}$$

Hence, the correct option is (C).

1.13 (B)

Given :

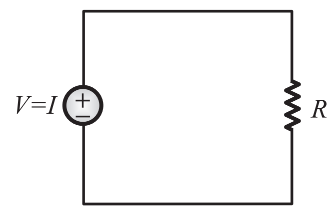
Case 1 :



Power consumed by resistor is given by,

$$P_1 = I^2R = 18\text{ W} \qquad \ldots\text{(i)}$$

Case 2 :



Power consumed by resistor is given by,

$$P_2 = \frac{V^2}{R} = \frac{I^2}{R} = 4.5\text{ W} \qquad \ldots\text{(ii)}$$

From equation (ii),

$$\frac{I^2}{R} = 4.5 \Rightarrow \frac{I^2\times R}{R\times R} = 4.5$$

$$\frac{I^2R}{R^2} = 4.5$$

$$I^2R = 18 \qquad \text{[From equation (i)]}$$

Therefore, $\dfrac{18}{R^2} = 4.5$

Hence, $R = 2\ \Omega$

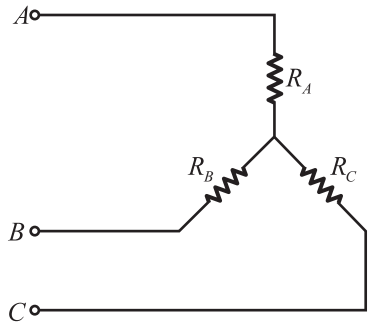
From equation (i),

$$I = \sqrt{\frac{18}{2}} = 3\text{ A}$$

Hence, the correct option is (B).

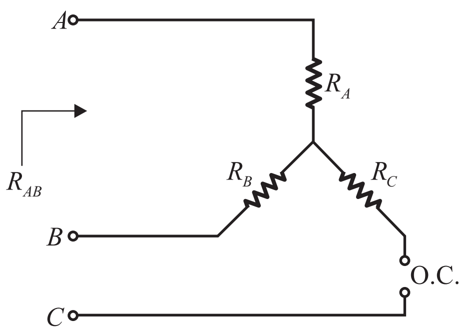
1.14 (B)

Given Y connection is shown below,



Case 1 :

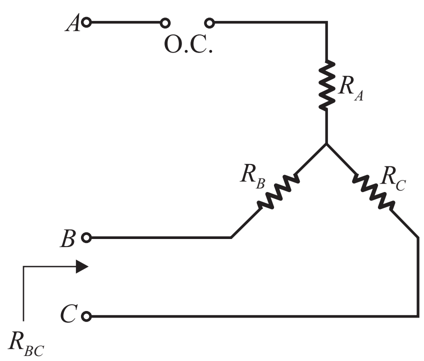
Resistance between terminal A and B with terminal C open is $6\ \Omega$.



Therefore, $R_{AB} = R_A + R_B = 6\ \Omega$ ...(i)

Case 2 :

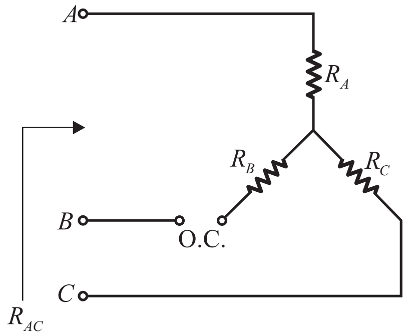
Resistance between terminal B and C with terminal A open is $11\ \Omega$.



Therefore, $R_{BC} = R_B + R_C = 11\ \Omega$ ...(ii)

Case 3 :

Resistance between terminal C and A with terminal B open is $9\ \Omega$.



Therefore, $R_{AC} = R_A + R_C = 9\ \Omega$ ...(iii)

Solving equations (i), (ii) and (iii),

$$R_A = 2\ \Omega,\ R_B = 4\ \Omega,\ R_C = 7\ \Omega$$

Hence, the correct option is (B).

1.15 (D)

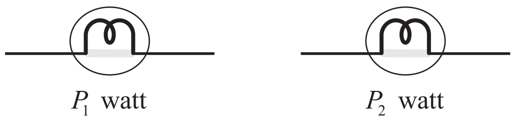
The Bulbs B_1 and B_2 are rated at $P_1 = 40\text{ W}$ and $P_2 = 60\text{ W}$ at same voltage V.

Key Point

1. If voltage rating of corresponding bulb is given, then we will use the corresponding voltage rating to calculate the resistance of bulb.

Then $R_1 = \dfrac{V_1^2}{P_1}$ Then $R_2 = \dfrac{V_2^2}{P_2}$

2. If voltage rating is not given, then we will consider same voltage rating for two bulbs.



Then $R_1 = \dfrac{V^2}{P_1}$ Then $R_2 = \dfrac{V^2}{P_2}$

If resistance of 40 W bulb is R_1 and resistance of 60 W bulb is R_2, then

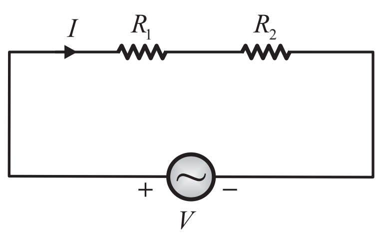
$$P_1 = \frac{V^2}{R_1} \Rightarrow R_1 = \frac{V^2}{P_1} = \frac{V^2}{40}$$

and $P_2 = \frac{V^2}{R_2} \Rightarrow R_2 = \frac{V^2}{P_2} = \frac{V^2}{60}$

As $P_1 < P_2 \Rightarrow R_1 > R_2$

Therefore, resistance of 40 W bulb is greater than resistance of 60 W bulb.

When they are connected in series, current will be same.



Current is given by, $I = \frac{V}{R_1 + R_2}$

Power consumed in series combination is given by,

$$P_{S_1} = I^2 R_1,\ P_{S_2} = I^2 R_2$$

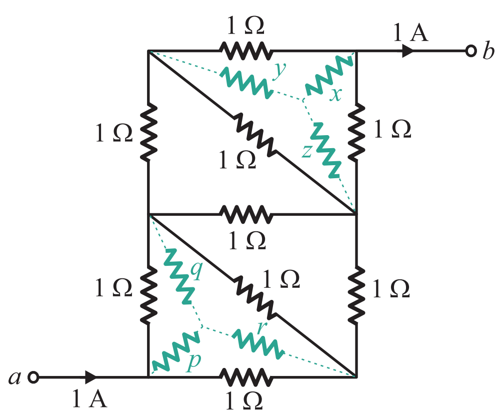
As $R_1 > R_2 \Rightarrow P_{S_1} > P_{S_2}$

Power consumed by 40 W bulb will be greater which results in brighter glow.

Hence, the correct option is (D).

1.16 (A)

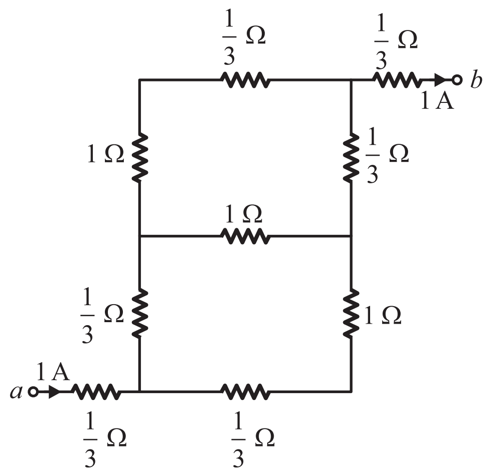
Given circuit is shown below,

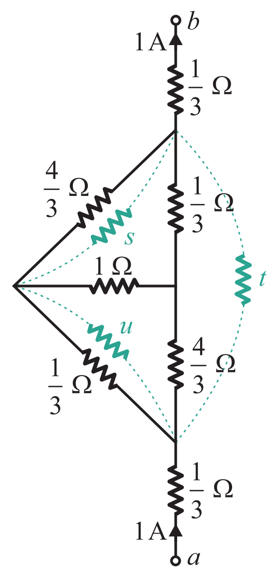


Applying delta to star conversion,

$$x = y = z = \frac{1 \times 1}{1+1+1} = \frac{1}{3}\ \Omega$$

$$p = q = r = \frac{1 \times 1}{1+1+1} = \frac{1}{3}\ \Omega$$



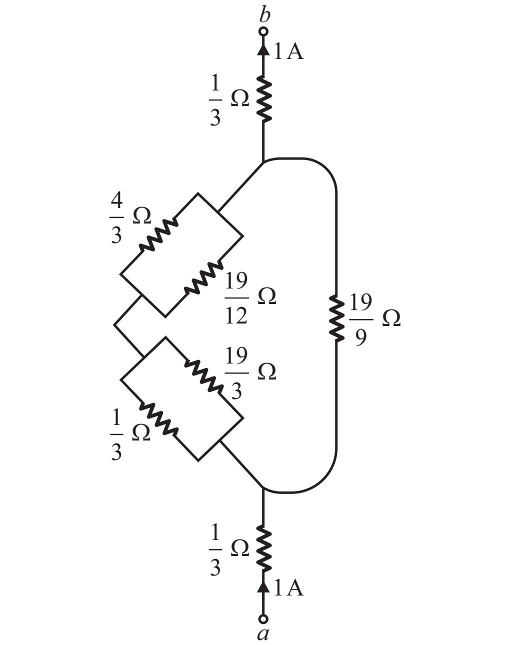


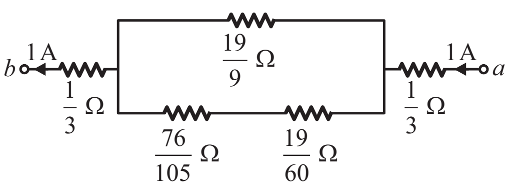
Applying star to delta conversion,

$$t = \frac{\frac{1}{3} \times \frac{4}{3} + 1 \times \frac{1}{3} + 1 \times \frac{4}{3}}{1} = \frac{19}{9}\ \Omega$$

$$s = \frac{\frac{1}{3} \times \frac{4}{3} + 1 \times \frac{1}{3} + 1 \times \frac{4}{3}}{\frac{4}{3}} = \frac{19}{12}\ \Omega$$

$$u = \frac{\frac{1}{3} \times \frac{4}{3} + 1 \times \frac{1}{3} + 1 \times \frac{4}{3}}{\frac{1}{3}} = \frac{19}{3}\ \Omega$$





$$R_{ab} = 1.3636\ \Omega$$

Voltage across between terminal a and b is,

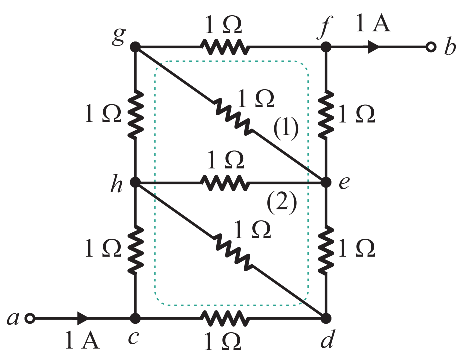
$$V_{ab} = 1.3636 \times 1 = 1.3636\ \text{V} \approx 1.4\ \text{V}$$

Hence, the correct option is (A).

☒ Avoid This Mistake

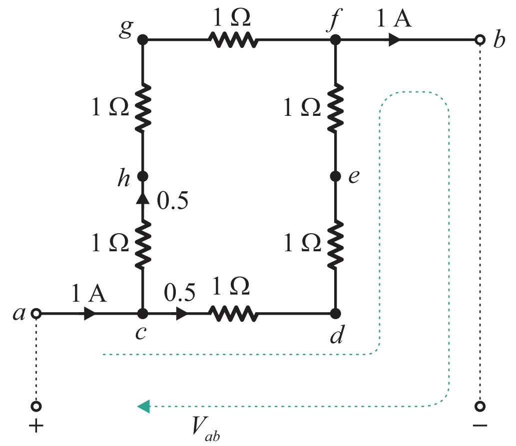
Direct Approach :

[On the basis of balanced Wheat-stone bridge.]



By looking the circuit, the dashed portion can be removed due to symmetry. This is what one can think by just seeing this circuit.

Hence, this circuit will look like as shown below,

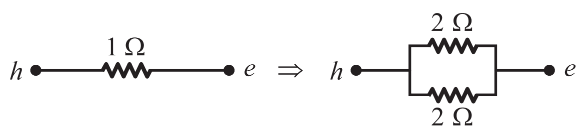


Applying KVL in the loop shown above,

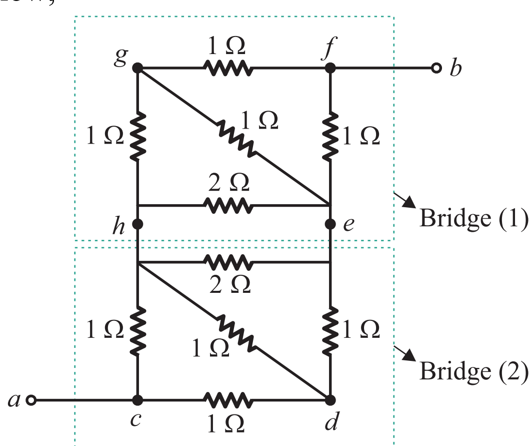
$$V_{ab} = 0.5 \times 1 + 0.5 \times 1 + 0.5 \times 1 = 1.5\ \text{V}$$

But this solution is totally wrong. This has been done here to just make you understand what exactly one can do such type of mistake. Here is the explanation i.e. why it's wrong.

The resistance between the terminals is common in two bridges i.e. 1 and 2. Now this has to be break into two resistances whose equivalent is only $1\ \Omega$ to get Wheat-stone bridge condition. Hence,



The given circuit can be drawn as shown below,



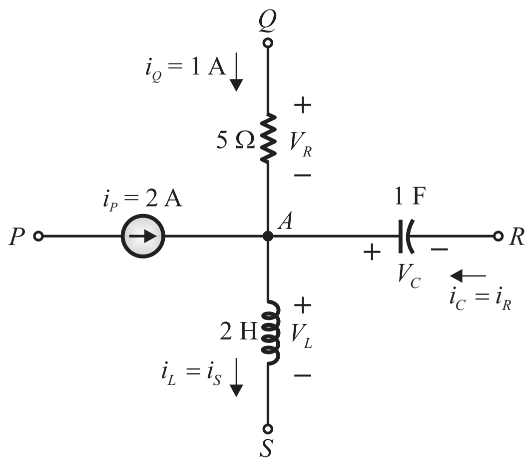
From the circuit, it is clear that the bridge 1 and bridge 2 are not balanced.

Since, $V_{ge} \neq 0$ V and $V_{hd} \neq 0$ V

So the above answer calculated by balance bridge is not possible. So, follow the star-delta or delta-star to get accurate result.

1.17 (B)

Given circuit is shown below,



Given : $V_C = 4\sin 2t$ V , $V_R = 5$ V

Current through capacitor is given by,

$$i_C = i_R = -C\frac{d}{dt}V_C = -\frac{d}{dt}4\sin 2t$$

$$i_C = i_R = -8\cos 2t$$

Applying KCL at node A,

$$-i_P - i_Q - i_R + i_S = 0$$

$$-2 - 1 + 8\cos 2t + i_L = 0$$

$$i_L = 3 - 8\cos 2t$$

Voltage across inductor is given by,

$$V_L = L\frac{di_L}{dt}$$

$$V_L = 2\times\frac{d}{dt}(3 - 8\cos 2t)$$

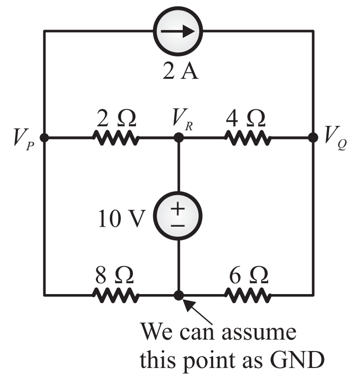
$$V_L = 2\times(-8)\times(2)(-\sin 2t)$$

$$V_L = 32\sin 2t \text{ V}$$

Hence, the correct option is (B).

1.18 (C)

Given circuit is shown below,



Given : $V_R = 10$ V

Applying KCL at node V_P,

$$\frac{V_P}{8} + 2 + \frac{V_P - 10}{2} = 0$$

$$\frac{V_P}{8} + \frac{V_P}{2} - 5 + 2 = 0$$

$$\frac{V_P + 4V_P}{8} = 3$$

$$\frac{5V_P}{8} = 3$$

$$V_P = 4.8 \text{ V}$$

Applying KCL at node V_Q,

$$\frac{V_Q}{6} + \frac{V_Q - 10}{4} - 2 = 0$$

$$\frac{V_Q}{6} + \frac{V_Q}{4} - \frac{10}{4} - 2 = 0$$

$$\frac{2V_Q + 3V_Q}{12} - \frac{18}{4} = 0$$

$$\frac{5V_Q}{12} = \frac{18}{4}$$

$$V_Q = 10.8 \text{ V}$$

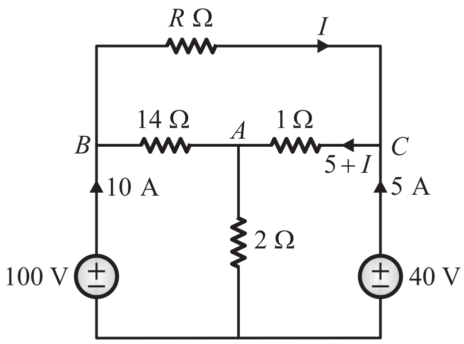
Potential difference between points V_P and V_Q is given by,

$$V_{PQ} = V_P - V_Q = 4.8 - 10.8 = -6 \text{ V}$$

Hence, the correct option is (C).

1.19 (D)

Given circuit is shown below,



Applying KCL at node A,

$$\frac{V_A-100}{14}+\frac{V_A-0}{2}+\frac{V_A-40}{1}=0$$

$$\frac{V_A}{14}+\frac{V_A}{2}+\frac{V_A}{1}-\frac{100}{14}-40=0$$

$$\frac{V_A+7V_A+14V_A}{14}-\frac{660}{14}=0$$

$$\frac{22V_A}{14}=\frac{660}{14}$$

$$V_A=30\text{ V}$$

Potential difference between point B and C is given by,

$$V_{BC}=V_B-V_C$$

where, $V_B=100$ V, $V_C=40$ V

$$V_{BC}=100-40=60\text{ V}$$

From figure,

$$5+I=\frac{V_C-V_A}{1}$$

$$5+I=40-30$$

$$I=5\text{ A}$$

Hence, $R=\dfrac{V_B-V_C}{I}=\dfrac{60}{5}=12\ \Omega$

Hence, the correct option is (D).

Scan for Video Solution

1.20 (C)

Given :

Practical inductor/coil $(R=1\ \Omega,\ L=2\text{ H})$

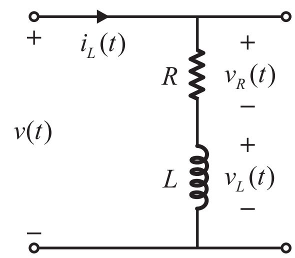
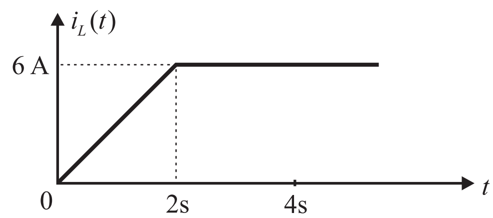


Fig. Practical inductor

Given waveform is shown below,

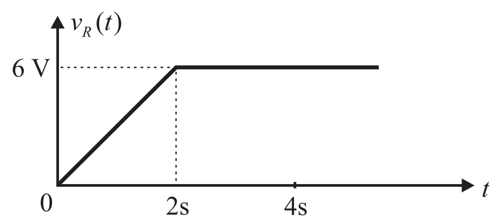


From the waveform, expression of $i_L(t)$ is given below,

$$i_L(t)=\begin{cases}3t\text{ A}; & 0\le t\le 2\\ 6\text{ A}; & t\ge 2\end{cases}$$

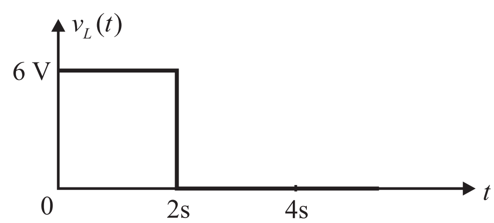
Voltage across resistor is given by,

$$v_R(t)=i_L(t)\times R=\begin{cases}3t\text{ V}; & 0\le t\le 2\\ 6\text{ V}; & t\ge 2\end{cases}$$



Voltage across inductor is given by,

$$v_L(t)=L\frac{d}{dt}i_L(t)=\begin{cases}6\text{ V}, & 0<t<2\\ 0\text{ V}, & t>2\end{cases}$$



Energy absorbed by inductor in first four sec

$= \text{Energy absorbed by resistance } (R = 1\ \Omega)$

$+ \text{Energy absorbed by inductor } (L = 2\text{ H})$

$$W_{\text{Inductor}} = W_R(0<t<2) + W_R(2<t<4) + W_L(0<t<2) + W_L(2<t<4)$$

(i) $W_R(0<t<2)$:

$$W_R(0<t<2) = \int_0^2 P_R(t)\,dt$$

$$W_R(0<t<2) = \int_0^2 v_R(t)\times i_L(t)\,dt$$

$$W_R(0<t<2) = \int_0^2 3t\times 3t\,dt$$

$$W_R(0<t<2) = 9\left[\frac{t^3}{3}\right]_0^2 = 24\text{ J}$$

(ii) $W_R(2<t<4)$:

$$W_R(2<t<4) = \int_2^4 P_R(t)\,dt$$

$$W_R(2<t<4) = \int_2^4 v_R(t)\times i_L(t)\,dt$$

$$W_R(2<t<4) = \int_2^4 6\times 6\,dt$$

$$W_R(2<t<4) = 36[t]_2^4 = 72\text{ J}$$

(iii) $W_L(0<t<2)$:

$$W_L(0<t<2) = \int_0^2 P_L(t)\,dt$$

$$W_L(0<t<2) = \int_0^2 v_L(t)\times i_L(t)\,dt$$

$$W_L(0<t<2) = \int_0^2 6\times 3t\,dt$$

$$W_L(0<t<2) = 18\left[\frac{t^2}{2}\right]_0^2 = 36\text{ J}$$

(iv) $W_L(2<t<4)$:

$$W_L(2<t<4) = \int_2^4 P_L(t)\,dt$$

$$W_L(2<t<4) = \int_2^4 v_L(t)\times i_L(t)\,dt$$

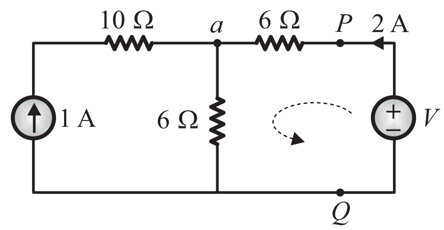
$$W_L(2<t<4) = \int_2^4 0\times 6\,dt = 0\text{ J}$$

$$W_{\text{Inductor}} = 24 + 72 + 36 + 0 = 132\text{ J}$$

Hence, the correct option is (C).

1.21 (C)

Given circuit is shown below,



Applying KCL at node a,

$$\frac{V_a}{6} - 1 - 2 = 0$$

$$V_a = 18\text{ V}$$

Applying KVL in loop shown in figure,

$$-V + 2\times 6 + V_a = 0$$

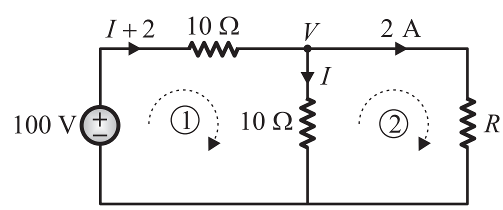
$$-V + 2\times 6 + 18 = 0$$

$$V = 18 + 12 = 30\text{ V}$$

Hence, the correct option is (C).

1.22 (B)

Given circuit is shown below,



Method 1

Using KVL :

Applying KVL in loop (1),

$$-100 + 10(I+2) + I\times 10 = 0$$

$$-100 + 20I + 20 = 0$$

$$-80 + 20I = 0$$

$$I = 4\text{ A}$$

Applying KVL in loop (2),

$$-4\times10+2\times R=0$$

$$R=20\ \Omega$$

Hence, the correct option is (B).

Method 2

Using KCL :

Applying KCL at node V,

$$\frac{V}{10}+\frac{V-100}{10}+2=0$$

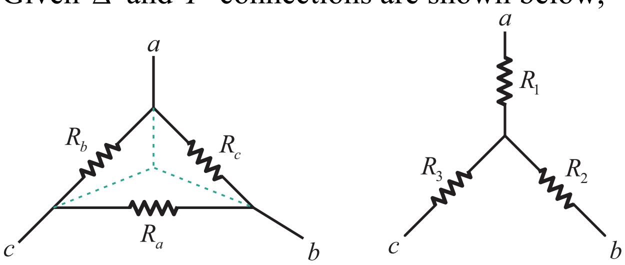
$$V=40\ \text{V}$$

Hence, $R=\dfrac{V}{2}=\dfrac{40}{2}=20\ \Omega$

Hence, the correct option is (B).

1.23 (A)

Given Δ and Y connections are shown below,



Given : $R_a=20\ \Omega$, $R_b=10\ \Omega$, $R_c=10\ \Omega$

Applying Δ to Y conversion,

$$R_1=\frac{R_b\times R_c}{R_a+R_b+R_c}=\frac{10\times10}{20+10+10}$$

$$R_1=\frac{100}{40}=2.5\ \Omega$$

$$R_2=\frac{R_a\times R_c}{R_a+R_b+R_c}=\frac{20\times10}{20+10+10}$$

$$R_2=\frac{200}{40}=5\ \Omega$$

$$R_3=\frac{R_b\times R_a}{R_a+R_b+R_c}=\frac{10\times20}{20+10+10}$$

$$R_3=\frac{200}{40}=5\ \Omega$$

Hence, the correct option is (A).

1.24 (D)

Given : $i(t)=10+20\sin\omega t$

RMS value of periodic signal is given by,

$$i(t)_{rms}=\sqrt{(I_{dc})^2+\left(\frac{I_{ac}}{\sqrt{2}}\right)^2}$$

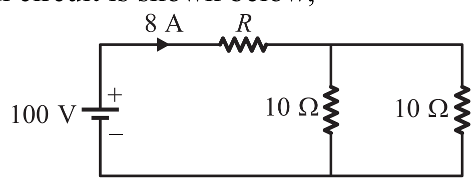
$$i(t)_{rms}=\sqrt{(10)^2+\left(\frac{20}{\sqrt{2}}\right)^2}$$

$$i(t)_{rms}=10\sqrt{3}=17.32\ \text{A}$$

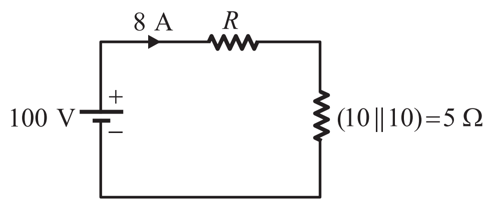
Hence, the correct option is (D).

1.25 (C)

Given circuit is shown below,



The above circuit can be redrawn as,



From figure,

$$8=\frac{100}{R+5}$$

$$8(R+5)=100$$

$$8R+40=100$$

$$8R=60$$

$$R=7.5\ \Omega$$

Hence, the correct option is (C).

1.26 (A)

Given : $u(t)=3+4\cos(3t)$

RMS value of periodic signal is given by,

$$u(t)_{rms}=\sqrt{u(t)_{dc}^2+\left(\frac{u(t)_{ac}}{\sqrt{2}}\right)^2}$$

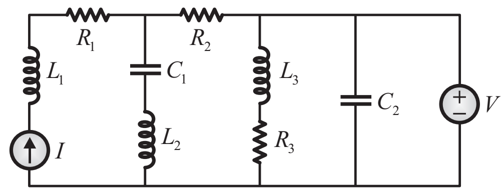
$$u(t)_{rms}=\sqrt{(3)^2+\left(\frac{4}{\sqrt{2}}\right)^2}=\sqrt{9+\frac{16}{2}}$$

$$u(t)_{rms}=\sqrt{17}\ \text{V}$$

Hence, the correct option is (A).

1.27 (D)

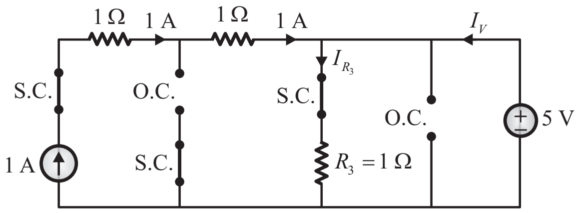
Given circuit is shown below,



Given : $I = 1\text{A},\ V = 5\text{V},\ C_1 = C_2 = 1\text{F}$

$$R_1 = R_2 = R_3 = 1\ \Omega,\ L_1 = L_2 = L_3 = 1\text{H}$$

In given circuit DC voltage is applied. Hence, in steady state inductor behaves as a short circuit and capacitor behaves as an open circuit.



Current through R_3 is given by,

$$I_{R_3} = \frac{5}{R_3} = \frac{5}{1} = 5\ \text{A}$$

Current through voltage source is given by,

$$I_V = I_{R_3} - 1 = 5 - 1 = 4\ \text{A}$$

Hence, the correct option is (D).

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Key Point

For DC source (At steady state),

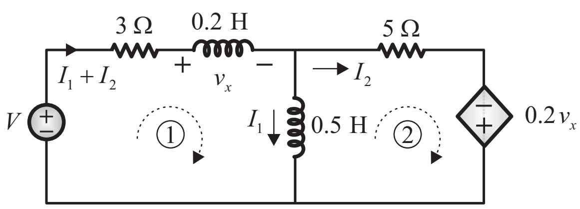
$$\omega = 0 \Rightarrow f = 0$$

Hence, $X_L = \omega L = 0$ (Short circuit)

and $X_C = \dfrac{1}{\omega C} = \infty$ (Open circuit)

1.28 (A)

Given circuit is shown below,



Applying KVL in loop (1),

$$-V + 3(I_1 + I_2) + v_x + 0.5\frac{dI_1}{dt} = 0 \quad \ldots\text{(i)}$$

Applying KVL in loop (2),

$$-0.5\frac{dI_1}{dt} + 5I_2 - 0.2v_x = 0$$

$$I_2 = \frac{0.5}{5}\frac{dI_1}{dt} + \frac{0.2}{5}v_x$$

$$I_2 = 0.1\frac{dI_1}{dt} + 0.04v_x \quad \ldots\text{(ii)}$$

Put the value of I_2 in equation (i),

$$-V + 3I_1 + 0.3\frac{dI_1}{dt} + 0.12v_x + v_x + 0.5\frac{dI_1}{dt} = 0$$

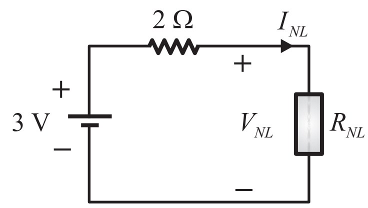
$$0.8\frac{dI_1}{dt} + 1.12v_x + 3I_1 - V = 0$$

$$\frac{dI_1}{dt} = -1.4v_x - 3.75I_1 + \frac{5}{4}V$$

Hence, the correct option is (A).

1.29 (A)

Given : $V_{NL} = I_{NL}^2 \quad \ldots\text{(i)}$



Applying KVL in the above circuit,

$$-3 + 2 \times I_{NL} + V_{NL} = 0$$

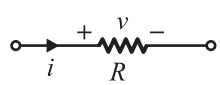
$$V_{NL} = 3 - 2I_{NL} \quad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$I_{NL}^2 + 2I_{NL} - 3 = 0$$

$$I_{NL} = 1\ \text{A},\ -3\ \text{A}$$

Passive sign convention :



$v = iR$

$$I_{NL} = 1\ \text{A} \quad (\text{Neglecting } I_{NL} = -3\ \text{A})$$

Power dissipated in the non-linear resistor is given by,

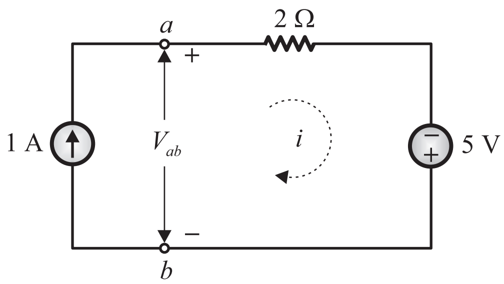
$$P = V_{NL} I_{NL} = I_{NL}^2 \times I_{NL}$$

$$P = I_{NL}^3 = (1)^3 = 1 \text{ W}$$

Hence, the correct option is (A).

1.30 (A)

Given circuit is shown below,



Given : $i = 1$ A

Applying KVL in the loop shown,

$$-V_{ab} + 2 \times i - 5 = 0$$

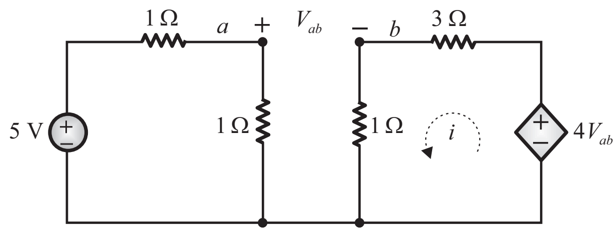
$$-V_{ab} + 1 \times 2 - 5 = 0$$

$$V_{ab} = -3 \text{ V}$$

Hence, the correct option is (A).

1.31 (B)

Given circuit is shown below,



$$V_a = 5 \times \frac{1}{1+1} \qquad \text{[By VDR]}$$

$$V_a = 2.5 \text{ V} \qquad \ldots\text{(i)}$$

$$V_b = 4V_{ab} \times \frac{1}{1+3} \qquad \text{[By VDR]}$$

$$V_b = V_{ab} = V_a - V_b$$

$$V_b = \frac{V_a}{2} = \frac{2.5}{2} \qquad \text{[From equation (i)]}$$

$$V_b = 1.25 \text{ V} \qquad \ldots\text{(ii)}$$

$$V_{ab} = V_a - V_b = 2.5 - 1.25 = 1.25 \text{ V}$$

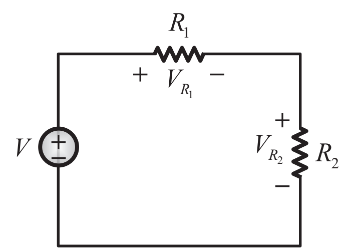
From figure,

$$i = \frac{4V_{ab}}{3+1} = V_{ab} = 1.25 \text{ A}$$

Hence, the correct option is (B).

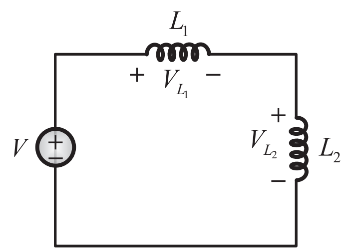
Key Point

1. Voltage division rule for resistor :



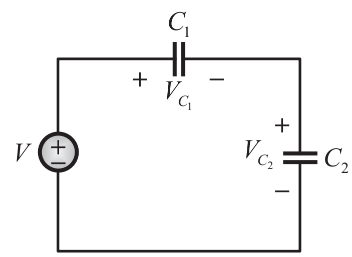
$$V_{R_1} = \frac{V \times R_1}{R_1 + R_2} \qquad V_{R_2} = \frac{V \times R_2}{R_1 + R_2}$$

2. Voltage division rule for inductor :



$$V_{L_1} = \frac{V \times L_1}{L_1 + L_2} \qquad V_{L_2} = \frac{V \times L_2}{L_1 + L_2}$$

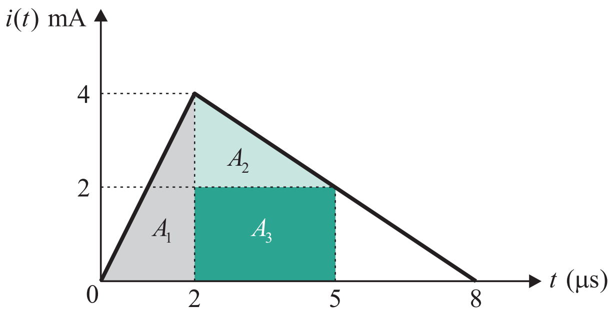
3. Voltage division rule for capacitor :



$$V_{C_1} = \frac{V \times C_2}{C_1 + C_2} \qquad V_{C_2} = \frac{V \times C_1}{C_1 + C_2}$$

1.32 (C)

Given waveform is shown below,



Charge stored in the capacitor upto $t = 5$ μsec is given by,

$$Q(0<t<5) = \text{ Area under curve } i-t$$

$$Q(0<t<5) = A_1 + A_2 + A_3$$

From figure,

$$A_1 = \frac{1}{2}\times 2\times 10^{-6}\times 4\times 10^{-3} = 4 \text{ nC}$$

$$A_2 = \frac{1}{2}\times(5-2)\times 10^{-6}\times(4-2)\times 10^{-3}$$

$$A_2 = 3 \text{ nC}$$

$$A_3 = (5-2)\times 10^{-6}\times 2\times 10^{-3} = 6 \text{ nC}$$

Therefore, $Q(0<t<5) = 4+3+6 = 13$ nC

Hence, the correct option is (C).

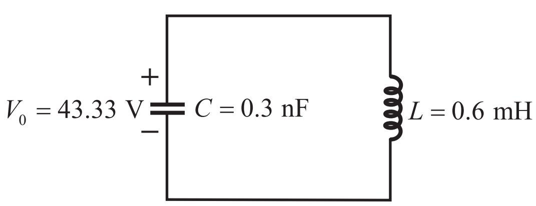
1.33 (D)

Total charge stored upto 5 μsec,

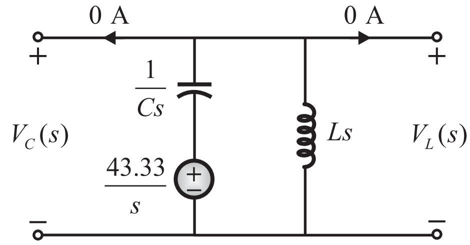
$$Q = 13 \text{ nC}$$

Voltage across capacitor before connecting to inductor is given by,

$$V_0 = \frac{Q}{C} = \frac{13\times 10^{-9}}{0.3\times 10^{-9}} = 43.33 \text{ V}$$



Transform domain :



From figure,

$$V_C(s) = V_L(s) = \frac{43.33}{s}\times\frac{Ls}{Ls+\frac{1}{Cs}}$$

$$V_C(s) = \frac{43.33\times L\times sC}{s^2LC+1}$$

$$V_C(s) = \frac{43.33\times LC\times s}{LC\left[s^2+\left(\frac{1}{\sqrt{LC}}\right)^2\right]} = \frac{43.33s}{(s^2+\omega_0^2)}$$

$$\left[\omega_0 = \frac{1}{\sqrt{LC}}\right]$$

$$V_C(t) = 43.33\cos\omega_0 t$$

Given : $L = 0.6\,\text{mH}$ and $C = 0.3\,\text{nF}$

$$\omega_0 = \sqrt{\frac{1}{0.6\times 0.3\times 10^{-9}\times 10^{-3}}}$$

$$\omega_0 = 2.35\times 10^6 \text{ rad/sec}$$

At $t = 1$ μsec,

$$\omega_0 t = 2.35 \text{ rad}$$

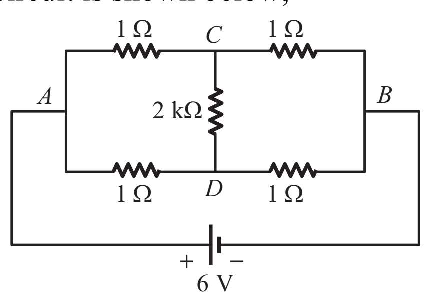
$$V_C(1\ \mu\text{s}) = 43.33\cos\left(2.35\times\frac{180}{\pi}\right)$$

$$V_C(1\ \mu\text{s}) = -30.6 \text{ V}$$

Hence, the correct option is (D).

1.34 (A)

Given circuit is shown below,



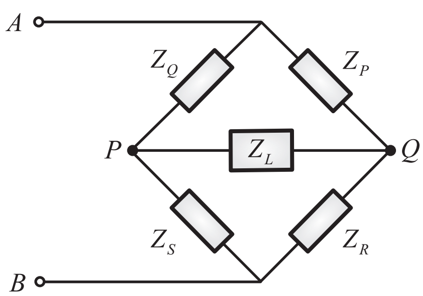
Since, the given bridge is balanced i.e. C and D are at same potential. Therefore, current through $2\ \text{k}\Omega$ resistor is zero.

Hence, the correct option is (A).

📖 Key Point

Condition for balanced bridge :

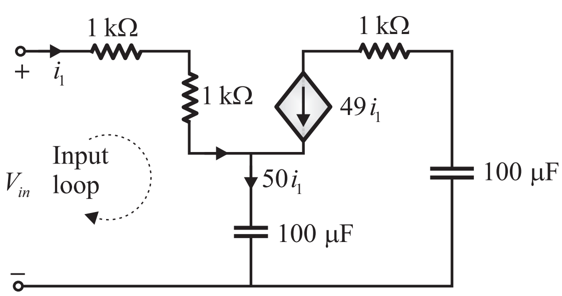
$$\frac{Z_P}{Z_Q} = \frac{Z_R}{Z_S}$$



For balanced bridge, current through load terminal i.e. Z_L will be zero and potential of P and Q terminals are equal.

1.35 (A)

Given circuit is shown below,



Applying KVL in input loop,

$$-V_{in} + i_1(1000+1000) + 50 i_1 \times \frac{-j}{\omega C} = 0$$

Input impedance is given by,

$$Z_{in} = \frac{V_{in}}{i_1} = 2000 - j50 \times \frac{1}{\omega \times 100 \times 10^{-6}}$$

$$Z_{in} = \frac{V_{in}}{i_1} = 2000 - j \times \frac{1}{\omega \times 2 \times 10^{-6}} \quad \ldots\text{(i)}$$

Input impedance of given RC network is given by,

$$Z_{in} = R_{in} - jX_{in} = R_{in} - j \times \frac{1}{\omega C_{in}} \quad \ldots\text{(ii)}$$

Comparing equation (i) and (ii),

$$R_{in} = 2000\ \Omega,\ C_{in} = 2 \times 10^{-6}\ \text{F}$$

$$C_{eq} = C_{in} = 2\ \mu\text{F}$$

Hence, the correct option is (A).

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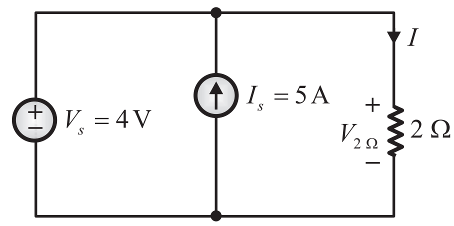
1.36 (B)

Given circuit is shown below,



Current through $2\ \Omega$ resistance is given by,

$$I = \frac{V_{2\,\Omega}}{2} = \frac{V_s}{2} = \frac{4}{2} = 2\ \text{A}$$

According to linearity rule, if current through $2\ \Omega$ resistance is to be doubled then the voltage source V_s has to be double.

i.e. $\quad V_s = 2 \times 4 = 8\ \text{V}$

Hence, the correct option is (B).

1.37 (D)

Consider 200 W / 220 V incandescent lamp :

Voltage rating $= V_1 = 220\ \text{V}$

Power rating $= P_1 = 200\ \text{W}$

Let resistance of each lamp is R_1, hence

$$R_1 = \frac{V_1^2}{P_1} = \frac{220^2}{200} = 242\ \Omega$$

Consider 100 W / 220 V incandescent lamp :

Voltage rating $= V_0 = 220\ \text{V}$

Power rating $= P_0 = 100\ \text{W}$

Let resistance of this lamp is R_0, hence

$$R_0 = \frac{V_0^2}{P_0} = \frac{220^2}{100} = 484\ \Omega$$

Let series combination of 'n' incandescent lamp (having rating 200 W / 220 V and $R_1 = 242\ \Omega$) is equivalent to incandescent lamp (having rating 100 W / 220 V and $R_0 = 484\ \Omega$), then

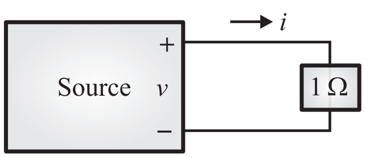
$$R_0 = nR_1$$

$$n = \frac{R_0}{R_1} = \frac{484}{242} = 2$$

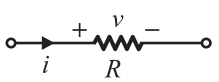
Hence, the correct option is (D).

1.38 (B)

Given : $v + i = 100$ …(i)



Passive sign convention :



$\boxed{v = iR}$

$$v = i \times 1 = i \quad \text{…(ii)}$$

From equation (i) and (ii),

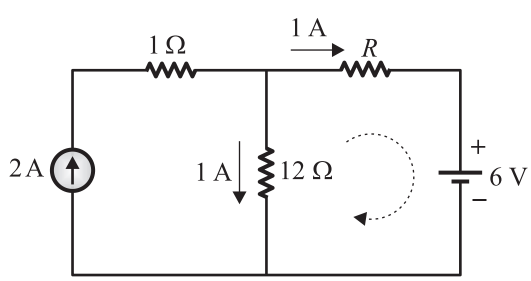
$$i + i = 100$$

$$i = 50\ \text{A}$$

Hence, the correct option is (B).

1.39 (B)

Given circuit is shown below,



Applying KVL in the loop shown above,

$$-12 \times 1 + R \times 1 + 6 = 0$$

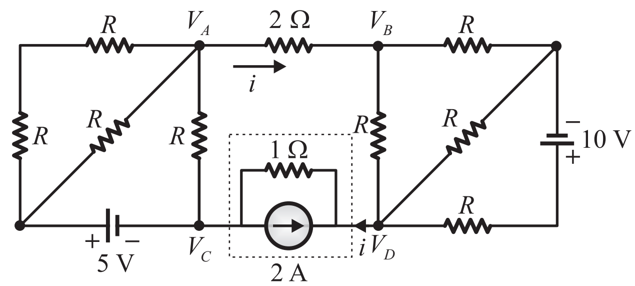
$$R = 6\ \Omega$$

Hence, the correct option is (B).

1.40 (A)

Given : $V_A - V_B = 6$ V

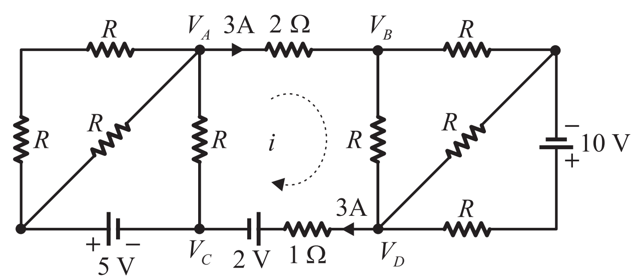
The circuit is shown below,



From the circuit, $i = \dfrac{V_A - V_B}{2} = \dfrac{6}{2} = 3$ A

The same current will flow through branch between node D and C.

Using source transformation :

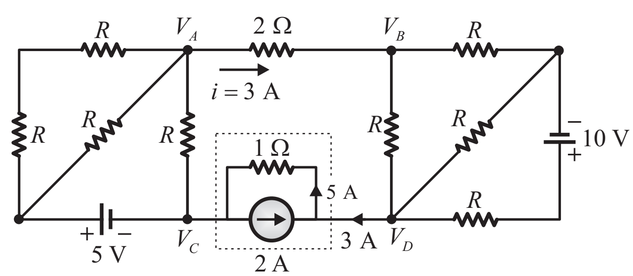


From the above circuit,

$$V_{DC} = 3 \times 1 + 2 = 5\ \text{V}$$

$$V_{CD} = V_C - V_D = -5\ \text{V}$$

Without using source transformation :



$$V_{DC} = 5 \times 1 = 5\ \text{V}$$

$$V_{CD} = -5\ \text{V}$$

Hence, the correct option is (A).

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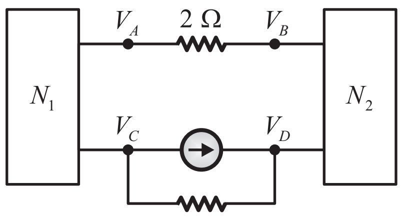
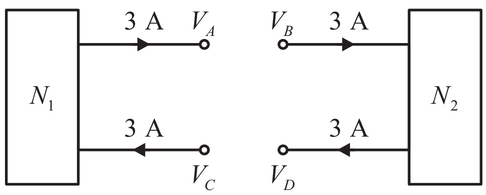
📖 Key Point



For a particular one port network,
Incoming current = Outgoing current
Hence,



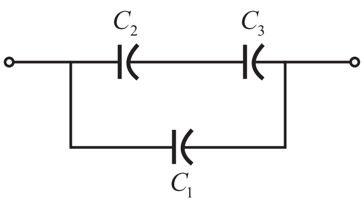
1.41 (C)

Given : $C_1 = 10\ \mu\text{F}$, $V_{10} = 10\ \text{V}$

$C_2 = 5\ \mu\text{F}$, $V_{20} = 5\ \text{V}$

$C_3 = 2\ \mu\text{F}$, $V_{30} = 2\ \text{V}$

Method 1



Charge in a capacitor is given by,

$$Q = CV$$

Capacitor	Breakdown voltage or maximum operating voltage	Maximum charge stored in capacitor
$C_1 = 10\ \mu\text{F}$	10 V	$Q_{1(\max)} = 100\ \mu\text{C}$
$C_2 = 5\ \mu\text{F}$	5 V	$Q_{2(\max)} = 25\ \mu\text{C}$
$C_3 = 2\ \mu\text{F}$	2 V	$Q_{3(\max)} = 4\ \mu\text{C}$

Since, C_2 and C_3 are in series, hence the charge on both capacitor will be same and equal to the min (Q_2, Q_3)

$$Q_{23} = 4\ \mu\text{C}$$

Equivalent capacitance of C_2 and C_3 is,

$$C_{23} = \frac{C_2 C_3}{C_2 + C_3} = \frac{5 \times 2}{5+2} = \frac{10}{7}\ \mu\text{F}$$

Equivalent voltage is,

$$V_{eq} = V_{23} = \frac{Q_{23}}{C_{23}} = \frac{4\ \mu\text{C}}{(10/7)\ \mu\text{F}} = 2.8\ \text{V}$$

In parallel, voltage will be same, hence 2.8 V will appear across C_1 also.

Charge stored in C_1 is given by,

$$Q_1 = C_1 V_{eq}$$

$$Q_1 = 10\ \mu\text{F} \times 2.8\ \text{V} = 28\ \mu\text{C}$$

In parallel, total charge is given by,

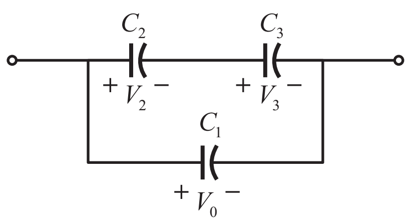
$$Q_T = Q_1 + Q_{23}$$

$$Q_T = (28+4)\mu\text{C} = 32\ \mu\text{C}$$

Hence, the correct option is (C).

Method 2

Capacitor	C_1	C_2	C_3
Value (in μF)	10	5	2
Breakdown Voltage (Volts)	10	5	2



From above figure, $V_0 = V_2 + V_3$

$$V_2 = \frac{C_3}{C_3 + C_2} V_0 = \frac{2}{5+2} V_0 = \frac{2V_0}{7}$$

$$V_3 = \frac{C_2}{C_2 + C_3} V_0 = \frac{5}{2+5} V_0 = \frac{5V_0}{7}$$

$$V_0 \le 10\ \text{V},\ V_2 \le 5\ \text{V and } V_3 \le 2\ \text{V}$$

(i) If $V_0 = 10$ V,

Then $V_2 = \frac{2\times10}{7} = \frac{20}{7} = 2.85$ V

So, $V_2 < 5$ V [No Breakdown]

$$V_3 = \frac{5\times10}{7} = \frac{50}{7} = 7.13 \text{ V}$$

So, $V_3 > 2$ V [Breakdown]

Hence, V_0 must be less than 10 V.

(ii) If $V_2 = 5$ V,

$$V_0 = \frac{7}{2}\times5 = \frac{35}{2} \text{ V}$$

$V_0 > 10$ V [Breakdown]

Hence, V_2 must be less than 5 V.

(iii) If $V_3 = 2$ V,

$$V_0 = \frac{7}{5}\times2 = 2.8 < 10 \text{ V}$$

[No Breakdown]

Also, $V_2 = \frac{2V_0}{7} = 0.8 \text{ V} < 5 \text{ V}$

[No Breakdown]

None of them are in breakdown.

Hence, maximum value of voltage across the combination can be $V_0 = 2.8$ V.

Total charge $Q = C_{eq}V_0$

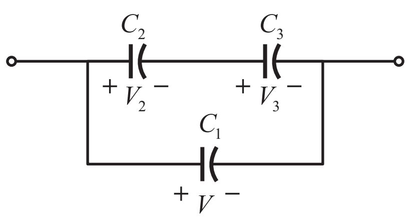
where, $C_{eq} = C_1 + \frac{C_2C_3}{C_2+C_3}$

$$C_{eq} = 10 + \frac{2\times5}{2+5} = \frac{80}{7} \ \mu\text{F}$$

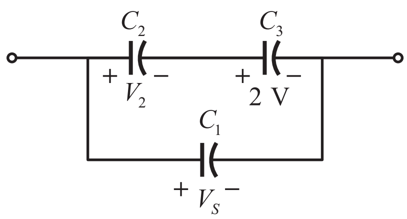
$$Q = \frac{80}{7}\times2.8 = 32 \ \mu\text{C}$$

Hence, the correct option is (C).

Method 3



Here, capacitors C_3 has minimum breakdown voltage and hence the breakdown voltage of capacitor C_3 will decide the maximum safe voltage V_S applied across the circuit as shown in figure.



$$V_3 = \frac{C_2}{C_2+C_3}\times V_S \quad \text{[By VDR]}$$

$$2 = \frac{5}{2+5}\times V_S$$

$$V_S = 2.8 \text{ V}$$

Also, total charge is given by,

$$Q = C_{eq}V_S$$

where, $C_{eq} = C_1 + \frac{C_2C_3}{C_2+C_3}$

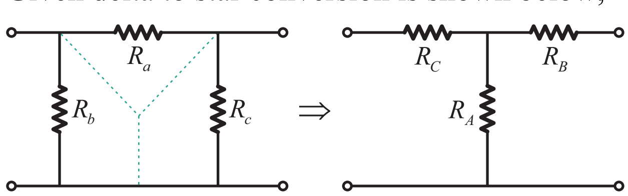
$$C_{eq} = 10 + \frac{2\times5}{2+5} = \frac{80}{7} \ \mu\text{F}$$

$$Q = \frac{80}{7}\times2.8 = 32 \ \mu\text{C}$$

Hence, the correct option is (C).

1.42 (B)

Given delta to star conversion is shown below,



Before scaling :

$$R_A = \frac{R_bR_c}{R_a+R_b+R_c}$$

If $R_a' = kR_a$, $R_b' = kR_b$, $R_c' = kR_c$

After scaling :

$$R_A' = \frac{R_b' R_c'}{R_a' + R_b' + R_c'}$$

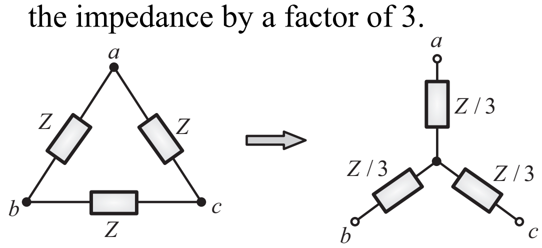
$$R_A' = \frac{(kR_b)(kR_c)}{kR_a + kR_b + kR_c} = \frac{k^2 R_b R_c}{k(R_a + R_b + R_c)}$$

$$R_A' = kR_A$$

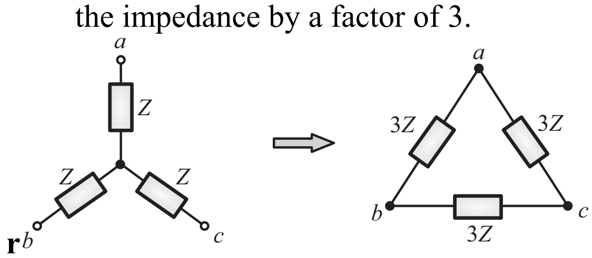
Hence, the correct option is (B).

🕮 Key Point

(i) If all the elements of Δ network are same, then Δ to Y conversion decreases the impedance by a factor of 3.



(ii) If all the elements of Y network are same, then Y to Δ conversion increases the impedance by a factor of 3.



1.43 330

🕮 Key Point

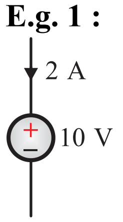
Concept of absorbed and delivered power :

(i) Power is absorbed by the electrical element if current enters into the positive terminal.

(ii) Power is delivered by the electrical element if current leaves from the positive terminal.

(iii) Absorbed power + Delivered power = 0

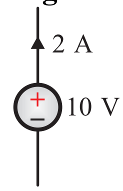
E.g. 1 :



$$P_{10\text{ V}} = P_{\text{absorb}} = 10 \times 2 = 20\text{ W}$$

$$P_{10\text{ V}} = P_{\text{deliver}} = -20\text{ W}$$

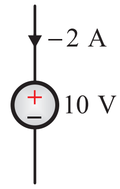
E.g. 2 :



$$P_{10\text{ V}} = P_{\text{deliver}} = 10 \times 2 = 20\text{ W}$$

$$P_{10\text{ V}} = P_{\text{absorb}} = -20\text{ W}$$

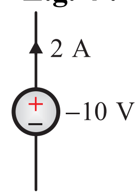
E.g. 3 :



$$P_{10\text{ V}} = P_{\text{absorb}} = 10 \times (-2) = -20\text{ W}$$

$$P_{10\text{ V}} = P_{\text{deliver}} = 20\text{ W}$$

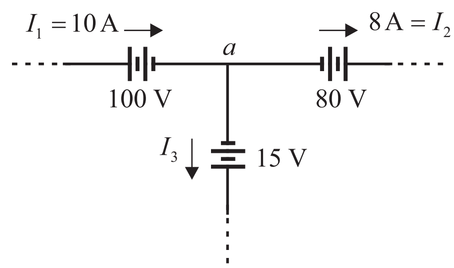
E.g. 4 :



$$P_{-10\text{ V}} = P_{\text{deliver}} = -10 \times 2 = -20\text{ W}$$

$$P_{-10\text{ V}} = P_{\text{absorb}} = 20\text{ W}$$

Given circuit is shown below,



Applying KCL at node a,

$$-I_1 + I_2 + I_3 = 0$$

$$I_3 = I_1 - I_2 = 10 - 8 = 2\text{ A}$$

(i) $P_{100\text{ V}} = P_{\text{absorb}} = 10 \times 100 = 1000\text{ W}$

$P_{100\text{ V}} = P_{\text{deliver}} = -1000\text{ W}$

(ii) $P_{80\text{ V}} = P_{\text{deliver}} = 8 \times 80 = 640\text{ W}$

$P_{80\text{ V}} = P_{\text{absorb}} = -640\text{ W}$

(iii) $P_{15\text{ V}} = P_{\text{deliver}} = 15 \times 2 = 30\text{ W}$

$P_{15\text{ V}} = P_{\text{absorb}} = -30\text{ W}$

Total power absorbed by the circuit is,

$$= 1000 - 640 - 30 = 330\text{ W}$$

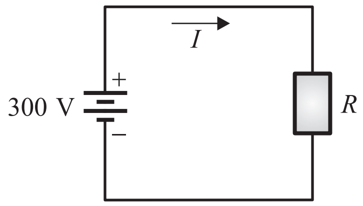
Total power delivered by the circuit is,

$$= -1000 + 640 + 30 = -330\text{ W}$$

Hence, the total power absorbed by the three circuit elements is **330 W.**

1.44 10

Given circuit is shown below,



Given : $R = \left(25 + \frac{I}{2}\right)\ \Omega$ …(i)

From the circuit,

$$R = \frac{300}{I} \quad \text{…(ii)}$$

From equation (i) and (ii),

$$25 + \frac{I}{2} = \frac{300}{I}$$

$$25I + \frac{I^2}{2} - 300 = 0$$

$$I^2 + 50I - 600 = 0$$

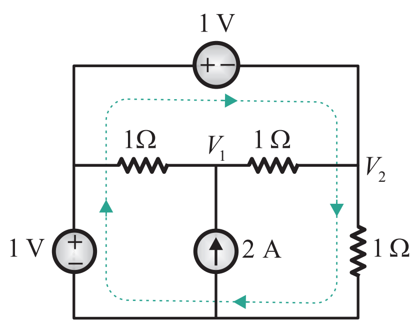
$$I = 10,\ -60$$

I	$R = 25 + \frac{I}{2}$	Status
10 A	30 Ω	Valid
– 60 A	–5 Ω	Invalid

Hence, the current I is **10 A.**

1.45 3

Given circuit is shown below,



Applying KVL in above loop,

$$-1 + 1 + V_2 = 0$$

$$V_2 = 0\ \text{V}$$

Applying KCL at node V_1,

$$\frac{V_1 - 1}{1} + \frac{V_1 - V_2}{1} - 2 = 0$$

$$\frac{V_1 - 1}{1} + \frac{V_1 - 0}{1} - 2 = 0$$

$$2V_1 = 3 \quad \Rightarrow \quad V_1 = \frac{3}{2}\ \text{V}$$

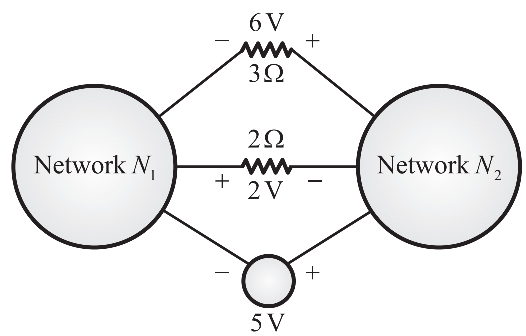
Power delivered by the current source is given by,

$$P_{2\,\text{A}} = P_{\text{deliver}} = 2 \times \frac{3}{2} = 3\ \text{W}$$

Hence, the power delivered by the current source is **3 W.**

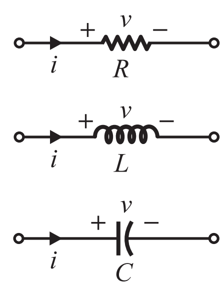
1.46 (A)

Given circuit is shown below,



Key Point

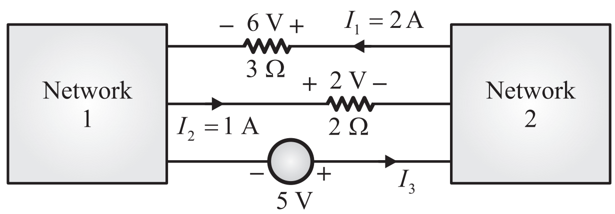
Passive sign convention :



1. $v = iR$

2. $v = L\frac{di}{dt}$

3. $i = C\frac{dv}{dt}$



Taking network 1 as a big node, same can be done by assuming network 2 as a big node.

Applying KCL in the network 1,

$$-I_1 + I_2 + I_3 = 0$$

$$I_3 = I_1 - I_2 = 2 - 1 = 1 \text{ A}$$

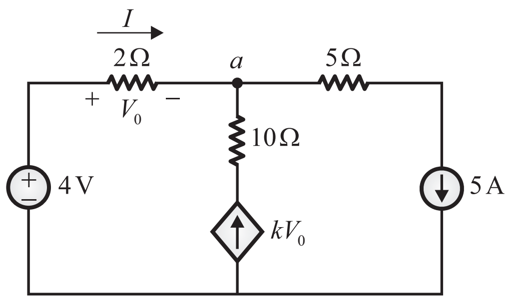
Power delivered by 5 V voltage source is,

$$P_{5\text{ V}} = P_{\text{deliver}} = 5 \times 1 = 5 \text{ Watt}$$

Hence, the correct option is (A).

1.47 **0.5**

Given circuit is shown below,



Applying KCL at node a,

$$-I - kV_0 + 5 = 0 \qquad \ldots\text{(i)}$$

Given : Power dissipated in 2 Ω resistor 12.5 Watts.

$$I^2 R = P$$

$$I^2 \times 2 = 12.5$$

$$I = 2.5$$

$$V_0 = I \times 2 = 2 \times 2.5 = 5 \text{ Volt}$$

From equation (i),

$$-2.5 - k \times 5 + 5 = 0$$

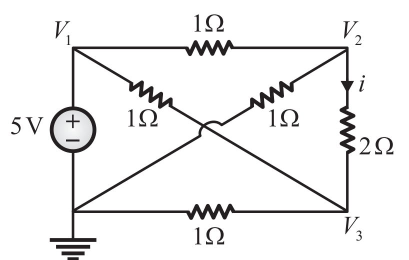
$$k = 0.5$$

Hence, the value of k is **0.5.**

1.48 **0**

Method 1

Given circuit is shown below,



From the circuit, $V_1 = 5\text{ V}$

Applying KCL at node V_2,

$$\frac{V_2 - V_1}{1} + \frac{V_2 - 0}{1} + \frac{V_2 - V_3}{2} = 0$$

$$\frac{5V_2}{2} - \frac{V_3}{2} = 5 \qquad \ldots\text{(i)}$$

Applying KCL at node V_3,

$$\frac{V_3 - V_1}{1} + \frac{V_3 - V_2}{2} + \frac{V_3 - 0}{1} = 0$$

$$\frac{5V_3}{2} - \frac{V_2}{2} = 5 \qquad \ldots\text{(ii)}$$

From equation (i) and equation (ii),

$$V_2 = \frac{5}{2}\text{V}, \quad V_3 = \frac{5}{2}\text{V}$$

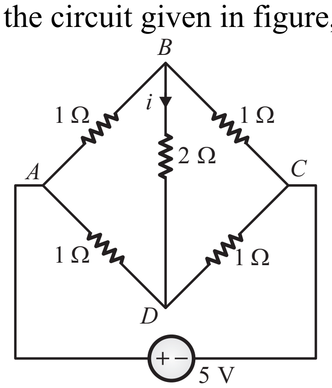
From the circuit,

$$i = \frac{V_2 - V_3}{2} = 0\text{ A}$$

Hence, the current i in the 2 Ω resistor is **0 A.**

Method 2

Redrawing the circuit given in figure,



Since, the given Wheat-stone bridge is balanced. i.e. node B and D are at same potential. Therefore, current 2 Ω resistor is zero.

Hence, the current i in the 2 Ω resistor is **0 A.**

1.49 **(D)**

Given circuits are shown below,

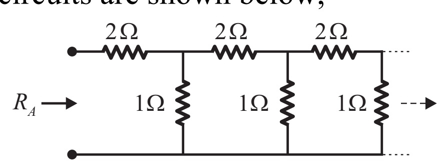


Fig. (i)

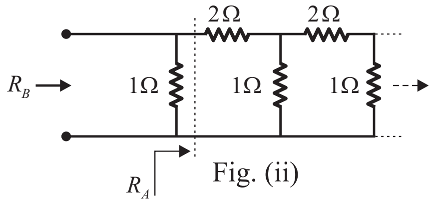
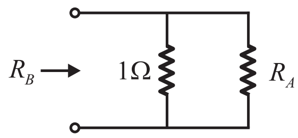


Fig. (ii)

Comparing figure (i) and (ii),



$$R_B = 1 \| R_A$$

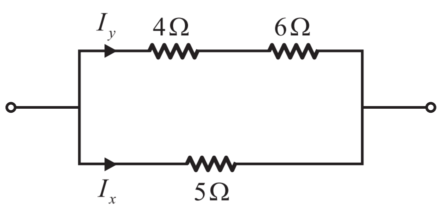
$$R_B = \frac{1 \times R_A}{1+R_A} = \frac{R_A}{1+R_A}$$

Hence, the correct option is (D).

1.50 **2**

Method 1

1 calorie per second = 4.184 W



Heat generated in 5Ω resistance

$$= 10 \text{ Cal/sec } = 10 \times 4.184 = 41.84 \text{ W}$$

Heat dissipated is given by,

$$I_x^2 \times 5 = 41.84$$

$$I_x = 2.892 \text{ A}$$

Voltage across resistances (5Ω) and $(4\Omega + 6\Omega)$ should be equal.

Therefore, $I_x \times 5 = I_y(4+6)$

$$I_y = \frac{2.892 \times 5}{10} = 1.446 \text{ A}$$

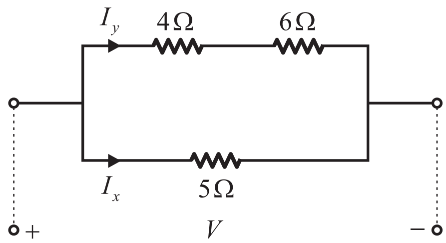
Therefore, heat generated by $4\ \Omega$ resistance is,

$$= 1.446^2 \times 4 = 8.3637 \text{ W}$$

$$= \frac{8.3637}{4.184} = 2 \text{ calories/sec}$$

Hence, the heat generated by the 4 Ω resistance is **2 calories/sec.**

Method 2



Heat $\propto I^2R$

From figure,

$$I_y = \frac{V}{4+6} = \frac{V}{10}$$

$$I_x = \frac{V}{5}$$

Heat generated in 5Ω resistance

$$= 10 \text{ Cal/sec}$$

Then, $$\frac{\text{Heat generated in } 5\ \Omega}{\text{Heat generated in } 4\ \Omega} = \frac{10}{H} = \frac{I_x^2 \times 5}{I_y^2 \times 4}$$

$$\left[\text{Since, Heat } \propto I^2R\right]$$

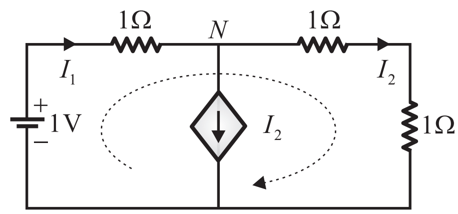
$$\frac{10}{H} = \frac{\left(\frac{V}{5}\right)^2 \times 5}{\left(\frac{V}{10}\right)^2 \times 4} \Rightarrow \frac{10}{H} = \frac{\frac{1}{25} \times 5}{\frac{1}{100} \times 4}$$

$$H = 2 \text{ calories/sec}$$

Hence, heat generated by the $4\ \Omega$ resistance is **2 calories/sec.**

1.51 **0.5**

Given circuit is shown below,



Applying KCL at node N,

$$-I_1 + I_2 + I_2 = 0$$

$$I_2 = \frac{I_1}{2} \qquad \ldots\text{(i)}$$

Applying KVL in the above shown loop,

$$-1+I_1\times 1+I_2\times 1+I_2\times 1=0 \qquad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$-1+I_1\times 1+\frac{I_1}{2}\times 1+\frac{I_1}{2}\times 1=0$$

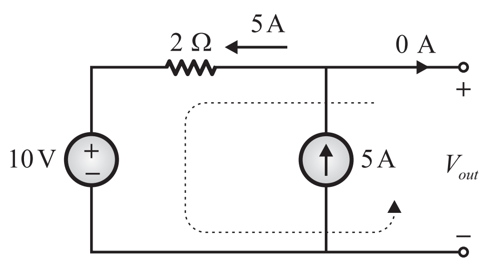
$$2I_1=1$$

$$I_1=\frac{1}{2}=0.5\text{ A}$$

Hence, the current supplied by the battery is **0.5 A.**

1.52　(D)

Given circuit is shown below,



Applying KVL in the given loop,

$$-V_{out}+5\times 2+10=0$$

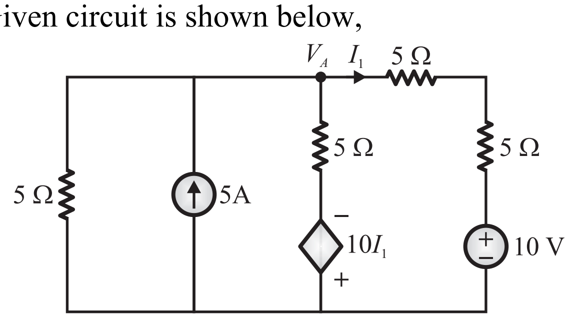
$$V_{out}=20\text{ V}$$

Voltage across current source $=V_{out}=20$ V

Hence, the correct option is (D).

1.53　11.428

Given circuit is shown below,



Applying KCL at node V_A,

$$\frac{V_A}{5}-5+\frac{V_A+10I_1}{5}+\frac{V_A-10}{10}=0$$

$$\frac{V_A}{5}+\frac{V_A}{5}+\frac{V_A}{10}+2I_1-5-1=0$$

$$\frac{4V_A+V_A}{10}+2I_1-6=0$$

$$\frac{V_A}{2}+2I_1=6 \qquad \ldots\text{(i)}$$

From the circuit,

$$I_1=\frac{V_A-10}{10} \qquad \ldots\text{(ii)}$$

Put the value of I_1 in equation (i),

$$\frac{V_A}{2}+2\left(\frac{V_A-10}{10}\right)=6$$

$$\frac{V_A}{2}+\frac{V_A}{5}-2=6$$

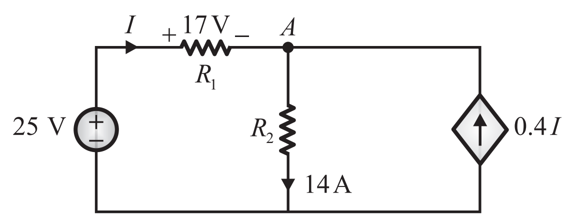
$$\frac{7V_A}{10}=8$$

$$V_A=\frac{8\times 10}{7}=11.428\text{ V}$$

Hence, the node voltage V_A is **11.428 V.**

1.54　250

Given circuit is shown below,



Applying KCL at node A,

$$I+0.4I-14=0$$

$$I=10\text{ A}$$

Power supplied by the 25 V source is given by,

$$P=VI=25\times 10=250\text{ W}$$

Hence, the power supplied by the 25 V source is **250 W.**

Scan for Video Solution

1.55 3

Given circuit is shown below,

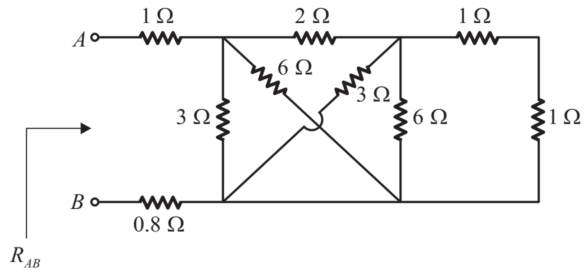


Fig. (a)

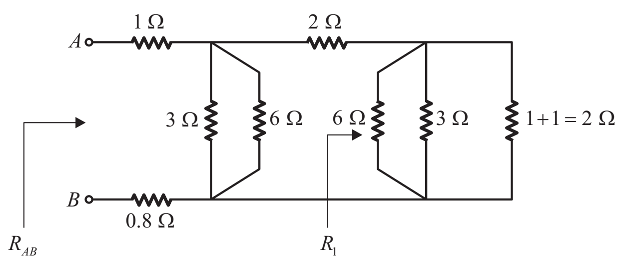


Fig. (b)

$$R_1 = 2 + (6 \,||\, 3 \,||\, 2) = 2 + 1 = 3\ \Omega$$

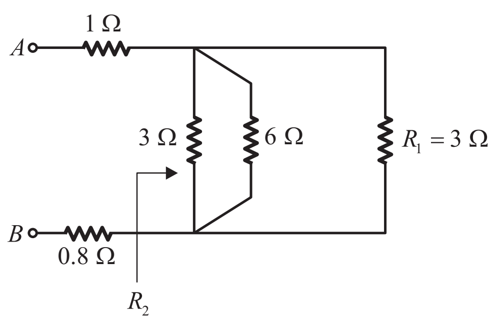


Fig. (c)

$$R_2 = 3 \,||\, 6 \,||\, R_1 = 3 \,||\, 6 \,||\, 3 = 1.2\ \Omega$$

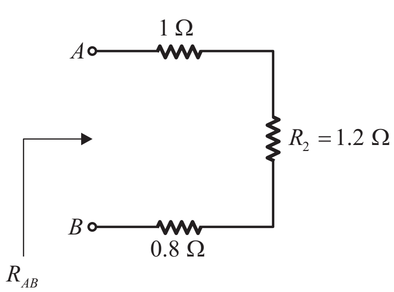


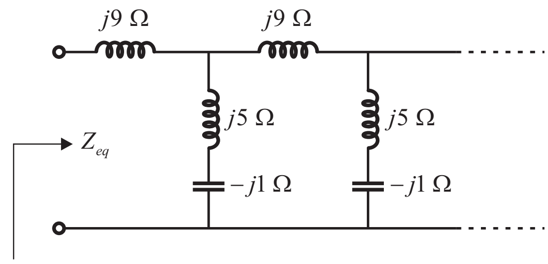
Fig. (d)

$$R_{AB} = 1 + 0.8 + 1.2 = 3\ \Omega$$

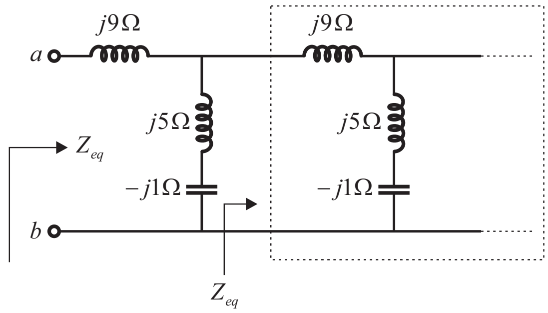
Hence, the equivalent resistance between the terminals A and B is **3 Ω.**

1.56 (A)

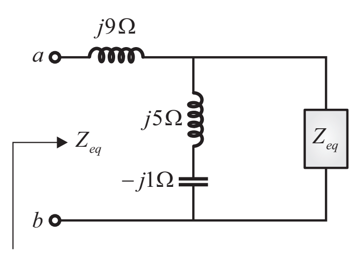
Given circuit is given below,



The given network consists only reactive element, the equivalent figure can be represented as,



The above circuit can be re-drawn as shown below,



The equivalent impedance (Z_{eq}) is,

$$Z_{eq} = \frac{j4 \times Z_{eq}}{j4 + Z_{eq}} + j9$$

$$Z_{eq} = \frac{(j4)Z_{eq} + j9(j4 + Z_{eq})}{j4 + Z_{eq}}$$

$$Z_{eq} = \frac{(j9)Z_{eq} + (j4)Z_{eq} - 36}{Z_{eq} + j4}$$

$$Z_{eq}^2 - (j9)Z_{eq} + 36 = 0$$

$$Z_{eq}^2 - (j12)Z_{eq} + (j3)Z_{eq} + 36 = 0$$

$$(Z_{eq}-j12)(Z_{eq}+j3)=0$$

$$Z_{eq}=j12$$

[$Z_{eq}=-j3$ is neglected as the whole circuit is a inductive nature]

Hence, the correct option is (A).

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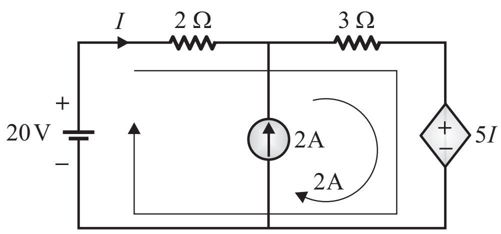
1.57 **1.4**

Given circuit is shown in figure,



Applying KVL in the loop shown,

$$-20+2I+3(I+2A)+5I=0$$

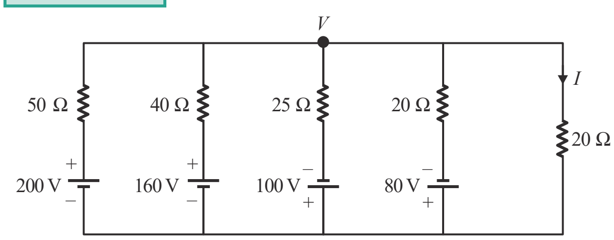
$$\Rightarrow \quad 10I=14$$

$$I=1.4\text{ A}$$

1.58 **0**

Given circuit is shown below

Method 1



Applying KCL at node

$$\frac{V-200}{50}+\frac{V-160}{40}+\frac{V+100}{25}+\frac{V+80}{20}+\frac{V}{20}=0$$

$$4V-800+5V-800+8V$$
$$+800+10V+800+10V=0$$

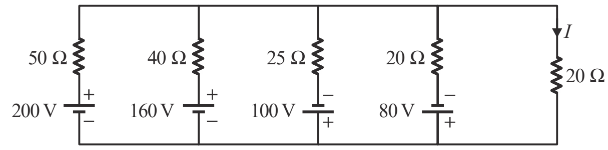
$$\Rightarrow \quad 37V=0$$

$$\therefore \quad V=0$$

Hence, $I=\dfrac{V-0}{20}=\dfrac{0}{20}=0$

Method 2

Using Millman's theorem :



By Millman's theorem,

$$E=\frac{\frac{200}{50}+\frac{160}{40}-\frac{100}{25}-\frac{80}{20}}{\frac{1}{50}+\frac{1}{40}+\frac{1}{25}+\frac{1}{20}}$$

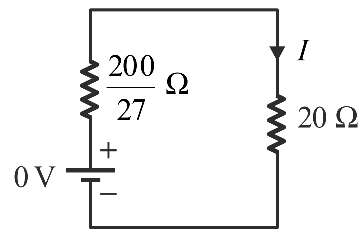
$$E=\frac{4+4-4-4}{\frac{1}{50}+\frac{1}{40}+\frac{1}{25}+\frac{1}{20}}=0$$

$$\frac{1}{R}=\frac{1}{50}+\frac{1}{40}+\frac{1}{25}+\frac{1}{20}$$

$$\frac{1}{R}=\frac{4+5+8+10}{200}=\frac{27}{200}$$

$$R=\frac{200}{27}$$

Equivalent circuit,

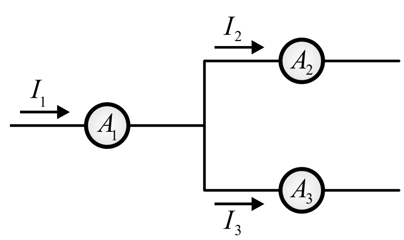


$$I=\frac{0}{20+\frac{200}{7}}=0\text{ A}$$

Hence, the current I flowing in above circuit will be **0 A.**

1.59 **1.732**

Given diagram is shown below



Applying KCL;

$$\vec{I}_1 = \vec{I}_2 + \vec{I}_3$$

$$\vec{I}_1 = 1\angle 10^0 + 1\angle 70^0$$

$$\vec{I}_1 = \cos 10^0 + j\sin 10^0 + \cos 70^0 + j\sin 70^0$$

$$\vec{I}_1 = 1.3268 + j1.113$$

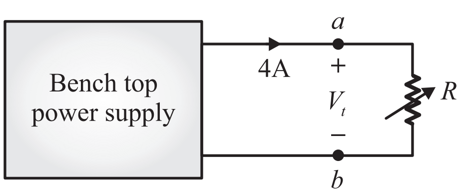
$$|\vec{I}_1| = 1.732 \text{ A}$$

1.60 (A)

Method 1

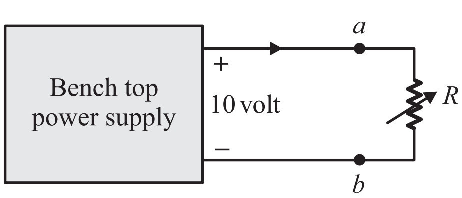
Case I :

Benchtop dc power supply will act as a ideal 4 A current source as long as it a terminal voltage is less than 10 V



Case II :

Benchtop dc power supply will act as a 10 V ideal voltage source for all load currents going down to 0 A



From **Case I** the maximum resistance offered by the ideal 4 A current source is

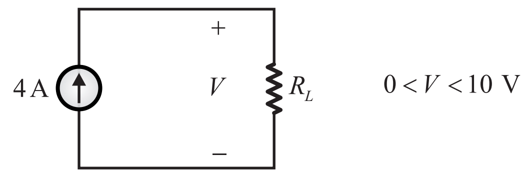
$$R = \frac{V}{I} = \frac{10}{4} = 2.5\,\Omega$$

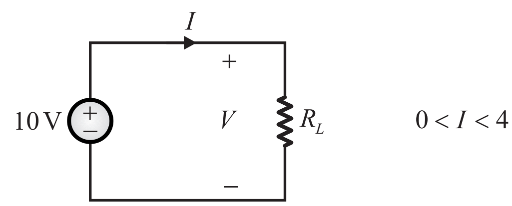
For maximum power transfer the value of load resistance should be equal to source resistance = 2.5

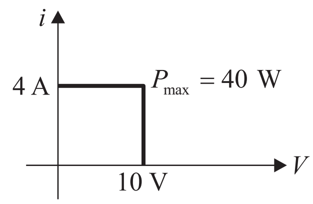
Current and voltage corresponding to load resistance of $2.5\,\Omega$ are 4 A and 10 Volt respectively.

Hence, the correct option is (A).

Method 2







From graph, it is clear that at maximum power $V = 10$ V and $i = 4$ A .

So, $\quad 4^2 \times R_L = 40 \qquad \dfrac{10^2}{R_L} = 40$

$$R_L = 2.5\ \Omega \qquad R_L = 2.5\ \Omega$$

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1 Basics of Signals

1994 IIT Kharagpur

1.1 The value of the integral $\int_{-5}^{6} e^{-2t}\delta(t-1)\,dt$ is equal to

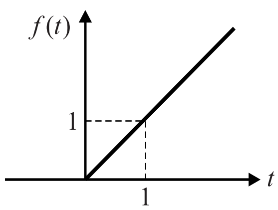
(A) 1 (B) e^{-2}

(C) 0 (D) e^{2}

1.2 Match the waveforms on the left-hand side with the correct mathematical description listed on the right hand side.

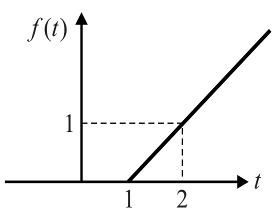
Waveform :

(A)



$(P)\ t.u(t-1)$

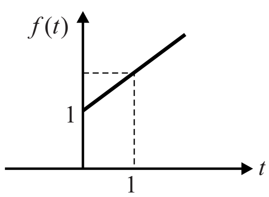
(B)



$(Q)\ (t+1).u(t-1)$

$(R)\ t.u(t)$

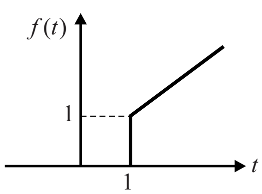
(C)



$(S)\ (t+1).u(t)$

$(T)\ (t-1).u(t)$

(D)



$(U)\ (t-1).u(t-1)$

$(V)\ (t-1).u(t+1)$

(A) A-R, B-S, C-U, D-P

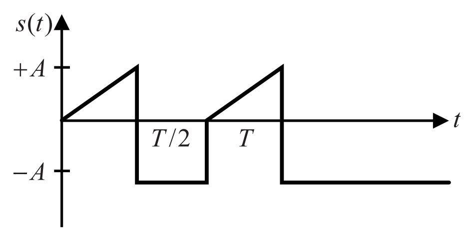
(B) A-R, B-U, C-S, D-P

(C) A-T, B-U, C-P, D-V

(D) A-P, B-T, C-S, D-U

1995 IIT Kanpur

1.3 The RMS value of the waveform $s(t)$ shown in fig is



(A) $\sqrt{\frac{3}{2}}A$ (B) $\sqrt{\frac{2}{3}}A$

(C) $\sqrt{\frac{1}{3}}A$ (D) $\sqrt{2}\,A$

1999 IIT Bombay

1.4 The RMS value of a half-wave rectifier symmetrical square wave current of $2A$ is

(A) $\sqrt{2}A$ (B) $1A$

(C) $\frac{1}{\sqrt{2}}A$ (D) $\sqrt{3}A$

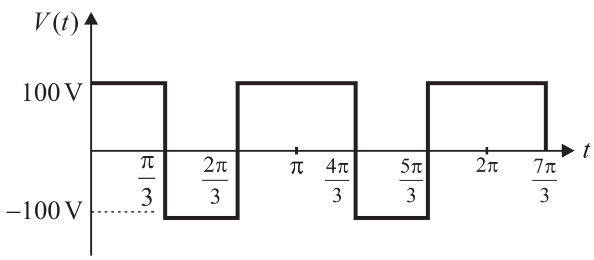
2002 IIT Kharagpur

1.5 If an a.c. voltage wave is corrupted with an arbitrary number of harmonics, then the overall voltage waveform differs from its fundamental frequency component in terms of

(A) only the peak values
(B) only the RMS values
(C) only the average values
(D) all the three measures (peak, RMS and average values)

2002 IISc Bangalore

1.6 What is the RMS value of the voltage waveform shown in the figures?



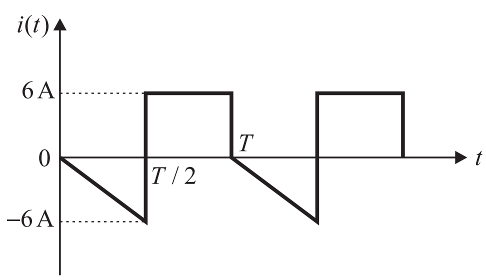
(A) $\left(\frac{200}{\pi}\right)$V (B) $\left(\frac{100}{\pi}\right)$V
(C) 200 V (D) 100 V

1.7 A current impulse $5\delta(t)$ is forced through a capacitor C. The voltage $V_c(t)$ across the capacitor is given by

(A) $5t$ (B) $5u(t)-C$
(C) $\frac{5}{C}t$ (D) $\frac{5u(t)}{C}$

2004 IIT Delhi

1.8 The RMS value of the periodic waveform given in figure is



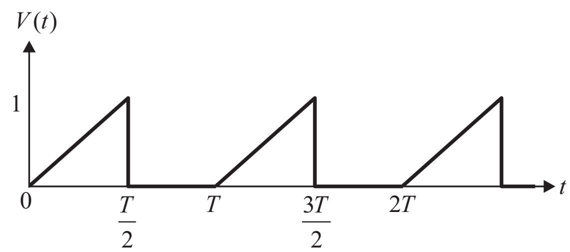
(A) $2\sqrt{6}$ A (B) $6\sqrt{2}$ A
(C) $\sqrt{\frac{4}{3}}$ A (D) 1.5 A

1.9 The rms value of the resultant current in a wire which carries a dc current of 10 A and a sinusoidal alternating current of peak value 20 is

(A) 14.1 A (B) 17.3 A
(C) 22.4 A (D) 30.0 A

2005 IIT Bombay

1.10 For the triangular waveform shown in the figure, the RMS value of the voltage is equal to



(A) $\sqrt{\frac{1}{6}}$ (B) $\sqrt{\frac{1}{3}}$
(C) $\frac{1}{3}$ (D) $\sqrt{\frac{2}{3}}$

2006 IIT Kharagpur

1.11 Which of the following is true?

(A) A finite signal is always bounded.
(B) A bounded signal always possesses finite energy.
(C) A bounded signal is always zero outside the interval $[-t_0, t_0]$ for some t_0.
(D) A bounded signal is always finite.

1.12 A continuous time system is described by $y(t) = e^{-|x(t)|}$, where $y(t)$ is the output and $x(t)$ is the input, $y(t)$ is bounded

(A) only when $x(t)$ is bounded.
(B) only when $x(t)$ is non-negative.
(C) only for $t \geq 0$ if $x(t)$ is bounded for $t \geq 0$.
(D) even when $x(t)$ is not bounded.

1.13 The running integrator given by

$$y(t)=\int_{-\infty}^{t}x(\tau)d\tau$$

(A) has no finite singularities in its double sided Laplace transform $Y(s)$.

(B) produces a bounded output for every causal bounded input.

(C) produces a bounded output for every anti causal bounded input.

(D) has no finite zeros in its double sided Laplace transform $Y(s)$.

2008 IISc Bangalore

1.14 Given a sequence $x[n]$ to generate the sequence $y[n]=x[3-4n]$, which one of the following procedures would be correct?

(A) First delay $x[n]$ by 3 samples to generate $z_1[n]$, then pick every 4^{th} sample of $z_1[n]$ to generate $z_2[n]$ and then finally time reverse $z_2[n]$ to obtain $y[n]$.

(B) First advance $x[n]$ by 3 samples to generate $z_1[n]$, then pick every 4^{th} sample of $z_1[n]$ to generate $z_2[n]$ and then finally time reverse $z_2[n]$ to obtain $y[n]$.

(C) First pick every fourth sample of $x[n]$ to generate $v_1[n]$, time-reverse $v_1[n]$ to obtain $v_2[n]$, and finally advance $v_2[n]$ by 3 sample to obtain $y[n]$.

(D) First pick every fourth sample of $x[n]$ to generate $v_1[n]$, time-reverse $v_1[n]$ to obtain $v_2[n]$, and finally delay $v_2[n]$ by 3 samples to obtain $y[n]$.

2010 IIT Guwahati

1.15 The period of the signal

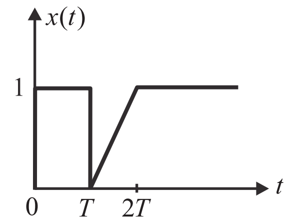
$$x(t)=8\sin\left(0.8\pi t+\frac{\pi}{4}\right) \text{ is}$$

(A) $0.4\pi s$　　(B) $0.8\pi s$

(C) $1.25\, s$　　(D) $2.5\, s$

2014 IIT Kharagpur

1.16 The function shown in the figure can be represented as **[Set - 01]**



(A) $u(t)-u(t-T)+\frac{(t-T)}{T}u(t-T)-\frac{(t-2T)}{T}u(t-2T)$

(B) $u(t)+\frac{t}{T}u(t-T)-\frac{t}{T}u(t-2T)$

(C) $u(t)-u(t-T)+\frac{(t-T)}{T}u(t)-\frac{(t-2T)}{T}u(t)$

(D) $u(t)+\frac{(t-T)}{T}u(t-T)-2\frac{(t-2T)}{T}u(t-2T)$

2015 IIT Kanpur

1.17 A moving average function is given by $y(t)=\frac{1}{T}\int_{t-T}^{t}u(\tau)d\tau$. If the input u is a sinusoidal signal of frequency $\frac{1}{2T}$ Hz, then in steady state, the output y will lag u (in degree) by ________. **[Set - 01]**

2016 IISc Bangalore

1.18 The value of $\int_{-\infty}^{+\infty} e^{-t}\delta(2t-2)dt$, where $\delta(t)$ is the Dirac delta function, is **[Set - 01]**

(A) $\frac{1}{2e}$ (B) $\frac{2}{e}$

(C) $\frac{1}{e^2}$ (D) $\frac{1}{2e^2}$

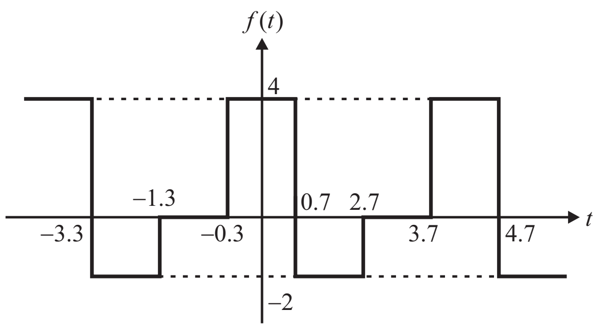
1.19 The value of the integral $2\int_{-\infty}^{\infty}\left(\frac{\sin 2\pi t}{\pi t}\right)dt$ is equal to **[Set - 02]**

(A) 0 (B) 0.5

(C) 1 (D) 2

2017 IIT Roorkee

1.20 The mean square value of the given periodic waveform $f(t)$ is **[Set - 02]**



2018 IIT Guwahati

1.21 Let f be the real valued function of a real variable defined $f(x) = x - [x]$, where $[x]$ denotes the largest integer less than or equal to x. The value of $\int_{0.25}^{1.25} f(x)dx$ is ______ (up to 2 decimal places).

1.22 Consider the two continuous-time signals defined below :

$$x_1(t) = \begin{cases} |t|, & -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} 1-|t|, & -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

These signals are sampled with a sampling period of $T = 0.25$ seconds to obtain discrete-time signals $x_1[n]$ and $x_2[n]$, respectively. Which one of the following statements is true?

(A) The energy of $x_1[n]$ is greater than the energy of $x_2[n]$

(B) The energy of $x_2[n]$ is greater than the energy of $x_1[n]$.

(C) $x_1[n]$ and $x_2[n]$ have equal energies.

(D) Neither $x_1[n]$ nor $x_2[n]$ is a finite-energy signal.

1.23 The signal energy of the continuous time signal

$$x(t) = [(t-1)u(t-1)] - [(t-2)u(t-2)] - [(t-3)u(t-3)] + [(t-4)u(t-4)]$$

is

(A) $\frac{11}{3}$ (B) $\frac{7}{3}$

(C) $\frac{1}{3}$ (D) $\frac{5}{3}$

2020 IIT Delhi

1.24 Which of the following is true for all possible non-zero choices of integers $m, n; m \ne n,$ or all possible non-zero choices of real numbers $p, q; p \ne q$, as applicable?

(A) $\lim_{\alpha\to\infty}\frac{1}{2a}\int_{-\alpha}^{\alpha}\sin p\theta \sin q\theta d\theta = 0$

(B) $\frac{1}{2\pi}\int_{-\pi}^{\pi}\sin p\theta \cos q\theta d\theta = 0$

(C) $\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2}\sin p\theta \sin q\theta d\theta = 0$

(D) $\frac{1}{\pi}\int_0^{\pi}\sin m\theta \sin n\theta d\theta = 0$

1.25 x_R and x_A are, respectively, the rms and average values of $x(t) = x(t-T)$, and similarly, y_R and y_A are respectively, the rms and average values of $y(t) = kx(t)$. k, T are independent of t. Which of the following is true ?

(A) $y_A = kx_A$; $y_R = kx_R$

(B) $y_A \neq kx_A$; $y_R \neq kx_R$

(C) $y_A \neq kx_A$; $y_R = kx_R$

(D) $y_A = kx_A$; $y_R \neq kx_R$

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Answers　Basics of Signals

1.1	**B**	**1.2**	**B**	**1.3**	**B**	**1.4**	**A**	**1.5**	**D**
1.6	**D**	**1.7**	**D**	**1.8**	**A**	**1.9**	**B**	**1.10**	**A**
1.11	**D**	**1.12**	**D**	**1.13**	**D**	**1.14**	**B**	**1.15**	**D**
1.16	**A**	**1.17**	**90**	**1.18**	**A**	**1.19**	**D**	**1.20**	**6**
1.21	**0.5**	**1.22**	**A**	**1.23**	**D**	**1.24**	**A), (B) and (D)**	**1.25**	**D**

Explanations　Basics of Signals

1.1　(B)

Given : $I = \int_{-5}^{6} e^{-2t}\delta(t-1)\,dt$　　...(i)

$$\left[\begin{array}{l}\text{By property of impulse function,}\\ \int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)\end{array}\right]$$

From equation (i),

$$I = e^{-2t}\big|_{t=1} = e^{-2}$$

Hence, the correct option is (B).

Key Point

$$\int_{t_1}^{t_2} x(t)\delta(t-t_0)dt = \begin{cases} x(t_0)\,; & t_1 < t_0 < t_2 \\ 0\,; & t_0 < t_1 \text{ and } t_0 > t_2 \\ \text{undefined}\,; & t_1 = t_0,\, t_2 = t_0 \end{cases}$$

where, $x(t)$ is continuous at t_0.

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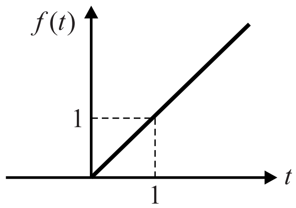
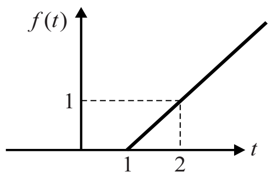
1.2 (B)

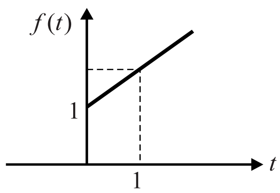
$f(t) = tu(t)$



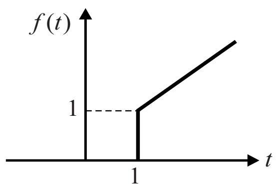
$f(t) = (t-1).u(t-1)$



$f(t) = (t+1).u(t)$



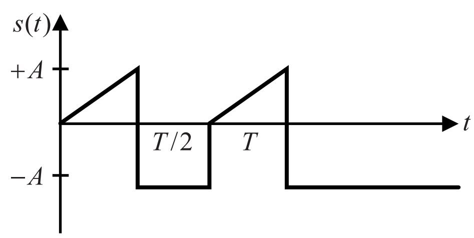
$f(t) = t.u(t-1)$



Hence, the correct option is (B).

1.3 (B)

Given :



Method 1

From the above figure,

$$s(t) = \begin{cases} \dfrac{2A}{T}t \; ; & 0 \le t \le \dfrac{T}{2} \\ -A \; ; & \dfrac{T}{2} \le t \le T \end{cases}$$

Time period of $s(t)$ is T.

RMS value of $s(t)$ is given by,

$$s_{rms}(t) = \sqrt{\frac{1}{T}\int_0^T s^2(t)dt}$$

$$s_{rms}(t) = \sqrt{\left[\frac{1}{T}\left\{\int_0^{\frac{T}{2}}\left(\frac{2At}{T}\right)^2 dt + \int_{\frac{T}{2}}^{T} A^2 dt\right\}\right]}$$

$$s_{rms}(t) = \sqrt{\left[\frac{1}{T}A^2\left[\frac{4}{3T^2}(t^3)_0^{T/2} + (t)_{T/2}^{T}\right]\right]}$$

$$s_{rms}(t) = \sqrt{\frac{A^2}{T}\left[\frac{4}{3T^2}\left(\frac{T^3}{8} - 0\right) + \left(T - \frac{T}{2}\right)\right]}$$

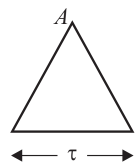
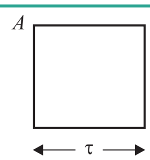
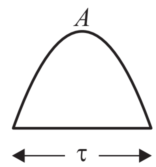
$$s_{rms}(t) = \sqrt{\frac{A^2}{T}\left[\frac{T}{6} + \frac{T}{2}\right]} = \sqrt{\frac{2}{3}}A$$

Hence, the correct option is (B).

Method 2

RMS value of a signal $x(t)$ is given by,

$$x_{rms} = \sqrt{\text{Power}} = \sqrt{\frac{\text{Energy}}{\text{Time period}}}$$

Signal	Energy
	$E = \dfrac{A^2\tau}{3}$
	$E = A^2\tau$
Sine wave 	$E = \dfrac{A^2\tau}{2}$

For duration 0 to $T/2$ the given waveform is triangular wave and from $T/2$ to T it is square wave.

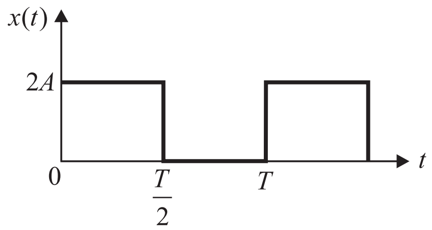
$$x_{rms} = \sqrt{\left[\frac{\frac{A^2}{3}\left(\frac{T}{2}\right) + A^2\left(\frac{T}{2}\right)}{T}\right]}$$

$$x_{rms} = \sqrt{\frac{A^2}{6} + \frac{A^2}{2}} = \sqrt{\frac{2}{3}}A$$

Hence, the correct option is (B).

1.4 (A)

The half-wave rectified square wave is shown below,



From the above figure,

$$x(t) = \begin{cases} 2A; & 0 \le t < \frac{T}{2} \\ 0; & \frac{T}{2} < t \le T \end{cases}$$

Time period of $x(t)$ is T.

RMS value of $x(t)$ is given by,

$$x_{rms}(t) = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt}$$

$$x_{rms}(t) = \sqrt{\left[\frac{1}{T}\left\{\int_0^{\frac{T}{2}} (2A)^2 dt + \int_{\frac{T}{2}}^{T} (0)^2 dt\right\}\right]}$$

$$x_{rms}(t) = \sqrt{\frac{1}{T}\left[4A^2\left(\frac{T}{2} - 0\right) + 0\right]}$$

$$x_{rms}(t) = \sqrt{\frac{4TA^2}{2T}} = \sqrt{2}A$$

Hence, the correct option is (A).

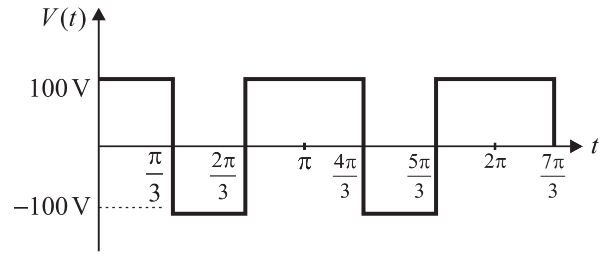
1.5 (D)

Waveform differs from its fundamental frequency component in terms of all the three measures (Peak, RMS, and average value).

Hence, the correct option is (D).

1.6 (D)

Given :



From the above figure,

$$V(t) = \begin{cases} 100\text{V} \ ; & 0 < t \le \frac{\pi}{3} \text{ and } \frac{2\pi}{3} < t \le \pi \\ -100\text{ V} \ ; & \frac{\pi}{3} < t \le \frac{2\pi}{3} \end{cases}$$

Time period of $V(t)$ is π.

RMS value of $V(t)$ is given by,

$$V_{rms}(t) = \sqrt{\frac{1}{T}\int_0^T V^2(t)dt}$$

$$V_{rms}(t) = \sqrt{\frac{1}{\pi}\left[\int_0^{\frac{\pi}{3}} 100^2 dt + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (-100)^2 dt + \int_{\frac{2\pi}{3}}^{\pi} 100^2 dt\right]}$$

$$V_{rms}(t) = \sqrt{\frac{100^2}{\pi}\left[\left(\frac{\pi}{3} - 0\right) + \left(\frac{2\pi}{3} - \frac{\pi}{3}\right) + \left(\pi - \frac{2\pi}{3}\right)\right]}$$

$$V_{rms}(t) = \sqrt{\frac{100^2}{\pi}\left(\frac{\pi}{3} + \frac{\pi}{3} + \frac{\pi}{3}\right)}$$

$$V_{rms}(t) = 100$$

Hence, the correct option is (D).

1.7 (D)

Given : $i_C(t) = 5\delta(t)$

The current across the capacitor is given by,

$$i_C(t) = C\frac{dV_C(t)}{dt}$$

$$5\delta(t) = C\frac{dV_C(t)}{dt}$$

$$\frac{5\delta(t)}{C} = \frac{dV_C(t)}{dt}$$

$$dV_C(t) = \frac{5}{C}\delta(t)dt$$

Integrating both sides,

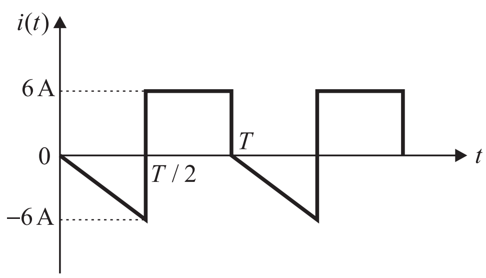
$$\int dV_C(t) = \frac{5}{C}\int \delta(t)dt$$

$$V_C(t) = \frac{5u(t)}{C}$$

Hence, the correct option is (D).

1.8 (A)

Given :



From the above figure,

$$i(t) = \begin{cases} -\frac{12}{T}t; & 0 \le t < \frac{T}{2} \\ 6\ ; & \frac{T}{2} \le t < T \end{cases}$$

Time period of $i(t)$ is T.

RMS value of $i(t)$ is given by,

$$i_{rms}(t) = \left[\sqrt{\frac{1}{T}\int_0^T \{i(t)\}^2 dt}\right]$$

$$i_{rms}(t) = \left[\sqrt{\frac{1}{T}\left\{\int_0^{T/2}\left(-\frac{12}{T}t\right)^2 dt + \int_{T/2}^{T}(6)^2 dt\right\}}\right]$$

$$i_{rms}(t) = \left[\sqrt{\frac{1}{T}\left\{\frac{144}{T^2}\left(\frac{t^3}{3}\right)_0^{T/2} + 36(t)_{T/2}^{T}\right\}}\right]$$

$$i_{rms}(t) = \left[\sqrt{\frac{1}{T}\left\{\frac{144}{T^2}\times\frac{T^3}{8\times 3} + 36\times\frac{T}{2}\right\}}\right]$$

$$i_{rms}(t) = \sqrt{\frac{1}{T}[6T + 18T]} = \sqrt{24} = 2\sqrt{6}\text{ A}$$

Hence, the correct option is (A).

Scan for Video Solution

1.9 (B)

Given : $i_{dc} = 10\text{ A}$, $i_{ac} = 20\cos\theta$

$$i(\theta) = i_{dc} + i_{ac} = 10 + 20\cos\theta$$

The RMS value of signal is,

$$i_{rms}(\theta) = \sqrt{i_{dc}^2 + \left(\frac{i_{m_{ac}}}{\sqrt{2}}\right)^2}$$

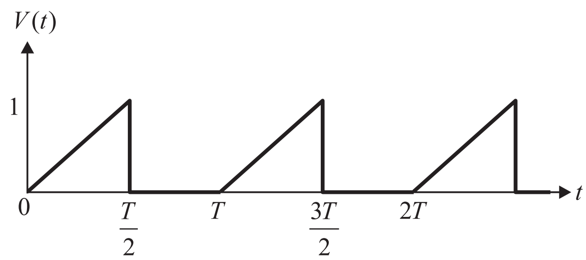
$$i_{rms}(\theta) = \sqrt{10^2 + \frac{1}{2}\times 20^2} = \sqrt{100+200}$$

$$i_{rms}(\theta) = \sqrt{300} = 10\sqrt{3} = 17.3\text{ A}$$

Hence, the correct option is (B).

1.10 (A)

Given :



From the above figure,

$$V(t) = \begin{cases} \frac{2}{T}t\ ; & 0 \le t < \frac{T}{2} \\ 0\ ; & \text{otherwise} \end{cases}$$

Time period of $V(t)$ is T.

RMS value of $V(t)$ is given by,

$$V_{rms}(t) = \sqrt{\frac{1}{T}\int_0^T V^2(t)dt} = \sqrt{\frac{1}{T}\int_0^{T/2}\left(\frac{2}{T}t\right)^2 dt}$$

$$V_{rms}(t) = \sqrt{\frac{1}{T}\times\frac{4}{T^2}\left(\frac{t^3}{3}\right)_0^{T/2}}$$

$$V_{rms}(t) = \sqrt{\frac{1}{T}\times\frac{4}{T^2}\times\frac{T^3}{8\times 3}} = \sqrt{\frac{1}{6}}$$

Hence, the correct option is (A).

Scan for Video Solution

1.11 (D)

Bounded signals : A signal is said to be bounded signal if it has finite amplitude i.e. bounded signal is bounded in amplitude. It is represented as shown below,

$$x(t) = \text{ Finite amplitude, } -\infty < t < \infty$$

Ex : $\sin \omega t$, $\cos \omega t$, $e^{-at}u(t)$,

$$u(t),\ \text{rect}\left(\frac{t}{\tau}\right),\ \text{tri}\left(\frac{t}{\tau}\right),\ \sin c(t)$$

Finite signals : A finite signal is said to be finite duration signal.

Ex : Impulse signal, $\delta(t)$

It has zero duration and infinite amplitude. Therefore, it is unbounded.

From option (A),

A finite signal is always bounded.

From the definition of finite signals, it is clear that statement in option (A) is wrong.

From option (B),

A bounded signal always possesses finite energy.

Unit step signal $u(t)$ is a bounded signal as its amplitude is bounded by 1. It is a power signal whose energy is infinite.

Another example is $e^{-at}u(t)$ which is also a bounded signal with finite energy.

Therefore, statement in option (B) is wrong.

From option (C),

A bounded signal is always zero outside the interval $[-t_0, t_0]$ for some t_0.

Bounded signal $u(t)$ has amplitude 1 from 0 to ∞.

Hence, statement in option (C) is also wrong.

From option (D),

A bounded signal is always finite.

From the definition of bounded signals, it is clear that a bounded signal always has finite amplitude.

Therefore, statement in option (D) is correct.

Hence, the correct option is (D).

1.12 (D)

Given : $y(t) = e^{-|x(t)|}$

$|x(t)|$ is always positive for positive as well as negative value of $x(t)$. Hence, power appearing in the exponential term will always be negative.

If $x(t) = \pm\infty$ for any value of t then

$$y(t) = e^{-|\pm\infty|} = e^{-\infty} = 0$$

$e^{-|x(t)|}$ is always convergent, even when $x(t)$ is not bounded.

Thus, $e^{-|x(t)|}$ is bounded even when $x(t)$ is not bounded.

Hence, the correct option is (D).

1.13 (D)

Given : $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

The double sided Laplace Transform for $y(t)$ is,

$$Y(s) = \frac{X(s)}{s}$$

So, it has singularity at origin i.e. at $s = 0$.

The output of integrator would be unbounded even for bounded input, if the duration of the signal is infinite.

So, the integrator output is unbounded, for causal signal i.e. $x(t) = u(t)$.

Integrator output is unbounded, even for anti-causal signal i.e. $x(t) = u(-t)$.

The double sided Laplace transform has a zero at $s = \infty$ .Thus, it has no finite zeros.

Hence, the correct option is (D).

1.14 (B)

Given : $y[n] = x[3-4n] = x[-4n+3]$

To obtain $y[n]$ the following steps must be taken :

Step-1 : Advance $x[n]$ by 3 samples,

i.e. $\quad z_1[n] = x[n+3]$

Step-2 : Take every fourth sample of $z_1[n]$ i.e.

$$z_2[n] = z_1[4n] = x[4n+3]$$

Step-3 : Time reverse of $z_2[n]$ will be

$$y[n] = z_2[-n] = x[-4n+3]$$

Hence, the correct option is (B).

Note : If we want to do time scaling first, we may proceed as given below

(i) Take every fourth sample of $x(n)$ i.e.

$$z_1[n] = x(4n)$$

(ii) Time reverse of $z_1[n]$ will be $z_2[n]$

$$z_2[n] = z_1[-n] = x[-4n]$$

(iii) Delay $z_2[n]$ by $\frac{3}{4}$ samples to obtain $y[n]$

$$y[n] = z_2\left[n - \frac{3}{4}\right] = x\left[-4\left(n - \frac{3}{4}\right)\right]$$

$$y[n] = x[-4n+3]$$

[But, delaying a discrete signal by 3/4 (fraction) samples is NOT possible]. So, above sequence of operation will give incorrect output**.**

1.15 (D)

Given : $x(t) = 8\sin\left(0.8\pi t + \frac{\pi}{4}\right)$

Comparing with standard equation,

$$x(t) = A\sin(\omega t + \phi),\ \omega = 0.8\pi$$

Time period is given by,

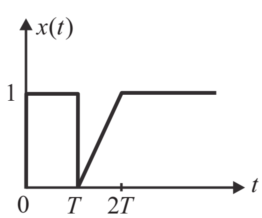
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.8\pi} = 2.5\,\text{s}$$

Hence, the correct option is (D).

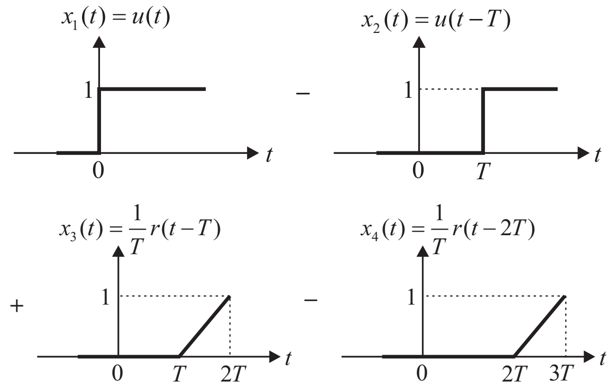
Scan for Video Solution

1.16 (A)

Given :



Let, $x(t) = x_1(t) - x_2(t) + x_3(t) - x_4(t)$



$$x(t) = u(t) - u(t-T) + \frac{1}{T}r(t-T) - \frac{1}{T}r(t-2T)$$

$$x(t) = u(t) - u(t-T) + \frac{1}{T}(t-T)u(t-T) - \frac{1}{T}(t-2T)u(t-2T)$$

Hence, the correct option is (A).

1.17 (90)

Given : $y(t) = \frac{1}{T}\int_{t-T}^{t} u(T)dT$

and $u(t)$ is sinusoidal signal of frequency f, i.e. $u(\tau) = A\sin\omega\tau$

$$f = \frac{1}{2T}$$

$$\omega = 2\pi f = 2\pi \times \frac{1}{2T} = \frac{\pi}{T}$$

Thus, $\omega T = \pi$

So, $y(t) = \frac{1}{T}\int_{t-T}^{t} A\sin\omega\tau\, d\tau$

$$y(t) = -\frac{A}{\omega T}[\cos\omega\tau]_{t-T}^{t}$$

$$y(t) = -\frac{A}{\pi}[\cos\omega t - \cos\omega(t-T)]$$

[Since, $\omega T = \pi$]

$$y(t) = -\frac{A}{\pi}[\cos\omega t - \cos(\omega t - \pi)]$$

$$y(t) = -\frac{A}{\pi}\cdot 2\cos\omega t$$

$$y(t) = -\frac{2A}{\pi}\cos\omega t = -\frac{2A}{\pi}\sin(90^0 - \omega t)$$

$$y(t) = \frac{2A}{\pi}\sin(\omega t - 90^0)$$

Thus, $y(t)$ lags $u(t)$ by $\theta = 90^0$.

Hence, the output y will lag u is $\mathbf{90^0}$.

1.18 (A)

Given : $I = \int_{-\infty}^{+\infty} e^{-t}\delta(2t-2)dt$

$$I = \int_{-\infty}^{+\infty} e^{-t}\delta[2(t-1)]\,dt \qquad \ldots(\text{i})$$

$$\left[\begin{array}{l}\text{By time scaling property of} \\ \text{impulse function, } \delta(at) = \dfrac{1}{|a|}\delta(t)\end{array}\right]$$

Thus from equation (i),

$$I = \int_{-\infty}^{\infty} e^{-t}\frac{1}{|2|}\delta(t-1)\,dt = \frac{1}{2}\int_{-\infty}^{\infty} e^{-t}\delta(t-1)\,dt$$

$$\left[\begin{array}{l}\text{By property of impulse function,} \\ \displaystyle\int_{-\infty}^{\infty} x(t)\,\delta(t-a)\,dt = x(a)\end{array}\right]$$

$$I = \frac{1}{2}e^{-t}\Big|_{t=1} = \frac{1}{2}e^{-1} = \frac{1}{2e}$$

Hence, the correct option is (A).

Scan for Video Solution

1.19 (D)

Given : $I = 2\int_{-\infty}^{\infty}\left(\frac{\sin 2\pi t}{\pi t}\right)dt$

$$I = 2\times 2\int_{-\infty}^{\infty}\frac{\sin 2\pi t}{2\pi t}dt = 4\int_{-\infty}^{\infty}\text{sinc}\,(2t)\,dt$$

$$\left[\begin{array}{l}\text{From definition of area,} \\ \displaystyle\int_{-\infty}^{\infty} x(t)dt = \text{Area of } x(t)\end{array}\right]$$

$$I = 4\int_{-\infty}^{\infty}\text{sinc}\,(2t)dt$$

$$I = 4\times\text{Area of sinc}\,(2t) \qquad \ldots(\text{i})$$

$$\left[\begin{array}{l}\text{Area of sinc function is unity,} \\ \text{i.e. } \displaystyle\int_{-\infty}^{\infty}\text{sinc}\,(t)dt = 1\end{array}\right]$$

$$\left[\begin{array}{l}\text{Using time scaling property,} \\ \displaystyle\int_{-\infty}^{\infty}\text{sinc}\,(kt)dt = \frac{1}{k}\end{array}\right]$$

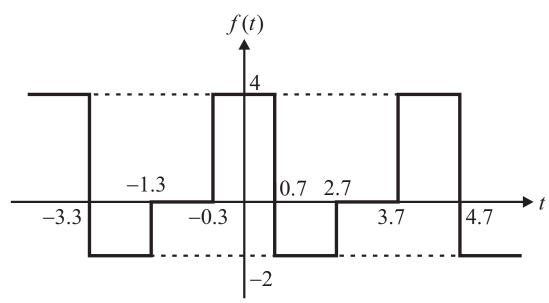
From equation (i),

$$I = 4\times\frac{1}{2} = 2$$

Hence, the correct option is (D).

1.20 6

Given :



For complete period, $f(t)$ may be mathematically defined as,

$$f(t) = \begin{cases} 4; & -0.3 < t < 0.7 \\ -2; & 0.7 < t < 2.7 \\ 0; & 2.7 < t < 3.7 \end{cases}$$

Mean square value of a periodic waveform is given by,

$$M = \frac{1}{T_0}\int_{T_0} f^2(t)\,dt$$

Time period of $f(t)$ is 4. i.e. $T_0 = 4$

$$M = \frac{1}{4}\int_{-0.3}^{3.7} f^2(t)dt$$

$$M = \frac{1}{4}\int_{-0.3}^{0.7} f^2(t)dt + \frac{1}{4}\int_{0.7}^{2.7} f^2(t)dt + \frac{1}{4}\int_{2.7}^{3.7} f^2(t)dt$$

$$M = \frac{1}{4}\left[\int_{-0.3}^{0.7}(4)^2\,dt + \int_{0.7}^{2.7}(-2)^2\,dt + 0\right]$$

$$M = \frac{1}{4}[16\{0.7-(-0.3)\}+4(2.7-0.7)]$$

$$M = \frac{1}{4}[16+8] = 6$$

Hence, the mean square value of the given periodic waveform $f(t)$ is **6.**

Scan for Video Solution

1.21 0.5

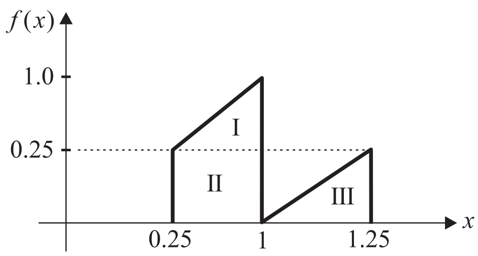
Given : $f(x) = x - [x]$

where, $[x]$ represents the largest integer less than or equal to x.

For $0 \le x < 1 \Rightarrow f(x) = x - [x] = x - 0 = x$

For $1 \le x < 2 \Rightarrow f(x) = x - [x] = x - 1$

Hence, $f(x) = \begin{cases} x; & 0.25 \le x < 1 \\ x-1; & 1 \le x < 1.25 \end{cases}$



$$\int_{0.25}^{1.25} f(x)dx = \text{Area of triangle I}$$
$$+ \text{Area of rectangle II} + \text{Area of triangle III}$$

$$\int_{0.25}^{1.25} f(x)dx = \left(\frac{1}{2}\times 0.75\times 0.75\right)+(0.75\times 0.25)$$
$$+\left(\frac{1}{2}\times 0.25\times 0.25\right)$$

$$\int_{0.25}^{1.25} f(x)dx = 0.28125+0.1875+0.03125 = 0.5$$

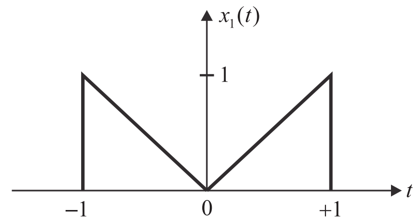
Hence, the value of $\int_{0.25}^{1.25} f(x)dx$ is **0.5.**

Scan for Video Solution

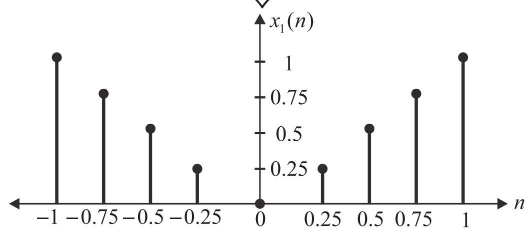
1.22 (A)

Given :

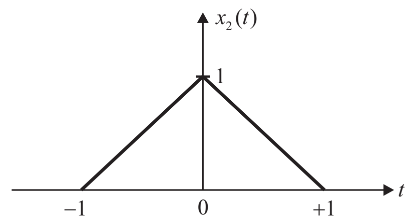
(a) $x_1(t) = \begin{cases} |t|, & -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$



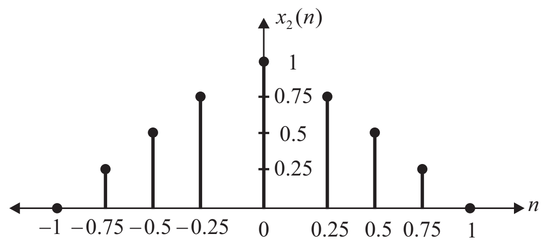
Sampling $\Downarrow$ $T_s = 0.25\,\text{Sec}$



(b) $x_2(t) = \begin{cases} 1-|t|, & -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$



Sampling $\Downarrow$ $T_s = 0.25$ sec



$$E_1 = \sum_{n=-\infty}^{\infty} |x_1(n)|^2$$

$$E_1 = 0^2 + 2\left[1^2+0.75^2+0.5^2+0.25^2\right]$$

$$E_1 = 3.75 \qquad \ldots\text{(i)}$$

$$E_2 = \sum_{n=-\infty}^{\infty} |x_2(n)|^2$$

$$E_2 = 1^2 + 2\left[0.75^2+0.5^2+0.25^2+0^2\right]$$

$$E_2 = 2.75 \qquad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$E_1 > E_2$$

Hence, the correct option is (A).

1.23 (D)

Given : $x(t) = [(t-1)u(t-1)] - [(t-2)u(t-2)]$

$$-[(t-3)u(t-3)] + [(t-4)u(t-4)]$$

Since, $r(t) = tu(t)$

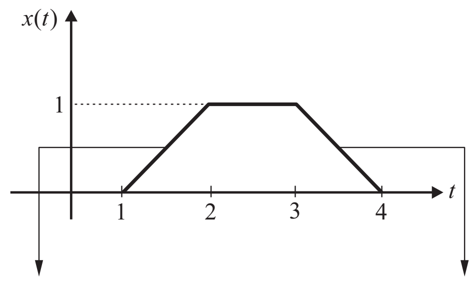
Put $t = t-1$, $r(t-1) = (t-1)u(t-1)$

Put $t = t-2$, $r(t-2) = (t-2)u(t-2)$

Put $t = t-3$, $r(t-3) = (t-3)u(t-3)$

Put $t = t-4$, $r(t-4) = (t-4)u(t-4)$

$$x(t) = r(t-1) - r(t-2) - r(t-3) + r(t-4)$$



$$x(t) - 0 = \frac{1-0}{2-1}(t-1) \qquad x(t) - 1 = \frac{0-1}{4-3}(t-3)$$

$$x(t) = t-1 \qquad x(t) = -t+3+1 = -t+4$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_1^2 (t-1)^2\, dt + \int_2^3 1^2\, dt + \int_3^4 (4-t)^2\, dt$$

$$E = \int_1^2 (t^2+1-2t)dt + \int_2^3 1^2\, dt + \int_3^4 (16+t^2-8t)dt$$

$$E = \left(\frac{t^3}{3} + t - t^2\right)_1^2 + (t)_2^3 + \left(16t + \frac{t^3}{3} - 4t^2\right)_3^4$$

$$E = \left(\frac{8}{3} + 2 - 4 - \frac{1}{3} - 1 + 1\right) + (3-2)$$

$$+\left(64 + \frac{64}{3} - 64 - 48 - 9 + 36\right)$$

$$E = \frac{5}{3}$$

Hence, the correct option is (D).

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1.24 (A), (B) and (D)

For this question, out of given choices, more than one options are conditionally true but none of the given option is correct if we consider $m = -n$ or $p = -q$ which satisfies the given conditions of $m \neq n$ and $p \neq q$ mentioned in the question. So this question must go in the category of MTA. For any conclusion, the statement that must be given in the question should be $|m| \neq |n|$ and $|p| \neq |q|$.

IIT has given one of the conditionally true option i.e. option (B) in their final answer key by changing their previously given option (A).

Checking the options for their validity :

Given : $m, n : m \neq n \rightarrow$ Integers

$p, q : p \neq q \rightarrow$ real numbers

From option (D)

From orthogonality between sine and sine functions

$$\frac{1}{L}\int_0^L \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right)dx = \begin{Bmatrix} 0;\ m \neq n \\ \frac{1}{2};\ m = n \end{Bmatrix} \quad \ldots\text{(i)}$$

$$\frac{1}{L}\int_{-L}^L \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right)dx = \begin{Bmatrix} 0;\ m \neq n \\ 1;\ m = n \neq 0 \end{Bmatrix} \quad \ldots\text{(ii)}$$

where m & n must be integers

For, $L = \pi$, from equation (i)

$$\frac{1}{\pi}\int_0^{\pi} \sin mx \sin nx dx = 0$$

So, option (D) is correct, which can be verified by evaluating the given integral as follows

$$I = \frac{1}{\pi}\int_0^{\pi} \sin m\theta \sin n\theta d\theta$$

$$I = \frac{1}{2\pi}\int_0^{\pi} -\cos(m+n)\theta\, d\theta + \int_0^{\pi} \cos(m-n)\theta\, d\theta$$

$$I = \frac{1}{2\pi}\left[-\frac{\sin(m+n)\theta}{(m+n)}\right]_0^{\pi} + \left[\frac{\sin(m-n)\theta}{(m-n)}\right]_0^{\pi}$$

$$I=\frac{1}{2\pi}\left[-\frac{\sin(m+n)\pi}{(m+n)}+\frac{\sin(m+n)\times 0}{(m+n)}+\frac{\sin(m-n)\pi}{(m-n)}-\frac{\sin(m-n)\times 0}{(m-n)}\right]$$

$$I=\frac{1}{2\pi}[-0+0+0-0]$$

for $|m|\neq|n|$ only, also $\sin m\pi=0,\ \forall m=1,2,\ldots..$

$$I=0$$

From option (A), that was given as correct option in first answer key by IIT.

$$I=\lim_{\alpha\to\infty}\frac{1}{4\alpha}\left[\int_{-\alpha}^{\alpha}\cos(p-q)\theta d\theta-\int_{-\alpha}^{\alpha}\cos(p+q)\theta d\theta\right]$$

$$I=\lim_{\alpha\to\infty}\frac{1}{4\alpha}\left[\frac{\sin(p-q)\theta}{(p-q)}-\frac{\sin(p+q)\theta}{(p+q)}\right]_{-\alpha}^{\alpha}$$

$$I=\lim_{\alpha\to\infty}\frac{1}{4\alpha}\left[\frac{2\sin(p-q)\alpha}{(p-q)}-\frac{2\sin(p+q)\alpha}{(p+q)}\right]$$

$$I=\lim_{\alpha\to\infty}\frac{1}{2\alpha}\left[\frac{\sin(p-q)\alpha}{(p-q)}-\frac{\sin(p+q)\alpha}{(p+q)}\right]$$

$$I=\lim_{\alpha\to\infty}\frac{1}{\alpha}\left[\frac{\sin(p-q)\alpha}{(p-q)\alpha}-\frac{\sin(p+q)\alpha}{(p+q)\alpha}\right]$$

$$\lim_{\alpha\to\infty}\frac{\sin\theta}{\alpha}=0$$

$\therefore \quad I=0$

For $|p|\neq|q|$ only.

So, option (A) is also conditionally true.

From option (B), which is given as correct option in final answer key by IIT,

$$I=\frac{1}{2\pi}\int_{-\pi}^{\pi}\sin p\theta\cos q\theta d\theta$$

Given integral can also conditionally give the result zero as the product of *sine* and *cos* functions will be an odd function and result of integration from negative to positive symmetric limits of an odd function is zero.

1.25 (D)

Given : $x(t-T)=x(t)$, $x(t)$ is periodic with period T

$\therefore$ Average value of $x(t)$

$$x_A=\frac{1}{T}\int_0^T x(t)dt \qquad \ldots\text{(i)}$$

RMS value of $x(t)$

$$x_R=\sqrt{\frac{1}{T}\int_0^T |x(t)^2|dt} \qquad \ldots\text{(ii)}$$

Given $y(t)=k\cdot x(t)$, period of $y(t)=$ Period of $x(t)=T$

$\therefore$ Average value of $y(t)$

$$y_A=\frac{1}{T}\int_0^T k\cdot x(t)dt$$

$$y_A=k\cdot\left[\frac{1}{T}\int_0^T x(t)dt\right]=k\cdot x_A$$

RMS value of $y(t)$

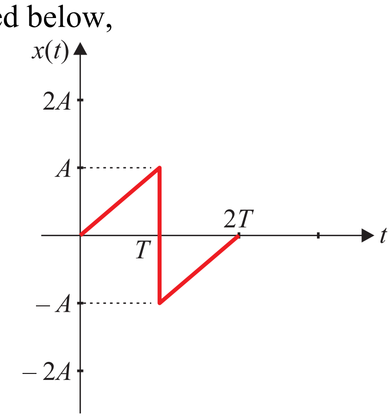
$$y_R=\sqrt{\frac{1}{T}\int_0^T |kx(t)|^2\,dt}$$

$$y_R=|k|\sqrt{\frac{1}{T}\int_0^T |x(t)^2|dt}=|k|\cdot x_R$$

As rms value can never be negative, so irrespective of the sign of k, y_R will always be positive. So if k is a negative constant then $y_R=k\cdot x_R$ is not true.

Hence, the correct option is (D).

It can be verified by a simple example, as explained below,



Consider a continuous time periodic signal $x(t)$ for which one period is shown in figure,

$$RMS=\sqrt{\text{Power}}$$

$$\text{Power}=\frac{\text{Energy in 1 period}}{\text{Time period}}$$

Energy of $x(t)$ in 1 period,

$$E_x = \frac{A^2T}{3} + \frac{A^2T}{3} = \frac{2A^2T}{3}$$

$\therefore$ Power of $x(t)$,

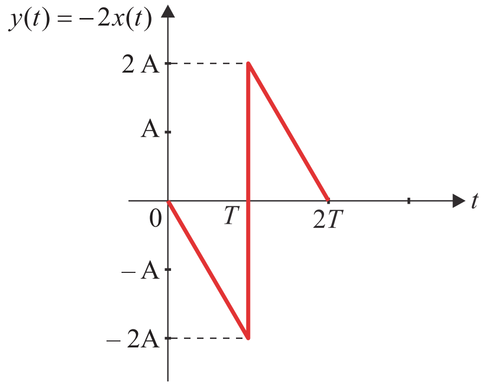
$$P_x = \frac{2A^2T}{3\times 2T} = \frac{A^2}{3}$$

$$\therefore \quad x(t)_{rms} = x_R = \sqrt{\frac{A^2}{3}} = \frac{A}{\sqrt{3}} \quad \ldots\text{(i)}$$

Given : $y(t) = kx(t)$

For $k = -2$, $y(t) = -2x(t)$

$y(t)$ is shown in figure,



$$\text{Energy of } y(t) = \frac{4A^2T}{3} + \frac{4A^2T}{3} = \frac{8A^2T}{3}$$

$$\therefore \quad \text{Power of } y(t) = \frac{8A^2T}{3} \times \frac{1}{2T} = \frac{4A^2}{3}$$

$$\therefore \quad y(t)_{rms} = y_R = \sqrt{\frac{4A^2}{3}} = 2\times\frac{A}{\sqrt{3}} \quad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$y_R \neq kx_R \text{ as } k = -2,\ k \neq 2$$

Hence, option (A) is not true for any negative value of k.

Given answer in IIT answer key : Option (A).

IIT should have given its correct option as option (D), but they have given option (A) only in their final answer key, which suggests that they have not considered the given relations for **negative values of *k***.

In general, option (D) is correct.

❖❖❖❖

1 Boolean Algebra & Minimization

1999 IIT Bombay

1.1 The logic function $f = \overline{(x.\bar{y}) + (\bar{x}.y)}$ is the same as

(A) $f = (x + y)(\bar{x} + \bar{y})$

(B) $f = \overline{(\bar{x} + \bar{y})(x + y)}$

(C) $f = (\overline{x.y})(\bar{x}.\bar{y})$

(D) None of the above

2000 IIT Kharagpur

1.2 The minimal product-of-sums function described by the K-map given in figure, is

C \ AB	00	01	11	10
0	1	1	ϕ	0
1	0	0	ϕ	0

(A) $\bar{A}\bar{C}$ (B) $\bar{A} + \bar{C}$

(C) $A + C$ (D) AC

2003 IIT Madras

1.3 The Boolean expression

$\bar{X}Y\bar{Z} + \bar{X}\bar{Y}Z + XY\bar{Z} + X\bar{Y}Z + XYZ$

can be simplified to

(A) $X\bar{Z} + \bar{X}Z + YZ$

(B) $XY + \bar{Y}Z + Y\bar{Z}$

(C) $\bar{X}Y + YZ + XZ$

(D) $\overline{XY} + Y\bar{Z} + \bar{X}Z$

2004 IIT Delhi

1.4 The simplified form of the Boolean expression $Y = (\bar{A}BC + D)(\bar{A}D + \bar{B}\bar{C})$ can be written as

(A) $\bar{A}D + \bar{B}\bar{C}D$

(B) $AD + B\bar{C}D$

(C) $(\bar{A} + D)(\bar{B}C + \bar{D})$

(D) $A\bar{D} + BC\bar{D}$

2007 IIT Kanpur

1.5 The Octal equivalent of the HEX number $AB.CD$ is

(A) 253.314 (B) 253.632

(C) 526.314 (D) 526.632

2010 IIT Guwahati

Statement for Linked Answer Questions 1.6 & 1.7

The following Karnaugh map represents a functions F

F: X \ YZ	00	01	11	10
0	1	1	1	0
1	0	0	1	0

1.6 A minimized form of the function F is

(A) $F=\bar{X}Y+YZ$

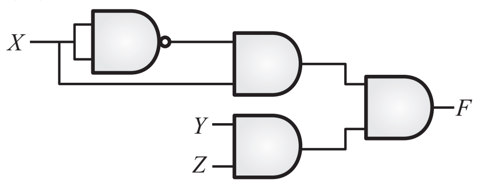
(B) $F=\bar{X}\bar{Y}+YZ$

(C) $F=\overline{XY}+Y\bar{Z}$

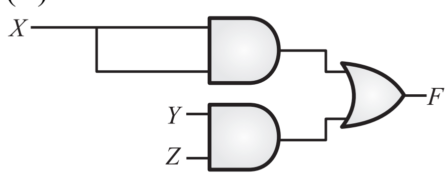
(D) $F=\overline{XY}+\bar{Y}Z$

1.7 Which of the following circuits is a realization of the above functions F?

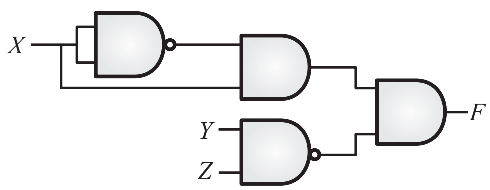
(A)



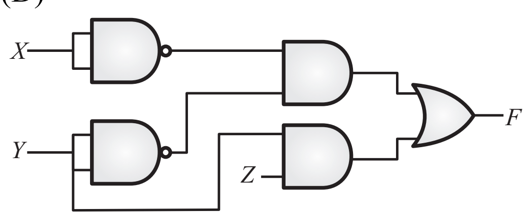
(B)



(C)



(D)



2012	IIT Delhi

1.8 In the sum of products function

$$f(X,Y,Z)=\sum m(2,3,4,5)$$

The prime implicants are

(A) $\bar{X}Y,\ X\bar{Y}$

(B) $\bar{X}Y,\ X\bar{Y}\bar{Z},\ X\bar{Y}Z$

(C) $\bar{X}Y\bar{Z},\ \bar{X}YZ,\ X\bar{Y}$

(D) $\bar{X}Y\bar{Z},\ \bar{X}YZ,\ X\bar{Y}\bar{Z},\ X\bar{Y}Z$

2014	IIT Kharagpur

1.9 Which of the following is an invalid state in 8 4 2 1 Binary Coded Decimal counter **[Set - 02]**

(A) 1000 (B) 1001

(C) 0011 (D) 1100

1.10 The SOP (sum of products) form of a Boolean functions is $\sum(0,1,3,7,11)$, where inputs are A, B, C, D, (A is MSB, and D is LSB). The equivalent minimized expression of the function is **[Set - 02]**

(A) $(\bar{B}+C)(\bar{A}+C)(\bar{A}+\bar{B})(\bar{C}+D)$

(B) $(\bar{B}+C)(\bar{A}+C)(\bar{A}+\bar{C})(\bar{C}+D)$

(C) $(\bar{B}+C)(\bar{A}+C)(\bar{A}+\bar{C})(\bar{C}+\bar{D})$

(D) $(\bar{B}+C)(A+\bar{B})(\bar{A}+\bar{B})(\bar{C}+D)$

2015	IIT Kanpur

1.11 $f(A,B,C,D)=\Pi M(0,1,3,4,5,7,9,11,12,13,14,15)$ is a maxterm representation of Boolean function $f(A,B,C,D)$ where A is the MSB and D is the LSB. The equivalent minimized representation of this function is **[Set - 01]**

(A) $(A+\bar{C}+D)(\bar{A}+B+D)$

(B) $A\bar{C}D+\bar{A}BD$

(C) $\bar{A}C\bar{D}+A\bar{B}C\bar{D}+A\bar{B}\bar{C}\bar{D}$

(D) $(B+\bar{C}+D)(A+\bar{B}+\bar{C}+D)$
$(\bar{A}+B+C+D)$

1.12 Consider the following Sum of Products expression, F.

$$F=ABC+\bar{A}\bar{B}C+A\bar{B}C+\bar{A}BC+\bar{A}\bar{B}\bar{C}$$

The equivalent Product of Sums expression is **[Set - 2]**

(A) $F=(A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$

(B) $F=(A+\bar{B}+\bar{C})(A+B+C)(\bar{A}+\bar{B}+\bar{C})$

(C) $F=(\bar{A}+B+\bar{C})(A+\bar{B}+\bar{C})(A+B+C)$

(D) $F=(\bar{A}+\bar{B}+C)(A+B+\bar{C})(A+B+C)$

2016 IISc Bangalore

1.13 The output expression for the Karnaugh map shown below is **[Set - 02]**

A \ BC	00	01	11	10
0	1	0	0	1
1	1	1	1	1

(A) $A+\overline{B}$ (B) $A+\overline{C}$

(C) $\overline{A}+\overline{C}$ (D) $\overline{A}+C$

1.14 The Boolean expression

$\overline{(a+\overline{b}+c+\overline{d})+(b+\overline{c})}$ simplifies to

[Set - 02]

(A) 1 (B) $\overline{a.b}$

(C) $a.b$ (D) 0

2017 IIT Roorkee

1.15 The Boolean expression $AB+A\overline{C}+BC$ simplifies to **[Set - 01]**

(A) $BC+A\overline{C}$ (B) $AB+A\overline{C}+B$

(C) $AB+A\overline{C}$ (D) $AB+BC$

1.16 The output expression for the Karnaugh map shown below is **[Set - 01]**

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	1	1
10	0	0	0	0

(A) $B\overline{D}+BCD$ (B) $B\overline{D}+AB$

(C) $\overline{B}D+ABC$ (D) $B\overline{D}+ABC$

2018 IIT Guwahati

1.17 Digital input signals *A, B, C* with A as the MSB and *C* as the LSB are used to realize the Boolean function $F=m_0+m_2+m_3+m_5+m_7$, where m_i denotes the i^{th} minterm. In addition. F has a don't care for m_1. The simplified expression for F is given by

(A) $\overline{A}\,\overline{C}+\overline{B}C+AC$

(B) $\overline{A}+C$

(C) $\overline{C}+A$

(D) $\overline{A}C+BC+A\overline{C}$

2019 IIT Madras

1.18 The output expression for the Karnaugh map shown below is

RS \ PQ				
	0	1	1	0
	1	1	1	1
	1	1	1	1
	0	0	0	0

(A) $QR+\overline{S}$ (B) $Q\overline{R}+S$

(C) $QR+S$ (D) $Q\overline{R}+\overline{S}$

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Answers — Boolean Algebra & Minimization

1.1	B	1.2	A	1.3	B	1.4	A	1.5	B
1.6	B	1.7	D	1.8	A	1.9	D	1.10	A
1.11	C	1.12	A	1.13	B	1.14	D	1.15	A
1.16	D	1.17	B	1.18	B				

Explanations — Boolean Algebra & Minimization

Concept of Essential Prime Implicants

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Concept of Karnaugh Map (K-Map) SOP & POS

Scan for Video Explanation

1.1 (B)

Given : $f=\overline{(x.\bar{y})+(\bar{x}.y)}$

From De-Morgan's law,

Law 1 : $\overline{A+B}=\bar{A}.\bar{B}$

Law 2 : $\overline{AB}=\bar{A}+\bar{B}$

$$f=\overline{(x.\bar{y})+(\bar{x}.y)}=(\overline{x\cdot\bar{y}})(\overline{\bar{x}\cdot y})$$

$$f=(\bar{x}+y)(x+\bar{y}) \qquad [x\cdot\bar{x}=0]$$

$$f=\bar{x}\,\bar{y}+x\,y=\overline{\overline{\bar{x}\,\bar{y}+x\,y}}$$

$$f=\overline{\left(\bar{\bar{x}}+\bar{\bar{y}}\right)(\bar{x}+\bar{y})}$$

$$f=\overline{(x+y)(\bar{x}+\bar{y})}$$

Hence, the correct option is (B).

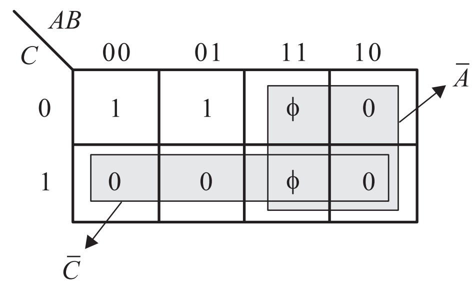
Key Point

Laws of Boolean algebra

1. $A\cdot 0=0$
2. $A\cdot 1=A$
3. $A\cdot\bar{A}=0$
4. $A+\bar{A}=1$
5. $A+A=A$
6. $A\cdot A=A$
7. $A+0=A$
8. $A+1=1$
9. $A+BC=(A+B)(A+C)$
10. $A+\bar{A}B=(A+\bar{A})(A+B)=A+B$

1.2 (A)

Given K-map is shown below,



Since 0's are grouped, so it is a case of POS form.

Thus, $f=\bar{A}\cdot\bar{C}$

Hence, the correct option is (A).

Key Point

Don't care can be denoted by "ϕ", "d" and "X".

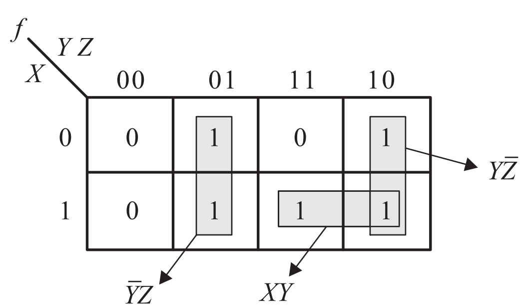
1.3 (B)

Given :

$$f=\bar{X}Y\bar{Z}+\bar{X}\bar{Y}Z+XY\bar{Z}+X\bar{Y}Z+XYZ$$

Method 1

K-map of function f in SOP form is,



The simplified form is

$$f = X\,Y + \bar{Y}\,Z + Y\,\bar{Z}$$

Hence, the correct option is (B).

Method 2

$$f(X,Y,Z) = \bar{X}\,Y\,\bar{Z} + \bar{X}\,\bar{Y}\,Z + X\,Y\,\bar{Z} + X\,\bar{Y}\,Z + X\,Y\,Z$$

$$f(X,Y,Z) = (\bar{X}\,Y\bar{Z} + XY\bar{Z}) + (\bar{X}\bar{Y}Z + X\bar{Y}Z) + (X\,Y\,\bar{Z} + X\,Y\,Z)$$

$$f(X,Y,Z) = Y\bar{Z}(X+\bar{X}) + \bar{Y}Z(X+\bar{X}) + XY(Z+\bar{Z}) \qquad \left[A+\bar{A}=1\right]$$

$$f(X,Y,Z) = Y\bar{Z} + \bar{Y}Z + XY$$

$$f(X,Y,Z) = XY + \bar{Y}Z + Y\bar{Z}$$

Hence, the correct option is (B).

1.4 (A)

Given : $Y = (\bar{A}\,B\,C + D)(\bar{A}\,D + \bar{B}\,\bar{C})$

$$Y = \bar{A}\bar{A}BCD + \bar{A}DD + \bar{A}BC\bar{B}\bar{C} + \bar{B}\bar{C}D \qquad \left[X\cdot\bar{X}=0,\ X\cdot X = X\right]$$

$$Y = \bar{A}\,B\,C\,D + \bar{A}\,D + \bar{B}\,\bar{C}\,D$$

$$Y = \bar{A}\,D(B\,C+1) + \bar{B}\,\bar{C}\,D \qquad [1+X=1]$$

$$Y = \bar{A}\,D + \bar{B}\,\bar{C}\,D$$

Hence, the correct option is (A).

1.5 (B)

Given : HEX number (*AB* . *CD*)

Binary code equivalent of HEX number $AB \cdot CD$ is given below.

HEX number	A	B	·	C	D
Binary number	1010	1011	·	1100	1101

To convert the integer part of above binary number into octal, form the group of three bit. Start to form the group from LSB as given below.

0 1 0　1 0 1　0 1 1　⇒ Integer part to binary number

0 added to form the group of 3

2　5　3　⇒ Integer part to octal number

To convert the fractional part of above binary number into octal number, form the group of three bit start to form the group from MSB.

0 added to form the group of 3

1 1 0　0 1 1　0 1 0　⇒ fractional part to binary number

6　3　2　⇒ fractional part to octal number

Thus, octal number is $(253\cdot 632)_8$.

Hence, the correct option is (B).

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1.6 (B)

Given K-map is shown below,

$X \backslash YZ$	00	01	11	10
0	1	1	1	0
1	0	0	1	0

F

$\bar{X}\,\bar{Y}$

YZ

The minimized form of the function F is,

$$F = \bar{X}\,\bar{Y} + Y\,Z$$

Hence, the correct option is (B).

📖 Key Point

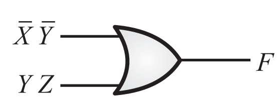
1. To find the minimized expression in POS form, 0's are grouped.
2. To find the minimized expression in SOP form, 1's are grouped.

1.7 (D)

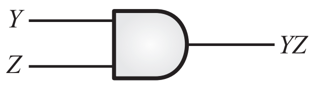
Method 1

$$F = \bar{X}\,\bar{Y} + Y\,Z$$

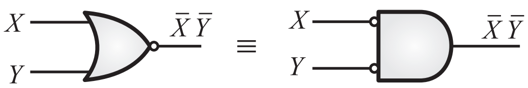
To realize F, an OR gate is needed to SUM UP the two term i.e. $\bar{X}\,\bar{Y}$ and $Y\,Z$.



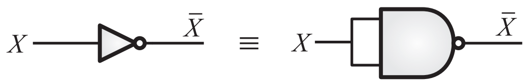
Product term YZ can be generated by an AND gate.



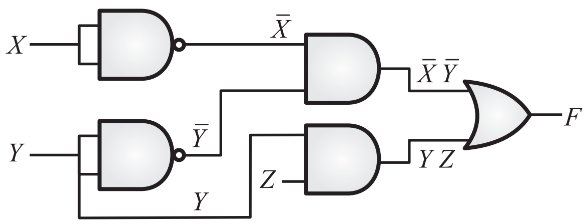
Product term $\overline{X}\,\overline{Y} = \overline{X+Y}$ can be generated by a NOR gate and NOR gate is equivalent to bubbled AND gate.



Where bubble denote the NOT gate and any NOT gate can be obtained from NAND gate by joining its two input together.



Hence, from above explanation F can be realized as shown below.



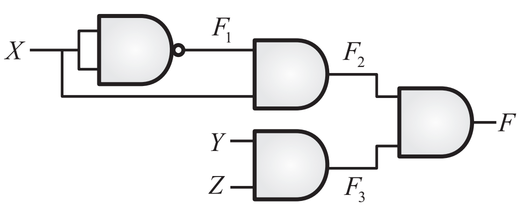
Hence, the correct option is (D).

Method 2

$$F = \overline{X}\,\overline{Y} + YZ$$

Checking from options :

(i) From option (A) :



$$F_1 = \overline{X \cdot X} = \overline{X}$$

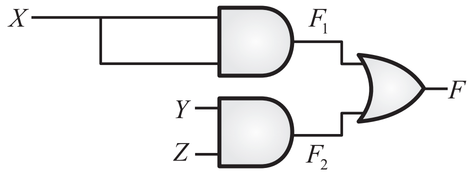
$$F_2 = F_1 \cdot X = \overline{X} \cdot X = 0$$

$$F_3 = YZ$$

$$F = F_2 F_3 = 0 \cdot F_3 = 0$$

Thus, option (A) is not correct.

(ii) From option (B) :



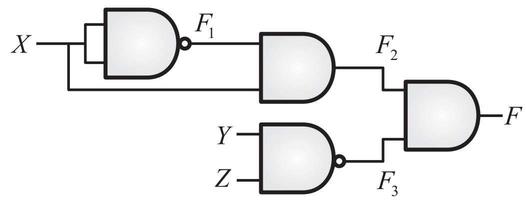
$$F_1 = X \cdot X = X$$

$$F_2 = Y \cdot Z$$

$$F = F_1 + F_2 = X + YZ$$

Thus, option (B) is not correct.

(iii) From option (C) :



$$F_1 = \overline{X \cdot X} = \overline{X}$$

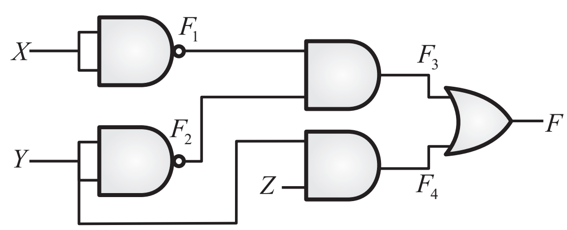
$$F_2 = F_1 \cdot X = \overline{X} \cdot X = 0$$

$$F_3 = \overline{Y \cdot Z}$$

$$F = F_2 F_3 = 0 \cdot \overline{YZ} = 0$$

Thus, option (C) is not correct.

(iv) From option (D) :



$$F_1 = \overline{X \cdot X} = \overline{X}$$

$$F_2 = \overline{Y \cdot Y} = \overline{Y}$$

$$F_3 = F_1 \cdot F_2 = \overline{X} \cdot \overline{Y}$$

$$F_4 = Y \cdot Z$$

$$F = F_3 + F_4 = \overline{X}\,\overline{Y} + YZ$$

Thus, option (D) is correct.

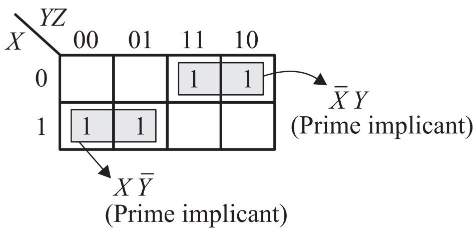
Hence, the correct option is (D).

1.8 (A)

Given : $f(X,Y,Z) = \Sigma m(2, 3, 4, 5)$

$$f(X,Y,Z) = \bar{X}\,Y\,\bar{Z} + \bar{X}\,Y\,Z + X\,\bar{Y}\,\bar{Z} + X\,\bar{Y}\,Z$$

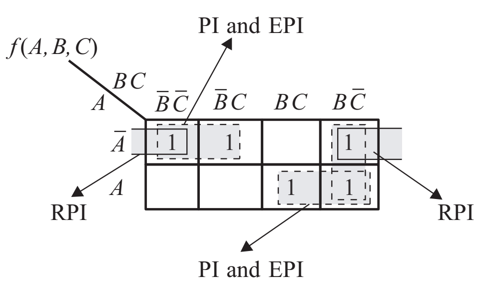
K-map for function $f(X,Y,Z)$ is shown below,



Hence, the correct option is (A).

Key Point

1. **Implicant :** Each individual min-term in canonical SOP form is called implicant.
2. **Prime Implicant (PI) :** Prime implicant is a min-term, which are obtained by combining maximum possible adjacent cells in the K-map.
3. **Essential Prime Implicant (EPI) :** It is a prime implicant which contains atleast one min-terms which is not covered by other prime implicant.
4. **Redundant Prime Implicant (RPI) :** The prime implicant whose each 1 is covered atleast by one EPI is called a redundant prime implicant.
5. **Example :**



Here total number of 1's = 5,

Hence, number of implicant = 5

Implicant $= (\bar{A}\,\bar{B}\,\bar{C}), (\bar{A}\,\bar{B}\,C), (A\,B\,C), (\bar{A}\,B\,\bar{C}), (A\,B\,\bar{C})$

Number of prime implicant = 4

Prime implicant $= \bar{A}\,\bar{B},\ \bar{A}\,\bar{C},\ A\,B,\ B\,\bar{C}$

Number of essential prime implicant = 2

Essential prime implicant $= \bar{A}\,\bar{B},\ A\,B$

1.9 (D)

In Binary coded Decimal 8 4 2 1,

Checking from options :

Options	8	4	2	1	Decimal	BCD
Option (A)	1	0	0	0	8	Valid
Option (B)	1	0	0	1	9	Valid
Option (C)	0	0	1	1	3	Valid
Option (D)	1	1	0	0	12	Invalid

BCD counter counts from 0 to 9 only, therefore from option (D) i.e. 1100 whose decimal equivalent is 12, is an invalid state.

Hence, the correct option is (D).

Scan for Video Solution

Key Point

BCD counter counts from 0 to 9 only hence the valid states in BCD code are 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001 and invalid states are 1010, 1011, 1100, 1101, 1110, 1111.

1.10 (A)

Given : The SOP (Sum of Product) form of a Boolean function,

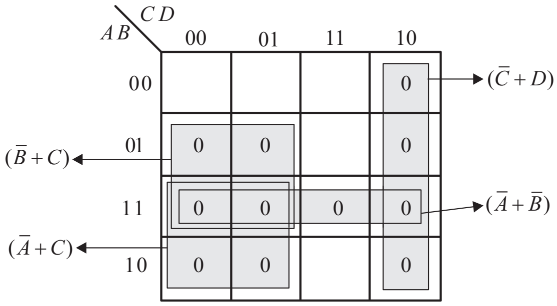
$$f(A,B,C,D) = \sum m(0,1,3,7,11)$$

(where A is MSB, D is LSB).

As the given options are in product of sum form, therefore convert it into canonical product of sum form.

$$f(A,B,C,D) = \Pi M(2,4,5,6,8,9,10,12,13,14,15)$$

Thus, K-map of function f in POS form is,



$$f = (\bar{B}+C)(\bar{A}+C)(\bar{A}+\bar{B})(\bar{C}+D)$$

Hence, the correct option is (A).

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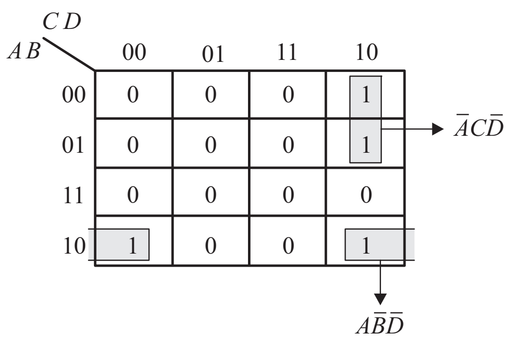
1.11 (C)

Given :

$$f(A,B,C,D) = \Pi M(0,1,3,4,5,7,9,11,12,13,14,15)$$

Method 1

K-map of function f in SOP form is,



Thus, Equivalent minimized representation is,

$$f = A\bar{B}\bar{D} + \bar{A}C\bar{D}$$

$$f = A\bar{B}(C+\bar{C})\bar{D} + \bar{A}C\bar{D}$$

$$f = A\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}C\bar{D}$$

$$f = \bar{A}C\bar{D} + A\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D}$$

Hence, the correct option is (C).

Method 2

$$f(A,B,C,D) = \Pi M(0,1,3,4,5,7,9,11,12,13,14,15)$$

The min term of given function is shown below,

$$f(A,B,C,D) = (2,6,8,10)$$

SOP expression of function f is,

$$f(A,B,C,D) = \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}$$

$$f(A,B,C,D) = \bar{A}C\bar{D}(B+\bar{B}) + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}$$

$$[B+\bar{B}=1]$$

$$f(A,B,C,D) = \bar{A}C\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}$$

Hence, the correct options is (C).

1.12 (A)

Given : $F = ABC + \bar{A}\bar{B}C + A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}\bar{C}$

$$F = \Sigma m(111,001,101,011,000)$$

$$F = \Sigma m(7,1,5,3,0)$$

$$F = \Sigma m(0,1,3,5,7)$$

$$F = \Pi M\,(2,4,6)$$

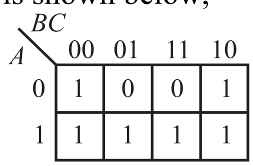
$$F = \Pi M\,(010,100,110)$$

$$F = (A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$$

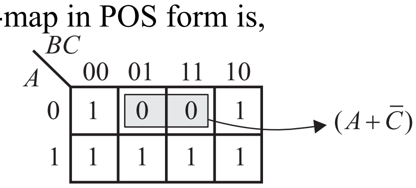
Hence, the correct option is (A).

1.13 (B)

Given K-map is shown below,



The K-map in POS form is,



The POS expression of function is,

$$Y = A+\bar{C}$$

Hence, the correct option is (B).

Scan for Video Solution

1.14 (D)

Given : $F = \overline{(a+\bar{b}+c+\bar{d})+(b+\bar{c})}$ ...(i)

Method 1

From De-morgan's law,

(i) $\overline{A\cdot B} = \bar{A}+\bar{B}$

(ii) $\overline{A+B} = \bar{A}\cdot\bar{B}$

Thus, $F = \overline{(a+\bar{b}+c+\bar{d})}\cdot\overline{(b+\bar{c})}$

$$F = (\bar{a}\cdot b\cdot\bar{c}\cdot d)(\bar{b}\cdot c) \quad [X\cdot\bar{X}=0]$$

$$F = 0$$

Hence, the correct option is (D).

Method 2

From equation (i),

$$F=\overline{(a+\overline{b}+c+\overline{d})+(b+\overline{c})}$$

$$F=\overline{a+\overline{b}+b+c+\overline{c}+\overline{d}}$$

$$F=\overline{a+\overline{d}+1+1} \qquad [x+\overline{x}=1]$$

$$F=\overline{a+\overline{d}+1} \qquad [1+x=1]$$

$$F=\overline{1}=0$$

Hence, the correct option is (D).

1.15 (A)

Given : $F(A,\ B,\ C)=AB+A\overline{C}+BC$

$$F(A,B,C)=AB(C+\overline{C})+A\overline{C}+BC$$

$$[C+\overline{C}=1]$$

$$F(A,B,C)=ABC+AB\overline{C}+A\overline{C}+BC$$

$$F(A,B,C)=(A+1)BC+(B+1)A\overline{C}$$

$$[A+1=1 \text{ and } B+1=1]$$

$$F(A,B,C)=BC+A\overline{C}$$

Hence, the correct option is (A).

Scan for Video Solution

Key Point

Consensus Law :

(i) Sum of Product Form,

$$AB+\overline{A}C+BC=AB+\overline{A}C$$

$$AC+A\overline{B}+BC=A\overline{B}+BC$$

$$\overline{A}\,\overline{B}+A\overline{C}+\overline{B}\,\overline{C}=A\overline{C}+\overline{A}\,\overline{B}$$

(ii) Product of Sum Form,

$$(A+B)(\overline{A}+C)(B+C)=(A+B)(\overline{A}+C)$$

$$(A+C)(A+\overline{B})(B+C)=(A+\overline{B})(B+C)$$

$$(\overline{A}+\overline{C})(\overline{A}+\overline{B})(\overline{B}+C)=(\overline{B}+C)(\overline{A}+\overline{C})$$

1.16 (D)

Given Karnaugh map is shown below,

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	1	1
10	0	0	0	0

$B\overline{D}$ ABC

Thus, the SOP expression of function F is,

$$F=B\overline{D}+ABC$$

Hence, the correct option is (D).

Scan for Video Solution

1.17 (B)

Given : $F=m_0+m_2+m_3+m_5+m_7$

Also, F has don't care at m_1,

Hence, $F(A,B,C)=\Sigma m(0,2,3,5,7)+\Sigma\phi(1)$

A \ BC	00	01	11	10
0	1	×	1	1
1		1	1	

$\overline{A}$ C

$$F=\overline{A}+C$$

Hence, the correct option is (B).

1.18 (B)

RS \ PQ	$\overline{P}\overline{Q}$	$\overline{P}Q$	PQ	$P\overline{Q}$
$\overline{R}\overline{S}$	0	1	1	0
$\overline{R}S$	1	1	1	1
RS	1	1	1	1
$R\overline{S}$	0	0	0	0

$Q\overline{R}$ S

$$y=Q\overline{R}+S$$

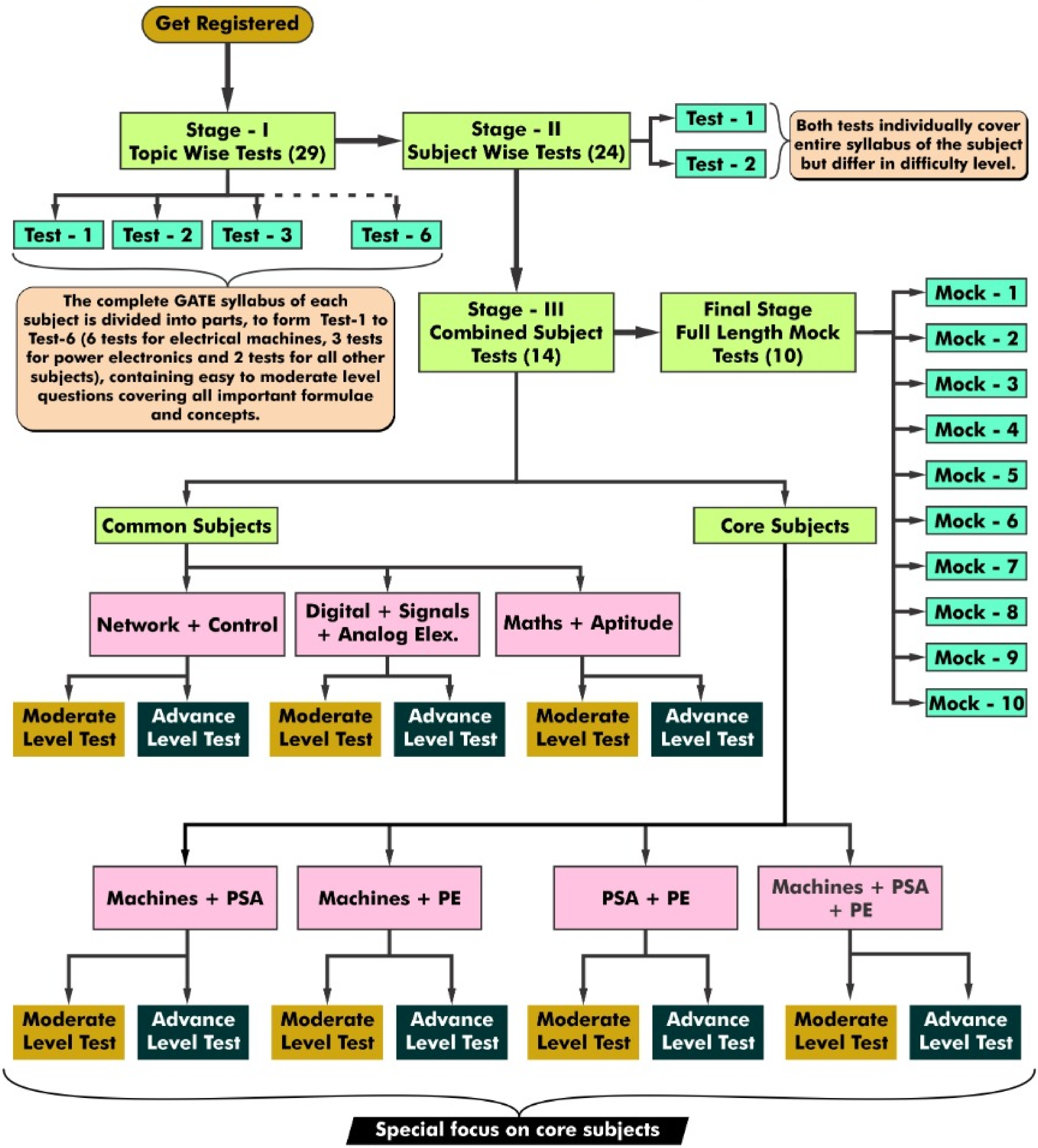
Hence, the correct option is (B).

❖❖❖❖

GATE ACADEMY

TEST SERIES STRUCTURE (EE)

(One Year Program)

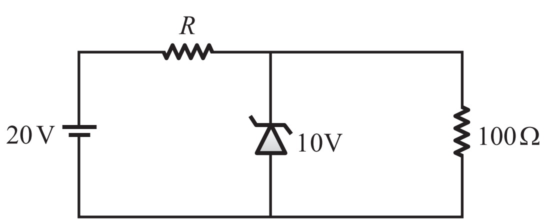


1 Diode Circuits & Applications

Topics : Characteristics of Diode, Clipper & Clamper Circuits and Zener Diode.

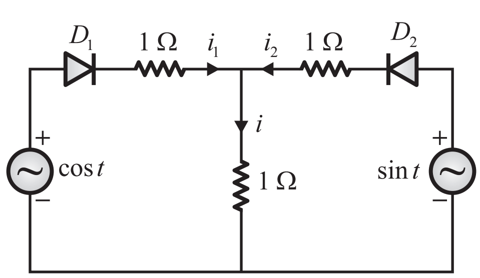
1991 IIT Madras

1.1 Figure shows an electronic voltage regulator. The Zener diode may be assumed to require a minimum current of 25 mA for satisfactory operations. The value of R (in ohms) required for satisfactory voltage regulation of the circuit is ________.



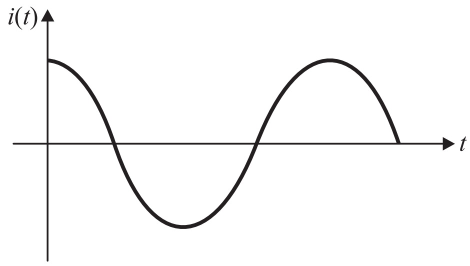
1992 IIT Delhi

1.2 In the circuit shown in figure the wave form of the current 'i' over one period of the input voltages is (Assume the diode to be ideal).

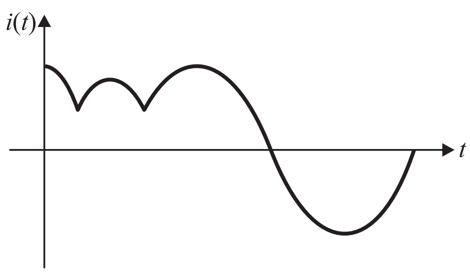


$i = i_1 + i_2$

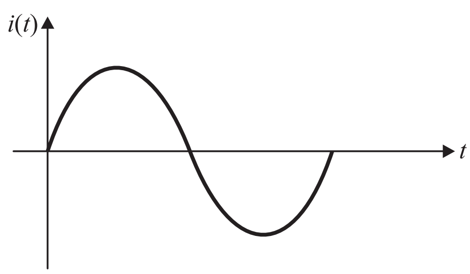
(A)



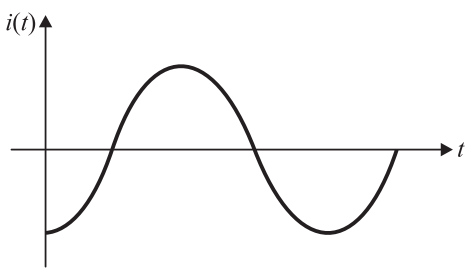
(B)



(C)



(D)



1996 IISc Bangalore

1.3 The depletion region or space charge region or transition region in a semiconductor *p-n* junction diode has

(A) electrons and holes.

(B) positive ions and electrons.

(C) positive and negative ions.

(D) negative ions and holes.

1999 IIT Bombay

1.4 As temperature is increased, the voltage across a diode carrying a constant current

(A) Increases

(B) Decreases

(C) Remain constant

(D) May increase and decrease depending upon the doping levels in the junction.

1.5 The mobility of an electron in a conductor expressed in terms of

(A) $\text{cm}^2/\text{V-sec}$ (B) $\text{cm}/\text{V-sec}$

(C) cm^2/V (D) cm^2/sec

2000 IIT Kharagpur

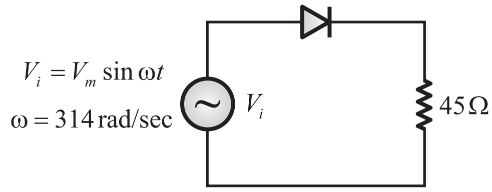
1.6 A diode whose terminal characteristics are related as $i_D = I_s e^{\frac{V}{V_T}}$, where I_s is the reverse saturation current and V_T is thermal voltage $(= 25\text{ mV})$ is biased at $i_D = 2\text{ mA}$. Its dynamic resistance is

(A) 25 Ω (B) 12.5 Ω

(C) 50 Ω (D) 100 Ω

2002 IISc Bangalore

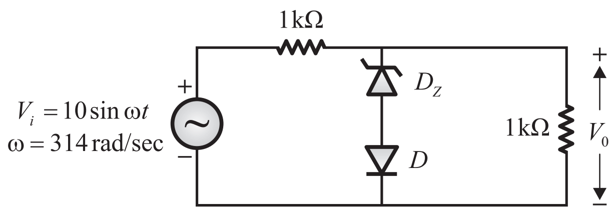
1.7 The forward resistance of the diode shown in figure is 5 Ω and the remaining parameters are same as those of ideal diode. The DC components of the source current is



(A) $\dfrac{V_m}{50\pi}$ (B) $\dfrac{V_m}{50\pi\sqrt{2}}$

(C) $\dfrac{V_m}{100\pi\sqrt{2}}$ (D) $\dfrac{2V_m}{50\pi\sqrt{2}}$

1.8 The cut-in voltage of both Zener diode D_z and diode D shown in figure is 0.7 V, while breakdown voltage in the Zener is 3.3 V and reverse break down of D is 50 V. The other parameters can be assumed to be the same as those of an ideal diode. The values of the peak output voltage (V_0) are



(A) 3.3 V in the positive half cycle and 1.4 V in the negative half cycle.

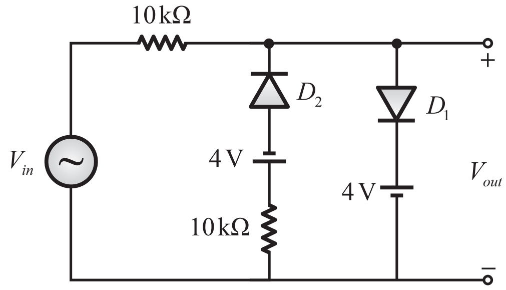
(B) 4 V in the positive half cycle and 5 V in the negative half cycle.

(C) 3.3 V in the both positive and negative half cycle.

(D) 4 V in the both positive and negative half cycle.

2003 IIT Madras

1.9 A voltage signal $10\sin\omega t$ is applied to the circuit with ideal diodes, as shown in figure. The maximum and minimum values of the output waveform V_{out} of the circuit are respectively.



(A) + 10 V and – 10 V

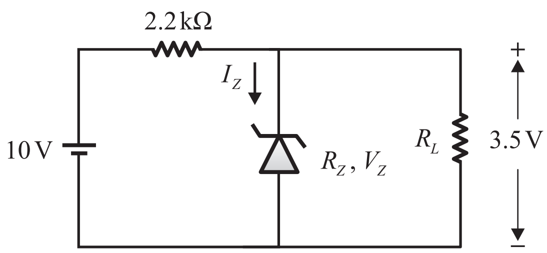
(B) + 4 V and – 4 V

(C) + 7 V and – 4 V

(D) + 4 V and – 7 V

2004 IIT Delhi

1.10 The current through the Zener diode in figure is

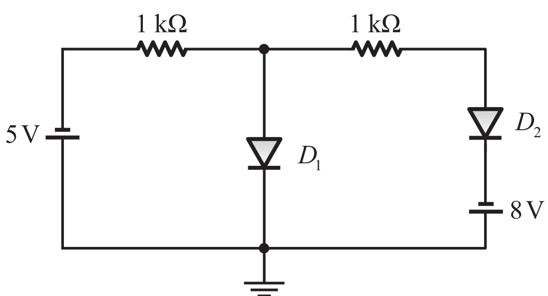


$R_Z = 0.1\ \text{k}\Omega$, $V_Z = 3.3$ V

(A) 3.3 mA (B) 4.3 mA

(C) 2 mA (D) 0 mA

1.11 Assuming that the diodes are ideal in figure, the current in diode D_1 is

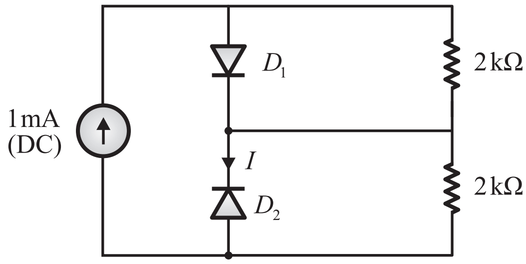


(A) 8 mA (B) 5 mA

(C) 0 mA (D) – 3 mA

2005 IIT Bombay

1.12 Assume that D_1 and D_2 in figure are ideal diodes. The value of current I is

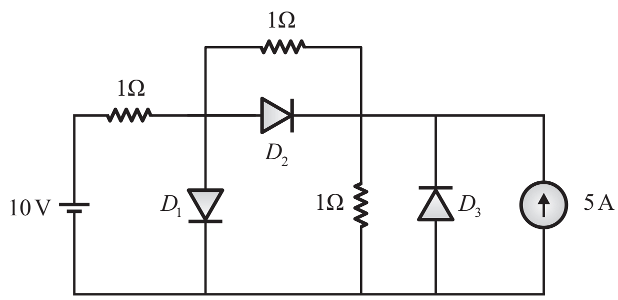


(A) 0 mA (B) 0.5 mA

(C) 1 mA (D) 2 mA

2006 IIT Kharagpur

1.13 What are the states of the three ideal diodes of the circuit shown in figure?



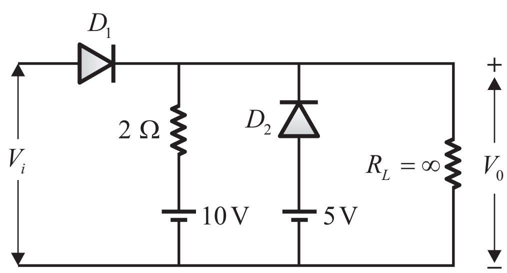
(A) D_1 ON, D_2 OFF, D_3 OFF

(B) D_1 OFF, D_2 ON, D_3 OFF

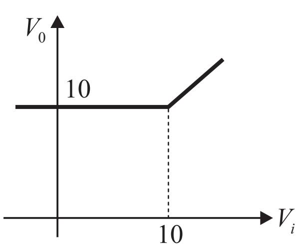
(C) D_1 ON, D_2 OFF, D_3 ON

(D) D_1 OFF, D_2 ON, D_3 ON

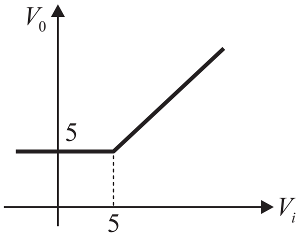
1.14 Assuming the diodes D_1 and D_2 of the circuit shown in figure to be ideal ones, the transfer characteristics of the circuit will be



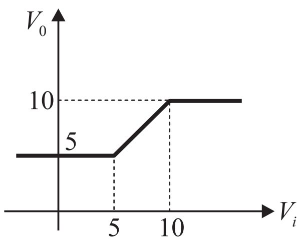
(A)



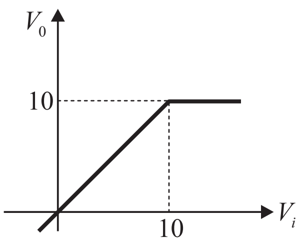
(B)



(C)



(D)



2008 IISc Bangalore

1.15 The equivalent circuits of a diode, during forward biased and reverse biased conditions, are shown in the figure.

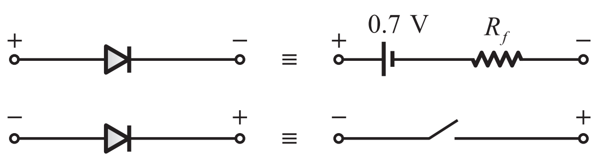


Fig. (a)

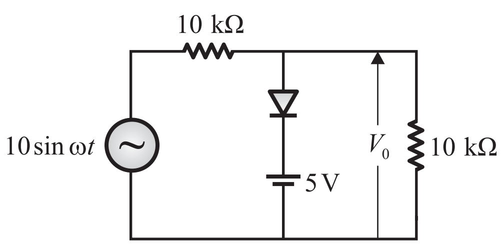
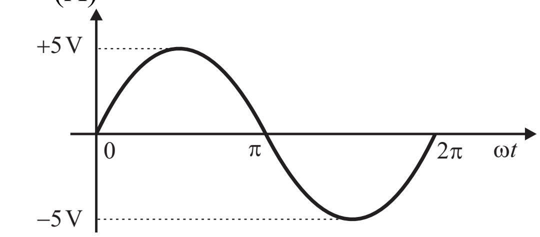


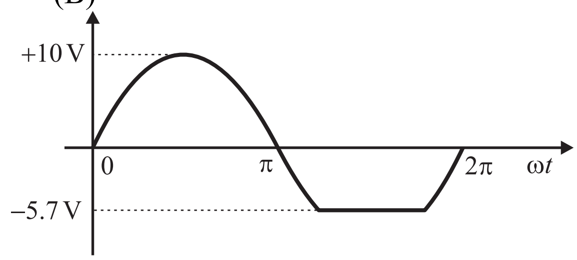
Fig. (b)

If such a diode is used in clipper circuit of figure given above, the output voltage (V_0) of the circuit will be

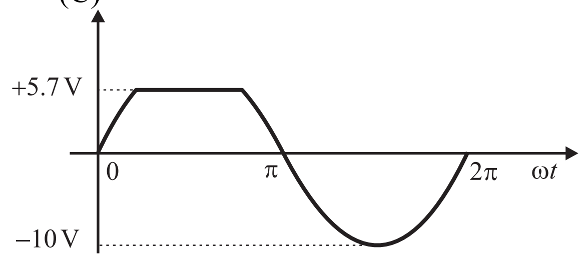
(A)



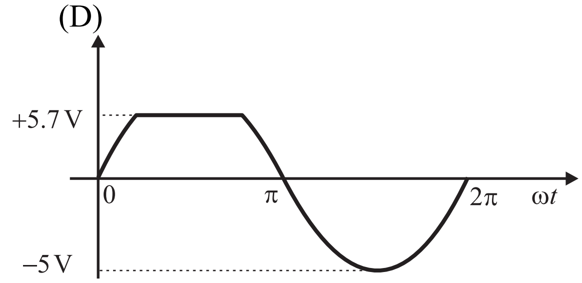
(B)



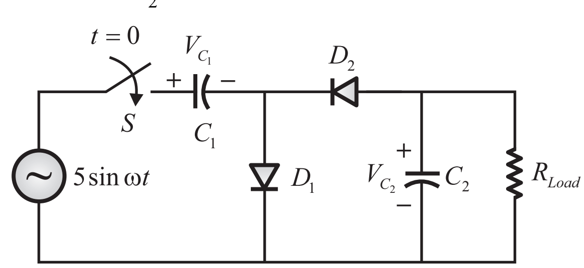
(C)



(D)



1.16 In the voltage double circuit shown in the figure, the switch 'S' is closed at $t=0$. Assuming diodes D_1 and D_2 to be ideal, load resistance to be infinite and initial capacitor voltages to be zero, the steady state voltage across capacitors C_1 and C_2 will be



(A) $V_{C_1} = 10$ V, $V_{C_2} = 5$ V

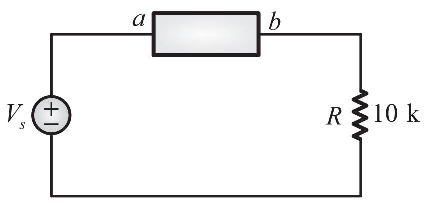
(B) $V_{C_1} = 10$ V, $V_{C_2} = -5$ V

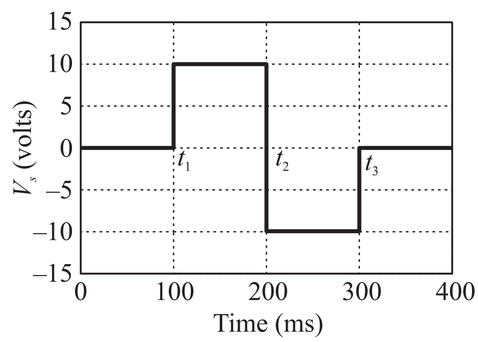
(C) $V_{C_1} = 5$ V, $V_{C_2} = 10$ V

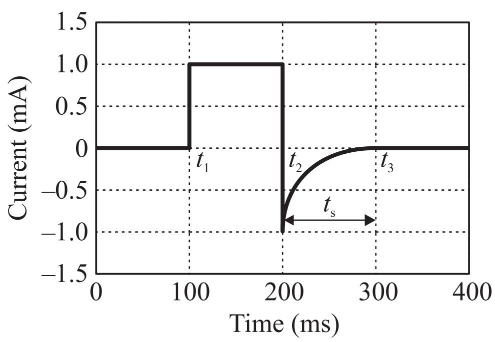
(D) $V_{C_1} = 5$ V, $V_{C_2} = -10$ V

2009 IIT Roorkee

1.17 The following circuit has a source voltage V_s as shown in the graph. The current through the circuit is also shown.



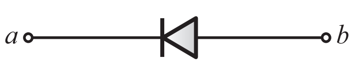


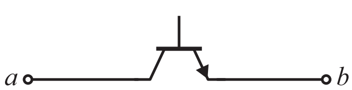


The element connected between a and b could be

(A) 

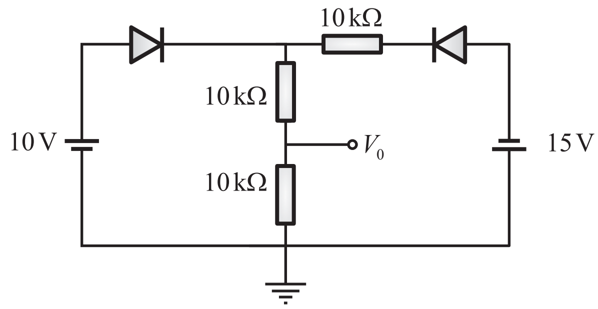
(B) 

(C) 

(D) 

2010 IIT Guwahati

1.18 Assuming that the diodes in the given circuit are ideal, the voltage V_0 is

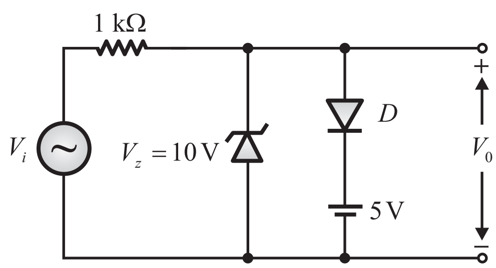


(A) 4 V (B) 5 V

(C) 7.5 V (D) 12.12 V

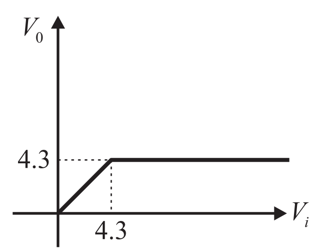
2011 IIT Madras

1.19 A clipper circuit is shown below

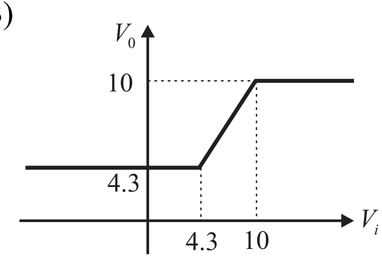


Assuming forward voltage drop of the diodes to be 0.7 V, the input-output transfer characteristics of the circuit is

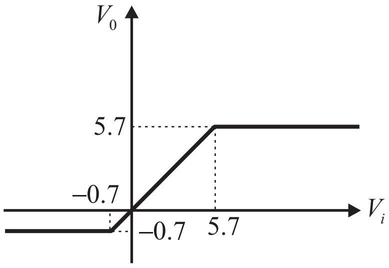
(A)



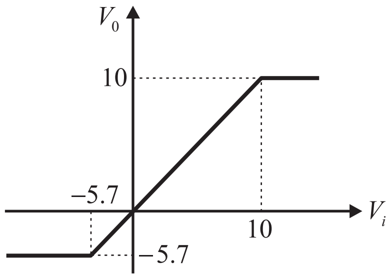
(B)



(C)



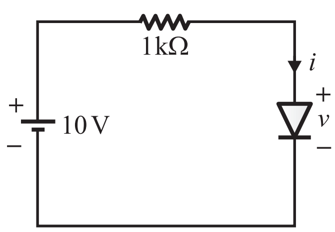
(D)



2012 IIT Delhi

1.20 The *i*-*v* characteristics of the diode in the circuit given below are

$$i = \begin{cases} 0 & v < 0.7 \text{ V} \\ \dfrac{v-0.7}{500}\text{A} & v \geq 0.7 \text{ V} \end{cases}$$



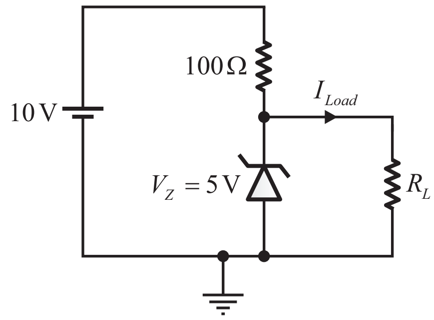
The current in the circuit is

(A) 10 mA (B) 9.3 mA

(B) 6.67 mA (D) 6.2 mA

2013 IIT Bombay

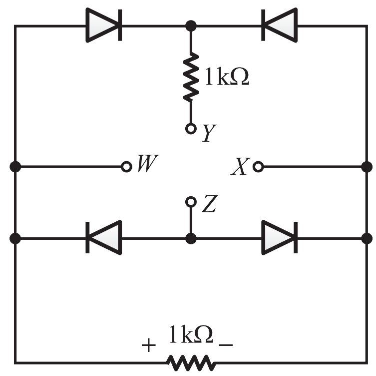
1.21 In the circuit shown below, the knee current of the ideal Zener diode is 10 mA. To maintain 5 V across R_L, the minimum value of R_L in Ω and the minimum power rating of the Zener diode in mW, respectively, are



(A) 125 and 125 (B) 125 and 250

(C) 250 and 125 (D) 250 and 250

1.22 A voltage $1000 \sin \omega t$ Volts is applied across *YZ*. Assuming ideal diodes, the voltage measured across *WX* in Volts, is



(A) $\sin \omega t$

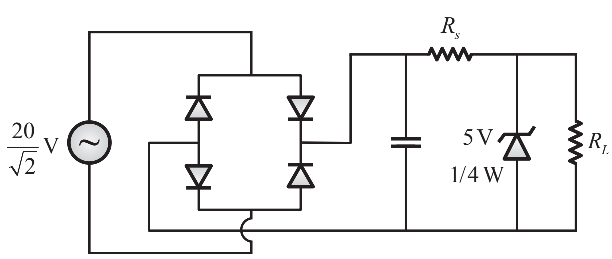
(B) $(\sin \omega t + |\sin \omega t|)/2$

(C) $(\sin \omega t - |\sin \omega t|)/2$

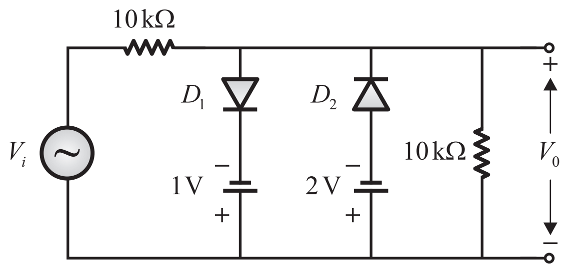
(D) 0 for all t

2014 IIT Kharagpur

1.23 The sinusoidal ac source in the figure has an rms value of $\dfrac{20}{\sqrt{2}}$ V. Considering all possible values of R_L, the minimum value of R_s in Ω to avoid burnout of the Zener diode is ________ **[Set - 02]**



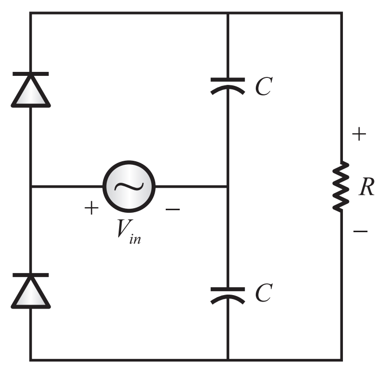
1.24 Assuming the diodes to be ideal in the figure, for the output to be clipped, the input voltage V_i must be outside the range **[Set - 02]**



(A) – 1 V and – 2 V (B) – 2 V to – 4 V
(C) + 1 V to – 2 V (D) + 2V to – 4 V

2015 IIT Kanpur

1.25 In the following circuit, the input voltage V_{in} is $100\sin(100\pi t)$. For $100\pi RC = 50$, the average voltage across R (in Volts) under steady-state is nearest to **[Set - 02]**



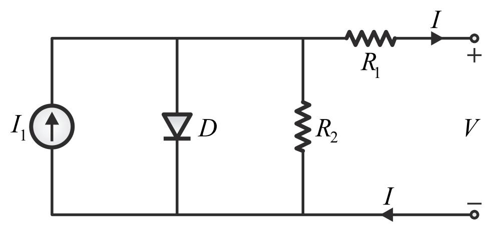
(A) 100 (B) 31.8
(C) 200 (D) 63.6

2020 IIT Delhi

1.26 A non-ideal diode is biased with a voltage of –0.03 V and a diode current of I_1 is measured. The thermal voltage is 26 mV and the ideality factor for the diode is $15/13$. The voltage, in V, at which the measured current increases to $1.5 I_1$ is closest to

(A) –4.50 (B) –0.09
(C) –1.50 (D) –0.02

1.27 Consider the diode circuit shown below. The diode D, obeys the current-voltage characteristics $I_D = I_S\left[\exp\left(\frac{V_D}{nV_T}\right) - 1\right]$, where $n > 1, V_T > 0, V_D$ is the voltage across the diode and I_D is the current through it. The circuit is biased so that voltage, $V > 0$ and current, $I < 0$. If you had to design this circuit to transfer maximum power from the current source (I_1) to a resistive load (not shown) at the output, what values of R_1 and R_2 would you choose?



(A) Small R_1 and small R_2
(B) Small R_1 and large R_2
(C) Large R_1 and large R_2
(D) Large R_1 and small R_2

❖❖❖❖

Answers Diode Circuits & Applications

1.1	80	1.2	B	1.3	C	1.4	B	1.5	A
1.6	B	1.7	A	1.8	B	1.9	D	1.10	C
1.11	C	1.12	A	1.13	A	1.14	A	1.15	A
1.16	D	1.17	A	1.18	B	1.19	C	1.20	D
1.21	B	1.22	D	1.23	300	1.24	B	1.25	C
1.26	B	1.27	B						

Explanations Diode Circuits & Applications

Concept of Clamper & Peak Detector Circuits

Scan for Video Explanation

Multiplier Circuit by using Clamper & Peak Detector Circuits

Scan for Video Explanation

1.1 80

Given :

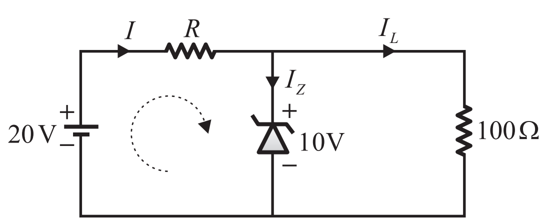
(i) Minimum Zener current,

$$I_{Z(\min)} = 25 \text{ mA}$$

(ii) Zener break down voltage,

$$V_Z = 10 \text{ V}$$

Given circuit is shown below,



Current in 100 Ω resistance is given by,

$$I_L = \frac{10}{100} = 0.1 \text{ A} = 100 \text{ mA}$$

Input current is given by,

$$I = I_L + I_{Z(\min)} = 100 + 25 = 125 \text{ mA}$$

Applying KVL in the loop shown,

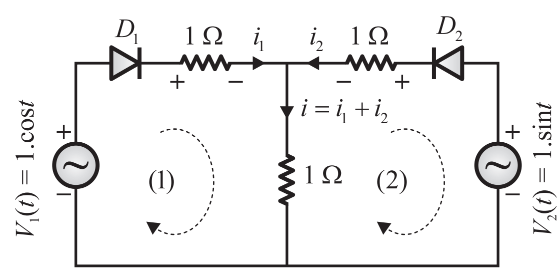
$$-20 + I \times R + 10 = 0$$

$$R = \frac{20-10}{125\times10^{-3}} = 80\ \Omega$$

Hence, the value of R required for satisfactory voltage regulation of the circuit is **80 Ω .**

1.2 (B)

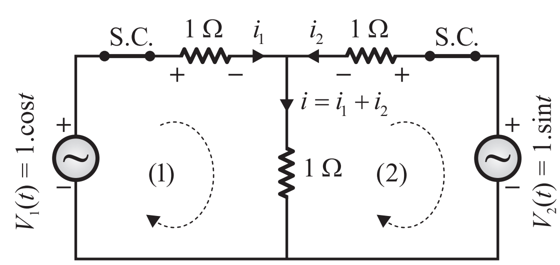
Given :



Frequency of $V_1(t)$ and $V_2(t)$, $\omega = 1$ rad/sec

i.e. time period $T = \frac{2\pi}{\omega} = 6.28$ sec

Assume both diodes D_1 and D_2 are ON.



Applying KVL in loop (1),

$$-V_1(t)+1\times i_1+1\times(i_1+i_2)=0$$

$$2i_1+i_2=V_1(t) \qquad \ldots (i)$$

Applying KVL in loop (2),

$$V_2(t)-1\times i_2-1\times(i_1+i_2)=0$$

$$i_1+2i_2=V_2(t) \qquad \ldots (ii)$$

By solving equation (i) and (ii),

$$i_1=\frac{2V_1(t)-V_2(t)}{3} \text{ and } i_2=\frac{2V_2(t)-V_1(t)}{3}$$

So, $i_1=\dfrac{2\cos t-\sin t}{3}$, $i_2=\dfrac{2\sin t-\cos t}{3}$

(i) For diode D_1 to be ON, i_1 should be greater than zero.

$$i_1=\frac{2\cos t-\sin t}{3}>0$$

$$\tan t<2 \quad \Rightarrow \quad t<1.11 \text{ sec}$$

For $0<t<1.11$ sec, D_1 is ON.

(ii) For diode D_2 to be ON, i_2 should be greater than zero.

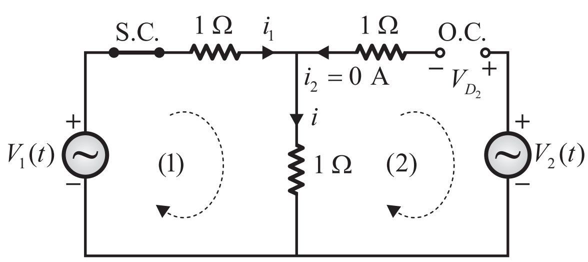
$$i_2=\frac{2\sin t-\cos t}{3}>0$$

$$\tan t>\frac{1}{2} \quad \Rightarrow \quad t>0.46 \text{ sec}$$

For $t>0.46$ sec, D_2 is ON.

For $0<t<0.46$ sec :

D_1 is ON and D_2 is OFF. Hence, modified circuit is shown below,

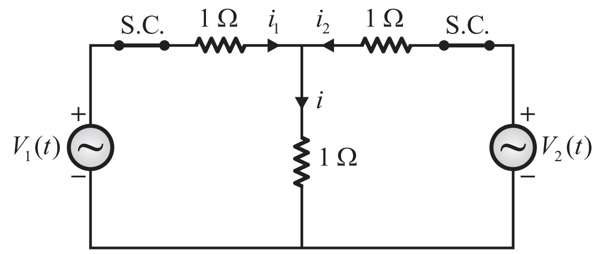


Applying KVL in loop (1),

$$i=i_1=\frac{V_1(t)}{2}=\frac{\cos t}{2}$$

For $0.46 \text{ sec}<t<1.11 \text{ sec}$:

Both diodes D_1 and D_2 are ON.



From figure,

$$i=i_1+i_2$$

$$i_1=\frac{2V_1(t)-V_2(t)}{3}>0,\ i_2=\frac{2V_2(t)-V_1(t)}{3}>0$$

$$i=i_1+i_2=\frac{V_1(t)+V_2(t)}{3}$$

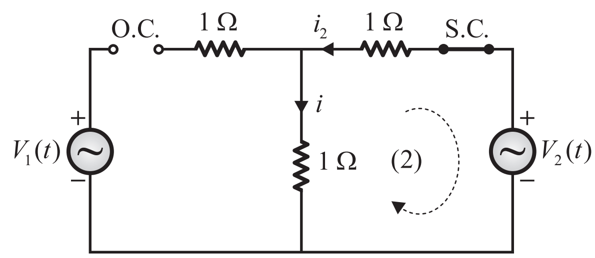
$$i=\frac{\sin t+\cos t}{3}=\frac{1\times\sin t+1\times\cos t}{3}$$

$$i=\frac{\sqrt{2}}{3}\left[\sin t\cos 45^0+\cos t\sin 45^0\right]$$

$$i=\frac{\sqrt{2}}{3}\sin(t+45^0)=\frac{\sqrt{2}}{3}\sin\left(t+\frac{\pi}{4}\right)$$

For $1.11<t<6.28$ sec :

For this range, only D_2 is ON.

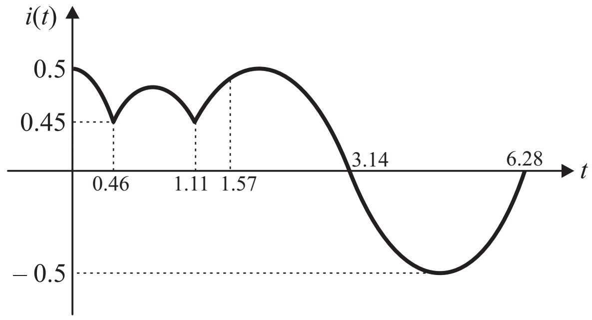


Applying KVL in loop (2),

$$i=\frac{V_2(t)}{1+1}=\frac{\sin t}{2}$$

For one cycle, $0<t<T$ ($T=6.28$ sec)

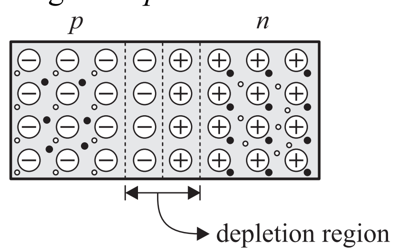
$$i=\begin{cases}\dfrac{1}{2}\cos t & ;\ 0<t<0.46 \text{ sec}\\[2mm] \dfrac{\sqrt{2}}{3}\sin\left(t+\dfrac{\pi}{4}\right) & ;\ 0.46<t<1.11\text{sec}\\[2mm] \dfrac{1}{2}\sin t & ;\ 1.11<t<6.28 \text{ sec}\end{cases}$$



Hence, the correct option is (B).

1.3 (C)

Depletion region in *p-n* diode is shown below,



From the above figure, it is clear that depletion region contains immobile negative ions on *p*-side and positive ions on *n*-side.

Hence, the correct option is (C).

1.4 (B)

The relation between voltage across diode and temperature is given by,

$$\frac{dV_D}{dT} = -2.3 \text{ mV}/^0\text{C}$$

Hence, the correct option is (B).

1.5 (A)

Mobility (μ) of an electron in a conductor is given by,

$$\mu = \frac{\text{Drift velocity}}{\text{Electric field}}$$

$$\text{Unit of mobility} = \frac{\text{Unit of drift velocity}}{\text{Unit of electric field}}$$

$$= \frac{\text{cm / sec}}{\text{V / cm}} = \text{cm}^2 / \text{V-sec}$$

Hence, the correct option is (A).

1.6 (B)

Given :

(i) Thermal voltage, $V_T = 25$ mV

(ii) Diode current, $i_D = 2$ mA

Dynamic resistance is given by,

$$r_D = \frac{\eta V_T}{i_D}$$

Here current voltage relation is given as,

$$I_d = e^{V/V_T}$$

Comparing it with approximated standard equation $I_d = e^{V/\eta V_T}$, $\eta = 1$

Therefore, $r_D = \dfrac{\eta V_T}{i_D} = \dfrac{25 \text{ mV}}{2 \text{ mA}} = 12.5\ \Omega$

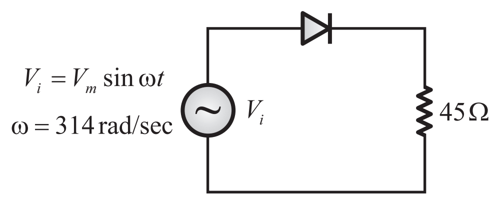
Hence, the correct option is (B).

1.7 (A)

Given :

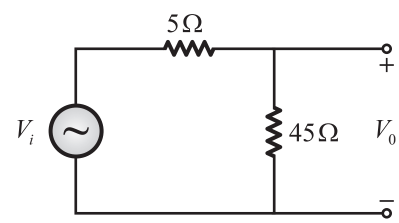
(i) Forward resistance of diode, $R_f = 5\ \Omega$

(ii) Given circuit is a half wave rectifier.



Case 1 : During positive half cycle,

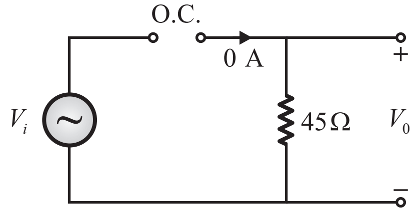
Diode is forward biased, hence it is ON and therefore, it can be replaced by $5\ \Omega$ resistance.



$$V_0 = \frac{45}{5+45} V_i = 0.9 V_i \quad \text{[By VDR]}$$

Case 2 : During negative half cycle,

Diode is reversed biased and hence, replaced by open circuit.

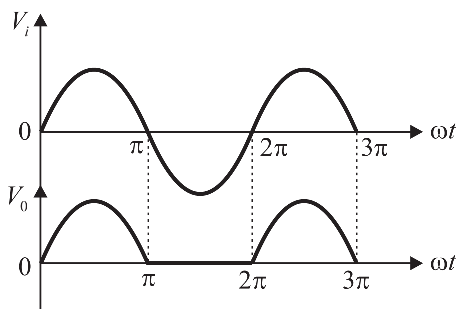


Hence, $V_0 = 0$ V

From above two cases, V_0 can be expressed as given below,

$$V_0 = \begin{cases} 0.9V_i\ ; & V_i > 0 \\ 0 \quad ; & V_i < 0 \end{cases}$$

Input and output waveform is shown below,



Average output voltage is given by,

$$V_{0(\text{avg})} = \frac{1}{2\pi}\int_0^{2\pi} V_0\, d\omega t$$

$$V_{0(\text{avg})} = \frac{1}{2\pi}\int_0^{\pi} 0.9V_m \sin\omega t\, d\omega t$$

$$V_{0(\text{avg})} = \frac{0.9V_m}{2\pi}[-\cos\omega t]_0^{\pi}$$

$$V_{0(\text{avg})} = \frac{0.9V_m}{2\pi}[1+1] = \frac{0.9\times 2V_m}{2\pi} = \frac{0.9V_m}{\pi}$$

$$I_{DC} = I_{(\text{avg})} = \frac{V_{0(\text{avg})}}{R} = \frac{V_{0(\text{avg})}}{45+5}$$

$$I_{DC} = I_{(\text{avg})} = \frac{0.9V_m}{50\pi} \approx \frac{V_m}{50\pi}$$

Hence, the correct option is (A).

1.8 (B)

Given :

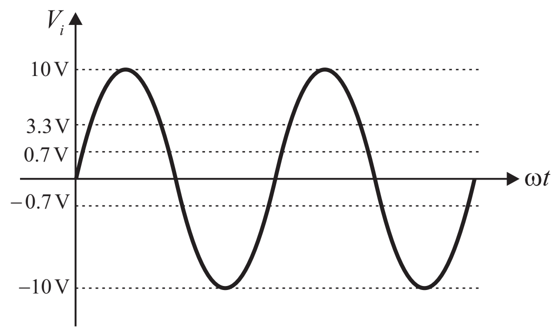
(i) Cut-in voltage of $D_z = 0.7$ V

(ii) Cut-in voltage of $D = 0.7$ V

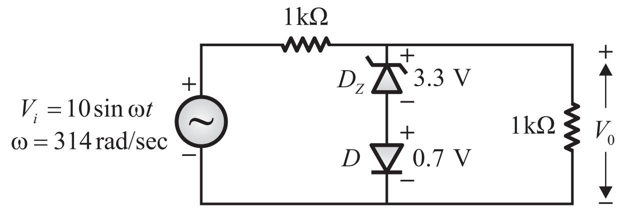
(iii) Break down voltage of $D_z = 3.3$ V

(iv) Break down voltage of $D = 50$ V

(v) $V_i = 10\sin\omega t$



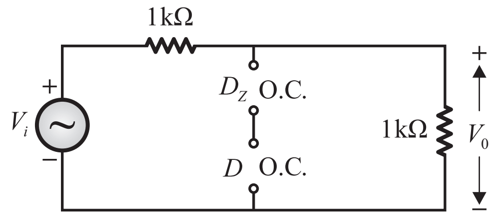
Given circuit is shown below,



Case 1 : During positive half cycle,

For $0 < V_i < 0.7$ V,

Zener diode will be in reverse bias region without breakdown and diode D will be in reverse bias.



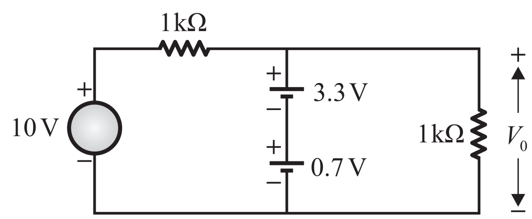
$$V_0 = \frac{V_i\times 1}{1+1} = \frac{V_i}{2} \qquad \text{[By VDR]}$$

For both diodes to be ON V_0 should be greater than 4 V ($V_Z + V_\gamma$).

Since, $V_0 = \dfrac{V_i}{2}$ therefore, for both diodes to be ON,

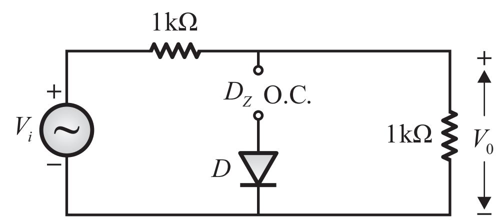
$$\frac{V_i}{2} > 4 \ \Rightarrow\ V_i > 8$$

For $V_i > 8$ V,



Hence, $V_0 = 0.7 + 3.3 = 4$ V

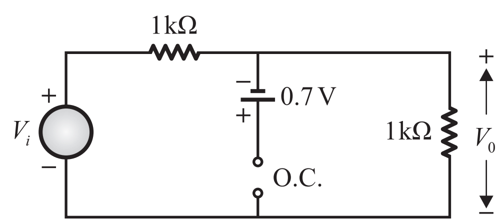
For $V_i < 8$ V,



$$V_0 = 1 \times \frac{V_i}{1+1} = \frac{V_i}{2} \quad \text{[By VDR]}$$

Case 2 : During negative half cycle,

In the negative half cycle, Zener diode will be in forward bias and diode D will be in reverse bias.



So, $\quad V_0 = V_i \times \frac{1}{1+1} \quad$ [By VDR]

$$V_0 = \frac{V_i}{2}$$

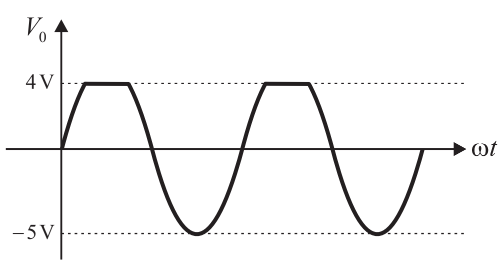


Fig. Output waveform

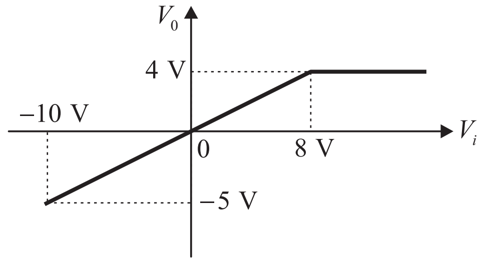
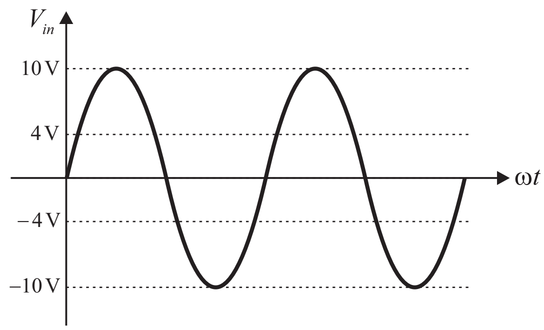


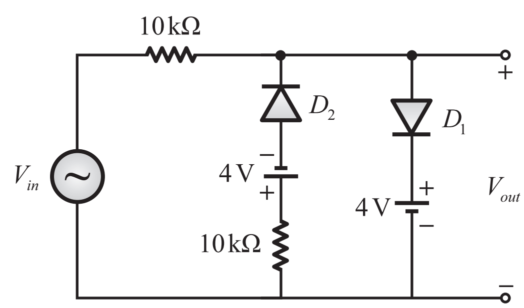
Fig. Transfer characteristics

Hence, the correct option is (B).

1.9 (D)

Given : $V_{in} = 10\sin\omega t$

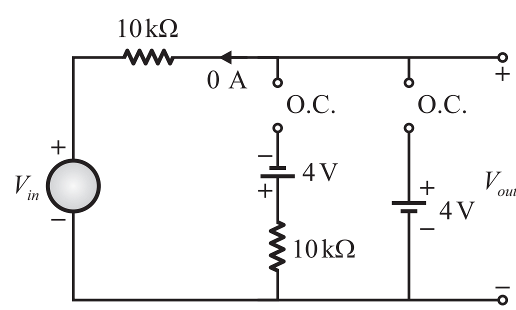




Method 1

Case 1 : $-4 < V_{in} < 4$,

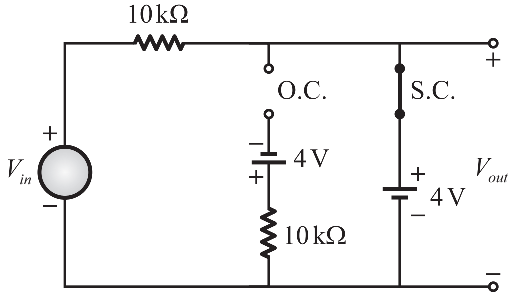
Both diodes are reverse biased i.e. OFF.



Hence, $V_{out} = V_{in}$

Case 2 : $V_{in} > 4$ V,

D_1 is forward biased i.e. ON and D_2 is reverse biased i.e. OFF.

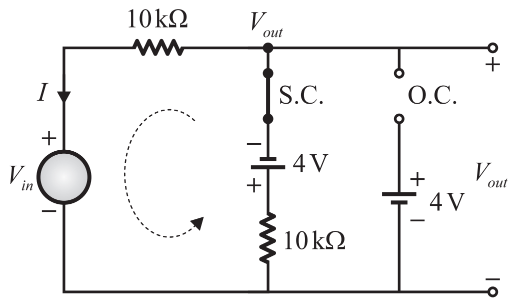


Hence, $V_{out} = 4$ V

Case 3 : $V_{in} < -4$ V,

Diode D_1 is reverse biased i.e. OFF.

Diode D_2 is forward biased i.e. ON.



Applying KVL in loop shown,

$$V_{in} + 10I + 4 + 10I = 0$$

$$I = \frac{-(V_{in}+4)}{20} \qquad \ldots \text{(i)}$$

From figure,

$$V_{out} = -10I - 4$$

From equation (i),

$$V_{out} = 10 \times \frac{(V_{in}+4)}{20} - 4$$

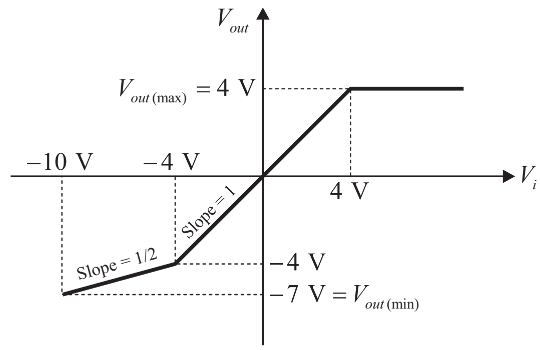
For minimum value of output,

$$V_{in(\min)} = -10 \text{ V}$$

Therefore, $V_{out(\min)} = 10 \times \dfrac{(-10+4)}{20} - 4$

$$V_{out(\min)} = -7 \text{ V}$$

Hence, the transfer characteristics is given by,

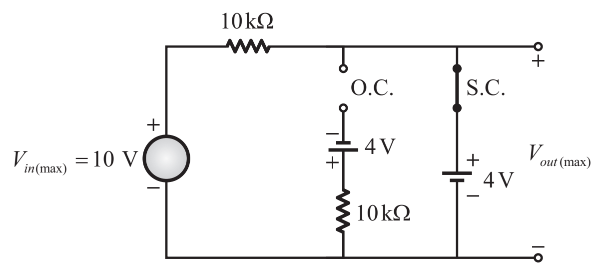


Hence, the correct option is (D).

Method 2

Case 1 : $V_{in(\max)} = 10$ V

D_1 is forward biased i.e. ON and D_2 is reverse biased i.e. OFF.

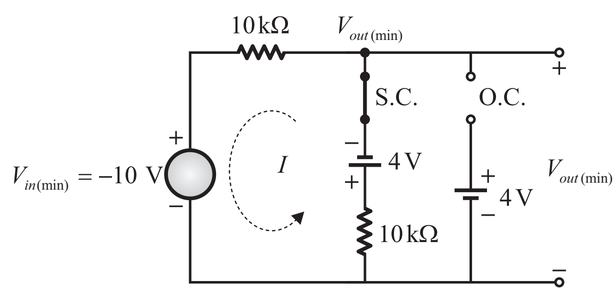


Hence, $V_{out(\max)} = 4$ V

Case 2 : $V_{in(\min)} = -10$ V

Diode D_1 is reverse biased i.e. OFF.

Diode D_2 is forward biased i.e. ON.



Applying KCL in above figure,

$$+V_{in(\min)} + 10I + 4 + 10I = 0$$

$$I = \frac{-V_{in(\min)} - 4}{20} = \frac{-(-10) - 4}{20} = \frac{3}{10}$$

$$V_{out(\min)} = -4 - 10I = -4 - 10\left(\frac{3}{10}\right)$$

$$V_{out(\min)} = -7\ \text{V}$$

Hence, the correct option is (D).

1.10 (C)

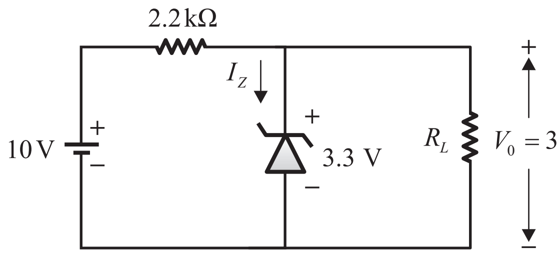
Given :

(i) $R_Z = 0.1\ \text{k}\Omega$

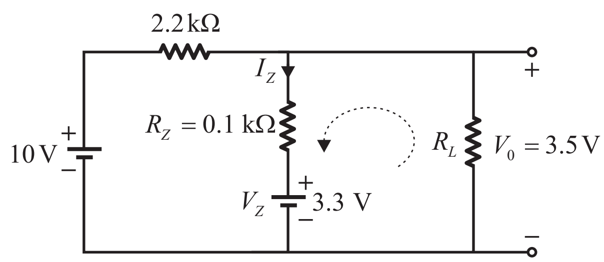
(ii) Zener breakdown voltage,

$V_Z = 3.3\ \text{V}$

Given circuit is shown below,



Since, output voltage is greater than breakdown voltage of Zener diode $(V_0 > V_z)$, therefore, Zener diode is in reverse breakdown region.



Applying KVL in the loop shown,

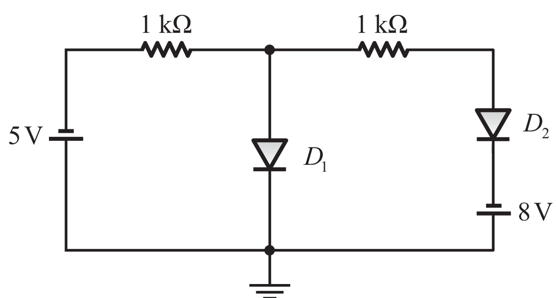
$$-3.5 + I_Z \times 0.1 + 3.3 = 0$$

$$I_Z = \frac{3.5 - 3.3}{0.1} = 2\ \text{mA}$$

Hence, the correct option is (C).

1.11 (C)

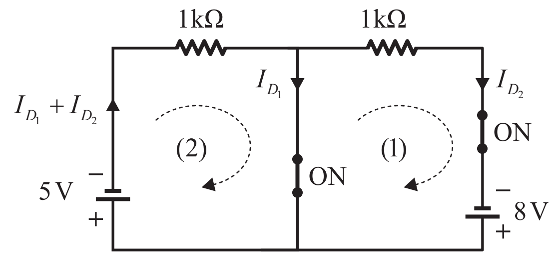
Given circuit is shown below,



Case 1 :

(i) Assume both diodes D_1 and D_2 are ON.

(ii) For both diodes to be ON, I_{D_1} and I_{D_2} should be greater than zero from p to n.



Applying KVL in loop (1),

$$1 \times I_{D_2} - 8 = 0$$

$$I_{D_2} = \frac{8}{1} = 8\ \text{mA}$$

Since, $I_{D_2} > 0$ A, our assumption about D_2 is correct i.e. D_2 is ON.

Applying KVL in loop (2),

$$5 + 1 \times (I_{D_1} + I_{D_2}) + 0 = 0$$

$$I_{D_1} + I_{D_2} = \frac{-5\ \text{V}}{1\ \text{k}\Omega} = -5\ \text{mA}$$

$$I_{D_1} = -5 - I_{D_2} = -5 - 8 = -13\ \text{mA}$$

Since, $I_{D_1} < 0$ A, our assumption about D_1 is wrong i.e. D_1 is OFF.

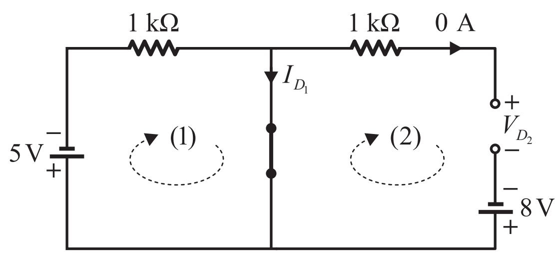
Hence, our whole assumption is wrong.

Case 2 :

(i) Assume diode D_1 is ON and D_2 is OFF.

(ii) For diode D_1 to be ON, current I_{D_1} should be greater than zero from p to n.

(iii) For diode D_2 to be OFF, voltage V_{D_2} should be less than zero from p to n.



Applying KVL in loop (2),

$$V_{D_2} - 8 + 0 \times 1 = 0$$

$$V_{D_2} = 8 \text{ V}$$

Since, $V_{D_2} > 0$ V, our assumption about D_2 is wrong i.e. D_2 is ON.

Applying KVL in loop (1),

$$5 + 1 \times I_{D_1} + 0 = 0$$

$$I_{D_1} = -5 \text{ mA}$$

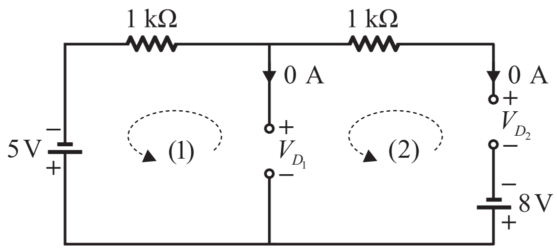
Since, $I_{D_1} < 0$ A, our assumption about D_1 is wrong i.e. D_1 is OFF.

Hence, our whole assumption is wrong.

Case 3 :

(i) Assume both diodes D_1 and D_2 are OFF.

(ii) For both diodes to be OFF, voltages V_{D_1} and V_{D_2} should be less than zero from *p* to *n*.



Applying KVL in loop (2),

$$-V_{D_2} + V_{D_1} + 8 = 0$$

$$-V_{D_2} - 5 + 8 = 0$$

$$V_{D_2} = 3 \text{ V}$$

Since $V_{D_2} > 0$ V, our assumption about D_2 is wrong i.e. D_2 is ON.

Applying KVL in loop (1),

$$-V_{D_1} - 5 = 0$$

$$V_{D_1} = -5 \text{ V}$$

Since $V_{D_1} < 0$ V, our assumption about D_1 is correct i.e. D_1 is OFF.

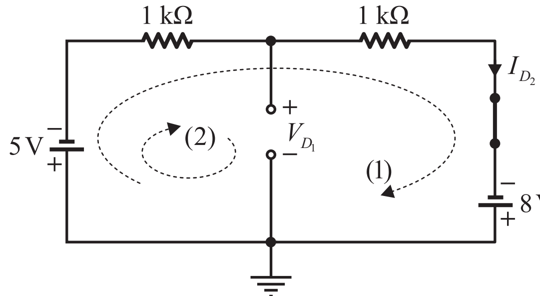
Hence, our whole assumption is wrong.

Case 4 :

(i) Assume diode D_2 is ON and D_1 is OFF.

(ii) For diode D_2 to be ON, current I_{D_2} should be greater than zero from *p* to *n*.

(iii) For diode D_1 to be OFF, voltage V_{D_1} should be less than zero from *p* to *n*.



Applying KVL in loop (1),

$$5 + 2I_{D_2} - 8 = 0$$

$$2I_{D_2} = 3$$

$$I_{D_2} = \frac{3}{2} \text{ mA}$$

Since, $I_{D_2} > 0$ A, our assumption about D_2 is correct i.e. D_2 is ON.

Applying KVL in loop (2),

$$5 + 1 \times I_{D_2} + V_{D_1} = 0$$

$$5 + 1 \times \frac{3}{2} + V_{D_1} = 0$$

$$\frac{13}{2} + V_{D_1} = 0$$

$$V_{D_1} = \frac{-13}{2}$$

Since $V_{D_1} < 0$ V, our assumption about D_1 is correct i.e. D_1 is OFF.

Therefore, current across D_1 is 0 A.

Hence, the correct option is (C).

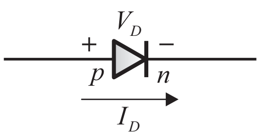
🕮 Key Point

(i) When there are two or more diodes in the circuit then our assumption about the diodes should be based on observation. In this question,

p terminal of D_1 is connected to –5 V and n terminal is grounded. Hence, we can assume D_1 is OFF.

p terminal of D_2 is connected to –5 V and n terminal is connected to –8 V i.e. at higher potential. Hence, we can assume D_2 is ON.

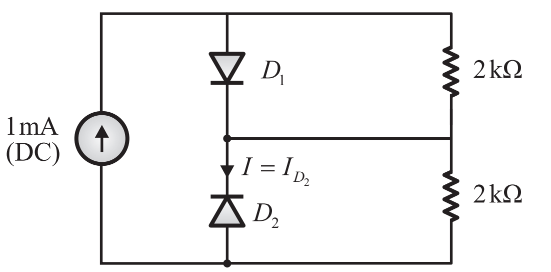
(ii) In all assumptions, direction of current and polarity of voltage across diode is taken from p to n.



For diode to be ON, V_D and I_D should be positive.

1.12 (A)

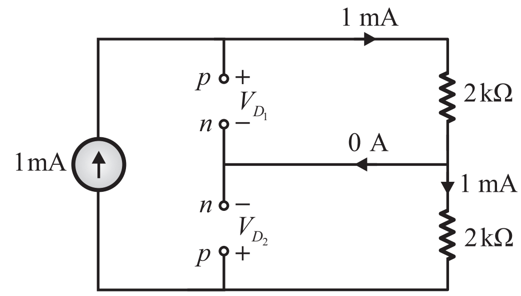
Given circuit is shown below,



Case 1 :

(i) Assume diodes D_1 and D_2 both are OFF.

(ii) For both diodes D_1 and D_2 to be OFF. V_{D_1} and V_{D_2} should be less than zero from p to n.



From figure,

$$V_{D_1} = 2\times 1 = 2 \text{ V}$$

$$V_{D_2} = -1\times 2 = -2 \text{ V}$$

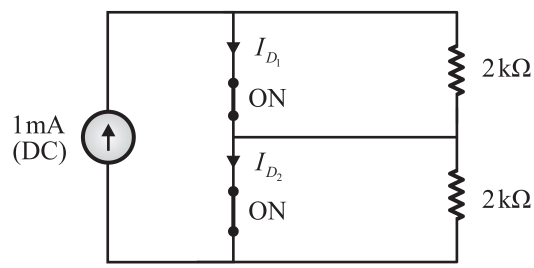
Hence, $V_{D_1} > 0$ V and $V_{D_2} < 0$ V.

Therefore, our assumption about D_2 is correct i.e. D_2 is OFF and our assumption about D_1 is wrong i.e. D_1 is ON.

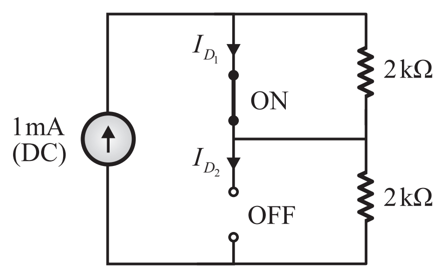
Hence, our whole assumption is wrong.

Case 2 :

Assume Diodes D_1 and D_2 both are ON.



Since, the direction of current in diode D_2 is n to p. Therefore, D_2 is OFF and D_1 is ON. Hence, the modified circuit is shown below,



Therefore, current, $I_{D_2} = 0$ mA

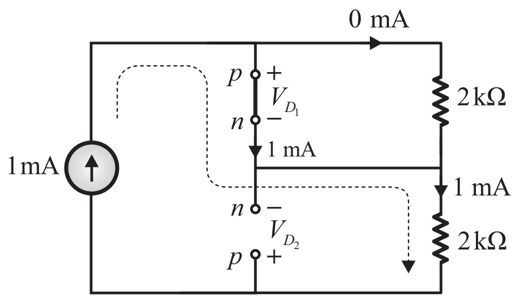
Hence, the correct option is (A).

Case 3 :

(i) Assume diode D_1 is ON and diode D_2 is OFF.

(ii) For diode D_1 to be ON, I_{D_1} should be greater than zero from p to n.

(iii) For diode D_2 to be OFF, V_{D_2} should be less than zero from p to n.



From figure, $I_{D_1} = 1$ mA

Since, $I_{D_1} > 0$ A therefore, our assumption about D_1 is correct i.e. D_1 is ON.

$$V_{D_2} = -1\times 2 = -2 \text{ V}$$

Since, $V_{D_2} < 0$ V therefore, our assumption about D_2 is correct i.e. D_2 is OFF.

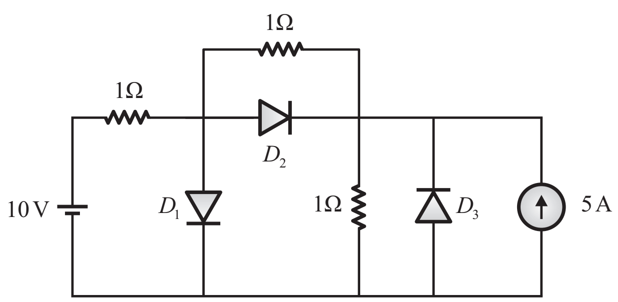
Therefore, $I_{D_2} = 0$ mA

Hence, the correct option is (A).

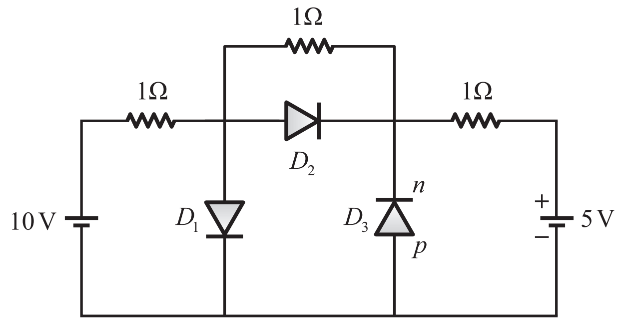
Scan for Video Solution

1.13 (A)

Given circuit is shown below,



Using source transformation, the given circuit can be redrawn as shown below,

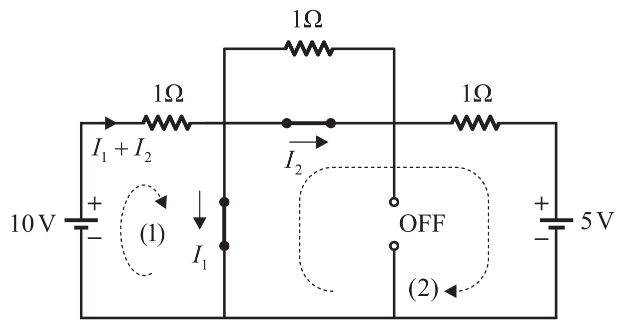


Case 1 :

Analytically, $D_3 = \text{OFF}$

(i) Assume both diodes D_1 and D_2 are ON and D_3 OFF.

(ii) For both diodes D_1 and D_2 to be ON, currents I_1 and I_2 should be greater than zero from p to n.



Applying KVL in loop (1),

$$-10 + (I_1 + I_2)\times 1 + 0 = 0$$

$$I_1 + I_2 = 10$$

$$I_1 = 10 - I_2 = 10 - (-5) = 15 \text{ A}$$

$$I_1 = 15 \text{ A}$$

Since, $I_1 > 0$ A, hence our assumption is correct i.e. D_1 is ON.

Applying KVL in loop (2),

$$0 + 0 + I_2 \times 1 + 5 = 0$$

$$I_2 = -5 \text{ A}$$

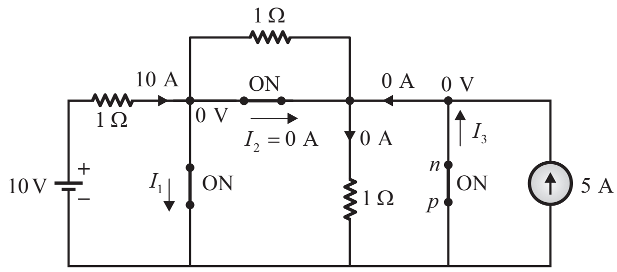
Since, $I_2 < 0$ A, hence our assumption is wrong i.e. D_2 is OFF.

Hence, our whole assumption is wrong.

Case 2 :

(i) Assume all diodes are ON.

(ii) For all diodes to be ON, I_1, I_2 and I_3 should be greater than zero from p to n.



From figure,

$$I_1 = 10\text{ A},\ I_2 = 0\text{ A and } I_3 = -5\text{ A}$$

Since, I_1 is greater than zero, therefore, our assumption about D_1 is correct i.e. D_1 is ON and since, $I_2 = 0$ hence, our assumption about D_2 is wrong i.e. D_2 is OFF.

Since, $I_3 < 0$ A hence, our assumption about D_3 is wrong i.e. D_3 is OFF.

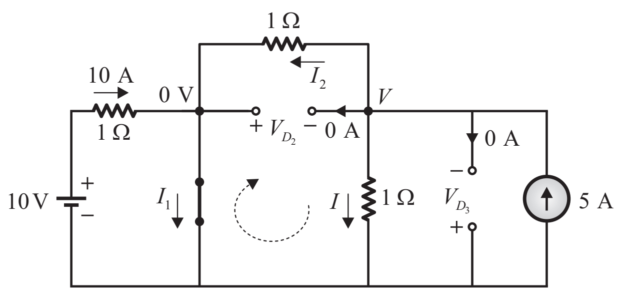
Hence, our whole assumption is wrong.

Case 3 :

(i) Assume D_1 is ON, D_2 and D_3 are OFF.

(ii) For diode D_1 to be ON, current I_D should be greater than zero from p to n.

(iii) For diode D_2 and D_3 to be OFF, voltage V_{D_2} and V_{D_3} should be less than zero from p to n.



Applying KCL at node V,

$$\frac{V}{1} + \frac{V-0}{1} - 5 = 0$$

$$V = 2.5\text{ V}$$

$$V_{D_3} = -V = -2.5\text{ V}$$

Since $V_{D_3} < 0$ V, hence our assumption about D_3 is correct i.e. D_3 is OFF.

From figure,

$$I_2 = \frac{V-0}{1} = 2.5\text{ A}$$

$$V_{D_2} = -I_2 \times 1 = -2.5 \times 1 = -2.5\text{ V}$$

Since $V_{D_2} < 0$ V, hence our assumption about D_2 is correct i.e. D_2 is OFF.

$$I_1 = 10 + I_2 = 10 + 2.5 = 12.5\text{ A}$$

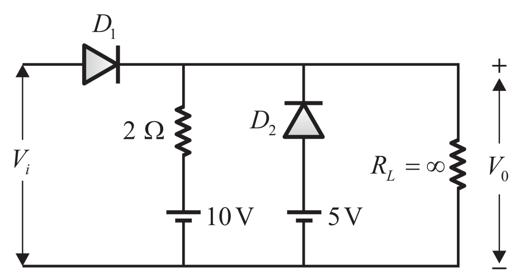
Since $I_1 > 0$ A, hence our assumption about D_1 is correct i.e. D_1 is ON.

Hence, the correct option is (A).

Scan for Video Solution

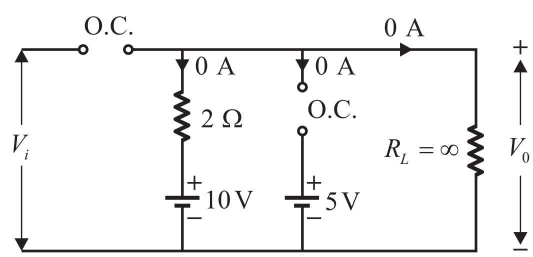
1.14 (A)

Given circuit is shown below,



For small value of input voltage (Either positive or negative),

Diode D_1 and D_2 are OFF due to 10 V voltage source.

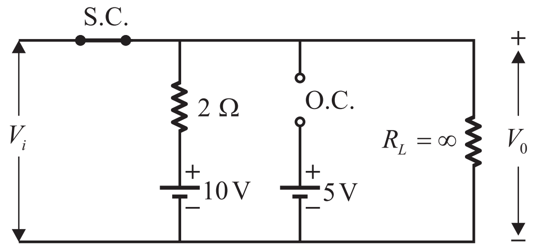


From figure, $V_0 = 10$ V

Condition for diode D_1 to be ON,

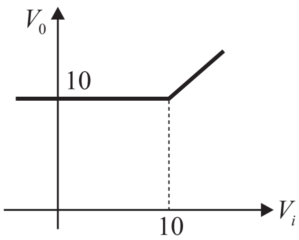
$$V_i > 10 \text{ V}$$

For this condition, D_2 will be OFF.



From figure, $V_0 = V_i$

Hence, transfer characteristic of the given circuit is shown below,



Hence, the correct option is (A).

1.15 (A)

Given :

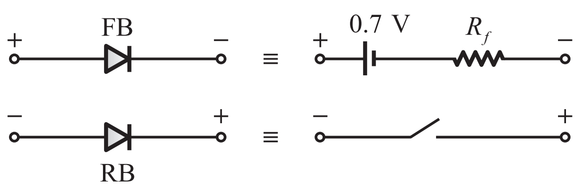


Fig. (a)

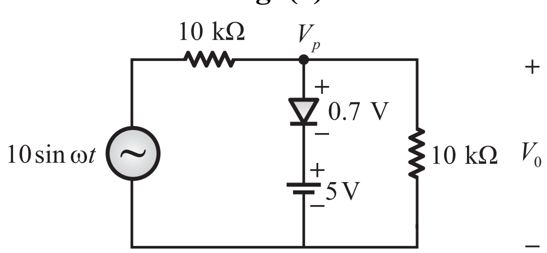


Fig. (b) Practical diode circuit

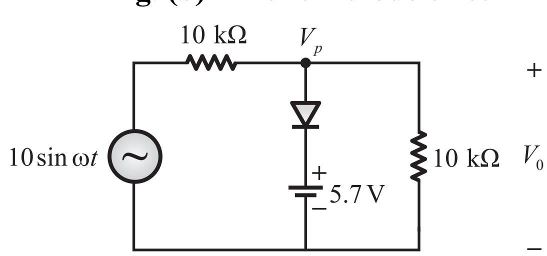
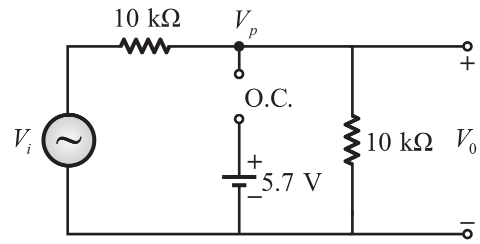


Fig. (c) Ideal diode circuit

Initially for small positive V_i, diode will be reverse biased and hence, it can be replaced by open circuit.



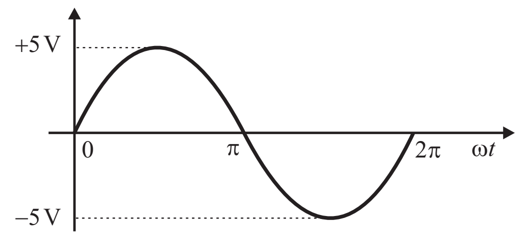
$$V_0 = V_p = \frac{V_i}{10+10} \times 10 = \frac{V_i}{2} \quad \text{[By VDR]}$$

Now, for diode to be in forward bias i.e. ON,

$$V_p > 5.7 \text{ V}$$

$$\frac{V_i}{2} > 5.7 \text{ V} \Rightarrow V_i > 11.4 \text{ V}$$

Hence, for diode to be ON, V_i must be more than 11.4 V which is never possible and therefore, diode will be reverse biased always. Hence, $V_0 = 5\sin\omega t$ at all times.



Hence, the correct option is (A).

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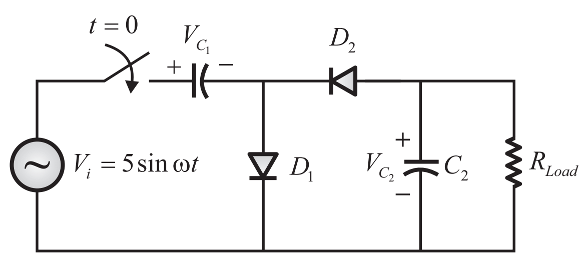
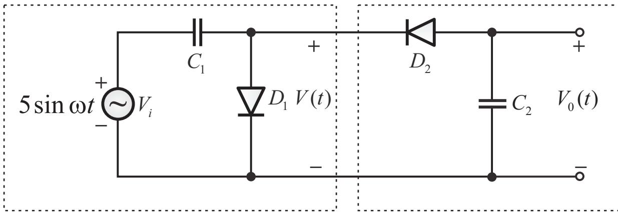
1.16 (D)

Given :



The given circuit can be redrawn as given below,



Circuit 1 Circuit 2

In above figure, circuit (1) represents ideal negative clamper circuit and circuit (2) represents ideal negative peak detector circuit. Hence, $V(t)$ is the output of negative clamper circuit and therefore, it will be shifted form of input in downward direction.

$$V(t) = V_i - V_m = 5\sin\omega t - 5$$

In negative clamper circuit, capacitor is charged upto maximum value of input.

Hence, $V_{C_1} = V_m = 5\text{ V}$

Output of negative peak detector circuit will be maximum negative value of $V(t)$.

$$V_0(t) = V(t)\big|_{\max} = -5 - 5 = -10\text{ V}$$

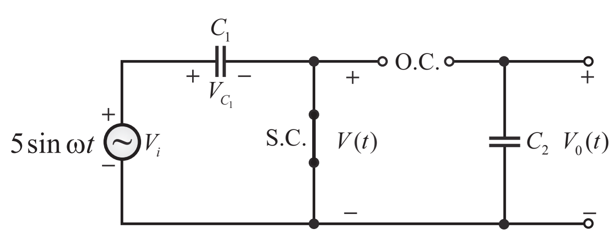
Hence, the correct option is (D).

Working :

Case 1 : During 1st positive half cycle,

Diode D_1 will be forward biased and diode D_2 will be reverse biased.

Hence, charging of capacitor C_1 will start and it will charge upto maximum value of input because charging time constant is zero.

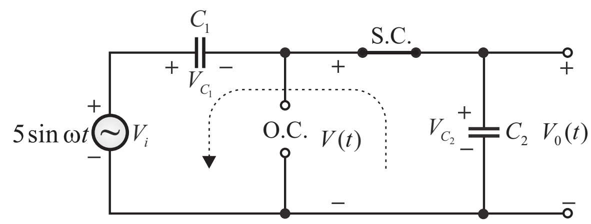


Hence, $V_{C_1} = |5\sin\omega t|_{\max} = 5\text{ V}$

Case 2 : During 1st negative half cycle,

Diode D_1 will be reverse biased and diode D_2 will be forward biased.

Hence, charging of capacitor C_2 will start and capacitor C_2 will charge upto maximum negative value of input, which appears across capacitor C_2.



Applying KVL in the loop shown,

$$V_i + 0 - V_{C_2} - V_{C_1} = 0$$

$$V_{C_2} = V_i - V_{C_1}$$

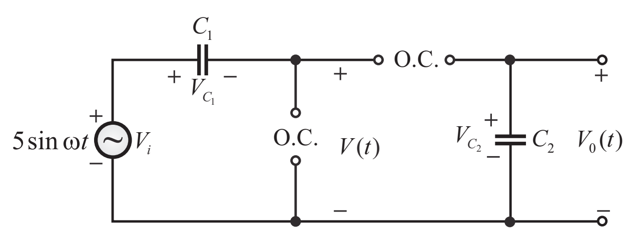
$$V_{C_2} = |5\sin\omega t|_{\min} - 5$$

$$V_{C_2} = -5 - 5 = -10\text{ V}$$

Case 3 : During 2nd positive half cycle,

Positive terminal of D_1 is at -5 V and positive terminal of D_2 is at -10 V. Now, diode D_1 and D_2 will be forward biased only when voltage at their negative terminal will be less than that of the positive terminal, which is never possible beyond first negative half cycle and first positive half cycle.

Hence, D_1 and D_2 will always be reverse biased.



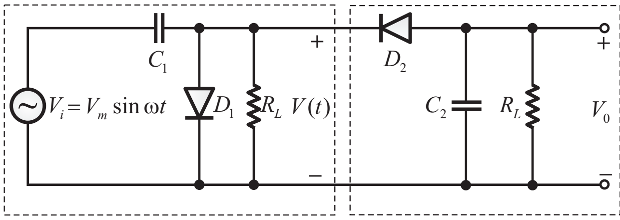
From above circuit,

$$V_0(t) = V_{C_2} = -10\text{ V}$$

Hence, the correct option is (D).

📖 Key Point

(1)



Circuit 1 **Circuit 2**

Circuit 1 : It is a ideal negative clamper circuit. Output of this circuit will be shifted form of input in downward direction. Hence, output $V(t)$ of this circuit is given by,

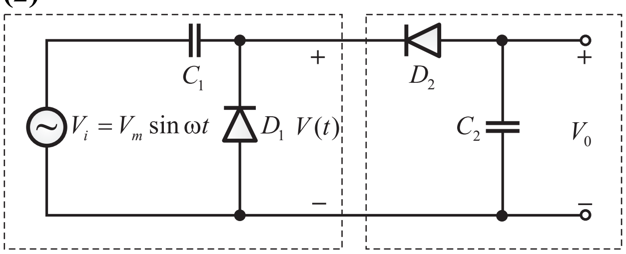
$$V(t) = V_i - V_m = V_m \sin \omega t - V_m$$

Circuit 2 : It is a ideal negative peak detector circuit.

Output of this circuit will be maximum negative value of $V(t)$. Hence, output $V_0(t)$ of this circuit is given by,

$$V_0(t) = |V_m \sin \omega t|_{\min} - V_m = -V_m - V_m = -2V_m$$

(2)



Circuit 1 **Circuit 2**

Circuit 1 : It is a ideal positive clamper circuit. Output of this circuit will be shifted form of input in upward direction. Hence, output $V(t)$ of this circuit is given by,

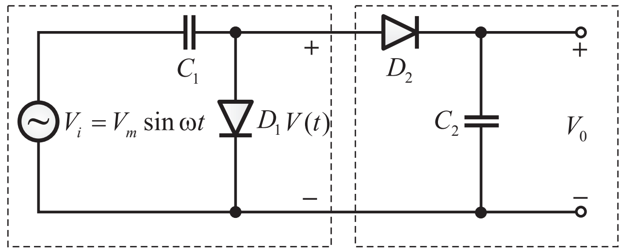
$$V(t) = V_i + V_m = V_m \sin \omega t + V_m$$

Circuit 2 : It is a ideal negative peak detector circuit.

Output of this circuit will be maximum negative value of $V(t)$. Hence, output $V_0(t)$ of this circuit is given by,

$$V_0(t) = |V_m \sin \omega t|_{\min} + V_m = -V_m + V_m = 0\,\text{V}$$

(3)



Circuit 1 **Circuit 2**

Circuit 1 : It is a ideal negative clamper circuit.

Output of this circuit will be shifted form of input in downward direction. Hence, output $V(t)$ of this circuit is given by,

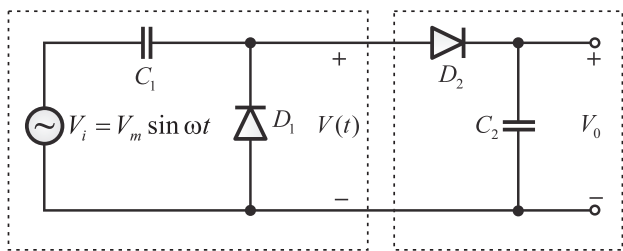
$$V(t) = V_i - V_m = V_m \sin \omega t - V_m$$

Circuit 2 : It is a ideal positive peak detector circuit.

Output of this circuit will be maximum positive value of $V(t)$. Hence, output $V_0(t)$ of this circuit is given by,

$$V_0(t) = |V_m \sin \omega t|_{\max} - V_m = V_m - V_m = 0\,\text{V}$$

(4)



Circuit 1 **Circuit 2**

Circuit 1 : It is a ideal positive clamper circuit.

Output of this circuit will be shifted form of input in upward direction. Hence, output $V(t)$ of this circuit is given by,

$$V(t) = V_i + V_m = V_m \sin \omega t + V_m$$

Circuit 2 : It is a ideal positive peak detector circuit.

Output of this circuit will be maximum positive value of $V(t)$. Hence, output $V_0(t)$ of this circuit is given by,

$$V_0(t) = |V_m \sin \omega t|_{\max} + V_m = V_m + V_m = 2V_m$$

1.17 (A)

Given circuit is shown below,

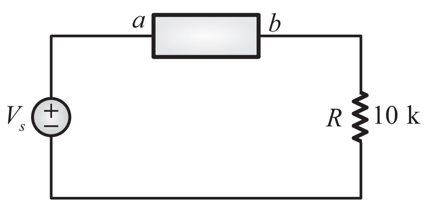


Fig. (a)

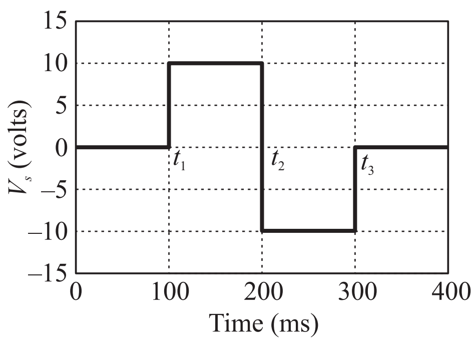


Fig. (b)

(i) When $0 < t < t_1$:

$$V_s = 0,\ i = \frac{V_s}{R} = 0 \text{ A}$$

(ii) When $t_1 < t < t_2$:

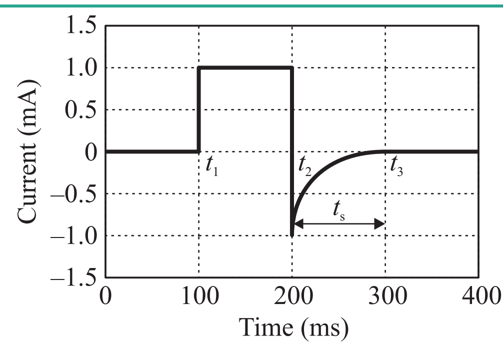
$$V_s = 10 \text{ V},\ i = \frac{V_s}{R} = \frac{10}{10} = 1 \text{ mA}$$

(iii) When $t_2 < t < t_3$:

$$V_s = -10 \text{ V}$$

$$i = \frac{V_s}{R} = \frac{-10}{10} = -1 \text{ mA}$$

(spike for very short duration and zero for remaining)



From figure, t_s = Storage time

Storage time is the time duration over which the forward current reverses and maintains constant level.

Or **storage time** is the time take by electron to move back from p-side to n-side and for holes to move back from n-side to p-side.

Element is not conducting for negative supply.

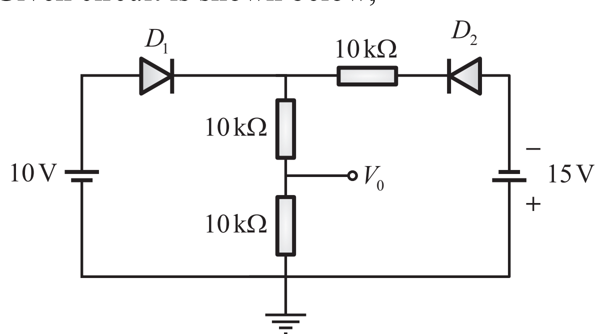
The element connected between 'a' and 'b' is allowing the flow of current when $V_a > V_b$ and not allowing the flow of current when $V_a < V_b$.

Hence, the element is a p-n junction diode with p-side at 'a' and n-side at 'b'.

Hence, the correct option is (A).

1.18 (B)

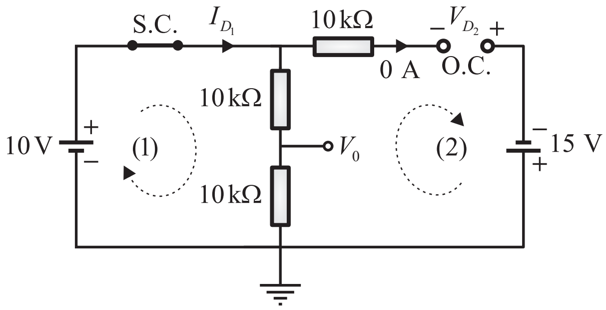
Given circuit is shown below,



(i) Assume diode D_1 is forward biased i.e. ON and diode D_2 is reversed biased i.e. OFF.

(ii) For diode D_1 to be ON, current I_{D_1} should be greater than zero from p to n.

(iii) For diode D_2 to be OFF, voltage V_{D_2} should be less than zero from p to n.



Applying KVL in loop (1),

$$-10+20I_{D_1}=0$$

$$I_{D_1}=\frac{10}{20}=0.5 \text{ mA}$$

Since, $I_{D_1}>0$ A, our assumption about diode D_1 is correct i.e. D_1 is ON.

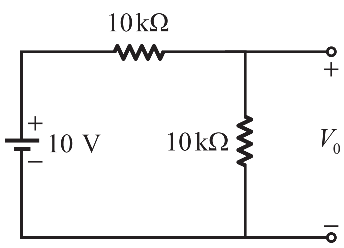
Applying KVL in loop (2),

$$-15-0.5\times 20+0\times 10-V_{D_2}=0$$

$$V_{D_2}=-15-10=-25 \text{ V}$$

Since $V_{D_2}<0$ V, our assumption about is also correct i.e. D_2 is OFF.

Hence, modified circuit can be drawn as shown below,



From the above circuit,

$$V_0=10\times\frac{10}{10+10} \quad \text{[By VDR]}$$

$$V_0=5 \text{ V}$$

Hence, the correct option is (B).

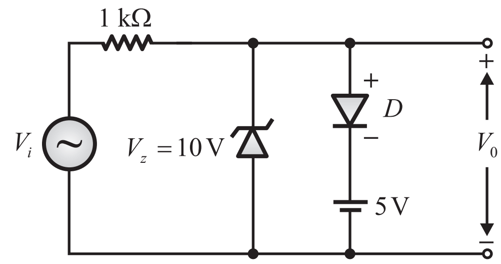
📖 Key Point

Always take the assumption about diode on the basis of observation.

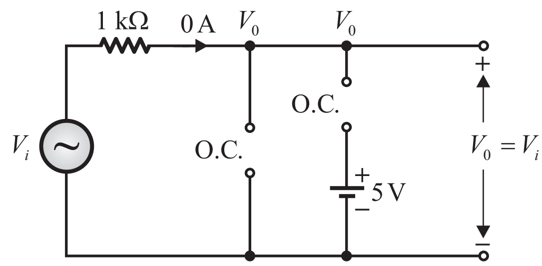
1.19 (C)

Given : Forward voltage drop of both diode is 0.7 V.

Given circuit is shown below,



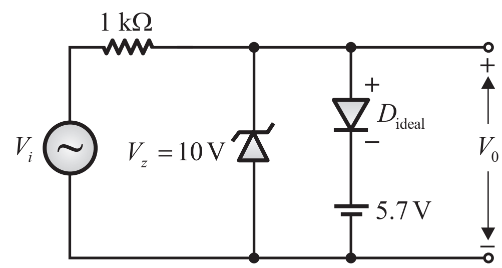
From observation of above figure, initially for positive half cycle diode D is OFF due to 5 V and Zener diode will be in reverse bias region without breakdown.



From figure, $V_0=V_i$

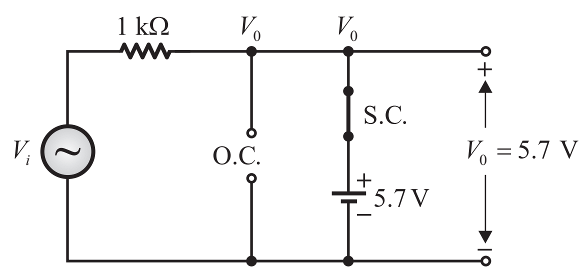
Diode D is a practical diode with cut-in voltage (V_γ) of 0.7 V.

Ideal diode D with $V_\gamma=0$ V is shown below,



For diode D to be ON, V_i should be greater than 5.7 V.

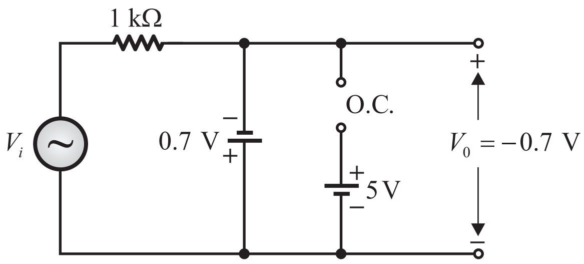
For $V_i>5.7$ V,



Zener diode will never breakdown as diode D is ON first and makes $V_0 = 5.7$ V. Now, the voltage across Zener diode is fixed at 5.7 V. Hence, Zener diode will never breakdown.

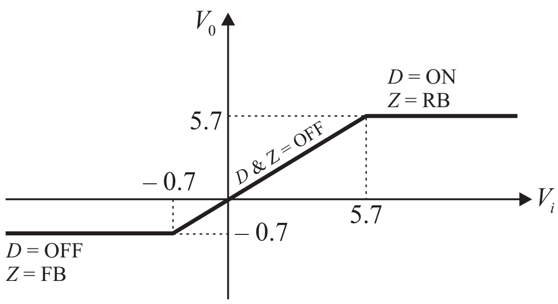
For $V_i < -0.7$ V (Negative half cycle),

Zener diode acts as a normal diode in forward bias and diode D will be reverse biased (for $V_i < 0$) and acts as open circuit.



$$V_0 = -0.7 \text{ V}$$

Hence, the transfer characteristic is given by,



Hence, the correct option is (C).

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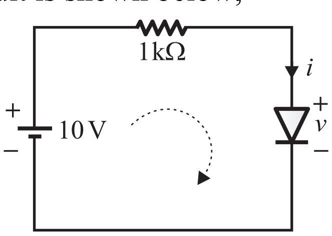
1.20 (D)

Given circuit is shown below,



Method 1

Applying KVL in the loop shown,

$$-10 + iR + v = 0$$

$$i = \frac{v - 0.7}{500}, R = 1\text{k}\Omega \qquad \text{[Given]}$$

$$10 = \frac{(v - 0.7)}{500} \times 1000 + v$$

$$10 = 2v - 1.4 + v$$

$$3v = 11.4$$

$$v = 3.8 \text{ V}$$

$$i = \left(\frac{3.8 - 0.7}{500}\right) \times 1000 \text{ mA}$$

$$i = 6.2 \text{ mA}$$

Hence, the correct option is (D).

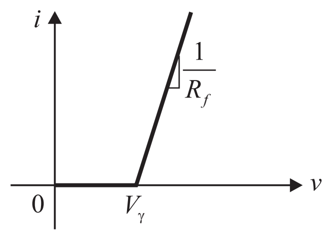
Method 2

From given i-v characteristics,

$$i = \frac{v - 0.7}{500} \qquad \ldots \text{(i)}$$

i-v characteristics of forward bias practical diode is given by,

$$i = \frac{v - V_\gamma}{R_f} \qquad \ldots \text{(ii)}$$



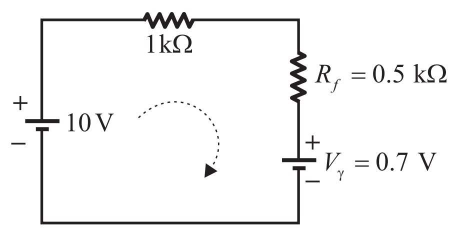
R_f = forward resistance of diode

V_γ = cut-in voltage (offset voltage)

From equation (i) and (ii),

$$R_f = 500\ \Omega = 0.5\ \text{k}\Omega,\ V_\gamma = 0.7 \text{ V}$$

The given circuit is replaced by its equivalent as shown below,



Appling KVL in the above loop,

$$-10+1\times i+0.5\times i+0.7=0$$

$$1.5\times i=9.3$$

$$i=\frac{9.3}{1.5}=6.2 \text{ mA}$$

Hence, the correct option is (D).

Scan for Video Solution

1.21 (B)

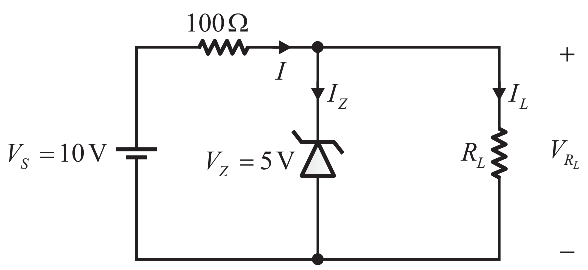
Given :

(i) Knee current of Zener diode,

$I_{Z(\min)}=10$ mA

(ii) Zener voltage, $V_Z=5$ Volt

Regulator circuit is shown below,



For variable load, input current I is given by,

$$I=I_{Z(\max)}+I_{L(\min)} \qquad \dots \text{(i)}$$

$$I=I_{Z(\min)}+I_{L(\max)} \qquad \dots \text{(ii)}$$

(i) Calculation for minimum load resistance :

Input current I is given by,

$$I=\frac{V_S-V_Z}{R}=\frac{10-5}{0.1}=50 \text{ mA}$$

From equation (ii),

$$I_{L(\max)}=I-I_{Z(\min)}=50-10=40 \text{ mA}$$

From figure,

$$V_{R_L}=I_LR_L=V_Z=5 \text{ V}$$

$$I_{L(\max)}R_{L(\min)}=5 \text{ V}$$

$$R_{L(\min)}=\frac{5}{I_{L(\max)}}=\frac{5}{40}=125\ \Omega$$

(ii) Calculation for minimum power rating :

Power dissipation across Zener diode is given by,

$$P_Z=V_ZI_Z$$

Minimum power rating of Zener diode represents maximum allowed power dissipation across Zener diode

$$P_{Z(\max)}=V_Z\,I_{Z(\max)} \qquad \dots \text{(iii)}$$

For no load condition $R_L=\infty$, $I_{L(\min)}=0$ A

From equation (i), $I_{Z(\max)}=I=50$ mA

From equation (iii), $P_{Z(\max)}=5\times 50=250$ mW

Hence, the correct option is (B).

☒ Avoid This Mistake

(i) The minimum power rating of the Zener diode is $P_{Z(\max)}=I_{Z(\max)}\times V_Z$. **[Correct]**

(ii) The minimum power rating of the Zener diode is $P_{Z(\min)}=I_{Z(\min)}\times V_Z$. **[Incorrect]**

(iii) The maximum power dissipation across Zener diode is $P_{Z(\max)}=I_{Z(\max)}\times V_Z$ **[Correct]**

(iv) The minimum power dissipation across Zener diode is $P_{Z(\min)}=I_{Z(\min)}\times V_Z$ **[Correct]**

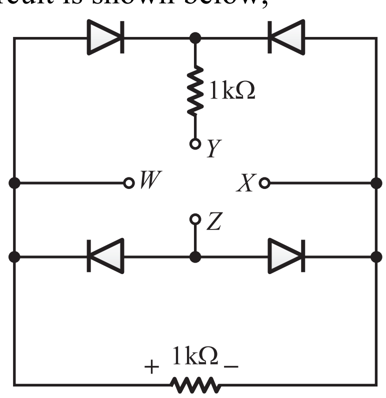
1.22 (D)

Given :

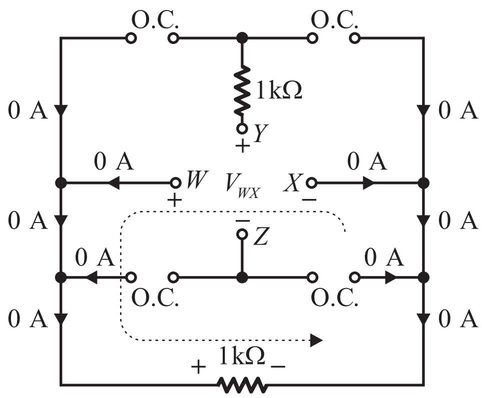
(i) $V_{YZ}=1000\sin\omega t$

(ii) All diodes are ideal.

Given circuit is shown below,



Case 1 : During positive half cycle,



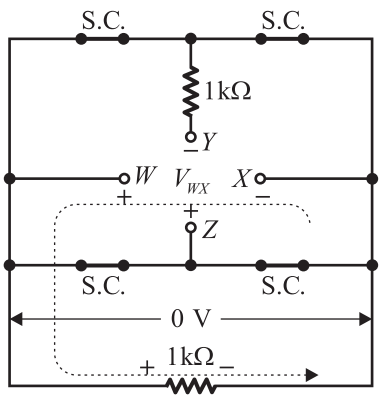
All the four diodes are reverse biased, so all the diodes will be open circuited.

Applying KVL in the loop shown,

$$-V_{WX} + 1 \times 0 = 0$$

Hence, $V_{WX} = 0$ V ... (i)

Case 2 : During negative half cycle,



All the four diodes are forward biased, so all the diodes will be short circuited.

Applying KVL in the loop shown,

$$-V_{WX} + 1 \times 0 = 0$$

$$V_{WX} = 0 \text{ V} \quad \dots \text{(ii)}$$

From equation (i) and (ii),

$$V_{WX} = 0 \text{ for all } t$$

Hence, the correct option is (D).

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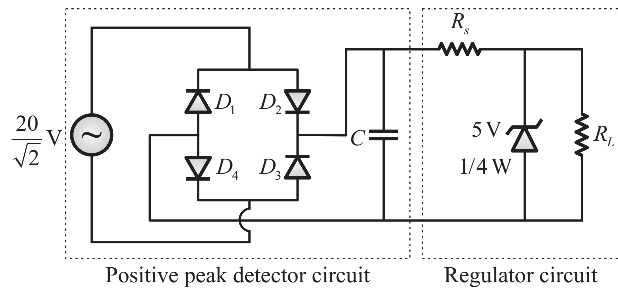
1.23 300

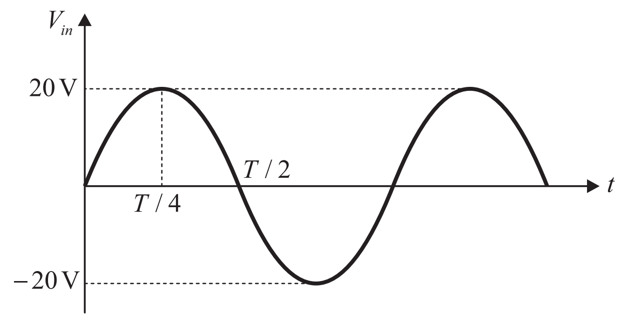
Given circuit is shown below,



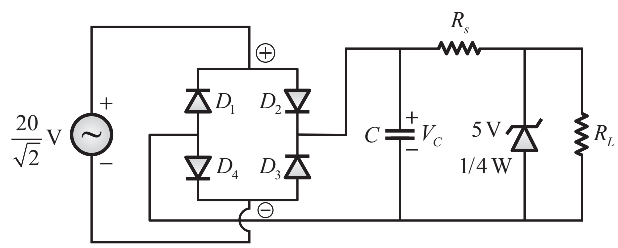
Initially the capacitor is uncharged.

i.e. $V_C = 0$ V

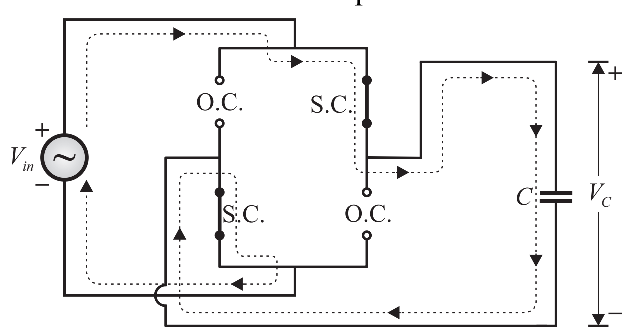
Input is given by,



Case 1 : $0 < t < \dfrac{T}{4}$,



Diode D_2 and D_4 is in forward bias and it will be short circuited and diode D_1 and D_3 is in reverse bias and it will be open circuited.

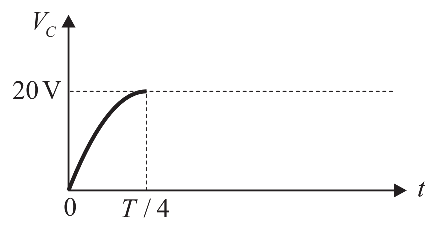


Applying KVL in shown in figure,

$$-V_{in}+0+V_C+0=0$$

$$V_C=V_{in}$$

At $t=\frac{T}{4}$, $V_{in}=20\text{ V}$, $V_C=20\text{ V}$

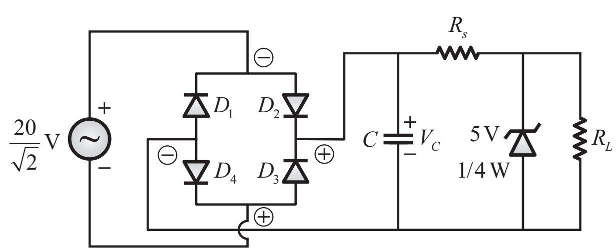


From above figure, it is observed that capacitor is charged up to 20 V for 0 to $\frac{T}{4}$.

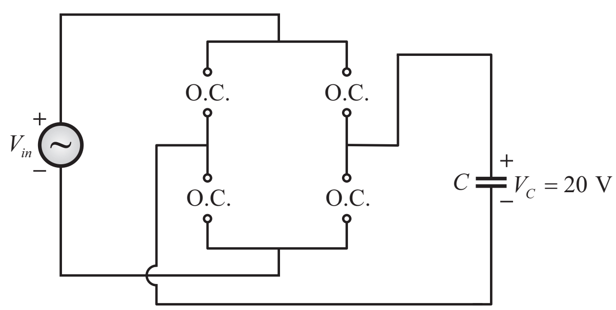
Case 2 : $t>\frac{T}{4}$,

$$V_i\left(\frac{T^+}{4}\right)<20\text{ V}$$

$$V_C\left(\frac{T^+}{4}\right)=20\text{ V}$$



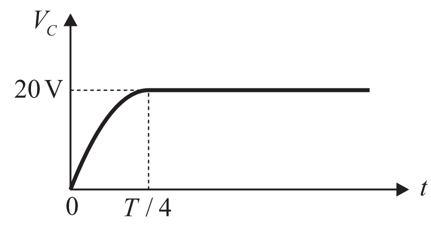
V_C is fixed at 20 V and for $t>\frac{T}{4}$, V_i is less than 20 V. Therefore, all four diodes are in reverse bias i.e. open circuited.

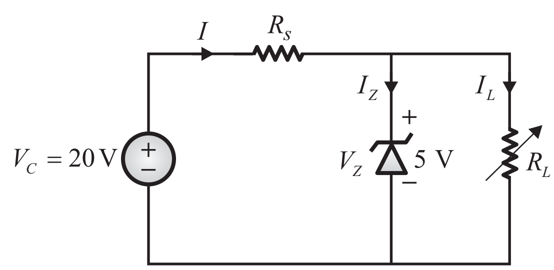


All the diodes are OFF therefore, capacitor will not get discharged.

Discharging time constant, $\tau_D=\infty$ sec

Hence, $V_C=20\text{ V}$





Given : Power rating of Zener diode, $P_Z=\frac{1}{4}$ W

$$\frac{1}{4}=V_Z\times I_{Z(\max)}$$

$$0.25=5\times I_{Z(\max)}$$

$$I_{Z(\max)}=\frac{0.25}{5}=0.05=50\text{ mA} \quad \ldots\text{(i)}$$

Considering all possible values of R_L i.e. we can consider,

$$R_{L(\max)}=\infty \text{ (No load)}$$

$$I_{L(\min)}=0\text{ A}$$

$$I_{in(\text{fixed})}=I_{Z(\max)}+I_{L(\min)}$$

$$I_{in(\text{fixed})}=I_{Z(\max)}+0$$

From equation (i),

$$I_{in(\text{fixed})}=50\text{ mA}$$

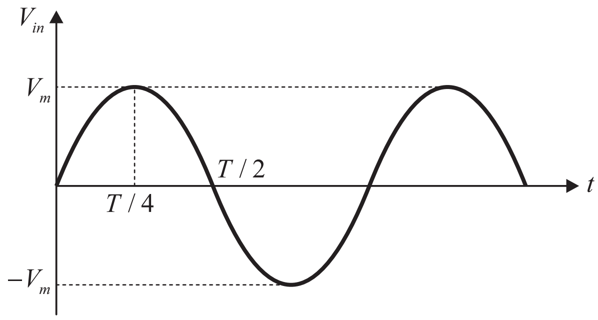
$$50=\frac{20-5}{R_S}$$

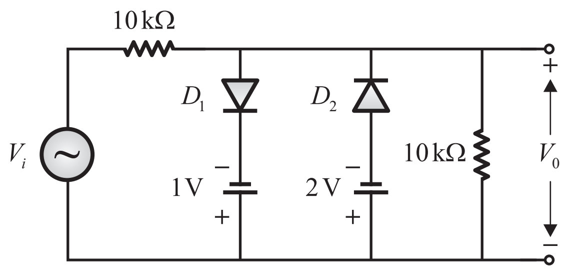
$$R_S=300\ \Omega$$

Hence, the minimum value of R_s to avoid burnout of the Zener diode is **300 Ω.**

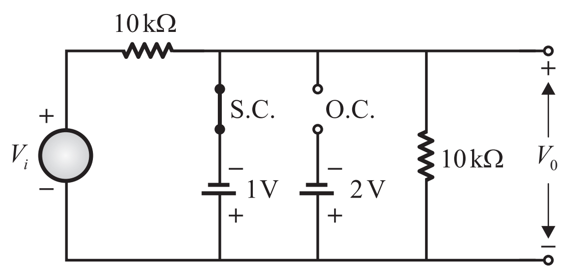
1.24 (B)

Given : The diodes in given figure are ideal i.e. $V_\gamma = 0\,\text{V}$.





During positive half cycle of input, diode D_1 will be ON due to –1 V and diode D_2 will be OFF due to –2 V.

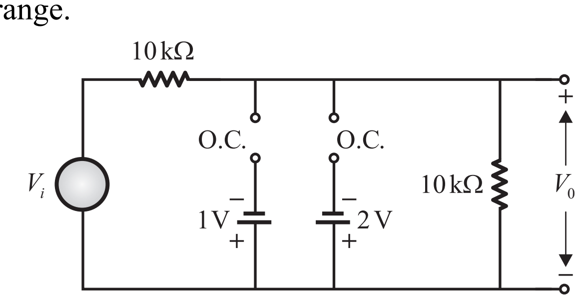


From figure, $V_0 = -1$ V

For diode D_1 to be OFF, V_0 should be less than -1 V.

For $-2\,\text{V} < V_0 < -1\,\text{V}$,

Both diodes D_1 and D_2 will be OFF for this range.



From figure,

$$V_0 = \frac{10}{10+10} V_i = \frac{V_i}{2} \quad \text{[By VDR]}$$

Hence, both diodes are OFF and the output is not clipped when $-2 < V_0 < -1$

i.e. $\quad -2 < \frac{V_i}{2} < -1 \quad \Rightarrow -4 < V_i < -2$.

Therefore, for output to be clipped, input voltage must be outside the range – 2 V to – 4 V.

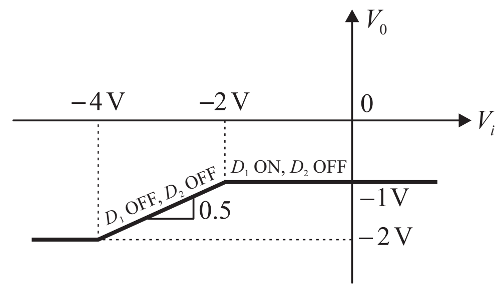


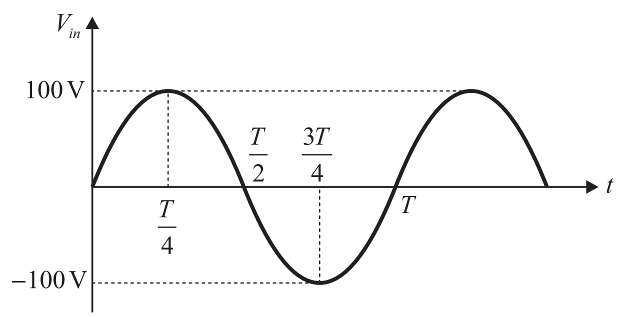
Fig. Transfer characteristics

Hence, the correct option is (B).

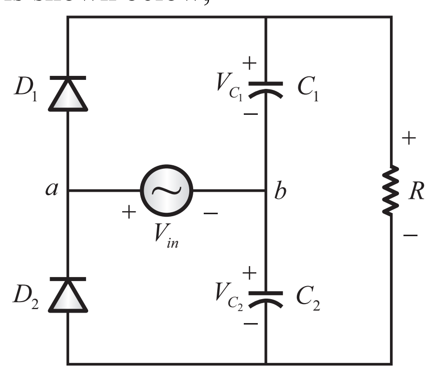
1.25 (C)

Given :

$$V_{in} = V_m \sin \omega t = 100 \sin(100\pi t)$$



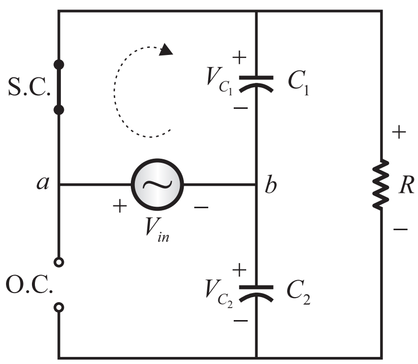
Circuit is shown below,



Case 1 : During positive half cycle,

(a) **For $0 < t < \frac{T}{4}$;**

Diode D_1 is forward biased i.e. ON and diode D_2 is reverse biased i.e. OFF. Capacitor C_1 charges to V_m almost instantly with zero time constant through diode D_1.



Applying KVL in above loop,

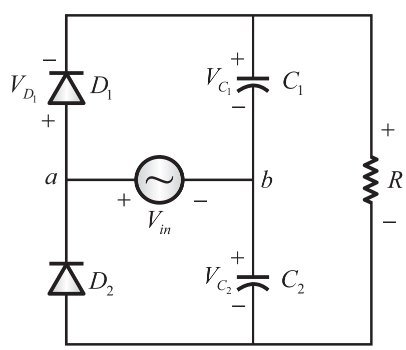
$$V_{C_1} - V_{in} = 0 \quad \Rightarrow \quad V_{C_1} = V_{in}$$

Hence,

$$V_{in}\left(\frac{T}{4}\right) = V_{C_1}\left(\frac{T}{4}\right) = V_m = 100\,\text{V}$$

$$V_{C_2}\left(\frac{T}{4}\right) = 0\ \text{V}$$

(b) **For $\frac{T}{4} < t < \frac{T}{2}$;**



From above figure,

$$V_{D_1} = V_{in} - V_{C_1}$$

Since, $V_{in}\left(\frac{T^+}{4}\right) < 100\ \text{V}$

From property of capacitor voltage,

$$V_{C_1}\left(\frac{T^+}{4}\right) = V_{C_1}\left(\frac{T}{4}\right) = 100\ \text{V}$$

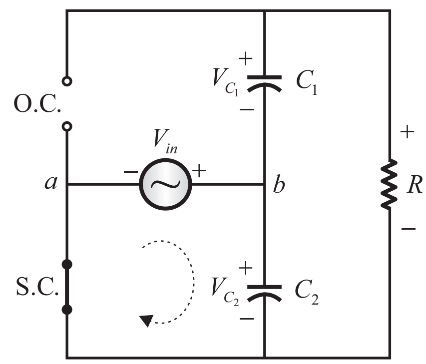
Therefore, $V_{D_1} < 0\,\text{V}$

Since, input voltage can never be greater than 100 V hence, diode D_1 will always remain OFF.

Case 2 : During negative half cycle,

(a) **For $\frac{T}{2} < t < \frac{3T}{4}$;**

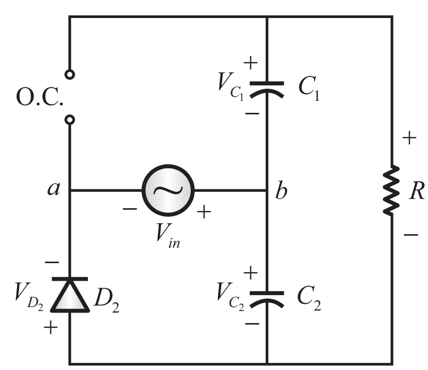
Diode D_1 will remain OFF and diode D_2 is forward biased i.e. ON. Capacitor C_2 charges to V_m almost instantly with zero time constant through diode D_2.



Hence, $V_{C_2}\left(\frac{3T}{4}\right) = 100\,\text{V}$

$$V_{C_1}\left(\frac{3T}{4}\right) = 100\,\text{V}$$

(b) **For $t > \frac{3T}{4}$;**



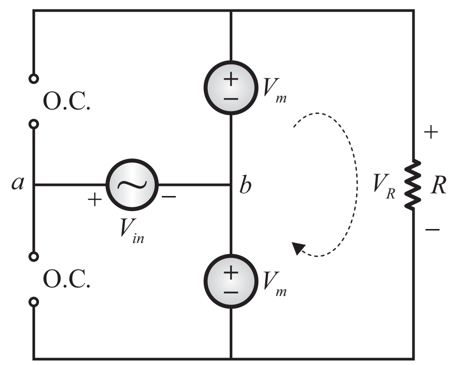
From above figure,

$$V_{D_2} = V_{in} - V_{C_2}$$

Since, $V_{C_2}\left(\frac{3T^+}{4}\right) = 100\text{ V}$ and input voltage can never be greater than 100 V hence, $V_{D_2} < 0\text{ V}$.

Therefore, diode D_2 will always remain OFF.

At $t > \frac{3T}{4}$, both diodes D_1 and D_2 are OFF and the circuit will be in steady state. Hence, capacitors will be replaced by voltage sources.



Thus, the average voltage across R under steady state is $V_R = 2V_m$.

$$V_R = 2\times 100 = 200\text{ V}$$

Therefore, the above circuit works as a voltage doubler circuit.

Hence the correct option is (C).

Key Point

In above question,

For a R-C circuit, time constant is given by,

$$\tau = RC = \frac{50}{100\pi} = 0.16\text{ sec} = 160\text{ msec}$$

Time constant τ is much larger than time period of V_{in}.

i.e. $$T = \frac{1}{f} = \frac{1}{50} = 0.02\text{ sec} = 20\text{ msec}$$

The capacitor discharges very little when corresponding diode is off.

1.26 (B)

Given : For non-ideal diode

$$I_D = I_0\left(e^{V/\eta V_T} - 1\right)$$

$$V_1 = -0.03\text{ V}$$

$$V_2 = ?$$

$$I_2 = 1.5 I_1$$

$$V_T = 26\text{ mV}$$

$$\eta = \frac{15}{13}$$

$$\frac{I_{D_2}}{I_{D_1}} = 1.5 = \frac{I_0\left(e^{V_2/\eta V_T} - 1\right)}{I_0\left(e^{V_1/\eta V_T} - 1\right)}$$

$$1.5e^{V_1/\eta V_T} - 1.5 = e^{V_2/\eta V_T} - 1$$

$$1.5e^{V_1/\eta V_T} - e^{V_2/\eta V_T} = 0.5$$

$$e^{V_2/\eta V_T} = 1.5e^{V_1/\eta V_T} - 0.5$$

$$e^{V_2/\eta V_T} = 1.5e^{\frac{-0.03}{\frac{15}{13}\times 0.026}} - 0.5$$

$$e^{V_2/\eta V_T} = 0.0518$$

$$\frac{V_2}{\eta V_T} = \ln 0.0518 = -2.9603$$

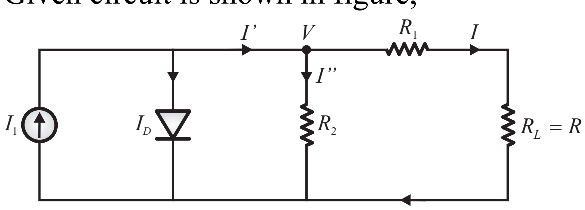
$$V_2 = -2.9603\times\frac{15}{13}\times 0.026$$

$$V_2 = -0.088 \approx -0.09$$

Hence, the correct option is (B).

1.27 (B)

Given circuit is shown in figure,



For maximum power transfer to the resistive load, the load current must be maximum.

From KCL at node V.

$$I' = I + I''$$

where, $I' = I_1 - I_D$

As I' is constant, so for load current I to be maximum, I'' must be minimum.

For I'' to be minimum, R_2 must be high.

$$I'' = \frac{V}{R_2}$$

For I to be maximum R_1 must be low.

$$I = \frac{V}{R_1 + R_L}$$

Hence, the correct option is (B).

❖❖❖❖

1 Basics of Control System

1995 IIT Kanpur

1.1 The closed-loop transfer function of a control system is given by,

$$\frac{C(s)}{R(s)} = \frac{2(s-1)}{(s+2)(s+1)}$$

For a unit step input the output is

(A) $-3e^{-2t} + 4e^{-t} - 1$

(B) $-3e^{-2t} - 4e^{-t} + 1$

(C) zero

(D) infinity

1996 IISc Bangalore

1.2 The unit-impulse response of a unity feedback control system is given by $c(t) = -te^{-t} + 2e^{-t}$, $(t \geq 0)$.

The open loop transfer function is equal to

(A) $\frac{2s+1}{(s+1)^2}$ (B) $\frac{2s+1}{s^2}$

(C) $\frac{s+1}{(s+2)^2}$ (D) $\frac{s+1}{s^2}$

1.3 The closed loop transfer function of a control system is given by,

$$\frac{C(s)}{R(s)} = \frac{1}{(1+s)}$$

For the input $r(t) = \sin t$, the steady state value of $c(t)$ is equal to

(A) $\frac{1}{\sqrt{2}}\cos t$

(B) 1

(C) $\frac{1}{\sqrt{2}}\sin t$

(D) $\frac{1}{\sqrt{2}}\sin\left(t - \frac{\pi}{4}\right)$

2000 IIT Kharagpur

1.4 A linear time invariant system initially at rest, when subjected to a unit step input, gives a response $y(t) = te^{-t}, t > 0$. The transfer function of the system is

(A) $\frac{1}{(s+1)^2}$ (B) $\frac{1}{s(s+1)^2}$

(C) $\frac{s}{(s+1)^2}$ (D) $\frac{1}{s(s+1)}$

1.5 Feedback control systems are

(A) Insensitive to both forward-path and feedback-path parameter changes.

(B) Less sensitive to feedback-path parameter changes than to forward-path parameter changes.

(C) Less sensitive to forward-path parameter changes than to feedback-path parameter changes.

(D) Equally sensitive to forward-path and feedback-path parameter changes.

2002 IISc Bangalore

1.6 The transfer function of the system described by $\frac{d^2y}{dt^2}+\frac{dy}{dt}=\frac{du}{dt}+2u$ with u as input and y as output is

(A) $\frac{(s+2)}{(s^2+s)}$ (B) $\frac{(s+1)}{(s^2+s)}$

(C) $\frac{2}{(s^2+s)}$ (D) $\frac{2s}{(s^2+s)}$

2003 IIT Madras

1.7 A control system with certain excitation is governed by the following mathematical equation

$$\frac{d^2x}{dt^2}+\frac{1}{2}\frac{dx}{dt}+\frac{1}{18}x=10+5e^{-4t}+2e^{-5t}$$

The natural time constant of the response of the system are

(A) $\frac{1}{4}$ sec and $\frac{1}{5}$ sec

(B) 3 sec and 6 sec

(C) 4 sec and 5 sec

(D) $\frac{1}{3}$ sec and $\frac{1}{6}$ sec

1.8 A control system is defined by the following mathematical relationship

$$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+5x=12(1-e^{-2t})$$

The response of the system as $t\to\infty$ is

(A) $x=6$ (B) $x=2$

(C) $x=2.4$ (D) $x=-2$

2008 IISc Bangalore

1.9 A function $y(t)$ satisfies the following differential equation

$$\frac{dy(t)}{dt}+y(t)=\delta(t)$$

where $\delta(t)$ is the delta function

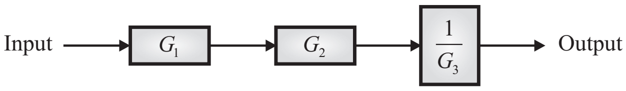
Assuming zero initial condition and denoting the unit step function by $u(t)$, $y(t)$ can be of the form

(A) e^t (B) e^{-t}

(C) $e^t u(t)$ (D) $e^{-t}u(t)$

2009 IIT Roorkee

1.10 The measurement system shown in the figure uses three sub-systems in cascade whose gains are specified as G_1, G_2 and $\frac{1}{G_3}$. The relative small errors associated with each respective subsystem G_1, G_2 and G_3 are ε_1, ε_2 and ε_3. The error associated with the output is

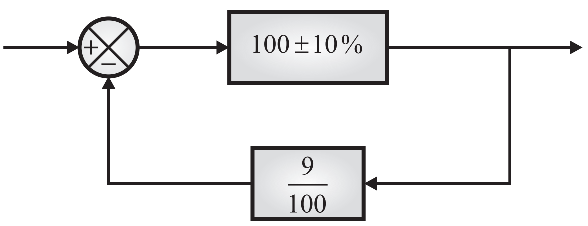


(A) $\varepsilon_1+\varepsilon_2+\frac{1}{\varepsilon_3}$ (B) $\frac{\varepsilon_1\,\varepsilon_2}{\varepsilon_3}$

(C) $\varepsilon_1+\varepsilon_2-\varepsilon_3$ (D) $\varepsilon_1+\varepsilon_2+\varepsilon_3$

2010 IIT Guwahati

1.11 As shown in the figure, a negative feedback system has an amplifier of gain 100 with ± 10% tolerance in the forward path, and an attenuator of value $\frac{9}{100}$ in the feedback path. The overall system gain is approximately :



(A) $10\pm1\%$ (B) $10\pm2\%$

(C) $10\pm5\%$ (D) $10\pm10\%$

2012　IIT Delhi

1.12 A system with transfer function

$$G(s)=\frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$$

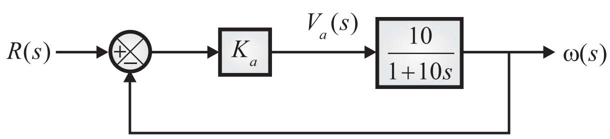
is excited by $\sin(\omega t)$. The steady state output of the system is zero at

(A) $\omega = 1$ rad/sec　(B) $\omega = 2$ rad/sec

(C) $\omega = 3$ rad/sec　(D) $\omega = 4$ rad/sec

2013　IIT Bombay

1.13 The open-loop transfer function of a dc motor is given as $\dfrac{\omega(s)}{V_a(s)}=\dfrac{10}{1+10s}$. When connected in feedback as shown below, the approximate value of K_a that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open-loop system is



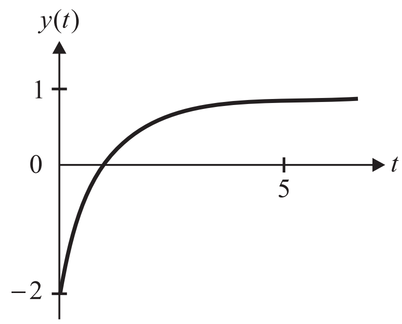
(A) 1　(B) 5

(C) 10　(D) 100

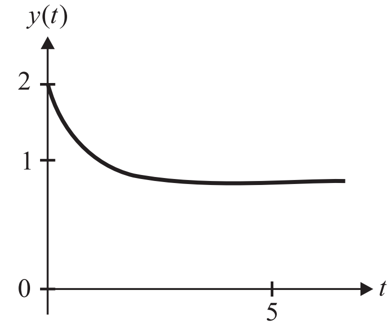
2015　IIT Kanpur

1.14 The unit step response of a system with the transfer function $G(s)=\dfrac{1-2s}{1+s}$ is given by which one of the following waveforms? **[Set - 01]**

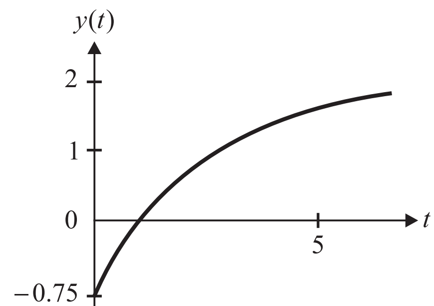
(A)



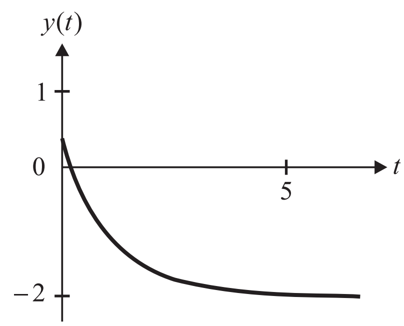
(B)



(C)



(D)



1.15 An open loop control system results in a response of $e^{-2t}(\sin 5t+\cos 5t)$ for a unit impulse input. The DC gain of the control system is ________. **[Set - 02]**

2020　IIT Delhi

1.16 ax^3+bx^2+cx+d is a polynomial on real x over real coefficients a, b, c, d where in $a \neq 0$. Which of the following statements is true?

(A) No choice of coefficients can make all roots identical.

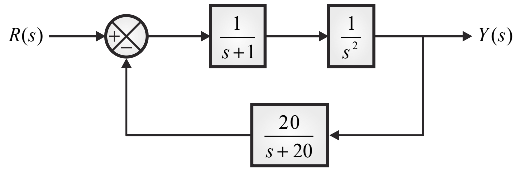
(B) a, b, c, d can be chosen to ensure that all roots are complex.

(C) d can be chosen to ensure that $x=0$ is a root for any given set a, b, c.

(D) c alone cannot ensure that all roots are real.

❖❖❖❖

1.17 Which of the following options is correct for the system shown below?



(A) 3rd order and unstable

(B) 4th order and stable

(C) 3rd order and stable

(D) 4th order and unstable

Answers Basics of Control System

1.1	**A**	**1.2**	**B**	**1.3**	**D**	**1.4**	**C**	**1.5**	**C**
1.6	**A**	**1.7**	**B**	**1.8**	**C**	**1.9**	**D**	**1.10**	**C**
1.11	**A**	**1.12**	**C**	**1.13**	**C**	**1.14**	**A**	**1.15**	**0.241**
1.16	**C**	**1.17**	**D**						

Explanations Basics of Control System

1.1 (A)

Given : Closed loop transfer function is,

$$\frac{C(s)}{R(s)} = \frac{2(s-1)}{(s+2)(s+1)}$$

For step input, $r(t) = u(t)$

Taking Laplace transform of $r(t)$,

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{2(s-1)}{s(s+2)(s+1)}$$

Applying partial fraction,

$$\frac{2(s-1)}{s(s+2)(s+1)} = \frac{A}{s} + \frac{B}{(s+2)} + \frac{C}{(s+1)}$$

$$A = s\,C(s)\big|_{s=0} = -1$$

$$B = (s+2)\,C(s)\big|_{s=-2} = -3$$

$$C = (s+1)\,C(s)\big|_{s=-1} = 4$$

Then, $$C(s) = \frac{-1}{s} + \frac{-3}{s+2} + \frac{4}{s+1}$$

Taking inverse Laplace transform of $C(s)$,

$$c(t) = \left[-1 - 3e^{-2t} + 4e^{-t}\right]u(t)$$

Hence, the correct option is (A).

1.2 (B)

Given : Unit-impulse response of unity feedback system is,

$$c(t) = -te^{-t} + 2e^{-t},\ (t \geq 0)$$

Input, $r(t) = \delta(t)$

Taking Laplace transform of $r(t)$ and $c(t)$,

$$R(s)=1$$

$$C(s)=-\frac{1}{(s+1)^2}+\frac{2}{(s+1)}$$

$$C(s)=\frac{2s+1}{(s+1)^2}$$

Transfer function is given by,

$$T(s)=\frac{C(s)}{R(s)}=\frac{2s+1}{(s+1)^2}=\frac{2s+1}{s^2+2s+1}$$

Closed loop transfer function of a unity negative feedback system is given by,

$$T(s)=\frac{G(s)}{1+G(s)}$$

where, $G(s)=$ open loop transfer function

$$G(s)=\frac{T(s)}{1-T(s)}=\frac{\frac{2s+1}{s^2+2s+1}}{1-\frac{2s+1}{s^2+2s+1}}$$

$$G(s)=\frac{2s+1}{s^2}$$

Hence, the correct option is (B).

1.3 (D)

Given : Closed loop transfer function is,

$$\frac{C(s)}{R(s)}=\frac{1}{1+s} \quad \ldots(i)$$

$$r(t)=\sin t$$

Method 1

Taking Laplace transform of $r(t)$,

$$R(s)=\frac{1}{s^2+1}$$

From equation (i),

$$C(s)=\frac{R(s)}{s+1}=\frac{1}{(s^2+1)(s+1)}$$

Applying partial fraction,

$$\frac{1}{(s+1)(s^2+1)}=\frac{A}{s+1}+\frac{Bs+C}{s^2+1}$$

$$1=As^2+A+Bs^2+Bs+Cs+C$$

Comparing the coefficient of s^2, s^1 and s^0,

$$A+B=0 \Rightarrow A=-B \quad \ldots(ii)$$

$$B+C=0 \Rightarrow C=-B \quad \ldots(iii)$$

$$A+C=1 \quad \ldots(iv)$$

Substituting equation (ii) and (iii) in (iv),

$$-B-B=1$$

$$B=-\frac{1}{2}$$

$$A=C=\frac{1}{2}$$

$$C(s)=\frac{1}{2(s+1)}+\frac{(-s+1)}{2(s^2+1)}$$

$$C(s)=\frac{1}{2}\left[\frac{1}{s+1}-\frac{s}{s^2+1}+\frac{1}{s^2+1}\right]$$

Taking inverse Laplace transform,

$$c(t)=\frac{1}{2}\left[e^{-t}-\cos t+\sin t\right]$$

Exponential term does not exist at steady state as,

$$\lim_{t\to\infty} e^{-t}=0$$

At steady state, the response is,

$$c(t)\big|_{ss}=\frac{1}{2}[\sin t-\cos t]$$

$$c(t)\big|_{ss}=\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\sin t-\frac{1}{\sqrt{2}}\cos t\right]$$

$$c(t)\big|_{ss}=\frac{1}{\sqrt{2}}\left[\sin t\cos 45^0-\cos t\sin 45^0\right]$$

$$c(t)\big|_{ss}=\frac{1}{\sqrt{2}}\sin(t-45^0)$$

Hence, the correct option is (D).

Method 2

Given : $r(t)=\sin t$ i.e. $\omega=1$

From equation (i),

$$C(s)=\frac{1}{s+1}R(s)$$

For sinusoidal input, we put $s=j\omega$,

$$C(j\omega)=\frac{1}{j\omega+1}R(j\omega)$$

Taking inverse Laplace transform both side

$$c(t)=\frac{1}{\sqrt{2}\angle 45^0}\sin t$$

$$c(t)=\frac{1}{\sqrt{2}}\sin(t-45^0)$$

Hence, the correct option is (D).

1.4 (C)

Given : $x(t)=u(t), \quad y(t)=te^{-t}u(t)$

Taking Laplace transform,

$$X(s)=L[u(t)]=\frac{1}{s}$$

$$e^{-t}u(t)\xleftrightarrow{\text{L.T.}}\frac{1}{s+1}$$

$$te^{-t}u(t)\xleftrightarrow{\text{L.T.}}\frac{-d}{ds}\left(\frac{1}{s+1}\right)=\frac{1}{(s+1)^2}$$

Property of Laplace transform

$$t^n x(t)\xleftrightarrow{\text{L.T.}}(-1)^n\frac{d^n}{ds^n}[X(s)]$$

$$Y(s)=L\left[te^{-t}u(t)\right]=\frac{1}{(s+1)^2}$$

Transfer function of the system is,

$$T(s)=\frac{Y(s)}{X(s)}=\frac{s}{(s+1)^2}$$

Hence, the correct option is (C).

1.5 (C)

Sensitivity of closed loop systems w.r.t. forward path parameter G is given by,

$$S_G^T=\frac{\partial T/T}{\partial G/G}=\frac{1}{1+GH}$$

Sensitivity of closed loop systems w.r.t. feedback path parameters H is given by,

$$S_H^T=\frac{\partial T/T}{\partial H/H}=\frac{-GH}{1+GH}$$

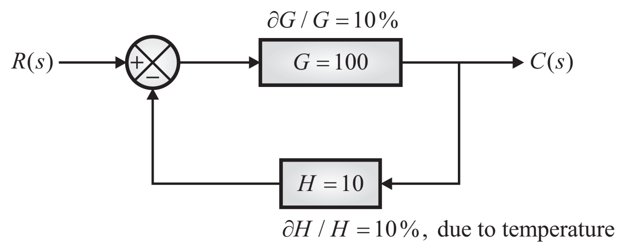
For feedback control system generally,

$$GH>>1$$

So, $\quad S_H^T\approx -1$

Therefore, feedback control systems are less sensitive to forward-path changes than to feedback-path parameter changes.

Example :



$$S_G^T=\frac{\partial T/T}{\partial G/G}=\frac{1}{1+GH}=\frac{1}{1+1000}\approx\frac{1}{1000}$$

$$\frac{\partial T}{T}=\frac{\partial G/G}{1000}=\frac{10\%}{1000}$$

$$\frac{\partial T}{T}=0.01\% \qquad \ldots(\text{i})$$

$$S_H^T=\frac{\partial T/T}{\partial H/H}=\frac{-GH}{1+GH}=\frac{-100\times 10}{1+100\times 10}\approx -1$$

$$\frac{\partial T}{T}=\frac{\partial H}{H}\times(-1)$$

$$\frac{\partial T}{T}=-10\% \qquad \ldots(\text{ii})$$

Note : In case of a percentage only magnitude is considered.

Therefore, from equation (i) and (ii), sensitivity with respect to forward path is less than the sensitivity with respect to feedback path.

Hence, the correct option is (C).

1.6 (A)

Given : $\dfrac{d^2y}{dt^2}+\dfrac{dy}{dt}=\dfrac{du}{dt}+2u$

where, u is the input and y is the output.

Taking Laplace transform of equation with initial condition zero,

$$s^2Y(s)+sY(s)=sU(s)+2U(s)$$

$$\frac{Y(s)}{U(s)}=\frac{s+2}{s^2+s}$$

Hence, the correct option is (A).

1.7 (B)

Given : $\dfrac{d^2x}{dt^2}+\dfrac{1}{2}\dfrac{dx}{dt}+\dfrac{1}{18}x=10+5e^{-4t}+2e^{-5t}$

The R.H.S. equation $10+5e^{-4t}+2e^{-5t}$ represents the input.

$$\frac{d^2x}{dt^2}+\frac{1}{2}\frac{dx}{dt}+\frac{1}{18}x=r(t)$$

Applying Laplace transform to the left hand side of the equation,

$$\left(s^2X(s)+\frac{sX(s)}{2}+\frac{X(s)}{18}\right)=R(s)$$

$$\left(s^2+\frac{s}{2}+\frac{1}{18}\right)X(s)=R(s)$$

Transfer function is given by,

$$T(s)=\frac{X(s)}{R(s)}=\frac{1}{s^2+\frac{1}{2}s+\frac{1}{18}}$$

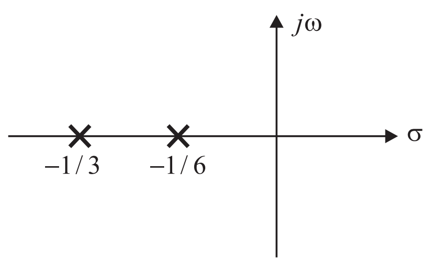
Characteristic equation is,

$$s^2+\frac{s}{2}+\frac{1}{18}=0$$

$$\left(s+\frac{1}{3}\right)\left(s+\frac{1}{6}\right)=0$$

$$s=-\frac{1}{3},-\frac{1}{6}$$

$$T(s)=\frac{18}{(6s+1)(3s+1)}$$



Reciprocal of magnitude of negative real roots represents time constants.

$$\tau_1=\left|\frac{1}{1/3}\right|=3\text{ sec},\quad \tau_2=\left|\frac{1}{1/6}\right|=6\text{ sec}$$

Hence, the correct option is (B).

☒ Avoid This Mistake

$$5e^{-4t}\rightarrow\tau=\frac{1}{4}\text{sec}$$

$$2e^{-5t}\rightarrow\tau=\frac{1}{5}\text{sec}$$

🕮 Key Point

(i) The forced response implies the response only due to input when initial conditions are zero.

(ii) The natural response implies the response only due to initial conditions when no input is applied.

1.8 (C)

Given : The mathematical model of control system is,

$$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+5x=12(1-e^{-2t})$$

where, $x(t)$ is the response of the system.

Taking Laplace transform of the system with initial condition assumed to be zero,

$$x(0)=0 \text{ and } x'(0)=0$$

$$\left[s^2X(s)-s\,x(0)-x'(0)\right]+6\left[sX(s)-x(0)\right]$$

$$+5X(s)=12\left[\frac{1}{s}-\frac{1}{s+2}\right]$$

$$(s^2+6s+5)X(s)=12\frac{2}{s(s+2)}$$

$$X(s)=\frac{24}{s(s+2)(s^2+6s+5)}$$

Applying final value theorem,

$$\lim_{t\to\infty}x(t)=\lim_{s\to0}sX(s)$$

$$\lim_{t\to\infty}x(t)=\lim_{s\to0}\frac{24}{(s+2)(s^2+6s+5)}$$

$$x(\infty)=2.4$$

Hence, the correct option is (C).

1.9 (D)

Given : $\frac{dy(t)}{dt}+y(t)=\delta(t),\ y(0)=0$

Taking Laplace transform of the equation with initial condition zero,

$$sY(s)-y(0)+Y(s)=1$$

$$Y(s)=\frac{1}{s+1}$$

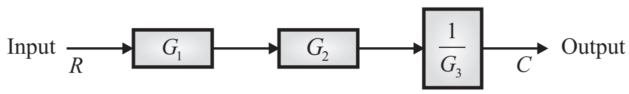
Taking inverse Laplace transform,

$$y(t)=e^{-t}u(t)$$

Hence, the correct option is (D).

1.10 (C)

Given :



Overall transfer function,

$$G(s)=\frac{C(s)}{R(s)}=G_1G_2\times\frac{1}{G_3}=\frac{G_1G_2}{G_3}$$

Output $C=\frac{G_1G_2}{G_3}\times R$

$$\ln C=\ln G_1+\ln G_2-\ln G_3+\ln R$$

Differentiating above equation,

$$\frac{1}{C}dC=\frac{1}{G_1}dG_1+\frac{1}{G_2}dG_2-\frac{1}{G_3}dG_3+\frac{1}{R}dR$$

As there is no error in R, so $dR=0$.

$$\frac{dC}{C}=\frac{dG_1}{G_1}+\frac{dG_2}{G_3}-\frac{dG_3}{G_3}$$

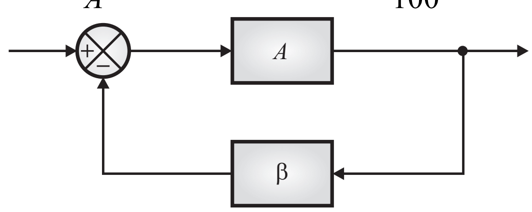
$$\varepsilon=\varepsilon_1+\varepsilon_2-\varepsilon_3$$

So, relative error in output $=\varepsilon_1+\varepsilon_2-\varepsilon_3$

Hence, the correct option is (C).

1.11 (A)

Given : $\frac{\partial A}{A}=10\%,\ A=100,\ \beta=\frac{9}{100}$



Sensitivity is given by,

$$S=\frac{\frac{\partial A_f}{A_f}}{\frac{\partial A}{A}}=\frac{1}{1+A\beta}$$

$$\frac{\partial A_f}{A_f}=\frac{\frac{\partial A}{A}}{1+A\beta}=\frac{10\%}{1+100\times\frac{9}{100}}$$

$$\frac{\partial A_f}{A_f}=1\%$$

Gain with feedback is given by,

$$A_f=\frac{A}{1+A\beta}=\frac{100}{1+100\times\frac{9}{100}}=10$$

Therefore, the overall system gain is,

$$A_f\pm\frac{\partial A_f}{A_f}=10\pm 1\%$$

Hence, the correct option is (A).

1.12 (C)

Given : $G(s)=\frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$ ...(i)

and $x(t)=\sin(\omega t)$

Laplace transform of $x(t)$ is,

$$X(s)=L[\sin(\omega t)]=\frac{\omega}{s^2+\omega^2} \quad \text{...(ii)}$$

$$G(s)=\frac{Y(s)}{X(s)} \Rightarrow Y(s)=G(s)X(s)$$

From equation (i) and (ii),

$$Y(s)=\frac{\omega(s^2+9)(s+2)}{(s^2+\omega^2)(s+1)(s+3)(s+4)}$$

Using final value theorem,

$$\lim_{t\to\infty}y(t)=\lim_{s\to 0}sY(s)$$

$$\lim_{t\to\infty}y(t)=\lim_{s\to 0}\frac{s\,\omega(s^2+9)(s+2)}{(s^2+\omega^2)(s+1)(s+3)(s+4)}$$

Final value theorem is applicable only for absolute stable system. But $(s^2+\omega^2)$ term in denominator is making system as a marginal stable system because

$$s^2+\omega^2=0$$

$$s=\pm j\omega \quad \text{[imaginary axis poles]}$$

For existence of final value theorem, $(s^2+\omega^2)$ should be cancelled out with (s^2+9).

$$\omega^2 = 9 \quad \Rightarrow \quad \omega = 3 \text{ rad/sec}$$

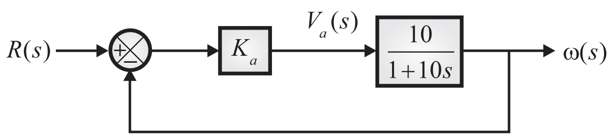
At $\omega = 3$ rad/sec, steady state output will be zero.

Hence, the correct option is (C).

1.13 (C)

Given : $\tau_{\text{closed loop}} = \dfrac{1}{100}\tau_{\text{open loop}}$

where, τ represents time constant.

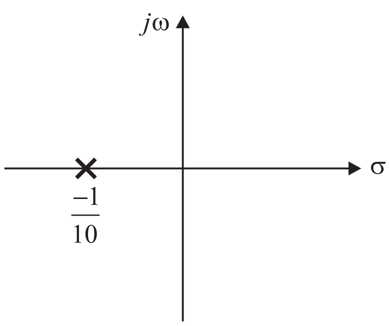


From figure,

Open loop transfer function of DC motor

$$= \frac{\omega(s)}{V_a(s)} = \frac{10}{1+10s}$$

Location of pole of open loop transfer function is shown below,



Time constant is defined as reciprocal of magnitude of negative real root.

Hence, $\tau_{\text{open loop}} = 10$ sec

$$\tau_{\text{closed loop}} = \frac{1}{100}\tau_{\text{open loop}} = 0.1 \text{ sec} \qquad \ldots\text{(i)}$$

Closed-loop transfer function for negative unity feedback is given by,

$$T(s) = \frac{G(s)}{1+G(s)}$$

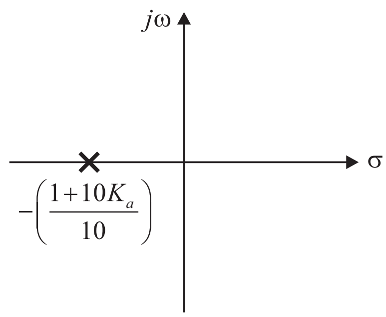
Here, $G(s) = K_a\left(\dfrac{10}{1+10s}\right)$

$$T(s) = \frac{\omega(s)}{R(s)} = \frac{K_a\left(\dfrac{10}{1+10s}\right)}{1+K_a\left(\dfrac{10}{1+10s}\right)}$$

$$T(s) = \frac{10K_a}{1+10s+10K_a}$$

$$= \frac{10K_a}{10s+(10K_a+1)}$$

Location of pole of closed loop transfer function is shown below,



From above figure,

$$\tau_{\text{closed loop}} = \frac{10}{10K_a+1} \qquad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$\frac{10}{10K_a+1} = 0.1 \text{ sec}$$

$$\frac{10}{10K_a+1} = \frac{1}{10}$$

$$10K_a + 1 = 100$$

$$10K_a = 99$$

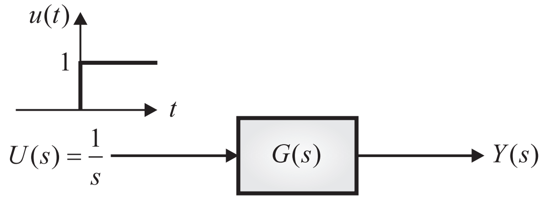
$$K_a = 9.9 \simeq 10$$

Hence, the correct option is (C).

Scan for Video Solution

1.14 (A)

Given : $G(s) = \dfrac{1-2s}{1+s}$



$$Y(s) = G(s)U(s)$$

$$Y(s) = \frac{(1-2s)}{(1+s)} \times \frac{1}{s}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s+1}$$

$$A = sY(s)\big|_{s=0}$$

$$A = \left.\frac{s(1-2s)}{s(1+s)}\right|_{s=0} = 1$$

$$B = (s+1)Y(s)\big|_{s=-1}$$

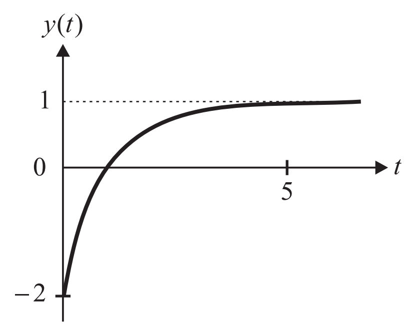
$$B = \left.\frac{1-2s}{3}\right|_{s=-1} = -3$$

$$Y(s) = \frac{1}{s} + \frac{-3}{s+1}$$

Taking inverse Laplace transform,

$$y(t) = u(t) - 3e^{-t}u(t)$$

$$y(t) = (1-3e^{-t})u(t)$$



Hence, the correct option is (A).

1.15 **0.241**

Given : $g(t) = e^{-2t}(\sin 5t + \cos 5t)$

$$x(t) = \delta(t)$$

Taking Laplace transform,

$$X(s) = 1$$

$$g(t) = e^{-2t}\sin 5t + e^{-2t}\cos 5t$$

Note : Laplace transform,

$$\sin bt \xleftrightarrow{\text{L.T.}} \frac{b}{s^2+b^2}$$

$$\cos bt \xleftrightarrow{\text{L.T.}} \frac{s}{s^2+b^2}$$

$$e^{-at}x(t) \xleftrightarrow{\text{L.T.}} X(s+a)$$

Taking Laplace transform of $g(t)$,

$$G(s) = \frac{5}{(s+2)^2+5^2} + \frac{s+2}{(s+2)^2+5^2}$$

DC gain means $|G(s)|_{s=0} = G(0)$

$$G(0) = \frac{5}{2^2+5^2} + \frac{2}{2^2+5^2} = \frac{7}{29} = 0.241$$

Hence, the DC gain of the control system is **0.241.**

1.16 **(C)**

Given : Polynomial is $f(x) = ax^3 + bx^2 + cx + d$ where x is real.

If the value of 'd' is chosen as 0, then $f(x)$ becomes

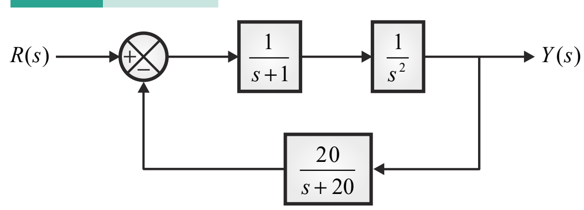
$$f(x) = ax^3 + bx^2 + cx$$

$$f(x) = x(ax^2 + bx + c)$$

$\therefore$ $x = 0$ will be one of the roots of $f(x)$ irrespective of the value of a, b, c

Hence, the correct option is (C).

1.17 **(D)**



Characteristic equation is given by,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{1}{(s+1)}\frac{1}{s^2}\frac{20}{(s+20)} = 0$$

$$s^2(s+1)(s+20)+20=0$$

$$s^2[s^2+21s+20]+20=0$$

$$s^4+21s^3+20s^2+20=0 \rightarrow 4^{th} \text{ order}$$

's' term missing hence it is unstable.

We can check using RH Rule,

$$\begin{array}{c|ccc} s^4 & 1 & 20 & 20 \\ s^3 & 21 & 0 & \\ s^2 & 20 & 20 & \\ s^1 & -21 & 0 & \\ s^0 & 20 & & \end{array}$$

4^{th} order and unstable

Hence, the correct option is (D).

❖❖❖❖

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1 Linear Algebra

1994 IIT Kharagpur

1.1 A 5×7 matrix has all its entries equal to -1. The rank of the matrix is

(A) 7 (B) 5

(C) 1 (D) zero

1.2 The eigen-values of the matrix $\begin{bmatrix} a & 1 \\ a & 1 \end{bmatrix}$ are

(A) $(a+1)$, 0 (B) a, 0

(C) $(a-1)$, 0 (D) 0, 0

1.3 The number of linearly independent solutions of the system of equations

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \text{ is equal to}$$

(A) 1 (B) 2

(C) 3 (D) 0

1995 IIT Kanpur

1.4 The inverse of the matrix $S = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is

(A) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 2 & -2 \\ -2 & 2 & -2 \\ 0 & 2 & 2 \end{bmatrix}$

(D) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$

1.5 Given the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$.

Its eigen values are __________.

1996 IISc Bangalore

1.6 The rank of the following $(n+1)\times(n+1)$ matrix, where 'a' is a real number is

$$\begin{bmatrix} 1 & a & a^2 & \cdots & a^n \\ 1 & a & a^2 & \cdots & a^n \\ \vdots & & & & \\ 1 & a & a^2 & \cdots & a^n \end{bmatrix}$$

(A) 1

(B) 2

(C) n

(D) depends on the value of a.

1997 IIT Madras

1.7 A square matrix is called singular if its

(A) Determinant is unity.

(B) Determinant is zero.

(C) Determinant is infinity.

(D) Rank is unity.

1.8 Express the given matrix,

$$A=\begin{bmatrix}2 & 1 & 5\\ 4 & 8 & 13\\ 6 & 27 & 31\end{bmatrix}$$

as a product of triangular matrices L and U where the diagonal elements of the lower triangular matrix L are unity and U is an upper triangular matrix.

1998 IIT Delhi

1.9 $A=\begin{bmatrix}2 & 0 & 0 & -1\\ 0 & 1 & 0 & 0\\ 2 & 0 & 3 & 0\\ -1 & 0 & 0 & 4\end{bmatrix}$.

The sum of the Eigen values of the matrix A is

(A) 10 (B) – 10

(C) 24 (D) 22

1.10 The vector $\begin{bmatrix}1\\ 2\\ -1\end{bmatrix}$ is an Eigen vector of

$$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}.$$

One of the Eigen value of A is

(A) 1 (B) 2

(C) 5 (D) –1

1.11 If $A=\begin{bmatrix}5 & 0 & 2\\ 0 & 3 & 0\\ 2 & 0 & 1\end{bmatrix}$ then $A^{-1}=$

(A) $\begin{bmatrix}1 & 0 & -2\\ 0 & \frac{1}{3} & 0\\ -2 & 0 & 5\end{bmatrix}$

(B) $\begin{bmatrix}5 & 0 & 2\\ 0 & -\frac{1}{3} & 0\\ 2 & 0 & 1\end{bmatrix}$

(C) $\begin{bmatrix}\frac{1}{5} & 0 & \frac{1}{2}\\ 0 & \frac{1}{3} & 0\\ \frac{1}{2} & 0 & 1\end{bmatrix}$

(D) $\begin{bmatrix}\frac{1}{5} & 0 & -\frac{1}{2}\\ 0 & \frac{1}{3} & 0\\ -\frac{1}{2} & 0 & 1\end{bmatrix}$

1.12 A set of linear equation is represented by the matrix equation $AX=B$. The necessary condition for the existence of a solution for this system is

(A) A must be invertible.

(B) B must be linearly dependent on the columns of A.

(C) B must be linearly independent of the columns of A.

(D) None.

2002 IISc Bangalore

1.13 The determinant of the matrix $\begin{bmatrix}1 & 0 & 0 & 0\\ 100 & 1 & 0 & 0\\ 100 & 200 & 1 & 0\\ 100 & 200 & 300 & 1\end{bmatrix}$ is

(A) 100 (B) 200

(C) 1 (D) 300

1.14 The Eigen values of the system represented by $X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ are

(A) 0, 0, 0, 0 (B) 1, 1, 1, 1

(C) 0, 0, 0, –1 (D) 1, 0, 0, 0

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1.15 In the matrix equation $PX = Q$ which of the following is a necessary condition for the existence of at least one solution for the unknown vector X

(A) Augmented matrix $[P:Q]$ must have the same rank as matrix P.

(B) Vector Q must have only non-zero elements.

(C) Matrix P must be singular.

(D) Matrix P must be square.

1.16 For the matrix $P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, one of the eigen values is equal to – 2. Which of the following is an eigen vector?

(A) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ (D) $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

1.17 If $R = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$ then the top row of R^{-1} is

(A) $\begin{bmatrix} 5 & 6 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & -3 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & -1 & 0 \end{bmatrix}$

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Statement for Linked Answer Questions 1.18 & 1.19

Given that three vector as

$$P = \begin{bmatrix} -10 \\ 1 \\ 3 \end{bmatrix}^T, Q = \begin{bmatrix} -2 \\ -5 \\ 9 \end{bmatrix}^T, R = \begin{bmatrix} 2 \\ -7 \\ 12 \end{bmatrix}^T$$

1.18 An orthogonal set of vectors having a span that contains P, Q, R is

(A) $\begin{bmatrix} -6 \\ -3 \\ 6 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$

(B) $\begin{bmatrix} -4 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \\ -11 \end{bmatrix} \begin{bmatrix} 8 \\ 2 \\ -3 \end{bmatrix}$

(C) $\begin{bmatrix} 6 \\ 7 \\ -1 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ -2 \end{bmatrix} \begin{bmatrix} 3 \\ 9 \\ -4 \end{bmatrix}$

(D) $\begin{bmatrix} 4 \\ 3 \\ 11 \end{bmatrix} \begin{bmatrix} 1 \\ 31 \\ 3 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$

1.19 The following vector is linearly dependent upon the solution to the previous problem

(A) $\begin{bmatrix} 8 \\ 9 \\ 3 \end{bmatrix}$ (B) $\begin{bmatrix} -2 \\ -17 \\ 30 \end{bmatrix}$

(C) $\begin{bmatrix} 4 \\ 4 \\ 5 \end{bmatrix}$ (D) $\begin{bmatrix} 13 \\ 2 \\ -3 \end{bmatrix}$

2007 IIT Kanpur

1.20 $X = [x_1 \; x_2 \; ..... \; x_n]^T$ is an n-tuple nonzero vector.

The $n \times n$ matrix $V = XX^T$

(A) has rank zero. (B) has rank 1.

(C) is orthogonal. (D) has rank n.

Statement for Linked Answer Questions 1.21 & 1.22

Cayley-Hamilton Theorem states that a square matrix satisfies its own characteristic equation. Consider a matrix

$$A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$$

1.21 A satisfies the relation

(A) $A + 3I + 2A^{-1} = 0$

(B) $A^2 + 2A + 2I = 0$

(C) $(A + I)(A + 2I) = 0$

(D) $\exp(A) = 0$

1.22 A^9 equals

(A) $511\ A + 510\ I$

(B) $309\ A + 104\ I$

(C) $154\ A + 155\ I$

(D) $\exp(9A)$

1.23 Let x and y be two vectors in a 3 dimensional space and $<x, y>$ denote their dot product. Then the determinant

$$\det \begin{bmatrix} <x,x> & <x,y> \\ <y,x> & <y,y> \end{bmatrix}$$

(A) is zero when x and y are linearly independent.

(B) is positive when x and y are linearly independent.

(C) is non-zero for all non-zero x and y.

(D) is zero only when either x or y is zero.

1.24 The linear operation $L(x)$ is defined by the cross product $L(x) = \overline{b} \times \overline{x}$, where $\overline{b} = [0 \;\; 1 \;\; 0]^T$ and $\overline{x} = [x_1 \; x_2 \; x_3]^T$ are three dimensional vectors. The 3×3 matrix M of this operation satisfies

$$L(x) = M \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Then the eigen values of M are

(A) 0, +1, −1 (B) 1, −1, 1

(C) i, $-i$, 1 (D) i, $-i$, 0

2008 IISc Bangalore

1.25 The characteristic equation of a (3×3) matrix P is defined as,

$$\alpha(\lambda) = |\lambda I - P| = \lambda^3 + \lambda^2 + 2\lambda + 1 = 0.$$

If I denotes identity matrix, then the inverse of matrix P will be

(A) $(P^2 + P + 2I)$

(B) $(P^2 + P + I)$

(C) $-(P^2 + P + I)$

(D) $-(P^2 + P + 2I)$

1.26 If the rank of a (5×6) matrix Q is 4, then which one of the following statements is correct?

(A) Q will have four linearly independent rows and four linearly independent columns.

(B) Q will have four linearly independent rows and five linearly independent columns.

(C) QQ^T will be invertible.

(D) Q^TQ will be invertible.

1.27 A is a $m \times n$ full rank matrix with $m > n$ and I is an identity matrix. Let matrix $A^+ = (A^T A)^{-1} A^T$. Then which one of the following statements is FALSE?

(A) $AA^+A = A$ (B) $(AA^+)^2 = AA^+$

(C) $A^+A = I$ (D) $AA^+A = A^+$

1.28 Let P be a 2×2 real orthogonal matrix and $\vec{x}$ is a real vector $[x_1 x_2]^T$ with length $\|\vec{x}\| = \left(x_1^2 + x_2^2\right)^{\frac{1}{2}}$. Then which one of the following statements is correct?

(A) $\|P\vec{x}\| \le \|\vec{x}\|$ where at least one vector satisfies $\|P\vec{x}\| < \|\vec{x}\|$.

(B) $\|P\vec{x}\| = \|\vec{x}\|$ for all vectors $\vec{x}$.

(C) $\|P\vec{x}\| \ge \|\vec{x}\|$ where at least one vector satisfies $\|P\vec{x}\| > \|\vec{x}\|$.

(D) No relationship can be established between $\|\vec{x}\|$ and $\|P\vec{x}\|$.

2009 IIT Roorkee

1.29 The trace and determinant of a 2×2 matrix are known to be – 2 and – 35 respectively. It's Eigen values are

(A) – 30 and – 5 (B) – 37 and – 1

(C) – 7 and 5 (D) 17.5 and – 2

2010 IIT Guwahati

1.30 An eigen vector of $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ is

(A) $[-1 \quad 1 \quad 1]^T$ (B) $[1 \quad 2 \quad 1]^T$

(C) $[1 \quad -1 \quad 2]^T$ (D) $[2 \quad 1 \quad -1]^T$

1.31 For the set of equations,

$$x_1 + 2x_2 + x_3 + 4x_4 = 2$$

and $3x_1 + 6x_2 + 3x_3 + 12x_4 = 6$.

Which of the following statements is true

(A) Only the trivial solution $x_1 = x_2 = x_3 = x_4 = 0$ exists.

(B) There are no solution.

(C) A unique non-trivial solution exists.

(D) Multiple non-trivial solutions exist.

2011 IIT Madras

1.32 The matrix $[A] = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ is decomposed into a product of a lower triangular matrix $[L]$ and an upper triangular matrix $[U]$. The properly decomposed $[L]$ and $[U]$ matrices respectively are

(A) $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 0 \\ 4 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 0 \\ 4 & -3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$

2012 IIT Delhi

1.33 Given that,

$$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

the value of A^3 is

(A) 15 A + 12 I (B) 19 A + 30 I

(C) 17 A + 15 I (D) 17 A + 21 I

2013 IIT Bombay

1.34 The equation $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has

(A) no solution.

(B) only one solution $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

(C) non-zero unique solution.

(D) multiple solution.

1.35 A matrix has Eigen values –1 and –2. The corresponding Eigen vectors are $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ respectively.

The matrix is

(A) $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

2014 IIT Kharagpur

1.36 Given a system of equations

$$x + 2y + 2z = b_1$$

$$5x + y + 3z = b_2$$

which of the following is true regarding its solutions? **[Set - 01]**

(A) The system has a unique solution for any given b_1 and b_2.

(B) The system will have infinitely many solutions for any given b_1 and b_2.

(C) Whether or not a solution exists depends on the given b_1 and b_2.

(D) The system would have no solution for any values of b_1 and b_2.

1.37 A system matrix is given as follows

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -6 & -11 & 6 \\ -6 & -11 & 5 \end{bmatrix}$$

The absolute value of the ratio of the maximum Eigen value to the minimum Eigen value is ______. **[Set - 01]**

1.38 Which one of the following statements is true for all real symmetric matrices? **[Set - 02]**

(A) All the eigen values are real.

(B) All the eigen values are positive.

(C) All the eigen values are distinct.

(D) Sum of all the eigen values is zero.

1.39 Two matrices A and B are given below,

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}; \quad B = \begin{bmatrix} p^2 + q^2 & pr + qs \\ pr + qs & r^2 + s^2 \end{bmatrix}$$

If the rank of matrix A is N, then the rank of matrix B is **[Set - 03]**

(A) $\frac{N}{2}$ (B) $N-1$

(C) N (D) 2 N

2015 IIT Kanpur

1.40 If the sum of the diagonal elements of a 2×2 matrix is – 6, then the maximum possible value of determinant of the matrix is ______. **[Set - 01]**

1.41 The maximum value of 'a' such that the matrix $\begin{bmatrix} -3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2 \end{bmatrix}$ has three linearly independent real eigenvectors is **[Set - 01]**

(A) $\frac{2}{3\sqrt{3}}$ (B) $\frac{1}{3\sqrt{3}}$

(C) $\frac{1+2\sqrt{3}}{3\sqrt{3}}$ (D) $\frac{1+\sqrt{3}}{3\sqrt{3}}$

1.42 We have a set of 3 linear equations and 3 unknowns. '$X = Y$' means X and Y are equivalent statements and '$X \neq Y$' means X and Y are not equivalent statements. **[Set - 02]**

***P* :** There is a unique solution.

***Q* :** The equations are linearly independent.

***R* :** All Eigen-values of the coefficient matrix are nonzero.

***S* :** The determinant of the coefficient matrix is nonzero.

Which one of the following is TRUE?

(A) $P = Q = R = S$

(B) $P = R \neq Q = S$

(C) $P = Q \neq R = S$

(D) $P \neq Q \neq R \neq S$

2016 IISc Bangalore

1.43 Consider a 3×3 matrix with every element being equal to 1. Its only non-zero eigenvalue is ______. **[Set - 01]**

1.44 Let the eigenvalues of a 2×2 matrix A be 1, -2 with eigenvectors x_1 and x_2 respectively. Then the eigenvalues and eigenvectors of the matrix $A^2 - 3A + 4I$ would respectively be **[Set - 01]**

(A) 2,14; x_1, x_2.

(B) 2,14; $x_1 + x_2, x_1 - x_2$.

(C) 2,0; x_1, x_2.

(D) 2,0; $x_1 + x_2, x_1 - x_2$.

1.45 Let A be a 4 × 3 real matrix with rank 2. Which one of the following statement is TRUE? **[Set - 01]**

(A) Rank of $A^T A$ is less than 2.

(B) Rank of $A^T A$ is equal to 2.

(C) Rank of $A^T A$ is greater than 2.

(D) Rank of $A^T A$ can be any number between 1 and 3.

1.46 A 3×3 matrix P is such that, $P^3 = P$. Then the eigenvalues of P are **[Set - 02]**

(A) $1, 1, -1$

(B) $1, 0.5 + j0.866, 0.5 - j0.866$

(C) $1, -0.5 + j0.866, -0.5 - j0.866$

(D) $0, 1, -1$

1.47 Let $P = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Consider the set S of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ such that $a^2 + b^2 = 1$ where $\begin{pmatrix} a \\ b \end{pmatrix} = P\begin{pmatrix} x \\ y \end{pmatrix}$. Then S is **[Set - 02]**

(A) a circle of radius $\sqrt{10}$.

(B) a circle of radius $\dfrac{1}{\sqrt{10}}$.

(C) an ellipse with major axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(D) an ellipse with minor axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

2017 IIT Roorkee

1.48 The matrix $A = \begin{bmatrix} \frac{3}{2} & 0 & \frac{1}{2} \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{3}{2} \end{bmatrix}$ has three distinct Eigen values and one of its Eigen vectors is $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Which one of the following can be another Eigen vector of A? **[Set - 01]**

(A) $\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

1.49 The Eigen value of the matrix given below are

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}$$ **[Set - 02]**

(A) (0, – 1, – 3) (B) (0, – 2, – 3)

(C) (0, 2, 3) (D) (0, 1, 3)

2018 IIT Guwahati

1.50 Consider a non-singular 2×2 square matrix A. If trace (A) = 4 and trace (A^2) = 5. The determinant of the matrix A is _______ (up to 1 decimal place).

1.51 Let $A=\begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

and $B = A^3 - A^2 - 4A + 5I$

where, I is the 3×3 identity matrix. The determinant of B is _______. (upto one decimal places).

2019 IIT Madras

1.52 M is a 2×2 matrix with eigenvalues 4 and 9. The eigenvalues of M^2 are

(A) 16 and 81 (B) 2 and 3

(C) 4 and 9 (D) – 2 and – 3

1.53 The rank of the matrix,

$$M=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \text{ is } \_\_\_\_\_\_\_.$$

1.54 Consider a 2×2 matrix $M=[v_1 \quad v_2]$, where v_1 and v_2 are the column vectors. Suppose $M^{-1}=\begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix}$, where u_1^T and u_2^T are the row vectors. Consider the following statements :

Statement 1 : $u_1^T v_1 = 1$ and $u_2^T v_2 = 1$

Statement 2 : $u_1^T v_2 = 0$ and $u_2^T v_1 = 0$

Which of the following options is correct

(A) Statement 1 is true and statement 2 is false

(B) Both the statements are false

(C) Statement 2 is true and statement 1 is false

(D) Both the statements are true

2020 IIT Delhi

1.55 The number of purely real elements in a lower triangular representation of the given 3×3 matrix, obtained through the given decomposition is

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}^T$$

(A) 5 (B) 8

(C) 6 (D) 9

❖❖❖❖

Answers Linear Algebra

1.1	C	1.2	A	1.3	A	1.4	D	1.5	–1,–2,–3
1.6	A	1.7	B	1.8	*	1.9	A	1.10	C
1.11	A	1.12	B	1.13	C	1.14	D	1.15	A
1.16	D	1.17	B	1.18	A	1.19	B	1.20	B
1.21	A, C	1.22	A	1.23	B	1.24	D	1.25	D
1.26	A	1.27	D	1.28	B	1.29	C	1.30	B
1.31	D	1.32	D	1.33	B	1.34	D	1.35	D
1.36	B	1.37	0.33	1.38	A	1.39	C	1.40	9
1.41	B	1.42	A	1.43	3.0	1.44	A	1.45	B
1.46	D	1.47	D	1.48	C	1.49	A	1.50	5.5
1.51	1	1.52	A	1.53	3	1.54	D	1.55	MTA

Explanations Linear Algebra

1.1 (C)

Given :

A 5×7 matrix has all its entries equal to -1,

$$A=\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}_{5\times 7}$$

Method 1

By elementary transformations,

$$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1,$$

$$R_4 \to R_4 - R_1 \text{ and } R_5 \to R_5 - R_1$$

$$A=\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{5\times 7}$$

There exists only one non-zero row.

So, $\rho(A) = 1$

Hence, the correct option is (C).

Method 2

The rank of a matrix is defined as the number of linearly independent rows/columns, whichever is minimum in the matrix.

In the given matrix, there exist linear relationships as given below,

$$R_2 = R_1,\ R_3 = R_1,$$

$$R_4 = R_1 \text{ and } R_5 = R_1$$

Thus, only row R_1 is an independent row.

So, $\rho(A) = 1$

Hence, the correct option is (C).

Key Point

These elementary transformations are done in order to convert matrix to its **Echelon form.**

The number of non-zero rows remaining after elementary transformations gives the rank of that matrix.

A matrix is in its Echelon form if :

(i) Leading non-zero elements of a row are behind the leading non-zero elements in its previous row.

(ii) All the zero rows should be below all the non-zero rows.

This method is also known as **Gauss elimination method**.

1.2 (A)

Given : $A = \begin{bmatrix} a & 1 \\ a & 1 \end{bmatrix}$

Method 1

The characteristics equation is given by,

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} a & 1 \\ a & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} a-\lambda & 1 \\ a & 1-\lambda \end{bmatrix} \right| = 0$$

$$(a-\lambda)(1-\lambda) - a = 0$$

$$a - a\lambda - \lambda + \lambda^2 - a = 0$$

$$\lambda^2 - \lambda(a+1) = 0$$

$$\lambda_1 = 0,\ \lambda_2 = a+1$$

The roots of characteristics equation of a matrix gives its eigen values.

Hence, the correct option is (A).

Method 2

Let λ_1 and λ_2 be the eigen values of the matrix A.

By the property of eigen value,

Trace of the matrix A

= Sum of leading diagonal elements

= sum of eigen values

$$\lambda_1 + \lambda_2 = a + 1$$

Only option (A) satisfies this value.

Hence, the correct option is (A).

1.3 (A)

Given : A system of equation is,

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

It is in form of a homogenous equation which is given by,

$$AX = 0$$

Order of matrix A is $(n) = 3$

By elementary transformations,

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - 2R_1$

$$[A] = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 0 & -2 & -4 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$[A] = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 2 = r$$

The number of linearly independent solutions is given by,

$$n - r = 3 - 2 = 1$$

Hence, the correct option is (A).

Key Point

For a homogenous equation :

$Ax = 0$

Unique solution
$|A| \neq 0$
Rank $(A) = n$

Infinitely many solutions
$|A| = 0$
Rank $(A) < n$
[$(n-r)$ linearly independent non-trivial solution]

Note : A homogenous equation is always consistent (at least one solution exists).

1.4 (D)

Given : $S = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Inverse of a matrix is given by,

$$S^{-1} = \frac{[\text{Adj}(S)]}{|S|} \qquad \ldots\text{(i)}$$

$$|S| = 1(1-0)+1(1-0)+0 = 2$$

$$\text{Adj}(S) = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

From equation (i),

$$S^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, the correct option is (D).

1.5 **–1, –2, –3**

Given : $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$

The characteristics equation is given by,

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 0-\lambda & 1 & 0 \\ 0 & 0-\lambda & 1 \\ -6 & -11 & -6-\lambda \end{vmatrix} = 0$$

$$(-\lambda)[6\lambda+\lambda^2+11]-1[0+6] = 0$$

$$\lambda^3+6\lambda^2+11\lambda+6 = 0$$

$$\lambda^3+\lambda^2+5\lambda^2+5\lambda+6\lambda+6 = 0$$

$$\lambda^2(\lambda+1)+5\lambda(\lambda+1)+6(\lambda+1) = 0$$

$$(\lambda+1)(\lambda^2+5\lambda+6) = 0$$

$$(\lambda+1)(\lambda+2)(\lambda+3) = 0$$

$$\lambda_1 = -1,\ \lambda_2 = -2 \text{ and } \lambda_3 = -3$$

The roots of characteristics equation of a matrix gives its eigen values.

Hence, the eigen values of A are **–1, –2 and –3.**

1.6 **(A)**

Given : An $(n+1)\times(n+1)$ matrix,

$$\begin{bmatrix} 1 & a & a^2 & \cdots & a^n \\ 1 & a & a^2 & \cdots & a^n \\ \vdots & & & & \\ 1 & a & a^2 & \cdots & a^n \end{bmatrix}$$

In the given matrix, there exists a linear relationships as given below,

$$R_2 = R_1,\ R_3 = R_1,$$

$$R_4 = R_1 \cdots R_n = R_1$$

Thus, only row R_1 is an independent row.

So, $\rho(A) = 1$

Hence, the correct option is (A).

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1.7 **(B)**

Given : Matrix is a square matrix.

For a square matrix to be singular, its determinant must be 0.

Hence, the correct option is (B).

Key Point

If the determinant $|A| = 0$, then A is a singular matrix.

1.8 *

Given : $A = \begin{bmatrix} 2 & 1 & 5 \\ 4 & 8 & 13 \\ 6 & 27 & 31 \end{bmatrix}$

L is a lower triangular matrix with its diagonal elements unity.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$$

U is an upper triangular matrix.

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

According to question, $A = LU$

$$\begin{bmatrix} 2 & 1 & 5 \\ 4 & 8 & 13 \\ 6 & 27 & 31 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 5 \\ 4 & 8 & 13 \\ 6 & 27 & 31 \end{bmatrix}$$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & u_{13}l_{21} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

Equating the corresponding elements on both sides : $u_{11} = 2,\ u_{12} = 1,\ u_{13} = 5$.

$$l_{21}u_{11} = 4 \Rightarrow l_{21} = \frac{4}{u_{11}} = \frac{4}{2} = 2$$

$$l_{21}u_{12} + u_{22} = 8$$

$$u_{22} = 8 - l_{21}u_{12} = 8 - (2)(1) = 6$$

$$l_{21}u_{13} + u_{23} = 13$$

$$u_{23} = 13 - l_{21}u_{13} = 13 - (2)(5) = 3$$

$$l_{31}u_{11} = 6 \Rightarrow l_{31} = \frac{6}{u_{11}} = \frac{6}{2} = 3$$

$$l_{31}u_{12} + l_{32}u_{22} = 27$$

$$l_{32} = (27 - l_{31}u_{12})\frac{1}{u_{22}} = \frac{(27 - (3)(1))}{6}$$

$$l_{32} = \frac{24}{6} = 4$$

$$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 31$$

$$u_{33} = 31 - l_{31}u_{13} - l_{32}u_{23}$$

$$u_{33} = 31 - (3)(5) - (4)(3) = 4$$

So, the matrices L and U are,

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$$

and $$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 5 \\ 0 & 6 & 3 \\ 0 & 0 & 4 \end{bmatrix}$$

Hence, the matrix A can be expressed as,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 1 & 5 \\ 0 & 6 & 3 \\ 0 & 0 & 4 \end{bmatrix}$$

1.9 (A)

Given : $$A = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 3 & 0 \\ -1 & 0 & 0 & 4 \end{bmatrix}$$

Trace of the matrix A

= Sum of leading diagonal elements

= sum of eigen values

= 2 + 1 + 3 + 4 = 10

Hence, the correct option is (A).

Key Point

The sum of eigen values of any square matrix is the sum of its main diagonal elements. This sum is also called as the **Trace** of that matrix.

1.10 (C)

Given : $$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$ is an Eigen vector of A.

For any Eigen vector $[X]$ of a matrix $[A]$ corresponding to Eigen value λ, the following equation satisfies,

$$[A - \lambda I][X] = 0$$

$$AX = \lambda X$$

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ 10 \\ -5 \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

On comparing both sides, $\lambda = 5$

Hence, the correct option is (C).

1.11 (A)

Given : $A = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

Inverse of a matrix is given by,

$$A^{-1} = \frac{[\text{Adj}(A)]}{|A|} \quad \ldots\text{(i)}$$

$$\text{Adj}(A) = \begin{bmatrix} 3 & 0 & -6 \\ 0 & 1 & 0 \\ -6 & 0 & 15 \end{bmatrix}$$

$$|A| = 5(3-0) + 2(0-6) = 15 - 12 = 3$$

From equation (i),

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 3 & 0 & -6 \\ 0 & 1 & 0 \\ -6 & 0 & 15 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & \frac{1}{3} & 0 \\ -2 & 0 & 5 \end{bmatrix}$$

Hence, the correct option is (A).

1.12 (B)

Given :

A set of linear equation is represented as,

$$AX = B$$

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$

and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

Then, $a_{11}x + a_{12}y = b_1$

$$a_{21}x + a_{22}y = b_2$$

This set of linear equation gives at least one solution if it satisfies the column picture,

$$x\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} + y\begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

So, this equation shows that B is linearly dependent on the columns of A.

Hence, the correct option is (B).

1.13 (C)

Given : $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 100 & 1 & 0 & 0 \\ 100 & 200 & 1 & 0 \\ 100 & 200 & 300 & 1 \end{bmatrix}$

Method 1

The determinant of matrix A is

$$|A| = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 100 & 1 & 0 & 0 \\ 100 & 200 & 1 & 0 \\ 100 & 200 & 300 & 1 \end{vmatrix}$$

$$|A| = 1\begin{vmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 200 & 300 & 1 \end{vmatrix} - 0 + 0 - 0$$

$$|A| = 1\times 1\begin{vmatrix} 1 & 0 \\ 300 & 1 \end{vmatrix} = 1\times 1\times(1^2 - 0) = 1$$

Hence, the correct option is (C).

Method 2

The given matrix has all the elements above the main diagonal equal to zero.

Thus, matrix A is a lower triangular matrix.

The determinant of A is given by,

$$|A| = \text{Product of leading diagonal elements}$$

$$|A| = 1\times 1\times 1\times 1 = 1$$

Hence, the correct option is (C).

📖 Key Point

A matrix with all the elements above main diagonal equal to zero is called a lower triangular matrix.

For any triangular matrix :

(i) The leading diagonal elements gives the Eigen values of that matrix.

(ii) Determinant is equal to the product of its leading diagonal elements.

1.14 (D)

Given : $X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Method 1

The characteristic equation is given by,

$$|X - \lambda I| = 0$$

$$\begin{vmatrix} 0-\lambda & 1 & 0 & 0 \\ 0 & 0-\lambda & 1 & 0 \\ 0 & 0 & 0-\lambda & 1 \\ 0 & 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$(-\lambda)\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$(-\lambda)(-\lambda)(-\lambda)(1-\lambda) = 0$$

$$\lambda_1 = \lambda_2 = \lambda_3 = 0 \text{ and } \lambda_4 = 1$$

The roots of characteristics equation of a matrix gives its eigen values.

Hence, the correct option is (D).

Method 2

In the given matrix, all the elements below main diagonal are zero.

So, matrix X is an upper triangular matrix.

For any triangular matrix, the Eigen values are the diagonal elements

$$\lambda_1 = \lambda_2 = \lambda_3 = 0 \text{ and } \lambda_4 = 1$$

Hence, the correct option is (D).

Method 3

Matrix X is an square matrix since, number of rows is equal to the number of columns.

Trace of the matrix A

= Sum of leading diagonal elements

= sum of eigen values

$= 0 + 0 + 0 + 1 = 1$

Checking from the options, only option (D) satisfies this condition.

Hence, the correct option is (D).

📖 Key Point

For any square matrix :

1. Sum of Eigen values = Trace of the matrix (sum of leading diagonal elements).
2. Product of Eigen values is equal to the determinant of the matrix.

1.15 (A)

Given : $PX = Q$

It is a non-homogenous equation.

So, for existence of at least one solution, the augmented matrix $[P:Q]$ must have the same rank as matrix P.

Hence, the correct option is (A).

📖 Key Point

For a non-homogenous equation $[PX = Q]$:

1. If rank $[P:Q] \neq \text{rank}[P]$
   $\Rightarrow$ No solution exists.
2. If rank $[P:Q] = \text{rank}[P]$
   $=$ Number of variables
   $\Rightarrow$ Unique solution exists.
3. If rank $[P:Q] = \text{rank}[P]$
   $<$ Number of variables
   $\Rightarrow$ Infinite numbers of solutions exist.

1.16 (D)

Given : $P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

One of Eigen values, $\lambda = -2$.

For any Eigen vector $[X]$ of a matrix $[P]$ corresponding to Eigen value λ, the following equation satisfies,

$$[P - \lambda I][X] = 0$$

$$PX = \lambda X$$

$$\begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -2\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$3x - 2y + 2z = -2x$$

$$5x - 2y + 2z = 0 \qquad \ldots\text{(i)}$$

$$-2y + z = -2y$$

$$z = 0 \qquad \ldots\text{(ii)}$$

On solving equations (i) and (ii),

$$5x - 2y = 0$$

$$\frac{x}{y} = \frac{2}{5}$$

Thus, possible eigen vector of P is $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

Hence, the correct option is (D).

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1.17 (B)

Given : $R = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$

The inverse of a matrix R is given by,

$$R^{-1} = \frac{\text{Adj}[R]}{|R|}$$

$\text{Adj}(R)$ is defined as a transpose of co-factor matrix for R.

So, $$R^{-1} = \frac{1}{|R|}\begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

[C_{ij} is the co-factor of the element in i^{th} row and j^{th} column]

Then the top row of R^{-1} is given by,

$$\begin{bmatrix} \frac{C_{11}}{|R|} & \frac{C_{21}}{|R|} & \frac{C_{31}}{|R|} \end{bmatrix}$$

Co-factors are given by,

$$C_{11} = 2 - (-3) = 5$$

$$C_{21} = -(0 - (-3)) = -3$$

$$C_{31} = [-(-1)] = 1$$

Determinant of matrix R,

$$|R| = (1)C_{11} + 2C_{21} + 2C_{31} = 5 - 6 + 2 = 1$$

Top row of $R^{-1} = [5 \quad -3 \quad 1]$

Hence, the correct option is (B).

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1.18 (A)

Given : Three vectors are : $P = [-10 \ 1 \ 3]$, $Q = [-2 \ -5 \ 3]$ and $R = [2 \ -7 \ 12]$

Checking from the options,

Option (A) :

$$v_1 = [-6 \ -3 \ 6]^T \text{ and } v_2 = [4 \ -2 \ 3]^T$$

$$-6 \times 4 + (-3)(-2) + 6 \times 3 = -24 + 6 + 18$$

$$-6 \times 4 + (-3)(-2) + 6 \times 3 = 0$$

Vectors v_1 and v_2 are orthogonal.

In option (B), (C) and (D), there are three vector in each. For three vectors to be a set of orthogonal vectors, they have to be mutually orthogonal.

In all the options except option (A), the vectors are not mutually orthogonal.

Hence, the correct option is (A).

Key Point

The two n-tuple vectors $x = \{x_1, x_2, ...x_n\}$ and $y = \{y_1, y_2, ... y_n\}$ are said to be orthogonal if, their dot product is zero i.e.,

$$x_1 y_1 + x_2 y_2 + x_3 y_3 + ... + x_n y_n = 0$$

1.19 (B)

Let the linearly dependent vector be,

$$A = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

For linearly dependent vectors, the determinant should be equal to zero.

$$\begin{vmatrix} -6 & -3 & 6 \\ 4 & -2 & 3 \\ a & b & c \end{vmatrix} = 0$$

$$-6(-2c - 3b) + 3(4c - 3a) + 6(4b + 2a) = 0$$

$$a + 14b + 8c = 0$$

Checking from the options,

Option (A) :

$$8 + 14(9) + 8(3) = 158 \neq 0$$

Option (B) :

$$-2 + 14(-17) + 8(30) = 0$$

Hence, the correct option is (B).

1.20 (B)

Given : $X = [x_1 \ \ x_2 \ \ ..... \ \ x_n]^T$

X is an n-tuple non-zero vector.

A vector is a matrix with only one row/column and therefore, it also contains of only one independent row/column.

So, $\rho(X) = 1$

The rank of X^T is also equal to 1 i.e.,

$$\rho(X^T) = 1$$

According to the question,

$$V = XX^T$$

Then, $\rho(V) = \rho(XX^T) \leq \min\left[\rho(X), \rho(X^T)\right]$

$$\rho(V) \leq \min(1, 1)$$

$$\rho(V) \leq 1 \Rightarrow \rho(V) = 0 \text{ or } 1$$

But, it is given that V is the product of two non-zero n-tuple matrices,

$$\rho(V) \neq 0$$

So, $\rho(V) = 1$

Hence, the correct option is (B).

Key Point

(i) Rank of transpose of a matrix is same as that of the matrix.

$$\rho(A^T) = \rho(A)$$

(ii) Rank of product of two matrices is less than or equal to minimum of the rank of individual matrices.

$$\rho(AB) \leq \min\left[\rho(A), \rho(B)\right]$$

1.21 (A) and (C)

Given : $A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$

The characteristic equation is given by,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -3-\lambda & 2 \\ -1 & -\lambda \end{vmatrix} = 0$$

$$(\lambda + 3)\lambda + 2 = 0$$

$$\lambda^2 + 3\lambda + 2 = 0$$

By Cayley Hamilton theorem, every square matrix satisfies its own characteristic equation.

$$A^2+3A+2I=0 \qquad \ldots(i)$$

$$(A+I)(A+2I)=0$$

Multiplying equation (i) with A^{-1}.

$$A^{-1}(A^2+3A+2I)=0$$

$$A+3I+2A^{-1}=0$$

Hence, the correct options are (A) and (C).

1.22 (A)

$$A^2+3A+2I=0$$

[From Question 1.21]

The characteristic equation is given by,

$$\lambda^2+3\lambda+2=0$$

$$\lambda_1=-1,\ \lambda_2=-2$$

The roots of characteristics equation of a matrix gives its eigen values.

So, according to Cayley-Hamilton theorem the eigen values of A^9 are,

$$(\lambda_1)^9=-1$$

$$(\lambda_2)^9=-512$$

Checking from the options,

Option (A) :

$$511(-1)+510=-1$$

$$511(-2)+510=-512$$

Other options do not satisfy the condition.

Hence, the correct option is (A).

1.23 (B)

Given : $A=\det\begin{bmatrix} <x,x> & <x,y> \\ <y,x> & <y,y> \end{bmatrix}$

Method 1

Assume that x and y are two vectors in 2-dimensional space (for simplicity of calculation),

Taking, $x=x_1\hat{i}+x_2\hat{j}$

$$y=y_1\hat{i}+y_2\hat{j}$$

Then, $x.x=x_1^2+x_2^2$

$$x.y=x_1y_1+x_2y_2=y.x$$

$$y.y=y_1^2+y_2^2$$

Putting these values in equation (i),

$$A=\begin{vmatrix} x_1^2+x_2^2 & x_1y_1+x_2y_2 \\ x_1y_1+x_2y_2 & y_1^2+y_2^2 \end{vmatrix}$$

$$A=\left(x_1^2+x_2^2\right)\left(y_1^2+y_2^2\right)-\left(x_1y_1+x_2y_2\right)^2$$

$$A=x_1^2y_1^2+x_1^2y_2^2+x_2^2y_1^2+x_2^2y_2^2-x_1^2y_1^2$$
$$-x_2^2y_2^2-2x_1x_2y_1y_2$$

$$A=x_1^2y_2^2+x_2^2y_1^2-2x_1x_2y_1y_2$$

$$A=\left(x_1y_2-x_2y_1\right)^2$$

If $A=0 \Rightarrow\ x_1y_2-x_2y_1=0$

$$\frac{x_1}{x_2}=\frac{y_1}{y_2}$$

The above expression shows that vectors x and y are linearly dependent.

Else, if $A\neq 0$

$$\left(x_1y_2-x_2y_1\right)^2\neq 0 \quad \text{[A positive number]}$$

$$\frac{x_1}{x_2}\neq\frac{y_1}{y_2}$$

The above expression shows that vectors x and y are linearly independent.

Therefore, it is concluded that for A to be positive, x and y are linearly independent.

Hence, the correct option is (B).

Method 2

$\bar{x}$ and $\bar{y}$ are 3-dimensional vectors.

Thus, the dot product is defined as,

$$\vec{x}\cdot\vec{y}=|\vec{x}||\vec{y}|\cos\alpha$$

$$\bar{x}.\bar{x} = |\bar{x}||\bar{x}|\cos 0 = |x|^2 = x^2$$

$$\bar{y}.\bar{y} = y^2 \cos 0 = y^2$$

$$\bar{x}.\bar{y} = \bar{y}.\bar{x} = |x||y|\cos\alpha = xy\cos\alpha$$

where, α is the angle between vectors $\bar{x}$ and $\bar{y}$.

$$|A| = \begin{vmatrix} \bar{x}.\bar{x} & \bar{x}.\bar{y} \\ \bar{y}.\bar{x} & \bar{y}.\bar{y} \end{vmatrix} = \begin{vmatrix} x^2 & xy\cos\alpha \\ xy\cos\alpha & y^2 \end{vmatrix}$$

$$|A| = xy\begin{vmatrix} x & y\cos\alpha \\ x\cos\alpha & y \end{vmatrix}$$

$$|A| = xy\left[xy - xy\cos^2\alpha\right] = x^2y^2\sin^2\alpha$$

$$|A| = x^2y^2\sin^2\alpha = 0$$

From option (A),

If vectors $\bar{x}$ and $\bar{y}$ are linearly independent means they are not parallel i.e., $\alpha \neq 0$.

$$|A| = (xy\sin\alpha^2) > 0 \text{ i.e. positive}$$

Hence, option (A) is incorrect.

From option (B),

When vector $\bar{x}$ and $\bar{y}$ are linearly independent then $|A| \Rightarrow$ Positive.

Hence, option (B) is correct.

From option (C),

If $x \neq 0$ and $y \neq 0$, then $|A|$ will depend on the value of α. For $\alpha = 0$,

$$|A| = 0$$

Hence, option (C) is also incorrect.

From option (D),

$|A|$ will be zero if $\alpha = 0$ or $x = 0$ or $y = 0$.

Hence, option (D) is also incorrect.

Hence, the correct option is (B).

1.24 (D)

Given :

(i) $L(x) = \bar{b} \times \bar{x}$

$b = [0 \quad 1 \quad 0]^T$ and $x = [x_1 \quad x_2 \quad x_3]^T$

So, $\bar{b} = 0.\hat{i} + 1.\hat{j} + 0.\hat{k}$

$$\bar{x} = x_1\hat{i} + x_2\hat{j} + x_3\hat{k}$$

Now, $L(x) = \bar{b} \times \bar{x}$

$$L(x) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 0 \\ x_1 & x_2 & x_3 \end{vmatrix}$$

$$L(x) = i[1.x_3 - 0.x_2] - \hat{j}[0.x_3 - 0.x_1] + \hat{k}[0.x_2 - 1.x_1]$$

$$L(x) = x_3\hat{i} + 0\hat{j} - x_1\hat{k}$$

$$L(x) = [x_3 \quad 0 \quad -x_1]^T = \begin{bmatrix} x_3 \\ 0 \\ -x_1 \end{bmatrix}$$

According to question,

$$L(x) = M\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \ldots\text{(i)}$$

$$\begin{bmatrix} x_3 \\ 0 \\ -x_1 \end{bmatrix} = M\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Let, $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

From equation (i),

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ 0 \\ -x_1 \end{bmatrix}$$

$ax_1 + bx_2 + cx_3 = x_3 \Rightarrow a = 0, b = 0, c = 1$

$dx_1 + ex_2 + fx_3 = 0 \Rightarrow d = e = f = 0$

$gx_1 + hx_2 + ix_3 = -x_1 \Rightarrow g = -1, h = i = 0$

So, $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

The characteristic equation of matrix M is given by,

$$|M - \lambda I| = 0$$

$$\left| \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -\lambda & 0 & 1 \\ 0 & -\lambda & 0 \\ -1 & 0 & -\lambda \end{vmatrix} = 0$$

$$-\lambda\left[\lambda^2\right] - \lambda = 0$$

$$\lambda(\lambda^2 + 1) = 0$$

$$\lambda_1 = 0, \lambda_2 = +i \text{ and } \lambda_3 = -i$$

The roots of characteristics equation of a matrix gives its eigen values.

Hence, the correct option is (D).

1.25 (D)

Given :

The characteristic equation of matrix P is defined as

$$\alpha(\lambda) = \lambda^3 + \lambda^2 + 2\lambda + 1 = 0$$

By Cayley Hamilton theorem, every square matrix satisfies its own characteristic equation.

$$\alpha(P) = 0$$

$$P^3 + P^2 + 2P + I = 0$$

$$\frac{1}{P}(P^3 + P^2 + 2P + I) = \frac{0}{P}$$

$$P^{-1} = -(P^2 + P + 2I)$$

Hence, the correct option is (D).

1.26 (A)

Given :

(i) A (5×6) matrix Q.

(ii) $\rho(Q) = 4$

Rank is defined as the number of linearly independent rows/columns in a matrix.

So, matrix Q has 4 linearly independent rows and columns.

Hence, the correct option is (A).

1.27 (D)

$$A^+ = (A^T A)^{-1} A^T$$

$$A^+ = A^{-1}(A^T)^{-1} A^T \quad [(AB)^{-1} = B^{-1}A^{-1}]$$

$$A^+ = A^{-1}$$

Checking from the options,

Option (A) :

$$AA^+A = AA^{-1}A = IA = A$$

Thus, option (A) is true.

Option (B) :

$$(AA^+)^2 = (AA^{-1})^2 = I^2 = I$$

$$AA^+ = AA^{-1} = I$$

Thus, option (B) is also true.

Option (C) :

$$A^+A = A^{-1}A = I$$

Thus, option (C) is also true.

Option (D) :

$$AA^+A = AA^{-1}A = IA = A$$

Thus, option (D) is false.

Hence, the correct option is (D).

1.28 (B)

Given :

(i) P is a 2×2 real orthogonal matrix

(ii) A real vector, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(iii) Magnitude or length, $\|\vec{x}\| = \sqrt{x_1^2 + x_2^2}$

Method 1

For orthogonal matrix P

$$PP^T = P^T P = I$$

Here, P is a 2×2 matrix an $\vec{x}$ is 2×1 matrix.

So, $P \cdot \vec{x}$ will be 2×1 matrix.

Let $P\cdot\vec{x}=\begin{bmatrix}y_1\\y_2\end{bmatrix}$

$$\|P\cdot\vec{x}\|=\sqrt{y_1^2+y_2^2} \quad \ldots(i)$$

$$(P\cdot\vec{x})^T(P\cdot\vec{x})=[y_1 \quad y_2]\begin{bmatrix}y_1\\y_2\end{bmatrix}$$

$$(P\cdot\vec{x})^T(P\cdot\vec{x})=y_1^2+y_2^2 \quad \ldots(ii)$$

From equation (i) and (ii),

$$\|P\cdot\vec{x}\|^2=(P\cdot\vec{x})^T(P\cdot\vec{x})=(\vec{x}^T\cdot P^T)(P\cdot\vec{x})$$

$$\|P\cdot\vec{x}\|^2=\vec{x}^T(P^T\cdot P)\vec{x}$$

$$\|P\cdot\vec{x}\|^2=\vec{x}^T\cdot I\cdot\vec{x}=[x_1 \quad x_2]\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

$$\|P\cdot\vec{x}\|^2=x_1^2+x_2^2=\|\vec{x}\|^2$$

$$\|P\cdot\vec{x}\|=\|\vec{x}\|$$

Hence, the correct option is (B).

Method 2

Let an orthogonal matrix,

$$P=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

$$P\cdot\vec{x}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

$$P\cdot\vec{x}=\begin{bmatrix}x_1\cos\theta-x_2\sin\theta\\ x_1\sin\theta+x_2\cos\theta\end{bmatrix}$$

$$\|P\cdot\vec{x}\|=\sqrt{(x_1\cos\theta-x_2\sin\theta)^2+(x_1\sin\theta+x_2\cos\theta)^2}$$

$\|P\cdot\vec{x}\|=\sqrt{x_1^2+x_2^2}=\|\vec{x}\|$ for all vector.

Hence, the correct option is (B).

1.29 (C)

Given : A 2×2 matrix with trace -2 and determinant -35.

Let, λ_1 and λ_2 be the Eigen values of the matrix.

By the property of a square matrix,

$$\lambda_1+\lambda_2=-2 \quad \ldots(i)$$

$$\lambda_1\times\lambda_2=-35 \quad \ldots(ii)$$

Method 1

From equation (i),

$$\lambda_1=-2-\lambda_2$$

Put the value of λ_1 in equation (ii),

$$(-2-\lambda_2)\lambda_2=-35$$

$$\lambda_2^2+2\lambda_2-35=0$$

$$\lambda_2=5,-7$$

Then, $\lambda_1=-7, 5$

Thus, the Eigen value of the matrix is -7 and 5.

Hence, the correct option is (C).

Method 2

Checking from the options,

Option (A) :

Trace $=(-30)+(-5)=-35\neq -2$

Incorrect option.

Option (B) :

Trace $=(-37)+(-1)=-38\neq -2$

Also, incorrect option.

Option (C) :

Trace $=(-7)+(5)=-2$

Determinant $=(-7)\times(5)=-35$

Hence, the correct option is (C).

1.30 (B)

Given : $P=\begin{bmatrix}1&1&0\\0&2&2\\0&0&3\end{bmatrix}$

The matrix P has all the elements below the main diagonal as zero. So, it is an upper triangular matrix.

For any triangular matrix, the diagonal elements give its Eigen values $\lambda_1 = 1, \lambda_2 = 2$ and $\lambda_3 = 3$.

Method 1

For any Eigen vector [X], of a matrix [P] corresponding to Eigen value λ, the following equation satisfies,

$$[P - \lambda I][X] = 0$$

For $\lambda = 3$,

$$\begin{bmatrix} -2 & 1 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

By solving,

$$-2x_1 + x_2 + 0x_3 = 0 \qquad \ldots\text{(i)}$$

$$0x_1 - x_2 + 2x_3 = 0 \qquad \ldots\text{(ii)}$$

$$0x_1 - 0x_2 + 0x_3 = 0 \qquad \ldots\text{(iii)}$$

Let $x_3 = k$, then by equation (ii),

$$x_2 = 2k \qquad \text{[From equation (i)]}$$

$$2x_1 = x_2 \Rightarrow 2x_1 = 2k \Rightarrow x_1 = k$$

$$x_1 = x_3 = k \text{ and } x_2 = 2k$$

Thus, Eigen vector is of the form,

$$X = [k \quad 2k \quad k]^T$$

where, $k =$ any non-zero number.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ 2k \\ k \end{bmatrix} = k \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Only **option (B)** satisfies the above equation.

Hence, the correct option is (B).

Method 2

For any Eigen vector [X], for a matrix [P] corresponding to Eigen value λ, the following equation satisfies,

$$[P - \lambda I][X] = 0$$

$$PX = \lambda X$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [\lambda] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

By the trial of given options,

$$X = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}^T \text{ for corresponding } \lambda = 3.$$

Hence, the correct option is (B).

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1.31 (D)

Given :

A set of equations is,

$$x_1 + 2x_2 + x_3 + 4x_4 = 2 \qquad \ldots\text{(i)}$$

$$3x_1 + 6x_2 + 3x_3 + 12x_4 = 6 \qquad \ldots\text{(ii)}$$

It is in form of a non-homogenous equation given by,

$$AX = B$$

where, $A = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 3 & 6 & 3 & 12 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

Number of variables $(n) = 4$

The augmented matrix is given by,

$$[A : B] = \begin{bmatrix} 1 & 2 & 1 & 4 & : & 2 \\ 3 & 6 & 3 & 12 & : & 6 \end{bmatrix}$$

By elementary transformation $R_2 \to R_2 - 3R_1$,

$$[A : B] = \begin{bmatrix} 1 & 2 & 1 & 4 & : & 2 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

There exists only one non-zero row.

So, $\rho(A) = 1, \ \rho(A : B) = 1$

For a non-homogenous equation,

If $[\rho(P:Q) = \rho(P)] < n$

Then, infinite numbers of solutions or multiple non-trivial solution exist.

Hence, the correct option is (D).

1.32 (D)

Given : $[A]=\begin{bmatrix}2 & 1\\4 & -1\end{bmatrix}$

According to question, $A = LU$

Checking from the options,

Option (A) :

$$LU=\begin{bmatrix}1 & 0\\4 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & -2\end{bmatrix}=\begin{bmatrix}1 & 1\\4 & 6\end{bmatrix}\neq A$$

Option (B) :

$$LU=\begin{bmatrix}2 & 0\\4 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}=\begin{bmatrix}2 & 2\\4 & 3\end{bmatrix}\neq A$$

Option (C) :

$$LU=\begin{bmatrix}1 & 0\\4 & 1\end{bmatrix}\begin{bmatrix}2 & 1\\0 & -1\end{bmatrix}=\begin{bmatrix}2 & 1\\8 & 3\end{bmatrix}\neq A$$

Option (D) :

$$LU=\begin{bmatrix}2 & 0\\4 & -3\end{bmatrix}\begin{bmatrix}1 & 0.5\\0 & 1\end{bmatrix}=\begin{bmatrix}2 & 1\\4 & -1\end{bmatrix}=A$$

Hence, the correct option is (D).

1.33 (B)

Given :

(i) Matrix $A=\begin{bmatrix}-5 & -3\\2 & 0\end{bmatrix}$

(ii) I is a 2×2 identity matrix.

The characteristic equation is given by,

$$|A-\lambda I|=0$$

$$\begin{vmatrix}-5-\lambda & -3\\2 & -\lambda\end{vmatrix}=0$$

$$5\lambda+\lambda^2+6=0$$

By Cayley Hamilton theorem, every square matrix satisfies its own characteristic equation,

$$5A+A^2+6I=0$$

$$A^2=-5A-6I \qquad \ldots\text{(i)}$$

Multiplying A in above equation,

$$A^3=-5A^2-6A$$

From equation (i),

$$A^3=-5(-5A-6I)-6A=19A+30I$$

Hence, the correct option is (B).

1.34 (D)

Given : $\begin{bmatrix}2 & -2\\1 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

Method 1

$$x_1-x_2=0$$

and $\quad 2x_1-2x_2=0\Rightarrow x_1=x_2=K$

where, K = any constant number

The matrix $\begin{bmatrix}x_1\\x_2\end{bmatrix}$ can be written as,

$$\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}K\\K\end{bmatrix}=K\begin{bmatrix}1\\1\end{bmatrix}$$

So there are infinite number of possible solutions.

Hence, the correct option is (D).

Method 2

In the given matrix, $A=\begin{bmatrix}2 & -2\\1 & -1\end{bmatrix}$

There exist a linear relationship as, $R_1=2R_2$

Thus, only one independent row R_2 exist.

So, $\quad \rho(A)=1$

For a homogenous equation,

If $\quad \rho(A)<n \qquad$ [n = number of variables]

Then, infinite number of solutions exist.

Hence, the correct option is (D).

1.35 (D)

Given :

(i) Eigen values are –1, –2.

(ii) Eigen vector are $\begin{bmatrix}1\\-1\end{bmatrix}$ and $\begin{bmatrix}1\\-2\end{bmatrix}$.

Let the matrix A be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Method 1

For any Eigen vector $[X]$, of a matrix $[A]$ corresponding to Eigen value λ, the following equation satisfies,

$$[A-\lambda I][X]=0$$

$$AX=\lambda X$$

For $\lambda=-1$ and $X=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}=(-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$a-b=-1 \quad \ldots\text{(i)}$$

$$c-d=1 \quad \ldots\text{(ii)}$$

For $\lambda=-2$ and $X=\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}=(-2)\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$a-2b=-2 \quad \ldots\text{(iii)}$$

$$c-2d=4 \quad \ldots\text{(iv)}$$

From equation (i) and (iii),

$$a=0 \text{ and } b=1$$

From equation (ii) and (iv),

$$c=-2 \text{ and } d=-3$$

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Hence, the correct option is (D).

Method 2

Modal matrix can be formed as,

$$[M]=[v_1 \quad v_2]$$

[where, v_1, v_2 are Eigen vectors]

$$[M]=\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$$

The matrix A can be formed as,

$$[A]=[M]\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}[M]^{-1}$$

$$[A]=\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\left(\frac{-1}{1}\begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix}\right)$$

$$[A]=\begin{bmatrix} -1 & -2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Hence, the correct option is (D).

Key Point

Concept of Diagonalization :

Any square matrix whose eigen values are distinct, can be represented as :

$$A=[M]\,[D]\,[M^{-1}]$$

where, D is a diagonal matrix whose diagonal elements are Eigen values of A.

M is a non-singular matrix whose columns are respective Eigen vectors of A.

Note : M is also referred to as **Modal matrix**.

Method 3

For any Eigen vector $[X]$ for a matrix $[A]$ corresponding to Eigen value λ, the following equation satisfies,

$$[A-\lambda I][X]=0$$

By the property of square matrix,

Sum of eigen values of a matrix $(\lambda_1+\lambda_2)$

= Trace of the matrix

Product of eigen values of a matrix $(\lambda_1\lambda_2)$

= Determinant of the matrix ($|A|$)

By these properties, options (A) and (B) are not possible.

Option (C) :

For $\lambda=-1$;

$$[A-\lambda I][X]=\begin{bmatrix} -1-(-1) & 0 \\ 0 & -2-(-1) \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$[A-\lambda I][X]=\begin{bmatrix}0 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1\\ -1\end{bmatrix}$$

$$[A-\lambda I][X]=\begin{bmatrix}0\\ 1\end{bmatrix}\neq\begin{bmatrix}0\\ 0\end{bmatrix}$$

Thus, option (C) is incorrect.

Option (D) :

For $\lambda=-1$;

$$[A-\lambda I][X]=\begin{bmatrix}0-(-1) & 1\\ -2 & -3-(-1)\end{bmatrix}\begin{bmatrix}1\\ -1\end{bmatrix}$$

$$[A-\lambda I][X]=\begin{bmatrix}0\\ 0\end{bmatrix}$$

For $\lambda=-2$;

$$[A-\lambda I][X]=\begin{bmatrix}0-(-2) & 1\\ -2 & -3-(-2)\end{bmatrix}\begin{bmatrix}1\\ -2\end{bmatrix}$$

$$[A-\lambda I][X]=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Hence, the correct option is (D).

1.36 (B)

Given : $x+2y+2z=b_1,\ 5x+y+3z=b_2$

It is in the form of a non-homogenous equation given by,

$$AX=B$$

where, $A=\begin{bmatrix}1 & 2 & 2\\ 5 & 1 & 3\end{bmatrix}_{2\times 3}$, $B=\begin{bmatrix}b_1\\ b_2\end{bmatrix}_{2\times 1}$

Number of variables (n) = 3

The augmented matrix is given by,

$$[A:B]=\begin{bmatrix}1 & 2 & 2 & : & b_1\\ 5 & 1 & 3 & : & b_2\end{bmatrix}$$

By elementary transformation $R_2\to R_2-5R_1$,

$$[A:B]=\begin{bmatrix}1 & 2 & 2 & : & b_1\\ 0 & -9 & -7 & : & b_2-5b_1\end{bmatrix}$$

There exist two non-zero rows irrespective of values of b_1 and b_2.

So, $\rho(A)=2,\ \rho(A:B)=2$

For a non-homogenous equation,

If $[\rho(P:Q)=\rho(P)]<n$

Then, infinite numbers of solutions exist.

Hence, the correct option is (B).

1.37 0.33

Given : $A=\begin{bmatrix}0 & 1 & -1\\ -6 & -11 & 6\\ -6 & -11 & 5\end{bmatrix}$

The characteristic equation is given by,

$$|A-\lambda I|=0$$

$$\begin{vmatrix}-\lambda & 1 & -1\\ -6 & -11-\lambda & 6\\ -6 & -11 & 5-\lambda\end{vmatrix}=0$$

$$-\lambda[(-11-\lambda)(5-\lambda)+66]-[(-6)(5-\lambda)+36]$$
$$-[66-(11+\lambda)6]=0$$

$$\lambda^3+6\lambda^2+11\lambda+6=0$$

$$\lambda^3+\lambda^2+5\lambda^2+5\lambda+6\lambda+6=0$$

$$(\lambda+1)(\lambda^2+5\lambda+6)=0$$

$$\lambda_1=-1,\ \lambda_2=-2 \text{ and } \lambda_3=-3$$

The roots of characteristics equation of a matrix gives its Eigen values.

Maximum Eigen value, $\lambda_{max}=-1$

Minimum Eigen value, $\lambda_{min}=-3$

So, the required absolute value or magnitude of ratio is given by,

$$\left|\frac{\lambda_{max}}{\lambda_{min}}\right|=\left|\frac{-1}{-3}\right|=\frac{1}{3}=0.33$$

Hence, the minimum Eigen value is **0.33.**

1.38 (A)

All the Eigen values of a real symmetric matrix are real but not necessarily distinct or positive.

Hence, the correct option is (A).

1.39 (C)

Given : $A=\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $\rho(A)=\text{N}$

$$B=\begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix}$$

Matrix B is obtained from matrix A by applying row/column operations.

So, $\rho(B)=\rho(A)=\text{N}$

Hence, the correct option is (C).

Key Point

Rank of a matrix is unaltered by the elementary transformations i.e. row/column operations.

1.40 9

Given :

A 2×2 matrix, $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Trace of matrix $=-6$

Let λ_1 and λ_2 are two eigen values of matrix A.

By the properties of square matrices,

Product of eigen values of a matrix

= Determinant of matrix

$$|A|=\lambda_1\lambda_2 \qquad \ldots\text{(i)}$$

Sum of eigen values of a matrix

= Trace of the matrix

$$\lambda_1+\lambda_2=-6 \qquad \ldots\text{(ii)}$$

From equations (i) and (ii),

$$|A|=\lambda_1(-6-\lambda_1)=-6\lambda_1-\lambda_1^2$$

For extreme value, the stationary points are given by,

$$\frac{d|A|}{d\lambda_1}=0$$

$$-6-2\lambda_1=0$$

$$\lambda_1=-3$$

Taking second derivative of $|A|$,

$$\frac{d^2|A|}{d\lambda_1^2}=-2 \qquad \textbf{[maxima]}$$

So, $|A|$ is maximum at $\lambda_1=-3$.

From equation (ii),

$$\lambda_2=-6+3=-3$$

Thus, the maximum value of determinant is,

$$|A|_{\max}=\lambda_1\times\lambda_2=(-3)\times(-3)=9$$

Hence, the maximum possible value of determinant of matrix is **9**.

1.41 (B)

Given : $A=\begin{bmatrix} -3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2 \end{bmatrix}$

Method 1

The characteristic equation is given by,

$$|A-\lambda I|=0$$

$$\begin{vmatrix} -3-\lambda & 0 & -2 \\ 1 & -1-\lambda & 0 \\ 0 & a & -2-\lambda \end{vmatrix}=0$$

$$(-3-\lambda)\begin{vmatrix} -1-\lambda & 0 \\ a & -2-\lambda \end{vmatrix}-2\begin{vmatrix} 1 & -1-\lambda \\ 0 & a \end{vmatrix}=0$$

$$-(\lambda+3)\begin{vmatrix} \lambda+1 & 0 \\ -a & \lambda+2 \end{vmatrix}-2\begin{vmatrix} 1 & -(1+\lambda) \\ 0 & a \end{vmatrix}=0$$

$$-(\lambda+3)[(\lambda+1)(\lambda+2)]-2[a]=0$$

$$(\lambda+1)(\lambda+2)(\lambda+3)+2a=0$$

$$\lambda^3+6\lambda^2+11\lambda+(2a+6)=0$$

$$a=-\frac{1}{2}\left[\lambda^3+6\lambda^2+11\lambda+6\right]$$

For extreme value, the stationary points are given by,

$$\frac{da}{d\lambda}=0$$

$$\frac{da}{d\lambda} = -\frac{1}{2}\left[3\lambda^2 + 12\lambda + 11\right] = 0$$

$$3\lambda^2 + 12\lambda + 11 = 0$$

$$\lambda = \frac{-12 \pm \sqrt{12^2 - 4\times3\times11}}{6} = \frac{-12 \pm \sqrt{12}}{6}$$

$$\lambda = \frac{-12 \pm 2\sqrt{3}}{6} = \frac{-2 \pm \sqrt{3}}{3}$$

$$\lambda_1 = -\left(\frac{2+\sqrt{3}}{3}\right),\ \lambda_2 = -\left(\frac{2-\sqrt{3}}{3}\right)$$

Taking second derivative of a,

$$\frac{d^2a}{d\lambda^2} = -\frac{1}{2}(6\lambda + 12) = -3(\lambda + 2)$$

For $\lambda_1 = -\left(\frac{2+\sqrt{3}}{3}\right)$, $\frac{d^2a}{d\lambda^2} = -\text{ve}$ **[maxima]**

For $\lambda_2 = -\left(\frac{2-\sqrt{3}}{3}\right)$, $\frac{d^2a}{d\lambda^2} = -\text{ve}$ **[maxima]**

So, for both λ_1 and λ_2 there is maxima of a

For $\lambda_1 = -\left(\frac{2+\sqrt{3}}{3}\right) = -1.24$

$$a_1 = -\frac{1}{2}\left[\lambda_1^3 + 6\lambda_1^2 + 11\lambda_1 + 6\right]$$

$$a_1 = -\frac{1}{2}\left[(-1.24)^3 + 6(-1.24)^2 + 11(-1.24) + 6\right]$$

$$a_1 = 0.16$$

For $\lambda_2 = -\left(\frac{2-\sqrt{3}}{3}\right) = -0.09$

$$a_2 = -\frac{1}{2}\left[\lambda_2^3 + 6\lambda_2^2 + 11\lambda_2 + 6\right]$$

$$a_2 = -\frac{1}{2}\left[(-0.09)^3 + 6(-0.09)^2 + 11(-0.09) + 6\right]$$

$$a_2 = -2.53$$

So, maximum value of $a = 0.16$.

Checking from the options,

Option (A) $= \frac{2}{3\sqrt{3}} = 0.385$

Option (B) $= \frac{1}{3\sqrt{3}} = 0.19$

Option (C) $= \frac{1+2\sqrt{3}}{3\sqrt{3}} = 0.859$

Option (D) $= \frac{1+\sqrt{3}}{3\sqrt{3}} = 0.525$

Therefore, option (B) gives the closest value of a.

Hence, the correct option (B).

Method 2

Let the given matrix be $A = \begin{bmatrix} -3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2 \end{bmatrix}$.

The characteristics equation of A is $|A - \lambda I| = 0$

$$\begin{vmatrix} -3-\lambda & 0 & -2 \\ 1 & -1-\lambda & 0 \\ 0 & a & -2-\lambda \end{vmatrix} = 0$$

$$(-3-\lambda)(-1-\lambda)(-2-\lambda) - 2a = 0$$

$$(\lambda+1)(\lambda+2)(\lambda+3) + 2a = 0 \quad \ldots\text{(i)}$$

If A has three distinct Eigen values then A has three linearly independent eigen vectors.

Let $f(\lambda) = (\lambda+1)(\lambda+2)(\lambda+3)$

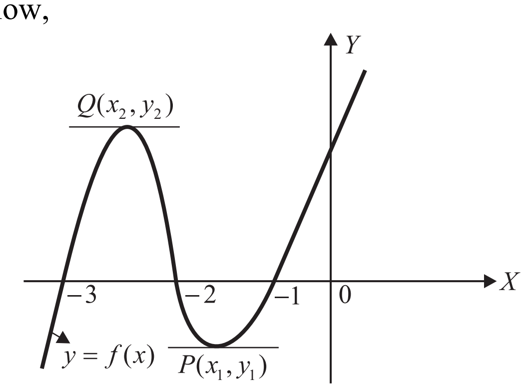
From equation (i),

$$f(\lambda) + 2a = 0$$

$$f(\lambda) = -2a \quad \ldots\text{(ii)}$$

Consider $f(x) = (x+1)(x+2)(x+3)$

The graph of $f(x)$ is as shown in the figure below,



The number of distinct real roots of an equation $F(x)=k$, (k is real) is same as that of the number of points of intersection of the curve $y=F(x)$ and the line $y=k$.

The curve $y=f(x)$ intersects at three points with a line $y=y_0$ only when $y_1 \le y_0 \le y_2$ i.e. for $f(x)+2a=0 \Rightarrow f(x)=-2a$, three distinct real roots exist for

$$y_1 \le -2a \le y_2 \qquad \ldots\text{(iii)}$$

i.e. $$y_1 \le f(x) \le y_2 \qquad \ldots\text{(iv)}$$

[From equation (ii)]

Now we will find y_1 and y_2 [i.e., the minimum and maximum values of $f(x)$]

$$f(x)=(x+1)(x+2)(x+3)$$

$$f(x)=x^3+6x^2+11x+6$$

$$f'(x)=3x^3+12x+11=0$$

$$f'(x)=0 \Rightarrow \quad 3x^3+12x+11=0$$

$$x=\frac{-6\pm\sqrt{3}}{3}$$

and $$f''(x)=6x+12$$

At $x=\dfrac{-6+\sqrt{3}}{3}$; $f''(x)=2\sqrt{3}>0$ and

At $x=\dfrac{-6-\sqrt{3}}{3}$; $f''(x)=-2\sqrt{3}<0$

Since, $f(x)$ has a maximum at $x=\dfrac{-6-\sqrt{3}}{3}$

and a minimum at $x=\dfrac{-6+\sqrt{3}}{3}$

The maximum value of $f(x)=y_2=f(x)=\dfrac{2}{3\sqrt{3}}$

at $$x=\frac{-6-\sqrt{3}}{3}$$

The minimum value of $f(x)=y_1=f(x)$

at $$x=\frac{-6-\sqrt{3}}{3}=\frac{-2}{3\sqrt{3}}$$

From equation (iv),

$$\frac{-2}{3\sqrt{3}} \le f(x) \le \frac{2}{3\sqrt{3}}$$

$$\frac{-2}{3\sqrt{3}} \le -2a \le \frac{2}{3\sqrt{3}} \quad \text{[From equation (iii)]}$$

$$\frac{1}{3\sqrt{3}} \ge a \ge \frac{-1}{3\sqrt{3}} \Rightarrow \frac{-1}{3\sqrt{3}} \le a \le \frac{1}{3\sqrt{3}}$$

Therefore, the maximum value of 'a' such that the matrix A has three real linearly independent eigen vectors is $\dfrac{1}{3\sqrt{3}}$.

Hence, the correct option (B).

Key Point

A continuous function $f(x)$ at an stationary point (given by the root of $f'(x)=0$) is :

(i) Maximum if $f''(x)=-\text{ve}$

(ii) Minimum if $f''(x)=+\text{ve}$

1.42 (A)

Given : A set of three linear equations with three variables.

$$[A]_{3\times3}[X]_{3\times1}=[B]_{3\times1}$$

Statement P : There is a unique solution.

In this case,

$$\rho(A)=\rho(A:B)=\text{Number of unknowns}$$

$$\rho(A)=\rho(A:B)=3$$

This means, three independent rows exist in matrix A irrespective of matrix B.

Also, $$|A|\ne 0 \qquad \ldots\text{(i)}$$

So, $$P=Q=S$$

Statement R : All Eigen-values of the coefficient matrix are nonzero.

Let λ_1, λ_2 and λ_3 are eigen values of matrix A.

By the properties of square matrix,

$$\lambda_1\times\lambda_2\times\lambda_3=|A| \qquad \ldots\text{(ii)}$$

From equations (i) and (ii),

$$\lambda_1 \times \lambda_2 \times \lambda_3 \neq 0$$

$$\lambda_1 \neq 0,\ \lambda_2 \neq 0 \text{ and } \lambda_3 \neq 0$$

Therefore, from the above discussion, it is inferred that,

$$P = Q = R = S$$

Hence, the correct option is (A).

1.43 3.0

Given : $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{3\times 3}$

The characteristic equation is given by,

$$|A - \lambda I| = 0 \qquad \text{...(i)}$$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

The value of determinant remains unchanged after any linear row/column operation.

For simplification, performing $C_2 \to C_2 - C_3$,

$$\begin{vmatrix} 1-\lambda & 0 & 1 \\ 1 & -\lambda & 1 \\ 1 & \lambda & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)[-\lambda(1-\lambda) - \lambda] + (\lambda + \lambda) = 0$$

$$(1-\lambda)(\lambda^2 - 2\lambda) + 2\lambda = 0$$

$$\lambda[(1-\lambda)(\lambda - 2) + 2] = 0$$

$$\lambda(-\lambda^2 + 3\lambda) = 0$$

$$\lambda^2(3-\lambda) = 0$$

$$\lambda_1 = \lambda_2 = 0 \text{ and } \lambda_3 = 3$$

The roots of the characteristics equation gives the eigen values.

Hence, the only non-zero Eigen value is **3**.

1.44 (A)

Given : Eigen values of 2×2 matrix A are $1, -2$.

Applying properties of Eigen value,

Eigen value of A^2 : λ^2

$$(1)^2,\ (-2)^2 = 1,\ 4$$

Eigen value of -3A : -3λ

$$(-3)(1),\ (-3)(-2) = -3,\ 6$$

Eigen value of $4I$: 4, 4

Eigen values of $A^2 - 3A + 4I$:

$$(1-3+4),\ (4+6+4) = 2, 14$$

Eigen vectors remains unchanged for any operation.

Hence, the correct option is (A).

1.45 (B)

Given : For real matrix A rank is $\rho(A_{4\times 3}) = 2$.

Also, $\rho(A^T_{3\times 4}) = 2$

Thus, $\rho(A^T A) \leq \min[\rho(A^T), \rho(A)]$

$$\rho(A^T A) \leq \min[2,\ 2]$$

$$\rho(A^T A) = 2$$

Hence, the correct option is (B).

1.46 (D)

Given : $P^3 = P$

By Cayley-Hamilton theorem, every square matrix satisfies its own characteristics equation.

$$\lambda^3 = \lambda$$

$$\lambda^3 - \lambda = 0$$

$$\lambda(\lambda^2 - 1) = 0$$

$$\lambda_1 = 0,\ \lambda_2 = 1 \text{ and } \lambda_3 = -1$$

Hence, the correct option is (D).

1.47 (D)

Given : $P=\begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}$ and $\begin{bmatrix}a\\ b\end{bmatrix}=P\begin{bmatrix}x\\ y\end{bmatrix}$

$$a^2+b^2=1 \qquad \ldots\text{(i)}$$

Thus, $\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$

$$3x+y=a \qquad \ldots\text{(ii)}$$

$$x+3y=b \qquad \ldots\text{(iii)}$$

Squaring and adding equations (ii) and (iii),

$$a^2+b^2=9x^2+y^2+6xy+x^2+9y^2+6xy$$

$$a^2+b^2=10x^2+10y^2+12xy$$

From equation (i),

$$10x^2+10y^2+12xy=1$$

The standard equation of an ellipse is given by,

$$ax^2+by^2+2hxy=0$$

On comparing : $a=10,\ b=10,\ h=6$

If $h^2-ab<0$, it represents an ellipse.

$$h^2-ab=36-100=-64$$

The condition is true.

The length of semi-axis are given by,

$$(ab-h^2)r^4-(a+b)r^2+1=0$$

$$64r^4-20r^2+1=0$$

$$r^2=\frac{1}{4} \text{ or } r^2=\frac{1}{16}$$

$$r_1=\frac{1}{2},\ r_2=\frac{1}{4}$$

Length of major axis $=2r_1=1$

Length of minor axis $=2r_2=\frac{1}{2}$

Equation of the major axis is given by,

$$\left(a-\frac{1}{r_1^2}\right)x+hy=0$$

$$(10-4)x+6y=0$$

$$x+y=0$$

Major axis exists along $y=-x$. So, minor axis exists along $y=x$, since both axes are perpendicular on each other.

The vector $\begin{bmatrix}1\\ 1\end{bmatrix}$ lies on the line $y=x$ (minor axis)

Hence, the correct option is (D).

1.48 (C)

Given :

(i) $A=\begin{bmatrix}\frac{3}{2} & 0 & \frac{1}{2}\\ 0 & -1 & 0\\ \frac{1}{2} & 0 & \frac{3}{2}\end{bmatrix}$ with all distinct Eigen values.

(ii) One of the Eigen vector is $x_1=\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$.

Transpose of matrix A is given by,

$$A^T=\begin{bmatrix}\frac{3}{2} & 0 & \frac{1}{2}\\ 0 & -1 & 0\\ \frac{1}{2} & 0 & \frac{3}{2}\end{bmatrix}=A$$

A matrix whose transpose is equal to itself is called a Symmetric matrix.

So, all its Eigen vectors must be orthogonal.

For orthogonal vectors, $x_1^T\cdot x_2=0$

Checking from the options,

Option (A) :

$$x_1^T\cdot x_2=\begin{bmatrix}1 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\ 0\\ -1\end{bmatrix}=0+0-1=-1$$

Hence, option (A) is incorrect option.

Option (B) :

$$x_1^T \cdot x_2 = [1 \quad 0 \quad 1]\begin{bmatrix}-1\\0\\0\end{bmatrix} = -1+0+0 = -1$$

Hence, option (B) is incorrect option.

Option (C) :

$$x_1^T \cdot x_2 = [1 \quad 0 \quad 1]\begin{bmatrix}1\\0\\-1\end{bmatrix} = 1+0-1 = 0$$

Hence, the correct option is (C).

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Key Point

For a Symmetric matrix $\left[A^T = A\right]$:

(i) All the eigen values are real.

(ii) All the eigen vectors corresponding to distinct eigen values are orthogonal.

1.49 (A)

Given : Matrix $A = \begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\0 & -3 & -4\end{bmatrix}$

Method 1

The characteristic equation of a matrix is given by,

$$|A-\lambda I| = 0$$

$$\begin{vmatrix}-\lambda & 1 & 0\\0 & -\lambda & 1\\0 & -3 & -4-\lambda\end{vmatrix} = 0$$

$$-\lambda(4\lambda+\lambda^2+3) = 0$$

$$\lambda(\lambda+3)(\lambda+1) = 0$$

$$\lambda_1 = 0,\ \lambda_2 = -1 \text{ and } \lambda_3 = -3$$

The roots of characteristics equation of a matrix gives its eigen values.

Hence, the correct option is (A).

Method 2

Matrix X is an square matrix because number of rows is equal to the number of columns.

Sum of Eigen values = Trace of matrix

$$= 0+0+(-4) = -4$$

Checking from the options, only option (A) satisfies this condition.

Hence, the correct option is (A).

1.50 5.5

Given :

(i) Matrix $[A]$ is a non-singular 2×2 square matrix.

(ii) Trace of matrix $[A]$ is 4.

(iii) Trace of matrix $[A^2]$ is 5.

Let Eigen values of matrix $[A]$ be λ_1 and λ_2.

Therefore, Eigen values of matrix $[A^2]$ will be λ_1^2 and λ_2^2.

Trace of matrix $[A] = \lambda_1+\lambda_2 = 4$

$$(\lambda_1+\lambda_2)^2 = 16$$

$$\lambda_1^2+\lambda_2^2+2\lambda_1\lambda_2 = 16 \qquad \ldots(i)$$

Trace of matrix $[A^2] = \lambda_1^2+\lambda_2^2 = 5 \qquad \ldots(ii)$

From equation (i) and (ii),

$$5+2\lambda_1\lambda_2 = 16$$

$$\lambda_1\lambda_2 = \frac{11}{2} = 5.5$$

Determinant of matrix A is given by,

$$|A| = \text{Product of Eigen values}$$

$$|A| = \lambda_1\lambda_2 = 5.5$$

Hence, the determinant of the matrix A is **5.5.**

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1.51 **1**

Given :

(i) $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

(ii) $B = A^3 - A^2 - 4A + 5I$

(iii) I is the 3×3 identity matrix.

Characteristics equation is given by,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ -1 & 2-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$$(-2-\lambda)(2-\lambda)(1-\lambda) = 0$$

$$\lambda = 1,\ 2,\ -2$$

The roots of characteristics equation of a matrix gives its Eigen values. According to Cayley-Hamilton theorem, every square matrix satisfies its own characteristics equation.

$$B = A^3 - A^2 - 4A + 5I$$

$$B = \lambda^3 - \lambda^2 - 4\lambda + 5$$

The eigen value of matrix B is given by,

$$\lambda_1 = 1^3 - 1^2 - 4 + 5 = 1$$

$$\lambda_2 = 2^3 - 2^2 - (4\times2) + 5 = 1$$

$$\lambda_3 = (-2)^3 - (-2)^2 - [4\times(-2)] + 5 = 1$$

Determinant of matrix B is given by,

$$|B| = \text{Product of Eigen values}$$

$$|B| = \lambda_1\lambda_2\lambda_3 = 1\times1\times1 = 1$$

Hence, the determinant of B is **1**.

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1.52 **(A)**

From the properties of Eigen values.

If $\lambda_1, \lambda_2, ...\lambda_m$ are Eigen values of matrix M, then $(\lambda_1)^n, (\lambda_2)^n, ...(\lambda_m)^n$ will be Eigen values of matrix M^n.

Given Eigen values of matrix M are 4 and 9.

So, Eigen values of M^2 will be $(4)^2$ and $(9)^2$.

Hence, the correct option is (A).

1.53 **3**

$$M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$|M| = 0 - 1(0-1) + 1(1-0) = 1 + 1 = 2$$

As determinant value of 3×3 matrix is non zero.

Hence, rank of the matrix is 3.

1.54 **(D)**

Given : $M = [v_1 \quad v_2]$, $M^{-1} = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix}$

Method 1

Since, $M^{-1}M = I$

$$\begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix}[v_1 \quad v_2] = \begin{bmatrix} u_1^T v_1 & u_1^T v_2 \\ u_2^T v_1 & u_2^T v_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By equating,

$$u_1^T v_1 = 1 \quad u_1^T v_2 = 0 \quad u_2^T v_1 = 0 \quad u_2^T v_2 = 1$$

So, both statements are true.

Method 2

Consider matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\Rightarrow \quad M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = [V_1 \quad V_2]$$

$$V_1 = \begin{bmatrix} a \\ c \end{bmatrix}, V_2 = \begin{bmatrix} b \\ d \end{bmatrix}$$

$$\Rightarrow \quad M^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix}$$

(i) $u_1^T = \begin{bmatrix} \dfrac{d}{ad-bc} & \dfrac{-b}{ad-bc} \end{bmatrix}$

(ii) $u_2^T = \begin{bmatrix} \dfrac{-c}{ad-bc} & \dfrac{a}{ad-bc} \end{bmatrix}$

Statement-I :

$$u_1^T V_1 = \begin{bmatrix} \dfrac{d}{ad-bc} & -\dfrac{b}{ad-bc} \end{bmatrix}\begin{bmatrix} a \\ c \end{bmatrix}$$

$$= \frac{ad}{ad-bc} - \frac{bc}{ad-bc} = \frac{ad-bc}{ad-bc} = 1$$

$$u_2^T V_2 = \begin{bmatrix} \dfrac{-c}{ad-bc} & \dfrac{a}{ad-bc} \end{bmatrix}\begin{bmatrix} b \\ d \end{bmatrix}$$

$$= \frac{-cd}{ad-bc} + \frac{ad}{ad-bc} = \frac{ad-bc}{ad-bc} = 1$$

Thus statement 1 true.

Statement-II :

$$u_1^T V_2 = \begin{bmatrix} \dfrac{d}{ad-bc} - \dfrac{-b}{ad-bc} \end{bmatrix}\begin{bmatrix} b \\ d \end{bmatrix}$$

$$= \frac{db}{ad-bc} - \frac{bd}{ad-bc} = \frac{db-db}{ad-bc} = 0$$

$$u_2^T V_2 = \begin{bmatrix} \dfrac{-c}{ad-bc} & \dfrac{a}{ad-bc} \end{bmatrix}\begin{bmatrix} a \\ c \end{bmatrix}$$

$$= \frac{-ac}{ad-bc} + \frac{ac}{ad-bc} = \frac{ac-ac}{ad-bc} = 0$$

Hence statement-II is also true.

Hence, the correct option is (D).

1.55 MTA

Given : $[P] = \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix}$

$$[Q] = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$[R]^T = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$[P] = [Q][R]^T$$

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11}^2 & a_{11}a_{12} & a_{11}a_{13} \\ a_{12}a_{11} & a_{12}^2 + a_{22}^2 & a_{12}a_{13} + a_{22}a_{23} \\ a_{13}a_{11} & a_{13}a_{12} + a_{23}a_{22} & a_{13}^2 + a_{23}^2 + a_{33}^2 \end{bmatrix}$$

Comparing the elements of matrix P with the matrix $Q.R^T$

$$a_{11}^2 = 2$$

$$a_{11} = \pm\sqrt{2}$$

$$a_{11}a_{12} = 3$$

$$a_{12} = \pm\frac{3}{\sqrt{2}}$$

$$a_{11}a_{13} = 3$$

$$a_{13} = \pm\frac{3}{\sqrt{2}}$$

$$a_{12}^2 + a_{22}^2 = 2$$

$$a_{22}^2 = 2 - a_{12}^2 = 2 - \left(\frac{9}{2}\right)$$

$$a_{22}^2 = -\frac{5}{2}$$

$$a_{22} = \pm j\sqrt{\frac{5}{2}}$$

$$a_{12}a_{13} + a_{22}a_{23} = 1$$

$$\left(\frac{3}{\sqrt{2}}\right)\left(\frac{3}{\sqrt{2}}\right) + j\sqrt{\frac{5}{2}}a_{23} = 1$$

$$a_{23}j\left(\sqrt{\frac{5}{2}}\right) = 1 - \frac{9}{2}$$

$$a_{23}\left(j\sqrt{\frac{5}{2}}\right)=\frac{-7}{2}$$

$$a_{13}^2+a_{23}^2+a_{23}^2=7$$

$$a_{23}^2=7-\frac{9}{2}+\frac{49}{10}$$

$$a_{33}=\pm\sqrt{9.4}$$

From this the lower triangular matrix can be written as,

$$\text{Lower triangular matrix}=\begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$a_{11}=\pm\sqrt{2}, \qquad a_{12}=a_{13}=\pm\frac{3}{\sqrt{2}}$$

$$a_{22}=+j\frac{\sqrt{5}}{\sqrt{2}}, \qquad a_{23}=\frac{j7}{\sqrt{10}}$$

$$a_{33}=\pm\sqrt{9.4}$$

In the above lower triangular matrix elements $[a_{11}, a_{13}, a_{23}, a_{33}, 0, 0, 0]$ are real elements

So, total number of real elements = 7

IIT has declared this questions as MTA.

As the correct answer for the question, i.e. total number of real elements equal to 7, was absent in the given options, so after facing challenge to their given option in first answer key, finally IIT has declared this question as **Marks to All.**

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