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GATE - 2022

Electronics & Comm. Engg.

This sample PDF of **GATE Previous Years Solution Book** contains randomly selected questions with solutions from some of the chapters of every subject along with part of concept refresher **Synopsis** of those chapters to let the aspirants have an idea about the content, style and appearance of the book.

Previous Years marks distribution analysis is also given in tabular form with index page of every subject, which contains analysis of GATE papers from 2003 onwards as GATE pattern has turned objective since 2003.

Volume 1 of Electronics & Communication Engineering GATE Previous Years Solution Book contains the common subjects of EC, EE and IN and hence it is equally advantageous for GATE aspirants of all these three branches.

GATE ACADEMY PUBLICATIONS®

GATE 2022

Electronics & Communication Engineering

(Volume - I)

TOPIC WISE GATE SOLUTIONS 1987-2021

Umesh Dhande

GATE ACADEMY®
steps to success...

I am thankful to

My wife ***Sakshi Dhande****, Son* ***Advait,***

who are my source of encouragement and inspiration

and

my dearest father ***Shri Hariram Dhande,***

mother ***Indu Dhande,***

who are responsible for what I am today.

Last but not the least special thanks to

my brother ***Dr. Rakesh Dhande***

and sisters ***Dr. Preeti Fendar, Punam Dhande***

who always motivate me to give my best.

To My Son Advait

IMPORTANCE of GATE

GATE examination has been emerging as one of the most prestigious competitive exam for engineers. Earlier it was considered to be an exam just for eligibility for pursuing PG courses, but now GATE exam has gained a lot of attention of students as this exam open an ocean of possibilities like :

1. **Admission into IISc, IITs, IIITs, NITs**

   A good GATE score is helpful for getting admission into IISc, IITs, IIITs, NITs and many other renowned institutions for M.Tech./M.E./M.S. An M.Tech graduate has a number of career opportunities in research fields and education industries. Students get ₹ 12,400 per month as stipend during their course.

2. **Selection in various Public Sector Undertakings** (**PSUs**)

   A good GATE score is helpful for getting job in government-owned corporations termed as **Public Sector Undertakings** (**PSUs**) in India like IOCL, BHEL, NTPC, BARC, ONGC, PGCIL, DVC, HPCL, GAIL, SAIL & many more.

3. Direct recruitment to Group A level posts in Central government, i.e., Senior Field Officer (Tele), Senior Research Officer (Crypto) and Senior Research Officer (S&T) in Cabinet Secretariat, Government of India, is now being carried out on the basis of GATE score.

4. **Foreign universities through GATE**

   GATE has crossed the boundaries to become an international level test for entry into postgraduate engineering programmes in abroad. Some institutes in two countries **Singapore** and **Germany** are known to accept GATE score for admission to their PG engineering programmes.

5. **National Institute of Industrial Engg. (NITIE)**

   - NITIE offers **PGDIE / PGDMM / PGDPM** on the basis of GATE scores. The shortlisted candidates are then called for group Discussion and Personal Interview rounds.
   - NITIE offers a Doctoral Level Fellowship Programme recognized by Ministry of HRD (MHRD) as equivalent to PhD of any Indian University.
   - Regular full time candidates those who will qualify for the financial assistance will receive ₹ 25,000 during 1st and 2nd year of the Fellowship programme and ₹ 28,000 during 3rd, 4th and 5th year of the Fellowship programme as per MHRD guidelines.

6. **Ph.D. in IISc/ IITs**

   - IISc and IITs take admissions for Ph.D. on the basis of GATE score.
   - Earn a Ph.D. degree directly after Bachelor’s degree through integrated programme.
   - A fulltime residential researcher (RR) programme.

7. **Fellowship Program in management (FPM)**

   - Enrolment through GATE score card
   - Stipend of ₹ 22,000 – 30,000 per month + HRA
   - It is a fellowship program
   - Application form is generally available in month of sept. and oct.

Note : In near future, hopefully GATE exam will become a mandatory exit test for all engineering students, so take this exam seriously. Best of LUCK !

GATE Exam Pattern

Section	Question No.	No. of Questions	Marks Per Question	Total Marks
General Aptitude	1 to 5	5	1	5
	6 to 10	5	2	10
Technical + Engineering Mathematics	1 to 25	25	1	25
	26 to 55	30	2	60
Total Duration : 3 hours		**Total Questions : 65**		**Total Marks : 100**
Note : 40 to 45 marks will be allotted to Numerical Answer Type Questions				

Pattern of Questions :

(i) **Multiple Choice Questions (MCQ)** carrying 1 or 2 marks each in all the papers and sections. These questions are objective in nature, and each will have a choice of four answers, out of which the candidate has to select (mark) the correct answer.

Negative Marking for Wrong Answers : For a wrong answer chosen in a MCQ, there will be negative marking. For **1-mark** MCQ, **1/3 mark** will be deducted for a wrong answer. Likewise for, **2-marks** MCQ, **2/3 mark** will be deducted for a wrong answer.

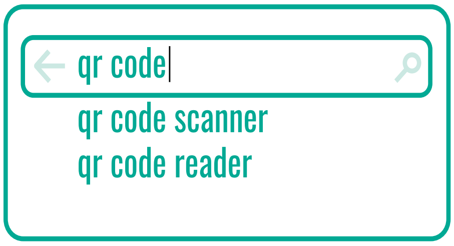
(ii) **Numerical Answer Type (NAT)** Questions carrying **1** or **2** marks each in all the papers and sections. For these questions, the answer is a signed real number, which needs to be entered by the candidate using the virtual numeric keypad on the monitor (Keyboard of the computer will be disabled). No choices will be shown for these type of questions. The answer can be a number such as **10** or **– 10** (an integer only). The answer may be in decimals as well, for example, **10.1** (one decimal) or **10.01** (two decimal) or **–10.001** (three decimal). These questions will be mentioned with, up to which decimal places, the candidates need to make an answer. Also, an appropriate range will be considered while evaluating the numerical answer type questions so that the candidate is not penalized due to the usual round-off errors. Wherever required and possible, it is better to give NAT answer up to a maximum of three decimal places.

Note : There is NO negative marking for a wrong answer in NAT questions.

What is special about this book ?

GATE ACADEMY Team took several years' to come up with the solutions of GATE examination. It is because we strongly believe in quality. We have significantly prepared each and every solution of the questions appeared in GATE, and many individuals from the community have taken time out to proof read and improve the quality of solutions, so that it becomes very lucid for the readers. Some of the key features of this book are as under :

- This book gives complete analysis of questions chapter wise as well as year wise.
- Video Solution of important conceptual questions has been given in the form of QR code and by scanning QR code one can see the video solution of the given question.
- Solutions has been presented in lucid and understandable language for an average student.
- In addition to the GATE syllabus, the book includes the nomenclature of chapters according to text books for easy reference.
- Last but not the least, author's 10 years experience and devotion in preparation of these solutions.
- Steps to Open Video solution through mobile.



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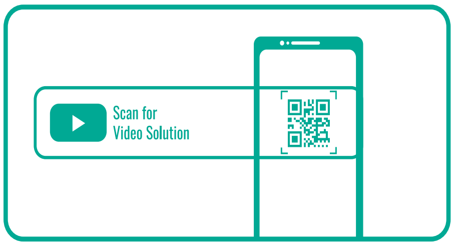
(2) Download & Install any QR Code Scanner App.



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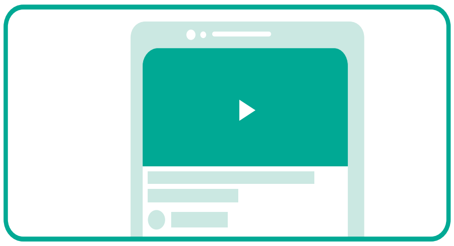

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ACKNOWLEDGEMENT

12 Years of recurring effort went into the volume which is now ready to cover the aptitude of GATE aspirants.

We are glad of this opportunity to acknowledge the views and to express with all the weaknesses of mere words the gratitude that we must always feel for the generosity of them.

We now express our gracious gratitude to the persons who have contributed a lot in order to put forth this into device. They are to be mentioned here and they are Anshuman Shukla, Ankita Gupta, Akansha Singh, Durshree Sharma, Gaurav Thakur, Gurpreet Kaur, K.L. Srinivas, Krishana Kumar, Koushalya Chandrawanshi, Mukesh Kumar Sahu, Neeraj Jagwani, Neelima Patel, Pritesh Patel, Pratima Patel, Reena Sahu, Rishabh Jain, Ravi Yadav, Saurabh Sinha, Shrikant Soni, Shubham Dabir, T. Pushpalata, Vikas Athe, Vivek Kumar, Vaibhav Bohra, and Vaishali Rathi.

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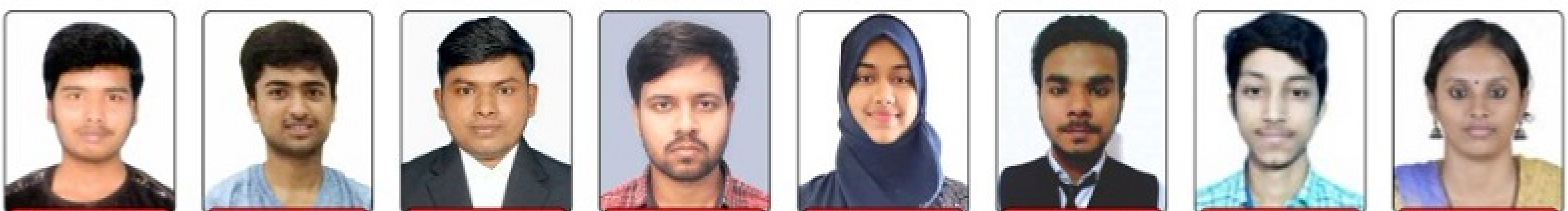

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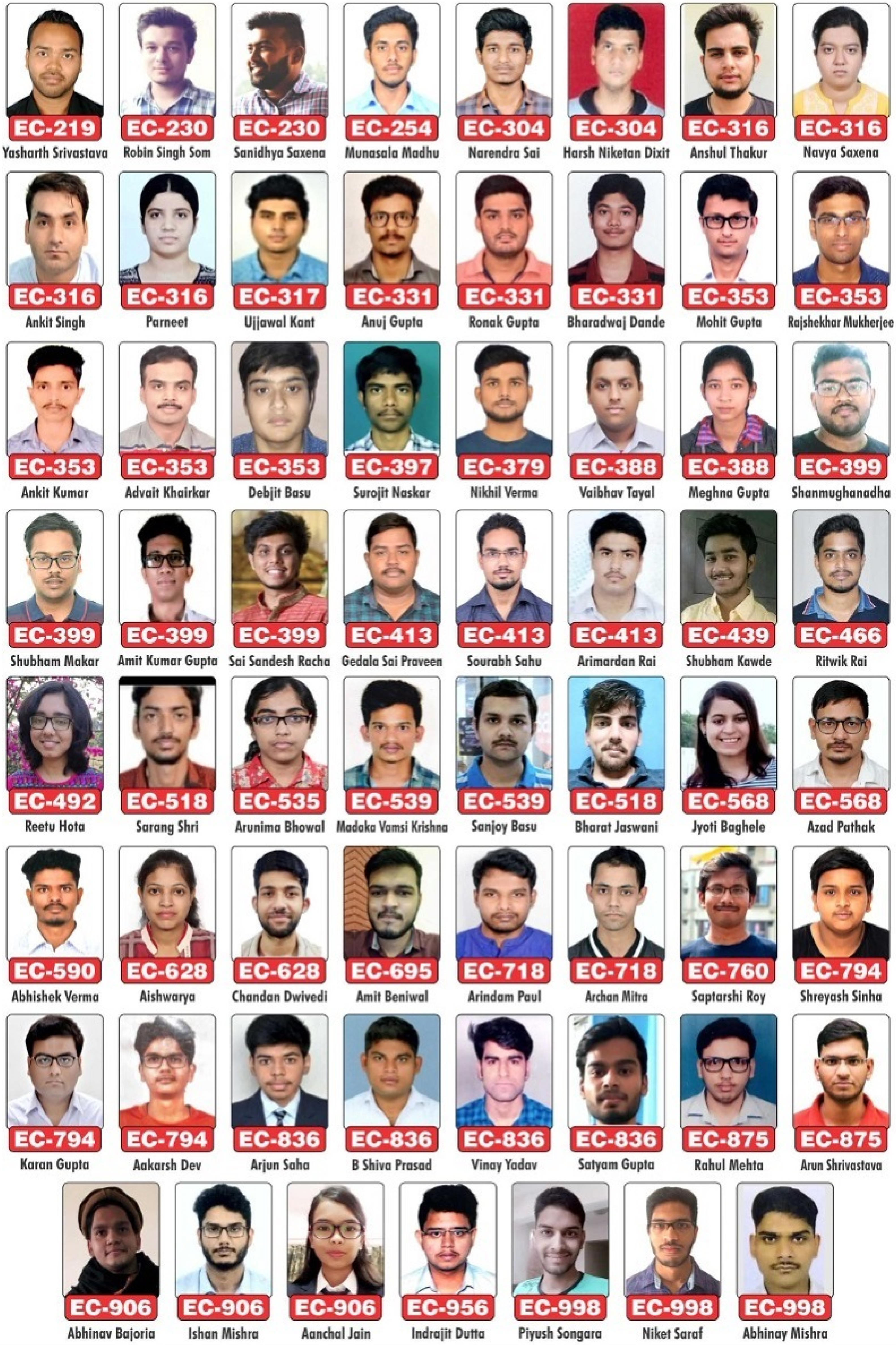

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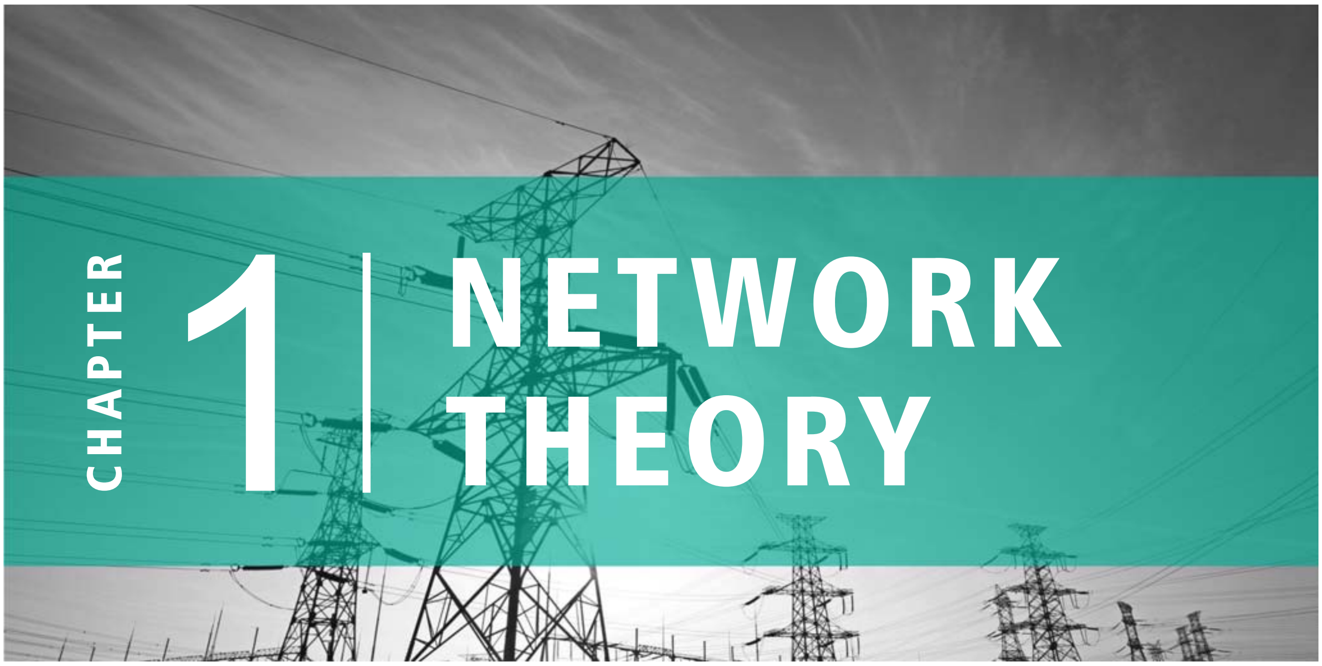


Marks Distribution of Network Theory in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	4	8	20
2004	5	5	15
2005	5	6	17
2006	6	–	6
2007	2	4	10
2008	2	7	16
2009	3	4	11
2010	2	4	10
2011	3	3	9
2012	4	4	12
2013	3	6	15
2014 Set-1	2	4	10
2014 Set-2	2	2	6
2014 Set-3	2	4	10

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-4	2	4	10
2015 Set-1	4	3	10
2015 Set-2	3	3	9
2015 Set-3	3	2	7
2016 Set-1	1	2	5
2016 Set-2	4	3	10
2016 Set-3	1	3	7
2017 Set-1	1	2	5
2017 Set-2	2	2	6
2018	1	3	7
2019	1	2	5
2020	3	2	7
2021	2	4	10

Syllabus : Network Theory

Circuit analysis: Node and mesh analysis, superposition, Thevenin's theorem, Norton's theorem, reciprocity. Sinusoidal steady state analysis: phasors, complex power, maximum power transfer. Time and frequency domain analysis of linear circuits: RL, RC and RLC circuits, solution of network equations using Laplace transform. Linear 2-port network parameters, wye-delta transformation.

Contents : Network Theory

1 Basic Concept of Networks

Partial Synopsis

Voltmeter

Voltmeter is a device which measure open circuited voltage between two points in an electrical and electronics circuit. It is always connected in shunt with the part of the circuit being measured. Ideally, internal resistance of voltmeter should be infinite and practically it should be as high as possible.

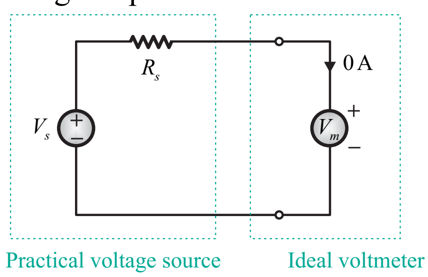


Fig. (a)

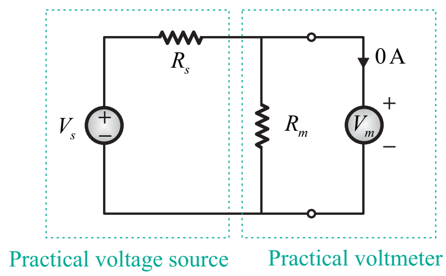


Fig. (b)

Reading of voltmeter in figure (a) is given by,

$$V_m = V_s$$

Reading of voltmeter in figure (b) is given by,

$$V_m = \frac{V_s \times R_m}{R_m + R_s}$$

$$V_m \approx V_s$$

Ammeter

Ammeter is a device which measures short circuit current in an electrical and electronics circuit. It is always connected in series with part of circuit, where line current is being measured. Ideally, internal resistance of ammeter should be zero and practically it should be as low as possible.

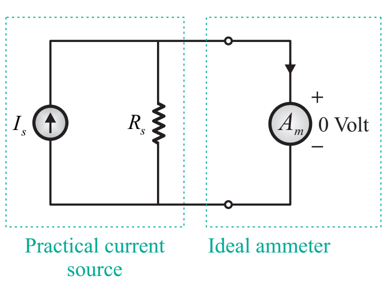


Fig. (a)

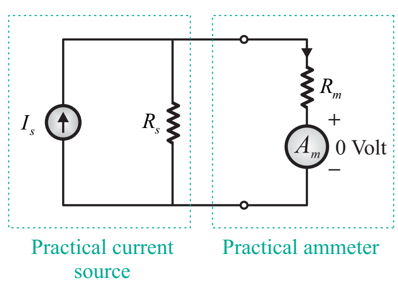


Fig. (b)

Reading of ammeter in figure (a) is given by,

$$A_m = I_s$$

Reading of ammeter in figure (b) is given by,

$$A_m = \frac{I_s \times R_s}{R_s + R_m}$$

$$A_m \approx I_s$$

Key Point

Source / device	Practically Internal Resistance	Ideally Internal Resistance
1. Voltage Source	R_s = small	$R_s = 0$ (S.C.)
2. Current Source	R_s = high	$R_s = \infty$ (O.C.)
3. Voltmeter	R_m = high	$R_m = \infty$ (O.C.)
4. Ammeter	R_m = small	$R_m = 0$ (S.C.)

Star to Delta conversion $[T$ network to π network$]$

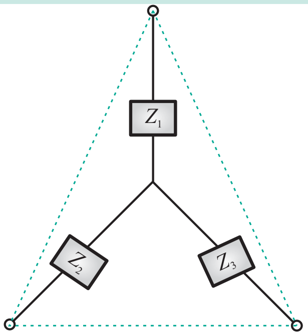


Fig. Star network

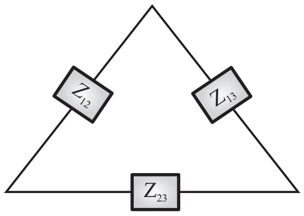


Fig. Delta network

- $Z_{12} = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_3}$ $\quad Z_{23} = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_1}$ $\quad Z_{13} = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_2}$
- If $Z_1 = Z_2 = Z_3 = Z$ then $Z_{12} = Z_{23} = Z_{13} = 3Z$
- In case of same impedance, star to delta conversion increases the impedance by a factor of 3 (Decreases by the same factor 3 if the element is capacitor).

$$\boxed{R_{eq} = 3R\ ,\ L_{eq} = 3L\ ,\ C_{eq} = C/3}$$

Delta to star conversion $[\pi$ network to T network$]$

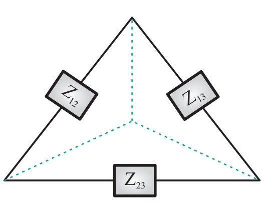


Fig. Delta network

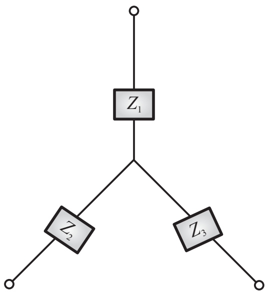


Fig. Star network

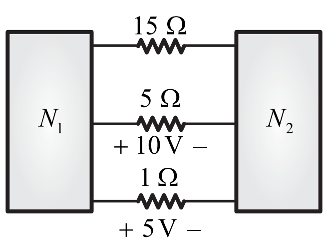
- $Z_1 = \dfrac{Z_{12}Z_{13}}{Z_{12}+Z_{23}+Z_{13}}$ $Z_2 = \dfrac{Z_{12}Z_{23}}{Z_{12}+Z_{23}+Z_{13}}$ $Z_3 = \dfrac{Z_{23}Z_{13}}{Z_{12}+Z_{23}+Z_{13}}$
- If $Z_{12} = Z_{23} = Z_{13} = Z$ then $Z_1 = Z_2 = Z_3 = \dfrac{Z}{3}$
- In case of same impedance, delta to star conversion decrease the impedance by a factor of 3 (Increases by the same factor 3 if the element is capacitor).

$$\boxed{R_{eq} = R/3,\; L_{eq} = L/3,\; C_{eq} = 3C}$$

➢ Sample Questions

1993	IIT Bombay

1.1 The two electrical sub networks N_1 and N_2 are connected through three resistors as shown in figure. The voltage across 5 ohm resistor and 1 ohm resistor are given to be 10 V and 5 V, respectively. Then voltage across 15 ohm resistor is

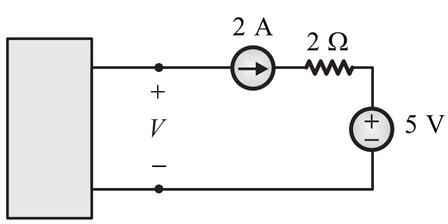


(A) – 105 V (B) + 105 V

(C) – 15 V (D) + 15 V

1997	IIT Madras

1.2 The voltage V in figure is always equal to

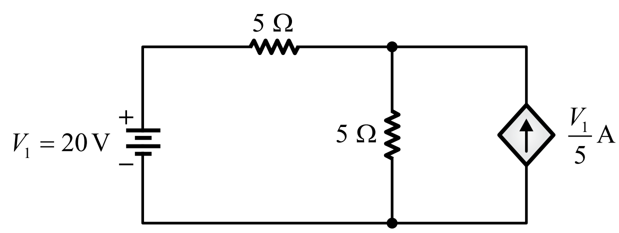


(A) 9 V (B) 5 V

(C) 1 V (D) None of these

2002	IISc Bangalore

1.3 The dependent current source shown in the figure

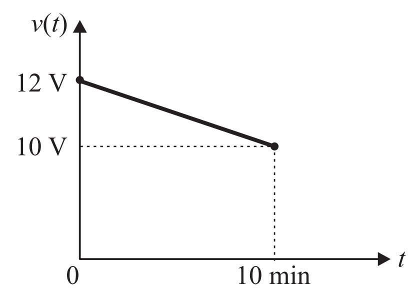


(A) delivers 80 W (B) absorbs 80 W

(C) delivers 40 W (D) absorbs 40 W

2009	IIT Roorkee

1.4 A fully charged mobile phone with a 12 V battery is good for a 10 minute talk-time. Assume that, during the talk-time, the battery delivers a constant current of 2 A and its voltage drop linearly varies from 12 V to 10 V as shown in the figure. How much energy does the battery deliver during this talk-time?

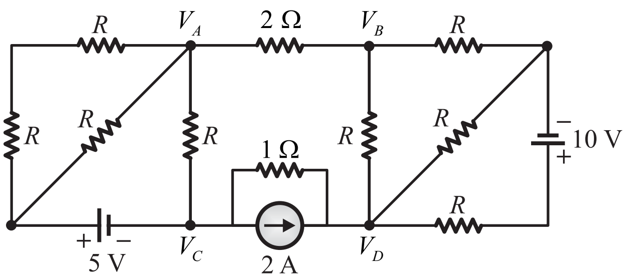


(A) 200 J (B) 12 kJ

(C) 13.2 kJ (D) 14.4 kJ

2012 IIT Delhi

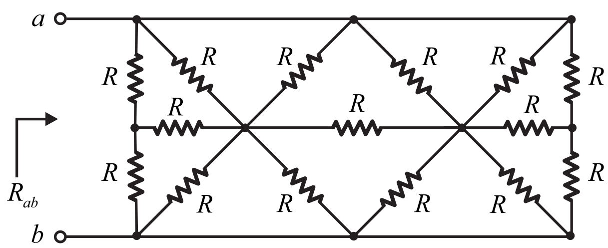
1.5 If $V_A - V_B = 6\text{ V}$, then $V_C - V_D$ is



(A) – 5 V (B) 2 V
(C) 3 V (D) 6 V

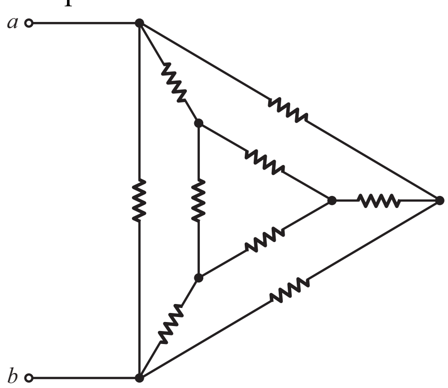
2015 IIT Kanpur

1.6 In the network shown in the figure, all resistors are identical with $R = 300\ \Omega$. The resistance R_{ab} (in Ω) of the network is __________. **[Set - 01]**



2016 IISc Bangalore

1.7 In the given circuit, each resistor has a value equal to $1\ \Omega$.

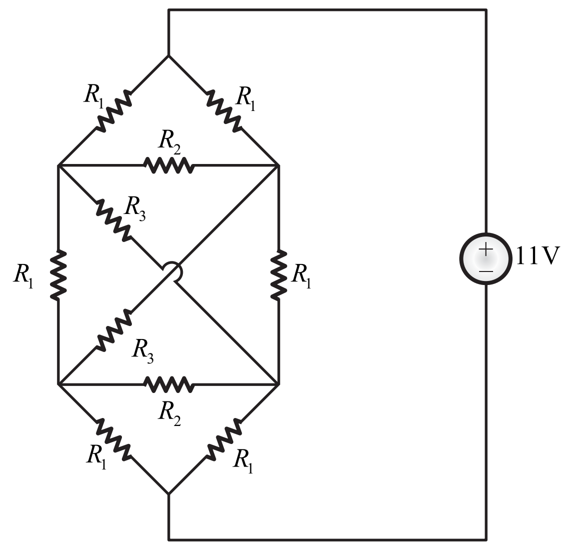


What is the equivalent resistance across the terminals a and b? **[Set - 02]**

(A) $1/6\ \Omega$ (B) $1/3\ \Omega$
(C) $9/20\ \Omega$ (D) $8/15\ \Omega$

2018 IIT Guwahati

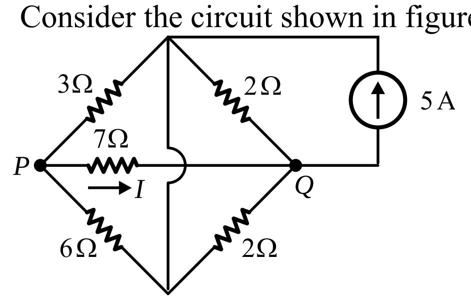
1.8 Consider the network shown below with $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 3\ \Omega$. The network is connected to a constant voltage source of 11V.



The magnitude of the current (in amperes, accurate to two decimal places) through the source is _______.

2021 IIT Bombay

1.9 Consider the circuit shown in figure.



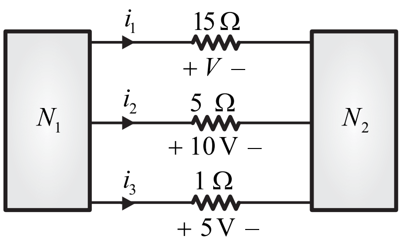
The current I flowing through the 7Ω resistor between P and Q (Round off to 1 decimal places) is _______.

❖❖❖❖

Explanations   Basic Concept of Networks

1.1 (A)

Given circuit is shown below,



From the figure,

$$i_2 = \frac{10}{5} = 2\text{ A},\quad i_3 = \frac{5}{1} = 5\text{ A}$$

Applying KCL at N_1,

$$i_1 + i_2 + i_3 = 0$$

$$i_1 = -(i_2 + i_3) = -7\text{ A}$$

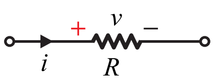
Voltage across 15 Ω resistor is,

$$V_{15\,\Omega} = i_1 \times 15 = 15 \times (-7) = -105\text{ V}$$

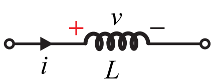
Hence, the correct option is (A).

Key Point

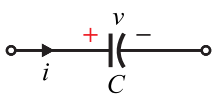
Passive sign convention :

1. 

$v = iR$

2. 

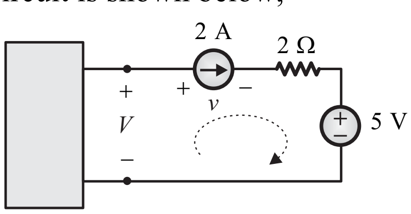
$v = L\dfrac{di}{dt}$

3. 

$i = C\dfrac{dv}{dt}$

1.2 (D)

Given circuit is shown below,



Applying KVL in the loop shown,

$$-V + v + 2 \times 2 + 5 = 0$$

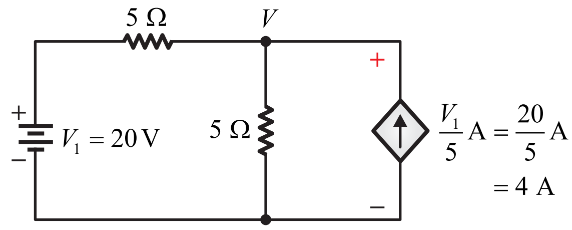
$$V = 9 + v$$

In above expression, voltage across the current source (v) is unknown. Hence, exact voltage cannot be determined.

Hence, the correct option is (D).

1.3 (A)

Given circuit is shown below,



Applying KCL at node V,

$$\frac{V-20}{5} + \frac{V}{5} - 4 = 0$$

$$\frac{2V}{5} = 8 \qquad \Rightarrow \qquad V = 20\text{ Volt}$$

Power across dependent current source is given by,

$$P_{\text{deliver}} = 20 \times 4 = 80\text{ W}$$

Dependent current source is delivering power of 80 W to the circuit.

Hence, the correct option is (A).

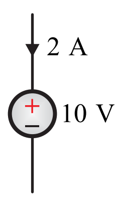
Key Point

Concept of absorbed and delivered power :

(i) Power is absorbed by the element if current enters into the positive terminal.

(ii) Power is delivered by the element if current leaves from the positive terminal.

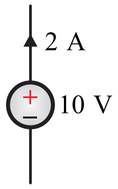
E.g. 1 :



$$P_{10\text{ V}} = P_{\text{absorb}} = 10 \times 2 = 20\text{ W}$$

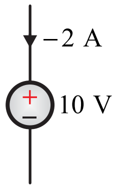
$$P_{10\text{ V}} = P_{\text{deliver}} = -20\text{ W}$$

E.g. 2 :



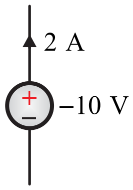
$$P_{10\text{ V}} = P_{\text{deliver}} = 10 \times 2 = 20 \text{ W}$$
$$P_{10\text{ V}} = P_{\text{absorb}} = -20 \text{ W}$$

E.g. 3 :



$$P_{10\text{ V}} = P_{\text{absorb}} = 10 \times (-2) = -20 \text{ W}$$
$$P_{10\text{ V}} = P_{\text{deliver}} = 20 \text{ W}$$

E.g. 4 :

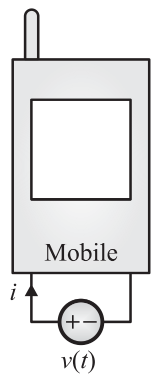


$$P_{-10\text{ V}} = P_{\text{deliver}} = -10 \times 2 = -20 \text{ W}$$
$$P_{-10\text{ V}} = P_{\text{absorb}} = 20 \text{ W}$$

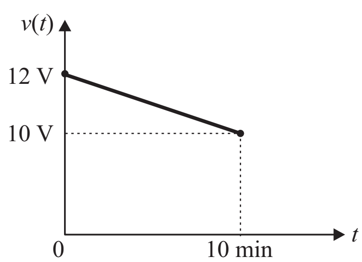
1.4 (C)

Given : Current supplied by the battery = 2 A.

Total time T = 10 min = 600 sec



Method 1



Equation of $v(t)$ is given by,

$$v(t) = \left[\frac{10-12}{10 \times 60}\right] t + 12 = \frac{-t}{300} + 12$$

where, t is in second

Power delivered is given by,

$$p(t) = v(t) \times i(t); \quad \text{where, } i(t) = 2 \text{ A}$$

$$p(t) = \left[\frac{-t}{300} + 12\right] 2$$

$$p(t) = \left(\frac{-t}{150} + 24\right) \text{ Watts}$$

Total energy delivered by the battery in 10 minutes i.e. 600 sec is given by,

$$E = \int_0^{600} p(t)\, dt$$

$$E = \int_0^{600} \left(\frac{-t}{150} + 24\right) dt$$

$$E = \frac{-1}{150}\left[\frac{t^2}{2}\right]_0^{600} + 24[t]_0^{600}$$

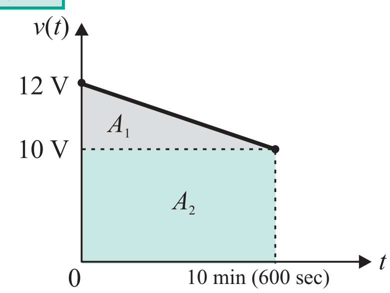
$$E = \frac{-1}{150 \times 2} \times 600 \times 600 + 24 \times 600$$

$$E = -1200 + 14400 = 13200 \text{ J}$$

$$E = 13.2 \text{ kJ}$$

Hence, the correct option is (C).

Method 2



The power delivered by the battery is given by,

$$p(t) = v(t) \times i(t) \quad \text{where, } i(t) = 2 \text{ A}$$

The energy delivered during time T (600 sec) is given by,

$$E = \int_0^T p(t)\, dt = \int_0^{600} v(t) \times i(t)\, dt$$

$$E = 2 \int_0^{600} v(t)\, dt$$

$$E = 2\,[\text{Area of } v(t) \text{ from 0 to 600 sec}]$$

$$E = 2[A_1 + A_2]$$

$$E = 2\left[\frac{1}{2}\times 2\times 600 + 10\times 600\right] = 13200\text{ J}$$

$$E = 13.2\text{ kJ}$$

Hence, the correct option is (C).

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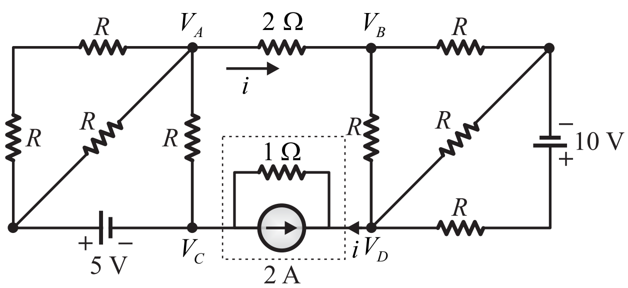
1.5 (A)

Given : $V_A - V_B = 6$ V

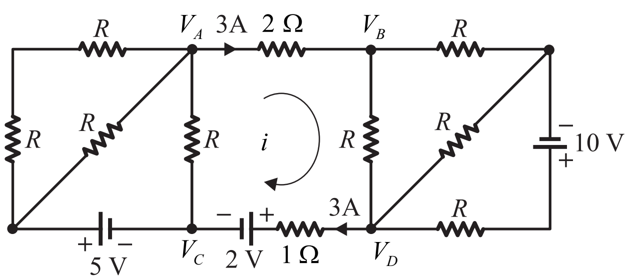
The circuit is shown below,



From figure, $i = \dfrac{V_A - V_B}{2} = \dfrac{6}{2}\text{ A} = 3\text{ A}$

The same current will flow through branch between node C and D.

Using source transformation :

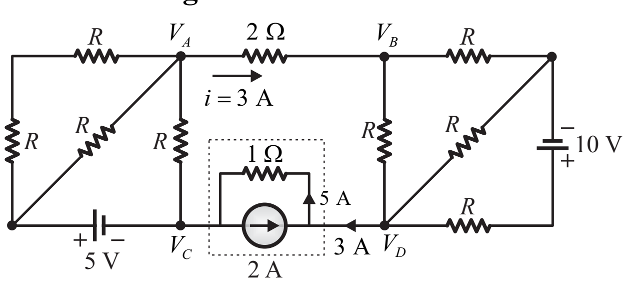


From above figure,

$$V_{DC} = 3\times 1 + 2 = 5\text{ V}$$

$$V_{CD} = V_C - V_D = -5\text{ V}$$

Without using source transformation :



$$V_{DC} = 5\times 1 = 5\text{ V}$$

$$V_{CD} = -5\text{ V}$$

Hence, the correct option is (A).

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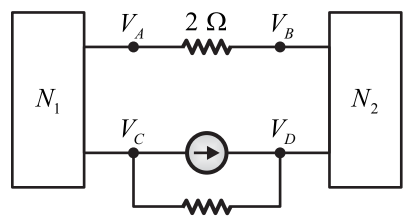
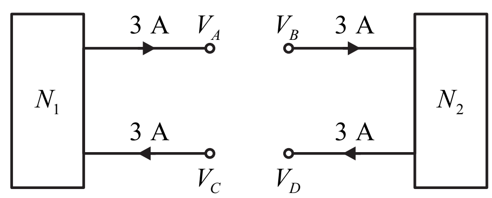
Key Point



For a particular one port network,

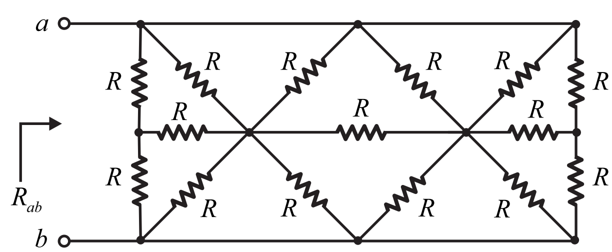
Incoming current = Outgoing current

Hence,



1.6 100

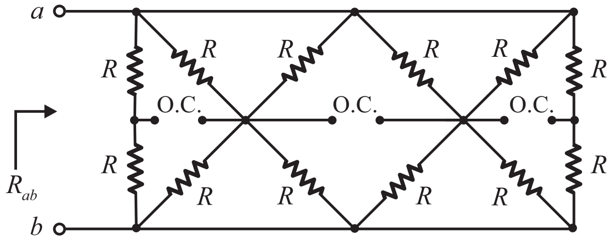
Given circuit is shown below,

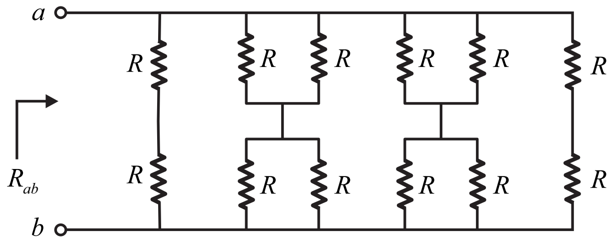


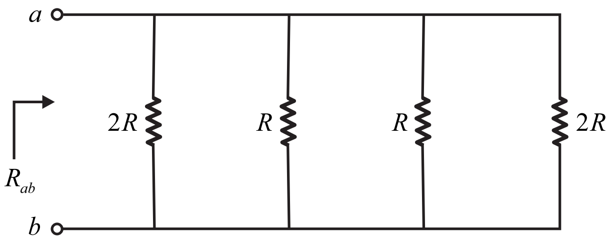
The given circuit has three balanced bridges. Hence, modify the given circuit as discussed below,

Method 1

Using concept of open circuit :







$$\frac{1}{R_{ab}} = \frac{1}{2R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{2R}$$

$$\frac{1}{R_{ab}} = \frac{3}{R}$$

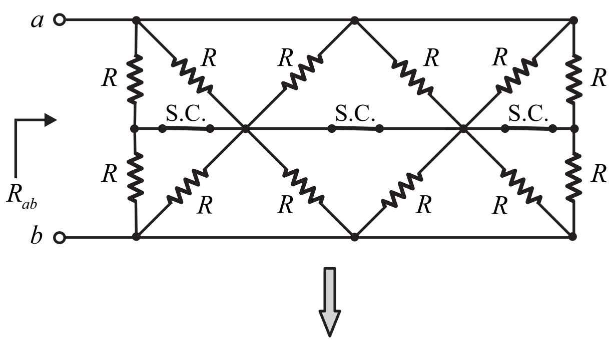
$$R_{ab} = \frac{R}{3} = \frac{300}{3} = 100\ \Omega$$

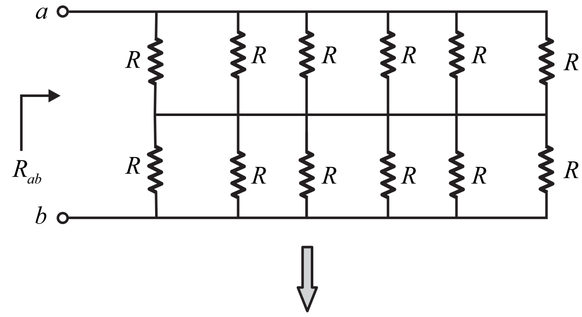
Hence, the resistance R_{ab} is **100 Ω.**

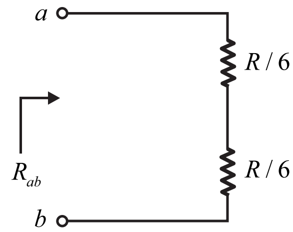
Method 2

Using concept of short circuit :

In case of resistor, the current through resistor is 0 A only when potential difference across the resistor is 0 V. i.e. resistor behaves as a short circuit.







$$R_{ab} = \frac{R}{6} + \frac{R}{6} = \frac{2R}{6}$$

$$R_{ab} = \frac{2 \times 300}{6} = 100\ \Omega$$

Hence, the resistance R_{ab} is **100 Ω.**

1.7 (D)

Given circuit is shown below,

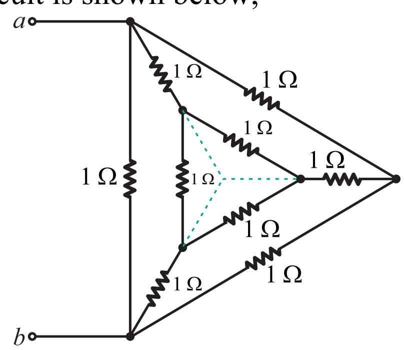


Fig. (a)

Applying Δ to Y conversion,

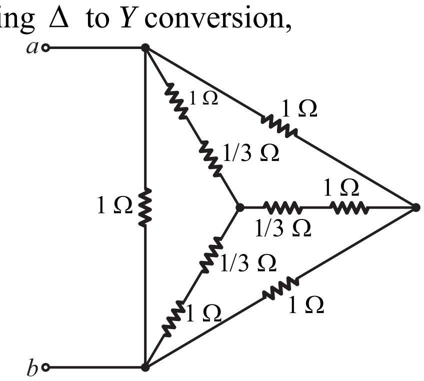


Fig. (b)

Modified figure is shown below,

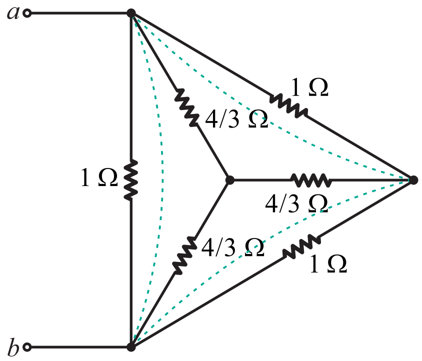


Fig. (c)

Applying Y to Δ conversion,

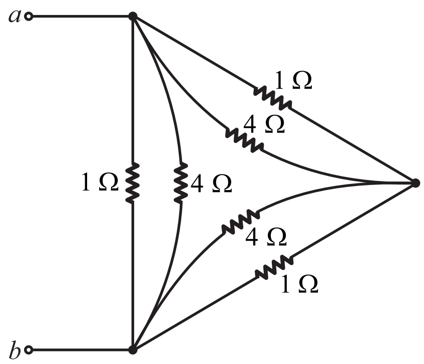


Fig. (d)

Modified figure is shown below,

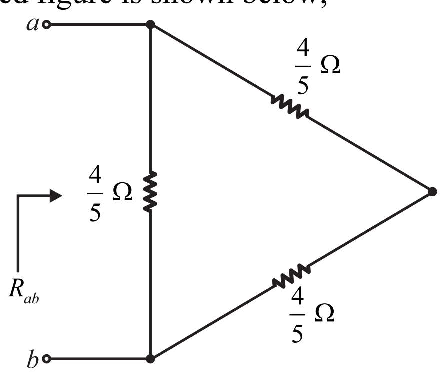


Fig. (e)

$$R_{ab} = \left(\frac{4}{5}+\frac{4}{5}\right) \bigg\| \frac{4}{5} = \frac{8}{15}\ \Omega$$

Hence, the correct option is (D).

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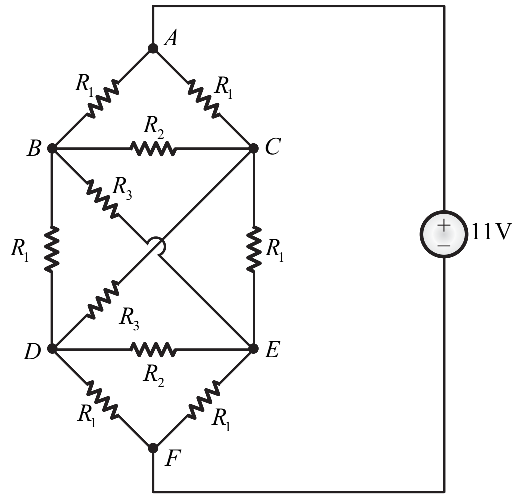
1.8	8

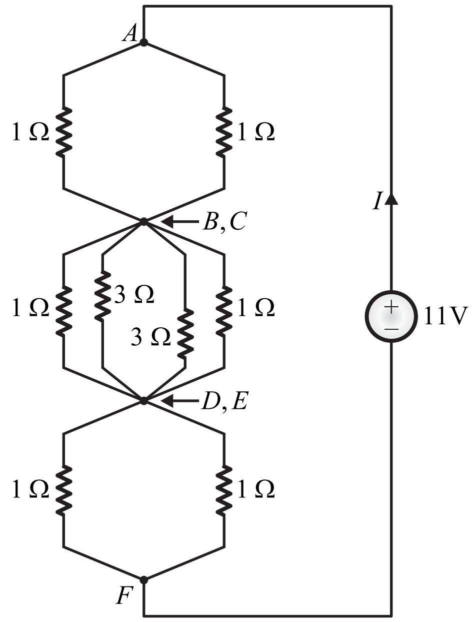
Given circuit is shown below,



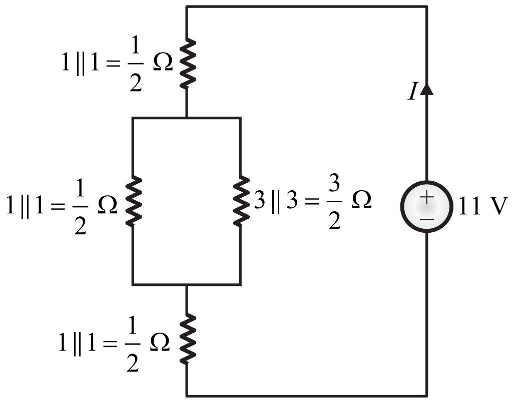
Given : $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 3\ \Omega$

From the symmetry of given circuit, we can conclude that point B and C, D and E are equipotential points since the resistive distance of both the points from terminals A as well as from F are equal.

We can remove the resistance connected between two equipotential points as current through it will always be zero and the simplified circuit is shown below,



Modified figure is shown below,



Current supplied by voltage source is given by,

$$I=\frac{V}{R_{eq}}$$

where, $R_{eq}=\frac{1}{2}+\frac{1}{2}+\left(\frac{1}{2}\|\frac{3}{2}\right)$

$$R_{eq}=1+\frac{\frac{1}{2}\times\frac{3}{2}}{\frac{1}{2}+\frac{3}{2}}=1+\frac{3/4}{2}$$

$$R_{eq}=\frac{11}{8}\Omega$$

Therefore, $I=\frac{V}{R_{eq}}=\frac{11}{11/8}=8\text{ A}$

Hence, the magnitude of the current through the source is **8 A.**

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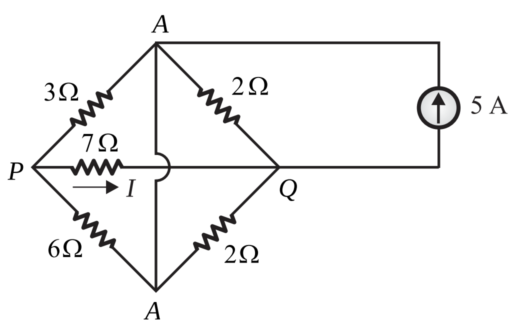
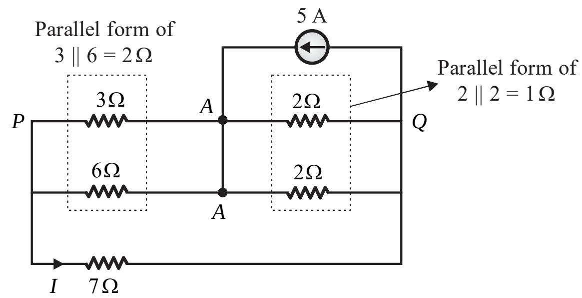
1.9 **0.5**

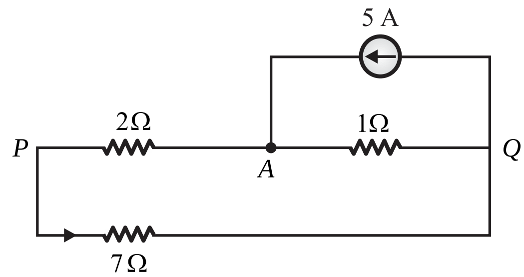
Given circuit is shown below,



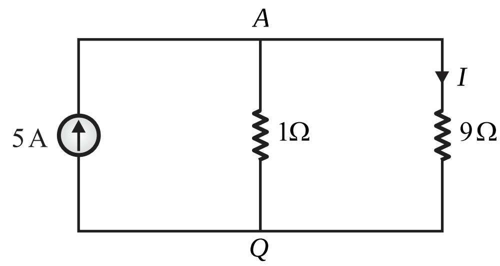
Re-arrange the above circuit as shown below,



Now above circuit reduces as,



Above circuit can be re-arranged as,



Apply current divider rule

$$I=\frac{5\times1}{1+9}=\frac{5}{10}=0.5\text{ A}$$

Hence, the correct answer is 0.5 A.

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2 Network Theorems

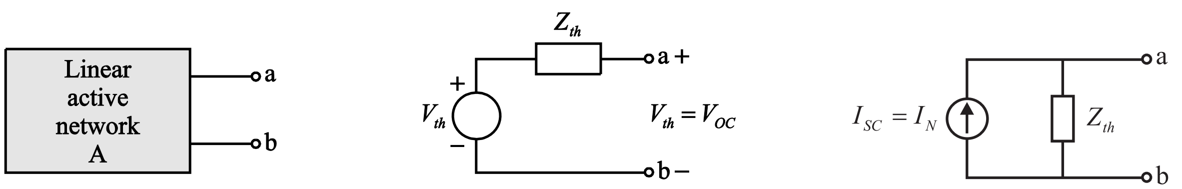
Partial Synopsis

Thevenin's and Norton's Theorem

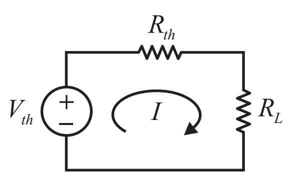
Statement of Thevenin's theorem: A linear network consisting of a number of voltage/current sources and resistances can be replaced by an equivalent network having a single voltage source called Thevenin's voltage (V_{th}) and a single resistance called Thevenin's resistance (R_{th}).

Statement of Norton's theorem : A linear network consisting of a number of voltage/current sources and resistances can be replaced by an equivalent network having a current source called Norton's current (I_N) and a single resistance called Norton's resistance (R_N).

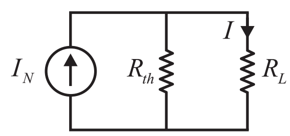
Thevenin's and Norton's equivalent circuits of complex network are shown below,



(a) General network (b) Its Thevenin's equivalent (c) Its Norton's equivalent



$$I = \frac{V_{th}}{R_{th} + R_L}$$



$$I = \frac{R_{th} I_N}{R_{th} + R_L}$$

Steps to find equivalent Thevenin's/Norton's resistance :

Remember :

Case 1 : Circuit containing only independent sources

Voltage sources are replaced by short circuits and current sources are replaced by open circuits. Calculate equivalent resistance seen from open circuited load terminals,

$$R_{th} = R_{eq}$$

Case 2 : Circuit containing independent as well as dependent sources

Replace all independent voltage sources by short circuits and current sources by open circuits but keep dependent sources, then

$$R_{th} = \frac{V_{dc}}{I_{dc}}$$ $$\left(\text{Also, } R_{th} = \frac{V_{OC}}{I_{SC}} = \frac{V_{th}}{I_N}\right)$$

Where $V_{dc} = dc$ voltage source applied across load terminals

And, $I_{dc} = dc$ current supplied by V_{dc}

Case 3 : Circuit containing only dependent sources

Keep dependent sources

$$R_{th} = \frac{V_{dc}}{I_{dc}}$$

Where $V_{dc} = dc$ voltage source applied across load terminals

And, $I_{dc} = dc$ current supplied by V_{dc}

In this case, Thevenin's equivalent voltage, $V_{th} = 0$ and $I_N = 0$ since, there is no independent source.

Sample Questions

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2.1 If an impedance Z_L is connected across a voltage source V with source impedance Z_S then for maximum power transfer, the load impedance must be equal to

(A) source impedance Z_S.

(B) complex conjugate of Z_S.

(C) real part of Z_S.

(D) imaginary part of Z_S.

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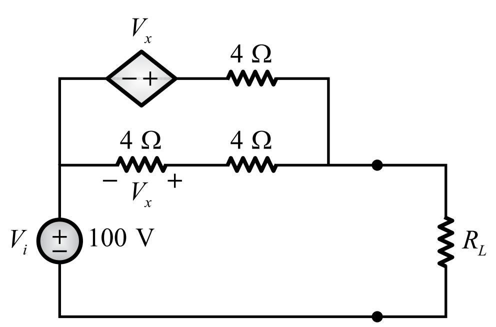
2.2 In the circuit shown, what value of R_L maximizes the power delivered to R_L?



(A) 2.4 Ω (B) 3.66 Ω

(C) 4 Ω (D) 6 Ω

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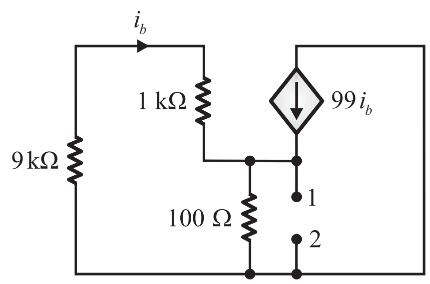
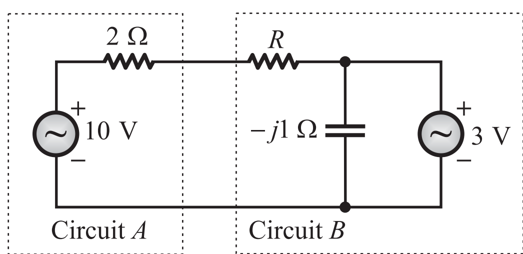
2.3 The impedance looking into nodes 1 and 2 in the given circuit is



(A) 50 Ω (B) 100 Ω

(C) 5 kΩ (D) 10.1 kΩ

2.4 Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is



(A) 0.8 Ω (B) 1.4 Ω

(C) 2 Ω (D) 2.8 Ω

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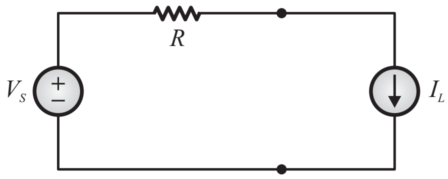
2.5 In the circuit shown below, V_S is a constant voltage source and I_L is a constant load current. **[Set - 02]**



The value of I_L that maximizes the power absorbed by the constant current load is

(A) $\frac{V_S}{4R}$ (B) $\frac{V_S}{2R}$

(C) $\frac{V_S}{R}$ (D) ∞

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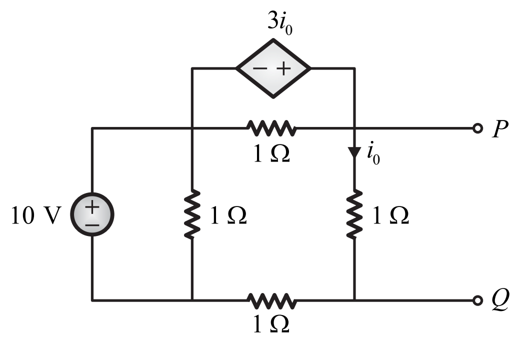
2.6 Consider the circuit shown in the figure.



The Thevenin equivalent resistance (in Ω) across PQ is _______. **[Set - 02]**

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Explanations — Network Theorems

2.1 (B)

According to maximum power transfer theorem, load impedance should be equal to complex conjugate of source impedance.

$$R_S = R_L \text{ and } X_L = -X_S$$

[when both R_L and X_L are varying]

$$Z_L = R_L + jX_L$$

$$Z_S^* = R_S - jX_S$$

i.e. $Z_L = Z_S^*$

Hence, the correct option is (B).

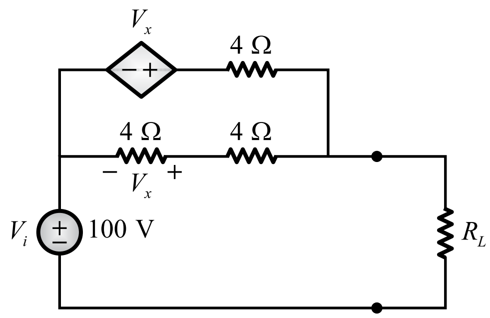
Table 2.1 : Maximum Power Transfer Theorem

S. No.	Z_S (Source Impedance)	Z_L (Load Impedance)	Condition for MPT	P_L(max) [Maximum power delivered to load]	% η (Efficiency)
1.	$R_S + j0$	$R_L + j0$	$R_L = R_S$	$\lvert I^2\rvert R_L = \frac{V_s^2}{4R_s}$	50%
2.	$R_S + jX_S$	$R_L + jX_L$	$R_L = \sqrt{R_S^2 + (X_S + X_L)^2}$	$\lvert I^2\rvert R_L$	< 50%
3.	$R_S + jX_S$	$R_L + jX_L$	$X_L + X_S = 0$	$\lvert I^2\rvert R_L = \frac{V_s^2 R_L}{(R_s + R_L)^2}$	< 50%
4.	$R_S + jX_S$	$R_L + jX_L$	$Z_L = Z_S^*$	$\lvert I^2\rvert R_L = \frac{V_s^2}{4R_s}$	50%

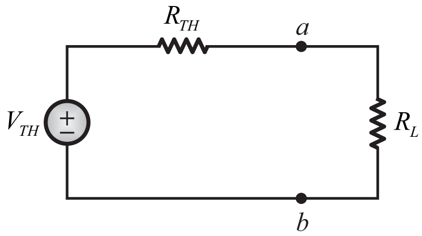
5.	$R_S + jX_S$	$R_L + j0$	$R_L = \sqrt{R_S^2 + X_S^2}$	$\lvert I^2 \rvert R_L$	$< 50\%$
6.	$R_S + j0$	$R_L + jX_L$	$R_L = \sqrt{R_S^2 + X_L^2}$	$\lvert I^2 \rvert R_L$	$< 50\%$

2.2 (C)

Given circuit is shown below,



Thevenin's equivalent circuit :



In case of resistive network, maximum power will be transferred to R_L, when

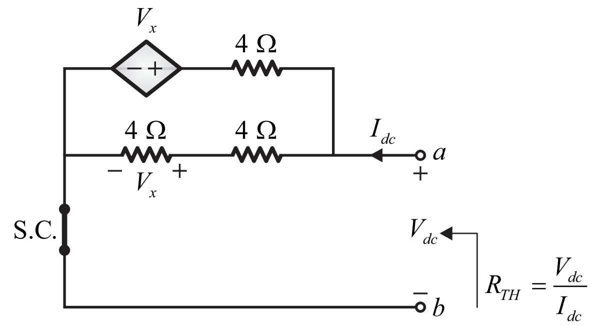
$$R_L = R_{TH}$$

Calculation of R_{TH} :

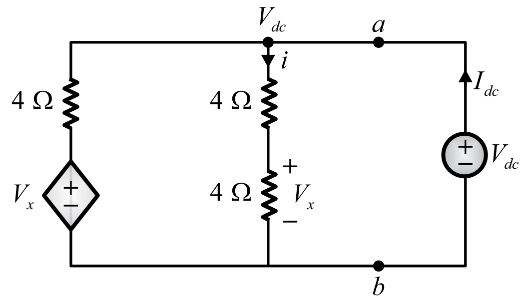
(when dependent sources are present)

(i) Apply a voltage source 'V_{dc}' at terminal ab and assume 'I_{dc}' current flowing to the network.

(ii) Replace all independent source by their internal resistance, i.e. short circuit all the independent voltage source $(R_{in} = 0)$ and open circuit all the independent current source $(R_{in} = \infty)$.



Modified circuit is shown in below figure,



Applying KCL at node a,

$$\frac{V_{dc} - V_x}{4} + \frac{V_{dc}}{8} - I_{dc} = 0$$

$$2(V_{dc} - V_x) + V_{dc} = 8\ I_{dc}$$

$$3V_{dc} - 2V_x = 8\ I_{dc} \qquad \ldots\text{(i)}$$

Current, $i = \dfrac{V_{dc}}{8} = \dfrac{V_x}{4}$ [From figure]

and $V_x = 4i = \dfrac{V_{dc}}{2}$ ...(ii)

From equation (i) and (ii),

$$3V_{dc} - V_{dc} = 8\,I_{dc}$$

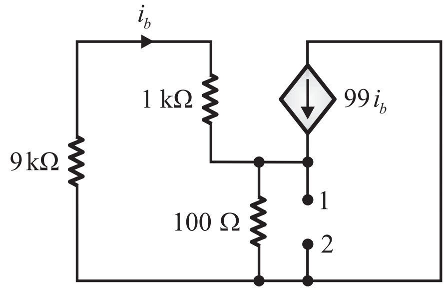
$$\frac{V_{dc}}{I_{dc}} = 4\ \Omega$$

So, $R_{TH} = 4\ \Omega$

Hence, the correct option is (C).

2.3 (A)

Given circuit is shown below,

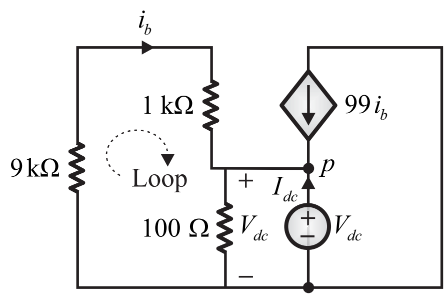


Calculation of R_{TH} :

(when dependent sources are present)

(i) Apply a voltage source 'V_{dc}' between terminal '1' and '2' and assume 'I_{dc}' current flowing to the network.

(ii) Replace all independent source by their internal resistance, i.e. short circuit all the independent voltage source $(R_{in} = 0)$ and open circuit all the independent current source $(R_{in} = \infty)$.



The Thevenin equivalent impedance is given by,

$$Z_{TH} = \frac{V_{dc}}{I_{dc}}$$

Applying KCL at node p,

$$-i_b + \frac{V_{dc}}{100} - 99i_b - I_{dc} = 0$$

$$\frac{V_{dc}}{100} - 100i_b = I_{dc} \quad \ldots\text{(i)}$$

Applying KVL in the above shown loop,

$$10\times10^3 i_b + V_{dc} = 0$$

$$i_b = \frac{-V_{dc}}{10\times10^3} \quad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$\frac{V_{dc}}{100} - 100\left[\frac{-V_{dc}}{10^4}\right] = I_{dc}$$

$$\frac{2V_{dc}}{100} = I_{dc}$$

$$\frac{V_{dc}}{I_{dc}} = 50\ \Omega$$

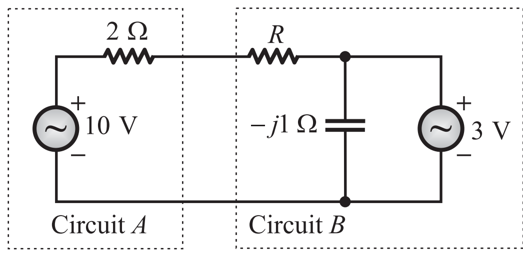
$$Z_{TH} = 50\ \Omega$$

Hence, the correct option is (A).

Scan for
Video Solution

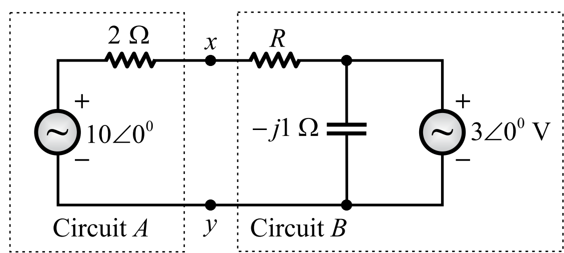
2.4 (A)

Given circuit is shown below,

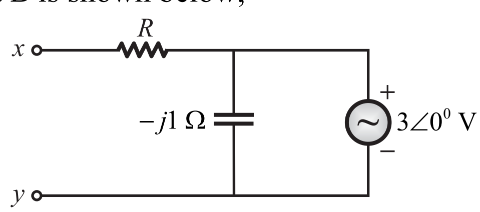


Method 1

Both the voltage sources are in same phase, let $10\angle0^0$ and $3\angle0^0$ V.



Circuit B is shown below,

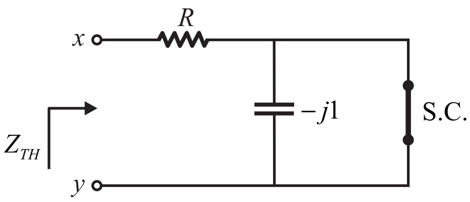


The Thevenin's or open circuit voltage across xy is given by,

$$V_{TH} = 3\angle0^0\ \text{V}$$

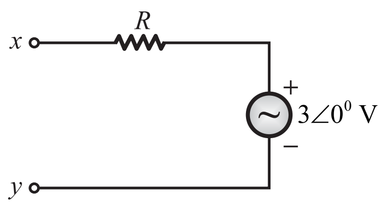
Calculation of Z_{TH} :

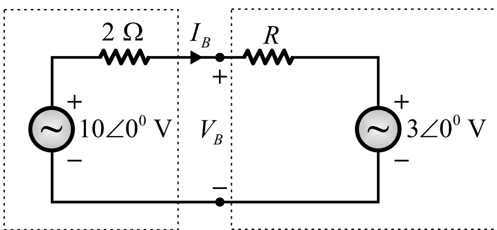
(when dependent sources are not present)
Replace all independent source by their internal resistance, i.e. short circuit all the independent voltage source $(R_{in} = 0)$ and open circuit all the independent current source $(R_{in} = \infty)$.



$Z_{TH} = R$

So, Thevenin's equivalent of circuit B is shown below,





Circuit A Circuit B

Current, $I_B = \dfrac{10-3}{R+2} = \dfrac{7}{(R+2)}\angle 0^0$

The voltage across circuit B is

$$V_B = I_B R + 3\angle 0^0 = \left(\frac{7R}{R+2} + 3\right)\angle 0^0$$

Power delivered to circuit B by circuit A,

$$P_B = V_B I_B \cos 0^0 = \frac{7}{(R+2)}\left[\frac{7R}{R+2} + 3\right]$$

$$P_B = \frac{7}{(R+2)}\left[\frac{10R+6}{R+2}\right] = 14\frac{(5R+3)}{(R+2)^2}$$

For maximum power $P_B(\max)$,

$$\frac{dP_B}{dR} = 0 \qquad \ldots(\text{i})$$

$$\frac{dP_B}{dR} = \frac{\left[14(R+2)^2 \times 5\right] - \left[14(5R+3)\times 2\times (R+2)\right]}{(R+2)^4}$$

From equation (i),

$$\frac{\left[14(R+2)^2 \times 5\right] - \left[14(5R+3)\times 2\times (R+2)\right]}{(R+2)^4} = 0$$

$$14(R+2)^2 \times 5 = 14(5R+3)\times 2\times (R+2)$$

$$70(R^2 + 4R + 4) = 28(5R^2 + 13R + 6)$$

$$70R^2 + 84R - 112 = 0$$

$$R = -2,\ 0.8\ \Omega$$

$R = -\text{ve}$ [not possible]

So, $R = 0.8\ \Omega$

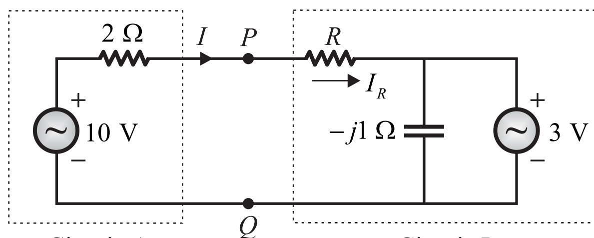
Hence, the correct option is (A).

Key Point

The impedance in parallel with voltage source and impedance in series with current source is **dummy impedance** i.e. does not affect the equivalent circuit.

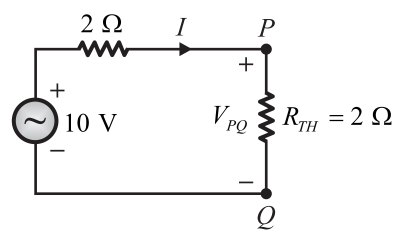
Method 2

Maximum power is transferred from circuit A to circuit B, if circuit B offers $2\ \Omega$ to circuit A.



Circuit A Circuit B

Thevenin equivalent of circuit A is given by,



Then, $I = \dfrac{10}{4} = 2.5\ \text{A}$

and $V_{PQ} = I \times 2 = 2.5 \times 2$

$V_{PQ} = 5\ \text{Volt}$

$$I_R = \frac{5-3}{R} = \frac{2}{R}$$

and $\quad I_R = I$ [From figure]

$$\frac{2}{R} = 2.5$$

$$R = \frac{2}{2.5} = 0.8\ \Omega$$

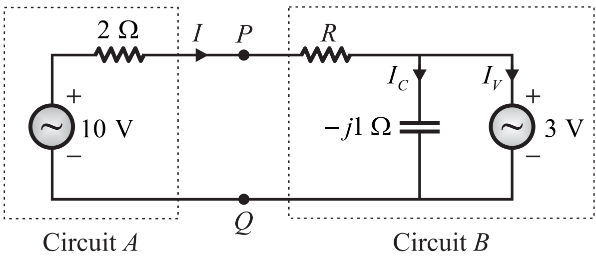
Hence, the correct option is (A).

Key Point

Circuit B is replaced by resistance of $2\ \Omega$ for maximum power transfer to circuit B by circuit A.

Method 3

Objective approach :



Circuit A　　Circuit B

Applying KVL in above figure,

$$I = \frac{10-3}{R+2} = \frac{7}{R+2}$$

Options	R (Ω)	I (A)
(A)	0.8	2.5
(B)	1.4	2.05
(C)	2	1.75
(D)	2.8	1.45

Circuit B will consume maximum power when maximum current will flow through the circuit and from the above table it is clear that maximum current is flowing when R is equal to $0.8\ \Omega$.

Hence, the correct option is (A).

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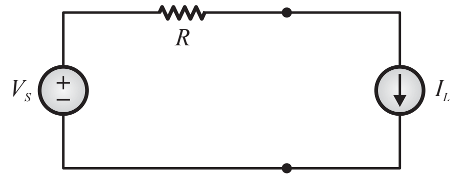
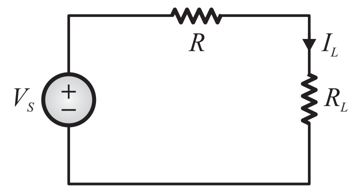
2.5 (B)

Given circuit is shown below,

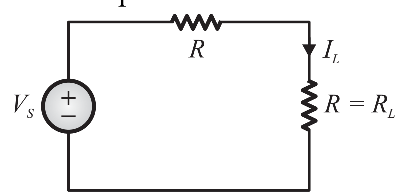


Method 1

We can replace constant current source I_L with equivalent resistance R_L and current flowing to the R_L would be equal to I_L.



For maximum power transfer to the constant current source I_L, resistance of constant current source must be equal to source resistance i.e. R.

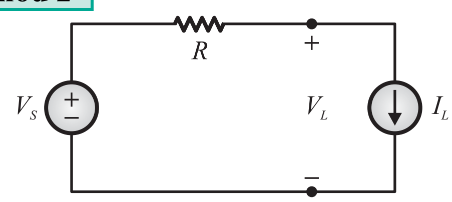


Applying KVL in above figure,

$$I_L = \frac{V_s}{2R}$$

Hence, the correct option is (B).

Method 2



Applying KVL in above figure,

$$V_L = V_s - I_L R$$

$$P_L = V_L I_L$$

where, P_L is power absorbed by constant current source I_L.

$$P_L = (V_s - I_L R) I_L$$

$$P_L = V_s I_L - I_L^2 R$$

For maximum power P_L,

$$\frac{dP_L}{dI_L} = 0$$

$$\frac{dP_L}{dI_L} = V_s - 2I_L R$$

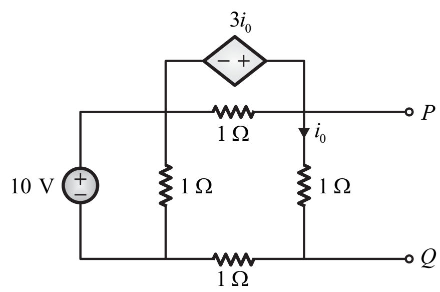
$$0 = V_s - 2I_L R$$

$$I_L = \frac{V_s}{2R}$$

Hence, the correct option is (B).

2.6 | – 1

Given circuit is shown below,

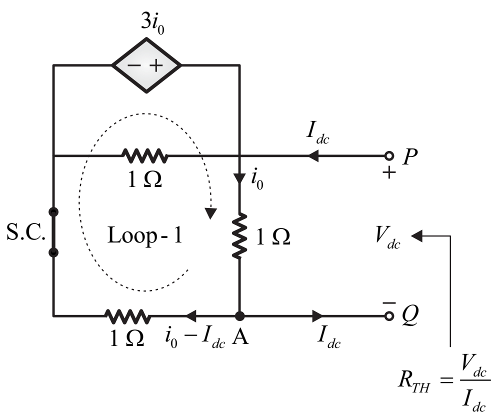


Calculation of R_{TH} :

(when dependent sources are present)

(i) Apply a voltage source 'V_{dc}' at terminal of R_L, and assume 'I_{dc}' current is flowing to the network.

(ii) Replace all independent source by their internal resistance, i.e. short circuit all the independent voltage source $(R_{in} = 0)$ and open circuit all the independent current sources $(R_{in} = \infty)$.



$$R_{TH} = \frac{V_{dc}}{I_{dc}} = \frac{i_0 \times 1}{I_{dc}} = \frac{i_0}{I_{dc}} \quad \ldots(i)$$

Applying KVL in loop (1),

$$-3i_0 + i_0 \times 1 + (i_0 - I_{dc}) \times 1 = 0$$

$$I_{dc} = -i_0$$

$$\frac{i_0}{I_{dc}} = -1$$

Hence, from equation (i),

$$R_{TH} = -1\ \Omega$$

Hence, the Thevenin equivalent resistance across PQ is **– 1 Ω.**

Key Point

In case of dependent source (active network), equivalent impedance can be negative.

❖❖❖❖

3 Two-Port Networks

➢ Partial Synopsis

Summary

Sr.	Parameter	Dependent variables	Independent variables	Equation	Matrix Form
1.	**Z - Parameter**	V_1, V_2	I_1, I_2	$V_1 = Z_{11}I_1 + Z_{12}I_2$ $V_2 = Z_{21}I_1 + Z_{22}I_2$	$[Z]_{2\times2} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$
2.	**Y - Parameter**	I_1, I_2	V_1, V_2	$I_1 = Y_{11}V_1 + Y_{12}V_2$ $I_2 = Y_{21}V_1 + Y_{22}V_2$	$[Y]_{2\times2} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$
3.	**h - parameter**	V_1, I_2	I_1, V_2	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$	$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$
4.	**g - Parameter**	I_1, V_2	V_1, I_2	$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$	$[g] = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}_{2\times2}$
5.	**ABCD Parameter**	V_1, I_1	$V_2, -I_2$	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$	$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{2\times2}$
6.	**ABCD Inverse/ T Inverse Parameter**	V_2, I_2	$V_1, -I_1$	$V_2 = A^{-1}V_1 - B^{-1}I_1$ $I_2 = C^{-1}V_1 - D^{-1}I_1$	$[ABCD]^{-1} = [T]^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Symmetricity & Reciprocity Conditions

Parameter	Condition for Symmetry	Condition for Reciprocity
Z	$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
Y	$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
h	$\Delta h = \lvert h \rvert = h_{11}h_{22} - h_{21}h_{12} = 1$	$h_{12} = -h_{21}$
g	$\Delta g = \lvert g \rvert = g_{11}g_{22} - g_{12}g_{21} = 1$	$g_{12} = -g_{21}$
$ABCD$	$A = D$	$\lvert T \rvert = AD - BC = 1$
$A^{-1}B^{-1}C^{-1}D^{-1}$	$A^{-1} = D^{-1}$	$A^{-1}D^{-1} - B^{-1}C^{-1} = 1$

Sample Questions

1988 IIT Kharagpur

3.1 Two two-port network A and B are connected in parallel. The combination is to be represented as a single two-port network. The parameters of this network are obtained by addition of the individual

(A) z-parameters

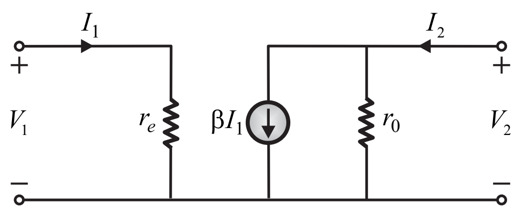
(B) h-parameters

(C) y-parameters

(D) $ABCD$ parameters

2006 IIT Kharagpur

3.2 In the two port network shown in the figure below, z_{12} and z_{21} are respectively



(A) r_e and βr_0 (B) 0 and $-\beta r_0$

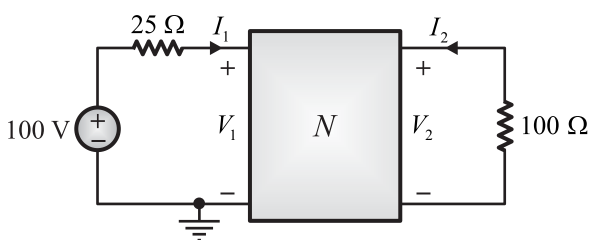
(C) 0 and βr_0 (D) r_e and $-\beta r_0$

2011 IIT Madras

3.3 In the circuit shown below, the network N is described by the following Y matrix

$$Y = \begin{bmatrix} 0.1\text{ S} & -0.01\text{ S} \\ 0.01\text{ S} & 0.1\text{ S} \end{bmatrix}$$

The voltage gain $\dfrac{V_2}{V_1}$ is

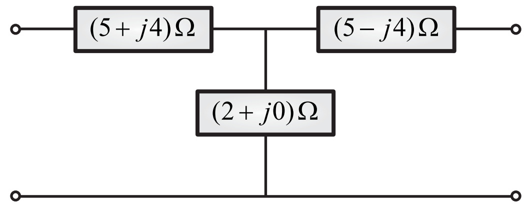


(A) 1/90 (B) –1/90

(C) –1/99 (D) –1/11

2015 IIT Kanpur

3.4 The $ABCD$ parameters of the following 2-port network are **[Set - 03]**



(A) $\begin{bmatrix} 3.5+j2 & 20.5 \\ 20.5 & 3.5-j2 \end{bmatrix}$

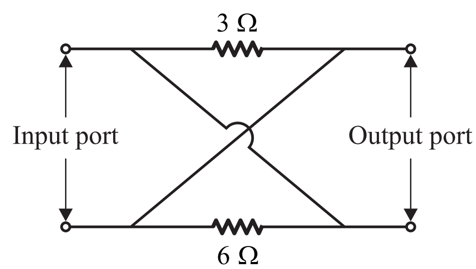
(B) $\begin{bmatrix} 3.5+j2 & 30.5 \\ 0.5 & 3.5-j2 \end{bmatrix}$

(C) $\begin{bmatrix} 10 & 2+j0 \\ 2+j0 & 10 \end{bmatrix}$

(D) $\begin{bmatrix} 7+j4 & 0.5 \\ 30.5 & 7-j4 \end{bmatrix}$

2016 IISc Bangalore

3.5 The z-parameter matrix $\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$ for the two-port network shown is **[Set - 03]**

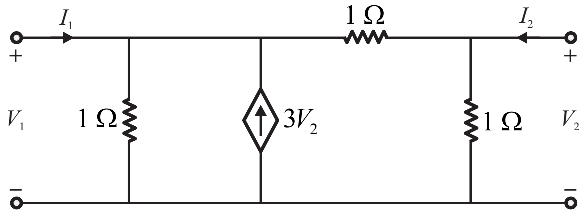


(A) $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 9 & -3 \\ 6 & 9 \end{bmatrix}$ (D) $\begin{bmatrix} 9 & 3 \\ 6 & 9 \end{bmatrix}$

2021 IIT Bombay

3.6 Consider the two port network shown in the figure.



The admittance parameters, in Siemens, are

(A) $Y_{11} = 2,\ Y_{12} = -4,\ Y_{21} = -4,\ Y_{22} = 2$

(B) $Y_{11} = 1,\ Y_{12} = -2,\ Y_{21} = -1, Y_{22} = 3$

(C) $Y_{11} = 2,\ Y_{12} = -4,\ Y_{21} = -1,\ Y_{22} = 2$

(D) $Y_{11} = 2,\ Y_{12} = -4,\ Y_{21} = -4,\ Y_{22} = 3$

Explanations Two-Port Networks

3.1 (C)

If two 2-port networks A and B are connected in parallel then equivalent y-parameter is the sum of individual y-parameters.

$$[Y] = [Y]_A + [Y]_B$$

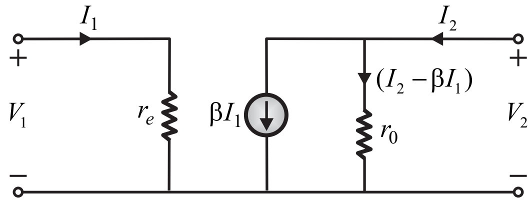
Hence, the correct option is (C).

Table 3.1 :

S. No.	2-port connection	Relation
1.	Series	Addition of z-parameter
2.	Parallel	Addition of y-parameter
3.	Series parallel	Addition of h-parameter
4.	Parallel series	Addition of g-parameter
5.	Cascade	Multiplication of *ABCD* parameters

3.2 (B)

Given circuit is shown below,



Standard z-parameter equation :

$$\boldsymbol{V_1 = z_{11}I_1 + z_{12}I_2} \quad \ldots\text{(i)}$$

$$\boldsymbol{V_2 = z_{21}I_1 + z_{22}I_2} \quad \ldots\text{(ii)}$$

From figure,

$$V_1 = I_1 r_e + 0 \times I_2 = I_1 r_e \quad \ldots\text{(iii)}$$

$$V_2 = (I_2 - \beta I_1) r_0$$

$$V_2 = -\beta r_0 I_1 + r_0 I_2 \quad \ldots\text{(iv)}$$

Comparing equation (iii) and (iv) with equation (i) and (ii),

Hence, $z_{11} = r_e,\ z_{12} = 0$

and $z_{21} = -\beta r_0,\ z_{22} = r_0$

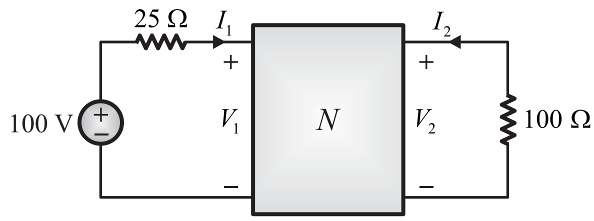
Hence, the correct option is (B).

3.3 (D)

Given : $Y = \begin{bmatrix} 0.1 & -0.01 \\ 0.01 & 0.1 \end{bmatrix}$ S

$$I_1 = 0.1V_1 - 0.01V_2 \quad \ldots\text{(i)}$$

$$I_2 = 0.01V_1 + 0.1V_2 \quad \ldots\text{(ii)}$$



Applying KVL in input port,

$$100 = 25I_1 + V_1 \quad \ldots\text{(iii)}$$

Applying KVL in output port,

$$V_2 = -100 I_2 \quad \ldots\text{(iv)}$$

From equation (iii) and (iv),

$$V_2 = -100(0.01V_1 + 0.1V_2)$$

$$V_2 = -V_1 - 10V_2$$

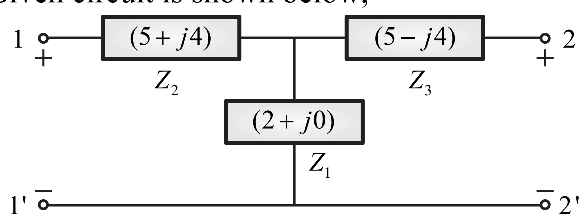
$$11V_2 = -V_1$$

$$\frac{V_2}{V_1} = \frac{-1}{11}$$

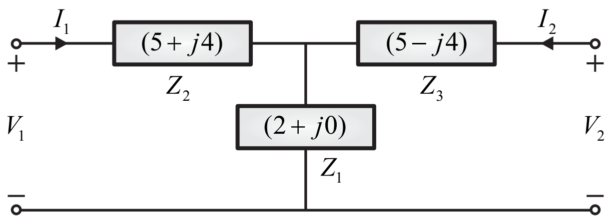
Hence, the correct option is (D).

3.4 (B)

Given circuit is shown below,



Method 1



z-parameter for standard T network is given by,

$$[Z] = \begin{bmatrix} Z_1 + Z_2 & Z_1 \\ Z_1 & Z_1 + Z_3 \end{bmatrix}$$

$$[Z] = \begin{bmatrix} (5 + j4) + (2) & 2 \\ 2 & (5 - j4) + (2) \end{bmatrix}$$

$$[Z] = \begin{bmatrix} 7 + j4 & 2 \\ 2 & 7 - j4 \end{bmatrix}$$

Hence, $V_1 = (7 + j4)I_1 + 2I_2$ ...(i)

$V_2 = 2I_1 + (7 - j4)I_2$ ...(ii)

Standard *ABCD* parameter equation :

$\boldsymbol{V_1 = AV_2 - BI_2}$ ...(iii)

$\boldsymbol{I_1 = CV_2 - DI_2}$ ...(iv)

From equation (ii),

$$2I_1 = V_2 - (7 - j4)I_2$$

$$I_1 = \frac{V_2}{2} - \left(\frac{7 - j4}{2}\right)I_2 \quad \text{...(v)}$$

$$I_1 = 0.5V_2 - (3.5 - j2)I_2$$

Put the value of I_1 in equation (i),

$$V_1 = (7 + j4)\left[\frac{V_2}{2} - \left(\frac{7 - j4}{2}\right)I_2\right] + 2I_2$$

$$V_1 = \left(\frac{7 + j4}{2}\right)V_2 - \frac{(7 + j4)(7 - j4)}{2}I_2 + 2I_2$$

$$V_1 = \left(\frac{7 + j4}{2}\right)V_2 - \frac{(49 + 16)}{2}I_2 + 2I_2$$

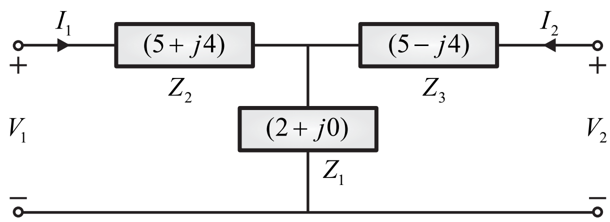
$$V_1 = (3.5 + j2)V_2 - 30.5I_2 \quad \text{...(vi)}$$

Comparing equation (vi) and (v) with equation (iii) and (iv),

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 3.5 + j2 & 30.5 \\ 0.5 & 3.5 - j2 \end{bmatrix}$$

Hence, the correct option is (B).

Method 2



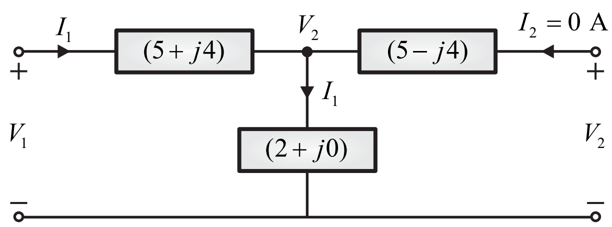
Standard *ABCD* parameter equation :

$\boldsymbol{V_1 = AV_2 - BI_2}$

$\boldsymbol{I_1 = CV_2 - DI_2}$

(i) Calculation of *A* :

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ = Reverse voltage gain, when output is open circuited.



$$V_2 = \frac{2 + j0}{(5 + j4) + (2 + j0)}V_1 \quad \text{[By VDR]}$$

$$V_2 = \frac{2}{7 + j4}V_1$$

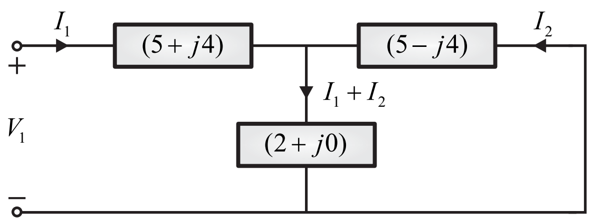
$$\frac{V_1}{V_2}=\frac{7+j4}{2}=3.5+j2$$

$$A=\left.\frac{V_1}{V_2}\right|_{I_2=0}=3.5+j2$$

(ii) Calculation of *B* :

$$B=\left.\frac{-V_1}{I_2}\right|_{V_2=0}= \text{Transfer impedance, when}$$

output is short circuited.



$$I_2=-\left(\frac{2+j0}{(5-j4)+(2+j0)}\right)I_1 \text{ [By CDR]}$$

$$I_1=\frac{-(7-j4)}{2}I_2$$

Applying KVL in input loop,

$$-V_1+(5+j4)I_1+(2+j0)(I_1+I_2)=0$$

$$V_1=(7+j4)I_1+2I_2$$

$$V_1=\frac{-(7+j4)(7-j4)}{2}I_2+2I_2$$

$$V_1=\frac{-65}{2}I_2+2I_2$$

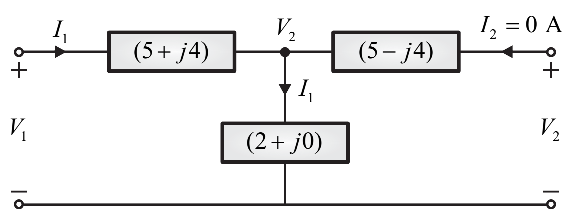
$$V_1=-30.5I_2$$

$$B=\left.\frac{-V_1}{I_2}\right|_{V_2=0}=30.5$$

(iii) Calculation of *C* :

$$C=\left.\frac{I_1}{V_2}\right|_{I_2=0}= \text{Transfer admittance, when}$$

output is open circuited.



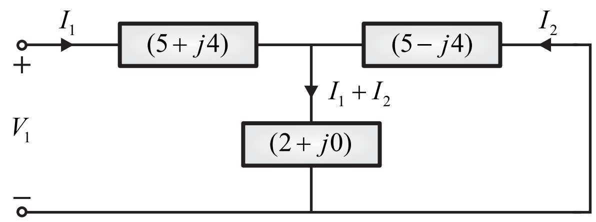
From figure, $V_2=(2+j0)I_1$

$$C=\left.\frac{I_1}{V_2}\right|_{I_2=0}=0.5$$

(iv) Calculation of *D* :

$$D=\left.\frac{-I_1}{I_2}\right|_{V_2=0}= \text{Reverse current gain, when}$$

output is short circuited.



$$I_2=-\left(\frac{2+j0}{(5-j4)+(2+j0)}\right)I_1 \text{ [By CDR]}$$

$$I_1=\frac{-(7-j4)}{2}I_2$$

$$\frac{-I_1}{I_2}=3.5-j2$$

$$D=\left.\frac{-I_1}{I_2}\right|_{V_2=0}=3.5-j2$$

$$T=\begin{bmatrix}A & B\\ C & D\end{bmatrix}=\begin{bmatrix}3.5+j2 & 30.5\\ 0.5 & 3.5-j2\end{bmatrix}$$

Hence, the correct option is (B).

Method 3

Given T-network is reciprocal network hence, for $ABCD$ parameter to be reciprocal.

$$\Delta T=1$$

From option (A),

$$\Delta T=-404$$

From option (B),

$$\Delta T=1$$

From option (C),

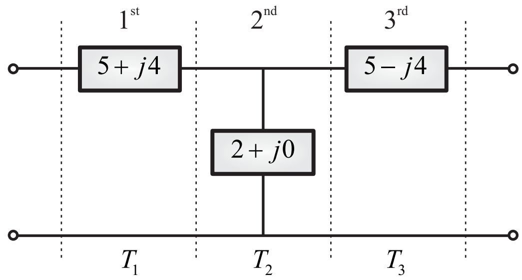
$$\Delta T=96$$

From option (D),

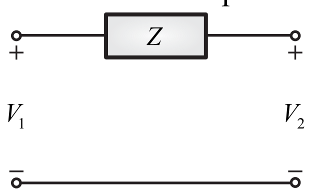
$$\Delta T=49.75$$

Hence, the correct option is (B).

Method 4

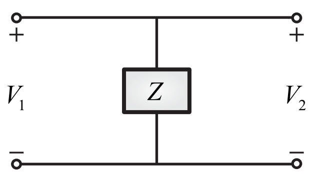


For standard network T parameter are



$$T=\begin{bmatrix}1 & Z\\ 0 & 1\end{bmatrix}$$

Fig. (a)



$$T=\begin{bmatrix}1 & 0\\ \frac{1}{Z} & 1\end{bmatrix}$$

Fig. (b)

For 1st section of network, T_1 parameter is given by,

$$T_1=\begin{bmatrix}1 & 5+j4\\ 0 & 1\end{bmatrix}$$

For 2nd section of network, T_2 parameter is given by,

$$T_2=\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}$$

For 3rd section of network, T_3 parameter is given by,

$$T_3=\begin{bmatrix}1 & 5-j4\\ 0 & 1\end{bmatrix}$$

Overall T-parameter of cascade network is multiplication of individual network

$$[T]=[T_1][T_2][T_3]$$

$$[T]=\begin{bmatrix}1 & 5+j4\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}\begin{bmatrix}1 & 5-j4\\ 0 & 1\end{bmatrix}$$

$$[T]=\begin{bmatrix}1+2.5+j2 & 5+j4\\ \frac{1}{2} & 1\end{bmatrix}\begin{bmatrix}1 & 5-j4\\ 0 & 1\end{bmatrix}$$

$$[T]=\begin{bmatrix}3.5+j2 & 5+j4\\ \frac{1}{2} & 1\end{bmatrix}\begin{bmatrix}1 & 5-j4\\ 0 & 1\end{bmatrix}$$

$$[T]=\begin{bmatrix}3.5+j2 & 17.5-j14+j10+8+5+j4\\ \frac{1}{2} & 2.5-j2+1\end{bmatrix}$$

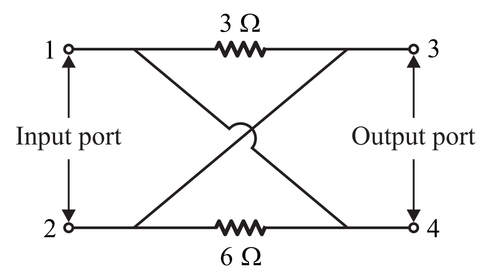
$$[T]=\begin{bmatrix}3.5+j2 & 30.5\\ \frac{1}{2} & 3.5-j2\end{bmatrix}$$

$$[T]=\begin{bmatrix}3.5+j2 & 30.5\\ 0.5 & 3.5-j2\end{bmatrix}$$

Hence, the correct option is (B).

3.5 (A)

Given circuit is shown below,

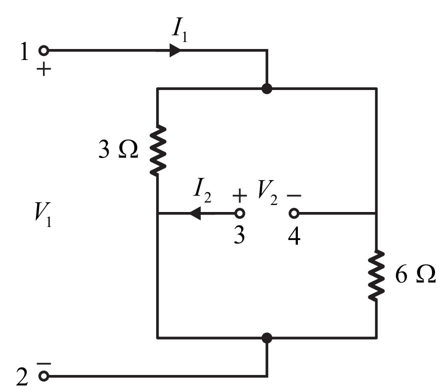


Standard z-parameter equation :

$$V_1=z_{11}I_1+z_{12}I_2$$

$$V_2=z_{21}I_1+z_{22}I_2$$

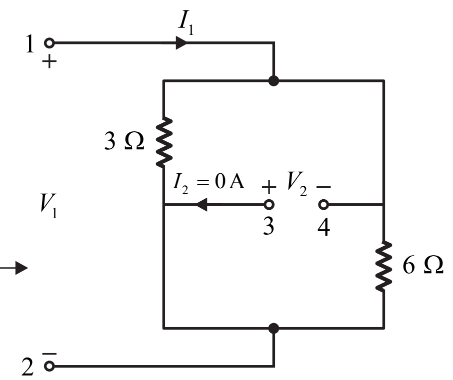
Given circuit can be redrawn as shown below,



(i) Calculation of z_{11} :

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \text{Driving point input impedance,}$$

when output is open circuited.



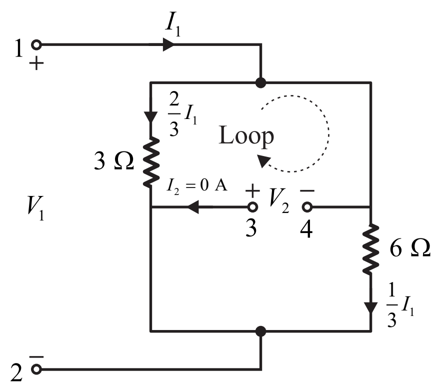
$z_{11} = 3 \parallel 6$

$$z_{11} = 3 \parallel 6 = 2\ \Omega$$

(ii) Calculation of z_{21} :

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \text{Transfer impedance, when}$$

output is open circuited.



$$I_{3\,\Omega} = \frac{I_1 \times 6}{3+6} = \frac{2}{3}I_1 \qquad \text{[By CDR]}$$

$$I_{6\,\Omega} = \frac{I_1 \times 3}{3+6} = \frac{1}{3}I_1 \qquad \text{[By CDR]}$$

Applying KVL in above loop,

$$-V_2 - 3 \times \frac{2}{3}I_1 = 0$$

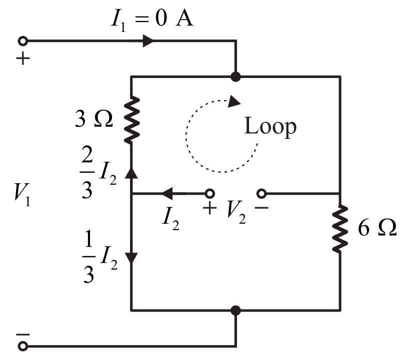
$$\frac{V_2}{I_1} = -2\ \Omega$$

$$z_{21} = \frac{V_2}{I_1} = -2\ \Omega$$

(iii) Calculation of z_{22} :

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \text{Driving point output impedance,}$$

when input is open circuited.



$$I_{3\,\Omega} = \frac{I_2 \times 6}{3+6} = \frac{2}{3}I_2 \qquad \text{[By CDR]}$$

Applying KVL in above shown loop,

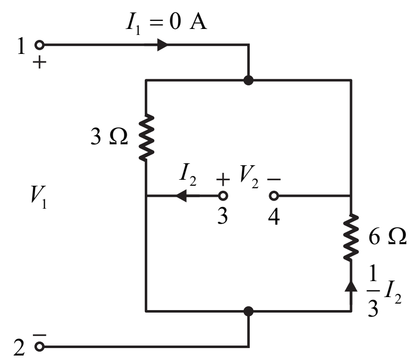
$$-V_2 + \frac{2}{3}I_2 \times 3 = 0$$

$$z_{22} = \frac{V_2}{I_2} = 2\ \Omega$$

(iv) Calculation of z_{12} :

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \text{Transfer impedance, when}$$

input is open circuited.



$$I_{6\,\Omega} = \frac{3 \times I_2}{3+6} = \frac{3}{9}I_2 \qquad \text{[By CDR]}$$

From figure,

$$-V_1 - 6 \times \frac{1}{3}I_2 = 0$$

$$V_1 = -6 \times I_2 \times \frac{3}{9} = -2I_2$$

$$z_{12} = \frac{V_1}{I_2} = -2\ \Omega$$

$$[z] = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

Hence, the correct option is (A).

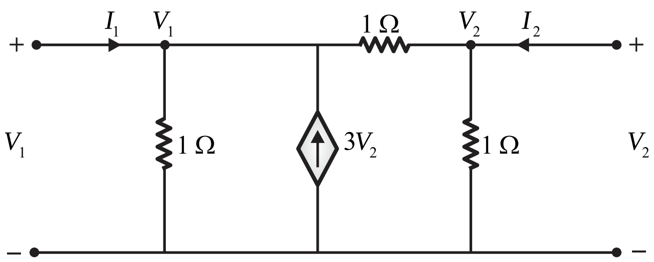
Key Point

All resistive network represent reciprocal network. Hence, $z_{12} = z_{21}$

Only option (A) and (B) satisfy above condition so, we can directly calculate z_{12} or z_{21}.

3.6 (C)

Given circuit is shown below



Nodal analysis at node V_1

$$-I_1 + \frac{V_1}{1} - 3V_2 + \frac{V_1 - V_2}{1} = 0$$

$$I_1 = V_1 - 3V_2 + V_1 - V_2$$

$$I_1 = 2V_1 - 4V_2$$

Compare I_1 with 1st equation of Y-parameter is,

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

Thus, $Y_{11} = 2$ ℧, $Y_{12} = -4$ ℧

Nodal analysis at node V_2

$$-I_2 + \frac{V_2}{1} + \frac{V_2 - V_1}{1} = 0$$

$$I_2 = 2V_2 - V_1$$

$$I_2 = -V_1 + 2V_2$$

Compare I_2 with 2nd equation of Y-parameter is,

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Thus, $Y_{21} = -1$ ℧, $Y_{22} = 2$ ℧

Thus admittance parameter of given network is,

$$[Y] = \begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$$

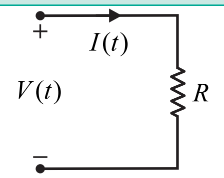
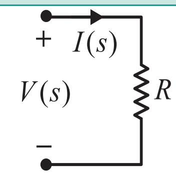
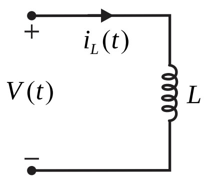
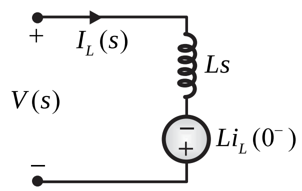
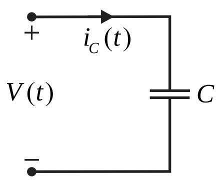
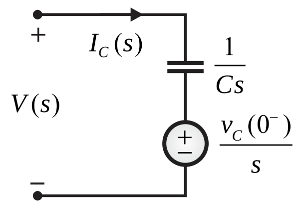
Hence, the correct option is (C).

❖❖❖❖

4 Transient Analysis

➢ Partial Synopsis

Transform Networks

Circuit Elements	Time Domain Equations	Time Domain Circuit	Frequency Domain Equations	Frequency Domain Circuit
Resistance (opposes the flow of current)	$V(t) = R\,I(t)$		$V(s) = R\,I(s)$	
Inductance (opposes the change of current) $I_L(0^-) = I_L(0)$ $= I_L(0^+)$	$V_L(t) = L\frac{d}{dt}I_L(t)$ $I_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L(t)\,dt$ $I_L(t) = I_L(0^-) + \frac{1}{L}\int_0^t V_L(t)\,dt$		$V_L(s) = LsI_L(s) - LI_L(0^-)$ $I_L(s) = \frac{V_L(s)}{Ls} + \frac{I_L(0^-)}{s}$	
Capacitance (opposes the change of voltage) $V_C(0^-) = V_C(0)$ $= V_C(0^+)$	$I_C(t) = C\frac{d}{dt}V_C(t)$ $V_C(t) = \frac{1}{C}\int_{-\infty}^{t} I_C(t)\,dt$ $V_C(t) = V_C(0^-) + \frac{1}{C}\int_0^t I_C(t)\,dt$		$V_C(s) = \frac{I_C(s)}{Cs} + \frac{V_C(0^-)}{s}$ $I_C(s) = CsV_C(s) - CV_C(0^-)$	

🕮 Remember :

Following equations can be used to find response of RL and RC circuits after switching :

1. If the switching occurs at $t = 0$.

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right]e^{-t/\tau} \qquad v(t) = v(\infty) + \left[v(0) - v(\infty)\right]e^{-t/\tau}$$

2. If the switching occurs at $t = t_0$.

$$i(t) = i(\infty) + \left[i(t_0) - i(\infty)\right]e^{-(t-t_0)/\tau} \qquad v(t) = v(\infty) + \left[v(t_0) - v(\infty)\right]e^{-(t-t_0)/\tau}$$

3. Both equation can be used only when R, L, C is constant not a function of time for example if $R(t)$ i.e. R is a function of time then we can't use above equation.

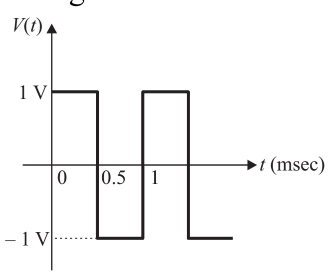
To find different parameters of RL circuit, following facts can be used.

Parameters to find for $t > 0$	$t = 0^+$	$t = \infty$	Requirements
$I_L(t)$	$I_L(0^+) = I_L(0^-)$	$I_L(\infty) =$ Current through inductor under short circuit condition	$I_L(t)$ for $t = \infty$ & $t = 0^-$
$V_L(t)$	$V_L(0^+) \neq V_L(0^-)$	$V_L(\infty) = 0$ volt always	$V_L(t)$ for $t = 0^-$ & $t = 0^+$
$I_R(t)$	$I_R(0^+) \neq I_R(0^-)$	$I_R(\infty) =$ to be calculated under steady state condition with inductor short circuited	$I_R(t)$ for $t = 0^-, 0^+$ & ∞
$V_R(t)$	$V_R(0^+) \neq V_R(0^-)$	$V_R(\infty) =$ to be calculated under steady state condition with inductor short circuited	$V_R(t)$ for $t = 0^-, 0^+$ & ∞
$V_C(t)$	$V_C(0^+) = V_C(0^-)$	$V_C(\infty) =$ voltage across capacitor under open circuit condition	$V_C(t)$ for $t = \infty$ & $t = 0^-$
$I_C(t)$	$I_C(0^-) \neq I_C(0^+)$	$I_C(\infty) = 0$ volt always	$I_C(t)$ for $t = 0^-$ & $t = 0^+$
$V_R(t)$	$V_R(0^-) \neq I_R(0^+)$	$V_R(\infty) =$ to be calculated under steady state condition with capacitor open circuited	$V_R(t)$ for $t = 0^-, 0^+$ & ∞
$I_R(t)$	$I_R(0^-) \neq V_R(0^+)$	$I_R(\infty) =$ to be calculated under steady state condition with inductor short circuited	$I_R(t)$ for $t = 0^-, 0^+$ & ∞

➢ Sample Questions

1987 IIT Bombay

4.1 A square waveform as shown in figure is applied across 1 mH ideal inductor. The current through inductor is a



(A) triangular wave with peak amplitude of 0.5 Amp.

(B) triangular wave with peak amplitude of 1.0 Amp.

(C) triangular wave with peak amplitude of 2.0 Amp.

(D) sine wave with peak amplitude of 0.5 Amp.

1989 **IIT Kanpur**

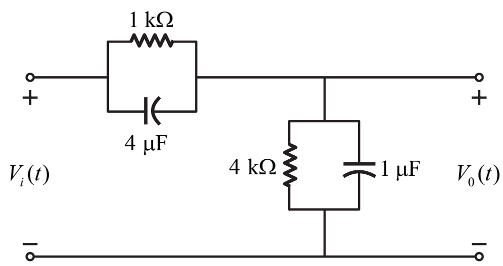
4.2 If the Laplace transform of the voltage across a capacitor of value of $\frac{1}{2}$ F is $V_C(s) = \frac{s+1}{s^3 + s^2 + s + 1}$, the value of current through the capacitor at $t = 0^+$ is

(A) 0 A (B) 2 A

(C) (1/2) A (D) 1 A

2006 **IIT Kharagpur**

4.3 In the figure shown below, assume that all the capacitors are initially uncharged. If $V_i(t) = 10u(t)$ volt, $V_0(t)$ is given by



(A) $8e^{-0.004t}$ Volts

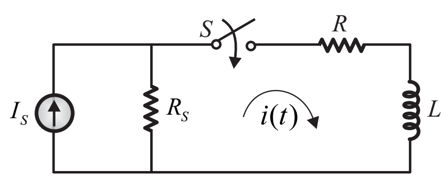
(B) $8(1 - e^{-0.004t})$ Volts

(C) $8u(t)$ Volts

(D) 8 Volts

2008 **IISc Bangalore**

4.4 In the given circuit, the switch S is closed at t = 0. The rate of change of current $\frac{d}{dt}i(0^+)$ is given by



(A) 0 (B) $\frac{R_S I_S}{L}$

(C) $\frac{(R + R_S)I_S}{L}$ (D) ∞

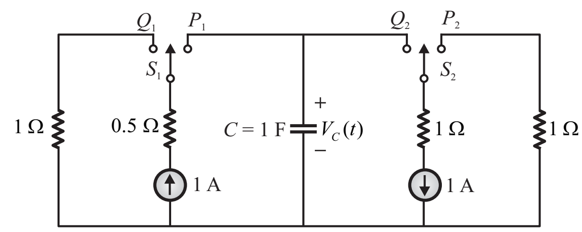
4.5 The circuit shown in the figure is used to charge the capacitor C alternately from two current sources as indicated. The switches S_1 and S_2 are mechanically coupled and connected as follows :

For $2nT \le t < (2n+1)T$,

$(n = 0, 1, 2, ...)$ S_1 to P_1 and S_2 to P_2

For $(2n+1)T \le t < (2n+2)T$,

$(n = 0, 1, 2, ...)$ S_1 to Q_1 and S_2 to Q_2.



Assume that the capacitor has zero initial charge. Given that $u(t)$ is a unit step function, the voltage $V_C(t)$ across the capacitor is given by

(A) $\sum_{n=0}^{\infty}(-1)^n tu(t - nT)$

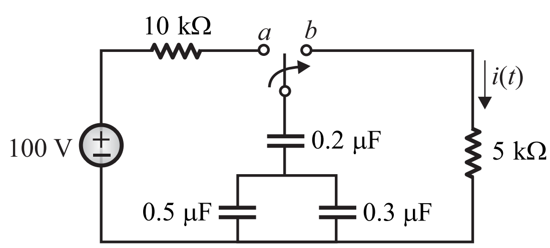
(B) $u(t) + 2\sum_{n=1}^{\infty}(-1)^n u(t - nT)$

(C) $tu(t) + 2\sum_{n=1}^{\infty}(-1)^n (t - nT)u(t - nT)$

(D) $\sum_{n=0}^{\infty}\left[0.5 - e^{-(t-2nT)} + 0.5e^{-(t-2nT-T)}\right]$

2009 **IIT Roorkee**

4.6 The switch in the circuit shown was on position a for a long time, and is moved to position b at time $t = 0$. The current $i(t)$ for $t > 0$ is given by



(A) $0.2e^{-125t}\,u(t)$ mA

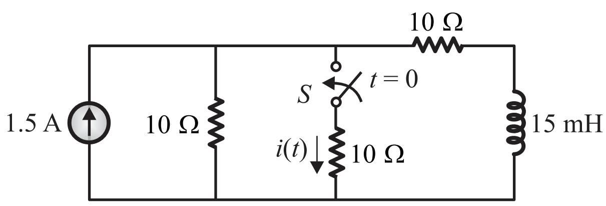
(B) $20e^{-1250t}\,u(t)$ mA

(C) $0.2e^{-1250t}\,u(t)$ mA

(D) $20e^{-1000t}\,u(t)$ mA

2010 IIT Guwahati

4.7 In the circuit shown, the switch S is open for a long time and is closed at $t = 0$. The current $i(t)$ for $t \geq 0^+$ is



(A) $i(t) = 0.5 - 0.125e^{-1000t}$ A

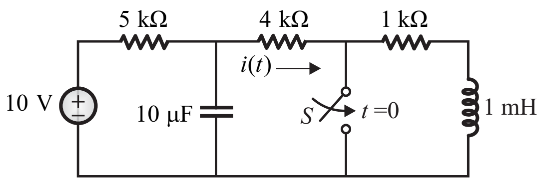
(B) $i(t) = 1.5 - 0.125e^{-1000t}$ A

(C) $i(t) = 0.5 - 0.5e^{-1000t}$ A

(D) $i(t) = 0.375e^{-1000t}$ A

2014 IIT Kharagpur

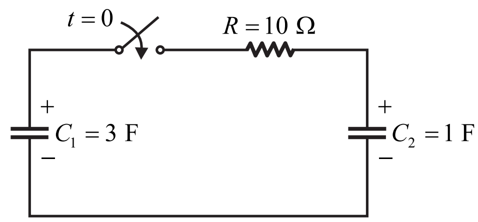
4.8 In the figure shown, the ideal switch has been open for a long time. If it is closed at $t = 0$, then the magnitude of the current (in mA) through the 4 kΩ resistor at $t = 0^+$ is ________. **[Set - 02]**



2015 IIT Kanpur

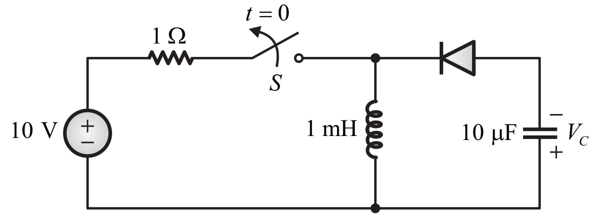
4.9 In the circuit shown, the initial voltages across the capacitors C_1 and C_2 are 1 V and 3 V, respectively. The switch is closed at time $t = 0$. The total energy dissipated (in Joules) in the resistor R until steady state is reached, is ________.

[Set - 02]



2016 IISc Bangalore

4.10 The switch S in the circuit shown has been closed for a long time. It is opened at time $t = 0$ and remains open after that. Assume that the diode has zero reverse current and zero forward voltage drop.

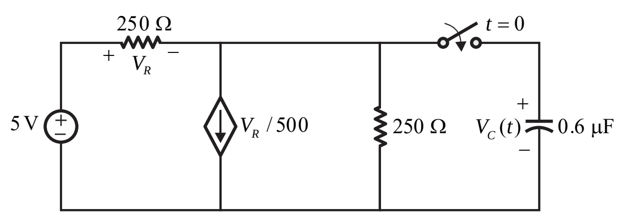


The steady state magnitude of the capacitor voltage V_C (in volts) is _____.

[Set - 02]

2021 IIT Bombay

4.11 In the circuit shown in figure, the switch is closed at time $t = 0$, while the capacitor is initially charged to – 5V [i.e. $V_C(0) = -5$ V]



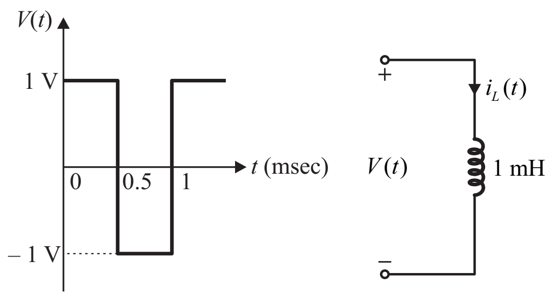
The time after which the voltage across the capacitor becomes zero is (Round off to 3 decimal places) ______ ms.

❖❖❖❖

Explanations Transient Analysis

4.1 (A)

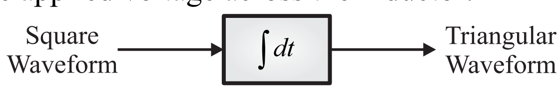
Given waveform is shown below,



Current through an inductor is given by,

$$i_L(t) = \frac{1}{L}\int_0^t V(t)\,dt$$

So, current through inductor is the integration of the applied voltage across the inductor.

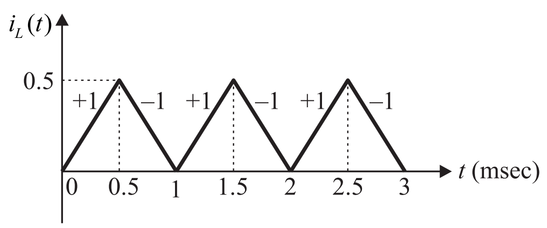


The square wave can be written as,

$$V(t) = u(t) - 2u(t-0.5) + 2u(t-1) + \cdots\cdots$$

Integrating the voltage to get current through the inductor,

$$i_L(t) = r(t) - 2r(t-0.5) + 2r(t-1) - \cdots\cdots$$



Peak amplitude of $i_L(t) = 0.5$ Amp .

Hence, the correct option is (A).

4.2 (C)

Given : $C = \frac{1}{2}$ F, $V_C(s) = \dfrac{s+1}{s^3+s^2+s+1}$

Method 1

For capacitor,

$$X_C = \frac{1}{Cs} = \frac{1}{\frac{1}{2}s} = \frac{2}{s}$$

Transform voltage of capacitor is given by,

$$V_C(s) = I_C(s) X_C$$

$$I_C(s) = \frac{V_C(s)}{X_C} = \frac{(s+1)s}{(s^3+s^2+s+1)2}$$

The value of current at $t = 0^+$, means it is the initial value.

By initial value theorem,

$$i_C(0^+) = \lim_{s\to\infty} s\, I_C(s)$$

$$i_C(0^+) = \lim_{s\to\infty} \frac{s \times s(s+1)}{2(s^3+s^2+s+1)}$$

$$i_C(0^+) = \lim_{s\to\infty} \frac{s^2(s+1)}{2(s^2+1)(s+1)}$$

$$i_C(0^+) = \lim_{s\to\infty} \frac{s^3\left(1+\frac{1}{s}\right)}{2s^2\left(1+\frac{1}{s^2}\right)s\left(1+\frac{1}{s}\right)}$$

$$i_C(0^+) = \frac{1}{2+0} = \frac{1}{2}\text{ A}$$

Hence, the correct option is (C).

Method 2

Given : $V_C(s) = \dfrac{s+1}{s^3+s^2+s+1}$

$$V_C(s) = \frac{(s+1)}{(s+1)(s^2+1)} = \frac{1}{(s^2+1)}$$

Taking inverse Laplace transform both sides,

$$v_C(t) = \sin t$$

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

Given, $C = \frac{1}{2}$ F

$$\therefore \quad i_C(t) = \frac{1}{2}\frac{d}{dt}(\sin t) = \frac{1}{2}\cos t$$

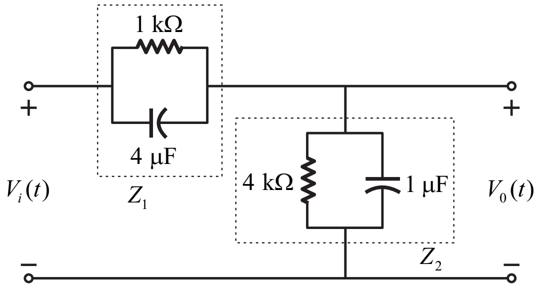
Hence, the current through the capacitor at $t = 0^+$ is given as

$$\lim_{t\to 0} i_C(t) = \lim_{t\to 0}\frac{1}{2}\cos t = \frac{1}{2}\text{ Amp}$$

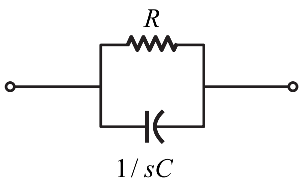
Hence, the correct option is (C).

4.3 (C)

Given circuit is shown below,



For a particular parallel *R-C* network,



$$Z(s) = \frac{R \times \frac{1}{sC}}{R + \frac{1}{sC}} = \frac{R}{1 + sCR}$$

For $R_1 = 1\text{ k}\Omega = 10^3\,\Omega$

$$C_1 = 4\ \mu\text{F} = 4 \times 10^{-6}\text{ F}$$

$$R_1C_1 = 4 \times 10^{-3} = 0.004\text{ sec}$$

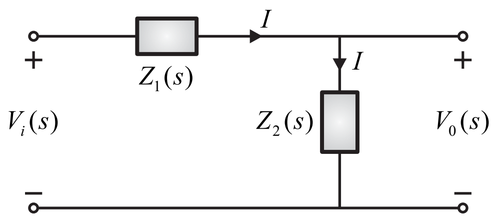
For $R_2 = 4\text{ k}\Omega = 4 \times 10^3\,\Omega$

$$C_2 = 1\ \mu\text{F} = 10^{-6}\text{ F}$$

$$R_2C_2 = 0.004\text{ sec}$$

Hence, impedances are given by,

$$Z_1(s) = \frac{1000}{1 + 0.004s},\quad Z_2(s) = \frac{4000}{1 + 0.004s}$$



The output voltage,

$$V_0(s) = \frac{V_i(s) \times Z_2(s)}{Z_1(s) + Z_2(s)} = \frac{V_i(s)}{1 + \frac{Z_1(s)}{Z_2(s)}}$$

[By VDR]

$$V_0(s) = \frac{V_i(s)}{1 + \frac{1}{4}} = 0.8V_i(s)$$

Taking inverse Laplace transform,

$$V_0(t) = 0.8V_i(t)$$

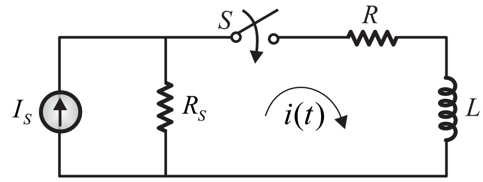
$$V_i(t) = 10u(t) \qquad \text{[Given]}$$

$$V_0(t) = 0.8 \times 10u(t) = 8u(t)\text{ V}$$

Hence, the correct option is (C).

4.4 (B)

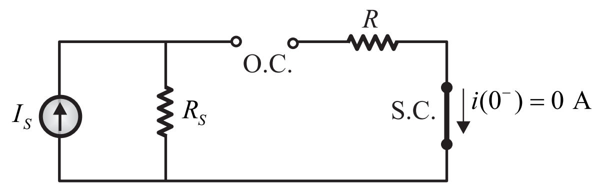
Given circuit is shown below,



Method 1

(i) At $t = 0^- / t < 0$ / steady state :

In steady state, inductor behaves as a short circuit.



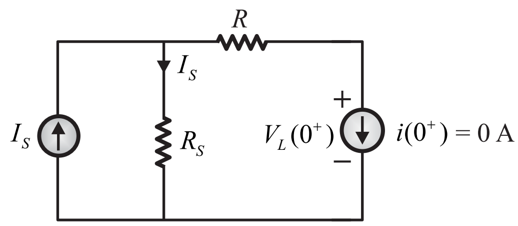
From property of inductor,

$$i(0^-) = i(0^+) = 0\text{ A}$$

(ii) At $t = 0^+$:

Inductor is replaced by a current source with initial value i.e.

$$i(0^-) = i(0^+) = 0\text{ A} \quad \text{(open circuit)}$$



From figure, inductor voltage is,

$$V_L(0^+) = I_S R_S$$

Derivative of inductor current at $t = 0^+$ is given by,

$$\frac{d}{dt}i(0^+) = \frac{V_L(0^+)}{L}$$

$$\frac{d}{dt}i(0^+) = \frac{I_S R_S}{L}$$

Hence, the correct option is (B).

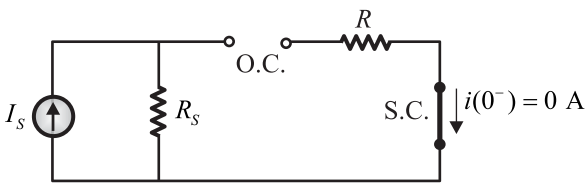
Method 2

Current through inductor is given by,

$$i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right]e^{-t/\tau} \quad \ldots\text{(i)}$$

(i) At $t = 0^-$ / $t < 0$ / steady state :

In steady state, inductor behaves as a short circuit.



From property of inductor,

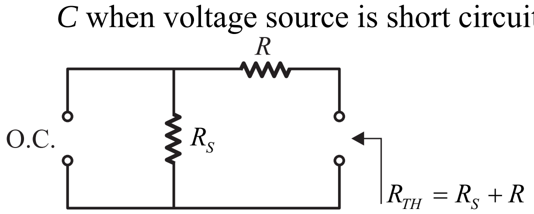
$$i(0^-) = i(0^+) = 0 \text{ A}$$

(ii) At $t \geq 0$ (Transient) :

For a *R-L* network,

Time constant, $\tau = \dfrac{L}{R_{TH}}$

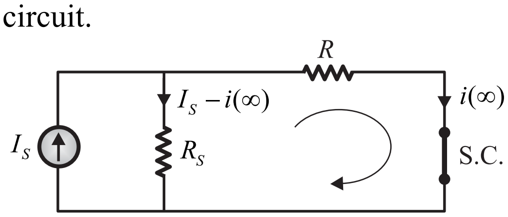
where, R_{TH} = Thevenin resistance across *C* when voltage source is short circuited.



$$\tau = \frac{L}{R_{TH}} = \frac{L}{R_S + R} \qquad \text{[From figure]}$$

(iii) At $t = \infty$/steady state :

In steady state, inductor behaves as short circuit.



Applying KVL in loop shown,

$$-\left[I_S - i(\infty)\right]R_S + R\,i(\infty) = 0$$

$$i(\infty)(R + R_S) = I_S R_S$$

$$i(\infty) = \frac{I_S R_S}{R + R_S}$$

Put the values of $i(0^+)$, $i(\infty)$ and τ in equation (i),

$$i(t) = \frac{I_S R_S}{R + R_S}\left[1 - e^{-t(R+R_S)/L}\right]$$

$$\frac{d}{dt}i(t) = \frac{I_S R_S}{R + R_S}\left[e^{-t(R+R_S)/L} \times \frac{R + R_S}{L}\right]$$

At $t = 0^+$,

$$\frac{d}{dt}i(0^+) = \frac{I_S R_S}{L} \text{ A/sec}$$

Hence, the correct option is (B).

4.5 (C)

Given : $Q(0^-) = 0$ C

$$V_C(0^-) = \frac{Q(0^-)}{C}$$

$$V_C(0^-) = 0 \text{ V}$$

From property of capacitor,

$$V_C(0^-) = V_C(0^+) = 0 \text{ V}$$

n	t	Position of switch	
0	$0 \leq t < T$	S_1 and P_1	S_2 and P_2
	$T \leq t < 2T$	S_1 and Q_1	S_2 and Q_2
1	$2T \leq t < 3T$	S_1 and P_1	S_2 and P_2
	$3T \leq t < 4T$	S_1 and Q_1	S_2 and Q_2
2	$4T \leq t < 5T$	S_1 and P_1	S_2 and P_2
	$5T \leq t < 6T$	S_1 and Q_1	S_2 and Q_2

Case 1 :

When S_1 is connected to P_1 and S_2 is connected to P_2.

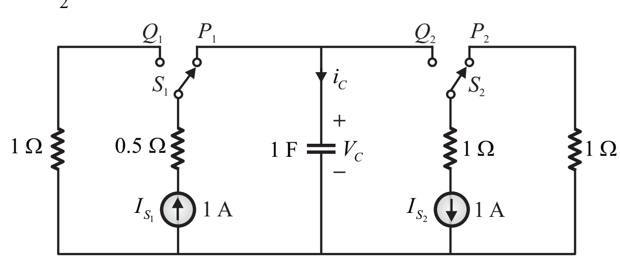


Fig. (a) : $2nT \leq t < (2n+1)T$

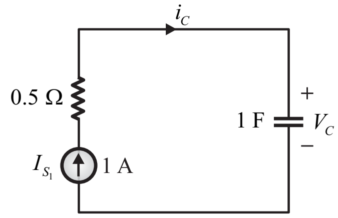


Fig. (b)

For $n = 0$ $[0 \le t < T]$

$$i_C(t) = 1\text{ A} = u(t)$$

$$V_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(t)\,dt$$

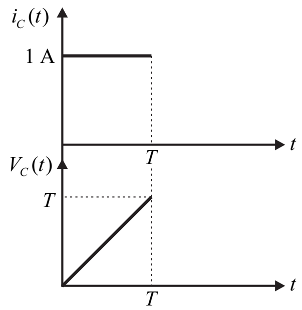
$$V_C(t) = V_C(0^-) + \frac{1}{C}\int_0^t i_C(t)\,dt$$

$$V_C(t) = 0 + \frac{1}{1}\int_0^t 1\,dt$$

$$V_C(t) = t \qquad \text{[Ramp signal]}$$

$$V_C(t=0) = 0\text{ V}$$

$$V_C(t=T) = T\text{ V}$$



This graph indicates the linear charging of capacitor as a constant current flows through capacitor.

Case 2 :

Similarly when S_1 is connected to Q_1 and S_2 is connected to Q_2.

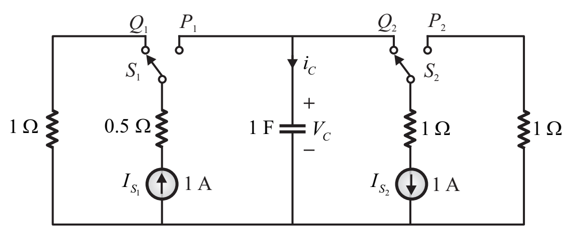


Fig. (c) : $(2n+1)T \le t < (2n+2)T$

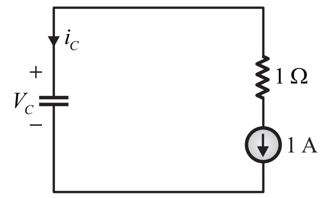


Fig. (d)

For $n = 0$ $[T \le t < 2T]$

$$i_C(t) = -1\text{ A}$$

$$i_C(t) = -u(t)$$

Constant current for a duration of T sec.

$$V_C(t) = \frac{1}{C}\int_{-\infty}^{t} I_C(t)\,dt$$

$$V_C(t) = \frac{1}{C}\int_{-\infty}^{0} I_C(t)\,dt + \frac{1}{C}\int_0^T I_C(t)\,dt + \frac{1}{C}\int_T^t I_C(t)\,dt$$

$$V_C(t) = 0 + T + \frac{1}{C}\int_T^t -1\,dt$$

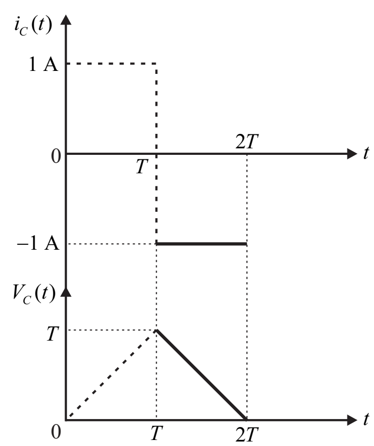
$$V_C(t) = 0 + T - \frac{1}{1}\int_T^t 1\,dt$$

$$V_C(t) = T - \frac{1}{1}(t - T)$$

$$V_C(t) = T - (t - T) = 2T - t$$

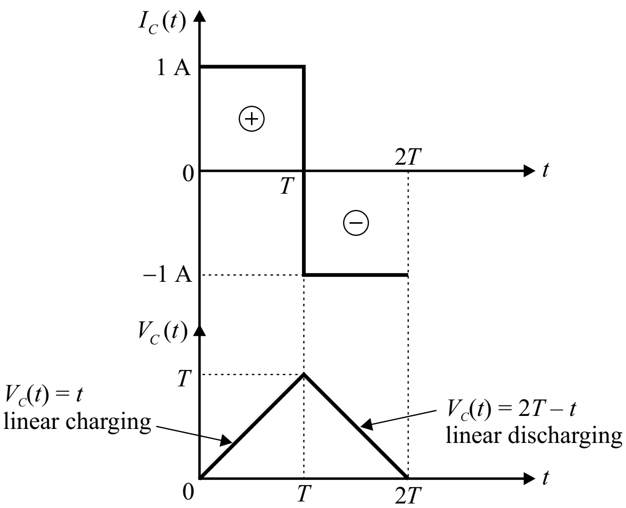
$$V_C(t=T) = T\text{ volt}$$

$$V_C(t=2T) = 0\text{ volt}$$

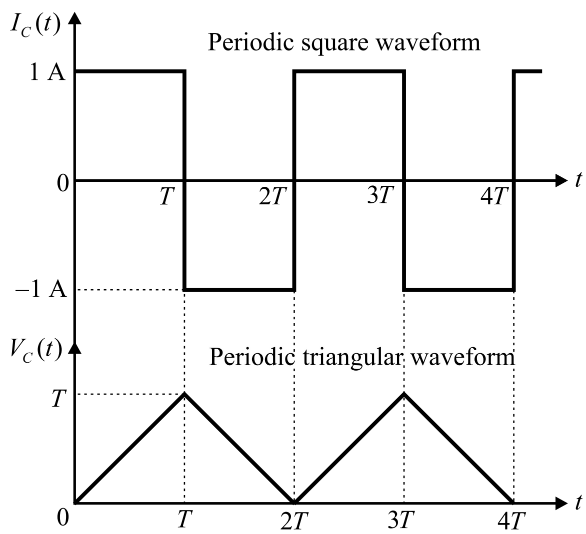


This graph indicates the linear discharging of capacitor as a constant current flows through capacitor.

Combining both the cases together,



V_C decreases linearly from T volt to 0 V similarly from $2T \le t < 3T$, V_C increases linearly from 0 V to T volt and from $3T \le t < 4T$, V_C decreases linearly from T volt to 0 V.



Using unit step functions,

$$V_C(t) = tu(t) - (t-T)u(t-T) - (t-T)u(t-T) + (t-2T)u(t-2T) + (t-2T)u(t-2T) - (t-3T)u(t-3T) - (t-3T)u(t-3T)\cdots\cdots$$

$$V_C(t) = tu(t) - 2(t-T)u(t-T) + 2(t-2T)u(t-2T) - 2(t-3T)u(t-3T)\cdots\cdots$$

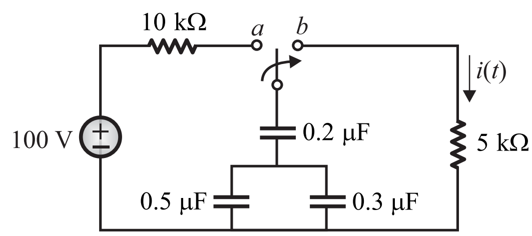
$$V_C(t) = tu(t) + 2\sum_{n=1}^{\infty}(-1)^n (t-nT)u(t-nT)$$

Hence, the correct option is (C).

Scan for Video Solution

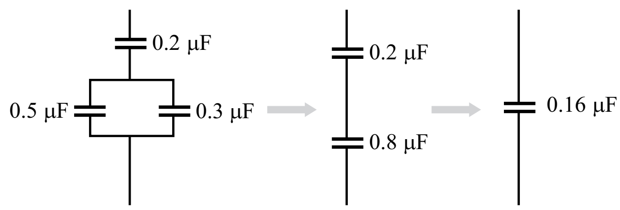
4.6 (B)

Given circuit is shown below,

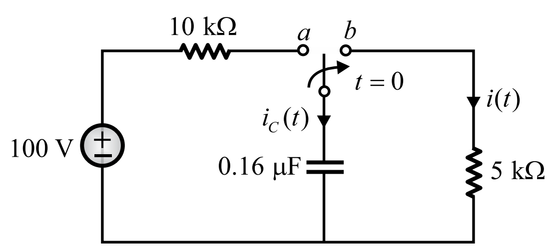


Method 1

Capacitor network can be simplified as,



The given network can be simplified as,



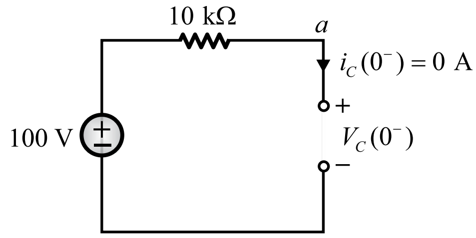
Voltage across capacitor is given by,

$$V_C(t) = V_C(\infty) + \left[V_C(0^+) - V_C(\infty)\right]e^{-t/\tau} \quad \ldots\text{(i)}$$

(i) At $t = 0^-$ / $t < 0$ / steady state :

Switch is at position a. In steady state, capacitor behaves as an open circuit.

The circuit will become as shown below,



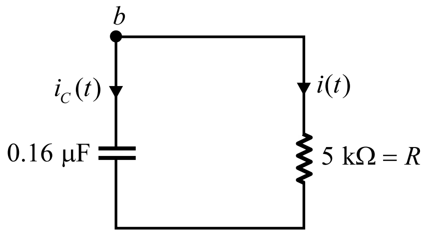
$V_C(0^-) = 100$ V

From property of capacitor,

$V_C(0^+) = V_C(0^-) = 100$ V

(ii) At $t \geq 0$ (Transient) :

Switch is at position *b*,



From figure,

$$i(t) = -i_C(t) \quad \ldots\text{(ii)}$$

For a *R-C* network, time constant,

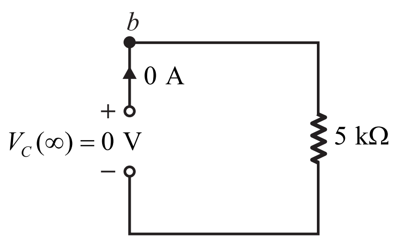
$$\tau = RC = 5\times10^{3}\times0.16\times10^{-6}$$

$$\tau = \frac{4}{5}\times10^{-3} = 0.8\times10^{-3}\text{ sec} = \frac{1}{1250}\text{sec}$$

(iii) At $t = \infty$ / steady state :

In steady state, capacitor behaves as an open circuit.

The circuit will become as shown below,



Put the values of $V_C(0^+), V_C(\infty)$ and τ in equation (i),

$$V_C(t) = 0 + [100 - 0]e^{-t/\tau} = 100e^{-t/\tau}$$

Capacitor current is given by,

$$i_C(t) = C\frac{d}{dt}V_C(t)$$

$$i_C(t) = C\frac{d}{dt}\left[100\,e^{-t/\tau}\right]$$

$$i_C(t) = -\frac{C}{\tau}100\,e^{-t/\tau} = -\frac{C}{RC}100e^{-t/\tau}$$

$$i_C(t) = -\frac{100}{R}e^{-t/\tau} \quad \ldots\text{(iii)}$$

Put the values of *R* and τ in equation (iii),

$$i_C(t) = \frac{-100}{5000}e^{\frac{-t\times5}{4\times10^{-3}}}$$

$$i_C(t) = -20e^{-1250t}u(t)\text{ mA}$$

From equation (ii),

$$i(t) = -i_C(t) = 20\,e^{-1250t}u(t)\text{ mA}$$

Hence, the correct option is (B).

Method 2

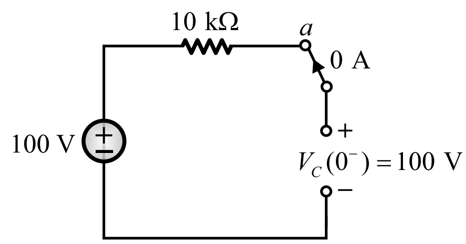
Current through capacitor is given by,

$$i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right]e^{-t/\tau} \quad \ldots\text{(i)}$$

(i) At $t = 0^-$ / $t < 0$ / steady state :

Switch is at position *a*, circuit will be in steady state and capacitor behaves as an open circuit.

The circuit will become as shown below,



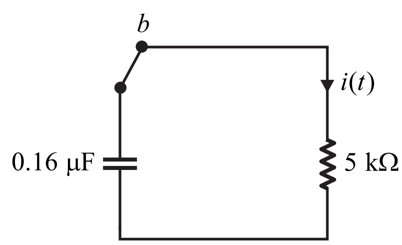
From property of capacitor,

$V_C(0^+) = V_C(0^-) = 100$ Volt

(ii) At $t \geq 0$ (Transient) :

Switch is at position '*b*'.

Circuit will become as shown below,



For a *R-C* network, Time constant,

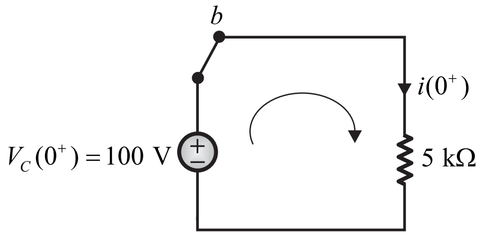
$$\tau = RC = 5\times10^{3}\times0.16\times10^{-6}$$

$$\tau = 0.8\times10^{-3}\ \text{sec}$$

(iii) At $t = 0^+$:

Switch is at position 'b'. Capacitors is replaced by a voltage source with initial value. i.e. $V_C(0^+) = V_C(0^-) = 100$ Volt

The circuit will become as shown below,



Applying KVL in the loop,

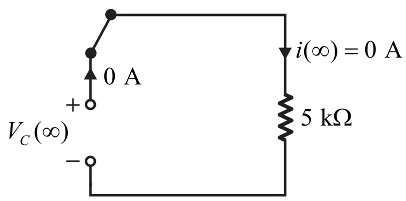
$$-100 + 5i(0^+) = 0$$

$$i(0^+) = 20\ \text{mA}$$

(iv) At $t = \infty$ / steady state :

In steady state, capacitor behaves as an open circuit.

The circuit will become as shown below,



Put the values of $i(0^+)$, $i(\infty)$ and τ in equation (i),

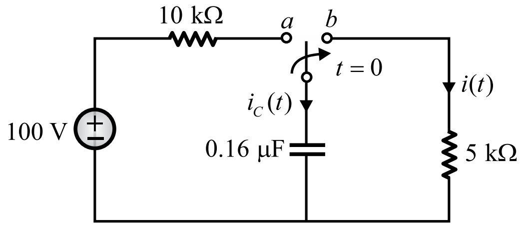
$$i_C(t) = i(t) = 0 + \left[20\times10^{-3} - 0\right]e^{\frac{-t}{0.8\times10^{-3}}}$$

$$i(t) = 20\,e^{-1250t}\,u(t)\ \text{mA}$$

Hence, the correct option is (B).

Method 3 : Direct calculation

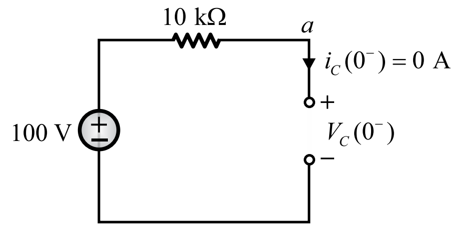
The given network can be simplified as,



(i) At $t = 0^-$ / $t < 0$ / steady state :

Switch is at position a, circuit will be in steady state and capacitor behaves as an open circuit.

The circuit will become as shown below,



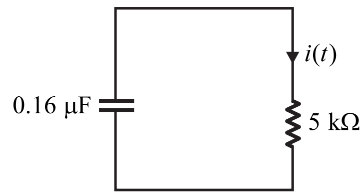
$$V_C(0^-) = 100\ \text{V}$$

From property of capacitor,

$$V_C(0^+) = V_C(0^-) = 100\ \text{V}$$

(ii) At $t \ge 0$ (Transient) :

Switch is moving to position b.

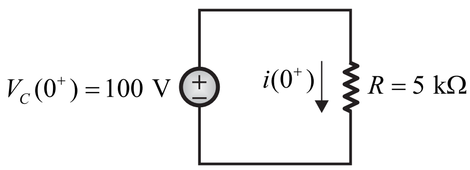


For R-C network, time constant

$$\tau = RC = 0.16\times5\times10^{-3} = \frac{1}{1250}\ \text{sec}$$

Hence, only option (B) or (C) is correct.

(iii) At $t = 0^+$:



$$i(0^+) = 20\ \text{mA}$$

Hence, the correct option is (B).

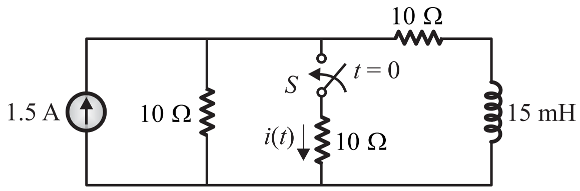
Key Point

(i) Current through the resistor does not follow the storing property i.e. $i(0^+) \neq i(0^-)$

(ii) Time constant is only defined for transient circuit i.e. for $t \ge 0$.

4.7 (A)

Given circuit is shown below,



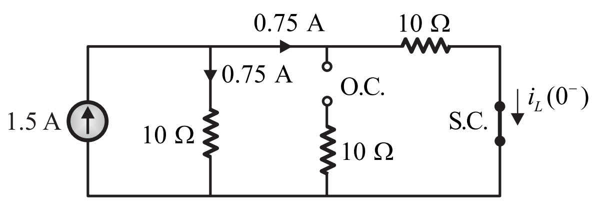
Current through 10 Ω resistor is given by,

$$i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right] e^{-t/\tau} \quad \ldots\text{(i)}$$

(i) At $t = 0^-$ / $t < 0$ / steady state :

In steady state, inductor behaves as a short circuit.

The circuit will become as shown below,



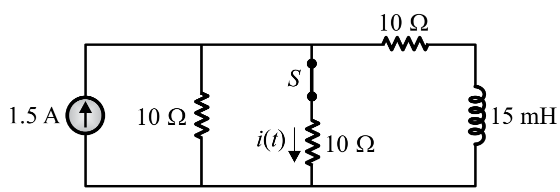
$$i_L(0^-) = \frac{10}{10+10} \times 1.5 = 0.75 \text{ A [By CDR]}$$

From property of inductor,

$$i_L(0^-) = i_L(0^+) = 0.75 \text{ A}$$

(ii) At $t \geq 0$ (Transient) :

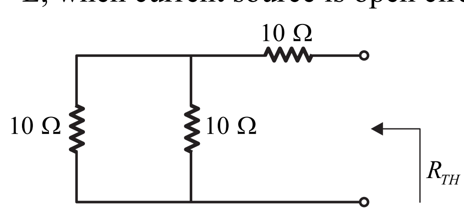
Circuit will become as shown below,



For a *R-L* network,

Time constant, $\tau = \dfrac{L}{R_{TH}}$

where, R_{TH} = Thevenin resistance across *L*, when current source is open circuited.



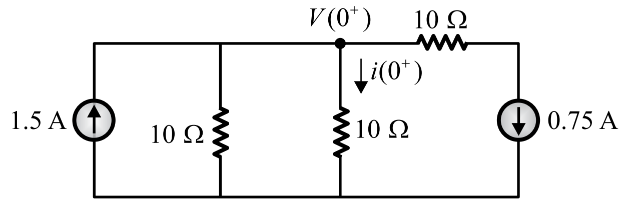
$$R_{TH} = (10 \parallel 10) + 10 = 15\ \Omega$$

$$\tau = \frac{15 \times 10^{-3}}{15} = 10^{-3} \text{ sec}$$

(iii) At $t = 0^+$:

Inductor is replaced by a current source with initial value i.e.

$$i_L(0^-) = i_L(0^+) = 0.75 \text{ A}$$



Applying KCL at node $V(0^+)$,

$$\frac{V(0^+)}{10} + \frac{V(0^+)}{10} - 1.5 + 0.75 = 0$$

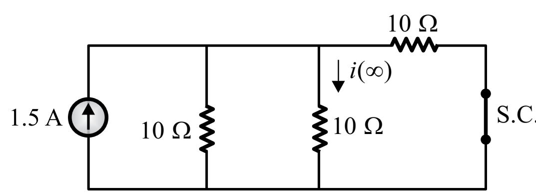
$$\frac{V(0^+)}{5} = 0.75$$

$$V(0^+) = 0.75 \times 5 = 3.75 \text{ V}$$

$$i(0^+) = \frac{V(0^+)}{10} = \frac{3.75}{10} = 0.375 \text{ A}$$

(iv) At $t = \infty$ / steady state :

Circuit will be in steady state and inductor behaves as a short circuited.



$$i(\infty) = \frac{1.5 \times 5}{10+5} = \frac{1.5 \text{ A}}{3} = 0.5 \text{ A [By CDR]}$$

Put the values of $i(0^+)$, $i(\infty)$ and τ in equation (i),

$$i(t) = 0.5 + \left[0.375 - 0.5\right] e^{-t/\tau}$$

$$i(t) = 0.5 - 0.125\, e^{\frac{-t \times 15}{15 \times 10^{-3}}}$$

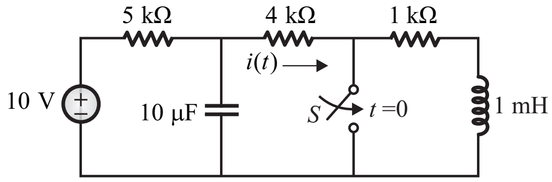
$$i(t) = 0.5 - 0.125\, e^{-1000t} \text{ A}$$

Hence, the correct option is (A).

Scan for Video Solution

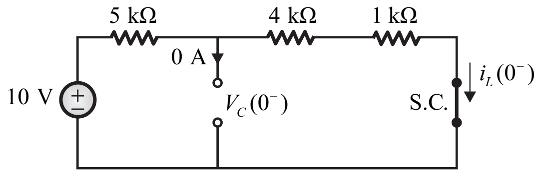
4.8　1.25

Given circuit is shown below,



(i)　At $t = 0^-$ / $t < 0$ / steady state :

In steady state, capacitor behaves as an open circuit and inductor behaves as a short circuit. Switch is in open position.



$$i_L(0^-) = \frac{10}{5+4+1} = 1 \text{ mA}$$

$$V_C(0^-) = \frac{5}{5+5} \times 10 = 5 \text{ volt}$$

From property of capacitor and inductor,

$$V_C(0^-) = V_C(0^+) = 5 \text{ V}$$

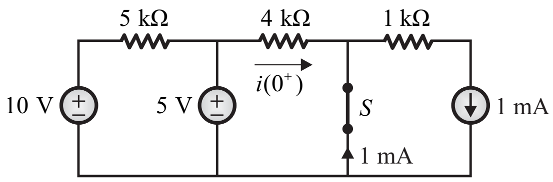
$$i_L(0^-) = i_L(0^+) = 1 \text{ mA}$$

(ii)　At $t = 0^+$:

Inductor is replaced by a current source with initial value and capacitor is replaced by a voltage source with initial value.

i.e. $V_C(0^-) = V_C(0^+) = 5$ V

$i_L(0^-) = i_L(0^+) = 1$ mA



Applying KVL in the loop shown,

$$-5 + 4 \times i(0^+) + 0 = 0$$

$$i(0^+) = \frac{5}{4} = 1.25 \text{ mA}$$

Hence, the magnitude of the current through the $4 \text{ k}\Omega$ resistor at $t = 0^+$ is **1.25 mA.**

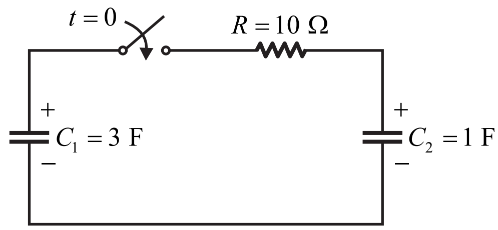
4.9　1.5

Given :

Initial voltage across C_1, $V_1(0^-) = 1$ V

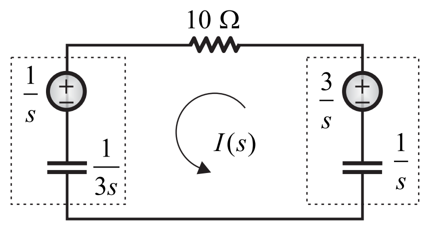
Initial voltage across C_2, $V_2(0^-) = 3$ V

Given circuit is shown below,



Method 1

Transform domain :



$$I(s) = \frac{\frac{3}{s} - \frac{1}{s}}{10 + \frac{1}{3s} + \frac{1}{s}} = \frac{\frac{2}{s}}{\frac{30s+1+3}{3s}}$$

$$I(s) = \frac{6}{30s+4} = \frac{6}{30\left(s + \frac{4}{30}\right)}$$

$$I(s) = \frac{0.2}{\left(s + \frac{2}{15}\right)}$$

Taking inverse Laplace transform,

$$i(t) = 0.2 e^{\frac{-2}{15}t}$$

Power dissipated in resistor is given by,

$$p(t) = i^2(t) R$$

$$p(t) = (0.2)^2 e^{\frac{-4t}{15}} \times 10 = 0.4 e^{\frac{-4t}{15}}$$

Total energy dissipated from $t = 0$ to $t = \infty$ is given by,

$$E = \int_0^\infty p\,dt = 0.4 \int_0^\infty e^{\frac{-4t}{15}}\,dt$$

$$E = 0.4\left[\frac{e^{\frac{-4t}{15}}}{\left(\frac{-4}{15}\right)}\right]_0^{\infty}$$

$$E = \frac{-0.4\times 15}{4}[0-1] = 1.5 \text{ Joules}$$

Hence, the total energy dissipated in the resistor R until steady state is reached, is **1.5 J.**

Method 2

Initial energy stored in capacitor,

$$= \frac{1}{2}(C_1V_1^2 + C_2V_2^2)$$

$$= \frac{1}{2}(3\times 1^2 + 1\times 3^2) = 6 \text{ J}$$

Final energy stored in capacitor,

$$= \frac{1}{2}(C_1 + C_2)V^2$$

[At steady state, voltage across two capacitor will be same]

Total charge before and after will be same.

$$Q(0^-) = Q(0^+)$$

i.e. $\quad C_1V_1 + C_2V_2 = (C_1 + C_2)V$

$$1\times 3 + 1\times 3 = (1+3)V$$

$$V = 1.5 \text{ V}$$

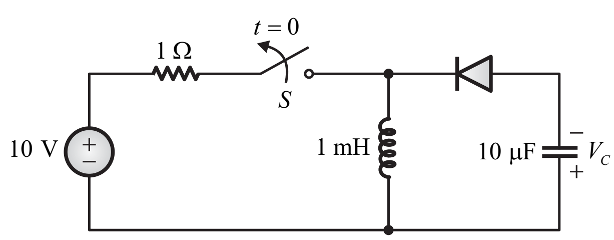
Final energy $= \frac{1}{2}(1+3)\times(1.5)^2 = 4.5$ J

Energy dissipated $= (6-4.5)$ J $= 1.5$ J

Hence, the total energy dissipated in the resistor R until steady state is reached, is **1.5 J.**

4.10 100

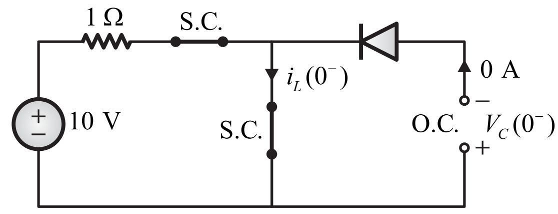
Given circuit is shown below,

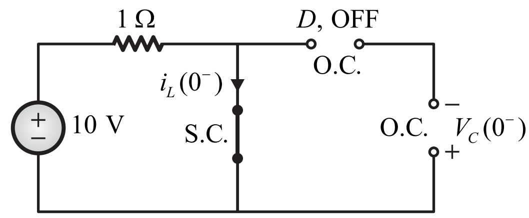


(i) At $t = 0^-$ / $t < 0$ / steady state :

Switch is at closed position.

In steady state, inductor will act as short circuit and capacitor will act as open circuit.





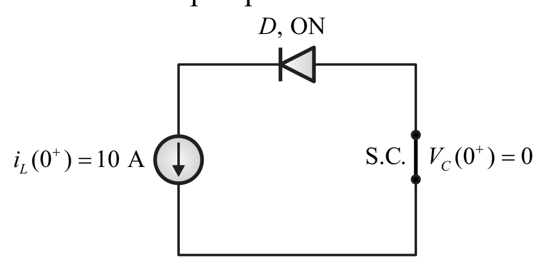
From figure,

$$i_L(0^-) = \frac{10}{1} = 10 \text{ A}$$

$$V_L(0^-) = 0 \text{ V and } V_C(0^-) = 0 \text{ V}$$

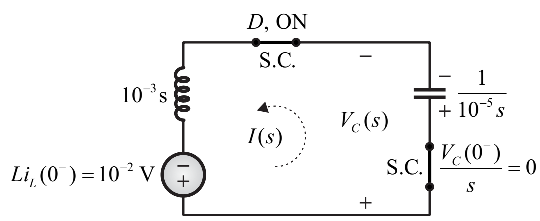
(ii) At $t = 0^+$:

Switch is at open position.



(iii) At $t \geq 0$ (Transient) :

Transform domain :



Applying KVL in above loop,

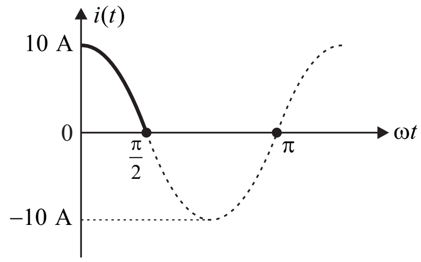
$$\frac{1}{10^{-5}s}I(s) + 10^{-3}sI(s) - 10^{-2} = 0$$

$$I(s) = \frac{10^{-2}}{10^{-3}s + \frac{10^5}{s}}$$

$$I(s) = \frac{10s}{s^2 + 10^8} \qquad \ldots\text{(i)}$$

Taking inverse Laplace transform,

$$i(t) = 10\cos 10^4 t$$



Diode will be ON for positive half cycle of $i(t)$

i.e. $0 < \omega t < \frac{\pi}{2}$.

After that at $\omega t = \frac{\pi}{2}$, as current becomes zero diode gets turned OFF.

$$V_C(s) = \frac{1}{10^{-5}s} \times I(s)$$

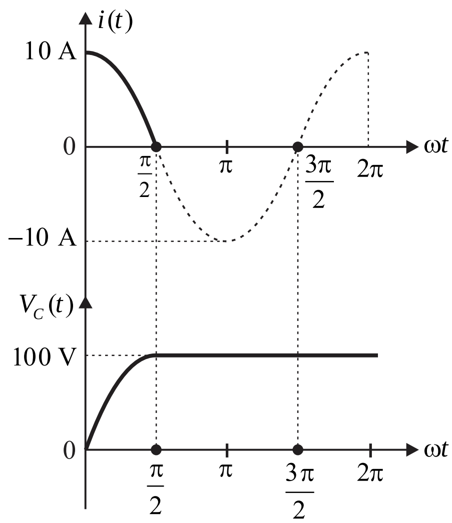
From equation (i),

$$V_C(s) = \frac{1}{10^{-5}s} \times \frac{10s}{s^2+10^8} = \frac{10^6}{s^2+10^8}$$

$$V_C(s) = \frac{10^2 \times 10^4}{s^2+(10^4)^2}$$

Taking inverse Laplace transform,

$$V_C(t) = 100\sin 10^4 t \text{ V}$$



At $\omega t = \frac{\pi}{2}$, when diode is turned OFF capacitor is charged to 100 V. Since there is no element present to discharge the capacitor, the capacitor remains charged to 100 V at steady state.

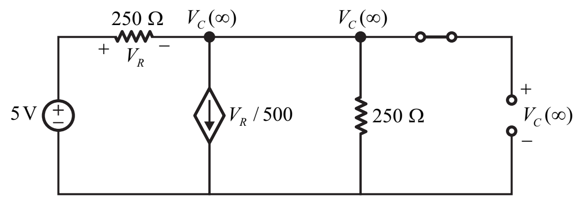
Hence, the steady state magnitude of the capacitor voltage V_C is **100 V.**

4.11 0.1386

Given, initial value of voltage across capacitor

$$V_C(0^-) = -5 \text{ V}$$

When switch is closed and circuit is in steady state (i.e. $t = \infty$, and capacitor becomes open circuit) So circuit becomes as,



(Circuit at steady state i.e. $t = \infty$)

Apply KCL at node $V_C(\infty)$ is,

$$\frac{V_C(\infty)}{250} + \frac{V_C(\infty)-5}{250} + \frac{V_R}{500} = 0$$

$$[\because V_R = V_C(\infty)]$$

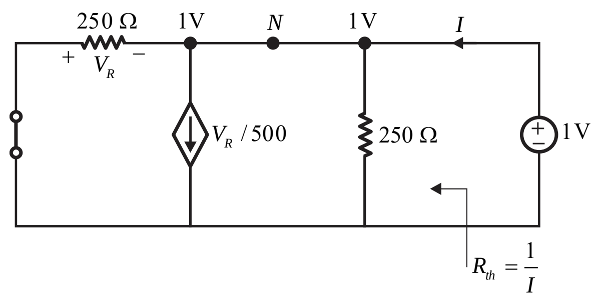
$$\frac{V_C(\infty)}{250} + \frac{V_C(\infty)-5}{250} + \frac{5-V_C(\infty)}{500} = 0$$

$$2V_C(\infty) + 2V_C(\infty) - 10 + 5 - V_C(\infty) = 0$$

$$3V_C(\infty) = 5$$

$$V_C(\infty) = \frac{5}{3} \text{ V} \quad \rightarrow \text{Steady state voltage of capacitor}$$

Now, for calculating time constant (τ), first we calculate R_{th} as shown below,



Apply KCL at node N,

$$\frac{1}{250} + \frac{1-0}{250} + \frac{V_R}{500} = I$$

$$I = \frac{1}{250} + \frac{1}{250} + \frac{-1}{500}$$

$$I = \frac{3}{500}$$

$$\therefore \qquad R_{th} = \frac{1}{I} = \frac{500}{3}\ \Omega$$

Thus, time constant, $\tau = R_{th} \times C$

$$\tau = \frac{500}{3} \times 0.6 \times 10^{-6}$$

$$\tau = 500 \times 0.2 \times 10^{-6} = 0.1 \text{ msec}$$

Apply transient equation for voltage across capacitor is,

$$V_C(t) = V_C(\infty) + [V_C(0) - V_C(\infty)]e^{-t/\tau},\ t \geq 0$$

$$V_C(t) = \frac{5}{3} + \left[-5 - \frac{5}{3}\right]e^{-t/\tau}$$

$$V_C(t) = \frac{5}{3} - \frac{20}{3}e^{-t/\tau} \qquad \ldots\text{(i)}$$

If $V_C(t) = 0$, then equations (i) becomes as,

$$0 = \frac{5}{3} - \frac{20}{3}e^{-t/\tau}$$

$$\frac{5}{3} = \frac{20}{3}e^{-t/\tau}$$

$$\frac{1}{4} = e^{-t/\tau}$$

$$e^{-t/\tau} = 0.25$$

$$\frac{-t}{\tau}\ln(e) = \ln(0.25)$$

$$t = -\tau \ln(0.25)$$

$$t = -0.1 \ln(0.25) = 0.1386 \text{ msec}$$

Hence, the correct answer of t is 0.1386 msec.

❖❖❖❖

7 Resonance

➢ Partial Synopsis

Series RLC Resonance Circuit

Applying KVL in the series RLC resonance circuit shown in figure,

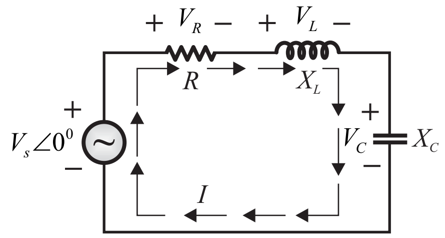


Fig. A series resonance circuit

$$V_s = V_R + j(V_L - V_C)$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Variation of current with change in frequency is shown in figure,

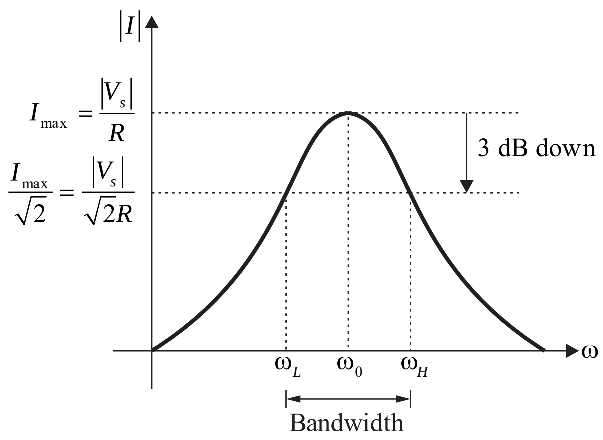


Fig. The variation of current amplitude with frequency for series RLC circuit

At resonance, voltage across resistor, inductor and capacitor are given respectively as,

$$V_R = I_R R = V_s \qquad V_L = j\omega_0 L I_R = j\omega_0 L\left(\frac{V_s}{R}\right)$$

$$V_C = \frac{1}{j\omega_0 C} I_R = -j\frac{1}{\omega_0 C}\left(\frac{V_s}{R}\right) \qquad V_L + V_C = 0$$

$$|V_L| = |V_C| = \frac{\omega_0 L |V_s|}{R} = \frac{|V_s|}{\omega_0 CR} \qquad |V_R| = |V_s|$$

At resonance, the voltage across inductor and capacitor are equal in magnitude and out of phase. Thus, the net voltage across inductor and capacitor combination is zero. Hence, the entire supply voltage appears across the resistance .

Impedance of a series resonant circuit :

The impedance of a series RLC circuit is given by,

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \qquad |Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Variation of impedance with change in frequency is shown in figure below,

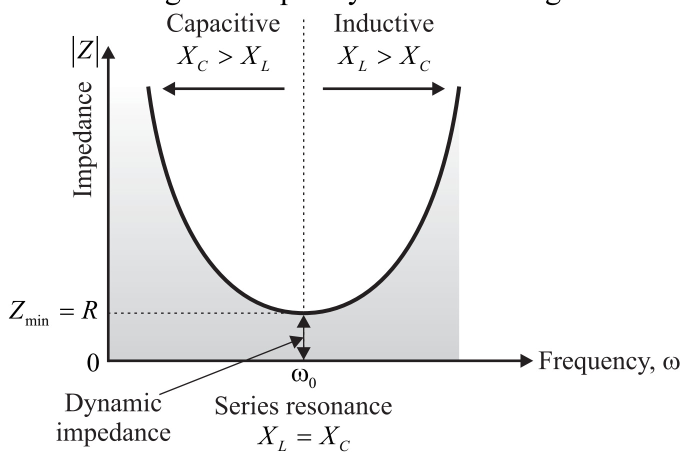


Fig. Variation of impedance *Z* with frequency in a series RLC circuit

The nature of circuit with change in frequency in case of series RLC circuit is given by,

Relationship between ω and ω_0	*R/L/C* behaviour	Power factor
$\omega < \omega_0$	*C*	Leading
$\omega = \omega_0$	*R*	Unity
$\omega > \omega_0$	*L*	Lagging

3 dB frequency :

The frequency at which current response will be 70.7% or $\frac{1}{\sqrt{2}}$ times or 0.707 times of its maximum value is referred as 3 dB frequency.

Higher and lower 3dB cut off frequencies of series RLC circuit are given by,

$$\omega_H = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \text{ rad/sec} \qquad \ldots\text{(i)}$$

$$\omega_L = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \text{ rad/sec} \qquad \ldots\text{(ii)}$$

ω	$\omega L-\dfrac{1}{\omega C}$	$\lvert Z\rvert=\sqrt{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}$	$\angle Z=\tan^{-1}\left(\dfrac{\omega L-\dfrac{1}{\omega C}}{R}\right)$	$I=\dfrac{V}{Z}$	PF
$\omega=\omega_H$ $\omega>\omega_0$	$\omega_H L-\dfrac{1}{\omega_H C}=+R$	$\lvert Z_H\rvert=\sqrt{2}R$	$\angle Z_H=45^0$	$I_H=\dfrac{V}{\sqrt{2}R}\angle-45^0$	0.707 lagging
$\omega=\omega_L$ $\omega<\omega_0$	$\omega_L L-\dfrac{1}{\omega_L C}=-R$	$\lvert Z_L\rvert=\sqrt{2}R$	$\angle Z_L=-45^0$	$I_L=\dfrac{V}{\sqrt{2}R}\angle 45^0$	0.707 leading
$\omega=\omega_0$	$\omega_0 L-\dfrac{1}{\omega_0 C}=0$	$Z_0=Z_{min}=R$	$\angle Z_0=0^0$	$I_{max}=\dfrac{V}{R}\angle 0^0$	UPF

➢ Sample Questions

2004 　IIT Delhi

8.1 Consider the following statements S1 and S2,

S1 : At the resonant frequency the impedance of a series *RLC* circuit is zero.

S2 : In a parallel *GLC* circuit, increasing the conductance *G* results in increase in its *Q* factor.

Which one of the following is correct ?

(A) S1 is FALSE and S2 is TRUE

(B) Both S1 and S2 are TRUE

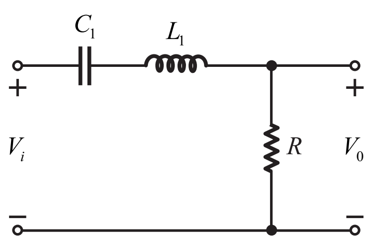
(C) S1 is TRUE and S2 is FALSE

(D) Both S1 and S2 are FALSE

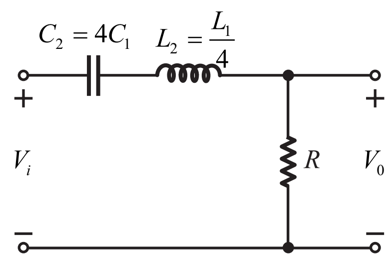
2007 　IIT Kanpur

8.2 Two series resonant filters are as shown in the figure. Let the 3-dB bandwidth of Filter 1 be B_1 and that of filter 2 be B_2.

The value of $\dfrac{B_1}{B_2}$ is



Filter 1



Filter 2

(A) 4　　(B) 1

(C) $\dfrac{1}{2}$　　(D) $\dfrac{1}{4}$

2015 　IIT Kanpur

8.3 An *LC* tank circuit consists of an ideal capacitors *C* connected in parallel with a coil of inductance *L* having an internal resistance *R*. The resonant frequency of the tank circuit is **[Set - 02]**

(A) $\dfrac{1}{2\pi\sqrt{LC}}$

(B) $\dfrac{1}{2\pi\sqrt{LC}}\sqrt{1-R^2\dfrac{C}{L}}$

(C) $\dfrac{1}{2\pi\sqrt{LC}}\sqrt{1-\dfrac{L}{R^2C}}$

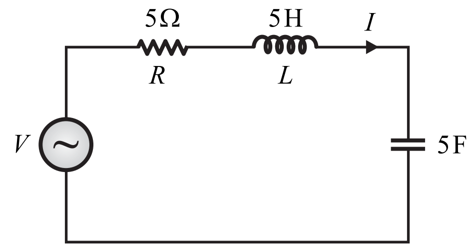
(D) $\dfrac{1}{2\pi\sqrt{LC}}\left(1-R^2\dfrac{C}{L}\right)$

2017 | IIT Roorkee

8.4 In the circuit shown V is a sinusoidal voltage source. The current I is in phase with voltage V. The ratio

$$\frac{\text{Amplitude of voltage across the capacitor}}{\text{Amplitude of voltage across the resistor}}$$

is _____. **[Set - 02]**



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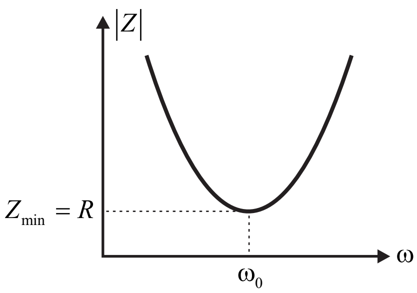
Explanations | Complex Power

8.1 (D)

Impedance characteristics of series RLC circuit shown below,



At resonance, $Z_{\min} = R$

So, S1 is false statement.

For parallel RLC/GLC circuit, Q-factor in terms of circuit elements (R, L and C) is given by,

$$Q = R\sqrt{\frac{C}{L}} = \frac{1}{G}\sqrt{\frac{C}{L}}$$

and $$Q \propto \frac{1}{G}$$

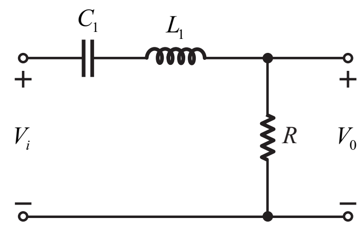
From above relation, when G increases then Q decreases if C and L are constant.

So, S2 is also false statement.

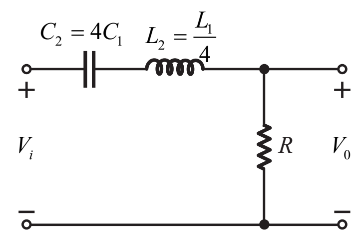
Hence, the correct option is (D).

8.2 (D)

Given circuit is shown below,



Filter 1



Filter 2

Bandwidth for series RLC circuit is given by,

$$B = \frac{R}{L} \text{ rad/sec}$$

For filter 1, $$B_1 = \frac{R}{L_1} \quad \ldots\text{(i)}$$

For filter 2, $$B_2 = \frac{R}{L_2}$$

Given : $$L_2 = \frac{L_1}{4}$$

$$B_2 = \frac{4R}{L_1} \quad \ldots\text{(ii)}$$

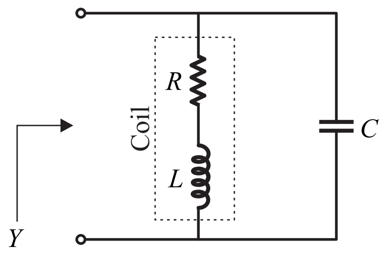
From equation (i) and (ii),

$$\frac{B_1}{B_2} = \frac{1}{4}$$

Hence, the correct option is (D).

8.3 (B)

According to the question, tank circuit is given by,



Admittance $Y = \dfrac{1}{R + j\omega L} + j\omega C$

$$Y = \frac{(R - j\omega L)}{R^2 + (\omega L)^2} + j\omega C$$

$$Y = \frac{R}{R^2 + (\omega L)^2} + j\omega\left[C - \frac{L}{R^2 + (\omega L)^2}\right]$$

At resonance, imaginary part of Y should be zero. Hence,

$$C - \frac{L}{R^2 + (\omega_0 L)^2} = 0$$

$$R^2 + (\omega_0 L)^2 = \frac{L}{C}$$

$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_0^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2$$

Resonance frequency is given by,

$$\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

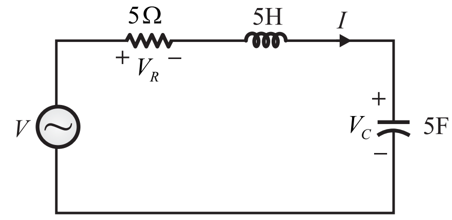
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{CR^2}{L}}$$

Hence, the correct option is (B).

8.4 0.2

Given circuit is shown below,



Method 1

Since, I is in phase with V, hence power factor angle ϕ will be zero (unity power factor) i.e. circuit is in resonance.

$$X_L = X_C \text{ (Condition of resonance)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25}} = \frac{1}{5} \text{ rad/sec}$$

$$X_C = \frac{1}{\omega_0 C} = \frac{1}{\frac{1}{5}\times 5} = 1\,\Omega$$

$$|V_C| = IX_C = I \text{ volts}$$

$$|V_R| = IR = 5I \text{ volts}$$

$$\left|\frac{V_C}{V_R}\right| = \frac{1}{5} = 0.2$$

Hence, the ratio $\left|\dfrac{V_C}{V_R}\right|$ is **0.2.**

Method 2

For series RLC resonance circuit,

$$V_C = QV\angle -90^0$$

$$V_R = V$$

$$\left|\frac{V_C}{V_R}\right| = Q$$

For series RLC network, Q-factor in terms of circuit elements (R, L and C) is given by,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Given : $R = 5\Omega$, $L = 5\,\text{H}$, $C = 5\,\text{F}$

$$Q = \frac{1}{5}\sqrt{\frac{5}{5}} = 0.2$$

$$Q = \left|\frac{V_C}{V_R}\right| = 0.2$$

Hence, the ratio $\left|\frac{V_C}{V_R}\right|$ is **0.2.**

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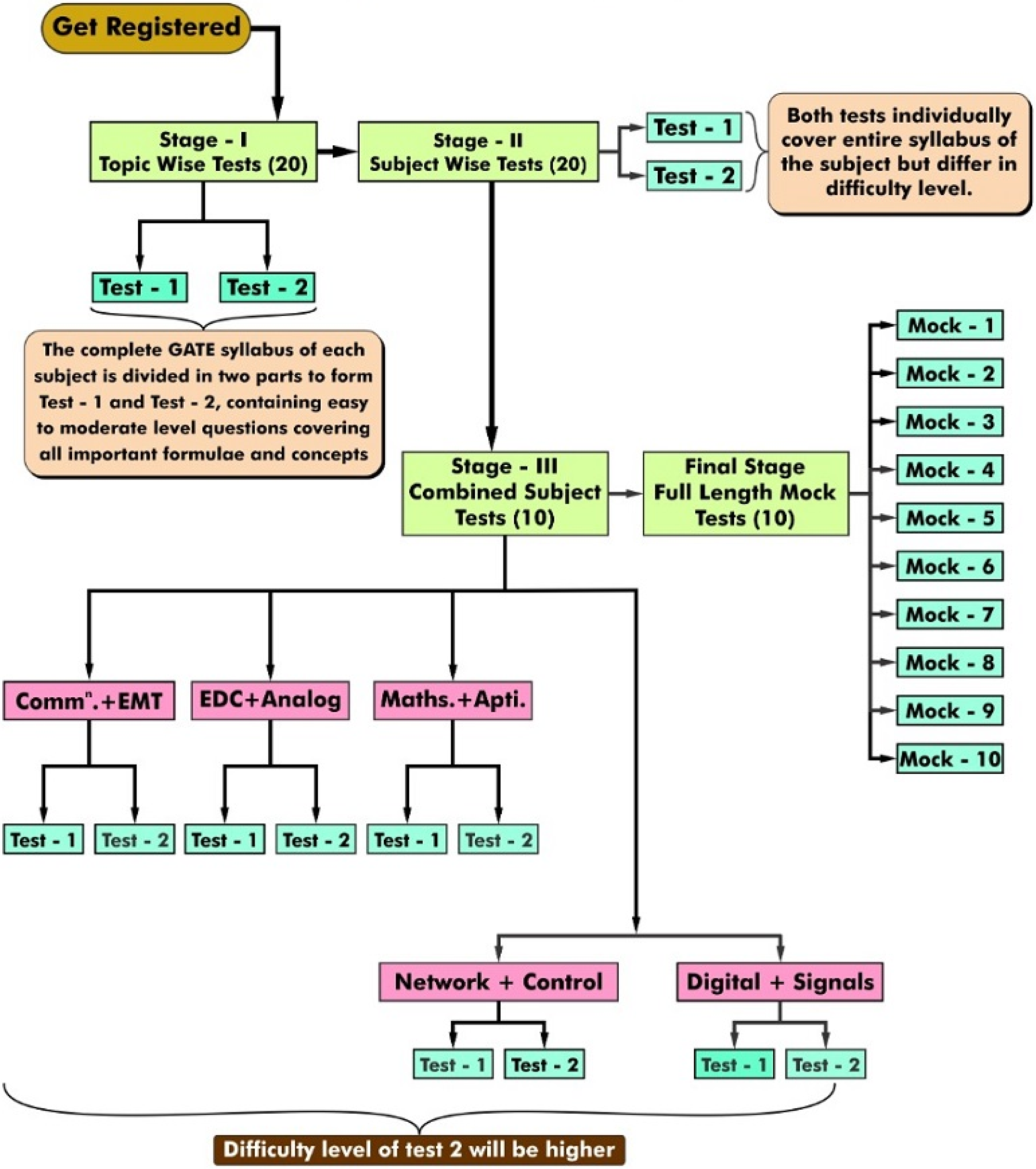
(One Year Program)



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Deepak Kumar


EE-27
Nilay Srivastava


EE-46
Ajay Kumar Gupta


EE-46
Ramraja Patel


EE-46
Rahul Chauhan


EE-53
Pritam Saurav

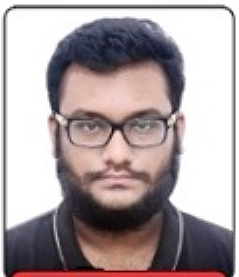

EE-53
Arihant Jain


EE-53
B Anurag


EE-70
Lokesh Garg


EE-86
Divakar Chaudhary


EE-86
Tushar Chaudhary


EE-93
Ashutosh Singh


EE-108
Sanket Tekawade


EE-127
Subham Dutta


EE-137
Mona Kumari

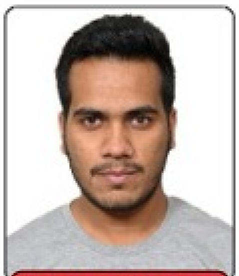

EE-146
Divyansh Trivedi


EE-146
Abantika Sengupta


EE-170
Hritik Jaiswal


EE-170
Aman Kumar


EE-170
Wasim Ahmed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Nikhit Yerne


EE-184
Anant Kumar


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Anagha Paul


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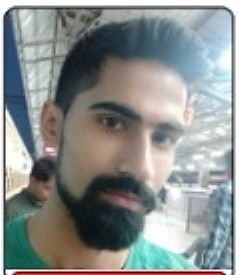
Ashwin Kumar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Ashish Jhamnani


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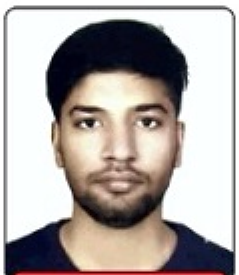
Nimesh Nahar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Shagun Garg


EE-259
Manikarna Indurthy


EE-268
K Srikanth

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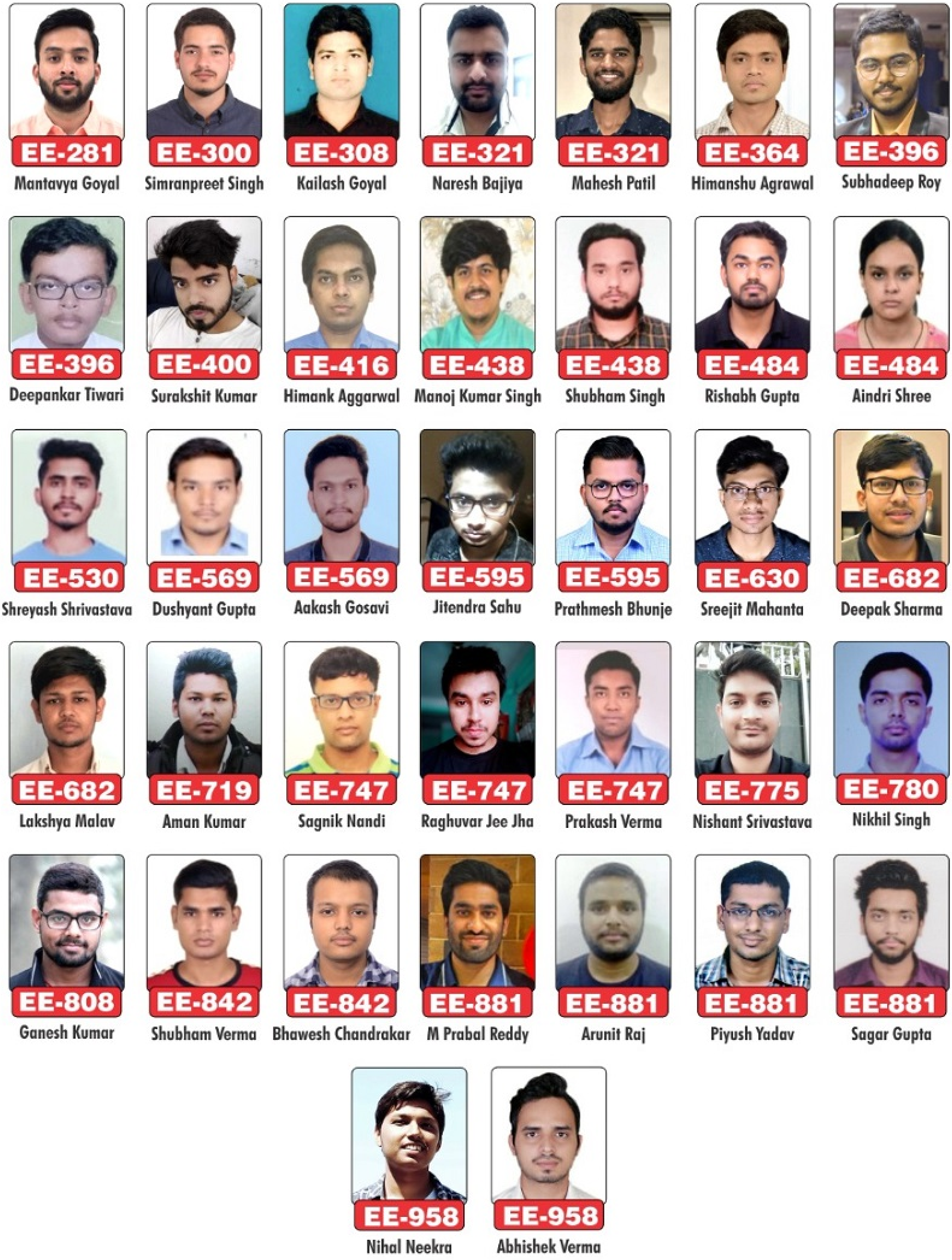


CHAPTER 2 | SIGNALS & SYSTEMS

Marks Distribution of Signals & Systems in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	4	4	12
2004	3	6	15
2005	6	6	18
2006	3	3	9
2007	1	4	9
2008	2	8	18
2009	3	5	13
2010	3	2	7
2011	3	4	11
2012	2	3	8
2013	6	3	12
2014 Set-1	3	4	11
2014 Set-2	3	3	9
2014 Set-3	4	3	10

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-4	4	4	12
2015 Set-1	2	3	8
2015 Set-2	5	4	13
2015 Set-3	4	5	14
2016 Set-1	4	4	12
2016 Set-2	1	2	5
2016 Set-3	3	4	11
2017 Set-1	3	4	11
2017 Set-2	2	4	10
2018	3	3	9
2019	3	3	9
2020	2	2	6
2021	2	3	8

Syllabus : Signals & Systems

Continuous-time signals : Fourier series and Fourier transform, sampling theorem and applications. Discrete-time signals : DTFT, DFT, z-transform, discrete-time processing of continuous-time signals. LTI systems : definition and properties, causality, stability, impulse response, convolution, poles and zeroes, frequency response, group delay, phase delay.

Contents : Signals & Systems

1 Basics of Signals

➢ Partial Synopsis

Energy signals : Energy signals are signals which have finite energy. They are mostly defined for non-periodic or finite duration signals.

In continuous domain,

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)\,dt = \int_{-\infty}^{\infty} x^2(t)\,dt \quad \text{where } x(t) \text{ is a real signal}$$

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2\,dt = \int_{-\infty}^{\infty} |x(t)|^2\,dt \quad \text{where } x(t) \text{ is a complex signal}$$

In discrete domain,

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} x^2(n) = \sum_{n=-\infty}^{\infty} x^2(n) \quad \text{where } x(n) \text{ is a real signal}$$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2 = \sum_{n=-\infty}^{\infty} |x(n)|^2 \quad \text{where } x(n) \text{ is a complex signal}$$

Power signals : Power signals are signals which have finite power. They are mostly defined for periodic signals. All periodic signals are power signals but all power signals are not periodic.

In continuous domain,

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt \quad \text{where } x(t) \text{ is a real signal}$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\,dt \quad \text{where } x(t) \text{ is a complex signal}$$

In discrete domain,

$$P = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=-N}^{N} x^2(n) \quad \text{where } x(n) \text{ is a real signal}$$

$$P = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=-N}^{N} |x(n)|^2 \quad \text{where } x(n) \text{ is a complex signal}$$

Relation between energy and power signals :

In continuous domain, $P = \lim_{T\to\infty} \frac{E}{T}$ In discrete domain, $P = \lim_{N\to\infty} \frac{E}{2N+1}$

RMS value $= \sqrt{\text{Power}} = \dfrac{\text{Energy in one period}}{\text{Time period}}$

Properties of energy signals :

1. Energy signals always consists finite energy, $0 < E < \infty$
2. Power of energy signal is zero, $P = 0$
3. If $x(t)$ is an even / odd function then energy in left and right halves of $x(t)$ will be same. i.e.

   Energy (L.H.S.) = Energy (R.H.S.), Total energy = 2Energy (L.H.S.) = 2Energy (R.H.S.)
4. $x(t) = x_e(t) + x_0(t)$ $E[x(t)] = E[x_e(t)] + E[x_0(t)]$

Properties of power signals :

1. Power signals always consists finite power, $0 < P < \infty$
2. Energy of power signal is infinite, $E = \infty$
3. For any periodic signal, $P[\text{L.H.S.}] = P[\text{R.H.S.}]$

Energy and Power of Some Important Signals

Signals	Energy	Power	Area	Nature
$e^{-at}u(t),\ e^{at}u(-t)$	$E = \frac{1}{2a}$	$P = 0$	$A = \frac{1}{a}$	Energy signal
$e^{-a\lvert t\rvert}$	$E = \frac{1}{a}$	$P = 0$	$A = \frac{2}{a}$	Energy signal
$u(t),\ u(n)$	$E = \infty$	$P = \frac{1}{2}$	$A = \infty$	Power signal
$A\,\text{rect}\left(\frac{t}{\tau}\right)$	$E = A^2\tau$	$P = 0$	$A = A\tau$	Energy signal
$A\,\text{tri}\left(\frac{t}{\tau}\right)$	$E = \frac{2}{3}A^2\tau$	$P = 0$	$A = A\tau$	Energy signal
$\text{sinc}(t)$	$E = 1$	$P = 0$	$A = 1$	Energy signal
$Sa(t)$	$E = \pi$	$P = 0$	$A = \pi$	Energy signal
$A\cos\omega t,\ A\sin\omega t,$ $A\cos(\omega t + \phi),\ A\sin(\omega t + \phi)$	$E = \infty$	$P = \frac{A^2}{2}$	$A = 0$	Power signal
$Ae^{j\omega t},\ Ae^{j(\omega t + \phi)}$	$E = \infty$	$P = A^2$	$A = 0$	Power signal
$e^{-\pi t^2}$	$E = \frac{1}{\sqrt{2}}$	$P = 0$	$A = 1$	Energy signal
$\delta(t)$	$E = \infty$	P = Undefined	$A = 1$	Neither Energy nor Power signal

➢ Sample Questions

1992 IIT Delhi

1.1 Which of the following signals is/are periodic?

(A) $s(t) = \cos 2t + \cos 3t + \cos 5t$

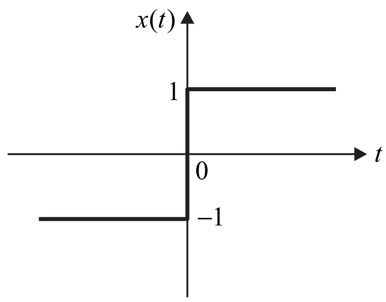
(B) $s(t) = \exp(j8\pi t)$

(C) $s(t) = \exp(-7t)\sin 10\pi t$

(D) $s(t) = \cos 2t \cos 4t$

2005 IIT Bombay

1.2 The function $x(t)$ is shown in the figure. Even and odd parts of a unit-step function $u(t)$ are respectively,



(A) $\frac{1}{2}, \frac{1}{2}x(t)$ (B) $-\frac{1}{2}, \frac{1}{2}x(t)$

(C) $\frac{1}{2}, -\frac{1}{2}x(t)$ (D) $-\frac{1}{2}, -\frac{1}{2}x(t)$

1.3 The power of the signal

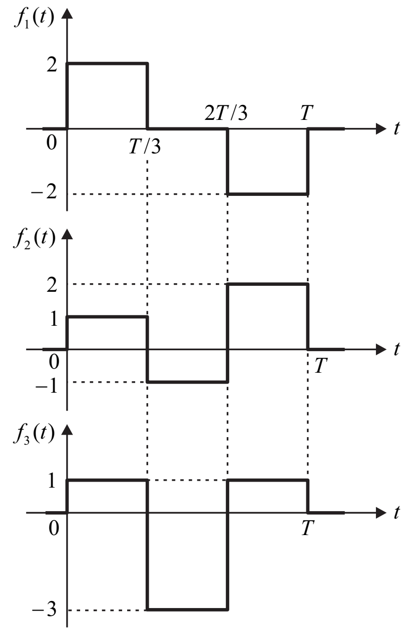
$$s(t) = 8\cos\left(20\pi t - \frac{\pi}{2}\right) + 4\sin(15\pi t) \text{ is}$$

(A) 40 (B) 41

(C) 42 (D) 82

2007 IIT Kanpur

1.4 Three functions $f_1(t)$, $f_2(t)$ and $f_3(t)$, which are zero outside the interval $[0, T]$ are shown in the figure. Which of the following statements is correct?



(A) $f_1(t)$ and $f_2(t)$ are orthogonal

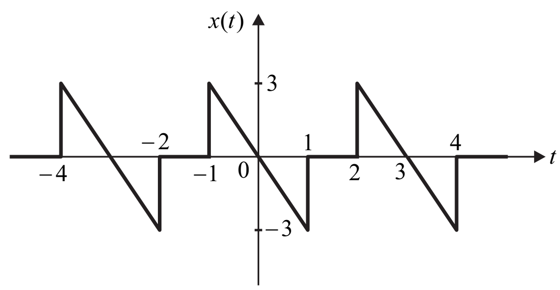
(B) $f_1(t)$ and $f_3(t)$ are orthogonal

(C) $f_2(t)$ and $f_3(t)$ are orthogonal

(D) $f_1(t)$ and $f_2(t)$ are orthonormal

2015 IIT Kanpur

1.5 The waveform of a periodic signal $x(t)$ is shown in the figure.



A signal $g(t)$ is defined by $g(t) = x\left(\frac{t-1}{2}\right)$. The average power of $g(t)$ is ________. **[Set - 01]**

❖❖❖❖

Explanations Basics of Signals

1.1 (A), (B) and (D)

Given :

Option (A) :

$$s(t) = \cos 2t + \cos 3t + \cos 5t$$

The combination of periodic signals will be periodic, if the ratio of any two frequencies (or any two time periods) is a rational number, i.e. ratio can be represented in the form of $\frac{p}{q}$, where p and q are integers.

Here, $\omega_1 = 2\,\text{rad/sec}$, $\omega_2 = 3\,\text{rad/sec}$,

$$\omega_3 = 5\,\text{rad/sec}$$

$$\frac{\omega_1}{\omega_2} = \frac{2}{3},\ \frac{\omega_1}{\omega_3} = \frac{2}{5},\ \frac{\omega_2}{\omega_3} = \frac{3}{5}$$

Hence, it is a periodic signal.

Option (B) :

$$s(t) = \exp(j8\pi t)$$

$$s(t) = \cos(8\pi t) + j\sin(8\pi t)$$

Here, $\omega_1 = 8\pi$, $\omega_2 = 8\pi$

$$\frac{\omega_1}{\omega_2} = \frac{8\pi}{8\pi} = 1$$

Hence, it is periodic.

Option (C) :

$$s(t) = \exp(-7t)\sin 10\pi t$$

This is a decreasing exponential signal.

Hence, obviously it is non periodic.

Option (D) :

$$s(t) = \cos 2t \cos 4t$$

Since, $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

So, $\cos 2t \cos 4t = \frac{1}{2}[\cos 2t + \cos 6t]$

Here, $\omega_1 = 2\,\text{rad/sec}$, $\omega_2 = 6$ rad/sec

$$\frac{\omega_1}{\omega_2} = \frac{2}{6}$$

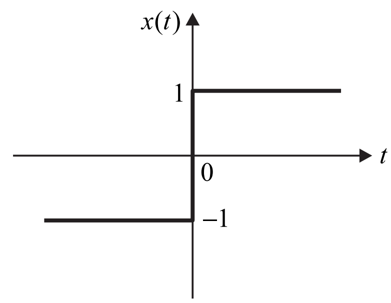
Thus, it is also a periodic signal.

Hence, the correct options are (A), (B) and (D).

Scan for Video Solution

1.2 (A)

Given :



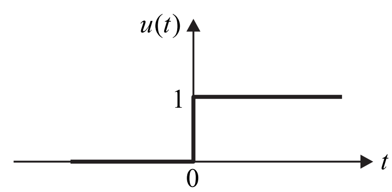
For any signal $y(t)$, even part and odd part is given by $y_e(t)$ and $y_0(t)$.

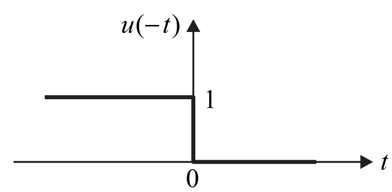
where, $y_e(t) = \frac{y(t) + y(-t)}{2}$

and $y_0(t) = \frac{y(t) - y(-t)}{2}$

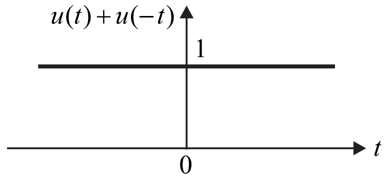
According to the question,

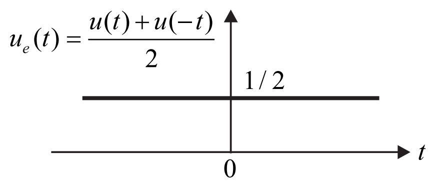
$$y(t) = u(t)$$





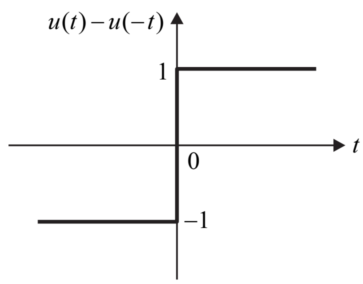
For even part, $u_e(t) = \frac{u(t) + u(-t)}{2}$

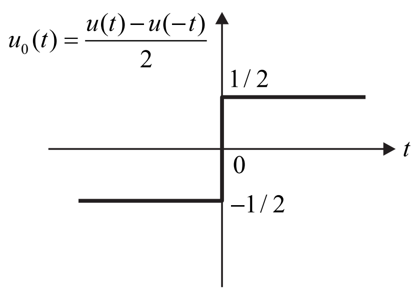


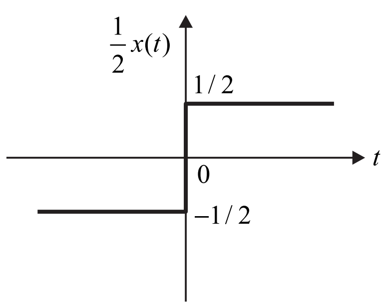


Thus, $u_e(t) = \dfrac{u(t)+u(-t)}{2} = \dfrac{1}{2}$

For odd part $u_0(t) = \dfrac{u(t)-u(-t)}{2}$







Therefore, $u_0(t) = \dfrac{u(t)-u(-t)}{2} = \dfrac{1}{2}x(t)$

Hence, the correct option is (A).

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1.3 (A)

Given :

$$s(t) = 8\cos\left(20\pi t - \frac{\pi}{2}\right) + 4\sin(15\pi t)$$

$$s(t) = 8\sin(20\pi t) + 4\sin(15\pi t)$$

Let $s(t) = x_1(t) + x_2(t)$

where, $x_1(t) = 8\sin(20\pi t)$

and $x_2(t) = 4\sin(15\pi t)$

Let power of $x_1(t)$ is P_1 and power of $x_2(t)$ is P_2. Signals with different frequencies are orthogonal to each other.

Thus, $x_1(t)$ and $x_2(t)$ are orthogonal to each other, therefore total power is given by addition of P_1 and P_2.

i.e. $P = P_1 + P_2$

where, $P_1 = \dfrac{A_1^2}{2} = \dfrac{8^2}{2} = 32$ Watt

$$P_2 = \frac{A_2^2}{2} = \frac{4^2}{2} = 8 \text{ Watt}$$

$$P = (32+8) = 40 \text{ Watt}$$

Hence, the correct option is (A).

Key Point

Signal	Power
$A\sin\omega t$ / $A\cos\omega t$	$P = \dfrac{A^2}{2}$
$A\sin(\omega t+\phi)$ / $A\cos(\omega t+\phi)$	$P = \dfrac{A^2}{2}$
$A_1\cos(\omega_1 t+\phi_1) + B\cos(\omega_2 t+\phi_2)$, $\omega_1 \neq \omega_2$	$P = \dfrac{A^2+B^2}{2}$
$A_1\cos(\omega_1 t+\phi_1) + B\cos(\omega_2 t+\phi_2)$, $\omega_1 = \omega_2$	$P = \dfrac{A^2+B^2+2AB\cos(\phi_1-\phi_2)}{2}$

1.4 (B)

Two functions $u(t)$ and $v(t)$ are said to be orthogonal in interval (a, b),

$$\int_a^b u(t)\,v^*(t)\,dt = 0$$

The function $f_1(t)$, $f_2(t)$ and $f_3(t)$ are real functions and defined only in the interval (0, T).

(i) To check $f_1(t)$ and $f_3(t)$:

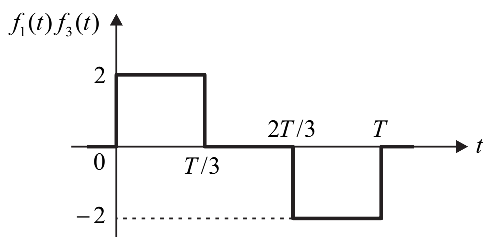
Since, $f_3(t)$ is a real function,

Therefore,

$$f_3^*(t) = f_3(t) \qquad \text{(Real function)}$$

$$I = \int_0^T f_1(t)\,f_3^*(t)\,dt = \int_0^T f_1(t)\,f_3(t)\,dt$$

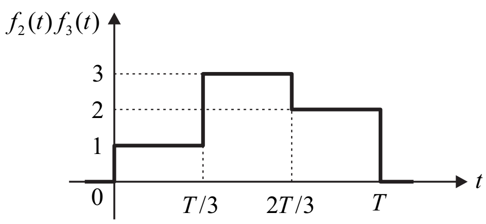
$I =$ [Area under the product of $f_1 f_3$ from 0 to T]



$$I = \left[2\times\frac{T}{3} - 2\times\left(T - \frac{2T}{3}\right)\right] = 0$$

Hence, $f_1(t)$ and $f_3(t)$ are orthogonal.

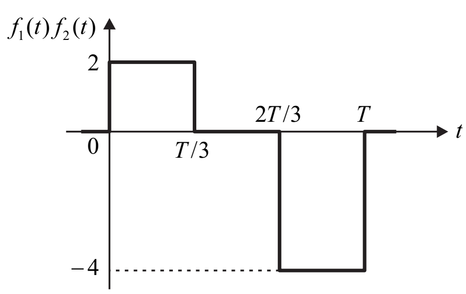
(ii) To check $f_2(t)$ and $f_3(t)$:



$$I = \left[1\times\frac{T}{3} + 3\times\frac{T}{3} + 2\times\frac{T}{3}\right] = 2T$$

Hence, $f_2(t)$ and $f_3(t)$ are not orthogonal.

(iii) To check $f_1(t)$ and $f_2(t)$:



$$I = \left[2\times\frac{T}{3} - 4\times\frac{T}{3}\right] = -\frac{2T}{3}$$

Hence, $f_1(t)$ and $f_2(t)$ are not orthogonal.

$$\int_0^T [f_1(t)]^2\,dt = \text{Energy of } f_1(t)$$

$$\int_0^T [f_1(t)]^2\,dt = 4\times\frac{T}{3} + 4\times\frac{T}{3} = \frac{8T}{3}$$

$$\int_0^T [f_2(t)]^2\,dt = \text{Energy of } f_2(t)$$

$$\int_0^T [f_2(t)]^2\,dt = 1\times\frac{T}{3} + 1\times\frac{T}{3} + 4\times\frac{T}{3} = 2T$$

$$\int_0^T [f_3(t)]^2\,dt = \text{Energy of } f_3(t)$$

$$\int_0^T [f_3(t)]^2\,dt = 1\times\frac{T}{3} + 9\times\frac{T}{3} + 1\times\frac{T}{3} = \frac{11T}{3}$$

But $\int_0^T [f_1(t)]^2 \neq 1$ and $\int_0^T [f_3(t)]^2 \neq 1$

So, they are not orthonormal.

Hence, the correct option is (B).

Key Point

(i) If $\int_a^b u(t)\,v^*(t)dt = 0$

Then, $u(t)$ and $v(t)$ are said to be orthogonal.

(ii) If $\int_a^b u(t)\,v^*(t)dt = 0$

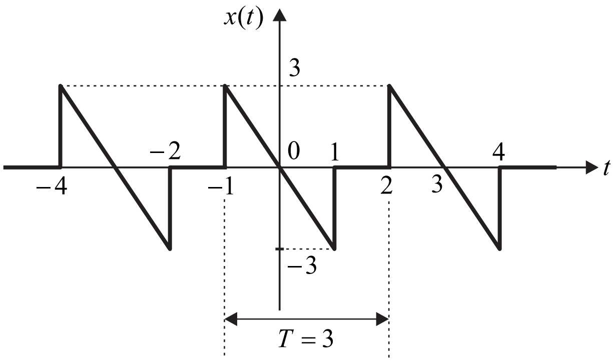
and $\int_a^b |u(t)|^2\,dt = \int_a^b |v(t)|^2\,dt = 1$

Then, $u(t)$ and $v(t)$ are said to be orthonormal.

1.5	2

Method 1

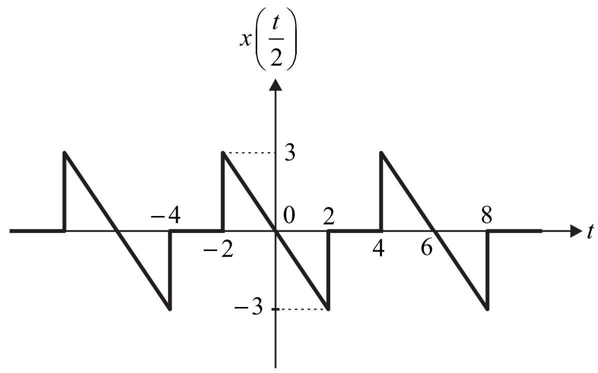
Given :



$$g(t) = x\left(\frac{t-1}{2}\right) = x\left[\frac{1}{2}(t-1)\right]$$

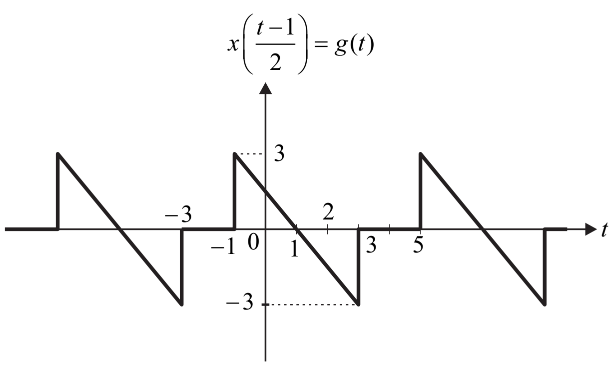
Using time scaling property :

$$x(t) \xrightarrow{t=t/2} x\left(\frac{t}{2}\right) \quad \text{[Time expansion]}$$



Using time shifting property :

$$x\left(\frac{t}{2}\right) \xrightarrow{t=t-1} x\left(\frac{t-1}{2}\right) \quad \text{[Right shift]}$$



$$P_{avg}[g(t)] = \frac{1}{6}\int_{-1}^{5}[g(t)]^2 dt$$

$$P_{avg}[g(t)] = \frac{1}{6}\int_{-1}^{3}\left[\frac{6}{4}(1-t)\right]^2 dt$$

$$P_{avg}[g(t)] = \frac{1}{6}\times\frac{36}{16}\int_{-1}^{3}(t^2-2t+1)\,dt$$

$$P_{avg}[g(t)] = \frac{3}{8}\left[\frac{t^3}{3}-\frac{2t^2}{2}+t\right]_{-1}^{3}$$

$$P_{avg}[g(t)] = \frac{3}{8}\left[9+\frac{1}{3}-9+1+3+1\right]$$

$$P_{avg}[g(t)] = \frac{3}{8}\left[\frac{16}{3}\right] = 2\ \text{W}$$

Hence, the average power of $g(t)$ is **2 W.**

Method 2

Power of any signal is only affected by amplitude scaling and is NOT affected by amplitude inversion, time inversion, time shifting and time scaling.

Thus, power of $g(t) =$ Power of $x(t)$

i.e. $$P\left[x\left(\frac{t-1}{2}\right)\right] = P[x(t)] \quad \ldots\text{(i)}$$

$$P[x(t)] = \frac{1}{T}\int_{-T/2}^{T/2}|x(t)|^2 dt$$

$$P[x(t)] = \frac{1}{3}\left[\int_{-1}^{1}\left[\frac{-6}{2}t\right]^2 dt + \int_{1}^{2}0^2\,dt\right]$$

$$P[x(t)] = \frac{1}{3}\left[9\int_{-1}^{1}t^2\,dt\right] = 3\left[\frac{t^3}{3}\right]_{-1}^{1}$$

$$P[x(t)] = 1+1 = 2\ \text{W}$$

From equation (i),

$$P\left[x\left(\frac{t-1}{2}\right)\right] = P[x(t)] = 2\ \text{W}$$

Hence, the average power of $g(t)$ is **2 W.**

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Video Solution

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3 Laplace Transform

➢ Partial Synopsis

9. Time Convolution : Convolution of two signals in time domain is equivalent to their multiplication in s-domain.

For both Bilateral and Unilateral Laplace transform :

$$x_1(t) \xleftrightarrow{LT} X_1(s) \qquad \text{ROC} : R_1$$
$$x_2(t) \xleftrightarrow{LT} X_2(s) \qquad \text{ROC} : R_2$$
$$x_1(t) \otimes x_2(t) \xleftrightarrow{LT} X_1(s).X_2(s) \qquad \text{ROC} : R_1 \cap R_2$$

10. S-domain Convolution : Multiplication of two signals in time domain is equivalent to $1/2\pi j$ times their convolution in s-domain.

For both Bilateral and Unilateral Laplace transform :

$$x_1(t) \xleftrightarrow{LT} X_1(s) \qquad \text{ROC} : R_1$$
$$x_2(t) \xleftrightarrow{LT} X_2(s) \qquad \text{ROC} : R_2$$
$$x_1(t).x_2(t) \xleftrightarrow{LT} \frac{1}{2\pi j}[X_1(s) \otimes X_2(s)] \qquad \text{ROC} : R_1 \cap R_2$$

11. Conjugate :

$$x(t) \xleftrightarrow{LT} X(s) \qquad \text{ROC} : R$$
$$x^*(t) \xleftrightarrow{LT} X^*(s^*) \qquad \text{ROC} : R$$

Basic Laplace Transform Pairs

S.	$x(t)$	$X(s)$	ROC
1.	$\delta(t)$	1	Entire s-plane
2.	$u(t)$	$\frac{1}{s}$	$\text{Re}(s) > 0$
3.	$-u(-t)$	$\frac{1}{s}$	$\text{Re}(s) < 0$
4.	$e^{-at}u(t)$	$\frac{1}{s+a}$	$\text{Re}(s) > -a$

5.	$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\text{Re}(s) < -a$
6.	$e^{at}u(t)$	$\frac{1}{s-a}$	$\text{Re}(s) > a$
7.	$-e^{at}u(-t)$	$\frac{1}{s-a}$	$\text{Re}(s) < a$
8.	$r(t) = t.u(t)$	$\frac{1}{s^2}$	$\text{Re}(s) > 0$
9.	$t^n.u(t)$	$\frac{n!}{s^{n+1}}$	$\text{Re}(s) > 0$
10.	$t.e^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\text{Re}(s) > -a$
11.	$t^n.e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$	$\text{Re}(s) > -a$
12.	$e^{-at}u(t-b)$	$\frac{e^{-(s+a)b}}{s+a}$	$\text{Re}(s) > -a$
13.	$e^{-a(t-b)}u(t-b)$	$\frac{e^{-sb}}{s+a}$	$\text{Re}(s) > -a$
14.	$\cos\omega_0 t.u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\text{Re}(s) > 0$
15.	$-\cos\omega_0 t.u(-t)$	$\frac{s}{s^2+\omega_0^2}$	$\text{Re}(s) < 0$
16.	$\sin\omega_0 t.u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\text{Re}(s) > 0$
17.	$-\sin\omega_0 t.u(-t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\text{Re}(s) < 0$
18.	$e^{-at}\cos\omega_0 t.u(t)$	$\frac{(s+a)}{(s+a)^2+\omega_0^2}$	$\text{Re}(s) > -a$
19.	$-e^{-at}\cos\omega_0 t.u(-t)$	$\frac{(s+a)}{(s+a)^2+\omega_0^2}$	$\text{Re}(s) < -a$
20.	$e^{-at}\sin\omega_0 t.u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\text{Re}(s) > -a$
21.	$-e^{-at}\sin\omega_0 t.u(-t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\text{Re}(s) < -a$
22.	1	Bilateral LT does not exist	No common ROC
23.	$\text{sgn}(t)$	Bilateral LT does not exist	No common ROC

Sample Questions

1987 IIT Bombay

3.1 Laplace transforms of the functions $t\,u(t)$ and $\sin t\,u(t)$ are respectively

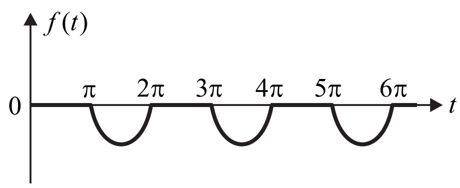
(A) $\frac{1}{s^2}, \frac{s}{s^2+1}$ (B) $\frac{1}{s}, \frac{1}{s^2+1}$

(C) $\frac{1}{s^2}, \frac{1}{s^2+1}$ (D) $s, \frac{s}{s^2+1}$

1993 IIT Bombay

3.2 The Laplace transform of the periodic function $f(t)$ described by the curve below, i.e.

$$f(t) = \begin{cases} \sin t, & \text{if } (2n-1)\pi \le t \le 2n\pi \quad (n = 1,2,3,....) \\ 0, & \text{otherwise} \end{cases}$$



(A) $\frac{-e^{-\pi s}}{(s^2+1)(1-e^{-\pi s})}$

(B) $\frac{e^{-\pi s}}{(s^2+1)(1+e^{-\pi s})}$

(C) $\frac{e^{\pi s}}{(s^2+1)(1+e^{\pi s})}$

(D) $\frac{-e^{\pi s}}{(s^2-1)(1-e^{-\pi s})}$

2015 IIT Kanpur

3.3 The value of the integral

$$\int_{-\infty}^{\infty} 12\cos(2\pi t)\frac{\sin(4\pi t)}{4\pi t}\,dt \text{ is } \_\_\_\_\_\_.$$

[Set - 02]

3.4 Consider the function

$$g(t) = e^{-t}\sin(2\pi t)u(t)$$

where $u(t)$ is the unit step function. The area under $g(t)$ is ______. **[Set - 03]**

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Explanations Laplace Transform

3.1 (C)

$$t\,u(t) \xleftrightarrow{L.T.} \frac{1}{s^2}$$

$$\sin at\,u(t) \xleftrightarrow{L.T.} \frac{a}{s^2+a^2}$$

Thus, $\sin t\,u(t) \xleftrightarrow{L.T.} \frac{1}{s^2+1^2} = \frac{1}{s^2+1}$

Hence, the correct option is (C).

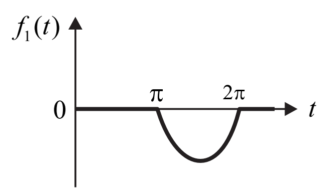
3.2 (A)

Given :

$$f(t) = \begin{cases} \sin t, & \text{if } (2n-1)\pi \le t \le 2n\pi \ (n = 1,2,3,....) \\ 0, & \text{otherwise} \end{cases}$$

Put $n = 1$,

$$f(t) = \begin{cases} \sin t, & \text{if } \pi \le t \le 2\pi \\ 0, & \text{otherwise} \end{cases} = f_1(t)$$



Waveform of $\sin t\,u(t)$, $\sin(t-\pi)u(t-\pi)$ and $\sin(t-2\pi)u(t-2\pi)$ are shown in figure (i), (ii) and (iii) respectively.

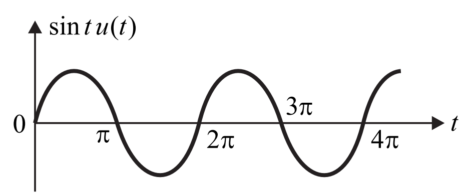


Fig. (i)

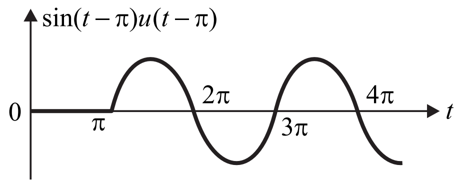


Fig. (ii)

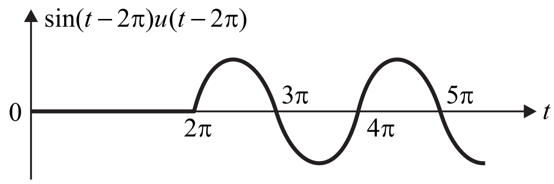


Fig. (iii)

So, $f_1(t) = -[\sin(t-\pi)u(t-\pi)$
$+\sin(t-2\pi)u(t-2\pi)]$ ...(i)

$$\sin t\, u(t) \xleftrightarrow{L.T.} \frac{1}{(s^2+1)}$$

$$\sin(t-\pi)u(t-\pi) \xleftrightarrow{L.T.} \frac{e^{-\pi s}}{(s^2+1)}$$

[By time shifting property]

$$\sin(t-2\pi)u(t-2\pi) \xleftrightarrow{L.T.} \frac{e^{-2\pi s}}{(s^2+1)}$$

Taking Laplace transform of equation (i),

$$F_1(s) = -\left[\frac{e^{-\pi s}}{(s^2+1)} + \frac{e^{-2\pi s}}{(s^2+1)}\right]$$

$$F_1(s) = -e^{-\pi s}\left[\frac{1+e^{-\pi s}}{s^2+1}\right]$$

Laplace transform of periodic signal with period T_0 is given by,

$$F(s) = \frac{F_1(s)}{1-e^{-T_0 s}}$$

where, $F_1(s)$ is Laplace transform of one period.

Here, $T_0 = 2\pi$

Therefore, $F(s) = \dfrac{F_1(s)}{1-e^{-2\pi s}}$

$$F(s) = -e^{-\pi s}\left[\frac{1+e^{-\pi s}}{s^2+1}\right] \times \frac{1}{(1-e^{-2\pi s})}$$

$$F(s) = -e^{-\pi s}\frac{(1+e^{-\pi s})}{(s^2+1)(1+e^{-\pi s})(1-e^{-\pi s})}$$

$$F(s) = \frac{-e^{-\pi s}}{(s^2+1)(1-e^{-\pi s})}$$

Hence, the correct option is (A).

3.3 **3**

Method 1

Given : $I = \displaystyle\int_{-\infty}^{\infty} 12\cos(2\pi t)\frac{\sin(4\pi t)}{4\pi t}dt$

Since, the term inside integration is even function.

Thus, $I = \dfrac{12}{4\pi} \times 2\displaystyle\int_0^{\infty} \frac{\cos 2\pi t \sin 4\pi t}{t}dt$

$$I = \frac{3}{\pi}\int_0^{\infty} \frac{2\cos 2\pi t \sin 4\pi t}{t}dt$$

$$I = \frac{3}{\pi}\left[\int_0^{\infty} \frac{\sin 6\pi t}{t}dt + \int_0^{\infty} \frac{\sin 2\pi t}{t}dt\right]$$

$[2\sin A\cos B = \sin(A+B) + \sin(A-B)]$

Taking Laplace transform,

$$I = \frac{3}{\pi}\left[L\left\{\frac{\sin 6\pi t}{t}\right\} + L\left\{\frac{\sin 2\pi t}{t}\right\}\right] \quad \text{with } s = 0$$

$$\left[X(s)\big|_{s=0} = \int_{-\infty}^{\infty} x(t)\,dt\right]$$

$$I = \frac{3}{\pi}\left[\int_s^{\infty} \frac{6\pi}{s^2+36\pi^2}ds + \int_s^{\infty} \frac{2\pi}{s^2+4\pi^2}ds\right] \text{ with } s = 0$$

[From the division by 't' property

$$L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} F(s)\,ds\,]$$

$$I = \frac{3}{\pi}\left[6\pi \times \frac{1}{6\pi}\tan^{-1}\left(\frac{s}{6\pi}\right) + 2\pi \times \frac{1}{2\pi}\tan^{-1}\left(\frac{s}{2\pi}\right)\right]_s^{\infty}$$

with $s = 0$

$$I = \frac{3}{\pi}\left[\tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{6\pi}\right) + \tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{2\pi}\right)\right]$$

with $s = 0$

$$I = \frac{3}{\pi}\left[\frac{\pi}{2} - \tan^{-1} 0 + \frac{\pi}{2} - \tan^{-1} 0\right]$$

$$I = \frac{3}{\pi}[\pi] = 3$$

Hence, the value of the integral is **3.**

📖 Key Point

Area in one domain = Amplitude at origin in other domain

$$X(s)\Big|_{s=0} = \int_{-\infty}^{\infty} x(t)\,dt$$

$$x(t)\Big|_{t=0} = \int_{-\infty}^{\infty} X(s)\,ds$$

Method 2

Given integral is

$$I = \int_{-\infty}^{\infty} 12\cos 2\pi t \cdot \frac{\sin 4\pi t}{4\pi t}\,dt$$

$$I = 3\int_{-\infty}^{\infty} \cos 2\pi t \cdot \frac{\sin 4\pi t}{\pi t}\,dt$$

$$I = 3\int_{-\infty}^{\infty} x(t)\,dt \qquad \ldots\text{(i)}$$

Where $x(t) = \cos 2\pi t \cdot \dfrac{\sin 4\pi t}{\pi t}$

$$\Rightarrow \qquad x(t) = x_1(t)\cdot x_2(t) \qquad \ldots\text{(ii)}$$

Where, $x_1(t) = \cos 2\pi t$

$$x_2(t) = \frac{\sin 4\pi t}{\pi t}$$

From analysis equation of Fourier transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt$$

At $\omega = 0$, this equation becomes

$$X(\omega)\Big|_{\omega=0} = X(0) = \int_{-\infty}^{\infty} x(t)\,dt = \text{Area under } x(t) \qquad \ldots\text{(iii)}$$

From equation (ii),

$$x(t) = x_1(t)\cdot x_2(t)$$

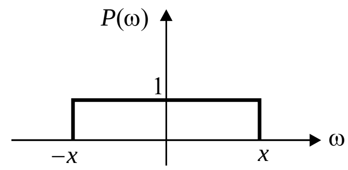
Applying Fourier transform both sides and using multiplication properly of Fourier transform,

$$X(\omega) = \frac{1}{2\pi}\left[X_1(\omega)\times X_2(\omega)\right] \qquad \ldots\text{(iv)}$$

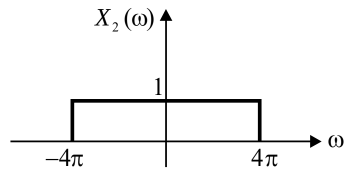
$$x_1(t) = \cos 2\pi t$$

$$X_1(\omega) = \pi\left[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)\right]$$

As for $p(t) = \dfrac{\sin xt}{\pi t} \leftrightarrow$



So, $x_2(t) = \dfrac{\sin 4\pi t}{\pi t} \leftrightarrow$

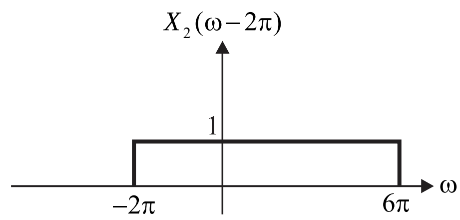


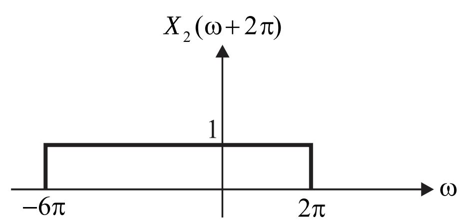
From equation (iv),

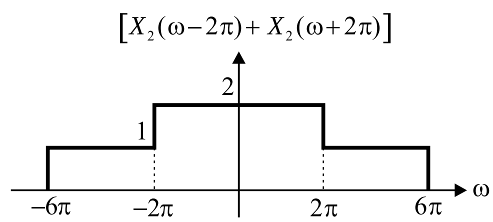
$$X(\omega) = \frac{1}{2\pi}\left[X_2(\omega)\otimes \pi\{\delta(\omega - 2\pi) + \delta(\omega + 2\pi)\}\right]$$

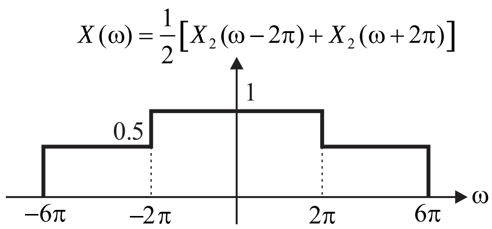
$$= \frac{1}{2}\left[X_2(\omega - 2\pi) + X_2(\omega + 2\pi)\right]$$

$X_2(\omega - 2\pi)$, $X_2(\omega + 2\pi)$ and their addition are shown below









Hence, $X(\omega)\big|_{\omega=0}=X(0)=1$

From equation (iii),

$$\int_{-\infty}^{\infty} x(t)dt = X(0)=1$$

So from equation, (i)

$$I=3\int_{-\infty}^{\infty} x(t)dt = 3\times 1 = 3$$

Hence, the value of the integral is **3.**

3.4 **0.155**

Given : $g(t)=e^{-t}\sin(2\pi t)u(t)$

Method 1

Area of the signal $g(t)$ is given by,

$$A=\int_{-\infty}^{\infty} g(t)\,dt$$

$$A=\int_{-\infty}^{\infty} e^{-t}\sin(2\pi t)u(t)\,dt$$

$$A=\int_{0}^{\infty} e^{-t}\sin(2\pi t)\,dt$$

Laplace transform of $\sin(2\pi t)$ is given by,

$$\int_{-\infty}^{\infty} e^{-st}\sin(2\pi t)\,dt=\frac{2\pi}{s^2+(2\pi)^2}$$

Putting $s = 1$, we get

$$A=\int_{-\infty}^{\infty} e^{-t}\sin(2\pi t)\,dt=\frac{2\pi}{1^2+(2\pi)^2}=0.155$$

Hence, the area under $g(t)$ is **0.155.**

Method 2

Laplace transform of function $g(t)$ is given by,

$$G(s)=\int_{-\infty}^{\infty} e^{-st}g(t)dt$$

Putting $s=0$,

$$G(0)=\int_{-\infty}^{\infty} g(t)dt = \text{Area of function } g(t).$$

$$g(t)=e^{-t}\sin(2\pi t)u(t)$$

Taking Laplace transform,

$$G(s)=\frac{2\pi}{(s+1)^2+(2\pi)^2}$$

[Using frequency shifting property of Laplace transform]

$$G(0)=\frac{2\pi}{1^2+(2\pi)^2}=0.155$$

Hence, the area under $g(t)$ is **0.155.**

❖❖❖❖

4 Continuous Time Convolution

Partial Synopsis

Convolution Integral

Convolution Integral : The output of any continuous time LTI system is the convolution of the input $x(t)$ with the impulse response $h(t)$ of the system.

$$y(t) = x(t) \otimes h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

Properties of Convolution Integral :

(1) Commutative : $x(t) \otimes h(t) = h(t) \otimes x(t)$

(2) Associative : $\{x(t) \otimes h_1(t)\} \otimes h_2(t) = x(t) \otimes \{h_1(t) \otimes h_2(t)\}$

(3) Distributive : $x(t) \otimes \{h_1(t) + h_2(t)\} = x(t) \otimes h_1(t) + x(t) \otimes h_2(t)$

(4) Convolution with impulse function :

(a) $x(t) \otimes \delta(t) = x(t)$

(b) $x(t) \otimes \delta(t-t_0) = x(t-t_0)$

(5) Convolution with step function :

(a) $x(t) \otimes u(t) = \int_{-\infty}^{t} x(\tau) d\tau$

(b) $x(t) \otimes u(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau) d\tau$

(6) If $y(t) = x(t) \otimes h(t)$ **then** $x(t-t_1) \otimes h(t-t_2) = y(t-t_1-t_2)$

(7) Area of $y(t) = [\text{Area of } x(t)] \cdot [\text{Area of } h(t)]$

Sample Questions

1990　IISc Bangalore

4.1 The impulse response and the excitation function of a linear time invariant causal system are shown in figure (a) and (b) respectively.

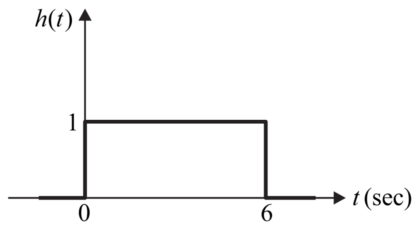


Fig. (a)

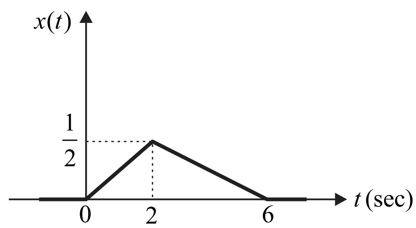


Fig. (b)

The output of the system at $t = 2$ sec. is equal to

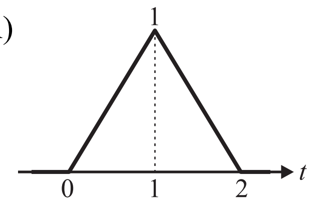
(A) 0　　(B) 1/2

(C) 3/2　　(D) 1

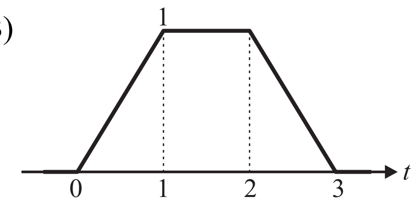
2000　IIT Kharagpur

4.2 Let $u(t)$ be the step function. Which of the waveforms in given figure corresponds to the convolution of $u(t) - u(t-1)$ with $u(t) - u(t-2)$.

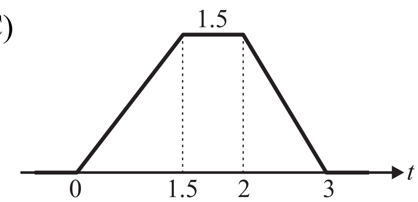
(A)



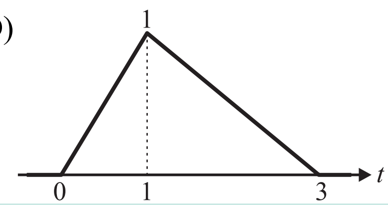
(B)



(C)



(D)



2015　IIT Kanpur

4.3 The result of the convolution $x(-t) \otimes \delta(-t - t_0)$ is　　**[Set - 01]**

(A) $x(t + t_0)$　　(B) $x(t - t_0)$

(C) $x(-t + t_0)$　　(D) $x(-t - t_0)$

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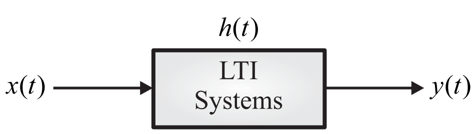
Explanations　Continuous Time Convolution

4.1　(B)

Given : System is causal and LTI system.

Method 1

Hence, $h(t) = 0$ for $t < 0$



For LTI system, output $y(t)$ is given by

$$y(t) = x(t) \otimes h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

$$y(t) = \int_{0}^{t} x(\tau) h(t-\tau)\, d\tau \quad \text{[System is causal]}$$

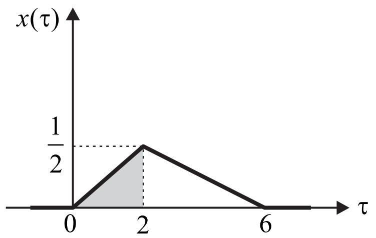
As $x(t) = 0$, for $t < 0$

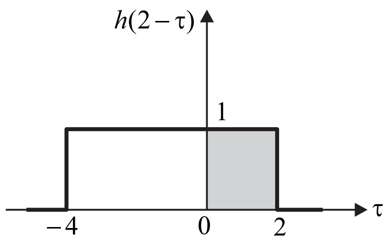
and $h(t) = 0$, for $t < 0$

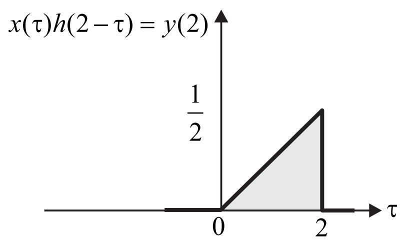
Put $t = 2\,\text{sec}$,

$$y(2) = \int_{-\infty}^{\infty} x(\tau)h(2-\tau)d\tau\,.$$

Signal $x(\tau)$ and $h(2-\tau)$ are shown in below figure.







$y(2) =$ Area of shaded region of $x(\tau)h(2-\tau)$

$$y(2) = \frac{1}{2}\times\left(2\times\frac{1}{2}\right)$$

$$y(2) = \frac{1}{2}$$

Hence, the correct option is (B).

Method 2

We have to find

$$h(t) * x(t) = y(t)$$

From the property based on linearity,

$$h'(t) * x(t) = y'(t)$$

$$h(t) = u(t) - u(t-6)$$

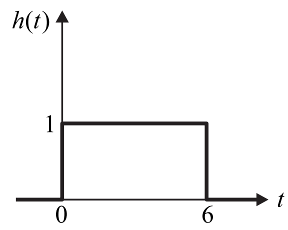
$$h'(t) = \delta(t) - \delta(t-6)$$

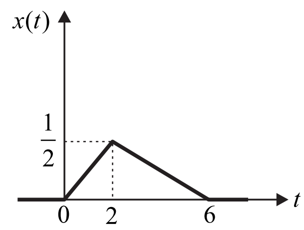
Hence, $y'(t) = h'(t) * x(t)$

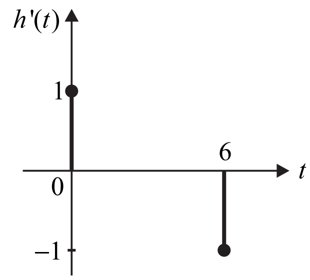
$$y'(t) = [\delta(t) - \delta(t-6)] * x(t)$$

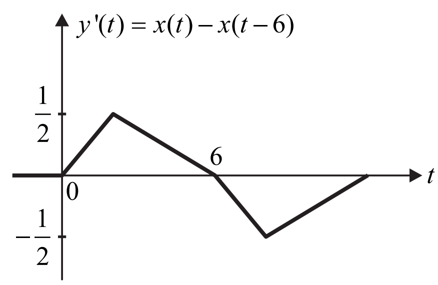
$$y'(t) = x(t) - x(t-6)$$

$h(t)$, $x(t)$, $h'(t)$ and $y'(t)$ are shown below,









Therefore, $y(t)\big|_{t=2\text{ sec}} = \int_0^2 y'(t)\,dt$

i.e. $\quad y(t)\big|_{t=2\text{ sec}} = \int_0^2 \frac{1}{4}t\,dt = \frac{1}{2}$

Hence, the correct option is (B).

Method 3

The impulse response can be written as,

$$h(t) = u(t) - u(t-6)$$

Taking Laplace transform,

$$H(s) = \frac{1}{s}\left[1 - e^{-6s}\right]$$

Also, $\quad x(t) = \frac{1}{4}r(t) - \frac{3}{8}r(t-2) + \frac{1}{8}r(t-6)$

Taking Laplace transform,

$$X(s)=\frac{1}{4s^2}-\frac{3}{8s^2}e^{-2s}+\frac{1}{8s^2}e^{-6s}$$

Since, $L[x(t)\otimes h(t)]=X(s)H(s)=Y(s)$

$$Y(s)=\frac{1}{s^3}\left[1-e^{-6s}\right]\left[\frac{1}{4}-\frac{3}{8}e^{-2s}+\frac{1}{8}e^{-6s}\right]$$

$$Y(s)=\frac{1}{s^3}\left[\frac{1}{4}-\frac{3}{8}e^{-2s}-\frac{1}{8}e^{-6s}+\frac{3}{8}e^{-8s}-\frac{1}{8}e^{-12s}\right]$$

Taking inverse Laplace transform,

$$y(t)=\frac{1}{8}t^2u(t)-\frac{3}{16}(t-2)^2u(t-2)$$
$$-\frac{1}{16}(t-6)^2u(t-6)+\frac{3}{16}(t-8)^2u(t-8)$$
$$-\frac{1}{16}(t-12)^2u(t-12)$$

Hence,

$$y(2)=\frac{1}{8}\times 4\times u(2)-\frac{3}{16}\times 0\times u(0)$$
$$-\frac{1}{16}\times(-4)^2\times u(-4)+\frac{3}{16}\times(-6)^2\times u(-6)$$
$$-\frac{1}{16}\times(-10)^2\times u(-10) \quad \ldots(i)$$

Since, we know $u(-\text{ve})=0$ and $u(+\text{ve})=1$.

From equation (i),

$$y(2)=\frac{1}{8}\times 4\times 1=\frac{1}{2}$$

Hence, the correct option is (B).

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4.2 (B)

Method 1

Let $x_1(t)=u(t)-u(t-1)$

Taking Laplace transform,

$$X_1(s)=\frac{1}{s}-\frac{e^{-s}}{s}$$

and $x_2(t)=u(t)-u(t-2)$.

Taking Laplace transform,

$$X_2(s)=\frac{1}{s}-\frac{e^{-2s}}{s}$$

and $y(t)=x_1(t)\otimes x_2(t)$.

Taking Laplace transform,

$$Y(s)=X_1(s)X_2(s)$$

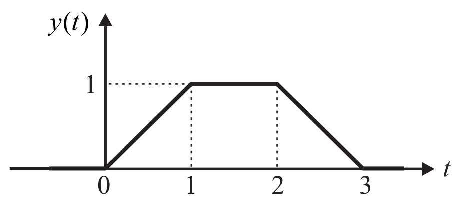
$$Y(s)=\left[\frac{1}{s}-\frac{e^{-s}}{s}\right]\left[\frac{1}{s}-\frac{e^{-2s}}{s}\right]$$

$$Y(s)=\frac{1}{s^2}\left[1-e^{-s}-e^{-2s}+e^{-3s}\right]$$

Taking Inverse Laplace transform,

$$y(t)=tu(t)-(t-1)u(t-1)-(t-2)u(t-2)+(t-3)u(t-3)$$

Waveform of $y(t)$ is shown below,



Hence, the correct option is (B).

Method 2

$$y(t)=x_1(t)\otimes x_2(t)$$
$$y(t)=[u(t)-u(t-1)]\otimes[u(t)-u(t-2)]$$
$$y(t)=u(t)\otimes u(t)-u(t)\otimes u(t-2)$$
$$-u(t-1)\otimes u(t)+u(t-1)\otimes u(t-2)$$

Key Point

Convolution of two unit step signal results in the ramp signal.

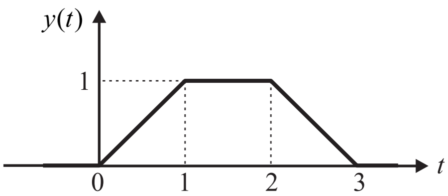
$$u(t-t_1)\otimes u(t-t_2)=r[t-(t_1+t_2)]$$

Then, $y(t)=r(t)-r(t-1)-r(t-2)+r(t-3)$

$$y(t)=t\,u(t)-(t-1)\,u(t-1)-(t-2)$$

$$u(t-2)+(t-3)u(t-3)$$

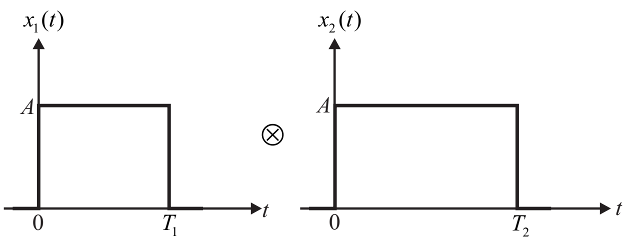
Waveform of $y(t)$ is shown below,

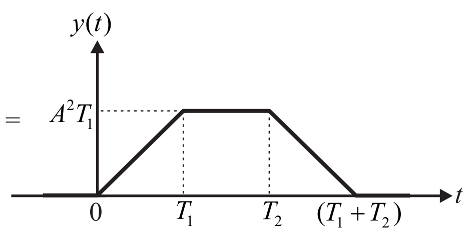


Hence, the correct option is (B).

Method 3

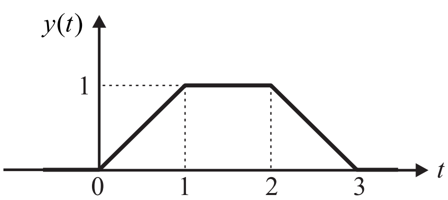
Convolution of two rectangular signal of unequal duration gives trapezoidal signal as shown below,





where, $T_1=1$, $T_2=2$, $A=1$

Then, $A^2T_1=1$



Hence, the correct option is (B).

4.3 (D)

Method 1

$\delta(t)$ is an even function,

i.e. $\delta(-t)=\delta(t)$.

Then, $\delta(-t-t_0)=\delta[-(t+t_0)]$

$$\delta(-t-t_0)=\delta(t+t_0)$$

$$x(-t)\otimes\delta(-t-t_0)$$

$$=x(-t)\otimes\delta(t+t_0)=x[-(t+t_0)]$$

$$x(-t)\otimes\delta(-t-t_0)=x(-t-t_0)$$

Hence, the correct option is (D).

Method 2

$\delta(t)$ is an even function,

i.e. $\delta(-t)=\delta(t)$.

Then, $\delta(-t-t_0)=\delta[-(t+t_0)]$

$$\delta(-t-t_0)=\delta(t+t_0)$$

Also, $L\left[x(-t)\right]=X(-s)$

[By time inversion property of Laplace transform]

Therefore,

$$L\left[x(-t)\otimes\delta(t+t_0)\right]=X(-s)e^{st_0}$$

$$x(t)\xleftrightarrow{\text{L.T.}}X(s)$$

$$x(-t)\xleftrightarrow{\text{L.T.}}X(-s)$$

$$x\left[-(t+t_0)\right]\xleftrightarrow{\text{L.T.}}X(-s)\,e^{st_0}$$

$$x(-t-t_0)\xleftrightarrow{\text{L.T.}}X(-s)\,e^{st_0}$$

Hence, $x(-t)\otimes\delta(t+t_0)=x(-t-t_0)$

[By time shifting property of Laplace transform]

Hence, the correct option is (D).

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5 Continuous Time Fourier Transform

➢ Partial Synopsis

(8) Frequency Differentiation or Multiplication by t : Frequency differentiation leads to multiplication by t in time domain.

$$-j2\pi t.x(t)\xleftrightarrow{FT}\frac{d}{df}X(f) \qquad t.x(t)\xleftrightarrow{FT}\frac{j}{2\pi}\frac{d}{df}X(f)$$

$$-jt.x(t)\xleftrightarrow{FT}\frac{d}{d\omega}X(\omega) \qquad t.x(t)\xleftrightarrow{FT}j\frac{d}{d\omega}X(\omega)$$

(9) Duality : Duality allows to obtain both the dual transform pairs from one equation.

$$x(t)\xleftrightarrow{FT}X(f) \quad\Rightarrow\quad X(t)\xleftrightarrow{FT}x(-f)$$

$$x(t)\xleftrightarrow{FT}X(\omega) \quad\Rightarrow\quad X(t)\xleftrightarrow{FT}2\pi.x(-\omega)$$

(10) Integration in Time domain :

$$x(t)\xleftrightarrow{FT}X(f) \quad\Rightarrow\quad \int_{-\infty}^{t}x(\tau)d\tau\xleftrightarrow{FT}\frac{X(f)}{j2\pi f}+\frac{1}{2}X(0)\delta(f)$$

$$x(t)\xleftrightarrow{FT}X(\omega) \quad\Rightarrow\quad \int_{-\infty}^{t}x(\tau)d\tau\xleftrightarrow{FT}\frac{X(\omega)}{j\omega}+\pi X(0)\delta(\omega)$$

(11) Conjugation and Conjugate symmetry :

Conjugation $\Rightarrow$ $x^*(t)\xleftrightarrow{FT}X^*(-j\omega)$

Conjugate symmetry $\Rightarrow$ If $x(t)$ is real, then $X(-j\omega)=X^*(j\omega)$

Basic Fourier Transforms

S.	$x(t)$	$X(f)$	$X(\omega)$
1.	$\delta(t)$	1	1
2.	$e^{-at}u(t),\ a>0$	$\frac{1}{a+j2\pi f}$	$\frac{1}{a+j\omega}$
3.	$e^{at}u(-t),\ a>0$	$\frac{1}{a-j2\pi f}$	$\frac{1}{a-j\omega}$
4.	$e^{-a\|t\|}$	$\frac{2a}{a^2+(2\pi f)^2}$	$\frac{2a}{a^2+\omega^2}$

5.	$A\,\text{rect}\left(\frac{t}{\tau}\right)$	$A\tau\,\text{sinc}(f\tau)$	$A\tau\,\text{Sa}\left(\frac{\omega\tau}{2}\right)$
6.	$A\,\text{tri}\left(\frac{t}{\tau}\right) = A\left[1-\frac{\lvert t\rvert}{\tau}\right]$	$A\tau\,\text{sinc}^2(f\tau)$	$A\tau\,\text{Sa}^2\left(\frac{\omega\tau}{2}\right)$
7.	$\text{sgn}(t)$	$\frac{1}{j\pi f}$	$\frac{2}{j\omega}$
8.	$e^{-a\lvert t\rvert}\text{sgn}(t)$	$\frac{-j4\pi f}{a^2+(2\pi f)^2}$	$\frac{-j2\omega}{a^2+\omega^2}$
9.	$t.e^{-at}u(t)$	$\frac{1}{(a+j2\pi f)^2}$	$\frac{1}{(a+j\omega)^2}$
10.	$t.e^{at}u(-t)$	$\frac{-1}{(a-j2\pi f)^2}$	$\frac{-1}{(a-j\omega)^2}$
11.	1	$\delta(f)$	$2\pi.\delta(\omega)$
12.	$u(t)$	$\frac{\delta(f)}{2}+\frac{1}{j2\pi f}$	$\pi\delta(\omega)+\frac{1}{j\omega}$
13.	$\cos 2\pi f_0 t$	$\frac{\delta(f-f_0)+\delta(f+f_0)}{2}$	$\pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$
14.	$\sin 2\pi f_0 t$	$\frac{\delta(f-f_0)-\delta(f+f_0)}{2j}$	$\frac{\pi}{j}\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$
15.	$e^{-at}\cos(2\pi f_0 t).u(t)$	$\frac{a+j2\pi f}{(a+j2\pi f)^2+(2\pi f_0)^2}$	$\frac{a+j\omega}{(a+j\omega)^2+(\omega_0)^2}$
16.	$e^{-at}\sin(2\pi f_0 t).u(t)$	$\frac{2\pi f_0}{(a+j2\pi f)^2+(2\pi f_0)^2}$	$\frac{\omega_0}{(a+j\omega)^2+(\omega_0)^2}$
17.	$te^{-a\lvert t\rvert}$	$\frac{-j8\pi f a}{\left[a^2+(2\pi f)^2\right]^2}$	$\frac{-j4a\omega}{\left[a^2+\omega^2\right]^2}$
18.	e^{jt}	$\delta\left(f-\frac{1}{2\pi}\right)$	$2\pi\delta(\omega-1)$

➢ Sample Questions

1992 IIT Delhi

5.1 If $G(f)$ represents the Fourier transform of a signal $g(t)$ which is real and odd symmetric in time, then

(A) $G(f)$ is complex.

(B) $G(f)$ is imaginary.

(C) $G(f)$ is real.

(D) $G(f)$ is real and non-negative.

2003 IIT Madras

Common Data for Questions 5.2 & 5.3

The system under consideration is an *RC* low-pass filter (*RC*-LPF) with $R = 1\,\text{k}\Omega$ and $C = 1\,\mu\text{F}$.

5.2 Let $H(f)$ denote the frequency response of the *RC*-LPF. Let f_1 be the highest frequency such that $0 \le |f| \le f_1$, $\frac{|H(f_1)|}{H(0)} \ge 0.95$. Then, f_1 (in Hz) is

(A) 327.8 (B) 163.9
(C) 52.2 (D) 104.4

5.3 Let $\tau_g(f)$ be the group delay function of the given *RC*-LPF and $f_2 = 100$ Hz. Then $\tau_g(f_2)$ in ms, is

(A) 0.717 (B) 7.17
(C) 71.7 (D) 4.505

2012 **IIT Delhi**

5.4 The Fourier transform of a signal $h(t)$ is $H(j\omega) = (2\cos\omega)(\sin 2\omega)/\omega$. The value of $h(0)$ is

(A) 1/4 (B) 1/2
(C) 1 (D) 2

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Explanations — Continuous Time Fourier Transform

5.1 **(B)**

Fourier Transform of real and odd symmetric signal is imaginary and odd function of frequency.

Table 5.1 : Symmetry Conditions of Fourier Transform

$x(t)$	$X(f)$	Example
Even	Even	$\text{rect}(t) \longleftrightarrow \text{sinc}(f)$
Odd	Odd	$\text{sgn}(t) \longleftrightarrow \frac{1}{j\pi f}$
Real and even	Real and even	$\text{rect}(t) \longleftrightarrow \text{sinc}(f)$
Imaginary and even	Imaginary and even	$j\,\text{rect}(t) \longleftrightarrow j\,\text{sinc}(f)$
Real and odd	Imaginary and odd	$\text{sgn}(t) \longleftrightarrow \frac{1}{j\pi f}$
Imaginary and odd	Real and odd	$j\,\text{sgn}(t) \longleftrightarrow \frac{1}{\pi f}$
Complex and even	Complex and even	$(1+j)\text{rect}(t) \longleftrightarrow (1+j)\text{sinc}(f)$
Complex and odd	Complex and odd	$(1+j)\text{sgn}(t) \longleftrightarrow \frac{1-j}{\pi f}$
Real even and imaginary odd (Conjugate symmetric)	Real	$\text{rect}(t) + j\,\text{sgn}(t) \longleftrightarrow \text{sinc}(f) + \frac{1}{\pi f}$

Real odd and imaginary even (Conjugate anti-symmetric)	Imaginary	$\text{sgn}(t)+j\,\text{rect}(t) \longleftrightarrow \dfrac{1}{j\pi f}+j\,\text{sinc}(f)$
Real	Real even and imaginary odd (Conjugate symmetric)	$\text{rect}(t)\longleftrightarrow\text{sinc}(f)$
Imaginary	Real odd and imaginary even (Conjugate anti-symmetric)	$j\,\text{rect}(t)\longleftrightarrow j\,\text{sinc}(f)$

Table 5.2 : Concept of Real and Imaginary / Conjugate symmetric and Conjugate anti symmetric in t-domain :

Real	Imaginary	Conjugate symmetric	Conjugate anti symmetric
$x(t)=x^*(t)$	$x(t)=-x^*(t)$	$x(t)=x^*(-t)$	$x(t)=-x^*(-t)$
$x_R(t)=\dfrac{x(t)+x^*(t)}{2}$	$x_I(t)=\dfrac{x(t)-x^*(t)}{2}$	$x_{C.S}(t)=\dfrac{x(t)+x^*(-t)}{2}$	$x_{C.A.S}(t)=\dfrac{x(t)-x^*(-t)}{2}$

Table 5.3 : Concept of Real and Imaginary / Conjugate symmetric and Conjugate anti symmetric in f-domain :

Real	Imaginary	Conjugate symmetric	Conjugate anti symmetric
$X(f)=X^*(f)$	$X(f)=-X^*(f)$	$X(f)=X^*(-f)$	$X(f)=-X^*(-f)$
$X_R(f)=\dfrac{X(f)+X^*(f)}{2}$	$X_I(f)=\dfrac{X(f)-X^*(f)}{2}$	$X_{C.S}(f)=\dfrac{X(f)+X^*(-f)}{2}$	$X_{C.A.S}(f)=\dfrac{X(f)-X^*(-f)}{2}$

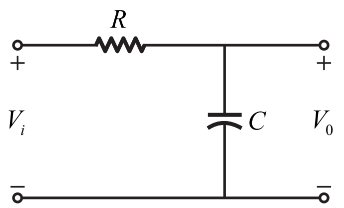
Hence, the correct option is (B).

Scan for Video Solution

5.2 (C)

Given : Frequency response of $RC-LPF$ is $H(f)$.

$R=1\text{k}\Omega$, $C=1.0\ \mu\text{F}$, $RC=10^{-3}$ sec



From figure,

$$H(s)=\frac{V_0(s)}{V_i(s)}=\frac{1}{1+RCs}$$

$$H(j\omega)=\frac{1}{1+j\omega RC}$$

$$|H(\omega)|=\frac{1}{\sqrt{1+\omega^2R^2C^2}}$$

[Putting $\omega=2\pi f$]

$$|H(\omega)|=\frac{1}{\sqrt{1+4\pi^2 f^2R^2C^2}}$$

Clearly, $|H(0)|=1$

Given, $\left|\dfrac{H(f)}{H(0)}\right|\geq 0.95$ for $0\leq f\leq f_1$

$$\frac{1}{\sqrt{1+4\pi^2 f_1^2R^2C^2}}=0.95$$

$$\frac{1}{1+4\pi^2 f_1^2R^2C^2}=(0.95)^2=0.90$$

$$1+4\pi^2 f_1^2R^2C^2=1.108$$

$$f_1^2 = \frac{0.108}{4\pi^2 \times (10^{-3})^2}$$

$$f_1 = 52.2\,\text{Hz}$$

Hence, the correct option is (C).

5.3 (A)

Given : $f_2 = 100$ Hz

Since, $H(j\omega) = \dfrac{1}{1 + j\omega RC}$

$$\theta(\omega) = -\tan^{-1}(\omega RC)$$

Group delay, $\tau_g = -\dfrac{d\theta(\omega)}{d\omega}$

$$\tau_g = \frac{RC}{1+\omega^2 R^2 C^2} = \frac{RC}{1+4\pi^2 f^2 R^2 C^2}$$

At $f = 100$ Hz ,

$$\tau_g = \frac{10^{-3}}{1+4\pi^2 \times 100^2 (10^{-3})^2}$$

$$\tau_g = 0.717 \text{ ms}$$

Hence, the correct option is (A).

5.4 (C)

Given : $H(j\omega) = \dfrac{(2\cos\omega)(\sin 2\omega)}{\omega}$

Method 1

Since, $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$$H(j\omega) = \frac{\sin 3\omega + \sin\omega}{\omega}$$

$$H(j\omega) = \left(\frac{\sin 3\omega}{\omega} \times \frac{3}{3}\right) + \frac{\sin\omega}{\omega}$$

$$H(j\omega) = 3\,Sa(3\omega) + Sa(\omega)$$

Fourier transform of rectangular signal is given by,

$$A\text{rect}\left[\frac{t}{\tau}\right] \xleftrightarrow{F.T.} A\tau\, Sa\left[\frac{\omega\tau}{2}\right]$$

To find inverse Fourier transform of $Sa(\omega)$

$$\frac{\tau}{2} = 1 \quad \Rightarrow \quad \tau = 2$$

and $\quad A\tau = 1 \Rightarrow \quad A \times 2 = 1 \quad \Rightarrow \quad A = \dfrac{1}{2}$

Hence, $Sa(\omega) \xleftrightarrow{I.F.T.} \dfrac{1}{2}\text{rect}\left[\dfrac{t}{2}\right] = h_1(t)$

To find inverse Fourier transform of $3Sa(3\omega)$

$$\frac{\tau}{2} = 3 \quad \Rightarrow \quad \tau = 6$$

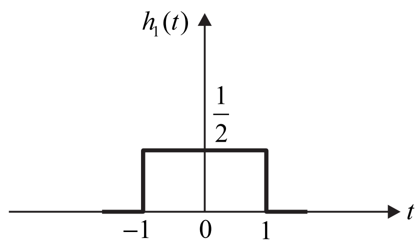
and $\quad A\tau = 3 \Rightarrow \quad A \times 6 = 3 \quad \Rightarrow \quad A = \dfrac{1}{2}$

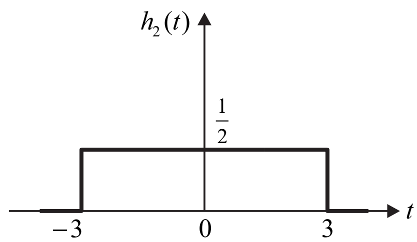
Hence, $3Sa(3\omega) \xleftrightarrow{I.F.T.} \dfrac{1}{2}\text{rect}\left[\dfrac{t}{6}\right] = h_2(t)$

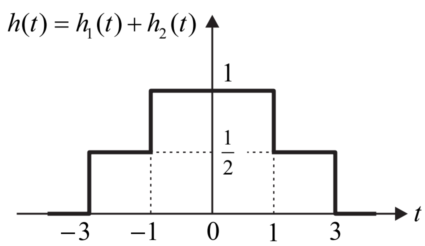
Hence,

$$Sa(\omega) + 3Sa(3\omega) \xleftrightarrow{I.F.T.} \frac{1}{2}\text{rect}\left[\frac{t}{2}\right] + \frac{1}{2}\text{rect}\left[\frac{t}{6}\right]$$

$$Sa(\omega) + 3Sa(3\omega) \xleftrightarrow{I.F.T.} h_1(t) + h_2(t) = h(t)$$







From above figure,

$$h(0) = 1$$

Hence, the correct option is (C).

Method 2

Since, $\displaystyle\int_{-\infty}^{\infty} \text{sinc}(\omega)\, d\omega = 1$

$$\int_{-\infty}^{\infty} \frac{\sin(\pi\omega)}{\pi\omega} d\omega = 1$$

Put $\pi\omega = \theta \Rightarrow \pi d\omega = d\theta$,

$$\int_{-\infty}^{\infty} \frac{\sin\theta}{\theta}\frac{d\theta}{\pi} = 1$$

$$\int_{-\infty}^{\infty} Sa(\theta)\frac{d\theta}{\pi} = 1$$

$$\int_{-\infty}^{\infty} Sa(\theta)\, d\theta = \pi \quad \ldots\text{(i)}$$

Put $\theta = 3\phi$,

$$\int_{-\infty}^{\infty} Sa(3\phi)\, 3\, d\phi = \pi$$

$$\int_{-\infty}^{\infty} Sa(3\phi)\, d\phi = \frac{\pi}{3} \quad \ldots\text{(ii)}$$

Since, $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$$H(j\omega) = \frac{\sin 3\omega + \sin\omega}{\omega}$$

$$H(j\omega) = \left(\frac{\sin 3\omega}{\omega}\times\frac{3}{3}\right) + \frac{\sin\omega}{\omega}$$

$$H(j\omega) = 3\,Sa(3\omega) + Sa(\omega)$$

From the definition of inverse Fourier transform,

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)\, e^{j\omega t} d\omega$$

Put $t = 0$,

$$h(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)\, d\omega$$

$$h(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 3\,Sa(3\omega) + Sa(\omega)\, d\omega$$

$$h(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 3\,Sa(3\omega)\, d\omega + \frac{1}{2\pi}\int_{-\infty}^{\infty} Sa(\omega)\, d\omega$$

Using equation (i) and (ii),

$$h(0) = \frac{1}{2\pi}\times 3\times\frac{\pi}{3} + \frac{1}{2\pi}\times\pi = 1$$

Hence, the correct option is (C).

Method 3

$$H(j\omega) = 2\cos\omega\left(\frac{2\sin 2\omega}{2\omega}\right)$$

$$H(j\omega) = (2\cos\omega)[2\,Sa(2\omega)]$$

$$H(j\omega) = H_1(j\omega)H_2(j\omega)$$

where, $H_1(j\omega) = 2\cos\omega$

$$H_1(j\omega) = 2\left[\frac{e^{j\omega} + e^{-j\omega}}{2}\right]$$

$$H_1(j\omega) = [e^{j\omega} + e^{-j\omega}]$$

and $H_2(j\omega) = 2Sa(2\omega)$

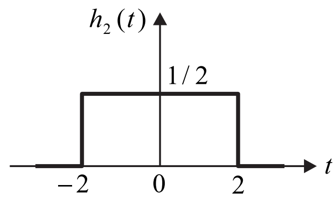
Fourier transform of rectangular signal is given by,

$$A\,rect\left[\frac{t}{\tau}\right] \xleftrightarrow{F.T.} A\tau\, Sa\left[\frac{\omega\tau}{2}\right]$$

Here, $\frac{\tau}{2} = 2 \Rightarrow \tau = 4$

and $A\tau = 2 \Rightarrow A\times 4 = 2 \Rightarrow A = \frac{1}{2}$

Hence, $2Sa(2\omega) \xleftrightarrow{I.F.T.} \frac{1}{2} rect\left[\frac{t}{4}\right] = h_2(t)$

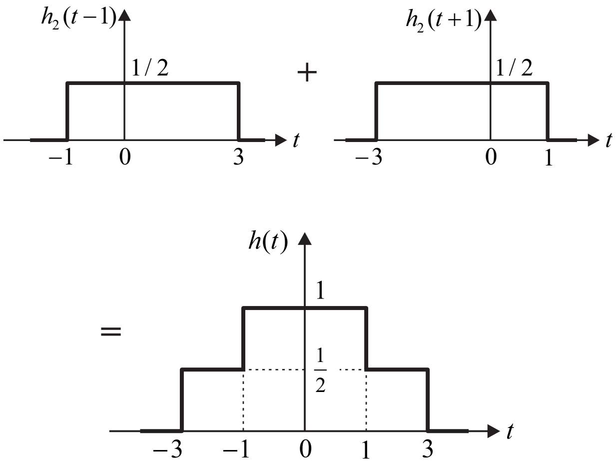


$$H(j\omega) = (e^{j\omega} + e^{-j\omega})H_2(j\omega)$$

$$H(j\omega) = e^{j\omega}H_2(j\omega) + e^{-j\omega}H_2(j\omega)$$

Taking inverse Fourier transform,

$$h(t) = h_2(t+1) + h_2(t-1)$$



From above figure,

$$h(0) = 1$$

Hence, the correct option is (C).

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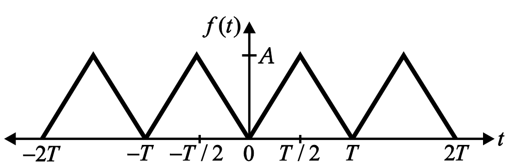
6 Continuous & Discrete Time Fourier Series

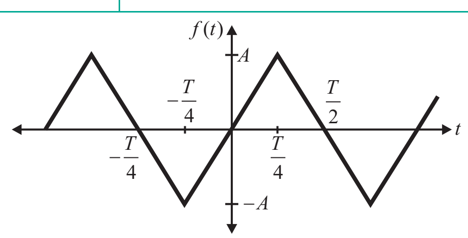
➢ Partial Synopsis

Trigonometric F.S. Coefficients for signals having symmetry

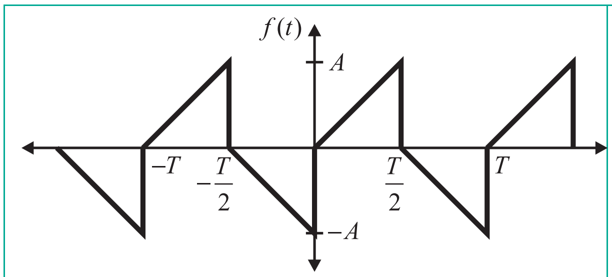
Symmetry	Coefficients			F.S. Representation
Even Symmetry $\boldsymbol{x(t) = x(-t)}$ The trigonometric Fourier series representation of even signals contains cosine terms only. The constant a_0 may or may not be zero.	$\boldsymbol{a_0 \neq 0}$ Or $\boldsymbol{a_0 = 0}$	$\boldsymbol{a_n \neq 0}$	$\boldsymbol{b_n = 0}$	$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t$ $a_0 = \frac{2}{T_0} \int_0^{T_0/2} x(t) dt$ $a_n = \frac{4}{T_0} \int_0^{T_0/2} x(t) \cos n\omega_0 t \, dt$
Odd Symmetry $x(t) = -x(-t)$ The trigonometric Fourier series representation of odd signals contains sine terms only. It has a zero average value, $a_0 = 0$.	$\boldsymbol{a_0 = 0}$	$\boldsymbol{a_n = 0}$	$\boldsymbol{b_n \neq 0}$	$x(t) = \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$ $b_n = \frac{4}{T_0} \int_0^{T_0/2} x(t) \sin n\omega_0 t \, dt$
Half-wave Symmetry $\boldsymbol{x(t) = -x\left(t \pm \frac{T_0}{2}\right)}$ For a signal having half-wave symmetry $a_0 = 0$ and a_n and b_n exists for odd values of n.	$\boldsymbol{a_0 = 0}$	$\boldsymbol{a_{2n} = 0,}$ $\boldsymbol{a_{2n+1} \neq 0}$	$\boldsymbol{b_{2n} = 0,}$ $\boldsymbol{b_{2n+1} \neq 0}$	$x(t) = \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$ $a_n = \frac{4}{T_0} \int_0^{T_0/2} x(t) \cos n\omega_0 t \, dt$, n odd $b_n = \frac{4}{T_0} \int_0^{T_0/2} x(t) \sin n\omega_0 t \, dt$, n odd



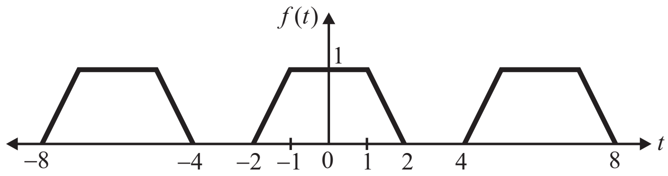
Waveform with even symmetry



Waveform with odd symmetry



Waveform with half-wave symmetry



Waveform with hidden half-wave symmetry

Some waveform show half wave symmetry (hidden) after subtraction of the dc component (a_0), such waveform is shown in figure. The trigonometric Fourier series representation of half-symmetric signals contains only odd harmonics of sine and cosine terms.

Note : RMS value of a periodic waveform $= \sqrt{\frac{1}{T}\int_0^T f^2(t)dt}$

Polar Fourier Series

The polar form or cosine form of Fourier series is expressed as follows

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t - \theta_n)$$

Where, $A_0 = a_0$ $\qquad A_n = \sqrt{a_n^2 + b_n^2}$ $\qquad \theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$

➢ Sample Questions

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6.1 A half-wave rectified sinusoidal waveform has a peak voltage of 10 V. Its average value and the peak value of the fundamental component are respectively given by

(A) $\frac{20}{\pi}$ V, $\frac{10}{\sqrt{2}}$ V (B) $\frac{10}{\pi}$ V, $\frac{10}{\sqrt{2}}$ V

(C) $\frac{10}{\pi}$ V, 5 V (D) $\frac{20}{\pi}$ V, 5 V

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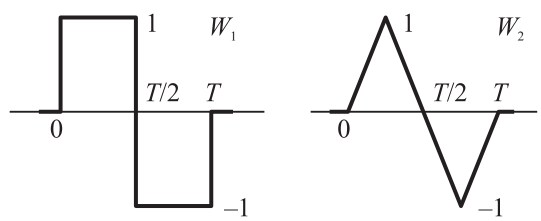
6.2 One period $(0, T)$ each of two periodic waveforms W_1 and W_2 are shown in figure. The magnitudes of the n^{th} Fourier series coefficients of W_1 and W_2 for $n \geq 1$, n odd are respectively proportional to



(A) $|n^{-3}|$ and $|n^{-2}|$

(B) $|n^{-2}|$ and $|n^{-3}|$

(C) $|n^{-1}|$ and $|n^{-2}|$

(D) $|n^{-4}|$ and $|n^{-2}|$

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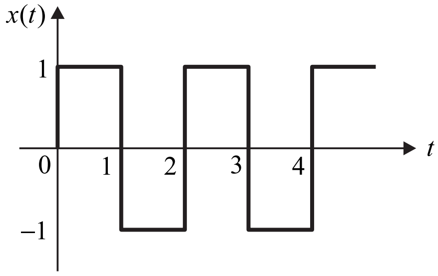
6.3 Consider the periodic square wave in the figure shown. **[Set - 02]**



The ratio of the power in the 7th harmonic to the power in the 5th harmonic for this waveform is closest in value to __________.

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6.4 The exponential Fourier series representation of continuous time periodic signal $x(t)$ is defined as

$$x(t)=\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where ω_0 is the fundamental angular frequency of $x(t)$ and the coefficient of series are a_k.

The following information is given about $x(t)$ and a_k.

(i) $x(t)$ is real and even having fundamental period of 6 sec.

(ii) Average value of $x(t)$ is 2.

(iii) $a_k=\begin{cases}k, & 1\le k\le 3\\ 0, & k>3\end{cases}$

The average power of the signal $x(t)$ (round of one decimal place) is _______.

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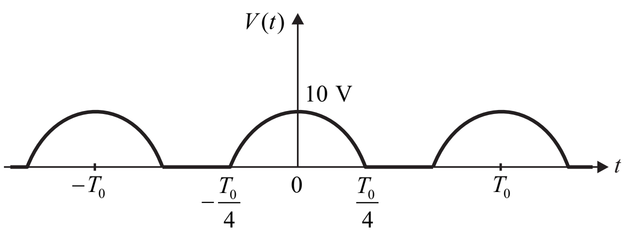
Explanations — Continuous & Discrete Time Fourier Series

6.1 (C)

Given : $V_m=10\text{ V}$

Half-wave rectified sinusoidal waveform is shown below,



Let $V(t)=V_m\cos(\omega_0 t)$; $|t|<\dfrac{T_0}{4}$

Time period $=T_0$

Fundamental frequency,

$$f_0=\frac{1}{T_0}=1^{\text{st}}\text{ harmonics}$$

Fourier series of $V(t)$ is given by,

$$V(t)=a_0+a_1\cos(\omega_0 t)+a_2\cos(2\omega_0 t)+\cdots$$

where, $a_0=\text{DC value}=\text{average value}$

$$a_0=\frac{1}{T_0}\int_{T_0}V(t)dt=\frac{1}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}}V_m\cos(\omega_0 t)dt$$

$$a_0=\frac{1}{T_0}\frac{V_m[\sin\omega_0 t]_{-T_0/4}^{T_0/4}}{\omega_0}$$

$$a_0=\frac{V_m}{T_0\times\frac{2\pi}{T_0}}\left[\sin\left(\frac{2\pi}{T_0}\times\frac{T_0}{4}\right)-\sin\left\{\frac{2\pi}{T_0}\times\left(\frac{-T_0}{4}\right)\right\}\right]$$

$$a_0=\frac{V_m}{2\pi}[1-(-1)]=\frac{V_m}{2\pi}\times 2$$

$$a_0=\frac{V_m}{\pi}$$

Here, $V_m=10\text{V}$

Then, $a_0=\dfrac{10}{\pi}\text{V}$

Fundamental component is given by,

$$a_1 = \frac{2}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} V(t)\cos(\omega_0 t)dt$$

$$a_1 = \frac{2}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} V_m\cos(\omega_0 t)\cos(\omega_0 t)dt$$

$$a_1 = \frac{2}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} V_m\cos^2(\omega_0 t)dt$$

$$a_1 = \frac{2V_m}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} \frac{(1+\cos 2\omega_0 t)}{2}dt$$

$$a_1 = \frac{2V_m}{2T_0}\left[\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} dt + \int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} \cos 2\omega_0 t\, dt\right]$$

$$a_1 = \frac{V_m}{T_0}\left[\frac{T_0}{4} - \left(-\frac{T_0}{4}\right)\right] + \frac{V_m}{T_0}\left[\frac{\sin 2\omega_0 t}{2\omega_0}\right]_{-T_0/4}^{T_0/4}$$

$$a_1 = \frac{V_m}{T_0}\left[\frac{T_0}{4} - \left(-\frac{T_0}{4}\right)\right] = \frac{V_m}{T_0}\times\frac{T_0}{2}$$

$$a_1 = \frac{V_m}{2}$$

$$a_1 = \frac{10}{2} = 5\text{ V}$$

Since, $V(-t) = V(t) \;\Rightarrow\; V(t)$ is an even signal.

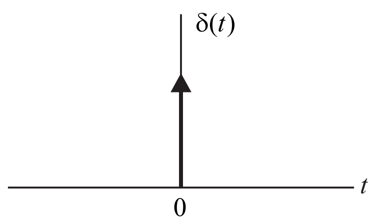
So, $b_n = 0$

Hence, the correct option is (C).

6.2 (C)

Exponential Fourier series coefficient C_n of different signals are,

(i)



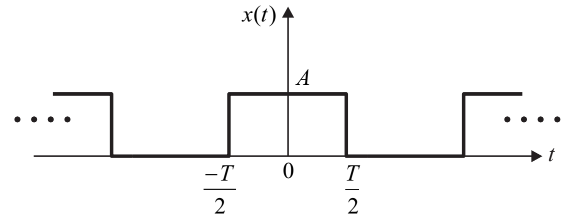
$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-j\omega_0 nt}dt$$

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)e^{-j\omega_0 nt}dt$$

From property of delta function,

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)\,dt = \frac{1}{T_0}$$

(ii)



Time period of above signal $x(t)$ is T_0.

$$C_n = \frac{1}{T_0}\int_{-T/2}^{T/2} x(t)e^{-j\omega_0 nt}dt = \frac{A}{T_0}\int_{-T/2}^{T/2} e^{-j\omega_0 nt}dt$$

$$C_n = \frac{A}{T_0}\left[\frac{e^{-j\omega_0 nt}}{-j\omega_0 n}\right]_{-T/2}^{T/2}$$

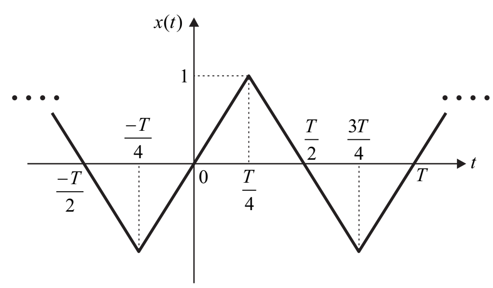
$$C_n = \frac{A}{T_0}\left[\frac{e^{-j\omega_0 n\frac{T}{2}} - e^{j\omega_0 n\frac{T}{2}}}{-j\omega_0 n}\right]$$

$$C_n = \frac{2A}{n\omega_0 T_0}\left[\frac{e^{j\omega_0 n\frac{T}{2}} - e^{-j\omega_0 n\frac{T}{2}}}{2j}\right]$$

$$C_n = \frac{2A}{T_0 n\omega_0}\sin\left(n\omega_0\frac{T}{2}\right)$$

$$C_n \propto \frac{1}{n}$$

(iii) Let the given signal be $x(t)$



Time period of above signal $x(t)$ is T.

$$x(t)=\begin{cases}\frac{4}{T}t & \frac{-T}{4}<t<\frac{T}{4}\\ \frac{-4}{T}t+2 & \frac{T}{4}<t<\frac{3T}{4}\end{cases}$$

$$C_n=\frac{1}{T}\int_{-T/4}^{T/4}\frac{4}{T}te^{-j\omega_0 nt}dt$$

$$+\frac{1}{T}\int_{T/4}^{3T/4}\left(\frac{-4}{T}t+2\right)e^{-j\omega_0 nt}dt$$

$$C_n=\left[\frac{4}{T^2}\left(\frac{te^{-j\omega_0 nt}}{-j\omega_0 n}-\frac{e^{-j\omega_0 nt}}{(-j\omega_0 n)^2}\right)_{-T/4}^{T/4}\right]$$

$$+\left[\frac{-4}{T^2}\left(\frac{te^{-j\omega_0 nt}}{-j\omega_0 n}-\frac{e^{-j\omega_0 nt}}{(-j\omega_0 n)^2}\right)_{T/4}^{3T/4}\right.$$

$$\left.+\frac{2}{T}\left(\frac{e^{-j\omega_0 nt}}{-j\omega_0 n}\right)_{T/4}^{3T/4}\right]$$

$$C_n=\left[\frac{4}{T^2}\left(\frac{te^{-j\omega_0 nt}}{-j\omega_0 n}+\frac{e^{-j\omega_0 nt}}{\omega_0^2 n^2}\right)_{-T/4}^{T/4}\right]$$

$$+\left[\frac{-4}{T^2}\left(\frac{te^{-j\omega_0 nt}}{-j\omega_0 n}+\frac{e^{-j\omega_0 nt}}{\omega_0^2 n^2}\right)_{T/4}^{3T/4}\right.$$

$$\left.+\frac{2}{T}\left(\frac{e^{-j\omega_0 nt}}{-j\omega_0 n}\right)_{T/4}^{3T/4}\right]$$

From above equation,

$$C_n\propto\frac{1}{n^2}$$

Hence, the correct option is (C).

Key Point

(i) For all rectangular signal,

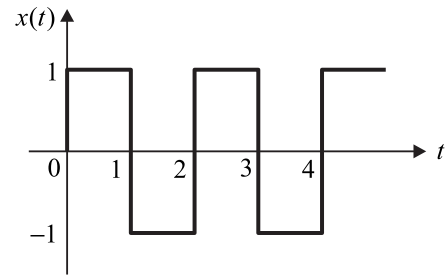
$$C_n\propto\frac{1}{n}$$

(ii) For all triangular signal,

$$C_n\propto\frac{1}{n^2}$$

6.3 **0.51**

Periodic sequence wave is shown below.



Peak value $V_m=1$ volt

Time period $T_0=2$ sec

Frequency $\omega_0=\frac{2\pi}{T_0}=\pi$ rad/sec

Method 1

Since, for the periodic waveform shown $x(t)=-x(-t)$ i.e. the above signal is odd and for an odd signal Fourier coefficient $a_n=0$, $a_0=0$ and $x(t)=-x\left(t\pm\frac{T}{2}\right)$ then the signal exhibits half wave symmetry. Therefore, only odd component of sine wave present in series.

and $$b_n=\frac{2}{T_0}\int_0^{T_0}x(t)\sin\omega_0 nt\,dt$$

Here, $\omega_0=\pi$ and $T_0=2$

Then, $$b_n=\int_0^1\sin n\pi t\,dt-\int_1^2\sin n\pi t\,dt$$

$$b_n=\left[-\frac{\cos n\pi t}{n\pi}\right]_0^1+\left[\frac{\cos n\pi t}{n\pi}\right]_1^2$$

$$b_n=\frac{-2\cos n\pi+2}{n\pi}$$

$$b_n=\frac{4}{n\pi}\quad[\text{For } n=\text{odd}]$$

$$x(t)=\sum_{n=1,3,5}^{\infty}\frac{4V_m}{n\pi}\sin n\omega t$$

$$x(t)=\sum_{n=1,3,5}^{\infty}\frac{4}{n\pi}\sin n\omega t$$

$$x(t)=\frac{4}{\pi}\sin\pi t+\frac{4}{3\pi}\sin 3\pi t+\frac{4}{5\pi}\sin 5\pi t+\frac{4}{7\pi}\sin 7\pi t+\cdots\cdots$$

The rms value of 5th harmonic is,

$$V_{5rms}=\frac{1}{\sqrt{2}}\left[\frac{4}{5\pi}\right]=\frac{2\sqrt{2}}{5\pi}$$

The rms value of 7th harmonic is,

$$V_{7rms}=\frac{1}{\sqrt{2}}\left[\frac{4}{7\pi}\right]=\frac{2\sqrt{2}}{7\pi}$$

Power and rms value are related by,

$$P\propto V_{rms}^2$$

$$\frac{P_7}{P_5}=\frac{V_{7rms}^2}{V_{5rms}^2}$$

$$\frac{P_7}{P_5}=\frac{\left(\frac{2\sqrt{2}}{7\pi}\right)^2}{\left(\frac{2\sqrt{2}}{5\pi}\right)^2}=\frac{25}{49}\cong 0.51$$

Hence, the ratio of the power in the 7th harmonic to the power in the 5th harmonic is **0.51.**

Method 2

Fourier series coefficient of rectangular signal

$$a_n\propto\frac{1}{n}$$

Hence, $a_5\propto\frac{1}{5}$ and $a_7\propto\frac{1}{7}$

We have power $p_n\propto|a_n|^2$

Hence, $\frac{P_7}{P_5}=\frac{|a_7|^2}{|a_5|^2}=\frac{(1/7)^2}{(1/5)^2}=0.51$

Hence, the ratio of the power in the 7th harmonic to the power in the 5th harmonic is **0.51.**

6.4 32

Given : $x(t)=\sum_{k=-\infty}^{\infty}a_k e^{jk\omega_0 t}$

(i) $x(t)$ is real and even with $T=6$ sec

(ii) Average value of $x(t)$=2

(iii) $a_k=\begin{cases}k, & 1\le k\le 3\\ 0, & k>3\end{cases}$

From symmetry conditions, as $x(t)$ is real and even, so its fourier series coefficients will also be real and even.

i.e. $a_k^*=a_k$ and $a_{-k}=a_k$

so if $a_1=1\Rightarrow a_{-1}=a_1=1$

$$a_2=2\Rightarrow a_{-2}=a_2=2$$

$$a_3=3\Rightarrow a_{-3}=a_3=3$$

$$a_k=0 \text{ for } k>3$$

$$a_{-k}=0 \text{ for } k<-3$$

Also, given that average value of $x(t)$

i.e. $\frac{1}{T}\int_0^T x(t)\,dt=a_0=2$

So, the complete set of Fourier series coefficients is given as

$$a_k=\{3,2,1,\underset{\uparrow}{2},1,2,3\}=\{a_{-3},a_{-2},a_{-1},a_0,a_1,a_2,a_3\}$$

Using Parseval's theorem, average power of $x(t)$ is given as

$$P_x=\frac{1}{T}\int_0^T|x(t)|^2\,dt=\sum_{k=-\infty}^{\infty}|a_k|^2$$

$$P_x=(3)^2+(2)^2+(1)^2+(2)^2+(1)^2+(2)^2+(3)^2$$

$$P_x=9+4+1+4+1+4+9$$

$$P_x=32\text{ W}$$

Hence, the average power of signal $x(t)$ is 32 W

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7 Z - Transform

➢ Partial Synopsis

Properties of Z-Transform

(1) Linearity property :

If $x_1[n] \xleftrightarrow{ZT} X_1(z)$	ROC : R_1
$x_2[n] \xleftrightarrow{ZT} X_2(z)$	ROC : R_2
Then $ax_1[n]+bx_2[n] \xleftrightarrow{ZT} aX_1(z)+bX_2(z)$	ROC : Atleast $R_1 \cap R_2$

(2) Time shifting property :

If $x[n] \xleftrightarrow{ZT} X(z)$ ROC : R

Then $x[n \pm k] \xleftrightarrow{ZT} z^{\pm k}\, X(z)$ ROC : R, except possible addition or deletion of $z = 0/\infty$

(3) Scaling property :

If $x[n] \xleftrightarrow{ZT} X(z)$	ROC : R
Then $a^n x[n] \xleftrightarrow{ZT} X\left(\frac{z}{a}\right)$	ROC : $\lvert a \rvert R$

(4) Time reversible property :

If $x[n] \xleftrightarrow{ZT} X(z)$	ROC : R
Then $x[-n] \xleftrightarrow{ZT} X(z^{-1})$	ROC : $\frac{1}{R}$

(5) Convolution property :

If $x_1[n] \xleftrightarrow{ZT} X_1(z)$	ROC : R_1
$x_2[n] \xleftrightarrow{ZT} X_2(z)$	ROC : R_2
Then $x_1[n] \otimes x_2[n] \xleftrightarrow{ZT} X_1(z)\; X_2(z)$	ROC : $R_1 \cap R_2$

(6) Multiplication by n :

If $x[n] \xleftrightarrow{ZT} X(z)$	ROC : R
Then $n\,x[n] \xleftrightarrow{ZT} -z\dfrac{d}{dz}X(z)$	ROC : R

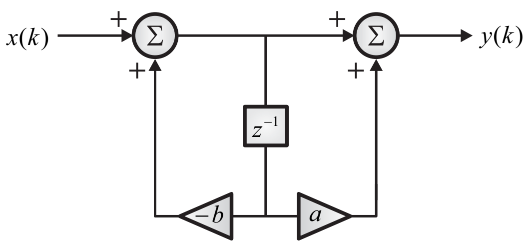
(7) Time differencing :

If $x[n] \xleftrightarrow{ZT} X(z)$	ROC : R
Then $x[n]-x[n-1] \xleftrightarrow{ZT} X(z)(1-z^{-1})$	ROC : $R \cap (\lvert z\rvert > 0)$

➢ Sample Questions

1988 IIT Kharagpur

7.1 Consider the system shown in the figure below. The transfer function $Y(z)/X(z)$ of the system is



(A) $\dfrac{1+az^{-1}}{1+bz^{-1}}$ (B) $\dfrac{1+bz^{-1}}{1+az^{-1}}$

(C) $\dfrac{1+az^{-1}}{1-bz^{-1}}$ (D) $\dfrac{1-bz^{-1}}{1+az^{-1}}$

2014 IIT Kharagpur

7.2 For an all-pass system $H(z)=\dfrac{(z^{-1}-b)}{(1-az^{-1})}$, where $\left|H\left(e^{-j\omega}\right)\right|=1$, for all ω. If Re $(a)\neq 0$, Im$(a)\neq 0$, then b equals **[Set - 03]**

(A) a (B) a^*

(C) $1/a^*$ (D) $1/a$

2015 IIT Kanpur

7.3 Two causal discrete-time signals $x[n]$ and $y[n]$ are related as $y[n]=\sum_{m=0}^{n} x[m]$. If the Z-Transform of $y[n]$ is $\dfrac{2}{z(z-1)^2}$, the value of $x[2]$ is ______. **[Set - 02]**

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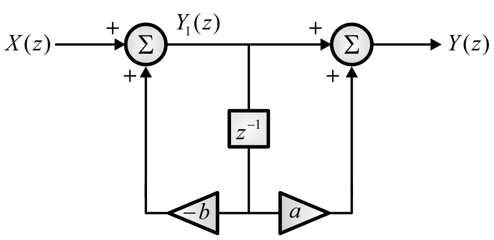
Explanations Z-Transform

7.1 (A)

Given system is shown below,



From above figure, $Y_1(z)$ can be written as,

$$Y_1(z)=X(z)-bz^{-1}Y_1(z)$$

$$Y_1(z)[1+bz^{-1}]=X(z)$$

$$Y_1(z)=\frac{X(z)}{(1+bz^{-1})} \quad \ldots(i)$$

$Y(z)$ can be written as,

$$Y(z)=Y_1(z)+az^{-1}Y_1(z)$$

$$Y(z) = Y_1(z)[1 + az^{-1}] \quad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$Y(z) = \frac{X(z)}{(1 + bz^{-1})}(1 + az^{-1})$$

Transfer function $H(z)$ is given by,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(1 + az^{-1})}{(1 + bz^{-1})}$$

Hence, the correct option is (A).

7.2 (B)

Given : $H(z) = \dfrac{(z^{-1} - b)}{(1 - az^{-1})}$

Method 1

Let, $a = a_1 + ja_2$ and $b = b_1 + jb_2$

So, $$H(j\omega) = \frac{e^{-j\omega} - (b_1 + jb_2)}{1 - (a_1 + ja_2)(e^{-j\omega})}$$

$$H(j\omega) = \frac{(\cos\omega - j\sin\omega) - (b_1 + jb_2)}{1 - (a_1 + ja_2)(\cos\omega - j\sin\omega)}$$

$$= \frac{(\cos\omega - b_1) - j(\sin\omega + b_2)}{1 - \left[\begin{array}{l}(a_1\cos\omega + a_2\sin\omega) \\ \qquad + j(a_2\cos\omega - a_1\sin\omega)\end{array}\right]}$$

$$|H(j\omega)|^2 = \frac{(\cos\omega - b_1)^2 + (\sin\omega + b_2)^2}{\left[\begin{array}{l}\{1 - (a_1\cos\omega + a_2\sin\omega)\}^2 \\ \qquad + (a_2\cos\omega - a_1\sin\omega)^2\end{array}\right]}$$

$$|H(j\omega)|^2 = 1$$

Hence,

$$\left[\cos^2\omega - 2b_1\cos\omega + b_1^2\right] + \left[\sin^2\omega + 2b_2\sin\omega + b_2^2\right]$$
$$= 1 + (a_1\cos\omega + a_2\sin\omega)^2 - 2(a_1\cos\omega + a_2\sin\omega) + (a_2\cos\omega - a_1\sin\omega)^2$$

$$1 + b_1^2 + b_2^2 + 2(b_2\sin\omega - b_1\cos\omega) = 1 + a_1^2 + a_2^2 - 2a_1\cos\omega - 2a_2\sin\omega$$

$$b_1^2 + b_2^2 - 2b_1\cos\omega + 2b_2\sin\omega = a_1^2 + a_2^2 - 2a_1\cos\omega - 2a_2\sin\omega$$

On comparing,

$$b_1 = a_1 \text{ and } b_2 = -a_2$$

Then, $b = b_1 + jb_2 = a_1 - ja_2 = a^*$

Hence, the correct option is (B).

Method 2

$$H(z) = \frac{(z^{-1} - b)}{(1 - az^{-1})}$$

Pole : $(1 - az^{-1}) = 0$

$$az^{-1} = 1$$
$$z = a$$

Zero : $(z^{-1} - b) = 0$

$$z = \frac{1}{b}$$

But, a is complex in nature.

Then, $a = |a| e^{j\angle a}$

Key Point

For an all pass system where $|H(e^{j\omega})| = K$ for all ω, if the pole lies at 'a' then the zero must be present at $\dfrac{1}{a^*}$.

Zero $= \dfrac{1}{b} = \dfrac{1}{a^*} = \dfrac{1}{|a| e^{-j\angle a}}$

Since, $b = a^*$

Hence, the correct option is (B).

7.3 0

Given : $y[n] = \sum_{m=0}^{n} x[m]$

Method 1

Taking Z-Transform,

$$Z\{y[n]\} = Z\left\{\sum_{m=0}^{n} x[m]\right\}$$

$$Y(z) = \frac{1}{1 - z^{-1}} X(z)$$

[By summation property of Z-Transform]

Also, $$Y(z) = \frac{2}{z(z-1)^2}$$

Therefore, $\dfrac{2}{z(z-1)^2} = \dfrac{z}{(z-1)} X(z)$

Thus, $X(z) = \dfrac{2}{z^2(z-1)} = z^{-2}\dfrac{2}{(z-1)}$

$$X(z) = 2z^{-3}\frac{z}{(z-1)}$$

Taking inverse Z-Transform,

$$x[n] = 2u[n-3]$$

[By shifting property of Z-Transform]

Put $n = 2$,

$$x[2] = 2u[2-3] = 2u[-1] = 0$$

Hence, the value of $x[2]$ is **0.**

Method 2

$$Y(z) = \frac{2}{z(z-1)^2} = \frac{2}{z^3(1-z^{-1})^2}$$

$$Y(z) = \frac{2}{z^3}(1-z^{-1})^{-2}$$

$$Y(z) = \frac{2}{z^3}(1+2z^{-1}+3z^{-2}+4z^{-3}+5z^{-4}+...)$$

[From binomial expansion]

$$Y(z) = 2z^{-3}+4z^{-4}+6z^{-5}+8z^{-6}+10z^{-7}+...$$

Taking inverse Z-Transform,

$$y[n] = (\underset{\uparrow}{0}, 0, 0, 2, 4, 6, 8, 10, ...)$$

and $y[n] = \sum_{m=0}^{n} x[m]$

At $n = 0$, $y[0] = x[0]$

Thus, $x[0] = 0$

At $n = 1$,

$$y[1] = \sum_{m=0}^{1} x[m] = x[0] + x[1]$$

$$0 = 0 + x[1]$$

Then, $x[1] = 0$

At $n = 2$,

$$y[2] = \sum_{m=0}^{2} x[m] = x[0] + x[1] + x[2]$$

$$0 = 0 + 0 + x[2]$$

$$x[2] = 0$$

Hence, the value of $x[2]$ is **0.**

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8 Discrete Time Convolution

Sample Questions

2012 IIT Delhi

8.1 Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = (1/2)^n u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = 1/2$, then $g[1]$ equals

(A) 0 (B) 1/2

(C) 1 (D) 3/2

2017 IIT Roorkee

8.2 Two discrete-time signals $x[n]$ and $h[n]$ are both non-zero only for $n = 0, 1, 2$ and are zero otherwise. It is given that

$$x[0]=1,\ x[1]=2,\ x[2]=1,\ h[0]=1$$

Let $y[n]$ be the linear convolution of $x[n]$ and $h[n]$. Given that $y[1]=3$ and $y[2]=4$, the value of the expression $(10\,y[3]+y[4])$ is _______.

[Set - 01]

2021 IIT Bombay

8.3 For a unit step input $u[n]$, a discrete time LTI system produces an output signal $[2\delta[n+1]+\delta[n]+\delta[n-1]]$. Let $y[n]$ be the output of system for an input $\left[\left(\frac{1}{2}\right)^n u[n]\right]$. The value of $y[0]$ is _______.

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Explanations **Discrete Time Convolution**

8.1 **(A)**

Given : $h[n]=\left(\frac{1}{2}\right)^n u[n]$

Method 1 : Convolution

$$y[0]=1 \text{ and } y[1]=\frac{1}{2}$$

$$y[n]=h[n]\otimes g[n]$$

$$y[n]=\sum_{k=-\infty}^{\infty} g[k]h[n-k] \qquad \text{...(i)}$$

$$y[n]=\sum_{k=0}^{\infty} g[k]h[n-k] \quad [g[n] \text{ is causal}]$$

Put $n=0$,

$$y[0]=\sum_{k=0}^{\infty} g[k]h[-k]$$

$$y[0]=g[0]h[0]+g[1]h[-1]+.....$$

$$1=g[0]\times 1+0+0+......$$

$[h[n]$ is causal]

$$g[0]=1$$

$$y[n]=\sum_{k=0}^{\infty} g[k]h[n-k]$$

Put $n=1$,

$$y[1]=\sum_{k=0}^{\infty} g[k]h[1-k]$$

$$y[1]=g[0]h[1]+g[1]h[0]+g[2]h[-1]+....$$

$$\frac{1}{2}=1\times\frac{1}{2}+g[1]\times 1+0+0+.......$$

$$g[1]=0$$

Hence, the correct option is (A).

Method 2 : Tabular Method

Output $y[n]$ is given by,

$$y[n]=g[n]\otimes h[n]$$

where, $h[n]=\left(\frac{1}{2}\right)^n u(n)$

$$h[n]=\left\{\underset{\uparrow}{1},\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},.......\right\}$$

and $\quad y[n]=\left\{\underset{\uparrow}{1},\ \frac{1}{2}\right\}.$

$g[n]$ \ $h[n]$	$\downarrow$ 1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
$\rightarrow g[0]$	$g[0]$	$\frac{1}{2}g[0]$	$\frac{1}{4}g[0]$	$\frac{1}{8}g[0]$
$g[1]$	$g[1]$	$\frac{1}{2}g[1]$	$\frac{1}{4}g[1]$	$\frac{1}{8}g[1]$

$$y[0]=g[0]=1$$

$$y[1]=g[1]+\frac{1}{2}g[0]$$

$$\frac{1}{2}=g[1]+\frac{1}{2}$$

$$g[1]=0$$

Hence, the correct option is (A).

Method 3 : Z-Transform

$$h[n]=\left(\frac{1}{2}\right)^n u[n]$$

Then, $\quad H(z)=\dfrac{1}{1-\frac{1}{2}z^{-1}}$

$$y[0]=1 \text{ and } y[1]=\frac{1}{2}$$

$$Y(z)=1+\frac{1}{2}z^{-1}$$

Since, $Y(z)=H(z)G(z)$

$$G(z)=\frac{Y(z)}{H(z)}$$

$$G(z)=\left(1+\frac{1}{2}z^{-1}\right)\div\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)$$

$$G(z)=\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)$$

$$G(z)=1-\frac{1}{4}z^{-2}$$

Thus, $g[0]=1$, $g[1]=0$ and

$$g[2]=-\frac{1}{4}$$

Hence, the correct option is (A).

Scan for Video Solution

8.2 **31**

Given :

$$x[n]=\{\underset{\uparrow}{1},\ 2,\ 1\} \text{ and } y[n]=\{y(0),\ \underset{\uparrow}{3},\ 4,.....\}$$

$$h[n]=\{\underset{\uparrow}{1},\ h_1,\ h_2\}$$

Method 1 : Z-Transform

Z-transform of $x[n]$ and $h[n]$ is given below,

$$X(z)=1+2z^{-1}+z^{-2}$$

$$H(z)=1+h_1z^{-1}+h_2z^{-2}$$

Since, $y[n]=x[n]\otimes h[n]$

Z-transform of $y[n]$ is given by,

$$Y(z) = X(z)H(z)$$

[By convolution property of Z-transform]

$$Y(z) = 1 + h_1 z^{-1} + h_2 z^{-2} + 2z^{-1} + 2h_1 z^{-2} + 2h_2 z^{-3} + z^{-2} + h_1 z^{-3} + h_2 z^{-4}$$

$$Y(z) = 1 + (h_1 + 2)z^{-1} + (h_2 + 2h_1 + 1)z^{-2} + (2h_2 + h_1)z^{-3} + h_2 z^{-4}$$

Inverse Z-transform of $Y(z)$ i.e. $y(n)$ is given below,

$$y[n] = \{\underset{\uparrow}{1}, \quad h_1 + 2, \quad h_2 + 2h_1 + 1, \quad 2h_2 + h_1, \quad h_2\}$$

$$y[1] = h_1 + 2 = 3$$

Then, $h_1 = 1$

$$y[2] = h_2 + 2h_1 + 1 = 4$$

Thus, $h_2 = 1$

$$y[3] = 2h_2 + h_1 = 3$$

$$y[4] = h_2 = 1$$

$$10y[3] + y[4] = 30 + 1 = 31$$

Hence, the value of the expression is **31.**

Method 2 : Tabular Method

Since, $y[n] = x[n] \otimes h[n]$

$h[n]$ \ $x[n]$	$\downarrow$ 1	2	1
$\rightarrow$ 1	1	2	1
h_1	h_1	$2h_1$	h_1
h_2	h_2	$2h_2$	h_2

Hence,

$$y[n] = \{\underset{\uparrow}{1}, \quad h_1 + 2, \quad h_2 + 2h_1 + 1, \quad 2h_2 + h_1, \quad h_2\}$$

$$y[1] = h_1 + 2 = 3$$

Then, $h_1 = 1$

$$y[2] = h_2 + 2h_1 + 1 = 4$$

Hence, $h_2 = 1$

$$y[3] = 2h_2 + h_1 = 3$$

$$y[4] = h_2 = 1$$

$$10y[3] + y[4] = 30 + 1 = 31$$

Hence, the value of the expression is **31.**

8.3 **0**

Given response of the system for step input, i.e. step response is,

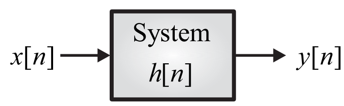
$$S[n] = 2\delta[n+1] + \delta[n] + \delta[n-1]$$

Impulse response of the system can be obtained by taking first difference of $S[n]$

$$h[n] = S[n] - S[n-1]$$

$$h[n] = 2\delta[n+1] + \delta[n] + \delta[n-1] - 2\delta[n] - \delta[n-1] - \delta[n-2]$$

$$h[n] = 2\delta[n+1] - \delta[n] - \delta[n-2]$$



For input, $x[n] = \left(\frac{1}{2}\right)^n u[n]$

Output, $y[n] = x[n] \otimes h[n]$

$$y[n] = \left(\frac{1}{2}\right)^n u[n] \otimes \{2\delta[n+1] - \delta[n] - \delta[n-2]\}$$

$$y[n] = 2\left(\frac{1}{2}\right)^{n+1} u[n+1] - \left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{2}\right)^{n-1} u[n-2]$$

$$(\because \quad x[n] \otimes \delta[n-n_0] = x[n-n_0])$$

Substituting $n = 0$ both sides,

$$y[0] = 2 \times \left(\frac{1}{2}\right)^1 u[1] - \left(\frac{1}{2}\right)^0 u[0] - \left(\frac{1}{2}\right)^{-2} u(-2)$$

$$y[0] = 2 \times \frac{1}{2} - 1 - 0 \qquad (\because \ u[-2] = 0)$$

$$\therefore \qquad y[0] = 1 - 1 = 0$$

Hence, the value of $y[0]$ is 0.

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9 DTFT and DFT

Partial Synopsis

Complex valued phase factor (Twiddle factor)

$$W_N = e^{\frac{-j2\pi}{N}}$$

$$W_N^{kn} = e^{\frac{-j2\pi kn}{N}}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\ W_N^{kn}$$

DFT : $X_N = [W_N] \cdot x_N$ where X_N and x_N are vectors representing N-point DFT $X(k)$ and N-point sequence $x[n]$ respectively and $[W_N]$ is $N \times N$ matrix.

4-point DFT $X(k)$ of 4-point sequence $x[n]$ can be evaluated as

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$$

Using periodicity property of twiddle factor $W_N^{K+N} = W_N^K$, above expression can be reduced to

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^0 & W_4^2 \\ W_4^0 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$$

IDFT : $[x[n]] = \frac{1}{N} [W_N]^{-1}\ [X[k]]$ where $[W_N]^{-1} = [W_N]^*$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} = \frac{1}{N}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix}$$

Properties of DFT

Important results :

$$X(0) = \sum_{n=0}^{N-1} x[n] = x[0] + x[1] + x[2] + x[3] ..... + x[n-1] \quad \text{...(i)}$$

$$X\left(\frac{N}{2}\right) = \sum_{n=0}^{N-1} (-1)^n x[n] = x[0] - x[1] + x[2] - x[3] ..... - x[n-1] \quad \text{...(ii)}$$

From equation (i) and (ii),

$$X(0) + X\left(\frac{N}{2}\right) = 2\left[x(0) + x(2) + x(4).....\right]$$

$$X(0) - X\left(\frac{N}{2}\right) = 2\left[x(1) + x(3) + x(5).....\right]$$

(1) Linearity property :

If $x_1[n] \xleftrightarrow{\text{N Point DFT}} X_1(k)$

$x_2[n] \xleftrightarrow{\text{N Point DFT}} X_2(k)$

Then $ax_1[n] + bx_2[n] \xleftrightarrow{DFT} aX_1[k] + bX_2[k]$

(2) Time shifting property :

If $x[n] \xleftrightarrow{\text{N Point DFT}} X(k)$

Then $x((n \pm l))_N \xleftrightarrow{DFT} e^{\pm\frac{j2\pi kl}{N}} X[k]$

(3) Time reversal property :

If $x[n] \xleftrightarrow{\text{N Point DFT}} X(k)$

Then $x((-n))_N \xleftrightarrow{DFT} X((-k))_N$

Where $x((-k))_N = X((N-k))_N$

(4) Circular convolution :

If $x_1[n] \xleftrightarrow{\text{N Point DFT}} X_1(k)$

$x_2[n] \xleftrightarrow{\text{N Point DFT}} X_2(k)$

Then $x_1[n] \; Ⓝ \; x_2[n] \xleftrightarrow{DFT} X_1[k] X_2[k]$

➢ Sample Questions

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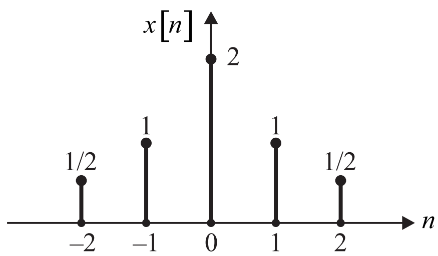
Statement for Linked Answer Questions 9.1 & 9.2

A sequence $x[n]$ has non-zero values as shown in the figure.

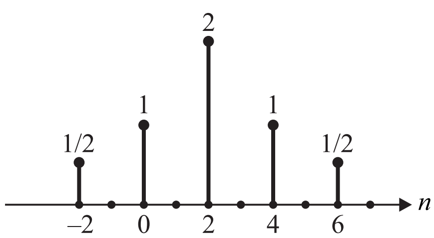


9.1 The sequence

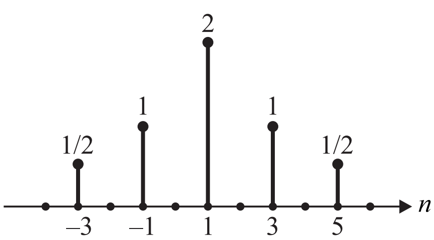
$$y[n]=\begin{cases} x\left[\frac{n}{2}-1\right]; & \text{for } n \text{ even} \\ 0; & \text{for } n \text{ odd} \end{cases}$$

will be

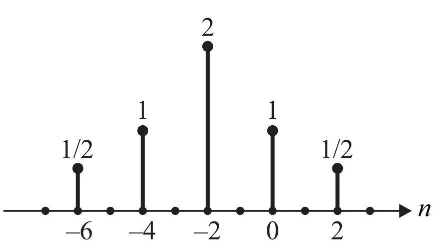
(A)



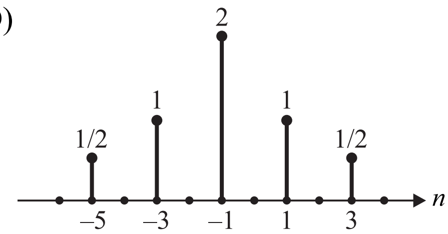
(B)



(C)



(D)



9.2 The Fourier transform of $y[2n]$ will be

(A) $e^{-2j\omega}[\cos 4\omega+2\cos 2\omega+2]$

(B) $[\cos 2\omega+2\cos 2\omega+2]$

(C) $e^{-j\omega}[\cos 2\omega+2\cos\omega+2]$

(D) $e^{\frac{-j\omega}{2}}[\cos 2\omega+2\cos\omega+2]$

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9.3 Let $X[k]=k+1,\ 0\le k\le 7$ be 8-point DFT of a sequence $x[n]$, where

$$X[k]=\sum_{n=0}^{N-1}x[n]e^{-j2\pi nk/N}$$

The value (correct to two decimal places) of $\sum_{n=0}^{3}x[2n]$ is _______.

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9.4 Consider the signal $x[n]=2^{n-1}u[-n+2]$ and $y[n]=2^{-n+2}u[n+1]$. Where $u[n]$ is the unit step sequence. Let $X(e^{j\omega})$ and $Y(e^{j\omega})$ be the discrete time Fourier transform of $x[n]$ and $y[n]$ respectively. The value of integral

$$\frac{1}{2\pi}\int_{0}^{2\pi}X(e^{j\omega})Y(e^{-j\omega})\,d\omega$$

(round off to one decimal places) is _______.

❖❖❖❖

Explanations DTFT and DFT

9.1 (A)

Given : $x[n]=\left\{\frac{1}{2},1,\underset{\uparrow}{2},1,\frac{1}{2}\right\}$

$$y[n]=\begin{cases} x\left[\frac{n}{2}-1\right]; & \text{for } n \text{ even} \\ 0; & \text{for } n \text{ odd}\end{cases}$$

$$y[-5]=y[-3]=y[-1]=y[1]=y[3].....=0$$

$$y[-4]=x\left[-\frac{4}{2}-1\right]=x[-3]=0$$

$$y[-2]=x\left[-\frac{2}{2}-1\right]=x[-2]=\frac{1}{2}$$

$$y[0]=x[0-1]=x[-1]=1$$

$$y[2]=x\left[\frac{2}{2}-1\right]=x[0]=2$$

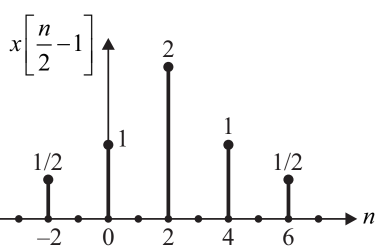
$$y[4]=x\left[\frac{4}{2}-1\right]=x[1]=1$$

$$y[6]=x\left[\frac{6}{2}-1\right]=x[2]=\frac{1}{2}$$

$$y[8]=x\left[\frac{8}{2}-1\right]=x[3]=0$$

All other values of $y[n]$ will be 0.

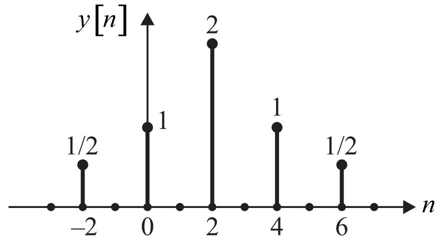
So, $\quad y[n]=\left\{\frac{1}{2},0,\underset{\uparrow}{1},0,2,0,1,0,\frac{1}{2}\right\}$

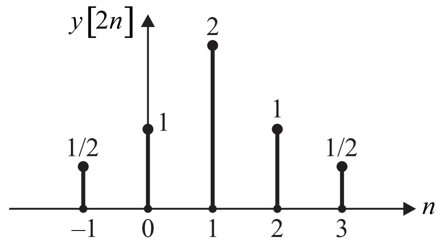


Hence, the correct option is (A).

9.2 (C)

$y[2n]$ is a time compression of $y[n]$.





$y[2n]$ can be written as,

$$y[2n]=\frac{1}{2}\delta[n+1]+\delta[n]+2\delta[n-1]$$
$$+\delta[n-2]+\frac{1}{2}\delta[n-3]$$

Consider $y[2n]=z[n]$

Taking Fourier transform,

$$y[2n]=z[n]\longleftrightarrow Z(e^{j\omega})$$

$$Z(e^{j\omega})=\frac{1}{2}e^{j\omega}+1+2e^{-j\omega}+e^{-j2\omega}+\frac{1}{2}e^{-j3\omega}$$

$$Z(e^{j\omega})=e^{-j\omega}\left(\frac{1}{2}e^{j2\omega}+e^{j\omega}+2+e^{-j\omega}+\frac{1}{2}e^{-j2\omega}\right)$$

$$Z(e^{j\omega})=e^{-j\omega}\left[\frac{1}{2}(e^{j2\omega}+e^{-j2\omega})+(e^{j\omega}+e^{-j\omega})+2\right]$$

$$Z(e^{j\omega})=e^{-j\omega}\left[\cos 2\omega+2\cos\omega+2\right]$$

Hence, the correct option is (C).

9.3 3

Given : $X[k]=k+1,\ 0\le k\le 7$,

$$X[k]=\{1, 2, 3, 4, 5, 6, 7, 8\} \qquad \ldots(i)$$

$$\sum_{n=0}^{3}x[2n]=x[0]+x[2]+x[4]+x[6] \ldots(ii)$$

N point Discrete Fourier Transform (DFT) given by,

$$X(k)=\sum_{n=0}^{N-1}x(n)e^{-j\frac{2\pi}{N}kn}$$

For $k=0$

$$X(0)=\sum_{n=0}^{7}x(n)=x(0)+x(1)+x(2)$$

$$+x(3)+x(4)+x(5)+x(6)+x(7) \quad \ldots\text{(iii)}$$

For $k=4$

$$X(4)=\sum_{n=0}^{7}x(n)e^{-j\pi n}$$

$$=\sum_{n=0}^{7}x(n)(-1)^n=x(0)-x(1)+x(2)-x(3)$$

$$+x(4)-x(5)+x(6)-x(7) \quad \ldots\text{(iv)}$$

Adding equation (iii) and (iv),

$$X(0)+X(4)=2\left[x(0)+x(2)+x(4)+x(6)\right]$$

$$x(0)+x(2)+x(4)+x(6)=\frac{X(0)+X(4)}{2}$$

From equation (i) and (ii),

$$\sum_{n=0}^{3}x[2n]=\frac{1+5}{2}=3$$

Hence, the value of $\sum_{n=0}^{3}x[2n]$ is **3.**

Scan for Video Solution

9.4 **8**

Given : $x[n]=2^{n-1}u[-n+2]\xrightarrow{DTFT}X(e^{j\omega})$

$$y[n]=2^{-n+2}u[n+1]\xrightarrow{DTFT}Y(e^{j\omega})$$

We have to evaluate the integral,

$$I=\frac{1}{2\pi}\int_0^{2\pi}X(e^{j\omega})Y(e^{-j\omega})\,d\omega \quad \ldots\text{(i)}$$

If $p[n]\xleftrightarrow{\text{DTFT}}P(e^{j\omega})$, then from synthesis equation of DTFT

$$p[n]=\frac{1}{2\pi}\int_0^{2\pi}P(e^{j\omega})e^{j\omega n}\,d\omega$$

$$p[0]=\frac{1}{2\pi}\int_0^{2\pi}P(e^{j\omega})\,d\omega \quad \ldots\text{(ii)}$$

From equation (i) and (ii),

If $\quad P(e^{j\omega})=X(e^{j\omega})Y(e^{-j\omega})$ then, $I=p[0]$ $\quad \ldots$(iii)

If $\quad y[n]\longleftrightarrow Y(e^{j\omega})$

Applying time reversal property,

$$y[-n]\longleftrightarrow Y(e^{-j\omega})$$

Let $y[-n]=z[n]$, then using convolution property,

$$x[n]\otimes z[n]\longleftrightarrow X(e^{j\omega})\cdot Y(e^{-j\omega})=P(e^{j\omega})$$

So, $\quad p[n]=x[n]\otimes z[n]$

Expand convolution as,

$$p[n]=\sum_{k=-\infty}^{\infty}x[k]z[n-k]$$

Here, $\quad x[n]=2^{n-1}u[-n+2]$

$$x[k]=2^{k-1}u[-k+2]$$

$$z[n]=y[-n]=2^{n+2}u[-n+1]$$

$$z[n-k]=2^{n-k+2}u[-n+k+1]$$

$$\therefore\ p[n]=\sum_{k=-\infty}^{\infty}2^{k-1}u[-k+2]\cdot 2^{n-k+2}u[-n+k+1]$$

As required integral, $I=p[n]\big|_{n=0}=p[0]$

$$p[0]=\sum_{k=-\infty}^{\infty}2^{k-1}u[-k+2]\cdot 2^{-k+2}u[k+1]$$

Sequence, $u[-k+2]=1$ only for $-\infty<k\le 2$, thus modifying limit of summation

$$p[0]=\sum_{k=-\infty}^{2}2^{k-1-k+2}\cdot 1\cdot u[k+1]$$

$$p[0]=\sum_{k=-\infty}^{2}2u[k+1]$$

Sequence $u[k+1]=1$ only for $-1\le k<\infty$, thus modifying lower limit of summation.

$$p[0] = \sum_{k=-1}^{2} 2 \times 1 = 2\sum_{k=-1}^{2} (1)^k$$

$$p[0] = 2[1+1+1+1] = 8$$

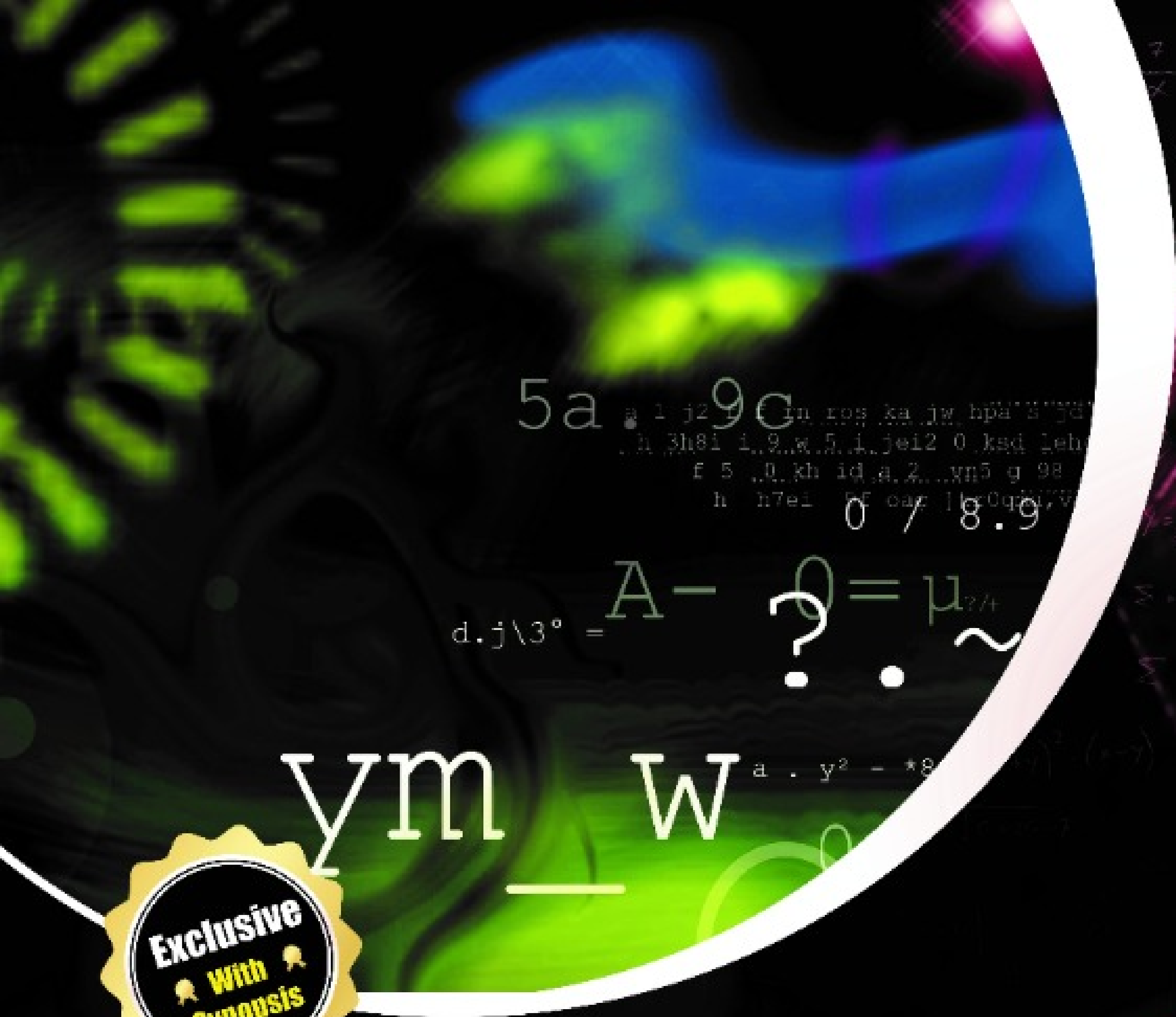
So, $$I = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega}) \cdot Y(e^{-j\omega})\, d\omega$$

$$I = \frac{1}{2\pi}\int_0^{2\pi} P(e^{j\omega})\, d\omega$$

$$I = p[0] = 8$$

Hence, the value of given integral is **8**.

❖❖❖❖



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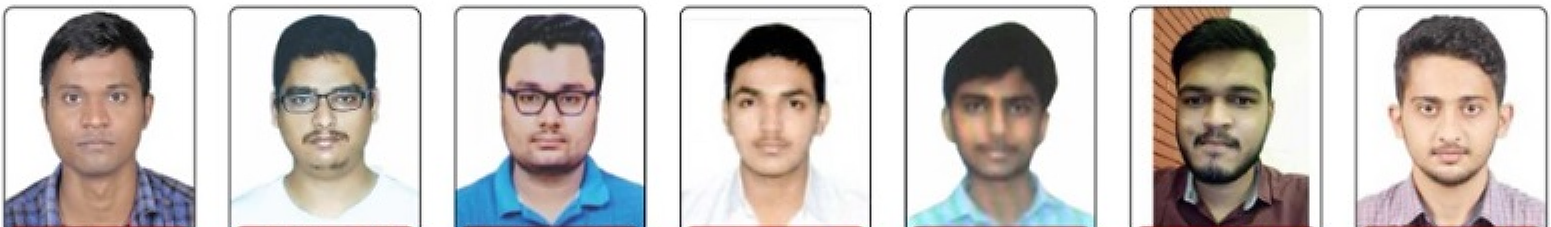

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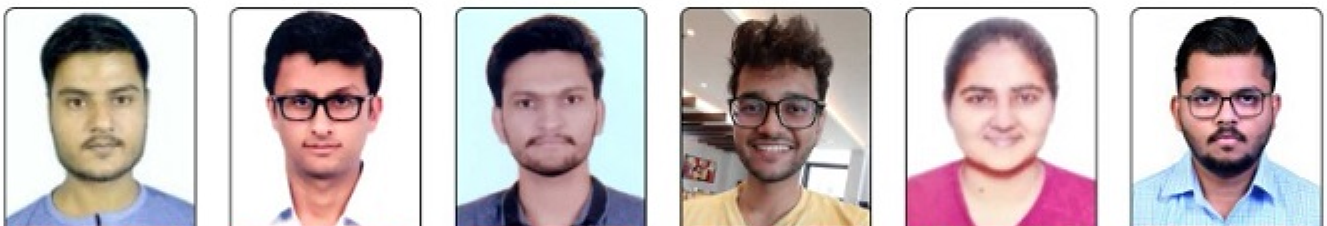

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CHAPTER 3 DIGITAL ELECTRONICS

Marks Distribution of Digital Electronics in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	5	7	19
2004	6	5	16
2005	2	6	14
2006	–	5	10
2007	2	7	16
2008	–	8	16
2009	1	6	13
2010	2	2	6
2011	3	2	7
2012	4	1	6
2013	1	1	3
2014 Set-1	3	3	9
2014 Set-2	3	2	7
2014 Set-3	2	3	8

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-4	2	2	6
2015 Set-1	1	3	7
2015 Set-2	2	3	8
2015 Set-3	2	3	8
2016 Set-1	1	2	5
2016 Set-2	3	2	7
2016 Set-3	2	3	8
2017 Set-1	2	3	8
2017 Set-2	3	3	9
2018	3	4	11
2019	4	2	8
2020	3	3	9
2021	3	3	9

Syllabus : Digital Electronics

Number representations: binary, integer and floating-point- numbers. Combinatorial circuits: Boolean algebra, minimization of functions using Boolean identities and Karnaugh map, logic gates and their static CMOS implementations, arithmetic circuits, code converters, multiplexers, decoders. Sequential circuits: latches and flip-flops, counters, shift-registers, finite state machines, propagation delay, setup and hold time, critical path delay. Data converters: sample and hold circuits, ADCs and DACs. Semiconductor memories: ROM, SRAM, DRAM. Computer organization: Machine instructions and addressing modes, ALU, data-path and control unit, instruction pipelining.

Contents : Digital Electronics

1 Number Systems

➢ Partial Synopsis

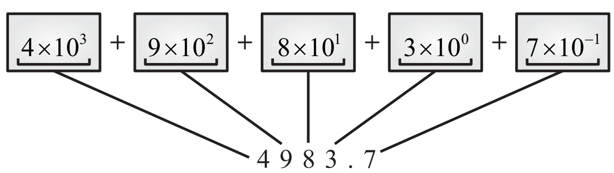
Number System

Decimal Number System :

When we write a decimal number, say 4983.7, we know that it can be represented as under :

$$4000 + 900 + 80 + 3 + 0.7 = 4983.7$$

The decimal number 4983.7 can also be written as $(4983.7)_{10}$ where the 10 subscript indicates the
radix or base.



In power of 10, we can write as below :

The leftmost digit which has the greatest weight is called the most significant digit and the rightmost digit which has the least weight is called the least significant digit.

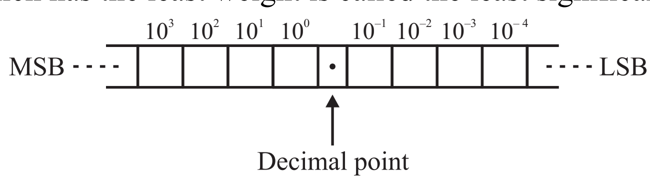
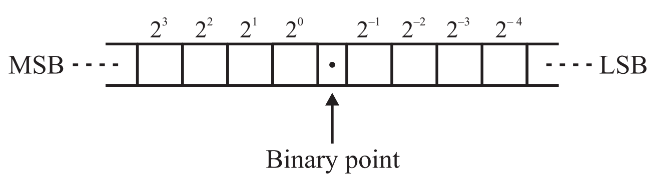


Fig. Decimal position values as powers of 10

Binary Number System :

Binary system with its two digits is a base-two system. The two binary digits (bits) are 1 and 0. Like digital system, in binary system, each binary digit commonly known as bit has its own value or weight. However, in binary system weight is expressed as a power of 2 as shown in figure.



Octal Number System :

The octal number system uses first eight digits of decimal number system : 0, 1, 2, 3, 4, 5, 6 and 7. As it uses 8 digits, its base is 8. Since its base is a power of $2(2^3)$, it is very easy to convert octal number to binary number and vice-versa.

Hexadecimal Number System :

The hexadecimal number system has a base of 16 having 16 digits : 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F. Since its base is a power of 2 (2^4), it is easy to convert hexadecimal numbers to binary and vice-versa.

Signed and Unsigned Binary Number

Unsigned Binary Numbers :

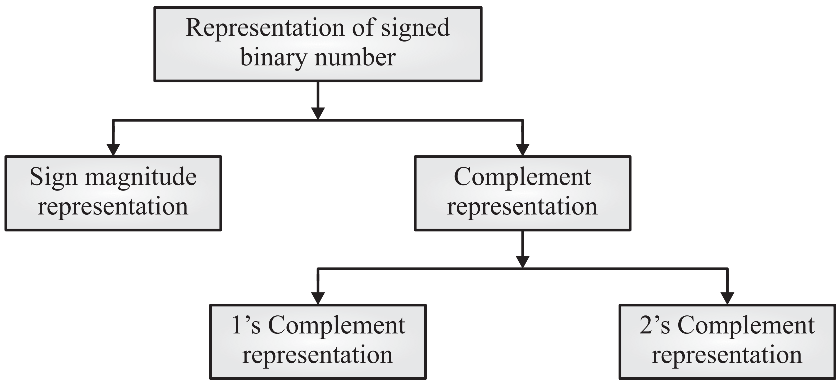
In some applications, all the data is either positive or negative. Then we can just forget about the $(+)$ or $(-)$ signs, and concentrate only on the magnitude (absolute value) of numbers. For example, the smallest 8 bit binary number is 0000 0000 i.e. all zeros, and the largest 8 bit binary number is 1111 1111. Therefore, the total range of unsigned 8 bit binary numbers is from $(00)_H$ to $(FF)_H$ or from $(0)_{10}$ to $(255)_{10}$.

Representation of unsigned binary numbers :

The numbers without positive or negative signs are known as unsigned numbers. The unsigned numbers are always considered as positive numbers.

Representation of singed binary numbers :

In the signed number system, the number may be positive or negative. Following are the possible representations for a signed binary number.



Sign Magnitude Representation :

In sign magnitude representation of a binary number, the MSB bit represents the sign and the remaining bits represents its magnitude. When the MSB is 1, the sign is negative and when it is 0 the sign is positive. For eg. In 7 bits, MSB bit represent sign and remaining 6 bits represent magnitude. And in 9 bits, MSB bit represent sign and remaining 8 bits represent magnitude.

Sign magnitude representations of + 42 and – 42 using 7 bits and 9 bits are shown below.

0	1	0	1	0	1	0	+ 42	0	0	0	1	0	1	0	1	0
1	1	0	1	0	1	0	+ 42	1	0	0	1	0	1	0	1	0

Using 7 bits   Using 9 bits

1's Complement Representation :

1's complement representation is obtained by complementing all the magnitude bits in sign magnitude representation. Like sign magnitude representation, in 1's complement representation also the MSB bit (sign bit) is '0' for a positive number and is '1' for a negative number. Sign magnitude and 1's complement representation for – 42 using 9 bit is shown below.

Sign magnitude representation for – 42	1	0	0	1	0	1	0	1	0
1's complement representation for – 42	1	1	1	0	1	0	1	0	1

Sign bit is always '1' for negative numbers

2's Complement Representation :

2's complement representation is obtained by adding 1 to the 1's complement representation at the LSB position. In previous topic, we have already obtained 1's complement representation of – 42 using 9 bits. Let us now find using complement representation of – 42 using 9 bits.

1's complement representation of – 42	1 1 1 0 1 0 1 0 1
	+ 1
2's complement representation of – 42	1 1 1 0 1 0 1 1 0

Note :

Above discussed process is valid for obtaining 1's complement representation and 2's complement representation for negative numbers only. For positive numbers, 1's complement representation and 2's complement representation is same as its sign magnitude representation.

Let us take example of + 42

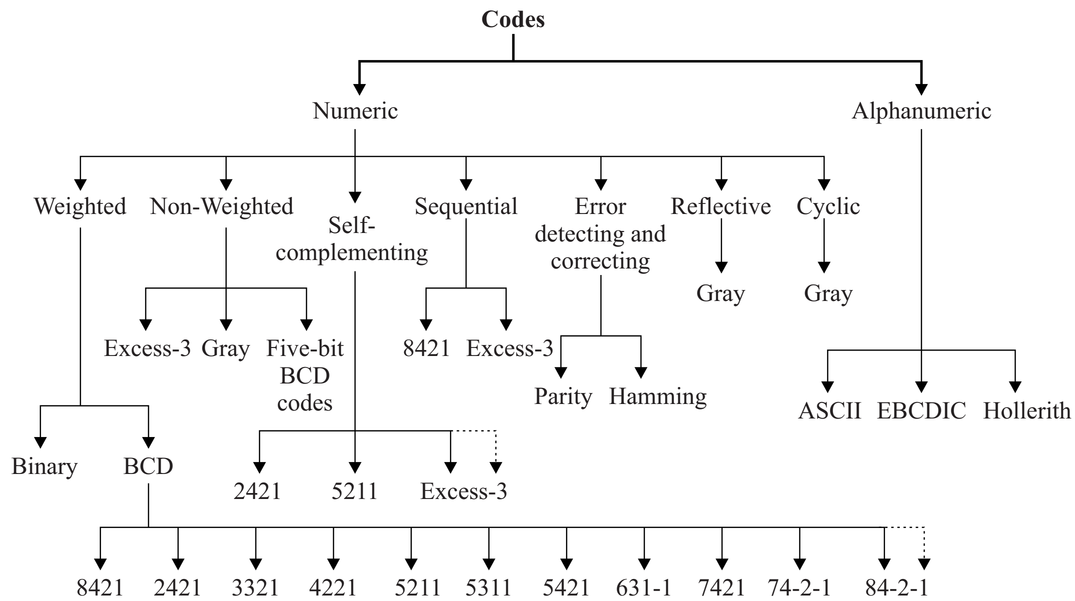
Sign magnitude representation of + 42 using 9 bits: $\underset{\text{sign bit}}{0}\ \underbrace{0\,0\,1\,0\,1\,0\,1\,0}_{\text{magnitude bits}}$

1's complement representation of + 42 using 9 bits: $\underset{\text{sign bit}}{0}\ \underbrace{0\,0\,1\,0\,1\,0\,1\,0}_{\text{magnitude bits}}$

2's complement representation of + 42 using 9 bits: $\underset{\text{sign bit}}{0}\ \underbrace{0\,0\,1\,0\,1\,0\,1\,0}_{\text{magnitude bits}}$

Binary Codes

Classification of Codes :



➢ Sample Questions

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1.1 The subtraction of a binary number Y from another binary number X, done by adding the 2's complement of Y to X, results in a binary number without overflow. This implies that the result is

(A) negative and is in normal form.

(B) negative and is in 2's complement form.

(C) positive and is in normal form.

(D) positive and is in 2's complement form.

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1.2 11001, 1001 and 111001 correspond to the 2's complement representation of which one of the following sets of number?

(A) 25, 9 and 57 respectively.

(B) – 6, – 6 and – 6 respectively.

(C) – 7, – 7 and – 7 respectively.

(D) – 25, – 9 and – 57 respectively.

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1.3 The two numbers represented in signed 2's complement form are

$P = 11101101$ and $Q = 11100110$

If Q is subtracted from P, the value obtained in signed 2's complement form is

(A) 100000111 (B) 00000111

(C) 11111001 (D) 111111001

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1.4 If $(1235)_x = (3033)_y$ where x and y indicate bases of the corresponding numbers, then

(A) $x = 7$ and $y = 5$ (B) $x = 8$ and $y = 6$

(C) $x = 6$ and $y = 4$ (D) $x = 9$ and $y = 7$

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Explanations   Number System

1.1 (B)

Subtraction of a binary number Y from another number X is given below,

$$X - Y = X + (-Y)$$

$$= X + 2\text{'s complement of } Y.$$

Example :

Let $X = 6$ and $Y = 9$

Now, $Z = X - Y$

$$= X + 2\text{'s complement of } Y.$$

Binary of $Y = 9 = 1\,0\,0\,1$

Sign magnitude representation of $-9 = 1\,1\,0\,0\,1$

1's complement representation of $-9 = 1\,0\,1\,1\,0$

2's complement representation of -9

$$= 1\,0\,1\,1\,0 + 1 = 1\,0\,1\,1\,1$$

And, sign magnitude representation of 6 using 5 bits is 0 0 1 1 0

$$Z = X + 2\text{'s complement of } Y.$$

$$\begin{array}{r} 0\,0\,1\,1\,0 \\ +\ 1\,0\,1\,1\,1 \\ \hline 1\,1\,1\,0\,1 \end{array}$$

Note : Carry not generated.

Since, MSB of Z is 1, so Z is a negative number and result Z is available in 2's complement form. To determine actual result, we have to find 2's complement of answer.

1's complement of 1 1 1 0 1 = 1 0 0 1 0

2's complement of 1 1 1 0 1 = 1 0 0 1 0 + 1

= 1 0 0 1 1

So, actual number is – 3.

We can conclude that if carry is not generated, result is negative and is in 2's complement form.

Hence, the correct option is (B).

Key Point

1. MSB of signed magnitude representation, 1's complement representation and 2's complement representation of any number denote its sign.
   (a) If MSB is 1, then number is – ve.
   (b) If MSB is 0, then number is + ve.
2. If MSB of 1's complement representation and 2's complement representation of any number is 1, then number will be – ve.
3. 1's complement representation, 2's complement representation and sign magnitude representation is known as “signed representation”.
4. During conversion from one signed representation to another, we will not change MSB.
5. Signed magnitude representation, 1's complement representation and 2's complement representation of any positive number will be same.

Example :

Representation	**+6**
Sign magnitude representation	0110
1's complement representation	0110
2's complement representation	0110

1.2 (C)

All numbers are in 2's complement form.

2's complement of (2's complement of a number) = Original number

(i) 11001 :

Magnitude sign bit (MSB) of 2’s complement is 1, hence number is negative.

$$\begin{array}{rl} \text{1's complement of } 11001 = & 10110 \\ & +\quad 1 \\ \hline \text{2's complement of } 11001 = & \underbrace{1}\ \underbrace{0111} \\ & \downarrow\quad\ \downarrow \\ \text{Original Number} = & -\quad 7 \end{array}$$

(ii) 1001 :

MSB of 2's complement is 1 hence, number is negative

$$\begin{array}{rl} \text{1's complement of } 1001 = & 1110 \\ & +\quad 1 \\ \hline \text{2's complement of } 1001 = & \underbrace{1}\ \underbrace{111} \\ & \downarrow\quad \downarrow \\ \text{Original Number} = & -\ 7 \end{array}$$

(iii) 111001 :

MSB of 2's complement is 1 hence, number is negative.

$$\begin{array}{rl} \text{1's complement of } 111001 = & 100110 \\ & +\quad 1 \\ \hline \text{2's complement of } 111001 = & \underbrace{1}\ \underbrace{00111} \\ & \downarrow\quad\ \downarrow \\ \text{Original Number} = & -\quad 7 \end{array}$$

Hence, the correct option is (C).

Note : If u see carefully, 2's complement number 111001 and 11001 are the extension of number 1001.

1 1 1 0 0 1

Extension of sign bit

1 1 0 0 1

Extension of sign bit

It means all 3-numbers 111001, 11001 and 1001 are same in decimal form.

Scan for Video Solution

1.3 (B)

Given : $P = 11101101$, $Q = 11100110$

Method 1

Both P and Q are in signed 2's complement form. The MSB denotes sign of given number i.e. if MSB is 0 then given number is positive and if MSB is 1 then given number is negative.

$$P = 11101101$$

Since MSB of P is 1, hence it is a negative number.

Original number P = 2's complement of P

Thus, the 2's complement of P will be

$$= 10010011$$
$$= -19$$

Similarly, the 2's complement of Q will be

$$\begin{array}{rr} = & 10011001 \\ + & 1 \\ \hline & 10011010 \end{array}$$
$$= -26$$

Thus, $P - Q = -19 - (-26) = +7$

Now, $+7 = 00000111$

The signed 2's complement of a positive number is identical to the sign magnitude form of the positive number.

$$+7 = 00000111$$

Hence, the correct option is (B).

Method 2

Given :

$$P = 11101101$$
$$Q = 11100110$$
$$P - Q = P + (-Q)$$
$$-Q = 2's\text{ complement of } Q = 00011010$$

$$\begin{array}{rr} P = & 1\,1\,1\,0\,1\,1\,0\,1 \\ -Q = & 0\,0\,0\,1\,1\,0\,1\,0 \\ \hline & 1\ 0\,0\,0\,0\,0\,1\,1\,1 \end{array}$$

Carry discarded (Rule of 2's complement addition)

Hence, the correct option is (B).

Scan for Video Solution

Key Point

Short trick to find 2's complement :
Move from right to left, keep bits till first '1' as it is, then complement each bit.

Example : To find 2's complement of 101011100100

$$\overset{\xleftarrow{\text{Move}}}{101011100100}$$

First '1'

2's complements = 010100011100

$$\underbrace{010100011}_{\text{Complemented}}\underbrace{100}_{\text{Same}}$$

1.4 **(B)**

Given : $(1235)_x = (3033)_y$

Converting both numbers into decimal number as,

$$(\underset{x^3}{1}\ \underset{x^2}{2}\ \underset{x^1}{3}\ \underset{x^0}{5})_x = (\underset{y^3}{3}\ \underset{y^2}{0}\ \underset{y^1}{3}\ \underset{y^0}{3})_y$$

$$x^3 + 2x^2 + 3x + 5 = 3y^3 + 3y + 3$$

Only option (C) $x = 8$, $y = 6$, satisfy the above equation.

Hence, the correct option is (B).

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2 Boolean Algebra

➢ Partial Synopsis

Laws of Boolean Algebra

Basic Laws

1. $0 \cdot 0 = 0$
2. $0 \cdot 1 = 0$
3. $1 \cdot 0 = 0$
4. $1 \cdot 1 = 1$
5. $0 + 0 = 0$
6. $0 + 1 = 1$
7. $1 + 0 = 1$
8. $1 + 1 = 1$
9. $\overline{1} = 0$
10. $\overline{0} = 1$

Complementation Laws

1. $\overline{0} = 1$
2. $\overline{1} = 0$
3. If $A = 0$ then $\overline{A} = 1$
4. If $A = 1$ then $\overline{A} = 0$
5. $\overline{\overline{A}} = A$

AND Laws

1. $A \cdot 0 = 0$
2. $A \cdot 1 = A$
3. $A \cdot A = A$
4. $A \cdot \overline{A} = 0$

OR Laws

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A + A = A$
4. $A + \overline{A} = 1$

Commutative Laws

1. $A + B = B + A$

   Can be extended to any number of variables

   $$A + B + C = B + C + A = C + A + B = B + A + C$$

2. $A \cdot B = B \cdot A$

   Can be extended to any number of variables

   $$A \cdot B \cdot C = B \cdot C \cdot A = C \cdot A \cdot B = B \cdot A \cdot C$$

Associative Laws

1. $(A + B) + C = A + (B + C)$

   Can be extended to any number of variables

$$A+(B+C+D)=(A+B+C)+D=(A+B)+(C+D)$$

2. $(A\cdot B)C = A(B\cdot C)$

   Can be extended to any number of variables

$$A(BCD)=(ABC)D=(AB)(CD)$$

Note : Universal gate does not follow associative law $(\overline{\overline{A+B}+C} \neq \overline{A+\overline{B+C}})$.

Distributive Laws

1. $A(B+C)=AB+AC$　　2. $A+BC=(A+B)(A+C)$
3. $A+\overline{A}B = A+B$

Absorption Laws

1. $A+A.B=A$　　2. $A(A+B)=A$

Consensus Theorem

1. $AB+\overline{A}C+BC = AB+\overline{A}C$　　2. $\overline{A}B+\overline{A}\,\overline{C}+\overline{B}\,\overline{C} = \overline{A}B+\overline{B}\,\overline{C}$
3. $(A+B)(A+\overline{C})(B+C)=(A+\overline{C})(B+C)$　　4. $(\overline{A}+B)(\overline{B}+\overline{C})(\overline{A}+\overline{C})=(\overline{A}+B)(\overline{B}+\overline{C})$

Transposition Theorem

1. $AB+\overline{A}C=(A+C)(\overline{A}+B)$

Demorgan's Theorem

1. $\overline{A+B}=\overline{A}\cdot\overline{B}$

   Can be extended to any number of variables

$$\overline{A+B+C+D+\ldots\ldots\ldots} = \overline{A}\cdot\overline{B}\cdot\overline{C}\cdot\overline{D}\ldots\ldots\ldots$$

2. $\overline{AB}=\overline{A}+\overline{B}$

   Can be extended to any number of variables

$$\overline{A\cdot B\cdot C\cdot D\cdot\ldots\ldots\ldots} = \overline{A}+\overline{B}+\overline{C}+\overline{D}+\ldots\ldots\ldots$$

Principle of Duality

If any given Boolean equation is valid, then it's dual will also be valid.

Procedure of finding Dual

- Change all '+' signs to '.' signs and all '.' signs to '+' signs.
- Change all 0s to 1s and all 1s to 0s.
- Do not complement the variables.

Example of Dual functions

Boolean Law/Equation,	Dual of Boolean Law/Equation,
$A\cdot 1 = A$	$A+0=A$
$0+1=1$	$1\cdot 0 = 0$
$A(B+C)=AB+AC$	$A+BC=(A+B)(A+C)$
$A+\overline{A}B=A+B$	$A(\overline{A}+B)=A\cdot B$

Note :

- If $F^D = F$ then function is called Self Dual Function.
- Number of unique function for n variable $= 2^{2^n}$.
- Number of Self Dual Function for n variable $= 2^{2^{n-1}}$.

Sample Questions

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2.1 The number of Boolean functions that can be generated by n variables is equal to

(A) $2^{2^{n-1}}$ (B) 2^{2^n}

(C) 2^{n-1} (D) 2^n

2004 IIT Delhi

2.2 The Boolean expression $AC + B\bar{C}$ is equivalent to

(A) $\bar{A}C + B\bar{C} + AC$

(B) $\bar{B}C + AC + B\bar{C} + \bar{A}C\bar{B}$

(C) $AC + B\bar{C} + \bar{B}C + ABC$

(D) $ABC + \bar{A}B\bar{C} + AB\bar{C} + A\bar{B}C$

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2.3 A function $F(A, B, C)$ defined by three Boolean variables A, B and C when expressed as sum of products is given by

$$F = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot\bar{C}$$

where, $\bar{A}, \bar{B}$ and $\bar{C}$ are the complements of the respective variables. The product of sum (POS) form of the function F is

(A) $F = (A+B+C)(A+\bar{B}+C)(\bar{A}+B+C)$

(B) $F = (\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(A+\bar{B}+\bar{C})$

(C) $F = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$

(D) $F = (\bar{A}+\bar{B}+C)(\bar{A}+B+C)(A+\bar{B}+C)(A+B+\bar{C})(A+B+C)$

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Explanations Boolean Algebra

Concept of Essential Prime Implicants

Scan for Video Explanation

Concept of Karnaugh Map (K-Map) SOP & POS

Scan for Video Explanation

2.1 (B)

The number of Boolean functions that can be generated by n variables $= 2^{2^n}$.

Hence, the correct option is (B).

Key Point

- Number of distinct Boolean function formed by n-variable is $\Rightarrow 2^{2^n}$.

- Number of distinct dual Boolean function formed by n-variable is $\Rightarrow 2^{2^n}$
- Number of distinct combination formed by n-variable is $\Rightarrow 2^n$.

2.2 (D)

Given : $Y = AC + B\overline{C}$

Method 1

$$Y = AC + B\overline{C}$$

Converting Y in standard canonical SOP form as,

$$Y = A(B+\overline{B})C + (A+\overline{A})B\overline{C}$$

$$Y = ABC + A\overline{B}C + AB\overline{C} + \overline{A}B\overline{C}$$

Hence, the correct option is (D).

Note : In the above question Y can also write in the form of minterms as,

$$Y = \underset{(111)}{ABC} + \underset{(101)}{A\overline{B}C} + \underset{(110)}{AB\overline{C}} + \underset{(010)}{\overline{A}B\overline{C}}$$

$$Y = \Sigma m(010, 101, 110, 111)$$

$$Y = \Sigma m(2, 5, 6, 7)$$

Method 2

By checking given option :

From option (A) :

$$Y = \overline{A}C + B\overline{C} + AC$$

$$Y = C(\overline{A}+A) + B\overline{C}$$

$$Y = C + B\overline{C} \qquad (\because A+\overline{A}=1)$$

By distributive law,

$$P + QR = (P+Q)(P+R)$$

So, $\quad Y = (C+B)(C+\overline{C}) \qquad (\because C+\overline{C}=1)$

$$Y = C + B$$

Hence, option (A) is incorrect.

From option (B) :

$$Y = \overline{B}C + AC + B\overline{C} + \overline{A}C\overline{B}$$

$$Y = \overline{B}C(1+\overline{A}) + B\overline{C} + AC \quad (\because 1+\overline{A}=1)$$

$$Y = \overline{B}C + B\overline{C} + AC$$

Hence, option (B) is incorrect.

From option (C) :

$$Y = AC + B\overline{C} + \overline{B}C + ABC$$

$$Y = AC[1+B] + B\overline{C} + \overline{B}C \quad (\because 1+B=1)$$

$$Y = AC + B\overline{C} + \overline{B}C$$

Hence, option (C) is incorrect.

From option (D) :

$$Y = ABC + \overline{A}B\overline{C} + AB\overline{C} + A\overline{B}C$$

$$Y = AC(B+\overline{B}) + B\overline{C}(A+\overline{A})$$

$$[\because (B+\overline{B})=1 \text{ and } (A+\overline{A})=1]$$

$$Y = AC + B\overline{C}$$

So, option (D) is correct.

Hence, the correct option is (D).

2.3 (C)

Given 3-variable function in SOP form as,

$$F(A,B,C) = \underbrace{\overline{A}\overline{B}\overline{C}}_{000} + \underbrace{\overline{A}B\overline{C}}_{010} + \underbrace{A\overline{B}\overline{C}}_{100}$$

$$F(A,B,C) = \sum m(000, 010, 100)$$

$$= \sum m(0,2,4) \rightarrow \text{SOP form}$$

Given expression is in SOP form. Therefore, its POS form can be given by,

$$F(A,B,C) = \pi M(1,3,5,6,7) \rightarrow \text{ POS form}$$

$$F(A,B,C) = \pi M(001, 011, 101, 110, 111)$$

$$F(A,B,C) = \underbrace{(A+B+\overline{C})}_{1}\underbrace{(A+\overline{B}+\overline{C})}_{3}\underbrace{(\overline{A}+B+\overline{C})}_{5}$$

$$\underbrace{(\overline{A}+\overline{B}+C)}_{6}\underbrace{(\overline{A}+\overline{B}+\overline{C})}_{7}$$

Hence, the correct option is (C).

Note : Many aspirants may think that, POS form is the complemented form of SOP or vice-versa but it is not true, SOP and POS form are the two different representation of any Boolean function (F).

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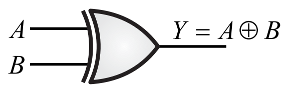
3 Logic Gates

➢ Partial Synopsis

Special Purpose Gates

1. Exclusive OR Gate (EX-OR or XOR) :

- **Symbol of 2-input X-OR gate :**



- **Truth table of 2-input X-OR gate :**

Inputs		Output
A	B	$Y = A\oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Boolean function of 2 input EX-OR gate is given as,

$Y = A\oplus B = \overline{A}B + A\overline{B} \rightarrow$ SOP form.

$Y = A\oplus B = (\overline{A}+\overline{B})(A+B) \rightarrow$ POS form.

> **Note :**
> **3 or more input EXOR gate does not exists practically but can be theoretically implemented.**

- **Anti-coincidence or inequality detector**

  Since an EXOR gate produces an output 1 when the input are not equal it is called anti-coincidence gate or inequality detector.

- **In EXOR operation :**

  For implementation of BUFFER circuit, any one of the inputs will be at logic 0,

  $A\oplus 0 = A$

For implementation of INVERSION circuit, any one of the inputs will be at logic 1,

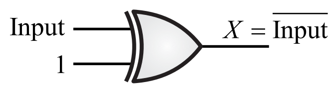
$$A \oplus 1 = \overline{A}$$

- It acts as an "odd" number of 1's detector
- **Properties of EXOR gate**

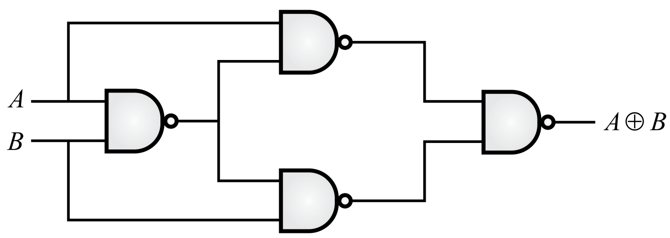
(a) $A \oplus 1 = \overline{A}$
(b) $A \oplus 0 = A$
(c) $A \oplus A = 0$
(d) $A \oplus \overline{A} = 1$
(e) $AB \oplus AC = A(B \oplus C)$
(f) If $A \oplus B = C$ and $A \oplus C = B$, $B \oplus C = A$ then $A \oplus B \oplus C = 0$

- **EXOR gate as an inverter**

An EXOR gate can be used as an inverter by connecting one of the two input terminals to logic 1 and applying the input signal to the other terminal.



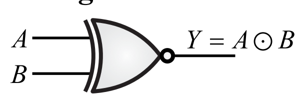
- **2-input EXOR gate using NAND gate**



Note :
(i) It is also called Stair case switch.
(ii) It is mostly used in Parity generation and detection.
(iii) It is also used in comparator circuit.

2. EX-NOR Gate :

- **Symbol of 2-input X-NOR gate :**



- **Truth Table of 2-input X-NOR gate :**

Inputs		Output
A	B	$Y = \overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

Boolean function of 2 input EX-NOR operation is given as :

$$Y = A \odot B = \overline{A \oplus B} = AB + \overline{A}\overline{B} \rightarrow \text{ SOP form.}$$

$$Y = A \odot B = \overline{A \oplus B} = (\overline{A} + B)(\overline{B} + A) \rightarrow \text{ PSO form.}$$

Note :
3 or more input EX-NOR gate does not exists practically.

- EX-NOR gate is a logic gate whose output is logic high when both the inputs are equal. Hence it is called equality detector.
- **In EX-NOR operation**

  For implementation of buffer circuit, any one of the inputs will be at logic 1,

  $$A \odot 1 = A$$

  For implementation of inversion circuit, any one of the inputs will be at logic 0,

  $$A \odot 0 = \overline{A}$$

- EX-NOR gate with even number of 1's detector for even numbers of input.
- EX-NOR gate with odd number of 1's detector for odd numbers of input.

  i.e. for odd inputs, $A \oplus B \oplus C = A \odot B \odot C$.

Note :
For number of Even input $\Rightarrow$ EXOR = $\overline{\text{EXNOR}}$
For number of Odd input $\Rightarrow$ EXOR = EXNOR

- 2-input EXOR and EX-NOR are dual as well as complimented to each other.
- **Some important results :**

  (a) $\overline{A} \oplus B = A \oplus \overline{B} = A \odot B$

  (b) $\overline{A} \odot B = A \odot \overline{B} = A \oplus B$

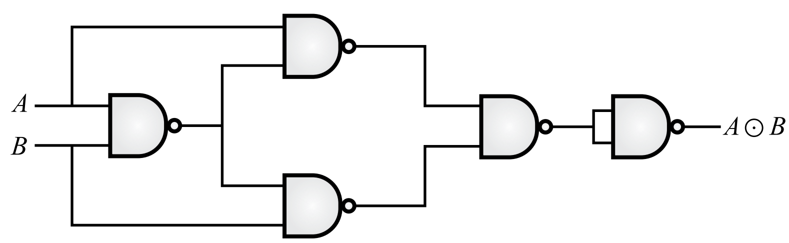
  (c) $\overline{A} \oplus B = \overline{A \oplus B} = A \odot B$

  (d) $\overline{A} \odot \overline{B} = A \odot B$

  (e) If $A \odot B = C$, Then

  $$A \odot B \odot C = 1$$

- **2-input EXNOR gate using NAND gate**



Sample Questions

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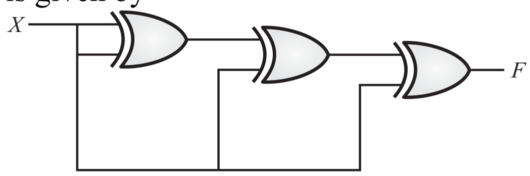
3.1 For the circuit shown below the output F is given by

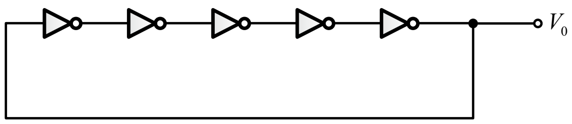


(A) $F = 1$ (B) $F = 0$

(C) $F = X$ (D) $F = \overline{X}$

2001 IIT Kanpur

3.2 For the ring oscillator shown in the figure, the propagation delay of each inverter is 100 pico-sec. What is the fundamental frequency of the oscillator output?

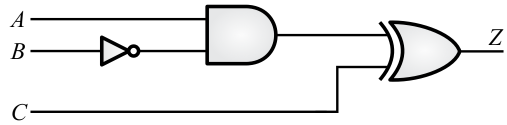


(A) 10 MHz (B) 100 MHz

(C) 1 GHz (D) 2 GHz

2015 IIT Kanpur

3.3 All the logic gates shown in the figure have a propagation delay of 20 ns. Let $A = C = 0$ and $B = 1$ unit time $t = 0$. At $t = 0$, all the inputs flip (i.e. $A = C = 1$ and $B = 0$) and remain in that state. For $t > 0$, output $Z = 1$ for a duration (in ns) of ________. **[Set - 01]**



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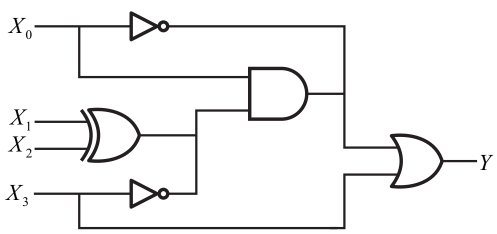
3.4 The logic gates shown in the digital circuit below use strong pull-down nMOS transistors for LOW logic level at the outputs. When the pull-downs are off, high-value resistors set the output logic levels to HIGH (i.e. the pull-ups are weak). Note that some nodes are intentionally shorted to implement "wired logic". Such shorted nodes will be HIGH only if the outputs of all the gates whose outputs are shorted are HIGH.



The number of distinct values of $X_3X_2X_1X_0$ (out of the 16 possible values) that give $Y = 1$ is ________.

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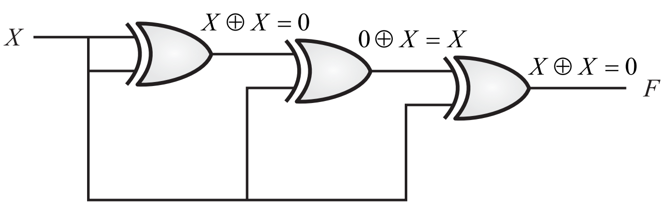
Explanations Logic Gates

3.1 (B)

Given circuit is shown below,



Output of 1st EX-OR gate,

$$F_1 = X \oplus X = 0$$

Output of 2nd EX-OR gate,

$$F_2 = 0 \oplus X = X$$

Output of 3rd EX-OR gate,

$$F = X \oplus X = 0$$

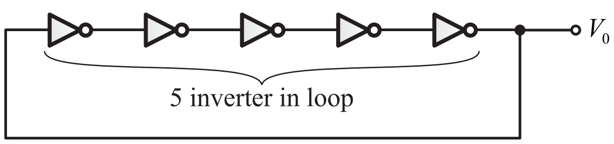
Hence, the correct option is (B).

Key Point

1. EX-OR and EX-NOR gate are called special purpose gate.
2. EX-OR gate is mostly used in "parity generation and detection".
3. 2-input EX-OR gate is also called anti-coincidence or inequality detector hence output of EX-OR detector will be 1, when two of its input will be unequal.
4. 2-input EX-NOR gate is called "Equivalence gate or coincidence gate" hence its output will be 1, when two of its input will be equal.

3.2 (C)

Ring oscillator is shown below,



Fundamental frequency of oscillator is given by,

$$f_{\text{CLK}} = \frac{1}{2nt_{pd}}$$

where, $n =$ Number of inverters used in loop and it should be odd number

$t_{pd} =$ Propagation delay of each inverter

$t_{pd} = 100\,\text{pico sec}$

Here, $t_{pd} = 100 \times 10^{-12}$ sec

$n = 5$

Hence, $f_{\text{CLK}} = \dfrac{1}{2 \times 5 \times 100 \times 10^{-12}}$ Hz

$f_{\text{CLK}} = 10^9$ Hz

$f_{\text{CLK}} = 1\,\text{GHz}$

Hence, the correct option is (C).

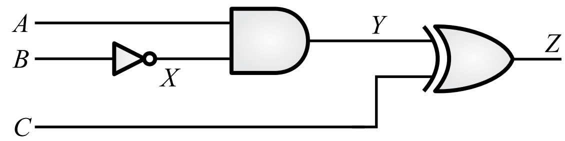
Key Point

For ring oscillator, the number of inverter must be odd.

3.3 40

Given : All the logic gates shown in the figure have a propagation delay of 20 ns. Let $A = C = 0$ and $B = 1$ unit at time $t = 0$. At $t = 0$, all the inputs flip $A = C = 1$ and $B = 0$.

Given logic circuit is shown below,



From above figure,

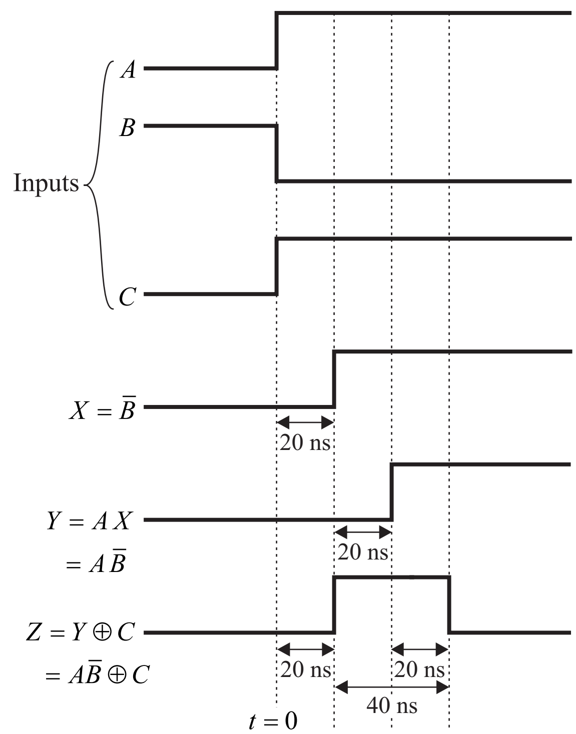
$$X = \overline{B}$$

$$Y = AX = A\overline{B}$$

$$Z = Y \oplus C$$

$$Z = AX \oplus C$$

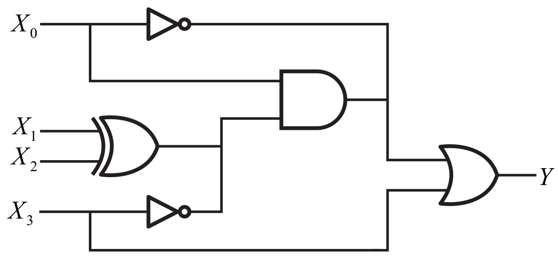
$$Z = A\overline{B} \oplus C$$



Hence, the output $Z = 1$ for duration of **40 ns**.

3.4 8

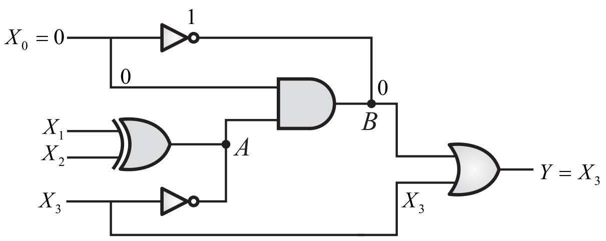
Given circuit is shown below,



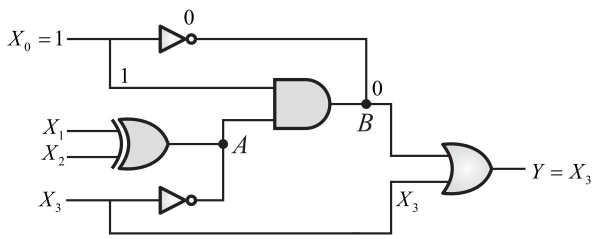
Method 1

According to question, given circuit have wired AND logic at node A and B, because node output is HIGH, when all the inputs came to node are HIGH.

Case 1 : When $X_0 = 0$ only, then output Y becomes as,



Case 2 : When $X_0 = 1$ only, then output Y becomes as,



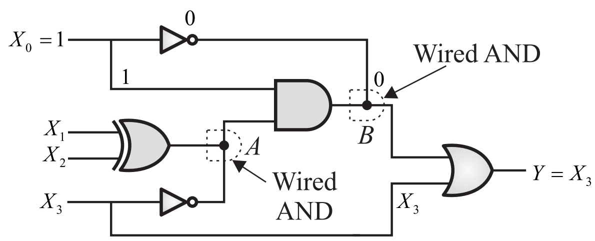
From above two cases it is clear that, $Y = X_3$ always, so truth table can be formed as shown below,

X_3	X_2	X_1	X_0	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

The number of distinct values of $X_3X_2X_1X_0$ (out of the 16 possible values) that give $Y = 1$ is **8**.

Method 2

According to question, given circuit have wired AND logic at node A and B, because node output is HIGH, when all the inputs came to node are HIGH.



From figure, $A = (X_1 \oplus X_2)\bar{X}_3$

$$B = (A \cdot X_0)\bar{X}_0$$

$$B = 0$$

Output $y = B + X_3 = X_3$

Output depends only on the value of X_3. Therefore output will be high for 8 combination out of total 16 combination.

Hence, the number of distinct values of $X_3X_2X_1X_0$ (out of the 16 possible values) that give $Y = 1$ is **8**.

Scan for Video Solution

4 Combinational Circuits

➢ Partial Synopsis

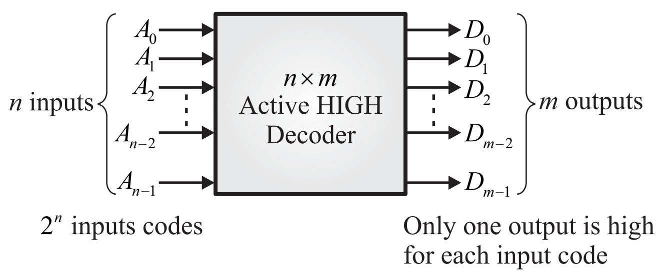
Decoder

- A decoder is a logic circuit that convert an n-bit binary input codes into m output lines such that only one output line is activated for each one of the possible combination of inputs.
- For n-input bits the maximum number of output lines m will be 2^n i.e. $m \le 2^n$.
- If the n-bit decoded information has unused or don't care combinations, the decoder output will have less than 2^n outputs.
- Decoder is used to convert binary data into other codes like binary to octal (3 : 8 decoder), binary to hexadecimal (4 : 16 decoder).
- **Types of decoder :**

  There are two types of decoder as given below :

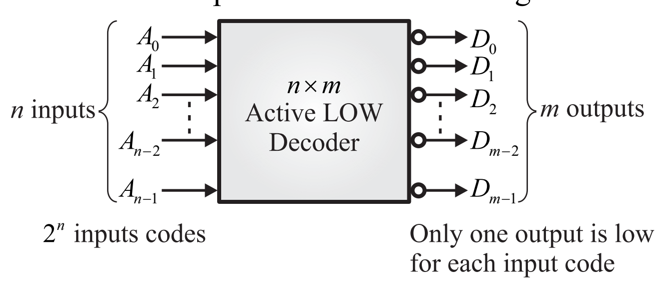
  (i) **Active high decoder :**

  For each input combination only one of the m outputs will be active (HIGH), all the other outputs will remain inactive (LOW).



  (ii) **Active low decoder :**

  For each input combination only one of the m output line will be active (LOW logic level), while all other outputs will be at HIGH logic level.



- **Example of 2x4 decoder :**

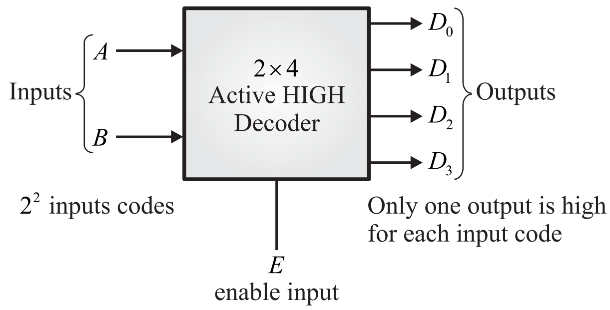
(i) Number of input lines = 2

Number of output lines = 4

Number of input codes $= 2^2 = 4$

(ii) Active high 2×4 decoder

(a) **Block diagram :**

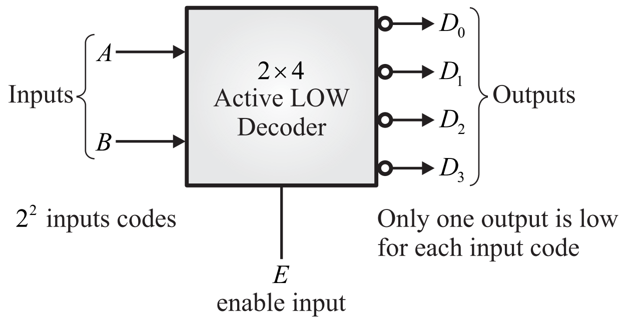


(b) **Truth table :**

Enable	Inputs		Outputs			
E	A	B	D_3	D_2	D_1	D_0
0	×	×	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0

(iii) Active low 2×4 decoder

(a) **Block diagram :**



(b) **Truth table :**

Enable	Inputs		Outputs			
E	A	B	D_3	D_2	D_1	D_0
0	×	×	1	1	1	1
1	0	0	1	1	1	0
1	0	1	1	1	0	1
1	1	0	1	0	1	1
1	1	1	0	1	1	1

Note :

- **2×4 decoder may act like 1×4 demultiplexer and vice-versa.**
- **Decoder and demultiplexer circuits are almost same.**
- **Decoder contains AND gates or NAND gates.**
- **A half adder or half subtractor circuit can be implemented by the 2×4 decoder and an OR gate.**

Comparator

- A comparator is a logic circuit used to compare the magnitudes of two binary numbers.
- Depending on the design, it may either simply provide an output that is active (goes HIGH for example) when the two numbers are equal, or additionally provide outputs that signify which of the number is greater when equality does not hold.
- The X-NOR gate is a basic comparator because its output is a 1 only if its two input bits are equal i.e. the output is a 1 if and only if the input bit coincide.
- **1-bit comparator :**

  Let the 1-bit numbers be $A = A_0$ and $B = B_0$.

  (i) **Truth table :**

A_0	B_0	$L(A<B)$	$E(A=B)$	$G(A>B)$
0	0	0	1	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	0

  L, E, G stands for lower, equal, greater respectively.

  (ii) **SOP expression :**

  $L = \overline{A}_0 B_0$ when $A < B$

  $E = A_0 \odot B_0$ when $A = B$

  $G = A_0 \overline{B}_0$ when $A > B$

- **2-bit comparator :**

  Let the 2-bit numbers are $A = A_1A_0$ and $B = B_1B_0$

  (i) **Truth table :**

Inputs				Outputs		
A_1	A_0	B_1	B_0	$L(A<B)$	$E(A=B)$	$G(A>B)$
0	0	0	0	0	1	0
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0

0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	1	0
1	0	1	1	1	0	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	1	0

(ii) **SOP expression :**

$$L = \bar{A}_1\,\bar{A}_0\,B_0 + \bar{A}_0\,B_1\,B_0 + \bar{A}_1\,B_1 \qquad \text{when } A < B$$

$$E = (A_0 \odot B_0)(A_1 \odot B_1) \qquad \text{when } A = B$$

$$G = A_0\,\bar{B}_1\,\bar{B}_0 + A_1\,\bar{B}_1 + A_1\,A_0\,\bar{B}_0 \qquad \text{when } A > B$$

Note :

For "*n*-bit" comparator

- **Number of combination which shows equal expression i.e. $A = B$ is 2^n.**
- **Number of combination which shows, greater expression i.e. $A > B$ or lower expression i.e. $A < B$ is $\dfrac{2^{2n} - 2^n}{2}$.**

Sample Questions

1990 IISc Bangalore

4.1 The minimal function that can detect a "divisible by 3" 8421 BCD code digit (representation is $D_8D_4D_2D_1$) is given by

(A) $D_8D_1 + D_4D_2 + \bar{D}_8D_2D_1$

(B) $D_8D_1 + D_4D_2\bar{D}_1 + \bar{D}_4D_2D_1 + \bar{D}_8\bar{D}_4\bar{D}_2\bar{D}_1$

(C) $D_8D_1 + D_4D_2 + \bar{D}_8\bar{D}_4\bar{D}_2\bar{D}_1$

(D) $D_4D_2\bar{D}_1 + D_4D_2D_1 + D_8\bar{D}_4D_2D_1$

1997 IIT Madras

4.2 A 2-bit binary multiplier can be implemented using

(A) 2 input AND gates only.

(B) Six 2-input AND gates and two XOR gates.

(C) Two 2-input NORs and one XNOR gate.

(D) XOR gates and shift registers.

2009 **IIT Kanpur**

Statement for Linked Answer Questions 4.3 & 4.4

Two products are sold from a vending machine, which has two push buttons P_1 and P_2. When a button is pressed, the price of the corresponding product is displayed in a 7-segment display.

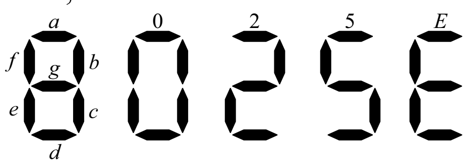
If no buttons are pressed, '0' is displayed, signifying 'Rs.0'.

If only P_1 is pressed, '2' is displayed, signifying 'Rs.2'.

If only P_2 is pressed, '5' is displayed, signifying 'Rs.5'.

If both P_1 and P_2 are pressed, 'E' is displayed, signifying 'Error'.

The names of the segments in the 7-segment display, and the glow of the display for '0', '2', '5' and 'E', are shown below.



Consider

(i) Push button pressed/not pressed is equivalent to logic 1/0 respectively.

(ii) A segment glowing/not glowing in the display is equivalent to logic 1/0 respectively.

4.3 If segments a to g are considered as functions of P_1 and P_2, then which of the following is correct ?

(A) $g = \bar{P}_1 + P_2,\ d = c + e$

(B) $g = P_1 + P_2,\ d = c + e$

(C) $g = \bar{P}_1 + P_2,\ e = b + c$

(D) $g = P_1 + P_2,\ e = b + c$

4.4 What are the minimum numbers of NOT gates and 2-input OR gates required to design the logic of the driver for this 7-segment display?

(A) 3 NOT and 4 OR

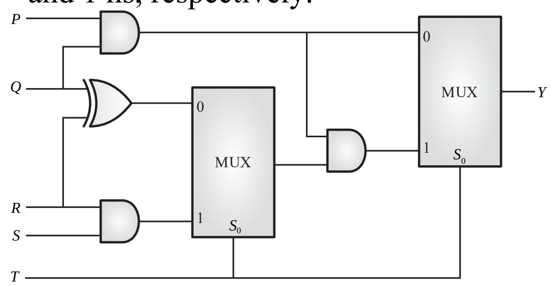
(B) 2 NOT and 4 OR

(C) 1 NOT and 3 OR

(D) 2 NOT and 3 OR

2021 **IIT Bombay**

4.5 The propagation delays of the XOR gate, AND gate and multiplexer (MUX) in the circuit shown in the figure are 4 ns, 2 ns and 1 ns, respectively.



If all the inputs P, Q, R, S and T are applied simultaneously and held constant, the maximum propagation delay of the circuit is

(A) 3 ns

(B) 5 ns

(C) 6 ns

(D) 7 ns

❖❖❖❖

Explanations **Combinational Circuits**

Introduction to Multiplexer

Scan for Video Explanation

Analysis of Multiplexer

Scan for Video Explanation

Designing of Multiplexer

Scan for Video Explanation

Designing of Higher Order MUX using Lower order Multiplexer

Scan for Video Explanation

4.1 (B)

Consider a truth table of a minimal function that can detect a "divisible by 3" 8421 BCD code. Truth table will be as shown below,

Decimal number	BCD Code				Output
	D_8	D_4	D_2	D_1	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	1

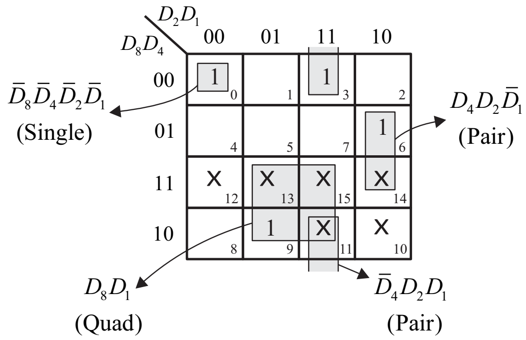
Output will be "1" for number 0, 3, 6 and 9.

(4-bit BCD exist only for 0 to 9 after that 10 to 15 work as don't care).

So, Y from above table in SOP from is,

$Y = \Sigma m(0, 3, 6, 9) + \Sigma d(10, 11, 12, 13, 14, 15)$

K-map for Y is shown below,



Minimal expression for output Y is given by,

$$Y = D_8D_1 + D_4D_2\bar{D}_1 + \bar{D}_4D_2D_1 + \bar{D}_8\,\bar{D}_4\,\bar{D}_2\,\bar{D}_1$$

Hence, the correct option is (B).

Key Point

(i) BCD is an abbreviation for binary coded decimal.

(ii) BCD is numeric code/weighted code in which each digit of a decimal number is represented by a separate group of bits.

(iii) Example of BCD code are : 8-4-2-1 BCD code, 2-4-2-1 BCD code, 4-2-2-1 BCD code, 7-4-2-1 BCD code etc.

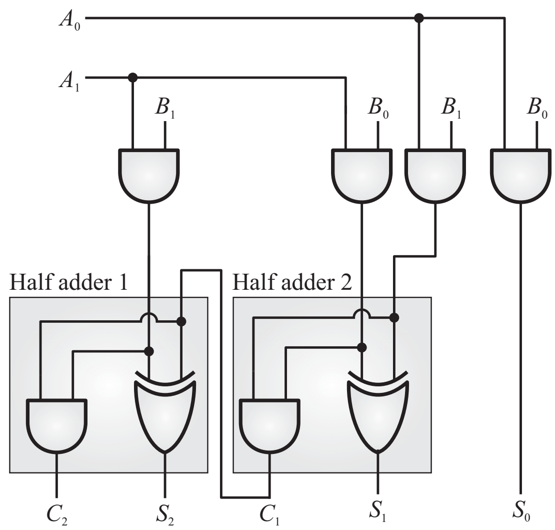
(iv) Many of aspirant thinks that, BCD code means 8-4-2-1 code but it is wrong. 8-4-2-1 code is a type of BCD code and most commonly used code.

4.2 (B)

2-bit multiplication can be represented as given below,

$$\begin{array}{cccc} & & B_1 & B_0 \\ & \times & A_1 & A_0 \\ \hline & & A_0B_1 & A_0B_0 \\ + & A_1B_1 & A_1B_0 & \vdots \\ \hline & S_2 & S_1 & S_0 \\ & C_2 & C_1 & \end{array}$$

The above expression can be represented by logic circuit as shown below,



Thus, 2 EX-OR and 6 AND gates are used.

Hence, the correct option is (B).

Key Point

In general, $n \times n$-bits combinational multiplier circuit requires,

(i) $n^2 \to$ AND gates (it does not contain gates of half adder).

(ii) $n \to$ Half adders.

(iii) $n^2 - 2n \to$ Full adder.

Propagation delay (τ) of the $n \times n$ multiplier can be expressed as,

$$\tau = (2n-3)\tau_{carry} + (n-1)\tau_{sum} + \tau_{AND}$$

Where, $n \to$ Number of bits used in multiplier.

4.3 (B)

Given : If no buttons are pressed then 0 is displayed,

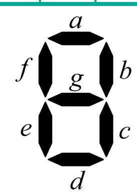
If only P_1 is pressed then 2 is displayed.

If only P_2 is pressed then 5 is displayed.

If both P_1 and P_2 are pressed then E is displayed.

Truth table for the above condition is shown below,

No.	P_1	P_2	a	b	c	d	e	f	g
0.	0	0	1	1	1	1	1	1	0
1.	0	1	1	0	1	1	0	1	1
2.	1	0	1	1	0	1	1	0	1
3.	1	1	1	0	0	1	1	1	1



Logical expression for *a b c d e f* and *g* are given by,

(1) $a = 1 = \Sigma(0, 1, 2, 3)$

$$a = \bar{P}_1\bar{P}_2 + \bar{P}_1 P_2 + P_1\bar{P}_2 + P_1 P_2$$

$$a = \bar{P}_1(\bar{P}_2 + P_2) + P_1(\bar{P}_2 + P_2) \quad (\because \bar{P}_2 + P_2 = 1)$$

$$a = \bar{P}_1 + P_1 \quad (\because \bar{P}_1 + P_1 = 1)$$

$$a = 1$$

(2) $b = \Sigma(0, 2)$

$$b = \bar{P}_1\,\bar{P}_2 + P_1\bar{P}_2$$

$$b = \bar{P}_2(\bar{P}_1 + P_1) \quad (\because \bar{P}_1 + P_1 = 1)$$

$$b = \bar{P}_2$$

(3) $c = \Sigma(0,1)$

$$c = \bar{P}_1\,\bar{P}_2 + \bar{P}_1 P_2$$

$$c = \bar{P}_1(\bar{P}_2 + P_2) \quad (\because \bar{P}_2 + P_2 = 1)$$

$$c = \bar{P}_1$$

(4) $d = \Sigma(0, 1, 2, 3)$

$$d = \bar{P}_1\bar{P}_2 + \bar{P}_1 P_2 + P_1\bar{P}_2 + P_1 P_2$$

$$d = \bar{P}_1(\bar{P}_2 + P_2) + P_1(\bar{P}_2 + P_2) \quad (\because \bar{P}_2 + P_2 = 1)$$

$$d = \bar{P}_1 + P_1 \quad (\because \bar{P}_1 + P_1 = 1)$$

$$d = 1$$

(5) $e = \Sigma(0, 2, 3)$

$$e = \bar{P}_1\bar{P}_2 + P_1\bar{P}_2 + P_1 P_2$$

$e = \bar{P}_1\bar{P}_2 + P_1(\bar{P}_2 + P_2)$

$(\because\ \bar{P}_2 + P_2 = 1)$

$e = \bar{P}_1\bar{P}_2 + P_1$

$e = (P_1 + \bar{P}_1)(P_1 + \bar{P}_2) \qquad (\because\ \bar{P}_1 + P_1 = 1)$

$e = P_1 + \bar{P}_2$

(6) $f = \Sigma(0, 1, 3)$

$f = \bar{P}_1\bar{P}_2 + \bar{P}_1P_2 + P_1P_2$

$f = \bar{P}_1(\bar{P}_2 + P_2) + P_1P_2 \qquad (\because\ \bar{P}_2 + P_2 = 1)$

$f = \bar{P}_1 + P_1P_2$

$f = (P_1 + \bar{P}_1)(\bar{P}_1 + P_2) \qquad (\because\ \bar{P}_1 + P_1 = 1)$

$f = \bar{P}_1 + P_2$

(7) $g = \Sigma(1, 2, 3)$

$g = \bar{P}_1P_2 + P_1\bar{P}_2 + P_1P_2$

$g = \bar{P}_1P_2 + P_1(\bar{P}_2 + P_2) \qquad (\because\ \bar{P}_2 + P_2 = 1)$

$g = \bar{P}_1P_2 + P_1$

$g = (P_1 + \bar{P}_1)(P_1 + P_2) \qquad (\because\ \bar{P}_1 + P_1 = 1)$

$g = P_1 + P_2$

and $c + e = \Sigma(0, 1) + \Sigma(0, 2, 3)$

$c + e = \Sigma(0, 1, 2, 3)$

$c + e = d$

Hence, the correct option is (B).

4.4 (D)

Table, for number of gates needed to implement this 7-segment display, is shown below,

LED	Function	No. of gates	
		NOT	OR
a	1	0	0
b	$\bar{P}_2$	1	0
c	$\bar{P}_1$	1	0
d	1	0	0
e	$P_1 + \bar{P}_2$	1	1
f	$\bar{P}_1 + P_2$	1	1
g	$P_1 + P_2$	0	1

Hence, number of OR gates needed = 3

Number of NOT gates needed = 2

(one NOT gate for complementing P_1 and other NOT gate for complementing P_2).

Hence, the correct option is (D).

4.5 (C)

Given :

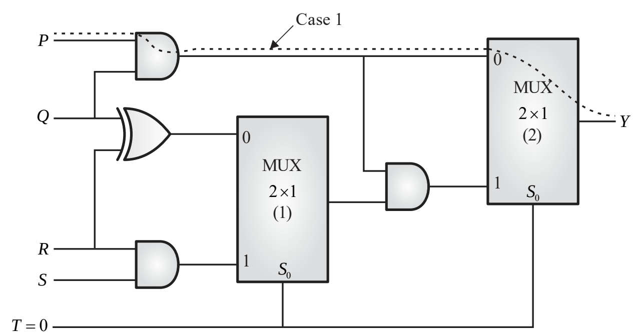
Delay of XOR gate $= 4$ns,

Delay of AND gate $= 2$ns

Delay of MUX gate $= 1$ns

Case 1 :

Assuming $T = 0$ then selection line of MUX $S_0 = 0$, so that MUX input '0' get enable so path followed by signal in the given circuit is shown by dotted lines as,



Let, total propagation delay τ_1 from input to output is,

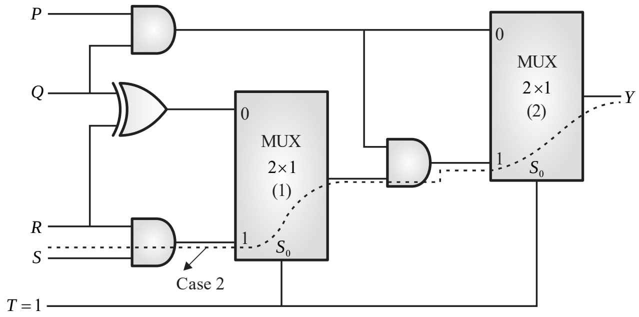
$$\tau_1 = (\text{Propagation delay of AND gate}) + (\text{Propagation delay of MUX-2})$$

$$\tau_1 = 2ns + 1ns = 3ns$$

Hence, when $S_0 = 0$, the MUX input '0' get enable and propagation delay of given circuit $\tau_1 = 3ns$

Case 2 :

Assuming $T = 1$ then selection line of MUX is $S_0 = 1$, so that MUX input '1' get enable so path followed by signal in the given circuit is shown by dotted lines as,



Let, total propagation delay τ_2 from input to output is,

$$\tau_2 = (\text{Propagation delay of AND gate}) + (\text{Propagation delay of MUX-1}) + (\text{Propagation delay of AND gate}) + (\text{Propagation delay of MUX-2})$$

$$\tau_2 = 2\,ns + 1ns + 2ns + 1ns = 6ns$$

Hence, when $S_0 = 1$, the MUX input '1' get enable and propagation delay of given circuit $\tau_2 = 6ns$

Hence maximum propagation delay of circuit is $\text{MAX}(\tau_1, \tau_2) = 6\,\text{ns}$

Hence, the correct option is (C).

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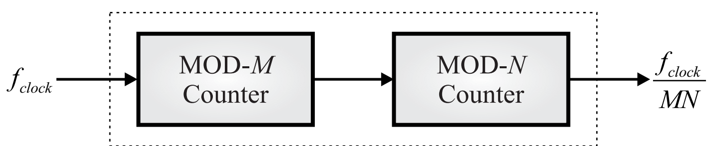
5 Sequential Circuits

➢ Partial Synopsis

Counters

- **Depending upon clock pulse, counter is of two types :**
  - (i) **Asynchronous counter :** In this type of counter flip-flops are connected in such a way that the output of 1st flip-flop drives the clock for the 2nd flip-flop, the output of 2nd flip-flop drives the clock for the 3rd flip-flop and so on.
  - (ii) **Synchronous counter :** In this type of counter there is no connection between the output of 1st flip-flop and clock input of next flip-flop and so on. In this type of counter all the flip-flops are connected to the same clock.
- **MOD number**
  - (i) Number of states present in a counter is known as modulus count or MOD number.
  - (ii) The MOD number indicates frequency division obtained from the last flip-flops. For MOD-N counter, the frequency of the output of last flip-flop is $\frac{f_{clock}}{N}$.

  - (iii) If two counters are cascaded, with MOD number M and MOD number N, then
    - (a) Overall modulus number of cascaded combinations of counter $= M \times N = MN$



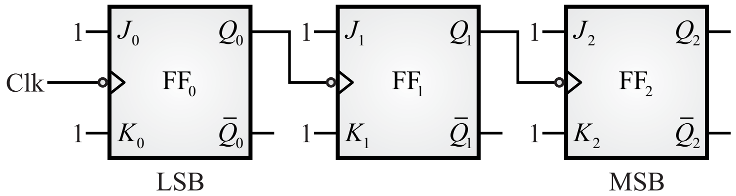
- Maximum decimal count of any counter $= N - 1$

Asynchronous Counter

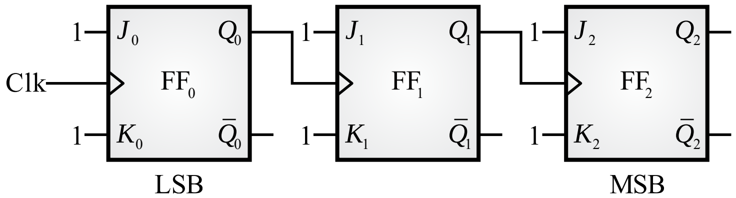
- It is also called ripple counter.
- In asynchronous counter, different clock pulse is applied to different flip-flops.
- In asynchronous counter, all flip-flops are operating in toggle mode.

- Example of asynchronous counter

(i) 3-bit binary ripple up counter :



(ii) 3-bit binary ripple down counter :

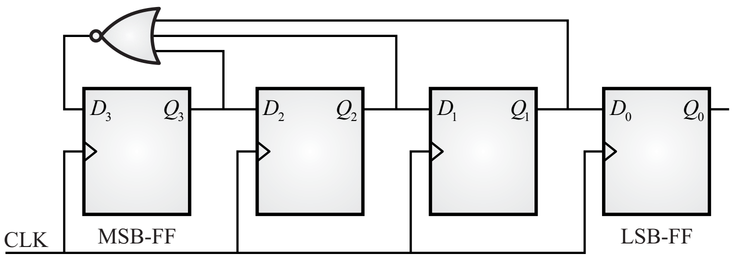


Synchronous Counter

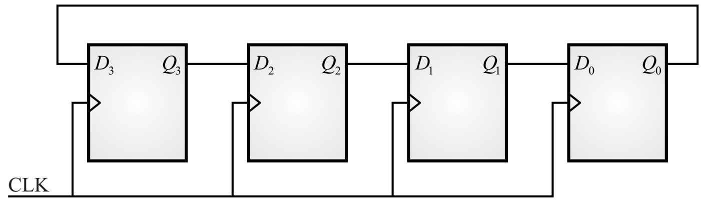
- The synchronous counters are classified as :
  - (i) Shift register counters
    - (a) Ring counter
    - (b) Twisted ring counter/Johnson counter
  - (ii) Series carry counter
  - (iii) Parallel carry counter

Ring Counter

- It is a **synchronous counter.** It is also called **serial-in serial-out (SISO) shift register.**
- In a ring counter, if feedback is used the number of states are reduced.
- An n-bit ring counter has,
  - (i) Number of flip-flop $=$ Mode of counter $= n$
  - (ii) Frequency of output of any flip-flop $= \dfrac{f_{CLK}}{n}$ where, f_{CLK} is frequency of clock signal.
  - (iii) Number of distinct states (or used states) $= n$
  - (iv) Number of unused states $= 2^n - n$
- For decoding ring counter no logic gates are required.
- **4-bit self starting ring counter :**



- **4-bit not self starting ring counter :**



➤ Sample Questions

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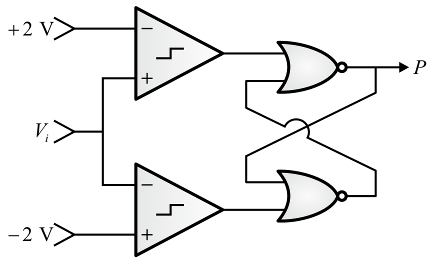
5.1 Choose the correct statements relating to the circuit of figure.



(A) For $V_i = -2$ V, $P = 0$

(B) For $V_i = +3$ V, $P = 0$

(C) $V_i = 0$ V, $P = 0$ always

(D) For $V_i = 0$ V, P can be either 0 or 1

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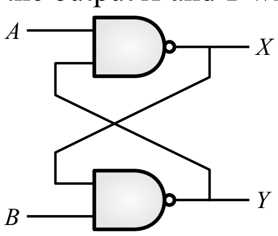
5.2 In figure A = 1 and B = 1, the input B is now replaced by a sequence 101010 ……, the output X and Y will be



(A) fixed at 0 and 1 respectively

(B) X = 1010 …. while Y = 0101 …

(C) X = 1010 …. while Y = 1010 …

(D) fixed at 1 and 0 respectively

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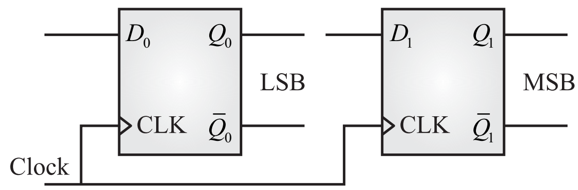
5.3 Two D flip-flops, as shown below, are to be connected as a synchronous counter that goes through the following $Q_1 Q_0$ sequence $00 \rightarrow 01 \rightarrow 11 \rightarrow 10 \rightarrow 00 \rightarrow \ldots$



The inputs D_0 and D_1 respectively should be connected as

(A) $\bar{Q}_1$ and Q_0

(B) $\bar{Q}_0$ and Q_1

(C) $\bar{Q}_1 Q_0$ and $\bar{Q}_1 Q_0$

(D) $\bar{Q}_1 \bar{Q}_0$ and $Q_1 Q_0$

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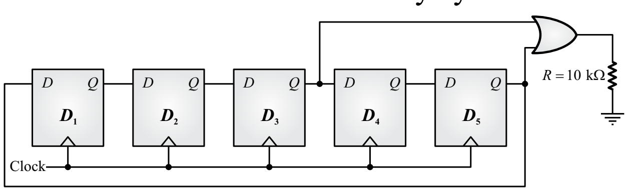
5.4 Assume that all the digital gates in the circuit shown in the figure are ideal, the resistor $R = 10\ \text{k}\Omega$ and the supply voltage is 5 V. The D flip-flops D_1, D_2, D_3, D_4 and D_5 are initialized with logic values 0, 1, 0, 1 and 0 respectively. The clock has a 30% duty cycle.



The average power dissipated (in mW) in the resistor R is ________.

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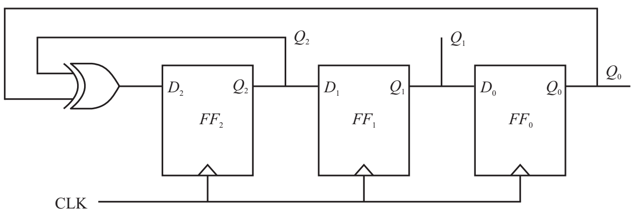
5.5 The propagation delay of the exclusive-OR (XOR) gate in the circuit is 3 ns. The propagation delay of all the flip-flops is assumed to be zero. The clock (CLK) frequency provided to the circuit is 500 MHz.



Starting from the initial value of the flip-flop outputs Q_2, Q_1, Q_0 = 111 with $D_2 = 1$, the minimum number of triggering clock edges after which the flip-flop outputs Q_2, Q_1, Q_0 becomes 100 (in integer) is _______.

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Explanations Sequential Circuits

Conclusion of Flip-Flops
Scan for Video Explanation

Concept of Asynchronous Counter
Scan for Video Explanation

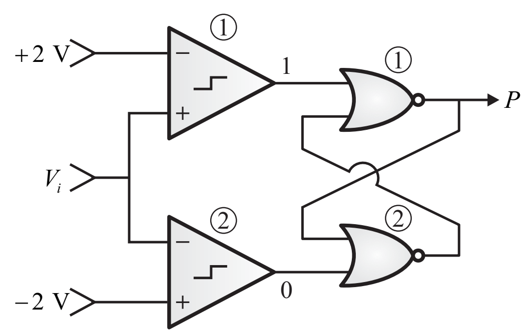
Concept of Asynchronous Clear & Preset input
Scan for Video Explanation

Concept of Mod N Asynchronous Counter (Up & Down)
Scan for Video Explanation

Concept of Synchronous Clear & Preset Input
Scan for Video Explanation

5.1 (B) and

Given logic circuit is shown below,



Method 1

Checking from options,

From option (A),

$$V_i = -2 \text{ V}$$

Then output of comparator 1 = logic 0

$$(\because V_- > V_+)$$

and output of comparator 2 = logic 0

$$(\because V_- = V_+)$$

Output P is given by,

$$P = \overline{0 + \text{output of second NOR gate}}$$

$$P = \overline{\text{output of second NOR gate}}$$

$$P = \overline{\overline{P}} = P$$

Thus, P can be either 0 or 1.

Hence, option (A) is incorrect.

From option (B),

$$V_i = +3 \text{ V}$$

Then output of comparator 1 = logic 1

$$(\because V_+ > V_-)$$

Output of comparator 2 = logic 0

$$(\because V_- > V_+)$$

Output P is given by,

$$P = \overline{1 + \text{output of second NOR gate}}$$

$$P = \overline{1}$$

$$P = 0$$

Hence, option (B) is correct.

From options (C) and (D),

$$V_i = 0$$

Then output of comparator 1 = logic 0

$$(\because V_- > V_+)$$

Output of comparator 2 = logic 0

$$(\because V_- > V_+)$$

Output P is given by,

$$P = \overline{0 + \text{output of second NOR gate}}$$

$$P = \overline{\text{output of second NOR gate}} = \overline{\overline{P}} = P$$

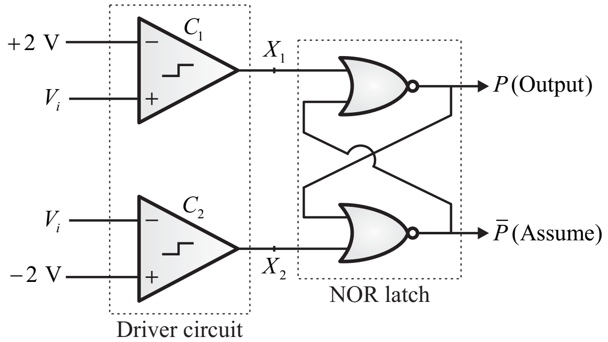
Thus, P can be either 0 or 1.

Hence, option (C) is incorrect but (D) is correct.

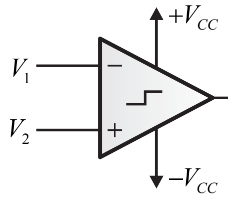
Hence, the correct options are (B) and (D).

Method 2

Given circuit is a NOR latch circuit along with comparators as shown below,



For comparator circuit,



If $V_1 > V_2 \rightarrow V_0 = -V_{CC}$ = Logic-0

$V_1 < V_2 \rightarrow V_0 = +V_{CC}$ = Logic-1

Case 1 : When $X_1 = 1$ and $X_2 = 0$ or 1 (Whatever may be) then,

$$P = 0$$

$$\overline{P} = \text{ Whatever may be}$$

Thus, $X_1 = 1$ is achieved only when $V_i > +2\text{V}$

So, option (B) satisfies.

Case 2 : When $X_2 = 1$ and $X_1 = 0$ or 1 (Whatever may be) then,

$$\overline{P} = 0$$

$$P = \text{May be 0 or 1 depend upon } X_1$$

Thus, $X_2 = 1$ is achieved only when $V_i < -2$ V

So, None of the option is satisfies.

Case 3 : When $X_1 = 0$ and $X_2 = 0$, both simultaneously then,

$P = 0$ or 1, depend upon previous condition of $\overline{P}$.

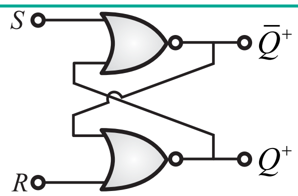
Thus, $X_1 = 0$ and $X_2 = 0$ is achieved only when $-2 < V_i < +2\text{ V}$.

So, option (D) is satisfies.

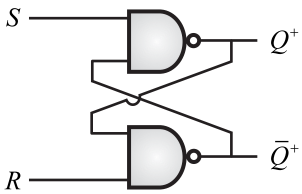
Hence, the correct options are (B) & (D).

Key Point

- In NOR latch if inputs are (0, 0), then outputs are previous state/hold [it may (0, 1) or (1, 0)].
- In NOR latch if inputs are (1, 1), then outputs are invalid state [it may (0, 0)].

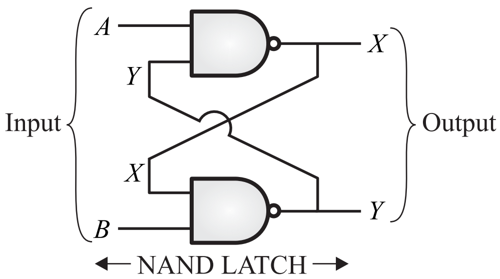


- In NAND latch if inputs are (1, 1), then outputs are previous states/hold [it may be (0, 1) or (1, 0)].
- In NAND latch if inputs are (0, 0), then outputs are invalid state [it may (1, 1)].



5.2 (A)

Given logic circuit is shown below,



Case 1 : When inputs, $A = B = 1$

Then outputs, $X = \bar{Y}$ and $Y = \bar{X}$

Case 2 : When inputs $A = 1$ and $B = 0$

Then outputs, $Y = 1$ and $X = \bar{Y} = \bar{1} = 0$.

Hence, $X = 0, Y = 1$

Case 3 : When inputs, $A = 1$ and $B = 1$

Then NAND latch holds previous states, so that

$$X = \bar{Y} = 0 \text{ and } Y = \bar{X} = 1$$

Therefore, output is fixed at 0 and 1 respectively.

Hence, the correct option is (A).

5.3 (A)

Count sequence for synchronous counter is given by,

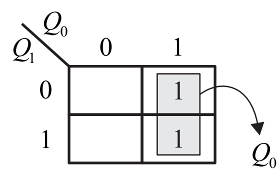
$00 \rightarrow 01 \rightarrow 11 \rightarrow 10 \rightarrow 00 \rightarrow$

State table for above sequence is shown below,

Present state		Next state		Inputs	
Q_1	Q_0	Q_1^+	Q_0^+	D_1	D_0
0	0	0	1	0	1
0	1	1	1	1	1
1	1	1	0	1	0
1	0	0	0	0	0

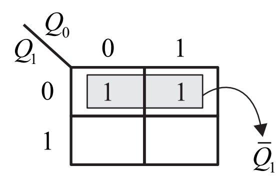
(Inputs D_1 and D_0 are sets according to the excitation table of D-flip-flop).

K-map for D_1 is shown below,



Hence, $D_1 = Q_0$

K-map for D_0 is shown below,

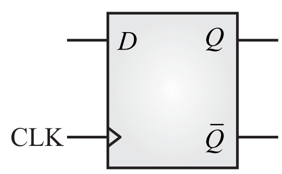


Thus, $D_0 = \bar{Q}_1$

Hence, the correct option is (A).

🕮 Key Point

(i) In D flip-flop output follows the input.



- Input = 0 then Output = 0
- Input = 1 then Output = 1

(ii) Most widely used of D-flip-flop is in shift registers.

5.4 1.5

Method 1

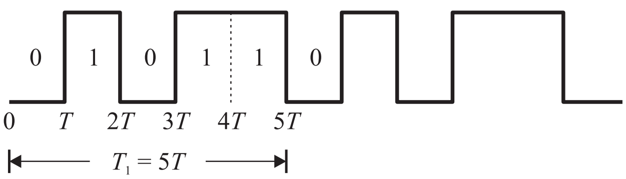
Average power dissipation is given by,

$$P_{avg} = \frac{V^2}{R} \times \frac{T_{ON}}{T}$$

CLK	Q_1	Q_2	Q_3	Q_4	Q_5	$Y = Q_3 + Q_5$
0	0	1	0	1	0	0
1	0	0	1	0	1	1
2	1	0	0	1	0	0
3	0	1	0	0	1	1
4	1	0	1	0	0	1
5	0	1	0	1	0	0

The waveform of the gate output

$Y = Q_3 + Q_5$



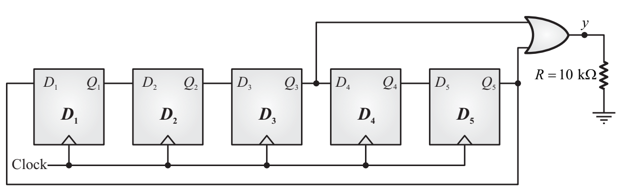
Thus, $P_{avg} = \frac{V^2}{R} \times \frac{T_{ON}}{T} = \frac{5^2}{10\text{ k}\Omega} \times \frac{3T}{5T}$

$$P_{avg} = 1.5 \text{ mW}$$

Hence, the average power dissipated in the resistor R is **1.5 mW**.

Method 2

Given circuit is synchronous ring counter with initial values are : $Q_1 = 0$, $Q_2 = 1$, $Q_3 = 0$, $Q_4 = 1$, $Q_5 = 0$ as, shown below,



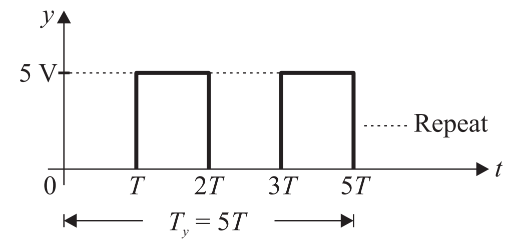
Here, logic-1 = 5 V

Logic-0 = 0 V,

So, its state table becomes as,

CLK	Q_1	Q_2	Q_3	Q_4	Q_5	$y = Q_3 + Q_5$
0	0	1	0	1	0	0
1	0	0	1	0	1	1
2	1	0	0	1	0	0
3	0	1	0	0	1	1
4	1	0	1	0	0	1
5	0	1	0	1	0	0

So, output waveform y can be formed as,



So, average power (P_{avg}) of periodic signal (y) is,

$$P_{avg} = \frac{1}{R}\left[\lim_{T_y \to \infty} \frac{1}{T_y} \int_0^{T_y} y^2(t)\, dt\right]$$

$$P_{avg} = \frac{1}{R}\left[\lim_{T_y \to \infty} \frac{1}{T_y}\left\{\int_0^{T} 0^2\, dt + \int_T^{2T} 5^2\, dt + \int_{2T}^{3T} 0^2\, dt + \int_{3T}^{5T} 5^2\, dt\right\}\right]$$

$$P_{avg} = \frac{1}{R}\left[\lim_{T_y \to \infty} \frac{1}{5T}\{25(2T - T) + 25(5T - 3T)\}\right]$$

$$P_{avg} = \frac{1}{R} \lim_{T_y \to \infty} \frac{1}{5T}[75T] = \frac{1}{R}\left[\frac{75}{5}\right]$$

$(\because R = 10 \text{ k}\Omega)$

$$P_{avg} = \frac{75}{10\text{k} \times 5} = 1.5 \text{ mW}$$

Hence, the average power dissipated in the resistor R is **1.5 mW**.

5.5 5

Given :

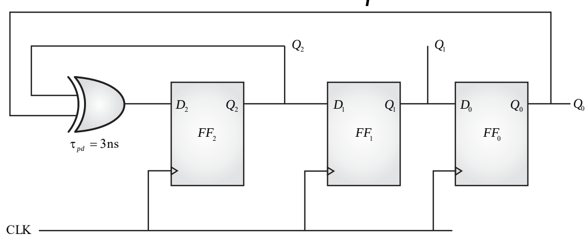
Delay of XOR gate $\tau_{pd} = 3ns$

Initially $Q_2Q_1Q_0 = 111$

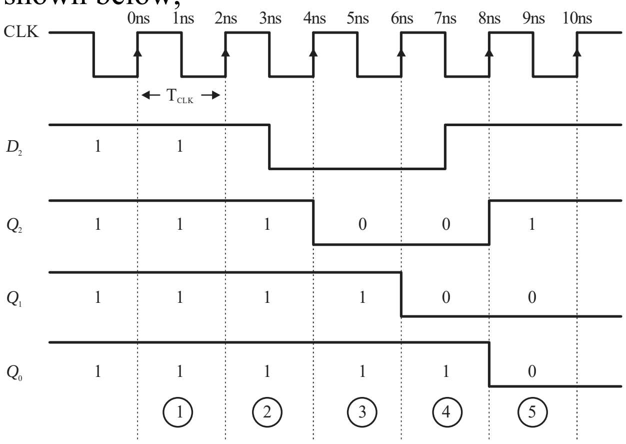
Initially $D_2 = 1$

Frequency of clock (*f*) = 500MHz

Time period of clock $T_{clock} = \frac{1}{f} = \frac{1}{500} = 2\,\text{ns}$.



The numbers of clock's to get $Q_2Q_1Q_0 = 100$ is shown below,



The minimum number of triggering clock edges after which the flip-flop output Q_2, Q_1, Q_0 becomes 100 is 5.

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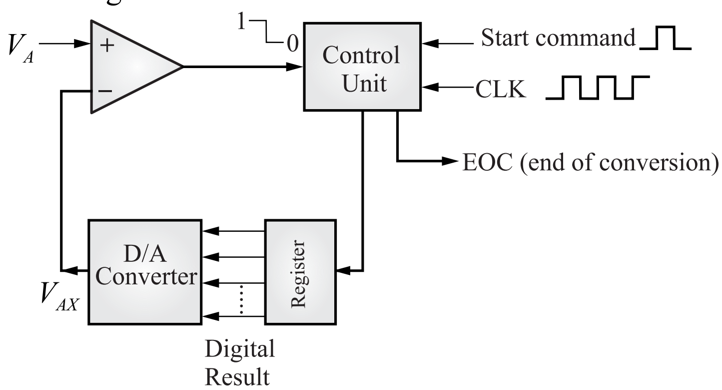
8 ADC & DAC

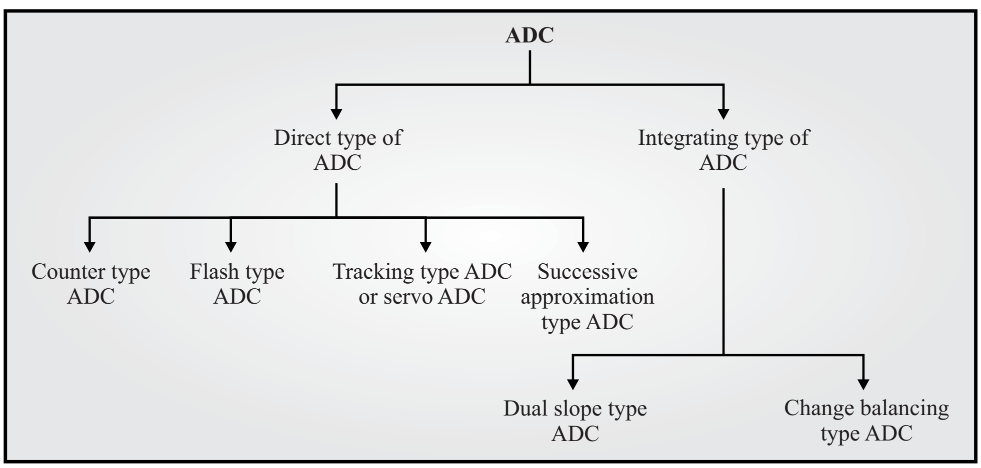
➢ Partial Synopsis

Analog to Digital Converters (ADC)

- It is often required that data taken in a physical system be converted into digital form. Thus, an analog to digital converter produces a digital output that is proportional to the value of input analog signal.
- ADCs are generally more complex and time consuming to design than DACs.
  An ADC takes an analog input voltage and after a certain amount of time produces a digital output code which represents the digital equivalent of the analog input. A general diagram of ADCs is shown in figure below.



Classification of ADC



Flash type/parallel comparator type ADC

- n-bit parallel ADC also called as flash type ADC/simultaneous type ADC.
- For *n*-bit flash type ADC requires,
  1. $2^n - 1$ number of comparator.
  2. 2^n number of resistors.
  3. $2^n \times n$ priority encoder.
  4. Number of priority encoder = 1
  5. Resolution $= \dfrac{V_{\max} - V_{\min}}{2^N - 1}$
- It is fastest type of ADC among all ADC's.
- One drawback with this ADC is, it requires more hardware if number of bits (*n*) increases more.

Maximum conversion time (T_c) for various n-bit ADC's

- Counter type ADC, $T_c = 2^n T_{clock}$
- Successive approximation type ADC, $T_c = nT_{clock}$
- Flash type ADC, $T_c = 1$ clock
- Dual slope integrating type ADC, $T_c = 2^{n+1} T_{clock}$

➢ Sample Questions

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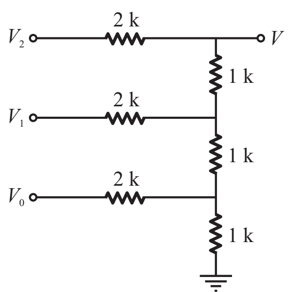
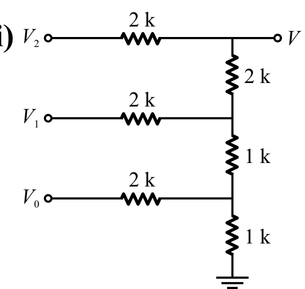
8.1 Which of the resistance networks of figure can be used as 3 bit *R*-2*R* ladder DAC. Assume V_0 corresponds to LSB.

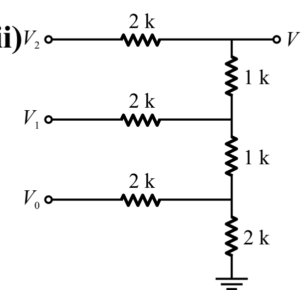
(i)



(ii)



(iii)



(A) Both (i) and (ii)
(B) Both (ii) and (iii)
(C) Only (iii)
(D) Only (ii)

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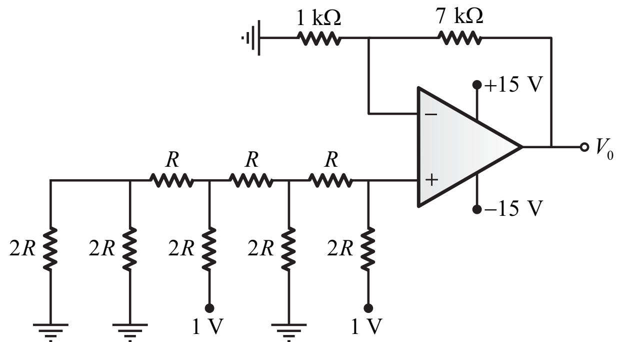
8.2 For the 4-bit DAC shown in figure, the output voltage V_0 is



(A) 10 V
(B) 5 V
(C) 4 V
(D) 8 V

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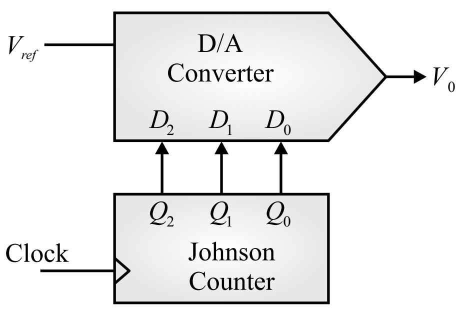
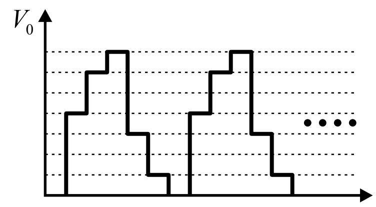
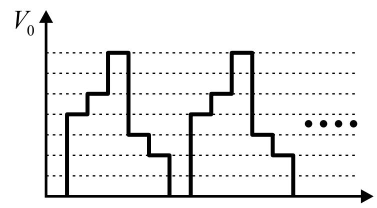
8.3 The output of a 3-stage Johnson (twisted ring) counter is fed to a digital-to-analog (D/A) converter as shown in the figure below. Assume all states of the counter to be unset initially. The waveform which represents the D/A converter output V_0 is



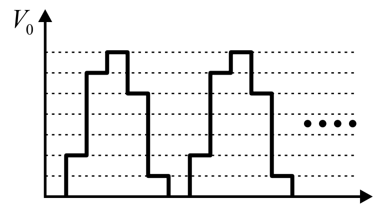
(A)



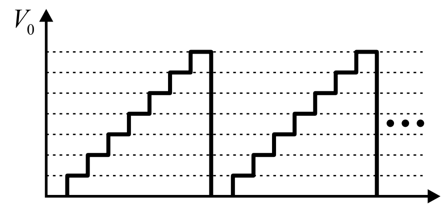
(B)



(C)



(D)



2021 IIT Bombay

8.4 An 8-bit unipolar (all analog output values are positive) digital-to-analog converter (DAC) has a full-scale voltage range from 0 V to 7.68 V. If the digital binary input code is 10010110 (the leftmost bit is MSB), then the analog output voltage of the DAC (rounded off to one decimal place) is ____________ V.

❖❖❖❖

Explanations ADC & DAC

Concept of DAC and Weighted Resistor DAC

Scan for Video Explanation

Concept of ADC and Flash ADC

Scan for Video Explanation

Concept of Successive approximation ADC

Scan for Video Explanation

Concept of Counter type ADC

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Concept of Resolution, Step size, Full scale output through question

Scan for Video Explanation

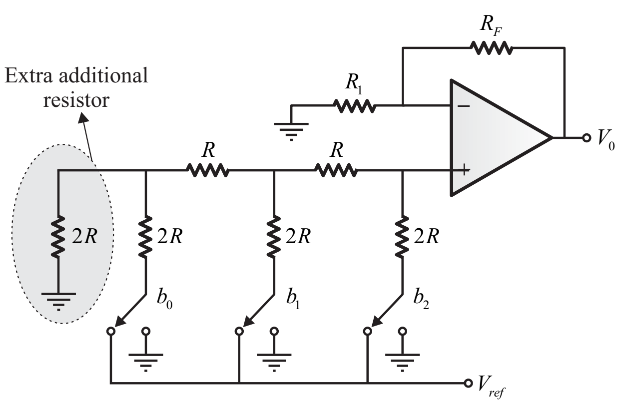
Concept of Dual Slope ADC

Scan for Video Explanation

8.1 (C)

For R-$2R$ ladder DAC, $2R$ resistor should grounded.

3-bit R-$2R$ ladder DAC using Op-Amp is shown below,



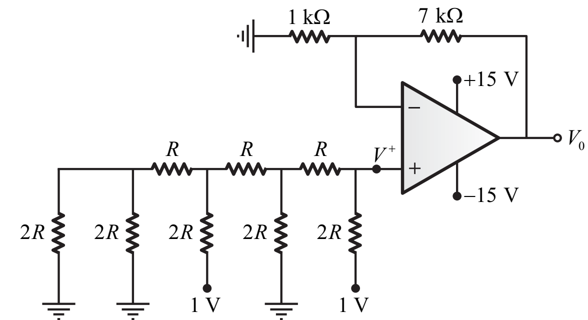
If we assume $R = 1\ \text{k}\Omega$

Then $2R = 2\ \text{k}\Omega$

Hence, the correct option is (C).

8.2 (B)

Given circuit is a 4-bit R-$2R$ ladder type DAC.



Here, Ground $\rightarrow$ Logic-0

1 V $\rightarrow$ Logic-1

Method 1

Output voltage V_0 is given by,

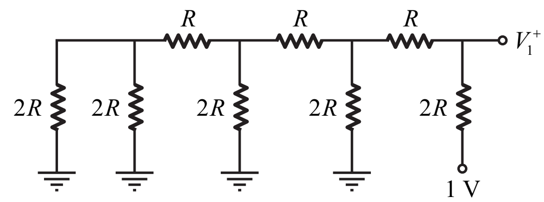
$$V_0 = \left(1 + \frac{R_f}{R_1}\right) V^+$$

$$V_0 = \left(1 + \frac{7\,\text{k}\Omega}{1\,\text{k}\Omega}\right) V^+$$

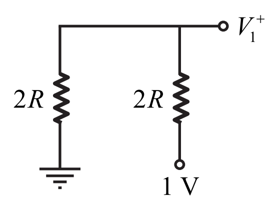
$$V_0 = 8V^+ \qquad \ldots\text{(i)}$$

For superposition theorem consider one source at a time.

Case 1 :

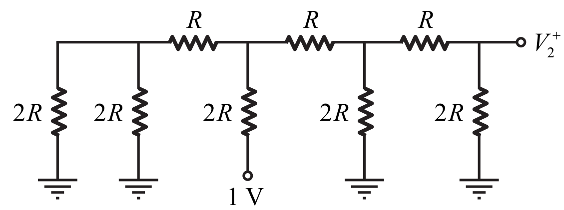


The equivalent circuit will be

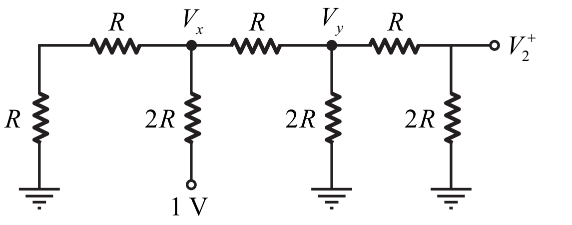


Hence, $V_1^+ = 0.5$ V

Case 2 :



The equivalent circuit can be draw as given below,



Applying KCL in above figure,

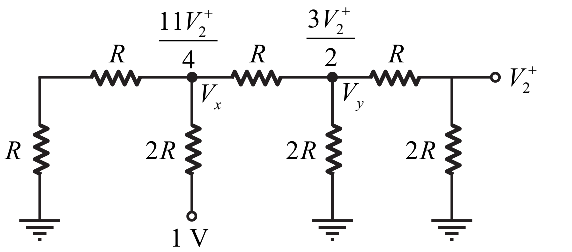
$$2V_x - V_y = 0.5 \qquad \ldots\text{(ii)}$$

$$\frac{5}{2}V_y - V_x - V_2^+ = 0 \qquad \ldots\text{(iii)}$$

$$\frac{3}{2}V_2^+ = V_y \qquad \ldots\text{(iv)}$$

From equation (iii) and (iv),

$$V_x = \frac{11}{4}V_2^+$$



Substituting values of V_x and V_y in terms of V_2^+ in equation (ii), we get

$$V_2^+ = \frac{1}{8}\text{ V}$$

Applying super position theorem, total voltage is given by,

$$V^+ = V_1^+ + V_2^+$$

$$V^+ = 0.5 + \frac{1}{8} = \frac{5}{8}\text{ V}$$

From equation (i),

$$V_0 = 8 \times \frac{5}{8} = 5\text{ V}$$

Hence, the correct option is (B).

Method 2

Output voltage for n-bit, R-$2R$ ladder DAC is given by,

$$V_0 = \frac{V_{ref}}{2^n} \times \left(1 + \frac{R_f}{R_1}\right) \times \text{(Decimal equivalent of binary digit)}$$

where, V_{ref} = Reference voltage = 1 V

$n = 4$

Binary number input = 1010

Decimal equivalent of (1010) = 10

$R_F = 7\text{ k}\Omega$ and $R_1 = 1\text{k}\Omega$

Hence, $V_0 = \dfrac{1}{2^4}\left(1 + \dfrac{7\text{ k}\Omega}{1\text{k}\Omega}\right) \times 10$

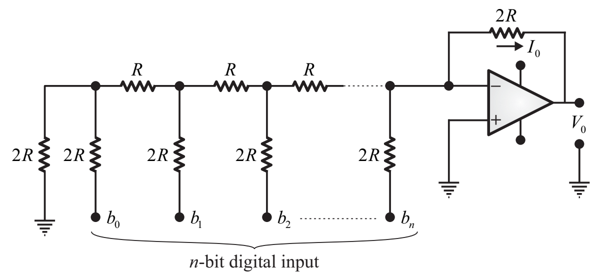
$$V_0 = \frac{1}{16} \times 8 \times 10$$

$$V_0 = 5\text{V}$$

Hence, the correct option is (B).

Key Point

(i) n-bit R-$2R$ ladder type DAC using inverting Op-amp,



Generalized expression for output V_0.

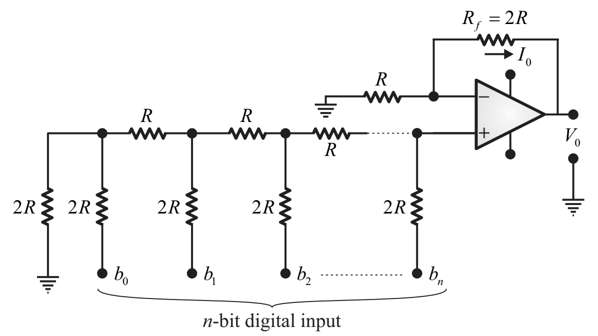
$$V_0 = -\left(\frac{R_f}{R}\right)\left(\frac{V_{ref}}{2^n}\right)(2^0 \times b_0 + 2^1 \times b_1 + \ldots.. + 2^n \times b_n)$$

$$V_0 = \begin{pmatrix}\text{Gain of}\\ \text{inverting}\\ \text{Op-Amp}\end{pmatrix}\begin{pmatrix}\text{Resolution}\\ \text{of DAC}\end{pmatrix}\begin{pmatrix}\text{Decimal}\\ \text{equivalent of}\\ \text{binary input}\\ \text{applied to}\\ \text{DAC}\end{pmatrix}$$

Generalize expression for output current (I_0),

$$I_0 = -\frac{V_0}{R_f} = -\frac{V_0}{2R}$$

(ii) n-bit R-$2R$ ladder type DAC using non-inverting Op-Amp,



Generalized expression for output (V_0)

$$V_0 = \left(1+\frac{R_f}{R}\right)\left(\frac{V_{fef}}{2^n}\right)(2^0 \times b_0 + 2^1 \times b_1 + \ldots.. + 2^n \times b_n)$$

$$V_0 = \begin{pmatrix}\text{Gain of}\\ \text{inverting}\\ \text{Op-Amp}\end{pmatrix}\begin{pmatrix}\text{Resolution}\\ \text{of DAC}\end{pmatrix}\begin{pmatrix}\text{Decimal}\\ \text{equivalent of}\\ \text{binary input}\\ \text{applied to}\\ \text{DAC}\end{pmatrix}$$

Generalized expression for output current (I_0),

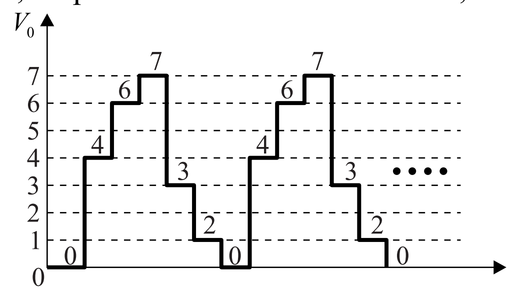
$$I_0 = -\frac{V_0}{R_f} = -\frac{V_0}{2R}$$

8.3 (A)

For the given 3-bit Johnson counter, state table is shown below,

Output of Johnson counter			Input of DAC			Output
Q_2	Q_1	Q_0	D_2	D_1	D_0	V_0
0	0	0	0	0	0	0
1	0	0	1	0	0	4
1	1	0	1	1	0	6
1	1	1	1	1	1	7
0	1	1	0	1	1	3
0	0	1	0	0	1	1
0	0	0	0	0	0	0

Thus, output waveform is shown below,



Hence, the correct option is (A).

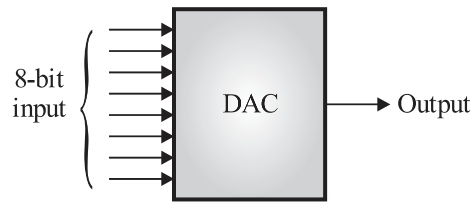
Scan for Video Solution

8.4 4.51

Given : Number of bits, $n = 8$,

Full scale voltage, $V_{FS} = 7.68$

Digital input $= (10010110)_2 \xrightarrow{\text{decimal}} (150)_{10}$



Thus, analog output from 8 bit unipolar DAC is,

V_{out} = Resolution × Decimal equivalent of digital binary input code

$$V_{out} = \left(\frac{V_{FS}}{2^n - 1}\right) \times 150 = \left(\frac{7.68}{2^8 - 1}\right) \times 150 = 4.517 \text{ Volt}$$

Hence, the analog output voltage of the 8 bit unipolar DAC is 4.517 Volt.

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CHAPTER 5 CONTROL SYSTEMS

Marks Distribution of Control Systems in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	2	6	14
2004	2	9	20
2005	3	6	15
2006	2	8	18
2007	1	9	19
2008	1	8	17
2009	2	4	10
2010	3	3	9
2011	3	3	9
2012	1	5	11
2013	2	5	12
2014 Set-1	2	3	8
2014 Set-2	2	3	8
2014 Set-3	1	3	7

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-4	2	3	8
2015 Set-1	3	3	9
2015 Set-2	3	3	9
2015 Set-3	3	4	11
2016 Set-1	2	3	8
2016 Set-2	2	3	8
2016 Set-3	2	3	8
2017 Set-1	3	3	9
2017 Set-2	3	3	9
2018	1	3	7
2019	1	4	9
2020	2	5	12
2021	2	2	6

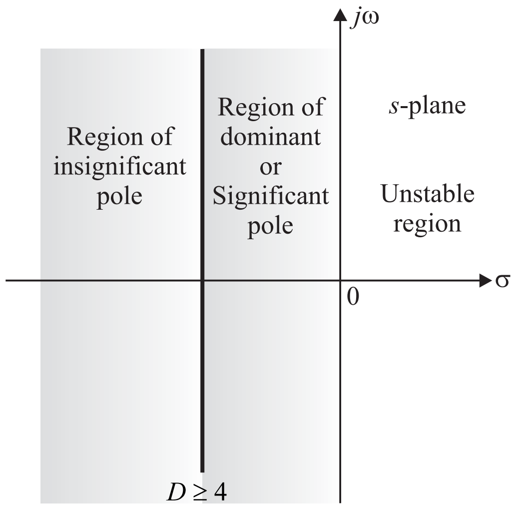
Syllabus : Control Systems

Basic control system components; Feedback principle; Transfer function; Block diagram representation; Signal flow graph; Transient and steady-state analysis of LTI systems; Frequency response; Routh-Hurwitz and Nyquist stability criteria; Bode and root-locus plots; Lag, lead and lag-lead compensation; State variable model and solution of state equation of LTI systems.

Contents : Control Systems

1 Basics of Control System

➢ Partial Synopsis



where, $D = \left|\dfrac{\text{Insignificant pole}}{\text{Significant pole}}\right|$

Fig. Regions of significant and insignificant poles in the s-plane

Remember

To find open loop transfer function from close loop transfer function,

$$\text{C.L.T.F.} = \frac{\text{Numerator}}{\text{Denominator}}\ ;\ \text{O.L.T.F.} = \frac{\text{Numerator}}{\text{Denominator} - \text{Numerator}}$$

To find closed loop transfer function from open loop transfer function,

$$\text{O.L.T.F.} = \frac{\text{Numerator}}{\text{Denominator}}\ ;\ \text{C.L.T.F.} = \frac{\text{Numerator}}{\text{Denominator} + \text{Numerator}}$$

Important Formulas

1. Transfer function :

$$T(s) = \frac{\text{Laplace transform of output}}{\text{Laplace transform of input}} \qquad [\text{Initial value} = 0]$$

2. Time constant :

$$\tau = \frac{1}{|\text{Negative real root}|} \qquad [\text{For first order system}]$$

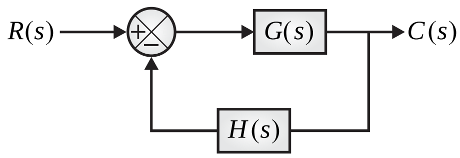
3. Bandwidth :

$$BW = \frac{1}{\tau} \qquad \text{[For first order system]}$$

4. Relation between impulse response $h(t)$ and step response $s(t)$:

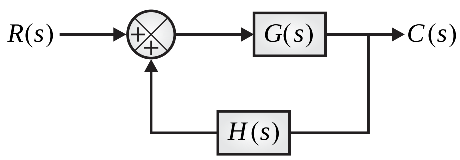
$$h(t) = \frac{d}{dt}s(t) \text{ or } s(t) = \int_{-\infty}^{t} h(t)\,dt$$

5. Negative feedback :



Closed loop transfer function is, $T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

6. Positive feedback :



Closed loop transfer function is, $T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)}$

7. Sensitivity :

$$S_B^A = \frac{\partial A/A \times 100\%}{\partial B/B \times 100\%} = \frac{\%\text{ change in } A}{\%\text{ change in } B}$$

(i) Open loop system, $S_G^T = 1$, where $T = 1$

(ii) Closed loop system, $S_G^T = \frac{1}{1+GH}$, where $T = \frac{G}{1+GH}$

$$S_H^T = \frac{-GH}{1+GH} \approx -1 \qquad \text{[Since, } GH >> 1 \text{ very high open loop transfer function]}$$

8. Initial value theorem :

$$x(0^+) = \lim_{t\to 0} x(t) = \lim_{s\to\infty} sX(s)$$

Valid for both unstable and stable system.

Exception :

(i) Initial value theorem is not valid for improper transfer function (number of zeros ≥ number of poles).

Sample Questions

1995 IIT Kanpur

1.1 The transfer function of a linear system is the

(A) ratio of the output $v_0(t)$ and the input $v_i(t)$.

(B) ratio of the derivatives of the output and the input.

(C) ratio of the Laplace transform of the output and that of the input with all initial conditions assumed to be zero.

(D) none of the above.

2001 IIT Kanpur

1.2 The open-loop DC gain of a unity negative feedback system with closed-loop transfer function $\dfrac{s+4}{s^2+7s+13}$ is

(A) $\dfrac{4}{13}$ (B) $\dfrac{4}{9}$

(C) 4 (D) 13

2007 IIT Kanpur

1.3 The transfer function of a plant is

$$T(s)=\frac{5}{(s+5)(s^2+s+1)}$$

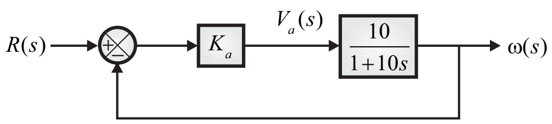
The second-order approximation of $T(s)$ using dominant pole concept is

(A) $\dfrac{1}{(s+5)(s+1)}$ (B) $\dfrac{5}{(s+5)(s+1)}$

(C) $\dfrac{5}{s^2+s+1}$ (D) $\dfrac{1}{s^2+s+1}$

2013 IIT Bombay

1.4 The open-loop transfer function of a dc motor is given as $\dfrac{\omega(s)}{V_a(s)}=\dfrac{10}{1+10s}$. When connected in feedback as shown below, the approximate value of K_a that will reduce the time constant of the c losed loop system by one hundred times as compared to that of the open-loop system is



(A) 1 (B) 5

(C) 10 (D) 100

2016 IISc Bangalore

1.5 A first-order low-pass filter of time constant T is excited with different input signals (with zero initial conditions up to $t = 0$). Match the excitation signals X, Y, Z with the corresponding time responses for $t \geq 0$ **[Set - 01]**

X : Impulse P : $1-e^{-t/T}$

Y : Unit step Q : $t-T(1-e^{-t/T})$

Z : Ramp R : $\dfrac{1}{T}e^{-t/T}$

(A) X → R, Y → Q, Z → P

(B) X → Q, Y → P, Z → R

(C) X → R, Y → P, Z → Q

(D) X → P, Y → R, Z → Q

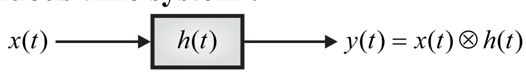
❖❖❖❖

Explanations — Basics of Control System

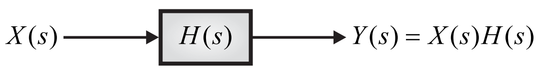
1.1 (C)

The transfer function of a linear time invariant system is the ratio of the Laplace transform of the output and that of the input with all initial conditions assumed to be zero.

Continuous time system :



Transform domain :



$$H(s) = \frac{Y(s)}{X(s)} = \text{Transfer function}$$

Hence, the correct option is (C).

1.2 (B)

Given : $\text{CLTF} = \dfrac{s+4}{s^2+7s+13}$

For unity negative feedback system the closed loop transfer function is given by,

$$\text{CLTF} = \frac{G(s)}{1+G(s)}$$

where, $G(s)$ = open loop transfer function

$$\frac{G(s)}{1+G(s)} = \frac{s+4}{s^2+7s+13}$$

$$\frac{1+G(s)}{G(s)} = \frac{s^2+7s+13}{s+4}$$

$$\frac{1}{G(s)} = \frac{s^2+7s+13}{s+4} - 1 = \frac{s^2+6s+9}{s+4}$$

$$G(s) = \frac{s+4}{s^2+6s+9}$$

For DC gain, $s = 0$

$$G(s)\big|_{s=0} = \frac{4}{9}$$

Hence, the correct option is (B).

Scan for Video Solution

1.3 (D)

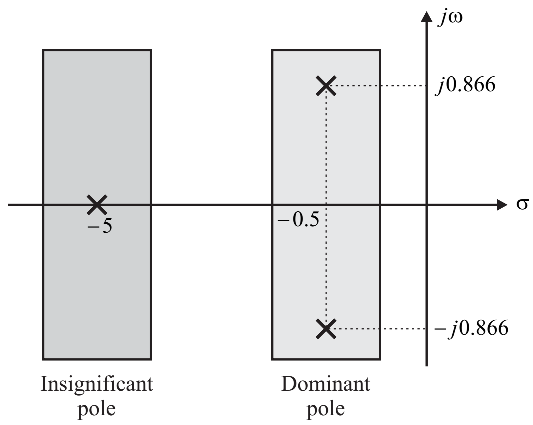
Given : $T(s) = \dfrac{5}{(s+5)(s^2+s+1)}$

$\left[3^{rd} \text{ order system}\right]$

In dominant pole concept, DC gain should be same after approximation.

DC gain before approximation is,

$$T(s=0) = \frac{5}{(0+5)(0+0+1)} = 1.0$$



Since, insignificant pole is more than five times of the dominant pole, hence insignificant pole can be neglected.

Therefore, second order approximation can be written as given below,

$$T(s) = \frac{5}{(s+5)(s^2+s+1)} \approx \frac{1}{(s^2+s+1)}$$

DC gain after approximation,

$$T(s)\big|_{s=0} = \frac{1}{1} = 1$$

Hence, the correct option is (D).

Scan for Video Solution

📖 Key Point

(i) Time constant due to dominant pole is higher than insignificant pole.

(ii) Dominant pole gives slower response compared to insignificant pole.

(iii) For approximation of transfer function we neglect insignificant poles.

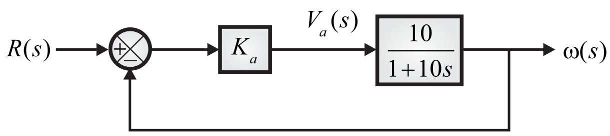
(iv) For existence of dominant pole, the ratio of insignificant pole to the significant pole (dominant pole) should be greater than or equal to 5.

(v) In dominant pole concept, overall time constant of system is given by dominant pole.

1.4 (C)

Given : $\tau_{\text{closed loop}} = \dfrac{1}{100}\tau_{\text{open loop}}$

where, τ represents time constant.

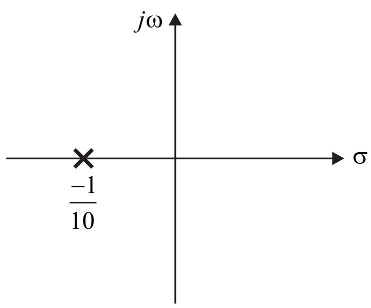


From figure,

Open loop transfer function of DC motor

$$= \frac{\omega(s)}{V_a(s)} = \frac{10}{1+10s}$$

Location of pole of open loop transfer function is shown below,



Time constant is defined as reciprocal of magnitude of negative real root.

Hence, $\tau_{\text{open loop}} = 10$ sec

Since, $\tau_{\text{closed loop}} = \dfrac{1}{100}\tau_{\text{open loop}}$

$$\tau_{\text{closed loop}} = \frac{10}{100} = 0.1 \text{ sec} \qquad \ldots\text{(i)}$$

Closed-loop transfer function for negative unity feedback is given by,

$$T(s) = \frac{G(s)}{1+G(s)}$$

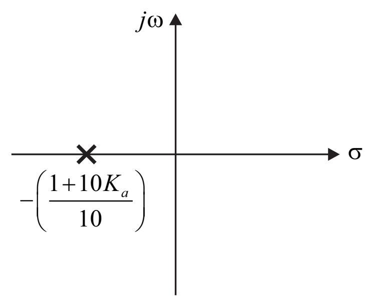
Here, $G(s) = K_a\left(\dfrac{10}{1+10s}\right)$

$$T(s) = \frac{\omega(s)}{R(s)} = \frac{K_a\left(\dfrac{10}{1+10s}\right)}{1+K_a\left(\dfrac{10}{1+10s}\right)}$$

$$T(s) = \frac{10K_a}{1+10s+10K_a}$$

$$T(s) = \frac{10K_a}{10s+(10K_a+1)}$$

Location of pole of closed loop transfer function is shown below,



From above figure,

$$\tau_{\text{closed loop}} = \frac{10}{10K_a+1} \qquad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$\frac{10}{10K_a+1} = 0.1 \text{ sec}$$

$$\frac{10}{10K_a+1} = \frac{1}{10}$$

$$10K_a + 1 = 100$$

$$10K_a = 99$$

$$K_a = 9.9 \simeq 10$$

Hence, the correct option is (C).

Scan for Video Solution

1.5 (C)

For first order system with time constant T,

$$G(s)=\frac{1}{sT}, \qquad H(s)=1$$

Block diagram of first order system is shown below,

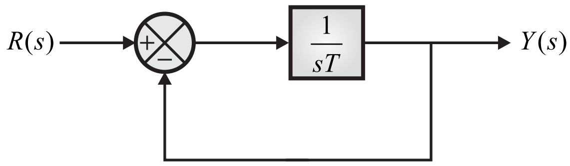


Fig. First order system

Closed loop transfer function of negative unity feedback system is given by,

$$\frac{Y(s)}{R(s)}=\frac{G(s)}{1+G(s)}$$

$$\frac{Y(s)}{R(s)}=\frac{1/sT}{1+\frac{1}{sT}}$$

$$Y(s)=\frac{1/sT}{1+\frac{1}{sT}}R(s)$$

$$Y(s)=\frac{1}{1+sT}R(s)$$

For impulse response, $R(s)=1$

$$y(t)=\frac{1}{T}e^{-t/T}, \qquad \text{for } t\geq 0$$

Hence, $X\rightarrow R$

For step response, $R(s)=\frac{1}{s}$

$$Y(s)=\frac{1}{s(1+sT)}=\frac{(1+sT)-(sT)}{s(1+sT)}$$

$$Y(s)=\frac{1}{s}-\frac{T}{(1+sT)}=\frac{1}{s}-\frac{T}{T\left(s+\frac{1}{T}\right)}$$

$$y(t)=(1-e^{-t/T}), \qquad \text{for } t\geq 0$$

Hence, $Y\rightarrow P$

For ramp response, $R(s)=\frac{1}{s^2}$

$$Y(s)=\frac{1}{s^2(1+sT)}=\frac{1}{s^2}-\frac{T}{s}+\frac{T}{s+\frac{1}{T}}$$

$$y(t)=t-T(1-e^{-t/T}), \text{ for } t\geq 0$$

Hence, $Z\rightarrow Q$

Hence, the correct option is (C).

Key Point

$$\frac{d}{dt}(\text{ramp response}) = \text{step response}$$

$$\frac{d}{dt}(\text{step response}) = \text{impulse response}$$

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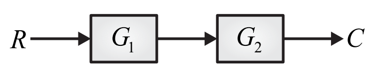
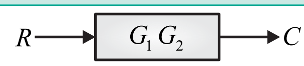
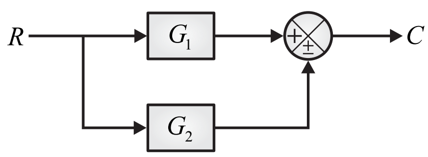
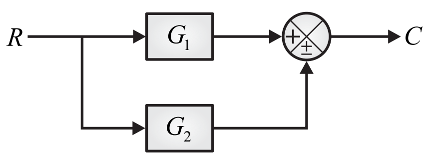
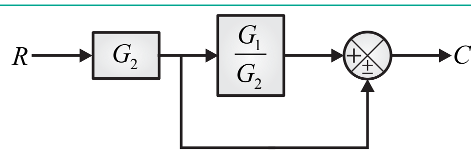
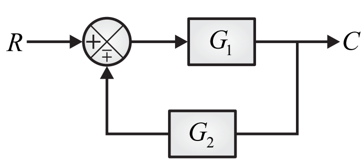
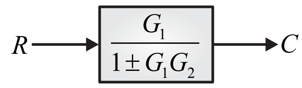
2 Block Diagram & Signal Flow Graph

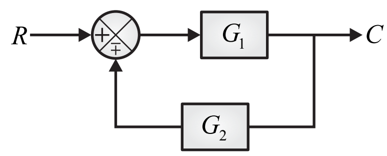
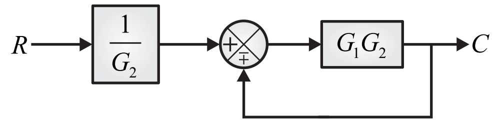
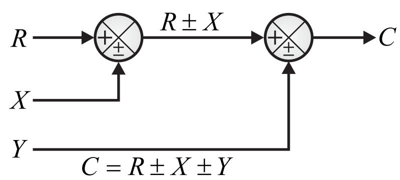
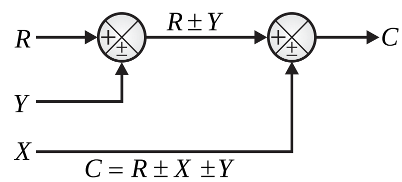
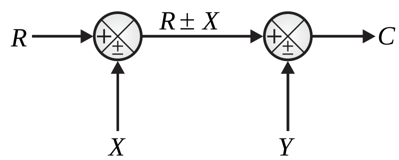
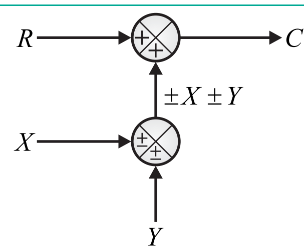
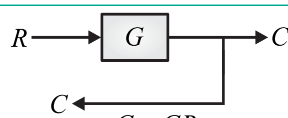
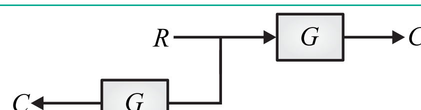
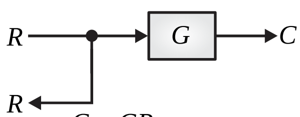
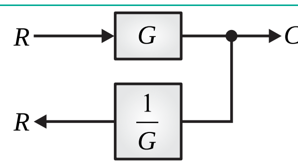
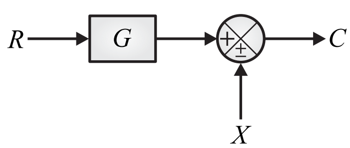
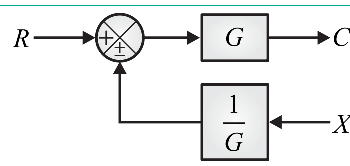
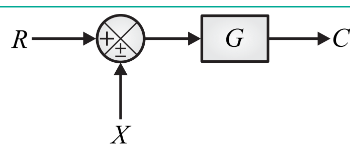
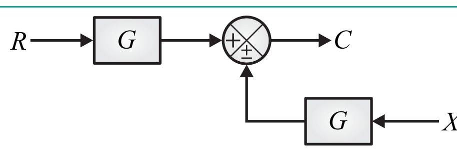
➢ Partial Synopsis

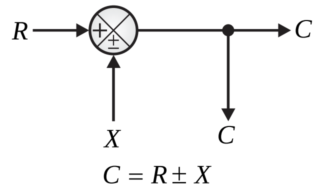
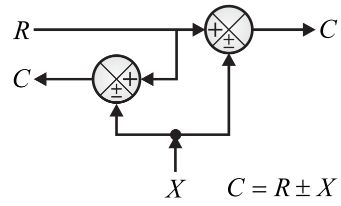
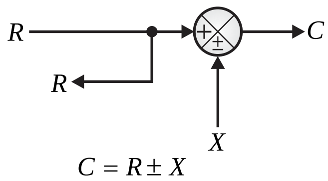
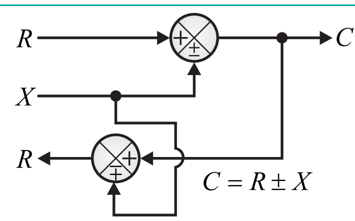
Block Diagram Reduction Rules

Block diagram reduction is used for simplifying (reducing) the block diagram, which is having many blocks, summing points and take off points and to obtain the overall transfer function.

Table : Block Diagram Reduction Rules

Transformation	Original Block Diagram	Equivalent Block Diagram
1. Combining blocks in cascade		 $C=(G_1G_2)R$
2. Combining blocks in parallel or eliminating a forward loop		 $C=(G_1\pm G_2)R$
3. Removing a block from a forward path	 $C=(G_1\pm G_2)R$	 $C=G_2\left(\frac{G_1}{G_2}\pm 1\right)R$ $C=(G_1\pm G_2)R$
4. Eliminating a feedback loop		 $C=\left(\frac{G_1}{1\pm G_1G_2}\right)R$

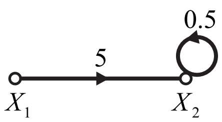
5. Removing a block from a feedback loop	 $C=\dfrac{G_1}{1\pm G_1G_2}R$	 $C=\left(\dfrac{G_1G_2}{1\pm G_1G_2}\right)\dfrac{1}{G_2}\times R$ $C=\dfrac{G_1}{1\pm G_1G_2}R$
6. Rearranging summing points	 $C=R\pm X\pm Y$	 $C=R\pm X\pm Y$
7. Rearranging summing points	 $C=R+(\pm X\pm Y)$	 $C=R+(\pm X\pm Y)$
8. Moving a take-off ahead of a block	 $C=GR$	 $C=GR$
9. Moving a take-off point beyond a block	 $C=GR$	 $C=GR$
10. Moving a summing point ahead of a block	 $C=RG\pm X$	 $C=G\left(R\pm\dfrac{1}{G}X\right)=RG\pm X$
11. Moving a summing point beyond a block	 $C=RG\pm XG=G(R\pm X)$	 $C=RG\pm GX=G(R\pm X)$

12. Moving a take-off point ahead of a summing point		
13. Moving a take-off point beyond of a summing point		

➢ Sample Questions

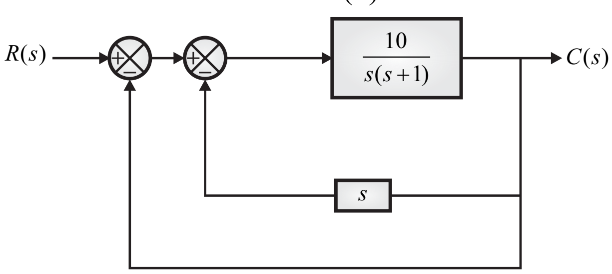
1987 IIT Bombay

2.1 In the signal flow graph shown in figure $X_2 = TX_1$ where T is equal to



(A) 2.5　　(B) 5000
(C) 5.5　　(D) 10

2.2 For the system shown in figure the transfer function $\frac{C(s)}{R(s)}$ is equal to

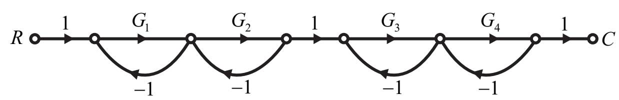


(A) $\frac{10}{s^2+s+10}$　　(B) $\frac{10}{s^2+11s+10}$

(C) $\frac{10}{s^2+9s+10}$　　(D) $\frac{10}{s^2+2s+10}$

1989 IIT Kanpur

2.3 The $\frac{C}{R}$ for the signal flow graph in the figure is



(A) $\frac{G_1G_2G_3G_4}{(1+G_1G_2)(1+G_3G_4)}$

(B) $\frac{G_1G_2G_3G_4}{(1+G_1+G_2+G_1G_2)(1+G_3+G_4+G_3G_4)}$

(C) $\frac{G_1G_2G_3G_4}{(1+G_1+G_2)(1+G_3+G_4)}$

(D) $\frac{G_1G_2G_3G_4}{(1+G_1+G_2+G_3+G_4)}$

2001 IIT Kanpur

2.4 An electrical system and its signal flow graph representations are shown in the figure (a) and (b) respectively. The value of G_2 and H, respectively are

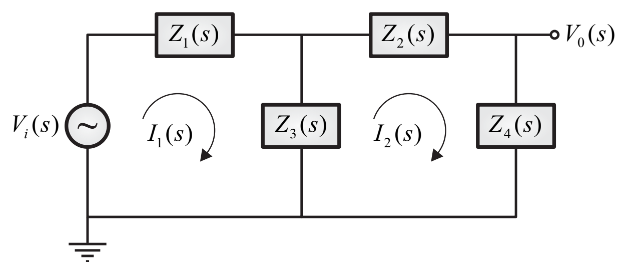


Fig. (a)

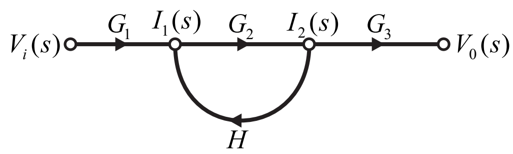


Fig. (b)

(A) $\dfrac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)}, \dfrac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

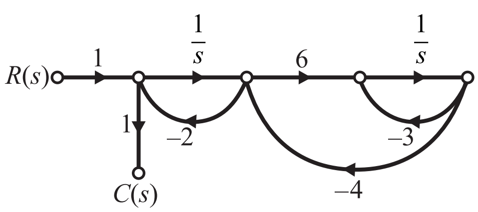
(B) $\dfrac{-Z_3(s)}{Z_2(s)-Z_3(s)+Z_4(s)}, \dfrac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

(C) $\dfrac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)}, \dfrac{Z_3(s)}{Z_1(s)+Z_3(s)}$

(D) $\dfrac{-Z_3(s)}{Z_2(s)-Z_3(s)+Z_4(s)}, \dfrac{Z_3(s)}{Z_1(s)+Z_3(s)}$

2003 IIT Madras

2.5 The signal flow graph of a system is shown in the below figure. The transfer function $\dfrac{C(s)}{R(s)}$ of the system is



(A) $\dfrac{6}{s^2+29s+6}$

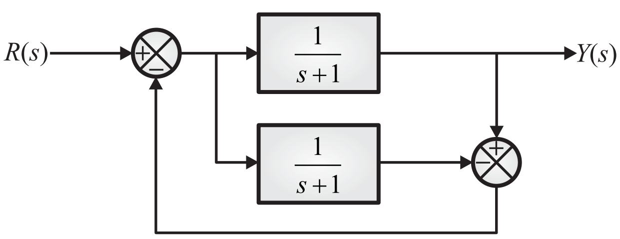
(B) $\dfrac{6s}{s^2+29s+6}$

(C) $\dfrac{s(s+2)}{s^2+29s+6}$

(D) $\dfrac{s(s+27)}{s^2+29s+6}$

2010 IIT Guwahati

2.6 The transfer function $\dfrac{Y(s)}{R(s)}$ of the system shown is



(A) 0

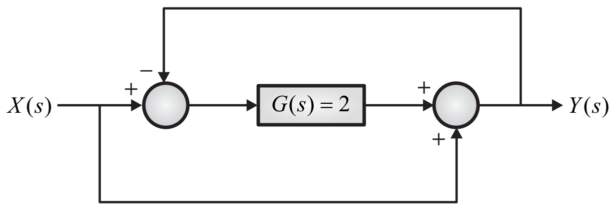
(B) $\dfrac{1}{s+1}$

(C) $\dfrac{2}{s+1}$

(D) $\dfrac{2}{s+3}$

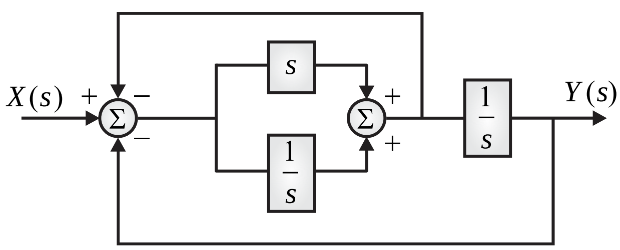
2017 IIT Roorkee

2.7 For the system shown in the figure, $\dfrac{Y(s)}{X(s)}=$ ______. **[Set - 02]**



2019 IIT Madras

2.8 The block diagram of a system is illustrated in the figure shown, where $X(s)$ is the input $Y(s)$ is the output. The transfer function $H(s)=\dfrac{Y(s)}{X(s)}$ is



(A) $H(s)=\dfrac{s^2+1}{s^3+2s^2+s+1}$

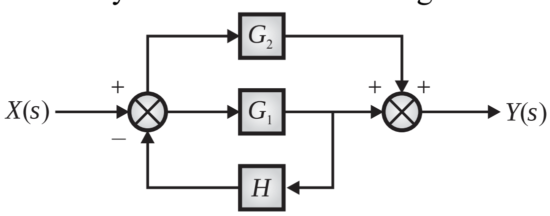
(B) $H(s)=\dfrac{s^2+1}{s^3+s^2+s+1}$

(C) $H(s)=\dfrac{s^2+1}{2s^2+1}$

(D) $H(s)=\dfrac{s+1}{s^2+s+1}$

2021　IIT Bombay

2.9 The block diagram of a feedback control system is shown in the figure.



The transfer function $\frac{Y(s)}{X(s)}$ of the system is

(A) $\frac{G_1+G_2+G_1G_2H}{1+G_1H}$

(B) $\frac{G_1+G_2}{1+G_1H+G_2H}$

(C) $\frac{G_1+G_2}{1+G_1H}$

(D) $\frac{G_1+G_2+G_1G_2H}{1+G_1H+G_2H}$

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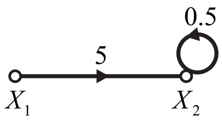
Explanations　Block Diagram & Signal Flow Graph

2.1　(D)

Given signal flow graph is shown below,



Method 1

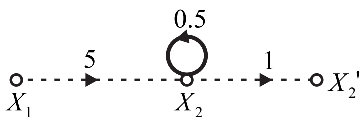
Forward path gain : $P_1 = 5$

Individual loop gain : $L_1 = 0.5$

Number of two non-touching loops : 0

Determinant : $\Delta = 1 - L_1 = 1 - 0.5 = 0.5$

Path factor :



The loop L_1 touches forward path P_1, hence

$$\Delta_1 = 1 - \text{ (isolated loop gain)}$$

$$\Delta_1 = 1 - 0 = 1$$

Using Mason's gain formula, transfer function is given by,

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

So, the transfer function is,

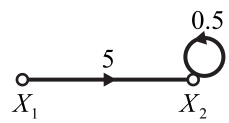
$$\frac{X_2'(s)}{X_1(s)} = \frac{P_1\Delta_1}{\Delta}$$

From figure, $X_2' = X_2$

$$\frac{X_2(s)}{X_1(s)} = \frac{5\times 1}{0.5} = 10$$

Hence, the correct option is (D).

Method 2



Writing transmittance equation directly from given signal flow graph,

$$X_2 = 5X_1 + 0.5X_2$$

$$(1-0.5)X_2 = 5X_1$$

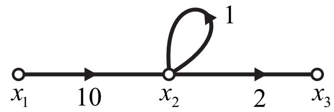
$$\frac{X_2}{X_1} = \frac{5}{1-0.5} = 10$$

Hence, the correct option is (D).

Scan for Video Solution

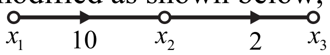
Key Point

(i) Mason's gain formula is invalid for self sustaining loop with unity gain.

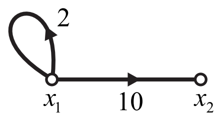


From above figure, $x_2 = 10x_1 + x_2$

Since, above equation is invalid hence the self loop shown in figure is invalid. Therefore, above signal flow graph can be modified as shown below,

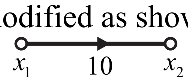


(ii) If self loop with any gain exist at input node, then this loop will be invalid.



From above figure, $x_1 = 2x_1$

Since, above equation is invalid, hence the self loop shown in figure is invalid.
Therefore, above signal flow graph can be modified as shown below,



2.2 (B)

The given block diagram is shown below,

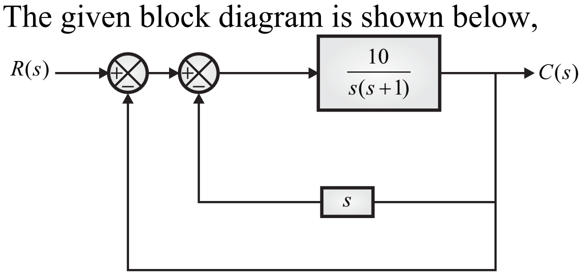


Fig. (a)

The block diagram can be redrawn as shown below,

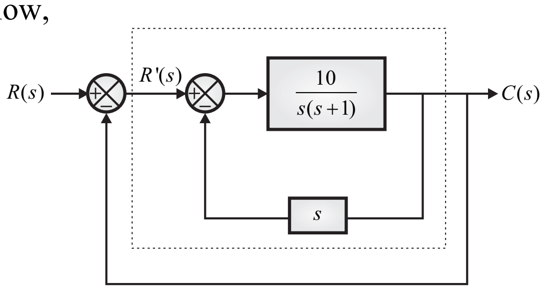


Fig. (b)

The transfer function of the unity negative feedback system is given by,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} \quad \ldots \text{(i)}$$

Inner loop shown by dotted lines in the figure gives,

$$\frac{C(s)}{R'(s)} = \frac{\frac{10}{s(s+1)}}{1+\frac{10}{s(s+1)}\times s} = \frac{10}{s^2+11s}$$

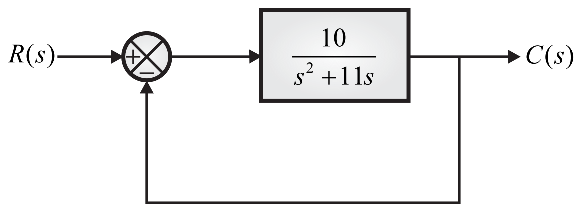


Fig. (c)

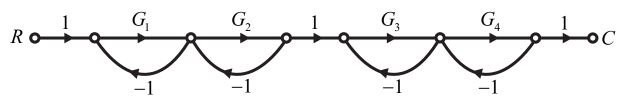
From equation (i),

$$\frac{C(s)}{R(s)} = \frac{\frac{10}{s^2+11s}}{1+\frac{10}{s^2+11s}\times 1} = \frac{10}{s^2+11s+10}$$

Hence, the correct option is (B).

2.3 (C)

The given signal flow graph is shown below,



Forward path gain : $P_1 = G_1G_2G_3G_4$

Individual loop gain : $L_1 = -G_1$,

$$L_2 = -G_2, L_3 = -G_3,\ L_4 = -G_4$$

Number of two non-touching loops :

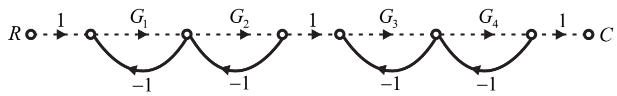
1. $(L_1L_3 = G_1G_3)$
2. $(L_1L_4 = G_1G_4)$
3. $(L_2L_3 = G_2G_3)$
4. $(L_2L_4 = G_2G_4)$

Determinant :

$$\Delta = 1-(L_1+L_2+L_3+L_4) + (L_1L_3+L_1L_4+L_2L_3+L_2L_4)$$

$$\Delta = 1-(-G_1-G_2-G_3-G_4) + (G_1G_3+G_1G_4+G_2G_3+G_2G_4)$$

Path factor :



All the loops touch forward path, hence

$$\Delta_1 = 1 - (\text{isolated loop gain})$$

$$\Delta_1 = 1 - 0 = 1$$

Using Mason's gain formula, transfer function is given by,

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

So, the transfer function is,

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1-[-G_1-G_2-G_3-G_4]+[G_1G_3+G_1G_4+G_2G_3+G_2G_4]}$$

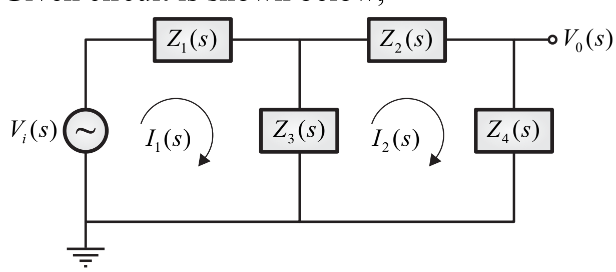
$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_1+G_2+G_3+G_4+G_1G_3+G_1G_4+G_2G_3+G_2G_4}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{(1+G_1+G_2)(1+G_3+G_4)}$$

Hence, the correct option is (C).

2.4 (C)

Given circuit is shown below,



Using KVL in loop (1),

$$-V_i(s) + I_1(s)Z_1(s) + [I_1(s) - I_2(s)]Z_3(s) = 0$$

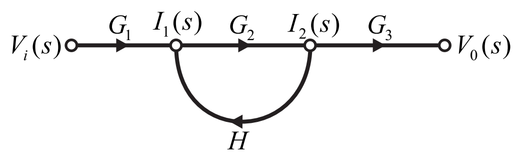
$$I_1(s) = \frac{V_i(s)}{Z_1(s)+Z_3(s)} + \frac{Z_3(s)}{Z_1(s)+Z_3(s)} I_2(s) \quad \ldots\text{(i)}$$

Using KVL in loop (2),

$$[I_2(s) - I_1(s)]Z_3(s) + I_2(s)[Z_2(s)+Z_4(s)] = 0$$

$$I_2(s) = \frac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)} I_1(s) \quad \ldots\text{(ii)}$$

Given signal flow graph is shown below,



From signal flow graph,

$$I_1(s) = G_1V_i(s) + HI_2(s) \quad \ldots\text{(iii)}$$

$$I_2(s) = G_2I_1(s) \quad \ldots\text{(iv)}$$

Compare equation (iii) with equation (i),

$$G_1 = \frac{1}{Z_1(s)+Z_3(s)}$$

and $$H = \frac{Z_3(s)}{Z_1(s)+Z_3(s)}$$

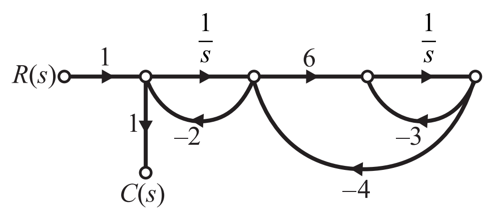
Compare equation (iv) with equation (ii),

$$G_2 = \frac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)}$$

Hence, the correct option is (C).

2.5 (D)

The given signal flow graph is shown below,



Forward path gain : $P_1 = 1 \times 1 = 1$

Individual loop gain :

$$L_1 = \frac{1}{s} \times (-3) = \frac{-3}{s}$$

$$L_2 = 6 \times \frac{1}{s} \times (-4) = \frac{-24}{s}$$

$$L_3 = \frac{1}{s} \times (-2) = \frac{-2}{s}$$

Number of two non-touching loops :

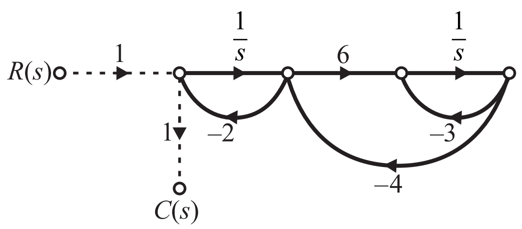
$$L_1L_3 = \frac{6}{s^2}$$

Determinant :

$$\Delta = 1-(L_1+L_2+L_3)+(L_1L_3)$$

$$\Delta = 1-\left(\frac{-3}{s}+\frac{-24}{s}+\frac{-2}{s}\right)+\frac{6}{s^2} = 1+\frac{29}{s}+\frac{6}{s^2}$$

Path factor :



Loop L_1 and loop L_2 does not touch forward path P_1, hence

$$\Delta_1 = 1-(L_1+L_2) = 1+\frac{27}{s}$$

Using Mason's gain formula, transfer function is given by,

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_k P_k\Delta_k$$

So, the transfer function is,

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{1\times(1+27/s)}{1+29/s+6/s^2}$$

$$\frac{C(s)}{R(s)} = \frac{s(s+27)}{s^2+29s+6}$$

Hence, the correct option is (D).

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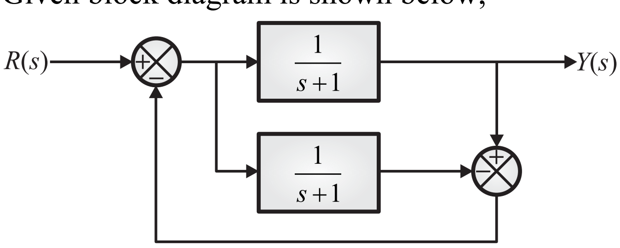
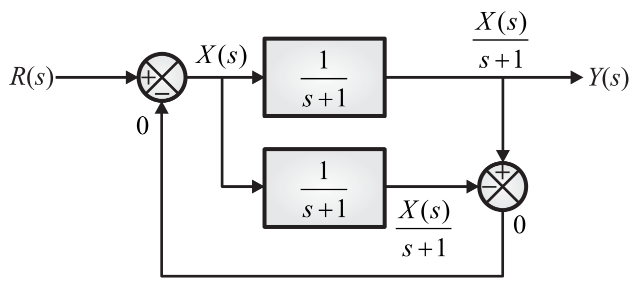
2.6 (B)

Given block diagram is shown below,



Method 1 : Block Diagram



From block diagram,

$$Y(s) = \frac{X(s)}{s+1} \qquad \ldots \text{(i)}$$

$$X(s) = R(s)-0 = R(s) \qquad \ldots \text{(ii)}$$

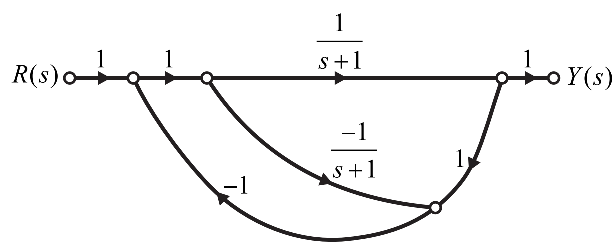
$$Y(s) = \frac{R(s)}{s+1}$$

$$\frac{Y(s)}{R(s)} = \frac{1}{s+1}$$

Hence, the correct option is (B).

Method 2 : SFG

The signal flow graph representation of the block diagram is shown below,



Forward path gain : $P_1 = \frac{1}{s+1}$

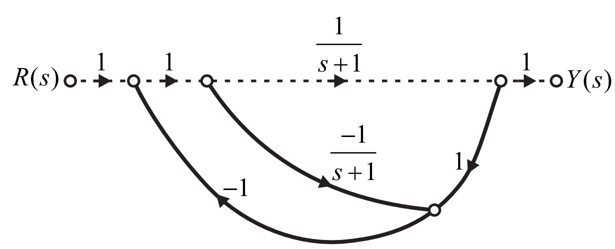
Individual loop gain : $L_1 = \frac{-1}{s+1},\; L_2 = \frac{1}{s+1}$

Number of two non-touching loops : 0

Determinant :

$$\Delta = 1-[L_1+L_2] = 1-\left[\frac{-1}{s+1}+\frac{1}{s+1}\right] = 1$$

Path factor :



All the loop touch forward path P_1, hence

$$\Delta_1 = 1 - \text{(isolated loop gain)}$$

$$\Delta_1 = 1 - 0 = 1$$

Using Mason's gain formula, transfer function is given by,

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

So, the transfer function is,

$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta}$$

$$\frac{Y(s)}{R(s)} = \frac{\frac{1}{s+1}\times 1}{1} = \frac{1}{s+1}$$

Hence, the correct option is (B).

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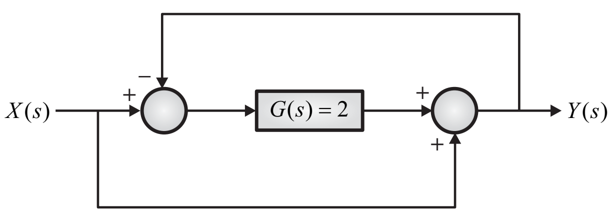
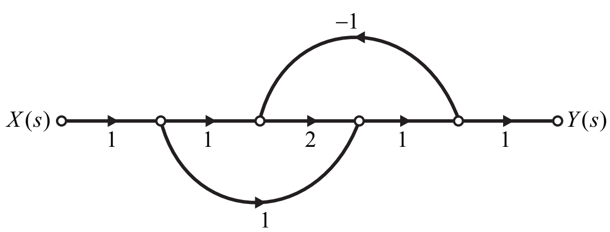
2.7　1

Given block diagram is shown below,



Method 1 : SFG

Signal flow graph of given block diagram is shown below,



Forward path gain :

$$P_1 = 1 \times 1 \times 2 \times 1 = 2,\ P_2 = 1 \times 1 \times 1 = 1$$

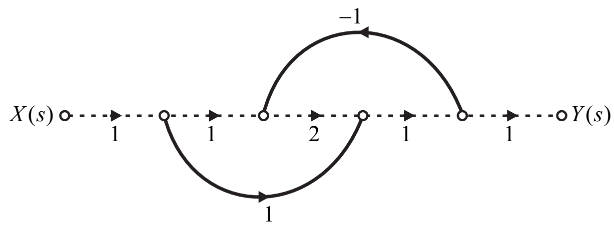
Individual loop gain : $L_1 = -2$

Number of two non-touching loop : 0

Determinate : $\Delta = 1 - L_1 = 1 + 2 = 3$

Path factor :

(i)

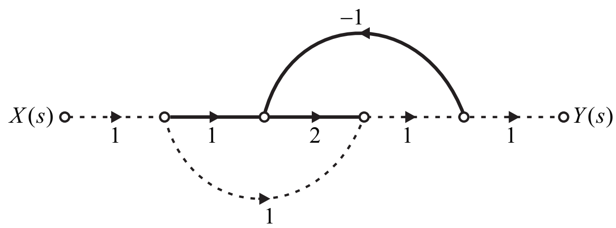


All the loops touch forward path P_1, hence

$$\Delta_1 = 1 - \text{(isolated loop gain)}$$

$$\Delta_1 = 1 - 0 = 1$$

(ii)



All the loops touch forward path P_2, hence

$$\Delta_2 = 1 - \text{(isolated loop gain)}$$

$$\Delta_2 = 1 - 0 = 1$$

Using Mason's gain formula, transfer function is given by,

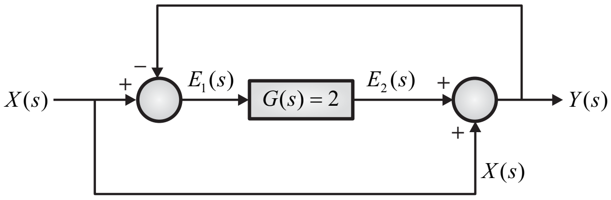
$$\frac{Y(s)}{X(s)} = \frac{1}{\Delta}\sum_{k=1}^{2} P_k \Delta_k$$

$$\frac{Y(s)}{X(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\frac{Y(s)}{X(s)} = \frac{(2\times 1)+(1\times 1)}{3} = 1$$

Hence, the value of $\frac{Y(s)}{X(s)}$ is **1.**

Method 2 : Block Diagram



From block diagram,

$$E_1(s) = X(s) - Y(s)$$

$$E_2(s) = 2E_1(s) = 2X(s) - 2Y(s)$$

$$Y(s) = E_2(s) + X(s)$$

$$Y(s) = 2X(s) - 2Y(s) + X(s)$$

$$3Y(s) = 3X(s)$$

$$\frac{Y(s)}{X(s)} = 1$$

Hence, the value of $\frac{Y(s)}{X(s)}$ is **1.**

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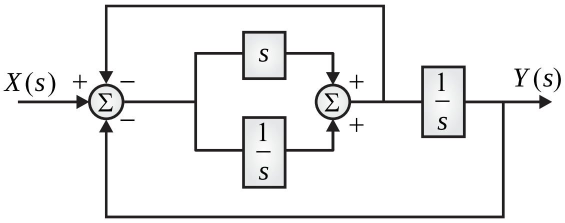
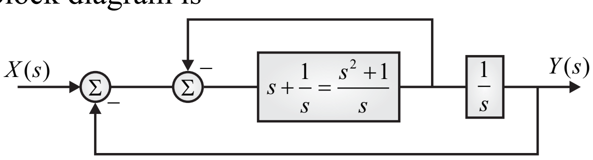
2.8 (A)

Method 1

Given block diagram is shown below,

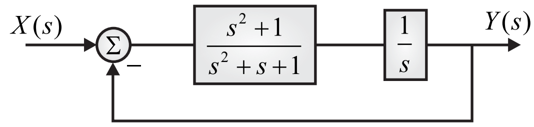


Solving the blocks in parallel and taking two summing points for the first adder, the equivalent block diagram is



Solving minor feedback loop,

$$\frac{\frac{s^2+1}{s}}{1+\frac{s^2+1}{s}} = \frac{s^2+1}{s^2+s+1}$$



Finally transfer function is given as

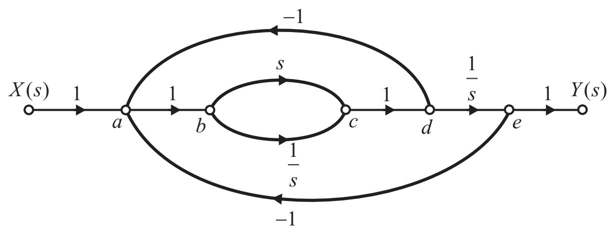
$$H(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{s^2+1}{s(s^2+s+1)}}{1+\frac{s^2+1}{s(s^2+s+1)}}$$

$$H(s) = \frac{s^2+1}{s^3+s^2+s+s^2+1} = \frac{s^2+1}{s^3+2s^2+s+1}$$

Hence, the correct option is (A).

Method 2

Converting the given block diagram into signal flow graph we get,



From above figure we have :

(i) Number of forward path : 2

(ii) Number of individual loops : 4

(iii) Pair of non-touching loop : 0

Forward path 1 gain

$$= P_1 = 1 \times s \times 1 \times \frac{1}{s} = 1$$

Forward path 2 gain

$$= P_2 = 1 \times \frac{1}{s} \times 1 \times \frac{1}{s} = \frac{1}{s^2}$$

Individual loop 1 gain

$$= 1 \times s \times 1 \times (-1) = -s$$

Individual loop 2 gain

$$= 1 \times \frac{1}{s} \times 1 \times (-1) = -\frac{1}{s}$$

Individual loop 3 gain

$$= 1 \times s \times 1 \times \left(\frac{1}{s}\right) \times (-1) = -1$$

Individual loop 4 gain

$$= 1 \times \frac{1}{s} \times 1 \times \frac{1}{s} \times (-1) = -\frac{1}{s^2}$$

Path factor for forward path 1 = 1

Path factor for forward path 2 = 1

From mason's gain formula, we have

$$\frac{Y(s)}{X(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\begin{array}{l}1-(\text{Sum of individual loop gain}) \\ \quad + (\text{Sum of gain products of} \\ \quad\quad \text{pair of non-touching loop}) - \ldots\ldots\end{array}}$$

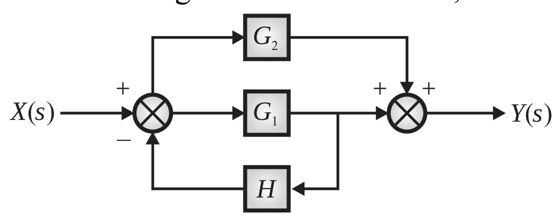
$$\frac{Y}{X} = \frac{1+\frac{1}{s^2}}{1-\left(-1-\frac{1}{s}-\frac{1}{s^2}-s\right)} = \frac{\frac{(s^2+1)}{s^2}}{\left(\frac{2s^2+s^3+s+1}{s^2}\right)}$$

$$\frac{Y}{X} = \frac{s^2+1}{s^3+2s^2+s+1}$$

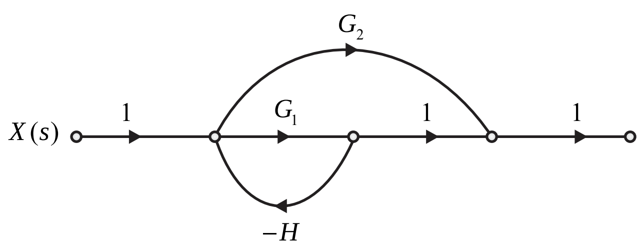
Hence, the correct option is (A).

2.9 (C)

Given block diagram as shown below,



The signal flow graph representation of block diagram is shown below,



Forward paths gain :

$$P_1 = G_1$$

$$P_2 = G_2$$

Individual loop gain :

$$L_1 = -G_1H$$

Determinant :

$$\Delta = 1-(L_1) = 1-(-G_1H) = 1+G_1H$$

Path factor :

$$\Delta_1 = 1-(\text{isolated loop gain})$$

$$\Delta_1 = 1-0 = 1$$

$$\Delta_2 = 1-(\text{isolated loop gain})$$

$$\Delta_2 = 1-0 = 1$$

From the Mason's gain formula the transfer function of this system is given by,

$$\frac{Y(s)}{X(s)} = \frac{1}{\Delta}\Sigma P_K\Delta_K$$

So, the transfer function is,

$$\frac{Y(s)}{X(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\therefore \quad \frac{Y(s)}{X(s)} = \frac{G_1+G_2}{1+G_1H}$$

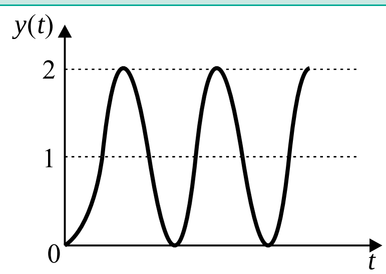
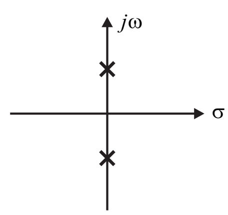
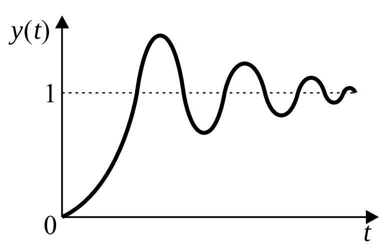
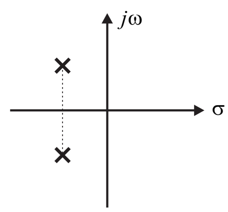
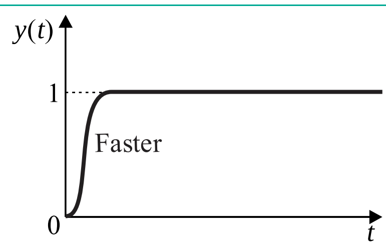
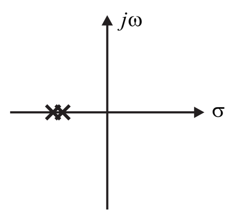
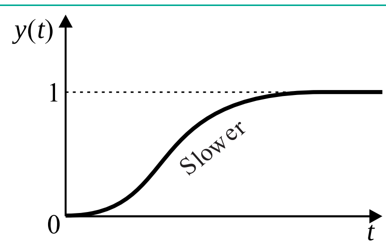
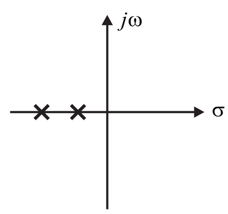
Hence, the correct option is (C).

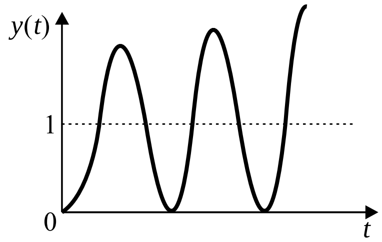
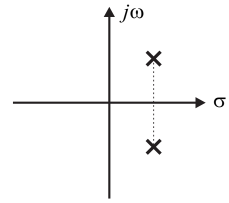
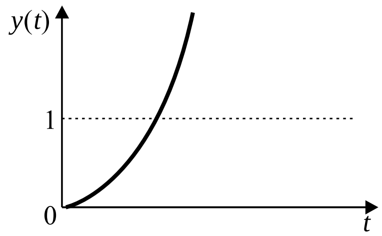
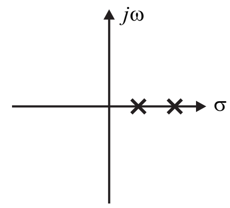
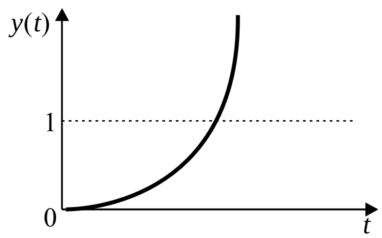
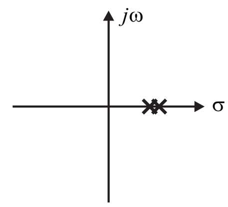
❖❖❖❖

4 Routh's Stability Criterion

➢ Partial Synopsis

Table : Step-response comparison for various characteristics-equation and root locations in the *s*-plane

Damping factor (ξ)	System behavior	Step Response	Stability	Characteristics Roots
$\xi = 0$	Un-damped		Marginally stable	Imaginary 
$0 < \xi < 1$	Under-damped		Stable	Complex Conjugate with negative real part 
$\xi = 1$	Critically damped		Stable	Real, Equal, Negative 
$\xi > 1$	Over-damped		Stable	Real, Unequal, Negative 

$-1 < \xi < 0$	Negative Under-damped		Unstable	Complex Conjugate with positive real part 
$\xi < -1$	Negative Over-damped		Unstable	Real, Unequal, Positive 
$\xi = -1$	Negative critical damped		Unstable	Real, Equal, Positive 

Damping Ratio for Series and Parallel RLC Circuit

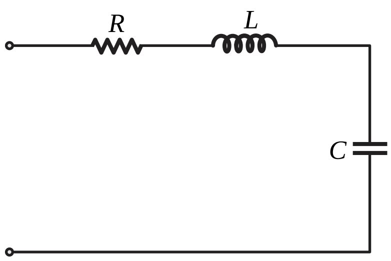


Fig. Series RLC circuit

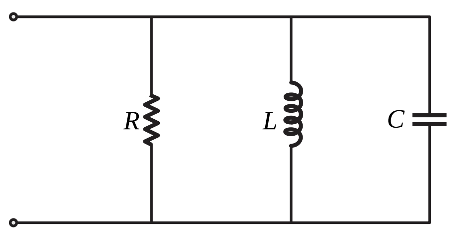


Fig. Parallel RLC circuit

Damping	ξ	Series	Parallel
Un-damped	$\xi = 0$	$\frac{R}{2}\sqrt{\frac{C}{L}} = 0$ $R = 0$	$\frac{1}{2R}\sqrt{\frac{L}{C}} = 0$ $R = \infty$
Under-damped	$0 < \xi < 1$	$0 < \frac{R}{2}\sqrt{\frac{C}{L}} < 1$ $0 < R < 2\sqrt{\frac{L}{C}}$	$0 < \frac{1}{2R}\sqrt{\frac{L}{C}} < 1$ $0 < \frac{1}{R} < 2\sqrt{\frac{C}{L}}$ $\frac{1}{2}\sqrt{\frac{L}{C}} < R < \infty$

Critical-damped	$\xi = 1$	$\frac{R}{2}\sqrt{\frac{C}{L}} = 1$ $R = 2\sqrt{\frac{L}{C}}$	$\frac{1}{2R}\sqrt{\frac{L}{C}} = 1$ $R = \frac{1}{2}\sqrt{\frac{L}{C}}$
Over-damped	$\xi > 1$	$\frac{R}{2}\sqrt{\frac{C}{L}} > 1$ $R > 2\sqrt{\frac{L}{C}}$	$\frac{1}{2R}\sqrt{\frac{L}{C}} > 1$ $R < \frac{1}{2}\sqrt{\frac{L}{C}}$

➢ Sample Questions

1988 IIT Kharagpur

4.1 Consider a characteristic equation given by $s^4 + 3s^3 + 5s^2 + 6s + K + 10$.

The condition for stability is

(A) $K > 5$ (B) $-10 < K$

(C) $K > -4$ (D) $-10 < K < -4$

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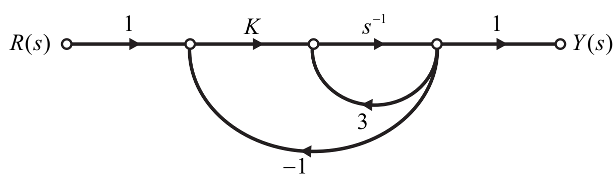
4.2 The system shown in the given figure remains stable when :



(A) $K < -1$ (B) $-1 < K < 1$

(C) $1 < K < 3$ (D) $K > 3$

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4.3 The number of open loop right half plane poles of

$$G(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3} \text{ is}$$

(A) 0 (B) 1

(C) 2 (D) 3

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4.4 Consider a transfer function $G_p(s) = \dfrac{ps^2 + 3ps - 2}{s^2 + (3+p)s + (2-p)}$ with 'p' a positive parameter. The maximum value of 'p' until which G_p remain stable is_________. **[Set - 04]**

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Explanations Routh's Stability Criterion

4.1 (D)

Given : The characteristic equation is,

$$s^4 + 3s^3 + 5s^2 + 6s + K + 10 = 0$$

Routh Tabulation :

s^4	1	5	$K+10$
s^3	3	6	0
s^2	3	$K+10$	0
s^1	$[6-(K+10)]$	0	0
s^0	$(K+10)$	0	0

For the system to be stable, all the roots must be in the left-half of s-plane, thus all the coefficients in the first column of Routh's tabulation must have the same sign. Therefore, the coefficient of first column of the Routh's table should be positive.

This leads to the following conditions :

(i) $6-(K+10)>0 \quad \Rightarrow \quad K<-4$

(ii) $K+10>0 \quad \Rightarrow \quad K>-10$

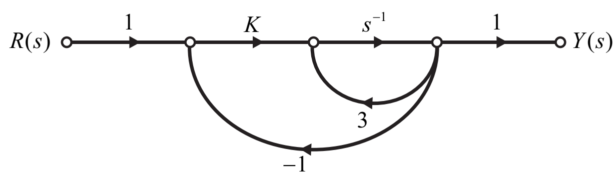
Therefore, the range of K for stability,

$$-10<K<-4$$

Hence, the correct option is (D).

4.2 (D)

The given signal flow graph for system is shown below,



Forward path gain : $P_1 = Ks^{-1}$

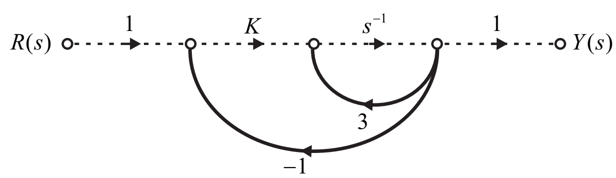
Individual loop gain : $L_1 = 3s^{-1}$, $L_2 = -Ks^{-1}$

Number of two non-touching loops : 0

Determinant :

$$\Delta = 1-(L_1+L_2) = 1-3s^{-1}+Ks^{-1}$$

Path factor :



All the loops touch forward path.

$$\Delta_1 = 1-(\text{isolated loop gain})$$

$$\Delta_1 = 1-0 = 1$$

Using Mason's gain formula, transfer function is given by,

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_k P_k\Delta_k$$

So, the transfer function is

$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta}$$

$$\frac{Y(s)}{R(s)} = \frac{Ks^{-1}}{1-3s^{-1}+Ks^{-1}} = \frac{K}{s-3+K}$$

Pole of this transfer function is,

$$s = 3-K$$

For the system to be stable, all the roots must be in the left-half of s-plane

$$3-K<0 \quad \Rightarrow \quad K>3$$

Hence, the correct option is (D).

4.3 (C)

Given : $G(s) = \dfrac{10}{s^5+2s^4+3s^3+6s^2+5s+3}$

The characteristic equation is,

$$s^5+2s^4+3s^3+6s^2+5s+3=0$$

Routh Tabulation :

s^5	1	3	5
s^4	2	6	3
s^3	0	$\frac{7}{2}$	0

Since, the first element of the s^3 row is zero, the all elements in the s^2 row would be infinite. To overcome this difficulty, we replace the zero in the s^3 row with a small positive variable ε and then proceed with the tabulation. Starting with the s^3 row, the results are as follows,

s^3	ε	$\frac{7}{2}$
s^2	$\frac{6\varepsilon-7}{\varepsilon}$	3
s^1	$\frac{7}{2}-\frac{3\varepsilon}{\left(\frac{6\varepsilon-7}{\varepsilon}\right)}$	0
s^0	3	0

From the Routh table,

$$\lim_{\varepsilon \to 0} \frac{6\varepsilon - 7}{\varepsilon} = \lim_{\varepsilon \to 0}\left(6 - \frac{7}{\varepsilon}\right) = -\infty$$

and $$\lim_{\varepsilon \to 0}\left(\frac{7}{2} - \frac{3\varepsilon}{6 - \frac{7}{\varepsilon}}\right) = \frac{7}{2}$$

Since, there are two sign changes in the first column of Routh's tabulation, then the equation has two roots in the right-half of s-plane.

Hence, the correct option is (C).

Scan for Video Solution

4.4 **2**

Given : $G_p(s) = \dfrac{ps^2 + 3ps - 2}{s^2 + (3+p)s + (2-p)}$

The characteristic equation is,

$$s^2 + (3+p)s + (2-p) = 0$$

Routh Tabulation :

s^2	1	$2-p$
s^1	$3+p$	0
s^0	$2-p$	0

For the system to be stable, all the roots must be in the left-half of s-plane, thus all the coefficients in the first column of Routh's tabulation must have the same sign. Therefore, first column of the Routh's table should be positive.

This leads to the following conditions :

(i) $3 + p > 0 \quad \Rightarrow \quad p > -3$

(ii) $2 - p > 0 \quad \Rightarrow \quad p < 2$

Therefore, the condition of p for the system to be stable is,

$$-3 < p < 2$$

The maximum value of p until which G_p remain stable is 2.

At $p = 2$, last row form ROZ, but there is no significance of ROZ in the last row.

Hence, the maximum value of p is **2.**

Scan for Video Solution

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5 Root Locus

➢ Partial Synopsis

Rules of Sketching Root Locus

1. Root locus is always symmetrical about the real axis (i.e. x-axis/ σ- axis).
2. Root locus always start from open loop pole $(K=0)$ and terminate either at finite open loop zero or at infinity that means virtual zero $(K=\infty)$.
3. **Existence of any point on root locus :**

   (i) The entire real axis of s-plane is occupied by the root locus for all values of K (i.e. $-\infty \le K \le \infty$).

   (ii) Root locus for $K \ge 0$ are found in the section if the sum of the number of open loop poles and zeros to the right of the section is odd.

   (iii)The remaining section of the real axis are occupied by the root locus for $K \le 0$ (i.e. complementary root locus).

   (iv)Open loop pole and zero are considered as the part of root locus, do not check even and odd concept at this point.
4. **Existence of root locus on real axis :**

   Root locus will exist only on that section of real axis, to the right of which sum of all poles and zeros is an odd number.
5. **Existence of root locus in complex plane/real axis :**

   (i) A point of s-plane will lie on root locus if the angle of $G(s)H(s)$ evaluated at that point is an odd integer multiple of $\pm 180^0$.

   (ii) Substitute the given complex location into the characteristic equation and then calculate the value of K.

   (iii)If K is real and positive for a given complex location then closed loop pole will exist at that location.

   (iv)If K is either negative or imaginary or complex for a given complex location then the location will be invalid and close loop pole will not exist at that location.

6. **Number of root locus branches :**

   (i) Number of root locus branches $= P(\text{if}, \ P > Z)$

   (ii) Number of root locus branches $= Z(\text{if}, \ Z > P)$

   (iii)Number of root locus branches $= P = Z(\text{if}, \ P = Z)$

   Note: Number of root locus branch $= \max(P, Z)$

7. **Break points/Saddle points :**

   (i) The point at which root locus branches meet or diverge is known as break point or saddle point.

   (ii) There are two types of break point :

   (a) Break away point (BAP)

   (b) Break in point (BIP)

   To find break point ;

   Step 1 : At first we have to find the characteristics equation, i.e., $1 + G(s)H(s) = 0$.

   Step 2 : Then we have to find the value of K in terms of s, i.e. $K = f(s)$.

   Step 3 : Find $\frac{dK}{ds} = 0$

   Step 4 : Valid roots of $\frac{dK}{ds} = 0$ will be the valid break points.

➢ Sample Questions

1988	IIT Kharagpur

5.1 Consider a closed-loop system shown in figure (a) below. The root locus for it is shown in figure (b). The closed loop transfer function for the system is

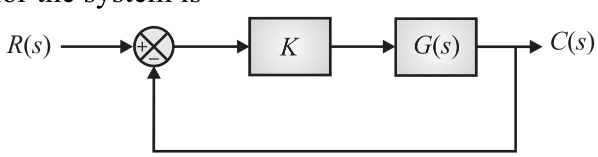


Fig. (a)

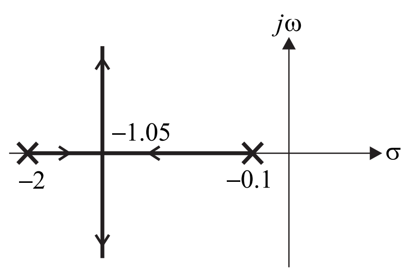


Fig. (b)

(A) $\dfrac{K}{1 + (0.5s + 1)(10s + 1)}$

(B) $\dfrac{K}{(s + 2)(s + 0.1)}$

(C) $\dfrac{K}{1 + K(0.5s + 1)(10s + 1)}$

(D) $\dfrac{K}{K + 0.2(0.5s + 1)(10s + 1)}$

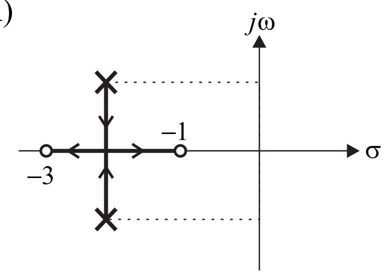
1989	IIT Kanpur

5.2 The OLTF of a feedback system is

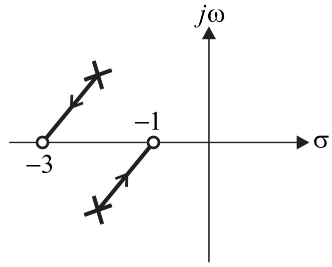
$$G(s)H(s) = \frac{K(s+1)(s+3)}{s^2 + 4s + 8}$$

The root locus for the same is

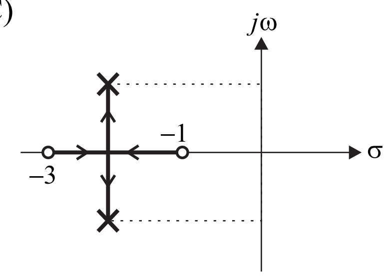
(A)



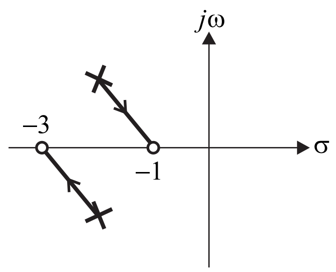
(B)



(C)



(D)



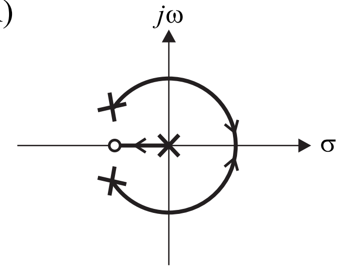
1990 IISc Bangalore

5.3 The transfer function of a closed loop system is

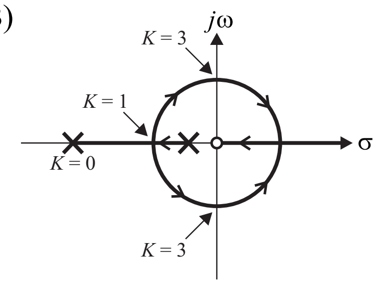
$$T(s) = \frac{K}{s^2 + (3-K)s + 1}$$

where, K is the path gain. The root locus plot of the system is

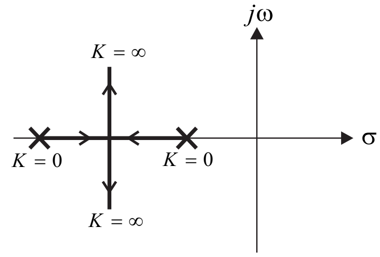
(A)



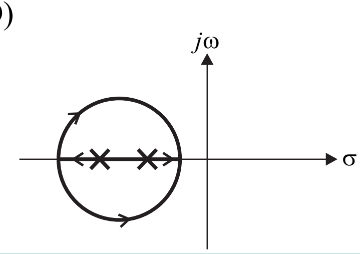
(B)



(C)



(D)



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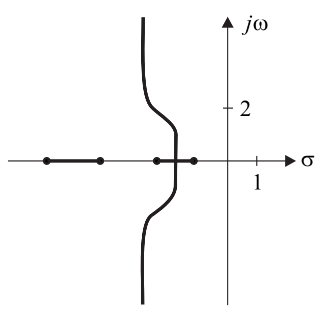
5.4 In the root locus plot shown in the figure, the pole/zero marks and the arrows have been removed. Which one of the following transfer functions has this root locus? **[Set - 03]**



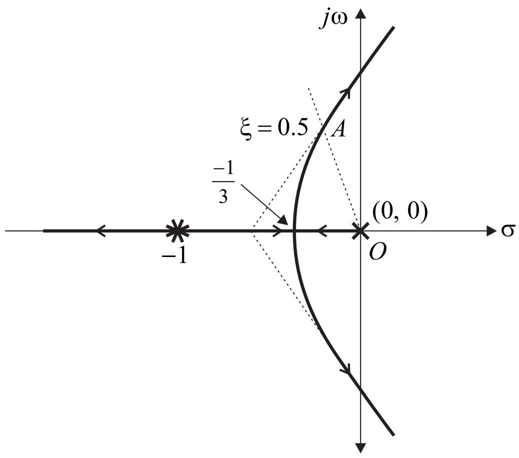
(A) $\dfrac{(s+1)}{(s+2)(s+4)(s+7)}$

(B) $\dfrac{(s+4)}{(s+1)(s+2)(s+7)}$

(C) $\dfrac{(s+7)}{(s+1)(s+2)(s+4)}$

(D) $\dfrac{(s+1)(s+2)}{(s+7)(s+4)}$

5.5 The characteristic equation of a unity negative feedback system is $1+KG(s)=0$. The open loop transfer function $G(s)$ has one pole at 0 and two poles at -1. The root locus of the system for varying K is shown in the figure. **[Set - 04]**



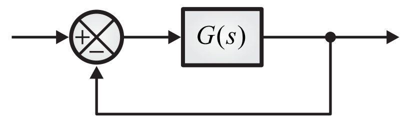
The constant damping ratio line, for $\xi = 0.5$, intersects the root locus at point A. The distance from the origin to point A is given as 0.5. The value of K at point A is________.

2017	**IIT Roorkee**

5.6 A linear time invariant (LTI) system with the transfer function

$$G(s)=\frac{K(s^2+2s+2)}{(s^2-3s+2)}$$

is connected in unity feedback configuration as shown in the figure.



For the closed loop system shown, the root locus for $0 < K < \infty$ intersects the imaginary axis for $K = 1.5$. The closed loop system is stable for **[Set - 01]**

(A) $K > 1.5$

(B) $1 < K < 1.5$

(C) $0 < K < 1$

(D) no positive value of K

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Explanations	**Root Locus**

5.1	**(D)**

Given :

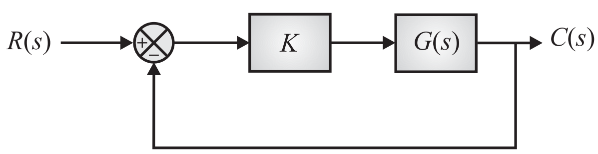


Fig. (a)

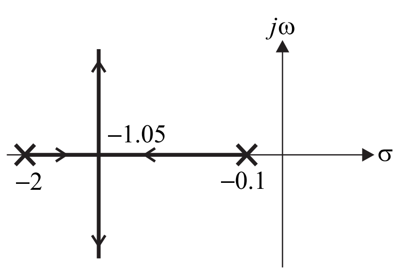


Fig. (b)

From root locus diagram, location of poles are $s=-0.1, -2$.

Hence, open loop transfer function is given by,

$$G'(s)=\frac{K}{(s+0.1)(s+2)}$$

where $G'(s)=KG(s)$ [From figure (a)]

Closed-loop transfer function with unity negative feedback is given by,

$$T(s)=\frac{G'(s)}{1+G'(s)}=\frac{\frac{K}{(s+0.1)(s+2)}}{1+\frac{K}{(s+0.1)(s+2)}}$$

$$T(s)=\frac{K}{(s+0.1)(s+2)+K}$$

$$T(s)=\frac{K}{K+0.1\times 2\left(\dfrac{s}{0.1}+1\right)\left(\dfrac{s}{2}+1\right)}$$

$$T(s)=\frac{K}{K+0.2(1+10s)(1+0.5s)}$$

Hence, the correct option is (D).

5.2 (A)

Method 1 : Procedure Based

Given : $G(s)H(s)=\dfrac{K(s+1)(s+3)}{s^2+4s+8}$

(i) Number of poles and zeros :

Number of zeros = 2

Number of poles = 2

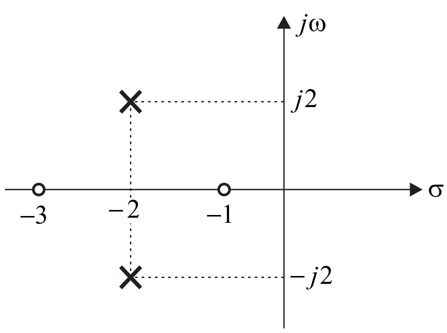
(ii) Location of poles and zeros :

Location of zeros,

$s=-1,-3$

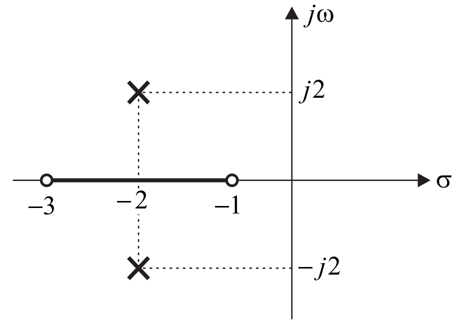
Location of poles,

$$s=\frac{-4\pm\sqrt{16-32}}{2}=-2\pm j2$$



(iii) Root locus branch on real axis :

Any section on real axis will be part of root locus branch, if the sum of the number of open loop poles and open loop zeros on real axis to the right of this section will be odd. Hence, root locus branch on real axis will lie between $-3<\sigma<-1$.



(iv) Number of branches (B) :

$B=P=Z=2$

(v) Number of asymptotes (A) :

$A=P-Z=2-2=0$

(vi) Break-away / Break-in point :

Characteristics equation is given by,

$1+G(s)H(s)=0$

$$1+\frac{K(s+1)(s+3)}{s^2+4s+8}=0$$

$$K=\frac{-(s^2+4s+8)}{(s+1)(s+3)}$$

$$\frac{dK}{ds}=0$$

$$\frac{d}{ds}\left[-\frac{s^2+4s+8}{s^2+4s+3}\right]=0$$

$$\frac{(2s+4)(s^2+4s+3)-(2s+4)(s^2+4s+8)}{(s^2+4s+3)^2}=0$$

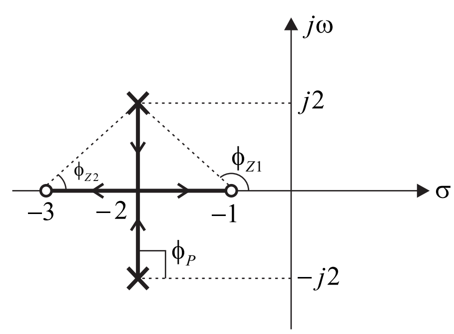
$2s+4=0$

$s=-2$

Since, point $s=-2$ is lying between two adjacent zeros on root locus branch of real axis, hence $s=-2$ is a **break-in point.**

(vii) Angle of departure (ϕ_d) :

$$\phi_d=180^0-(\Sigma\phi_P-\Sigma\phi_Z)$$



where, $\Sigma\phi_Z$ is sum of all the angles subtended by zeros to any particular pole.

$\Sigma\phi_P$ is sum of all the angles subtended by other poles to any particular pole.

$\phi_P = 90^0$

$$\phi_{Z_1} = 180^0 - \tan^{-1}\left(\frac{2}{1}\right) = 116.56^0$$

$$\phi_{Z_2} = \tan^{-1}(2) = 63.43^0$$

$$\Sigma\phi_Z = \phi_{Z_1} + \phi_{Z_2} = 116.56^0 + 63.43^0 = 180^0$$

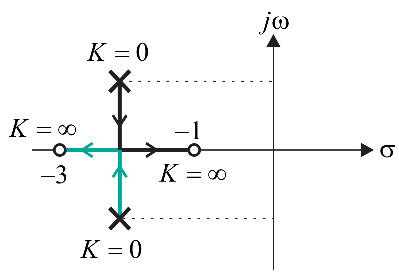
Hence,

$$\phi_d = 180^0 - \left[\phi_P - (\phi_{Z_1} + \phi_{Z_2})\right]$$

$$\phi_d = 180^0 - (90^0 - 180^0)$$

$$\phi_d = 270^0 \text{ or } -90^0$$

(viii) Root locus diagram :



Hence, the correct option is (A).

Method 2 : Concept Based

Option (B) and (D) are wrong because root locus is not symmetrical about real axis.

Option (C) is wrong because root locus direction is from zero to pole.

Hence, the correct option is (A).

Key Point

(i) Direction of root locus is always from open loop pole to open loop zero.

(ii) Root locus is always symmetrical about real axis.

Number of branches :

1. For $P > Z$ (Proper transfer function)
   $N = P$ (Number of branches = Number of poles)

2. For $Z > P$ (Improper transfer function)
   $N = Z$ (Number of branches = Number of zeros)

3. For $Z = P$ (Strictly proper transfer function)
   $N = P = Z$

5.3 (B)

Given : Closed loop transfer function is given below,

$$T(s) = \frac{K}{s^2 + (3-K)s + 1}$$

$$T(s) = \frac{K}{s^2 + 3s - Ks + 1} = \frac{\dfrac{K}{s^2+3s+1}}{1 - \dfrac{Ks}{s^2+3s+1}} \quad \ldots\text{(i)}$$

Transfer function for negative feedback system is given by,

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \quad \ldots\text{(ii)}$$

Comparing equation (i) and (ii),

$$G(s) = \frac{K}{s^2 + 3s + 1}, \qquad H(s) = -s$$

Hence, open loop transfer function is,

$$G(s)H(s) = \frac{-Ks}{s^2 + 3s + 1}$$

Due to negative sign, it is based on inverse root locus (IRL) or complementary root locus.

Method 1 : Procedure Based

(i) Number of poles and zeros :

Number of zero = 1

Number of poles = 2

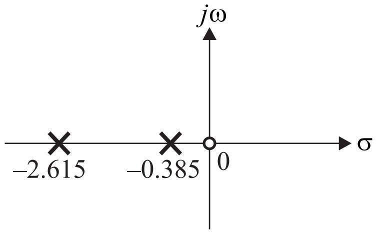
(ii) Location of poles and zeros :

Location of zero : $s = 0$

Location of poles :

$$s = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm 2.23}{2}$$

$$s = -2.615, -0.385$$



(iii) Root locus branch on real axis :

Any section on real axis will be part of root locus, if sum of number of open loop poles and open loop zeros to right of that section will be even **(complimentary root locus).** Hence, $\sigma > 0$ and $-2.615 < \sigma < -0.385$ will be part of root locus.

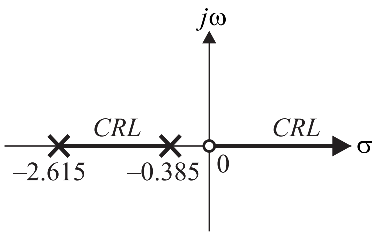


Fig. Pole-zero diagram

(iv) Number of branches (B)

$$B = P = 2 \qquad (P > Z)$$

(v) Number of asymptotes (A) :

$$A = P - Z = 2 - 1 = 1 \quad (P > Z)$$

(vi) Angle of asymptotes ($\angle A$) :

Angle of asymptotes is given by,

$$\angle A = \frac{2\alpha \times 180^0}{P - Z}$$

where, $\alpha = 0, 1, 2, ...(P - Z - 1)$

$$\alpha = 0$$

$$\angle A = 0^0$$

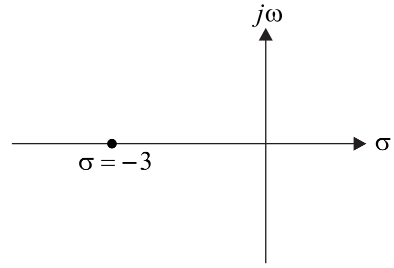
(vii) Centroid (σ) :

The intersection point of asymptotes on the real axis is called centroid.

Centroid is given by,

$$\sigma = \frac{\sum \text{Re}(\text{Poles}) - \sum \text{Re}(\text{Zeros})}{P - Z}$$

$$\sigma = \frac{-2.615 - 0.385}{2 - 1} = -3$$



(viii) Break-away / break-in point :

Characteristics equation is given by,

$$1 + G(s)H(s) = 0$$

$$1 - \frac{Ks}{s^2 + 3s + 1} = 0$$

$$K = \frac{s^2 + 3s + 1}{s}$$

$$\frac{dK}{ds} = 0$$

Hence, $\dfrac{(2s + 3)s - (s^2 + 3s + 1)}{s^2} = 0$

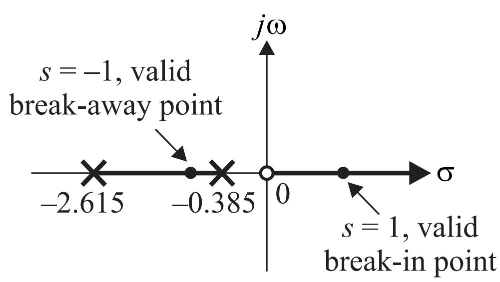
$$2s^2 + 3s - s^2 - 3s - 1 = 0$$

$$s^2 - 1 = 0$$

$$s = \pm 1$$

Since, point $s = +1$ is lying between one physical zero and one virtual zero on root locus branch of real axis, hence $s = +1$ will be **break-in point.**

Since, point $s = -1$ is lying between two adjacent poles on root locus branch of real axis, hence $s = -1$ will be **break-away point.**



(ix) Intersection with imaginary axis :

The value of K at point where root locus branch crosses the imaginary axis is determined by applying Routh Hurwitz criterion to the characteristic equation.

From characteristics equation,

$$1+G(s)H(s)=0$$

$$1-\frac{Ks}{s^2+3s+1}=0$$

$$s^2+(3-K)s+1=0$$

Routh Tabulation :

s^2	1	1
s^1	$3-K$	0 ← Row of zeros
s^0	1	0

The intersection of root locus plot with imaginary axis is given by the value of K obtained by solving the following equation.

$$3-K=0 \quad \Rightarrow \quad K=3$$

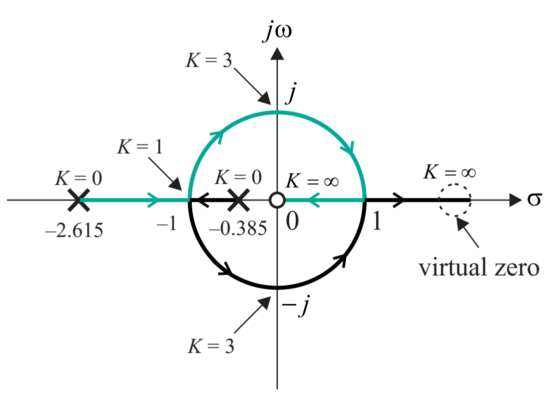
The auxiliary equation can be formed using the row above the ROZ.

$$A(s)=s^2+1=0$$

$$s=\pm j$$

Hence, root locus will intersect $j\omega$ axis at $\pm j$ and at these point value of K is 3.

(x) Root locus diagram :

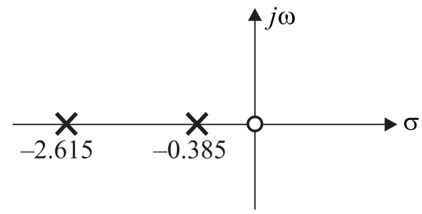


Hence, the correct option is (B).

Method 2 : Concept Based

$$G(s)H(s)=\frac{Ks}{s^2+3s+1}$$

Pole-zero location of OLTF is shown below,



Hence, from the given options, only option (B) will satisfy to pole-zero location shown above.

Hence, the correct option is (B).

Key Point

(i) In case of valid root locus branch, Break-away point exists between two adjacent poles and break-in point exists between two adjacent zeros.

(ii) The intersection point of asymptotes on real axis is called centroid.

(ii) Centroid is also referred as center of gravity.

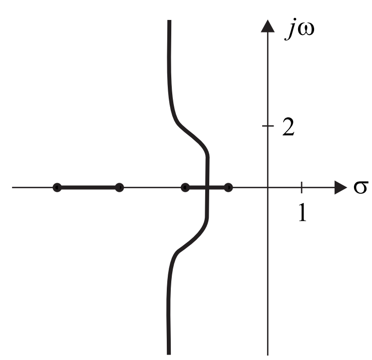
(iv) Centroid may be located anywhere on the real axis.

(v) The origin of asymptotic line is centroid.

(vi) This question can be solved either by taking positive feedback or negative feedback (value of $H(s)$ is negative).

5.4 (B)

The given root locus plot is shown below,



(i) For option (A), (B) and (C) :

Since $P > Z$ branches will start from each open loop poles and terminate at open loop zeros. The remaining $P - Z$ branches will approaches to infinity.

Angle of asymptotes is given by,

$$\angle A = \frac{(2\alpha + 1) \times 180^0}{P - Z}$$

where, $\alpha = 0, 1, 2, ... (P - Z - 1)$

$$\alpha = 0, 1$$

$$\angle A = 90^0, 270^0$$

Root locus branches will approaches to infinity with angle 90^0 and 270^0.

The intersection point of asymptotes on the real axis is called centroid.

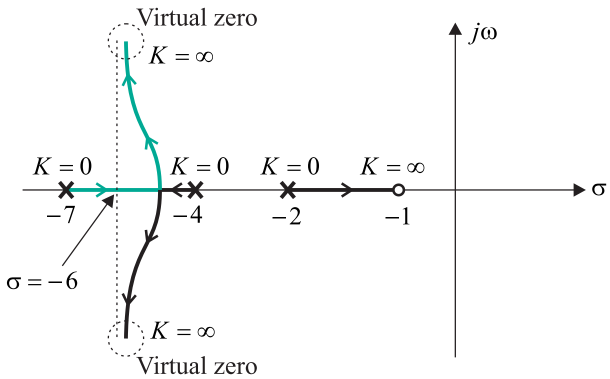
Centroid is given by,

$$\sigma = \frac{\sum \text{Re(Poles)} - \sum \text{Re(Zeros)}}{P - Z}$$

(ii) For option (A),

$$\sigma = \frac{-2 - 4 - 7 + 1}{3 - 1} = -6$$

The root locus can be drawn as shown below,

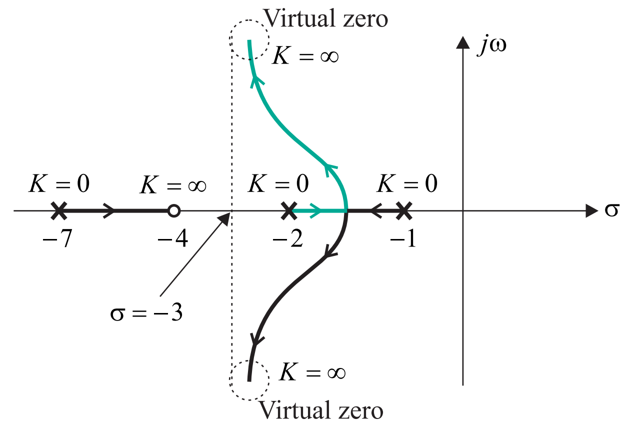


Thus, option (A) is incorrect.

(iii) For option (B),

$$\sigma = \frac{-1 - 2 - 7 + 4}{3 - 1} = -3$$

The root locus can be drawn as shown below,

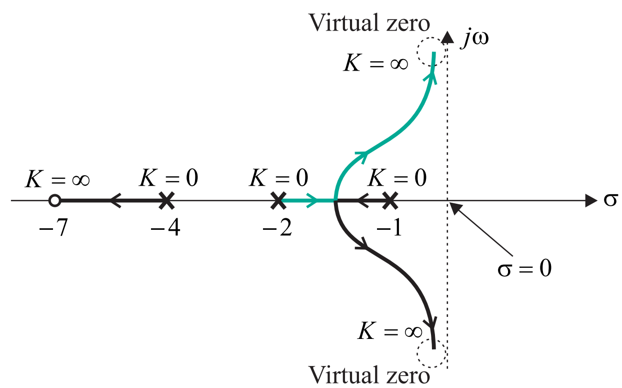


Thus, option (B) is correct.

(iv) For option (C),

$$\sigma = \frac{-1 - 2 - 4 + 7}{3 - 1} = 0$$

The root locus can be drawn as shown below,



Thus, option (C) is incorrect.

(v) For option (D),

Number of poles, $P = 2$

Number of zeros, $Z = 2$

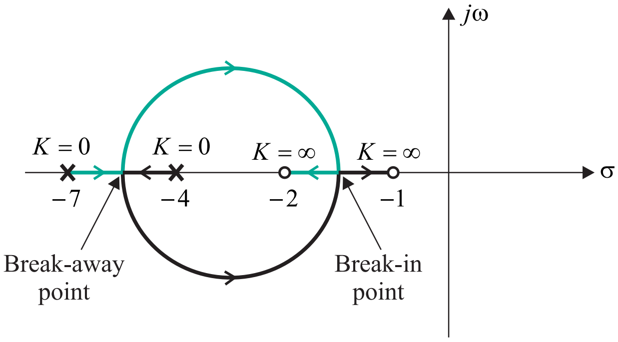
Number of branches, $B = 2$

Number of asymptotes, $A = 0$

Angle of asymptotes does not exist.

Centroid does not exist.

The root locus can be drawn as shown below,



Thus, option (D) is incorrect.

Hence, the correct option is (B).

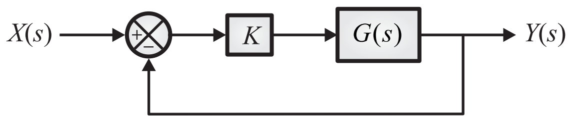
Scan for Video Solution

5.5 0.375

Given :

(i) Open loop transfer function $G(s)$ has one pole at 0 and two poles at -1.

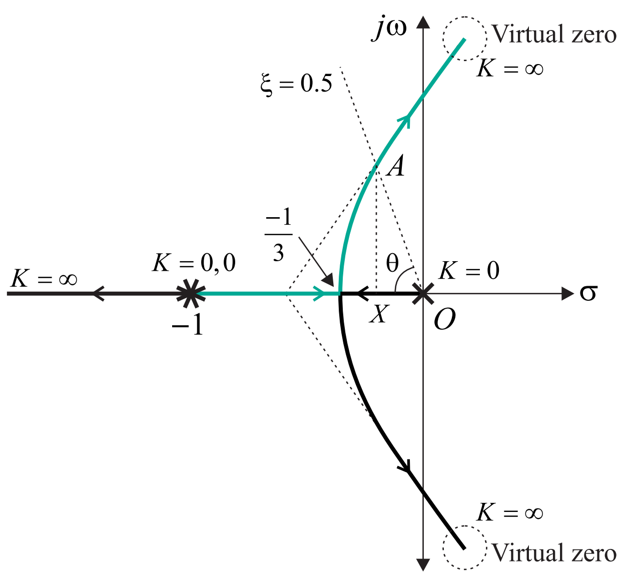
(ii) $\xi = 0.5,\ \ OA = 0.5$



$$G'(s) = KG(s) = \frac{K}{s(s+1)^2}$$

Method 1

The given root locus for varying K is shown below,



$$\theta = \cos^{-1}\xi \quad \Rightarrow \quad \theta = 60^0$$

In $\Delta AXO,\ \cos\theta = \dfrac{OX}{OA}$

$$OX = OA\cos 60^0 = 0.5 \times \frac{1}{2} = \frac{1}{4}$$

$$\sin\theta = \frac{AX}{AO}$$

$$AX = AO\sin 60^0 = 0.5 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

Hence, the coordinate of A is $\dfrac{-1}{4} + j\dfrac{\sqrt{3}}{4}$.

At $s_1 = \dfrac{-1}{4} + j\dfrac{\sqrt{3}}{4}$

$$G'(s_1) = \frac{K}{\left(\frac{-1}{4} + j\frac{\sqrt{3}}{4}\right)\left[\left(\frac{-1}{4} + j\frac{\sqrt{3}}{4}\right)+1\right]^2}$$

$$G'(s_1) = \frac{K}{\left(\frac{-1}{4} + j\frac{\sqrt{3}}{4}\right)\left(\frac{3}{4} + j\frac{\sqrt{3}}{4}\right)^2}$$

$$|G'(s_1)| = \frac{K}{\left(\frac{1}{16}+\frac{3}{16}\right)^{\frac{1}{2}}\left(\frac{9}{16}+\frac{3}{16}\right)^{2\times\frac{1}{2}}}$$

$$|G'(s_1)| = \frac{K}{\frac{1}{2}\times\frac{12}{16}} = \frac{16K}{6} \qquad \dots \text{(i)}$$

Since, point s_1 is lying on the root locus, hence from the magnitude condition,

$$|G'(s_1)| = 1 \qquad \dots \text{(ii)}$$

Note : There is no need to check angle condition.

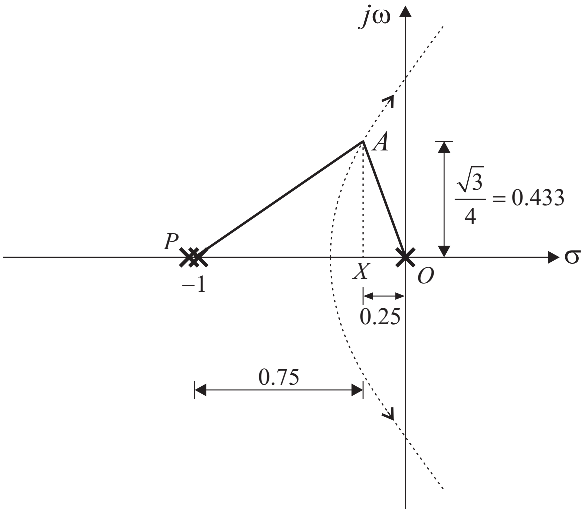
From equation (i) and (ii),

$$\frac{16K}{6} = 1 \quad \Rightarrow \quad K = \frac{3}{8} = 0.375$$

Hence, the value of K at point A is **0.375.**

Method 2

Graphical calculation of K :



From figure,

$$AP=\sqrt{(0.75)^2+(0.433)^2}=0.866$$

and $OA=0.5$

$$K=\frac{\text{Product of vector lengths drawn from the poles of } G(s)H(s) \text{ to } A}{\text{Product of vector lengths drawn from the zeros of } G(s)H(s) \text{ to } A}$$

Note : If open loop transfer function does not consist any zeros then product of vector lengths drawn from the zeros of $G(s)H(s)$ to point A will be unity.

$$K=\frac{0.866\times0.866\times0.5}{1}=0.375$$

Hence, the value of K at point A is **0.375.**

Method 3

Vector length :

$$OA=|OA|\angle OA=0.5\angle120^0$$

$$OA=0.5e^{j120}=0.5(\cos120+j\sin120)$$

$$OA=0.5\left[\frac{-1}{2}+j\frac{\sqrt{3}}{2}\right]=-\frac{1}{4}+j\frac{\sqrt{3}}{4}$$

Hence, the coordinate of A is $\frac{-1}{4}+j\frac{\sqrt{3}}{4}$.

From root locus diagram,

$$G(s)=\frac{K}{s(s+1)^2}$$

At $s_1=\frac{-1}{4}+j\frac{\sqrt{3}}{4}$

$$G'(s_1)=\frac{K}{\left(\frac{-1}{4}+j\frac{\sqrt{3}}{4}\right)\left[\left(\frac{-1}{4}+j\frac{\sqrt{3}}{4}\right)+1\right]^2}$$

$$G'(s_1)=\frac{K}{\left(\frac{-1}{4}+j\frac{\sqrt{3}}{4}\right)\left(\frac{3}{4}+j\frac{\sqrt{3}}{4}\right)^2}$$

$$|G'(s_1)|_{\frac{-1}{4}+j\frac{\sqrt{3}}{4}}=\frac{K}{\left(\frac{1}{16}+\frac{3}{16}\right)^{\frac{1}{2}}\left(\frac{9}{16}+\frac{3}{16}\right)^{2\times\frac{1}{2}}}$$

$$|G'(s_1)|_{\frac{-1}{4}+j\frac{\sqrt{3}}{4}}=\frac{K}{\frac{1}{2}\times\frac{12}{16}}=\frac{16K}{6} \qquad \ldots\text{(i)}$$

Point s_1 is lying at the root locus, hence from the magnitude condition,

$$|G(s_1)|_{\frac{-1}{4}+j\frac{\sqrt{3}}{4}}=1 \qquad \ldots\text{(ii)}$$

Note : There is no need to check angle condition.

Hence, from equation (i) and (ii),

$$\frac{16K}{6}=1$$

$$K=\frac{3}{8}=0.375$$

Hence, the value of K at point A is **0.375.**

Method 4

Given open loop transfer function is,

$$G(s)=\frac{K}{s(s+1)^2}$$

The closed loop transfer function of a unity negative feedback system is given by,

$$T(s)=\frac{G(s)}{1+G(s)}$$

$$T(s)=\frac{K}{s(s+1)^2+K}=\frac{K}{s^3+2s^2+s+K} \qquad \ldots\text{(i)}$$

The transfer function for standard third order system is given by,

$$T(s)=\frac{\omega_n^2}{(s^2+2\xi\omega_n s+\omega_n^2)(s+p)}$$

Where, $\xi =$ Damping ratio

$\omega_n =$ Natural angular frequency

$$T(s)=\frac{\omega_n^2}{s^3+ps^2+2\xi\omega_n s^2+2\xi\omega_n sp+\omega_n^2 s+\omega_n^2 p}$$

$$T(s)=\frac{\omega_n^2}{s^3+s^2(p+2\xi\omega_n)+s(2\xi\omega_n p+\omega_n^2)+\omega_n^2 p} \quad \ldots\text{(ii)}$$

On comparing equation (i) and (ii),

$$\omega_n^2 p = K \quad \ldots\text{(iii)}$$

$$2 = p+2\xi\omega_n$$

$$p+2\times\frac{1}{2}\omega_n = 2 \quad\Rightarrow\quad p+\omega_n = 2 \quad \ldots\text{(iv)}$$

$$1 = 2\xi\omega_n p+\omega_n^2$$

$$1 = \omega_n(\omega_n+p)$$

From equation (iv),

$$\omega_n(2)=1 \quad\Rightarrow\quad \omega_n=\frac{1}{2}$$

$$p = 2-\frac{1}{2}=\frac{3}{2}$$

From equation (iii),

$$K=\frac{1}{4}\times\frac{3}{2}=\frac{3}{8}=0.375$$

Hence, the value of K at point A is **0.375.**

Scan for Video Solution

5.6 (A)

Given : Open loop transfer function for a unity feedback system is given by,

$$G(s)=\frac{K(s^2+2s+2)}{(s^2-3s+2)}$$

The characteristics equation is given by,

$$1+G(s)=0$$

$$1+\frac{K(s^2+2s+2)}{s^2-3s+2}=0$$

$$s^2-3s+2+K(s^2+2s+2)=0$$

$$s^2(1+K)+s(-3+2K)+2+2K=0$$

Routh tabulation :

s^2	$1+K$	$2+2K$
s^1	$-3+2K$	0
s^0	$2+2K$	0

For the system to be stable, all the roots must be in the left-half of s-plane, thus all the coefficients in the first column of Routh's tabulation must have the same sign. Therefore, the coefficient of first column of the Routh's table should be positive.

This leads to the following conditions :

(i) $-3+2K>0 \quad\Rightarrow\quad K>1.5$

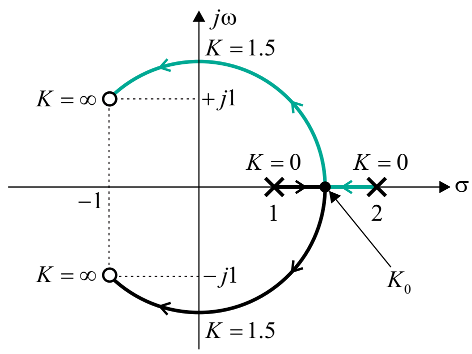
(ii) $2+2K>0$

$1+K>0 \quad\Rightarrow\quad K>-1$

On combining above two conditions,

Thus, for overall stability, $K>1.5$

Root locus diagram is shown below,



Hence, the correct option is (A).

Note : There is no need to calculate the value of K_0.

Range of K	Damping	Location of closed loop poles	Stability
$0<K_0<K$	Negative Over-damping	Real and unequal	Unstable
$K=K_0$	Negative Critical-damping	Real and repeated	Unstable
$K_0<K<1.5$	Negative Under-damping	Complex conjugate	Unstable
$K=1.5$	Undamped	Complex conjugate on imaginary axis	Marginal stable
$1.5<K<\infty$	Under-damping	Complex conjugate	Stable

❖❖❖❖

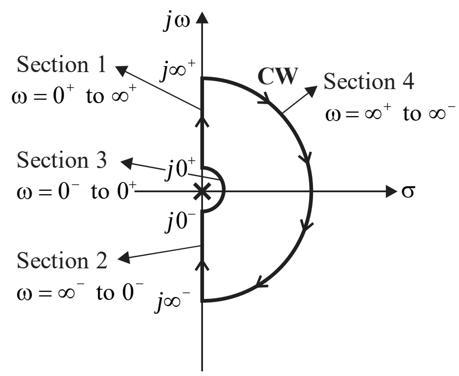
7 Nyquist Stability Criterion

➢ Partial Synopsis

Principle of argument:

This condition is useful for improper transfer function (specifically number of open loop poles = number of open loop zeros). It is related number of open loop poles and zeros in RHS of s-plane with number of encirclement

Case 1 : If we assume Nyquist contour in clockwise direction then according to principle of argument,



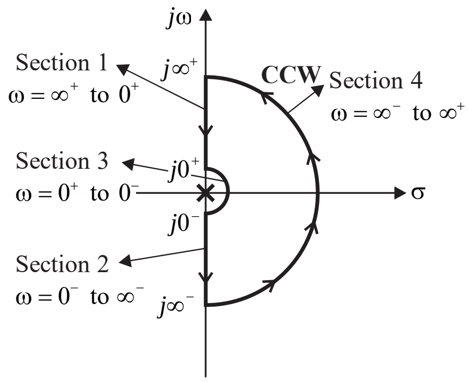
$$N = P - Z$$

where, N = Number of encirclements of origin in anti-clockwise direction

P = Number of right hand poles of open loop transfer function i.e., $G(s)H(s)$.

Z = Number of right hand zeros of open loop transfer function i.e., $G(s)H(s)$.

Case 2 : If we assume Nyquist contour in anticlockwise direction then according to principle of argument,



$$N = P - Z$$

where, N = Number of encirclements of origin in clockwise direction

P = Number of right hand poles of open loop transfer function i.e., $G(s)H(s)$.

Z = Number of right hand zeros of open loop transfer function i.e., $G(s)H(s)$.

Note : The above conditions is only valid for Improper transfer function (i.e. number of poles = number of zeros)

Nyquist Stability Criteria

Case 1 : If we assume Nyquist contour in clockwise direction then according to Nyquist stability criterion of a closed loop system

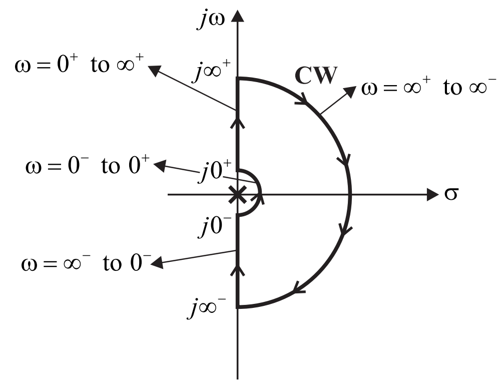
$$N = P - Z$$

where,

$N =$ Number of encirclements of critical point $(-1 + j\cdot 0)$ in anti-clockwise direction.

$P =$ Number of right hand poles of characteristics equation = Number of right half poles of open loop transfer function.

$Z =$ Number of right hand zeros of characteristics equation = Number of right half poles of close loop transfer function.



(characteristic equation $= 1 + G(s)H(s) = F(s) = \dfrac{N(s)}{D(s)}$)

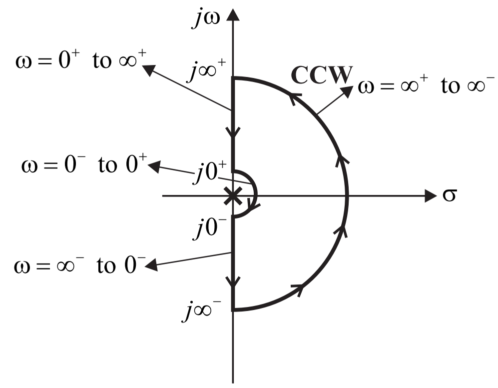
Case 2 : If we assume Nyquist contour in anticlockwise direction then according to Nyquist stability criterion of a closed loop system

$$N = P - Z$$

where,

$N =$ Number of encirclements of critical point $(-1 + j\cdot 0)$ in clockwise direction.

$P =$ Number of right hand poles of characteristics equation = Number of right half poles of open loop transfer function.



$Z =$ Number of right hand zeros of characteristics equation = Number of right half poles of close loop transfer function.

Note : The Nyquist stability criteria is valid for any type of transfer function whether it is proper or improper transfer function.

➢ Sample Questions

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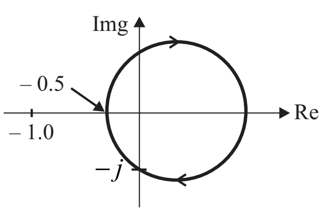
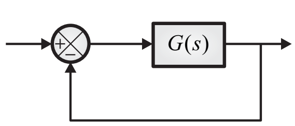
Common Data for Questions 7.1 & 7.2

The Nyquist plot of a stable transfer function $G(s)$ is shown in the figure. We are interested in the stability of the closed loop system in the feedback configuration shown below,





7.1 Which of the following statements is true ?

(A) $G(s)$ is an all-pass filter.

(B) $G(s)$ has a zero in the right-half plane.

(C) $G(s)$ is the impedance of a passive network.

(D) $G(s)$ is marginally stable.

7.2 The gain and phase margins of $G(s)$ for closed loop stability are

(A) 6 dB and 180^0 (B) 3 dB and 180^0

(C) 6 dB and 90^0 (D) 3 dB and 90^0

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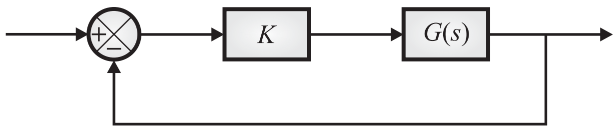
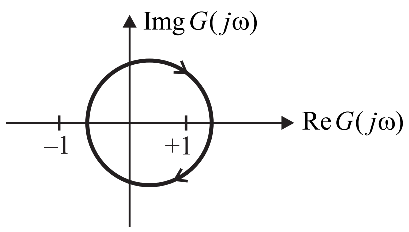
7.3 Consider the feedback system shown in the figure. The Nyquist plot of $G(s)$ is also shown. Which one of the following conclusions is correct? **[Set - 01]**





(A) $G(s)$ is an all-pass filter.

(B) $G(s)$ is a strictly proper transfer function.

(C) $G(s)$ is a stable and minimum-phase transfer function.

(D) The closed-loop system is unstable for sufficiently large and positive K.

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7.4 The Nyquist plot of the transfer function

$$G(s)=\frac{K}{(s^2+2s+2)(s+2)}$$

does not encircle the point $(-1+j0)$ for $K = 10$ but does encircle the point $(-1+j0)$ for $K = 100$. Then the closed loop system (having unity gain feedback) is **[Set - 01]**

(A) stable for $K=10$ and stable for $K=100$.

(B) stable for $K=10$ and unstable for $K=100$.

(C) Unstable for $K=10$ and stable for $K=100$.

(D) unstable for $K=10$ and unstable for $K=100$.

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Explanations Nyquist Stability Criterion

7.1 (B)

Given :

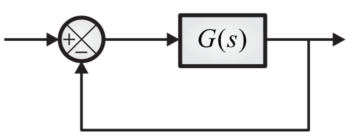


Fig. (a)

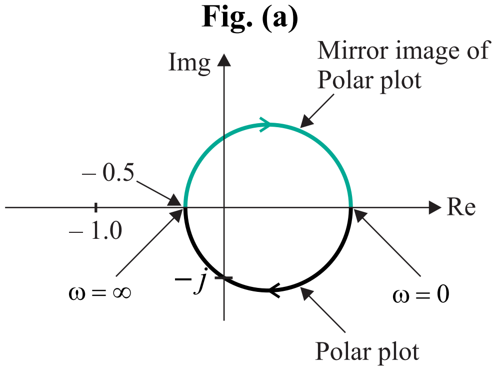


Fig. (b) Nyquist plot (first possibility)

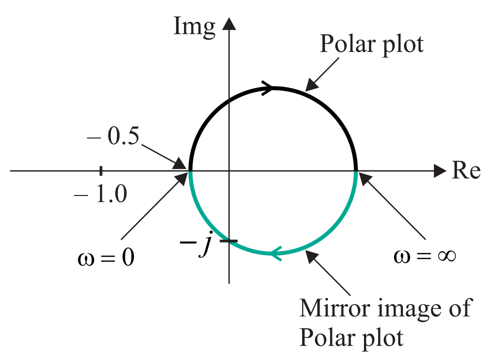


Fig. (c) Nyquist plot (second possibility)

(i) **Option (A) :** It is given that $G(s)$ is all pass filter, and we know that all pass filter has constant (fixed) magnitude for all frequency so its Nyquist plot should be a circle of constant radius with center at origin.

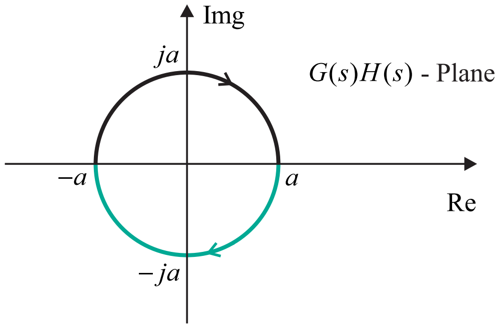


Fig. Nyquist plot of all pass filter

Thus, option (A) is incorrect.

(ii) **Option (B) :** It is given that $G(s)$ has a zero in the right half of s-plane (non-minimum phase system)

Assume $G(s) = K\left(\dfrac{s-z_1}{s+p_1}\right)$

where $z_1 > 0$ and $p_1 > 0$

Put $s = j\omega$ in above equation,

$$G(j\omega) = K\left(\frac{j\omega - z_1}{j\omega + p_1}\right)$$

Magnitude of open loop transfer function $G(j\omega)$ is given by,

$$|G(j\omega)| = K\left(\frac{\sqrt{\omega^2 + z_1^2}}{\sqrt{\omega^2 + p_1^2}}\right)$$

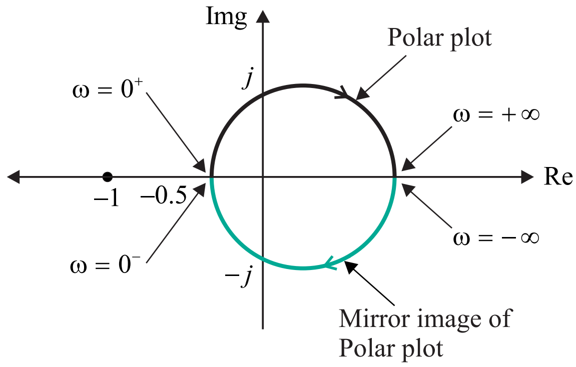
Phase angle of open loop transfer function $G(j\omega)$ is given by,

$$\phi(\omega) = \angle G(j\omega)$$

$$\phi(\omega) = 180^0 - \tan^{-1}\left(\frac{\omega}{z_1}\right) - \tan^{-1}\left(\frac{\omega}{p_1}\right)$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	$\dfrac{Kz_1}{p_1}$	180^0
∞	K	0^0

This will satisfy the Nyquist plot given in the question.



Comparing the table with given Nyquist plot,

$$\frac{Kz_1}{p_1} = 0.5 \qquad \ldots \text{(v)}$$

(iii) **Option (C) :** It is given that $G(s)$ is impedance of a passive network. Assuming a passive network having series combination of R and C, its impedance will be

$$G(s) = R + \frac{1}{Cs} \qquad \text{where } R > 0,\ C > 0$$

$$G(s) = \frac{RCs + 1}{Cs}$$

$$G(s) = \frac{RC\left[s + \dfrac{1}{RC}\right]}{Cs} = \frac{R\left[s + \dfrac{1}{RC}\right]}{s}$$

Put $s = j\omega$ in above equation,

$$G(j\omega) = \frac{R\left[j\omega + \dfrac{1}{RC}\right]}{j\omega}$$

Magnitude of open loop transfer function $G(j\omega)$ is given by,

$$|G(j\omega)| = R\frac{\sqrt{\omega^2 + \left(\dfrac{1}{RC}\right)^2}}{\omega}$$

Phase angle of open loop transfer function $G(j\omega)$ is given by,

$$\phi(\omega) = \angle G(j\omega)$$

$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{\frac{1}{RC}}\right) - \tan^{-1}\left(\frac{\omega}{0}\right)$$

$$\phi(\omega) = -90^0 + \tan^{-1}(\omega RC)$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	∞	-90^0
∞	R	0^0

Here at $\omega = 0$ magnitude is ∞ but for given Nyquist plot all the frequencies have finite magnitude thus option (C) is incorrect.

(iv) **Option (D) :** It is given that $G(s)$ is marginally stable but in the question it is mentioned that $G(s)$ is stable. Therefore, option (D) is incorrect.

Hence, the correct option is (B).

Key Point

Impedance of passive network represents minimum phase system i.e. left hand poles and zeros in s-plane. In minimum phase system, there is no possibility of negative sign of $G(j\omega)$ at $\omega = 0$ or $\omega = \infty$.

7.2 (C)

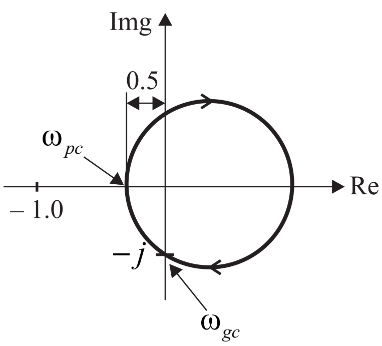


Fig. For stable system, $\omega_{pc} > \omega_{gc}$

From figure magnitude at phase crossover frequency is 0.5.

$$|G(j\omega_{pc})| = 0.5$$

Gain margin is given by,

$$\text{GM} = \frac{1}{|G(j\omega_{pc})|} = \frac{1}{0.5} = 2$$

In dB, $\text{GM} = 20\log \text{GM} = 20\log 2$

$$\textbf{GM = 6.02 dB}$$

From figure, phase at gain crossover frequency is -90^0.

$$\text{PM} = 180^0 + \angle G(j\omega_{gc}) = 180^0 - 90^0$$

$$\textbf{PM} = \mathbf{90^0}$$

Hence, the correct option is (C).

7.3 (D)

Given feedback system and Nyquist plot is shown below,

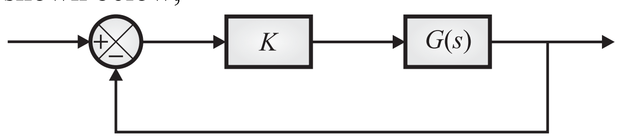


Fig. (a)

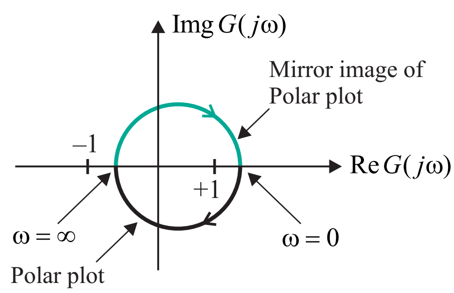


Fig. (b) Nyquist plot (first possibility)

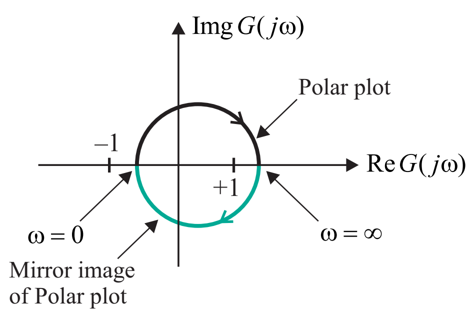


Fig. (c) Nyquist plot (second possibility)

Option (A) : It is given that $G(s)$ is all pass filter and we know that all pass filter has constant (fixed) magnitude for all frequency. So its Nyquist plot should be a circle of constant radius with center at origin. In given Nyquist plot the circle does not have constant radius with center at origin. Hence, option (A) is not correct.

Option (B) : In case of strictly proper transfer function (Number of poles are greater than number of zeros), at $\omega = \infty$, $|G(j\omega)| = 0$.

In the given Nyquist plot of $|G(j\omega)|$ is not zero at $\omega = \infty$, hence $G(s)$ is not a strictly proper transfer function.

Option (C) :

(i) In minimum phase transfer function, all poles and zeros lie in left half of s-plane i.e. minimum phase transfer function represents stable system.

Let, $G(s) = \dfrac{1+0.5s}{0.25+s}$

$G'(s) = KG(s)$

Assume, $K = 1$

$$G'(j\omega) = \frac{1+j0.5\omega}{0.25+j\omega}$$

Magnitude of open loop transfer function $G'(j\omega)$ is given by,

$$|G'(j\omega)| = \sqrt{\frac{1+(0.5\omega)^2}{(0.25)^2+\omega^2}}$$

Phase angle of open loop transfer function $G'(j\omega)$ is given by,

$$\angle G'(j\omega) = \tan^{-1}(0.5\omega) - \tan^{-1}\left(\frac{\omega}{0.25}\right)$$

ω	$\|G'(j\omega)\|$	$\angle G'(j\omega)$
0	4.0	0^0
∞	0.5	0^0

From above table, for minimum phase transfer function, $\angle G'(j\omega)$ at $\omega = 0$ and $\omega = \infty$ will be 0^0.

From given Nyquist plot,

Nyquist plot	$\angle G'(j\omega = 0)$	$\angle G'(j\omega = \infty)$
Fig. (b)	0^0	-180^0
Fig. (c)	-180^0	0^0

From above table, none of the given Nyquist plot has $\angle G'(j\omega) = 0^0$ at $\omega = 0$ and $\omega = \infty$, hence given $G'(s)$ is not a minimum phase transfer function.

(ii) In non-minimum phase and stable transfer function, all poles lie in left half of s-plane and atleast one zero lie in right half of s-plane.

Let, $G(s) = \dfrac{1-0.5s}{0.25+s}$

$G'(s) = KG(s)$

Assume, $K = 1$

$$G'(j\omega) = \frac{1-j0.5\omega}{0.25+j\omega}$$

Magnitude of open loop transfer function $G'(j\omega)$ is given by,

$$|G'(j\omega)| = \sqrt{\frac{1+(0.5\omega)^2}{(0.25)^2+\omega^2}}$$

Phase angle of open loop transfer function $G'(j\omega)$ is given by,

$$\angle G'(j\omega) = -\tan^{-1}(0.5\omega) - \tan^{-1}\left(\frac{\omega}{0.25}\right)$$

ω	$\|G'(j\omega)\|$	$\angle G'(j\omega)$
0	4.0	0^0
∞	0.5	-180^0

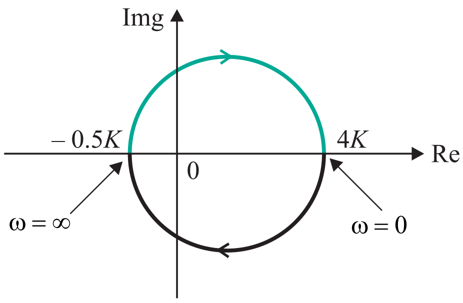
Since, assumed $G'(s)$ is matched with the given Nyquist plot i.e. given Nyquist plot represents non-minimum phase and stable transfer function.

Option (D) :

Let, $G'(s) = K\left(\frac{1-0.5s}{0.25+s}\right)$

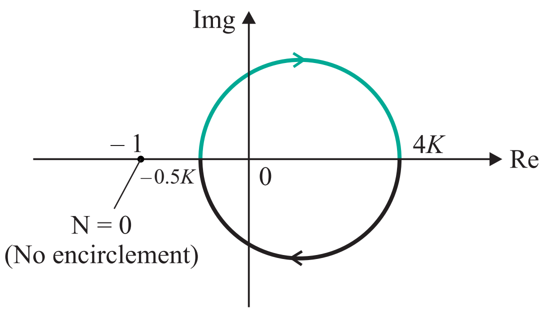
ω	$\lvert G'(j\omega)\rvert$	$\angle G'(j\omega)$
0	$4.0K$	0^0
∞	$0.5K$	-180^0

Hence, Nyquist plot for $G'(s)$ is shown below,



From assumed $G'(s)$, number of right hand pole of OLTF is $P = 0$

Case 1 : When $0.5K < 1$ i.e. $K < 2$

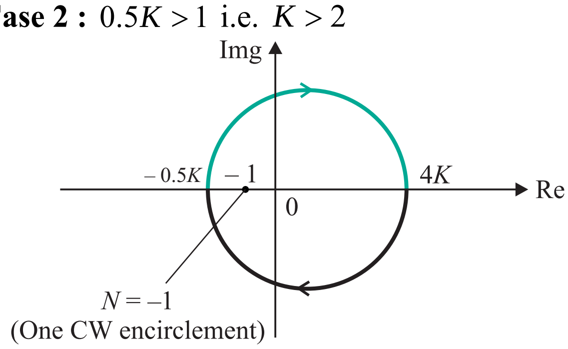


Nyquist stability criteria is given by,

$$N = P - Z$$
$$0 = 0 - Z$$
$$Z = 0$$

Thus, no pole of closed loop transfer function will lie in RHS and therefore system is stable.

Case 2 : $0.5K > 1$ i.e. $K > 2$



Nyquist stability criteria is given by,

$$N = P - Z$$
$$-1 = 0 - Z$$
$$Z = 1$$

Thus, one pole of closed loop transfer function will lie in RHS and therefore system is unstable. So, here system is conditional stable system i.e. system will be stable only for $K < 2$ and when sufficiently large value of K is provided then system becomes unstable.

Hence, the correct option is (D).

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Video Solution

7.4 **(B)**

Given : $G(s)H(s) = \frac{K}{(s^2+2s+2)(s+2)}$

(i) If K is 10, Nyquist plot do not enclose $-1 + j0$.

(ii) If K is 100, Nyquist plot enclose $-1 + j0$.

Method 1 : Nyquist Plot

1. For $K = 10$

$$G(s)H(s) = \frac{10}{(s^2+2s+2)(s+2)}$$

Put $s = j\omega$ in above equation,

$$G(j\omega)H(j\omega) = \frac{10}{(-\omega^2+2j\omega+2)(j\omega+2)}$$

Magnitude of $G(j\omega)$ is given by,

$$\lvert G(j\omega)H(j\omega)\rvert = \frac{10}{\sqrt{(2-\omega^2)^2+(2\omega)^2}\,\sqrt{\omega^2+4}}$$

Phase angle of $G(j\omega)$ is given by,

$$\angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{2\omega}{2-\omega^2} - \tan^{-1}\frac{\omega}{2}$$

ω	$\lvert G(j\omega)H(j\omega)\rvert$	$\angle G(j\omega)H(j\omega)$
0	2.5	0^0
∞	0	-270^0

To calculate ω_{pc}, equating imaginary part of $G(j\omega)$ to zero.

$$\text{Img}\left[-j\omega^3 + 6j\omega\right] = 0$$

$$-\omega_{pc}^3 + 6\omega_{pc} = 0$$

$$\omega_{pc}\left[6 - \omega_{pc}^2\right] = 0$$

$$\omega_{pc} = 0$$

and $$(6 - \omega_{pc}^2) = 0$$

$$\omega_{pc}^2 = 6$$

$$\omega_{pc} = \pm\sqrt{6} \text{ rad/sec}$$

Since, ω_{pc} is a positive real frequency.

Therefore, $\omega_{pc} = \sqrt{6}$ rad/sec

Magnitude at phase crossover frequency is given by,

$$|G(j\omega_{pc})| = \frac{10}{4 - 4\omega_{pc}^2}$$

$$|G(j\omega_{pc})| = \frac{10}{4 - 4(6)} = -0.5$$

2. For $K = 100$

$$G(s)H(s) = \frac{100}{(s^2 + 2s + 2)(s + 2)}$$

Put $s = j\omega$ in above equation,

$$G(j\omega)H(j\omega) = \frac{100}{(-\omega^2 + 2j\omega + 2)(j\omega + 2)}$$

Magnitude of $G(j\omega)$ is given by,

$$|G(j\omega)H(j\omega)| = \frac{100}{\sqrt{(2-\omega^2)^2 + (2\omega)^2}\,\sqrt{\omega^2 + 4}}$$

Phase angle of $G(j\omega)$ is given by,

$$\angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{2\omega}{2-\omega^2} - \tan^{-1}\frac{\omega}{2}$$

ω	$\lvert G(j\omega)H(j\omega)\rvert$	$\angle G(j\omega)H(j\omega)$
0	2.5	0^0
∞	0	-270^0

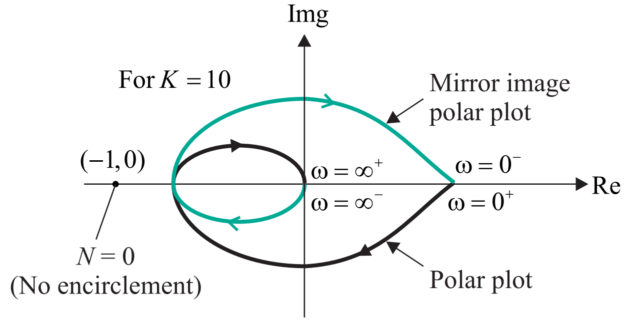


Fig. (i)

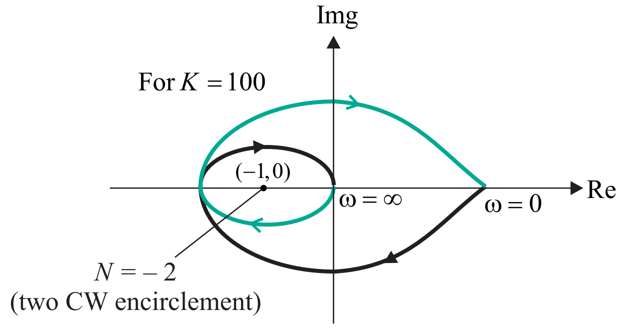


Fig. (ii)

Nyquist stability criterion is given by,

$$N = P - Z \qquad \ldots \text{(i)}$$

where,

N = Number of encirclement about critical point $(-1 + j0)$ in counter clockwise (anti clockwise) direction.

P = Number of right hand poles of open loop transfer function i.e. $G(s)H(s)$.

Z = Number of right hand poles of closed loop transfer function.

For the system to be stable : $Z = 0$

Characteristic equation of open loop transfer function is given by,

$$s^3 + 4s^2 + 6s + 4 = 0$$

Routh tabulation :

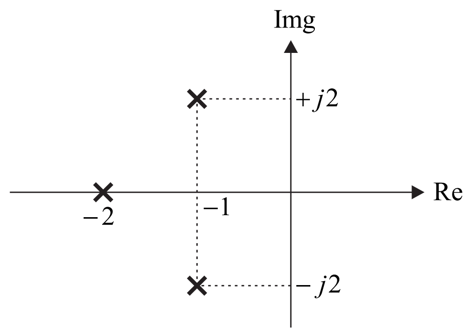
s^3	1	6
s^2	4	4
s^1	5	0
s^0	4	0

In the above Routh table, there is no sign change in first column. So number of right hand poles of OLTF, $P = 0$.

OR

$$G(s) = \frac{K}{(s^2 + 2s + 2)(s + 2)}$$

Location of poles, $s = -2, -1 \pm j2$



As shown in above figure, number of right hand poles of OLTF is zero, hence $P = 0$.

From figure (i), there is no encirclement about critical point $(-1 + j0)$.

Thus, $N = 0$

From equation (i),

$$0 = 0 - Z$$

$$Z = 0$$

Thus, no pole of closed loop transfer function will lie in RHS and therefore system is stable.

From figure (ii), number of encirclement about critical point $(-1 + j0)$ is two times in clockwise direction.

Thus, $N = -1 - 1 = -2$

From equation (i),

$$-2 = 0 - Z$$

$$Z = 2$$

Thus, two poles of closed loop transfer function will lie in RHS and therefore system is unstable.

Hence, the correct option is (B).

Method 2 : Routh Hurwitz Criteria

Characteristic equation is given by,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{(s^2 + 2s + 2)(s + 2)} = 0$$

$$s^3 + 4s^2 + 6s + 4 + K = 0$$

Routh tabulation :

s^3	1	6
s^2	4	$4 + K$
s^1	$\frac{20 - K}{4}$	0
s^0	$4 + K$	0

For system to be stable,

$$\frac{20 - K}{4} > 0 \quad \text{and} \quad 4 + K > 0$$

$$K < 20 \quad \text{and} \quad K > -4$$

Hence, system will be stable for $-4 < K < 20$ i.e. stable for $K = 10$ and unstable for $K > 20$ i.e. for $K = 100$.

Hence, the correct option is (B).

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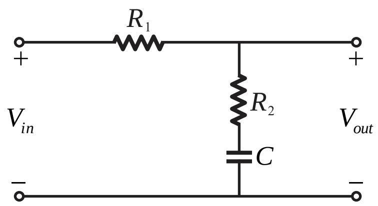
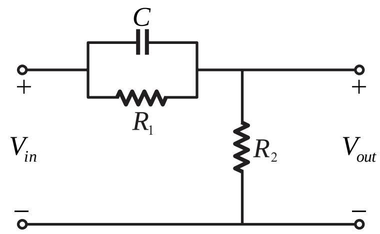
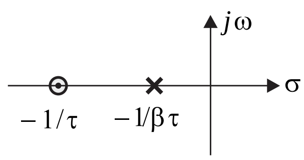
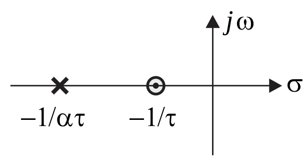
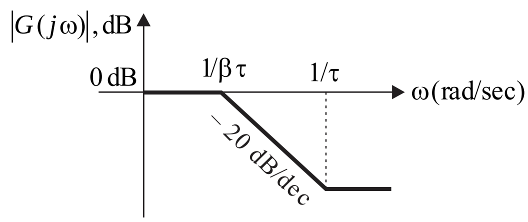
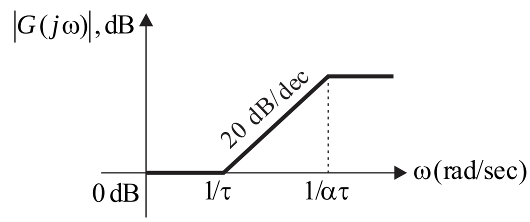
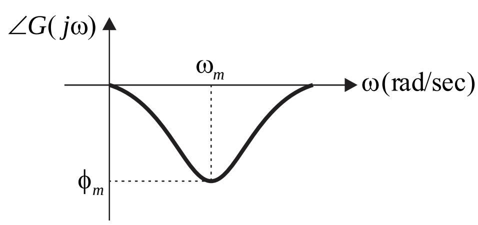
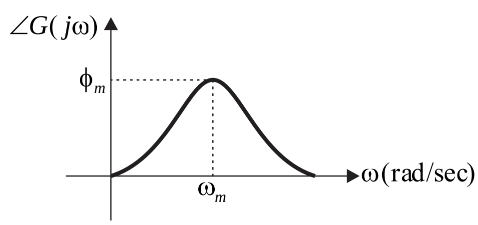
10 Controllers & Compensators

➢ Partial Synopsis

Proportional Controller	Derivative Controller	Integral Controller
$p(t) \propto e(t)$	$p(t) \propto \frac{d}{dt} e(t)$	$p(t) \propto \int_0^t e(t)\,dt$
$p(t) = K_P e(t)$	$p(t) = K_D \frac{d}{dt} e(t)$	$p(t) = K_I \int_0^t e(t)\,dt$
$P(s) = K_P E(s)$	$P(s) = K_D\, s.E(s)$	$P(s) = K_I . \frac{E(s)}{s}$
$\frac{P(s)}{E(s)} = K_P$	$\frac{P(s)}{E(s)} = sK_D$	$\frac{P(s)}{E(s)} = \frac{K_I}{s}$
$K_P \rightarrow$ Proportional controller gain. $(K_P > 1)$, $K_D \rightarrow$ Derivative Controller Gain $(K_D > 1)$, $K_I \rightarrow$ Integral Controller Gain$(K_I > 1)$.		
Proportional Derivative Controller (PD Controller)	**Proportional Integral Controller (PI Controller)**	**Proportional Integral Derivative Controller (PID controller)**
$p(t) \propto \frac{d}{dt} e(t)$ and $p(t) \propto e(t)$	$p(t) \propto \int_0^t e(t)\,dt$ and $p(t) \propto e(t)$	$p(t) \propto e(t)$ & $p(t) \propto \frac{d}{dt} e(t)$ and $p(t) \propto \int_0^t e(t)\,dt$
$p(t) = K_D \frac{d}{dt} e(t) + K_p e(t)$	$p(t) = K_I \int_0^t e(t)\,dt + K_p e(t)$	$p(t) = K_P e(t) + K_D \frac{d}{dt} e(t) + K_I \int_0^t e(t)\,dt$
$P(s) = (sK_D + K_P)E(s)$	$P(s) = \left(\frac{K_I}{s} + K_P\right) E(s)$	$P(s) = \left(K_P + sK_D + \frac{K_I}{s}\right) E(s)$
$\frac{P(s)}{E(s)} = sK_D + K_P$	$\frac{P(s)}{E(s)} = \frac{K_I + s.K_P}{s}$	$\frac{P(s)}{E(s)} = K_P + sK_D + \frac{K_I}{s}$

Comparison of Phase Lag and Lead Compensator

Characteristics	Phase Lag Compensator	Phase Lead Compensator
Electrical Network	 $\tau = R_2 C$ sec $\beta = \dfrac{R_1 + R_2}{R_2}, \beta > 1$	 $\tau = R_1 C$ sec $\alpha = \dfrac{R_2}{R_1 + R_2}, \alpha < 1$
Transfer function	$\dfrac{1+\tau s}{1+\beta\tau s}$	$\dfrac{\alpha(1+s\tau)}{(1+\alpha s\tau)}$
Pole - zero plot		
Filter	LPF	HPF
Magnitude plot		
Phase Plot		
ω_m	$\dfrac{1}{\tau\sqrt{\beta}}$ rad/sec	$\dfrac{1}{\tau\sqrt{\alpha}}$ rad/sec
ϕ_m	$\sin^{-1}\left(\dfrac{1-\beta}{1+\beta}\right)$	$\sin^{-1}\left(\dfrac{1-\alpha}{1+\alpha}\right)$
Response	It improves the steady state response.	It improves the transient response.
Steady State Error	It reduces the steady state error.	No significant reduction in steady state error.

➤ Sample Questions

1990 IISc Bangalore

9.1 The transfer function of a simple *RC* network function as a controller is

$$G_C(s) = \frac{s + Z_1}{s + P_1}$$

The condition for the *RC* network to act as a phase lead compensator is

(A) $P_1 < Z_1$ (B) $P_1 = 0$

(C) $P_1 = Z_1$ (D) $P_1 > Z_1$

1992 IIT Delhi

9.2 A process with open-loop model $G(s) = \frac{Ke^{-sT_d}}{s\tau + 1}$ is controlled by a PID controller. For this process

(A) the integral mode improves transient performance.

(B) the integral mode improves steady state performance.

(C) the derivative mode improves transient performance.

(D) the derivative mode improves steady state performance.

2005 IIT Bombay

9.3 A double integrator plant $G(s) = \frac{K}{s^2}$, $H(s) = 1$ is to be compensated to achieve the damping ratio $\xi = 0.5$ and an undamped natural frequency $\omega_n = 5$ rad/sec. Which one of the following compensator $G_C(s)$ will be used?

(A) $\frac{s+3}{s+9.9}$ (B) $\frac{s+9.9}{s+3}$

(C) $\frac{s-6}{s+8.33}$ (D) $\frac{s+6}{s}$

2007 IIT Kanpur

9.4 The open-loop transfer function of a plant is given as $G(s) = \frac{1}{(s^2 - 1)}$. If the plant is operated in a unity feedback configuration, then the lead compensator that can stabilize this control system is

(A) $\frac{10(s-1)}{(s+2)}$ (B) $\frac{10(s+4)}{(s+2)}$

(C) $\frac{10(s+2)}{(s+10)}$ (D) $\frac{2(s+2)}{(s+10)}$

2017 IIT Roorkee

9.5 Which of the following statements is **INCORRECT**? **[Set - 01]**

(A) Lead compensator is used to reduce the settling time.

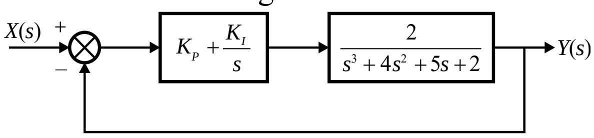
(B) Lag compensator is used to reduce the steady state error.

(C) Lead compensator may increase the order of a system.

(D) Lag compensator always stabilizes an unstable system.

2021 IIT Bombay

9.6 A unity feedback system that uses proportional-integral (PI) control is shown in the figure.



The stability of the overall system is controlled by tuning the PI control parameters K_P and K_I. The maximum value of K_I that can be chosen so as to keep the overall system stable or in the worst case, marginally stable (round off to three decimal places) is ________.

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Explanations **Controllers & Compensators**

Introduction of State Space Analysis

Scan for Video Explanation

9.1 **(D)**

Given : $G_C(s) = \dfrac{s+Z_1}{s+P_1}$

Put $s = j\omega$ in above equation,

$$G_C(j\omega) = \frac{j\omega + Z_1}{j\omega + P_1}$$

Phase angle of $G_C(j\omega)$ is given by,

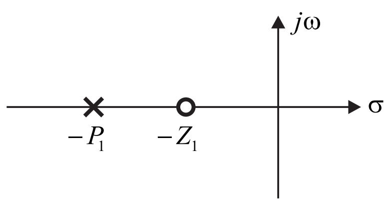
$$\angle G_C(j\omega) = \tan^{-1}\frac{\omega}{Z_1} - \tan^{-1}\frac{\omega}{P_1}$$

Phase angle $\angle G_C(j\omega)$ is positive for phase lead compensator. Hence

$$\tan^{-1}\frac{\omega}{Z_1} - \tan^{-1}\frac{\omega}{P_1} > 0$$

$$\frac{\omega}{Z_1} > \frac{\omega}{P_1}$$

$$Z_1 < P_1$$



Hence, the correct option is (D).

Key Point

(i) In lead compensator, zero is nearer to origin than pole in the left hand side of s-plane.

(ii) In lag compensator, pole is nearer to origin than zero in the left hand side of s-plane.

9.2 **(B) and (C)**

Given : $G(s) = \dfrac{Ke^{-sT_d}}{s\tau + 1}$

Effect of Integral mode :

(i) It introduces a pole in the system.
(ii) It increases type of system.
(iii) It increases gain of system.
(iv) **It decreases steady state error i.e. it improves steady state response.**
(v) It decreases bandwidth of system.
(vi) It decreases speed of response of system.
(vii) It increases settling time of system.
(viii) It increases peak overshoot of system.
(ix) It decreases damping factor ξ of system.

Effect of derivative mode :

(i) It introduces a zero in the system.
(ii) It decreases type of system.
(iii) It decreases gain of system.
(iv) It increases steady state error.
(v) It increases bandwidth of system.
(vi) It increases speed of response of system.
(vii) It decreases settling time of system.
(viii) It decreases peak overshoot of system.
(ix) **It improves transient performance of system.**
(x) It increases damping factor ξ of system.

Hence, the correct options are (B) and (C).

9.3 **(A)**

Given : For uncompensated system,

$$G(s) = \frac{K}{s^2}, H(s) = 1$$

For compensated system,

$$\xi = 0.5,\ \omega_n = 5 \text{ rad/sec}$$

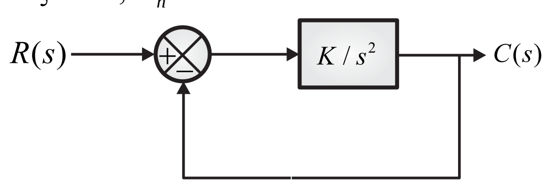


Fig. Uncompensated system

Transfer function of system is given by,

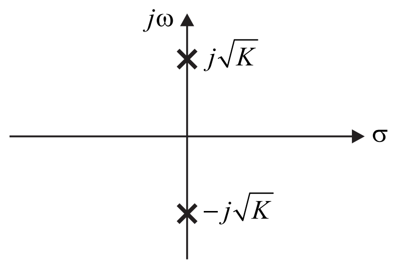
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K/s^2}{1+K/s^2}$$

$$\frac{C(s)}{R(s)} = \frac{K}{s^2+K}$$

The characteristic equation of uncompensated system is given by,

$$s^2 + K = 0 \qquad \ldots \text{(i)}$$

$$s = \pm j\sqrt{K}$$



The standard characteristic equation of second order system is given by,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \qquad \ldots\text{(ii)}$$

Comparing equation (i) and (ii),

$$\omega_n^2 = K \qquad \Rightarrow \qquad \omega_n = \sqrt{K}$$

and $\quad 2\xi\omega_n = 0$

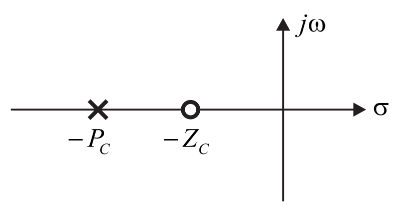
$$\xi = 0$$

Since $\xi = 0$, so it is an undamped system.

For compensated system, $\xi = 0.5$ which is underdamped system. The compensator should be selected such that it changes the system from undamped to underdamped by increasing damping ratio from 0 to 0.5.

The damping ratio of a system can be increased by using a lead compensator. The transfer function of a lead compensator is given by,

$$G_C(s) = \frac{s+Z_C}{s+P_C}, \qquad Z_C < P_C$$



Among the given option, the transfer function representing lead compensator with $Z_C < P_C$ is,

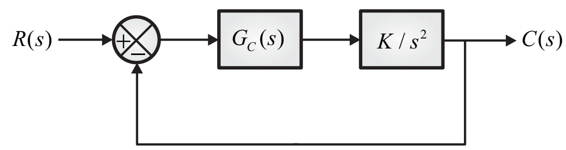
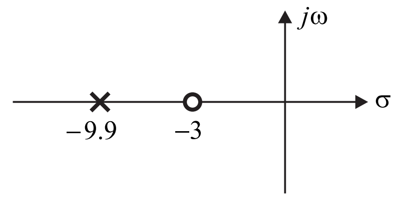


Fig. Compensated system

$$G_C(s) = \frac{s+3}{s+9.9}$$



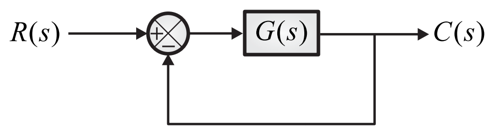
Overall transfer function is

$$G_T(s) = \frac{K(s+3)}{s^2(s+9.9)}$$

Hence, the correct option is (A).

9.4 (C)

Given : $G(s) = \dfrac{1}{s^2-1}$



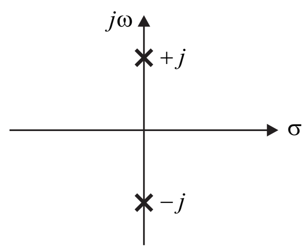
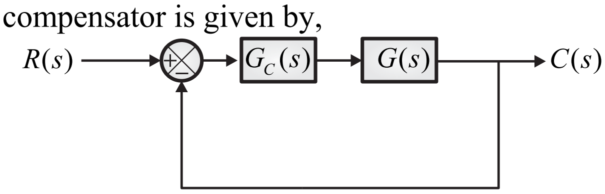


Fig. Marginal stable system

Block diagram of negative unity feedback with compensator is given by,



The characteristic equation of compensated system is,

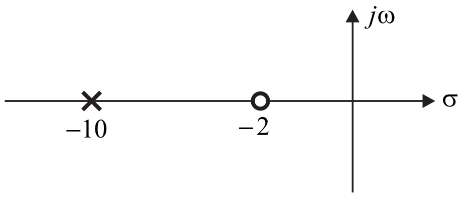
$$1 + G_C(s)G(s) = 0$$

$$1 + G_C(s)\frac{1}{s^2-1} = 0 \qquad \ldots \text{(i)}$$

(i) A lead compensator has zero nearer to origin than the pole in left half of s-plane, therefore transfer function of option (A) and (B) can not be lead compensator.

(ii) **For option (C) :** $G_C(s) = \dfrac{10(s+2)}{s+10}$

$G_C(s)$ represents lead compensator.



Putting above $G_C(s)$ in equation (i),

$$1 + \frac{10(s+2)}{(s+10)(s^2-1)} = 0$$

$$s^3 - s + 10s^2 - 10 + 10s + 20 = 0$$

$$s^3 + 10s^2 + 9s + 10 = 0$$

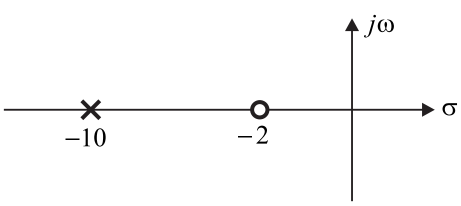
Routh Tabulation :

s^3	1	9
s^2	10	10
s^1	8	0
s^0	10	0

Since, all poles of compensated system lies in left half of s-plane, so given compensator with transfer function $G_C(s) = \dfrac{10(s+2)}{s+10}$ can stabilize the system.

(iii) **For option (D) :** $G_C(s) = \dfrac{2(s+2)}{s+10}$

$G_C(s)$ represents lead compensator.



Putting above $G_C(s)$ in equation (i),

$$1 + \frac{2(s+2)}{(s+10)(s^2-1)} = 0$$

$$s^3 - s + 10s^2 - 10 + 2s + 4 = 0$$

$$s^3 + 10s^2 + s - 6 = 0$$

Routh Tabulation :

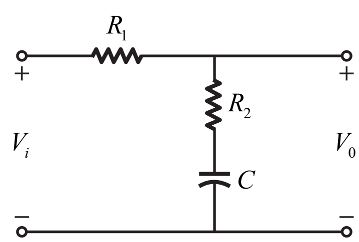
s^3	1	1
s^2	10	– 6
s^1	1.6	0
s^0	– 6	0

Since, there is one sign change in first column of Routh tabulation then the compensated system has one root in the right half of s-plane. Therefore, lead compensator with $G_C(s) = \dfrac{2(s+2)}{s+10}$ can not be used to stabilize the system.

Hence, the correct option is (C).

9.5 **(D)**

Lag compensator :

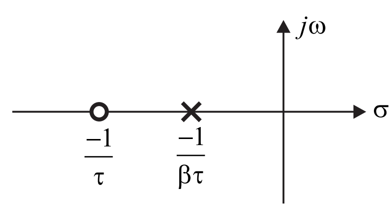


$$\frac{V_0(s)}{V_i(s)} = \frac{(1+s\tau)}{1+s\beta\tau}$$

where, $\beta > 1$, $\beta = \dfrac{R_1 + R_2}{R_2}$, $\tau = R_2 C$

β is lag compensator coefficient.

(i) Pole zero location of lag compensator is shown below,



(ii) The maximum phase lag ϕ_{max} occurs at mid frequency ω_m between upper and lower corner frequencies.

$$\omega_m = \frac{1}{\tau\sqrt{\beta}}$$

$$\phi_{max} = \sin^{-1}\left(\frac{1-\beta}{1+\beta}\right)$$

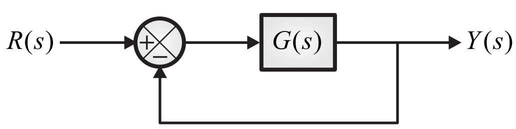
(iii) In lag compensator, pole is much nearer to origin than zero in left half of *s*-plane.

(iv) It acts as low pass filter.

(v) It may increase the order of system.

Example :

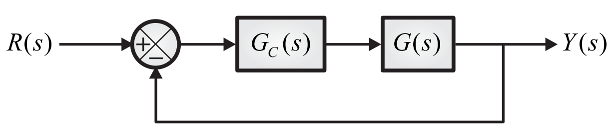
Block diagram of uncompensated system is shown below,



Let, $G(s) = \dfrac{1}{s(s+2)(s+4)}$

Type-1, order - 3

Block diagram of compensated system is shown below,



(a) If $G_C(s) = \dfrac{s+4}{s+1} \Rightarrow$ Lag compensator

$$G(s)G_C(s) = \frac{1}{s(s+1)(s+2)}$$

Type-1, order - 3

(b) If $G_C(s) = \dfrac{s+5}{s+3} \Rightarrow$ Lag compensator

$$G(s)G_C(s) = \frac{s+5}{s(s+4)(s+2)(s+3)}$$

Type-1, order - 4

Key Point

By the same way as explained above, we can perform analysis for lead compensator.

Lead compensator may increase order of system.

(vi) It increases the gain of system.

(vii) It decreases the steady state error of system i.e. it improves steady state response of system.

(viii) It decreases the bandwidth of system.

(ix) It decreases gain crossover ω_{gc} frequency.

(x) It decreases the speed of response of system.

(xi) It increases the settling time of system.

(xii) It increases the peak overshoot of system.

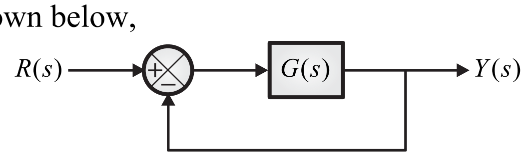
(xiii) It decreases damping factor ξ.

(xiv) It decreases natural frequency ω_n.

(xv) Lag compensator may stabilize an unstable system.

Example 1 :

Block diagram of uncompensated system is shown below,



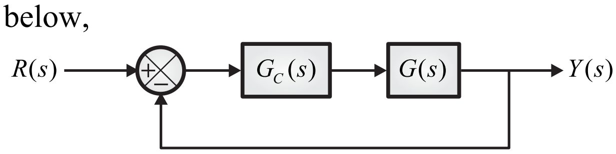
Let, $G(s) = \dfrac{1}{(s+2)(s-1)}$

OLTF - Unstable

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{1}{s^2+s-1}$$

CLTF - Unstable

Block diagram of compensated system is shown below,



If $G_C(s)=\dfrac{s+4}{s+3}\Rightarrow$ Lag compensator

$$T_C(s)=\frac{G(s)G_C(s)}{1+G(s)G_C(s)} \qquad \ldots \text{(i)}$$

$$T_C(s)=\frac{s+4}{(s+3)(s^2+s-1)+s+4}$$

$$T_C(s)=\frac{s+4}{s^3+4s^2+3s+1}$$

Characteristic equation is given by,

$$s^3+4s^2+3s+1$$

Routh Tabulation :

s^3	1	3
s^2	4	1
s^1	1/4	0
s^0	1	0

Since, there is no sign changes in the first column of Routh table.

Hence, compensated CLTF is stable

Example 2 :

Let, $G(s)=\dfrac{1}{(s-2)(s-1)}$

OLTF - Unstable

$$T(s)=\frac{G(s)}{1+G(s)}=\frac{1}{s^2-3s+3}$$

CLTF - Unstable

From equation (i),

$$T_C(s)=\frac{s+4}{(s+3)(s^2-3s+2)+s+4}$$

$$T_C(s)=\frac{s+4}{s^3-6s+10}$$

Characteristic equation is given by,

$$s^3-6s+10$$

Routh Tabulation :

s^3	1	-6
s^2	0	10

Since, the first element of the s^2 row is zero, the all elements in the s^1 row would be infinite.

To overcome this difficulty, we replace the zero in the s^2 row with a small positive variable ε and then proceed with the tabulation. Starting with the s^2 row, the results are as follows,

s^2	ε	10
s^1	$\dfrac{-6\varepsilon-10}{\varepsilon}$	0
s^0	10	0

From the Routh table,

$$\lim_{\varepsilon\to 0}\frac{-6\varepsilon-10}{\varepsilon}=\lim_{\varepsilon\to 0}\left(-6-\frac{10}{\varepsilon}\right)=-\infty$$

Since, there are two sign changes in the first column of Routh's tabulation, then the equation has two roots in the right-half of s-plane.

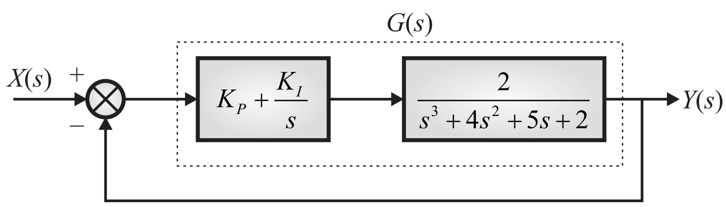
Hence, compensated CLTF is unstable.

Therefore, Lag compensator may or may not stabilizes an unstable system

Hence, the correct option is (D).

9.6 **3.125**

Given unity feedback system is shown below,



Characteristic equation of unity feedback system is,

$$1+G(s)=0$$

$$1+\left(K_P+\frac{K_I}{s}\right)\left(\frac{2}{s^3+4s^2+5s+2}\right)=0$$

$$1+\frac{K_P s+K_I}{s}\,\frac{2}{s^3+4s^2+5s+2}=0$$

$$s(s^3+4s^2+5s+2)+2sK_P+2K_I=0$$

$$s^4+4s^3+5s^2+2s+2sK_P+2K_I=0$$

$$s^4+4s^3+5s^2+(2+2K_p)s+2K_I=0$$

For keeping all coefficient, the necessary (but not sufficient) condition for the system to be stable is positive.

$$K_P > -1, \qquad K_I > 0$$

Routh tabulation,

s^4	1	5
s^3	4	$2+2K_P$
s^2	$\dfrac{18-2K_P}{4}$	$2K_I$
s^1	$\dfrac{\left(\dfrac{18-2K_P}{4}\right)(2+2K_P)-8K_I}{\dfrac{18-2K_P}{4}}$	
s^0	$2K_I$	

For the system to be stable, all the rats must be in the left-half of s-plane, thus all the coefficient in the first column must be positive in same sign or positive.

(i) $\dfrac{18-2K_P}{4} > 0$

$K_P < 9$

Thus, K_P must varies from,

$-1 < K_P < 9$

(ii) $$\frac{\left(\dfrac{18-2K_P}{4}\right)(2+2K_P)-8K_I}{\dfrac{18-2K_P}{4}} = 0$$

$$(18-2K_P)(2+2K_P)-4\times 8K_I = 0$$

$$4[(9-K_P)(1+K_P)]-4\times 8K_I = 0$$

$$(9-K_P)(1+K_P) = 8K_I$$

$$K_I = \frac{(9-K_P)(1+K_P)}{8}$$

Thus, K_I is a function of K_P.

From maxima or minima, $\dfrac{d}{dK_P}K_I = 0$

$$\frac{d}{dK_P}\left[\frac{(1+K_P)(9-K_P)}{8}\right] = 0$$

$$\frac{1}{8}[(1+K_P)(-1)+9-K_P] = 0$$

$$9-K_P = 1+K_P$$

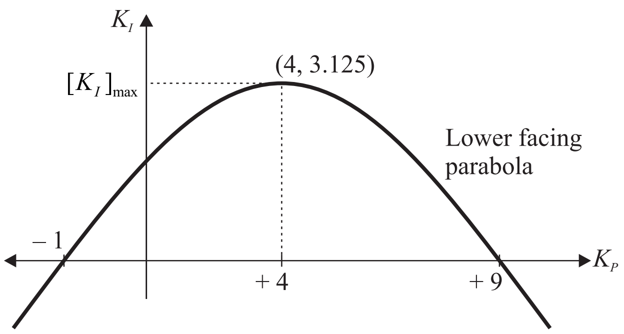
$$2K_P = 8$$

$$K_P = 4$$

Now, double derivative of K_I is,

$$\frac{d^2}{dK_P^2}K_I = -\frac{1}{4} < 0$$

It means, K_I has maxima at $K_P = 4$.



Thus, maximum value of K_I at $K_P = 4$ will be,

$$[K_I]_{\max} = \frac{(1+4)(9-4)}{8} = \frac{25}{8} = 3.125$$

Hence, the correct answer is 3.125.

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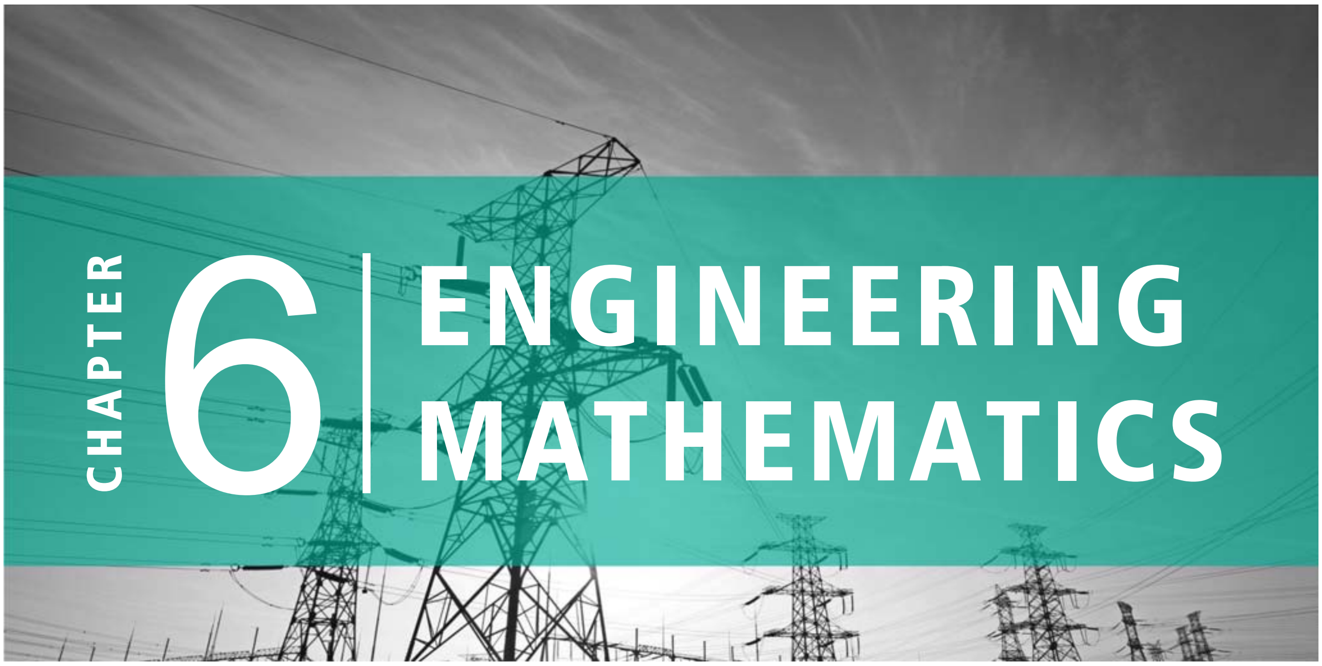

Marks Distribution of Engineering Mathematics in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	1	–	1
2004	–	–	–
2005	3	7	17
2006	6	8	22
2007	5	7	19
2008	6	5	16
2009	3	4	11
2010	3	6	15
2011	3	3	9
2012	3	5	13
2013	5	1	7
2014 Set-1	4	3	10
2014 Set-2	5	4	13
2014 Set-3	5	4	13

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-4	5	4	13
2015 Set-1	5	4	13
2015 Set-2	3	4	11
2015 Set-3	3	3	9
2016 Set-1	5	4	13
2016 Set-2	5	4	13
2016 Set-3	5	4	13
2017 Set-1	4	5	14
2017 Set-2	4	5	14
2018	6	4	14
2019	7	3	13
2020	4	3	10
2021	3	4	11

Syllabus : Engineering Mathematics

Linear Algebra: Vector space, basis, linear dependence and independence, matrix algebra, eigenvalues and eigenvectors, rank, solution of linear equations- existence and uniqueness. Calculus: Mean value theorems, theorems of integral calculus, evaluation of definite and improper integrals, partial derivatives, maxima and minima, multiple integrals, line, surface and volume integrals, Taylor series. Differential Equations: First order equations (linear and nonlinear), higher order linear differential equations, Cauchy's and Euler's equations, methods of solution using variation of parameters, complementary function and particular integral, partial differential equations, variable separable method, initial and boundary value problems. Vector Analysis: Vectors in plane and space, vector operations, gradient, divergence and curl, Gauss's, Green's and Stokes’ theorems. Complex Analysis: Analytic functions, Cauchy’s integral theorem, Cauchy’s integral formula, sequences, series, convergence tests, Taylor and Laurent series, residue theorem. Probability and Statistics: Mean, median, mode, standard deviation, combinatorial probability, probability distributions, binomial distribution, Poisson distribution, exponential distribution, normal distribution, joint and conditional probability.

Contents : Engineering Mathematics

1 Linear Algebra

Sample Questions

1994 IIT Kharagpur

1.1 Solve the following system of equations

$$x_1 + x_2 + x_3 = 3$$
$$x_1 - x_2 = 0$$
$$x_1 - x_2 + x_3 = 1$$

(A) Unique solution
(B) No solution
(C) Infinite number of solutions
(D) Only one solution

2006 IIT Kharagpur

1.2 The eigen values and the corresponding eigen vectors of a 2×2 matrix are given by

Eigen value	Eigen vector
$\lambda_1 = 8$	$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\lambda_2 = 4$	$v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The matrix is

(A) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$

2009 IIT Roorkee

1.3 The eigen values of the following matrix are

$$\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

(A) $3, 3+j5, 6-j$
(B) $-6+j5, 3+j, 3-j$
(C) $3+j, 3-j, 5+j$
(D) $3, -1+j3, -1-j3$

2014 IIT Kharagpur

1.4 The maximum value of the determinant among all 2×2 real symmetric matrices with trace 14 is ______. **[Set - 02]**

2018 IIT Guwahati

1.5 Consider matrix $A = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$ and vector $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. The number of distinct real values of k for which the equation $AX = 0$ has infinitely many solutions is ______.

2021 IIT Bombay

1.6 A real 2×2 non-singular matrix A with repeated Eigen value is given as

$$A=\begin{bmatrix} x & -3.0 \\ 3.0 & 4.0 \end{bmatrix}$$

where x is a real positive number. The value of x (round off to one decimal place) is ______.

1.7 If the vectors (1.0, –1.0, 2.0), (7.0, 3.0, x) and (2.0, 3.0, 1.0) in $\Re^3$ are linearly dependent, the value of x is ______.

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Explanations Linear Algebra

1.1 (A)

Given :

The system of equations are,

$$x_1 + x_2 + x_3 = 3$$

$$x_1 - x_2 = 0$$

$$x_1 - x_2 + x_3 = 1$$

Method 1

It is in the form of a non-homogeneous equation is given by,

$$AX = B$$

Where, $A=\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$, $B=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Number of unknowns $(n)=3$.

The augmented matrix is given by,

$$[A:B]=\begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 1 & -1 & 0 & : & 0 \\ 1 & -1 & 1 & : & 1 \end{bmatrix}$$

By elementary transformation

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$,

$$[A:B]=\begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & -2 & -1 & : & -3 \\ 0 & -2 & 0 & : & -2 \end{bmatrix}$$

$R_3 \to R_3 - R_2$,

$$[A:B]=\begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & -2 & -1 & : & -3 \\ 0 & 0 & 1 & : & 1 \end{bmatrix}$$

There exists three non-zero rows,

So, $\rho(A)=3,\ \rho(A:B)=3$

$$\rho(A)=\rho(A:B)=n=3$$

This shows that the given system of non-homogeneous equations has a unique solution.

Hence, the correct option is (A).

Method 2

Here we can see that the determinant of coefficient matrix is not equal to 0. The point is to be noted if $|A|\neq 0$ then we have unique solution.

Hence, the correct option is (A).

Key Point

For a non-homogenous equation $PX=Q$:

1. If rank $[P:Q]\neq$ rank$[P]$

   $\Rightarrow$ No solution exist.

2. If rank $[P:Q]=$ rank$[P]$ = Number of variables

   $\Rightarrow$ Unique solution exist.

3. If rank $[P:Q]=$ rank$[P]$ < Number of variables

   $\Rightarrow$ Infinite numbers of solutions exist.

1.2 (A)

Given :

For a 2×2 matrix A (Let),

Eigen values are $\lambda_1=8$ and $\lambda_2=4$

Eigen vectors are $v_1=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $v_2=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Method 1

By the property of eigen values,

$$\lambda_1 + \lambda_2 = 8 + 4 = 12 = \text{Trace of matrix A}$$

and $\lambda_1 \cdot \lambda_2 = 8 \times 4 = 32 = |A|$

By checking all options, only option (A) has sum of diagonal elements equal to 12.

Hence, the correct option is (A).

Method 2

Modal matrix for any square matrix is given by,

$$[M] = [v_1 \quad v_2] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

From the concept of diagonalization of a matrix, the matrix A is given by,

$$A = [M]\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}[M]^{-1}$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 8 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1}$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 8 & 0 \\ 0 & 4 \end{bmatrix}\frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A = \frac{-1}{2}\begin{bmatrix} -12 & -4 \\ -4 & -12 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$$

Hence, the correct option is (A).

📖 Key Point

Concept of diagonalization :

Any square matrix with all its Eigen values are distinct can be represented as,

$$A = [M]\,[D]\,[M^{-1}]$$

where, D is a diagonal matrix whose diagonal elements are Eigen values of A.

M is a non-singular matrix whose columns are respective Eigen vectors of A.

Note : M is also referred to as **Modal matrix**.

1.3　(D)

Given : $A = \begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$

Method 1

$$A = \begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

The characteristic equation is given by,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} -1-\lambda & 3 & 5 \\ -3 & -1-\lambda & 6 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$(\lambda - 3)(\lambda^2 + 2\lambda + 10) = 0$$

$$\lambda = 3$$

$$\lambda = \frac{-2 \pm \sqrt{4-40}}{2}$$

$$\lambda = \frac{-2 \pm j6}{2} = -1 \pm j3$$

Thus, the eigen value of the matrix are $+3, -1+j3, -1-j3$ respectively.

Hence, the correct option is (D).

Method 2

The characteristic equation for a 3×3 matrix is given by,

$$\lambda^3 - (\text{trace of } A)\lambda^2 + (M_{11} + M_{22} + M_{33})\lambda - |A| = 0$$

Here, $|A| = (-1)(-3-0) - 3(-9-0) + 5(0)$

$$|A| = 30$$

Trace of $A = (-1) + (-1) + 3 = 1$

Minor $M_{11} = (-1\times 3 - 0\times 6) = -3$

$$M_{22} = (-1\times 3 - 0\times 5) = -3$$

$$M_{33} = [(-1\times -1) - (3\times -3)] = 10$$

So, $\lambda^3 - \lambda^2 + 4\lambda - 30 = 0$

$$\lambda = 3, -1 \pm j3$$

Thus, the eigen value of the matrix are $+3, -1+j3, -1-j3$ respectively.

Hence, the correct option is (D).

Method 3

Here, the point is to be noted that the complex roots occur always in conjugate pair, the eigen values are the roots of characteristics equation, here by checking the options we can see that option (D) is correct because two roots are in the form of complex conjugate pair and one is real.

Hence, the correct option is (D).

Scan for Video Solution

1.4 **49**

Given : A 2×2 real symmetric matrix.

Trace of matrix = 14

Method 1

A symmetric matrix is a matrix whose transpose is equal to the matrix itself.

$$A^T = A$$

Let $$A=\begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

$$|A| = ac-b^2 \qquad \ldots\text{(i)}$$

According to question,

$$a+c=14$$

$$c=14-a$$

From equation (i),

$$|A| = a(14-a)-b^2$$

$$|A| = 14a-a^2-b^2 \qquad \ldots\text{(ii)}$$

This is a function in two variable *a* and *b*.

First order partial derivatives are,

$$p=\frac{\partial|A|}{\partial a}=14-2a$$

$$q=\frac{\partial|A|}{\partial b}=-2b$$

The strationary points are given by,

$$p=0 \Rightarrow a=7$$

$$q=0 \Rightarrow b=0$$

Second order partial derivatives are,

$$r=\frac{\partial^2|A|}{\partial a^2}=-2,\ s=\frac{\partial^2|A|}{\partial a\partial b}=0,$$

$$t=\frac{\partial^2|A|}{\partial b^2}=-2$$

In case r is a negative number, check the value of $rt-s^2$.

$$rt-s^2=4-0=4$$

Hence, $rt-s^2>0$ and $r<0$ (Maxima occurs)

Thus, maximum value of $|A|$ occurs at stationary point (7, 0).

From equation (ii),

$$|A|_{\max} = 14(7)-7^2-0^2$$

$$|A|_{\max} = 49$$

Hence, the maximum value of the determinant is **49.**

Method 2

Let $$A=\begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

$$|A| = ac-b^2 \qquad \ldots\text{(i)}$$

For maximum value of determinant *A*, *b* = 0

From equation (i),

$$|A| = ac$$

According to question,

$$a+c=14$$

By the trail the posible values of *a* and *c* for example (10, 4), (9, 5), (8, 6), (7, 7) (11, 3) etc. (7, 7) gives maximum determinat which is 49.

Hence, the maximum value of the determinant is **49.**

1.5 **2**

Given : $A=\begin{bmatrix} k & 2k \\ k^2-k & k^2 \end{bmatrix}$ $\quad X=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

It is in the form of homogenous equation given by,

$$AX = 0 \qquad \ldots(\text{i})$$

Homogenous linear equations are always consistent.

If $\rho(A) =$ Number of variables,

Then the system have unique solution when $|A| \neq 0$.

$\rho(A) <$ Number of variables then the system has infinite number of solution

Here the system must have infinitely many solutions.

Hence, $\rho(A) <$ Number of variables

Which is possible only if

$$|A| = 0$$

$$\begin{vmatrix} k & 2k \\ k^2 - k & k^2 \end{vmatrix} = 0$$

$$k^3 - 2k(k^2 - k) = 0$$

$$k^3 - 2k^3 + 2k^2 = 0$$

$$2k^2 - k^3 = 0$$

$$k^2(2 - k) = 0$$

$$k = 0, 0, 2$$

distinct

same

$$k = 0,\ 0,\ 2$$

Hence, number of distinct values of K for infinite number of solutions is **2**.

Scan for Video Solution

1.6 **10**

Given :

$$A = \begin{bmatrix} x & -3.0 \\ 3.0 & 4.0 \end{bmatrix}$$

It is given that eigen value of matrix A is repeated

Let the repeated eigen values are

$$\lambda_1 = \lambda$$

$$\lambda_2 = \lambda$$

We know that, sum of the eigen value of matrix $[A]$ is equal to trace of matrix

$\lambda_1 + \lambda_2$ = Trace (A) = Sum of diagonal elements of matrix $[A]$

$$\lambda + \lambda = x + 4$$

$$2\lambda = x + 4$$

$$\lambda = \frac{x+4}{2}$$

We know that, product of eigen value is equal to determinant of matrix $[A]$

$$\lambda_1 \times \lambda_2 = |A|$$

$$\lambda \times \lambda = 4x + 9$$

$$\lambda^2 = 4x + 9$$

Putting $\lambda = \dfrac{x+4}{2}$ into equation (ii)

$$\left(\frac{x+4}{2}\right)^2 = 4x + 9$$

$$x^2 + 8x + 16 = 16x + 36$$

$$x^2 - 8x - 20 = 0$$

$$x^2 - 10x + 2x - 20 = 0$$

$$x(x - 10) + 2(x - 10) = 0$$

$$(x - 10)(x + 2) = 0$$

$$x = 10,\ x = -2$$

It is given that x is positive real number, therefore we will select $x = 10$.

Hence, the correct answer is 10.

1.7 **8**

Given :

According to Question given vectors are (1.0, –1.0, 2.0), (7.0, 3.0, x) and (2.0, 3.0, 1.0) are linearly dependent.

We know that the determinant of linearly dependent vectors are zero.

$$\begin{vmatrix} 1 & -1 & 2 \\ 7 & 3 & x \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$1(3-3x)+1(7-2x)+2(21-6)=0$$

$$3-3x+7-2x+30=0$$

$$5x=40$$

$$x=8$$

Hence, the correct answer is 8.

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2 Differential Equations

Sample Questions

1994 **IIT Kharagpur**

2.1 Match each of the items A, B, C with an appropriate item from 1, 2, 3, 4 and 5.

A. $a_1\dfrac{d^2y}{dx^2}+a_2y\dfrac{dy}{dx}+a_3y=a_4$

B. $a_1\dfrac{d^3y}{dx^3}+a_2y=a_3$

C. $a_1\dfrac{d^2y}{dx^2}+a_2y\dfrac{dy}{dx}+a_3x^2y=0$

(1) Non–linear differential equation

(2) Linear differential equation with constant coefficient

(3) Linear homogeneous differential equation

(4) Non-linear homogeneous differential equation

(5) Non-linear first order differential equation

(A) A – 1, B – 2, C – 4

(B) A – 3, B – 4, C – 2

(C) A – 2, B – 5, C – 3

(D) A – 3, B – 1, C – 2

2007 **IIT Kanpur**

2.2 The solution of differential equation $k^2\dfrac{d^2y}{dx^2}=y-y_2$ under the boundary conditions

(i) $y=y_1$ at $x=0$

(ii) $y=y_2$ at $x=\infty$

where k, y_1 and y_2 are constants, is

(A) $y=(y_1-y_2)\exp\left(\dfrac{-x}{k^2}\right)+y_2$

(B) $y=(y_2-y_1)\exp\left(\dfrac{-x}{k}\right)+y_1$

(C) $y=(y_1-y_2)\sin h\left(\dfrac{x}{k}\right)+y_1$

(D) $y=(y_1-y_2)\exp\left(\dfrac{-x}{k}\right)+y_2$

2008 **IISc Bangalore**

2.3 Which of the following is a solution to the differential equation, $\dfrac{d}{dt}x(t)+3x(t)=0,\ \ x(0)=2$?

(A) $x(t)=3e^{-t}$ (B) $x(t)=2e^{-3t}$

(C) $x(t)=-\dfrac{3}{2}t^2$ (D) $x(t)=3t^2$

2009 IIT Roorkee

2.4 Match each differential equation in Group I to its family of solution curves from Group II.

Group I		Group II	
P.	$\frac{dy}{dx}=\frac{y}{x}$	**1.**	Circle
Q.	$\frac{dy}{dx}=-\frac{y}{x}$	**2.**	Straight lines
R.	$\frac{dy}{dx}=\frac{x}{y}$	**3.**	Hyperbolas
S.	$\frac{dy}{dx}=-\frac{x}{y}$		

(A) P – 2, Q – 3, R – 3, S – 1
(B) P – 1, Q – 3, R – 2, S – 1
(C) P – 2, Q – 1, R – 3, S – 3
(D) P – 3, Q – 2, R – 1, S – 2

2014 IIT Kharagpur

2.5 Which ONE of the following is a linear non-homogenous differential equation, where x and y are the independent and dependent variables respectively?

[Set - 03]

(A) $\frac{dy}{dx}+xy=e^{-x}$ (B) $\frac{dy}{dx}+xy=0$

(C) $\frac{dy}{dx}+xy=e^{-y}$ (D) $\frac{dy}{dx}+e^{-y}=0$

2015 IIT Kanpur

2.6 The solution of the differential equation

$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0$ with

$y(0)=y'(0)=1$ is

[Set - 01]

(A) $(2-t)e^{t}$ (B) $(1+2t)e^{-t}$

(C) $(2+t)e^{-t}$ (D) $(1-2t)e^{t}$

2018 IIT Guwahati

2.7 A curve passes through the point $(x=1, y=0)$ and satisfies the differential equation $\frac{dy}{dx}=\frac{x^2+y^2}{2y}+\frac{y}{x}$. The equation that describes the curve is

(A) $\ln\left(1+\frac{y^2}{x^2}\right)=x-1$

(B) $\frac{1}{2}\ln\left(1+\frac{y^2}{x^2}\right)=x-1$

(C) $\ln\left(1+\frac{y}{x}\right)=x-1$

(D) $\frac{1}{2}\ln\left(1+\frac{y}{x}\right)=x-1$

2021 IIT Bombay

2.8 Consider the differential equation given below.

$$\frac{dy}{dx}+\frac{x}{1-x^2}y=x\sqrt{y}$$

The integrating factor of differential equation is

(A) $(1-x^2)^{-\frac{1}{2}}$ (B) $(1-x^2)^{-\frac{3}{4}}$

(C) $(1-x^2)^{-\frac{1}{4}}$ (D) $(1-x^2)^{-\frac{3}{2}}$

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Explanations Differential Equations

2.1 (A)

The generalized form of linear differential equation is,

$$F(y)=\frac{d^n y}{dx}+A_1(x)\frac{d^{n-1}y}{dx^{n-1}}+A_2(x)\frac{d^{n-2}y}{dx^{n-2}}+\ldots A_n(x)=0$$

Option (A),

$$a_1\frac{d^2y}{dx^2}+a_2y\frac{dy}{dx}+a_3y=a_4$$

The dependent variable y is in the product with its derivative itself, it satisfies the criteria of nonlinearity. Thus, it is a non-linear equation.

Option (B),

$$a_1\frac{d^3y}{dx^3}+a_2y=a_3$$

It is in the generalized form of linear differential equation with constant coefficient.

Option (C),

$$a_1\frac{d^2y}{dx^2}+a_2y\frac{dy}{dx}+a_3x^2y=0$$

It does not satisfies the criteria of linearity and all the constant term are zero. Thus, it is non-linear homogeneous differential equation.

Hence, the correct option is (A).

Key Point

Criteria for linearity :

(i) The dependent variable y and its derivatives are of first degree. (For system to be linear, the degree of derivative should be 1).

(ii) Each coefficient depends only on the independent variable.

Criteria for non-linearity :

(i) If the coefficient of derivative of dependent variable is the dependent variable itself, then the differential equation will be non-linear.

(ii) If the power of derivative of dependent variable is more than one, then the differential equation will be non-linear.

2.2 (D)

Given : $k^2\frac{d^2y}{dx^2}=y-y_2$

$$(k^2D^2-1)y=-y_2$$

This is in form of a non-homogeneous linear differential equation.

$$[f(D)]y=\phi$$

The auxiliary equation is given by,

$$f(m)=0$$

$$k^2m^2-1=0$$

$$m_1=\frac{1}{k},\ m_2=-\frac{1}{k}$$

The roots are real and distinct. So, the complementary function is given by,

$$\text{C.F.}=C_1e^{m_1x}+C_2e^{m_2x}$$

$$\text{C.F.}=C_1e^{\frac{x}{k}}+C_2e^{\frac{-x}{k}}$$

The particular integral is given by,

$$\text{P.I.}=\frac{1}{f(D)}\times(-y_2)$$

$$\text{P.I.}=\frac{1}{(k^2D^2-1)}(-y_2)=\frac{1}{(1-k^2D^2)}(y_2)$$

$$\text{P.I.}=\left(1-k^2D^2\right)^{-1}y_2$$

Using Binomial expansion,

$$\text{P.I.}=\left(1+k^2D^2+.....\infty\right)y_2$$

Here, y_2 is a constant.

$$\text{P.I.}=y_2+0+0.....=y_2$$

The complete solution is given by,

$$y=\text{C.F.}+\text{P.I.}$$

$$y=C_1e^{\frac{x}{k}}+C_2e^{\frac{-x}{k}}+y_2$$

Using boundary conditions :

(i) $y = y_1$ at $x = 0$

$$y_1 = C_1 + C_2 + y_2$$

(ii) $y = y_2$ at $x = \infty$

$$y_2 = C_1 e^{\infty} + C_2 e^{-\infty} + y_2$$

$$C_1 e^{\infty} = 0$$

But for this to be true, C_1 has to be 0

So, $C_2 = (y_1 - y_2)$

Thus, $y = (y_1 - y_2) e^{\frac{-x}{k}} + y_2$

Hence, the correct option is (D).

2.3 (B)

Given : $\dfrac{dx(t)}{dt} + 3x(t) = 0$ … (i)

Initial condition, $x(0) = 2$.

Method 1

$$\frac{dx(t)}{dt} + 3x(t) = 0$$

$$\frac{dx}{dt} = -3x$$

$$\int \frac{dx}{x} = -3\int dt + c$$

$$\log x = -3t + c$$

$$x = e^{-3t+c}$$

$$x = C_1 e^{-3t} \qquad [C_1 = e^c]$$

Using boundary conditions :

At $t = 0,\ x(t) = 2$

So, $2 = C_1 e^0 \Rightarrow C_1 = 2$

$$x(t) = 2e^{-3t}$$

Hence, the correct option is (B).

Method 2

$$(D+3)x = 0$$

This is in form of a homogeneous linear differential equation.

$$[f(D)]x(t) = 0$$

The auxiliary equation is given by,

$$f(m) = 0$$

$$m + 3 = 0$$

$$m = -3$$

The complementary function is given by,

$$\text{C.F.} = Ce^{mt}$$

$$\text{C.F.} = Ce^{-3t}$$

Particular integral (P.I.) is 0, because it is a homogeneous equation.

The complete solution is,

$$x(t) = \text{C.F.} + \text{P.I.}$$

$$x(t) = Ce^{-3t}$$

Using boundary conditions :

At $t = 0,\ x(t) = 2$

So, $2 = Ce^0 \Rightarrow C = 2$

$$x(t) = 2e^{-3t}$$

Hence, the correct option is (B).

2.4 (A)

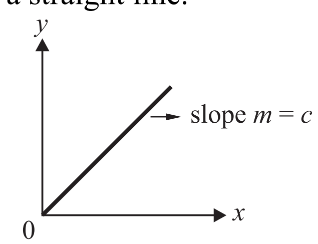
(P) $\dfrac{dy}{dx} = \dfrac{y}{x} \Rightarrow \dfrac{dy}{y} = \dfrac{dx}{x}$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\log y = \log x + \log c = \log xc$$

$$y = xc$$

This is a straight line.



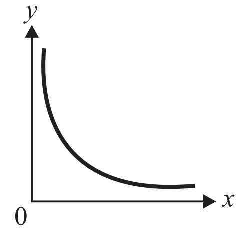
(Q) $\dfrac{dy}{dx} = \dfrac{-y}{x} \Rightarrow \dfrac{dy}{y} = -\dfrac{dx}{x}$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\log y = -\log x + \log c$$

$$\log(xy) = \log c \Rightarrow xy = c$$

This is a rectangular hyperbola.

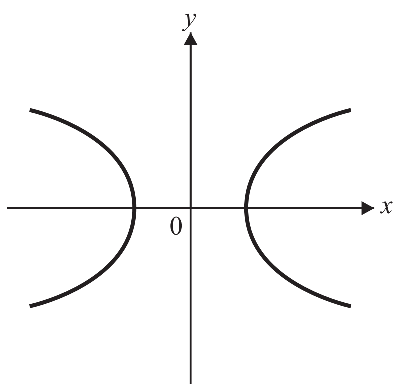


(R) $\frac{dy}{dx}=\frac{x}{y} \Rightarrow \int y\,dy=\int x\,dx$

$$\frac{y^2}{2}=\frac{x^2}{2}+c$$

$$\frac{y^2}{2}-\frac{x^2}{2}=c$$

This is also a hyperbola.

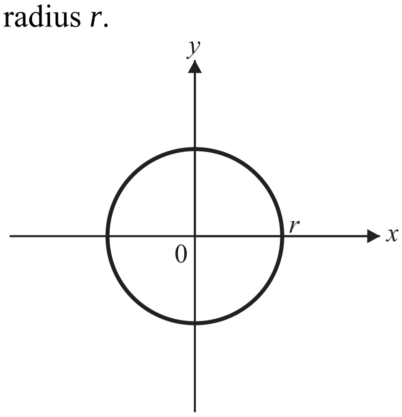


(S) $\frac{dy}{dx}=\frac{-x}{y} \Rightarrow \int y\,dy=-\int x\,dx$

$$\frac{y^2}{2}=\frac{-x^2}{2}+c$$

$$x^2+y^2=2c \Rightarrow x^2+y^2=r^2$$

This is a circle, with center at origin and radius r.



Thus, the appropriate match of given items is

P – 2, Q – 3, R – 3, S – 1

Hence, the correct option is (A).

2.5 (A)

In option (A) : $\frac{dy}{dx}+xy=e^{-x}$

All the derivatives are of first degree.

All the coefficients are functions of independent variable x only.

The constant term is not equal to zero.

So, the given equation is a linear non-homogeneous differential equation.

In option (B) : $\frac{dy}{dx}+xy=0$

All the derivatives are of first degree.

All the coefficients are functions of independent variable x only.

The constant term is equal to zero.

So, the given equation is a linear homogeneous differential equation.

In option (C) : $\frac{dy}{dx}+xy=e^{-y}$

All the derivatives are of first degree.

Here e^{-y} is having the power of y more than one in its expansion. So, the given equation is a non-linear differential equation.

In option (D) : $\frac{dy}{dx}+e^{-y}=0$

All the derivatives are of first degree.

Here e^{-y} is having the power of y more than one in its expansion. So, the given equation is a non-linear homogenous differential equation.

Hence, the correct option is (A).

Scan for Video Solution

2.6 (B)

Given : The differential equation is,

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0$$

Initial conditions :

$$y(0) = y'(0) = 1$$

Method 1

$$(D^2 + 2D + 1)y = 0$$

This is in form of a homogeneous linear differential equation.

$$[f(D)]y = 0$$

The auxiliary equation is given by,

$$f(m) = 0$$

$$m^2 + 2m + 1 = 0$$

$$m_1 = m_2 = -1$$

The roots are real and equal. So, the complementary function is given by,

$$\text{C.F.} = (C_1 + C_2 t)e^{m_1 t}$$

$$\text{C.F.} = (C_1 + C_2 t)e^{-t}$$

Particular integral (P.I.) is 0 since it is a homogeneous equation.

The complete solution is given by,

$$y = \text{C.F.} + \text{P.I.}$$

$$y = (C_1 + C_2 t)e^{-t}$$

Using boundary condition :

At $t = 0 \Rightarrow y = 1$,

$$y(0) = (C_1 + 0) \times 1$$

$$C_1 = 1$$

Differentiating y with respect to x.

$$y' = -(C_1 + C_2 t)e^{-t} + C_2 e^{-t}$$

$$y' = (C_2 - C_1)e^{-t} - C_2 t e^{-t}$$

At $t = 0 \Rightarrow y' = 1$,

$$y'(0) = (C_2 - C_1)1$$

$$1 = C_2 - C_1$$

$$C_2 - C_1 = 1$$

$$C_2 = 2$$

Thus, the particular solution is,

$$y(t) = (1 + 2t)e^{-t}$$

Hence, the correct option is (B).

Method 2

Taking Laplace transform on both sides of given equation,

$$L\left[\frac{d^2 y}{dt^2} + 2\frac{dy}{dt} + y\right] = 0$$

$$\left[s^2 Y(s) - sy(0) - y'(0)\right] + 2\left[sY(s) - y(0)\right] + Y(s) = 0$$

$$(s^2 + 2s + 1)Y(s) - s \times 1 - 1 - 2 \times 1 = 0$$

$$Y(s) = \frac{s+3}{s^2 + 2s + 1} = \frac{s+3}{(s+1)^2}$$

Taking inverse Laplace transform,

$$y(t) = L^{-1}\left[\frac{(s+1)+2}{(s+1)^2}\right] = e^{-t} L^{-1}\left[\frac{s+2}{s^2}\right]$$

$$y(t) = e^{-t}\left\{L^{-1}\left[\frac{1}{s}\right] + 2L^{-1}\left[\frac{1}{s^2}\right]\right\}$$

$$y(t) = (1 + 2t)e^{-t}$$

Hence, the correct option is (B).

2.7 (A)

Given : $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{2y} + \dfrac{y}{x}$

Initial condition : $y(1) = 0$

$$\frac{dy}{dx} = \frac{x^2}{2y} + \frac{y}{2} + \frac{y}{x} \qquad \ldots\text{(i)}$$

Taking $\dfrac{y}{x} = t$, $y = xt$ ...(ii)

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$

From equation (i),

$$t + x\frac{dt}{dx} = \frac{x}{2t} + \frac{tx}{2} + t$$

$$x\frac{dt}{dx} = \frac{x}{2t} + \frac{tx}{2}$$

$$x\frac{dt}{dx} = x\left(\frac{1+t^2}{2t}\right)$$

By variable separation method,

$$\frac{2t}{1+t^2}dt = dx$$

Integrating on both sides,

$$\ln(1+t^2) = x + C$$

From equation (ii),

$$t = \frac{y}{x}$$

$$\ln\left(1+\frac{y^2}{x^2}\right) = x + C \qquad \ldots\text{(iii)}$$

Using boundary condition :

When $x = 1,\ y = 0$

$$\ln\left(1+\frac{0}{1}\right) = 1 + C$$

$$\ln(1) = 1 + C$$

$$0 = 1 + C$$

$$C = -1$$

From equation (iii),

$$\ln\left(1+\frac{y^2}{x^2}\right) = x - 1$$

Hence, the correct option is (A).

2.8 (C)

Given : $\dfrac{dy}{dx} + \dfrac{x}{1-x^2}y = x\sqrt{y}$

Dividing both side with $\sqrt{y}$

$$y^{-\frac{1}{2}}\frac{dy}{dx} + \frac{x}{1-x^2}y.y^{-\frac{1}{2}} = x$$

$$y^{-\frac{1}{2}}\frac{dy}{dx} + \frac{x}{1-x^2}y^{\frac{1}{2}} = x \qquad \ldots\text{(i)}$$

Let $y^{\frac{1}{2}} = t$

Differentiating equation (i), both side with respect to x,

$$\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = \frac{dt}{dx}$$

$$y^{-\frac{1}{2}}\frac{dy}{dx} = \frac{2dt}{dx} \qquad \ldots\text{(ii)}$$

Putting equation (ii) into equation (i)

$$2\frac{dt}{dx} + \frac{x}{1-x^2}t = x$$

$$\frac{dt}{dx} + \frac{x}{2(1-x^2)}t = \frac{x}{2}$$

Integrating factor (I.F) is given by,

$$\text{I.F} = e^{\int P\,dx} = e^{\frac{1}{2}\int\frac{x}{1-x^2}dx}$$

$$\text{I.F} = e^{-\frac{1}{4}\int\frac{-2x}{1-x^2}dx}$$

$$\text{I.F} = e^{-\frac{1}{4}\log_e(1-x^2)}$$

$$\text{I.F} = e^{\log_e(1-x^2)^{-1/4}}$$

$$\text{I.F} = (1-x^2)^{-\frac{1}{4}}$$

Hence, the correct option is (C).

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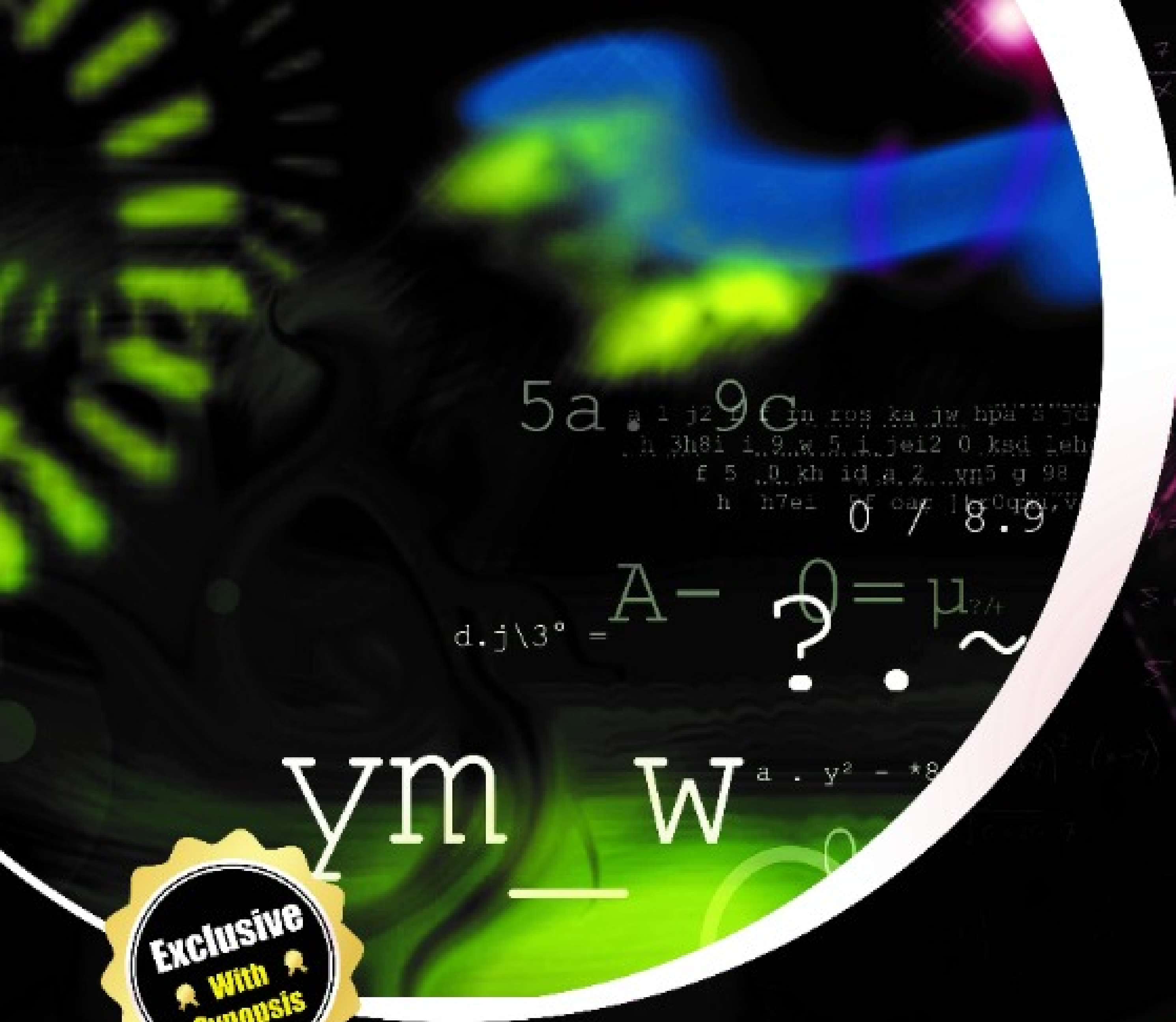

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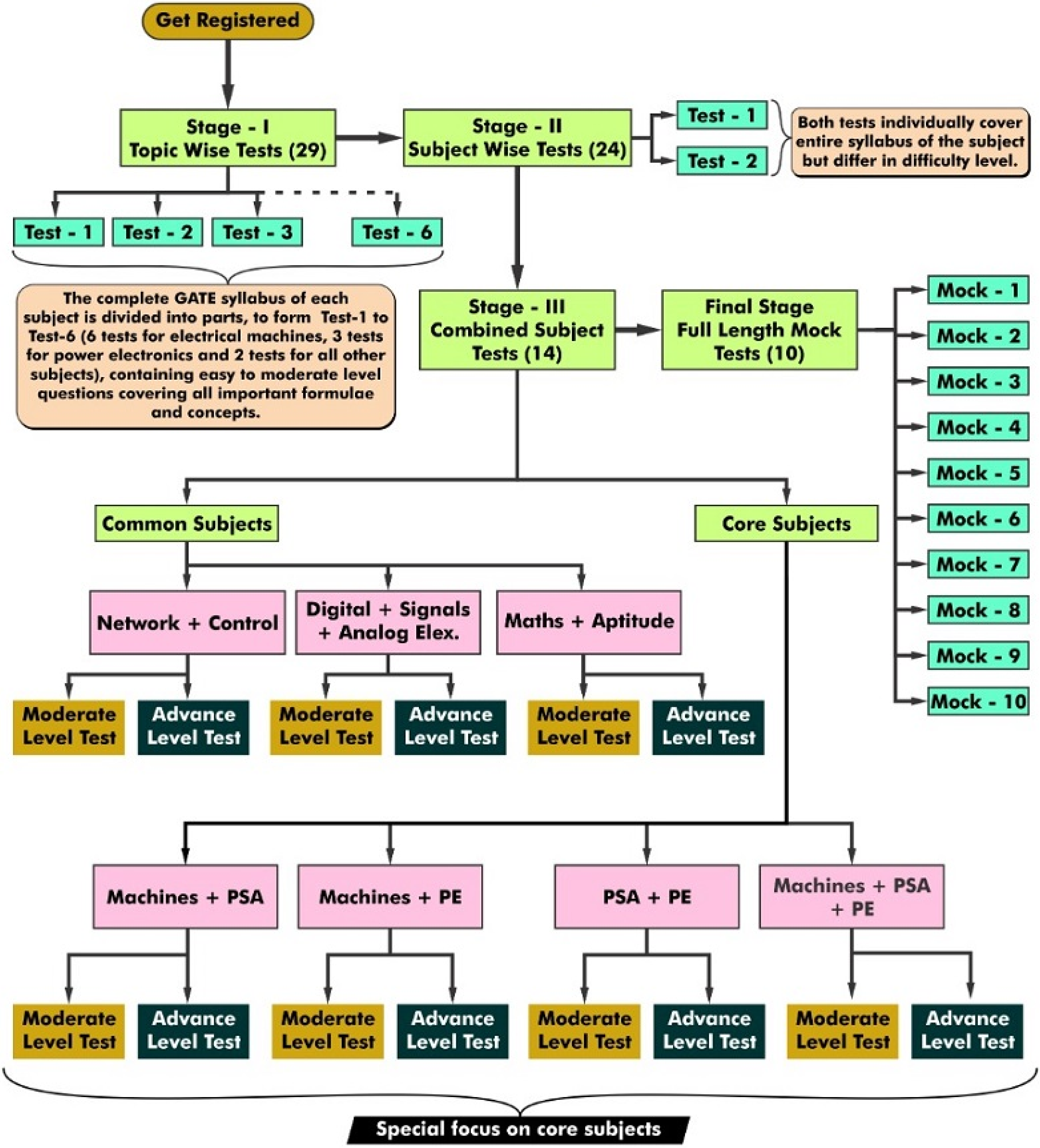

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