29 YEARS GATE SOLVED PAPERS

4 Years ESE Solved Papers

Exclusive With Synopsis

GATE 2022

UMESH DHANDE

ENGINEERING MATHEMATICS

EE CE CH EC ME IN PI

Topic-wise Previous 29 Years GATE Solutions

- Diligent solutions of GATE previous year questions (1993-2021)
- Multi method approach for a single problem to develop crystal clear concepts
- This book is also a value addition for ESE/PSUs/DRDO/ISRO
- Video Solution for Conspicuous Questions to Enhance problem solving Skills

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GATE Since 2004

Engineering Mathematics

Branches : CE / CH / EC / EE / IN / ME / PI

ESE (Prelims)

TOPIC WISE GATE SOLUTIONS
1993 - 2021

Umesh Dhande

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To My Son Advait

IMPORTANCE of GATE

GATE examination has been emerging as one of the most prestigious competitive exam for engineers. Earlier it was considered to be an exam just for eligibility for pursuing PG courses, but now GATE exam has gained a lot of attention of students as this exam open an ocean of possibilities like :

1. **Admission into IISc, IITs, IIITs, NITs**

   A good GATE score is helpful for getting admission into IISc, IITs, IIITs, NITs and many other renowned institutions for M.Tech./M.E./M.S. An M.Tech graduate has a number of career opportunities in research fields and education industries. Students get ₹ 12,400 per month as stipend during their course.

2. **Selection in various Public Sector Undertakings** (**PSUs**)

   A good GATE score is helpful for getting job in government-owned corporations termed as **Public Sector Undertakings** (**PSUs**) in India like IOCL, BHEL, NTPC, BARC, ONGC, PGCIL, DVC, HPCL, GAIL, SAIL & many more.

3. Direct recruitment to Group A level posts in Central government, i.e., Senior Field Officer (Tele), Senior Research Officer (Crypto) and Senior Research Officer (S&T) in Cabinet Secretariat, Government of India, is now being carried out on the basis of GATE score.

4. **Foreign universities through GATE**

   GATE has crossed the boundaries to become an international level test for entry into postgraduate engineering programmes in abroad. Some institutes in two countries **Singapore** and **Germany** are known to accept GATE score for admission to their PG engineering programmes.

5. **National Institute of Industrial Engg. (NITIE)**
   - NITIE offers **PGDIE / PGDMM / PGDPM** on the basis of GATE scores. The shortlisted candidates are then called for Group Discussion and Personal Interview rounds.
   - NITIE offers a Doctoral Level Fellowship Programme recognized by Ministry of HRD (MHRD) as equivalent to Ph.D. of any Indian University.
   - Regular full time candidates those who will qualify for the financial assistance will receive ₹ 25,000 during 1st and 2nd year of the Fellowship programme and ₹ 28,000 during 3rd, 4th and 5th year of the Fellowship programme as per MHRD guidelines.

6. **Ph.D. in IISc/ IITs**
   - IISc and IITs take admissions for Ph.D. on the basis of GATE score.
   - Earn a Ph.D. degree directly after Bachelor’s degree through integrated programme.
   - A fulltime residential researcher (RR) programme.

7. **Fellowship Program in management (FPM)**
   - Enrolment through GATE score card
   - Stipend of ₹ 22,000 – 30,000 per month + HRA
   - It is a fellowship program
   - Application form is generally available in month of Sept. and Oct.

Note : In near future, hopefully GATE exam will become a mandatory exit test for all engineering students, so take this exam seriously. Best of LUCK !

GATE Exam Pattern

Section	Question No.	No. of Questions	Marks Per Question	Total Marks
General Aptitude	1 to 5	5	1	5
	6 to 10	5	2	10
Technical + Engineering Mathematics	1 to 25	25	1	25
	26 to 55	30	2	60
Total Duration : 3 hours		**Total Questions : 65**		**Total Marks : 100**

Note :

(i) 40 to 45 marks will be allotted to Numerical Answer Type Questions.

(ii) MSQ also added from GATE 2021 for which **no negative** marking.

Pattern of Questions :

GATE 2021 would contain questions of THREE different types in all the papers :

(i) **Multiple Choice Questions (MCQ)** carrying 1 or 2 marks each, in all the papers and sections. These questions are objective in nature, and each will have choice of four answers, out of which ONLY ONE choice is correct.

Negative Marking for Wrong Answers : For a wrong answer chosen in a MCQ, there will be negative marking. For 1-mark MCQ, 1/3 mark will be deducted for a wrong answer. Likewise, for 2-mark MCQ, 2/3 mark will be deducted for a wrong answer.

(ii) **Multiple Select Questions (MSQ)** carrying 1 or 2 marks each in all the papers and sections. These questions are objective in nature, and each will have choice of four answers, out of which ONE or MORE than ONE choice(s) are correct.

Note : There is **NO negative** marking for a wrong answer in MSQ questions. However, there is NO partial credit for choosing partially correct combinations of choices or any single wrong choice.

(iii) **Numerical Answer Type (NAT)** Questions carrying 1 or 2 marks each in most of the papers and sections. For these questions, the answer is a signed real number, which needs to be entered by the candidate using the virtual numeric keypad on the monitor (keyboard of the computer will be disabled). No choices will be shown for these types of questions. The answer can be a number such as 10 or -10 (an integer only). The answer may be in decimals as well, for example, 10.1 (one decimal) or 10.01 (two decimals) or -10.001 (three decimals). These questions will be mentioned with, up to which decimal places, the candidates need to present the answer. Also, for some NAT type problems an appropriate range will be considered while evaluating these questions so that the candidate is not unduly penalized due to the usual round-off errors. Candidates are advised to do the rounding off at the end of the calculation (not in between steps). Wherever required and possible, it is better to give NAT answer up to a maximum of three decimal places.

Example : If the wire diameter of a compressive helical spring is increased by 2%, the change in spring stiffness (in %) is _ (correct to two decimal places).

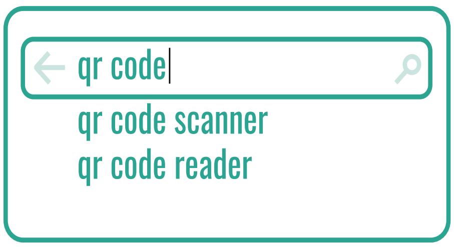
Note : *There is NO negative marking for a wrong answer in NAT questions.*

Also, there is NO partial credit in NAT questions.

What is special about this book ?

GATE ACADEMY Team took several years to come up with the solutions of GATE examination. It is because we strongly believe in quality. We have significantly prepared each and every solution of the questions appeared in GATE, and many individuals from the community have taken time out to proof read and improve the quality of solutions, so that it becomes very lucid for the readers. Some of the key features of this book are as under :

- ☞ This book gives complete analysis of questions chapter wise as well as year wise.
- ☞ Video Solution of important conceptual questions has been given in the form of QR code and by scanning QR code one can see the video solution of the given question.
- ☞ Solutions has been presented in lucid and understandable language for an average student.
- ☞ In addition to the GATE syllabus, the book includes the nomenclature of chapters according to text books for easy reference.
- ☞ Last but not the least, author's 10 years experience and devotion in preparation of these solutions.
- ☞ Steps to Open Video solution through mobile.



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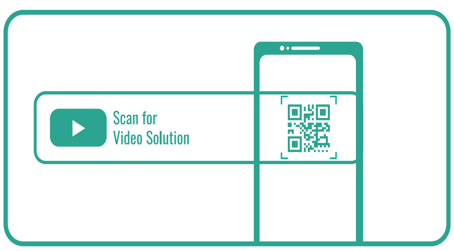

(3) Scan the given QR Code for particular question.

(4) Visit the link generated & you'll be redirect to the video solution.

Note : For recent updates regarding minor changes in this book, visit - www.gateacademy.co.in. We are always ready to appreciate and help you.

GATE SYLLABUS

Electronics & Communication (EC) :

Linear Algebra: Vector space, basis, linear dependence and independence, matrix algebra, eigenvalues and eigenvectors, rank, solution of linear equations- existence and uniqueness.

Calculus: Mean value theorems, theorems of integral calculus, evaluation of definite and improper integrals, partial derivatives, maxima and minima, multiple integrals, line, surface and volume integrals, Taylor series. **Differential Equations:** First order equations (linear and nonlinear), higher order linear differential equations, Cauchy's and Euler's equations, methods of solution using variation of parameters, complementary function and particular integral, partial differential equations, variable separable method, initial and boundary value problems.

Vector Analysis: Vectors in plane and space, vector operations, gradient, divergence and curl, Gauss's, Green's and Stokes' theorems. **Complex Analysis:** Analytic functions, Cauchy's integral theorem, Cauchy's integral formula, sequences, series, convergence tests, Taylor and Laurent series, residue theorem. **Probability and Statistics:** Mean, median, mode, standard deviation, combinatorial probability, probability distributions, binomial distribution, Poisson distribution, exponential distribution, normal distribution, joint and conditional probability.

Electrical Engineering (EE) :

Linear Algebra: Matrix Algebra, Systems of linear equations, Eigenvalues, Eigenvectors. **Calculus:** Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, Partial Derivatives, Maxima and minima, Multiple integrals, Fourier series, Vector identities, Directional derivatives, Line integral, Surface integral, Volume integral, Stokes's theorem, Gauss's theorem, Divergence theorem, Green's theorem. **Differential equations:** First order equations (linear and nonlinear), Higher order linear differential equations with constant coefficients, Method of variation of parameters, Cauchy's equation, Euler's equation, Initial and boundary value problems, Partial Differential Equations, Method of separation of variables. **Complex variables:** Analytic functions, Cauchy's integral theorem, Cauchy's integral formula, Taylor series, Laurent series, Residue theorem, Solution integrals. **Probability and Statistics:** Sampling theorems, Conditional probability, Mean, Median, Mode, Standard Deviation, Random variables, Discrete and Continuous distributions, Poisson distribution, Normal distribution, Binomial distribution, Correlation analysis, Regression analysis.

Mechanical Engineering (ME) :

Linear Algebra: Matrix algebra, systems of linear equations, eigenvalues and eigenvectors. **Calculus:** Functions of single variable, limit, continuity and differentiability, mean value theorems, indeterminate forms; evaluation of definite and improper integrals; double and triple integrals; partial derivatives, total derivative, Taylor series (in one and two variables), maxima and minima, Fourier series; gradient, divergence and curl, vector identities, directional derivatives, line, surface and volume integrals, applications of Gauss, Stokes and Green's theorems. **Differential equations:** First order equations (linear and nonlinear); higher order linear differential equations with constant coefficients; Euler-Cauchy equation; initial and boundary value problems; Laplace transforms; solutions of heat, wave and Laplace's equations. Complex variables: Analytic functions; Cauchy-Riemann equations; Cauchy's integral theorem and integral formula; Taylor and Laurent series. **Probability and Statistics:** Definitions of probability, sampling theorems, conditional probability; mean, median, mode and standard deviation; random variables, binomial, Poisson and normal distributions. **Numerical Methods:** Numerical solutions of linear and non-linear algebraic equations; integration by trapezoidal and Simpson's rules; single and multi-step methods for differential equations.

Civil Engineering (CE) :

Linear Algebra: Matrix algebra; Systems of linear equations; Eigen values and Eigen vectors. **Calculus:** Functions of single variable; Limit, continuity and differentiability; Mean value theorems, local maxima and minima; Taylor series; Evaluation of definite and indefinite integrals, application of definite integral to obtain area and volume; Partial derivatives; Total derivative; Gradient, Divergence and Curl, Vector identities; Directional derivatives; Line, Surface and Volume integrals. **Ordinary Differential Equation (ODE):** First order (linear and non-linear) equations; higher order linear equations with constant coefficients; Euler-Cauchy equations; initial and boundary value problems. **Partial Differential Equation (PDE):** Fourier series; separation of variables; solutions of one- dimensional diffusion equation; first and second order one-dimensional wave equation and two-dimensional Laplace equation. **Probability and Statistics:** Sampling theorems; Conditional probability; Descriptive statistics – Mean, median, mode and standard deviation; Random Variables – Discrete and Continuous, Poisson and Normal Distribution; Linear regression. **Numerical Methods:** Error analysis. Numerical solutions of linear and non-linear algebraic equations; Newton's and Lagrange polynomials; numerical differentiation; Integration by trapezoidal and Simpson's rule; Single and multi-step methods for first order differential equations.

Instrumentation Engineering (IN) :

Linear Algebra: Matrix algebra, systems of linear equations, consistency and rank, Eigen value and Eigen vectors. **Calculus:** Mean value theorems, theorems of integral calculus, partial derivatives, maxima and minima, multiple integrals, Fourier series, vector identities, line, surface and volume integrals, Stokes, Gauss and Green's theorems. **Differential equations:** First order equation (linear and nonlinear), second order linear differential equations with constant coefficients, method of variation of parameters, Cauchy's and Euler's equations, initial and boundary value problems, solution of partial differential equations: variable separable method. **Analysis of complex variables:** Analytic functions, Cauchy's integral theorem and integral formula, Taylor's and Laurent's series, residue theorem, solution of integrals.

Probability and Statistics: Sampling theorems, conditional probability, mean, median, mode, standard deviation and variance; random variables: discrete and continuous distributions: normal, Poisson and binomial distributions. **Numerical Methods:** Matrix inversion, solutions of non-linear algebraic equations, iterative methods for solving differential equations, numerical integration, regression and correlation analysis.

Chemical Engineering (CH) :

Linear Algebra: Matrix algebra, Systems of linear equations, Eigen values and eigenvectors. **Calculus:** Functions of single variable, Limit, continuity and differentiability, Taylor series, Mean value theorems, Evaluation of definite and improper integrals, Partial derivatives, Total derivative, Maxima and minima, Gradient, Divergence and Curl, Vector identities, Directional derivatives, Line, Surface and Volume integrals, Stokes, Gauss and Green's theorems. **Differential equations:** First order equations (linear and nonlinear), Higher order linear differential equations with constant coefficients, Cauchy's and Euler's equations, Initial and boundary value problems, Laplace transforms, Solutions of one dimensional heat and wave equations and Laplace equation. **Complex variables:** Complex number, polar form of complex number, triangle inequality. **Probability and Statistics:** Definitions of probability and sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Poisson, Normal and Binomial distributions, Linear regression analysis. **Numerical Methods:** Numerical solutions of linear and non-linear algebraic equations. Integration by trapezoidal and Simpson's rule. Single and multi-step methods for numerical solution of differential equations.

Production and Industrial Engineering (PI) :

Linear Algebra: Matrix algebra, Systems of linear equations, Eigen values and Eigen vectors. **Calculus:** Functions of single variable, Limit, continuity and differentiability, Mean value theorems, Evaluation of definite and improper integrals, Partial derivatives, Total derivative, Maxima and minima, Gradient, Divergence and Curl, Vector identities, Directional derivatives; Line, Surface and Volume integrals; Stokes, Gauss and Green's theorems. **Differential Equations:** First order equations (linear and nonlinear), Higher order linear differential equations with constant coefficients, Cauchy's and Euler's equations, Initial and boundary value problems, Laplace transforms. **Complex Variables:** Analytic functions, Cauchy's integral theorem, Taylor series. **Probability and Statistics:** Definitions of probability and sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Linear regression, Random variables, Poisson, normal, binomial and exponential distributions. **Numerical Methods:** Numerical solutions of linear and nonlinear algebraic equations, Integration by trapezoidal and Simpson's rules, Single and multi-step methods for differential equations.

Electrical Engineering (EE) :

Matrix theory, Eigen values & Eigen vectors, system of linear equations, Numerical methods for solution of non-linear algebraic equations and differential equations, integral calculus, partial derivatives, maxima and minima, Line, Surface and Volume Integrals. Fourier series, linear, non-linear and partial differential equations, initial and boundary value problems, complex variables, Taylor's and Laurent's series, residue theorem, probability and statistics fundamentals, Sampling theorem, random variables, Normal and Poisson distributions, correlation and regression analysis.

PREFACE

It is our pleasure, that we insist on presenting **"Engineering Mathematics"** authored for **GATE 2022** to all of the aspirants and career seekers. The prime objective of this book is to respond to tremendous amount of ever growing demand for error free, flawless and succinct but conceptually empowered solutions to all the question over the period 1993 - 2021.

This book serves to the best supplement the texts for GATE 2022 (CE/CH/EC/EE/IN/ME/PI) but shall be useful to a larger extent for other discipline as well.

Simultaneously having its salient features the book comprises :

- Step by step solution to all questions.
- Complete analysis of questions, i.e. chapter wise as well as year wise.
- Detailed explanation of all the questions.
- Solutions are presented in simple and easily understandable language.
- Video solutions available for good questions.
- It covers all GATE questions from 1993 to 2021 (29 years).

The authors do not sense any deficit in believing that this title will in many aspects, be different from the similar titles within the search of student.

We would like to express our sincere appreciation to **Mrs. Sakshi Dhande Mam** (Co-Director, GATE ACADEMY Learning Pvt. Ltd.) for her constant support and constructive suggestions and comments in reviewing the script.

In particular, we wish to thank **GATE ACADEMY** expert team members for their hard work and consistency while designing the script.

The final manuscript has been prepared with utmost care. However, going a line that, there is always room for improvement in anything done, we would welcome and greatly appreciate the suggestions and corrections for further improvement.

GATE ACADEMY

ACKNOWLEDGEMENT

13 Years of recurring effort went into the volume which is now ready to cover the aptitude of GATE aspirants.

We are glad of this opportunity to acknowledge the views and to express with all the weaknesses of mere words the gratitude that we must always feel for the generosity of them.

We now express our gracious gratitude to Pratima Patel who has contributed a lot in order to put forth this into device.

In this new addition of our book, for making solutions more lucid and better thanks to Mr. Sonal Kumar Agrawal for giving his unconditional efforts.

Special thanks to **Mr. Gurupal S. Chawla,** who has been involved in this project from the beginning and has given his best effort on his part. This book was not possible without his unconditional effort and we are grateful to him for preparing video solutions for QR code.

Lastly, we take this opportunity to acknowledge the service of the total team of publication and everyone who collaborated in producing this work.

Umesh Dhande

(Director, GATE ACADEMY Learning Pvt. Ltd.)

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3 Integral & Differential Calculus

Introduction

(i) The word calculus comes from Latin meaning "small stone". Because it is like understanding something by looking at small pieces.

(ii) Differential Calculus cuts something into small pieces to find how it changes.

(iii) Integral Calculus joins (integrates) the small pieces together to find how much there is.

Some Useful Formulae for Differentiation

1.	$\frac{d}{dx}(c) = 0$; c = Constant	**11.**	$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
2.	$\frac{d}{dx}(x) = 1$	**12.**	$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$
3.	$\frac{d}{dx}(x^n) = nx^{n-1}$	**13.**	$\frac{d}{dx}(\sec x) = \sec x \tan x$
4.	$\frac{d}{dx}(e^x) = e^x$	**14.**	$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
5.	$\frac{d}{dx} a^x = a^x \log_e a$	**15.**	$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
6.	$\frac{d}{dx}(\log x) = \frac{1}{x}$	**16.**	$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
7.	$\frac{d}{dx}(\log_a x) = \frac{1}{x}\log_a e$	**17.**	$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
8.	$\frac{d}{dx}(\sin x) = \cos x$	**18.**	$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
9.	$\frac{d}{dx}(\cos x) = -\sin x$	**19.**	$\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$
10.	$\frac{d}{dx}(\tan x) = \sec^2 x$	**20**	$\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$

Definite Integration

Some useful properties of Definite Integrals

1. $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

2. $\int_a^a f(x)dx = 0$

3. $\int_a^b [f(x) \pm g(x)]\, dx = \int_a^b f(x)\, dx \pm \int_a^b g(x)\, dx$

4. $\int_a^b f(x)dx = \int_a^b f(t)dt$

5. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

6. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ where $a < c < b$

7. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

8. $\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)\, dx, & \text{if } f(-x) = f(x); \quad \text{Even function} \\ 0, & \text{if } f(-x) = -f(x); \quad \text{Odd function} \end{cases}$

9. $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$

10. $\int_0^{na} f(x)dx = n\int_0^a f(x)dx$ if $f(x) = f(x+a)$

11. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

10. $\int_a^b x\, f(x)dx = \frac{b-a}{2}\int_a^b f(x)dx$ if $f(a+b-x) = f(x)$

12. $\int f^n . g dx = f^{n-1}g - f^{n-2}g' + f^{n-3}g'' - ......(-1)^n \int fg^n dx$

    [Generalized form of integration by parts]

Some Useful Formulas for Integration

Algebraic Functions

1.	$\int k\, dx = kx + C$	**4.**	$\int \frac{1}{x} dx = \ln(x) + C$ {for positive values of x only}
2.	$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$	**5.**	$\int \frac{c}{ax+b} dx = \frac{c}{a}\ln(ax+b) + C$

3.	$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$ (for $n \neq -1$)		

Exponential Functions

1.	$\int e^x\, dx = e^x + C$	**2.**	$\int a^x\, dx = \frac{a^x}{\ln a} + C$

Logarithms Functions

1.	$\int \ln x dx = x \ln x - x + C$	**2.**	$\int \log_a x\, dx = x \log_a x - \frac{x}{\log a} + C$

Trigonometric Functions

1.	$\int \sin x\, dx = -\cos x + C$	**2.**	$\int \cos x\, dx = \sin x + C$
3.	$\int \tan x\, dx = -\ln(\cos x) + C$ $= \ln(\sec x) + C$	**4.**	$\int \cot x\, dx = \ln(\sin x) + C$
5.	$\int \sec x\, dx = \ln(\sec x + \tan x) + C$ $= \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + C$	**6.**	$\int \text{cosec}\, x\, dx = -\ln(\text{cosec}\, x + \cot x) + C$ $= \log \tan \frac{x}{2}$
7.	$\int \sec^2 x\, dx = \tan x + C$	**8.**	$\int \text{cosec}^2 x\, dx = -\cot x + C$
9.	$\int \sec x \tan x\, dx = \sec x + C$	**10.**	$\int \text{cosec}\, x \cot x\, dx = -\text{cosec}\, x + C$

Inverse of trigonometric Functions

1.	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C$	**2.**	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) = \frac{-1}{a} \cot^{-1}\left(\frac{x}{a}\right)$
3.	$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C$	**4.**	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left(\frac{a+x}{a-x}\right) = \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right)$
5.	$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1}\left(\frac{x}{a}\right)$	**6.**	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right)$
7.	$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2}) + C$	**8.**	$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}) + C$
9.	$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{(a^2 + x^2)} + \frac{a^2}{2} \sinh^{-1}\left(\frac{x}{a}\right)$	**10.**	$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{(a^2 - x^2)} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right)$

Hyperbolic Functions

1.	$\int \sinh x dx = \cosh x + C$	**4.**	$\int \text{cosec}\,h x dx = \ln(\tanh \frac{x}{2}) + C$
2.	$\int \cosh x dx = \sinh x + C$	**5.**	$\int \text{sech}\, x dx = \tan^{-1}(\sinh x) + C$
3.	$\int \tanh x dx = \ln(\cosh x) + C$	**6.**	$\int \coth x dx = \ln(\sinh x) + C$

Composed Functions

1.	$\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx) + C$
2.	$\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx) + C$
3.	$\int \cos ax \cosh bx\, dx = \frac{1}{a^2+b^2}(a \sin ax \cosh bx + b \cos ax \sinh bx) + C$
4.	$\int \sin ax \cosh bx\, dx = \frac{1}{a^2+b^2}(b \sin ax \sinh bx - a \cos ax \cosh bx) + C$

Some Important Results of Definite Integrals

1.	$\int_0^{\infty} \sqrt{x} e^{-x}\, dx = \frac{1}{2}\sqrt{\pi}$	**2.**	$\int_0^{\infty} e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$
3.	$\int_0^{\frac{\pi}{2}} \sin^n x dx = \int_0^{\frac{\pi}{2}} \cos^n x dx$ $= \frac{(n-1)(n-3).....}{n(n-2)(n-4).....} \times \frac{2}{3}\left(\text{or } \frac{1}{2}\right) K$ where, $K = \begin{cases} \frac{\pi}{2}, & \text{if } n \text{ is even} \\ 1, & \text{Otherwise} \end{cases}$	**4.**	$\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx$ $= \frac{(m-1)(m-3).....(n-1)(n-3).....}{(m+n)(m+n-2).....} K$ where, $K = \begin{cases} \frac{\pi}{2}, & \text{if } m \text{ and } n \text{ is even} \\ 1, & \text{Otherwise} \end{cases}$

Special Functions (Gamma and Beta Functions)

Gamma Function

Gamma function denoted by $\Gamma(n)$ is defined by the improper integral which is dependent on the parameter n,

$$\Gamma(n) = \int_0^{\infty} e^{-t} t^{n-1} dt, \quad (n > 0)$$

Key Point

Standard results of Gamma function :

(i) $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$

(ii) $\Gamma(n)=(n-1)!$

(iii) $\Gamma(n+1)=\begin{cases} n\Gamma(n), & \text{if } n \text{ is any fraction} \\ n!, & \text{if } n \text{ is an integer} \end{cases}$

(iv) $\Gamma\left(-\frac{1}{2}\right)=-2\sqrt{\pi}$

Beta Function

Beta function $\beta(m,n)$ defined by

$$\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx,\ (m>0, n>0)$$

Key Point

Standard results of Gamma function :

(i) $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

(ii) $\displaystyle\int_0^{\frac{\pi}{2}}\sin^m\cos^n x.dx=\frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$

(iii) $\displaystyle\int_0^{\frac{\pi}{2}}\sin^n x\,dx=\frac{1}{2}\beta\left(\frac{n+1}{2},\frac{1}{2}\right)=\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)}\cdot\frac{\sqrt{\pi}}{2}$

(iv) $\displaystyle\int_0^{\frac{\pi}{2}}\cos^n x\,dx=\frac{1}{2}\beta\left(\frac{1}{2},\frac{n+1}{2}\right)=\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)}\frac{\sqrt{\pi}}{2}$

Application of Definite Integral (Area, Length and Volume)

Applications		Formula
Area or Quadrature	Cartesian form	$\displaystyle\int_{x_1}^{x_2}\int_{y_1}^{y_2} dydx$
	Polar form $r=f(\theta)$	$\displaystyle\int_\alpha^\beta \frac{r^2}{2}d\theta$ or $\displaystyle\iint_S r dr d\theta$

Length of Curve	Cartesian form $y = f(x)$	$\int_a^b \sqrt{\left[1+\left(\frac{dy}{dx}\right)^2\right]}\, dx$
	Polar form $r = f(\theta)$	$\int_\alpha^\beta \sqrt{\left[r^2+\left(\frac{dr}{d\theta}\right)^2\right]}\, d\theta$
Volume of Revolution	Volume revolved by x - axis	$\int_a^b \pi\, y^2(x)\, dx$
	Volume revolved by y - axis,	$\int_a^b \pi\, x^2(y)\, dy$
	About the initial line ($\theta = 0$)	$\int_\alpha^\beta \frac{2\pi}{3} r^3 \sin\theta\, d\theta$
	About the line $\left(\theta = \frac{\pi}{2}\right)$	$\int_\alpha^\beta \frac{2\pi}{3} r^3 \cos\theta\, d\theta$
Volume as Double and Triple Integral	Triple integral	Cartesian: $\iiint_V dx\, dy\, dz$ Cylindrical: $\iiint r dr d\phi dz$ Spherical : $\iiint r^2 \sin\theta dr d\theta d\phi$
	Double integral	$\iint_s f(x, y) dx\ dy$

Key point

Area bounded by the curve $y = f(x)$ in the x-axis ordinates $x = a$ and $x = b$ is,

$$\int_a^b y dx = \int_a^b f(x)\, dx$$

❖❖❖❖

Electronics & Communication : EC

2005 IIT Bombay

3.1 The value of the integral $I = \frac{1}{\sqrt{2\pi}}\int_0^{\infty} \exp\left(\frac{-x^2}{8}\right) dx$ is

(A) 1 (B) π

(C) 2 (D) 2π

2006 IIT Kharagpur

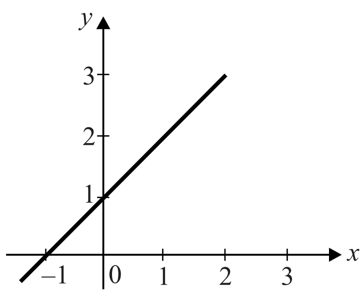
3.2 The integral $\int_0^{\pi} \sin^3 \theta \, d\theta$ is given by

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{4}{3}$ (D) $\frac{8}{3}$

2007 IIT Kanpur

3.3 The following plot shows a function y which varies linearly with x. The value of the integral $I = \int_1^2 y \, dx$ is



(A) 1.0 (B) 2.5

(C) 4.0 (D) 5.0

2008 IISc Bangalore

3.4 Consider points P and Q in the x-y plane, with $P = (1, 0)$ and $Q = (0, 1)$. The line integral $2\int_P^Q (xdx + ydy)$ along the semicircle with the line segment PQ as its diameter

(A) is – 1.

(B) is 0.

(C) is 1.

(D) depends on the direction (Clockwise or anti-clockwise) of the semicircle.

3.5 The value of the integral of the function $g(x, y) = 4x^3 + 10y^4$ along the straight line segment from the point (0, 0) to the point (1, 2) in the x-y plane is

(A) 33 (B) 35

(C) 40 (D) 56

2010 IIT Guwahati

3.6 The integral $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{\frac{-x^2}{2}} dx$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) 1 (D) ∞

2014 IIT Kharagpur

3.7 The volume under the surface $z(x, y) = x + y$ and above the triangle is the x-y plane defined by $\{0 \le y \le x$ and $0 \le x \le 12\}$ is **[Set - 01]**

2015 IIT Kanpur

3.8 The value of the integral

$$\int_{-\infty}^{\infty} 12\cos(2\pi t)\frac{\sin(4\pi t)}{4\pi t} dt \text{ is } \_\_\_\_\_\_.$$

[Set - 02]

3.9 The contour on the xy plane, where the partial derivative of $x^2 + y^2$ with respect to y is equal to the partial derivative of $6y + 4x$ with respect to x is **[Set - 03]**

(A) $y = 2$ (B) $x = 2$

(C) $x = y = 4$ (D) $x - y = 0$

2016 IISc Bangalore

3.10 The integral $\frac{1}{2\pi}\iint_D (x+y+10)\,dx\,dy$, where D denotes the disc : $x^2+y^2 \le 4$, evaluates to _______. **[Set - 01]**

3.11 The region specified by

$$\left\{(\rho,\ \phi,\ z): 3 \le \rho \le 5,\ \frac{\pi}{8} \le \phi \le \frac{\pi}{4},\ 3 \le z \le 4.5\right\}$$

in cylindrical coordinates has volume of _______. **[Set - 01]**

3.12 Suppose C is the closed curve defined as the circle $x^2+y^2=1$ with C oriented anti-clockwise. The value of $\oint (xy^2dx + x^2y\,dy)$ over the curve C equals _______. **[Set - 02]**

3.13 The integral $\int_0^1 \frac{dx}{\sqrt{(1-x)}}$ is equal to _______. **[Set - 03]**

3.14 A triangle in the xy plane is bounded by the straight lines $2x=3y$, $y=0$ and $x=3$. The volume above the triangle and under the plane $x+y+z=6$ is_______. **[Set - 03]**

2017 IIT Roorkee

3.15 A three dimensional region R of finite volume is described by $x^2+y^2 \le z^3$; $0 \le z \le 1$ where x, y, z are real. The volume of R (up to two decimal places)______. **[Set - 01]**

3.16 Let $I = \int_C (2z\,dx + 2y\,dy + 2x\,dz)$, where x, y, z are real, and let C be the straight line segment from point A : (0, 2, 1) to point B : (4, 1, –1). The value of I is ______. **[Set - 01]**

3.17 The values of the integrals

$$\int_0^1\left(\int_0^1 \frac{x-y}{(x+y)^3}dy\right)dx$$

and $\int_0^1\left(\int_0^1 \frac{x-y}{(x+y)^3}dx\right)dy$ are **[Set - 02]**

(A) same and equal to 0.5.
(B) same and equal to – 0.5.
(C) 0.5 and – 0.5, respectively.
(D) – 0.5 and 0.5, respectively.

2018 IIT Guwahati

3.18 Let $f(x,y) = \frac{ax^2+by^2}{xy}$, where a and b are constants. If $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$ at $x=1$ and $y=2$, then the relation between a and b is

(A) $a = \frac{b}{4}$ (B) $a = \frac{b}{2}$
(C) $a = 2b$ (D) $a = 4b$

3.19 Let $r = x^2+y-z$ and $z^3 - xy + yz + y^3 = 1$. Assume that x and y are independent variables. At $(x,y,z) = (2,-1,1)$, the value (correct to two decimal places) of $\frac{\partial r}{\partial x}$ is _______.

2019 IIT Madras

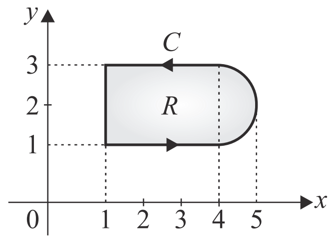
3.20 The value of the integral $\int_0^{\pi}\int_y^{\pi} \frac{\sin x}{x}dxdy$, is equal to _____

3.21 Consider the line integral

$$\oint_c (x\,dy - y\,dx)$$

the integral being taken in a counterclockwise direction over the closed curve C that forms the boundary of the region R shown in the figure below.

The region R is the area enclosed by the union of a 2 x 3 rectangle and a semi-circle of radius 1. The line integral evaluates to



(A) $8+\pi$ (B) $12+\pi$

(C) $16+2\pi$ (D) $6+\frac{\pi}{2}$

Electrical Engineering : EE

2005 IIT Bombay

3.1 If $S=\int_1^{\infty} x^{-3}dx$, then S has the value

(A) $\frac{-1}{3}$ (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) 1

2006 IIT Kharagpur

3.2 A surface $S(x, y)=2x+5y-3$ is integrated once over a path consisting of the points that satisfy

$$(x+1)^2+(y-1)^2=\sqrt{2}.$$

The integral evaluates to

(A) $17\sqrt{2}$ (B) $\frac{17}{\sqrt{2}}$

(C) $\frac{\sqrt{2}}{17}$ (D) 0

3.3 The expression $V=\int_0^H \pi R^2\left(1-\frac{h}{H}\right)^2 dh$ for the volume of a cone is equal to,

(A) $\int_0^R \pi R^2\left(1-\frac{h}{H}\right)^2 dr$

(B) $\int_0^R \pi R^2\left(1-\frac{h}{H}\right)^2 dh$

(C) $\int_0^R 2\pi rH\left(1-\frac{r}{R}\right)dh$

(D) $\int_0^R 4\pi rH\left(1-\frac{r}{R}\right)^2 dr$

2007 IIT Kanpur

3.4 The integral $\frac{1}{2\pi}\int_0^{2\pi}\sin(t-\tau)\cos\tau\, d\tau$ equals

(A) $\sin t\cos t$ (B) 0

(C) $\left(\frac{1}{2}\right)\cos t$ (D) $\left(\frac{1}{2}\right)\sin t$

2009 IIT Roorkee

3.5 $f(x, y)$ is a continuous function defined over $(x, y)\in[0,1]\times[0,1]$. Given the two constraints, $x>y^2$ and $y>x^2$, the volume under $f(x, y)$ is

(A) $\int_{y=0}^{y=1}\int_{x=y^2}^{x=\sqrt{y}} f(x,y)\,dxdy$

(B) $\int_{y=x^2}^{y=1}\int_{x=y^2}^{x=1} f(x,y)\,dxdy$

(C) $\int_{y=0}^{y=1}\int_{x=0}^{x=1} f(x,y)\,dxdy$

(D) $\int_{y=0}^{y=\sqrt{x}}\int_{x=0}^{x=\sqrt{y}} f(x,y)\,dxdy$

2010 IIT Guwahati

3.6 The value of the quantity, where

$$P=\int_0^1 xe^x\,dx$$

(A) 0 (B) 1

(C) e (D) $\frac{1}{e}$

3.7 A function $y = 5x^2 + 10x$ is defined over an open interval $x \in (1, 2)$. At least at one point in this interval, $\frac{dy}{dx}$ is exactly

(A) 20 (B) 25
(C) 30 (D) 35

2014 IIT Kharagpur

3.8 A particle, starting from origin at $t = 0$ s, is traveling along x-axis with velocity

$$v = \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right) \text{m/s}.$$

At $t = 3$ s, the difference between the distance covered by the particles and the magnitude of displacement from the origin is______. **[Set - 02]**

3.9 To evaluate the double integral

$$\int_0^8 \left(\int_{y/2}^{(y/2)+1} \left(\frac{2x - y}{2} \right) dx \right) dy,$$

we make the substitution $u = \left(\frac{2x - y}{2} \right)$ and $v = \frac{y}{2}$. The integral will reduce to **[Set - 02]**

(A) $\int_0^4 \left(\int_0^2 2u\, du \right) dv$

(B) $\int_0^4 \left(\int_0^1 2u\, du \right) dv$

(C) $\int_0^4 \left(\int_0^1 u\, du \right) dv$

(D) $\int_0^4 \left(\int_0^2 u\, du \right) dv$

2016 IISc Bangalore

3.10 The value of the line integral

$$\int_c (2xy^2 dx + 2x^2 y dy + dz)$$

along a path joining the origin (0, 0, 0) and the point (1, 1, 1) is **[Set - 02]**

(A) 0 (B) 2
(C) 4 (D) 6

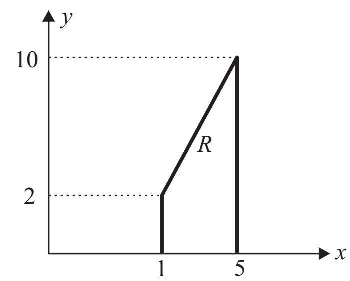
3.11 The value of the integral $2\int_{-\infty}^{\infty} \left(\frac{\sin 2\pi t}{\pi t} \right) dt$ is equal to **[Set - 02]**

(A) 0 (B) 0.5
(C) 1 (D) 2

3.12 The volume enclosed by the surface $f(x, y) = e^x$ over the triangle bounded by the lines $x = y$; $x = 0$; $y = 1$ in the xy plane is ______. **[Set - 02]**

2017 IIT Roorkee

3.13 Let $I = C\iint_R xy^2 dx\, dy$, where R is the region shown in the figure and $C = 6 \times 10^{-4}$. The value of I equals ________. (Give the answer up to two decimal places.) **[Set - 01]**



3.14 Consider a function $f(x, y, z)$ given by $f(x, y, z) = (x^2 + y^2 - 2z^2)(y^2 + z^2)$. The partial derivative of this function with respect to x at the point, $x = 2$, $y = 1$ and $z = 3$ is ______. **[Set - 02]**

2018 IIT Guwahati

3.15 As shown in the figure, C is the arc from the point (3, 0) to the point (0, 3) on the circle $x^2 + y^2 = 9$. The value of the integral

$$\int_C (y^2 + 2xy)\, dx + (x^2 + 2xy)\, dy$$

Is _____ (up to 2 decimal places).

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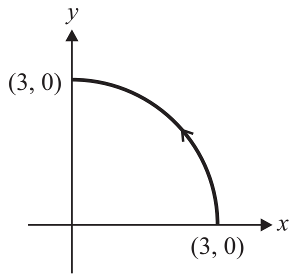


2019 IIT Madras

3.16 If $f = 2x^3 + 3y^2 + 4z$, the value of line integral $\int_C \text{grad } f \cdot dr$ evaluated over contour C formed by the segments $(-3,-3,2) \to (2,-3,2) \to (2,6,2) \to (2,6,-1)$ is ________.

2020 IIT Delhi

3.17 The value of the following complex integral, with C representing the unit circle centered at origin in the counterclockwise sense, is

$$\int_C \frac{z^2+1}{z^2-2z}\, dz$$

(A) $8\pi i$ (B) πi

(C) $-8\pi i$ (D) $-\pi i$

2021 IIT Bombay

3.18 Suppose the circles $x^2 + y^2 = 1$ and $(x-1)^2 + (y-1)^2 = r^2$ intersect each other orthogonally at the point (u, v). Then $u + v =$ ________.

Mechanical Engineering : ME

1994 IIT Kharagpur

3.1 The value of $\int_0^\infty e^{-y^3} y^{\frac{1}{2}}\, dy$ is

(A) $\frac{1}{2}\sqrt{\pi}$ (B) $\frac{1}{3}\sqrt{\pi}$

(C) $\frac{\sqrt{\pi}}{2}$ (D) $3\sqrt{\pi}$

1995 IIT Kanpur

3.2 The area bounded by the parabola $2y = x^2$ and the lines $x = y - 4$ is equal to

(A) 6 (B) 18

(C) ∞ (D) None

3.3 Given $y = \int_1^{x^2} \cos t\, dt$ then $\frac{dy}{dx}$ is

(A) $2\cos\frac{x^2}{2}$ (B) $2x\cos\frac{x^2}{2}$

(C) $2x\cos x^2$ (D) $x^2\cos x$

1997 IIT Madras

3.4 The curve given by the equation $x^2 + y^2 = 3axy$ is,

(A) Symmetrical about x-axis.

(B) Symmetrical about y-axis.

(C) Symmetrical about the line $y = x$.

(D) Tangential to $x = y = \frac{a}{3}$.

1998 IIT Delhi

3.5 If $\phi(x) = \int_0^{x^2} \sqrt{t}\, dt$, then $\frac{d\phi}{dx}$ is

(A) $2x^2$ (B) $\sqrt{x}$

(C) 0 (D) 1

2000 IIT Kharagpur

3.6 $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(x+y)\, dx\, dy$

(A) 0 (B) π

(C) $\frac{\pi}{2}$ (D) 2

2003 IIT Madras

3.7 The area enclosed between the parabola $y = x^2$ and the straight line $y = x$ is

(A) 1/8 (B) 1/6

(C) 1/3 (D) 1/2

2004 IIT Delhi

3.8 If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ then $\frac{dy}{dx}$ is

(A) $\sin\frac{\theta}{2}$ (B) $\cos\frac{\theta}{2}$

(C) $\tan\frac{\theta}{2}$ (D) $\cot\frac{\theta}{2}$

3.9 The volume of the object expressed in spherical co-ordinates is given by

$$V = \int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^1 r^2 \sin\phi \, dr d\phi d\theta$$

The value of the integral is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{4}$

2005 IIT Bombay

3.10 $\int_{-a}^{a} (\sin^6 x + \sin^7 x)\, dx$ is equal to

(A) $2\int_0^a \sin^6 x \, dx$

(B) $2\int_0^a \sin^7 x \, dx$

(C) $2\int_0^a (\sin^6 x + \sin^7 x)\, dx$

(D) zero

3.11 Changing the order of the integration in the double integral leads to

$$I = \int_0^8 \int_{\frac{x}{4}}^2 f(x, y) dy\, dx \text{ leads to}$$

$$I = \int_r^s \int_p^q f(x, y) dy\, dx \text{. What is } q?$$

(A) $4y$ (B) $16y^2$

(C) x (D) 8

2006 IIT Kharagpur

3.12 Assuming $i = \sqrt{-1}$ and t is a real number, $\int_0^{\frac{\pi}{3}} e^{it} dt$ is

(A) $\frac{\sqrt{3}}{2} + i\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2} - i\frac{1}{2}$

(C) $\frac{1}{2} + i\left(\frac{\sqrt{3}}{2}\right)$ (D) $\frac{1}{2} + i\left(1 - \frac{\sqrt{3}}{2}\right)$

2008 IISc Bangalore

3.13 The length of the curve $y = \frac{2}{3}x^{3/2}$ between $x = 0$ and $x = 1$ is

(A) 0.27 (B) 0.67

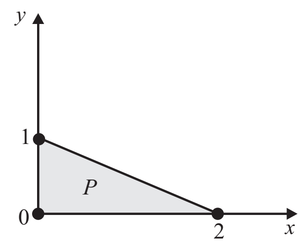
(C) 1 (D) 1.22

3.14 Which of the following integrals is unbounded?

(A) $\int_0^{\frac{\pi}{4}} \tan x \, dx$ (B) $\int_0^{\infty} \frac{1}{x^2+1} dx$

(C) $\int_0^{\infty} xe^{-x} \, dx$ (D) $\int_0^1 \frac{1}{1-x} dx$

3.15 Consider the shaded triangular region P shown in the figure. What is $\iint_P xy dx dy$?



(A) $\frac{1}{6}$ (B) $\frac{2}{9}$

(C) $\frac{1}{16}$ (D) 1

2009 IIT Roorkee

3.16 The area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ is

(A) $\frac{16}{3}$ (B) 8

(C) $\frac{32}{3}$ (D) 16

2010 IIT Guwahati

3.17 The parabolic arc $y=\sqrt{x}$, $1\le x\le 2$ is revolved around the x-axis. The volume of the solid of revolution is

(A) $\pi/4$ (B) $\pi/2$

(C) $3\pi/4$ (D) $3\pi/2$

3.18 The value of the integral $\int_{-\infty}^{\infty}\frac{dx}{1+x^2}$ is

(A) $-\pi$ (B) $-\pi/2$

(C) $\pi/2$ (D) π

2011 IIT Madras

3.19 If $f(x)$ is an even function and 'a' is a positive real number, then $\int_{-a}^{a} f(x)\,dx$ equals

(A) 0 (B) a

(C) $2a$ (D) $2\int_0^a f(x)\,dx$

2012 IIT Delhi

3.20 The area enclosed between the straight line $y=x$ and the parabola $y=x^2$ in the x-y plane is

(A) 1/6 (B) 1/4

(C) 1/3 (D) 1/2

2013 IIT Bombay

3.21 The value of the definite integral

$$\int_1^e \sqrt{x}\ln(x)\,dx \text{ is}$$

(A) $\frac{4}{9}\sqrt{e^3}+\frac{2}{9}$ (B) $\frac{2}{9}\sqrt{e^3}-\frac{4}{9}$

(C) $\frac{2}{9}\sqrt{e^3}+\frac{4}{9}$ (D) $\frac{4}{9}\sqrt{e^3}-\frac{2}{9}$

2014 IIT Kharagpur

3.22 The value of the integral

$$\int_0^2 \frac{(x-1)^2\sin(x-1)}{(x-1)^2+\cos(x-1)}dx \text{ is}$$ **[Set - 04]**

(A) 3 (B) 0

(C) – 1 (D) – 2

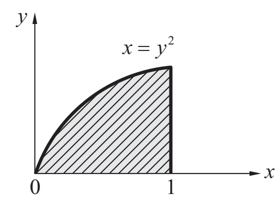
3.23 The value of the integral $\int_0^2\int_0^x e^{x+y}\,dy\,dx$ is, **[Set - 04]**

(A) $\frac{1}{2}(e-1)$ (B) $\frac{1}{2}(e^2-1)^2$

(C) $\frac{1}{2}(e^2-e)$ (D) $\frac{1}{2}\left(e-\frac{1}{e}\right)^2$

2019 IIT Madras

3.24 A parabola $x=y^2$ with $0\le x\le 1$ as shown in figure. The volume of the solid of rotation obtained by rotating the shaded area of 360^0 around the x-axis is



(A) π (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) 2π

3.25 The value of the following definite integral is ______ (round off to three decimal places).

$$\int_1^e x(\ln x)\,dx$$

3.26 The transformation matrix for mirroring a point in x-y plane about the line $y=x$ is given by

(A) $\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$ (B) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

(C) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (D) $\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$

2020 IIT Delhi

3.27 Let $I = \int_{x=0}^{1} \int_{y=0}^{x^2} xy^2 dydx$. Then, I may also be expressed as **[Set - 02]**

(A) $\int_{y=0}^{1} \int_{x=\sqrt{y}}^{1} yx^2 dxdy$

(B) $\int_{y=0}^{1} \int_{x=0}^{\sqrt{y}} yx^2 dxdy$

(C) $\int_{y=0}^{1} \int_{x=0}^{\sqrt{y}} xy^2 dxdy$

(D) $\int_{y=0}^{1} \int_{x=\sqrt{y}}^{1} xy^2 dxdy$

2021 IIT Bombay

3.28 The value of $\int_{0}^{\frac{\pi}{2}} \int_{0}^{\cos\theta} r\sin\theta\, dr\, d\theta$ is **[Set-2]**

(A) 0 (B) π

(C) $\frac{1}{6}$ (D) $\frac{4}{3}$

Civil Engineering : CE

1994 IIT Kharagpur

3.1 The volume generated by revolving the area bounded by the parabola $y^2 = 8x$ and the line $x = 2$ about y axis is,

(A) $\frac{128\pi}{5}$

(B) $\frac{5}{128\pi}$

(C) $\frac{127}{5\pi}$

(D) None of the above

3.2 The integration of $\int \log x\, dx$ has the value

(A) $(x\log x - 1)$

(B) $\log x - x$

(C) $x(\log x - 1)$

(D) None of the above

1995 IIT Kanpur

3.3 The area bounded by the parabola $2y = x^2$ and the line $x = y - 4$ is equal to

(A) 6 (B) 18

(C) ∞ (D) None of these

3.4 By reversing the order of integration $\int_0^2 \int_{x^2}^{2x} f(x,y)\, dy\, dx$ may be represented as _______.

(A) $\int_0^2 \int_{x^2}^{2x} f(x,y)\, dy\, dx$

(B) $\int_0^2 \int_{y}^{\sqrt{y}} f(x,y)\, dx\, dy$

(C) $\int_0^4 \int_{\frac{y}{2}}^{\sqrt{y}} f(x,y)\, dx\, dy$

(D) $\int_{x^2}^{2x} \int_0^2 f(x,y)\, dy\, dx$

1997 IIT Madras

3.5 Area bounded by the curve $y = x^2$ and lines $x = 4$ and $y = 0$ is given by

(A) 64 (B) $\frac{64}{3}$

(C) $\frac{128}{3}$ (D) $\frac{128}{4}$

1999 IIT Bombay

3.6 The function $f(x) = e^x$ is

(A) Even

(B) Odd

(C) Neither even nor odd

(D) None of the above

2001 IIT Kanpur

3.7 Value of the integral $I=\int_0^{\frac{\pi}{4}}\cos^2 x dx$ is

(A) $\frac{\pi}{8}+\frac{1}{4}$ (B) $\frac{\pi}{8}-\frac{1}{4}$

(C) $-\frac{\pi}{8}-\frac{1}{4}$ (D) $-\frac{\pi}{8}+\frac{1}{4}$

2002 IISc Bangalore

3.8 The value of the following improper integral is $\int_0^1 x\ln x\, dx$

(A) $\frac{1}{4}$ (B) 0

(C) $\frac{-1}{4}$ (D) 1

3.9 The value of the following definite integral $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{\sin 2x}{1+\cos x}dx$ is,

(A) –2 ln2 (B) 2

(C) 0 (D) $(\ln 2)^2$

2003 IIT Madras

3.10 If P, Q and R are three points having coordinates (3, –2, –1), (1, 3, 4), (2, 1, –2) in *XYZ* space, then the distance from point *P* to plane *OQR* (*O* being the origin of the coordinate system) is given by

(A) 3 (B) 5

(C) 7 (D) 9

2005 IIT Bombay

3.11 Value of the integral $\oint_C (xydy-y^2dx)$, where, C is the square cut from the first quadrant by the line x = 1 and y = 1will be (Use Green's theorem to change the line integral into double integral)

(A) $\frac{1}{2}$ (B) 1

(C) $\frac{3}{2}$ (D) $\frac{5}{3}$

2006 IIT Kharagpur

3.12 What is the area common to the circles $r=a$ and $r=2a\cos\theta$?

(A) $0.524\,a^2$ (B) $0.614\,a^2$

(C) $1.047\,a^2$ (D) $1.228\,a^2$

2007 IIT Kanpur

3.13 Evaluate $\int_0^\infty \frac{\sin t}{t}dt$

(A) π (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$

2008 IISc Bangalore

3.14 The value of $\int_0^3\int_0^x (6-x-y)\,dx\,dy$ is

(A) 13.5 (B) 27.0

(C) 40.5 (D) 54.0

2010 IIT Guwahati

3.15 A parabolic cable is held between two supports at the same level. The horizontal span between the supports is *L*. The sag at the mid-span is *h*. The equation of the parabola is $y=4h\frac{x^2}{L^2}$, where x is the horizontal coordinate and y is the vertical coordinate with the origin at the centre of the cable. The expression for the total length of the cable is

(A) $\int_0^L\sqrt{1+64\frac{h^2x^2}{L^4}}dx$

(B) $2\int_0^{L/2}\sqrt{1+64\frac{h^3x^2}{L^4}}dx$

(C) $\int_0^{L/2}\sqrt{1+64\frac{h^2x^2}{L^4}}dx$

(D) $2\int_0^{L/2}\sqrt{1+64\frac{h^2x^2}{L^4}}dx$

2011 IIT Madras

3.16 What is the value of the definite integral $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$?

(A) 0 (B) $\frac{a}{2}$

(C) a (D) $2a$

2013 IIT Bombay

3.17 The value of $\int_0^{\frac{\pi}{6}}\cos^4 3\theta\sin^3 6\theta\, d\theta$

(A) 0 (B) $\frac{1}{15}$

(C) 1 (D) $\frac{8}{3}$

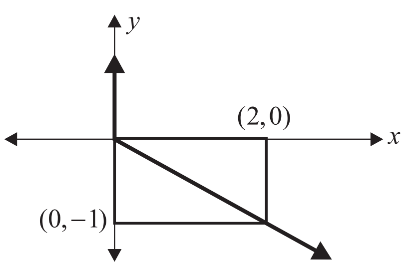
2015 IIT Kanpur

3.18 Given $i=\sqrt{-1}$, the value of the definite integral, $I=\int_0^{\frac{\pi}{2}}\frac{\cos x+i\sin x}{\cos x-i\sin x}dx$ is **[Set - 02]**

(A) 1 (B) – 1

(C) i (D) $-i$

3.19 Choose the most appropriate equation for the function drawn as a thick line, in the plot below. **[Set - 02]**



(A) $x=y-|y|$ (B) $x=-(y-|y|)$

(C) $x=y+|y|$ (D) $x=-(y+|y|)$

2016 IISc Bangalore

3.20 In a process, the number of cycles to failure decreases exponentially with an increase in load. At a load of 80 units, it takes 100 cycles for failure. When the load is halved, it takes 10000 cycles for failure. The load for which the failure will happen in 5000 cycles is **[Set - 01]**

(A) 40.00 (B) 46.02

(C) 60.01 (D) 92.02

3.21 The value of $\int_0^{\infty}\frac{1}{1+x^2}dx+\int_0^{\infty}\frac{\sin x}{x}dx$ is **[Set - 01]**

(A) $\frac{\pi}{2}$ (B) π

(C) $\frac{3\pi}{2}$ (D) 1

3.22 The area of the region bounded by the parabola $y=x^2+1$ and the straight line $x+y=3$ is **[Set - 01]**

(A) $\frac{59}{6}$ (B) $\frac{9}{2}$

(C) $\frac{10}{3}$ (D) $\frac{7}{6}$

3.23 The angle of intersection of the curves $x^2=4y$ and $y^2=4x$ at point (0, 0) is **[Set - 02]**

(A) 0^0 (B) 30^0

(C) 45^0 (D) 90^0

3.24 The area between the parabola $x^2=8y$ and the straight line $y=8$ is ______. **[Set - 02]**

3.25 If $f(x)=2x^7+3x-5$, which of the following is a factor of $f(x)$? **[Set - 02]**

(A) (x^3+8) (B) $(x-1)$
(C) $(2x-5)$ (D) $(x+1)$

2017 IIT Roorkee

3.26 Let x be a continuous variable defined over the interval $(-\infty,\infty)$ and $f(x)=e^{-x-e^{-x}}$. **[Set - 02]**

The integral $g(x)=\int f(x)dx$ is equal to

(A) $e^{e^{-x}}$ (B) $e^{-e^{-x}}$
(C) $e^{-e^{x}}$ (D) e^{-x}

3.27 Consider the following definite integral

$$I=\int_0^1 \frac{(\sin^{-1}x)^2}{\sqrt{1-x^2}}dx$$

The value of the integral is, **[Set - 02]**

(A) $\frac{\pi^3}{24}$ (B) $\frac{\pi^3}{12}$
(C) $\frac{\pi^3}{48}$ (D) $\frac{\pi^3}{64}$

2018 IIT Guwahati

3.28 The value of the integral $\int_0^{\pi} x\cos^2 x.dx$ is **[Set - 01]**

(A) $\frac{\pi^2}{8}$ (B) $\frac{\pi^2}{4}$
(C) $\frac{\pi^2}{2}$ (D) π^2

2020 IIT Delhi

3.29 The area of an ellipse represented by an equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is **[Set - 01]**

(A) πab (B) $\frac{\pi ab}{4}$
(C) $\frac{4\pi ab}{3}$ (D) $\frac{\pi ab}{2}$

2021 IIT Bombay

3.30 The value (round off to one decimal place) of $\int_{-1}^{1} xe^{|x|}dx$ is ______. **[Set-2]**

3.31 The volume determined from

$$\iiint_V 8xyz\,dV \text{ for } V=[2,3]\times[1,2]\times[0,1]$$

will be (in integer) _______. **[Set-1]**

Instrumentation Engineering : IN

2005 IIT Bombay

3.1 The value of the integral $\int_{-1}^{1}\frac{1}{x^2}dx$ is

(A) 2 (B) does not exist
(C) – 2 (D) ∞

3.2 The curves, for which the curvature ρ at any point is equal to $\cos^3\theta$, where θ is the angle made by the tangent at that point with the positive direction of the x-axis, are (given $\rho=y''/[1+(y')^2]^{3/2}$, where y' and y'' are the first and second derivatives of y with respect to x)

(A) circles (B) parabolas
(C) ellipses (D) hyperbolas

2007 IIT Kanpur

3.3 The value of the integral $\int_0^{\infty}\int_0^{\infty} e^{-x^2}e^{-y^2}dxdy$ is

(A) $\sqrt{\pi/2}$ (B) $\sqrt{\pi}$
(C) π (D) $\pi/4$

2008 IISc Bangalore

3.4 Given $y=x^2+2x+10$ the value of $\left.\frac{dy}{dx}\right|_{x=1}$ is equal to

(A) 0 (B) 4

(C) 12 (D) 13

3.5 The expression $e^{-\ln(x)}$ for $x > 0$ is equal to

(A) $-x$ (B) x

(C) x^{-1} (D) $-x^{-1}$

2009 IIT Roorkee

3.6 $\int_0^{\frac{\pi}{4}} \left(\frac{1-\tan x}{1+\tan x}\right) dx$ evaluates to

(A) 0 (B) 1

(C) ln 2 (D) $\frac{1}{2}\ln 2$

2015 IIT Kanpur

3.7 The double integral $\int_0^a \int_0^y f(x,y)\,dx\,dy$ is equivalent to

(A) $\int_0^x \int_0^y f(x,y)\,dx\,dy$ (B) $\int_0^\alpha \int_x^y f(x,y)\,dx\,dy$

(C) $\int_0^a \int_x^a f(x,y)\,dx\,dy$ (D) $\int_0^a \int_0^a f(x,y)\,dx\,dy$

2019 IIT Madras

3.8 The curve $y = f(x)$ is such that the tangent to the curve at every point (x, y) has a $Y-axis$ intercept c, given by $c = -y$. Then, $f(x)$ is proportional to

(A) x^{-1} (B) x^2

(C) x^3 (D) x^4

2020 IIT Delhi

3.9 A straight line drawn on an x-y plane intercepts the x-axis at –0.5 and the y-axis at 1. The equation that describes this line is ________.

(A) $y = x - 0.5$ (B) $y = 0.5x - 1$

(C) $y = 2x + 1$ (D) $y = -0.5x + 1$

Chemical Engineering : CH

2000 IIT Kharagpur

3.1 For an even function $f(x)$

(A) $\int_{-a}^{a} f(x)\,dx = 0$

(B) $\int_{-a}^{a} f(-x)\,dx = 0$

(C) $f(x) = -f(-x) = 0$

(D) $f(x) = f(-x)$

3.2 The line integral of

$$\int_C \left\{\frac{y}{x^2+y^2}dx - \frac{x}{x^2+y^2}dy\right\},$$

where C is the unit circle around the origin traversed once in the counter-clockwise direction, is

(A) -2π (B) 0

(C) 2π (D) π

2004 IIT Delhi

3.3 Value of the integral $\int_{-2}^{2} \frac{dx}{x^2}$ is

(A) 0 (B) 0.25

(C) 1 (D) ∞

2006 IIT Kharagpur

3.4 The derivative of $|x|$ with respect to x when $x \neq 0$ is

(A) $\frac{|x|}{x}$ (B) -1

(C) 1 (D) Undefined

2007 IIT Kanpur

3.5 Evaluate the following integral $(n \neq 0)$

$$\int \left(-xy^n dx + x^n y dy\right)$$

Within the area of a triangle with vertices (0, 0) (1, 0) and (1, 1) (counter clockwise)

(A) 0 (B) $\frac{1}{n+1}$

(C) $\frac{1}{2}$ (D) $\frac{n}{2}$

3.6 The family of curves that is orthogonal to $xy = c$ is

(A) $y = c_1 x$ (B) $y = \frac{c_1}{x}$

(C) $y^2 x^2 = c_1$ (D) $y^2 - x^2 = c_1$

2011 IIT Madras

3.7 The value of the improper integral $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)}$ is,

(A) -2π (B) 0

(C) π (D) 2π

3.8 Which one of the following functions $y(x)$ has the slope of its tangent equal to $\frac{ax}{y}$? **Note :** a and b are real constants

(A) $y = \frac{x+b}{a}$ (B) $y = ax + b$

(C) $y = \frac{\sqrt{x^2+b}}{a}$ (D) $y = \sqrt{ax^2+b}$

2012 IIT Delhi

3.9 If a is a constant, then the value of the integral $a^2 \int_0^{\infty} x e^{-ax} dx$ is,

(A) $\frac{1}{a}$ (B) a

(C) 1 (D) 0

2013 IIT Bombay

3.10 Evaluate $\int \frac{dx}{e^x - 1}$

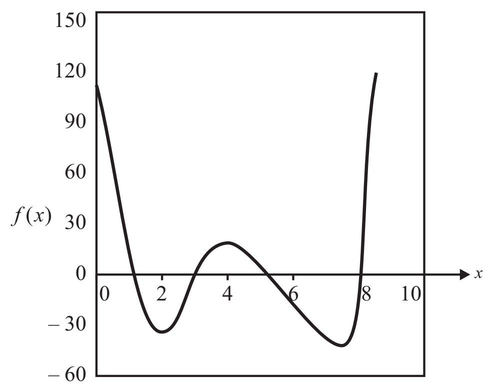
(Note : C is a constant of integration)

(A) $\frac{e^x}{e^x - 1} + C$ (B) $\frac{\ln(e^x - 1)}{e^x} + C$

(C) $\ln\left(\frac{e^x}{e^x - 1}\right) + C$ (D) $\ln(1 - e^{-x}) + C$

2017 IIT Roorkee

3.11 The number of positive roots of the function $f(x)$ shown below in the range $0 < x < 6$ is _____.



Production & Industrial : PI

2008 IISc Bangalore

3.1 The value of the integral $\int_{-\pi/2}^{\pi/2} x \cos x \, dx$ is,

(A) 0 (B) $\pi - 2$

(C) π (D) $\pi + 2$

2009 IIT Roorkee

3.2 The total derivative of the function 'xy' is,

(A) $x\,dy + y\,dx$ (B) $x\,dx + y\,dy$

(C) $dx + dy$ (D) $dx - dy$

2010 IIT Guwahati

3.3 The integral $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-x^2}{2}} dx$ is equal to

(A) $\frac{1}{2}$ (B) $1/\sqrt{2}$

(C) 1 (D) ∞

3.4 If $f(x) = \sin|x|$ then the value of $\frac{df}{dx}$ at $x = \frac{-\pi}{4}$ is

(A) 0 (B) $\frac{1}{\sqrt{2}}$

(C) $-\frac{1}{\sqrt{2}}$ (D) 1

2012 IIT Delhi

3.5 The area enclosed between the straight line $y = x$ and the parabola $y = x^2$ in the x-y plane is

(A) 1/6 (B) 1/4

(C) 1/3 (D) 1/2

2013 IIT Bombay

3.6 The value of the definite integral $\int_1^e \sqrt{x}\ln(x)\,dx$ is

(A) $\frac{4}{9}\sqrt{e^2} + \frac{2}{9}$ (B) $\frac{2}{9}\sqrt{e^3} - \frac{4}{9}$

(C) $\frac{2}{9}\sqrt{e^3} + \frac{4}{9}$ (D) $\frac{4}{9}\sqrt{e^3} - \frac{2}{9}$

2017 IIT Roorkee

3.7 The improper integral

$$\int_0^\infty e^{-2t}\,dt \text{ converges to}$$

(A) 0 (B) 1.0

(C) 0.5 (D) 2.0

2019 IIT Madras

3.8 The solution of $\int_1^a \int_1^b \frac{dxdy}{xy}$ is

(A) log (ab) (B) log (a/b)

(C) log (a)+ In (b) (D) log (a) In (b)

Answers Integral & Differential Calculus

Electronics & Communication : EC

3.1	A	3.2	C	3.3	B	3.4	B	3.5	A
3.6	C	3.7	864	3.8	3	3.9	A	3.10	20
3.11	4.712	3.12	0	3.13	2	3.14	10	3.15	0.785
3.16	– 11	3.17	C	3.18	D	3.19	4.5	3.20	2
3.21	B								

Electrical Engineering : EE

3.1	C	3.2	D	3.3	D	3.4	D	3.5	A
3.6	B	3.7	B	3.8	2	3.9	B	3.10	B
3.11	D	3.12	0.718	3.13	0.99	3.14	40	3.15	0
3.16	139	3.17	D	3.18	1				

Mechanical Engineering : ME

3.1	B	3.2	B	3.3	C	3.4	C	3.5	A
3.6	D	3.7	B	3.8	C	3.9	A	3.10	A
3.11	A	3.12	A	3.13	D	3.14	D	3.15	A
3.16	A	3.17	D	3.18	D	3.19	D	3.20	A
3.21	C	3.22	B	3.23	B	3.24	C	3.25	2.097
3.26	B	3.27	D	3.28	C				

Civil Engineering : CE

3.1	A	3.2	C	3.3	B	3.4	C	3.5	B
3.6	C	3.7	A	3.8	C	3.9	C	3.10	A
3.11	C	3.12	D	3.13	B	3.14	A	3.15	D
3.16	B	3.17	B	3.18	C	3.19	B	3.20	B
3.21	B	3.22	B	3.23	D	3.24	85.33	3.25	B
3.26	B	3.27	A	3.28	B	3.29	A	3.30	0
3.31	15								

Instrumentation Engineering : IN

3.1	B	3.2	B	3.3	D	3.4	B	3.5	C
3.6	D	3.7	C	3.8	B	3.9	C		

Chemical Engineering : CH

3.1	D	3.2	A	3.3	D	3.4	A	3.5	C
3.6	D	3.7	C	3.8	D	3.9	C	3.10	D
3.11	3								

Production & Industrial : PI

3.1	A	3.2	A	3.3	C	3.4	C	3.5	A
3.6	C	3.7	C	3.8	D				

Explanations **Integral & Differential Calculus**

Electronics & Communication : EC

3.1 (A)

Given : $I = \frac{1}{\sqrt{2\pi}} \int_0^\infty \exp\left(\frac{-x^2}{8}\right) dx \quad \ldots \text{(i)}$

$$I = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{\frac{-x^2}{8}} dx$$

Putting $\frac{x^2}{8} = t \Rightarrow x = 2\sqrt{2}\, t^{1/2}$

Differentiating on both sides,

$$dx = \frac{\sqrt{2}}{\sqrt{t}} dt$$

Put all these values in equation (i),

$$I = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-t} \frac{\sqrt{2}}{\sqrt{t}} dt$$

$$I = \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-t} t^{\frac{-1}{2}} dt$$

$$\left[\text{By Gamma function, } \Gamma n = \int_0^\infty e^{-x} x^{n-1} dx\right]$$

Here, $n - 1 = \frac{-1}{2} \Rightarrow n = \frac{1}{2}$

$$n = \frac{1}{2}$$

Thus, $I = \frac{1}{\sqrt{\pi}} \Gamma \frac{1}{2} \qquad \left[\Gamma \frac{1}{2} = \sqrt{\pi}\right]$

$$I = \frac{1}{\sqrt{\pi}} \sqrt{\pi} = 1$$

Hence, the correct option is (A).

3.2 (C)

Given : $I = \int_0^\pi \sin^3\theta \, d\theta$

$$I = \int_0^\pi (1 - \cos^2\theta) \sin\theta \, d\theta \quad \ldots \text{(i)}$$

Let $\cos\theta = t$

Differentiating on both sides,

$$-\sin\theta \, d\theta = dt,$$

At $\theta = 0$, $t = \cos 0 = 1$

At $\theta = \pi$, $t = \cos \pi = -1$

Then, $I = -\int_1^{-1} (1 - t^2) dt$

$$I = \int_{-1}^{1} (1 - t^2) dt$$

$$I = \left[t - \frac{t^3}{3}\right]_{-1}^{1}$$

$$I = \left[\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)\right]$$

$$I = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

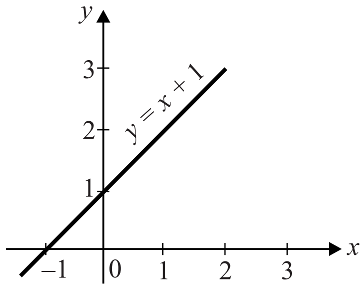
Hence, the correct option is (C).

3.3 (B)

Given : $I = \int_1^2 y \, dx$

Method 1

From the given plot, the equation of the straight line for y can be written as $y = x + 1$



$y = mx + C$ [where, $m = 1$ and $C = 1$]

$$I = \int_1^2 y dx = \int_1^2 (x+1)\, dx$$

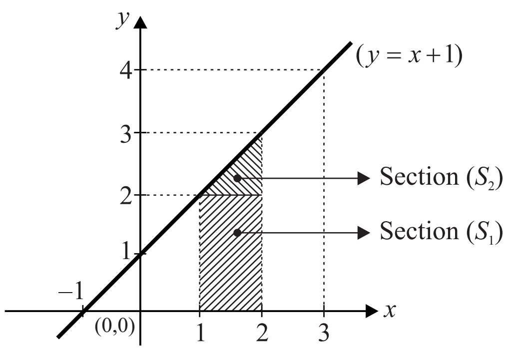
$$I = \left[\frac{x^2}{2} + x\right]_1^2 = 4 - \frac{3}{2} = \frac{5}{2} = 2.5$$

Hence, the correct option is (B).

Method 2

$$I = \int_1^2 y\, dx$$

Here, I represent the area covered by the curve given by the function 'y' over x-axis from limits [1, 2].



In this figure, the area I is comprised of two sections S_1 and S_2.

So, I = Area (S_1) + Area (S_2)

$$I = 1\times 2 + \frac{1}{2}\times 1\times 1$$

$$I = 2 + 0.5 = 2.5 \text{ square unit}$$

Hence, the correct option is (B).

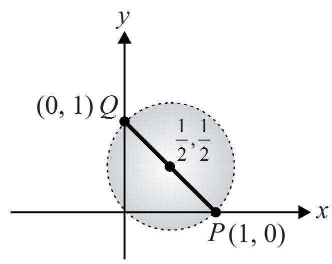
Scan for Video Solution

3.4 (B)

Given : $I = 2\int_P^Q (x\,dx + y\,dy)$ ...(i)

Method 1

Two points P and Q are in xy plane, where $P(1, 0)$ and $Q(0, 1)$.



$$I = 2\int_p^Q (x\,dx + y\,dy)$$

$$I = 2\int_1^0 x\,dx + 2\int_0^1 y\,dy$$

$$I = 2\left[\frac{x^2}{2}\right]_1^0 + 2\left[\frac{y^2}{2}\right]_0^1 = 0$$

Hence, the correct option is (B).

Method 2

Let, $f(x, y) = \phi dx + \psi dy$

On comparing $f(x, y)$ with equation (i),

$$I = 2\int_P^Q f(x, y)$$

Where, $\phi = x$, $\psi = y$

So, $\frac{d\phi}{dy} = 0$, $\frac{d\psi}{dx} = 0$

According to Green's theorem,

$$\oint(\phi dx + \psi dy) = \iint\left(\frac{d\psi}{dx} - \frac{d\phi}{dy}\right)dxdy$$

$$I = \iint\left(\frac{d\psi}{dx} - \frac{d\phi}{dy}\right)dxdy$$

$$I = \iint(0 - 0)\,dxdy$$

$$I = 0$$

Hence, the correct option is (B).

3.5 (A)

Given : $g(x, y) = 4x^3 + 10y^4$ ...(i)

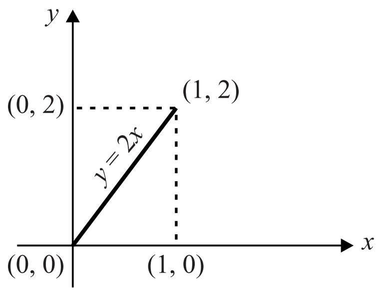
The straight line passing (0, 0) to (1, 2) is given by,

$$y = mx + c$$

Intercept of y-axis, $c = 0$.

Slope, $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-0}{1-0} = 2$

So, $y = 2x$ ...(ii)



From equations (i) and (ii),

$$g(x, y) = 4x^3 + 10(2x)^4$$

Since, the two variable function $g(x, y)$ has now been converted to a single variable function [say $H(x)$].

Therefore, the required integral can be computed simply by a single integration with respect to x over the limits (0, 1).

$$H(x) = 4x^3 + 160x^4$$

$$I = \int_0^1 H(x)\,dx$$

$$I = \int_0^1 (4x^3 + 160x^4)\,dx$$

$$I = 4\int_0^1 x^3 dx + 160\int_0^1 x^4 dx$$

$$I = 4\left[\frac{x^4}{4}\right]_0^1 + 160\left[\frac{x^5}{5}\right]_0^1$$

$$I = 1 + 32 = 33$$

Hence, the correct option is (A).

3.6 (C)

Given : $f(x) = \dfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{\frac{-x^2}{2}} dx$ ... (i)

Put $\dfrac{x^2}{2} = t \Rightarrow x = \sqrt{2t}$

Differentiating on both sides,

$$xdx = dt$$

$$dx = \frac{1}{x}dt$$

$$dx = \frac{1}{\sqrt{2}} t^{-1/2} dt$$

Put all these values in equation (i),

$$f(x) = \frac{1}{\sqrt{2\pi}} \times 2\int_0^{\infty} e^{-t} \frac{1}{\sqrt{2}} t^{-1/2}\,dt$$

$$f(x) = \frac{1}{\sqrt{\pi}}\int_0^{\infty} e^{-t} t^{\frac{1}{2}-1} dt$$

[For even function $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$]

By Gamma function,

$$\Gamma n = \int_0^{\infty} e^{-x} x^{n-1} dx$$

Here, $\dfrac{-1}{2} = n - 1 \Rightarrow n = \dfrac{1}{2}$

$$f(x) = \frac{1}{\sqrt{\pi}}\Gamma\left(\frac{1}{2}\right) \quad \left[\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}\right]$$

$$f(x) = \frac{\sqrt{\pi}}{\sqrt{\pi}} = 1$$

Hence, the correct option is (C).

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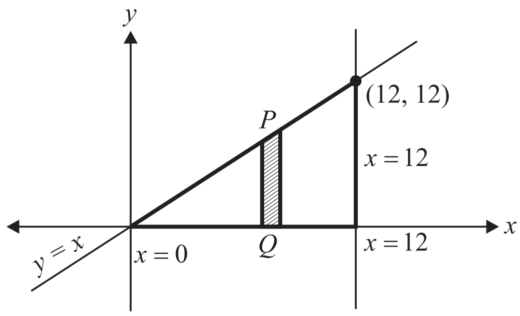
3.7 864

Given : $z(x, y) = x + y$

and $0 \le y \le x$, $0 \le x \le 12$



Thus, the volume as double integral, under the surface,

$$V = \iint_s z\,dxdy \quad (\text{where, } 0 < z < x + y)$$

$$V = \int_{x=0}^{12}\int_{y=0}^{x} (x+y)dxdy$$

$$V = \int_{x=0}^{12}\left(xy + \frac{y^2}{2}\right)_0^x dx$$

$$V = \int_{x=0}^{12}\left(x^2 + \frac{x^2}{2}\right)dx = \int_{x=0}^{12}\frac{3x^2}{2}dx$$

$$V = \frac{3}{2}\times\left(\frac{x^3}{3}\right)_0^{12} = \frac{3}{2}\times\frac{1}{3}\times 12^3$$

$$V = 864$$

Hence, the volume under the surface is **864**.

3.8 **3**

Given : $I = \int_{-\infty}^{\infty} 12\cos(2\pi t)\frac{\sin(4\pi t)}{4\pi t}dt$

$\left[\because\ 12\cos(2\pi t)\frac{\sin(4\pi t)}{4\pi t} \text{ is a even function}\right]$

$$I = \frac{12}{4\pi}\times 2\int_0^{\infty}\frac{\cos 2\pi t \sin 4\pi t}{t}dt$$

$$I = \frac{3}{\pi}\int_0^{\infty}\frac{2\cos 2\pi t\sin 4\pi t}{t}dt$$

$$[2\cos A\sin B = \sin(A+B) - \sin(A-B)]$$

$$I = \frac{3}{\pi}\times\int_0^{\infty}\frac{\sin(6\pi t) - \sin(-2\pi t)}{t}dt$$

$$I = \frac{3}{\pi}\times\int_0^{\infty}\frac{\sin(6\pi t) + \sin(2\pi t)}{t}dt$$

Let $\quad \dfrac{\sin(6\pi t) + \sin(2\pi t)}{t} = h(t)$

Then, $\quad I = \dfrac{3}{\pi}\times\displaystyle\int_0^{\infty} h(t)\,dt$

By property of Laplace transforms,

$$\int_0^{\infty} h(t)\,dt = H(s)\big|_{s=0}$$

So, $\quad I = \dfrac{3}{\pi}[H(s)]_{s=0}$ $\quad\quad$ … (i)

Taking Laplace transform of $h(t)$,

$$H(s) = L\left\{\frac{\sin 6\pi t}{t}\right\} + L\left\{\frac{\sin 2\pi t}{t}\right\}$$

By the property,

$$L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} F(s)\,ds$$

So, $\quad H(s) = \displaystyle\int_s^{\infty}\frac{6\pi}{s^2 + 36\pi^2}ds + \int_s^{\infty}\frac{2\pi}{s^2 + 4\pi^2}ds$

$$H(s) = 6\pi\left[\tan^{-1}\left(\frac{s}{6\pi}\right)\times\frac{1}{6\pi}\right]_s^{\infty} + 2\pi\left[\tan^{-1}\left(\frac{s}{2\pi}\right)\times\frac{1}{2\pi}\right]_s^{\infty}$$

$$H(s) = \left[\tan^{-1}\infty - \tan^{-1}\left(\frac{s}{6\pi}\right)\right] + \left[\tan^{-1}\infty - \tan^{-1}\left(\frac{s}{2\pi}\right)\right]$$

$$H(s) = \pi - \tan^{-1}\left(\frac{s}{6\pi}\right) - \tan^{-1}\left(\frac{s}{2\pi}\right)$$

From equation (i),

$$I = \frac{3}{\pi}\times\left[\pi - \tan^{-1}\left(\frac{s}{6\pi}\right) - \tan^{-1}\left(\frac{s}{2\pi}\right)\right]_{s=0}$$

$$I = \frac{3}{\pi}\times[\pi - 0 - 0]$$

$$I = 3$$

Hence, the value of the integral is **3.**

3.9 (A)

Given : $f_1(x, y) = x^2 + y^2$

$$f_2(x, y) = 6y + 4x$$

According to the question,

$$\frac{\partial}{\partial y} f_1(x, y) = \frac{\partial}{\partial x} f_2(x, y)$$

$$\frac{\partial}{\partial y}[x^2 + y^2] = \frac{\partial}{\partial x}[6y + 4x]$$

$$2y = 4$$

$$y = 2$$

It shows the contour of constant line parallel to x-axis at a distance of 2 above the x-axis.

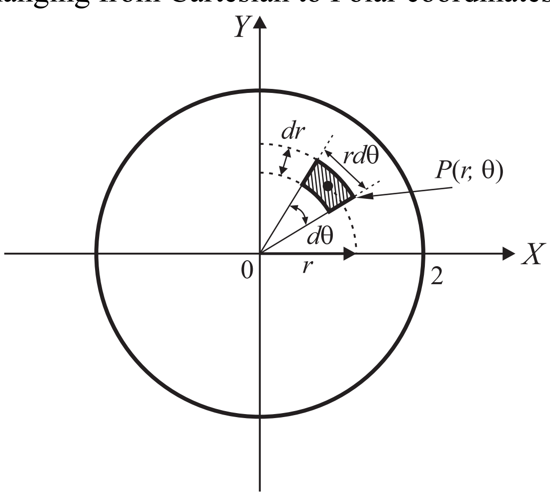
Hence, the correct option is (A).

3.10 20

Given : $I = \frac{1}{2\pi}\iint_D (x + y + 10)\, dxdy$ ...(i)

where, D denotes the disc $x^2 + y^2 \le 4$.

Changing from Cartesian to Polar coordinates.



Put $x = r\cos\theta,\ y = r\sin\theta,$

$$dxdy = rdrd\theta$$

$$r^2\cos^2\theta + r^2\sin^2\theta = 4$$

$$r = 2$$

r varies from 0 to 2 and θ varies from 0 to 2π.

From equation (i),

$$I = \frac{1}{2\pi}\int_{\theta=0}^{2\pi}\int_{r=0}^{2}[r(\cos\theta + \sin\theta) + 10]\, r\, dr\, d\theta$$

$$I = \frac{1}{2\pi}\int_{\theta=0}^{2\pi}\int_{r=0}^{2}\left[r^2(\cos\theta + \sin\theta) + 10r\right] dr\, d\theta$$

$$I = \frac{1}{2\pi}\int_{\theta=0}^{2\pi}(\cos\theta + \sin\theta)\left[\frac{r^3}{3}\right]_{r=0}^{2} d\theta + \frac{10}{2\pi}\int_{\theta=0}^{2\pi}\left[\frac{r^2}{2}\right]_{r=0}^{2} d\theta$$

$$I = \frac{1}{2\pi}\int_{\theta=0}^{2\pi}\frac{8}{3}(\cos\theta + \sin\theta)\, d\theta + \frac{1}{2\pi}\int_{\theta=0}^{2\pi}5(4)\, d\theta$$

$$I = \frac{1}{2\pi}\left[\frac{8}{3}(\sin\theta - \cos\theta)\right]_0^{2\pi} + \frac{1}{2\pi}20[\theta]_0^{2\pi}$$

$$I = \frac{1}{2\pi}\times\frac{8}{3}[(0-1)-(0-1)] + \frac{1}{2\pi}\times 20\times(2\pi)$$

$$I = 20$$

Hence, the value of integral is **20.**

3.11 4.712

Given : A Region is specified as,

$$3 \le \rho \le 5,\ \frac{\pi}{8} \le \phi \le \frac{\pi}{4},\ 3 \le z \le 4.5$$

The volume of cylindrical coordinates is given by,

$$V = \iiint_V \rho\, d\rho\, d\phi\, dz$$

$$V = \int_{\rho=3}^{5}\int_{\phi=\pi/8}^{\pi/4}\int_{z=3}^{4.5}\rho\, d\rho\, d\phi\, dz$$

$$V = \int_{z=3}^{4.5}\int_{\phi=\pi/8}^{\pi/4}\left[\frac{\rho^2}{2}\right]_{\rho=3}^{5} d\phi\, dz$$

$$V = \int_{z=3}^{4.5}\int_{\phi=\pi/8}^{\pi/4}\left[\frac{25-9}{2}\right] d\phi\, dz$$

$$V = \int_{z=3}^{4.5}\int_{\phi=\pi/8}^{\pi/4} 8\, d\phi\, dz = 8[\phi]_{\pi/8}^{\pi/4}[z]_3^{4.5}$$

$$V = 8\left[\frac{\pi}{4} - \frac{\pi}{8}\right]\times(4.5-3) = 8\times\frac{\pi}{8}\times 1.5$$

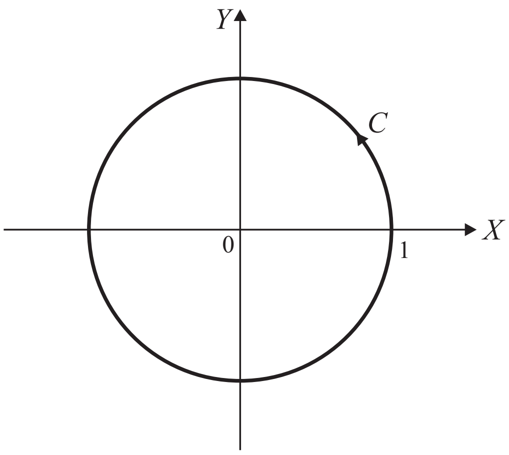
$$V = 4.712$$

Hence, the volume of cylindrical coordinates is **4.712.**

3.12　0

Given : C is the closed curve defined as the circle $x^2 + y^2 = 1$ with C oriented anticlockwise.

Method 1



According to Green's theorem,

$$\oint (\phi dx + \psi dy) = \iint \left(\frac{d\psi}{dx} - \frac{d\phi}{dy} \right) dxdy$$

Here, $\phi = xy^2$, $\psi = x^2 y$

$$\int xy^2 dx + x^2 y\, dy$$

$$= \iint_R \left[\frac{d}{dx}(x^2 y) - \frac{d}{dy}(xy^2) \right] dxdy$$

$$\int xy^2 dx + x^2 y\, dy = \iint_R (2xy - 2xy)\, dxdy$$

$$\int xy^2 dx + x^2 y\, dy = 0$$

Hence, the value of $\oint (xy^2 dx + x^2 y\, dy)$ over the curve C is **0.**

Method 2

$$\oint xy^2 dx + x^2 y\, dy = \oint (xy^2 \hat{a}_x + x^2 y\, \hat{a}_y) \cdot d\vec{l}$$

From strokes theorem,

$$\oint \vec{F} \cdot d\vec{l} = \iint (\nabla \times \vec{F})\, ds$$

$$\nabla \times \vec{F} = \nabla \times (xy^2 \hat{a}_x + x^2 y\, \hat{a}_y) = 0$$

Hence, the value of $\oint (xy^2 dx + x^2 y\, dy)$ over the curve C is **0.**

3.13　2

Given : $I = \int_0^1 \frac{dx}{\sqrt{(1-x)}}$　　…(i)

Method 1

$$I = \int_0^1 \frac{dx}{\sqrt{(1-x)}}$$

Let, $x = \sin^2 \theta$

$dx = 2\sin\theta\cos\theta\, d\theta$

When $x = 0$, $\sin^2\theta = 0$

$\theta = 0^0$

When $x = 1$, $\sin^2\theta = 1$

$\theta = \pi/2$

From equation (i),

$$I = \int_0^{\pi/2} \frac{2\sin\theta\cos\theta\, d\theta}{\sqrt{(1-\sin^2\theta)}} = \int_0^{\pi/2} \frac{2\sin\theta\cos\theta\, d\theta}{\cos\theta}$$

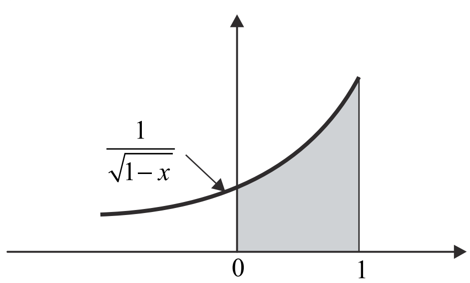
$$I = \int_0^{\pi/2} 2\sin\theta\, d\theta = 2[-\cos\theta]_0^{\pi/2}$$

$$I = 2$$

Hence, the value of integral is **2.**

Method 2

$I = \int_0^1 \frac{1}{\sqrt{(1-x)}} dx$ is a form of definite integral.



Let, $(1-x) = t$

$-dx = dt$

At $x = 0 \rightarrow t = 1$

At $x = 1 \rightarrow t = 0$

$$I = \int_1^0 \frac{-dt}{\sqrt{t}} = \int_0^1 \frac{dt}{\sqrt{t}}$$

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$$I=\int_0^1 t^{-1/2}dt=\left[\frac{t^{\frac{-1}{2}+1}}{-\frac{1}{2}+1}\right]_0^1$$

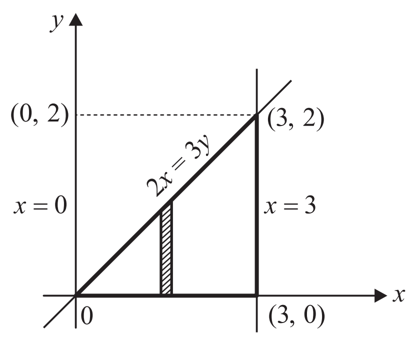
$$I=2\left[\sqrt{t}\right]_0^1=2$$

Hence, the value of integral is **2.**

3.14 10

Given : Straight lines : $2x=3y$, $y=0$, $x=3$

Plane : $x+y+z=6$



Required volume as double integral is given by,

$$V=\iint_s z\,dxdy$$

$$V=\int_0^3\int_0^{2x/3}(6-x-y)\,dx\,dy$$

$$V=\int_0^3\left[6y-xy-\frac{y^2}{2}\right]_0^{2x/3}dx$$

$$V=\int_0^3\left[4x-\frac{2x^2}{3}-\frac{2x^2}{9}\right]dx$$

$$V=\int_0^3\left[4x-\frac{8x^2}{9}\right]dx=\left[\frac{4x^2}{2}-\frac{8}{9}\frac{x^3}{3}\right]_0^3$$

$$V=\frac{4}{2}\times3^2-\frac{8}{9}\times\frac{3^3}{3}=18-8=10$$

Hence, the volume is **10.**

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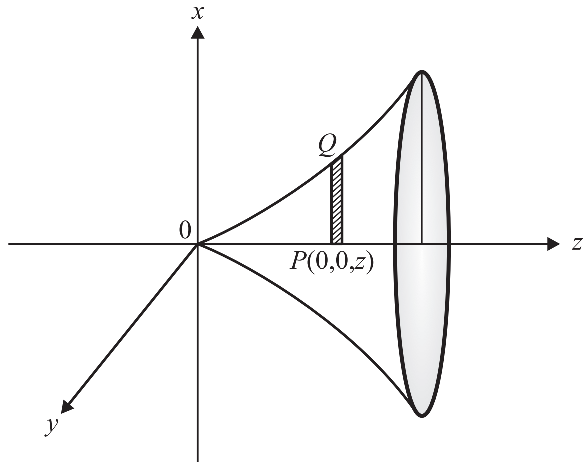
3.15 0.785

Given : $x^2+y^2\le z^3;\ 0\le z\le 1$

Let $x^2+y^2=l^2\Rightarrow l=\sqrt{x^2+y^2}$

The revolution is about z-axis.

The volume of revolution is given by,



$$V=\int_0^1 \pi l^2 dz$$

Putting, $l^2=x^2+y^2=z^3$

$$V=\int_0^1 \pi z^3 dz$$

$$V=\left[\frac{\pi z^4}{4}\right]_0^1=\frac{\pi}{4}=0.7853$$

Hence, the volume of R is **0.785.**

Note : Here in the question volume of region R is asked i.e. it is nothing but the volume forming by revolving the region in 3D about z-axis.

3.16 - 11

Given : $I=\int_C(2z\,dx+2y\,dy+2x\,dz)$ ... (i)

Method 1

The equation of straight line passing through (x_1, y_1, z_1) to (x_2, y_2, z_2) is given by,

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}=t$$

At point (0, 2, 1) to (4, 1, –1),

$$\frac{x-0}{4-0}=\frac{y-2}{1-2}=\frac{z-1}{-1-1}=t$$

$$x=4t,\ y=-t+2,\ z=-2t+1$$

$$dx=4td,\ dy=-dt,\ dz=-2dt$$

For $x=0,\ y=2,\ z=1$

$$t=0$$

For $x=4,\ y=1,\ z=-1$

$$t=1$$

From equation (i),

$$I=\int_C 2(-2t+1)4dt+2(-t+2)(-dt)+2(4t)(-2dt)$$

$$I=\int_0^1(-16t+8+2t-4-16t)\,dt$$

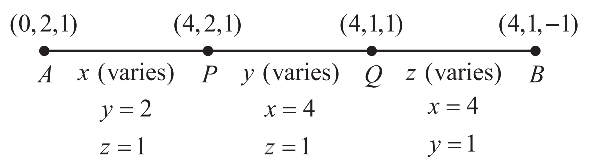
$$I=\int_0^1(4-30t)\,dt$$

$$I=\left|4t-\frac{30t^2}{2}\right|_0^1=4-15=-11$$

Hence, the value of I is **–11.**

Method 2

Integrate $2zdx+2ydy+2xdz$ along path from point $A(0,2,1)$ to $B(4,1,-1)$.



Step I : Integrate along line joining $A(0,2,1)$ to $P(4,2,1)$

$$dy=dz=0 \text{ and } z=1$$

$$I_1=\int_0^4 2zdx=8$$

Step II : Integrate along line joining $P(4,2,1)$ to $Q(4,1,1)$

$$dx=dz=0$$

$$I_2=\int_2^1 2ydy=\left[y^2\right]_2^1=-3$$

Step III : Integrate along line joining $Q(4,1,1)$ to $B(4,1,-1)$

$$dx=dy=0 \text{ and } x=4$$

$$I_3=\int_1^{-1}2xdz=\int_1^{-1}8dz=-16$$

Thus, $I=I_1+I_2+I_3=8-3-16=-11$

Hence, the value of I is **–11.**

3.17 (C)

Given : $I_1=\int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dy\right)dx$

and $I_2=\int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dx\right)dy$

Solving for I_1,

$$I_1=\int_0^1\int_0^1\left(\frac{x-y+x-x}{(x+y)^3}\right)dy\,dx$$

$$I_1=\int_0^1\int_0^1\left(\frac{2x-(x+y)}{(x+y)^3}\right)dy\,dx$$

$$I_1=\int_0^1\left[\int_0^1\left(\frac{2x}{(x+y)^3}-\frac{1}{(x+y)^2}\right)dy\right]dx$$

$$I_1=\int_0^1\left[2x\left(\frac{-1}{2(x+y)^2}\right)+\left(\frac{1}{x+y}\right)\right]dx\Bigg|_0^1$$

$$I_1=\int_0^1\left.\frac{-x+x+y}{(x+y)^2}\right|_0^1dx=\int_0^1\left.\frac{y}{(x+y)^2}\right|_0^1dx$$

$$I_1=\int_0^1\frac{1}{(x+1)^2}dx=-\left[\frac{1}{x+1}\right]\Bigg|_0^1=0.5$$

Now, solving for I_2,

$$I_2=\int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dx\right)dy$$

$$I_2=\int_0^1\left(\int_0^1\frac{x-y+y-y}{(x+y)^3}\right)dx\,dy$$

$$I_2=\int_0^1\left(\int_0^1\frac{1}{(x+y)^2}-\frac{2y}{(x+y)^3}dx\right)dy$$

$$I_2 = \int_0^1 \left[\frac{-1}{(x+y)} + \frac{2y}{2(x+y)^2} \right]_0^1 dy$$

$$I_2 = \int_0^1 \left[\frac{-x-y+y}{(x+y)^2} \right]_0^1 dy = \int_0^1 \left[\frac{-x}{(x+y)^2} \right]_0^1 dy$$

$$I_2 = \int_0^1 \frac{-1}{(1+y)^2} dy = -\left[\frac{-1}{y+1} \right] \Bigg|_0^1 = -0.5$$

Hence, the correct option is (C).

3.18 (D)

Given :

(i) $$f(x,y) = \frac{ax^2+by^2}{xy} \quad \ldots \text{(i)}$$

(ii) $$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} \text{ for } x=1,\ y=2 \quad \ldots \text{(ii)}$$

Taking partial derivative of equation (i) with respect to x,

$$\frac{\partial f}{\partial x} = \frac{1}{y}\left[\frac{x(2ax)-(ax^2+by^2)\times 1}{x^2} \right]$$

$$\left.\frac{\partial f}{\partial x}\right|_{x=1,y=2} = \frac{1}{2}\left[\frac{2a-(a+4b)}{1} \right] = \frac{a-4b}{2}$$

Taking partial derivative of equation (i) with respect to y,

$$\frac{\partial f}{\partial y} = \frac{1}{x}\left[\frac{y(2by)-(ax^2+by^2)\cdot 1}{y^2} \right]$$

$$\left.\frac{\partial f}{\partial y}\right|_{x=1,y=2} = \frac{1}{1}\left[\frac{8b-(a+4b)}{4} \right] = \frac{4b-a}{4}$$

From equation (ii)

$$\frac{a-4b}{2} = \frac{4b-a}{4}$$

$$2(a-4b) = 4b-a$$

$$2a-8b = 4b-a$$

$$3a = 12b$$

$$a = 4b$$

Hence, the correct option is (D).

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3.19 4.5

Given : $r = x^2 + y - z \quad \ldots \text{(i)}$

$$z^3 - xy + yz + y^3 = 1 \quad \ldots \text{(ii)}$$

Here x, y are independent and z is dependent.

Taking partial derivative of equation (i) with respect to x,

$$\frac{\partial r}{\partial x} = \frac{\partial}{\partial x}(x^2+y-z) = \frac{\partial x^2}{\partial x} + \frac{\partial y}{\partial x} - \frac{\partial z}{\partial x}$$

$$\frac{\partial r}{\partial x} = 2x + 0 - \frac{\partial z}{\partial x} \quad \ldots \text{(iii)}$$

[Since y is independent]

Taking partial derivative of equation (ii) with respect to x,

$$3z^2\frac{\partial z}{\partial x} - y + y\frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x}(3z^2+y) = y$$

$$\frac{\partial z}{\partial x} = \frac{y}{3z^2+y} \quad \ldots \text{(iv)}$$

From equation (iii) and (iv),

$$\frac{\partial r}{\partial x} = 2x - \frac{y}{3z^2+y}$$

At point $x=2,\ y=-1,\ z=1$

$$\frac{\partial r}{\partial x} = 4 - \frac{(-1)}{3(1)^2+(-1)} = 4 + \frac{1}{2}$$

$$\left.\frac{\partial r}{\partial x}\right|_{x=2,y=-1,z=1} = 4.5$$

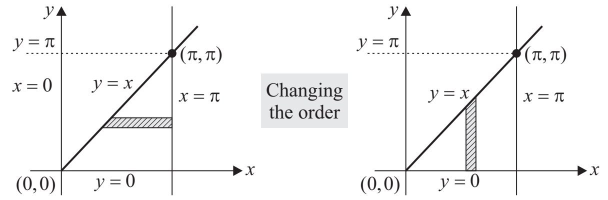
Hence, the value of $\frac{\partial r}{\partial x}$ is **4.5.**

3.20 2

Given : $$I = \int_0^{\pi}\int_y^{\pi} \frac{\sin x}{x} dx\, dy$$

Limits : $x = y,\ x = \pi$ and $y = 0,\ y = \pi$ tracing the limits we get

The strip is horizontal changing into vertical.



$$\int_0^{\pi}\int_y^{\pi}\frac{\sin x}{x}\,dx\,dy=\int_0^{\pi}\int_0^{x}\frac{\sin x}{x}\,dy\,dx$$

$$I=\int_0^{\pi}\left(\frac{\sin x}{x}\right)[y]_0^x\,dx=\int_0^{\pi}\frac{\sin x}{x}\cdot x\,dx$$

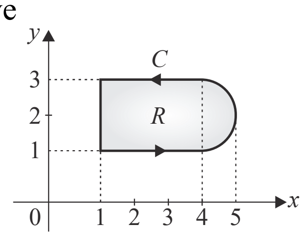
$$I=-[\cos x]_0^{\pi}=-(-1-1)=2$$

Hence, the value of the given integral is **2.**

3.21　B

Given : $\oint_c x\,dy-y\,dx$　　　... (i)

c is the curve



Here, the curve is closed.

So, we can apply Green's theorem :

$$\oint_c \phi\,dx+\psi\,dy=\iint_s\left(\frac{\partial\psi}{\partial x}-\frac{\partial\phi}{\partial y}\right)dx\,dy\quad\ldots\text{(ii)}$$

Comparing equation (i) and (ii) we get,

$$\phi=-y\quad\Rightarrow\quad\frac{\partial\phi}{\partial y}=-1$$

$$\psi=x\quad\Rightarrow\quad\frac{\partial\psi}{\partial x}=1$$

From equation (ii),

$$=\iint_s(1-(-1))\,dx\,dy=2\iint_s dx\,dy$$

= 2 (Area of closed curve)

= 2 [Area of rectangle + Area of half circle]

$$=2\left[\text{Base}\times\text{Hieght}+\frac{\pi r^2}{2}\right]$$

$$=2\left[3\times 2+\frac{\pi(1)^2}{2}\right]$$

$$=12+\pi$$

Hence, the correct option is (B).

Electrical Engineering : EE

3.1　(C)

Given : $S=\int_1^{\infty}x^{-3}dx$

$$S=\left[\frac{x^{-3+1}}{-3+1}\right]_1^{\infty}\qquad\left[\int x^n dx=\frac{x^{n+1}}{n+1}\right]$$

$$S=-\frac{1}{2}\left[\frac{1}{x^2}\right]_1^{\infty}=-\frac{1}{2}\left[\frac{1}{\infty}-1\right]$$

$$S=-\frac{1}{2}[0-1]=\frac{1}{2}$$

Hence, the correct option is (C).

3.2　(D)

Given :

The equation of surface is,

$$S(x,\ y)=2x+5y-3\qquad\ldots\text{(i)}$$

The path of integration is,

$$(x+1)^2+(y-1)^2=\sqrt{2}$$

This represents a circle with centre $(-1,1)$ and radius $2^{\frac{1}{4}}$ units.

Let, $x+1=X\Rightarrow x=X-1$

$y-1=Y\Rightarrow y=Y+1$

From equation (i),

$$S=2(X-1)+5(Y+1)-3$$

$$S=2X+5Y$$

The integral of surface S over a path is given by,

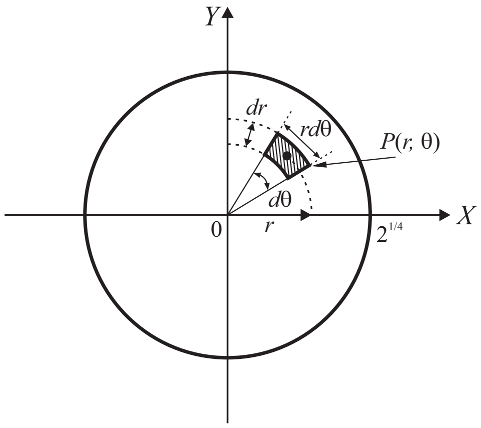
$$I=\int\int S(X,\ Y)\,dX\,dY\qquad\ldots\text{(ii)}$$

Converting Cartesian into polar co-ordinates,

$X=r\cos\theta$ and $Y=r\sin\theta$

Area of elementary portion P,

$$dXdY=r\,dr\,d\theta$$



So, $S(r,\theta)=2r\cos\theta+5r\sin\theta$

Limit of θ **:** $0<\theta<2\pi$

Limit of r **:** $0<r<2^{\frac{1}{4}}$

Let, $\alpha=2^{\frac{1}{4}}$

From equation (ii),

$$I=\int_0^{2\pi}\int_0^{\alpha} r(2\cos\theta+5\sin\theta)r\,dr\,d\theta$$

$$I=\int_0^{2\pi}\left[(2\cos\theta+5\sin\theta)\left(\frac{r^3}{3}\right)_0^{\alpha}\right]d\theta$$

$$I=\frac{\alpha^3}{3}\int_0^{2\pi}(2\cos\theta+5\sin\theta)\,d\theta$$

$$I=\frac{\alpha^3}{3}[2\sin\theta-5\cos\theta]_0^{2\pi}$$

$$I=\frac{\alpha^3}{3}[(2\sin 2\pi-5\cos 2\pi)-(2\sin 0-5\cos 0)]$$

$$I=\frac{\alpha^3}{3}[-5+5]=0$$

Hence, the correct option is (D).

3.3 (D)

Given :

The volume of a cone is,

$$V=\int_0^H \pi R^2\left(1-\frac{h}{H}\right)^2 dh$$

$$V=\frac{\pi R^2}{H^2}\int_0^H (H-h)^2\,dh$$

Take $(H-h)=t$

Differentiating both sides,

$$-dh=dt \Rightarrow dh=-dt$$

On changing the limits,

$$h=0\Rightarrow t=H$$

And $\quad h=H\Rightarrow t=0$

So, $\quad V=-\dfrac{\pi R^2}{H^2}\displaystyle\int_H^0 t^2\,dt$

$$V=-\frac{\pi R^2}{H^2}\times\frac{1}{3}[t^3]_H^0$$

$$V=-\frac{\pi R^2}{H^2}\times\frac{1}{3}[0-H^3]$$

$$V=\frac{1}{3}\pi R^2 H \qquad \ldots(\text{i})$$

Checking from the options, **(A)** and **(C)** are easily eliminated because the integrating variable and the function's variable are different. Let V_B and V_D are volumes calculated from respective options.

Option (B) :

$$V_B=\int_0^R \pi R^2\left(1-\frac{h}{H}\right)^2 dh$$

$$V_B=\frac{\pi R^2}{H^2}\int_0^R (H-h)^2\,dh$$

$$V_B=\frac{\pi R^2}{H^2}\left(-\frac{1}{3}\right)\left[(H-h)^3\right]_0^R$$

$$V_B=-\frac{\pi R^2}{3H^3}\left[(H-R)^3-H^3\right]$$

$$V_B=\frac{\pi R^2}{3}\left[1-\left(1-\frac{R}{H}\right)^3\right]\neq V$$

So, this is an incorrect option.

Option (D) :

$$V_D=\int_0^R 4\pi rH\left(1-\frac{r}{R}\right)^2 dr$$

$$V_D=\frac{4\pi H}{R^2}\int_0^R r(R^2+r^2-2rR)\,dr$$

$$V_D=\frac{4\pi H}{R^2}\int_0^R (rR^2+r^3-2r^2R)\,dr$$

$$V_D = \frac{4\pi H}{R^2}\left[R^2\frac{r^2}{2}+\frac{r^4}{4}-2R\frac{r^3}{3}\right]_0^R$$

$$V_D = \frac{4\pi H}{R^2}\left[R^4\left(\frac{1}{2}+\frac{1}{4}-\frac{2}{3}\right)-0\right]$$

$$V_D = \frac{4\pi H}{R^2}\times R^4\times\frac{1}{12} = \frac{1}{3}\pi R^2 H = V$$

Hence, the correct option is (D).

3.4 (D)

Given : $I = \frac{1}{2\pi}\int_0^{2\pi}\sin(t-\tau)\cos\tau\,d\tau \quad \ldots\text{(i)}$

Method 1

$$I = \frac{1}{2\pi}\int_0^{2\pi}\sin(t-\tau)\cos\tau\,d\tau$$

$$I = \frac{1}{2\pi}\int_0^{2\pi}(\sin t\cos\tau-\cos t\sin\tau)\cos\tau\,d\tau$$

$$I = I_1 + I_2 \qquad \ldots\text{(ii)}$$

where, $I_1 = \left(\frac{1}{2\pi}\sin t\int_0^{2\pi}\cos^2\tau\,d\tau\right)$

$$I_2 = -\left(\frac{\cos t}{2\pi}\int_0^{2\pi}\sin\tau\cos\tau\,d\tau\right)$$

Taking, $I_1 = \frac{1}{2\pi}\sin t\int_0^{2\pi}\cos^2\tau\,d\tau$

$$I_1 = \frac{1}{2\pi}\sin t\int_0^{2\pi}\left(\frac{1+\cos 2\tau}{2}\right)d\tau$$

$$I_1 = \frac{1}{2\pi}\sin t\left[\int_0^{2\pi}\frac{1}{2}d\tau+\int_0^{2\pi}\frac{\cos 2\tau}{2}d\tau\right]$$

$$I_1 = \frac{1}{2\pi}\sin t\left[\frac{1}{2}\times 2\pi\right]-0 = \frac{1}{2}\sin t$$

Now solving for I_2,

$$I_2 = -\left(\frac{\cos t}{2\pi}\int_0^{2\pi}\sin\tau\cos\tau\,d\tau\right)$$

$$I_2 = -\frac{\cos t}{4\pi}\int_0^{2\pi}\sin 2\tau\,d\tau$$

$$\left[\int_0^{2\pi}\cos 2\tau\,d\tau = 0, \int_0^{2\pi}\sin 2\tau\,d\tau = 0\right]$$

$$I_2 = 0$$

From equation (ii),

$$I = I_1 + I_2 = \frac{1}{2}\sin t + 0 = \frac{1}{2}\sin t$$

Hence, the correct option is (D).

Method 2

From the property of integral,

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$I = \frac{1}{2\pi}\int_0^{2\pi}\sin\{t-(2\pi-\tau)\}\cos(2\pi-\tau)d\tau$$

$$I = \frac{1}{2\pi}\int_0^{2\pi}\sin(t+\tau)\cos\tau\,d\tau \qquad \ldots\text{(iii)}$$

From given equation (i) and (iii),

$$I + I = \frac{1}{2\pi}\int_0^{2\pi}[\sin(t-\tau)+\sin(t+\tau)]\cos\tau\,d\tau$$

$$2I = \frac{1}{2\pi}\int_0^{2\pi}2\sin t\cos\tau\cos\tau\,d\tau$$

$$I = \frac{\sin t}{2\pi}\int_0^{2\pi}\cos^2\tau\,d\tau$$

$$I = \frac{\sin t}{4\pi}\int_0^{2\pi}(1+\cos 2\tau)d\tau = \frac{1}{2}\sin t$$

Hence, the correct option is (D).

Key point

From the property of definite integrals,

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

3.5 (A)

Given : $f(x,y)$ is defined over $0 \le x, y \le 1$

The region of integration between the parabolas,

$$y^2 = x \qquad \ldots\text{(i)}$$

$$x^2 = y \quad \Rightarrow x = \sqrt{y} \qquad \ldots\text{(ii)}$$

Note : $x > y^2 \Rightarrow$ The region on the right-hand side of the parabola $y^2 = x$

$y > x^2 \Rightarrow$ The region above the parabola $x^2 = y$

From equations (i) and (ii),

$$y^2 = \sqrt{y}$$

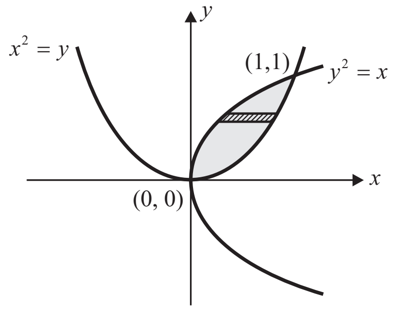
$$y^4 - y = 0$$

$$y(y^3-1)=0$$
$$y=0,1,1,1$$

If $y=0$ then $x=0$

If $y=1$ then $x=1$

Thus the point is (0, 0) and (1, 1)



Limit for x : $x=y^2$ to $x=\sqrt{y}$

Limit for y : $y=0$ to $y=1$

Volume (volume as double integral) under the curve $f(x,y)$ is given by,

$$V=\int_{y=0}^{y=1}\int_{x=y^2}^{x=\sqrt{y}} f(x,y)\,dx\,dy$$

Hence, the correct option is (A).

3.6 (B)

Given : $P=\int_0^1 xe^x\,dx$

Applying integration by parts,

$$\int uv\,dx = u\int v\,dx-\int\left(\frac{d}{dx}u\int v\,dx\right)dx$$

$$P=\int_0^1 xe^x\,dx=\left[xe^x-\int e^x\,dx\right]_0^1$$

$$P=[xe^x-e^x]_0^1$$

$$P=(1\times e^1-e^1)-(0\times e^0-e^0)$$

$$P=0-(-1)=1$$

Hence, the correct option is (B).

3.7 (B)

Given : $y=5x^2+10x,\quad x\in(1,2)$

Differentiating above equation with respect to x,

$$\frac{dy}{dx}=10x+10$$

$$\left.\frac{dy}{dx}\right|_{x=1}=20 \qquad \left.\frac{dy}{dx}\right|_{x=2}=30$$

Since, $1<x<2$

$$20<\frac{dy}{dx}<30$$

Only, the number 25 satisfies the above condition.

Hence, the correct option is (B).

3.8 2

Given : The velocity of a moving particle,

$$v=\frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)$$

Displacement of a moving particle is given by,

$$x=\int v\,dt$$

The particle is started from origin at $t=0$ s .

At $t=0 \Rightarrow x=0$

After $t=3$s,

$$x=\int_0^3 v\,dt=\frac{\pi}{2}\int_0^3\cos\frac{\pi t}{2}\,dt$$

$$x=\frac{\pi}{2}\left(\frac{\sin\frac{\pi t}{2}}{\frac{\pi}{2}}\right)_0^3=\left(\sin\frac{\pi t}{2}\right)_0^3$$

$$x=\sin\frac{3\pi}{2}-\sin 0=-1\text{ m}$$

Negative sign implies that, the particle has moved '1' unit to the left of origin.

But $x=\sin\left(\frac{\pi t}{2}\right)$ is oscillating.

The respective distance travelled by the particle is given in the table below,

Time (t)	Position (x)	Distance covered
0	0	0
1	1	1
2	0	2
3	–1	3

Total distance covered = 3 m

Magnitude of net displacement $= |-1| = 1$ m

Required difference is given by, $3-1=2$ m

Hence, difference between the distance covered by the particles and the magnitude of displacement from the origin is **2**.

3.9 (B)

Given :

(i) $$I=\int_0^8\left(\int_{y/2}^{(y/2)+1}\left(\frac{2x-y}{2}\right)dx\right)dy$$

(ii) $$u=\frac{2x-y}{2}\Rightarrow u=x-\frac{y}{2} \text{ and } du=dx$$

On changing the limits,

At $x=\frac{y}{2}$, $u=\dfrac{\frac{2y}{2}-y}{2}=0$

At $x=\frac{y}{2}+1$, $u=\dfrac{2\left(\frac{y}{2}+1\right)-y}{2}=\dfrac{y+2-y}{2}=1$

Also, $v=\frac{y}{2}\Rightarrow dv=\frac{dy}{2}$

$dy=2dv$

On changing the limits,

At, $y=0\Rightarrow v=0$

At, $y=8\Rightarrow v=4$

Then, the integral I reduces to,

$$I=\int_0^4\left[\int_0^1 2u\,du\right]dv$$

Hence, the correct option is (B).

3.10 (B)

Given : $I=\int_c (2xy^2dx+2x^2ydy+dz)$

Is along the path joining origin $(0,0,0)$ and $(1,1,1)$.

The equation of straight line passing through (x_1,y_1,z_1) to (x_2,y_2,z_2) is given by,

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}=t$$

$$\frac{x-0}{1-0}=\frac{y-0}{1-0}=\frac{z-0}{1-0}=t\,(\text{say})$$

Then $x=t,\ y=t,\ z=t$

$dx=dt,\ dy=dt,\ dz=dt$

Limits of t are from $t=0$ to $t=1$

Thus, $I=\int_c (2xy^2dx+2x^2ydy+dz)$

$$I=\int_{t=0}^{t=1}(4t^3+1)\,dt=\left|\frac{4t^4}{4}+t\right|_0^1=1+1=2$$

Hence, the correct option is (B).

3.11 (D)

Given : $I=2\int_{-\infty}^{\infty}\left(\frac{\sin 2\pi t}{\pi t}\right)dt$

Check if given function is an even function.

$$f(t)=\frac{\sin 2\pi t}{t}$$

$$f(-t)=\frac{-\sin 2\pi t}{-t}=\frac{\sin 2\pi t}{t}$$

$$f(t)=f(-t)$$

So, given function is an even function.

For any even function, the following property is applicable :

$$\int_{-\infty}^{\infty}f(t)dt=2\int_0^{\infty}f(t)\,dt$$

Then, the integral I is calculated as,

$$I=\frac{2}{\pi}\times 2\int_0^{\infty}\left(\frac{\sin 2\pi t}{t}\right)dt$$

$$I=\frac{4}{\pi}\int_0^{\infty}\left(\frac{\sin 2\pi t}{t}\right)dt \qquad \ldots(i)$$

By definition of Laplace transform,

$$L\{f(t)\}=\int_0^{\infty}e^{-st}f(t)\,dt$$

From equation (i),

$$\frac{4}{\pi}\int_0^{\infty}\left(\frac{\sin 2\pi t}{t}\right)dt=\frac{4}{\pi}\int_0^{\infty}e^{-0t}\left(\frac{\sin 2\pi t}{t}\right)dt$$

$$\frac{4}{\pi}\int_0^{\infty}\left(\frac{\sin 2\pi t}{t}\right)dt = \frac{4}{\pi}L\left(\frac{\sin 2\pi t}{t}\right)\Bigg|_{s=0} \quad \ldots\text{(ii)}$$

Let $f(t) = \sin 2\pi t$,

$$L[f(t)] = \frac{2\pi}{(2\pi)^2 + s^2} = F(s)$$

Properties of Laplace transform is,

$$L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} F(s)\,ds$$

From equation (ii), we have

$$I = \frac{4}{\pi}\left[\int_s^{\infty}\frac{2\pi}{s^2+(2\pi)^2}ds\right]_{s=0}$$

$$I = \frac{4}{\pi}\times\frac{2\pi}{4\pi^2}\left[\int_s^{\infty}\frac{1}{1+\left(\frac{s}{2\pi}\right)^2}ds\right]_{s=0}$$

$$I = \frac{2}{\pi^2}\left[\int_s^{\infty}\frac{1}{1+\left(\frac{s}{2\pi}\right)^2}ds\right]_{s=0}$$

$$I = \frac{2}{\pi^2}\times 2\pi\left[\tan^{-1}\left(\frac{s}{2\pi}\right)\right]_s^{\infty}\Bigg|_{s=0}$$

$$I = \frac{4}{\pi}\left[\tan^{-1}\infty - \tan^{-1}\left(\frac{s}{2\pi}\right)\right]_{s=0}$$

$$I = \frac{4}{\pi}\left[\frac{\pi}{2} - \tan^{-1}(0)\right] = \frac{4}{\pi}\times\frac{\pi}{2} = 2$$

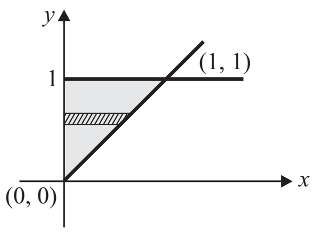
Hence, the correct option is (D).

3.12 0.718

Given : Surface, $f(x, y) = e^x$

A triangle is bounded in x-y plane by the following lines :

$x = y$	[tangent of 45^0]
$x = 0$	[y-axis]
$y = 1$	[parallel to x-axis]



Volume (volume as double integral) enclosed by a surface $f(x, y)$ is given by,

$$V = \iint f(x, y)\,dx\,dy$$

By integration or surface integration,

$$V = \int_{y=0}^{1}\int_{x=0}^{y} e^x\,dx\,dy$$

$$V = \int_{y=0}^{1}[e^x]\Big|_{x=0}^{y}dy = \int_{y=0}^{1}(e^y - 1)dy$$

$$V = (e^y - y)\Big|_0^1 = (e-1)-(1-0)$$

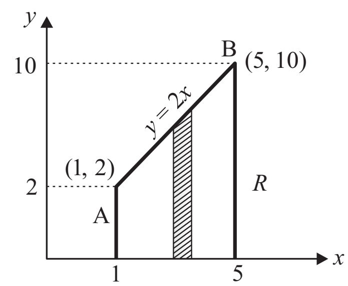
$$V = e-1-1 = e-2 = 0.71828$$

Hence, the volume is **0.718**.

3.13 0.99

Given : $I = C\iint_R xy^2\,dx\,dy$

where, $C = 6\times10^{-4}$ cm



From the figure, the equation of line AB is given by,

$$y - 2 = \frac{10-2}{5-1}(x-1)$$

$$y - 2 = \frac{8}{4}(x-1)$$

$$y = 2x - 2 + 2$$

$$y = 2x \quad \ldots\text{(i)}$$

Limits of x : $x = 1$ to $x = 5$

Limits of y : $y = 0$ to $y = 2x$

So, $I = C\iint_R xy^2\,dy\,dx$

$$I = C\int_1^5\int_0^{2x} xy^2\,dydx = C\int_1^5 x\left(\frac{y^3}{3}\right)_0^{2x}dx$$

$$I = C\int_1^5 \frac{x}{3}(2x)^3\,dx = C\times\frac{8}{3}\int_1^5 x^4\,dx$$

$$I = C\frac{8}{3}\left[\frac{x^5}{5}\right]_1^5 = C\times\frac{8}{15}(5^5-1)$$

$$I = 6\times10^{-4}\times\frac{8}{15}\times(5^5-1) = 0.999$$

Hence, value of I is **0.99**.

3.14 40

Given : $f(x,y,z) = (x^2+y^2-2z^2)(y^2+z^2)$

Taking partial derivative of above equation with respect to x,

$$\frac{\partial f}{\partial x} = (y^2+z^2)(2x)$$

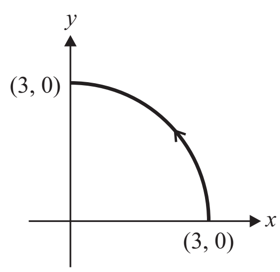
At point $(2, 1, 3)$,

$$\left.\frac{\partial f}{\partial x}\right|_{x=2,\ y=1,\ z=3} = (1^2+3^2)(2\times2) = 40$$

Hence, the partial derivative is **40**.

3.15 0

Given : $I = \int\limits_C (y^2+2xy)\,dx + (x^2+2xy)\,dy$



Equation $x^2+y^2=9$, is given for circle in the first quadrant.

From Green's theorem,

$$\oint\limits_C (\phi dx + \psi dy) = \iint\limits_S \left(\frac{\partial\psi}{\partial x} - \frac{\partial\phi}{\partial y}\right) dx\,dy \quad \ldots\text{(i)}$$

Comparing equation (i) with given integral,

$$\phi = y^2+2xy \text{ and } \psi = x^2+2xy$$

So, $\quad \dfrac{\partial\psi}{\partial x} = 2x+2y$ and $\dfrac{\partial\phi}{\partial y} = 2y+2x$

Put the above values in equation (i),

$$\oint\limits_C (\phi dx + \psi dy) = \iint\limits_S \left[(2x+2y)-(2x+2y)\right] dx\,dy$$

$$\oint\limits_C (\phi dx + \psi dy) = 0$$

Hence, the value of integral is **0.**

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3.16 139

Given : $f = 2x^3+3y^2+4z$

$$\therefore \quad \text{grad } f = \frac{\partial f}{\partial x}\hat{a}_x + \frac{\partial f}{\partial y}\hat{a}_y + \frac{\partial f}{\partial z}\hat{a}_z$$

$$\text{grad } f = 6x^2\hat{a}_x + 6y\hat{a}_y + 4\hat{a}_z \quad \ldots\text{(i)}$$

Given line integral can be evaluated in three parts

$$\int \text{grad } f\cdot d\hat{r} = \int\limits_{-3,-3,2}^{2,-3,2} \text{grad } f\cdot d\hat{r} + \int\limits_{2,-3,2}^{2,6,2} \text{grad } f\cdot d\hat{r} + \int\limits_{2,6,2}^{2,6,-1} \text{grad } f\cdot d\hat{r}$$

For line $(-3,-3,2)$ to $(2,-3,2)$, we have to move along x-axis, so $d\hat{r} = dx\,\hat{a}_x$

$$\text{grad } f . d\hat{r} = (6x^2\hat{a}_x + 6y\hat{a}_y + 4\hat{a}_z)\cdot dx\hat{a}_x = 6x^2 dx$$

$$\int\limits_{(1)} \text{grad } f\cdot d\hat{r} = \int_{-3}^{2} 6x^2 dx = 6\left[\frac{x^3}{3}\right]_{-3}^{2} = 2[8+27] = 70$$

For line (2, –3, 2) to (2, 6, 2), we have to move along y-axis, so $d\hat{r} = dy\,\hat{a}_y$

$$\therefore \quad \text{grad } f\cdot d\hat{r} = (6x^2\,\hat{a}_x + 6y\,\hat{a}_y + 4\,\hat{a}_z)\cdot dy\,\hat{a}_y = 6y\,dy$$

$$\int\limits_{(2)} \text{grad } f\cdot d\hat{r} = \int_{-3}^{6} 6y\,dy = 6\left[\frac{y^2}{2}\right]_{-3}^{6}$$

$$= 3[36-9] = 81$$

For line (2, 6, 2) to (2, 6, –1), we have to move along z-axis, so $d\hat{r} = dz\,\hat{a}_z$

$\therefore$ $$\text{grad} f \cdot d\hat{r} = (6x^2\,\hat{a}_x + 6y\,\hat{a}_y + 4\,\hat{a}_z) \cdot dz\,\hat{a}_z = 4dz$$

$$\int_{(3)} \text{grad} f \cdot d\,\hat{r} = \int_2^{-1} 4\,dz = 4[z]_2^{-1}$$

$$= 4 \times (-3) = -12$$

$\therefore$ $$\int \text{grad} f \cdot d\,\hat{r}$$

$$= \int_{(1)} \text{grad} f \cdot d\,\hat{r} + \int_{(2)} \text{grad} f \cdot d\,\hat{r}$$

$$\int \text{grad} f \cdot d\,\hat{r} = 70 + 81 - 12 = 139$$

Hence, the value of line integral is **139.**

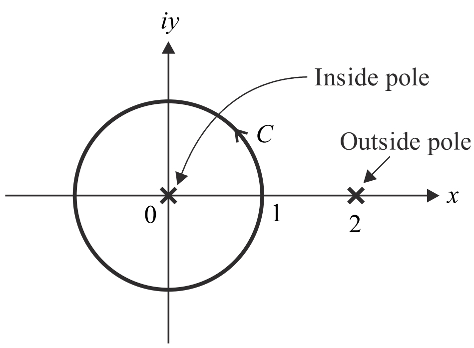
3.17 (D)

Given : Complex integral

$\therefore$ $$I = \int_C \frac{z^2+1}{z^2-2z}\,dz$$

$\Rightarrow$ $$I = \int_C \frac{z^2+1}{z(z-2)}\,dz$$

The singular points of $f(z)$ are $z=0$ and $z=2$.



Since, the curve 'C' is a circle of radius 1, the point $z=0$ will lie inside the circle and the point $z=2$ will lie outside the circle.

$\therefore$ Residue of $f(z)$ is given by,

$$\text{Res}(f(z)) = \lim_{z\to 0}(z-0)\frac{z^2+1}{z(z-2)}$$

$$= \frac{0+1}{0-2} = \frac{-1}{2}$$

Using Cauchy's residue theorem,

$$\int_C \frac{z^2+1}{z(z-2)} = 2\pi i\ [\text{Res}(f(z))]$$

$$= 2\pi i \times \frac{-1}{2} = -\pi i$$

Hence, the correct option is (D).

3.18 (1)

Given : $x^2 + y^2 = 1$ ...(i)

$$(x-1)^2 + (y-1)^2 = r^2 \quad \text{...(ii)}$$

Differentiating equation (i) with respect to x,

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

Differentiating equation (ii) with respect to x,

$$2(x-1) + 2(y-1)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(x-1)}{(y-1)}$$

Let m_1 be the slope of equation (i)

$$m_1 = \frac{dy}{dx} = \frac{-x}{y}$$

Let m_2 be the slope of equation (ii)

$$m_2 = \frac{dy}{dx} = \frac{-(x-1)}{(y-1)}$$

Since, equation (i) and (ii) intersect orthogonally each other at the point (u, v)

Therefore, slope of equation (i) and (ii) must be satisfied at the point (u, v)

Thus, $m_1 = \dfrac{-u}{v}$

$$m_2 = \frac{-(u-1)}{(v-1)} = \frac{1-u}{v-1}$$

From the concept of straight line

$$m_1 \times m_2 = -1$$

$$\left(\frac{-u}{v}\right)\left(\frac{1-u}{v-1}\right)=-1$$

$$\frac{u^2-u}{v^2-v}=-1$$

$$u-u^2=v^2-v$$

$$u+v=v^2+u^2 \quad \ldots\text{(iii)}$$

It is given that

$u^2+v^2=1$ (Since the point (u, v) is the point of intersection and hence will satisfy the both equation of circles)

Therefore, from equation (iii),

$$u+v=1$$

Hence, the correct answer is 1.

Key Point

From the concept of straight line, if two line intersect each other orthogonally them product of their slope will be

$$-\tan\theta\times\cot\theta=m_1\times m_2=-1.$$

Mechanical Engineering : ME

3.1 (B)

Given : $I=\int_0^\infty e^{-y^3}y^{\frac{1}{2}}\,dy$

Putting $y^3=t$

Differentiating both the sides with respect to t,

$$3y^2dy=dt$$

$$y^{\frac{1}{2}}dy=\frac{1}{3}y^{\frac{-3}{2}}dt=\frac{1}{3}t^{\frac{-1}{2}}dt$$

$$I=\int_0^\infty e^{-t}\frac{1}{3}t^{\frac{-1}{2}}dt$$

Using the property of gamma function,

$$\int_0^\infty e^{-t}t^{n-1}dt=\Gamma n$$

Here, $n-1=\dfrac{-1}{2} \quad\Rightarrow\quad n=\dfrac{1}{2}$

$$I=\frac{1}{3}\Gamma n$$

$$I=\frac{1}{3}\Gamma\frac{1}{2}=\frac{1}{3}\sqrt{\pi}$$

Hence, the correct option is (B).

3.2 (B)

Given : $2y=x^2$ ...(i)

and $x=y-4$ ...(ii)

Point of intersection is given from equations (i) and (ii),

$$2(x+4)=x^2$$

$$2x+8=x^2$$

$$x^2-2x-8=0$$

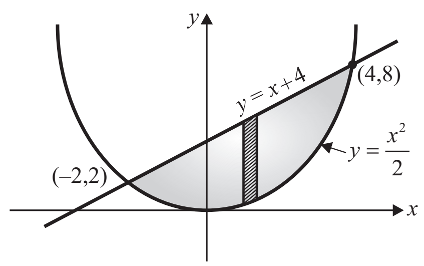
$$(x-4)(x+2)=0$$

$$x=-2,4$$

From equation (ii),

$$y=2 \text{ at } x=-2$$

$$y=8 \text{ at } x=4$$



Take vertical strip

Limit of x : $x=-2$ to $x=4$

Limit of y : $y=\dfrac{x^2}{2}$ to $y=x+4$

Area enclosed by the curve is

$$A=\int_{x_1}^{x_2}\int_{y_1}^{y_2}dx\,dy$$

$$A=\int_{-2}^{4}\int_{x^2/2}^{x+4}dx\,dy$$

$$A=\int_{-2}^{4}|y|_{x^2/2}^{x+4}dx=\int_{-2}^{4}\left(x+4-\frac{x^2}{2}\right)dx$$

$$A=\left[\frac{x^2}{2}+4x-\frac{x^3}{6}\right]_{-2}^{4}$$

$$A = 8 + 16 - \frac{32}{3} - 2 + 8 - \frac{4}{3}$$

$$A = 18$$

Hence, the correct option is (B).

3.3 (C)

Given : $y = \int_1^{x^2} \cos t\, dt \quad \Rightarrow \quad y = [\sin t]_1^{x^2}$

$$y = \sin x^2 - \sin 1$$

Differentiating with respect to x,

$$\frac{dy}{dx} = 2x \cos x^2$$

Hence, the correct option is (C).

3.4 (C)

Given : $f(x, y) = x^2 + y^2 = 3axy$

$$f(y, x) = y^2 + x^2 = 3ayx$$

Therefore, $f(x, y) = f(y, x)$

The curve is symmetric.

Hence, the correct option is (C).

Key Point

By interchanging x and y if there is no change then, curve is symmetric about the line $y = x$.

3.5 (A)

Given : $\phi(x) = \int_0^{x^2} \sqrt{t}\, dt$

$$\phi(x) = \left[\frac{2}{3} t^{\frac{3}{2}} \right]_0^{x^2} = \frac{2}{3} x^3$$

Differentiating both sides with respect to x,

Thus, $\frac{d\phi}{dx} = \frac{2}{3} \times 3x^2 = 2x^2$

Hence, the correct option is (A).

3.6 (D)

Given : $I = \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(x + y)\, dx\, dy$

Integrating with respect to x,

$$I = \int_0^{\frac{\pi}{2}} \left[-\cos(x + y) \right]_0^{\frac{\pi}{2}} dy$$

$$I = \int_0^{\frac{\pi}{2}} \left[-\cos\left(\frac{\pi}{2} + y \right) + \cos(0 + y) \right] dy$$

$$[\cos(90 + \theta) = -\sin\theta]$$

Integrating with respect to y,

$$I = \int_0^{\frac{\pi}{2}} (\sin y + \cos y)\, dy$$

$$I = [-\cos y + \sin y]_0^{\pi/2}$$

$$I = [-0 - (-1) + (1 - 0)]$$

$$I = 1 + 1 = 2$$

Hence, the correct option is (D).

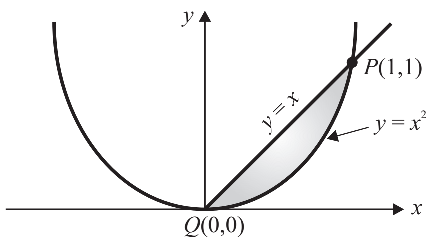
3.7 (B)

Given : Straight line $y = x$ ...(i)

and Parabola $y = x^2$ ...(ii)

Method 1

The area enclosed is shown below as shaded :



The coordinates of points P and Q is obtained by solving,

From equations (i) and (ii),

i.e., $x = x^2$

$$x(x - 1) = 0$$

$$x = 0, x = 1$$

$$y = 0 \text{ when } x = 0$$

$$y = 1 \text{ when } x = 1$$

Therefore area enclosed is given by,

$$A = \int (y_1 - y_2)\, dx$$

$$A = \int_0^1 x\, dx - \int_0^1 x^2 dx$$

$$A=\left[\frac{x^2}{2}\right]_0^1-\left[\frac{x^3}{3}\right]_0^1$$

$$A=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$$

Hence, the correct option is (B).

Method 2

The area bounded in xy-plane by the following line

$$y=x \quad \text{[Tangent of } 45^0\text{]} \quad \ldots(i)$$

$$y=x^2 \quad \ldots(ii)$$

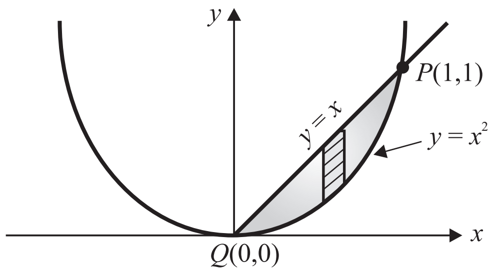
[Parabola symmetry about y-axis]

From equations (i) and (ii),

i.e., $x=x^2$

$$x(x-1)=0$$

$$x=0, x=1$$



Limit of y : $y=x^2$ to $y=x$

Limit of x : $x=0$ to $x=1$

Area enclosed by the surface is given by,

$$A=\iint dx\,dy$$

$$A=\int_0^1\int_{x^2}^{x} dx\,dy$$

$$A=\int_0^1 |y|_{x^2}^{x}\,dx=\int_0^1 (x-x^2)\,dx$$

$$A=\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1=\frac{1}{2}-\frac{1}{3}$$

$$A=\frac{1}{6}$$

Hence, the correct option is (B).

3.8 (C)

Given : $x=a(\theta+\sin\theta)$ ...(i)

and $y=a(1-\cos\theta)$ ...(ii)

Differentiating equations (i) and (ii) w.r.t. θ on both sides,

$$\frac{dx}{d\theta}=a(1+\cos\theta),\ \frac{dy}{d\theta}=a\sin\theta$$

By chain rule,

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}$$

$$\frac{dy}{dx}=\frac{a\sin\theta}{a(1+\cos\theta)}=\frac{\sin\theta}{1+\cos\theta}$$

$$\frac{dy}{dx}=\frac{2\sin\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\theta}{2}\right)}=\tan\frac{\theta}{2}$$

Hence, the correct option is (C).

3.9 (A)

Given : $V=\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^1 r^2\sin\phi\,dr\,d\phi\,d\theta$

Integrating with respect to 'r',

$$V=\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\left[\frac{r^3}{3}\right]_0^1\sin\phi\,d\phi\,d\theta$$

$$V=\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\left[\frac{1}{3}\right]\sin\phi\,d\phi\,d\theta$$

Integrating with respect to ϕ,

$$V=\frac{1}{3}\int_0^{2\pi}[-\cos\phi]_0^{\frac{\pi}{3}}d\theta$$

$$V=\frac{1}{3}\int_0^{2\pi}\left[\frac{1}{2}\right]d\theta$$

Integrating with respect to θ,

$$V=\frac{1}{3}\times\frac{1}{2}[2\pi-0]$$

$$V=\frac{1}{3}\times\frac{1}{2}\times 2\pi=\frac{\pi}{3}$$

Hence, the correct option is (A).

3.10 (A)

Given : $I=\int_{-a}^{a}(\sin^6 x+\sin^7 x)\,dx$

$$I=\int_{-a}^{a}\sin^6 x\,dx+\int_{-a}^{a}\sin^7 x\,dx \qquad \ldots(i)$$

By the property of definite integrals,

$$\int_{-a}^{a}f(x)\,dx=\begin{cases}2\int_0^a f(x)\,dx & ;\ f(x)\text{ is even}\\ 0 & ;\ f(x)\text{ is odd}\end{cases}$$

$\sin^6 x$ is even function and $\sin^7 x$ is odd function.

Therefore, $\int_{-a}^{a}\sin^7 x\,dx=0$

$$\int_{-a}^{a}\sin^6 x\,dx=2\int_0^a \sin^6 x\,dx$$

From equation (i),

$$I=2\int_0^a \sin^6 x\,dx$$

Hence, the correct option is (A).

3.11 (A)

Given : $I=\int_0^8\int_{\frac{x}{4}}^{2}f(x,y)dy\,dx \qquad \ldots(i)$

$$I=\int_r^s\int_p^q f(x,y)dy\,dx \qquad \ldots(ii)$$

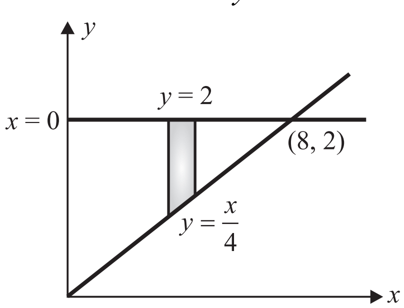
From equation (i),

Limits of integration :

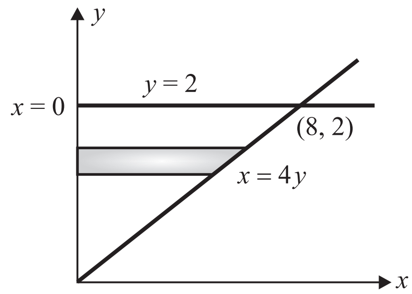
Limit of x : $x=0$ to $x=8$

Limit of y : $y=\frac{x}{4}$ to $y=2$

It gives variable limits for y. Therefore



For changing the limits take horizontal strip,



Therefore, limit of integration :

Limit of x : $x=0$ to $x=4y$

Limit of y : $y=0$ to $y=2$

$$I=\int_0^2\int_0^{4y}f(x,y)\,dy\,dx \qquad \ldots(iii)$$

Comparing equations (ii) and (iii),

$s=2,\ p=0,\ q=4y$ and $r=0$

Hence, the correct option is (A).

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3.12 (A)

Given : $I=\int_0^{\frac{\pi}{3}}e^{it}dt$

$$I=\left[\frac{e^{it}}{i}\right]_0^{\pi/3}$$

[By Euler's theorem, $e^{i\theta}=\cos\theta+i\sin\theta$]

$$I=\left[\frac{\cos t+i\sin t}{i}\right]_0^{\pi/3}$$

$$I=\frac{1}{i}\left[\frac{1}{2}+i\frac{\sqrt{3}}{2}-1\right]$$

$$I=\left[-\frac{1}{2i}+\frac{\sqrt{3}}{2}\right]=\left[\frac{\sqrt{3}}{2}+i\frac{1}{2}\right]$$

Hence, the correct option is (A).

3.13 (D)

Given : $y=\frac{2}{3}x^{3/2} \qquad \ldots(i)$

Length of the curve is given by,

$$L = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx \qquad \text{...(ii)}$$

Differentiating equation (i) with respect to x on both sides,

$$\frac{dy}{dx} = \left[\frac{2}{3}\times\frac{3}{2}x^{\frac{3}{2}-1}\right] = x^{1/2}$$

From equation (ii),

$$L = \int_0^1 \sqrt{x+1}\, dx$$

Let $\quad x+1 = y$

$\quad dx = dy$

For $\quad x = 0,\ y = 1$

$\quad x = 1,\ y = 2$

Therefore, $\int_1^2 y^{1/2} dy = \frac{2}{3}\left[y^{3/2}\right]_1^2 = 1.22$

Hence, the correct option is (D).

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3.14 (D)

Checking from the options,

From option (A) :

$$\int_0^{\frac{\pi}{4}} \tan x\, dx = \log\left|\sec x\right|_0^{\pi/4}$$

$$= \log\left|\sec 45^0 - \sec 0^0\right| = \log\left|\sqrt{2}-1\right|$$

[Bounded]

From option (B) :

$$\int_0^{\infty} \frac{dx}{x^2+1} = \left[\tan^{-1} x\right]_0^{\infty}$$

$$= \tan^{-1}\infty - \tan^{-1} 0 = \frac{\pi}{2}$$

[Bounded]

From option (C) :

$$\int_0^{\infty} x e^{-x} dx = \left[-e^{-x}(x+1)\right]_0^{\infty} = 1$$

[Bounded]

From option (D) :

$$\int_0^1 \frac{dx}{(1-x)} = \ln(1-x)\Big|_0^1 = \ln(0) - \ln(1) = -\infty$$

[Unbounded]

Since ln (0) is unbounded therefore, option (D) satisfies.

Hence, the correct option is (D).

3.15 (A)

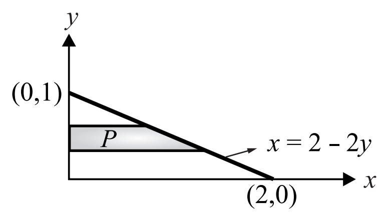
Given : $I = \iint_P xy\, dx\, dy$

Equation of line passing through $(0,1)$ to $(2,0)$

$$y-1 = \frac{0-1}{2-0}(x-0)$$

$$y-1 = \frac{-1}{2}x$$

$$x = 2-2y$$



Taking horizontal strip therefore limit of integration is,

Limit of *x* : $x = 0$ to $x = 2-2y$

Limit of *y* : $y = 0$ to $y = 1$

So, the required integral is,

$$I = \int_0^1 \int_0^{(2-2y)} (xy\, dx)\, dy$$

$$I = \int_0^1 \left\{\left[\frac{yx^2}{2}\right]_0^{2-2y}\right\} dy$$

$$I = \int_0^1 \frac{y}{2}(2-2y)^2 dy$$

$$I = \int_0^1 2y(1-y)^2\, dy$$

$$I = \int_0^1 2y(1+y^2-2y)\, dy$$

$$I = \int_0^1 (2y+2y^3-4y^2)\, dy$$

$$I = \left[y^2 + \frac{2y^4}{4} - \frac{4y^3}{3} \right]_0^1 = 1 + \frac{1}{2} - \frac{4}{3}$$

$$I = \frac{1}{6}$$

Hence, the correct option is (A).

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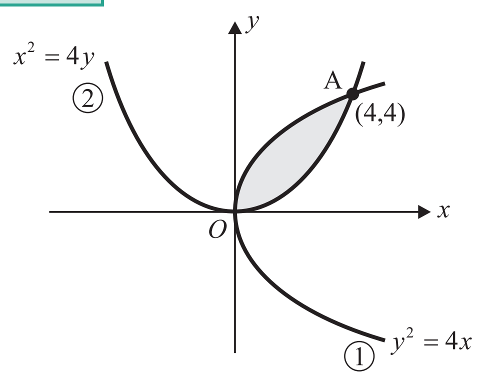
3.16 (A)

Given : Curve 1 : $y^2 = 4x$

Curve 2 : $x^2 = 4y$

Method 1



Intersection points of curve 1 and 2 :

$$y^2 = 4x = 4\sqrt{4y} = 8\sqrt{y}$$

$$y^4 = 8 \times 8y \Rightarrow y(y^3 - 64) = 0$$

$$y = 4 \text{ and } y = 0$$

Then $\quad x = 4$ when $y = 4$

$\quad x = 0$ when $y = 0$

Therefore intersection point are A (4, 4) and O (0, 0).

The area enclosed between curves 1 and 2 is given by,

$$I = \int_{x_1}^{x_2} (y_1 - y_2)\, dx$$

$$I = \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4} \right) dx$$

$$I = \left[2\frac{x^{3/2}}{3/2} - \frac{x^3}{4 \times 3} \right]_0^4$$

$$I = \frac{4}{3}(4)^{3/2} - \frac{(4)^3}{3 \times 4}$$

$$I = \frac{16}{3}$$

Hence, the correct option is (A).

Method 2

Area enclosed in xy-plane by the following curve,

$$y^2 = 4x \qquad \ldots\text{(i)}$$

[Parabola symmetry about x-axis]

$$x^2 = 4y \qquad \ldots\text{(ii)}$$

[Parabola symmetry about y-axis]

From equations (i) and (ii),

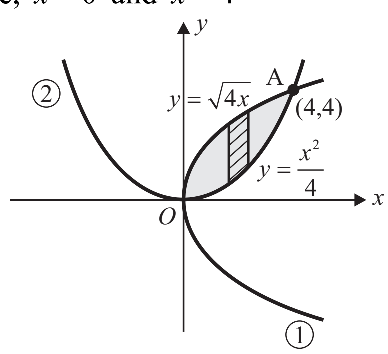
$$x^2 = 4y$$

$$x^2 = 4\sqrt{4x}$$

$$x^4 = 16 \times 4x$$

$$x(x^3 - 64) = 0$$

Therefore, $x = 0$ and $x = 4$



Taking vertical strip therefore limit of integration is,

Limit of x : $x = 0$ to $x = 4$

Limit of y : $y = \frac{x^2}{4}$ to $y = \sqrt{4x}$

Area enclosed by the curve is given by,

$$I = \int_{x_1}^{x_2} \int_{y_1}^{y_2} dx\, dy$$

$$I = \int_0^4 \int_{x^2/4}^{\sqrt{4x}} dx\, dy$$

$$I = \int_0^4 |y|_{x^2/4}^{\sqrt{4x}}\, dx = \int_0^4 2\sqrt{x} - \frac{x^2}{4} dx$$

$$I = \left|\frac{2x^{3/2}}{3/2} - \frac{x^3}{12}\right|_0^4$$

$$I = \left[\frac{4}{3}(2)^3 - \frac{64}{12}\right] = \frac{16}{3}$$

Hence, the correct option is (A).

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3.17 **(D)**

Given : $y = \sqrt{x}$

The volume of a solid generated by revolution about the x-axis, of the area bounded by curve $y = f(x)$, the x-axis and the ordinates $x = a$, $y = b$ is,

$$V = \int_a^b \pi y^2 dx$$

Here, $a = 1, b = 2$ and $y = \sqrt{x} \Rightarrow y^2 = x$

$$V = \int_1^2 \pi x \, dx$$

$$V = \pi\left[\frac{x^2}{2}\right]_1^2 = \frac{\pi}{2}\left[x^2\right]_1^2$$

$$V = \frac{\pi}{2}[2^2 - 1^2] = \frac{3}{2}\pi$$

Hence, the correct option is (D).

3.18 **(D)**

Given : $I = \int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \left[\tan^{-1} x\right]_{-\infty}^{\infty}$

$$I = \tan^{-1}(\infty) - \tan^{-1}(-\infty)$$

$$I = \frac{\pi}{2} - \left[\frac{-\pi}{2}\right] = \pi$$

Hence, the correct option is (D).

3.19 **(D)**

For any even function $f(x)$, property of integral is,

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx, \quad \text{for even}$$

$$= 0, \quad \text{for odd}$$

Hence, the correct option is (D).

3.20 **(A)**

Refer Solution 3.7 [Mechanical]

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3.21 **(C)**

Given :

$$I = \int_1^e \sqrt{x}\ln(x)\,dx = \int_1^e \underset{\text{(I)}}{\ln(x)}\underset{\text{(II)}}{\sqrt{x}}\,dx$$

Integrating by parts,

$$I = \left[\left(\ln(x)\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) - \int \frac{1}{x}\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}dx\right]_1^e$$

$$I = \left[\frac{2}{3}x^{\frac{3}{2}}\ln(x) - \frac{2}{3}\int x^{\frac{1}{2}}dx\right]_1^e$$

$$I = \left[\frac{2}{3}x^{\frac{3}{2}}\ln(x) - \frac{2}{3}\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^e$$

$$I = \left[\frac{2}{3}x^{\frac{3}{2}}\ln(x) - \frac{4}{9}x^{\frac{3}{2}}\right]_1^e$$

$$I = \left(\frac{2}{3}e^{\frac{3}{2}} - \frac{4}{9}e^{\frac{3}{2}}\right) - \left(0 - \frac{4}{9}\right)$$

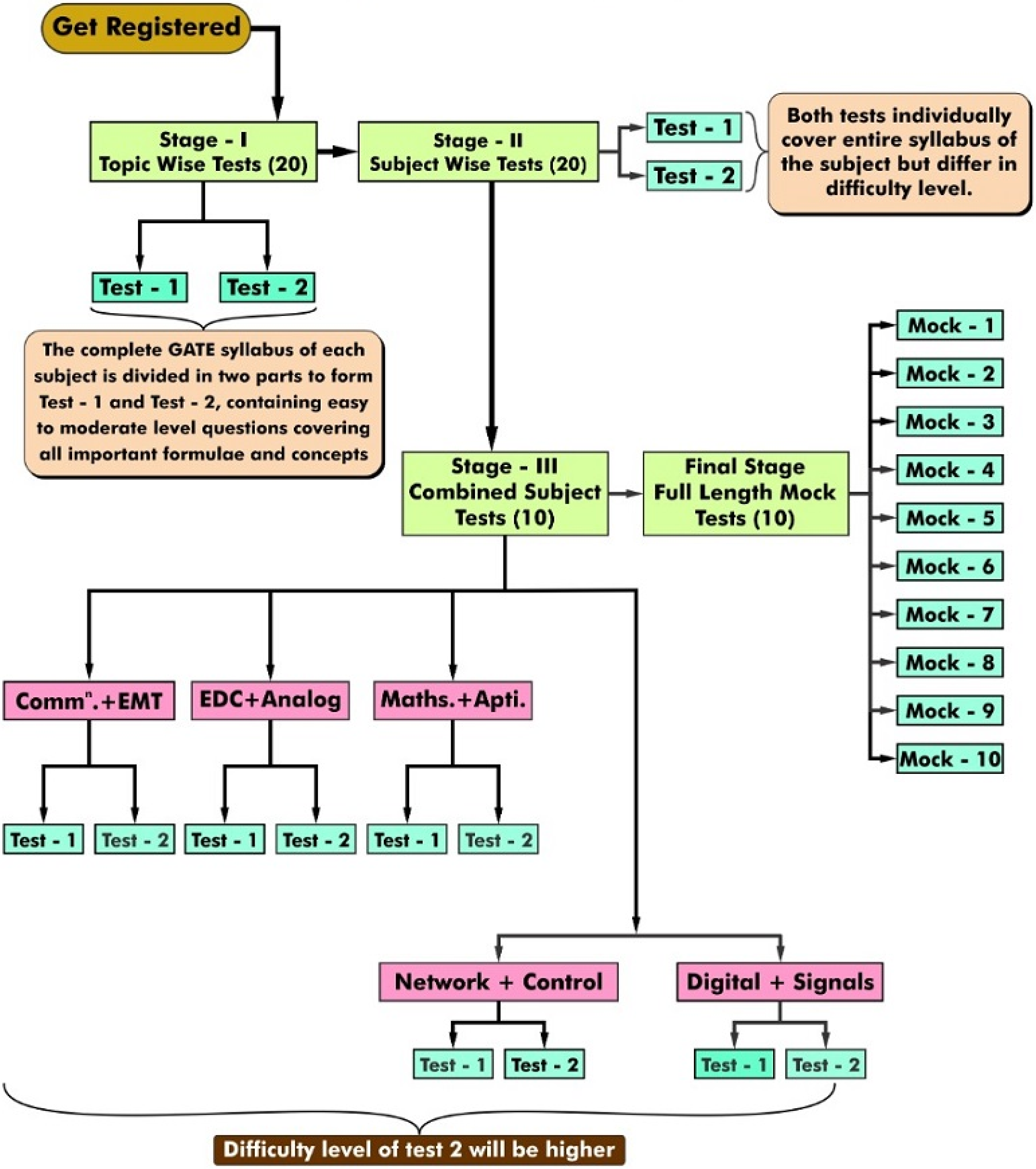
$$I = \frac{2}{9}\sqrt{e^3} + \frac{4}{9}$$

Hence, the correct option is (C).

3.22 **(B)**

Given : $I = \int_0^2 \frac{(x-1)^2\sin(x-1)}{(x-1)^2 + \cos(x-1)}dx \quad \ldots(i)$

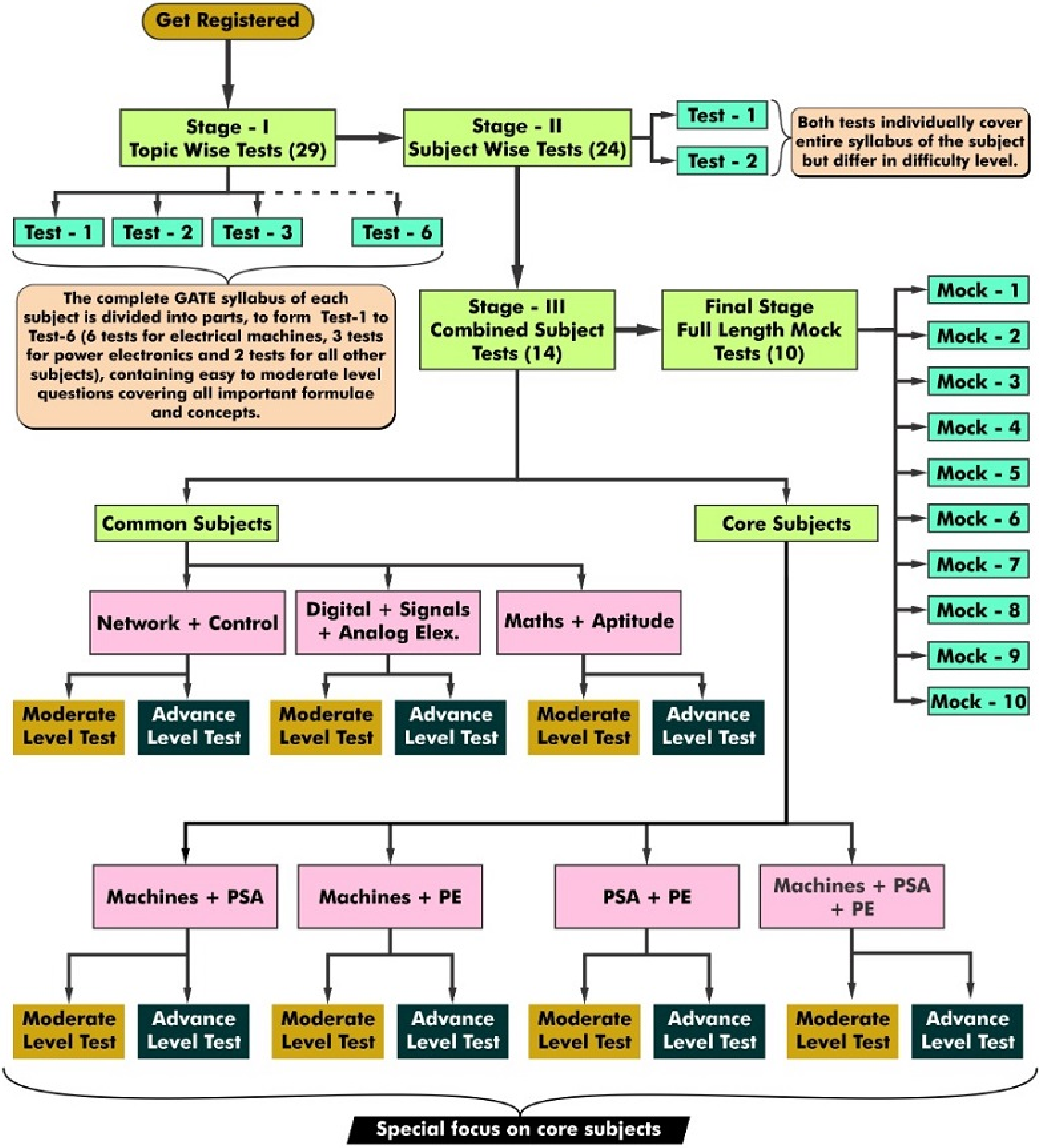
GATE ACADEMY
TEST SERIES STRUCTURE (EC)
(One Year Program)



GATE ACADEMY

TEST SERIES STRUCTURE (EE)

(One Year Program)



Method 1

Let, $(x-1)=t$

$x=(t+1)$

At $x=0$, $t=-1$

At $x=2$, $t=1$

$$I=\int_{-1}^{1}\left(\frac{t^2\sin t}{t^2+\cos t}\right)dt$$

$[(t^2\sin t)/(t^2+\cos t)$ is an odd function, therefore integral is 0]

$$I=\int_{-1}^{1}\frac{t^2\sin t}{t^2+\cos t}dt=0$$

Hence, the correct option is (B).

Method 2

By the properties of definite integrals,

$$\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$$

So, $$I=\int_0^2\frac{(2-x-1)^2\sin(2-x-1)}{(2-x-1)^2+\cos(2-x-1)}dx$$

$$I=\int_0^2\frac{(1-x)^2\sin(1-x)}{(1-x)^2+\cos(1-x)}dx \quad \ldots\text{(ii)}$$

From equations (i) and (ii),

$I=-I$

$I+I=0 \quad\Rightarrow I=0$

Hence, the correct option is (B).

Scan for Video Solution

3.23 (B)

Given : $I=\int_0^2\int_0^x e^{x+y}\,dydx$

$$I=\int_0^2 e^x\left(\int_0^x e^y\,dy\right)dx$$

$$I=\int_0^2 e^x\left[e^y\right]_0^x dx$$

$$I=\int_0^2 e^x(e^x-1)\,dx$$

$$I=\int_0^2 (e^{2x}-e^x)\,dx$$

$$I=\left(\frac{e^{2x}}{2}-e^x\right)_0^2$$

$$I=\frac{e^4}{2}-e^2-\frac{1}{2}+1$$

$$I=\frac{1}{2}(e^4-2e^2+1)=\frac{1}{2}(e^2-1)^2$$

Hence, the correct option is (B).

3.24 (C)

Given : $x=y^2$, $0\le x\le 1$

Volume of revolution about x-axis is,

$$V=\int_a^b \pi y^2\,dx$$

Here, $y^2=x$, $a=0$, $b=1$.

$$\therefore \quad V=\int_0^1 \pi x\,dx=\pi\left[\frac{x^2}{2}\right]_0^1=\frac{\pi}{2}[1-0]=\frac{\pi}{2}$$

Hence, the correct option is (C).

3.25 2.097

Given : Integral is $I=\int_1^e x(\ln x)\,dx$

$$\int_1^e x(\ln x)\,dx=\left[\ln x\times\frac{x^2}{2}\right]_1^e-\int_1^e\frac{1}{x}\times\frac{x^2}{2}dx$$

[Applying Integration by parts]

$$=\left[\ln_e e\times\frac{e^2}{2}-\ln_e 1\times\frac{1}{2}\right]-\frac{1}{2}\left[\frac{x^2}{2}\right]_1^e$$

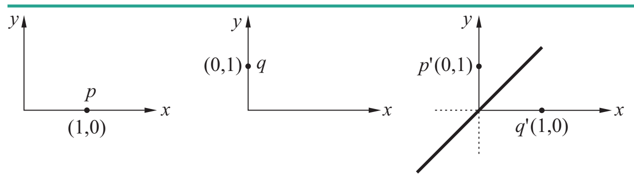
$$\int_1^e x(\ln x)\,dx=\frac{e^2}{2}-\frac{1}{4}[e^2-1]=\frac{e^2}{2}-\frac{e^2}{4}+\frac{1}{4}$$

$$=\frac{e^2}{4}+\frac{1}{4}=2.097$$

Hence, the value of the definite integral is **2.097**.

3.26 (B)

To find the transformation matrix for mirroring a point along the line $y=x$, considering two points, first on x-axis point p with coordinates (1, 0) and other on y-axis point q with coordinates (0, 1).



When we take the mirrors of these base factors about the line $y = x$, then we get the coordinates of mirror of point P as (0, 1) and the coordinates of mirror of point q as (1, 0). Hence, the transformation matrix is obtained as

$$[T] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Column 1 $\rightarrow$ Coordinates of mirror of p

Column 2 $\rightarrow$ Coordinates of mirror of q

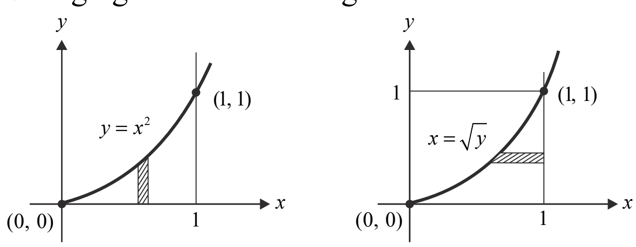
when this matrix is multiples with any shape in XY-plane then it will give the coordinates of mirror of that shape about the line $y = x$.

Hence, the correct option is (B).

3.27 (D)

Given : $I = \int_{x=0}^{1}\int_{y=0}^{x^2} xy^2\, dy dx$

Changing the order of integration



New limits after changing the order of integration are given by,

$$x = \sqrt{y} \text{ to } x = 1$$

and, $\quad y = 0$ to $y = 1$

$$\therefore \quad I = \int_{y=0}^{1}\int_{x=\sqrt{y}}^{1} xy^2\, dx dy$$

Hence, the correct option is (D).

3.28 (C)

Given : $I = \int_{0}^{\pi/2}\int_{0}^{\cos\theta} r\sin\theta\, dr\, d\theta$

$$I = \int_{0}^{\pi/2}\left[\int_{r=0}^{\cos\theta} r\, dr\right]\sin\theta d\theta$$

$$I = \int_{\theta=0}^{\pi/2}\left(\frac{r^2}{2}\right)_0^{\cos\theta} \sin\theta d\theta$$

$$I = \frac{1}{2}\int_{\theta=0}^{\pi/2} \sin\theta\cos^2\theta d\theta$$

Now, using Gamma function with $m = 1,\ n = 2$.

$$= \frac{1}{2}\times\frac{\Gamma\left(\frac{1+1}{2}\right)\Gamma\left(\frac{2+1}{2}\right)}{2\Gamma\left(\frac{1+2+2}{2}\right)} = \frac{1}{4}\times\frac{\Gamma 1\,\Gamma\frac{3}{2}}{\Gamma\frac{5}{2}} = \frac{1}{4}\times\frac{1\times\Gamma\frac{3}{2}}{\frac{3}{2}\Gamma\frac{3}{2}} = \frac{1}{6}$$

Hence, the correct option is (C).

📖 Key Point

$$I = \int_{\theta=0}^{\pi/2} \sin^m\theta\cos^n\theta d\theta = \frac{\Gamma\left(\frac{m+1}{2}\right).\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$$

$\Gamma(n+1) = n\Gamma n$ if n is fraction

Civil Engineering : CE

3.1 (A)

Given : Parabola, $y^2 = 8x$ ...(i)

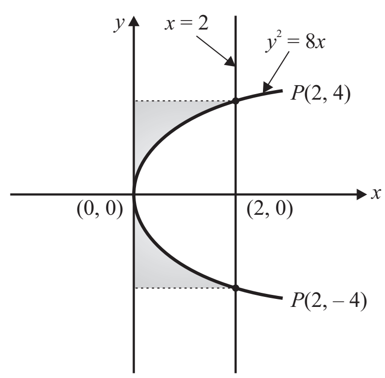
Line, $x = 2$ ...(ii)

Solving equation (i) and (ii),

$$y^2 = 8\times 2 \quad\Rightarrow\quad y^2 = 16$$

$$y = \pm 4$$

Thus common points are $(2, 4)$ and $(2, -4)$.



$V_1 =$ Volume generated by revolving $y^2 = 8x$ between $y = -4$ and $y = 4$ about y axis.

$$V_1 = \int_{y_1}^{y_2} \pi x^2 dy = \int_{-4}^{4} \pi \left[\frac{y^2}{8}\right]^2 dy$$

$$V_1 = \int_{-4}^{4} \pi \left[\frac{y^4}{8^2}\right] dy = \frac{\pi}{8^2}\left[\frac{y^5}{5}\right]_{-4}^{4}$$

$$V_1 = \frac{\pi}{8^2 \times 5} \times \left[4^5 + 4^5\right] = \frac{32\pi}{5}$$

$V_2 =$ Volume generated by revolving $x = 2$ between $y = -4$ and $y = 4$ about y-axis.

$$V_2 = \int_{y_1}^{y_2} \pi x^2 dy = \int_{-4}^{4} \pi [2]^2\, dy$$

$$V_2 = 4\pi[y]_{-4}^{4} = 4\pi[4+4] = 32\pi$$

Required volume $= V_2 - V_1 = 32\pi - \frac{32\pi}{5} = \frac{128\pi}{5}$

Hence, the correct option is (A).

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Key Point

(i) Volume revolved by x-axis,

$$V_x = \int_a^b \pi y^2 dx$$

(ii) Volume revolved by y-axis,

$$V_y = \int_a^b \pi x^2 dy$$

3.2 (C)

Given : $I = \int \log x\, dx$

$$I = \int \log x \times 1\, dx$$

Applying integration by parts,

$$\int \underset{\text{(I)}}{u}\ \underset{\text{(II)}}{v}\, dx = u\int v\, dx - \int\left(\frac{d}{dx}(u)\int v\, dx\right)dx$$

$$I = \int \underset{\text{(I)}}{\log x} \times \underset{\text{(II)}}{1}\ dx$$

$$I = \log x \int 1\, dx - \int\left(\frac{d}{dx}(\log x) \times \int 1\, dx\right)dx$$

$$I = \log x \times x - \int \left(\frac{1}{x} \times x\right)dx$$

$$I = x\log x - \int dx$$

$$I = x\log x - x + C \qquad \text{[Since, } C = 0\text{]}$$

$$I = x(\log x - 1)$$

Hence, the correct option is (C).

Key Point

Applying integration by parts,

$$\int \underset{\text{(I)}}{u}\ \underset{\text{(II)}}{v}\, dx = u\int v\, dx - \int\left(\frac{d}{dx}(u)\int v\, dx\right)dx\,]$$

By ILATE's rule preference of function is decided.

$I =$ Inverse function ($\sin^{-1} x$, $\cos^{-1} x$)

$L =$ Log function [$\log x$, $\log(1+x)$]

$A =$ Algebraic function ($x^2 + 1$)

$T =$ Trigonometric function ($\sin x, \cos x$)

$E =$ Exponential function (e^x, e^{-x}).

3.3 (B)

Given : Parabola, $2y = x^2$ ...(i)

Line, $x = y - 4$ ...(ii)

$$x + 4 = y$$

Put the value of y in equation (i),

$$2(x+4) = x^2$$

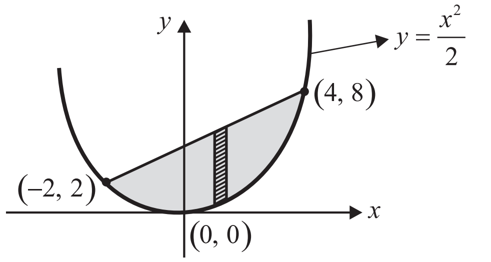
$$x^2 - 2x - 8 = 0$$

$$x = 4, -2$$

When $x = 4$ then $y = 8$.

When $x = -2$ then $y = 2$.

Thus, the common points are $(4, 8), (-2, 2)$.



$$\text{Area} = \iint dy\,dx = \int_{-2}^{4}\int_{y=\frac{x^2}{2}}^{x+4} dy\,dx = \int_{-2}^{4}[y]_{\frac{x^2}{2}}^{x+4}dx$$

$$\text{Area} = \int_{-2}^{4}\left[x+4-\frac{x^2}{2}\right]dx = \left[\frac{x^2}{2}+4x-\frac{1}{2}\times\frac{x^3}{3}\right]_{-2}^{4}$$

$$\text{Area} = \left[\frac{4^2-(-2)^2}{2}+4\left[4-(-2)\right]-\frac{1}{6}\left[4^3-(-2)^3\right]\right]$$

$$\text{Area} = [6+24-12] = 18$$

Hence, the correct option is (B).

3.4 (C)

Given :

$$\int_0^2\int_{x^2}^{2x} f(x,y)dy\,dx = \int_{x=0}^{2}\int_{y=x^2}^{y=2x} f(x,y)\,dydx$$

$$y = 2x \qquad \ldots(i)$$

$$y = x^2 \qquad \ldots(ii)$$

Solving equation (i) and equation (ii),

$$x^2 = 2x$$

$$x^2 - 2x = 0$$

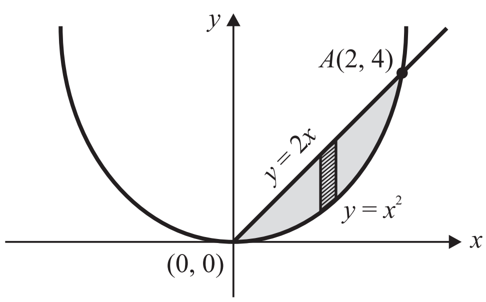
$$x(x-2) = 0$$

$$x = 0,\ x = 2$$

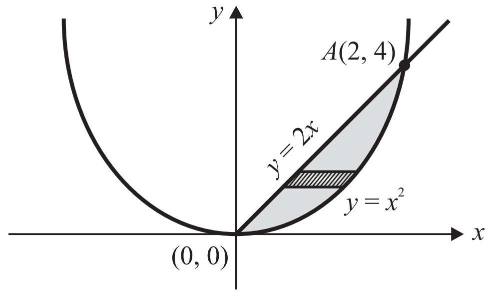
When $x = 0$ then $y = 0$

When $x = 2$ then $y = 4$

Thus, the common points are $(0,0)$ and $(2,4)$.



To change the order of integration, change the horizontal strip to vertical strip or vice-versa.



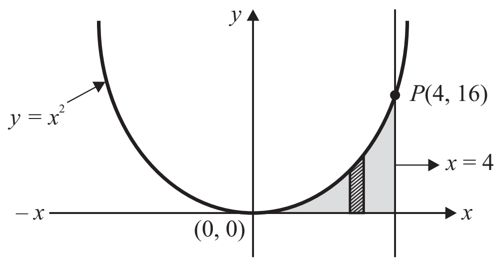
Therefore,

$$\int_0^2\int_{x^2}^{2x} f(x,y)dy\,dx = \int_{y=0}^{y=4}\int_{x=\frac{y}{2}}^{x=\sqrt{y}} f(x,y)\,dx\,dy$$

Hence, the correct option is (C).

3.5 (B)

Given : $y = x^2,\ x = 4,\ y = 0$



At $x = 4,\ y = x^2 = 4^2 = 16$

Thus, the common point is $P(4, 16)$.

Area bounded by curves $= \int_0^4\int_{y=0}^{y=x^2} dx\,dy$

$$\int_0^4 [y]_0^{x^2}dx = \int_0^4 x^2\,dx = \left[\frac{x^3}{3}\right]_0^4 = \frac{64}{3}$$

Hence, the correct option is (B).

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3.6 (C)

Given : $f(x) = e^x$

$$f(-x) = e^{-x}$$

For odd function, $f(x) = -f(-x)$

For even function $f(x) = f(-x)$

Therefore, $f(x)$ is neither even nor odd.

Hence, the correct option is (C).

3.7 (A)

Given : $I=\int_0^{\frac{\pi}{4}}\cos^2 x dx$

$$\left[\cos 2x = 1-2\sin^2 x = 2\cos^2 x - 1\right]$$

$$I=\int_0^{\frac{\pi}{4}}\frac{1+\cos 2x}{2}dx=\frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]_0^{\pi/4}$$

$$I=\frac{1}{2}\left[\frac{\pi}{4}+\frac{\sin 2\times\frac{\pi}{4}}{2}\right]$$

$$I=\frac{1}{2}\left[\frac{\pi}{4}+\frac{1}{2}\right]=\frac{\pi}{8}+\frac{1}{4}$$

Hence, the correct option is (A).

3.8 (C)

Given : $I=\int_0^1 x\ln x\, dx=\int_0^1 \underset{(\mathrm{I})}{\ln x}\times \underset{(\mathrm{II})}{x}\, dx$

Applying integration by parts,

$$I=\ln x\times\int x\,dx-\int\left(\frac{d}{dx}(\ln x)\times\int x dx\right)dx$$

$$I=\left[\ln x\times\frac{x^2}{2}-\int\frac{1}{x}\times\frac{x^2}{2}dx\right]_0^1$$

$$I=\left[\ln x\times\frac{x^2}{2}-\frac{1}{2}\int x\,dx\right]_0^1$$

$$I=\left[\ln x\times\frac{x^2}{2}-\frac{1}{2}\times\frac{x^2}{2}\right]_0^1$$

$$I=\left[\ln x\times\frac{x^2}{2}-\frac{x^2}{4}\right]_0^1$$

$$I=\ln(1)\times\frac{1}{2}-\frac{1}{4}-0-0$$

$$I=0\times\frac{1}{2}-\frac{1}{4}=-\frac{1}{4}$$

Hence, the correct option is (C).

3.9 (C)

Given : $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin 2x}{1+\cos x}dx$

$$f(x)=\frac{\sin 2x}{1+\cos x}$$

$$f(-x)=\frac{\sin(2\times(-x))}{1+\cos(-x)}$$

$$f(-x)=\frac{\sin(-2x)}{1+\cos(-x)}=\frac{-\sin 2x}{1+\cos x}$$

$$\left[\begin{array}{ll}\text{Since,} & \sin(-\theta)=-\sin\theta\\ & \cos(-\theta)=\cos\theta\end{array}\right]$$

Since, $f(x)=-f(-x)$

Hence, it is a odd function.

Therefore, $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin 2x}{1+\cos x}dx=0$

Hence, the correct option is (C).

Key Point

By the property of integrals,

$$\int_{-a}^{a}f(x)dx=\begin{cases}2\int_0^a f(x)dx; & \text{if } f(-x)=f(x)\\ & \text{Even function}\\ 0\quad ; & \text{if } f(-x)=-f(x)\\ & \text{Odd function}\end{cases}$$

3.10 (A)

Given : $P(3,-2,-1)$, $Q(1,3,4)$, $R(2,1,-2)$, $O(0,0,0)$

Equation of plane OQR is given by,

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ x_3-x_1 & y_3-y_1 & z_3-z_1\end{vmatrix}=0$$

$$\begin{vmatrix}x-0 & y-0 & z-0\\ 1 & 3 & 4\\ 2 & 1 & -2\end{vmatrix}=0$$

$$x(-6-4)-y(-2-8)+z(1-6)=0$$

$$-10x+10y-5z=0$$

$$-2x+2y-z=0$$

$$2x-2y+z=0$$

Distance from point $P(3,-2,-1)$ to the plane OQR is given by,

$$\left|\frac{2\times 3-2\times(-2)+(-1)}{\sqrt{2^2+(-2)^2+1^2}}\right|=\left|\frac{6+4-1}{\sqrt{9}}\right|=3$$

Hence, current option is (A).

Key Point

Distance of a point (x_0, y_0, z_0) from a plane $ax+by+cz=d$ is given by,

$$D=\frac{ax_0+by_0+cz_0+d}{\sqrt{a^2+b^2+c^2}}$$

3.11 (C)

Given : $I=\oint_C (xydy-y^2dx)=\oint_C(-y^2dx+xydy)$

According to Green's Theorem,

$$\oint_C(\phi dx+\psi dy)=\iint_C\left(\frac{\partial\psi}{\partial x}-\frac{\partial\phi}{\partial y}\right)dx\,dy \quad \ldots\text{(i)}$$

where, $\psi=xy,\ \phi=-y^2$

$$\frac{\partial\psi}{\partial x}=y,\frac{\partial\phi}{\partial y}=-2y$$

$$I=\oint_C(-y^2dx+xydy)$$

$$I=\int_{y=0}^{1}\int_{x=0}^{1}[y-(-2y)]\,dx\,dy$$

$$\oint_C(-y^2dx+xydy)=\int_{y=0}^{1}\int_{x=0}^{1}3y\,dx\,dy$$

$$\oint_C(-y^2dx+xydy)=\int_{y=0}^{1}3y[x]_0^1\,dy$$

$$\oint_C(-y^2dx+xydy)=\int_{y=0}^{1}3y\,dy$$

$$\oint_C(-y^2dx+xydy)=3\left[\frac{y^2}{2}\right]_0^1=\frac{3}{2}$$

Hence, the correct option is (C).

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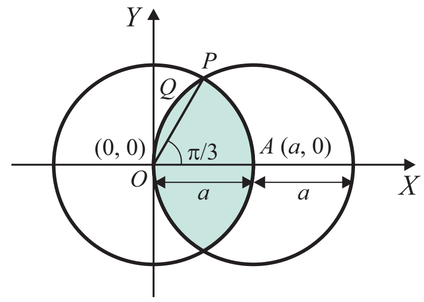
3.12 (D)

Given : $r=a$ …(i)

$r=2a\cos\theta$ …(ii)

Equation (i) represents a circle with centre $(0,0)$ and radius equals to a.

Equation (ii) represents a circle symmetrical about OX, with centre $(0,a)$ and radius equals to a.



Solving equation (i) and (ii),

$$a=2a\cos\theta$$

$$\frac{1}{2}=\cos\theta$$

$$\theta=\frac{\pi}{3}$$

Required area

$=2\times$ Area of $OAPQ$ [by symmetry]

$=2[$Area of $OAP+$ Area of $OPQ]$

$$=2\left[\frac{1}{2}\int_0^{\frac{\pi}{3}}r^2d\theta+\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}r^2d\theta\right]$$

$$=\int_0^{\frac{\pi}{3}}a^2d\theta+\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}4a^2\cos^2\theta d\theta$$

$$=a^2[\theta]_0^{\frac{\pi}{3}}+4a^2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\cos^2\theta d\theta$$

$$=a^2\left[\frac{\pi}{3}-0\right]+4a^2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\left(\frac{1+\cos 2\theta}{2}\right)d\theta$$

Required area

$$= \frac{a^2\pi}{3} + \frac{4a^2}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \frac{a^2\pi}{3} + 2a^2\left[\frac{\pi}{2} + \frac{\sin 2\times\frac{\pi}{2}}{2} - \frac{\pi}{3} - \frac{\sin\left(2\times\frac{\pi}{3}\right)}{2}\right]$$

$$= \frac{a^2\pi}{3} + 2a^2\left[\frac{\pi}{2} + 0 - \frac{\pi}{3} - \frac{\frac{\sqrt{3}}{2}}{2}\right]$$

$$= \frac{a^2\pi}{3} + 2a^2\left[\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right]$$

$$= a^2\left[\frac{\pi}{3} + \frac{\pi}{3} - \frac{\sqrt{3}}{2}\right] = a^2\left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right]$$

$$= 1.228a^2$$

Hence, correct option is (D).

3.13 (B)

Given : $I = \int_0^\infty \frac{\sin t}{t} dt$

Laplace transform of $\sin t$ is,

$$L[\sin t] = \frac{1}{s^2+1}$$

$$L\left[\frac{\sin t}{t}\right] = \int_s^\infty \frac{1}{s^2+1} ds = \left[\tan^{-1} s\right]_s^\infty$$

$$L\left[\frac{\sin t}{t}\right] = \tan^{-1}\infty - \tan^{-1} s$$

$$L\left[\frac{\sin t}{t}\right] = \frac{\pi}{2} - \tan^{-1} s = \cot^{-1} s$$

$$L\left[\frac{\sin t}{t}\right] = \int_0^\infty e^{-st}\frac{\sin t}{t} dt = \cot^{-1} s$$

Put $s = 0$,

$$\int_0^\infty e^{-0t}\frac{\sin t}{t} dt = \cot^{-1} 0$$

$$\int_0^\infty \frac{\sin t}{t} dt = \frac{\pi}{2}$$

Hence, the correct option is (B).

Key Point

(i) $L(\sin at) = \dfrac{a}{s^2+a^2}$

(ii) $L\left[\dfrac{x(t)}{t}\right] = \int_s^\infty X(s)\, ds$

(iii) $\int \dfrac{1}{x^2+a^2} dx = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right).$

3.14 (A)

$$I = \int_0^3\int_0^x (6-x-y)\, dx\, dy$$

$$I = \int_0^3\left[\int_0^x (6-x-y)\, dy\right] dx$$

$$I = \int_0^3\left[6y - xy - \frac{y^2}{2}\right]_0^x dx$$

$$I = \int_0^3\left[6x - x^2 - \frac{x^2}{2}\right] dx$$

$$I = \left[6\times\frac{x^2}{2} - \frac{x^3}{3} - \frac{1}{2}\times\frac{x^3}{3}\right]_0^3$$

$$I = \left[6\times\frac{3^2}{2} - \frac{3^3}{3} - \frac{1}{2}\times\frac{3^3}{3}\right]$$

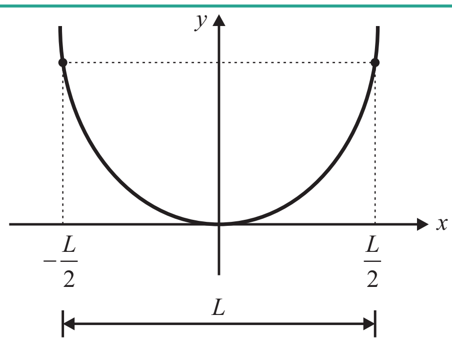
$$I = 27 - 9 - \frac{9}{2} = 18 - 4.5 = 13.5$$

Hence, the correct option is (A).

3.15 (D)

Given : $y = 4h\dfrac{x^2}{L^2}$ …(i)

The above equation is a parabola. i.e. symmetric about y-axis. Hence the length of cable is symmetrically distributed about x-axis (i.e. even function).



Hence, length $L = 2\int_0^{\frac{L}{2}} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

Since, $y = 4h\frac{x^2}{L^2}$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = \frac{4h}{L^2} \times 2x = \frac{8hx}{L^2}$$

Therefore, $L = 2\int_0^{\frac{L}{2}} \sqrt{1+64\frac{h^2x^2}{L^4}}\, dx$

Hence, the correct option is (D).

Key Point

(i) The length of the arc of the curve $y = f(x)$ between the points where $x = a$ and $x = b$ is

$$\int_a^b \sqrt{\left[1+\left(\frac{dy}{dx}\right)^2\right]}\, dx$$

(ii) The length of the arc of the curve $x = f(y)$ between the points where $y = a$ and $y = b$ is

$$\int_a^b \sqrt{\left[1+\left(\frac{dx}{dy}\right)^2\right]}\, dy$$

3.16 (B)

Given : $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\, dx$ ...(i)

By property of integral,

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}\, dx \quad \text{...(ii)}$$

Adding equation (i) and (ii),

$$I + I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\, dx + \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}\, dx$$

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}\, dx$$

$$2I = \int_0^a dx = [x]_0^a = a$$

$$I = \frac{a}{2}$$

Hence, the correct option is (B).

3.17 (B)

Given : $I = \int_0^{\frac{\pi}{6}} \cos^4 3\theta \sin^3 6\theta\, d\theta$

$$I = \int_0^{\frac{\pi}{6}} \cos^4 3\theta (2\sin 3\theta \cos 3\theta)^3 d\theta$$

$$[\sin 2A = 2\sin A\cos A]$$

$$I = \int_0^{\frac{\pi}{6}} 8\cos^7 3\theta \sin^3 3\theta\, d\theta$$

Let $3\theta = t$, $3d\theta = dt$

When $\theta = 0$, $t = 0$

When $\theta = \frac{\pi}{6}$, $t = \frac{\pi}{2}$

$$I = 8\int_0^{\frac{\pi}{2}} \cos^7 t \sin^3 t \frac{dt}{3}$$

$$I = \frac{8}{3}\int_0^{\frac{\pi}{2}} \sin^3 t \cos^7 t\, dt$$

Here, $m = 3$, $n = 7$

$$I = \frac{8}{3}\left[\frac{(3-1)\times(7-1)(7-3)(7-5)}{(3+7)(3+7-2)(3+7-4)(3+7-6)(3+7-8)}\right]$$

$$I = \frac{8}{3} \times \frac{2\times 6\times 4\times 2}{10\times 8\times 6\times 4\times 2} = \frac{1}{15}$$

Hence, the correct option is (B).

Key Point

By the property of integrals,

$$\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx$$

$$= \frac{(m-1)(m-3).....(n-1)(n-3)(n-5)}{(m+n)(m+n-2)......}$$

3.18 (C)

Given : $I = \int_0^{\pi/2} \frac{\cos x + i\sin x}{\cos x - i\sin x} dx$

$$I = \int_0^{\pi/2} \frac{e^{ix}}{e^{-ix}} dx = \int_0^{\pi/2} e^{2ix} dx$$

$$I = \frac{1}{2i}\left[e^{2ix}\right]_0^{\pi/2} = \frac{1}{2i}\left[e^{i\pi} - 1\right]$$

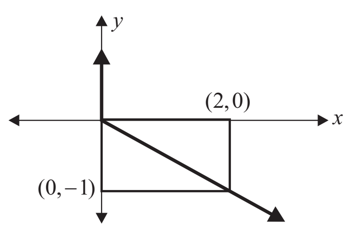
$$I = \frac{1}{2i}[\cos\pi + i\sin\pi - 1]$$

$$I = \frac{1}{2i}[-1+0-1] = \frac{-2}{2i} = i$$

Hence, the correct option is (C).

3.19 (B)

Given :



From figure,

$$x = 0,\ y = 0$$

$$x = 2,\ y = -1$$

From options (A) :

$$x = y - |y|$$

Put $y = -1$,

$$x = -1 - |-1| = -2 \Rightarrow (-2, -1)$$

From option (B) :

$$x = -(y - |y|)$$

Put $y = -1$,

$$x = -(-1 - |-1|) = 2 \Rightarrow (2, -1)$$

From option (C) :

$$x = y + |y|$$

Put $y = -1$,

$$x = -1 + |-1| = 0 \Rightarrow (0, -1)$$

From option (D) :

$$x = -(y + |y|)$$

Put $y = -1$,

$$x = -(-1 + |-1|) = -(0) = 0 \Rightarrow (0, -1)$$

Only option (B) is valid option for given figure.

Hence, the correct option is (B).

3.20 (B)

Since, the number of cycles to failure decreases exponentially with an increase in load. The general equation is given by,

$$y = n e^{-mx}$$

where, $y =$ number of cycle failure and x is load

Given : $y = 100$ and $x = 80$

Therefore, $100 = n e^{-80m}$ ...(i)

When load is halved, it take 10000 cycle for failure.

$$10000 = n e^{-40m} \quad ...(ii)$$

From equation (i) and (ii)

$$100 = e^{40m}$$

$$m = \frac{\ln 100}{40} \quad ...(iii)$$

For 5000 cycles to failure,

$$5000 = n e^{-xm} \quad ...(iv)$$

From equation (ii) and (iv),

$$2 = e^{m(x-40)}$$

$$m(x-40) = \ln 2$$

From equation (iii),

$$x - 40 = \frac{\ln 2}{m}$$

$$x-40=\frac{\ln 2}{\ln 100}\times 40$$

$$x-40=\frac{0.693}{4.605}\times 40$$

$$x=46.02$$

Hence, the correct option is (B).

3.21 (B)

Given : $I=\int_0^\infty \frac{1}{1+x^2}dx+\int_0^\infty \frac{\sin x}{x}dx$

Let $I=I_1+I_2$,

where, $I_1=\int_0^\infty \frac{1}{1+x^2}dx=[\tan^{-1}x]_0^\infty$

$$I_1=\tan^{-1}\infty-\tan^{-1}0$$

$$I_1=\frac{\pi}{2}-0=\frac{\pi}{2}$$

$$I_2=\int_0^\infty \frac{\sin x}{x}dx$$

$$L[\sin x]=\frac{1}{s^2+1}$$

$$L\left[\frac{\sin x}{x}\right]=\int_s^\infty \frac{1}{s^2+1}dx=[\tan^{-1}s]_s^\infty$$

$$L\left[\frac{\sin x}{x}\right]=\tan^{-1}\infty-\tan^{-1}s$$

$$L\left[\frac{\sin x}{x}\right]=\frac{\pi}{2}-\tan^{-1}s=\cot^{-1}s$$

$$L\left[\frac{\sin x}{x}\right]=\int_0^\infty e^{-sx}\frac{\sin x}{x}dx=\cot^{-1}s$$

Put $s=0$

$$\int_0^\infty e^{-0x}\frac{\sin x}{x}dx=\cot^{-1}(0)$$

$$I_2=\int_0^\infty \frac{\sin x}{x}dx=\frac{\pi}{2}$$

Hence, $I=I_1+I_2=\frac{\pi}{2}+\frac{\pi}{2}=\pi$

Hence, the correct option is (B).

3.22 (B)

Given : $y=x^2+1$ ...(i)

$x+y=3$ ...(ii)

From equation (ii),

$$y=3-x$$

Put the above value in equation (i),

$$3-x=x^2+1$$

$$x^2+x-2=0$$

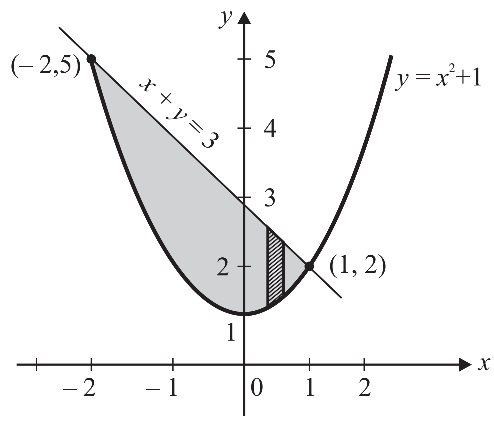
$$(x+2)(x-1)=0$$

$$x=-2,\ 1$$

When $x=-2,\ y=5$

When $x=1,\ y=2$

Thus, the common points are $(-2,\ 5)$ and $(1,2)$.



$$\text{Area}=\iint dydx=\int_{x=-2}^{1}\int_{y=x^2+1}^{3-x}dydx$$

$$\text{Area}=\int_{x=-2}^{1}[y]_{y=x^2+1}^{3-x}dx=\int_{x=-2}^{1}(2-x-x^2)dx$$

$$\text{Area}=\left[2x-\frac{x^2}{2}-\frac{x^3}{3}\right]_{-2}^{1}$$

$$\text{Area}=\left(2-\frac{1}{2}-\frac{1}{3}\right)-\left(-4-2+\frac{8}{3}\right)$$

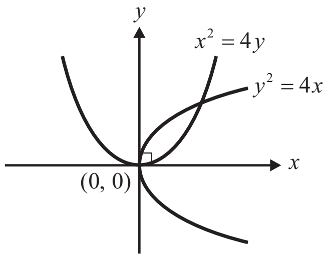
$$\text{Area}=\left(\frac{12-3-2}{6}\right)-\left(\frac{-10}{3}\right)$$

$$\text{Area}=\frac{7}{6}+\frac{10}{3}=\frac{27}{6}=\frac{9}{2}$$

Hence, the correct option is (B).

3.23 (D)

Given : Curves $x^2=4y$ and $y^2=4x$



Angle between the curves is angle between the tangents at the point of intersection. i.e. 90^0

Hence, the correct option is (D).

3.24 85.33

Given : $x^2 = 8y$ ...(i)

$y = 8$ ...(ii)

From equation (i) and equation (ii),

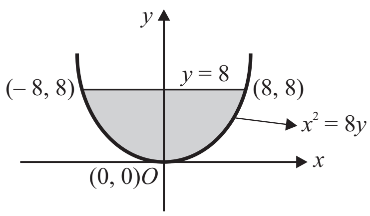
$$x^2 = 8\times 8 = 64$$

$$x = \pm 8$$

When $x = 8$, $y = 8$

When $x = -8$, $y = 8$

Thus, common points are $(8,8)$ and $(-8,8)$.



$$\text{Area} = \int_{-8}^{8}\int_{y=\frac{x^2}{8}}^{8} dy\,dx = \int_{-8}^{8} [y]_{y=\frac{x^2}{8}}^{8}\,dx$$

$$\text{Area} = \int_{-8}^{8}\left(8-\frac{x^2}{8}\right)dx = \left[8x-\frac{1}{8}\times\frac{x^3}{3}\right]_{-8}^{8}$$

$$\text{Area} = 8\times 8-\frac{1}{8}\times\frac{8^3}{3}-\left[8\times(-8)-\frac{1}{8}\times\frac{(-8)^3}{3}\right]$$

$$\text{Area} = 64-\frac{64}{3}-\left[-64+\frac{64}{3}\right]$$

$$\text{Area} = 128-\frac{128}{3} = 85.33$$

Hence, the area between the curves is **85.33**.

3.25 (B)

Given : $f(x) = 2x^7 + 3x - 5$

Checking from the options,

From option (B) :

Put $x = 1$,

$$f(1) = 2\times 1^7 + 3\times 1 - 5 = 0$$

Since, $x = 1$ satisfies the given function $f(x)$.

Therefore, $(x-1)$ is a factor of given function $f(x)$.

Hence, the correct option is (B).

3.26 (B)

Given : $f(x) = e^{-x-e^{-x}}$

$$g(x) = \int f(x)dx = \int e^{-x-e^{-x}}\,dx$$

$$g(x) = \int e^{-e^{-x}} e^{-x} dx \qquad \ldots\text{(i)}$$

Put $e^{-x} = t$,

Differentiating on both sides,

$$e^{-x}(-1)dx = dt$$

$$e^{-x}dx = -dt$$

From equation (i),

$$g(t) = \int e^{-t}(-1)\,dt$$

$$g(t) = \int -e^{-t}dt = -(e^{-t})(-1)$$

$$g(t) = e^{-t}$$

Put $t = e^{-x}$ in above equation,

$$g(x) = e^{-e^{-x}}$$

Hence, the correct option is (B).

3.27 (A)

Given : $I = \int_0^1 \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}}dx$ ...(i)

Let $\sin^{-1} x = t$

$$\frac{1}{\sqrt{1-x^2}}dx = dt$$

When $x = 0$, $t = \sin^{-1} 0 = 0$

When $x = 1$, $t = \sin^{-1} 1 = \frac{\pi}{2}$

From equation (i),

$$I=\int_0^{\frac{\pi}{2}} t^2\,dt$$

$$I=\left[\frac{t^3}{3}\right]_0^{\pi/2}=\frac{1}{3}\left[\left(\frac{\pi}{2}\right)^3-0^3\right]$$

$$I=\frac{1}{3}\times\frac{\pi^3}{8}=\frac{\pi^3}{24}$$

Hence, the correct option is (A).

3.28 (B)

Given : $I=\int_0^{\pi} x\cos^2 x.dx$

Method 1

$$I=\int_0^{\pi} x\left(\frac{1+\cos 2x}{2}\right).dx$$

$$I=\int_0^{\pi}\left(\frac{x}{2}+\frac{x}{2}\cos 2x\right).dx$$

$$I=\int_0^{\pi}\frac{x}{2}dx+\int_0^{\pi}\frac{x}{2}\cos 2x\,dx$$

$$I=\frac{1}{2}\left[\frac{x^2}{2}\right]_0^{\pi}+\frac{1}{2}\int_0^{\pi}\underset{\text{(I)}}{x}\ \underset{\text{(II)}}{\cos 2x}\,dx$$

$$I=\frac{\pi^2}{4}+\frac{1}{2}\left[x\times\frac{\sin 2x}{2}-\int\left(\frac{d}{dx}(x)\times\int\cos 2x\right)dx\right]$$

$$I=\frac{\pi^2}{4}+\frac{1}{2}\left[\frac{x\times\sin 2x}{2}+\frac{\cos 2x}{4}\right]_0^{\pi}$$

$$I=\frac{\pi^2}{4}+\frac{1}{2}\left[\frac{1}{4}-\frac{1}{4}\right]=\frac{\pi^2}{4}$$

Hence, the correct option is (B).

Method 2

$$I=\int_0^{\pi} x\cos^2 x\,dx$$

$$I=\int_0^{\pi}(\pi-x)\cos^2(\pi-x)dx$$

[By definite integral property]

$$I=\pi\int_0^{\pi}\cos^2 x\,dx-\int_0^{\pi}x\cos^2 x\,dx$$

$$I=\pi\int_0^{\pi}\cos^2 x\,dx-I$$

$$2I=\pi\int_0^{\pi}\cos^2 x\,dx$$

$$2I=2\pi\int_0^{\pi/2}\cos^2 x\,dx$$

$$2I=2\pi\left[\frac{\pi}{2}\times\frac{(2-1)}{2}\right]=2\pi\left[\frac{\pi}{4}\right]=\frac{\pi^2}{2}$$

$$I=\frac{\pi^2}{4}$$

Hence, the correct option is (B).

Key Point

By the property of integrals,

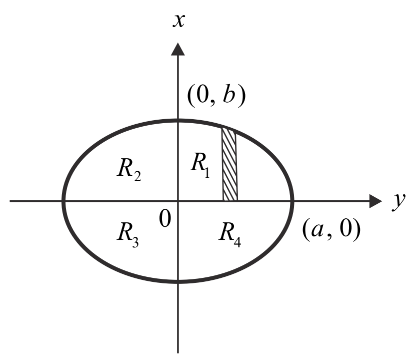
(i) $\int_0^{2a} f(x)dx=\begin{cases}2\int_0^{a} f(x)dx, & \text{if } f(2a-x)=f(x)\\ 0, & \text{if } f(2a-x)=-f(x)\end{cases}$

(ii) $\left[\int_0^{\pi/2}\cos^n x\,dx=\frac{\pi}{2}\times\frac{(n-1)(n-3)......}{n(n-2)......}\right]$

[if $n=$even]

3.29 (A)

Given : Equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



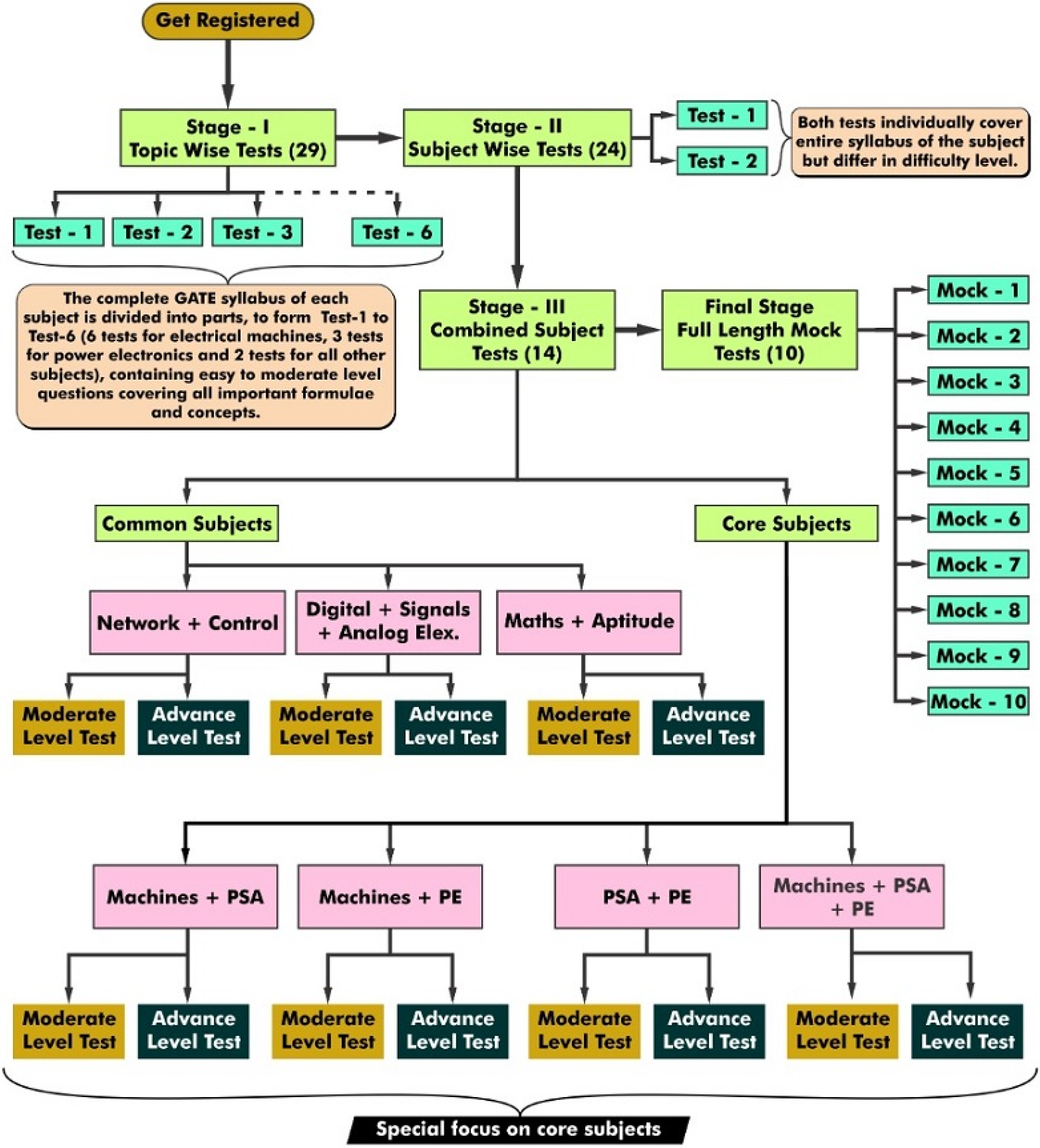
We know that, Area $=\iint_R dxdy$

$$\text{Area}=\int_0^{a}\int_0^{\frac{b}{a}\sqrt{a^2-x^2}} dy\,dx$$

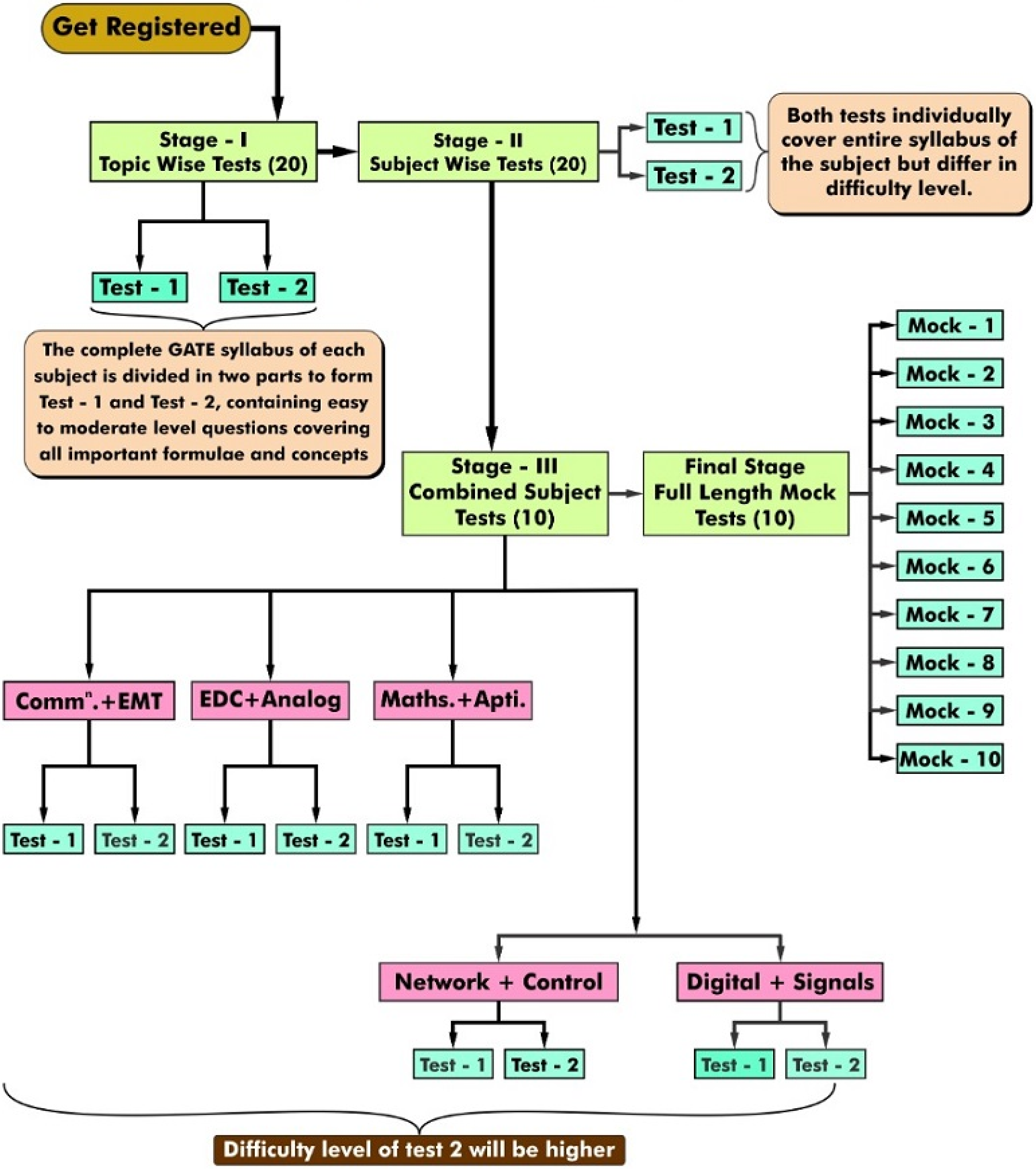
GATE ACADEMY

TEST SERIES STRUCTURE (EE)

(One Year Program)



GATE ACADEMY
TEST SERIES STRUCTURE (EC)
(One Year Program)



$$\text{Area of } R_1 = \int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\, dx$$

Area of

$$R_1 = \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$\text{Area of } R_1 = \frac{b}{a}\times\frac{a^2}{2}\times\frac{\pi}{2}$$

$$\text{Area of } R_1 = \frac{\pi ab}{4}$$

∴ Area of ellipse

$$= 4\times(\text{Area of } R_1) = \frac{\pi ab}{4}\times 4 = \pi ab$$

Hence, the correct option is (A).

3.30 0

Given : $I = \int_{-1}^{1} xe^{|x|}dx$

Let $f(x) = xe^{|x|}$

$$f(-x) = -xe^{|-x|} = -xe^{|x|}$$

∴ $f(x) = -f(-x)$ implies $f(x)$ is an odd function

So, from properties of definite integral

$\int_{-a}^{a} f(x)dx = 0$, if $f(x)$ is an odd function

$$I = \int_{-1}^{1} xe^{|x|} = 0$$

Hence, the correct answer is 0.

3.31 15

Given : $V = [2,3]\times[1,2]\times[0,1]$

$x \to 2$ to 3, $y \to 1$ to 2 and $z \to 0$ to 1

Volume

$$= \iiint_V 8xyzdxdydz = 8\int_2^3 xdx \times \int_1^2 ydy \times \int_0^1 zdz$$

$$V = 8\times\left[\frac{x^2}{2}\right]_2^3 \times \left[\frac{y^2}{2}\right]_1^2 \times \left[\frac{z^2}{2}\right]_0^1$$

$$V = 8\left[\frac{9-4}{2}\right]\left[\frac{4-1}{2}\right]\left[\frac{1-0}{2}\right]$$

$$V = \frac{8\times5\times3\times1}{8}$$

$$V = 15$$

Hence, the correct answer is 15.

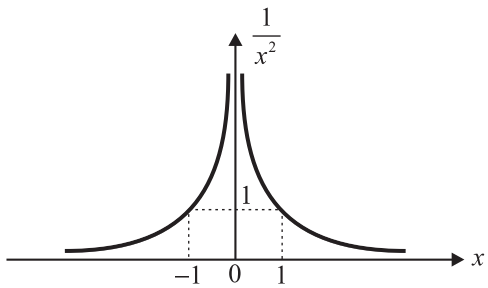
Instrumentation Engineering : IN

3.1 (B)

Given : $\int_{-1}^{1} \frac{1}{x^2}dx$

Method 1

Graph of given function $\frac{1}{x^2}$ is shown below,



From the above graph, it is observed that the function $\frac{1}{x^2}$ is discontinuous at $x = 0$.

Since, the limit of integration consists $x = 0$, therefore value of integral $\int_{-1}^{1}\frac{1}{x^2}dx$ does not exist.

Hence, the correct option is (B).

Method 2

$$\lim_{m\to 0^-}\int_{-1}^{m}\frac{1}{x^2}dx + \lim_{m\to 0^+}\int_{m}^{1}\frac{1}{x^2}dx$$

Function is discontinuous and hence its value does not exist.

Hence, the correct option is (B).

3.2 (B)

Given : $\rho = \cos^3\theta$ ...(i)

where, $\theta =$ Angle made by the tangent with the positive direction of the x-axis.

So, $\dfrac{dy}{dx} = y' = \tan\theta$ ...(ii)

and $\rho = \dfrac{y''}{[1+(y')^2]^{3/2}}$

From equation (i) and (ii),

$$\cos^3\theta = \frac{y''}{[1+\tan^2\theta]^{3/2}}$$

$$\cos^3\theta = \frac{y''}{[\sec^2\theta]^{3/2}} = y''\cos^3\theta$$

$$y'' = \frac{d^2y}{dx^2} = 1$$

Integrating both sides,

$$\frac{dy}{dx} = x + C$$

Integrating both sides again,

$$y = \frac{x^2}{2} + Cx + D$$

Above equation represents family of parabola.

Hence, the correct option is (B).

3.3 (D)

Given : $I = \int_0^\infty\int_0^\infty e^{-x^2}e^{-y^2}\,dxdy$

$$I = \int_0^\infty e^{-y^2}\left(\int_0^\infty e^{-x^2}dx\right)dy$$

$$I = \int_0^\infty e^{-x^2}dx \times \int_0^\infty e^{-y^2}dy$$

Let $I_1 = \int_0^\infty e^{-x^2}dx$ and $I_2 = \int_0^\infty e^{-y^2}dy$

$I = I_1 \times I_2$ ...(i)

Calculation of I_1 :

$$I_1 = \int_0^\infty e^{-x^2}dx$$

Let $x^2 = t \;\Rightarrow\; 2xdx = dt$

$$dx = \frac{dt}{2\sqrt{t}} = \frac{1}{2}t^{\frac{-1}{2}}dt$$

Changing the limits,

$x = 0 \Rightarrow t = 0$

$x = \infty \Rightarrow t = \infty$

Then, $I_1 = \int_0^\infty \frac{1}{2}t^{\frac{-1}{2}}e^{-t}dt = \frac{1}{2}\int_0^\infty e^{-t}t^{\frac{-1}{2}}dt$ ...(ii)

By the definition of Gamma function,

$\Gamma n = \int_0^\infty e^{-x}x^{n-1}dx$ ...(iii)

Comparing equation (ii) and (iii),

$$n-1 = -\frac{1}{2} \quad\Rightarrow\quad n = \frac{1}{2}$$

So, $I_1 = \frac{1}{2}\times\Gamma n = \frac{1}{2}\times\Gamma\frac{1}{2}$

$$I_1 = \frac{\sqrt{\pi}}{2} \qquad \left[\text{Since, } \Gamma\frac{1}{2} = \sqrt{\pi}\right]$$

Similarly, $I_2 = \dfrac{\sqrt{\pi}}{2}$

From equation (i),

$$I = I_1 \times I_2$$

$$I = \frac{\sqrt{\pi}}{2}\times\frac{\sqrt{\pi}}{2} = \frac{\pi}{4}$$

Hence, the correct option is (D).

3.4 (B)

Given : $y = x^2 + 2x + 10$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = 2x + 2$$

At $x = 1$,

$$\left.\frac{dy}{dx}\right|_{x=1} = (2\times 1) + 2 = 4$$

Hence, the correct option is (B).

3.5 (C)

Given : $e^{-\ln(x)}$; $x > 0$

From the property of logarithm,

$$a\log_e m = \log_e(m^a)$$

So, $$e^{-\log_e x} = e^{\log_e x^{-1}} = e^{\log_e \frac{1}{x}} = \frac{1}{x} = x^{-1}$$

Hence, the correct option is (C).

3.6 (D)

Given : $$I = \int_0^{\frac{\pi}{4}} \left(\frac{1-\tan x}{1+\tan x}\right) dx$$

Method 1

$$I = \int_0^{\frac{\pi}{4}} \left(\frac{1-\dfrac{\sin x}{\cos x}}{1+\dfrac{\sin x}{\cos x}}\right) dx$$

$$I = \int_0^{\frac{\pi}{4}} \left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) dx$$

Let $\cos x + \sin x = t$, $(-\sin x + \cos x)dx = dt$

Changing the limits,

$$x = 0 \Rightarrow t = 1$$

$$x = \frac{\pi}{4} \Rightarrow t = \sqrt{2}$$

Then, $$I = \int_1^{\sqrt{2}} \frac{1}{t} dt$$

$$I = [\ln t]_1^{\sqrt{2}} = \ln(\sqrt{2}) - \ln(1)$$

$$I = \ln\sqrt{2} = \ln(2)^{\frac{1}{2}} = \frac{1}{2}\ln 2$$

Hence, the correct option is (D).

Method 2

$$I = \int_0^{\pi/4} \frac{\tan\dfrac{\pi}{4} - \tan x}{1+\tan\dfrac{\pi}{4}\times\tan x} dx$$

$$I = \int_0^{\pi/4} \tan\left(\frac{\pi}{4} - x\right) dx$$

$$I = \int_0^{\pi/4} \tan x \, dx$$

$$I = [\log\sec x]_0^{\pi/4}$$

$$I = \log\sec\frac{\pi}{4} - \log\sec 0$$

$$I = \log\sqrt{2} - \log 1 = \frac{1}{2}\ln 2$$

Hence, the correct option is (D).

Scan for Video Solution

3.7 (C)

Given : $$\int_{y=0}^{a}\int_{x=0}^{y} f(x,y)\,dx\,dy$$

Limits of x :

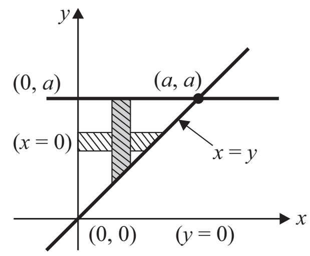
Lower limit, $x = 0$

Upper limit, $x = y$

Limits of y :

Lower limit, $y = 0$

Upper limit, $y = a$



The given limits in the question are according to horizontal strip.

By changing the order of integration, according to the vertical strip, new limits are given by,

Limits of y :

Lower limit, $y = x$

Upper limit, $y = a$

Limits of x :

Lower limit, $x = 0$

Upper limit, $x = a$

So, $$I = \int_{x=0}^{a}\int_{y=x}^{a} f(x,y)\,dy\,dx$$

Hence, the correct option is (C).

Scan for Video Solution

3.8 (B)

Given Y-axis intercept $c = -y$

Slope $m = \dfrac{dy}{dx} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$y = mx + c$$

$$y = \frac{dy}{dx}(x) + (-y)$$

$$x\frac{dy}{dx} = 2y$$

$$\frac{dy}{y} = \frac{2dx}{x}$$

Integrating on both side,

$$\int \frac{dy}{y} = \int \frac{2dx}{x}$$

$$\ln y = 2\ln x + \ln k$$

[Where, k = arbitrary constant]

$$\ln y = \ln(x^2 k)$$

$$y = x^2 k$$

$$y \propto x^2$$

$$\therefore \quad y = kx^2$$

Hence, the correct option is (B).

3.9 (C)

Given : x-axis intercept $= a = -0.5$

y-axis intercept $= b = 1$

The two coordinates where the line passes from are (x_1, y_1) and (x_2, y_2)

The equation of line passing from two points is given by,

$$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)} \cdot (x - x_1)$$

Putting the values of x_1, y_1, x_2, y_2 in above equation, we get

$$(y - 0) = \frac{(1 - 0)}{(0 + 0.5)} \cdot (x + 0.5)$$

$$y - 1 = 2x$$

$$y = 2x + 1$$

Hence, the correct option is (C)

Chemical Engineering : CH

3.1 (D)

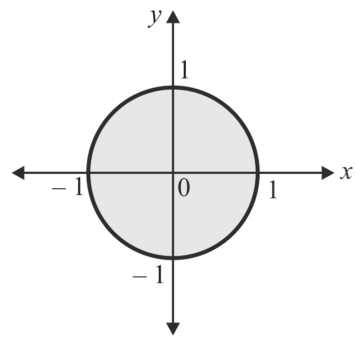
For an even function, $f(x) = f(-x)$

Hence, the correct option is (D).

3.2 (A)

Given : $I = \oint_C \left\{ \dfrac{y}{x^2 + y^2} dx - \dfrac{x}{x^2 + y^2} dy \right\}$

The contour C is, unit circle around origin



Put $x = \cos\theta$

$$dx = -\sin\theta\, d\theta$$

$$y = \sin\theta$$

$$dy = \cos\theta\, d\theta$$

Where, θ varies from 0 to 2π.

$$I = \oint_C \left\{ \frac{\sin\theta}{\sin^2\theta + \cos^2\theta}(-\sin\theta)d\theta - \frac{\cos\theta}{\sin^2\theta + \cos^2\theta}(\cos\theta)\, d\theta \right\}$$

So, $$I = \oint_C \left\{ \frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta + \cos^2\theta} \right\} d\theta$$

$$I = \oint_C \left\{ \frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta + \cos^2\theta} \right\} d\theta$$

$$I = -\oint_C \left\{ \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta + \cos^2\theta} \right\} d\theta$$

$$I = -\oint d\theta \qquad \{\text{since, } \sin^2\theta + \cos^2\theta = 1\}$$

$$I = -[\theta]_0^{2\pi} = -2\pi$$

Hence, the correct option is (A).

3.3 (D)

Given : $I = \int_{-2}^{2} \frac{dx}{x^2}$

$$f(x) = x^2$$

$$f(-x) = (-x)^2 = x^2$$

Since, $f(x) = f(-x)$

Therefore, $f(x)$ is an even function

$$I = 2\int_0^2 \frac{dx}{x^2} = 2\left[\frac{x^{-2+1}}{-2+1}\right]_0^2$$

$$I = -2\left[\frac{1}{x}\right]_0^2 = -2\left[\frac{1}{2} - \frac{1}{0}\right]$$

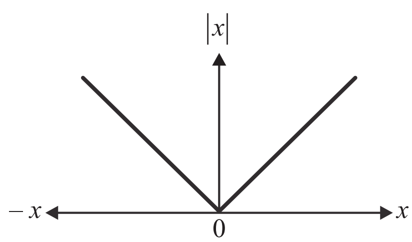
$$I = -2\left[\frac{1}{2} - \infty\right] = \infty$$

Hence, the correct option is (D).

3.4 (A)

Given : $|x|$

$|x|$ can be represented as,

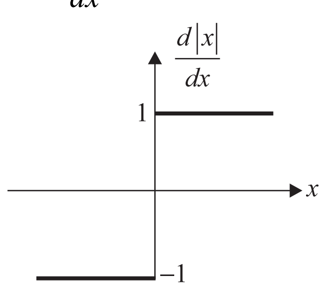


$$|x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

So, derivative of $|x|$,

$$\frac{d}{dx}|x| = \frac{dx}{dx} = 1,\ x > 0$$

$$\frac{d}{dx}|x| = \frac{-dx}{dx} = -1,\ x < 0$$



which can represented as $\frac{|x|}{x}$

Thus, $\frac{d}{dx}|x| = \frac{|x|}{x}$

Hence, the correct option is (A).

3.5 (C)

Given : $I = \int (-xy^n dx + x^n y\, dy)$

By Green's theorem,

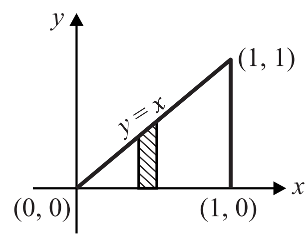
$$\int F_1\, dx + \int F_2\, dy = \iint_s \left[\left(\frac{\partial F_2}{\partial x}\right) - \left(\frac{\partial F_1}{\partial y}\right)\right] dx\, dy$$

where, $F_1 = -xy^n$, $F_2 = x^n y$

$$\frac{\partial F_2}{\partial x} = \frac{\partial}{\partial x}(x^n y) = nx^{n-1} y$$

$$\frac{\partial F_1}{\partial y} = \frac{\partial}{\partial y}(-xy^n) = -nxy^{n-1}$$

So, $I = \iint \left(nx^{n-1}y + nxy^{n-1}\right) dxdy$



So, $I = n \int_{x=0}^{1} \int_{y=0}^{x} \left(x^{n-1}y + xy^{n-1}\right) dx\, dy$

$$I = n\int_0^1 \left(\frac{x^{n-1}y^2}{2} + \frac{xy^n}{n}\right)_0^x dx$$

$$I = n\int_0^1 \left[\frac{x^{n-1}x^2}{2} + \frac{xx^n}{n}\right]$$

$$I = n\int_0^1 \left(\frac{x^{n+1}}{2} + \frac{x^{n+1}}{n}\right) dx$$

$$I = n\left[\frac{x^{n+2}}{2(n+2)} + \frac{x^{n+2}}{n(n+2)}\right]_0^1$$

$$I = n\left[\frac{1^{n+2}}{2(n+2)} + \frac{1^{n+2}}{n(n+2)}\right]$$

$$I = n\left[\frac{n+2}{2n(n+2)}\right] = \frac{1}{2}$$

Hence, the correct option is (C).

3.6 (D)

Given : $xy = c$ ...(i)

Differentiating equation (i),

$$x\,dy + y\,dx = 0$$

$$m_1 = \frac{dy}{dx} = \frac{-y}{x}$$

{where, m_1 = slope of curve $xy = c$ }

From option (D) :

$$y^2 - x^2 = c_1 \quad \text{...(ii)}$$

Differentiating equation (ii),

$$2y\,dy - 2x\,dx = 0$$

$$m_2 = \frac{dy}{dx} = \frac{x}{y}$$

{where, m_2 = slope the curve $y^2 - x^2 = c_1$ }

For the curves to be orthogonal,

$$m_1 \cdot m_2 = -1$$

$$\left(-\frac{y}{x}\right)\cdot\left(\frac{x}{y}\right) = -1$$

$$-1 = -1$$

Hence the correct option is (D).

3.7 (C)

Refer Solution 3.18 [Mechanical]

3.8 (D)

Given : A function $y(x)$

Checking from options,

From option (A) :

$$y = \frac{x+b}{a}$$

$$\frac{dy}{dx} = \frac{1}{a}$$

Hence, option (A) is incorrect.

From option (B) :

$$y = ax + b$$

$$\frac{dy}{dx} = a$$

Hence, option (B) is incorrect.

From option (C) :

$$y = \frac{\sqrt{x^2+b}}{a}$$

$$\frac{dy}{dx} = \frac{1}{a} \times \frac{1\times 2x}{2\sqrt{x^2+b}} = \frac{x}{a\sqrt{x^2+b}}$$

$$\frac{dy}{dx} = \frac{x}{a(ay)}$$

Hence, option (C) is incorrect.

From option (D) :

$$y = \sqrt{ax^2+b} \quad \text{...(i)}$$

Differentiating equation (i) with respect to x,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{ax^2+b}}2ax = \frac{ax}{\sqrt{ax^2+b}}$$

$$\frac{dy}{dx} = \frac{ax}{y}$$

Hence, the correct option is (D).

3.9 (C)

Given : $I = a^2\int_0^\infty xe^{-ax}\,dx$,

Let $-ax = t \Rightarrow dt = -a\,dx$

When $x = 0,\ t = 0$ and $x = \infty,\ t = -\infty$

Therefore, $I = a^2\int_0^{-\infty}\left(\frac{-t}{a}\right)e^t\,\frac{dt}{-a}$

$$I = \int_0^{-\infty} te^t\,dt = [e^t(t-1)]_0^{-\infty} = 1$$

Hence, the correct option is (C).

3.10 (D)

Given : $I = \int \frac{dx}{e^x - 1}$

$$I = \int \frac{dx}{e^x - 1} = \int \frac{e^{-x}}{1 - e^{-x}} dx$$

Let $1 - e^{-x} = t$

Differentiating on both sides,

$$e^{-x} dx = dt$$

$$I = \int \frac{1}{t} dt = \ln(t) + C$$

$$I = \ln(1 - e^{-x}) + C$$

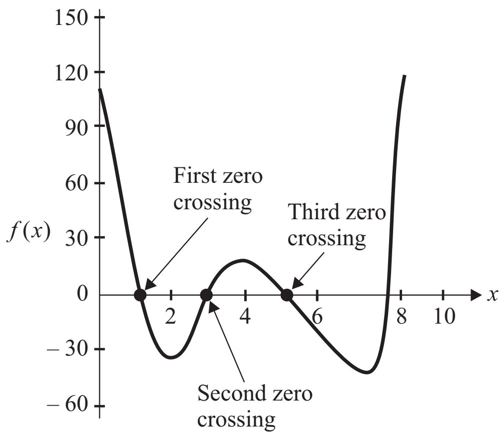
Hence, the correct option is (D).

3.11 3

Given : A function $f(x)$

Since, there are 3 zero cross points between $0 < x < 6$

Therefore, number of positive roots of the function $f(x)$ is 3.



Hence, the number of positive roots of the function $f(x)$ is **3**.

Production & Industrial : PI

3.1 (A)

Given : $I = \int_{-\pi/2}^{\pi/2} x \cos x dx$

$$f(x) = x \cos x$$

$$f(-x) = -x \cos x$$

$$f(-x) = -f(x)$$

Therefore, $f(x)$ is an odd function.

By the property of definite integral,

$$\int_{-a}^{+a} f(x) dx = \begin{cases} 0 & ; \text{ if } f(x) = \text{odd} \\ 2\int_0^a f(x) dx ; & \text{ if } f(x) = \text{even} \end{cases}$$

$$I = \int_{-\pi/2}^{\pi/2} x \cos x dx = 0$$

Hence, the correct option is (A).

3.2 (A)

Given : $f(x, y) = xy$

Total derivative is given by,

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

$$df = \frac{\partial xy}{\partial x} dx + \frac{\partial xy}{\partial y} dy$$

$$df = y\, dx + x\, dy$$

Hence, the correct option is (A).

3.3 (C)

Refer Solution 3.6 [Electronics]

3.4 (C)

Given : $f(x) = \sin|x|$ ...(i)

$$\sin x = \begin{cases} -\sin x & x < 0 \\ \sin x & x > 0 \end{cases}$$

Differentiating equation (i) with respect to x,

Hence, $\left.\frac{d}{dx}\right|_{x=-\pi/4} \sin|x|$

$$= \left.\frac{d}{dx}(-\sin x)\right|_{x=-\pi/4} = -\cos(-\pi/4)$$

$$= -\cos(\pi/4) = \frac{-1}{\sqrt{2}}$$

Hence, the correct option is (C).

3.5 (A)

Refer Solution 3.7 [Mechanical]

3.6 (C)

Given : $I=\int_1^e \sqrt{x}\ln(x)\,dx$

Let $\ln(x)=u$,

So, $\frac{1}{x}dx=du \Rightarrow dx=x\,du$

$dx=e^u\,du$ [Since, $u=\ln(x)$]

At, $x=1 \Rightarrow u=0$

$x=e^1 \Rightarrow u=1$

So, $I=\int_0^1 e^{u/2}\,u\,e^u\,du$

$$I=\int_0^1 u\,e^{3u/2}\,du$$

$$I=\left[\frac{2}{3}ue^{3u/2}-\frac{4}{9}e^{3u/2}\right]_0^1$$

$$I=\frac{2}{3}e^{3/2}-\frac{4}{9}e^{3/2}+\frac{4}{9}$$

$$I=\frac{2}{9}e^{3/2}+\frac{4}{9}$$

So, $I=\frac{2}{9}\sqrt{e^3}+\frac{4}{9}$

Hence, the correct option is (C).

3.7 (C)

Given : $I=\int_0^\infty e^{-2t}dt$

$$I=\left[\frac{e^{-2t}}{-2}\right]_0^\infty=\left[0+\frac{1}{2}\right]=\left[\frac{1}{2}\right]=0.5$$

Therefore, integral converges to 0.5.

Hence, the correct option is (C).

3.8 (C)

Given : Integral $I=\int_1^a\int_1^b \frac{dxdy}{xy}$

$$I=\int_1^a\int_1^b \frac{dx\ dy}{xy}$$

$$I=\int_1^a\int_1^b \frac{dx}{x}\cdot\frac{dy}{y}$$

Firstly, integrating wrt x,

$$=\int_1^a \frac{1}{y}(\ln x)_1^b\,dy$$

$$=(\ln x)_1^b\int_1^a \frac{dy}{y}$$

Now, integrating wrt y,

$$=[\ln b-\ln(1)](\ln y)_1^a \qquad (\because\ \ln(1)=0)$$

$$=\ln b\,[\ln a-\ln(1)]$$

$$=\ln b\cdot\ln a$$

Hence, the correct option is (D).

❖❖❖❖

Space for Notes

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