Technical Section

Question 1 **Signals & Systems (2M)**

The causal signal with z-transform $z^2(z-a)^{-2}$ is

($u[n]$ is the unit step signal)

(A) $a^{2n}u[n]$ (B) $(n+1)a^n u[n]$ (C) $n^{-1}a^n u[n]$ (D) $n^2 a^n u[n]$

Ans. **B**

Sol. **Given :** Z-transform of a causal signal is,

$$X(z) = z^2(z-a)^{-2} = \frac{z^2}{(z-a)^2} \qquad \ldots(i)$$

The Z transform pair for $a^n u[n]$ signal is given by

$$a^n u[n] \longleftrightarrow \frac{z}{z-a}$$

Using differentiation in z-domain property,

$$na^n u[n] \longleftrightarrow -z\frac{d}{dz}\left(\frac{z}{z-a}\right) \Rightarrow -z\left[\frac{(z-a)\times 1 - z\times 1}{(z-a)^2}\right]$$

$$na^n u[n] \longleftrightarrow \frac{az}{(z-a)^2}$$

Using time shifting property,

$$(n+1)a^{n+1}u[n+1] \longleftrightarrow \frac{az}{(z-a)^2}z$$

$$(n+1)a^n u[n+1] \longleftrightarrow \frac{z^2}{(z-a)^2} \qquad \ldots(ii)$$

Comparing equations (i) and (ii), required inverse of given transform is,

$$x[n] = (n+1)a^n u[n+1]$$

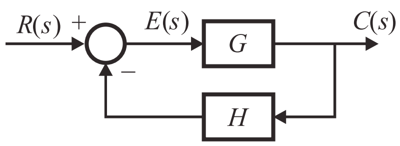
Sequence $u[n+1]$ exist for $-1 \le n < \infty$, but the factor $(n+1)$ is zero for $n = -1$, so $x[n]$ may be expressed as a causal sequence.

$$x[n] = (n+1)a^n u[n]$$

Hence, the correct option is (B).

Question 2 **Control Systems (1M)**

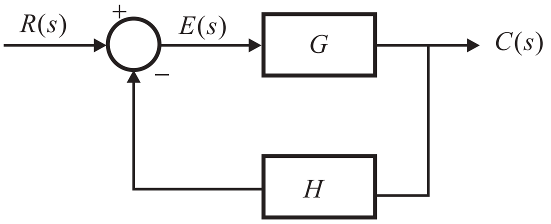
For the closed-loop system shown, the transfer function $\frac{E(s)}{R(s)}$ is



(A) $\frac{G}{1+GH}$ (B) $\frac{GH}{1+GH}$ (C) $\frac{1}{1+GH}$ (D) $\frac{1}{1+G}$

Ans. **C**

Sol.



Closed loop transfer function for negative feedback system is given by,

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} \qquad \ldots(i)$$

From the figure, $E(s)$ is input to G and $C(s)$ is output of G

$$\frac{C(s)}{E(s)} = G$$

$$C(s) = GE(s) \qquad \ldots(ii)$$

From equation (i) and (ii),

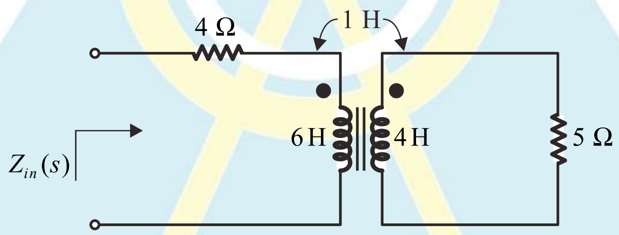
$$\frac{GE(s)}{R(s)} = \frac{G}{1+GH}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+GH}$$

Hence, the correct option is (C).

Question 3 **Control System (2M)**

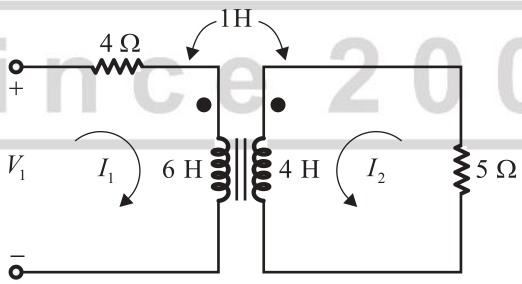
The input impedance, $Z_{in}(s)$, for the network shown is



(A) $\frac{23s^2+46s+20}{4s+5}$ (B) $6s+4$ (C) $7s+4$ (D) $\frac{25s^2+46s+20}{4s+5}$

Ans. A

Sol. **Method 1 :**



Applying KVL in the loop 1,

$$V_1 = 4I_1 + 6sI_1 + sI_2$$

$$V_1 = (4+6s)I_1 + sI_2 \qquad \ldots(i)$$

Applying KVL in the loop 2,

$$5I_2 + 4sI_2 + sI_1 = 0$$

$$I_2 = -\frac{sI_1}{4s+5} \qquad \ldots\text{(ii)}$$

So,
$$V_1 = (4+6s)I_1 + s\left[-\frac{sI_1}{4s+5}\right]$$

$$\frac{V_1}{I_1} = 4 + 6s - \frac{s^2}{4s+5}$$

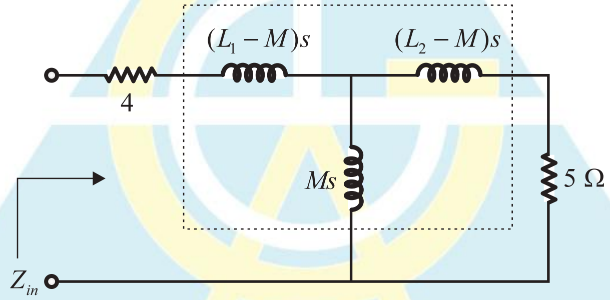
$$\frac{V_1}{I_1} = \frac{24s^2 + 16s + 20 + 30s - s^2}{4s+5}$$

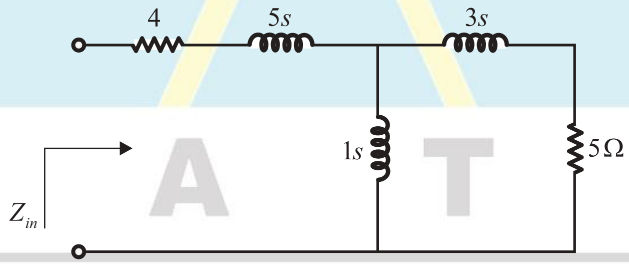
$$\frac{V_1}{I_1} = Z_{in}(s) = \frac{23s^2 + 46s + 20}{4s+5}$$

Hence, the correct option is (A).

Method 2 :

By taking the effect of dots into account the modified circuit diagram is as shown below,





$$Z_{in} = (4+5s) + \frac{(5+3s)s}{4s+5} = \frac{23s^2 + 46s + 20}{4s+5}$$

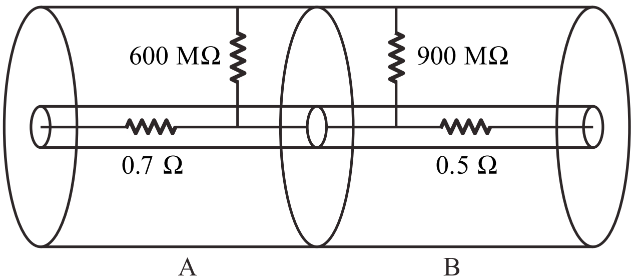
Hence, the correct option is (AB).

Question 4 **Power System (1M)**

Two single-core power cables have total conductor resistances of $0.7\ \Omega$ and $0.5\ \Omega$ respectively and their insulation resistances (between core and sheath) are $600\ \text{M}\Omega$ and $900\ \text{M}\Omega$ respectively. When the two cables are joined in series, the ratio of insulation resistance to conductor resistance is _______ $\times 10^6$

Ans. **300**

Sol. **Note :** When two cables are connected in series, conductor resistances will be in series and insulation resistances will be in parallel.



$$R_{conductor} = 0.7 + 0.5 = 1.2\Omega$$

$$R_{insulation} = \frac{600 \times 900}{600 + 900} = 360\ \text{M}\Omega$$

$$\frac{R_{insulation}}{R_{conductor}} = \frac{360\,\text{M}\Omega}{1.2\Omega} = 300 \times 10^6$$

Hence, the correct answer is 300.

Question 5 **Electrical Machine (2M)**

In a single phase transformer, the total iron loss is 2500 W at nominal voltage of 440 V and frequency 50 Hz. The total iron loss is 850 W at 220 V and 25 Hz. Then, at nominal voltage and frequency, the hysteresis loss and eddy current loss respectively are

(A) 1600 W and 900 W
(B) 900 W and 1600 W
(C) 250 W and 600 W
(D) 600 W and 250 W

Ans. **B**

Sol. **Given :** Single phase transformer

Case 1 : $V_1 = 440\text{V}, f_1 = 50\text{Hz}$

Total iron loss, $P_1 = 2500\text{W}$

$$\frac{V}{f} = \frac{V_1}{f_1} = \frac{440}{50} = 8.8$$

Case 2 : $V_2 = 220\text{V}, f_2 = 25\text{Hz}$

Total iron loss, $P_2 = 850\text{W}$

$$\frac{V}{f} = \frac{V_2}{f_2} = \frac{220}{25} = 8.8$$

Here, $\frac{V}{f} =$ constant

Hysteresis losses is given by

$$P_h = K_h (B_m)^x f\, v$$

$K_h =$ Hysteresis constant

$x =$ Steinmetz constant

$B_m =$ Maximum flux density

f = Supply frequency

v = volume of the core

Eddy current loss is given by

$P_e = K_e (B_m)^2 f^2 v$

K_e = Eddy current constant

x = Steinmetz constant

B_m = Maximum flux density

f = Supply frequency

v = volume of the core

Core loss is defined as,

$P_i = P_h + P_e$

Hysteresis loss, $P_h = K_h B_m^{1.6} f$

Eddy current loss, $P_e = K_e B_m^2 f^2$

$P_e = K_e f^2$

$P_h = K_h f$ and at $V_2 = 220\text{V}$,

$f_2 = 25\text{Hz}, P_2 = 850\text{W}$

As ratio of $\frac{V}{f}$ is constant in both case.

So, $\left(B_m \propto \frac{V}{f}\right)$ is also constant.

$\because$ $V \approx E = 4.44 f \phi_m N$

$B_m = \frac{\phi_m}{A} = \frac{V}{4.44 fNA}$ [N and A = Constant]

Case 1 : At normal voltage $V_1 = 440\text{V}, f_1 = 50\text{Hz}, P_i = 2500\text{W}$

$P_1 = K_h f + K_e f^2$

$2500 = 50K_h + (50)^2 K_e$

$K_h + 50K_e = 50$ ...(i)

Case 2 : At $V_2 = 220\text{V}, f_2 = 25\text{Hz}, P_2 = 850\text{W}$

$P_2 = K_h f_2 + K_e f_2^2$

$850 = 25K_h + (25)^2 K_e$

$K_h + 25K_e = 34$ ...(ii)

By equation (i) and equation (ii),

$$K_h + 50K_e = 50$$
$$-K_h + 25K_e = 50$$
$$25K_e = 16$$
$$K_e = 0.64$$

Putting the value of K_e in equation (i),

$$K_h + 50(0.64) = 50$$
$$K_h = 18$$

At nominal voltage $V = 440,\ f = 50\,\text{Hz}$

Hysteresis loss, $P_h = K_h f$

$$P_h = 18 \times 50 = 900\,\text{W}$$

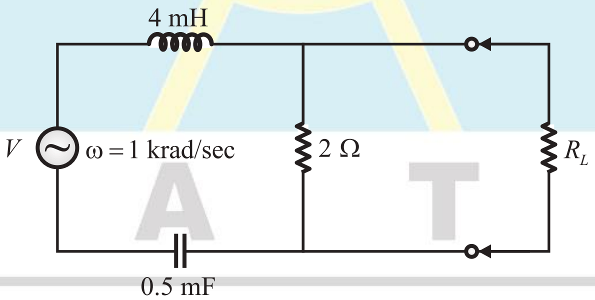
Eddy current loss, $P_e = K_e f^2$

$$P_e = (0.64) \times 50^2 = 1600\,\text{W}$$

Hence, the correct option is (B).

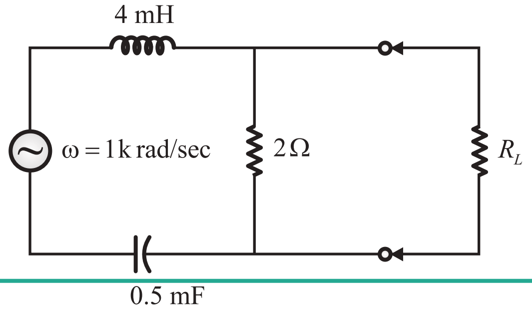
Question 6 **Network Theory (2M)**

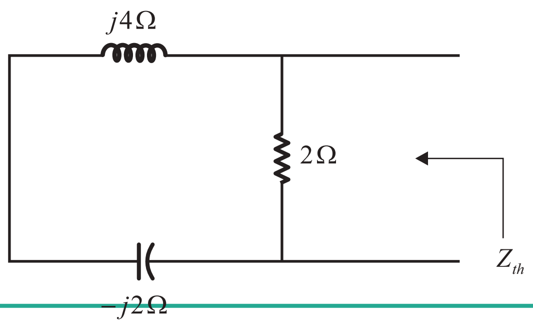
In the given circuit, for maximum power to be delivered to R_L, its value should be ______ Ω (Round off to 2 decimal places).



Ans. **1.414**

Sol. Given circuit is as shown below,





$$Z_{th} = \frac{2\times j2}{2+j2}$$

$$|Z_{th}| = \frac{2\times j2}{\sqrt{2^2+2^2}} = \frac{4}{2\sqrt{2}} = \sqrt{2}\,\Omega$$

For maximum power transfer,

$$R_L = |Z_{th}| = \sqrt{2} = 1.414\,\Omega$$

Hence, the correct answer is 1.414.

Question 7 **Electrical Machine (1M)**

Consider the table given :

Constructional feature	Machine type	Mitigation
P. Damper bars	S. Induction motor	X. Hunting
Q. Skewed rotor slots	T. Transformer	Y. Magnetic locking
R. Compensating winding	U. Synchronous machine	Z. Armature reaction
	V. DC machine	

The correct combination that relates the constructional feature, machine type and mitigation is

(A) P-V-X, Q-U-Z, R-T-Y
(B) P-U-X, Q-S-Y, R-V-Z
(C) P-T-Y, Q-V-Z, R-S-X
(D) P-U-X, Q-V-Y, R-T-Z

Ans. **B**

Sol. (i) Damper bars are used in synchronous machines to eliminate hunting (P-U-X)

(ii) Skewed rotor slots are used in induction machine to reduce magnetic locking (cogging) (Q-S-Y)

(iii)Compensating winding is used in DC machine to neutralize armature reaction (R-V-Z).

Hence, the correct option is (B).

Question 8 **Engineering Mathematics (1M)**

Let p and q be real numbers such that $p^2+q^2=1$. The eigen values of the matrix $\begin{bmatrix} p & q \\ q & -p \end{bmatrix}$ are

(A) 1 and 1 (B) 1 and -1 (C) j and $-j$ (D) pq and $-pq$

Ans. **B**

Sol. **Given :** $p^2+q^2=1$

$$\begin{bmatrix} p & q \\ q & -p \end{bmatrix}$$

$$\begin{bmatrix} p-\lambda & q \\ q & -p-\lambda \end{bmatrix} = 0$$

$$(p-\lambda)(-p-\lambda)-q^2=0$$

$$-p^2 - p\lambda + \lambda p + \lambda^2 - q^2 = 0$$

$$\lambda^2 - p^2 - q^2 = 0 \qquad (\because\ p^2 + q^2 = 1)$$

$$\lambda^2 - 1 = 0$$

$$\lambda = \pm 1$$

Eigen value of given matrix would be 1 and -1.

Hence, the correct option is (B).

Question 9 **Engineering Mathematics (2M)**

Let A be a 10×10 matrix such that A^5 is null matrix and let I be the 10×10 identity matrix. The determinant of $A+I$ is _____.

Ans. **1**

Sol. **Given :** $[A]_{10\times10}$

$$A^5 = 0$$

$$\lambda^5 = 0$$

$$\lambda = 0$$

Eigen value of $[A+I]' = \lambda + 1$

$$0 + 1 = 1$$

Determinant of $[A+I] = 1$

Hence, the correct answer is 1.

Question 10 **Engineering Mathematics (1M)**

Let $f(x)$ be a real-valued function such that $f'(x_0) = 0$ for some $x_0 \in (0, 1)$ and $f''(x_0) > 0$ for all $x \in (0,1)$. Then $f(x)$ has

(A) No local minimum in (0, 1)

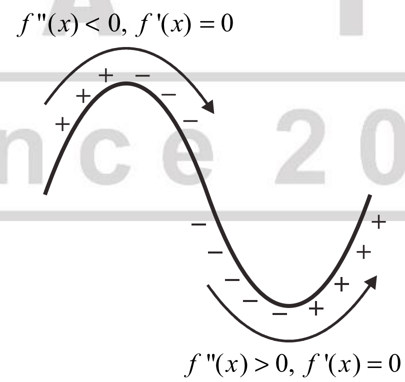
(B) One local maximum in (0, 1)

(C) Exactly one local minimum in (0, 1)

(D) Two distinct local minimum in (0, 1)

Ans. **C**

Sol.



(i) When value will go from positive to negative then gradient will give negative number.

(ii) When value will go from negative to positive then gradient will give positive number.

So, $f(x)$ has exactly one local minimum in (0, 1).

Hence, the correct option is (C)

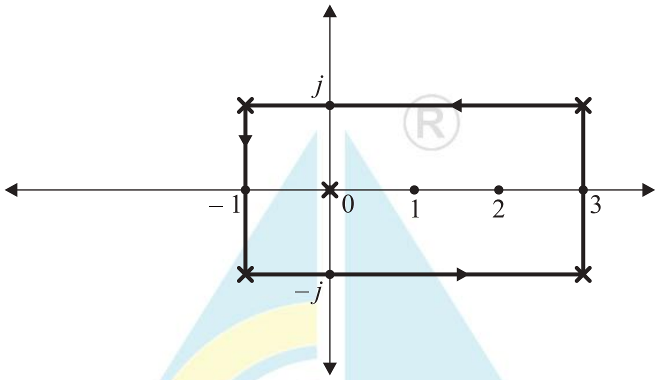
Question 11 **Engineering Mathematics (2M)**

Let $(-1-j)$, $(3-j)$, $(3+j)$ and $(-1+j)$ be the vertices of rectangle C in the complex plane. Assuming that C is traversed in counter-clockwise direction, the value of contour integral $\oint_C \frac{dz}{z^2(z-4)}$ is

(A) $\frac{j\pi}{2}$ (B) 0 (C) $\frac{-j\pi}{8}$ (D) $\frac{j\pi}{16}$

Ans. C

Sol. **Given :** Vertices of rectangle = $(-1-j)$, $(3-j)$, $(3+j)$ and $(-1+j)$



Poles = 0, 0, 4

$z=0$ lie inside contour and $z=4$ lie outside of contour.

$$R=\frac{1}{(n-1)!}\frac{d^{n-1}}{dz^{n-1}}(z-z_0)^n f(z)$$

$$R=\frac{1}{(2-1)!}\frac{d}{dz}z^2\frac{1}{z^2(z-4)}$$

$$R=1\times\frac{d}{dz}\frac{1}{(z-4)}$$

$$R=\frac{-1}{(z-4)^2}=\frac{-1}{16}$$

$$\therefore \quad \oint_C \frac{dz}{z^2(z-4)}=2\pi i\left(\frac{-1}{16}\right)=\frac{-j\pi}{8}$$

Hence, the correct option is (C).

Question 12 **Engineering Mathematics (1M)**

Let $P(z)=z^3+(1+j)z^2+(2+j)z+3$, where z is complex number. Which one of the following is true?

(A) Conjugate $\{P(z)\}=P$ (Conjugate $\{z\}$) for all z

(B) The sum of the roots of $P(z)=0$ is a real number

(C) The complex roots of the equation $P(z)=0$ come in conjugate pairs.

(D) All the roots cannot be real

Ans. D

Sol. **Given :** $P(z)=z^3+(1+j)z^2+(2+j)z+3$

For standard 3rd order equation of the from

$$\alpha x^3+\beta x^2+\gamma x+\delta=0$$

If the roots are a, b and c then

Sum of roots $a+b+c=\frac{-\beta}{\alpha}$

Product of roots $a\cdot b=\frac{-\delta}{\alpha}$

and $ab+bc+ca=\frac{\gamma}{\alpha}$

For given equation,

$$a+b+c=\frac{-\beta}{\alpha}=\frac{-(1+j)}{1}=-(1+j)$$

Hence all the roots cannot be real.

$$a\cdot b\cdot c=\frac{-\delta}{\alpha}=\frac{-3}{1}=-3$$

Complex roots can't be conjugate because sum of complex conjugate roots must be real.

Hence, the correct option is (D).

Question 13 **Engineering Mathematics (2M)**

Suppose the probability that a coin toss shows "head" is p, where $0<p<1$. The coin is tossed repeatedly until the first "head" appears. The expected number of tosses required is

(A) $\frac{p}{1-p}$ (B) $\frac{1-p}{p}$ (C) $\frac{1}{p}$ (D) $\frac{1}{p^2}$

Ans. C

Sol.

Number of tosses	1	2	3	4	5
Probability of outcomes	p	$p(1-p)$	$p(1-p)^2$	$p(1-p)^3$	$p(1-p)^4$

∴ The expected number of tosses required is,

$$E[X]=1\times p+2p(1-p)+3p(1-p)^2+4p(1-p)^3+......$$

$$E[X]=p[1+2(1-p)+3(1-p)^2+4(1-p)^3+.....]$$

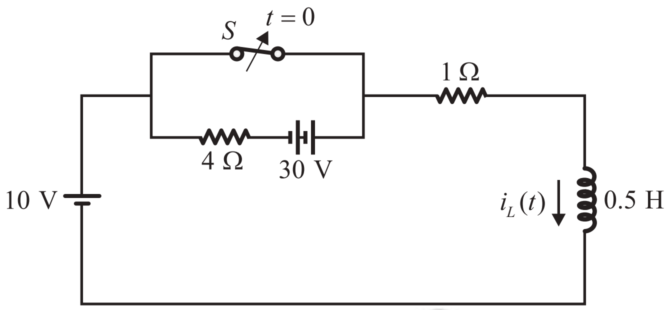
$$E[X]=p\times(1-1+p)^{-2}=p(p)^{-2}$$

$$E[X]=\frac{p}{p^2}=\frac{1}{p}$$

Hence, the correct option is (C).

Question 14 **Network Theory (2M)**

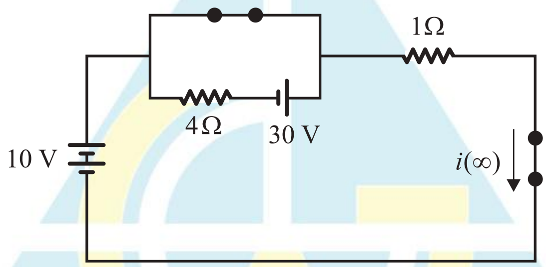
In the circuit, switch 'S' is in the closed position for a very long time. If the switch is opened at time $t=0$, then $i_L(t)$ in Amperes, for $t \geq 0$ is



(A) $8e^{-10t}$ (B) 10 (C) $8+2e^{-10t}$ (D) $10(1-e^{-2t})$

Ans. C

Sol. **(i) For t < 0 ,**

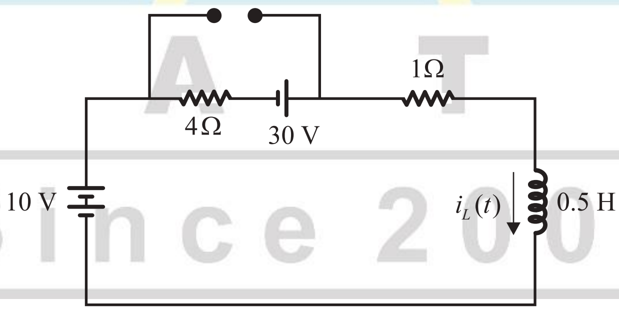


The current flowing through inductor before the switch is opened is given by

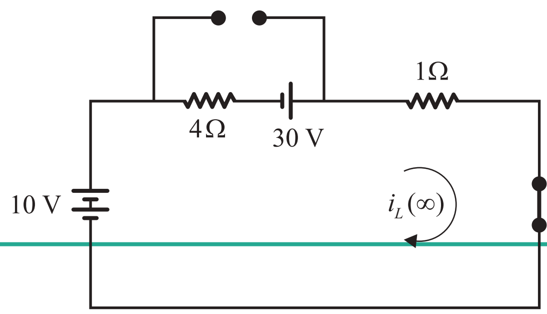
$$i(0^-) = \frac{10}{1} = 10\text{ A} = i_L(0^+)$$

(ii) For $t > 0$,

The circuit diagram after the switch is opened is given by



(iii) For $t = \infty$,



The current flowing through inductor in steady state is given by

$$i_L(\infty) = \frac{30+10}{5} = \frac{40}{5} = 8$$

$$R_{eq} = 5\ \Omega \text{ and } L = 0.5\ \text{H}$$

Time constant of the above circuit is given by,

$$\tau = \frac{L}{R} = 0.1\ \text{sec}$$

The charging equation of first order RL circuit is given by,

$$i_L(t) = i(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$i_L(t) = i(\infty) + [i_L(0^+) - i_L(\infty)]e^{-Rt/L}$$

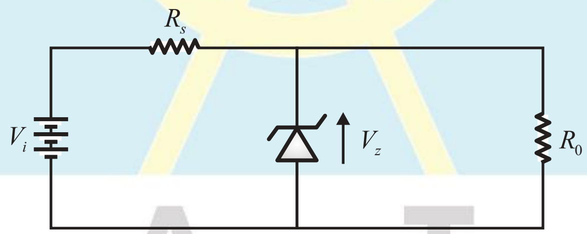
$$i_L(t) = 8 + (10-8)e^{-10t}$$

$$i_L(t) = 8 + 2e^{-10t}$$

Hence, the correct option is (C).

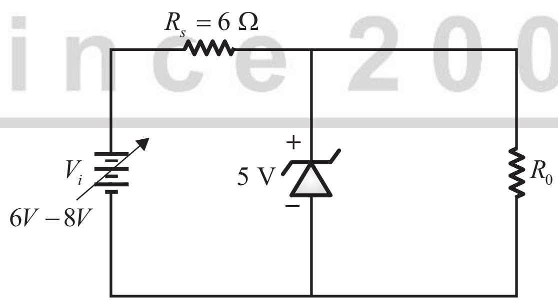
Question 15 **Analog Electronics (1M)**

In the circuit shown, a 5 V Zener diode is used to regulate the voltage across load R_0. The input is an unregulated DC voltage with a minimum value of 6 V and a maximum value of 8 V. The value of R_s is $6\ \Omega$. The Zener diode has a maximum rated power dissipation of 2.5 W. Assuming the Zener diode to be ideal, the minimum value of R_0 is ______ Ω.



Ans. **30**

Sol.



Given:

(i) $R_s = 6\ \Omega$

(ii) Minimum value of DC input

Voltage = 6 V

(iii) Maximum value of

DC input voltage = 8 V

Maximum rated power dissipation = 2.5 W

$$P_{z(max)} = P_{rating}$$

$$P_{rating} = 2.5 \text{ W} = V_z I_{z_{max}}$$

$$I_{z_{max}} = \frac{2.5}{5}$$

$$I_{z_{max}} = 0.5 \text{ A}$$

As load is fixed (i.e. R_0 =fixed)

So I_L is fixed.

$$I_0(\text{varying}) = I_z(\text{varying}) + I_L(\text{fixed})$$

So,

$$I_{in(min)} = I_{z(min)} + I_L$$

$$I_{in(max)} = I_{z(max)} + I_L$$

$$I_{in(max)} = \frac{V_{in(max)} - V_z}{R_s}$$

$$I_{in(max)} = \frac{8-5}{6} = 0.5 \text{ A}$$

$$I_{in(min)} = \frac{V_{in(min)} - V_z}{R_s} = \frac{6-5}{6} = \frac{1}{6} \text{A}$$

Now,

$$I_{in(min)} = I_{z(min)} + I_L$$

$$\frac{1}{6} = 0 + I_L$$

$$I_L = \frac{1}{6} A$$

Now,

$$V_0 = I_L R_0$$

$$5 = \frac{1}{6} \times R_0$$

$$R_0 = 30\,\Omega$$

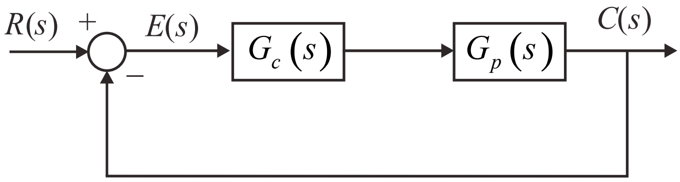
Hence, the correct answer is 30.

Question 16 **Control system (2M)**

Consider a closed loop system as shown.

$$G_p(s) = \frac{14.4}{s(1+0.1s)}$$

is the plant transfer function and $G_c(s) = 1$ is the compensator. For a unit-step input, the output response has damped oscillations. The damped natural frequency is____ rad/s. (Round off to 2 decimal places).



Ans. **10.90**

Sol. **Given :** $G_p(s) = \dfrac{14.4}{s(1+0.1s)}$

$$G_p(s) = \frac{14.4}{0.1s(s+10)}$$

$$G_p(s) = \frac{144}{s(s+10)}$$

and $G_c(s) = 1$

$\therefore$ $$G_p(s)_{CLTF} = \frac{144}{s^2+10s+144} \quad \ldots(i)$$

Now, comparing with the standard equation,

$\Rightarrow$ $$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad \ldots(ii)$$

From equation (i) and (ii),

$$\omega_n = 12$$

$$2\xi\omega_n = 10$$

$$\xi = \frac{10}{2\times 12} = \frac{5}{12}$$

Now, $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_d = 12\sqrt{1-\left(\frac{5}{12}\right)^2} = 12\sqrt{\frac{144-25}{144}}$$

$$\omega_d = 12\sqrt{\frac{119}{144}} = 10.90$$

Hence, the correct answer is 10.90.

Question 17 **Digital Electronics (1M)**

A 16-bit synchronous binary up-counter is clocked with a frequency f_{clk}. The two most significant bits are ORed together to form an output Y. Measurements shows that Y is periodic and the duration for which Y the remains high in each period is 24 ms. The clock frequency f_{clk} is ______ MHz. (Round off to 2 decimal places)

Ans. **2.05**

Sol. Given counter = 16 bit

$$2^{16}\text{combination}\begin{cases} \begin{matrix} Q_{15} & Q_{14} & Q_{13} & & Q_1 & Q_0 \\ 0 & 0 & 0 & \ldots\ldots\ldots & 0 & 0 \\ & & & & & \\ 1 & 1 & 1 & \ldots\ldots\ldots & 1 & 1 \end{matrix} \end{cases}$$

If we fix $Q_{15}\,Q_{14}$ bit then $(Q_{13}$ to $Q_0)$ will take 2^{14} combination

$Y = Q_{15} + Q_{14}$	Q_{15}	Q_{14}	$Q_{13} \ldots\ldots\ldots Q_1\,Q_0$
0	0 0	0 0	2^{14} combination
1	0 0	1 1	2^{14} combination
1	1 1	0 0	2^{14} combination
1	1 1	1 1	2^{14} combination

So, $Y = 0$ for 2^{14} times. [25 %]

$Y = 1$ for 3×2^{14} times. [75 %]

Given that, $Y = 1$ for 24 ms

So, $Y = 0$ for $\dfrac{24 \times 25}{75} = 8$ ms

So, total time period $= 8 + 24 = 32$ ms

Time period of clock, $T_{clk} = \dfrac{32\text{ ms}}{2^{16}}$

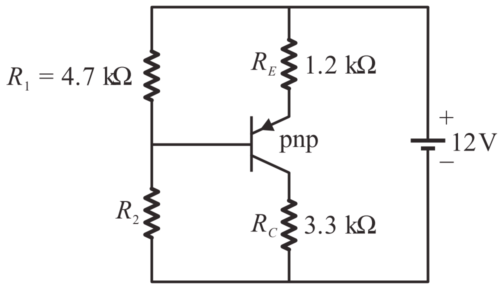
$$f_{clk} = \frac{2^{16}}{32 \times 10^{-3}}$$

$$f_{clk} = 2.048\text{MHz} \approx 2.05\text{MHz}$$

Hence, the correct answer is 2.05.

Question 18 **Analog Electronics (1M)**

In the BJT circuit shown, beta of the PNP transistor is 100. Assume $V_{BE} = -0.7\text{V}$. The voltage across R_C will be 5 V when R_2 is ________ $\text{k}\Omega$. (Round off to 2 decimal places)



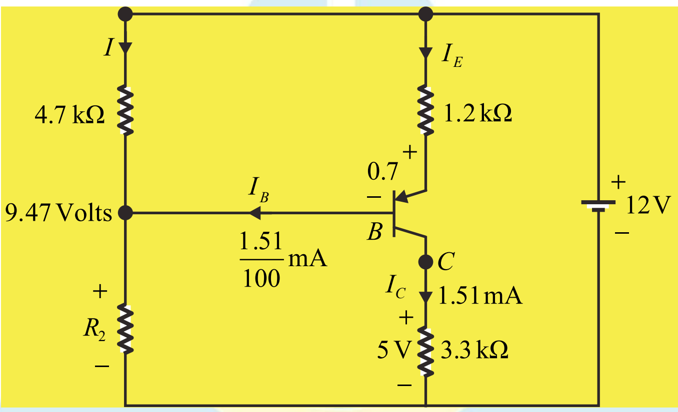
Ans. **18.099**

Sol. **Given :**

(i) $\beta = 100$

(ii) $V_{BE} = -0.7\text{V}$

(iii) $V_s = 12\text{ V}$



$$I_C = \beta I_B$$

$$1.51 = 100 I_B$$

$$I_B = 1.51 \times 10^{-2}\text{ mA}$$

$$I_E = (1+\beta) I_B$$

$$I_E = 1.52\text{mA}$$

$$12 - 1.2 \times 1.5251 - 0.7 - V_{R_2} = 0$$

$$V_{R_2} = 9.46988$$

$$I = \frac{12 - 9.46988}{4.7} = 0.53\text{ mA}$$

$$I_{R_2} = 0.53 - \frac{1.51}{100} = 0.52322\text{ mA}$$

$$R_2 = \frac{V_{R_2}}{I_{R_2}} = \frac{9.46988}{0.52322} = 18.099\text{ k}\Omega$$

Question 19 **Electromagnetic Theory (1M)**

Which one of the following vector functions represents a magnetic field $\vec{B}$?

($\hat{x}, \hat{y}$, and $\hat{z}$ are unit vectors along *x*-axis, *y*-axis and *z*-axis respectively)

(A) $10x\hat{x} + 20y\hat{y} - 30z\hat{z}$ (B) $10y\hat{x} + 20x\hat{y} - 10z\hat{z}$

(C) $10z\hat{x} + 20y\hat{y} - 30x\hat{z}$ (D) $10x\hat{x} - 30z\hat{y} + 20y\hat{z}$

Ans. **A**

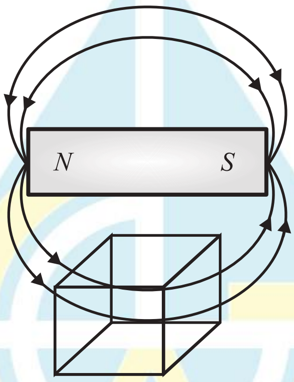
Sol. The vector functions represents a magnetic field $\vec{B}$ if $\oint\!\!\oint \vec{B} \cdot ds = 0$

Where, $\vec{B} =$ Magnetic flux density

By applying Divergence theorem,

$$\iiint (\nabla \cdot \vec{B})\, ds = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$



Checking from options,

Option (A) : $\vec{\nabla} \cdot \vec{B} = 10 + 0 + 20 = 30$

Option (B) : $\vec{\nabla} \cdot \vec{B} = 10 + 20 - 30 = 0$

Option (C) : $\vec{\nabla} \cdot \vec{B} = 0 + 0 - 10 = -10$

Option (D) : $\vec{\nabla} \cdot \vec{B} = 0 + 20 + 0 = 20$

$\vec{B} = 10x\hat{x} + 20y\hat{y} - 30z\hat{z}$ represents the magnetic field

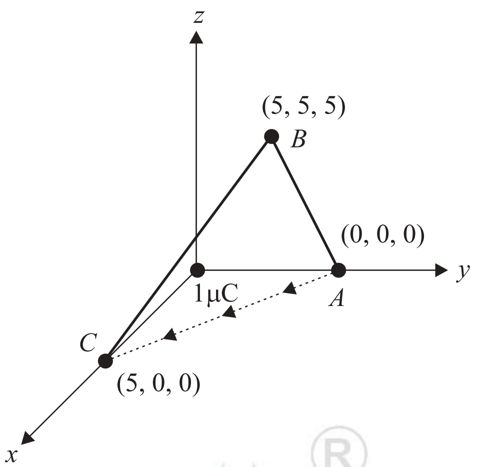
Hence, the correct option is (A).

Question 20 **Electromagnetic Theory (1M)**

A $1\ \mu C$ point charge is held at the origin of a cartesian coordinate system. If a second point charge of $10\ \mu C$ is moved from (0, 10, 0) to (5, 5, 5) and subsequently to (5, 0, 0), then the total work done is ___mJ. (Round off to 2 decimal places). Take $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ in SI units. All coordinates are in meters.

Ans. **9**

Sol. **Given :**



Now electric field because of $1\mu C$ is,

$$\frac{1\times10^{-6}}{4\pi\varepsilon_0\times r^2}\hat{a}_r=\vec{E}_r$$

$$\vec{\nabla}\times\vec{E}_r=\begin{vmatrix}\hat{a}_r & \hat{a}_\theta & a_\phi \\ \frac{\partial}{\partial_r} & \frac{\partial}{\partial_\theta} & \frac{\partial}{\partial_\phi} \\ F(r) & 0 & 0\end{vmatrix}$$

$\vec{\nabla}\times\vec{E}=0$ as electric field is curl free, path independent, irrotational.
As electric field is path independent so instead of going from A to B to C.
We can go from A to C directly.

So, $$V_{AC}=\frac{1\times10^{-6}}{4\pi\varepsilon_0 10}-\frac{1\times10^{-6}}{4\pi\varepsilon_0 5}$$

$$V_{AC}=\frac{1\times10^{-6}}{4\pi\varepsilon_0}\left\{\frac{1}{10}-\frac{1}{5}\right\}=\frac{1\times10^{-6}}{4\pi\varepsilon_0}\times\frac{1}{10}$$

So now work done $=qV$

$$W=10\times10^{-6}\times\frac{1\times10^{-6}}{4\pi\varepsilon_0\times10}$$

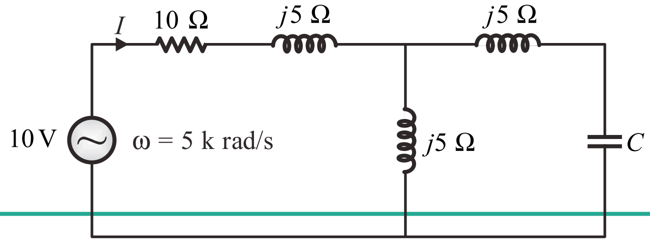
$$W=10\times10^{-12}\times9\times10^{9}\times10^{-1}$$

Work done $=0.009$ Joules $=9$ mJ

Hence, the correct answer is 9.

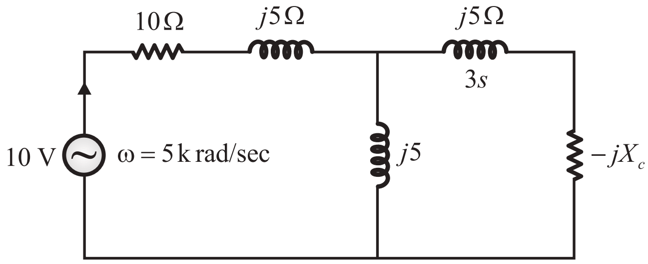
Question 21 **Network Theory (1M)**

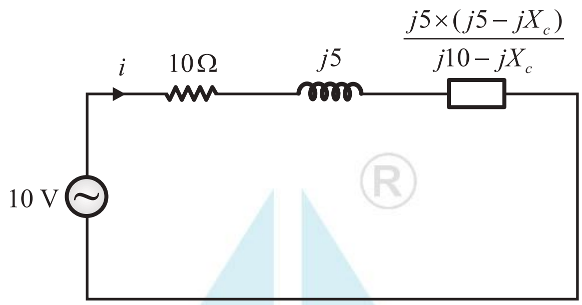
In the given circuit, the value of capacitor C that makes current $I=0$ is ________ μF.



Ans. **20**

Sol. Given circuit is as shown below,





For $i=0$, Z must be infinite,

For Z to be infinite X_c must be equals to $10\,\Omega$

$$X_c = 10$$

$$\frac{1}{2\pi f C} = 10$$

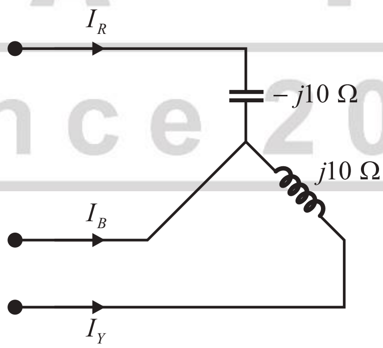
$$10 = \frac{1}{5000\,C}$$

$$C = \frac{1}{50000} = 20\,\mu\text{F}$$

Hence, the correct answer is 20.

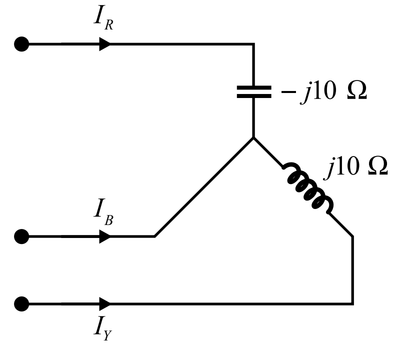
Question 22 **Power System (2M)**

A three-phase balanced voltage is applied to the load shown. The phase sequence is RYB. The ratio $\left|\frac{I_B}{I_R}\right|$ is ______.

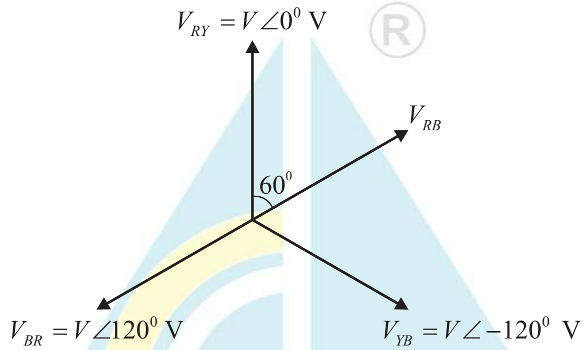


Ans. 1

Sol. Given circuit is as shown below,



Voltage across capacitor is V_{RB} and voltage across inductor is V_{YB} (where, V_{RB}, V_{YB} are line voltages)



$$V_{RY} = V\angle 0^0 \text{V}$$

$$V_{YB} = V\angle -120^0 \text{V}$$

$$V_{BR} = V\angle 120^0 \text{V}$$

$$I_R = \frac{V_{BR}}{-j10\,\Omega} = \frac{V\angle -60^0}{-j10} = \frac{V}{10}\angle 30^0 \text{ A}$$

$$I_Y = \frac{V_{YB}}{j10} = \frac{V\angle -120^0}{j10} = \frac{V}{10}\angle -210^0 \text{ A}$$

From the above circuit the value of I_B is given by,

$$I_B = -(I_R + I_y)$$

$$I_B = -\left[\frac{V}{10}\angle 30^0 + \frac{V}{10}\angle -210^0\right] \text{A} = -\frac{V}{10}\angle 90^0 \text{ A}$$

$$|I_R| = \frac{V}{10}$$

$$|I_B| = \frac{V}{10}$$

$$\left|\frac{I_B}{I_Y}\right| = 1$$

Hence, the correct answer is 1.

Question 23 **Power System (1M)**

Two generators have cost functions F_1 and F_2. Their incremental-cost characteristics are

$$\frac{dF_1}{dP_1} = 40 + 0.2P_1 \text{ and } \frac{dF_2}{dP_2} = 32 + 0.4P_2$$

They need to deliver a combined load of 260 MW. Ignoring the network losses, for economic operation, the generations P_1 and P_2 (in MW) are

(A) $P_1 = P_2 = 130$ (B) $P_1 = 160, P_2 = 100$

(C) $P_1 = 140, P_2 = 120$ (D) $P_1 = 120, P_2 = 140$

Ans. **B**

Sol. **Given :**

Incremental cost, $\frac{dF_1}{dP_1} = 40 + 0.2P_1$

$$\frac{dF_2}{dP_2} = 32 + 0.4P_2$$

Total load demand, $P_D = 260$ MW

For minimum cost (without considering loss) (I.C) of all the units must be same.

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$40 + 0.2P_1 = 32 + 0.4P_2$$

$$0.2P_1 - 0.4P_2 = -8 \quad \ldots\text{(i)}$$

and $$P_1 + P_2 = 260$$

$$0.4P_1 + 0.4P_2 = 104 \quad \ldots\text{(ii)}$$

Solving equation (i) and (ii),

$$0.2P_1 - 0.4P_2 = -8$$

$$0.4P_1 + 0.4P_2 = 104$$

$$0.6P_1 = 96$$

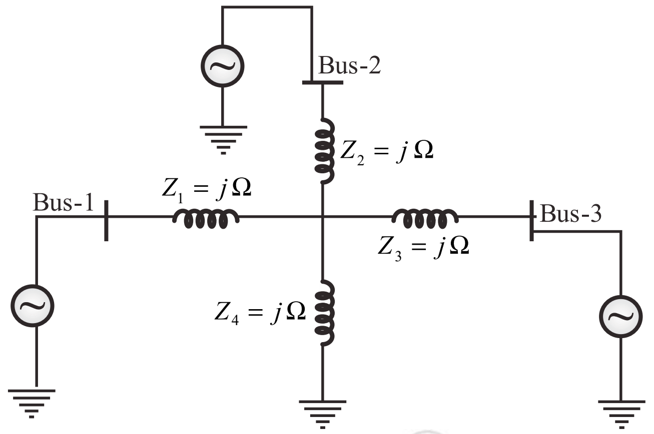
$$P_1 = 160\text{MW}$$

$$P_2 = (260 - 160)\text{MW} = 100\text{MW}$$

Hence, the correct option is (B).

Question 24 **Power System (2M)**

A 3-bus network is shown. Consider generators as ideal voltage sources. If rows 1, 2 and 3 of the Y_{bus} matrix correspond to bus 1, 2 and 3 respectively, then Y_{bus} of the network is



(A) $\begin{bmatrix} -4j & j & j \\ j & -4j & j \\ j & j & -4j \end{bmatrix}$

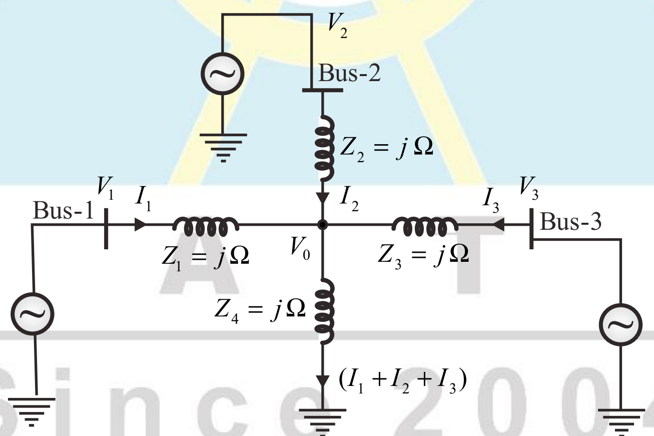
(B) $\begin{bmatrix} -4j & 2j & 2j \\ 2j & -4j & 2j \\ 2j & 2j & -4j \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{3}{4}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{3}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & \frac{-3}{4}j \end{bmatrix}$

(D) $\begin{bmatrix} \frac{-1}{2}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{1}{2}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & \frac{-1}{2}j \end{bmatrix}$

Ans. **C**

Sol.



From the above circuit diagram, output voltage is given by,

$$V_0 = j[I_1 + I_2 + I_3]$$

Applying KVL at bus 1,

$$V_1 - I_1[j] = j[I_1 + I_2 + I_3]$$

$$j2I_1 + jI_2 + jI_3 = V_1 \quad \text{...(i)}$$

Similarly at bus 2,

$$jI_1 + j2I_2 + jI_3 = V_2 \quad \text{...(ii)}$$

Similarly at bus 3,

$$jI_1 + jI_2 + j2I_3 = V_3 \quad \text{...(iii)}$$

Solving equation (i), (ii) and (iii),

$$I_1 = -\frac{3}{4}jV_1 + \frac{1}{4}jV_2 + \frac{1}{4}jV_3$$

$$I_2 = \frac{1}{4}jV_1 - \frac{3}{4}jV_2 + \frac{1}{4}jV_3$$

$$I_3 = \frac{1}{4}jV_1 + \frac{1}{4}jV_2 - \frac{3}{4}jV_3$$

Hence, $$Y\text{bus} = \begin{bmatrix} -\frac{3}{4}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{-3}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & \frac{-3}{4}j \end{bmatrix}$$

Hence, the correct option is (C).

Question 25 **Signals & Systems (2M)**

Let $f(t)$ be an even function, i.e., $f(-t) = f(t)$ for all t. Let the Fourier transform of $f(t)$ be defined as

$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$. Suppose $\frac{dF(\omega)}{d\omega} = -\omega F(\omega)$ for all ω and $F(0) = 1$. Then

(A) $f(0) < 1$ (B) $f(0) > 1$ (C) $f(0) = 1$ (D) $f(0) = 0$

Ans. **A**

Sol. **Given :**

(i) $f(t)$ is even i.e. $f(-t) = f(t)$

(ii) $\frac{dF(\omega)}{d\omega} = -\omega F(\omega)$ for all ω

(iii) $F(0) = 1$

The only even function, whose differentiation contains itself is a Gaussian function represented as $e^{-a\omega^2}$.

Selecting a Gaussian function which satisfies all mentioned condition,

$$F(\omega) = e^{-\frac{\omega^2}{2}}$$

$$\frac{d}{d\omega}F(\omega) = -\frac{1}{2} \times 2\omega \times e^{-\frac{\omega^2}{2}} = -\omega F(\omega)$$

$$F(0) = e^{-\frac{\omega^2}{2}}\Big|_{\omega=0} = 1$$

So, the required Fourier transform is,

$$f(t) \longleftrightarrow e^{-\frac{\omega^2}{2}} = e^{-\frac{4\pi^2 f^2}{2}} = e^{-2\pi^2 f^2}$$

$$f(t) \longleftrightarrow e^{-\pi(\sqrt{2\pi} f)^2} \quad \ldots\text{(i)}$$

From standard result, Fourier transform only Gaussian function results in Gaussian, i.e.

$$e^{-\pi t^2} \longleftrightarrow e^{-\pi f^2}$$

To obtain a transform as given in equation (i), applying time scaling property, replacing $'t'$ by $\frac{t}{\sqrt{2\pi}}$, we get

$$e^{-\pi\left(\frac{t}{\sqrt{2\pi}}\right)^2} \longleftrightarrow \frac{1}{1/\sqrt{2\pi}} e^{-\pi\left(\frac{f}{1/\sqrt{2\pi}}\right)^2}$$

$$e^{-\frac{t^2}{2}} \longleftrightarrow \sqrt{2\pi}\, e^{-2\pi^2 f^2}$$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \longleftrightarrow e^{-2\pi^2 f^2}$$

Comparing with equation (i),

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$$

$\therefore$ $$f(0) = \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}}\Big|_{t=0} = \frac{1}{\sqrt{2\pi}}$$

$\therefore$ $$f(0) < 1$$

Hence, the correct option is (A).

Question 26 **Signals & Systems (2M)**

Consider a continuous time signal $x(t)$ defined by

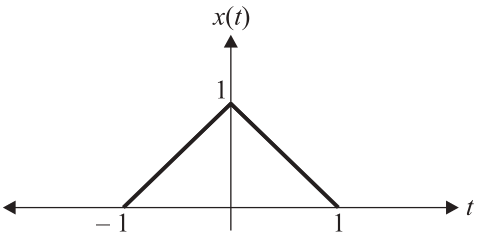
$x(t) = 0$ for $|t| > 1$ and $x(t) = 1 - |t|$ for $|t| \le 1$

Let the Fourier transform of $x(t)$ be defined as $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$. The maximum magnitude of $X(\omega)$ is ____.

Ans. 1

Sol. **Given :** $x(t) = \begin{cases} 0, & |t| > 1 \\ 1 - |t|, & |t| \le 1 \end{cases}$

Comparing $x(t)$ with standard definition of triangular function,



$$A\text{tri}\left(\frac{t}{\tau}\right)=\begin{cases}0, & |t|>\tau\\ A\left[1-\frac{|t|}{\tau}\right], & |t|\le\tau\end{cases}$$

So, given $x(t)$ is a triangular function of amplitude $A=1$ and half width $\tau=1$ as shown in figure,

$$x(t)=\text{tri}(t)$$

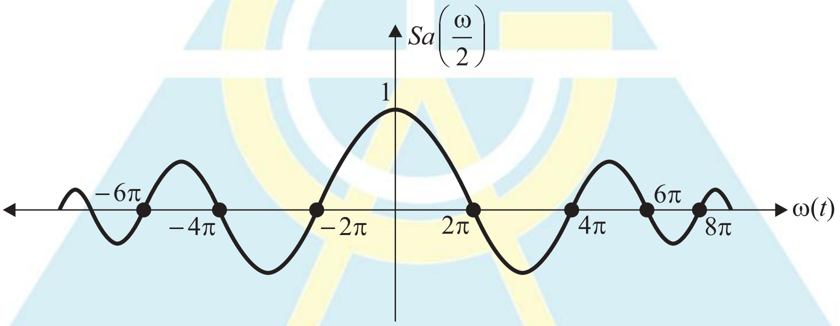
From standard result,

$$A\text{tri}\left(\frac{t}{\tau}\right)\longleftrightarrow A\tau\,\text{sinc}^2(f\tau)\text{ or } A\tau Sa^2\left(\frac{\omega\tau}{2}\right)$$

$$\text{tri}(t)\xleftrightarrow[\tau=1]{A=1}\text{sinc}^2(f)\text{ or } Sa^2\left(\frac{\omega}{2}\right)$$

$\therefore$ $$X(\omega)=Sa^2\left(\frac{\omega}{2}\right)=\left\{Sa\left(\frac{\omega}{2}\right)\right\}^2$$

$$Sa\left(\frac{\omega}{2}\right)=\frac{\sin\left(\frac{\omega}{2}\right)}{\left(\frac{\omega}{2}\right)}$$



Peak value of $Sa\left(\frac{\omega}{2}\right)$ is 1 which occurs at $\omega=0$ as,

$$\lim_{t\to 0}\frac{\sin t}{t}=1$$

As, $$X(\omega)=\left[Sa\left(\frac{\omega}{2}\right)\right]^2$$

Hence, maximum magnitude of $X(\omega)$ is,

$$|X(\omega)|_{max}=(1)^2=1$$

Hence, the maximum magnitude of $X(\omega)$ is 1.

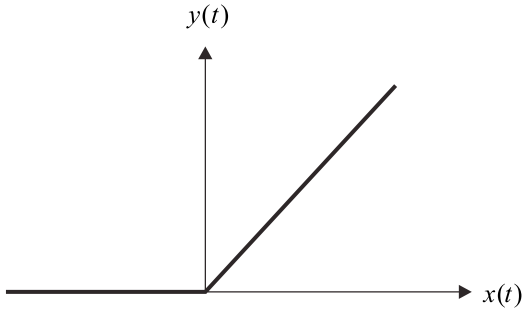
Question 27 **Signals & Systems (1M)**

If the input $x(t)$ and output $y(t)$ of a system are related as $y(t)=\max[0,\ x(t)]$, then the system is

(A) Linear and time-variant
(B) Linear and time-invariant
(C) Non-linear and time-variant
(D) Non-linear and time-invariant

Ans. D

Sol. Given input output relationship is



$$y(t) = \text{Max}\,[0, x(t)]$$

$$y(t) = \begin{cases} 0, & x(t) \le 0 \\ x(t), & x(t) > 0 \end{cases}$$

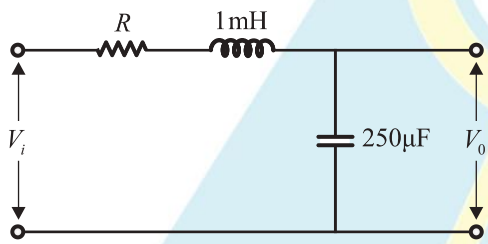
As output is splitted not in time but for values of $x(t)$, hence the system will not follow superposition, so it is not a linear system.

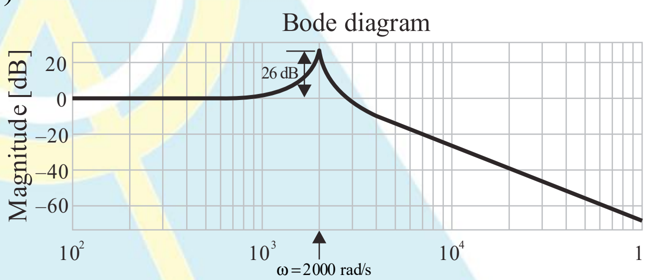
As there is no time scaling or any coefficient multiplied that is function A time and no extra terms other than $x(t)$ and $y(t)$, so the system is time invariant.

Hence, the correct option is (D).

Question 28 **Control System (1M)**

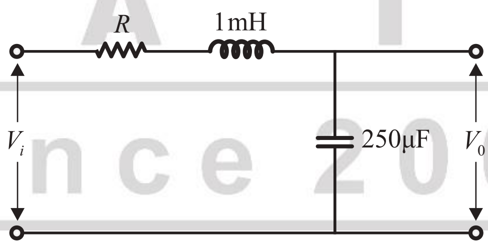
The Bode magnitude plot for the transfer function $\frac{V_0(s)}{V_i(s)}$ of the circuit is as shown. The value of R is _______ Ω. (Round off to 2 decimal places)



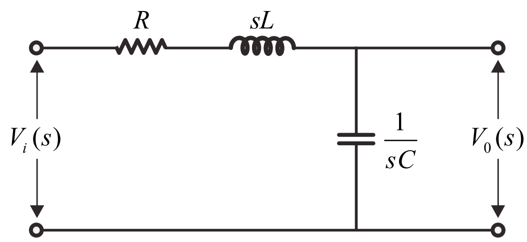


Ans. **0.1**

Sol. Given circuit is as shown below,



Transforming the above circuit in Laplace domain



Here, $L = 1$ mH, $C = 250$ μF

The transfer function $T(s)$ for above circuit is given by,

$$T(s)=\frac{V_0(s)}{V_i(s)}=\frac{\frac{1}{sC}}{R+sL+\frac{1}{sC}}$$

$$T(s)=\frac{V_0(s)}{V_i(s)}=\frac{\frac{1}{sC}}{\frac{s^2LC+sCR+1}{sC}}$$

$$T(s)=\frac{V_0(s)}{V_i(s)}=\frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

So, characteristic equation of above transfer function is,

$$s^2+\frac{R}{L}s+\frac{1}{LC}=0 \quad \ldots \text{(i)}$$

Comparing equation (i) with standard characteristics equation, $s^2+2\xi\omega_n s+\omega_n^2=0$ then

$$\therefore \quad \omega_n=\frac{1}{\sqrt{LC}}$$

$$\xi=\frac{R}{2}\sqrt{\frac{C}{L}} \quad \ldots \text{(ii)}$$

Now from the Bode plot, resonant peak M_r is 26 dB at $\omega_r=2000$ rad/sec ,

$$(M_r)_{\text{dB}}=26\text{ dB}=20\log M_r$$

$$M_r=19.95\approx 20$$

So, $$M_r=\frac{1}{2\xi\sqrt{1-\xi^2}}=20$$

Solving above equation, we get $\xi=0.025$

Now from equation (ii),

$$\xi=\frac{R}{2}\sqrt{\frac{C}{L}}$$

$$0.025=\frac{R}{2}\sqrt{\frac{250\times10^{-6}}{1\times10^{-3}}}$$

$$0.000625=\frac{R^2}{4}\times0.25$$

$$R^2=0.01\ \Omega$$

$$R=0.1\ \Omega$$

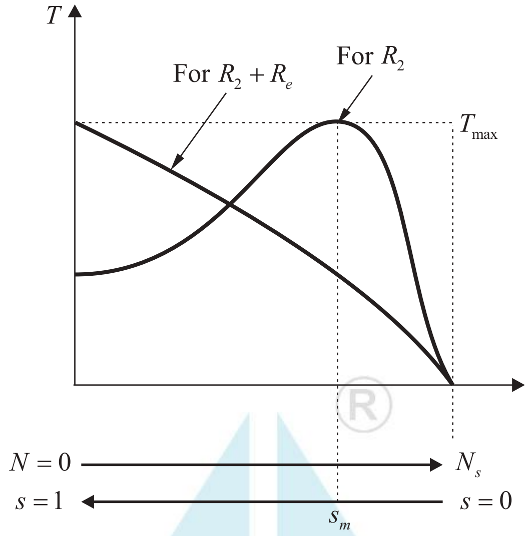
Hence, the correct answer is 0.1.

Question 29 **Electrical Machine (2M)**

An 8 pole, 50 Hz, 3 phase, slip-ring induction motor has an effective rotor resistance of $0.08\,\Omega$ per phase. Its speed at maximum torque is 650 rpm. The additional resistance per phase that must be inserted in the rotor to achieve maximum torque at start is ______ Ω. (Round off to 2 decimal places). Neglect magnetizing current and stator leakage impedance. Consider equivalent circuit parameters referred to stator.

Ans. 0.52

Sol. From torque-slip characteristic of $3\text{-}\phi$ induction motor,



Given : Three phase induction motor

(i) Poles, $P = 8$

(ii) Frequency, $f = 50$ Hz

(iii)Rotor resistance, $R_2 = 0.08\ \Omega/\text{phase}$

(iv)Speed corresponding to maximum torque $= 650$ rpm

$$N_s = \frac{120 f}{p} = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

Speed corresponding to maximum torque,

$$s_m = \frac{N_s - N_r}{N_s}$$

$$s_m = \frac{750 - 650}{750} = 0.133$$

Rotor resistance/phase, $R_2 = 0.08\Omega$

$$s_m = \frac{R_2}{X_2} \qquad \ldots(i)$$

The additional resistance per phase that must be inserted in the rotor circuit to achieve maximum torque at start is R_{ext}.

To obtain the maximum torque at starting, maximum slip, $s_m = 1$.

$$s_m = \frac{R_2 + R_{ext}}{X_2} \qquad \ldots(ii)$$

From equation (i) and (ii),

$$\frac{R_2}{s_m} = \frac{R_2 + R_{ext}}{s_{m\,(\text{at starting})}}$$

$$\frac{0.08}{0.133} = \frac{0.08 + R_{ext}}{1}$$

$$R_{ext} = 0.52\Omega$$

Hence, the correct answer is 0.52.

Question 30 **Electrical Machine (1M)**

The power input to a 500 V, 50 Hz, 6 pole, 3 phase induction motor running at 975 rpm is 40 kW. The stator losses are 1 kW. If the total friction and windage losses are 2.025 kW, then the efficiency is ______%.

Ans. 90

Sol. Given :

(i) Input power, $P_{in} = 40\text{kW}$

(ii) Voltage, $V = 500\text{ V}$

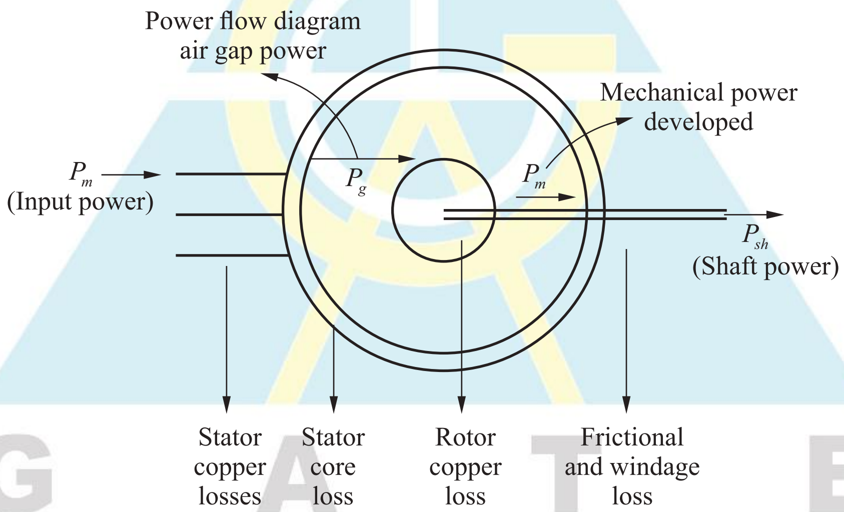
(iii)Frequency $f = 50\text{ Hz}$

(iv)Pole $P = 6$

(v) Rotor speed, $N_r = 975$ rpm

(vi) The total stator losses are $1\text{kW} = 1000\text{ W}$

(vii) Total friction and windage loss $= 2.025\text{kW} = 2025\text{ W}$



Air gap power, $P_g = P_{in} - \text{Stator losses}$

$$P_g = 40000 - 1000 = 39000\text{ W}$$

Rotor copper losses, $P_{cu} = sP_g$

So, $P_m = P_g - sP_g$

$$P_m = (1-s)P_g$$

Synchronous speed, $N_s = \dfrac{120 f}{P}$

$$N_s = \frac{120\times 50}{6} = 1000\text{ rpm}$$

So, slip $s = \dfrac{N_s - N_r}{N_s} = \dfrac{1000-975}{1000} = 0.025$

So, $P_m = (1-0.025)\times 39000 = 38025\text{ W}$

Shaft power, $P_{sh} = P_m - \text{Frictional and windage losses}$

$P_{sh} = 38025 - 2025 = 36000\text{ W}$

Efficiency, $\eta = \frac{P_0}{P_{in}} = \frac{P_{sh}}{P_{in}}$

$\%\eta = \frac{36000}{40000}\times 100 = 90\%$

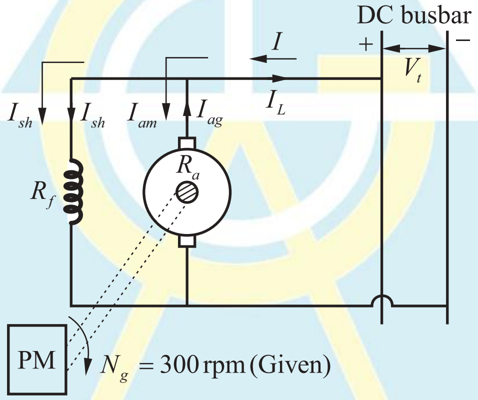
Hence, the correct answer is 90.

Question 31 **Electrical Machine (2M)**

A belt-driven DC shunt generator running at 300 rpm delivers 100 kW to a 200 V DC grid. It continues to run as a motor when the belt breaks, taking 10 kW from the DC grid. The armature resistance is 0.025 Ω, field resistance is 50 Ω and brush drop is 2 V. Ignoring armature reaction, the speed of the motor is _____ rpm. (Round off to 2 decimal places.)

Ans. 275.00 to 275.25 (275.186)

Sol.



Case 1 : When machine acts as generator,

(i) Power output, $P_0 = 100\text{kW}$

(ii) Terminal voltage, $V_t = 200\text{ V}$

Load current, $I_L = \frac{P_0}{V_t} = \frac{100\text{ kW}}{200} = 500\text{ A}$

$I_{sh} = \frac{V_t}{R_{sh}} = \frac{200}{50} = 4\text{ A}$

$I_a = I_L + I_{sh} = 500 + 4 = 504\text{ A}$

Generated voltage, $E_g = V_t + I_a R_a + V_{BD}$

$E_g = 200 + (504\times 0.025) + 2$

$E_g = 214.6\text{V}$

Case 2 : When machine continue to run as a motor,

(i) Power input, $P_{in} = 10\text{kW}$

(ii) Terminal voltage of busbar, $V_t = 200\,\text{V}$

Supply current, $I_L = \dfrac{P_{in}}{V_t} = \dfrac{10\,\text{kW}}{200} = 50\,\text{A}$

$$I_a = I_L - I_{sh} = 50 - 4 = 46$$

$\therefore$ Back emf of motor, $E_b = V - I_a R_a - V_b$

$$E_b = 200 - (46 \times 0.025) - 2 = 196.85\,\text{V}$$

When machine acts as generator, $E_g \propto \phi_{fg} N_g$

$\therefore$ Speed of generator, $N_g \propto \dfrac{E_g}{\phi_{fg}}$ …(i)

When machine acts as motor, $E_b \propto \phi_{fm} N_m$

$\therefore$ Speed of motor, $N_m \propto \dfrac{E_b}{\phi_{fm}}$ …(ii)

From equation (i) and (ii), we get

$$\frac{N_m}{N_g} = \frac{E_b}{E_g} \times \frac{\phi_{fg}}{\phi_{fm}}$$

$$\frac{N_m}{N_g} = \frac{E_b}{E_g} \times \frac{I_{fg}}{I_{fm}} = \frac{E_b}{E_g} \times \frac{\frac{V}{R_f}}{\frac{V}{R_f}}$$

$$\therefore \quad \frac{N_m}{N_g} = \frac{E_b}{E_g}$$

$$\therefore \quad N_m = N_g \times \frac{E_b}{E_g} = 300 \times \frac{196.85}{214.6} = 275.186$$

$\therefore$ Speed of motor, $N_m = 275.186\text{rpm}$

Hence, the correct answer is 275.186

Question 32 **Digital Electronics (2M)**

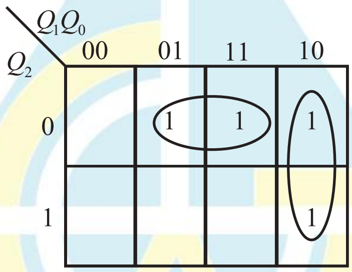
A counter is constructed with three D flip-flops. The input-output pairs are named as (D_0, Q_0), (D_1, Q_1) and (D_2, Q_2), where the subscript 0 denotes LSB. The output sequence is desired to be Gray-code sequence 000, 001, 011, 010, 110, 111, 101 and 100, repeating periodically. Note that the bits are listed in the $Q_2 Q_1 Q_0$ format. The combination logic expression for D_1 is

(A) $Q_2 Q_1 Q_0$ (B) $Q_2 Q_0 + Q_1 \bar{Q}_0$ (C) $\bar{Q}_2 Q_0 + Q_1 \bar{Q}_0$ (D) $Q_2 Q_1 + \bar{Q}_2 \bar{Q}_1$

Ans. **C**

Sol. **Given :** $000 \rightarrow 001 \rightarrow 011 \rightarrow 010 \rightarrow 100 \rightarrow 111 \rightarrow 101 \rightarrow 100$

Present state			Next state			Input		
Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	D_2	D_1	D_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	1	0	1	1
0	1	1	0	1	0	0	1	0
0	1	0	1	0	0	1	0	0
1	0	0	1	1	1	1	1	1
1	1	1	1	0	1	1	0	1
1	0	1	1	0	0	1	0	0
1	0	0	0	0	0	0	0	0



$D_1 = \bar{Q}_2 Q_0 + Q_1 \bar{Q}_0$

Hence, the correct answer is (C).

Question 33 **Power System (1M)**

An alternator with an internal voltage of $1\angle\delta_1$ pu and synchronous reactance of 0.4 pu is connected by a transmission line of reactance 0.1 pu to a synchronous motor having synchronous reactance 0.35 pu and internal voltage of $0.85\angle\delta_2$ pu. If the real power supplied by the alternator is 0.866 pu, then $(\delta_1 - \delta_2)$ is ______ degrees (Round off to 2 decimal places). (Machines are of non-salient type. Neglect resistances)

Ans. **60**

Sol. **Given :**

(i) $E_g\angle\delta_1 = 1\angle\delta_1$

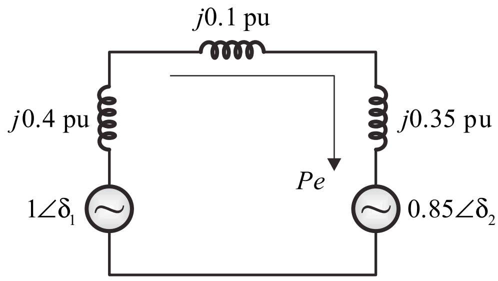
(ii) $X_g = j0.4\text{pu}$

(iii) $X_l = j0.1\text{pu}$

(iv) $X_m = j0.35\text{pu}$

(v) $E_m\angle\delta_2 = 0.85\angle\delta_2\ \text{pu}$

Active power transfered from generator to motor is 0.866 pu.



$$P_e = \frac{E_g \times V_t}{X_T} \sin(\delta_1 - \delta_2)$$

$$0.866 = \frac{1 \times 0.85}{(0.4 + 0.1 + 0.35)} \sin(\delta_1 - \delta_2)$$

$$0.866 = \frac{0.85}{0.85} \sin(\delta_1 - \delta_2)$$

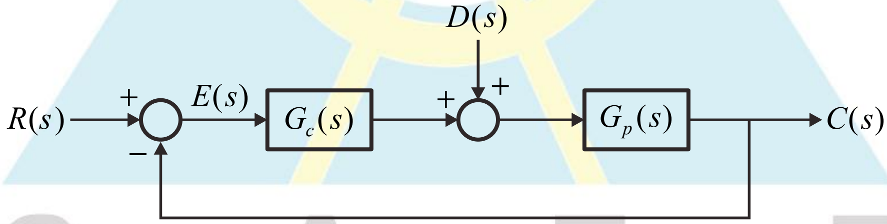
$$\delta_1 - \delta_2 = \sin^{-1}(0.866) = 60^0$$

Hence, the correct answer is 60.

Question 34 **Control System (2M)**

In the given figure, plant $G_p(s) = \frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}$ and compensator $G_c(s) = K\left[\frac{1+T_1 s}{1+T_2 s}\right]$.

The external disturbance input is $D(s)$. It is desired that when the disturbance is a unit step, the steady state error should not exceed 0.1 unit. The minimum value of K is ______. (Round off to 2 decimal places)

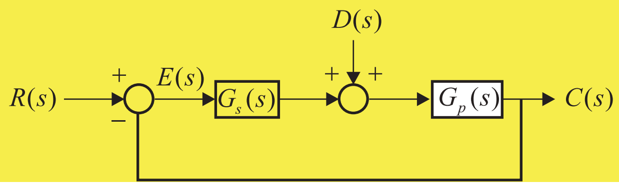


Ans. **10.45 (MTA)**

Sol. **Method 1 :**

Given : $G_p(s) = \frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}$

$$G_c(s) = K\left[\frac{1+T_1 s}{1+T_2 s}\right]$$



Put $R(s) = 0$, $C(s) = G_p[E(s)G_c(s) + D]$

$$-E = G_p G_c E + G_p D$$

$$-E[1 + G_p G_c] = G_p D$$

$$E(s)=\frac{-G_pD}{1+G_pG_c}$$

$$e_{ss}=\lim_{s\to 0}SE(s)$$

$$e_{ss}=\frac{-2.2}{1+2.2K}$$

$$e_{ss}\le 0.1$$

$$\frac{-22}{1+22K}\le 0.1$$

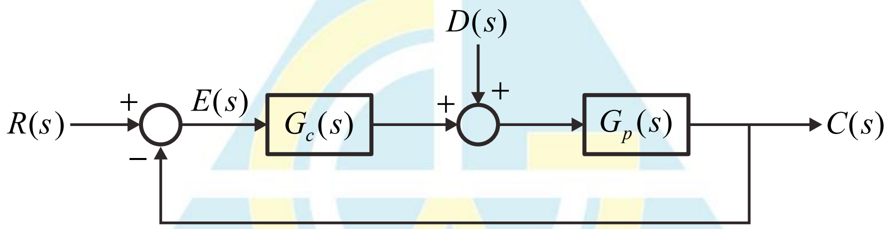
$$-2.2\le 0.1+0.22K$$

$$0.22K\ge -2.3$$

$$K\ge -10.45$$

$$K_{min}=-10.45$$

Method 2 :



$$G(s)-C(s)=E(s)$$

$$G(s)=0$$

$$-C(s)=E(s)$$

$$[E(s)\times G_c(s)+D(s)]G_D(s)=-E(s)$$

$$D(s)G_P(s)+E(s)\times G_c(s)\times h_p(s)+E(s)=1$$

$$E(s)=\left\{\frac{1-D(s)G_p(s)}{1+G_C(s)G_P(s)}\right\}$$

$$=\frac{1-\frac{1}{s}\left(\frac{2.2}{(1+0.15)(1+0.45)(1+1.25)}\right)}{1+k\left[\frac{1+T_1s}{1+T_2s}\right]\left[\frac{2.2}{(1+0.15)(1+0.45)(1+1.25)}\right]}$$

$$\lim_{s\to 0}sE(s)=0.1$$

$$\frac{-2.2}{1+2.2k}=0.1$$

$$-2.2=0.1+0.22k$$

$$\frac{-2.2-0.1}{0.22}=k$$

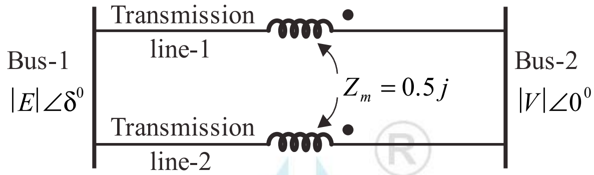
$$\frac{-2.3}{0.22}=k=-10.45$$

$|k| = 10.45$

Hence, the correct answer is 10.45.

Question 35 **Power System (2M)**

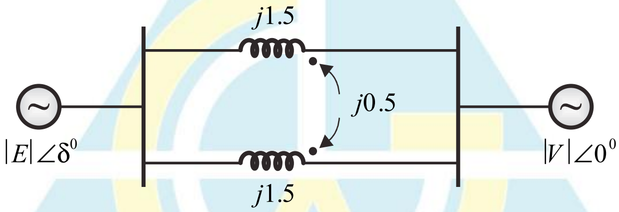
In the figure shown, self-impedances of the two transmission lines are $1.5j$ pu each and $Z_m = 0.5j$ pu is the mutual impedance. Bus voltages shown in the figure are in pu. Given that $\delta > 0$, the maximum steady state real power that can be transferred in pu from bus-1 to bus-2 is



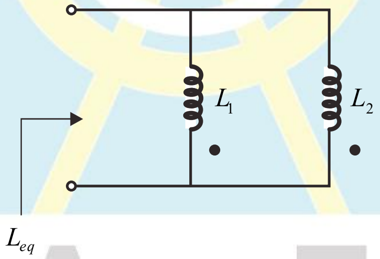
(A) $|E||V|$ (B) $\frac{|E||V|}{2}$ (C) $2|E||V|$ (D) $\frac{3|E||V|}{2}$

Ans. A

Sol. **Given :**



Equivalent inductance for a parallel magnetic circuit with mutual inductance M is given by,



$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

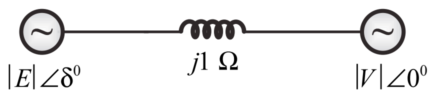
$$j\omega L_{eq} = \frac{(j\omega)^2 \left[L_1 L_2 - M^2\right]}{j\omega[L_1 + L_2 - 2M]}$$

$$j\omega L_{eq} = \frac{(j\omega L_1)(j\omega L_2) - (j\omega M)^2}{j\omega L_1 + j\omega L_2 - 2(j\omega M)}$$

$$j\omega L_{eq} = \frac{1.5 \times 1.5 - (0.5)^2}{1.5 + 1.5 - 2 \times 0.5}$$

$$j\omega L_{eq} = j1$$

So, $jX_{eq} = j1\Omega$



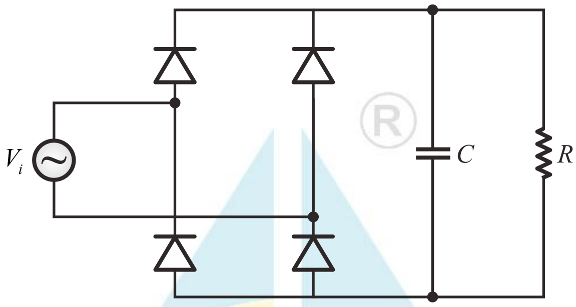
So, maximum power transfered in steady state

$$= \frac{|E||V|}{1} = |E||V| = |EV|$$

Hence, the correct option is (A).

Question 36 **Power Electronics (1M)**

In the circuit shown, the input V_i is a sinusoidal AC voltage having an rms value of 230 V$\pm$ 20%. The worst case peak-inverse voltage seen across any diode is _______ V. (Round off to 2 decimal places)



Ans. **390.266**

Sol. **Given :**

Supply voltage, $V_{s(rms)} = (230 \pm 20\%)\text{ V}$

$$\Delta V = \pm\frac{20 \times 230}{100} = \pm 46$$

$$V_{s(rms)} = (230 \pm 46)\text{ V}$$

$$V_{s(rms)} = (184 \text{ to } 276)\text{ V}$$

Supply voltage will vary in between $184\sqrt{2}\sin\omega t$ to $276\sqrt{2}\sin\omega t$.

In given converter, the discharging time of capacitor will be 4τ and capacitor will charge for every less time than $\frac{\pi}{2\omega}$ sec.

Let us assume, in the range of $0-\pi/2$ $D_1 D_2$ are ON, so $D_3 D_3$ will be OFF.

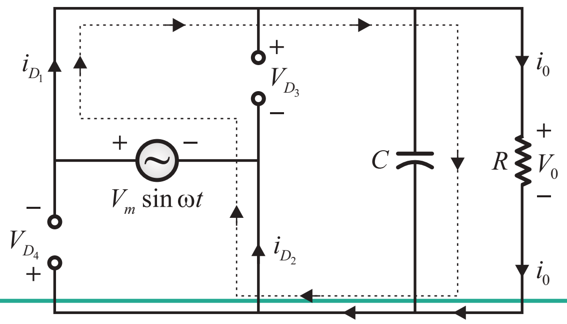
Mode I : $0 < \omega t < \pi$

Diode D_1, D_2 will be forward biased and diode D_3, D_4 will be reversed biased.

Mode II : $\pi < \omega t < 2\pi$

Diode D_3, D_4 will be forward biased and diode D_1, D_2 will be reversed biased.

From mode-I,



Applying KVL in the above circuit,

$$V_m \sin\omega t - V_{D_3} = 0$$

$$V_{D_3} = V_m \sin\omega t$$

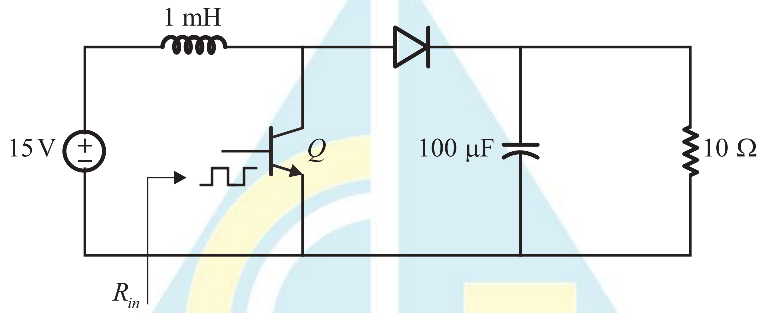
The peak inverse voltage across each diode is given by,

$$V_D\big|_{max} = 276\sqrt{2}\text{ V} = 390.32\text{ V}$$

Hence, the correct answer is 390.32.

Question 37 **Power Electronics (2M)**

Consider the boost converter shown. Switch Q is operating at 25 kHz with a duty cycle of 0.6. Assume the diode and switch to be ideal. Under steady-state condition, the average resistance R_{in} as seen by the source is ________ Ω. (Round off to 2 decimal places)



Ans. **1.6**

Sol. **Given :** Boost convertor

(i) Switching frequency, $f_s = 25\text{ kHz}$

(ii) Duty cycle, $\delta = 0.6$

Method 1 :

As the switches are lossless,

$$P_{in} = P_{out}$$

$$V_s i_s = V_0 i_0$$

$$i_s = \frac{i_0}{(1-\delta)}$$

$\therefore$
$$V_{0(avg)} = \frac{15}{(1-0.6)} = 37.5\text{ V}$$

Average value of load current is given by,

$$i_{0(avg)} = \frac{V_0}{R} = \frac{37.5}{10} = 3.75\text{ Amp}$$

Average value of source current,

$$i_{s(avg)} = i_{L(avg)} = \frac{10}{(1-\delta)} = 9.375\text{ Amp}$$

Ripple component of inductor current is given by,

$$\Delta i_L = \frac{\delta V_s}{fL} = \frac{0.6\times 15}{25\text{k}\times 1\text{m}} = 0.36\text{ Amp}$$

∴ Minimum current through inductor,

$$i_{L_{\min}} = i_{L_{(avg)}} - \frac{\Delta i_L}{2}$$

$$i_{L_{\min}} = 9.375 - \frac{0.36}{2} = 9.195 \text{ Amp}$$

As $i_{L_{\min}}$ is not zero, hence inductor current will be continuous

Input resistance seen from supply side is given by,

$$R_{in} = \frac{V_s}{i_s} = \frac{15}{9.375} = 1.6\ \Omega$$

Method 2 :

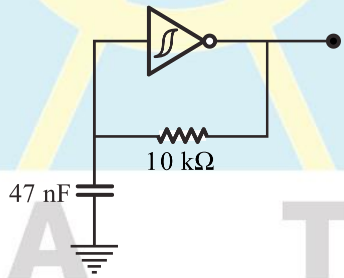
$$R_{in} = \frac{V_s}{i_s} = \frac{\frac{V_0}{(1-\delta)}}{\frac{i_0}{(1-\delta)}} = \left(\frac{V_0}{i_0}\right)(1-\delta)^2$$

$$R_{in} = R_0(1-\delta)^2 = 10(1-0.6)^2 = 1.6\Omega$$

Hence, the correct answer is 1.6.

Question 38 **Analog Electronics (2M)**

A CMOS Schmitt-trigger inverter has a low output level of 0 V and a high output level of 5 V. It has input thresholds of 1.6 V and 2.4 V. The input capacitance and output resistance of the Schmitt trigger are negligible. The frequency of the oscillator shown in the figure is ______ Hz. (Round off to 2 decimal places)



Ans. **3158.12**

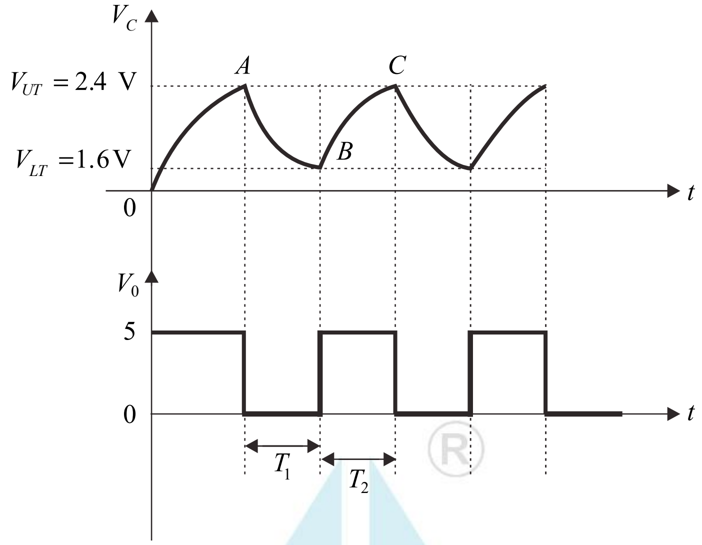
Sol. **Given :**

(i) $V_{LT} = 1.6\text{ V}$

(ii) $V_{UT} = 2.4\text{ V}$

(iii) $+V_{sat} = 5\text{ V}$

(iv) $-V_{sat} = 0\text{ V}$



Total time period is given by,

$$T = T_1 + T_2$$

Where, T_1 is OFF time

T_2 is ON time

$$f = \frac{1}{T}$$

Calculation of T_1 :

$$V_c(0^-) = 2.4 \text{ V}$$

$$V_c(\infty) = 0 \text{ V}$$

$$V_c(t) = 0 + (2.4 - 0)e^{-t/RC}$$

At $t = T_1$; $V_c = 1.6$ V

$$1.6 = 2.4e^{-T_1/RC}$$

$$e^{-T_1/RC} = \frac{1.6}{2.4}$$

$$e^{T_1/RC} = \frac{24}{16}$$

$$T_1 = RC\ln(1.5)$$

Calculation of T_2 :

$$V_c(0^-) = 1.6 \text{ V}$$

$$V_c(\infty) = 5 \text{ V}$$

The charging equation of the first order RC circuit is given by

$$V_c(t) = 5 + (16.5)e^{-t/RC}$$

At $t = T_2$

$V_c = 2.4$ V

$$2.4 = 5 + (16.5)e^{-T_2/RC}$$

$$e^{-T_2/RC} = \frac{34}{26}$$

$$T_2 = RC\log(1.307)$$

Calculation of T :

$$\boldsymbol{T = T_1 + T_2}$$

$$T = RC\ln(1.5\times 1.307)$$

$$T = 10\times 10^3 \times 47\times 10^{-9}\times 0.673$$

$$T = 316.644\ \mu\text{sec}$$

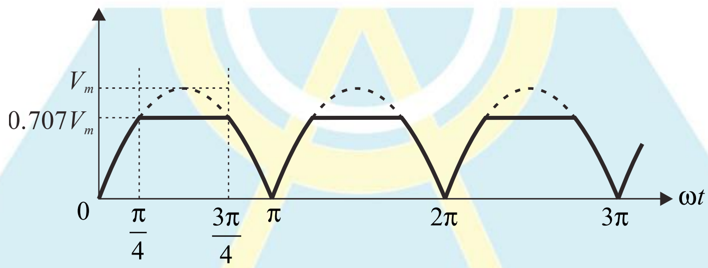
$$f = \frac{1}{T}$$

$$f = 3158.12\ \text{Hz}$$

Hence, the correct answer is 3158.12.

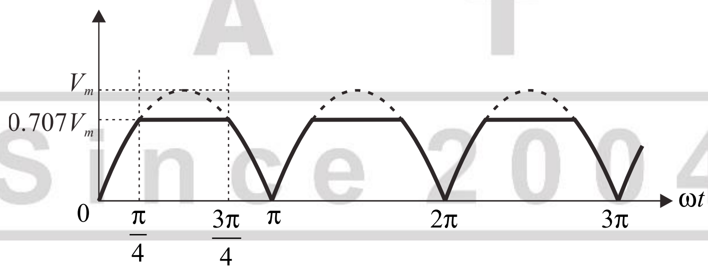
Question 39 **Analog Electronics (2M)**

The waveform shown in solid line is obtained by clipping a full-wave rectified sinusoid (shown dashed). The ratio of the rms value of the full-wave rectified waveform to the rms value of the clipped waveform is ________. (Round off to 2 decimal places,)



Ans. **1.21**

Sol. Given waveform,



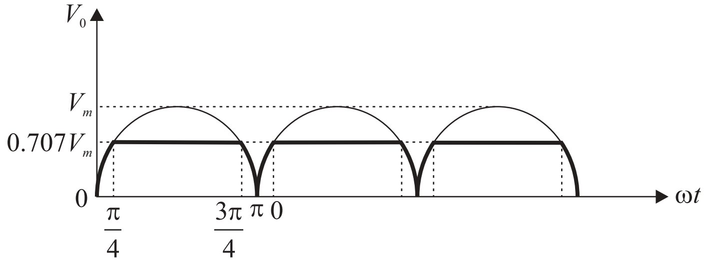
RMS value of above signal given by,

$$V_{rms} = \left[\frac{1}{\pi}\int_0^{\pi}(\sin\omega t)^2\, d\omega t\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{\pi}\int_0^{\pi}\frac{(1-\cos 2\omega t)}{2}d\omega t\right]^{\frac{1}{2}}$$

$$= V_{rms}\left[\frac{1}{2\pi}\left((\pi - 0) - \frac{\sin 2\omega t}{2}\Bigg|_0^{\pi}\right)\right]^{\frac{1}{2}}$$

$$= \frac{V_m}{\sqrt{2}} = 0.707\,V_m$$

RMS value of clipped signal is given by,



$$= \left[\frac{1}{\pi}\int_0^{\frac{\pi}{4}}(V_m \sin\omega t)^2\,d\omega t + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}(0.707V_m)^2\,d\omega t + \int_{\frac{3\pi}{4}}^{\pi}(V_m \sin\omega t)^2 d\omega t\right]^{\frac{1}{2}}$$

$$= \left\{\frac{1}{\pi}\left[V_m^2\int_0^{\frac{\pi}{4}}\left(\frac{1-\cos 2\omega t}{2}\right) + 0.707^2 V_m^2 \times \left(\frac{3\pi}{4}-\frac{\pi}{4}\right)\right] + V_m^2\int_{\frac{3\pi}{4}}^{\pi}\left(\frac{1-\cos 2\pi\omega t}{2}\right)\right\}^{\frac{1}{2}}$$

$$= \frac{V_m^2}{2\pi}\left[\frac{\pi}{4} - \left(\frac{\sin 2\times\frac{\pi}{4} - \sin 2\times 0}{2}\right) + \left(\frac{3\pi}{4}-\frac{\pi}{4}\right) + \left(\pi - \frac{3\pi}{4}\right) - \sin 2\pi - \sin 2\times\frac{3\pi}{4}\right]^{\frac{1}{2}}$$

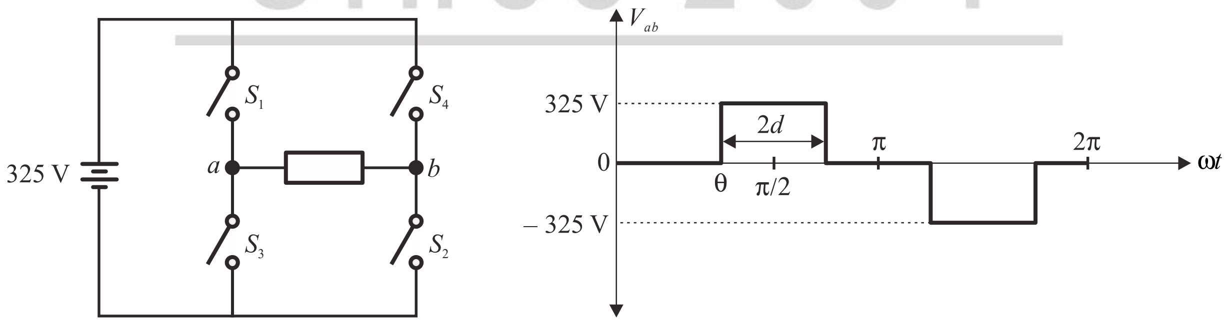
$$= \frac{V_m^2}{2\pi}\left[\frac{\pi}{4} - \frac{1}{2} + \frac{\pi}{2} + \frac{\pi}{4} - \frac{1}{2}\right]^{\frac{1}{2}} = V_m\left(\frac{\pi-1}{2\pi}\right)^{\frac{1}{2}} = 0.5839V_m$$

$$\frac{V_{(rms)}\ \text{sinewave}}{V_{(rms)}\ \text{clipped wave}} = \frac{0.707\,V_m}{0.5839\,V_m} = 1.21$$

Hence, the correct answer is 1.21.

Question 40 **Power Electronics (2M)**

A single-phase full bridge inverter fed by a 325 V DC produces a symmetric quasi-square waveform across "*ab*" as shown. To achieve a modulation index of 0.8, the angle θ expressed in degrees should be ________. (Round off to 2 decimal places)



(Modulation index is defined as the ratio of the peak of the fundamental component of V_{ab} to the applied DC value)

Ans. **51.07**

Sol. Fourier series expression for single pulse width modulation is given by,

$$V_0(nt) = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin(nd) \sin\left(\frac{n\pi}{2}\right) \sin n\omega t$$

Fundamental component of single pulse width modulation is given by,

$$V_{0_1}(t) = \frac{4V_s}{\pi} \sin(d) \qquad \ldots(i)$$

Peak amplitude of fundamental component of single pulse with a modulation index of 0.8 is

$$V_{0_1} = mV_s$$

$$V_{0_1} = 0.8V_s \qquad \ldots(ii)$$

Comparing equation (i) and (ii),

$$\frac{4V_s}{\pi} \sin(d) = 0.8V_s$$

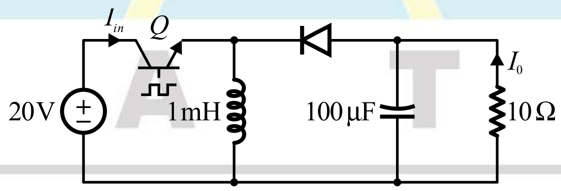
$$\sin d = 0.62832$$

Therefore, $\theta = \left(\frac{\pi}{2} - d\right) = 51.07^0$

Hence, the correct answer is 51.07.

Question 41 **Power Electronics (2M)**

Consider the buck-boost converter shown. Switch Q is operating at 25 kHz and 0.75 duty-cycle. Assume diode and switch to be ideal. Under steady-state condition, the average current flowing through the inductor is _______ A.



Ans. **24**

Sol. **Given :** Buck-boost converter

(i) Switching frequency, $f_s = 25\text{ kHz}$

(ii) Duty cycle, $\delta = 0.75$

Method 1 :

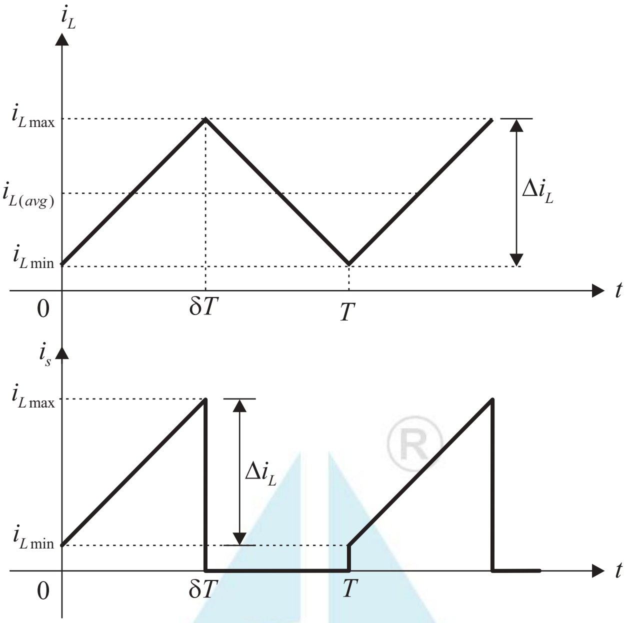


Fig. Inductor current and source current waveform

Average value of inductor current is given by,

$$i_{L(avg)} = \frac{\frac{1}{2}\Delta i_L T + i_{L\min} T}{T}$$

$$i_{L(avg)} = \left(\frac{i_{L\max} + i_{L\min}}{2}\right)$$

Average value of source current is given by,

$$i_{s(\text{avg})} = \frac{\frac{1}{2}\Delta i_L \delta T + i_{L\min}\delta T}{T}$$

$$i_{s(\text{avg})} = \frac{1}{2}(i_{L\max} + i_{L\min})\delta \qquad [i_{s(avg)} = \delta i_{L(avg)}]$$

Output voltage for buck-boost convertor is given by,

$$V_0 = \frac{\delta V_s}{(1-\delta)} = \frac{0.75\times 20}{0.25} = 60 \text{ V}$$

$$i_{0(avg)} = \frac{V_0}{R} = 6 \text{ A}$$

As the switches are lossless,

$$P_{in} = P_{out}$$

$$V_s i_s = V_0 i_0$$

$$\frac{V_0}{V_s} = \frac{i_s}{i_0} = \frac{\delta}{(1-\delta)}$$

$$\therefore \quad i_{s(avg)} = \frac{i_0\delta}{(1-\delta)} = \frac{6\times 0.75}{0.25} = 18 \text{ A}$$

Average value of inductor current,

$$i_{L(avg)} = \frac{i_{s(avg)}}{\delta} = \frac{18}{0.75} = 24\text{ A}$$

Ripple in inductor current is given by,

$$\Delta i_L = \frac{\delta V_s}{fL} = \frac{0.75\times 20}{25\text{k}\times 1\text{m}} = 0.6\text{ A}$$

Minimum value of inductor current will be,

$$i_{L\min} = i_{L(avg)} - \frac{\Delta i_L}{2} = 24 - \frac{0.6}{2} = 23.7\text{ Amp}$$

As $i_{L(\min)}$ is greater than zero, so i_L will be continuous.

So, $i_{L(avg)} = 24\text{ Amp}$

Hence, the correct answer is 24.

Method 2 :

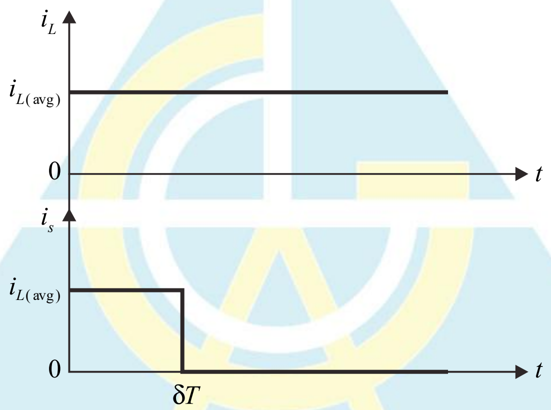


Fig. inductor current and Source current waveform

Average value of switch current is given by,

$$I_{SW(avg)} = \delta \times I_{L(avg)}$$

$$I_{L(avg)} = \frac{I_{SW(avg)}}{\delta}$$

Average value of inductor current is given by,

As the switches are lossless,

$$P_{in} = P_{out}$$

$$V_s i_s = V_0 i_0$$

$$i_s = \frac{i_0}{(1-\delta)} \qquad \text{(For Boost Converter, } I_{L(avg)} = i_{s(avg)})$$

$$I_{L(avg)} = \frac{I_0}{1-\delta} = \frac{20}{1-0.75} = 24\text{ A}$$

Hence, the correct answer is 24.

Question 42 **Engineering Mathematics (1M)**

Suppose the circles $x^2+y^2=1$ and $(x-1)^2+(y-1)^2=r^2$ intersect each other orthogonally at the point (u, v). Then $u+v=$ _______.

Ans. 1

Sol. **Given :** $x^2+y^2=1$ ...(i)

and $(x-1)^2+(y-1)^2=r^2$ ...(ii)

Differentiating equation (i) w.r.t. x,

$$2x+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-x}{y}$$

Differentiating equation (ii) w.r.t x,

$$2(x-1)+2(y-1)\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-(x-1)}{(y-1)}$$

Let, $M_1=\frac{dy}{dx}=\frac{-x}{y}$

$$M_2=\frac{dy}{dx}=\frac{-(x-1)}{(y-1)}$$

Now, $M_1\times M_2=-1|_{\text{at }(x,y)=(u,v)}$

$$\left(\frac{-u}{v}\right)\left(\frac{1-u}{v-1}\right)=-1$$

$$\frac{u-u^2}{v^2-v}=1$$

$$u-u^2=v^2-v$$

$$u+v=v^2+u^2 \qquad \{\because x^2+y^2=1\Rightarrow u^2+v^2=1\}$$

$$u+v=1$$

Hence, the correct answer is 1.

Question 43 **Power System (1M)**

Consider a power system consisting of N number of buses. Buses in this power system are categorized into slack bus. PV buses and PQ buses for load flow study. The number of PQ buses is N_L. The balanced Newton-Raphson method is used to carry out load flow study in polar form H, S, M and R are sub-matrices of the Jacobian matrix J as shown below:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}=J\begin{bmatrix}\Delta\delta\\ \Delta\gamma\end{bmatrix},\text{ where } J=\begin{bmatrix}H & S\\ M & R\end{bmatrix}$$

The dimension of the sub matrix M is

(A) $N_L \times (N-1)$ (B) $(N-1)\times(N-1-N_L)$

(C) $N_L \times (N-1+N_L)$ (D) $(N-1)\times(N-1+N_L)$

Ans. A

Sol. Jacobian matrix is given by,

$$J = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial \gamma} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial \gamma} \end{bmatrix}$$

Comparing it with Jacobian matrix given in question

$$J = \begin{bmatrix} H & S \\ M & R \end{bmatrix}$$

So, $M = \frac{\partial Q}{\partial \delta}$

$Q \rightarrow$ Known

$\delta \rightarrow$ Unknown

So , size of sub-Jacobian matrix M is given by,

$$= N_L \times (N-1)$$

Hence, the correct option is (A).

Question 44 **Engineering Mathematics (2M)**

In the open interval $(0, 1)$, the polynomial $p(x) = x^4 - 4x^3 + 2$ has

(A) Two real roots (B) One real root

(C) Three real roots (D) No real roots

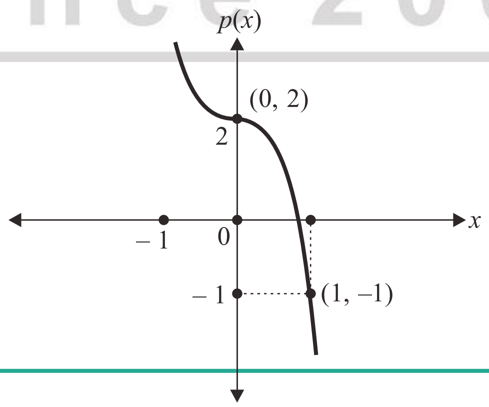
Ans. B

Sol. **Given :** $p(x) = x^4 - 4x^3 + 2$

$$p(0) = 0 - 0 + 2 = 2$$

$$p(1) = 1 - 4 + 2 = -1$$

$$p(-1) = 1 + 4 + 2 = 7$$



Since, $p(x)$ intersect x-axis, one time in the interval (0, 1).

Therefore, $p(x)$ has 1 real root in (0, 1).

Hence, the correct option is (B).

Question 45 **Electromagnetic Theory (2M)**

One Coulomb of point charge moving with a uniform velocity $10\,\hat{x}$ m/s enters the region $x \geq 0$ having a magnetic flux density $\vec{B} = (10\,y\hat{x} + 10\,x\hat{y} + 10\hat{z})\,\text{T}$. The magnitude of force on the charge at $x = 0^+$ is ______ N. ($\hat{x}$, $\hat{y}$ and $\hat{z}$ are unit vectors along x-axis, y-axis and z-axis respectively)

Ans. **100**

Sol. **Given :** Magnetic flux density, $\vec{B} = (10\,y\hat{x} + 10\,x\hat{y} + 10\hat{z})\,\text{T}$

Velocity, $\vec{V} = 10\,\hat{x}\,\text{m/s}$

Force, $\vec{F} = (\vec{V} \times \vec{B})$

$$\vec{F} = \left(10\,\hat{x} \times (10\,y\hat{x} + 10\,x\hat{y} + 10\hat{z})\right)$$

$$|\vec{F}| = |-100| = 100\,\text{N}$$

Hence, the correct answer is 100.

Question 46 **Signals & Systems (1M)**

Two discrete-time linear time-invariant systems with impulse responses $h_1[n] = \delta[n-1] + \delta[n+1]$ and $h_2[n] = \delta[n] + \delta[n-1]$ are connected in cascade. Where $\delta[n]$ in the Kronecker delta. The impulse response of the cascade system

(A) $\delta[n-2] + \delta[n+1]$

(B) $\delta[n-1]\delta[n] + \delta[n+1]\delta[n-1]$

(C) $\delta[n-2] + \delta[n-1] + \delta[n] + \delta[n+1]$

(D) $\delta[n]\delta[n-1] + \delta[n-2]\delta[n+1]$

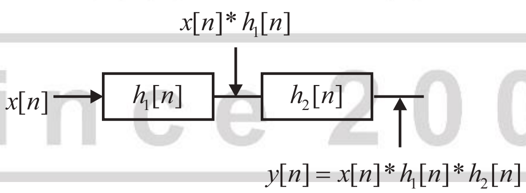
Ans. **C**

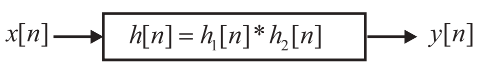
Sol. **Given :** The two systems being cascaded are having impulse responses

$$h_1[n] = \delta[n-1] + \delta[n+1]$$

$$h_2[n] = \delta[n] + \delta[n+1]$$

Impulse response of the cascaded equivalent system is given as





$$h[n] = h_1[n] * h_2[n] = \{\delta[n-1] + \delta[n+1]\} * \{\delta[n] + \delta[n-1]\}$$

$$h[n] = \delta[n-2] + \delta[n-1] + \delta[n] + \delta[n+1]$$

$$h[n] = \delta[n-1] + \delta[n] + \delta[n-1] + \delta[n-2]$$

Hence, the correct option is (C).

Question 47 **Control system (2M)**

The state space representation of a first-order system is given as

$$\dot{x} = -x + u$$
$$y = x$$

Where, x is the state variable, u is the control input and y is the controlled output. Let $u = -kx$ be the control law, where K is the controller gain. To place a closed loop pole at – 2, the value of k is _____.

Ans. 1

Sol. **Given :** $\dot{x} = -x + u$

$y = x$

$\dot{x} = -x + u$

Let $u = -Kx$

$\dot{x} = -x - Kx$

Method 1 :

$$\dot{x} = x(-1-K)$$
$$|sI - A| = 0$$
$$|sI - (-1-K)| = 0$$
$$s + 1 + K = 0$$
$$\because \quad s = -2$$
$$\therefore \quad -2 + 1 + K = 0$$
$$K = 1$$

Hence, the correct answer is $K = 1$.

Method 2 :

$$\dot{x} = -x + u$$
$$y = x$$

Let $u = -Kx$

$$sx(s) = -x(s) - kx(s)$$
$$y(s) = x(s)$$
$$sx(s) + x(s) + Kx(s) = 1$$
$$x(s)[s + 1 + K] = 1$$

Characteristic equation $= [s + 1 + K]$ to place close loop pole at – 2

$$1 + K = 2$$
$$K = 1$$

Hence, the correct answer is 1.

Question 48 **Measurement (1M)**

Inductance is measured by,

(A) Schering bridge (B) Maxwell bridge (C) Kelvin bridge (D) Wien bridge

Ans. B

Sol. (i) Schering bridge is used for the measurement of capacitance.

(ii) Wien bridge is used for the measurement of frequency.

(iii) Kelvin bridge is used for the measurement of resistance.

(iv) Maxwell bridge is used for the measurement of inductance.

Hence, the correct option is (B).

Question 49 **Network Theory (1M)**

A signal generator having a source resistance of 50 Ω is set to generate a 1 kHz sinewave. Open circuit terminal voltage is 10 V peak-to-peak. Connecting a capacitor across the terminals reduces the voltage to 8 V peak-to-peak. The value of this capacitor is ________μF. (Round off to 2 decimal places)

Ans. 2.38

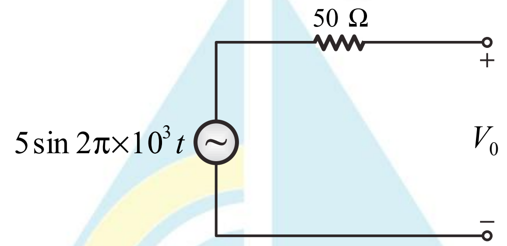
Sol. Given :

(i) $V_s = 10$ volts peak-to-peak

(ii) $R_s = 50\,\Omega$

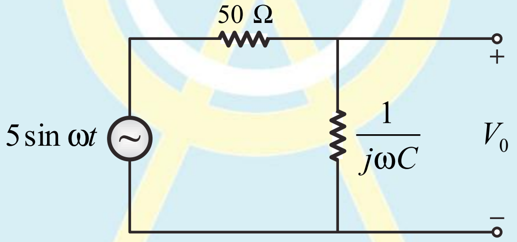
(iii) $f = 1000$ Hz

Case 1 : Under open circuited conduction



$\therefore \quad V_0 = 5\sin(2\pi\times10^3 t)$

Case 2 : When the capacitor is connected across the terminals, voltage across terminals reduces to 8 volts peak to peak.



Applying voltage division rule, voltage across capacitor is given by,

$$V_C = \frac{\frac{1}{j\omega C}}{50 + \frac{1}{j\omega C}} \times V_{in} = \frac{V_{in}}{1 + j\omega C\times 50}$$

$$4 = \frac{5}{1 + j\omega C\times 50}$$

$$0.8 = \frac{1}{\sqrt{1 + R^2\omega^2 C^2}}$$

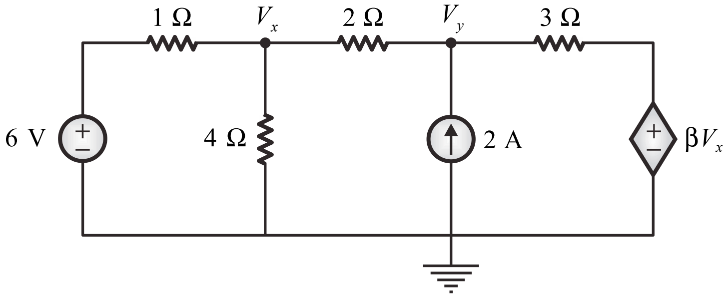
$$0.8 = \frac{1}{\sqrt{1 + R^2\times(2\pi\times10^3)^2 C^2}}$$

$C = 2.38\ \mu\text{F}$

Hence, the correct answer is 2.38.

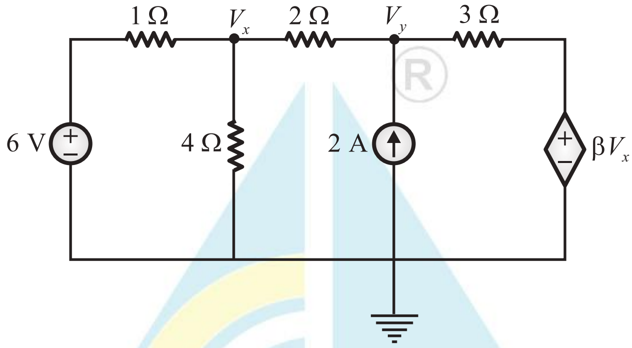
Question 50 **Network Theory (1M)**

In the given circuit, for voltage V_y to be zero, the value of β should be ________.(Round off to 2 decimal places)



Ans. **– 3.25**

Sol. Given circuit is as shown below,



Applying nodal analysis at node V_x,

$$\frac{V_x}{4}+\frac{V_x-6}{1}+\frac{V_x-0}{2}=0 \quad \text{(From the given condition, } V_y = 0\text{)}$$

$$\frac{3V_x}{4}+V_x=6$$

$$7V_x=24$$

$$V_x=\frac{24}{7}$$

Applying nodal analysis at node V_y,

$$2=\frac{0-V_x}{2}+\frac{0-\beta V_x}{3}$$

$$12=-3V_x-2\beta V_x$$

$$12=-(3+2\beta)V_x$$

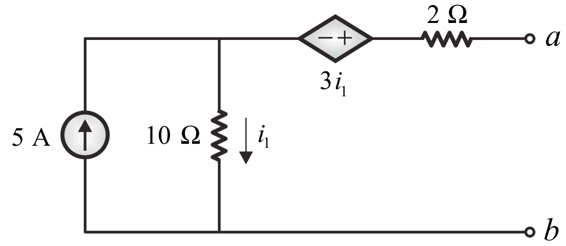
$$12=-(3+2\beta)\frac{24}{7}$$

$$\beta=-3.25$$

Hence, the correct answer is – 3.25.

Question 51 **Network Theory (1M)**

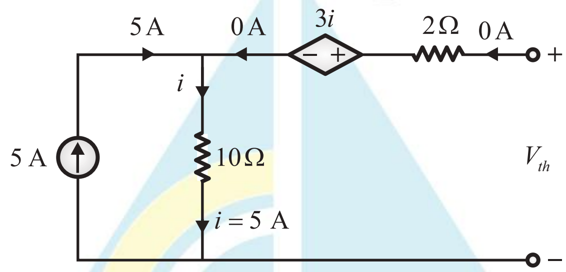
For the network shown, the equivalent Thevenin voltage and Thevenin impedance as seen across terminals '*ab*' is



(A) 10 V in series with 12 Ω
(B) 65 V in series with 15 Ω
(C) 50 V in series with 2 Ω
(D) 35 V in series with 2 Ω

Ans. **B**

Sol. **Calculation of V_{th} :**



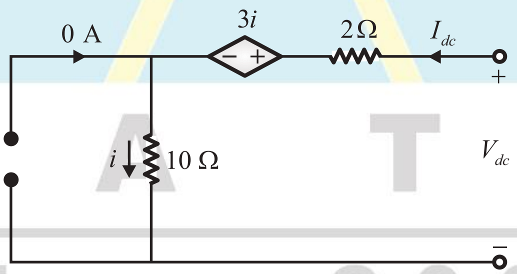
From the circuit diagram,

$$i = 5 \text{ A}$$

$$V_{th} = 3i + 10i = 13i$$

$$V_{th} = 65 \text{ V}$$

Calculation of R_{th} :



From the circuit diagram,

$$i = I_{dc}$$

$$V_{dc} = 2I_{dc} + 3i + 10i$$

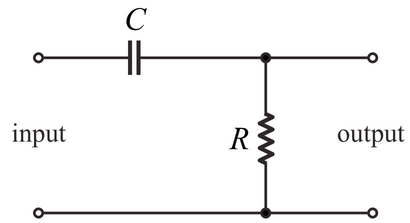
$$V_{dc} = 2I_{dc} + 3I_{dc} + 10I_{dc}$$

$$\frac{V_{dc}}{I_{dc}} = 15 \ \Omega$$

Hence, the correct option is (B).

Question 52 **Analog Electronics (2M)**

A 100 Hz square wave, switching between 0 V and 5 V, is applied to a CR high-pass filter circuit as shown. The output voltage waveform across the resistor is 6.2 V peak-to-peak. If the resistance R is 820 Ω, then the value C is ______ μF. (Round off to 2 decimal places.)



Ans. **12.75**

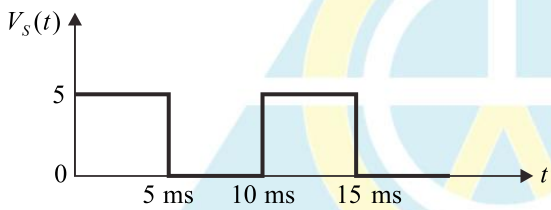
Sol. **Given :**

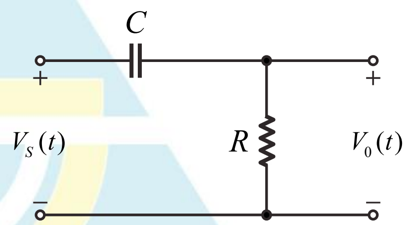
(i) Square waveform of 0 to 5 V amplitude.

(ii) Frequency, $f = 100$ Hz

(iii) Resistance, $R = 820\ \Omega$

(iv) Peak-to-peak voltage across resistance, $R = 6.2$ V





From 0 to 5 msec, capacitor charge exponentially

$$\therefore \quad V_C(\infty) = 5\text{ V}$$

$$V_C(0) = 0\text{ V} \qquad \text{(As capacitor is initially discharged)}$$

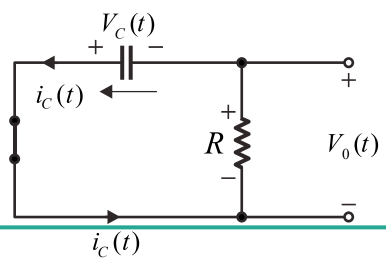
Charging equation of first order RC circuit is given by,

$$\therefore \quad V_C(t) = V_C(\infty) + \left[V_C(0^+) - V_C(\infty)\right]e^{-t/\tau}$$

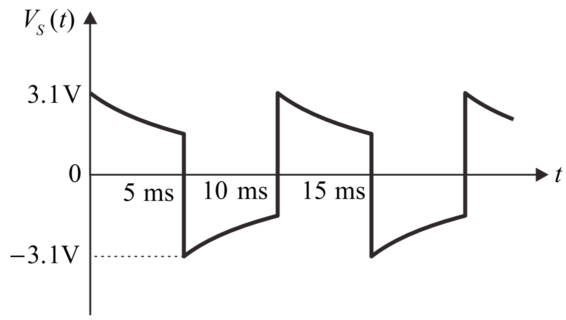
$$V_C(t) = 5 + (0-5)e^{-t/\tau}$$

$$V_C(t) = 5(1 - e^{-t/\tau})$$

From 5 msec to 10 msec, capacitor discharges through resistance.



As the voltage across resistance varies between 3.1 V to –3.1 V (6.2 V peak to peak)



Voltage across resistance is given by,

$$V_R(t) = V_S - V_C(\tau) = 5 - \left[5(1-e^{-t/\tau})\right] = 5e^{-t/\tau}$$

$$V_R(t) = -3.1 \text{ at } t = 5 \text{ msec}$$

$$-3.1 = 5e^{\frac{-5\times10^{-3}}{820\times C}}$$

$$C = 12.75\ \mu\text{F}$$

Hence, the value of C is **12.75 μF**.

Question 53 **Power System (2M)**

Suppose I_A, I_B and I_C are a set of unbalanced current phasors in a three-phase system. The phase-B zero-sequence current $I_{B0} = 0.1\angle 0^0$ p.u. If phase-A current $I_A = 1.1\angle 0^0$ p.u and phase-C current $I_C = (1\angle 120^0 + 0.1)$ p.u., then I_B in p.u is

(A) $1\angle 240^0 - 0.1\angle 0^0$

(B) $1.1\angle 240^0 - 0.1\angle 0^0$

(C) $1.1\angle -120^0 + 0.1\angle 0^0$

(D) $1\angle -120^0 + 0.1\angle 0^0$

Ans. **D**

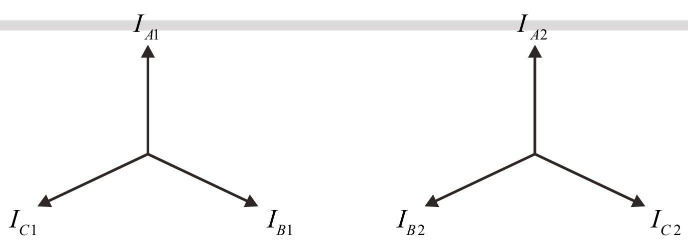
Sol. **Given :** $I_{B0} = 0.1\angle 0^0$

$$I_A = 1.1\angle 0^0$$

$$I_C = (1\angle 120^0 + 0.1) \text{ pu}$$

$$I_B = ?$$

Breaking line currents into sequence currents



$$I_B = I_{B0} + I_{B1} + I_{B2}$$

$$I_B = 0.1\angle 0^0 + I_{B1} + I_{B2}$$

$$I_A = I_{A0} + I_{A1} + I_{A2}$$

$$1.1\angle 0^0 = 0.1\angle 0^0 + I_{B1}\angle 120^0 + I_{B2}\angle 240^0 \qquad \ldots \text{(i)}$$

$$I_C = I_{C0} + I_{C1} + I_{C2}$$

$$1\angle 120^0 + 0.1 = 0.1\angle 0^0 + I_{B1}\angle 240^0 + I_{B2}\angle 120^0$$

$$1\angle 120^0 = I_{B1}\angle 240^0 + I_{B2}\angle 120^0 \qquad \ldots \text{(ii)}$$

From equation (i),

$$1\angle 0^0 = I_{B1}\angle 120^0 + I_{B2}\angle 240^0 \qquad \ldots \text{(iii)}$$

Adding equation (ii) and (iii),

$$1\angle 0^0 + 1\angle 120^0 = I_{B1}(\angle 120^0 + \angle 240^0) + I_{B2}(\angle 120^0 + \angle 240^0)$$

$$(I_{B1} + I_{B2})\angle 180^0 = 1\angle 0^0 + 1\angle 120^0$$

$$I_{B1} + I_{B2} = 1\angle -180^0 + 1\angle -60^0$$

So,

$$I_B = I_{B0} + I_{B1} + I_{B2}$$

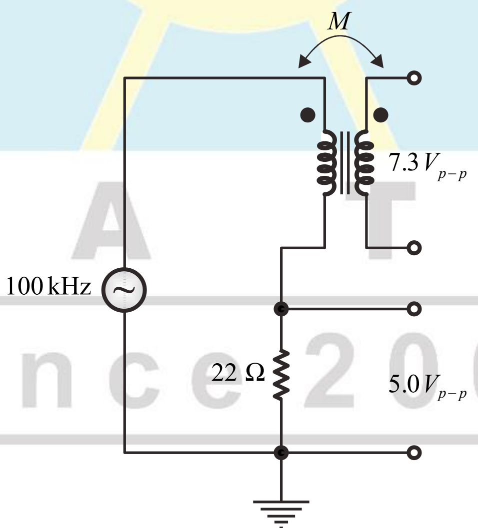
$$I_B = 0.1\angle 0^0 + 1\angle -180^0 + 1\angle -60^0$$

$$I_B = 0.1\angle 0^0 + 1\angle -120^0$$

Hence, the correct option is (D).

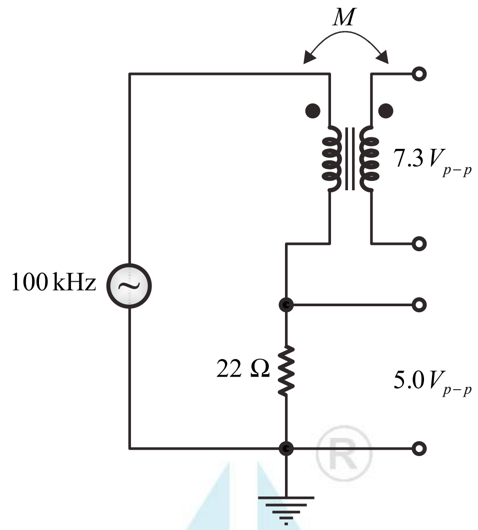
Question 54 **Electrical Machine (2M)**

An air-core radio-frequency transformer as shown has a primary winding and a secondary winding. The mutual inductance M between the windings of the transformer is ________ μH. (Round off to 2 decimal places)



Ans. **51.12**

Sol. Given circuit diagram is as shown below,



Voltage across secondary winding of air core transformer is given by,

$$V_2 = \omega M I_1$$

$$V_2 = (2\pi f) M I_1$$

From above circuit diagram, current of primary of air core transformer I_1 is given by,

$$I_1 = \frac{5}{22} \text{ A}$$

Mutual inductance, $M = \dfrac{V_2}{2\pi f \times I_1}$

$$M = \frac{7.3}{2\pi \times 10^5 \times \frac{5}{22}} = 51.12 \ \mu\text{H}$$

Hence, the correct answer is 51.12.

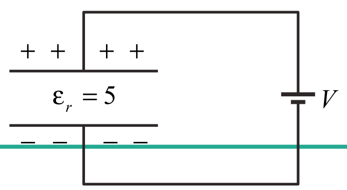
Question 55 **Electromagnetic Theory (2M)**

Consider a large parallel plate capacitor. The gap d between the two plates is filled entirely with a dielectric slab of relative permittivity 5. The plates are initially charged to a potential difference of V volts and then disconnected from the source. If the dielectric slab is pulled out completely, then the ratio of the new electric field E_2 in the gap to the original electric field E_1 is ________.

Ans. **5**

Sol. Large parallel plate capacitor indicates, fringing effect can be neglected and had E-field is same as infinite surface charge.

Initial case :



$$E_{old} = \frac{V}{l} = \frac{\rho}{\varepsilon_0 \times 5}$$

$$\rho_{s_{old}} = \frac{V}{l} \times 5\varepsilon_0 = E_{old} \times 5\varepsilon_0$$

After disconnection charge will be conserved.

$$E_{new} = \frac{\rho_{s_{old}}}{\varepsilon_0} = \frac{\frac{V}{l} \times 5\varepsilon_0}{\varepsilon_0}$$

$$E_{new} = 5E_{old}$$

$$\frac{E_2}{E_1} = 5$$

Hence, the correct answer is 5.

General Aptitude

Question 56 **General Aptitude (1M)**

For a regular polygon having 10 sides, the interior angle between the sides of the polygon, in degrees, is

(A) 396 (B) 324 (C) 216 (D) 144

Ans. **D**

Sol. **Given :** Sides of polygon, $n = 10$

Since, interior angle between the sides of the polygon

$$= \frac{(n-2)}{n} \times 180^0$$

$$= \frac{(10-2)}{10} \times 180^0 = 144^0$$

Hence, the correct option is (D).

Question 57 **General Aptitude (2M)**

Seven Cars P, Q, R, S, T, U and V are parked in a row not necessarily in that order. The cars T and U should be parked next to each other. The cars S and V also should be parked next to each other, whereas P and Q can't be parked next to each other. Q and S must be parked next to each other. R is parked to the immediate right of V. T is parked to the left of U.

Based on the above statements, the only **incorrect** option given below is

(A) There are two cars parked in between Q and V.

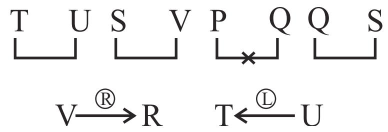
(B) Q and R are not parked together.

(C) V is the only car parked in between S and R.

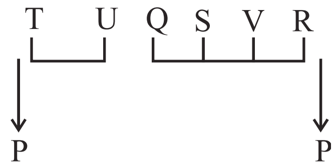
(D) Car P is parked at extreme end.

Ans. **A**

Sol. Step 1. Make proper drafting of given info.



Step 2. Combine drafting :



Only one car is in between Q and V.

Hence, the correct option is (A).

Question 58 **General Aptitude (1M)**

The people __________ were at the demonstration were from all sections of society.

(A) whose (B) which (C) who (D) whom

Ans. **C**

Sol. Since **who** refers to the subject.

Hence, the correct option is (C).

Question 59 **General Aptitude (1M)**

Oasis is to sand as island is to __________.

Which one of the following options maintains a similar logical relation in the above sentence?

(A) Stone (B) Land (C) Water (D) Mountain

Ans. **C**

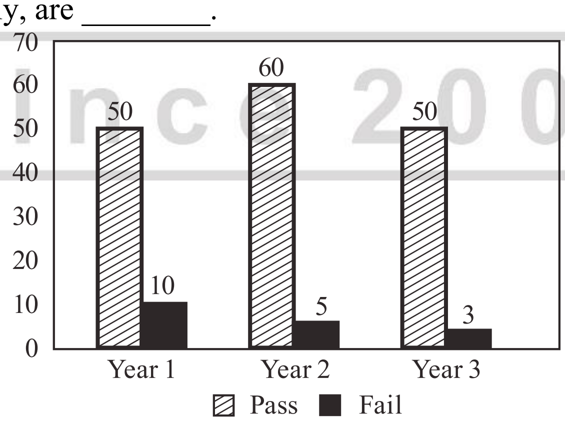
Sol. Oasis is a fertile land surrounded by sand.

Island is a fertile land surrounded by water.

Hence, the correct option is (C).

Question 60 **General Aptitude (2M)**

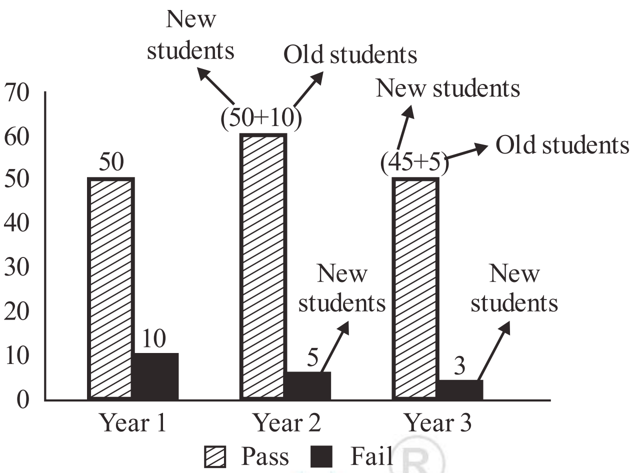
The number of student passing or failing in an exam for a particular subject are presented in the bar chart below. Students who pass the exam cannot appear for the exam again. Students who fails the exam in the first attempt must appear for the exam in the following year. Students always pass the exam in their second attempt. The numbers of students who took the exam for the first time in the year 2nd the year 3 respectively, are _________.



(A) 65 and 53 (B) 60 and 50 (C) 55 and 53 (D) 55 and 48

Ans. **D**

Sol.



$\therefore$ The number of students who took exam for 1st time in year 2 $\Rightarrow$ $50+5=55$

The number of students who took exam for 1st time in year 3 $\Rightarrow$ $45+3=48$

Hence, the correct option is (D).

Question 61 **General Aptitude (1M)**

Which one of the following numbers is exactly divisible by $(11^{13}+1)$?

(A) $11^{26}+1$ (B) $11^{33}+1$ (C) $11^{39}-1$ (D) $11^{52}-1$

Ans. **D**

Sol. $\because \dfrac{(x^n-a^n)}{(x+a)}$ for n is even

$x=(11)^{13}$

$a=(1)^{13}$

$$\therefore \frac{x^n-a^n}{(x+a)}=\frac{\{(11)^{13}\}^n-\{(1)^{13}\}^n}{(11)^{13}+(1)^{13}}$$

$$=\frac{\{(11)^{13}\}^4-\{(1)^{13}\}^4}{(11)^{13}+(1)^{13}} \quad \text{for } n=4 \text{ (even)}$$

$$=\frac{\left[\{(11)^{13}\}^2-\{(1)^{13}\}^2\right]\left[\{(11)^{13}\}^2+\{(1)^{13}\}^2\right]}{(11)^{13}+(1)^{13}}$$

$\because a^2-b^2=(a-b)(a+b)$

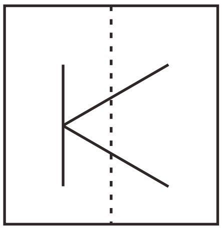
$$\frac{x^n-a^n}{(x+a)}=\frac{\left[(11^{13}-1^{13})(11^{13}+1^{13})\right]\left[(11^{13})^2+(1^{13})^2\right]}{11^{13}+1^{13}}$$

$\therefore (11^{52}-1)$ is divisible by $(11^{13}+1)$.

Hence, the correct option is (D).

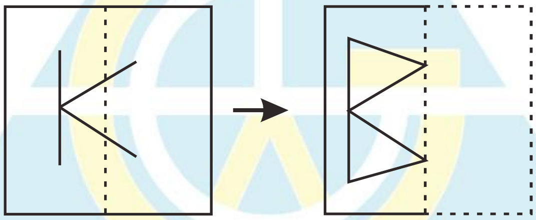
Question 62 **General Aptitude (1M)**

A transparent square sheet shown below is folded along the dotted line. The folded sheet will look like.



Ans. C

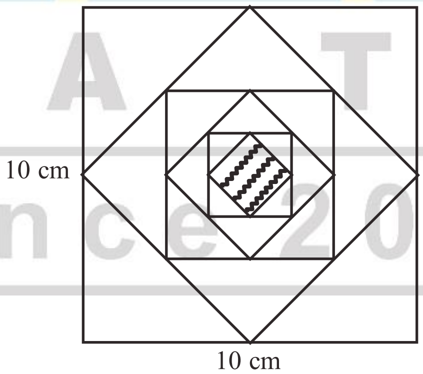
Sol.



Hence, the correct option is (C).

Question 63 **General Aptitude (2M)**

In the figure shown above, each inside square is formed by joining the mid points of the sides of the next larger square. The area of the smallest square (shaded) as shown, in cm^2 is

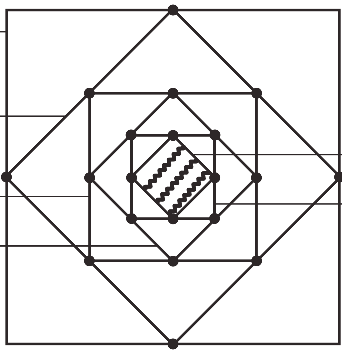


(A) 12.50 (B) 6.25 (C) 3.125 (D)1.5625

Ans. C

Sol. From figure, side of outer square = diagonal of inside square

$\therefore$ Side of square $\Rightarrow$



$\because$ Side of 6th square (smallest square) $= \dfrac{10}{4\sqrt{2}}$

$\therefore$ Area $= \dfrac{100}{32} = 3.125\ \text{cm}^2$

Hence, the correct option is (C).

Question 64 **General Aptitude (2M)**

Let X is a continuous random variable denoting the temperature measured. The range of temperature is [0, 100] degree Celsius and let the probability density function of X be $f(x) = 0.01$ for $0 \le X \le 100$. The mean of X is ___________.

(A) 2.5 (B) 5.0 (C) 25.0 (D) 50.0

Ans. **D**

Sol. **Given :** PDF $f(x) = 0.01; \quad 0 \le x \le 100$

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$\because$ It is uniform probability distribution

$$\therefore \quad E(x) = \frac{0+100}{2} = 50$$

Hence, the correct option is (D).

Question 65 **General Aptitude (2M)**

The importance of sleep is often overlooked by students when they are preparing for exams. Research has consistently shown that sleep deprivation greatly reduces the ability to recall the material learnt. Hence, cutting down on sleep to study longer hours can be counterproductive.

Which one of the following statements is the **CORRECT** inference from the above passage?

(A) Sleeping well alone is enough to prepare for an exam. Studying has lesser benefit.

(B) Students are efficient and are not wrong in thinking that sleep is a waste of time.

(C) If a student is extremely well prepared for an exam, he needs little or no sleep.

(D) To do well in an exam, adequate sleep must be part of the preparation.

Ans. **D**