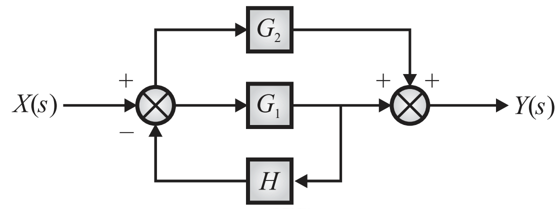
Technical Section

Question 1 **Control System (1M)**

The block diagram of a feedback control system is shown in the figure,



The transfer function $\frac{Y(s)}{X(s)}$ of the system is

(A) $\frac{G_1+G_2+G_1G_2H}{1+G_1H}$

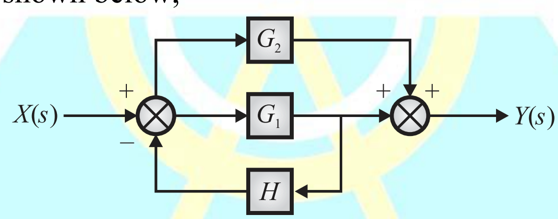
(B) $\frac{G_1+G_2}{1+G_1H+G_2H}$

(C) $\frac{G_1+G_2}{1+G_1H}$

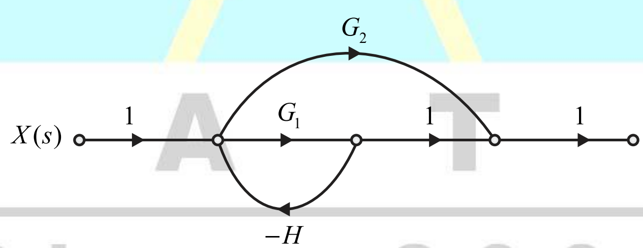
(D) $\frac{G_1+G_2+G_1G_2H}{1+G_1H+G_2H}$

Ans. C

Sol. Given block diagram is as shown below,



The signal flow graph is shown below,



Number of forward paths and its associate path factor is,

$$P_1 = G_1 \quad \Delta_1 = 1$$

$$P_2 = G_2 \quad \Delta_2 = 1$$

Number of individual loops,

$$L_1 = -G_1H$$

Determinant of SFG is,

$$\Delta = 1-(L_1) = 1-(-G_1H) = 1+G_1H$$

From the Mason's gain formula the transfer function of this system can be given as

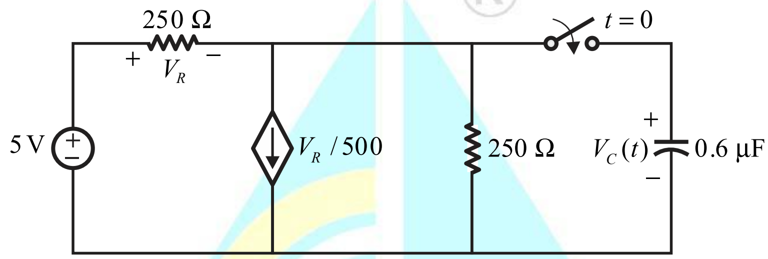
$$\frac{Y(s)}{X(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\therefore \quad \frac{Y(s)}{X(s)} = \frac{G_1 + G_2}{1 + G_1 H}$$

Hence, the correct option is (C).

Question 2 **Network Theory (2M)**

In the circuit shown in figure, the switch is closed at time $t = 0$, while the capacitor is initially charged to – 5V [i.e. $V_C(0) = -5\text{ V}$]



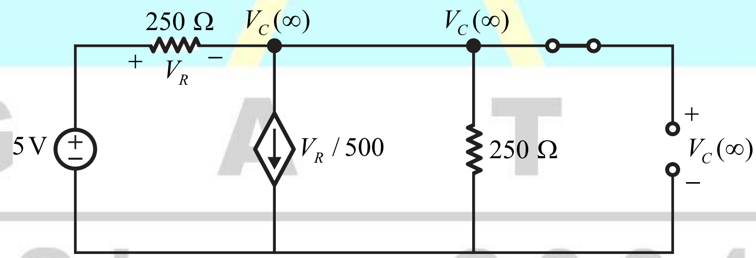
The time after which the voltage across the capacitor becomes zero is (Round off to 3 decimal places) _______ ms.

Ans. **0.1386**

Sol. Given, initial value of voltage across capacitor

$$V_C(0^-) = -5\text{ V}$$

When switch is closed and circuit is in steady state (i.e. $t = \infty$, and capacitor becomes open circuit) So circuit becomes as,



(Circuit at steady state i.e. $t = \infty$)

Apply KCL at node $V_C(\infty)$ is,

$$\frac{V_C(\infty)}{250} + \frac{V_C(\infty) - 5}{250} + \frac{V_R}{500} = 0 \qquad (\because V_R = V_C(\infty))$$

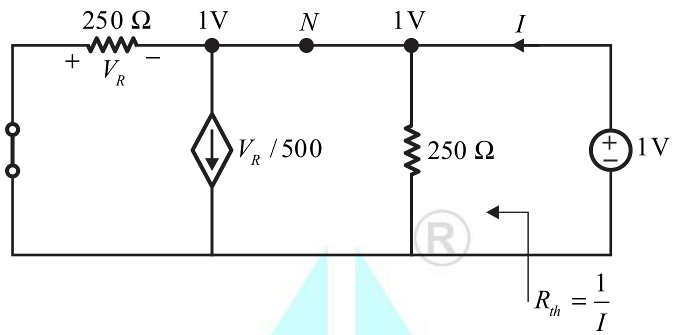
$$\frac{V_C(\infty)}{250} + \frac{V_C(\infty) - 5}{250} + \frac{5 - V_C(\infty)}{500} = 0$$

$$2V_C(\infty) + 2V_C(\infty) - 10 + 5 - V_C(\infty) = 0$$

$$3V_C(\infty) = 5$$

$$V_C(\infty)=\frac{5}{3}\text{ V} \quad \rightarrow \text{Steady state voltage of capacitor}$$

Now, for calculating time constant (τ), first we calculate R_{th} as shown below,



Apply KCL at node N,

$$\frac{1}{250}+\frac{1-0}{250}+\frac{V_R}{500}=I$$

$$I=\frac{1}{250}+\frac{1}{250}+\frac{-1}{500}$$

$$I=\frac{3}{500}$$

$\therefore$ $$R_{th}=\frac{1}{I}=\frac{500}{3}\ \Omega$$

Thus, time constant, $\tau=R_{th}\times C$

$$\tau=\frac{500}{3}\times 0.6\times 10^{-6}$$

$$\tau=500\times 0.2\times 10^{-6}=0.1\text{ msec}$$

Apply transient equation for voltage across capacitor is,

$$V_C(t)=V_C(\infty)+[V_C(0)-V_C(\infty)]e^{-t/\tau},\ t\geq 0$$

$$V_C(t)=\frac{5}{3}+\left[-5-\frac{5}{3}\right]e^{-t/\tau}$$

$$V_C(t)=\frac{5}{3}-\frac{20}{3}e^{-t/\tau} \qquad \text{...(i)}$$

If $V_C(t)=0$, then equations (i) becomes as,

$$0=\frac{5}{3}-\frac{20}{3}e^{-t/\tau}$$

$$\frac{5}{3}=\frac{20}{3}e^{-t/\tau}$$

$$\frac{1}{4}=e^{-t/\tau}$$

$$e^{-t/\tau}=0.25$$

$$\frac{-t}{\tau}\ln(e)=\ln(0.25)$$

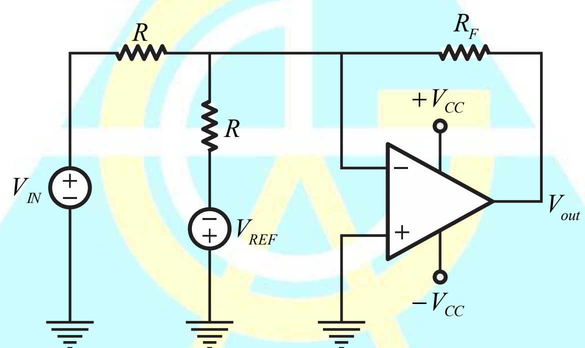
$$t=-\tau\ln(0.25)$$

$$t=-0.1\ln(0.25)=0.1386\text{ msec}$$

Hence, the correct answer of t is 0.1386 msec.

Question 3 **Analog Electronics (1M)**

Consider the circuit with an ideal Op-Amp shown in figure,

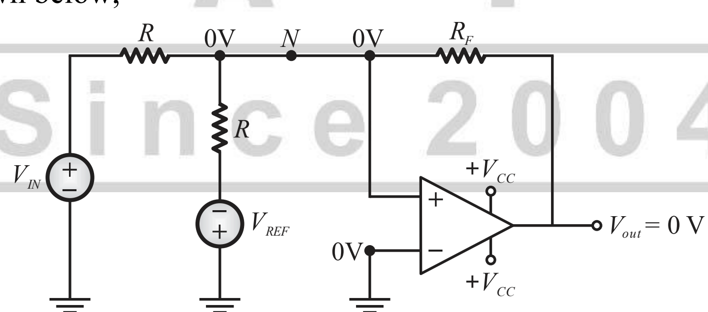


Assuming $|V_{IN}| << |V_{CC}|$ and $|V_{REF}| << |V_{CC}|$. The condition at which V_{out} equal to zero is

(A) $V_{IN}=V_{REF}$ (B) $V_{IN}=0.5V_{REF}$ (C) $V_{IN}=2V_{REF}$ (D) $V_{IN}=2+V_{REF}$

Ans. **A**

Sol. Given circuit is shown below,



According to question, $|V_{IN}| << |V_{CC}|$ and $|V_{REF}| << |V_{CC}|$

Here, Op-Amp is ideal and -ve feedback is present so virtual ground concept is applicable.

So, $V_+=V_-=0$

Thus applying KCL at node N,

$$\frac{0-V_{IN}}{R}+\frac{0+V_{REF}}{R}+\frac{0-V_{out}}{R_F}=0$$

$$\frac{V_{out}}{R_F}=\frac{V_{REF}-V_{IN}}{R}$$

$$V_{out}=\frac{R_F}{R}[V_{REF}-V_{IN}]$$

According to question, $V_{out}=0$

$$0=\frac{R_F}{R}[V_{REF}-V_{IN}]$$

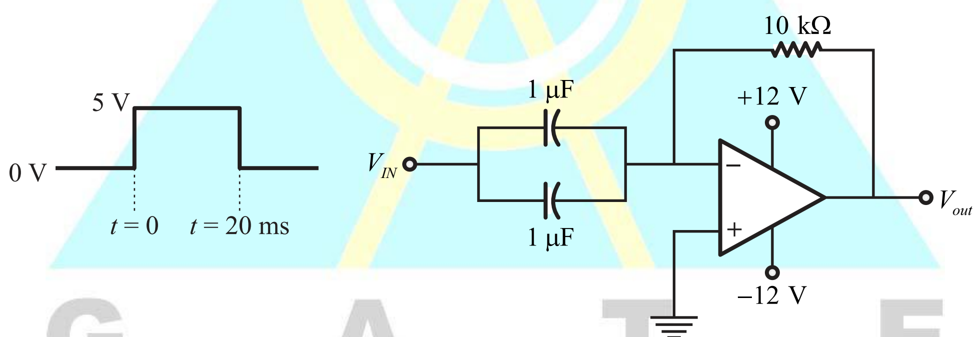
$$V_{REF}-V_{IN}=0$$

$$V_{REF}=V_{IN}$$

Hence, the correct answer is option (A).

Question 4 **Analog Electronics (2M)**

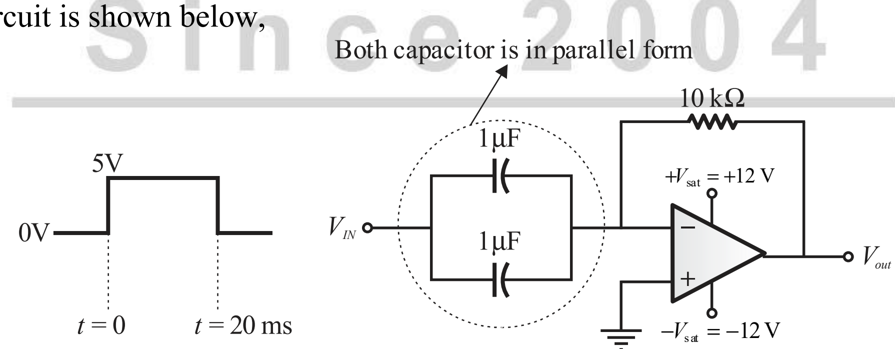
A circuit with an ideal OPAMP is shown in the figure. A pulse V_{IN} of 20 ms duration is applied to the input. The capacitors are initially uncharged.



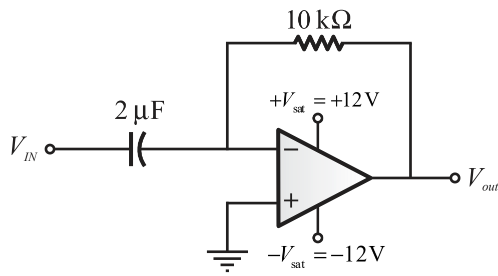
The output voltage V_{out} of this circuit at $t=0^+$ (in integer) is ______ V.

Ans. **–12**

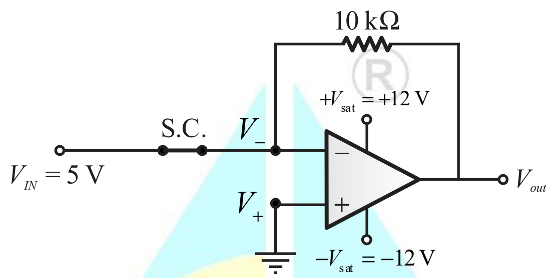
Sol. The given circuit is shown below,



Above figure can be redrawn as



According to transient theory, at $t = 0^+$, capacitor will be short circuit, so above circuit becomes as,



From above circuit it is clear that,

$$V_- = V_{IN} = +5\text{ V}$$

$$V_+ = 0\text{ V}$$

Here, $V_- \neq V_+$

It means, voltage at both terminals of op-amp are unequal and fixed so virtual ground concept is not valid here and op-amp work as a comparator.

Thus, $V_- > V_+$

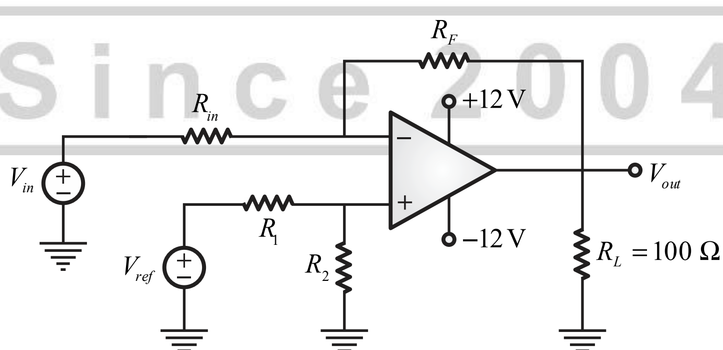
So, $V_{out} = -V_{sat}$

$V_{out} = -12\text{ V}$

Hence, the output voltage V_{out} of this circuit at $t = 0^+$ is -12 V.

Question 5 **Analog Electronics (1M)**

For the circuit with an ideal Op-Amp shown in the figure. V_{REF} is fixed.

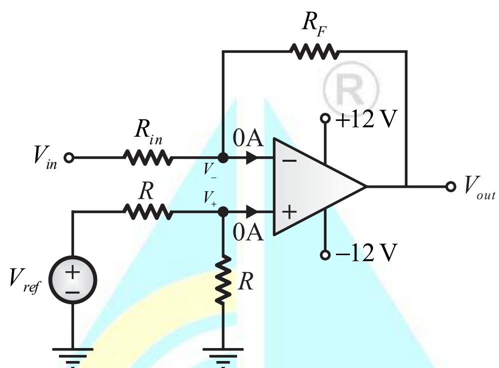


If $V_{out} = 1$ volt, for $V_{in} = 0.1$ volt and $V_{out} = 6$ volt, for $V_{in} = 1$ volt. Where V_{out} is measured across R_L load connected at the output of this Op-Amp, the value of $\frac{R_F}{R_{in}}$ is

(A) 3.285 (B) 2.860 (C) 3.825 (D) 5.555

Ans. – 5.55 (According to IIT Bombay option D is correct)

Sol. The given circuit is shown below,

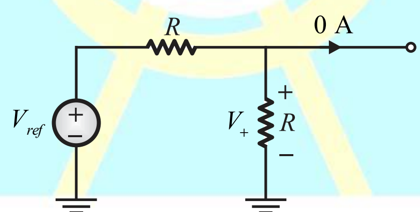


Given :

(i) $V_{out} = 1\text{ V}$ when $V_{in} = 0.1\text{ V}$

(ii) $V_{out} = 6\text{ V}$ when $V_{in} = 1\text{ V}$

By voltage divider rule, voltage at non-inverting terminal can be found as



Thus, $V_+ = \left(\frac{R}{R+R}\right) V_{ref}$

$$V_+ = \frac{V_{ref}}{2}$$

Here virtual ground concept is applicable because –ve feedback is present and op-amp is ideal so due to virtual ground concept,

$$V_- = V_+ = \frac{V_{ref}}{2}$$

Applying KCL at V_-,

$$\frac{V_- - V_{in}}{R_{in}} + \frac{V_- - V_{out}}{R_F} = 0$$

$$\frac{\frac{V_{ref}}{2}-V_{in}}{R_{in}}+\frac{\frac{V_{ref}}{2}-V_{out}}{R_F}=0$$

$$\frac{V_{ref}}{2}\left[\frac{1}{R_{in}}+\frac{1}{R_F}\right]-\frac{V_{in}}{R_{in}}=\frac{V_{out}}{R_F}$$

$$\frac{V_{ref}}{2}\left[\frac{R_{in}+R_F}{R_{in}R_F}\right]-\frac{V_{in}}{R_{in}}=\frac{V_{out}}{R_F}$$

$$\frac{V_{ref}}{2}\left[1+\frac{R_F}{R_{in}}\right]-\left(\frac{R_F}{R_{in}}\right)V_{in}=V_{out} \quad \ldots\text{(i)}$$

Condition 1 : If $V_{out}=1\text{ V}$ when $V_{in}=0.1\text{ V}$ then equation (i) becomes as

$$\frac{V_{ref}}{2}\left[1+\frac{R_F}{R_{in}}\right]-(0.1)\left(\frac{R_F}{R_{in}}\right)=1 \quad \ldots\text{(ii)}$$

Condition 2 : If $V_{out}=6\text{ V}$ when $V_{in}=1\text{ V}$ then equation (ii) becomes as

$$\frac{V_{ref}}{2}\left[1+\frac{R_F}{R_{in}}\right]-(1)\left(\frac{R_F}{R_{in}}\right)=6 \quad \ldots\text{(iii)}$$

Assume $\frac{V_{ref}}{2}\left[1+\frac{R_F}{R_{in}}\right]=x$, then subtracting equation (iii) from (ii) then,

$$\left(x-0.1\frac{R_F}{R_{in}}\right)-\left(x-1\frac{R_F}{R_{in}}\right)=1-6$$

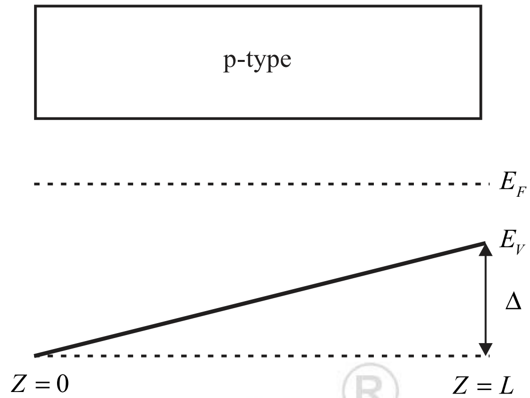
$$0.9\frac{R_F}{R_{in}}=-5$$

$$\frac{R_F}{R_{in}}=-\frac{5}{0.9}=-5.55$$

Note : As per the given data of question we get negative answer so none of the option is matched.

Question 6 **Electronic Devices (1M)**

The energy band diagram of a p-type semiconductor bar of length L under equilibrium condition (i.e. the Fermi energy levels E_F is constant) is shown in the figure. The valance band E_V is sloped since doping is non-uniform along the bar. The difference between the energy levels of the valance band at the two edges of the bar is Δ.



If the charge of an electron is q then the magnitude of the electric field developed inside this semiconductor bar is

(A) $\frac{\Delta}{qL}$ (B) $\frac{2\Delta}{qL}$ (C) $\frac{\Delta}{2qL}$ (D) $\frac{3\Delta}{2qL}$

Ans. A

Sol. From questions it is clear that, change in energy level / difference in energy level of E_C and E_V is Δ.

$$E=-\frac{dV}{dx}$$

$\because$ $$E_C=-qV$$

$$E=+\frac{1}{q}\frac{dE_C}{dx} \quad \ldots \text{(i)}$$

$dE_C \rightarrow$ Change in energy level $=\Delta$

$dx \rightarrow$ Length of bar $=L$

Hence, equation (i) becomes as

$$E=\frac{+\Delta}{qL}$$

Hence, the correct option is (A).

Question 7 **Electronic Devices (1M)**

A bar of silicon is doped with boron concentration of 10^{16} cm^{-3} and assumed to be fully ionized. It is exposed to light such that electron-hole pairs are generated throughout the volume of the bar at the rate of 10^{20} cm^{-3} s^{-1}. If the recombination lifetime is 100 µs and intrinsic carrier concentration of silicon is 10^{10}

cm^{-3} and assuming 100% ionization of boron, then the approximate product of steady-state electron and hole concentration due to this light exposure is

(A) $10^{20}\,cm^{-6}$ (B) $2\times10^{20}\,cm^{-6}$ (C) $10^{32}\,cm^{-6}$ (D) $2\times10^{32}\,cm^{-6}$

Ans. D

Sol. **Given :**

Boron concentration $N_A = 10^{16}\,cm^{-3}$

So, $N_A = p_{p_0} = 10^{16}\,cm^{-3}$

Generation Rate $G_p = 10^{20}\,cm^{-3}s^{-1}$

Recombination life time $\tau_p = 100\ \mu sec$

Intrinsic carrier concentration $n_i = 10^{10}\ cm^{-3}$

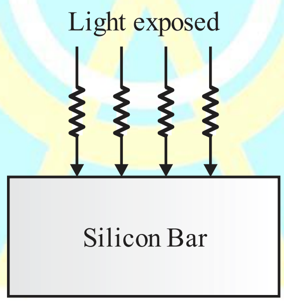
Using mass action law under thermal equilibrium

$$n_{p_0} = \frac{n_i^2}{p_{p_0}} = \frac{n_i^2}{N_A} = \frac{(10^{10})^2}{10^{16}}$$

$$n_{p_0} = 10^4\,cm^{-3} \rightarrow \text{under no light exposed.}$$

So minority carrier concentration n_{p_0} when light is not exposed is $10^4\ cm^{-3}$

When light exposed,



So after exposing light, excess carrier concentration is

$$\Delta n = \Delta p$$

So, generation rate G_p is

$$G_p = \frac{\Delta p}{\tau_p}$$

$$\Delta p = G_p \times \tau_p = 10^{20} \times 100\,\mu sec$$

$$\Delta p = 10^{16}\,cm^{-3}$$

$$\Delta n = 10^{16}\,cm^{-3}$$

So after exposing light, under equilibrium,

1. Electrons concentration :

$$n_{p_0} + \Delta n = (10^4 + 10^{16})\,cm^{-3} \cong 10^{16}\,cm^{-3}$$

2. Hole concentration :

$$p_{p_0} + \Delta_p = (10^{16} + 10^{16})\,\text{cm}^{-3} = 2\times 10^{16}\,\text{cm}^{-3}$$

Thus, product of hole concentration and electron concentration after exposing the light is

$$(n_{p_0} + \Delta_n)(p_{p_0} + \Delta_p) = 10^{16} \times 2\times 10^{16} = 2\times 10^{32}\,\text{cm}^{-6}$$

Hence, the correct option is (D).

Question 8 **Electronic Devices (2M)**

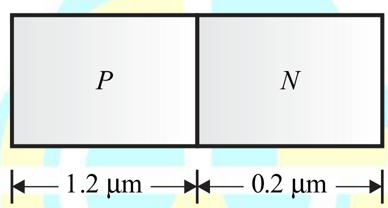
A silicon P-N junction is shown in the figure. The doping in the P region is $5\times 10^{16}\,\text{cm}^{-3}$ and doping in the N region is $10\times 10^{16}\,\text{cm}^{-3}$. The parameters given are

Built in voltage $(\phi_{bi}) = 0.8$ V

Electron charge $(q) = 1.6\times 10^{-19}$ C

Vacuum permittivity $(\varepsilon_0) = 8.85\times 10^{-12}$ F/m

Relative permittivity of silicon $(\varepsilon_{si}) = 12$



The magnitude of reverse bias voltage that would completely deplete one of the two regions (P or N) prior to the other (rounded off to one decimal place) is________ V.

Ans. **107.67**

Sol. **Given:**

Doping in P-region $N_A = 5\times 10^{16}\,\text{cm}^{-3}$

Doping in N-region $N_D = 10^{17}\,\text{cm}^{-3}$

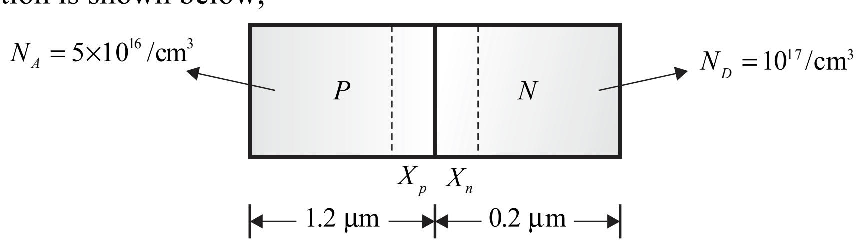
Built in voltage $(\phi_{bi}) = 0.8$ V

Electron charge $(q) = 1.6\times 10^{-19}$ C

Vacuum permittivity $(\varepsilon_0) = 8.85\times 10^{-12}$ F/m

Relative permittivity of silicon $(\varepsilon_{si}) = 12$

Given p-n junction is shown below,



So width of depletion region, $W = X_n + X_p$

From charge equality concept,

$$X_n.N_D = X_p.N_A$$

$$X_n.N_D = (W - X_n)N_A$$

$$X_n(N_D + N_A) = W.N_A$$

$$X_n = \frac{WN_A}{N_A + N_D} = \frac{W \times 5\times10^{16}}{(5\times10^{16}) + (10\times10^{16})}$$

$$X_n = \frac{W\times(5\times10^{16})}{15\times10^{16}} = \frac{W}{3}$$

So, $X_p = W - X_n = W - \dfrac{W}{3}$

$$X_p = \frac{2}{3}\text{ W}$$

Suppose when N-is depleted completely, during reverse bias so,

$$X_n = 0.2\ \mu\text{m}$$

$\therefore$ $$W = 3X_n = 0.6\ \mu\text{m}$$

$$X_p = \frac{2}{3}\times W = \frac{1.2}{3} = 0.4\ \mu\text{m}$$

i.e. when N is depleted completely (i.e. 0.2 μm) then and only then *P* side is depleted only by 0.4 μm only. Thus

$$\left.\begin{aligned} X_n &= 0.2\ \mu\text{m} \\ W &= 0.6\ \mu\text{m} \end{aligned}\right\} \rightarrow (N\text{ region depleted completely})$$

So, total depletion width (W) under reverse bias is,

$$W = \sqrt{\frac{2\varepsilon(V_0+V_R)}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}$$

$$W^2 = \frac{2\varepsilon(V_0+V_R)}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)$$

$$(0.6\times10^{-6}\times100)^2 = \frac{2\times12\times8.85\times10^{-14}}{1.6\times10^{-19}}\left(\frac{1}{5\times10^{16}}+\frac{1}{10\times10^{16}}\right)(V_0+V_R)$$

$$(0.6\times10^{-4})^2 = 3.9825\times10^{-10}\ (V_0+V_R)$$

$$V_0 + V_R = 9.0395$$

$$V_R = 9.0395 - V_0$$

$$V_R = 9.0395 - 0.8$$

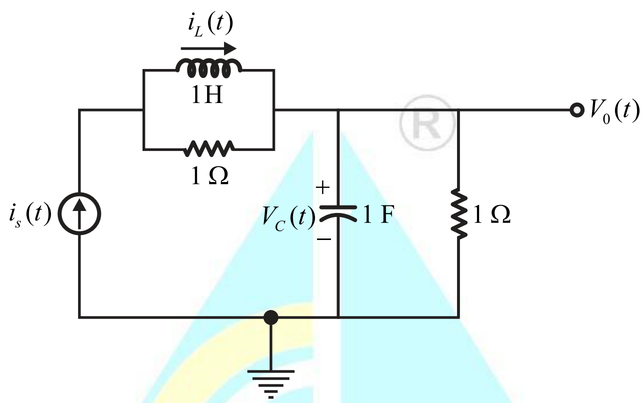
$$V_R = 8.2395\text{ V} \approx 8.24\text{ V}$$

Thus, the magnitude of reverse bias voltage that would completely deplete N region is 8.24 V.

Hence, the correct answer for V_R is 8.24 V.

Question 9 **Control Systems (2M)**

The electrical system shown in figure, converts input source current $i_s(t)$ to output voltage $V_0(t)$.



Current $i_L(t)$ in the inductor and voltage $V_C(t)$ across the capacitor are taken as the state variables, both assumed to be initially equal to zero i.e. $i_L(0) = 0$ and $V_C(0) = 0$. The system is

(A) Completely state controllable as well as completely observable.

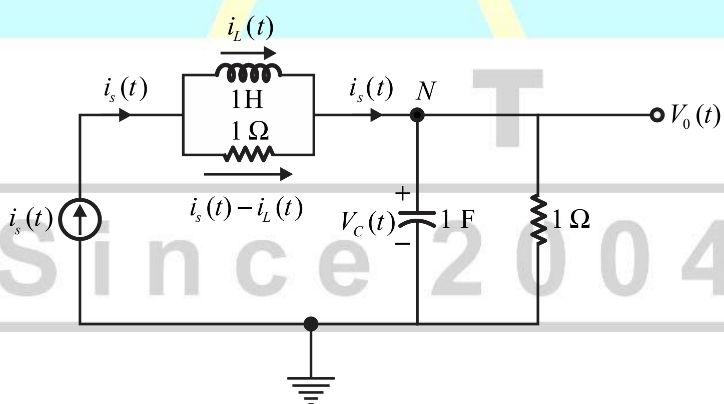
(B) Completely state controllable but not observable.

(C) Completely state observable but not state controllable.

(D) Neither state controllable nor observable.

Ans. **D**

Sol. Given circuit is shown below,



Assume, $i_L(t) = x_1 \rightarrow$ First state variable

$V_C(t) = x_2 \rightarrow$ Second state variable

$i_s(t) = u \rightarrow$ Input of system

and $V_0(t) = y \rightarrow$ Output of system

Apply KCL at node N,

$$i_s(t)=\frac{d}{dt}V_C(t)+\frac{V_C(t)}{1}$$

$$u=\frac{d}{dt}x_2+x_2$$

$$u=\dot{x}_2+x_2$$

$$\dot{x}_2=-x_2+u \qquad \ldots(\text{i})$$

KVL in the loop between inductor and resistor,

$$i_s(t)-i_L(t)=\frac{d}{dt}i_L(t)$$

$$u-x_1=\frac{d}{dt}x_1$$

$$u-x_1=\dot{x}_1$$

$$\dot{x}_1=-x_1+u \qquad \ldots(\text{ii})$$

Output, $V_0(t)=V_C(t)$

$$y=x_2 \qquad \ldots(\text{iii})$$

From equation (i), (ii) and (iii), state variable modal can be written as,

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 1\end{bmatrix}u$$

$$y=[0 \quad 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Thus matrices A, B, and C are,

$$A=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix},\ B=\begin{bmatrix}1\\ 1\end{bmatrix} \text{ and } C=[0 \quad 1]$$

$$AB=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}=\begin{bmatrix}-1\\ -1\end{bmatrix}$$

Condition of controllability,

$\therefore$ $$[Q_C]=[B \quad AB]$$

$$[Q_C]=\begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}$$

$\therefore$ $$|Q_C|=-1+1=0$$

Thus, system is uncontrollable.

$$A^T C^T = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Condition of observability,

$$[Q_0] = \begin{bmatrix} C^T & A^T C^T \end{bmatrix}$$

$$[Q_0] = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}$$

$$|Q_0| = 0$$

Thus, system is un-observable.

So, the system is neither controllable nor observable.

Hence, the correct option is (D).

Question 10 **Electromagnetic Theory (2M)**

An antenna with a directive gain of 6 dB is radiating a total power of 16 kW. The amplitude of electric field in free space at a distance of 8 km from the antenna in the direction of 6 dB gain is (Round off to 3 decimal places) ______ V/m.

Ans. 0.2443

Sol. **Given :** Directive gain $G_d(\theta, \phi) = 6$ dB

Radiation power $P_{rad} = 16\,\text{kW}$

Distance $r = 8\,\text{km}$

Directive gain $G_d(dB) = 6 = 10\log G_d$

$$G_d = 10^{0.6} = 3.981$$

So maximum electric field amplitude in free space at distance r from antenna is,

$$|E_s|_{\max} = \eta\left[\frac{G_d\,P_{rad}}{2\pi r^2}\right] = \frac{120\pi\times 3.981\times 16\times 10^3}{2\pi(8\times 10^3)^2} = 0.2443\,\text{V/m}$$

Hence, the correct answer for $|E_s|_{\max}$ is $0.2443\,\text{V/m}$.

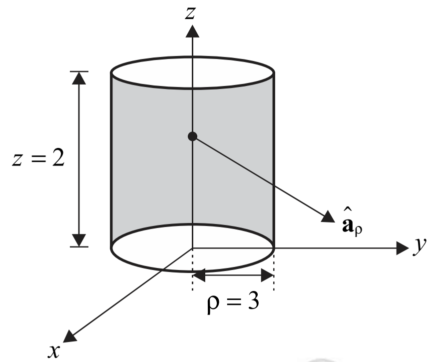
Question 11 **Electromagnetic Theory (2M)**

For a vector field $\vec{\mathbf{D}} = \rho\cos^2\phi\,\hat{\mathbf{a}}_\rho + z^2\sin^2\phi\,\hat{\mathbf{a}}_\phi$ in a cylindrical coordinate system (ρ, ϕ, z) with unit vector $\hat{\boldsymbol{a}}_\rho, \hat{\boldsymbol{a}}_\phi$ and $\hat{\boldsymbol{a}}_z$, the net flux of $\vec{\mathbf{D}}$ leaving the closed surface of the cylinder $(\rho = 3, 0 \le z \le 2)$ (Round off to 2 decimal places) is ______.

Ans. 56.548

Sol. **Given :** $\vec{\mathbf{D}} = \rho\cos^2\phi\,\hat{\mathbf{a}}_\rho + z^2\sin^2\phi\,\hat{\mathbf{a}}_\phi$

Cylinder $(\rho = 3, 0 \le z \le 2)$ is shown below,



From this given cylinder,

$$\rho = 3\,\text{m},\ z = 0\ \text{to}\ 2\,\text{m},\ \phi = 0\ to\ 2\pi$$

Shaded portion shows closed surface of cylinder and unit vector that is perpendicular this surface is $\hat{\boldsymbol{a}}_\rho$ thus, net flux (ψ) leaving the closed surface of cylinder is

$$\psi = \iint \vec{\mathbf{D}}.\vec{ds} = \int_{\phi=0}^{2\pi}\int_{z=0}^{2} (\rho\cos^2\phi\ \hat{\mathbf{a}}_\rho + z^2\sin^2\phi\ \hat{\mathbf{a}}_\phi)\cdot(\rho d\phi dz\ \hat{\mathbf{a}}_\rho)$$

$$\psi = \int_{\phi=0}^{2\pi}\int_{z=0}^{2} \rho^2\cos^2\phi\, d\phi\, dz\Big|_{\rho=3} \qquad (\because\ \hat{a}_\rho.\hat{a}_\rho = 1 \text{ and } \hat{a}_\rho.\hat{a}_\phi = 0)$$

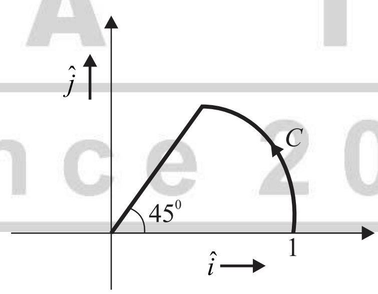
$$\psi = 3^2 \times (2-0) \times \frac{2\pi}{2} = 18\pi\ \text{C}$$

$$\psi = 18\pi = 56.548\ \text{C}$$

Hence, total flux ψ living closed surface of the given cylinder as 56.548 C.

Question 12 **Electromagnetic Theory (1M)**

The vector function $\mathbf{F}(\mathbf{r}) = -x\hat{i} + y\hat{j}$ is defined over a circular are C shown in the figure,



The line integral of $\int_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$ is

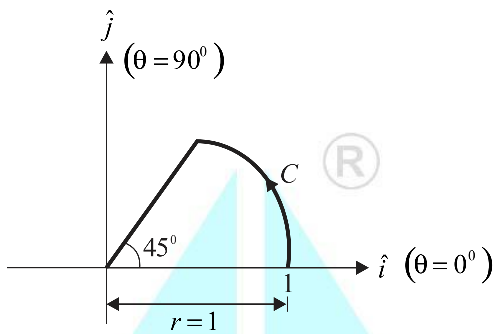
(A) 1/2 (B) 1/4 (C) 1/6 (D) 1/3

Ans. **A**

Sol. **Given :** $\mathbf{F}(\mathbf{r}) = -x\hat{i} + y\hat{j}$

So, $$\int_C \mathbf{F(r)}\,d\mathbf{r} = \int_C (-x\hat{i} + y\hat{j}).(dx\ \hat{i} + dy\ \hat{j})$$

$$\int_C \mathbf{F(r)}\,d\mathbf{r} = \int_C (-x.dx + y.dy) \quad \ldots \text{(i)} \quad \left(\because\ \hat{i}.\hat{i} = \hat{j}.\hat{j} = 1\right)$$



Assume, $x = r\cos\theta,\ y = r\sin\theta$

$\because \quad r = 1$

$x = \cos\theta$ and $y = \sin\theta$

Here, θ, Variation from 0 to 45^0.

Put $x = \cos\theta,\ y = \sin\theta$

$dx = -\sin\theta\, d\theta$

$dy = \cos\theta\, d\theta$

Thus equation (i) becomes as

$$\int_C \mathbf{F(r)}\,d\mathbf{r} = \int_{\theta=0}^{\pi/4} -\cos\theta(-\sin\theta\, d\theta) + \sin\theta(\cos\theta d\theta)$$

$$\int_C \mathbf{F(r)}\,d\mathbf{r} = \int_{\theta=0}^{\pi/4} (\cos\theta\sin\theta + \sin\theta\cos\theta)\, d\theta$$

$$\int_C \mathbf{F(r)}\,d\mathbf{r} = \int_{\theta=0}^{\pi/4} \sin 2\theta\, d\theta = \left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/4}$$

$$\int_C \mathbf{F(r)}\,d\mathbf{r} = \frac{-1}{2}\left[\cos\frac{\pi}{2} - \cos 0\right] = \frac{-1}{2}(-1)$$

$$\int_C \mathbf{F(r)}\,d\mathbf{r} = \frac{1}{2}$$

Hence, the correct option is (A).

Question 13 **Electromagnetic Theory (1M)**

Consider the vector field $\overline{\mathbf{F}} = \hat{\mathbf{a}}_x(4y - c_1 z) + \hat{\mathbf{a}}_y(4x + 2z) + \hat{\mathbf{a}}_z(2y + z)$ in a rectangular coordinate system (x, y, z) with unit vectors $\hat{\mathbf{a}}_x, \hat{\mathbf{a}}_y, \hat{\mathbf{a}}_z$. If the field **F** is irrotational (conservative), then the constant c_1 (in integer) is ______.

Ans. **0**

Sol. **Given :** $\overline{\mathbf{F}} = \hat{\mathbf{a}}_x(4y - c_1 z) + \hat{\mathbf{a}}_y(4x + 2z) + \hat{\mathbf{a}}_z(2y + z)$

Here, $F_x = 4y - c_1 z,\ F_y = 4x - 2z,\ F_z = 2y + z,$

For a vector field to be irrotational its curl must be zero,

i.e. $\nabla \times \overline{F} = 0$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = 0$$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 4y - c_1 z & 4x + 2z & 2y + z \end{vmatrix} = 0$$

$$\hat{a}_x[2-2] - \hat{a}_y[0 + c_1] + \hat{a}_z[4-4] = 0$$

$$0\hat{a}_x - c_1\hat{a}_y + 0\hat{a}_z = 0$$

$$-c_1\hat{a}_y = 0$$

It shows $\hat{a}_y \neq 0$ because it is unit vector.

Thus, $c_1 = 0$ then the above equation will be satisfied.

Hence, the correct answer is zero.

Question 14 **Electromagnetic Theory (1M)**

The refractive indices of the core and cladding of an optical fiber are 1.50 and 1.48 respectively. The critical propagation angle, which is defined as the maximum angle that the light beam makes with the axis of the optical fiber to achieve the total internal reflection (round off to two decimal places) is ______ degree.

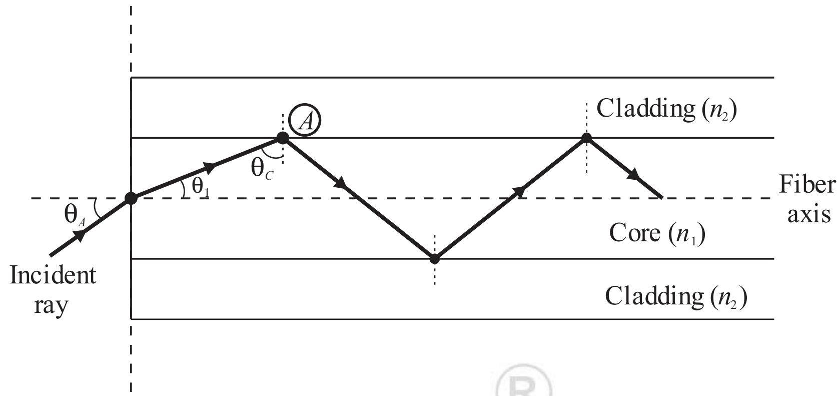
Ans. **9.32**

Sol. **Given :**

Refractive index of core $n_1 = 1.50$

Refractive index of cladding $n_2 = 1.48$

According to the definition of critical propagation angle given in statement of question, θ_1, which is made with optical fiber axis and leads total internal reflection (TIR) in optical fiber cable, so we can calculate θ_1 as,



Here, θ_A = Acceptance angle

θ_C = Critical angle

θ_1 = Critical propagation angle

$$\cos\theta_C = \sqrt{1-\left[\frac{n_2}{n_1}\right]^2}$$

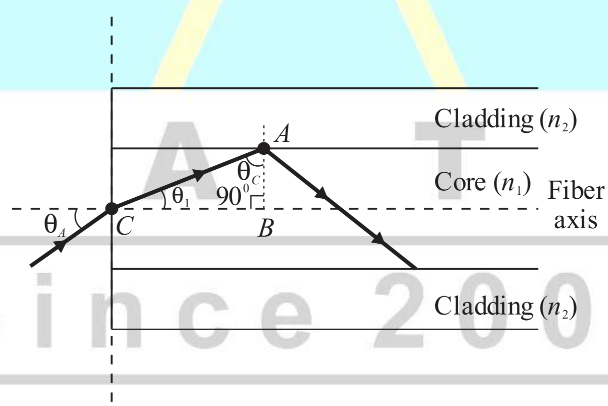
Where, n_1 = refractive index of core = 1.5

n_2 = refractive index of cladding = 1.48

$$\cos\theta_C = \sqrt{1-\left[\frac{1.48}{1.50}\right]^2}$$

$$\theta_C = \cos^{-1}[0.1627]$$

$$\theta_C = 80.636^0$$



So, from triangle ABC,

$$\theta_1 = 90^0 - \theta_C = 90^0 - (80.636^0)$$

$$\theta_1 = 9.36^0$$ **Ans.**

Hence, the value of critical propagation angle that leads total internal reflection in optical fiber is 9.36^0

Question 15 **Electromagnetic Theory (1M)**

Consider a rectangular coordinate system (x, y, z) with unit vector $\mathbf{a}_x, \mathbf{a}_y$ and $\mathbf{a}_z$. A plane wave travelling in the region $z \geq 0$ with electric field vector $\mathbf{E} = 10\cos(2\times10^8 t + \beta z)\,\hat{\mathbf{a}}_y$ is incident normally on the plane at $z = 0$, where β is the phase constant. The region $z \geq 0$ is in free space and the region $z < 0$ is filled with a lossless medium (permittivity $\varepsilon = \varepsilon_0$ permeability $\mu = 4\mu_0$, where $\varepsilon_0 = 8.85\times10^{-12}$ F/m and $\mu_0 = 4\pi\times10^{-7}$ H/m). The value of reflection coefficient is

(A) $\frac{1}{3}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $\frac{2}{3}$

Ans. **A**

Sol. **Given : Medium 1** $(z \geq 0)$

(i) Medium 1 is free space.

(ii) $\mu_1 = \mu_0,\ \varepsilon_1 = \varepsilon_0$

(iii) $\mathbf{E}_1 = 10\cos(2\times10^8 t + \beta z)\,\hat{\mathbf{a}}_y$

(iv) $\omega = 2\times10^8$ rad/sec, $(E_1)_m = 10$ V/m

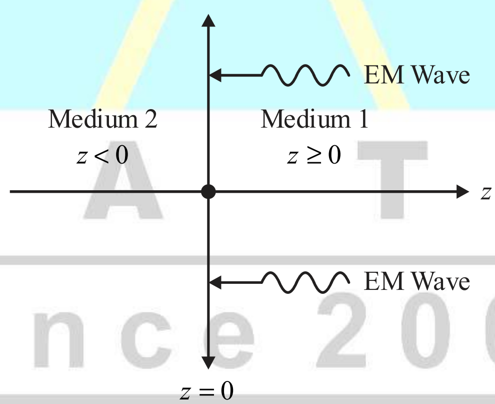
(v) $\eta_1 = \sqrt{\dfrac{\mu_1}{\varepsilon_1}}$

Medium 2 $(z < 0)$

(i) Medium 2 is lossless

(ii) $\mu_2 = 4\mu_0,\ \varepsilon_2 = \varepsilon_0$

(iii) $\eta_2 = \sqrt{\dfrac{\mu_2}{\varepsilon_2}}$



Reflection coefficient $(\Gamma) = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

$$\Gamma = \frac{\sqrt{\dfrac{\mu_2}{\varepsilon_2}} - \sqrt{\dfrac{\mu_1}{\varepsilon_1}}}{\sqrt{\dfrac{\mu_2}{\varepsilon_2}} + \sqrt{\dfrac{\mu_1}{\varepsilon_1}}}$$

Since, $\varepsilon_2 = \varepsilon_1$

$$\Gamma = \frac{\sqrt{\mu_2} - \sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}}$$

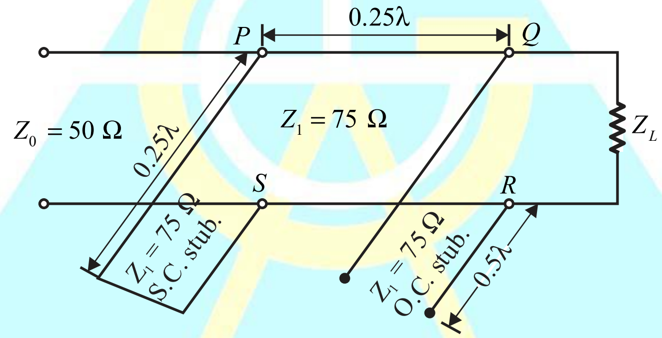
$$\Gamma = \frac{\sqrt{4\mu_0} - \sqrt{\mu_0}}{\sqrt{4\mu_0} + \sqrt{\mu_0}}$$

$$\Gamma = \frac{\sqrt{4} - \sqrt{1}}{\sqrt{4} + \sqrt{1}} = \frac{1}{3}$$

Hence, the correct option is (A).

Question 16 **Electromagnetic Theory (1M)**

The impedance matching network shown in the figure is to match a lossless line having characteristic impedance $Z_0 = 50\ \Omega$ with a load impedance Z_L. A quarter-wave line having a characteristic impedance $Z_1 = 75\ \Omega$ is connected to Z_L. Two stubs having characteristic impedance of $75\ \Omega$ each are connected to this quarter-wave line. One is a short-circuited (S.C.) stub of length 0.25λ connected across PS and the other one is an open-circuited (O.C.) stub of length 0.5λ connected across QR.



The impedance matching is achieved when the real part of Z_L is

(A) 112.5 Ω (B) 75.0 Ω (B) 50.0 Ω (D) 33.3 Ω

Ans. **A**

Sol. **Given :**

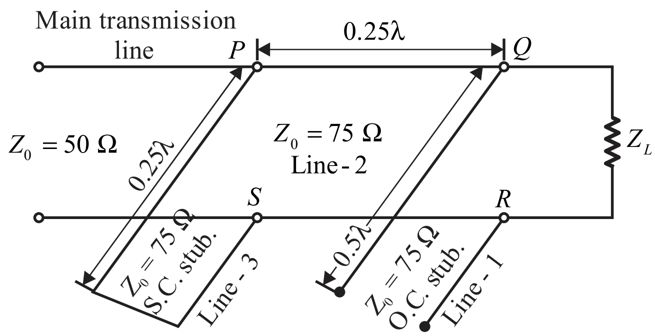
Lossless line characteristic impedance $Z_0 = 50\Omega$

Open circuit stub characteristic impedance $(Z_1)_{oc} = 75\Omega$

Short circuit stub characteristic impedance $(Z_1)_{sc} = 75\Omega$

Method 1 :

The given arrangement of transmission line is shown below,



Here, $(Z_L)_{Line-1} = (Z_L)_{oc} = \infty$

$(Z_L)_{Line-3} = (Z_L)_{sc} = 0$

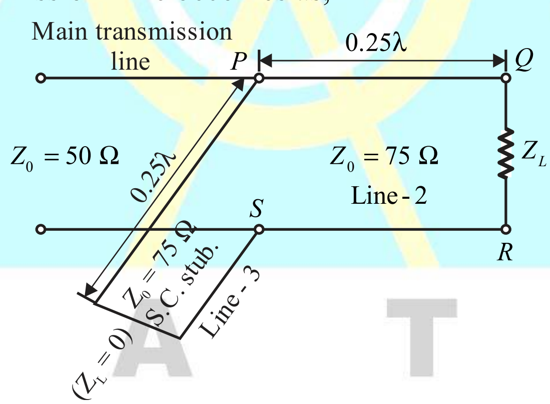
Input Impedance of $\frac{\lambda}{2}$ long line-1,

$$Z_{in}\left[l = \frac{\lambda}{2}\right]_{\text{Line-1}} = (Z_L)_{\text{Line-1}} = \infty \rightarrow \text{open circuit}$$

Thus total impedance at terminal QR is,

$$Z_{QR} = Z_L \,||\, (Z_{in})_{\text{Line-1}} = Z_L \,||\, \infty = Z_L$$

Now arrangement of Transmission Line becomes as,



Thus Input Impedance of line 2 is,

$$Z_{in}\left[l = \frac{\lambda}{4}\right]_{\text{Line-2}} = \frac{Z_0^2}{Z_L} = \frac{75^2}{Z_L}$$

Input Impedance of Line 3 is,

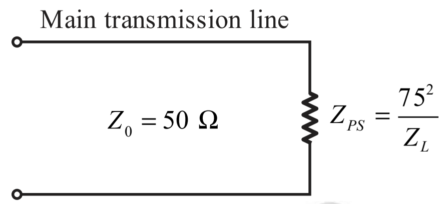
$$Z_{in}\left[l = \frac{\lambda}{4}\right]_{\text{Line-3}} = \frac{Z_0^2}{(Z_L)_{Line-3}} = \frac{75^2}{0} = \infty$$

Thus total impedance at terminal PS is,

$$Z_{PS} = (Z_{in})_{Line-2} \,||\, (Z_{in})_{Line-3}$$

$$Z_{PS}=\frac{75^2}{Z_L}\parallel\infty=\left(\frac{75^2}{Z_L}\right)$$

Now Transmission Line arrangement becomes as-



Hence Z_{PS} work as load for main Transmission line.

For matching of main Transmission line with load (Z_{PS}),

$$Z_{PS}=Z_o$$

$$\frac{75^2}{Z_L}=50 \Rightarrow Z_L=\frac{75^2}{50}=112.5\Omega$$

Hence, the correct option is (A)

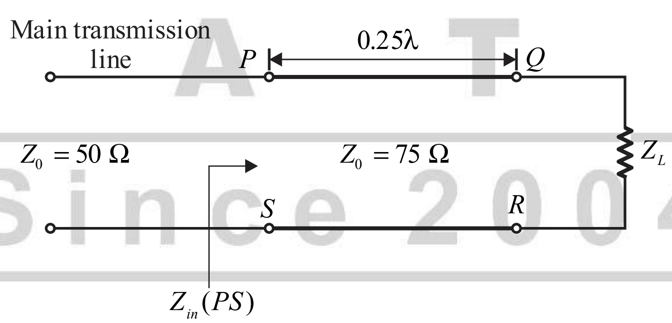
Method 2 :

From given arrangement, it is clear that,

$$Z_{in}\left[l=\frac{\lambda}{2}\right]_{Line-1}=(Z_L)_{Line-1}=\infty$$

$$Z_{in}\left[l=\frac{\lambda}{4}\right]_{Line-3}=\frac{Z_0^2}{(Z_L)_{Line-3}}=\frac{Z_0^2}{0}=\infty$$

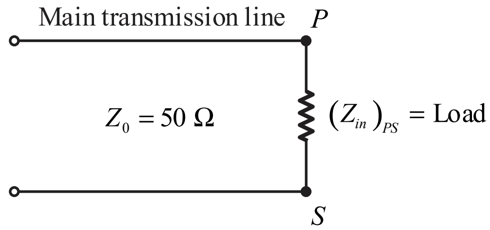
So, input impedance of both line-1 and line-3 are ∞ (i.e. open circuit) so it does not make any effect on main transmission line, so given transmission line configuration becomes as,



Thus Input Impedance at terminal PS is,

$$(Z_{in})_{PS}=\frac{Z_0{}^2}{Z_L}=\frac{75^2}{Z_L}\Omega$$

So $(Z_{in})_{PS}$ work as load for main transmission line so arrangement of transmission line becomes as,



Hence $(Z_{in})_{PS}$ work as load for main Transmission line so the condition, for matching the main Transmission line with load $(Z_{in})_{PS}$ is

$$(Z_{in})_{PS} = Z_0$$

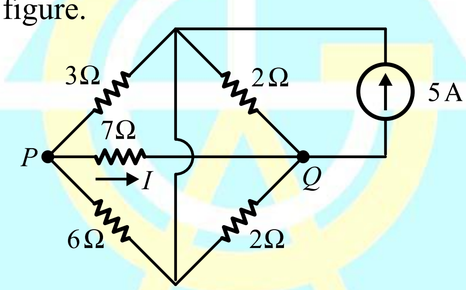
$$\frac{75^2}{Z_L} = 50$$

$$Z_L = \frac{75^2}{50} = 112.5\Omega$$

Hence, the correct option is (A)

Question 17 **Network Theory (1M)**

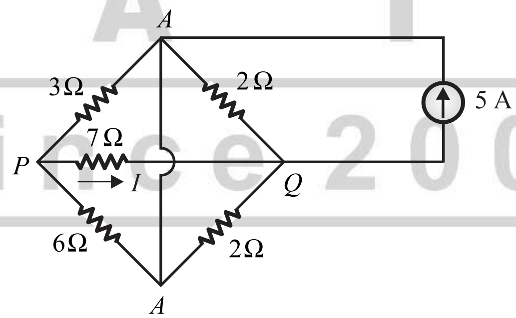
Consider the circuit shown in figure.



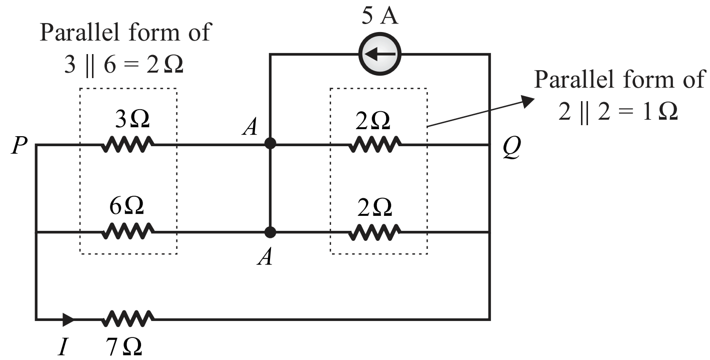
The current I flowing through the 7Ω resistor between P and Q (Round off to 1 decimal places) is ________.

Ans. **0.5**

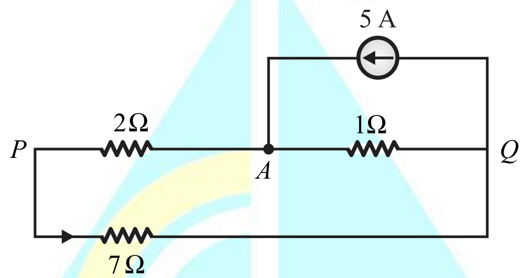
Sol. Given circuit is shown below,



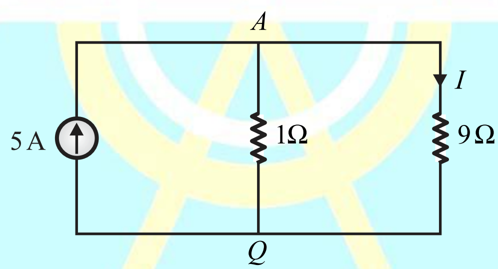
Re-arrange the above circuit as shown below,



Now above circuit reduces as,



Above circuit can be re-arranged as,



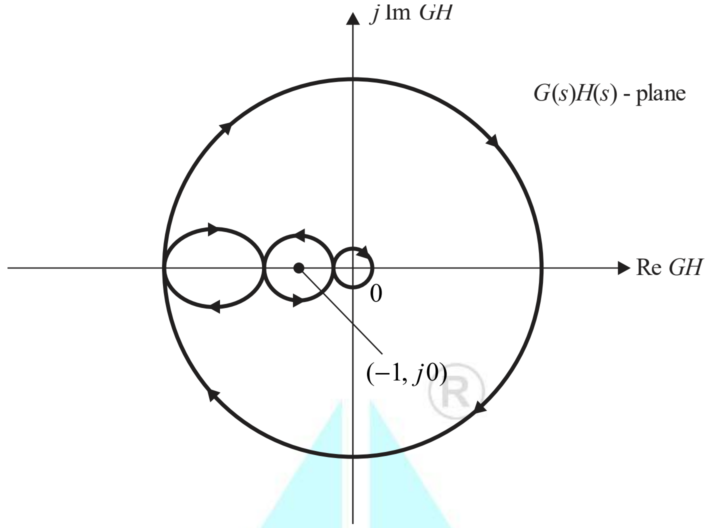
Apply current divider rule

$$I = \frac{5 \times 1}{1+9} = \frac{5}{10} = 0.5\text{ A}$$

Hence, the correct answer is 0.5 A.

Question 18 **Control Systems (1M)**

The complete Nyquist plot of the open loop transfer function $G(s)H(s)$ of a feedback control system shown in figure,

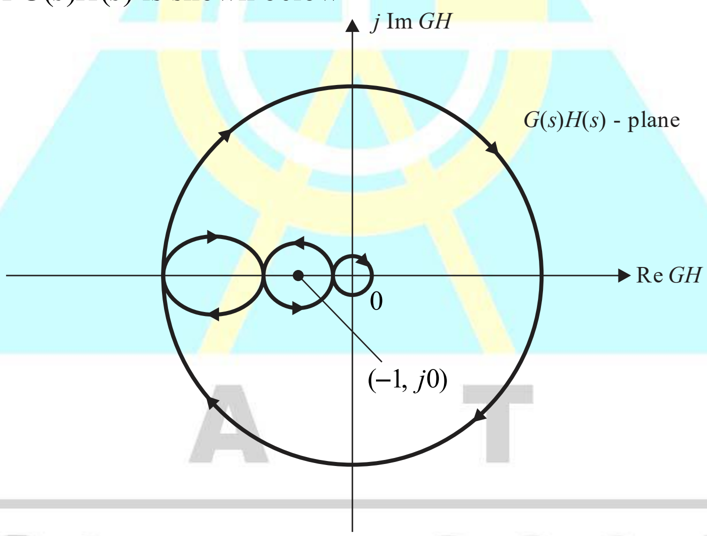


If $G(s)H(s)$ has one zero in right half of the s-plane, the number of poles that, the closed loop system will have in right half of s-plane is

(A) 0 (B) 1 (C) 4 (D) 3

Ans. **D (According to IIT Bombay Answer is D)**

Sol. Given Nyquist plot of $G(s)H(s)$ is shown below



Case 1 :

Here, $G(s)H(s)$ has one zero in the right half of s-plane i.e. $Z = 1$.

According to principle of argument

$N = P - Z$ (Anti clockwise direction) ...(i)

Where,

N = Number of encirclements of Nyquist plot of $G(s)H(s)$ about origin in anti-clockwise direction

P = Number of poles of $G(s)H(s)$ in right half of s-plane.

Z = Number of zeros of $G(s)H(s)$ in right half of s-plane.

Here, $N = -2$ (anti clockwise direction)

$Z = 1$

From equation (i)

$-2 = P - 1$

$P = -1$ ...(ii)

So, poles cannot be negative in numbers so no more further discussion on this.

Case 2 : If we assume Nyquist contour in anti clockwise direction then according to principle of argument

$N = P - Z$ (Clockwise direction) ...(iii)

Where,

N = Number of encirclements of Nyquist plot of $G(s)H(s)$ about origin in clockwise direction

P = Number of poles of $G(s)H(s)$ in right half of s-plane.

Z = Number of zeros of $G(s)H(s)$ in right half of s-plane.

Here, $N = 2$ (Clockwise direction)

$Z = 1$

From equation (iii)

$2 = P - 1$

$P = 3$ ...(iv)

According to Nyquist stability criteria

$N = P - Z$ (Anti clockwise direction) ...(v)

Where, N = Number of encirclements about $(-1+0j)$ in clockwise direction

Z = Number of closed loop poles in right half of s-plane.

P = Number of open loop poles or poles of $G(s)H(s)$ in right half of s-plane.

From given Nyquist plot number of encirclements about $(-1+0j)$ is

$N = 1 - 1 = 0$

Using equation (v)

$0 = P - Z$

$Z = P$

$Z = 3$

So here number of poles in right half of s-plane is 3.

Hence, the correct option is (D).

Question 19 **Digital Electronics (1M)**

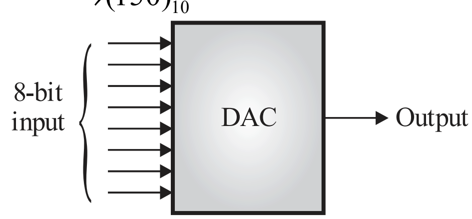
An 8-bit unipolar (all analog output values are positive) digital-to-analog converter (DAC) has a full-scale voltage range from 0 V to 7.68 V. If the digital input code is 10010110 (the leftmost bit is MSB), then the analog output voltage of the DAC (rounded off to one decimal place) is ________V.

Ans. 4.51

Sol. Given : Number of bits, $n = 8$,

Full scale voltage, $V_{FS} = 7.68$

Digital input $= (10010110)_2 \xrightarrow{\text{decimal}} (150)_{10}$



Thus, analog output from 8 bit unipolar DAC is,

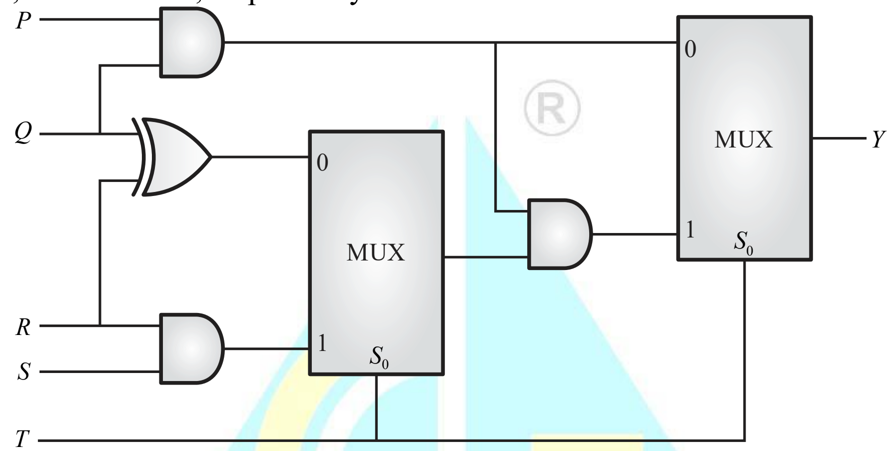
$$V_{out} = (\text{Resolution}) \times (\text{Decimal equivalent of digital input})$$

$$V_{out} = \left(\frac{V_{FS}}{2^n - 1}\right) \times 150 = \left(\frac{7.68}{2^8 - 1}\right) \times 150 = 4.517 \text{ Volt}$$

Hence, the analog output voltage of the 8 bit unipolar DAC is 4.517 Volt

Question 20 **Digital Electronics (2M)**

The propagation delays of the XOR gate, AND gate and multiplexer (MUX) in the circuit shown in the figure are 4 ns, 2 ns and 1 ns, respectively.



If all the inputs P, Q, R, S and T are applied simultaneously and held constant, the maximum propagation delay of the circuit is

(A) 3 ns (B) 5 ns (C) 6 ns (D) 7 ns

Ans. C

Sol. **Given :**

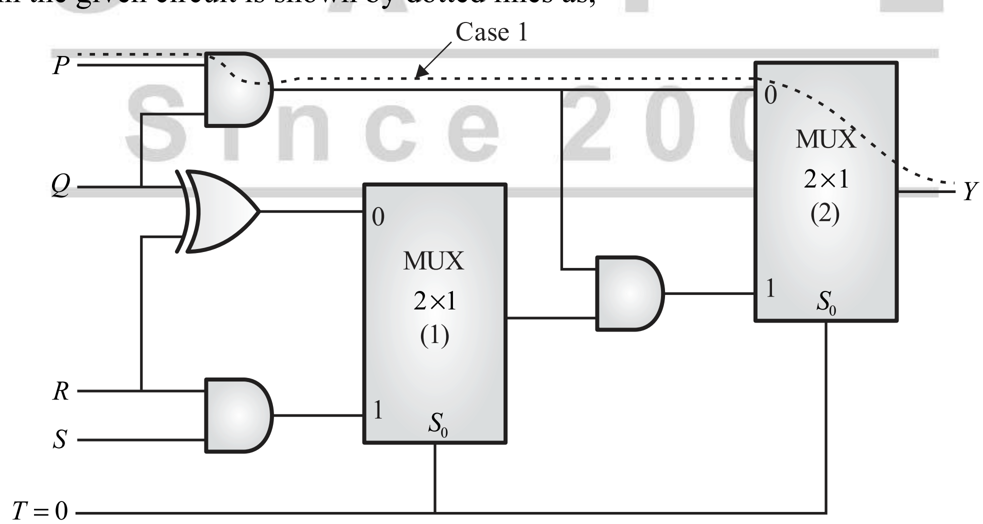
Delay of XOR gate $= 4$ns

Delay of AND gate $= 2$ns

Delay of MUX $= 1$ns

Case 1 :

Assuming $T = 0$ then selection line of MUX $S_0 = 0$, so that MUX input '0' get enable so path followed by signal in the given circuit is shown by dotted lines as,



So total propagation delay τ_1 from input to output is,

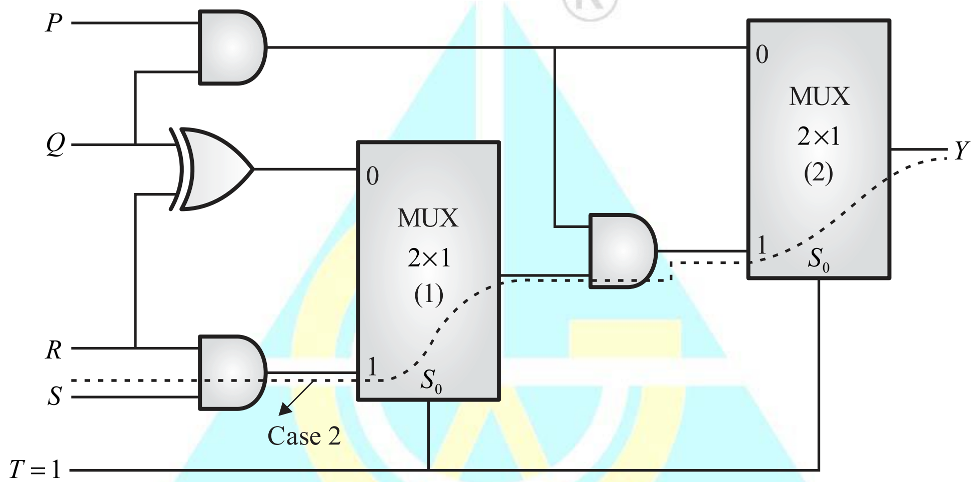
$\tau_1 = (\text{Propagation delay of AND gate}) + (\text{Propagation delay of MUX-2})$

$\tau_1 = 2\,ns + 1ns = 3ns$

Hence MUX input '0' get enable then propagation delay of given circuit $\tau_1 = 3ns$

Case 2 :

Assuming $T = 1$ then selection line of MUX is $S_0 = 1$, so that MUX input '1' get enable so path followed by signal in the given circuit is shown by dotted lines as,



So total propagation delay τ_2 from input to output is,

$\tau_2 = (\text{Propagation delay of AND gate}) + (\text{Propagation delay of MUX-1}) +$
$(\text{Propagation delay of AND gate}) + (\text{Propagation delay of MUX-2})$

$\tau_2 = 2\,ns + 1ns + 2ns + 1ns = 6ns$

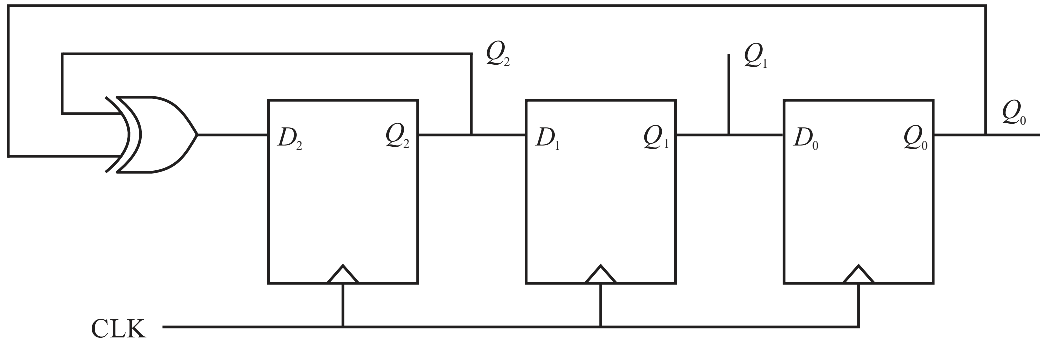
Hence MUX input '1' get enable then propagation delay of given circuit $\tau_2 = 6ns$

Hence maximum delay of circuit is $\text{MAX}(\tau_1, \tau_2) = \text{MAX}(3\text{ns}, 6\text{ns}) = 6\,\text{ns}$

Hence, the correct option is (C).

Question 21 **Digital Electronics (2M)**

The propagation delay of the exclusive-OR (XOR) gate in the circuit is 3 ns. The propagation delay of all the flip-flops is assumed to be zero. The clock (Clk) frequency provided to the circuit is 500 MHz.



Starting from the initial value of the flip-flop outputs Q_2, Q_1, Q_0 = 111 with $D_2 = 1$, the minimum number of triggering clock edges after which the flip-flop outputs Q_2, Q_1, Q_0 becomes 100 (in integer) is_____

Ans. **5**

Sol.

Given:

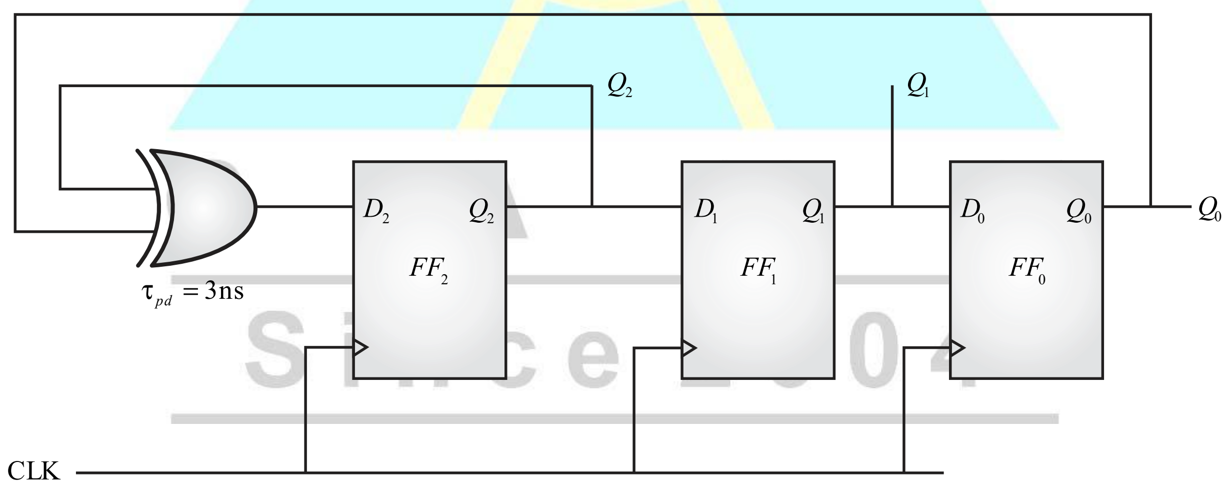
Delay of XOR gate $\tau_{pd} = 3ns$

Initially $Q_2Q_1Q_0 = 111$

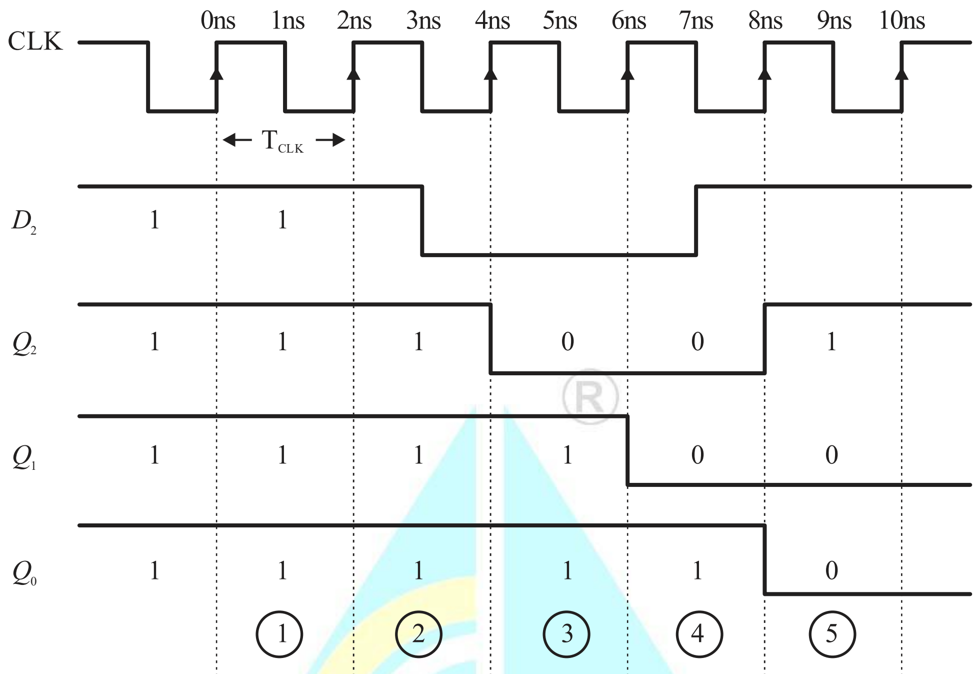
Initially $D_2 = 1$

Frequency of clock f = 500MHz

Time period of clock $T_{clock} = \frac{1}{f} = \frac{1}{500} = 2$ ns.



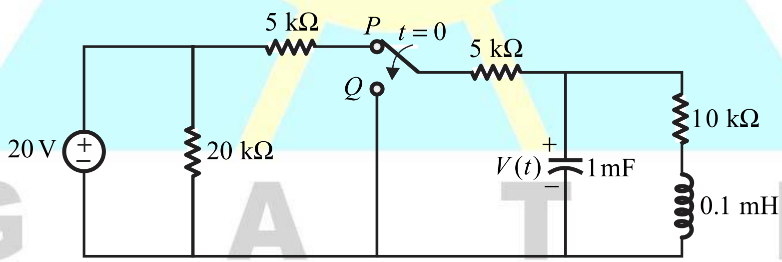
The numbers of clock's to get $Q_2Q_1Q_0 = 100$ is shown below,



The minimum number of triggering clock edges after which the flip-flop output $Q_2\ Q_1\ Q_0$ becomes 100 is 5.

Question 22 **Network Theory (2M)**

The switch in the circuit in the figure is in position P for a long time and then moved to position Q at time $t = 0$.

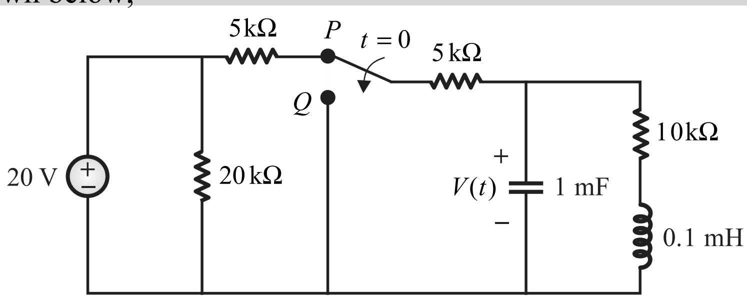


The value of $\frac{d}{dt}V(t)$ at $t = 0^+$ is,

(A) 0 V/sec (B) 3 V/sec (C) – 3 V/sec (D) – 5 V/sec

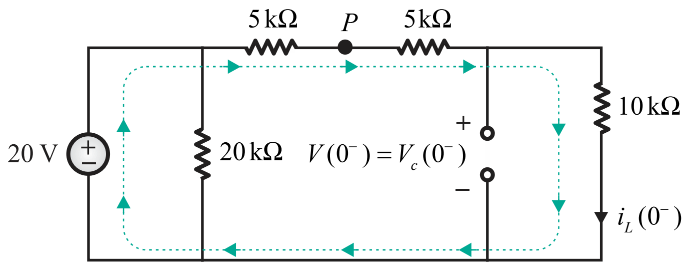
Ans. **C**

Sol. Given circuit is shown below,



(i) At $t = 0^-$ / $t < 0$/ steady state:

Switch is at position P, inductor behaves as short circuit and capacitor behaves as open circuit. So circuit becomes as,



Apply KVL in loop shown by dotted line,

$$20-5\times i_L(0^-)-5\times i_L(0^-)-10\times i_L(0^-)=0$$

$$i_L(0^-)=\frac{20}{5+5+10}$$

$$i_L(0^-)=1\,\text{mA}$$

So voltage $V_C(0^-)$ is

$$V_C(0^-)=10\times i_L(0^-)$$

$$V_C(0^-)=10\times 1$$

$$V_C(0^-)=10\,\text{V}$$

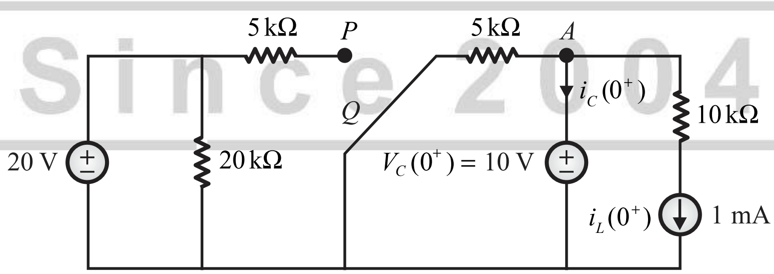
Thus current and voltage across inductor and capacitor at $t=0^+$ is,

$$i_L(0^-)=i_L(0^+)=1\,\text{mA}$$

$$V_C(0^-)=V_C(0^+)=10\,\text{V}$$

(ii) At $t=0^+$:

Switch is at position Q and Inductor is replaced by a current source with initial value $i_L(0^+)$ and capacitor is replaced by voltage source with initial value $V_C(0^+)$ so circuit becomes as,



Apply nodal analysis at node A,

$$\frac{10}{5}+i_C(0^+)+1=0$$

$$2+i_C(0^+)+1=0$$

$$i_C(0^+) = -3\,\text{mA}$$

So voltage across capacitor at $t = 0^+$ is,

$$i_C(0^+) = C\frac{d}{dt}V_C(0^+)$$

$$\frac{d}{dt}V_C(0^+) = \frac{i_C(0^+)}{C} = \frac{-3}{1} = -3 \text{ V/sec}$$

Hence, the correct option is (C).

Question 23 **Computer Organization (2M)**

The content of the registers are $R_1 = 25$ H, $R_2 = 30\,\text{H}$ and $R_3 = 40\,\text{H}$. The following machine instructions are executed

PUSH $\{R_1\}$

PUSH $\{R_2\}$

PUSH $\{R_3\}$

POP $\{R_1\}$

POP $\{R_2\}$

POP $\{R_3\}$

After execution, the content of registers R_1, R_2, R_3 are

(A) $R_1 = 40$ H, $R_2 = 30$ H, $R_3 = 25$ H (B) $R_1 = 25$ H, $R_2 = 30$ H, $R_3 = 40$ H

(C) $R_1 = 30$ H, $R_2 = 40$ H, $R_3 = 25$ H (D) $R_1 = 40$ H, $R_2 = 25$ H, $R_3 = 30$ H

Ans. **A**

Sol. **Given :** $R_1 = 25\,\text{H}$, $R_2 = 30\,\text{H}$, $R_3 = 40\,\text{H}$

PUSH $\{R_1\}$

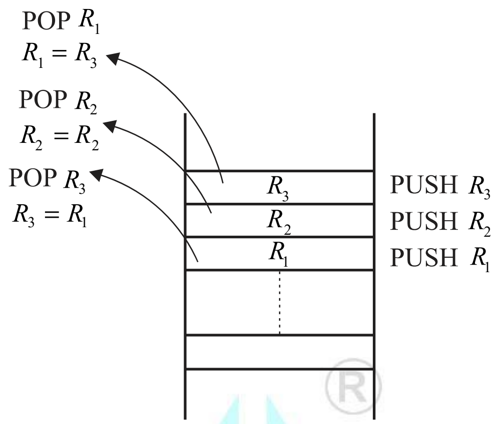
PUSH $\{R_2\}$

PUSH $\{R_3\}$

POP $\{R_1\}$

POP $\{R_2\}$

POP $\{R_3\}$



Hence, $R_1 = 40\,\text{H}$, $R_2 = 30\,\text{H}$, $R_3 = 25\,\text{H}$

Hence, the correct option is (A).

Question 24 **Signals & Systems (2M)**

The exponential Fourier series representation of continuous time periodic signal $x(t)$ is defined as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where ω_0 is the fundamental angular frequency of $x(t)$ and the coefficient of series are a_k.

The following information is given about $x(t)$ and a_k.

(i) $x(t)$ is real and even having fundamental period of 6 sec.

(ii) Average value of $x(t)$ is 2.

(iii) $a_k = \begin{cases} k, & 1 \le k \le 3 \\ 0, & k > 3 \end{cases}$

The average power of the signal $x(t)$ (round of one decimal place) is ________.

Ans. **32**

Sol. **Given :**

$$x(t) = \sum_{k=\infty}^{\infty} a_k e^{jk\omega_0 t}$$

(i) $x(t)$ is real and even with $T = 6$ sec

(ii) Average value of $x(t) = 2$

(iii) $a_k = \begin{cases} k, & 1 \le k \le 3 \\ 0, & k > 3 \end{cases} \Rightarrow a_k = \{........1, 2, 3\}$

From symmetry conditions, as $x(t)$ is real and even, so its fourier series coefficients will also be real and even.

i.e. $a_k^* = a_k$ and $a_{-k} = a_k$

so if $a_1 = 1 \Rightarrow a_{-1} = a_1 = 1$

$a_2 = 2 \Rightarrow a_{-2} = a_2 = 2$

$a_3 = 3 \Rightarrow a_{-3} = a_3 = 3$

$a_k = 0$ for $k > 3$

$a_{-k} = 0$ for $k < -3$

Also, given that average value of $x(t)$

i.e. $$\frac{1}{T}\int_0^T x(t)\,dt = a_0 = 2$$

So, the complete set of Fourier series coefficients is given as

$$a_x = \{3, 2, 1, \underset{\uparrow}{2}, 1, 2, 3\} = \{a_{-3}, a_{-2}, a_{-1}, a_0, a_1, a_2, a_3\}$$

Using Parseval's theorem, average power of $x(t)$ is given as

$$P_x = \frac{1}{T}\int_0^T |x(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

$$P_x = (3)^2 + (2)^2 + (1)^2 + (2)^2 + (1)^2 + (2)^2 + (3)^2$$

$$P_x = 9 + 4 + 1 + 4 + 1 + 4 + 9$$

$$P_x = 32 \text{ W}$$

Hence, the average power of signal $x(t)\,is$ 32 W

Question 25 **Signals & Systems (1M)**

Consider a real-valued base-band signal $x(t)$, band limited to 10 kHz. The Nyquist rate for the signal $y(t) = x(t)\cdot x\left(1+\frac{t}{2}\right)$ is

(A) 15 kHz (B) 30 kHz (C) 60 kHz (D) 20 kHz

Ans. **B**

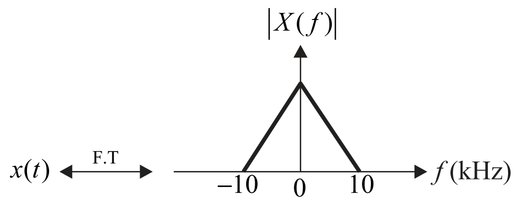
Sol. **Given :**

$x(t)$ is band limited to 10 kHz

$$y(t) = x(t)\cdot x\left(1+\frac{t}{2}\right) = x(t)\cdot x\left[\frac{1}{2}(t+2)\right]$$

Method 1 :

Assuming magnitude spectrum of x(t) is shown below,

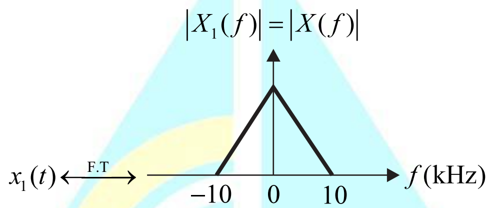


Let $\quad x(t) \xleftrightarrow{\text{F.T.}} X(f)$

$$x_1(t) = x(t+1) \xleftrightarrow{\text{F.T.}} e^{j2\pi f} X(f) = X_1(f)$$

Magnitude spectrum of $x_1(t)$,

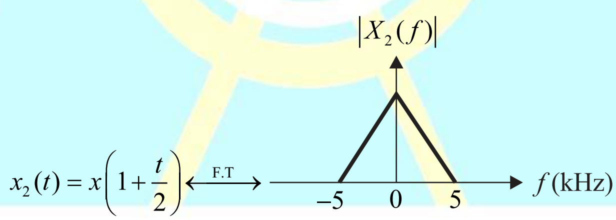
$$|X_1(f)| = |X(f)|$$



$$x_2(t) = x_1\left(\frac{t}{2}\right) = x\left(\frac{t}{2}+1\right) \xleftrightarrow{\text{F.T.}} 2e^{j2\pi f} X(2f) = X_2(f)$$

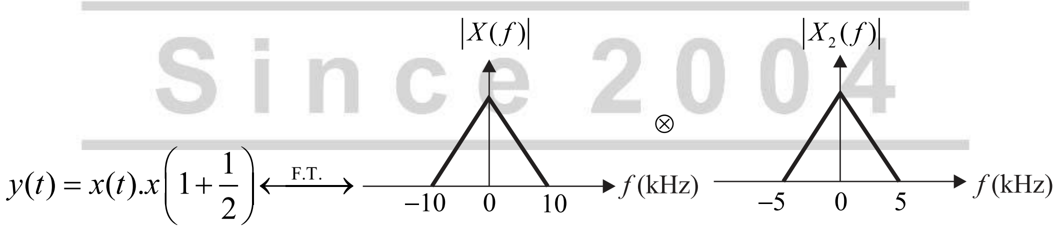
Magnitude spectrum of $x_2(t)$,

$$|X_2(f)| = 2|X(2f)|$$

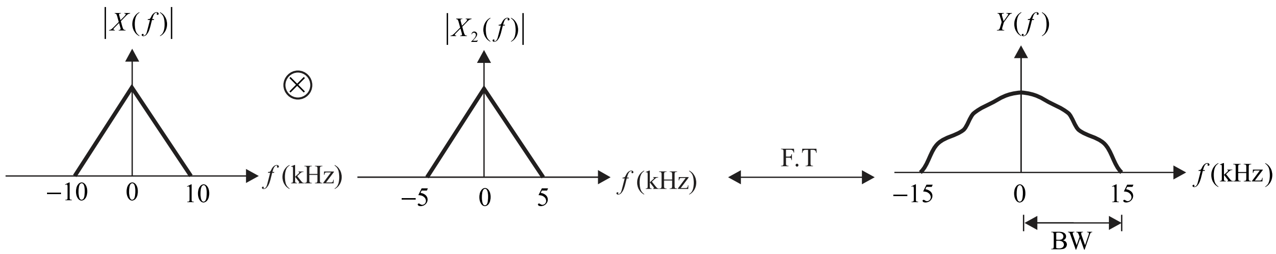


Thus applying the property of convolution,

$$y(t) = x(t)\cdot x_2(t) \xleftrightarrow{\text{F.T.}} Y(f) = X(f) \otimes X_2(f)$$



So that spercturm of Y(*f*) is shown below,



Bandwidth (BW) of $y(t) = 15\,\text{kHz}$

So, the nyquist rate of $y(t) = 2\times 15 = 30\,\text{kHz}$

Hence, the correct option is (B).

Method 2 :

Given : $y(t) = x(t)\cdot x\left(1+\frac{t}{2}\right)$

Where maximum frequency component of x(t) is $f_{m_1} = 10\,\text{kHz}$

To find Nyquist rate of sampling for $y(t)$ we need to find maximum frequency component in $y(t)$

As $y(t)$ is obtained by multiplication of two functions in time domain so Fourier transform of $y(t)$, i.e. $Y(\omega)$ will be the convolution of individual transforms of multiplied signals.

Let, $$x\left(\frac{t}{2}+1\right) = x_1(t)$$

$$y(t) = x(t).x_1(t)$$

$$Y(\omega) = \frac{1}{2\pi}X(\omega)\cdot X_1(\omega) = X(f)\cdot X_1(f) \quad \ldots\text{(i)}$$

When two functions are convolved, then the left and right most limits of existence of resultant function will be addition of left and right most frequencies of $X(f)$ and $X_1(f)$

Given, $X(f)$ has maximum frequency of 10 kHz

$$x(t) \to f_{m_1} = 10\,\text{kHz}$$

As time shifting do not have any effect on the existence duration of its spectrum

So $$x(t+1) \to f_m' = 10\,\text{kHz}$$

As expansion in time domain results in compression in frequency domain, so after applying time scaling

$$x\left(\frac{t}{2}+1\right) \to f_{m_2} = \frac{10}{2} = 5\,\text{kHz}$$

So, the maximum frequency component in the convolved signal $y(t)$ is

$$f_m = f_{m1} + f_{m2} = 15 \text{ kHz}$$

So, the Nyquist sampling rate is given by,

$$f_s = 2f_m = 2\times 15\,\text{kHz} = 30\,\text{kHz}$$

Hence, the correct option is (B).

Question 26 **Signals & Systems (2M)**

For a unit step input $u[n]$, a discrete time LTI system produces an output signal $[2\delta[n+1]+\delta[n]+\delta[n-1]]$. Let $y[n]$ be the output of system for an input $\left[\left(\frac{1}{2}\right)^n u[n]\right]$. The value of $y[0]$ is _______.

Ans. **0**

Sol. Given response of the system for step input, i.e. step response is,

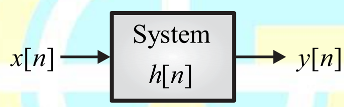
$$S[n]=2\delta[n+1]+\delta[n]+\delta[n-1]$$

Impulse response of the system can be obtained by taking first difference of $S[n]$

$$h[n]=S[n]-S[n-1]$$

$$h[n]=2\delta[n+1]+\delta[n]+\delta[n-1]-2\delta[n]-\delta[n-1]-\delta[n-2]$$

$$h[n]=2\delta[n+1]-\delta[n]-\delta[n-2]$$



For input, $x[n]=\left(\frac{1}{2}\right)^n u[n]$

Output, $y[n]=x[n]\otimes h[n]$

$$y[n]=\left(\frac{1}{2}\right)^n u[n]\otimes\{2\delta[n+1]-\delta[n]-\delta[n-2]\}$$

$$y[n]=2\left(\frac{1}{2}\right)^{n+1} u[n+1]-\left(\frac{1}{2}\right)^n u[n]-\left(\frac{1}{2}\right)^{n-2} u[n-2] \quad (\because\ x[n]\otimes\delta[n-n_0]=x[n-n_0])$$

Substituting $n=0$ both sides,

$$y[0]=2\times\left(\frac{1}{2}\right)^1 u[1]-\left(\frac{1}{2}\right)^0 u[0]-\left(\frac{1}{2}\right)^{-2} u(-2)$$

$$y[0]=2\times\frac{1}{2}-1-0 \quad (\because\ u[-2]=0)$$

$\therefore$ $$y[0]=1-1=0$$

Hence, the value of $y[0]$ is 0.

Question 27 **Signals & Systems (2M)**

Consider the signal $x[n]=2^{n-1}u[-n+2]$ and $y[n]=2^{-n+2}u[n+1]$. Where $u[n]$ is the unit step sequence. Let $X(e^{j\omega})$ and $Y(e^{j\omega})$ be the discrete time Fourier transform of $x[n]$ and $y[n]$ respectively. The value of integral

$$\frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})\,d\omega$$

(round off to one decimal places) is _______.

Ans. **8**

Sol. **Given :** $x[n]=2^{n-1}u[-n+2]\xrightarrow{DTFT}X(e^{j\omega})$

$y[n]=2^{-n+2}u[n+1]\xrightarrow{DTFT}Y(e^{j\omega})$

We have to evaluate the integral,

$$I=\frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})\,d\omega \qquad \ldots\text{(i)}$$

If $p[n]\xleftrightarrow{\text{DTFT}}P(e^{j\omega})$, then from synthesis equation of DTFT

$$p[n]=\frac{1}{2\pi}\int_0^{2\pi} P(e^{j\omega})e^{j\omega n}\,d\omega$$

$$p[0]=\frac{1}{2\pi}\int_0^{2\pi} P(e^{j\omega})\,d\omega \qquad \ldots\text{(ii)}$$

From equation (i) and (ii),

If $P(e^{j\omega})=X(e^{j\omega})Y(e^{-j\omega})$ then, $I=p[0]$ ...(iii)

If $y[n]\longleftrightarrow Y(e^{j\omega})$

Applying time reversal property,

$$y[-n]\longleftrightarrow Y(e^{-j\omega})$$

Let $y[-n]=z[n]$, then using convolution property,

$$x[n]\otimes z[n]\longleftrightarrow X(e^{j\omega})\cdot Y(e^{-j\omega})=P(e^{j\omega})$$

So, $p[n]=x[n]\otimes z[n]$

Expend convolution as,

$$p[n]=\sum_{k=-\infty}^{\infty} x[k]z[n-k]$$

Here, $x[n]=2^{n-1}u[-n+2]$

$$x[k]=2^{k-1}u[-k+2]$$

$$z[n]=y[-n]=2^{n+2}u[-n+1]$$

$$z[n-k]=2^{n-k+2}u[-n+k+1]$$

$\therefore$ $$p[n]=\sum_{k=-\infty}^{\infty}2^{k-1}u[-k+2]\cdot 2^{n-k+2}u[-n+k+1]$$

As required integral, $I=p[n]\big|_{n=0}=p[0]$

$$p[0]=\sum_{k=-\infty}^{\infty}2^{k-1}u[-k+2]\cdot 2^{-k+2}u[k+1]$$

Sequence, $u[-k+2]=1$ only for $-\infty<k<2$, thus modifying limit of summation

$$p[0]=\sum_{k=-\infty}^{2}2^{k-1-k+2}\cdot 1\cdot u[k+1]$$

$$p[0]=\sum_{k=-\infty}^{2}2u[k+1]$$

Sequence $u[k+1]=1$ only for $-1<k<\infty$, thus modifying lower limit of summation.

$$p[0]=\sum_{k=-1}^{2}2\times 1=2\sum_{k=-1}^{2}(1)^k$$

$$p[0]=2[1+1+1+1]=8$$

So, $$I=\frac{1}{2\pi}\int_0^{2\pi}X(e^{j\omega})\cdot Y(e^{-j\omega})\,d\omega=\frac{1}{2\pi}\int_0^{2\pi}P(e^{j\omega})\,d\omega$$

$$I=p[0]=8$$

Hence, the value of given integral is 8.

Question 28 **Signals & Systems (1M)**

Consider two 16 point sequences $x[n]$ and $h[n]$. Let the linear convolution $x[n]$ and $h[n]$ be denoted by $y[n]$, while $z[n]$ denotes the 16 point inverse discrete Fourier transform (IDFT) of the product of the 16 point DFTs of $x[n]$ and $h[n]$. The value(s) of k for which $z[k]=y[k]$ is/are

(A) $k=0,1,2,.....,15$ (B) $k=0$ (C) $k=15$ (D) $k=0$ or 15

Ans. **C**

Sol. **Given :** $x[n]$ and $h[n]$ are 16 point sequences

and $$y[n]=x[n]\otimes h[n] \quad \ldots\text{(i)}$$

Also, $$z[n]=IDFT[X(k)\cdot H(k)]$$

Where, $$x[n]\xleftrightarrow[\text{16 point}]{DFT}X(k)$$

$$h[n] \xleftrightarrow[16\text{point}]{DFT} H(k)$$

From circular convolution property of DFT.

$$x[n] \; Ⓝ \; h[n] \xleftrightarrow{DFT} X(k)\cdot H(k)$$

So, $$z[n] = x[n] \; Ⓝ \; h[n] \qquad \ldots\text{(ii)}$$

We have to find value of k, for which $y(k) = z(k)$.

i.e., value of n, for which linear and circular convolutions of 16 point sequences $x[n]$ and $h[n]$ are same.

From definitions of linear convolution,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\cdot h[n-k]$$

From definitions of circular convolution,

$$z[n] = \sum_{k=0}^{N-1} x(k)\cdot h[(n-k)]_N \quad \text{where, } N = 16$$

Assuming two 16 point sequences as,

$$x[n] = \{\underset{\uparrow}{1}, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$$

$$h[n] = \{\underset{\uparrow}{a}, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p\}$$

So, linear convolution $y[n]$ is,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\cdot h[n-k]$$

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]\cdot h[-k]$$

$$x[k] = \{\underset{\uparrow}{1}, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$$

$$h[-k] = \{p, o, n, m, l, k, j, i, h, g, f, e, d, c, b, \underset{\uparrow}{a}\}$$

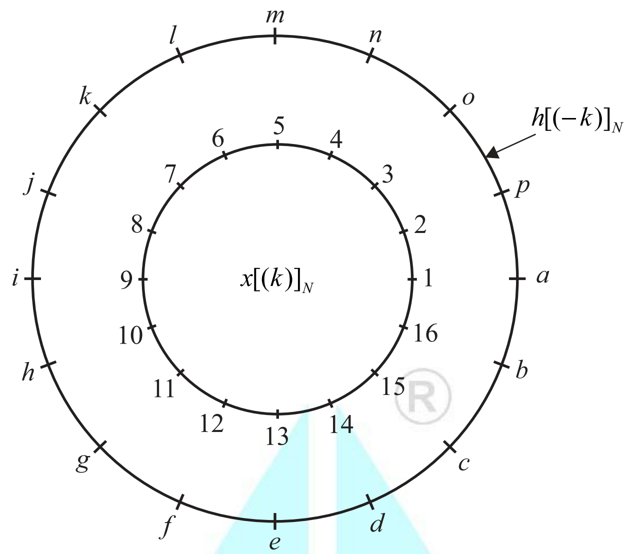
$\therefore$ $$y[0] = (1\times a) + 0+0+0+0+0+0+0+0+0+0+0+0+0+0+0$$

$$y[0] = a \qquad \ldots\text{(i)}$$

From 16 point circular convolution,

$$z[n] = \sum_{K=0}^{15} x(k)\cdot h[(n-k)]_{16}$$

$$z[0] = \sum_{K=0}^{15} x(k)\cdot h[(-k)]_{16}$$



Thus, $z(0)=(a\times1)+(b\times2)+(c\times3)+(d\times4)+(e\times5)+(f\times6)+(g\times7)$
$+(h\times8)+(i\times9)+(j\times10)+(k\times11)+(l\times12)+(m\times13)+(n\times14)$
$+(o\times15)+(p\times16)$ ...(ii)

From equations (i) and (ii),

$$z(0)\neq y(0)$$

So, $z(k)=y(k)$ is not satisfied for $k=0$ and hence options (A), (B) and (D) are wrong.

Now we are choosing option (C) and verify it as follows,

Linear convolution, $y[n]=\sum_{k=-\infty}^{\infty}x[k]\cdot h[n-k]$

Taking $n=15$, $y[15]=\sum_{k=-\infty}^{\infty}x[k]\cdot h[15-k]$

To find $h[15-k]$, shifting sequence $h(-k)$ 15 times towards right hand side.

So, $x(k)=\{\underset{\uparrow}{1}, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$

$h[-k]=\{p, o, n, m, l, k, j, i, h, g, f, e, d, c, b, \underset{\uparrow}{a}\}$

$h[15-k]=\{\underset{\uparrow}{p}, o, n, m, l, k, j, i, h, g, f, e, d, c, b, a\}$

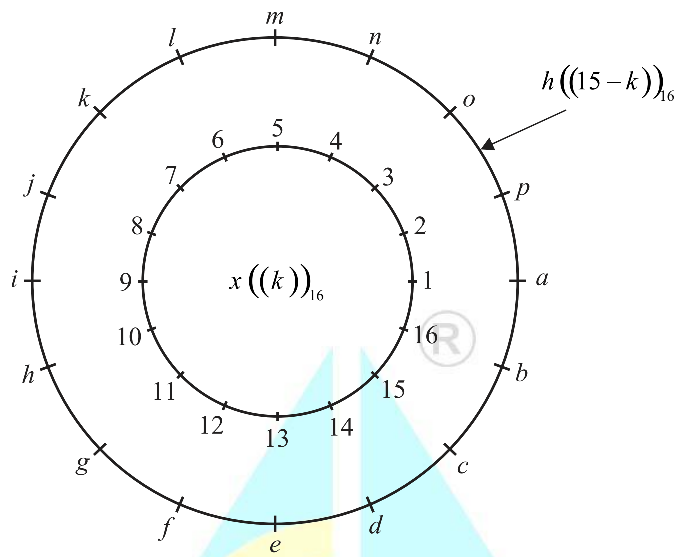
$\therefore$ $y(15)=p+2o+3n+4m+5l+6k+7j+8i+9h+10g+11f+12e$
$+13d+14c+15b+16a$ ...(iii)

Circular convolution,

$$z[n]=\sum_{k=0}^{15}x(k)\cdot h\left[(n-k)\right]_{16}$$

$$z[15]=\sum_{k=0}^{15}x(k)\cdot h\left[(15-k)\right]_{16}$$

To find $h[(15-k)]_{16}$, rotating $h[(-k)]_{16}$ in anticlockwise direction 15 times. So,



$\therefore$ $$z(15) = p + 2o + 3n + 4m + 5l + 6k + 7j + 8i + 9h + 10g + 11f + 12e + 13d + 14c + 15b + 16a \quad \ldots\text{(iv)}$$

From equation (iii) and (iv),

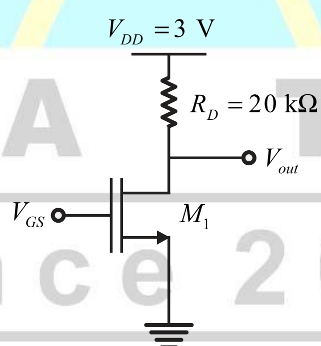
$$y(15) = z(15)$$

Hence, $y(k) = z(k)$ for $k = 15$ is satisfied.

Hence, the correct option is (C).

Question 29 **Electronic Devices (2M)**

For the transistor M_1 in the circuit shown in the figure, $\mu_n C_{ox} = 100\,\mu\text{A/V}^2$ and $(W/L) = 10$ where μ_n is the mobility of electron, C_{ox} is the oxide capacitance per unit area, W is the width and L is the length.



The channel length modulation coefficient is ignored. If the gate-to-source voltage V_{GS} is 1 V to keep the transistor at the edge of saturation, then the threshold voltage of the transistor (rounded off to one decimal place) is _________V.

Ans. **0.5**

Sol. **Given :**

(i) $\mu_n C_{ox} = 100\,\mu\text{A/V}^2$

(ii) $\frac{W}{L}=10$

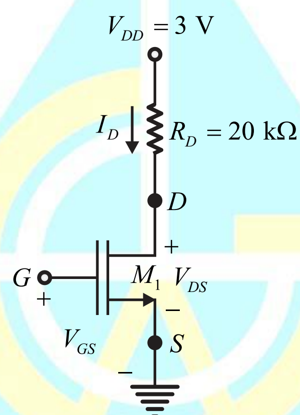
(iii) $V_{GS}=1\text{ V}$

Drain to source current (I_{DS}) when N-MOS is saturation is given by,

$$I_D = I_{DS} = \frac{\mu_n C_{ox}}{2}\times\frac{W}{L}(V_{GS}-V_T)^2 \qquad \text{(For NMOS in saturation)}$$

$$I_D = I_{DS} = \frac{100\times10^{-6}}{2}\times 10(1-V_T)^2$$

$$I_D = \frac{1}{2}(1-V_T)^2\ \text{mA}$$



Apply KVL at outer loop of N-MOS,

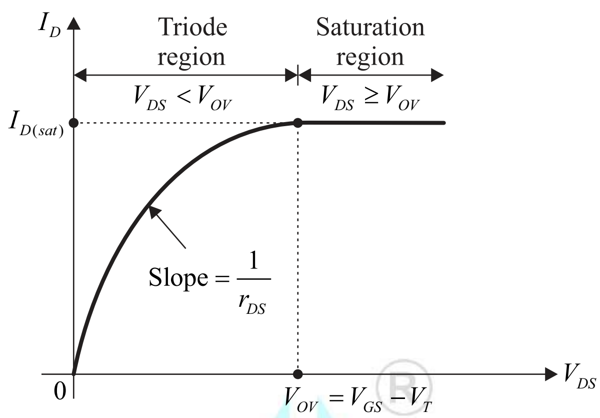
$$V_{DS}=3-20\times I_{DS}$$

Put the value of I_{DS}, in above equation we get

$$V_{DS}=3-\frac{20}{2}(1-V_T)^2$$

$$V_{DS}=3-10(1-V_T)^2$$

MOSFET operates in saturation if over drive voltage $V_{DS}\geq V_{OV}$ as shown below,



So, $\quad V_{DS} \geq V_{OV} = V_{GS} - V_T$

So, we take, $\quad V_{DS} = V_{GS} - V_T$

$$V_{GS} - V_T = 3 - 10(1 - V_T)^2$$

$$1 - V_T = 3 - 10(1 - V_T)^2$$

Let, $\quad 1 - V_T = x$

$$3 - 10x^2 = x$$

$$10x^2 + \times - 3x = 0$$

We get, $\quad x = -\frac{1 \pm \sqrt{1+120}}{20}$

$$x = \frac{1 \pm 11}{20}$$

$\therefore \quad x = 0.5$ and -0.6

When $x = 0.5$	When $x = -0.6$
$1 - V_T = 0.5$ $V_T = 0.5\text{ V}$	$1 - V_T = -0.6$ $V_T = 1.6\text{ V}$

For MOSFET to be in saturation

If $\quad V_T < V_{GS}$

i.e., $\quad V_T < 1$

$\therefore \quad V_T = 0.5\text{ V}$

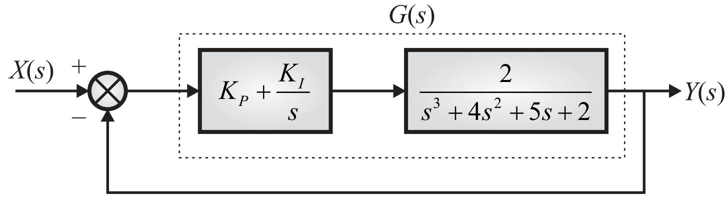
Hence, the threshold voltage of the transistor to be at the edge of saturation is **0.5 V**.

Key Point

If we choose $V_T = 1.6$ V, then the MOSFET goes in cut-off region, because $V_T > V_{GS}$.

Question 30 **Control System (2M)**

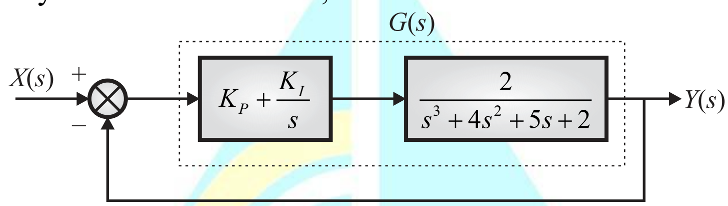
A unity feedback system that uses proportional-integral (PI) control is shown in the figure.



The stability of the overall system is controlled by tuning the PI control parameters K_P and K_I. The maximum value of K_I that can be chosen so as to keep the overall system stable or in the worst case, marginally stable (round off to three decimal places) is ______.

Ans. **3.125**

Sol. Given unity feedback system is shown below,



Characteristic equation of unity feedback system is,

$$1+G(s)=0$$

$$1+\left(K_P+\frac{K_I}{s}\right)\left(\frac{2}{s^3+4s^2+5s+2}\right)=0$$

$$1+\frac{K_Ps+K_I}{s}\,\frac{2}{s^3+4s^2+5s+2}=0$$

$$s(s^3+4s^2+5s+2)+2sK_P+2K_I=0$$

$$s^4+4s^3+5s^2+2s+2sK_P+2K_I=0$$

$$s^4+4s^3+5s^2+(2+2K_p)s+2K_I=0$$

From the above equation it is clear that, the sufficient condition for the system to be stable is,

$$K_P>(-1) \text{ and } K_I>0$$

The Routh table for above characteristic equation,

s^4	1	5	$2K_I$
s^3	4	$2+2K_P$	
s^2	$\frac{18-2K_P}{4}$	$2K_I$	
s^1	$\frac{\left(\frac{18-2K_P}{4}\right)(2+2K_P)-8K_I}{\frac{18-2K_P}{4}}$		
s^0	$2K_I$		

For stability or marginal stability of given system, the coefficient of first column of Routh table should be positive or same sign so,

(i) $\frac{18-2K_P}{4} > 0$

$K_P < 9$

Thus, K_P must varies from, $(-1) < K_P < 9$

(ii) $\dfrac{\left(\frac{18-2K_P}{4}\right)(2+2K_P) - 8K_I}{\frac{18-2K_P}{4}} = 0$

$$2+2K_P - \frac{8K_I \times 4}{18-2K_P} = 0$$

$$1+K_P = \frac{16K_I}{18-2K_P}$$

$$1+K_P = \frac{8K_I}{9-K_P}$$

$$K_I = \frac{(1+K_P)(9-K_P)}{8}$$

Here, K_I is a function of K_P.

For maxima or minima, $\frac{d}{dK_P}K_I = 0$

$$\frac{d}{dK_P}\left[\frac{(1+K_P)(9-K_P)}{8}\right] = 0$$

$$\frac{1}{8}[(1+K_P)(-1)+9-K_P] = 0$$

$$9-K_P = 1+K_P$$

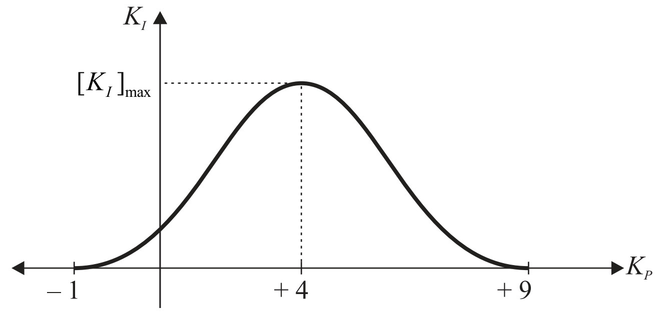
$$2K_P = 8$$

$$K_P = 4$$

Now, double derivative of K_I is,

$$\frac{d^2}{dK_P^2}K_I = -\frac{1}{4} < 0$$

It means, K_I has maxima at $K_P = 4$.



Thus, maximum value of K_I at $K_P = 4$ will be,

$$[K_I]_{max} = \frac{(1+4)(9-4)}{8} = 3.125$$ **Ans.**

Question 31 **Engineering Mathematics (2M)**

A box contains the following three coins

(i) A fair coin with head on one face and tail on other face.

(ii) A coin with heads on both faces.

(iii) A coin with tail on both faces.

A coin is picked randomly from box and tossed. Out of the two remaining coins in the box, one coin is then picked randomly and tossed. If the first toss result in a head, the probability of getting a head in the second toss is

(A) 2/5 (B) 1/3 (C) 1/2 (D) 2/3

Ans. **B**

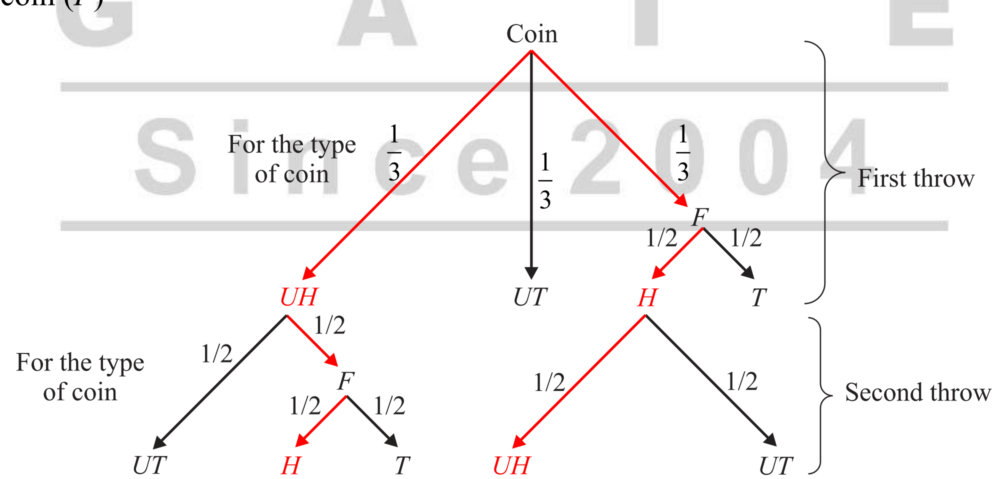
Sol. **Method 1 :**

There are three types of coins

(i) Unfair coin with head on both side (UH)

(ii) Unfair coin with tail on both side (UT)

(iii) Fair coin (F)



$P(A) =$ probability for head in first throw

$P(B) =$ probability for head in second throw

We need to find $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{\left(\frac{1}{3}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{3}\times\frac{1}{2}\times\frac{1}{2}\right)}{\frac{1}{3}+\left(\frac{1}{3}\times\frac{1}{2}\right)}$

$$P\left(\frac{B}{A}\right)=\frac{\frac{1}{12}+\frac{1}{12}}{\frac{3}{6}}=\frac{\frac{1}{6}}{\frac{3}{6}}=\frac{1}{3}$$

Hence, the correct option is (B).

Method 2 :

Step 1 : Find the probability of selecting a coin and getting heads after tossing it

$$P\text{ (coin 1 with head and tail)}=\frac{1}{3}\times\frac{1}{2}=\frac{1}{6} \qquad \ldots\text{(i)}$$

$$P\text{ (coin 2 with two heads)}=\frac{1}{3}\times 1=\frac{1}{3} \qquad \ldots\text{(ii)}$$

$$P\text{ (coin 3 with two tails)}=\frac{1}{3}\times 0=0 \qquad \ldots\text{(iii)}$$

Step 2 : Find the probability of getting heads in second toss,

If equation (i) happens, then probability of getting heads in second toss = 1

If equation (ii) happens, then probability of getting heads in second toss $=\frac{1}{2}$

If equation (iii) happens, then probability of getting heads in second toss = 0.

$\therefore$ Required probability $=\frac{1}{6}\times 1+\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$

Hence, the correct option is (B).

Question 32 **Engineering Mathematics (1M)**

Consider the differential equation given below

$$\frac{dy}{dx}+\frac{x}{1-x^2}y=x\sqrt{y}$$

The integrating factor of differential equation is

(A) $(1-x^2)^{-\frac{3}{4}}$ (B) $(1-x^2)^{-\frac{1}{4}}$ (C) $(1-x^2)^{-\frac{3}{2}}$ (D) $(1-x^2)^{-\frac{1}{2}}$

Ans. B

Sol. **Given :** First order deferential equation,

$$\frac{dy}{dx}+\frac{x}{1-x^2}y=x\sqrt{y}$$

$$\frac{1}{\sqrt{y}}\frac{dy}{dx}+\frac{x}{1-x^2}\left(\frac{y}{\sqrt{y}}\right)=x$$

Dividing both side with $\sqrt{y}$

$$y^{-\frac{1}{2}}\frac{dy}{dx}+\frac{x}{1-x^2}y.y^{-\frac{1}{2}}=x$$

$$y^{-\frac{1}{2}}\frac{dy}{dx}+\frac{x}{1-x^2}y^{\frac{1}{2}}=x \qquad \ldots\text{(i)}$$

Let, $y^{\frac{1}{2}}=t$ …(ii)

Differentiating equation (ii), both side with respect to x,

$$\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx}=\frac{dt}{dx}$$

$$y^{-\frac{1}{2}}\frac{dy}{dx}=2\frac{dt}{dx} \qquad \ldots\text{(iii)}$$

Putting equation (iii) into equation (i),

$$2\frac{dt}{dx}+\frac{x}{1-x^2}t=x$$

$$\frac{dt}{dx}+\frac{1}{2}\left[\frac{x}{(1-x^2)}\right]t=\frac{x}{2} \qquad \ldots\text{(iv)}$$

Compare equation (iv) with Labntiz first order differential equation,

$$\frac{dt}{dx}+Pt=Q$$

So, $P=\frac{1}{2}\left[\frac{x}{1-x^2}\right],\ Q=\frac{x}{2}$

Integrating factor (I.F) is given by,

$$\text{I.F}=e^{\int P\,dx}=e^{\frac{1}{2}\int\frac{x}{1-x^2}dx}$$

$$\text{I.F}=e^{-\frac{1}{4}\int\frac{-2x}{1-x^2}dx}$$

$$\text{I.F}=e^{-\frac{1}{4}\log_e(1-x^2)}$$

$$\text{I.F}=e^{\log_e(1-x^2)^{-1/4}}$$

$$\text{I.F}=(1-x^2)^{-\frac{1}{4}}$$

Hence, the correct option is (B).

Question 33 **Computer Organization (1M)**

Addressing of a $32\ \text{K}\times16$ memory is realized using a single decoder. The minimum number of AND gate required for the decoder is

(A) 2^8 (B) 2^{32} (C) 2^{15} (D) 2^{19}

Ans. **C**

Sol. **Given :**

Addressing of memory $= 32\ \text{K}\times16$

Addressing of memory $= 2^5\times2^{10}\times16$ $(\because\ K=2^{10})$

Addressing of memory $= 2^{15}\times16$

So $32\ \text{K}\times16$ memory needs 15 address lines

So it will need decoder of size 15×2^{15}, which will need 2^{15} AND gates.

Hence, the correct option is (C).

Question 34 **Engineering Mathematics (2M)**

A real 2×2 non-singular matrix A with repeated Eigen value is given as

$$A=\begin{bmatrix} x & -3.0 \\ 3.0 & 4.0 \end{bmatrix}$$

where x is a real positive number. The value of x (round off to one decimal place) is ______.

Ans. **10**

Sol. **Given :** $A=\begin{bmatrix} x & -3.0 \\ 3.0 & 4.0 \end{bmatrix}$

It is given that Eigen value of matrix A is repeated

Let the repeated Eigen values are

$$\lambda_1=\lambda$$

$$\lambda_2=\lambda$$

We know that, sum of the Eigen value of matrix $[A]$ is equal to trace of matrix

$$\lambda_1+\lambda_2 = \text{Trace }(A) = \text{Sum of diagonal elements of matrix } [A]$$

$$\lambda+\lambda=x+4$$

$$2\lambda=x+4$$

$$\lambda=\frac{x+4}{2} \qquad \ldots\text{(i)}$$

We know that, product of Eigen value is equal to determinant of matrix $[A]$

$$\lambda_1\times\lambda_2=|A|$$

$$\lambda\times\lambda=4x+9$$

$$\lambda^2 = 4x+9 \qquad \ldots(ii)$$

Putting $\lambda = \frac{x+4}{2}$ into equation (ii)

$$\left(\frac{x+4}{2}\right)^2 = 4x+9$$

$$x^2+8x+16 = 16x+36$$

$$x^2-8x-20=0$$

$$x^2-10x+2x-20=0$$

$$x(x-10)+2(x-10)=0$$

$$(x-10)(x+2)=0$$

$$x=10,\ x=-2$$

It is given that x is positive real number, therefore we will select $x=10$.

Hence, the correct answer is 10.

Question 35 **Engineering Mathematics (1M)**

Two continuous random variables X and Y are related as

$$Y = 2X+3$$

Let σ_X^2, σ_Y^2 denote the variance of X and Y respectively. The variance are related as,

(A) $\sigma_Y^2 = 2\sigma_X^2$ (B) $\sigma_Y^2 = 4\sigma_X^2$ (C) $\sigma_Y^2 = 5\sigma_X^2$ (D) $\sigma_Y^2 = 25\sigma_X^2$

Ans. **B**

Sol. **Given :** $\text{Var}[X] = \sigma_X^2$

$$\text{Var}[Y] = \sigma_Y^2$$

$$Y = 2X+3$$

Taking variance on both sides,

$$\text{Var}(Y) = \text{Var}[2X+3]$$

$$\text{Var}(Y) = \text{Var}(2X) + \text{Var}(3) \qquad \ldots(i)$$

Since, from the property of variance, for any constant "a"

$$\text{Var}[aX] = a^2\text{Var}[X]$$

$$\text{Var}[a] = 0$$

So, equation (i) becomes as,

$$\text{Var}(Y) = 2^2\ \text{Var}(X) + 0$$

$$\text{Var}(Y) = 4\ \text{Var}(X)$$

$$\sigma_Y^2 = 4\sigma_X^2$$

Hence, the correct option is (B)

Key Point

(i) $\text{Var}[aX] = a^2 \text{Var}[X]$

(ii) $\text{Var}[a] = 0$

Where, a is constant.

Question 36 **Engineering Mathematics (2M)**

Consider the integral

$$\oint_C \frac{\sin x}{x^2(x^2+4)} dx$$

where C is the counter clockwise oriented circle defined as $|x-i| = 2$. The value of the integral is

(A) $-\frac{\pi}{8}\sin 2i$ (B) $\frac{\pi}{8}\sin 2i$ (C) $-\frac{\pi}{4}\sin 2i$ (D) $\frac{\pi}{4}\sin 2i$

Ans. * **(According to IIT Bombay its Answer A)**

Sol. **Given :** $f(x) = \frac{\sin x}{x^2(x^2+4)}$

Poles of $f(x)$ are given by,

$x^2(x^2+4) = 0$

$x_1 = 0$ (multiple pole of order 2)

$x_2 = 2i$ (simple pole)

$x_3 = -2i$ (simple pole)

Contour C is given by $|x-i| = 2$

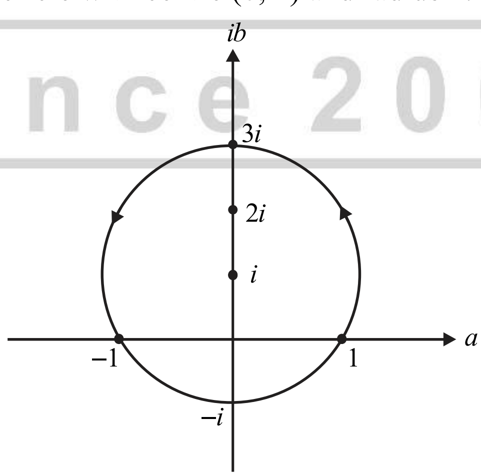
$|a+ib-i| = 2$

$\sqrt{a^2+(b-1)^2} = 2$

Squaring both sides

$a^2+(b-1)^2 = 4$

So, the contour C represent a circle with centre (0, 1) and radius 2.



Pole $x_1 = 0$ and $x_2 = 2i$ lie inside the circle but pole $x_3 = -2i$ lies outside of the circle.

Thus we will find residue at pole $x_1 = 0$ and $x_2 = 2i$.

Residue of $f(x)$ with multiple pole at $x = a$ with order m is given by,

$$R(x)\Big|_{x=a} = \frac{1}{(m-1)!}\left[\frac{d^{m-1}}{dx^{m-1}}(x-a)^m f(x)\right]_{x=a}$$

Residue at $x_1 = 0$ is given by,

$$R(x_1) = \frac{1}{(2-1)!}\left[\frac{d}{dx}x^2 \frac{\sin x}{x^2(x+4)}\right]_{x=0}$$

$$R(x_1) = \frac{d}{dx}\left[\frac{\sin x}{(x^2+4)}\right]_{x=0}$$

$$R(x_1) = \left[\frac{(x^2+4)\cos x - 2x\sin x}{(x^2+4)^2}\right]_{x=0}$$

$$R(x_1) = \frac{4-0}{16} = \frac{1}{4}$$

Residue at $x_2 = 2i$ is given by

$$R(x_2) = \left[\frac{(x-2i)\sin x}{x^2(x+2i)(x-2i)}\right]_{x=2i}$$

$$R(x_2) = \frac{\sin 2i}{(2i)^2 \times 4i} = \frac{-\sin 2i}{16i}$$

By residue theorem

$$\oint_C f(x)\,dx = 2\pi i \times \left[\Sigma R(x_i)\right]$$

$$\oint_C \frac{\sin x}{x^2(x^2+4)} = 2\pi i\left[R(x_1)+R(x_2)\right]$$

$$\oint_C \frac{\sin x}{x^2(x^2+4)} = 2\pi i\left[\frac{1}{4} - \frac{\sin 2i}{16i}\right]$$

$$\oint_C \frac{\sin x}{x^2(x^2+4)} = \frac{\pi i}{2} - \frac{\pi \sin 2i}{8}$$

Hence, none of the option is matching.

This question is not in proper form as per their options, so this question must be considered as marks to all (MTA).

Question 37 **Digital Electronics (1M)**

If $(1235)_x = (3033)_y$ where x and y indicate bases of the corresponding numbers, then

(A) $x = 7$ and $y = 5$ (B) $x = 8$ and $y = 6$ (C) $x = 6$ and $y = 4$ (D) $x = 9$ and $y = 7$

Ans. **B**

Sol. **Given :** $(1235)_x = (3033)_y$

Converting both numbers into decimal number as,

$$(\underset{x^3}{1}\ \underset{x^2}{2}\ \underset{x^1}{3}\ \underset{x^0}{5})_x = (\underset{y^3}{3}\ \underset{y^2}{0}\ \underset{y^1}{3}\ \underset{y^0}{3})_y$$

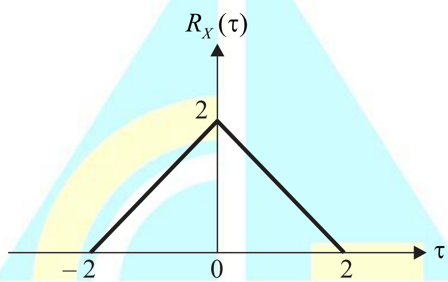
$$x^3 + 2x^2 + 3x + 5 = 3y^3 + 3y + 3$$

Only option (C) $x = 8$, $y = 6$, satisfy the above equation.

Hence, the correct option is (B).

Question 38 **Communications System (1M)**

The autocorrelation function $R_X(\tau)$ of a wide-sense stationary random process $X(t)$ is shown in the figure,

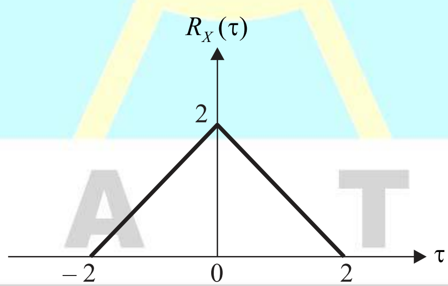


The average power of $X(t)$ is ______.

Ans. **2**

Sol. **Method 1 :**

Given, autocorrelation function $R_X(\tau)$ of wide sense stationary random process $X(t)$ is shown in figure.



Here, $R_X(\tau = 0) = 2$

Average power of $X(t)$ is given as mean square value of $X(t)$, i.e.,

$$P_x = E\left[X^2(t)\right] = E[X(t)X(t)] \quad \ldots\text{(i)}$$

Autocorrelation function of $X(t)$ in defined as,

$$R_X(\tau) = E\left[X(t)\cdot X(t+\tau)\right]$$

$$R_X(\tau = 0) = E[X(t)X(t+0)]$$

$$R_X(0) = E[X(t)X(t)] \quad \ldots\text{(ii)}$$

From equations (i) and (ii),

$$P_x = R_X(0) = 2\text{ W}$$

Method 2 :

Given $R_X(\tau)$ can be represented as,

$$R_X(\tau) = 2tri\left(\frac{\tau}{2}\right)$$

Power spectral density of $X(t)$ is related to $R_X(\tau)$ as,

$$S_X(f) = \text{F.T.}\left[R_X(\tau)\right]$$

$$2tri\left(\frac{\tau}{2}\right) \xleftrightarrow{\text{F.T.}} 4\,\text{sinc}^2(2f)$$

$\therefore$ $$S_X(f) = 4\,\text{sinc}^2(2f)$$

$\therefore$ Average power of $X(t)$

$$P_x = \int_{-\infty}^{\infty} S_X(f)\,df = 4\int_{-\infty}^{\infty} \text{sinc}^2(2f)\,df$$

$$P_x = 4\cdot\left[\text{Energy of sinc}(2f)\right] \qquad \ldots\text{(iii)}$$

From standard result,

$$\text{Energy of sinc}(f) = 1$$

Using scaling property,

$$\text{Energy of sinc}(2f) = \frac{1}{2}$$

From equation (iii),

$$P_x = 4\times\frac{1}{2} = 2\,\text{W}$$

Hence, average power of $X(t)$ is 2 W.

Question 39 **Communications System (2M)**

A message signal having peak-to-peak value of 2 V, root mean square value of 0.1 V and bandwidth of 5 kHz is sampled and fed to a pulse code modulation (PCM) system that uses a uniform quantizer. The PCM output is transmitted over a channel that can support a maximum transmission rate of 50 kbps. Assuming that the quantization error is uniformly distributed, the maximum signal to quantization noise ratio that can be obtained by the PCM system (rounded off to two decimal places) is ______.

Ans. **30.72**

Sol. **Given :** Peak to peak value of message signal $m(t)$

$$V_{P-P} = V_H - V_L = 2\,\text{V}$$

RMS value of message signal, $m(t) = 0.1\,\text{V}$

Bandwidth of message signal or maximum frequency component of $m(t)$

$$f_m = 5\,\text{kHz}$$

Bit rate of PCM system $R_b = 50\,\text{kbps}$

$\therefore$ Power in message signal $m(t)$

$$P_s = (\text{RMS value})^2 = (0.1)^2 = 0.01\,\text{W}$$

As nothing in mentioned about sampling frequency, so taking Nyquist rate of sampling, i.e.

$$f_s = 2f_m = 10\,\text{kHz}$$

Bit rate of PCM is, $R_b = nf_s = 50\,\text{kbps}$ ($\because$ $n =$ Number of bits)

$$\therefore \quad n = \frac{R_b}{f_s} = 5$$

Given that the PCM system uses uniform quantization and quantization error is uniformly distributed, so Quantization noise power is given as,

$$P_{nq} = \frac{s^2}{12}$$

Where, $s =$ Step size $= \dfrac{V_{P-P}}{L}$

$L =$ Number of level $= 2^n$

So,

$$P_{nq} = \frac{\left(\dfrac{V_P - V_P}{L}\right)^2}{12}$$

$$P_{nq} = \frac{\left(\dfrac{V_H - V_L}{2^n}\right)^2}{12} = \frac{\left(\dfrac{2}{32}\right)^2}{12} = \frac{4}{32\times32\times12} = \frac{1}{3072}$$

So, the signal to quantization noise power ratio,

$$SN_qR = \frac{P_s}{P_{nq}} = \frac{0.01}{\dfrac{1}{3072}}$$

$$SN_qR = 30.72$$

Hence, the value of signal to quantization noise ratio is 30.72.

Question 40 **Communications System (2M)**

Consider a polar non-return to zero (NRZ) waveform, using + 2 V and – 2 V for representing binary '1' and '0' respectively, is transmitted in the presence of additive zero-mean white Gaussian noise with variance 0.4 V^2. If the a priori probability of transmission of a binary '1' is 0.4, the optimum threshold voltage for a maximum a posteriori (MAP) receiver (rounded off to two decimal places) is _______ V.

Ans. 0.04

Sol. Given a polar non-return to zero waveform, using +2 V and –2 V for representing binary ‘1’ and ‘0’ respectively is transmitted in presence of white Gaussian noise with variance $0.4V^2$.

$$P(1)=0.4, \qquad P(0)=1-P(1)=0.6$$

Average probability of error in given as,

$$P_e = P(0)P\left(\frac{1}{0}\right)+P(1)P\left(\frac{0}{1}\right)$$

$$P_e = P(0)\int_{V_{th}}^{\infty} f\left(\frac{z}{0}\right)dz + P(1)\int_{-\infty}^{V_{th}} f\left(\frac{z}{1}\right)dz$$

For probability of error to be minimum, $\dfrac{dP_e}{dz}=0$.

$$P(0)\left[f\left(\frac{z}{0}\right)\right]_{V_{th}}^{\infty} + P(1)\left[f\left(\frac{z}{1}\right)\right]_{-\infty}^{V_{th}} = 0$$

$$-P(0)\,f\left(\frac{z}{0}\right)\Bigg|_{z=V_{th}} + P(1)\,f\left(\frac{z}{1}\right)\Bigg|_{V_{th}} = 0 \qquad \left[\text{As } f\left(\frac{z}{0}\right)\Bigg|_{z=\infty}=0,\ f\left(\frac{z}{1}\right)\Bigg|_{z=-\infty}=0\right]$$

$$P(0)\,f\left(\frac{z}{0}\right)\Bigg|_{z=V_{th}} = P(1)\,f\left(\frac{z}{1}\right)\Bigg|_{z=V_{th}}$$

$$P(0)\cdot\frac{1}{\sqrt{2\pi\sigma_n^2}}e^{-\frac{(z-a_2)^2}{2\sigma_n^2}}\Bigg|_{z=V_{th}} = P(1)\cdot\frac{1}{\sqrt{2\pi\sigma_n^2}}e^{-\frac{(z-a_1)^2}{2\sigma_n^2}}\Bigg|_{z=V_{th}}$$

where, given $a_1=+2\,\text{V}, a_{12}=-2\,\text{V}$

$$\frac{P(0)}{P(1)} = e^{-\frac{(z-a_2)^2}{2\sigma_n^2}} + e^{-\frac{(z-a_2)^2}{2\sigma_n^2}}\Bigg|_{z=V_{th}}$$

On solving, the optimum threshold for MAP receiver is obtained as,

$$V_{th} = \frac{a_1+a_2}{2}+\frac{\sigma_n^2}{a_1-a_2}\ln\left(\frac{P(0)}{P(1)}\right) \qquad \ldots\text{(i)}$$

Equation (i) may be used as standard equation to find optimum threshold for minimum probability of error $(P_e)_{\min}$ in MAP receiver : Also, if $P(0)=P(1)$ then $V_{th}=\dfrac{a_1+a_2}{2}$.

Substituting value of a_1, a_2, $P(0)$ and $P(1)$ in equation (i),

$$V_{th} = \frac{2+(-2)}{2}+\frac{0.4}{2-(-2)}\ln\left(\frac{0.6}{0.4}\right)$$

$$V_{th} = \frac{1}{10}\ln(1.5) = 0.0405$$

Hence, the minimum probability of error occurs for the optimum threshold of $V_{th} = 0.0405\,\text{V}$.

Question 41 **Communications System (2M)**

A digital transmission system uses a (7, 4) systematic linear Hamming code for transmitting data over a noisy channel. If three of the message code-word pairs in this code $(\boldsymbol{m_i}; \boldsymbol{c_i})$, where, $\boldsymbol{c_i}$ is the code-word corresponding to the i^{th} message $\boldsymbol{m_i}$, are known to be $(1100; 0101100)$, $(1110; 0011110)$, $(0110; 1000110)$, then which of the following is a **valid code-word** in this code?

(A) 1101001 (B) 1011010 (C) 0001011 (D) 0110100

Ans. C

Sol. Given message and corresponding code-words are,

Message					Code-word							
$\boldsymbol{m_i}$	$\boldsymbol{m_1}$	$\boldsymbol{m_2}$	$\boldsymbol{m_3}$	$\boldsymbol{m_4}$	$\boldsymbol{P_1}$	$\boldsymbol{P_2}$	$\boldsymbol{P_3}$	$\boldsymbol{m_1}$	$\boldsymbol{m_2}$	$\boldsymbol{m_3}$	$\boldsymbol{m_4}$	$\boldsymbol{C_i}$
m_{12}	1	1	0	0	0	1	0	1	1	0	0	C_{12}
m_{14}	1	1	1	0	0	0	1	1	1	1	0	C_{14}
m_6	0	1	1	0	1	0	0	0	1	1	0	C_6

Here, m_{12} means, $m_1\, m_2\, m_3\, m_4 = 1100$ and its decimal equivalent is 12

Here, m_{14} means, $m_1\, m_2\, m_3\, m_4 = 1110$ and its decimal equivalent is 14

Here, m_6 means, $m_1\, m_2\, m_3\, m_4 = 0110$ and its decimal equivalent is 6

From four parity generator we have to select any three equation,

$$P_A = m_1 \oplus m_2 \oplus m_3$$

$$P_B = m_1 \oplus m_2 \oplus m_4$$

$$P_C = m_2 \oplus m_3 \oplus m_4$$

$$P_D = m_1 \oplus m_3 \oplus m_4$$

From comparing with all three given code-words i.e., C_{12}, C_{14} and C_6

$$P_1 = P_B,\ P_2 = P_C,\ P_3 = P_A$$

So, for message $m_9 = 1001,\ C_9 = 0111001$

Option (A) is wrong.

$$m_{10} = 1010,\ C_{10} = 1101010$$

Option (B) is wrong.

$$m_{11} = 1011,\ C_{11} = 0001011$$

Option (C) is correct.

$$m_4 = 0100,\ C_4 = 1110100$$

Option (D) is wrong.

Hence, the correct option is (C).

Question 42 **Communications System (1M)**

A 4 kHz sinusoidal message signal having amplitude 4 V is fed to a delta modulator (DM) operating at a sampling rate of 32 kHz. The minimum step size required to avoid slope overload noise in the DM (rounded off to two decimal places) is _______ V.

Ans. 3.14

Sol. Given : Message signal frequency, $f_m = 4\,\text{kHz}$

Message signal amplitude, $A_m = 4\,\text{V}$

Sampling rate, $f_s = 32\,\text{kHz}$

Assume message signal, $m(t) = A_m \cos 2\pi f_m t$

Slope of $m(t) = \dfrac{d}{dt} m(t) = (2\pi f_m) A_m \cos 2\pi f_m t$

Maximum slope of $m(t)$ is,

$$\left.\frac{d}{dt} m(t)\right|_{\max} = 2\pi f_m A_m \qquad [\because \text{Maximum value of } (\cos 2\pi f_m t) = 1] \ \ldots\text{(i)}$$

To avoid slope overload distortion,

$$s f_s \ge \left.\frac{d}{dt} m(t)\right|_{\max} \qquad \text{where, } s = \text{Step size.}$$

$$s f_s \ge 2\pi f_m A_m$$

$$s \ge \frac{2\pi f_m A_m}{f_s}$$

So, minimum value of step size s is,

$$s_{\min} = \frac{2\pi f_m A_m}{f_s}$$

$$s_{\min} = \frac{2\pi \times 4 \times 4}{32} = \pi = 3.14\,\text{V}$$

Hence, the minimum step size required to avoid slope overload is 3.14 V.

Question 43 **Communications System (1M)**

A speech signal, band limited to 4 kHz, is sampled at 1.25 times the nyquist rate. The speech samples, assumed to be statistically independent and uniformly distributed in the range – 5 V to + 5 V, are subsequently quantized in an 8-bit uniform quantizer and then transmitted over a voice-grade AWGN telephone channel. If the ratio of transmitted signal power to channel noise power is 26 dB, the minimum channel bandwidth required to ensure reliable transmission of the signal with arbitrarily small probability of transmission error (rounded off to two decimal places) is _______ kHz.

Ans. 9.26

Sol. **Given :**

Speech signal/message signal frequency, $f_m = 4$ kHz

Sampling frequency, $f_s = 1.25 f_{NR}$

Nyquist rate $f_{NR} = 2f_m = 2\times4 = 8$ kHz

Thus, sampling frequency, $f_s = 1.25 f_{NR} = 1.25\times8$ kHz $= 10$ kHz

Speech samples are uniformly distributed in the range -5 V to 5 V and subsequently quantized in 8-bit uniform quantizer and transmitted over AWGN channel, so number of bits used by quantizer,

i.e., $n = 8$

Bit rate is given as, $R_b = nf_s = 8\times10 = 8$ kbps

According to channel capacity theorem,

$$(R_b)_{\max} \le C$$

$$(R_b)_{\max} = C = B\log_2(1+SNR)$$

$$80\times10^3 = B\log_2(1+SNR) \qquad \ldots\text{(i)}$$

Here, $(SNR)_{dB} = 26$ dB

$$10\log_{10}(SNR) = 26$$

$$SNR = (10)^{2.6} = 398.1$$

From equation (i),

$$80\times10^3 = B\log_2(1+398.1)$$

$$\Rightarrow \quad B = \frac{80\times10^3}{\log_2(399.1)} = 9.259 \text{ kHz} \approx 9.26 \text{ kHz}$$

Hence, minimum bandwidth required for reliable transmission is 9.259 kHz.

Question 44 **Communications System (2M)**

A sinusoidal message signal having root mean square value of 4 V and frequency of 1 kHz is fed to phase modulator with phase deviation constant 2 rad/volt. If the carrier signal is $c(t) = 2\cos(2\pi10^6 t)$, the maximum instantaneous frequency of the phase modulated signal (rounded off to one decimal place) is ________ Hz.

Ans. **1011313.71**

Sol. **Given :**

Message signal frequency, $f_m = 1$ kHz

Carrier signal, $c(t) = 2\cos(2\pi10^6 t)$

Phase sensitivity factor, $k_P = 2$ rad/V

Assuming message signal, $m(t) = A_m\sin(2\pi f_m t)$

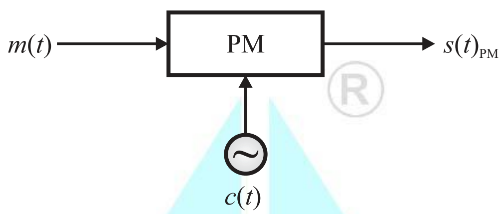
RMS value of message signal $m(t)$ is,

$$m(t)\big|_{\text{RMS}} = \frac{A_m}{\sqrt{2}} = 4\text{ V}$$

$$A_m = 4\sqrt{2}\ \text{Volt,}$$

So message signal $m(t)$ is,

$$m(t) = A_m \cos 2\pi f_m t = 4\sqrt{2}\cos 2\pi 10^3 t\text{V}$$



So, phase modulation equation is,

$$s(t)_{PM} = A_C \cos\Big[2\pi f_c t + \underbrace{k_p m(t)}_{\theta_i(t)}\Big]$$

Instantaneous frequency is given as,

$$f_i(t)\big|_{PM} = f_c + \frac{1}{2\pi}\frac{d}{dt}\theta_i(t)$$

$$f_i(t)\big|_{PM} = f_c + \frac{1}{2\pi}\frac{d}{dt}[k_p m(t)]$$

$$f_i(t)\big|_{PM} = f_c + \frac{k_p}{2\pi}\frac{d}{dt}(4\sqrt{2}\sin 2\pi.10^3 t)$$

$$f_i(t)\big|_{PM} = f_c + \frac{2}{2\pi}4\sqrt{2}(2\pi.10^3)(\cos 2\pi.10^3 t)$$

$$f_i(t)\big|_{PM} = f_c + 8\sqrt{2}\times 10^3 \cos 2\pi.10^3 t$$

So, maximum value of instantaneous frequency is,

$$f_i(t)\big|_{\max} = [1000 + 8\sqrt{2}]\times 10^3$$

$$f_i(t)\big|_{\max} = 1011.3137\times 10^3$$

$$f_i(t)\big|_{\max} = 1011313.7\text{ Hz}$$

Hence, the maximum instantaneous frequency of the phase modulated signal is **1011313.7 Hz**.

Question 45 **Communications System (1M)**

Consider a carrier signal which is amplitude modulated by a single-tone sinusoidal message signal with a modulation index of 50%. If the carrier and one of the sidebands are suppressed in the modulated signal, the percentage of power saved (rounded off to one decimal place) is _______.

Ans. **94.44**

Sol. Given signal is single tone sinusoidal message signal with modulation index,

$$\%m_a = 50\%$$

$m_a = 0.5$

According to question, carrier and one side band is suppressed, so saved power is,

Power saved = Carrier Power + One side band Power

$$= P_c + P_c \frac{m_a^2}{4} = P_c\left[1 + \frac{m_a^2}{4}\right]$$

Total power of AM signal $= P_c\left[1 + \frac{m_a^2}{2}\right]$

So, percentage saving in power is given as,

$$\%\text{ Power saving } = \frac{\text{Power saved}}{\text{Total power}} \times 100\%$$

$$\%\text{ Power saving } = \frac{P_c\left[1 + \frac{m_a^2}{4}\right]}{P_c\left[1 + \frac{m_a^2}{2}\right]} \times 100\%$$

$$\%\text{ Power saving} = \frac{1 + \left(\frac{1}{2}\right)^2 \frac{1}{4}}{1 + \left(\frac{1}{2}\right)^2 \frac{1}{2}} \times 100\%$$

$$\%\text{ Power saving } = \frac{1 + \frac{1}{16}}{1 + \frac{1}{8}} \times 100\%$$

$$\%\text{ Power saving} = 94.44\%$$

Hence, the percentage of power saved is 94.44%.

Question 46 **Communications System (2M)**

Consider a super heterodyne receiver tuned to 600 kHz. If the local oscillator feeds a 1000 kHz signal to the mixer, the image frequency (in integer) is _______ kHz.

Ans. **1400**

Sol. **Given :**

$f_{RF} = 600\,\text{kHz}$

$f_{LO} = 1000\,\text{kHz}$

For $f_{LO} > f_{RF}$, image frequency is given by,

$$f_{image} = f_{RF} + 2 f_{IF}$$

$$f_{image} = 2 f_{LO} - f_{RF} \qquad (\because\ f_{IF} = f_{LO} - f_{RF})$$

$$f_{image} = 2\times1000-600 = 1400 \text{ kHz}$$

Hence, the image frequency is 1400 kHz.

Question 47 **Engineering Mathematics (2M)**

In a high school having equal number of boy students and girl students. 75% of students study science and remaining 25% students study commerce. Commerce students are two times more likely to be a boy than are science students. The amount of information gained in knowing the a randomly selected girl student studies commerce (rounded off to 3 decimal places) is __________ bits.

Ans. **3.322**

Sol. **Method 1 :**

Given :

50% of the total students are boys so probability of boys $P(B)=\frac{1}{2}$

50% of the total students are girls so probability of girls $P(G)=\frac{1}{2}$

75% students studies science, so probability of student studies science, $P(S)=\frac{3}{4}$

25% students studies commerce, so probability of student studies commerce, $P(C)=\frac{1}{4}$

Given commerce students are two times more likely to be a boy than are science students.

Let the probability that a science student is boy i.e., $P\left(\frac{B}{S}\right)=p$.

Then as given, the probability that a commerce student is boy i.e., $P\left(\frac{B}{C}\right)=2p$.

From total probability theorem, the probability of a selected student to be a boy is

$$P(B)=P(C)P\left(\frac{B}{C}\right)+P(S)P\left(\frac{B}{S}\right)$$

$$\frac{1}{2}=\left(\frac{1}{4}\times2p\right)+\left(\frac{3}{4}\times p\right)$$

$$\frac{1}{2}=\frac{5p}{4}$$

$$p=\frac{2}{5}$$

So, $P\left(\frac{B}{S}\right)=p=\frac{2}{5}$ and $P\left(\frac{B}{C}\right)=2p=\frac{4}{5}$

Hence probability of a selected commerce student is a girl,

$$P\left(\frac{G}{C}\right)=1-P\left(\frac{B}{C}\right)=1-\frac{4}{5}=\frac{1}{5}$$

Probability of randomly selected girl studies commerce is given as

$$P\left(\frac{C}{G}\right)=\frac{P(G\cap C)}{P(G)}=\frac{P(C)P\left(\frac{G}{C}\right)}{P(G)}$$

$$\Rightarrow \quad P\left(\frac{C}{G}\right)=\frac{\frac{1}{4}\times\frac{1}{5}}{\frac{1}{2}}=\frac{2}{20}=\frac{1}{10}$$

Hence, information in knowing that a randomly selected girl studies commerce is given as

$$I=\log_2\left[\frac{1}{P\left(\frac{C}{G}\right)}\right]\text{bits}=\log_2 10=3.32 \text{ bits}$$

Hence, the amount of information gained in knowing the a randomly selected girl student studies commerce is 3.32 bits.

Method 2 :

Let total number of students is 100.

Given, there are equal number of boys and girl students.

$\therefore$ Total number of boys = 50

Total number of girls = 50

Given commerce students are two times more likely to be a boy than are science students.

Let Probability of science student to be boy = p.

$\therefore$ Probability of science students to be girl $=1-p$

p is defined as,

$$p=\frac{\text{Number of boys in science}}{\text{Total students in science}}=\frac{B_s}{75}$$

$\therefore$ Number of boys in science $=B_s=75p$

$\therefore$ Number of girls in science $=75-75p$

Number of boys in commerce $=50-75p$

Number of girls in commerce $=25-(50-75p)=75p-25$

Given, $\dfrac{50-75p}{25}=2p$

$50-75p=50p$

$125p=50$

$$p=\frac{50}{125}=\frac{2}{5}$$

$\therefore$ Number of girls in commerce $= 75p - 25 = 75 \times \frac{2}{5} - 25 = 30 - 25 = 5$

$\therefore$ Probability that if a randomly selected student out of 100 students is a girl then she studies commerce is,

$$P\left(\frac{C}{G}\right) = \frac{\text{Number of girls in commerce}}{\text{Total girl students}}$$

$$= \frac{5}{50} = \frac{1}{10}$$

Hence, Information in knowing that a randomly selected student is girl and studies commerce is

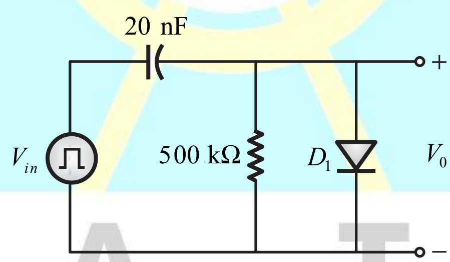
$$I = \log_2 \frac{1}{P\left(\frac{G}{C}\right)} \text{ bits}$$

$$I = \log_2 10 = 3.32 \text{ bits}$$

Hence, the amount of information gained in knowing the a randomly selected girl student studies commerce is 3.32 bits.

Question 48 **Analog Electronics (2M)**

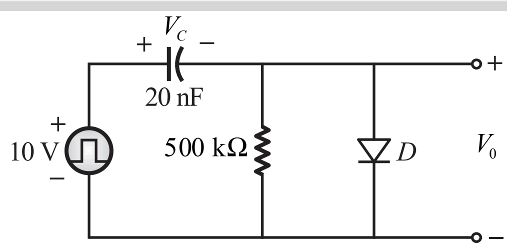
An asymmetrical periodic pulse train V_{in} of 10 V amplitude with on-time $T_{ON} = 1$ ms and off-time $T_{OFF} = 1$ μs is applied to the circuit shown in figure. The diode D_1 is ideal.

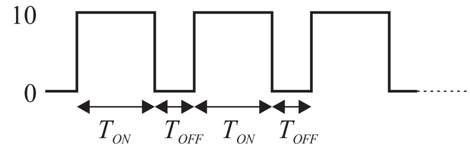


The difference between the maximum voltage and minimum voltage of the output wave form V_0 (in integer) is _____V.

Ans. **10**

Sol. Given circuit and asymmetrical periodic input V_{in} is shown below,

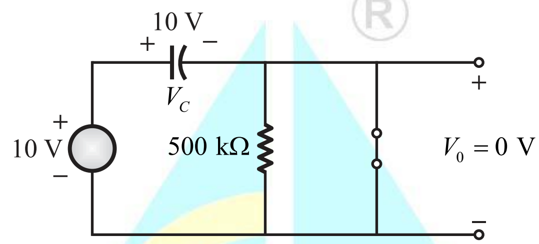




Here, $T_{ON} = 1$ msec, $T_{OFF} = 1$ μsec and initially capacitor C is uncharged and diode D is ideal diode.

(i) For $0 < t < T_{ON}$:

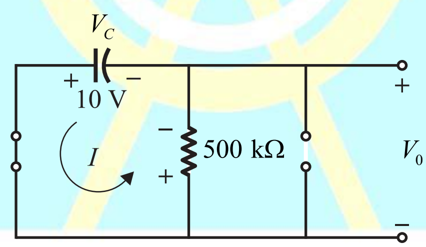
Maximum value of V_{in} is 10 V and diode D is ON and will be short circuit, so capacitor will get charged instantly to 10 V as shown below,



Thus, capacitor voltage $V_C = 10$ V and voltage $V_0 = 0$ volt.

(ii) For $T_{ON} < t < T_{OFF}$:

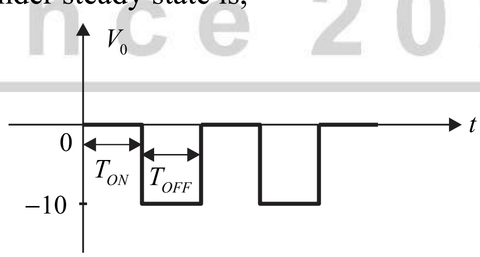
Given input voltage $V_{in} = 0$ it means short circuit and diode is OFF and will be open circuit, so circuit becomes as,



Current, $I = \frac{-10}{500} = -20$ mA

Voltage, $V_0 = I \times 500 = (-20) \times 500 = -10$ V

Thus output waveform under steady state is,



Here, V_0 (maximum) = 0 V, V_0 (minimum) = – 10 V

Thus, difference between maximum and minimum output voltages,

$V_0(\text{maximum}) - V_0(\text{minimum}) = 0 - (-10) = +10$ V

Hence, the difference between the maximum voltage and minimum voltage of the output wave form V_0 is + 10 V.

Question 49 **Engineering Mathematics (1M)**

If the vectors (1.0, –1.0, 2.0) (7.0, 3.0, x) (2.0, 3.0, 1.0) in $\mathbb{R}^3$ are linearly dependent, the value of x is _____.

Ans. **8**

Sol. **Given :**

According to Question given vectors are (1.0, –1.0, 2.0), (7.0, 3.0, x) and (2.0, 3.0, 1.0) are linearly dependent.

We know that the determinant of linearly dependent vectors are zero.

$$\begin{vmatrix} 1 & -1 & 2 \\ 7 & 3 & x \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$1(3-3x)+1(7-2x)+2(21-6)=0$$

$$3-3x+7-2x+30=0$$

$$5x=40$$

$$x=8$$

Hence, the correct answer is 8.

Question 50 **Electronic Devices (2M)**

For an n channel silicon MOSFET with 10 nm gate oxide thickness, the substrate sensitivity $\left(\partial V_T / \partial |V_{BS}|\right)$ is found to be 50 mV/V at a substrate voltage $|V_{BS}| = 2V$, where V_T is the threshold voltage of the MOSFET. Assume that $|V_{BS}| >> 2\phi_B$, where $q\phi_B$ is the separation between the fermi energy level E_F and the intrinsic level E_i in the bulk. Parameters given are :

Electron charge $(q) = 1.6\times10^{-19} C$

Vacuum permittivity $(\varepsilon_0) = 8.85\times10^{-12}$ F / m

Relative permittivity of silicon $(\varepsilon_{si}) = 12$

Relative permittivity of oxide $(\varepsilon_{ox}) = 4$

The doping concentration of the substrate is

(A) $7.37\times10^{15}\text{cm}^{-3}$

(B) $4.37\times10^{15}\text{cm}^{-3}$

(C) $2.37\times10^{15}\text{cm}^{-3}$

(D) $9.37\times10^{15}\text{cm}^{-3}$

Ans. **A**

Sol. **Given :**

Gate-oxide thickness, $t_{ox} = 10$ nm

Substrate sensitivity, $\dfrac{\partial V_T}{\partial |V_{BS}|} = 50 \text{ mV/V}$

Substrate voltage, $|V_{BS}| = 2 \text{ V}$

Electron charge $(q) = 1.6\times10^{-19} C$

Vacuum permittivity $(\varepsilon_0) = 8.85\times10^{-12} \text{F / m}$

Relative permittivity of silicon $(\varepsilon_{si}) = 12$

Relative permittivity of oxide $(\varepsilon_{ox}) = 4$

Threshold voltage when body effect is consider,

$$V_T = V_{To} + \gamma\left[\sqrt{|2\phi_F| + |V_{SB}|} - \sqrt{|2\phi_F|}\right] \quad \ldots\text{(i)}$$

Where, $\gamma = \dfrac{\sqrt{2qN_A\varepsilon_{si}}}{C_{ox}} = \dfrac{\sqrt{2qN_A\varepsilon_{si}}}{\varepsilon_{ox}} t_{ox}$ $\quad \left[\because \; C_{ox} = \dfrac{\varepsilon_{ox}}{t_{ox}}\right]$

As per the question, $\phi_F = \phi_B \rightarrow$ Both are same

Differentiate equation (i), with respect to V_{BS},

$$\frac{\partial V_T}{\partial V_{BS}} = 0 + \gamma\frac{1}{2}\left(|2\phi_F| + |V_{SB}|\right)^{-\frac{1}{2}} \quad \ldots\text{(ii)}$$

Since $|\text{V}_{\text{SB}}| >> |2\phi_\text{F}|$, so equation (ii) becomes as,

$$\frac{\partial \text{V}_\text{T}}{\partial \text{V}_\text{BS}} = \gamma\left(\frac{1}{2}\right)\left(|\text{V}_\text{SB}|\right)^{-\frac{1}{2}}$$

$$50\times10^{-3} = \gamma\left(\frac{1}{2}\right)(2)^{-\frac{1}{2}}$$

$$50\times10^{-3} = \gamma\frac{1}{2\sqrt{2}}$$

$$\gamma = 2\sqrt{2}\times50\times10^{-3} = 0.14142$$

As $\quad \gamma = \sqrt{2qN_A\varepsilon_{si}}\left[\dfrac{t_{ox}}{\varepsilon_{ox}}\right]$

$$\sqrt{2qN_A\varepsilon_{si}} \times \frac{t_{ox}}{\varepsilon_{ox}} = 0.14142$$

$$2q\,N_A\,\varepsilon_{si} = \left[0.14142\times\left(\frac{\varepsilon_{ox}}{t_{ox}}\right)\right]^2$$

$$N_A = \frac{1}{2q\varepsilon_{si}}\left[0.14142\times\left(\frac{\varepsilon_{ox}}{t_{ox}}\right)\right]^2$$

$$N_A = \frac{1}{2q(12\varepsilon_0)} \times \left[0.14142 \times \left(\frac{4\varepsilon_0}{t_{ox}}\right)\right]^2 \qquad (\because\ \varepsilon_{si} = 12\varepsilon_0,\ \varepsilon_{ox} = 4\varepsilon_0)$$

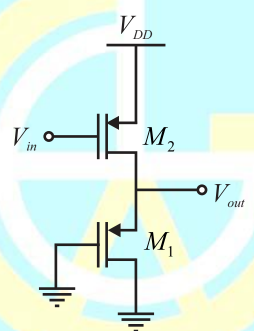
$$N_A = \frac{(0.14142)^2 \times \left(4 \times 8.85 \times 10^{-14}\right)^2}{2 \times 1.6 \times 10^{-19} \times 12 \times 8.55 \times 10^{-14} \times \left(10 \times 10^{-9} \times 100\right)^2}$$

$$N_A = 7.37 \times 10^{15}\ \text{cm}^{-3}$$

Hence, the correct option is (A).

Question 51 **Analog Electronics (1M)**

In the circuit shown in the figure, the transistors M_1 and M_2 are operating in saturation. The channel length modulation coefficients of both the transistors are non-zero. The transconductance of the MOSFETs M_1 and M_2 are g_{m1} and g_{m2}, respectively, and the internal resistance of the MOSFETs M_1 and M_2 are r_{01} and r_{02}, respectively.



Ignoring the body effect, the ac small signal voltage gain $\left(\partial V_{out} / \partial V_{in}\right)$ of the circuit is

(A) $-g_{m2}\left(r_{01} \,||\, r_{02}\right)$

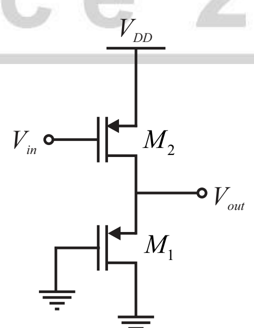
(B) $-g_{m2}\left(\frac{1}{g_{m1}} \,||\, r_{02}\right)$

(C) $-g_{m1}\left(\frac{1}{g_{m2}} \,||\, r_{01} \,||\, r_{02}\right)$

(D) $-g_{m2}\left(\frac{1}{g_{m1}} \,||\, r_{01} \,||\, r_{02}\right)$

Ans. **D**

Sol. Given circuit is shown below,



Method 1 :

Channel length modulation coefficient $\lambda \neq 0$, it means internal resistance of MOSFET M_1 and M_2 are not zero.

i.e. $r_{01} \neq 0$, $r_{02} \neq 0$

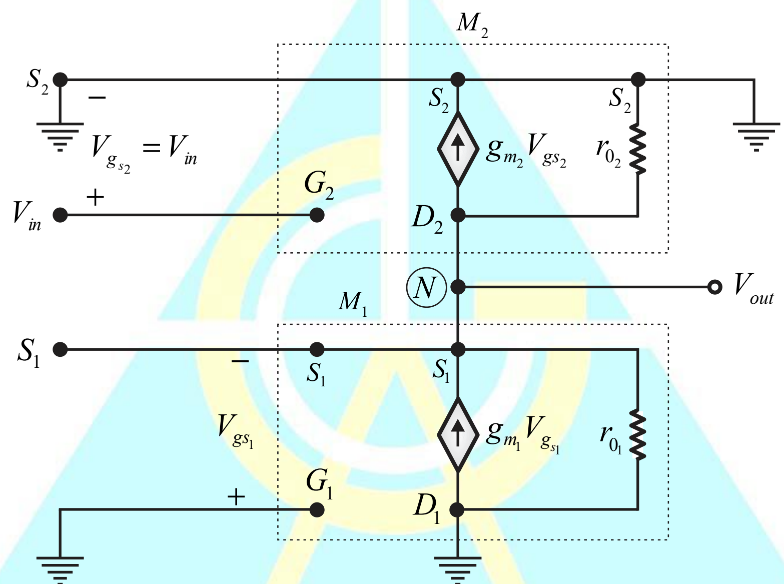
From figure $V_{g_1} = 0$ V, $V_{s_2} = 0$ V and $V_{d_1} = 0$ V,

$V_{s_1} = V_{d_2} = V_{out}$, $V_{g_2} = V_{in}$.

So, $V_{gs_1} = V_{g_1} - V_{s_1} = 0 - V_{out} = -V_{out}$

$V_{gs_2} = V_{g_2} - V_{s_2} = V_i - 0 = V_i$

The AC equivalent model of given circuit is shown below,



Applying KCL at node N,

$$\frac{V_{out}}{r_{01}} - g_{m_1}V_{gs_1} + g_{m_2}V_{gs_2} + \frac{V_{out}}{r_{02}} = 0 \quad \text{...(i)}$$

Here, $$V_{gs_1} = -V_{out} \quad \text{...(ii)}$$

$$V_{gs_2} = V_{in} \quad \text{...(iii)}$$

So, put equation (ii) and (iii) into equation (i),

$$V_{out}\left[\frac{1}{r_{01}} + \frac{1}{r_{02}} + g_{m_1}\right] = -g_{m_2}V_i$$

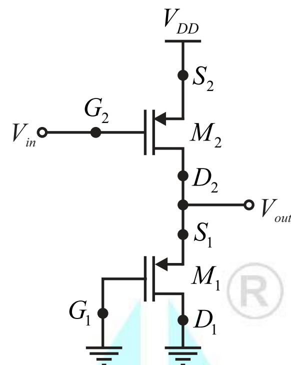
So, $$\frac{V_{out}}{V_i} = -g_{m_2}\left[\frac{1}{g_{m_1}} \parallel r_{01} \parallel r_{02}\right]$$

Hence, $$\frac{\partial V_{out}}{\partial V_i} = -g_{m_2}\left[\frac{1}{g_{m_1}} \parallel r_{01} \parallel r_{02}\right]$$

Hence, the correct option is (D).

Method 2 :

Given MOSFET arrangement is shown below,

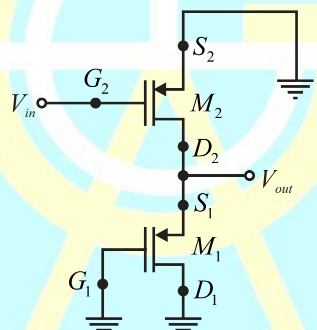


Channel length modulation coefficient $\lambda \neq 0$, it means internal resistance of MOSFET M_1 and M_2 are not zero i.e., $r_{01} \neq 0$, $r_{02} \neq 0$.

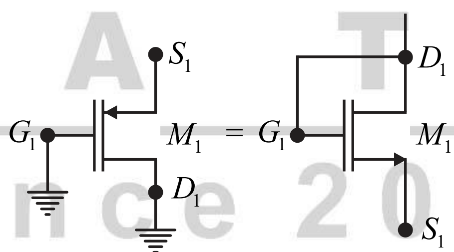
AC equivalent circuit :

(i) All capacitors are short circuited.

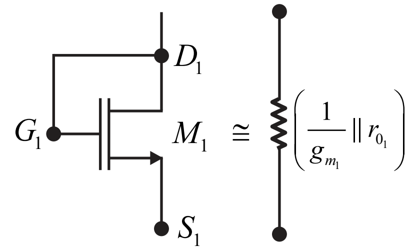
(ii) All DC voltage sources replaced by ground.



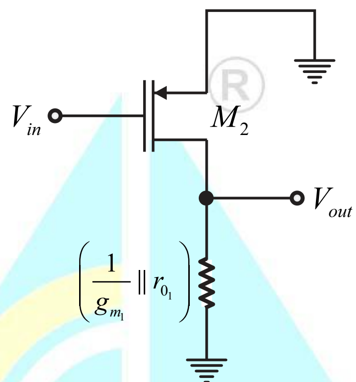
Here, drain (D_1) and gate (G_1) of MOSFET (M_1) are grounded, so it can be drawn as,



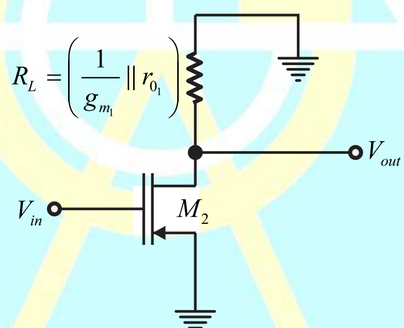
When drain and gate of MOSFET M_1 are shorted to each other then it can be replaced by resistance of value $\left(\dfrac{1}{gm_1} \| r_{01}\right)$ as shown below,



So, given circuit becomes as,



Above MOS circuit can redraw as,



Here, $\frac{1}{g_{m_1}} \| r_{01} = R_L$ work as load for MOSFET (M_2), so small signal voltage gain of MOSFET (M_2) is

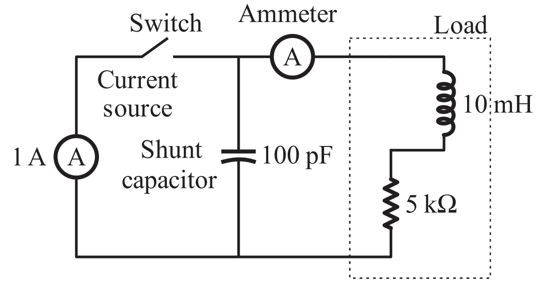
$$(A_V)_{M_2} = -g_{m_2}(R_L \| r_{0_2})$$

$$(A_V)_{M_2} = -g_{m_2}\left[\left(\frac{1}{g_{m_1}} \| r_{0_1}\right) \| r_{0_2}\right]$$

Hence, the correct option is (D).

Question 52 **Network Theory (2M)**

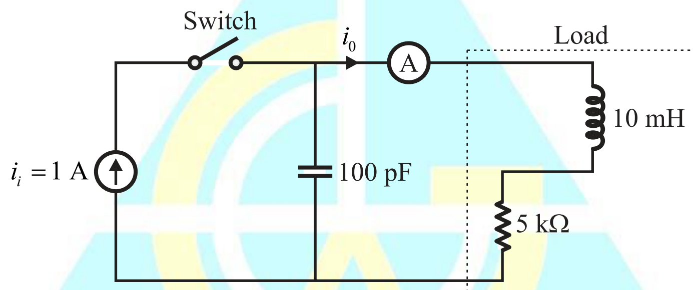
The circuit in the figure contains a current source driving a load having an inductor and a resistor in series, with a shunt capacitor across the load. The ammeter is assumed to have zero resistance. The switch is closed at time $t = 0$.



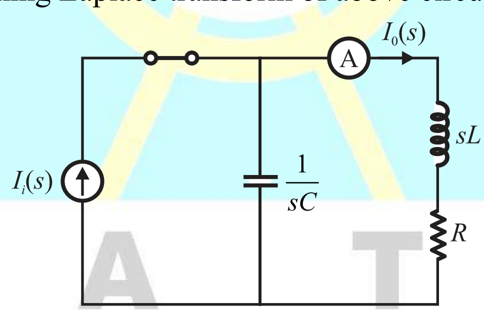
Initially when the switch is open, the capacitor is discharged and ammeter reads zero ampere. After the switch is closed, the ammeter reading keeps fluctuating for some time till it settles to a final steady value. The maximum ammeter reading that one will observe after the switch is closed (rounded off to two decimal places) is _______ A.

Ans. **1.44**

Sol. The given circuit is shown below,



When switch is closed and taking Laplace transform of above circuit,



Given : $L = 10$ mH, $C = 100$ pF, $R = 5$ kΩ.

Applying current divider rule in the above circuit,

$$I_0(s) = \frac{I_i(s) \times \left(\frac{1}{sC}\right)}{\frac{1}{sC} + R + sL}$$

$$I_0(s) = \frac{I_i(s)}{LCs^2 + RCs + 1}$$

$$I_0(s)=\frac{I_i(s)\times\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

$$\frac{I_0(s)}{I_i(s)}=\frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}} \quad \ldots \text{(i)}$$

Equation (i) shows current transfer function, so we can compare denominator of equation (i) with characteristic equation of second order system $s^2+2\xi\omega_n s+\omega_n^2=0$, then we get

(i) $\omega_n^2=\frac{1}{LC}$

$$\omega_n^2=\frac{1}{10\times10^{-3}\times100\times10^{-12}}$$

$$\omega_n=\sqrt{10^{12}}=10^6 \text{ rad/sec}$$

(ii) $2\xi\omega_n=\frac{R}{L}$

$$2\xi\times10^6=\frac{5000}{10\times10^{-3}}$$

$$\xi=\frac{1}{4}=0.25$$

Here $\xi<1$ so system is underdamped system.

(iii)Steady state value of $i_0(t)$ is,

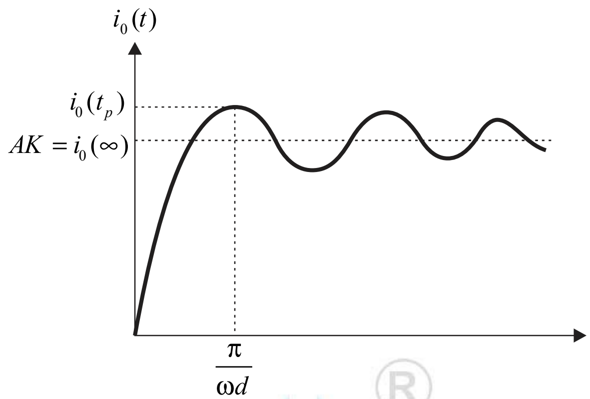
$$\lim_{t\to\infty} i_0(t)=i_0(\infty)=\lim_{s\to0} sI_0(s)$$

$$\lim_{t\to\infty} i_0(t)=i_0(\infty)=\lim_{s\to0} s\left(\frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}\right)I_i(s)$$

$$\lim_{t\to\infty} i_0(t)=i_0(\infty)=\lim_{s\to0} s\left(\frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}\right)\left(\frac{1}{s}\right)$$

$$\lim_{t\to\infty} i_0(t)=i_0(\infty)=1$$

Input $I_i(s)=\frac{1}{s}$ is the step function and $\xi<1$, so response $i_0(t)$ will be an underdamped response for unit step signal as shown below,



So, peak value of $i_0(t)$ is,

$$i_0(t_p) = AK\left[1 + e^{-\pi\cot\theta}\right] \quad \ldots\text{(ii)}$$

Where, $AK =$ Steady state value of $i_0(t)$

$$AK = i_0(\infty) = 1$$

Thus, $$i_0(t_p) = 1 + e^{-\pi\cot\theta} \quad \ldots\text{(iii)}$$

$$\cos\theta = \xi = 0.25$$

$$\theta = \cos^{-1}(0.25) = 75.52^0$$

$$\cot(75.52^0) = 0.258$$

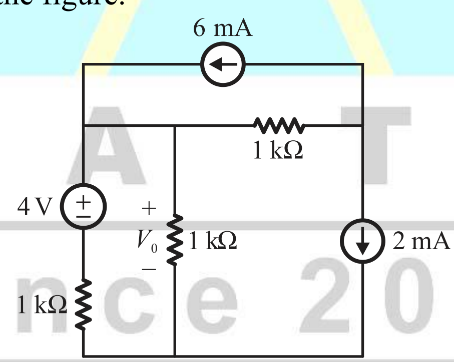
So, equation (iii) becomes

$$i_0(t_p) = 1 + e^{-\pi\times 0.258} = 1.44$$

Hence, the maximum ammeter reading that one will observe after the switch is closed is1.44.

Question 53 **Network Theory (1M)**

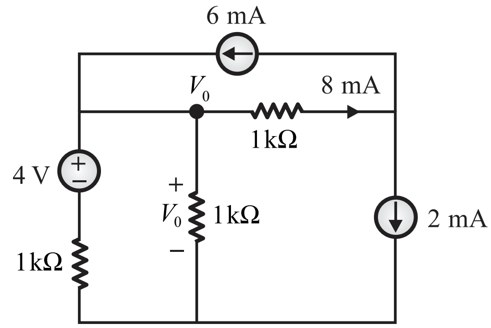
Consider the circuit shown in the figure.



The value of V_0 (rounded off to 1 decimal place) is _______ V.

Ans. 1

Sol. Given circuit is shown below,



Nodal analysis at node V_0

$$-6+\frac{V_0-4}{1}+\frac{V_0}{1}+8=0$$

$$2V_0-10+8=0$$

$$2V_0=2$$

$$V_0=1\text{V}$$

Hence, the value of V_0 is 1 V.

Question 54 **Electromagnetic Theory (2M)**

A standard air filled rectangular waveguide with dimensions $a=8$ cm , $b=4$ cm operates at 3.4 GHz. For the dominant mode of wave propagation, the phase velocity of the signal is v_p . The value (rounded off to two decimal places) of v_p/c , where c denotes the velocity of light is _______.

Ans. 1.198

Sol. **Given :** $a=8$ cm, $b=4$ cm,

For a standard rectangular waveguide $a>b$

So, cut-off frequency for dominant mode (TE_{10}) is

$$f_C=\frac{c}{2a}$$

$$f_C=\frac{3\times10^8}{2\times8\times10^{-2}}=\frac{30\times10^7}{16\times10^{-2}}=1.875\times10^9\text{ Hz}$$

$$f_C=1.875\text{ GHz}$$

Given operating frequency is, $f=3.4$ GHz

So, phase velocity in rectangular waveguide,

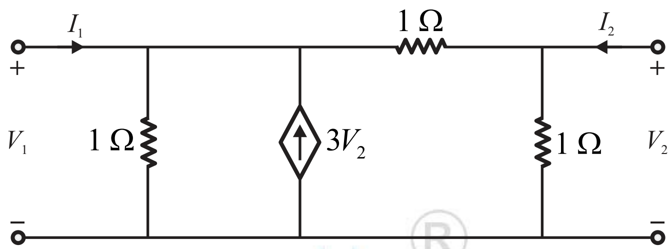
$$v_p=\frac{c}{\sqrt{1-\left(\frac{f_C}{f}\right)^2}}$$

$$\frac{v_p}{c}=\frac{1}{\sqrt{1-\left(\frac{1.875}{3.4}\right)^2}}=1.198$$

Hence, the value of v_p / c is 1.198.

Question 55 **Network Theory (2M)**

Consider the two port network shown in the figure.



The admittance parameters, in Siemens, are

(A) $Y_{11} = 2,\ Y_{12} = -4,\ Y_{21} = -4,\ Y_{22} = 2$

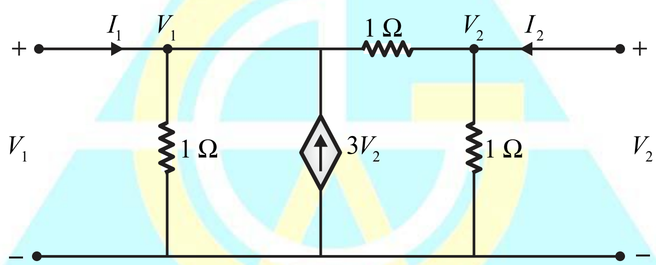
(B) $Y_{11} = 1,\ Y_{12} = -2,\ Y_{21} = -1, Y_{22} = 3$

(C) $Y_{11} = 2,\ Y_{12} = -4,\ Y_{21} = -1,\ Y_{22} = 2$

(D) $Y_{11} = 2,\ Y_{12} = -4,\ Y_{21} = -4,\ Y_{22} = 3$

Ans. **C**

Sol. Given circuit is shown below



Nodal analysis at node V_1

$$-I_1 + \frac{V_1}{1} - 3V_2 + \frac{V_1 - V_2}{1} = 0$$

$$I_1 = V_1 - 3V_2 + V_1 - V_2$$

$$I_1 = 2V_1 - 4V_2$$

Compare I_1 with 1st equation of Y-parameter is,

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

Thus, $Y_{11} = 2$ ℧, $Y_{12} = -4$ ℧

Nodal analysis at node V_2

$$-I_2 + \frac{V_2}{1} + \frac{V_2 - V_1}{1} = 0$$

$$I_2 = 2V_2 - V_1$$

$$I_2 = -V_1 + 2V_2$$

Compare I_2 with 2nd equation of Y-parameter is,

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Thus, $Y_{21} = -1$ ℧, $Y_{22} = 2$ ℧

Thus admittance parameter of given network is,

$$[Y]=\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$$

Hence, the correct option is (C).

General Aptitude

Question 56 **General Aptitude (1M)**

p and q are positive integers and $\frac{p}{q}+\frac{q}{p}=3$, then, $\frac{p^2}{q^2}+\frac{q^2}{p^2}=$

(A) 3 (B) 7 (C) 9 (D) 11

Ans. **B**

Sol. **Given :** $\frac{p}{q}+\frac{q}{p}=3$

Squaring on both sides, we get

$$\frac{p^2}{q^2}+\frac{q^2}{p^2}+2\frac{p}{q}\frac{q}{p}=9$$

$$\frac{p^2}{q^2}+\frac{q^2}{p^2}+2=9$$

$$\frac{p^2}{q^2}+\frac{q^2}{p^2}=9-2=7$$

Hence, the correct option is (B).

Question 57 **General Aptitude (1M)**

The current population of a city is 11,02,500. If it has been increasing at the rate of 5% per annum, what was its population 2 years ago?

(A) 9, 92, 500 (B) 9, 95, 006 (C) 10, 00, 000 (D) 12, 51, 506

Ans. **C**

Sol. **Given :**

Current population of city = 1102500

Rate of increment of population = 5% per year

Method 1 :

Let the population of the city 2 years ago be x. As the population has been increasing at the rate of 5% per annum, so after 2 years x becomes as,

$$(x\times 105\%)\times 105\% = 1102500$$

$$x\times\frac{105}{100}\times\frac{105}{100}=1102500$$

$$x=\frac{1102500\times 100\times 100}{105\times 105}$$

$$x = 10,00,000$$

Hence, the correct option is (C).

Method 2 :

Suppose 2 years ago population of city was "100"

$\therefore$

100

5% inc.

After 1^{st} year $\boxed{105}$

After 2nd years
i.e. present population $\boxed{105 + 5.25 = 110.25}$

$$\underset{\text{(Suppose population)}}{110.25} \longrightarrow \underset{\text{(Original population)}}{1102500}$$

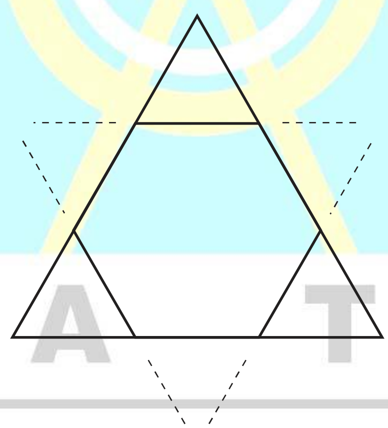
$$\therefore \quad 1 = \frac{1102500}{110.25} = 10000$$

Hence, 2 years ago population was

$$100 \times 10000 = 1000000$$

Hence, the correct option is (C).

Question 58 **General Aptitude (2M)**



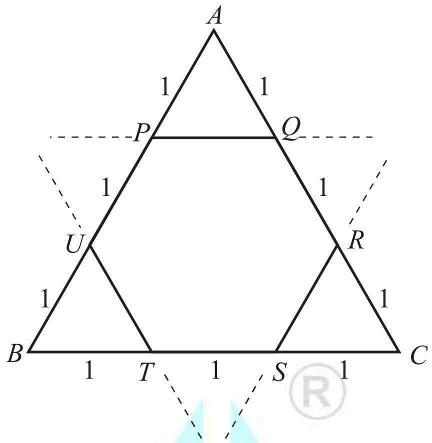
Corners are cut from an equilateral triangle to produce a regular convex hexagon as shown in the figure above.

The ratio of the area of the regular convex hexagon to the area of original equilateral triangle is

(A) 2 : 3 (B) 3 : 4 (C) 4 : 5 (D) 5 : 6

Ans. **A**

Sol. **Given :** Equilateral triangle



$\because$ Sides of Hexagon formed by an equilateral triangle to the same equilateral triangle $= 1:3$

$\therefore$ The ratio of the area of the regular convex Hexagon ($PQRSTU$) to the area of original equilateral triangle is

$$\Rightarrow \quad \frac{3\sqrt{3}}{2}(1)^2 : \frac{\sqrt{3}}{4}(3)^2$$

$$\Rightarrow \quad \frac{3\sqrt{3}}{2} : \frac{9\sqrt{3}}{4}$$

$$\Rightarrow \quad 2:3$$

Hence, the correct option is (A).

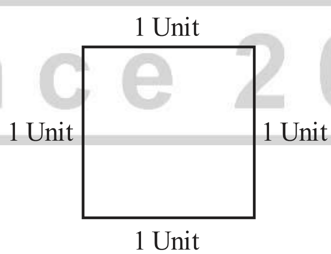
Question 59 **General Aptitude (2M)**

Consider a square sheet of side 1 unit. In the first step, it is cut along the main diagonal to get two triangles. In the next step, one of the cut triangles is revolved about its short edge to from solid cone. The volume of a resulting cone, in a cube units is __________.

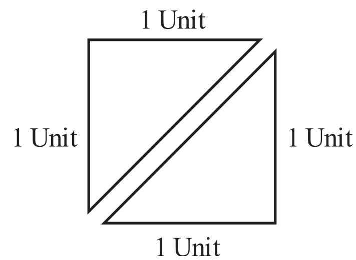
(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{3\pi}{2}$ (D) 3π

Ans. **A**

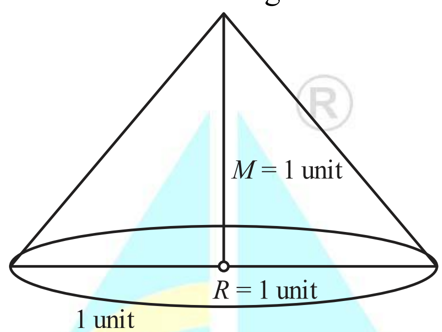
Sol. **Given :** Square sheet of side 1 unit.



It is cut along the main diagonal to get two triangles



One of the cut triangle is revolved about its short edge to form solid cone



$\because$ The volume of cone $= \dfrac{1}{3}\pi R^2 M$

$\therefore$ The volume of a resulting cone $= \dfrac{1}{3}\pi(1)^2(1) = \dfrac{\pi}{3}$

Hence, the correct option is (A).

Question 60 **General Aptitude (1M)**

Nostalgia is to anticipation as _______ is to _______.

Which one of the following options maintains a similar logical relation in the above sentence?

(A) Present, Past (B) Future, Past (C) Past, Future (D) Future, Present

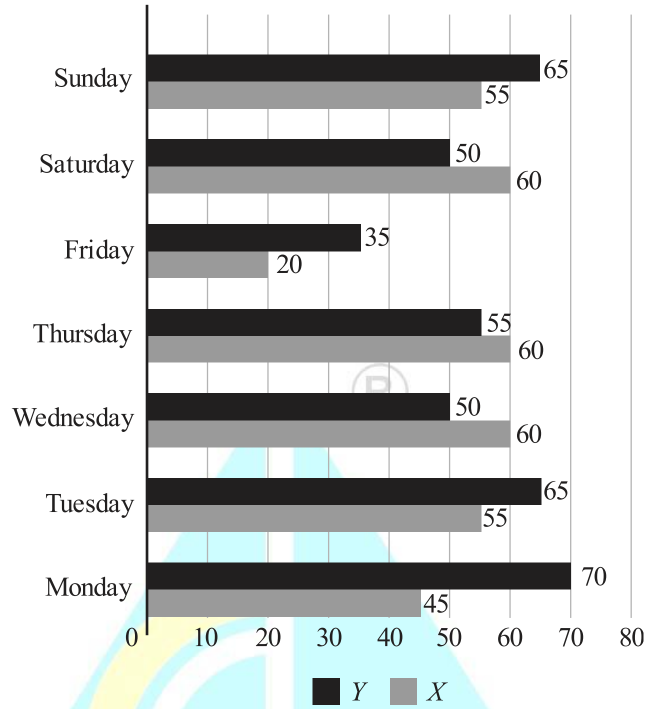
Ans. **(C)**

Sol. **Nostalgia :** Recollection of a period in the past.

Anticipation : Expectation or prediction to future.

Hence, the correct option is (C).

Question 61 **General Aptitude (2M)**



The number of minutes spent by two Students, X and Y, exercising every day in a given week are shown in the bar chart above.

The number of days in a given week in which one of the students spent a minimum of 10% more than the other student, on a given day, is

(A) 4 (B) 5 (C) 6 (D) 7

Ans. **C**

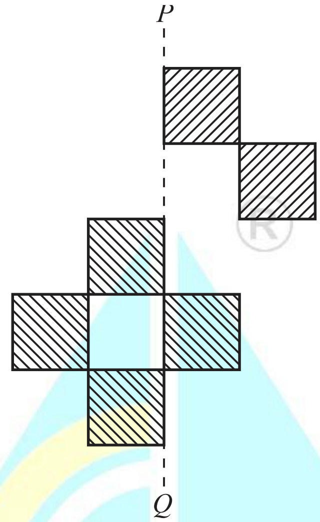
Sol. According to the given data in question we can formed the table as shown below,

Days	Y	X	%more than the other students
Sunday	65	55	$\frac{65-55}{55}\times 100\% = 18.18\%$
Saturday	50	60	$\frac{60-50}{50}\times 100\% = 20\%$
Friday	35	20	$\frac{35-20}{20}\times 100\% = 75\%$
Thursday	55	60	$\frac{60-55}{55}\times 100\% = 9.09\%$
Wednesday	50	60	$\frac{60-50}{50}\times 100\% = 20\%$
Tuesday	65	55	$\frac{65-55}{55}\times 100\% = 18.18\%$
Monday	70	45	$\frac{70-45}{45}\times 100\% = 55.55\%$

Except Thursday, there are total 6 days in which one of the students spent a minimum of 10% more than the other student.

Hence, the correct option is (C).

Question 62 **General Aptitude (1M)**

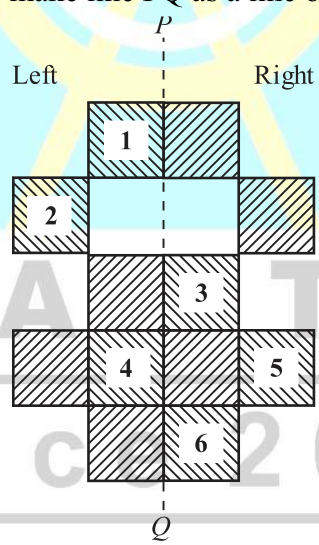


The least number of squares that must be added so that the line *P*-*Q* become the line of symmetry is ______.

(A) 4 (B) 3 (C) 6 (D) 7

Ans. **C**

Sol. We have to add six square blocks to make line PQ as a line of symmetry as shown below,



Hence, the correct option is (C).

Question 63 **General Aptitude (2M)**

Computers are ubiquitous. They are used to improve efficiency in almost all fields from agriculture to space exploration. Artificial intelligence (AI) is currently a hot topic. AI enables computer to learn, given enough training data. For humans, sitting in front of a computer for long hours can lead health issues.

Which of the following can be deduced from the above passage?

(i) Nowadays computers are present in almost all places.

(ii) Computers cannot be used for solving problems in engineering.

(iii) For humans, there are positive and negative effects of using computers.

(iv) Artificial intelligence can be done without data.

(A) (ii) and (iii) (B) (ii) and (iv) (C) (i), (ii) and (iv) (D) (i) and (iii)

Ans. **D**

Question 64 **General Aptitude (2M)**

Given below are two statements and two conclusions.

Statement :

Statement 1 : All purple are green.

Statement 2 : All Black are green.

Conclusion :

Conclusion I : Some black are purple

Conclusion II : No black is purple

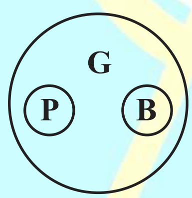
Based on the above statements and conclusions, which one of the following options is logically **CORRECT**?

(A) Only conclusion I is correct. (B) Only conclusion II is correct.

(C) Either conclusion I or II is correct (D) Both conclusion I & II are correct

Ans. **C**

Sol. According to the statement and conclusion given in the question we can formed Venn diagram as shown below,



1. can't say (wrong)
2. can't say (wrong)

Both the conclusion are wrong by can't say condition and variable of the conclusion are same. Also one conclusion is positive and another is negative. Hence they full fill the condition of "either or" case.

Hence, the correct option is (C).

Question 65 **General Aptitude (1M)**

Consider the following sentences :

(i) I woke up from sleep.

(ii) I woked up from sleep.

(iii) I was woken up from sleep.

(iv) I was wokened up from sleep.

Which of the above sentences are grammatically **CORRECT**?

(A) (i) and (ii) (B) (i) and (iii) (C) (ii) and (iii) (D) (i) and (iv)

Ans. **B**

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