Technical Section

Question 1 **Control System (NAT)**

Consider a system with transfer-function $G(s) = \frac{2}{s+1}$. A unit step function $\mu(t)$ is applied to the system, which results in an output $y(t)$. If $e(t) = y(t) - \mu(t)$, then $\lim_{t\to\infty} e(t)$ is ______. **[2 Marks]**

Ans. **1**

Sol. **Given :**

Transfer function $G(s) = \frac{2}{s+1} = \frac{Y(s)}{R(s)}$

Here, input $r(t)$ is step function,

i.e. $r(t) = \mu(t)$

So, $R(s) = \frac{1}{s}$

Thus, output $Y(s)$ is,

$$Y(s) = G(s)R(s)$$

$$Y(s) = \left(\frac{2}{s+1}\right)R(s) = \frac{2}{s+1} \times \frac{1}{s}$$

By final value theorem,

$$\lim_{t\to\infty} y(t) = y(\infty) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\left[\frac{2}{s(s+1)}\right] = \lim_{s\to 0} \frac{2}{s+1} = 2$$

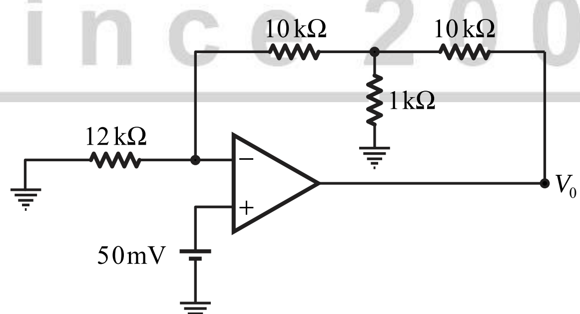
Given $e(t) = y(t) - \mu(t)$

So, $\lim_{t\to\infty} e(t) = \lim_{t\to\infty} y(t) - \lim_{t\to\infty} \mu(t)$

$$e(\infty) = y(\infty) - \mu(\infty) = 2 - 1 = 1$$

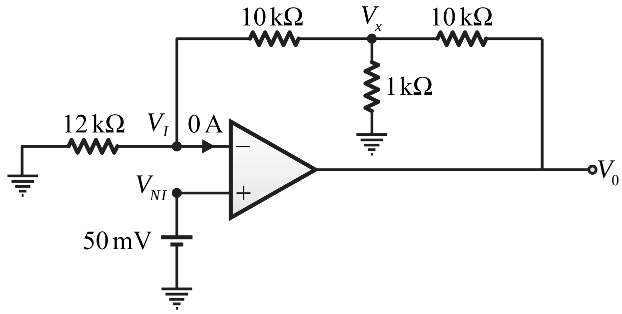
Question 2 **Analog Electronics (NAT)**

The circuit shown below uses an ideal OpAmp. Output V_0 in volt is ________ (rounded off to one decimal place). **[2 Marks]**



Ans. **1.05**

Sol. The given circuit is shown below,



By virtual ground concept $V_I = V_{NI} = 50$ mV

Applying KCL at (V_I) inverting terminal,

$$\frac{V_I - 0}{12\ K} + \frac{V_I - V_x}{10\,K} = 0$$

$$\frac{50\,\text{mV} - 0}{12\,K} + \frac{50\,\text{mV} - V_x}{10\,K} = 0$$

$$\frac{50}{12}\text{mV} + \frac{50}{10}\text{mV} = \frac{V_x}{10}$$

$$V_x = 50\,\text{mV} + \frac{500}{12}\text{mV}$$

$$V_x = 91.66\,\text{mV}$$

Apply KCL at V_x,

$$\frac{V_x - 50}{10} + \frac{V_x}{1} + \frac{V_x - V_0}{10} = 0$$

$$V_x - 50 + 10V_x + V_x - V_0 = 0$$

$$12V_x - 50 = V_0$$

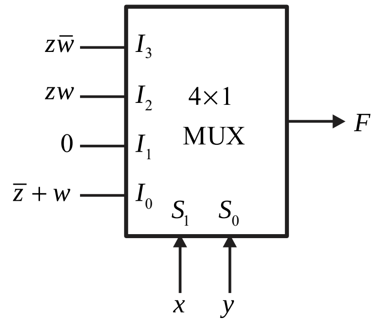
$$V_0 = 12\left[50\,\text{mV} + \frac{500\,\text{mV}}{12}\right] - 50$$

$$V_0 = 600\,\text{mV} + 500\,\text{mV} - 50\,\text{mV}$$

$$V_0 = 1050\,\text{mV} = 1.05\ \text{V}$$

Question 3 **Digital Electronics (MCQ)**

A 4×1 multiplexer with two selection line is used to realize Boolean function F having four Boolean variable x, y, z and w as shown in below, S_0 and S_1 denote the least significant bit (LSB) and most significant bit (MSB) of the selector lines of multiplexer respectively. I_0, I_1, I_2, I_3 are input lines of the multiplexer.



The canonical sum of product representation of F is **[2 Marks]**

(A) $F(x, y, z, w) = \Sigma m(2, 5, 9, 11, 14)$

(B) $F(x, y, z, w) = \Sigma m(0, 1, 3, 14, 15)$

(C) $F(x, y, z, w) = \Sigma m(0, 1, 3, 11, 14)$

(D) $F(x, y, z, w) = \Sigma m(1, 3, 7, 9, 15)$

Ans. C

Sol. **Given** 4×1 MUX have,

Input lines : $I_0 = \bar{z} + w,\ I_1 = 0$

$I_2 = zw,\ I_3 = z\bar{w}$

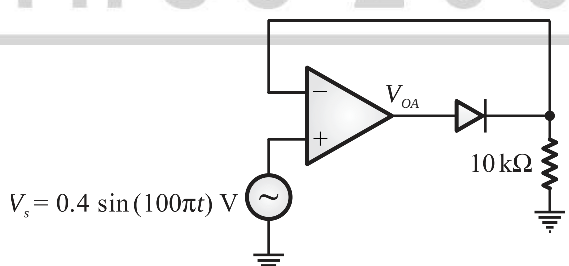
Selection lines : $S_1 = x,\ S_0 = y$

Output equation of 4×1 MUX

$$F = \bar{S}_1 \bar{S}_0 I_0 + \bar{S}_1 S_0 I_1 + S_1 \bar{S}_0 I_2 + S_1 S_0 I_3$$
$$F = \bar{x}\,\bar{y}(\bar{z}+w) + \bar{x}\,y \cdot 0 + x\,\bar{y}\,z\,w + x\,y\,z\,\bar{w}$$
$$F = \bar{x}\,\bar{y}\,\bar{z} + \bar{x}\,\bar{y}\,w + 0 + x\,\bar{y}\,z\,w + x\,y\,z\,\bar{w}$$
$$F = \Sigma m(000X,\ 00X1,\ 1011,\ 1110)$$
$$F = \Sigma m(0, 1, 1, 3, 11, 14)$$
$$F = \Sigma m(0, 1, 3, 11, 14)$$

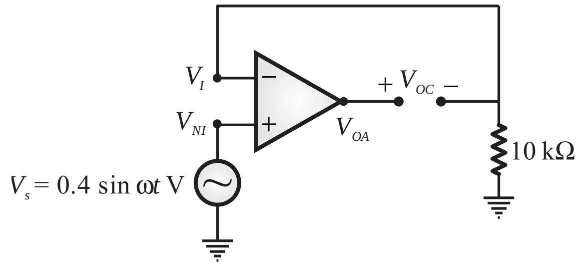
Question 4 **Analog Electronics (NAT)**

The diode used in the circuit has a fixed voltage drop of 0.6 V when forward biased. A signal V_s is given to the ideal OpAmp as shown. When V_s is at its positive peak, the output (V_{OA}) of the Op-Amp in volts is _______. **[1 Mark]**



Ans. 1

Sol. Assuming diode to be open circuited, the given circuit becomes as,



Now Op-Amp work as a comparator circuit,

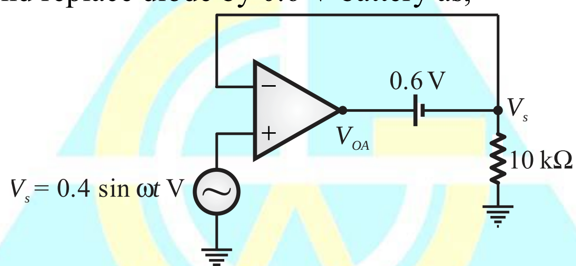
If, $V_s > 0$

$V_{NI} > V_I$

$V_{OA} = +V_{sat}$

So, $V_{OC} > 0$

It means, diode is ON and virtual ground concept is valid.
Now, equivalent circuit, and replace diode by 0.6 V battery as,



Thus, $V_{OA} = 0.6 + V_s$

$V_{OA(\max)} = 0.6 + (V_S)_{\max}$

$V_{OA(\max)} = 0.6 + 0.4 = 1\text{ V}$

Question 5 **Measurement (NAT)**

A household fan consumes 60 W and draws a current of 0.3125 A (rms) when connected to a 230 V (rms) ac, 50 Hz single phase mains. The reactive power drawn by the fan in VAR is ________ (rounded off to the nearest integer). **[2 Marks]**

Ans. **39.53**

Sol. **Given :** True power $P = 60$ W

$I_{rms} = 0.3125$ A and $V_{rms} = 230$ V

Reactive power, $Q = ?$

So true power, $P = V_{rms} I_{rms} \cos\phi$

$$60 = 230 \times 0.3125 \cos\phi$$

$$\cos\phi = \frac{60}{230 \times 0.3125} = 0.835$$

We can calculate $\sin\phi$ as,

$$\sin\phi = \sqrt{1 - \cos^2\phi}$$

$$\sin\phi = \sqrt{1-0.835^2} = 0.55$$

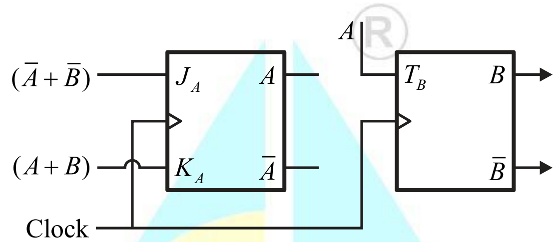
Thus reactive power Q,

$$Q = V_{rms} I_{rms} \sin\phi$$

$$Q = 230 \times 0.3125 \times 0.55 = 39.53 \text{ VAR}$$

Question 6 **Digital Electronics (MCQ)**

Given below is the diagram of a synchronous sequential circuit with one J-K flip-flop and one T flip-flop with their outputs denoted as A and B respectively, with $J_A = (\bar{A}+\bar{B})$, $K_A = (A+B)$ and $T_B = A$,



Starting from the initial state ($AB = 00$), the sequence of states (AB) visited by the circuit is

[2 Marks]

(A) $00 \to 10 \to 11 \to 01 \to 00$.....

(B) $00 \to 01 \to 10 \to 11 \to 00$.....

(C) $00 \to 10 \to 01 \to 11 \to 00$.....

(D) $00 \to 01 \to 11 \to 00$ .....

Ans. **C**

Sol. Given circuit is synchronous sequential circuit. Thus next state equation of FF A,

$$Q_{A+1} = J_A \bar{Q}_A + \bar{K}_A Q_A$$

$$Q_{A+1} = (\bar{Q}_A + \bar{Q}_B)\bar{Q}_A + (\overline{Q_A + Q_B})Q_A$$

$$Q_{A+1} = \bar{Q}_A + \bar{Q}_A \bar{Q}_B + \bar{Q}_A \bar{Q}_B Q_A$$

$$Q_{A+1} = \bar{Q}_A$$

Next state equation of FF B,

$$Q_{B+1} = T_B \oplus Q_B$$

$$Q_{B+1} = Q_A \oplus Q_B$$

State transition table,

Clock	Present state		Next State	
	Q_A	Q_B	$Q_{A+1} = \bar{Q}_A$	$Q_{B+1} = Q_A \oplus Q_B$
1	0	0	1	0
2	0	1	1	1
3	1	0	0	1
4	1	1	0	0

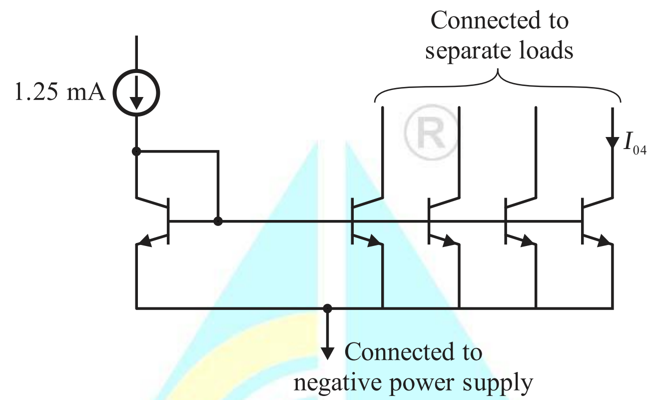
State sequence is as follows,

00, 10, 01, 11, 00 ……..

Hence, the correct answer is (C).

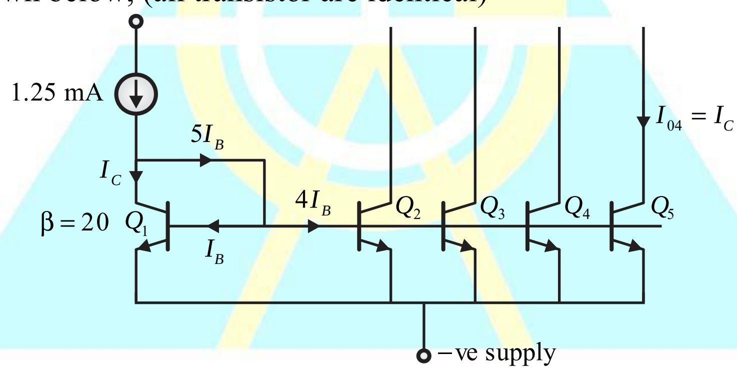
Question 7 **Analog Electronics (NAT)**

All the transistors used in the circuit are matched and have a current gain β of 20. Neglecting the Early effect, the current I_{04} in milliampere is _______. **[2 Marks]**



Ans. **1**

Sol. Given circuit is shown below, (all transistor are identical)



From figure, it is clear that,

$$1.25 \text{ mA} = I_C + 5I_B \quad \ldots(i)$$

$$V_{BE1} = V_{BE2} = V_{BE3} = V_{BE4} = V_{BE5}$$

$$I_C = \beta I_B$$

$$I_B = \frac{I_C}{\beta}$$

Now equation (i) becomes as,

$$\therefore \quad 1.25 \text{ mA} = I_C + 5 \times \frac{I_C}{\beta} = I_C\left[1 + \frac{5}{\beta}\right]$$

$$\therefore \quad I_C = \frac{1.25 \text{ mA}}{\left(1 + \frac{5}{\beta}\right)}$$

$\Rightarrow \qquad I_C = \frac{1.25}{1.25} = 1 \text{ mA}$

Question 8 **Digital Electronics (MCQ)**

A Boolean function F of three variables x, y and z is given as

$$F(x,\ y,\ z) = (x'+y+z)\cdot(x+y'+z')\cdot(x'+y+z')\cdot(x'y'z'+x'yz'+xyz')$$

Which one of the following is true? **[2 Marks]**

(A) $F(x,\ y,\ z) = (x'+y)(x+y'+z')$

(B) $F(x,\ y,\ z) = (x+y+z')(x'+y'+z')$

(C) $F(x,\ y,\ z) = x'y'z + xyz$

(D) $F(x,\ y,\ z) = x'z' + yz'$

Ans. **D**

Sol. **Method 1 :**

Given 3-variable Boolean function,

$$F[x,y,z,] = (\bar{x}+y+z)(x+\bar{y}+\bar{z})(\bar{x}+y+z)(\bar{x}\,\bar{y}\,\bar{z}+\bar{x}y\bar{z}+xy\bar{z})$$

$$F[x,y,z,] = (\bar{x}+y+z)(x+\bar{y}+\bar{z})[\bar{x}\,\bar{z}(y+\bar{y})+y\bar{z}(x+\bar{x})] \qquad [\because y+\bar{y} = x+\bar{x} = 1]$$

$$F[x,y,z] = [\bar{x}+y+z][x+\bar{y}+\bar{z}](\bar{x}+y+z)[\bar{x}\,\bar{z}+y\bar{z}] \qquad \ldots\text{(i)}$$

Converting $\bar{x}\,\bar{z}+y\,\bar{z}$ in standard POS format as,

$$\bar{x}\,\bar{z}+y\,\bar{z} = (\bar{x}+y+x)(\bar{x}+y+\bar{z})(x+y+\bar{z})(x+\bar{y}+\bar{z})\ (\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+\bar{z})$$

So, equation (i) become as,

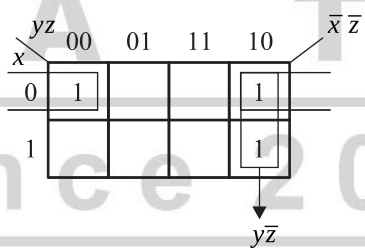
$$F[x,y,z] = (\bar{x}+y+z)(x+\bar{y}+\bar{z})(\bar{x}+y+\bar{z})(\bar{x}+y+z)$$
$$(\bar{x}+y+\bar{z})(x+y+\bar{z})(x+\bar{y}+\bar{z})(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+\bar{z})\,z$$

So, $F[x,y,z]$ can also be written as,

$F[x,y,z] = \pi M[1,3,4,5,7] \rightarrow$ POS format

$F[x,y,z] = \Sigma m[0,2,6] \rightarrow$ SOP format.

So k-map for SOP format is,



So, minimized function (F) is, $F[x,y,z] = \bar{x}\,\bar{z} + y\bar{z}$

Hence, the correct option is (D).

Method 2 :

Given : $F(x,\ y,\ z) = (x'+y+z)\cdot(x+y'+z')\cdot(x'+y+z')\cdot(x'y'z'+x'yz'+xyz')$

Assume, $\quad F_1[x,\ y,\ z] = [\bar{x}+y+z][x+\bar{y}+\bar{z}][\bar{x}+y+\bar{z}]$ (POS format)

$\qquad F_2[x,\ y,\ z] = [\bar{x}\,\bar{y}\,\bar{z}+\bar{x}\,y\,\bar{z}+xy\bar{z}]$ (SOP format)

Thus, $F[x, y, z] = F_1[x, y, z] \cdot F_2[x, y, z]$ ...(i)

Converting $F_1[x, y, z]$ in SOP format as,

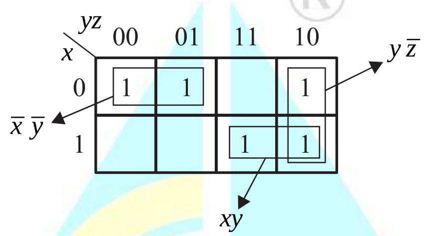
$F_1[x, y, z] = [\bar{x} + y + z][x + \bar{y} + \bar{z}][\bar{x} + y + \bar{z}]$ (POS format)

$F_1[x, y, z] = \pi m[100, 011, 101]$ (POS format)

$F_1[x, y, z] = \pi m[3, 4, 5]$ (POS format)

$F_1[x, y, z] = \Sigma m[0, 1, 2, 6, 7]$ (SOP format)

So, minimized form of $F_1[x, y, z]$, using K-map is,



Thus, $F_1[x, y, z] = [\bar{x}\,\bar{y} + xy + y\bar{z}]$ ...(ii)

Here, $F_2[x, y, z] = [\bar{x}\,\bar{y}\,\bar{z} + \bar{x}y\bar{z} + xy\bar{z}]$

$F_2[x, y, z] = [\bar{x}\,\bar{z}(\bar{y} + z) + y\bar{z}(\bar{x} + x)]$ $[\because y + \bar{y} = x + \bar{x} = 1]$

$F_2[x, y, z] = [\bar{x}\,\bar{z} + y\bar{z}]$ ...(iii)

Put F_1 and F_2 in from equation (ii) and (iii) to equation (i) as,

$F[x, y, z] = [\bar{x}\,\bar{y} + xy + y\bar{z}][\bar{x}\,\bar{z} + y\bar{z}]$

$F[x, y, z] = \bar{x}\,\bar{\bar{x}}\,\bar{y}\,\bar{z} + \bar{x}xy\bar{z} + \bar{x}y\bar{z}\,\bar{z} + \bar{x}\,\bar{y}y\bar{z} + xyy\bar{z} + y\bar{z}y\bar{z}$

$F[x, y, z] = \bar{x}\,\bar{y}\,\bar{z} + 0 + \bar{x}y\bar{z} + 0 + xy\bar{z} + y\bar{z}$

$F[x, y, z] = \bar{x}\,\bar{z}[\bar{y} + y] + y\bar{z}[x + 1]$ $[\because \bar{y} + y = 1 \text{ and } 1 + x = 1]$

$F[x, y, z] = \bar{x}\,\bar{z} + y\bar{z}$

Hence, the correct option is (D).

Method 3 :

Given : $F(x, y, z) = (x' + y + z) \cdot (x + y' + z') \cdot (x' + y + z') \cdot (x'y'z' + x'yz' + xyz')$

After multiplication and simplification,

$F[x, y, z] = \bar{x}\,\bar{y}\,\bar{z} + \bar{x}\,y\,\bar{z} + xy\bar{z}$

$F[x, y, z] = \bar{x}\,\bar{z}[\bar{y} + y] + y\bar{z}[\bar{x} + x]$ $[\because \bar{y} + y = \bar{x} + x = 1]$

$F[x, y, z] = \bar{x}\,\bar{z} + y\bar{z}$

Hence, the correct option is (D).

Question 9 **Digital Electronics (MSQ)**

For a 4-bit flash type Analog to Digital Convertor (ADC) with full scale input voltage range "V", which of the following statement(s) is/are true? **[1 Mark]**

(A) A change in input voltage by $\frac{V}{16}$ will always flip MSB of the output.

(B) A change in input-voltage by $\frac{V}{16}$ will always flip LSB of the output.

(C) The ADC requires one 4 to 2 priority encoder and 4 comparators.

(D) The ADC requires 15 comparators.

Ans. B, D

Sol. n-bit flash type ADC requires $2^n - 1$ comparators.

So, 4-bit flash type ADC requires

$2^4 - 1 = 15$ comparators

So, option (D) is correct.

Also, a change in input voltage by $\frac{V}{2^n}$ will always flip LSB of the output of n-bit flash ADC.

So, a change in input voltage by $\frac{V}{16}$ will always flip LSB of the output of 4-bit flash ADC.

Input of ADC	Output of ADC			
	MSB			LSB
$0 < V_{in} < \frac{V}{16}$	0	0	0	0
$\frac{V}{16} < V_{in} < \frac{2V}{16}$	0	0	0	1
$\frac{2V}{16} < V_{in} < \frac{3V}{16}$	0	0	1	0
$\frac{3V}{16} < V_{in} < \frac{4V}{16}$	0	0	1	1
$\frac{4V}{16} < V_{in} < \frac{5V}{16}$	0	1	0	0
$\frac{5V}{16} < V_{in} < \frac{6V}{16}$	0	1	0	1
$\frac{6V}{16} < V_{in} < \frac{7V}{16}$	0	1	1	0
$\frac{7V}{16} < V_{in} < \frac{8V}{16}$	0	1	1	1
$\frac{8V}{16} < V_{in} < \frac{9V}{16}$	1	0	0	0

$\frac{9V}{16} < V_{in} < \frac{10V}{16}$	1	0	0	1
$\frac{10V}{16} < V_{in} < \frac{11V}{16}$	1	0	1	0
$\frac{11V}{16} < V_{in} < \frac{12V}{16}$	1	0	1	1
$\frac{12V}{16} < V_{in} < \frac{13V}{16}$	1	1	0	0
$\frac{13V}{16} < V_{in} < \frac{14V}{16}$	1	1	0	1
$\frac{14V}{16} < V_{in} < \frac{15V}{16}$	1	1	1	0
$\frac{15V}{16} < V_{in} < \frac{16V}{16}$	1	1	1	1

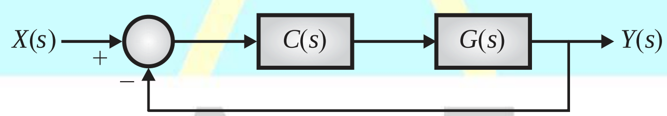
From the above table it is clear that, LSB will always flip from 0 to 1 or 1 to 0 when input voltage in changed by $\frac{V}{16}$. Hence B is also correct option.

Question 10 **Control System (MCQ)**

Consider a unity feedback configuration with a plant and a PID controller as shown in the figure.

$$G(s)=\frac{1}{(s+1)(s+3)} \text{ and } C(s)=K\frac{(s+3-j)(s+3+j)}{s}$$

with K being scalar. The closed loop is **[2 Marks]**



(A) Only stable for $K>0$
(B) Only stable for K between –1 and +1
(C) Stable for all values of K
(D) Only stable for $K<0$

Ans. **A**

Sol. **Given :** $G(s)=\frac{1}{(s+1)(s+3)}$

For PID controller,

$$C(s)=\frac{K(s+3-j)(s+3+j)}{s}$$

$$C(s)=K\left[\frac{(s+3)^2+1}{s}\right]$$

$$C(s)=K\left[\frac{s^2+6s+10}{s}\right]=K\left[6+\frac{10}{s}+s\right]$$

$$C(s) = K\left[6 + \frac{10}{s} + s\right]$$

Proportional term (6), Integral term ($\frac{10}{s}$), Derivative term (s)

Thus, $G'(s) = G(s)C(s)$

$$G'(s) = \left[\frac{1}{(s+1)(s+3)}\right]\left[K\left(6 + \frac{10}{s} + s\right)\right]$$

$$G'(s) = K\left[\frac{s^2 + 6s + 10}{s^3 + 4s^2 + 3s}\right]$$

So characteristic equation is,

$$1 + G'(s) = 0$$

$$s^3 + (4+K)s^2 + (6K+3)s + 10K = 0$$

Apply Routh-Hurwitz criteria,

s^3	1	$6K+3$
s^2	$4+K$	$10K$
s^1	$\frac{6K^2 + 27K + 12 - 10K}{4+K}$	0
s^0	$10K$	0

For stability, $10K > 0$

$K > 0$

It shows K is positive always.

So, for any positive value of K, system is stable.

Hence, the correct option is (A).

Shortcut method :

Internal product $(IP) = (4+K)(6K+3)$

External product $(EP) = 10K$

So, $IP > EP$

$K > 0$ (Stable system)

Hence, the correct option is (A).

Question 11 **Digital Electronics (MCQ)**

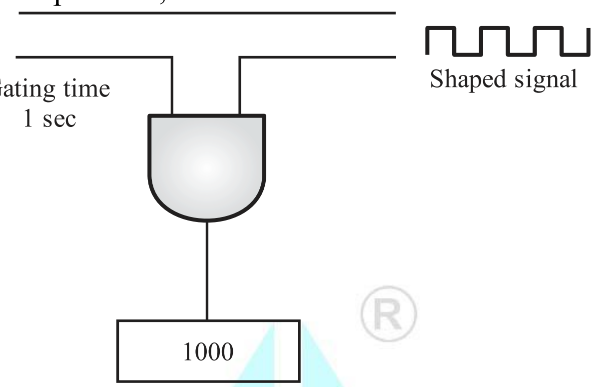
A $10\frac{1}{2}$ digital Counter-timer is set in the "frequency mode" of operation (with $T_s = 1\text{s}$). For a specific input, the rating obtained is 1000. Without disconnecting this input, the Counter-timer is changed to operate in the "Period mode" and the range selected is microseconds (μs, with $f_s = 1$ MHz). The counter will then display **[2 Marks]**

(A) 1000 (B) 100 (C) 0 (D) 10

Ans. A

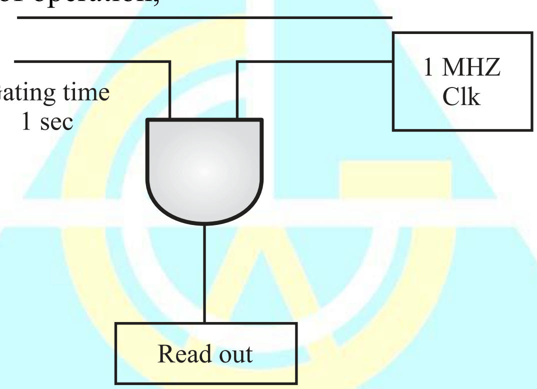
Sol. **Method 1 :**

Case 1 : Frequency mode of operation,



$f_{signal} = 1000\,\text{Hz}$

Case 2 : Time period mode of operation,



$$\text{Reading } = \frac{1\,\text{msec}}{1\,\mu\text{sec}} = 1000$$

Hence, the correct option is (A).

Method 2 :

In frequency mode,

$T_s =$ Gating internal $= 1\,\text{sec}$

Reading $= 1000$

Reading of counter = Gating interval × Frequency

$$1000 = 1 \times f$$

$$\therefore \quad f = 1000\,\text{Hz}$$

In period mode,

Frequency of pulse passing through gate $= f_s = 1\,\text{MHz}$

$$\text{Gating interval } = \frac{1}{f} = \frac{1}{1000} = 1\,\text{m sec}$$

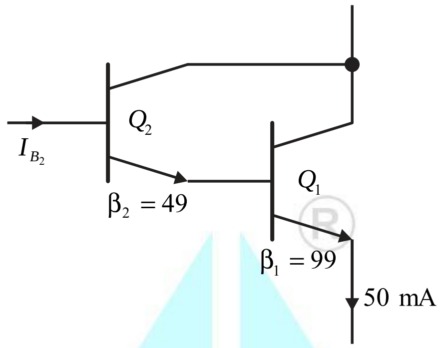
Reading = Gating interval × Frequency of pulse

$$\text{Reading } = 10^{-3} \times 10^{6} = 1000$$

Hence, the correct option is (A).

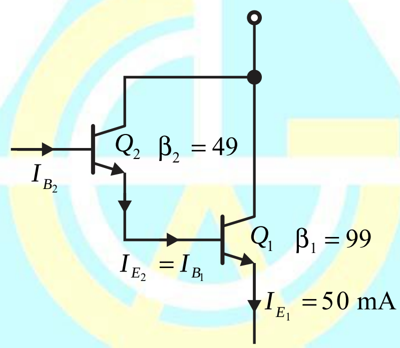
Question 12 **Analog Electronics (NAT)**

The transistor Q_1 has a current gain $\beta_1 = 99$ and the transistor Q_2 has a current gain $\beta_2 = 49$. The current I_{B_2} in microampere is _________. **[1 Mark]**



Ans. **10**

Sol. Given circuit is shown below,



For BJT Q_1, $I_{E_1} = (1+\beta_1) I_{B_1}$

$$I_{B_1} = \frac{I_{E_1}}{(1+\beta_1)}$$

$$I_{B_1} = \frac{50}{1+99} \text{ mA} = 0.5 \text{ mA}$$

From above figure it is clear that,

$\therefore$ $I_{E_2} = I_{B_1} = 0.5$ mA

For BJT Q_2, $I_{E_2} = (1+\beta_2) I_{B_2}$

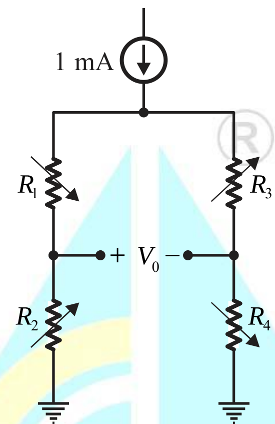
$$I_{B_2} = \frac{I_{E_2}}{(1+\beta_2)}$$

$$I_{B_2} = \frac{0.5}{(1+49)} \text{ mA} = \frac{0.5}{50} \text{ mA}$$

$$I_{B_2} = 10\ \mu\text{A}$$

Question 13 **Sensor & Industrial Instrumentation (NAT)**

For the full bridge made of linear strain gages with gage factor 2 as shown in the diagram $R_1 = R_2 = R_3 = R_4 = 100\ \Omega$ at 0^0C and strain is 0. The temperature coefficient of resistance of the strain gages used is 0.005 per^{0}C. All strain gages are made of same material and exposed to same temperature. While measuring a strain of 0.01 at a temperature of 50^0C, the output V_0 in millivolt is _______ (rounded off to two decimal places). **[2 Marks]**



Ans. **2.5**

Sol. At $T = 0^0\text{C} \Rightarrow R_1 = R_2 = R_3 = R_4 = 100\ \Omega = R$

At $T = 50^0\text{C} \Rightarrow R_{50} = (1+\alpha\Delta T)R$

$\Rightarrow \quad R_{50} = (1+0.005\times 50)100$

$\Rightarrow \quad R_{50} = (1.25)_{50} = 125\ \Omega$

So effective nominal resistance due to temperature change $= 125\ \Omega$

Now, assume tensile and compressive strength on arms R_2, R_3 and R_1, R_4 respectively

$\therefore \quad \Delta R$ due to ε

$\Rightarrow \quad 125\times 2\times 0.01 = 2.5\ \Omega$

$\therefore \quad R_2$ and $R_3 \Rightarrow 125 + 2.5 = 127.5\ \Omega$

$\therefore \quad R_1$ and $R_4 \Rightarrow 125 - 2.5 = 122.5\ \Omega$

$\therefore \quad V_{\text{output}} = \frac{1}{2}\times(127.5 - 122.5)$

$V_{\text{output}} = 2.5\ \text{mV}$

Question 14 **Engg. Mathematics (NAT)**

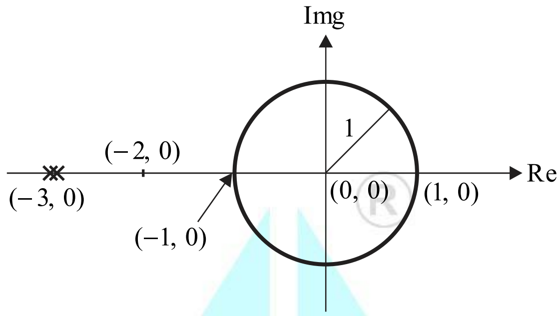
Let $f(z) = \frac{1}{z^2+6z+9}$ defined in complex plane. This integral $\oint_C f(z)\,dz$ over the contour of a circle c with center at the origin and unit radius is ______. **[1 Mark]**

Ans. **0**

Sol. **Given :** $f(z)=\dfrac{1}{z^2+6z+9}=\dfrac{1}{(z+3)^2}$

$f(z)$ has pole at $z=-3$ of order 2.

Given we have a unity radius circle with center at origin.



Since, no pole lie inside or on the contour.

Therefore, $\oint_C f(z)\,dz=0$

Question 15 **Engg. Mathematics (NAT)**

The determinant of the matrix M shown below is _______. **[1 Mark]**

$$M=\begin{bmatrix}1&2&0&0\\3&4&0&0\\0&0&4&3\\0&0&2&1\end{bmatrix}$$

Ans. **4**

Sol. **Given :** $$M=\begin{bmatrix}1&2&0&0\\3&4&0&0\\0&0&4&3\\0&0&2&1\end{bmatrix}$$

Now, $R_2\to R_2-3R_1$

$$\therefore \quad M=\begin{bmatrix}1&2&0&0\\0&-2&0&0\\0&0&4&3\\0&0&2&1\end{bmatrix}$$

Determinant of matrix, $|M|=1\begin{vmatrix}-2&0&0\\0&4&3\\0&2&1\end{vmatrix}$

$$|M|=-2\begin{vmatrix}4&3\\2&1\end{vmatrix}$$

$$|M|=1[-2(4-6)-0+0]$$

$$|M| = 4$$

Question 16 **Engg. Mathematics (NAT)**

Given $A = \begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix}$. The value of the determinant $\left|A^4 - 5A^3 + 6A^2 + 2I\right| =$ ________. **[2 Marks]**

Ans. **4**

Sol. **Given :** $A = \begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix}$

Using Cayley Hamilton theorem,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 5 \\ 0 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

Replacing λ by A,

$$A^2 - 5A + 6I = 0 \quad \ldots(\text{i})$$

Now, $A^4 - 5A^3 + 6A^2 + 2I = A^2(A^2 - 5A + 6) + 2I$

$$A^4 - 5A^3 + 6A^2 + 2I = A^2 \times 0 + 2I = 2I$$

Now, $\left|A^4 - 5A^3 + 6A^2 + 2I\right| = \left|2I_{2\times 2}\right| = 4$

Hence, the correct answer is 4.

Question 17 **Sensor & Industrial Instrumentation (NAT)**

A strain gage having nominal resistance of $1000\,\Omega$ has a gage factor of 2.5. If the strain applied to the gage is $100\,\mu\text{m/m}$, its resistance in ohm will change to _______ (rounded off to two decimal places).

[1 Mark]

Ans. **1000.25**

Sol. **Given :** $R = 1000\ \Omega$, Gauge factor $(G.F) = 2.5$

Strain $= 100\ \mu\text{m/m}$

According to question we have to calculate the value of $R + \Delta R$.

Here, $$G.F = \frac{\frac{\Delta R}{R}}{\varepsilon}$$

$$G.F\ \varepsilon R = \Delta R$$

$$2.5 \times 100 \times 10^{-6} \times 1000 = \Delta R$$

$$\Delta R = 0.25\ \Omega$$

$\therefore$ Resistance will change to $R + \Delta R = 1000 + 0.25 = 1000.25\,\Omega$

Question 18 **Sensor & Industrial Instrumentation (NAT)**

Given : Density of mercury is 13600 kg/m^3 and acceleration due to gravity is 9.81 m/s^2. Atmosphere pressure is 101 kPa. In a mercury U-tube manometer, the difference between heights of the liquid in U-tube is 1 cm. The differential pressure being measured in pascal is ________ (rounded off to the nearest integer). **[1 Mark]**

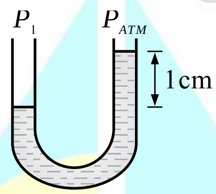
Ans. **1334.16**

Sol. **Given :** $\rho_m = 13600\,\text{kg/m}^3$

$g = 9.81\,\text{m/s}^2$

$P_{ATM} = 101\,\text{kPa}$

$h_m = 1\,\text{cm}$



Differential pressure,

$$\Delta P = P_1 - P_{ATM}$$

$$\Delta P = \rho_m\, g\, h_m$$

$$\Delta P = 13600 \times 9.81 \times 10^{-2}$$

$$\Delta P = 133416 \times 10^{-2}\,\text{Pa}$$

$$\Delta P = 1334.16\,\text{Pa}$$

Question 19 **Engg. Mathematics (MCQ)**

$f(z) = (z-1)^{-1} - 1 + (z-1) - (z-1)^2 + ...$ is the series expansion of **[2 Marks]**

(A) $\dfrac{1}{(z-1)^2}$ for $|z-1| < 1$

(B) $\dfrac{1}{z(z-1)}$ for $|z-1| < 1$

(C) $\dfrac{-1}{z(z-1)}$ for $|z-1| < 1$

(D) $\dfrac{-1}{(z-1)}$ for $|z-1| < 1$

Ans. **B**

Sol. **Given :**

$$f(z) = (z-1)^{-1} - 1 + (z-1) - (z-1)^2 + ...$$

$$f(z) = (z-1)^{-1}\left[1 - \frac{1}{(z-1)^{-1}} + \frac{(z-1)}{(z-1)^{-1}} - \frac{(z-1)^2}{(z-1)^{-1}} + ...\right]$$

$$f(z) = (z-1)^{-1}\left[1 - (z-1) + (z-1)^2 - (z-1)^3 + ...\right]$$

Now, $(1+x)^{-1} = 1 - x + x^2 - x^3 + ...;$ for $|x| < 1$

$\therefore$ $f(z) = \dfrac{1}{(z-1)}[1 + (z-1)]^{-1}$, for $|z-1| < 1$

$\Rightarrow$ $f(z) = \dfrac{1}{(z-1)}[z^{-1}]$, for $|z-1| < 1$

$\Rightarrow \qquad f(z) = \frac{1}{z(z-1)}$, for $|z-1| < 1$

Hence, the correct option is (B).

Question 20 **Digital Electronics (NAT)**

A 10-bit ADC has a full-scale of 10.230 V, when the digital output is $(11\,1111\,1111)_2$. The quantization error of the ADC in millivolt is ______. **[1 Mark]**

Ans. 5

Sol. Full scale voltage (V_{fs}) = 10.23 V

Number of bits (n) = 10

Number of steps $= 2^{10} - 1 = 1023$

We know that,

Full scale voltage = Resolution × Number of steps

$$10.23\text{ V} = \text{Resolution} \times 1023$$

$$\text{Resolution} = \frac{10.23}{1023} = \frac{1}{100} = 10\text{ mV}$$

Maximum quantization error of *ADC* is given as,

$$= \frac{\text{Resolution}}{2} = \frac{10\text{ mV}}{2} = 5\text{ mV}$$

Question 21 **Measurement (MCQ)**

In an ac main, the rms voltage V_{ac}, rms current I_{ac} and power W_{ac} are measured as, $V_{ac} = 100\text{ V} \pm 1\%$, $I_{ac} = 1\text{A} \pm 1\%$ and $W_{ac} = 50\text{ W} \pm 2\%$ (errors are with respect to readings). The percentage error in calculating the power factor measured using these readings is **[1 Mark]**

(A) 1% (B) 4% (C) 3% (D) 2%

Ans. B

Sol. **Given :** $V_{ac} = 100\text{ V} \pm 1\%$

$I_{rms} = 1\text{A} \pm 1\%$

$W_{ac} = 50\text{ W} \pm 2\%$

Now, we know that

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}}$$

and $\qquad W = VI\cos\phi$

or $\qquad \cos\phi = \frac{W}{VI}$

$$\%\varepsilon_{PF} = \pm[(\%\varepsilon_W) + (\%\varepsilon_V) + (\%\varepsilon_I)]$$

$$\%\varepsilon_{PF} = (\pm 2\%) + (\pm 1\%) + (\pm 1\%) = \pm 4\%$$

Hence, the correct option is (B).

Question 22 **Optical Instrumentation (NAT)**

When the movable arm of a Michelson interferometer in vacuum $(n=1)$ is moved by $325\,\mu\text{m}$, the number of fringe crossings is 1000. The wavelength of the laser used in nanometers is _______. **[1 Mark]**

Ans. 650

Sol. **Given :** Number of fringe crossing $(n) = 100$

$\eta_c = 1 \;\rightarrow\;$ For vacuum

$x = 325\ \mu\text{m}$

Thus, path difference of moveable are = $2x$ … (i)

Actual part difference due to fringe crossing $= n\lambda$ … (ii)

Equating equation (i) and (ii),

$$2x = n\lambda$$
$$2\times x = (1000)\,\lambda$$
$$2\times 325\,\mu\text{m} = 1000\ \lambda$$
$$650\,\mu\text{m} = 1000\ \lambda$$
$$650\,\text{nm} = \lambda$$

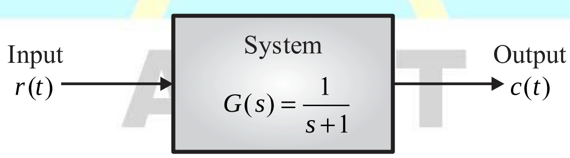
Question 23 **Control System (NAT)**

A sinusoidal $(\sqrt{2}\sin t)\mu(t)$, where $\mu(t)$ is the step input, is applied to a system with transfer-function $G(s)=\dfrac{1}{s+1}$. The amplitude of the steady state output is _______. **[2 Marks]**

Ans. 1

Sol. **Given :** $G(s)=\dfrac{1}{s+1}$

input $r(t) = A\sin\omega t = \sqrt{2}\sin t$



Thus output is $c(t)=\sqrt{2}\,|G(j\omega)|\sin(t+\phi)$ … (i)

Where $\phi = \angle G(j\omega)$

From input $r(t)=\sqrt{2}\sin t$, it is clear that $\omega=\omega_0=1$ rad/sec .

Here, $G(s)=\dfrac{1}{s+1}$

Put $s=j\omega$, $G(j\omega)=\dfrac{1}{j\omega+1}$

$$G(j\omega)\big|_{\omega=\omega_0=1}=\frac{1}{j+1}$$

$$|G(j\omega)|_{\omega=\omega_0=1} = \frac{1}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$$

$$\angle G(j\omega)|_{\omega=\omega_0=1} = \tan^{-1}\left(\frac{1}{1}\right) = 45^0$$

So, final output $c(t)$ under steady state at $\omega = \omega_0 = 1$ rad/sec is (from equation (i))

$$c(t) = 1\sin(t + 45^0)$$

Thus, amplitude of $c(t)$ under steady state at $\omega = \omega_0 = 1$ rad/sec is 1.

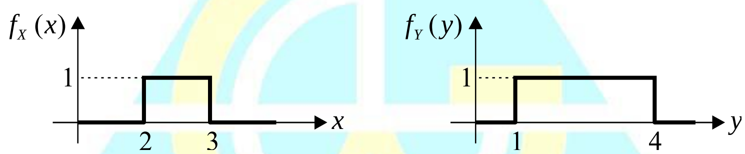
Question 24 **Communication System (NAT)**

Consider that x and y are independent continuous valued random variables with uniform PDF given by $x \sim U(2,3)$ and $y \sim U(1,4)$. Then $P(Y \le X)$ is equal to _______(rounded off to two decimal places).

[2 Marks]

Ans. **0.5**

Sol. Given pdf of random variable x and y is



$$P[x < X < x+dx,\ y < Y < y+dy] = f_{XY}(x,y)dx\,dy$$

$$P[2 < X < 3,\ 1 < Y < x] = \int_2^3\int_1^x f_{XY}(x,y)dx\,dy$$

$$P[Y \le x] = \int_2^3\int_1^x f_X(x)\,f_Y(y)\,dx\,dy$$

$$P[Y \le x] = \frac{1}{3}\times 1\int_2^3\int_1^x dy\cdot dx$$

$$P[Y \le x] = \frac{1}{3}\int_2^3 [y]_1^x\,dx$$

$$P[Y \le x] = \frac{1}{3}\int_2^3 (x-1)dx$$

$$P[Y \le x] = \frac{1}{3}\left[\frac{x^2}{2} - x\right]_2^3$$

$$P[Y \le x] = \frac{1}{3}\,[(4.5-3)-(2-2)]$$

$$P[Y \le x] = \frac{1}{3}\times 1.5 = 0.5$$

Question 25 **Communication System (NAT)**

An amplitude modulation (AM) scheme uses tone modulation, with modulation index of 0.6. The power efficiency of the AM scheme is _______ % (rounded off to one decimal place) **[1 Mark]**

Ans. 15.25

Sol. **Given :** $\mu_a = 0.6$

So efficiency of amplitude modulation is,

$$\%\eta = \frac{\mu_a^2}{2+\mu_a^2}\times 100\%$$

$$\%\eta = \frac{(0.6)^2}{2+(0.6)^2}\times 100\% = 15.25\%$$

Question 26 **Communication System (NAT)**

A signal having a bandwidth of 5 MHz is transmitted using the Pulse code modulation (PCM) scheme as follows. The signal is sampled at a rate 50% above the Nyquist rate and quantized into 256 levels. The binary pulse rate of the PCM signal in Mbits per second is _______. **[2 Marks]**

Ans. 120

Sol. **Given :** $BW = f_m = 5$ MHz

Nyquist rate (N.R) $= 2\times BW = 2f_m = 2\times 5 = 10$ MHz

Sampling frequency, $f_s = 1.5\times N.R$

$f_s = 1.5\times 10 = 15$ MHz

Given, $L = 256 = 2^n$ → Numbers of levels

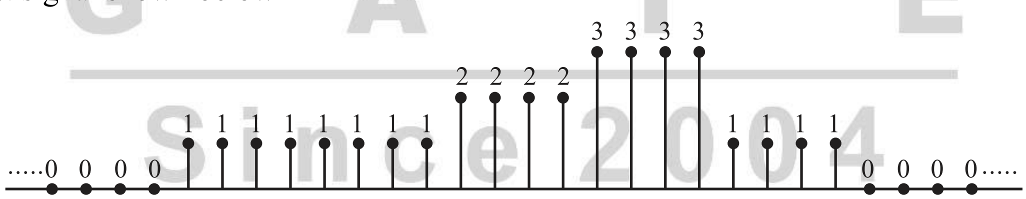
$n = 8$ bit → it should be integer always

Bit rate of PCM, $R_b = nf_s = 8\times 15$

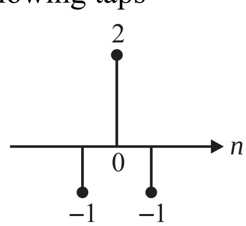
$R_b = 120$ Mbps

Question 27 **Signals & Systems (NAT)**

The input signal shown below



is passed through the filter with the following taps



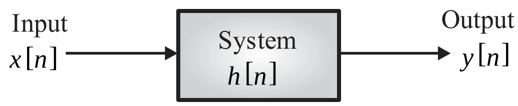
The number of non-zero output samples is ________. **[2 Marks]**

Ans. 10

Sol. **Given :**

$$x[n]=\{1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 1, 1, 1, 1\}$$

$$h[n]=\{-1, \underset{\uparrow}{2}, -1\}$$



Output of filter $y[n]$ is given as

$$y[n]=x[n]*h[n]$$

$$=x[n]*[-\delta[n+1]+2\delta[n]-\delta[n-1]]$$

$$y[n]=-x[n+1]+2x[n]-x[n-1] \qquad \ldots\text{(i)}$$

Origin for input is not mentioned in the question but as we just have to find number of non-zero samples of $y[n]$, so we can take origin at any sample.

Taxing origin at the left most sample and finding terms of R.H.S. of equation (i),

$$x[n]=\{\underset{\uparrow}{1},1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 1, 1, 1, 1\}$$

$$-x[n+1]=\{-1, -\underset{\uparrow}{1}, -1, -1, -1, -1, -1, -1, -2, -2, -2, -2, -3, -3, -3, -3, -1, -1, -1, -1, 0\}$$

$$2x[n]=\{0, 2, 2, 2, 2, 2, 2, 2, 2, 4, 4, 4, 4, 6, 6, 6, 6, 2, 2, 2, 2\}$$

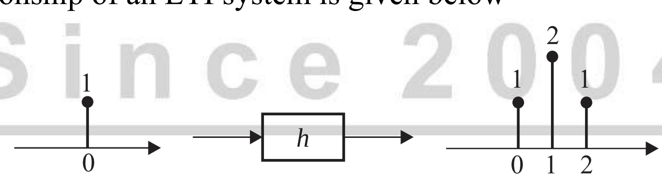
$$-x[n-1]=\{0, \underset{\uparrow}{0}, -1, -1, -1, -1, -1, -1, -1, -1, -2, -2, -2, -2, -3, -3, -3, -3, -1, -1, -1, -1\}$$

$$y[n]=\{-1, \underset{\uparrow}{1}, 0, 0, 0, 0, 0, 0, -1, -1, 0, 0, -1, -1, 0, 0, 2, -2, 0, 0, 1, -1\}$$

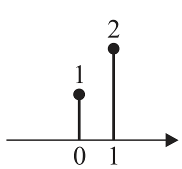
Hence, the number of non-zero samples in output $y[n]$ is 10.

Question 28 **Signals & Systems (MCQ)**

The input-output relationship of an LTI system is given below



For an input $x[n]$ shown below,



the peak value of the output when $x[n]$ is passed through h is ______. **[1 Mark]**

(A) 4 (B) 5 (C) 2 (D) 6

Ans. **B**

Sol. Given input-output relation and input x [n] are shown in figure

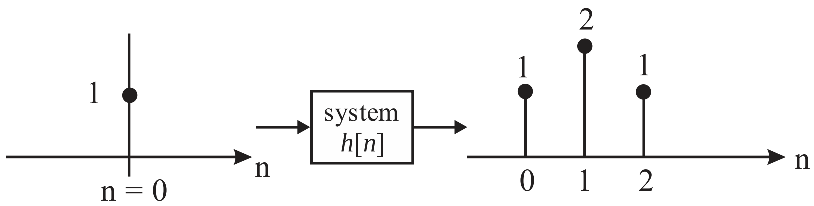


Figure-I

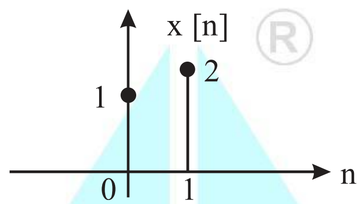


Figure-II

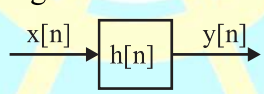
From figure-I, as the input applied is $\delta[n]$ i.e. impulse function, so its response is nothing but the impulse response of the given LTI system

$\therefore$ $h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

From figure-II,

$X[n] = \delta[n] + 2\,\delta[n-1]$

Response of the system for input x[n] is given as



$y[n] = x[n] * h[n]$

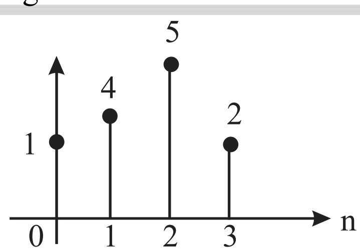
$$= \{\delta[n]+2\delta[n-1]\} * \{\delta[n]+2\delta[n-1]+s[n-2]\}$$

$\because$ $x[n] * \delta[n] = x[0]$

$$y[n]=\delta[n]+2\delta[n-1]+\delta[n-2]+2\delta[n-1]+4\delta[n-2]+2\delta[n-3]$$

$$y[n]=\delta[n]+4\delta[n-1]+5\delta[n-2]+2\delta[n-1]$$

Output y[n] is plotted as shown in figure.



Hence, the peak value of y[n] is 5 & the correct options (B)

Hence, the correct option is (B).

Question 29 **Signals & Systems (MCQ)**

Consider the sequence $x_n = 0.5\, x_{n-1} + 1$, $n = 1,2,\ldots\ldots$ with $x_0 = 0$. Then $\lim\limits_{n\to\infty} x_n$ is **[1 Mark]**

(A) ∞ (B) 0 (C) 2 (D) 1

Ans. **C**

Sol. **Given :** $x_n = 0.5 x_{n-1} + 1$, $n = 1,2,3\ldots..$

& $x_0 = 0$

Method 1 :

For limit, $(n-1) \to \infty$

So, $n \to \infty$

$$\therefore \quad \lim_{x\to\infty} x_n = \lim_{x\to\infty} 0.5 \quad x_{n-1} + 1$$

$$x_\infty = 0.5 \quad x_\infty + 1$$

$$= 0.5\ x_\infty = 1$$

$$\therefore \quad x_\infty = \frac{1}{0.5} = 2$$

Hence, the correct option is (C)

Method 2 :

Given $x_n = 0.5 x_{n-1} + 1$

$$x_1 = 0.5 x_0 + 1 = 1$$

$$x_2 = 0.5 x_1 + 1 = 1.5$$

$$x_3 = 0.5 x_2 + 1 = 1.75$$

$$x_4 = 0.5 x_3 + 1 = 1.875$$

$$x_5 = 0.5 x_4 + 1 = 1.9375$$

$$x_6 = 0.5 x_5 + 1 = 1.96875$$

$$x_7 = 0.5 x_6 + 1 = 1.984375$$

$$x_8 = 0.5 x_7 + 1 = 1.9921875$$

$$x_9 = 0.5 x_8 + 1 = 1.99609375 \approx 2$$

$$x_{10} = 0.5 x_9 + 1 = 2$$

$$x_{11} = 0.5 x_{10} + 1 = 2$$

If we continue the procedure further, then all of the next samples of x_n will have value equal to 2. So the steady state value of x_n is 2.

i.e. $\lim_{x \to \infty} x_n = 2$

Hence, the correct option in (C).

Question 30 **Signals & Systems (MCQ)**

The signal $\sin(\sqrt{2}\pi t)$ is **[1 Mark]**

(A) Periodic with period $T = 4\pi^2$

(B) Periodic with period $T = \sqrt{2}\pi$

(C) Not periodic

(D) Periodic with period $T = 2\pi$

Ans. **C**

Sol. **Given :** $x(t) = \sin\sqrt{2\pi t}$

For a non DC signal to be periodic, there must exist a constant 'T', such that,

$$x(t \pm T) = x(t) \quad \ldots(i)$$

Equation (i), can be satisfied only if the power of $'t'$ in $x(t)$ is 1.

As here the power of $'t'$ in $x(t)$ is $\frac{1}{2}$.

So, the signal is not periodic.

Hence, the correct option is (C).

Question 31 **Signals & Systems (NAT)**

Given $y(t) = e^{-3t}u(t) * u(t+3)$, where * denotes convolution operation. The value of $y(t)$ as $t \to \infty$ is ________ (rounded off to two decimal places). **[2 Marks]**

Ans. **0.33**

Sol. **Given :** $y(t) = e^{-3t}u(t) * u(t+3)$

Let $e^{-3t}u(t) = x(t)$ and $u(t+3) = h(t)$

Then, $y(t) = x(t) * h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

As $x(t) = e^{-3t}u(t)$, $h(t) = u(t+3)$

$x(\tau) = e^{-3\tau}u(\tau)$, $h(t-\tau) = u(t-\tau+3) = u(-\tau+t+3)$

$$\therefore \quad y(t) = \int_{-\infty}^{\infty} e^{-3\tau}u(-\tau+t+3)\,d\tau$$

$$y(t) = \int_{-\infty}^{t+3} e^{-3\tau}u(\tau)\,d\tau$$ [As $u(-\tau+t+3)$ is 1 only for $-\infty < \tau < t+3$]

$$\Rightarrow \qquad y(t)=\int_0^{t+3} e^{-3\tau}d\tau=\left[\frac{e^{-3\tau}}{-3}\right]_0^{t+3}$$

$$y(t)=\frac{1}{3}[1-e^{-3(t+3)}]$$

$$\lim_{t\to\infty} y(t)=\lim_{t\to\infty}\frac{1}{3}[1-e^{-3(t+3)}]$$

$$\lim_{t\to\infty} y(t)=\frac{1}{3}[1-e^{-\infty}]=\frac{1}{3}[1-0]=\frac{1}{3}=0.33$$

Question 32 **Signals & Systems (MCQ)**

Let $u(t)$ denote the unit step function. The bilateral Laplace transform of the function $f(t)=e^t u(-t)$ is

[1 Mark]

(A) $\frac{1}{s-1}$ with real part of $s>1$

(B) $\frac{1}{s-1}$ with real part of $s<1$

(C) $\frac{-1}{s-1}$ with real part of $s>1$

(D) $\frac{-1}{s-1}$ with real part of $s<1$

Ans. D

Sol. Method 1 :

$$u(t)\xleftrightarrow{\text{L.T}}\frac{1}{s},\ \text{Re}\{s\}>0$$

$$u(-t)\xleftrightarrow{\text{L.T}}\frac{-1}{s},\ \text{Re}\{s\}<0 \qquad \text{[Using time reversal property]}$$

$$e^t u(-t)\xleftrightarrow{\text{L.T}}\frac{-1}{s-1},\ \text{Re}\{s\}<1 \qquad \text{[Using frequency shifting property]}$$

Hence, the correct option is (D).

Method 2 :

$$f(t)=e^t u(-t)$$

$$F(s)=\int_{-\infty}^{\infty} f(t)e^{-st}dt$$

$$F(s)=\int_{-\infty}^{\infty} e^t u(-t)e^{-st}dt=\int_{-\infty}^{0} e^t e^{-st}dt$$

$$F(s)=\int_{-\infty}^{0} e^{(1-s)t}\,dt=\left[\frac{e^{(1-s)t}}{(1-s)}\right]_{-\infty}^{0}=\frac{1-0}{1-s}$$

$$F(s)=\frac{-1}{s-1}$$

As $f(t)$ is left sided, hence, ROC of rational Laplace transform will be left to the left most pole.

$\therefore \quad F(s) = \frac{-1}{s-1}, \ \text{Re}\{s\} < 1$

Hence, the correct option is (D).

Question 33 **Control System (NAT)**

Taking N as positive for clockwise encirclement, otherwise negative, the number of encirclements N of $(-1, 0)$ in the Nyquist plot of $G(s) = \frac{3}{s-1}$ is _______. **[1 Mark]**

Ans. **–1**

Sol. **Given :** $G(s) = \frac{3}{s-1}$

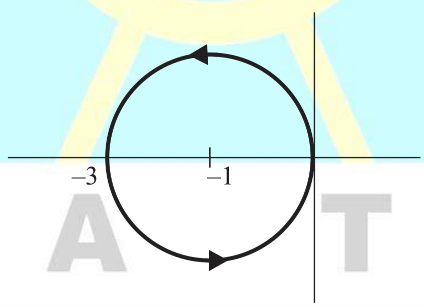
Magnitude, $|G(j\omega)| = \frac{3}{\sqrt{\omega^2 + 1}}$

Phase angle, $\angle G(j\omega) = -180^0 + \tan^{-1}\left(\frac{\omega}{1}\right)$

Magnitude and phase table is given by,

ω	Magnitude $\|G(j\omega)\|$	Phase angle $\angle G(j\omega)$
0	3	-180^0
∞	0	-90^0

Nyquist plot for $G(s)$ is shown below,



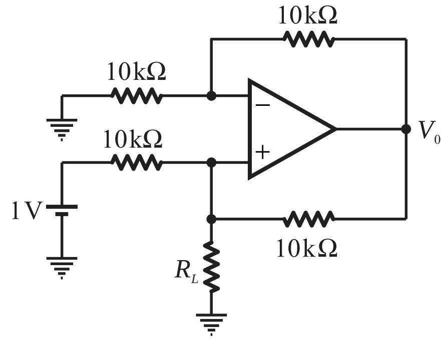
Since, N is the positive for clockwise encirclement and negative for anticlockwise encirclement.

Here, encirclement of critical point $(-1 + j0)$ is in anticlockwise direction.

So that, $N = -1$

Question 34 **Analog Electronics (MCQ)**

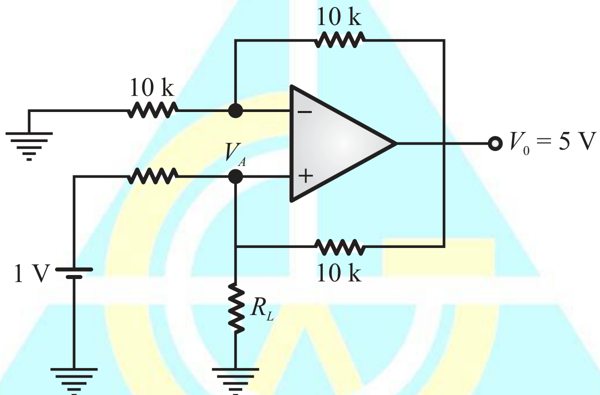
The output V_0 of the ideal OpAmp used in circuit shown below is 5 V. Then the value of resistor R_L in kilo ohm $(\text{k}\Omega)$ is **[2 Marks]**



(A) 2.5 (B) 25 (C) 5 (D) 50

Ans. **B**

Sol. Given circuit is shown below,



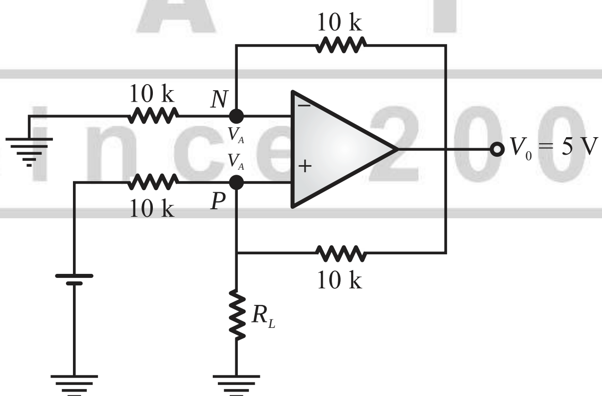
Given that the Op-Amp is ideal, therefore virtual ground conditions will hold true.

Let the potential at R_L is V_A

Now, by voltage divider rule,

$$V_A = V_0 \times \frac{R_L}{R_L + 10} \qquad \text{...(i)}$$

By virtual ground concept V_A will also appear at the inverting terminal.



Thus, KCL at point N,

$$\frac{V_A - 0}{10} + \frac{V_A - V_0}{10} = 0 \qquad \text{...(ii)}$$

$$\frac{V_A}{10} = \frac{5 - V_A}{10} \quad (\text{As } V_0 = 5 \text{ V})$$

$\therefore$ $$2V_A = 5 \text{ V}$$

$$V_A = 2.5 \text{ V}$$

Now, apply KCL at point P,

$$\frac{V_A - 1}{10} + \frac{V_A}{R_L} + \frac{V_A - V_0}{10} = 0$$

$$\frac{1.5}{10} - \frac{2.5}{10} = -\frac{2.5}{R_L}$$

As $V_0 = 5$ V and $V_A = 2.5$ V

$$\frac{-1.0}{10} = \frac{-2.5}{R_L}$$

$\therefore$ $$R_L = 25 \text{ k}\Omega$$

Hence, the correct option is (B).

Question 35 **Electrical Machine (NAT)**

A single-phase transformer has a magnetizing inductance of 250 mH and a core loss resistance of 300 Ω referred to primary side. When excited with a 230 V, 50 Hz sinusoidal supply at the primary, the power factor of the input current drawn with secondary on open circuit, is ______ (rounded off to two decimal places) **[1 Mark]**

Ans. **0.253**

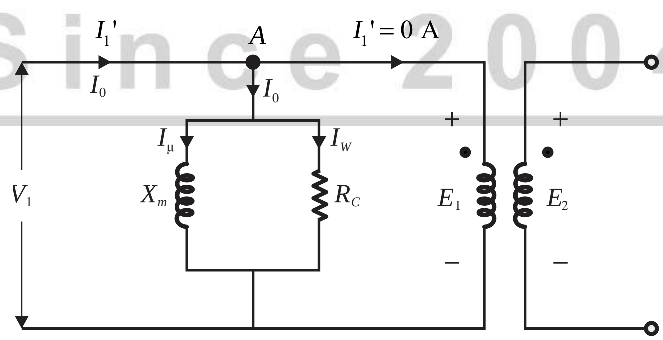
Sol. **Method 1 :**

As we know from equivalent circuit of transformer under no load condition,

$$\vec{I}_1 = \vec{I}_0 + \vec{I}_1'$$

Under no load condition, $\vec{I}_1' = 0$ A

So, $$\vec{I}_1 = \vec{I}_0$$



Applying nodal analysis at node A,

$$\frac{V}{jX_m}+\frac{V}{R_C}-I_0=0$$

$$I_0=\frac{V}{jX_m}+\frac{V}{R_C}$$

$$\bar{I}_0=\frac{230\angle 0^0}{j2\pi\times 50\times 250\times 10^{-3}}+\frac{230\angle 0^0}{300}$$

$$\bar{I}_0=-j0.2929+0.766$$

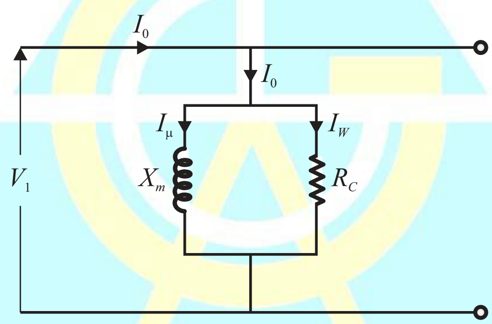
$$\bar{I}_0=0.766-j0.2929$$

$$\bar{I}_0=3.02\angle -75.34^0 \text{ A}$$

$$\cos\phi=\cos(-75.34^0)=0.253 \text{ lag}$$

Method 2 :

Equivalent circuit under noload condition is given by,



$$\bar{I}_0=I_\mu-jI_W$$

$$\angle I_0=\angle -\tan^{-1}\left(\frac{I_W}{I_\mu}\right)$$

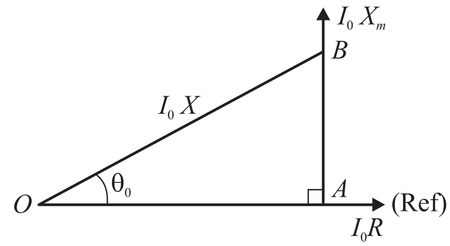
$$\cos(\angle I_0)=\cos\left[-\tan^{-1}\left(\frac{I_W}{I_\mu}\right)\right]$$

Where $\quad I_W=\frac{X}{jX_m},\ I_\mu=\frac{V}{R_C}$

$$\cos(\angle I_0)=\cos\left[-\tan^{-1}\left(\frac{R_C}{X_m}\right)\right]$$

$$\cos(\angle I_0)=\cos\left[-\tan^{-1}\left(\frac{300}{2\pi\times 50\times 250\times 10^{-3}}\right)\right]$$

$$\cos(\angle I_0)=0.253 \text{ lag}$$



From ΔOAB,

$$\tan\theta_0 = \frac{I_0 X_m}{I_0 R_C} = \frac{|X_m|}{R_C} = \frac{|2\pi f L|}{R_C}$$

$$\tan\theta_0 = \frac{2\pi\times 50\times 250\times 10^{-3}}{300}$$

$\therefore$ No load power factor, $\cos\theta_0 = \cos(\tan^{-1}\theta_0)$

$$\cos\theta_0 = \left[\tan^{-1}\left(\frac{2\pi\times 50\times 250\times 10^{-3}}{300}\right)\right]$$

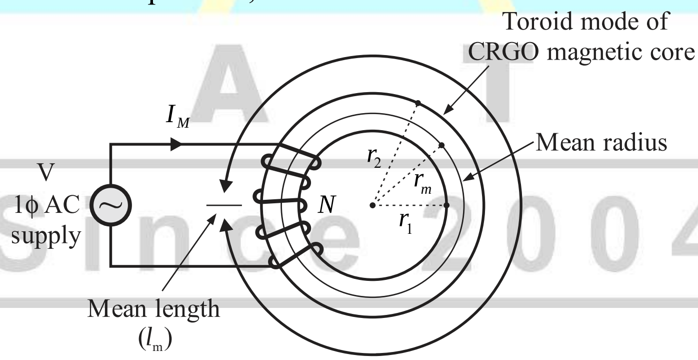
$$\cos\theta_0 = 0.253 \text{ lag}$$

Question 36 **Electrical Machine (NAT)**

A toroid made of CRGO has an inner diameter of 10 cm and an outer diameter of 14 cm. The thickness of the toroid is 2 cm. 200 turns of copper wire is wound on the core, $\mu_0 = 4\pi\times 10^{-7}$ H/m and μ_R of CRGO is 3000. When a current of 5 mA flows though the winding, the flux density in the core in millitesla is __________. **[2 Marks]**

Ans. 10

Sol. As per the given information is question,



Let $r_1 \rightarrow$ radius of inner circle $= \frac{d_1}{2} = \frac{10\,\text{cm}}{2} = 5\,\text{cm}$

$r_2 \rightarrow$ radius of outer circle $= \frac{d_2}{2} = \frac{14\,\text{cm}}{2} = 7\,\text{cm}$

Mean radius, $r_m = \frac{r_1 + r_2}{2} = \frac{5+7}{2} = 6\,\text{cm}$

Mean length, $l_m = 2\pi r_m = 2\pi \times 6\,\text{cm}$

$$l_m = 2\pi \times 6 \times 10^{-2}\ \text{m}$$

We know, in any magnetic core,

$$B = \mu H$$

$$B = \mu_0 \mu_r \left(\frac{NI}{l_{mean}}\right)$$

$$B = 4\pi \times 10^{-7} \times 3000 \times \left(\frac{200 \times 5 \times 10^{-3}}{2\pi \times 6 \times 10^{-2}}\right)$$

$$B = 10\,\text{m Tesla}$$

Question 37 **Electrical Machine (MCQ)**

A slip-ring induction motor is expected to be started by adding extra resistance in the rotor circuit. The benefit that is derived by adding extra resistance in the rotor circuit in comparison to the rotor being shorted is **[2 Marks]**

(A) The power factor at start will be lower.

(B) The starting torque would be higher.

(C) The losses at starting will be lower.

(D) The starting current is higher.

Ans. B

Sol. From option (A) :

Rotor power factor $B = \cos\theta_2 = \dfrac{R_2}{\sqrt{R_2^2 + (SX_2)^2}}$

Rotor power factor under standstill condition,

$$\cos\theta_{st} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \qquad \ldots\text{(i)}$$

Generally, order of rotor resistance/ph for SRIM $= 0.2\,\Omega$

order of standstill reactance/ph for SRIM $= 2\,\Omega$

In equation (i),

As $R_2^2 <<< X_2^2$, so R_2^2 is neglected.

$$\therefore \qquad \cos\theta_{st} = \frac{R_2}{\sqrt{0 + X_2^2}} = \frac{R_2}{X_2}$$

As rotor resistance increases due to external resistance, power factor at starting will be higher.

Conclusion : Option (A) is not correct.

From option (B) :

Starting torque,

$$T_{st} = \frac{180}{2\pi N_S} \times \frac{E_2^2 R_2}{R_2^2 + X_2^2} \qquad \ldots\text{(ii)}$$

In equation (ii),

As $R_2^2 <<< X_2^2$, so R_2^2 is neglected.

$$\therefore \qquad T_{st} = \frac{180}{2\pi N_S} \times \frac{E_2^2 R_2}{X_2^2}$$

$$T_{st} \propto R_2$$

As rotor resistance increases, starting toque will be increased.

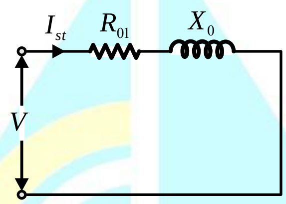
Conclusion : Option (B) is correct.

From option (C) :

Rotor current is very small at standstill so rotor copper loss is also lower.

Conclusion : Option (C) is correct.

From option (D) :



Starting current, $I_{st} = \dfrac{V}{R_{01} + jX_{01}}$

As resistance is increased, starting circuit of motor will be reduced.

Conclusion : Option (D) is not correct.

Note : This is a MCQ question hence, the most appropriate answer is option (B).

Question 38 **Engg. Mathematics (NAT)**

Consider the function $f(x) = -x^2 + 10x + 100$. The minimum value of the function in the interval [5, 10] is ________. **[1 Mark]**

Ans. 100

Sol. **Given :** $f(x) = -x^2 + 10x + 100$

Stationary point (extrema)

$$f'(x) = -2x + 10 = 0$$

$$x = 5$$

Double derivative,

$$f''(x) = -2$$

Means $x = 5$ is the point of maxima

Minimum $f(x), x \in [5, 10]$

Minimum $[f(5), f(10)] = 100$

Hence, the correct answer is 100.

Question 39 **Engg. Mathematics (MCQ)**

Consider the row vectors $V = (1, 0), W = (2, 0)$. The rank of the matrix $M = 2V^T V + 3W^T W$, where the superscript the transpose, is **[1 Mark]**

(A) 4 (B) 2 (C) 1 (D) 3

Ans. C

Sol. Given vector are, $V=(1,0), W=(2,0)$

$$\rho\left[2V^TV+3W^TW\right]=?$$

Assuming the vector is a column matrix so,

$$M=2V^TV+3W^TW$$

$$M=2\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}+3\begin{bmatrix}2 & 0\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix}=[14]$$

$$M=[14]$$

Range of $M=[14]$ is 1.

Assuming the vector is a row matrix so

$$M=2V^TV+3W^TW$$

$$M=2\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix}+3\begin{bmatrix}2\\0\end{bmatrix}\begin{bmatrix}2 & 0\end{bmatrix}$$

$$M=\begin{bmatrix}2 & 0\\0 & 0\end{bmatrix}+\begin{bmatrix}12 & 0\\0 & 0\end{bmatrix}$$

$$M=\begin{bmatrix}14 & 0\\0 & 0\end{bmatrix}$$

Rank of $M=\begin{bmatrix}14 & 0\\0 & 0\end{bmatrix}$ is 1.

Hence, the correct option is (C).

Question 40 **Sensors (NAT)**

A piezoresistive pressure sensor has a sensitivity of 1 (mV/V)/kPa. The sensor is excited with a dc supply of 10 V and the output is read using a 3½ digit 200 mV full-scale digital multimeter. The resolution of the measurement set-up, in pascal is ________. **[1 Mark]**

Ans. 10

Sol. **Given :** Sensitivity $=1\,\text{mV/V/kPa}$

An example of piezo resistive transducer is the strain gauge

$$V_B=V_s\times\frac{\Delta R}{R} \quad \ldots(i)$$

Where, $\dfrac{\Delta R}{R}=GF\times\varepsilon$ ...(ii)

and $\varepsilon=\dfrac{S}{Y}$ ...(iii)

Where, $V_s=$ Supply voltage, $V_B=$ Output voltage, $R=$ Nominal resistance,

$\Delta R=$ Change in resistance, $GF=$ Gauge factor, $\varepsilon=$ Strain applied

$Y=$ Young's Modulus and $S=$ Stress applied.

Based on equations (i), (ii) and (iii) and sensitivity data provided we conclude

$$V_B = 1\,\text{mV}$$

$$V_s = 1\text{V}$$

$$S = 10^3\,\text{Pa}$$

$\therefore$ For 1 V applied and stress of $10^3\,\text{Pa}$ we get output voltage $=1\times10^{-3}\,\text{V}$

Since, actual V_s is 10 V.

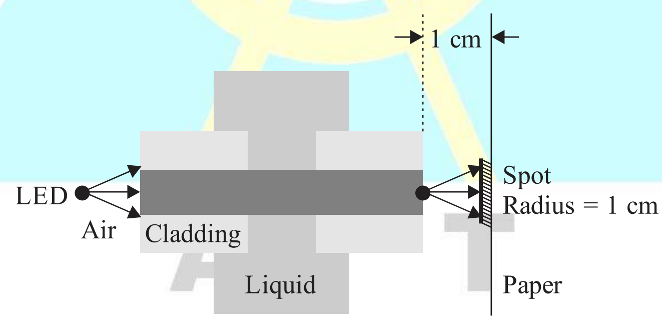
$\therefore$ Actual senstivity $=10\,\text{mV/V/kPa}$

$$\text{Resolution of the DVM} = \frac{\text{FSV}}{\text{Number of counts}} = \frac{200}{2000} = 0.1\text{ mV} \qquad \text{....(iv)}$$

$$\therefore \text{ Resolution in terms of applied stress} = \frac{0.1\text{ mV}}{10\text{ mV/V/kPa}} = 10\text{ Pa}$$

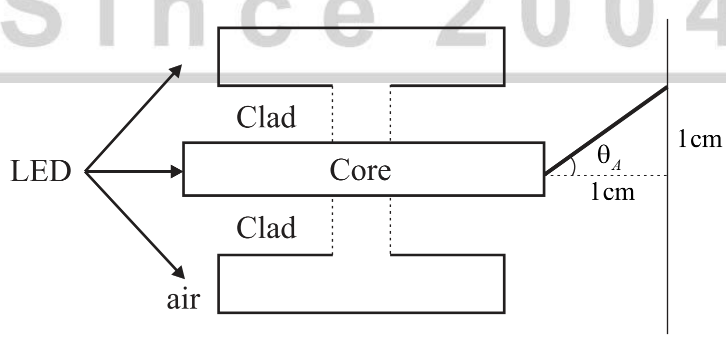
Question 41 **Optical & Communication (NAT)**

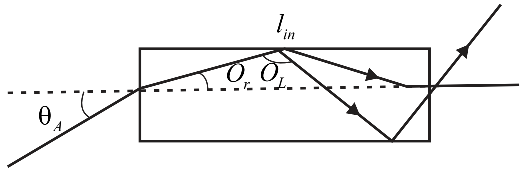
In the figure shown, a large multimode fiber with $n_{\text{core}}=1.5$ and $n_{\text{clad}}=1.2$ is used for sensing. A portion with the cladding removed passes through a liquid with refractive index n_{liquid}. An LED is used to illuminate the fiber from one end and a paper is placed on the other end, 1 cm from the end of the fiber. The paper shows a spot with radius 1 cm. The refractive index n_{liquid} of the liquid (rounded off to two decimal places) is ________. **[2 Marks]**



Ans. 1.322

Sol.





$$\frac{\sin\theta_A}{\sin\theta_r} = \frac{\mu_{core}}{\mu_{air}}$$

$$\sin\theta_A = \frac{\mu_{core}}{\mu_{air}}\sin\theta_A \qquad \ldots\text{(i)}$$

Now at total internal reflection,

$$\theta_r + \theta_c + 90^0 = 180^0$$

$$\theta_r + \theta_c = 90^0$$

$$\theta_r = 90^0 - \theta_c$$

$$\therefore \qquad \sin\theta_A = \frac{\mu_{core}}{\mu_{air}}\sin(\mu_0^0 - \theta_c) = \frac{\mu_{coe}}{\mu_{air}}\cos\theta_c \qquad \ldots\text{(ii)}$$

And we know that,

$$\cos\theta_c = \sqrt{1-\sin^2\theta_c} = \sqrt{1-\frac{\mu_{liq}^2}{\mu_{core}^2}} = \sqrt{\frac{\mu_{core}^2 - \mu_{liq}^2}{\mu_{core}2}}$$

$$\therefore \qquad \sin\theta_A = \frac{\mu_{core}}{\mu_{air}}\sqrt{\frac{\mu_{core}^2 - \mu_{liq}^2}{\mu_{core}}} = \sqrt{\frac{\mu_{core}^2 - \mu_{liq}^2}{\mu_{air}}} \qquad \left(\because \frac{\mu_{core}}{\mu_{core}} = 1\right)$$

Or

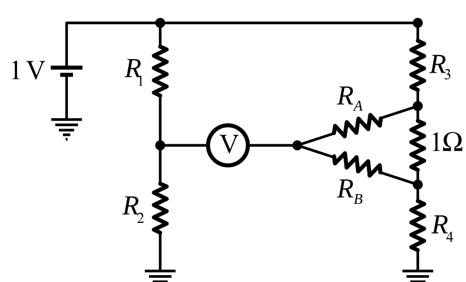
$$\frac{1}{\sqrt{2}} = \frac{\sqrt{(1.5)^2 - \mu_{liq}^2}}{1}$$

$$\mu_{liq} = \sqrt{(1.5)^2 - (0.707)^2}$$

$$\mu_{liq} = \sqrt{2.25 - 0.5} = 1.322$$

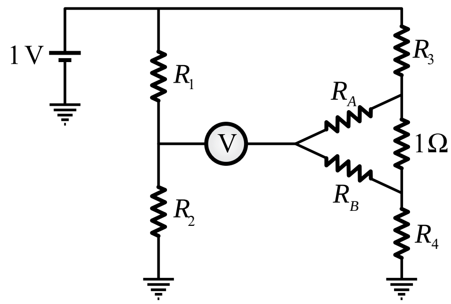
Question 42 **Network Theory (NAT)**

In the bridge circuit shown, the voltmeter V showed zero when the value of the resistors are : $R_1 = 100\,\Omega$, $R_2 = 110\,\Omega$ and $R_3 = 90\,\Omega$. If $\left(\frac{R_1}{R_2}\right) = \left(\frac{R_A}{R_B}\right)$, the value of R_4 in ohm is ________. **[2 Marks]**

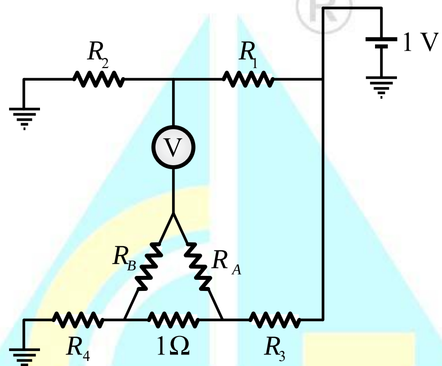


Ans. 99

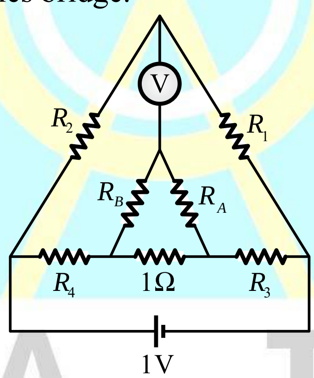
Sol. Given circuit is shown below,



Now by rotating the bridge by 90^0 in the clockwise direction.



The circuit looks like a Kelvin doubles bridge.



Here, $P = R_2,\ Q = R_1,\ p = R_B,\ q = R_A$

and $r = 1\ \Omega,\ R = R_4,\ S = R_3$

Now for balance of the Kelvin doubles bridge.

$$R = \frac{P}{Q} \times S + \frac{qr}{p+q+r}\left[\frac{P}{Q} - \frac{p}{q}\right]$$

$$R_4 = \frac{R_2}{R_1} \times R_3 + \frac{R_A \cdot 1}{R_B + R_A + 1}\left[\frac{R_2}{R_1} - \frac{R_B}{R_A}\right]$$

Now, given $\frac{R_2}{R_1} = \frac{R_B}{R_A}$

$\therefore$ $R_4 = \frac{R_2}{R_1} \times R_3$

$$R_4 = \frac{110}{100} \times 90 = \frac{9900}{100}$$

$$R_4 = 99\,\Omega$$

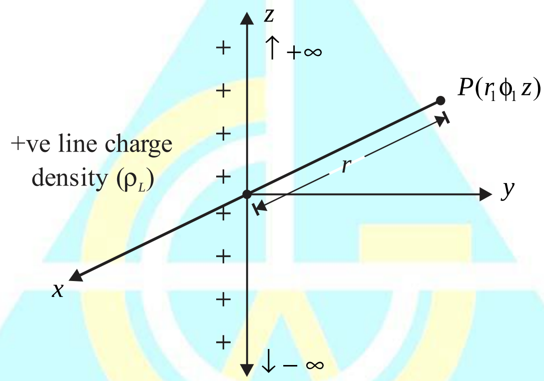
Question 43 **Electromagnetic Theory (MCQ)**

An infinitely long line, with uniform positive charge density, lies along the z-axis. In cylindrical coordinates (r, ϕ, z), at any point $\vec{P}$ not on the z-axis, the direction of the electric field is **[1 Mark]**

(A) $\hat{z}$ (B) $\frac{\hat{r}+\hat{z}}{\sqrt{2}}$ (C) $\hat{r}$ (D) $\hat{\phi}$

Ans. **C**

Sol. The expression of Electric field ($\vec{E}$)-at any arbitrary point P, from initially long line change is,



$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{r}$$

If infinite line charge is kept on z-axis, hence the direction would be in the direction of $\hat{r}$.

Hence, the correct option is (C).

Question 44 **Control System (MSQ)**

The step response of a circuit is seen to have an oscillatory behaviour at the output with oscillations dying down after some time. The correct inference(s) regrading the transfer function from input to output is/are **[1 Mark]**

(A) that it is of at least second order.

(B) that it has at least one pole-pair that is underdamped.

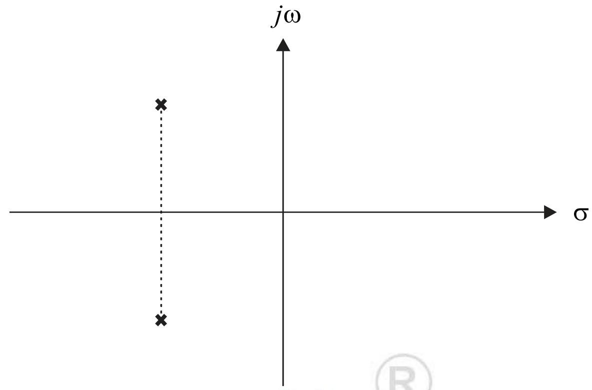
(C) that it does not have a real pole.

(D) that it is a first order system.

Ans. **A, B, C**

Sol. For oscillation there should be atleast two energy storing element i.e. order must be atleast 2.

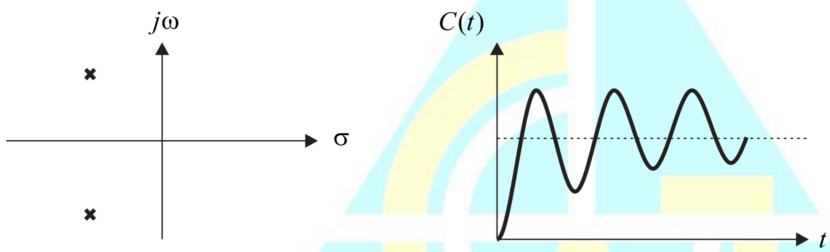
As given in question step response has oscillatory behaviour at output with oscillation dying after some time. This happens when we have complex conjugate poles (i.e. response is under damped in nature i.e. $0 < \xi < 1$.



If poles are not conjugate pair and are lying in axis either real or imaginary axis we do not get oscillations.

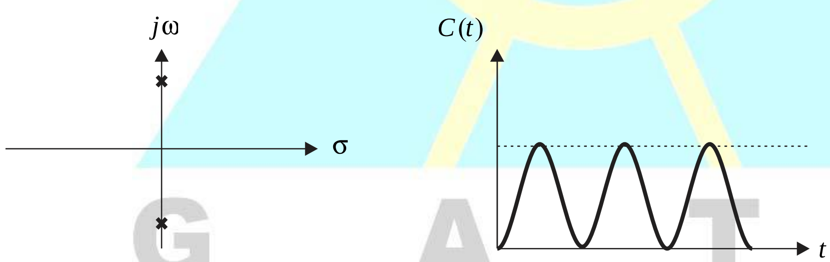
For second order system to unit step input

(i)



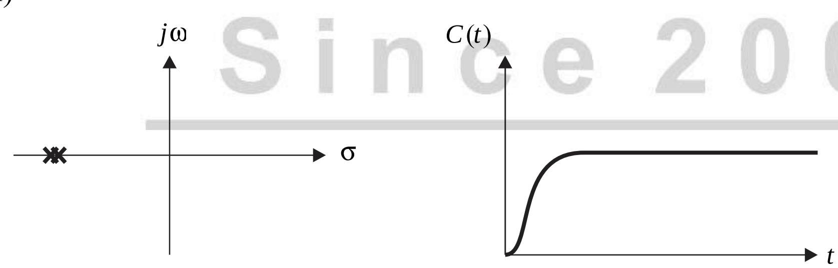
(Underdamped)

(ii)



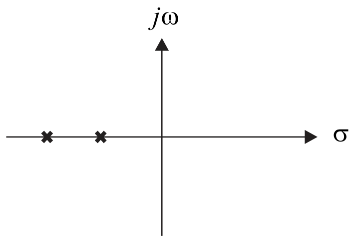
(Undamped)

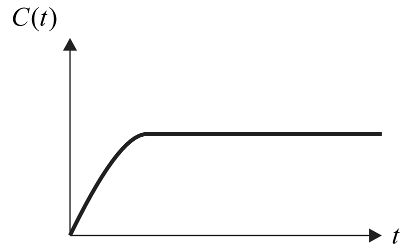
(iii)



(Critically damped)

(iv)



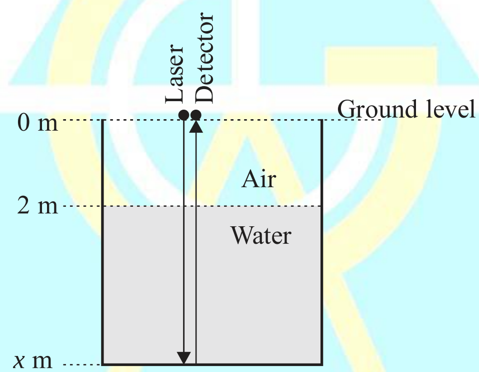


(Over damped)

Hence, the correct option is (A), (B) and (C).

Question 45 **Optical & Instrumentation (NAT)**

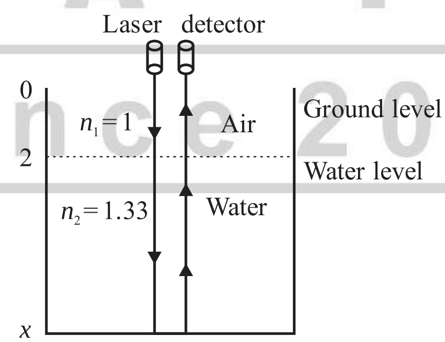
A laser pulse is sent from ground level to the bottom of a concrete water tank at normal incidence. The tank is filled with water up to 2 m below the ground level. The reflected pulse from the bottom of the tank travels back and hits the detector. The round-trip time elapsed between sending the laser pulse, the pulse hitting the bottom of the tank, reflecting back and sensed by the detector is 100 ns. The depth of the tank from ground level marked as x in metre is _______. (Refractive index of water $n_{water} = 1.3$ and velocity of light in air $C_{air} = 3\times10^8$ m/s) **[2 Marks]**



(A) 10 (B) 11 (C) 12 (D) 9

Ans. **C**

Sol. Given arrangement is shown below,



$$V = \frac{d}{t}$$

$$t = \frac{d}{V}$$

Refractive index $(\mu) \propto \dfrac{1}{\text{Velocity of Ray in medium}}$

$$\mu_{water} \propto \frac{1}{v_{water}}$$

$$\mu \propto \frac{1}{V}$$

$$\frac{\mu_1}{\mu_2} = \frac{v_2}{v_1}$$

$$\frac{\mu_{air}}{\mu_{water}} = \frac{v_{water}}{v_{air}}$$

$$\frac{1}{1.3} = \frac{v_{wter}}{v_{air}}$$

$$v_{water} = 2.31\times10^8 \text{ m/sec}$$

$$\text{Time } = \frac{2d_{air}}{V_{air}} + \frac{2d_{water}}{V_{water}}$$

$$100\times10^{-9} = 2\times\frac{2}{3\times10^8} + \frac{2(x-2)}{2.31\times10^8}$$

$$100\times10^{-9} = \frac{1}{10^8}\left[\frac{4}{3} + \frac{2x}{2.31} - \frac{4}{2.31}\right]$$

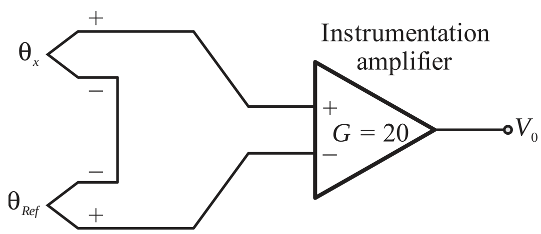
$$10 = 1.33 + 0.87x - 1.73$$

$$11.73 - 1.33 = 0.87\,x$$

$$x = \frac{10.4}{0.87} = 11.95 \simeq 12\,\text{m}$$

Question 46 **Sensors (MCQ)**

A J-type thermocouple has an output voltage $V_\theta = (13650 + 50\theta_x)\mu\text{V}$, where θ_x is the junction temperature in Celsius (^0C). The thermocouple is used with reference junction compensation, as shown in the figure. The instrumentation amplifier used has a gain $G = 20$. If θ_{Ref} is 1 ^{0}C, for an input θ_x of 100^0C, the output V_0 of the instrumentation amplifier in millivolt is **[2 Marks]**



(A) 98 mV (B) 100 mV (C) 101 mV (D) 99 mV

Ans. D

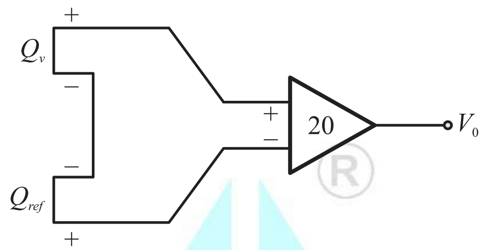
Sol. **Method 1 :**

Given :

Instrumentation amplifier gain $= 20$

Reference junction temperature $= 1^0\text{C}$

Hot junction temperature $= 100^0\text{C}$



Now, output voltage of the hot junction,

$$V_1 = (13650 + 50 \times 100)\,\mu\text{V}$$

$$V_1 = 18650\,\mu\text{V}$$

Output voltage of reference junction,

$$V_2 = (13650 + 50 \times 1)\,\mu\text{V}$$

$$V_1 = 13700\,\mu\text{V}$$

Now, $V_0 = A(V_1 - V_2)$

$$V_0 = 20(18650 - 13700)$$

$$V_0 = 20 \times 4950 = 99000\,\mu\text{V}$$

$$V_0 = 99\,\text{mV}$$

Method 2 :

Given : $V_\theta = (13650 + 50\theta_x)\mu_v$

Sensitivity $= S = \dfrac{dV_\theta}{d\theta_x} = 50\,\mu\text{V}/^0\text{C}$

From diagram, $V_0 = GV_{T.C} = 20 \times \dfrac{S}{TC}[T_H - T_C]$

$$V_0 = 20 \times 50\frac{\mu v}{C}\left[\theta - \theta_{ref}\right]$$

$$V_0 = 20 \times 50\frac{\mu v}{C}[100^0 - 1]^0$$

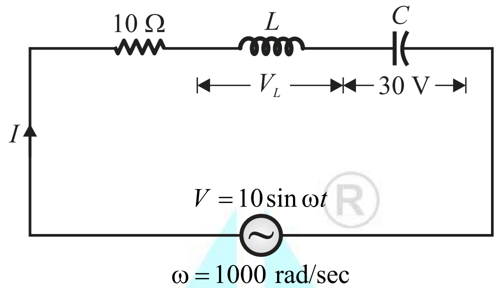
$$V_0 = 20 \times 50 \times 10^{-6} \times 99 = 99\,\text{mV}$$

Question 47 **Network Theory (NAT)**

An air core coil having a winding resistance of $10\,\Omega$ is connected in series with a variable capacitor C_x. The series circuit is excited by a 10 V sinusoidal voltage source of angular frequency 1000 rad/s. As the value of the capacitor is varied, a maximum voltage of 30 V was observed across it. Neglecting skin-effect, the value of the inductance of the coil in millihenry is _______. **[2 Marks]**

Ans. **30**

Sol. According to question, given RLC circuit is shown below,



Voltage across capacitor and inductor becomes maximum only when circuit is in resonance.

In resonance $|V_C| = Q|V|$

So, $$Q = \frac{|V_C|}{|V|} = \frac{30}{10} = 3$$

Also, $$Q = \frac{\omega_0 L}{R} = \frac{1000\times L}{10}$$

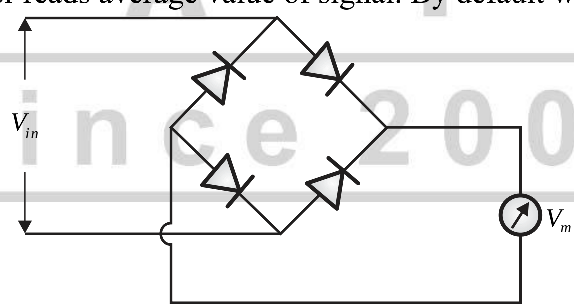
$$L = \frac{30}{1000}\text{ H} = 30\text{ mH}$$

Question 48 **Measurement (NAT)**

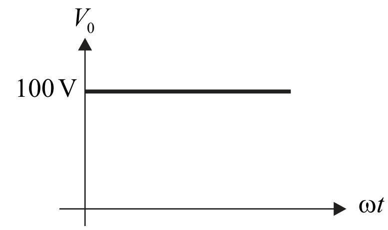
A 3½ digit rectifier type digital meter is set to read in its 2000 V range. A symmetrical square wave of frequency 50 Hz and amplitude ± 100 V is measured using the meter. The meter will read _______. **[2 Marks]**

Ans. **100**

Sol. Rectifier type digital meter reads average value of signal. By default we consider full wave rectifier



Output W/f of rectifier,



∴ Voltage read = 100 V.

Question 49 **Microprocessor (MCQ)**

A 16-bit microprocessor has twenty address lines (A_0 to A_{19}) and 16 data lines. The higher eight significant lines of the data bus of the processor are tied to the 8 data lines of a 16 Kbyte memory that can store 1-byte in each of its 16 K address location. The memory chip should map onto contiguous memory locations and occupy only 16 Kbyte of memory space. Which of the following statement(s) is/are correct with respect to the above design? **[1 Mark]**

(A) If the 16 Kbyte of memory chip is mapped with a starting address of 80000H, then the ending address will be 83FFFH.

(B) The active high chip-select needed to map 16 Kbyte memory with starting address at F0000H is given by the logic expression ($A_{19}.A_{18}.A_{17}.A_{16}$).

(C) The 16 Kbyte memory cannot be mapped with continuous address location with a starting address as 0F000H using only A_{19} to A_{14} for generating chip select.

(D) The above chip cannot be interfaced as the width of the data bus of the processor and memory chip differs.

Ans. **A, B, C**

Sol. 16 kB memory $= 2^4 k \times 8 = 2^4 \times 10^{10} \times 8 = 2^{14} \times 8$

∴ 14 address lines and 8 data lines are required.

Since, 8 data lines of processor is available for memory. So connection is possible.

So, option (D) is wrong

A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0000 H
1	1	1	1	1	1	1	1	1	1	1	1	1	1	3FFFH

Offset = 3FFFH – 0000H =3FFFH

Now if starting address is 80000H, then

Ending address = Starting address + Offset

$$= 80000\,H + 3\,FFFH = 83FFFH$$

So, option (A) is correct.

Given in option (B) active high chip select (Cs) is needed.

If $A_{19}.A_{18}.A_{17}.A_{16}$ is the input to CS then to enable chip this function should be 1. Now starting address is F0000H and ending address will be (F0000H + 3FFFH = F3FFFH)

A_{19}	A_{18}	A_{17}	A_{16}	A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Now, for given address $A_{10}.A_{18}.A_{17}.A_{16} = 1$ (always). Hence this expression can be used for chip select.

So, option (B) is correct.

If starting address is 0F000H, then ending address $=$ 0F000H + 3 FFFH $=$ 12 FFFH

A_{16}	A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0
0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1

For the given address range we need A_0 to A_{15} address lines. Remaining address lines only can be used for chip select.

Hence, option (C) is also correct.

Hence, the correct options are (A), (B) & (C).

Question 50 **Measurement (NAT)**

A bar primary current transformer of rating $\frac{1000}{1}$ A . 5 VA, UPF has 995 secondary turns. It exhibits zero ratio error and phase error of 30 minutes at 1000 A with rated burden. The watt loss component of the primary excitation current in ampere is _______ (rounded off to one decimal place). **[2 Marks]**

Ans. **5**

Sol. **Given :**

$N_p = 1$

$R = \text{True ratio } = \frac{1000}{1}$

$k = \text{Nominal ratio } = ?$

$R = k$

$\cos\phi = \text{UPF} = 1$

$N_s = 995$

Ratio error $= 0$

Phase error = 30 min

$\delta = 0$

$\cos\delta = 1$

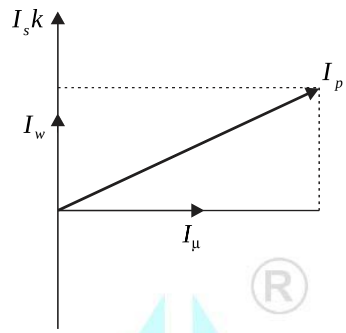
$\sin\delta = 0$

$$n = \frac{N_s}{N_p}$$

$$\text{Percentage ratio} = \frac{\text{Measured value} - \text{True value}}{\text{True value}} \times 100 = \frac{k-R}{R} \times 100$$

Where, k = Nominal ratio, R = True ratio, Ratio error $e_{rr} = 0$.

$$k = R = 1000$$



$$R = n + \left[\frac{I_w \cos\delta + I_\mu \sin\delta}{I_s}\right]$$

$$1000 = \frac{N_s}{N_p} + \left[\frac{I_w}{I_s} + 0\right]$$

$$1000 = \frac{995}{1} + I_w$$

$$I_w = 5\text{ A}$$

Question 51 **Measurement (NAT)**

A 300 V, 5 A, LPF wattmeter has full scale of 300 W. The wattmeter can be used for loads supplied by 300 V ac mains with a maximum power factor of ________ (rounded off to one decimal place)

[1 Mark]

Ans. **0.2**

Sol. **Given :**

(i) $V = 300\text{ V}$

(ii) $I = 5\text{ A}$

(iii) $P_m = 300\text{ W}$

$$P = VI\cos\phi$$

Maximum power factor, $\cos\phi = \frac{P_m}{VI} = \frac{300}{300\times 5} = 0.2$

$$\cos\phi = 0.2$$

Question 52 **Sensors & Transducers (MCQ)**

Input-output characteristic of a temperature sensor is exponential for a **[1 Mark]**

(A) Mercury thermometer
(B) Resistive temperature device (RTD)
(C) Thermistor
(D) Thermocouple

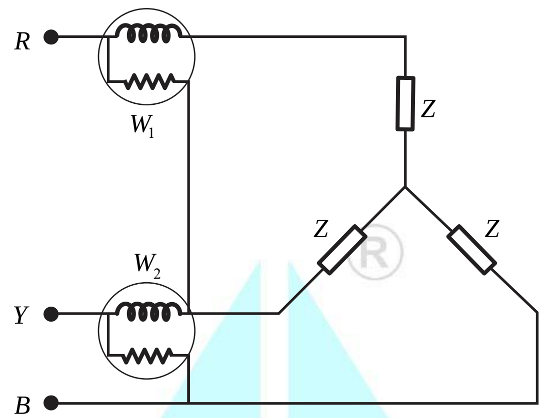
Ans. **C**

Sol. Input-output characteristics is exponential for thermistor.

Hence, the correct option is (C).

Question 53 **Measurement (NAT)**

The power in a 400 V (rms, line-line) three-phase, three-wire RYB sequence system is measured using the two wattmeters, as shown. The R-line current is $5\angle 60^0$ A. Wattmeter W_1 in the R-line read (in watt) _______. **[2 Marks]**

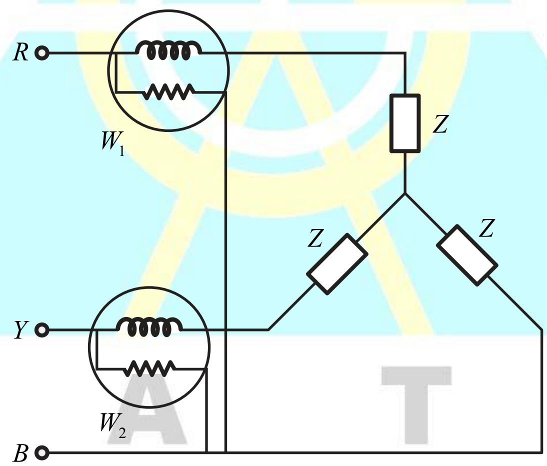


Ans. 0

Sol. **Given :**

(i) $(V_L)_{rms} = 400\,\text{V}$

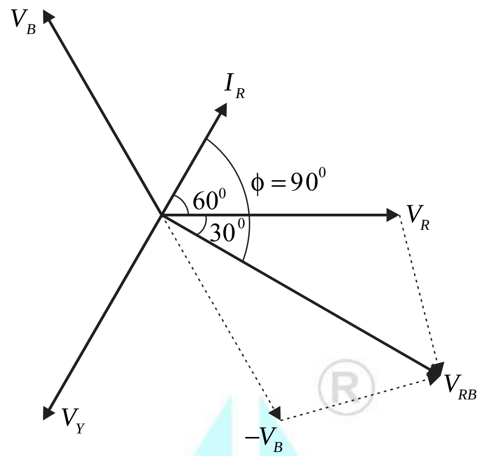
(ii) $I_R = 5\angle 60^0$



Reading of wattmeter 1 is given by,

$$W_1 = V_L I_L \cos\phi$$

$$W_1 = V_{RB} I_R \cos\phi$$

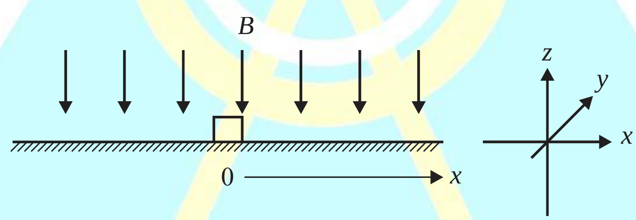


$$\phi = 90^0$$

$\therefore \quad W_1 = 0 \text{ watt}$

Question 54 **Electromagnetics (NAT)**

The figure below shows an electrically conductive bar of square cross-section resting on a plane surface. The bar of mass of 1 kg has depth of 0.5 m along the y direction. The coefficient of friction between the bar and the surface is 0.1. Assume the acceleration due to gravity to be 10 m/s^2. The system faces a uniform flux density $B = -1\hat{z}T$. At time $t = 0$, a current of 10 A is switched onto the bar and it maintained.



When the bar has moved by 1 m, its speed in metre per second is ________ (rounded off to one decimal place). **[2 Marks]**

Ans. **2.828**

Sol. Equation of motion,

$$v^2 - u^2 = 2as$$

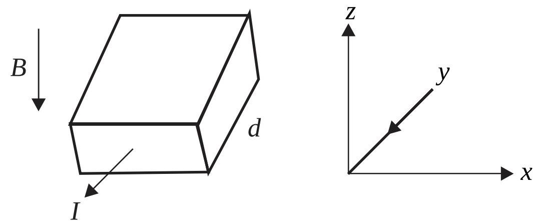
$$v^2 = 2as \qquad (\because \text{ Initial velocity } u = 0)$$

Two forces are experienced by bar

1. Magnetic force by current carrying conductor (F_B).
2. Kinetic friction force against the direction of movement (F_k).

$$F_{total} = F_B - F_k$$

As F_k is against the F_B,



Lets assume current is flowing in direction of $-\hat{a}_y$

Calculation of F_B :

Method 1 : $F_B = |Id||B|\sin\theta$

$\theta =$ Angle between I and $B = 90^0$

$F_B = 10\times 0.5\times 1\times \sin 90^0 = 5$ N

Method 2 : $F_B = I\vec{d}\times\vec{B}$

$F_B = 10\times 0.5(-\hat{a}_y)\times(-1\hat{a}_z) = 5$ N

Calculation of F_k :

$F_k = \mu_k mg$

Where, $\mu_k =$ Coefficient of friction

$m =$ Mass of bar and

$g =$ Gravity due to earth.

$F_k = 0.1\times 1\times 10 = 1\,\text{N}$

$F_{total} = F_B - F_k = 5 - 1 = 4\,\text{N}$

So that, $V^2 = 2as$

$\because$ $a = \dfrac{F_{total}}{\text{Mass}} = \dfrac{4}{1} = 4$ m/sec^2

$V^2 = 2\times 4\times 1$ $(\because s = 1\text{ m})$

$V = \sqrt{8} = 2.82842$ m/s

Question 55 **Electrical Machines (MCQ)**

A single-phase transformer has maximum efficiency of 98%. The core losses are 80 W and the equivalent winding resistance as seen from the primary side is $0.5\ \Omega$. The rated current on the primary side is 25 A. The percentage of the rated input current at which the maximum efficiency occurs is **[2 Marks]**

(A) 35.7% (B) 80.5% (C) 50.6% (D) 100%

Ans. **C**

Sol. **Given :**

(i) Core loss $= 80$ W

(ii) $R_1 = 0.5\Omega$

(iii) $I_{1(\text{rated})} = 25$ A

(iv) $\eta = 98\% = 0.98$

Transformer efficiency is given by,

$$\eta = \frac{xS\cos\phi}{xS\cos\phi + P_{core} + x^2 P_{copper}}$$

For maximum efficiency, $x^2 P_{copper} = P_{core}$

$$x^2 I_1^2 R_1 = P_{core}$$

$$x^2 \times 25^2 \times 0.5 = 80$$

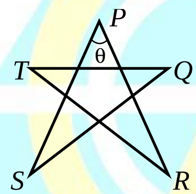
$$x = 0.5059$$

$$\%x = 0.5059 \times 100 = 50.6\%$$

General Aptitude

Question 56 **Aptitude**

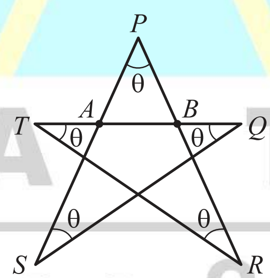
Following shape has equal length segments *PR*, *PS*, *QS*, *TR* and *TQ* are of equal length, what will be the angle θ? **[2 Marks]**



(A) 36 (B) 108 (C) 72 (D) 45

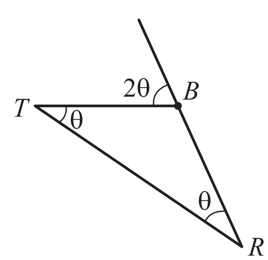
Ans. (A)

Sol. Here, star shape will all sequent are equal is shown below,



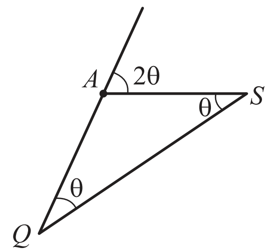
$\therefore$ $\angle P = \angle T = \angle S = \angle R = \angle Q = \theta$ (Equal sides of triangle have equal angle)

In ΔTBR,



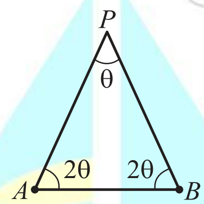
$\text{Ext}\angle B = 2\theta$ … (i) ($\because$ Sum of opposite interior angle = Exterior angle)

In ΔQAS ,



$$\text{Ext}\angle A = 2\theta \quad \text{...(ii)}$$

In ΔPAB ,



$\because$ Sum of all interior angles in a $\Delta = 180^0$.

$\therefore$ $$\theta + 2\theta + 2\theta = 180^0$$

$$\theta = 36^0$$

Hence, the correct option is (A)

Question 57 **Aptitude**

Let $\oplus$ and $\odot$ are two operators on numbers p and q such that $p \oplus q = \frac{p^2 + q^2}{pq}$ and $p \odot q = \frac{p^2}{q}$, if $x \oplus y = 2 \odot 2$, then x will be equal to **[1 Mark]**

(A) y (B) 2y (C) y/2 (D) 3y/2

Ans. **A**

Sol. **Given :** $$p \oplus q = \frac{p^2 + q^2}{pq}$$

$\therefore$ $$x \oplus y = \frac{x^2 + y^2}{xy} \quad \text{...(i)}$$

and $$p \odot q = \frac{p^2}{q}$$

$\therefore$ $$2 \odot 2 = \frac{2^2}{2} = 2 \quad \text{...(ii)}$$

From equation (i) and (ii),

$$\frac{x^2 + y^2}{xy} = 2$$

$\therefore$ $$x^2 + y^2 - 2xy = 0$$

$$(x-y)^2 = 0$$

$\therefore$ $x = y$ satisfy the condition.

Question 58 **Aptitude**

In a company, 35% employees drink coffee, 40% drink tea and 10% drink both tea and coffee. Then how much % of employees will neither drink tea nor coffee? **[1 Mark]**

(A) 25 (B) 35 (C) 40 (D) 15

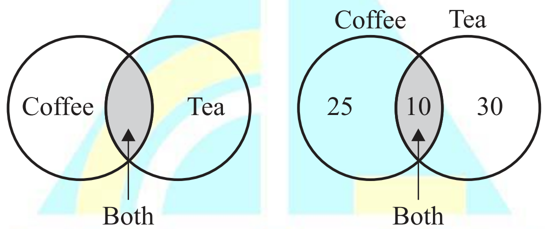
Ans. B

Sol. **Given :** Employees drink coffee = 35%

Employees drink tea = 40%

Employees drink both tea and coffee = 10%

So from above data we can easily sketch Venn diagram,



$25+10+30 = 65\%$ employees are those who either takes coffee or tea or both.

$\therefore$ $(100-65)\% = 35\%$ are those who neither take coffee nor tea.

Question 59 **Aptitude**

What will be the mirror image of the following word TRIANGLE if mirrored along x-axis.

[1 Mark]

The mirror image of the above text about the X-axis is

(A)  (B)  (C)  (D) 

Ans. B

Sol. If mirror is placed on *x*-axis, then the correct image is

Question 60 **Aptitude**

Statements : Either P marries Q or X marries Y. Among the given option below, the logical negation of the above statements is, **[2 Marks]**

(A) Neither P marries Q nor X marries Y. (B) P does not marry Q and X marries Y

(C) P marries Q and X marries Y. (D) X does not marries Y and P marries Q.

Ans. A

Sol. As we are directed to do logical negation, of given statement, situation of "either or" will becomes "Neither nor"

as : $P = Q$ $X = Y$

Negation : $P \neq Q$ $X \neq Y$

Hence, the correct option is (A).

Question 61 **Aptitude**

Four persons P, Q, R and S are to be seated in a row, facing same direction, but not necessary in the same order. P and R can not be seated adjacent to each other, S should be seated to the right of Q, then how many distinct seating arrangement is possible? **[1 Mark]**

(A) 4 (B) 2 (C) 8 (D) 6

Ans. D

Sol. **Condition 1 :** P and R can not sit adjacent to each other.

Condition 2 : S should be seated to the right of Q,

According to this all possible cases will be

1. Q P S R
2. Q R S P
3. P Q S R
4. R Q S P
5. P Q R S
6. R Q P S

Hence, the correct option is (D).

Question 62 **Aptitude**

Consider two rectangular sheets, M and N of identical dimensions of 6×4 cm each

Folding operation (i) : The sheet is folded into half by joining the short edges of the current shape.

Folding operation (ii) : The sheet is folded into half by joining the long edges of the current shape.

Folding operation (i) is carried out on sheet M 3 times.

Folding operation (ii) is carried out on sheet N 3 times.

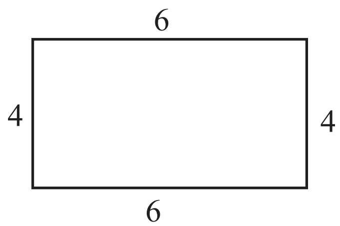
The ratio of perimeters of the final folded shape of sheet N to the final folded shape of sheet M is.

[2 Marks]

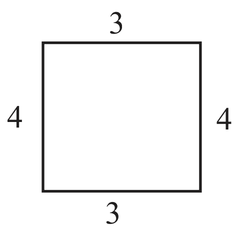
(A) 3 : 2 (B) 7 : 5 (C) 13 : 7 (D) 5 : 13

Ans. C

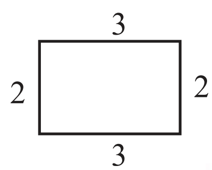
Sol. According to given data, we can proceed step by step as given below, M



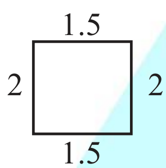
1st folding: (Respective of short edges)



2nd folding: (Respective of short side)

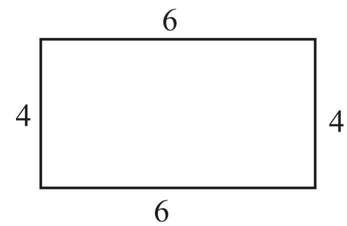


3rd folding: (Respective of short side)

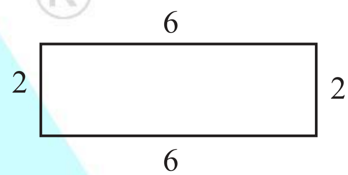


Perimeter of folded shape M =7

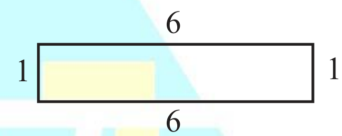
N



1st folding: (Respective of long edges)



2nd folding: (Respective of long side)



3rd folding: (Respective of long side)

Perimeter of folded shape N = 13

$\therefore$ Ratio of perimeters of the final folded shape N to the final folded shape of sheet M is **13:7.**

Question 63 **Aptitude**

A function $\lambda(p, q)$ is defined by

$$\lambda(p, q) = \begin{cases} (p-q)^2, & \text{if } p \geq q \\ p+q & \text{if } p < q \end{cases}$$

The value of expression $\dfrac{\lambda(-(-3+2),(-2+3)}{[-(-2+1)]}$ will be **[2 Marks]**

(A) 0 (B) $\dfrac{16}{3}$ (C) 16 (D) -1

Ans. **A**

Sol. Given expression easily solved as,

$$\frac{\lambda[-(-3+2), (-2+3)]}{-(-2+1)} = \frac{\lambda(1, 1)}{1} = \lambda(1, 1)$$

$\because \quad p = q = 1$

$\therefore \quad \lambda(1, 1) = (1-1)^2 = 0$

Hence, the correct option is (A).

Question 64 **Aptitude**

Getting to the top is ______ than staying in top **[1 Mark]**

(A) Easier (B) More Easier (C) Much Easier (D) Easiest

Ans. A

Sol. Getting to the top is ***easier*** than staying in top.

Hence, the correct option is (A).

Question 65 **Aptitude**

Human have the ability to construct worlds entirely in their minds, which does not exist in the physical world. So far as we know, no other species possess this ability. This skill is so important that we have different words to refer to its different factors, such as imagination, invention and innovation. With respect to above, which of the following is correct? **[2 Marks]**

(A) The terms imagination, invention and innovation refer to unrelated skills.

(B) No species possess the ability to construct worlds in their mind.

(C) Imagination, invention and innovation are un-related to the ability to construct mental worlds.

(D) We don't know of any species other than humans who posses the ability to construct mental worlds.

Ans. D

Sol. As given in the above passage "so far as we know, no other species posses this ability". By this we can conclude option D is correct.

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