## Technical Section

## Question 1

Two reservoirs are connected through a homogeneous and isotropic aquifer having hydraulic conductivity $(K)$ of $25 \mathrm{~m} /$ day and effective porosity ( $n$ ) of 0.3 as shown in the figure (not to scale). Ground water is flowing is the aquifer at the steady state


If water in Reservoir 1 is contaminated then the time (in days, round off to one decimal place) taken by the contaminated water to reach to Reservoir 2 will be $\qquad$ .
Ans. 2400
Sol.

$\Delta H=20 \mathrm{~m}$
$n=0.3$
$k=25 \mathrm{~m} /$ day
Time $($ in days $)=$ ?

$$
\begin{aligned}
\because \quad V_{s} & =\frac{V}{n}=\frac{k i}{n}=\frac{25 \times\left[\frac{20}{2000}\right]}{0.3} \\
V_{s} & =0.833 \mathrm{~m} / \text { day }
\end{aligned}
$$

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$V_{s}=\frac{D}{t}$

$$
t=\frac{2000}{0.833}=2400 \text { days }
$$

## Question 2

A partially-saturated soil samples has natural moisture content of $25 \%$ and bulk unit weight of $18.5 \mathrm{kN} / \mathrm{m}^{3}$. The specific gravity of soil solids is 2.65 and unit weight of water is $9.81 \mathrm{kN} / \mathrm{m}^{3}$. The unit weight of soil sample on full saturation is
(A) $20.12 \mathrm{kN} / \mathrm{m}^{3}$
(B) $19.03 \mathrm{kN} / \mathrm{m}^{3}$
(C) $21.12 \mathrm{kN} / \mathrm{m}^{3}$
(D) $18.50 \mathrm{kN} / \mathrm{m}^{3}$

Ans. B
Sol. Given :
$w=0.25$
$\gamma_{T}=18.5 \mathrm{kN} / \mathrm{m}^{3}$
$G=2.65$
$\gamma_{\omega}=9.81 \mathrm{kN} / \mathrm{m}^{3}$

$$
\begin{aligned}
& \gamma_{\text {saturated }}=\frac{(G+e) \gamma_{w}}{1+e} \\
& \gamma_{\text {saturated }}=\frac{(2.65+0.7565) \times 9.81}{1+0.7565} \\
& \gamma_{\text {saturated }}=19.03 \mathrm{kN} / \mathrm{m}^{3}
\end{aligned}
$$

Hence, the correct option is (B).

## Question 3

The shape of the most commonly deigned highway vertical curve is
(A) Spiral
(B) Parabolic
(C) Circular (same radius)
(D) Circular (different radius)

Ans. B
Sol. The shape of most commonly designed highway vertical curve is parabolic in nature. Hence, the correct option is (B).

## Question 4

A water sample is analyzed for coliform organisms by the multiple-tube fermentation method. The results of confirmed test are follows:

| Sample size (mL) | Number of positive <br> results out of $\mathbf{5}$ tubes | Number of negative <br> results out of $\mathbf{5}$ tubes |
| :---: | :---: | :---: |
| 0.01 | 5 | 0 |
| 0.001 | 3 | 2 |
| 0.0001 | 1 | 4 |

The most probable number (MPN) of coliform organisms for the above results is to be obtained using the following MPN Index.

| MPN Index for Various Combinations of Positive Results when <br> Five Tubes used per Dilution of $10.0 \mathrm{~mL}, 1.0 \mathrm{~mL}$ and 0.1 mL |  |
| :--- | :--- |
| Combination of positive tubes | MPN Index per 100mL |
| $0-2-4$ | 11 |
| $1-3-5$ | 19 |
| $4-2-0$ | 22 |
| $5-3-1$ | 110 |

The MPN of coliform organisms per 100 mL is
(A) 110
(B) 1100000
(C) 110000
(D) 1100

Ans. C
Sol. We will search for combination of + ve tubes as $5-3-1$ in $0.01,0.001$ and 0.0001 ml from the table.
Since, the MPN chart given MPN value of 110 per 100 ml for $10 \mathrm{ml}, 1 \mathrm{ml}$ and 0.1 ml dilutions then the result so obtained is multiplied by $\frac{10 \mathrm{ml}}{0.01 \mathrm{ml}}=1000$ to obtain MPN per 100 ml for $0.01,0.001,0.0001 \mathrm{ml}$ dilutions $\Rightarrow$ MPN per $100 \mathrm{ml}=110 \times 1000=110000$
Hence, the correct option is (C).

## Question 5

The volume determined from $\iiint_{V} 8 x y z d V$ for $V=[2,3] \times[1,2] \times[0,1]$ will be (in integer) $\qquad$ .

Ans. 15
Sol. Volume $=\iiint 8 x y z d x d y d z$

$$
V=8\left(\int_{0}^{1} z \int_{1}^{2} y \int_{2}^{3} x d x d y d z\right)
$$

$$
V=8\left(\int_{0}^{1} z d z \int_{1}^{2} y d y \int_{2}^{3} x d x\right)
$$

$$
V=8\left(\frac{z^{2}}{2}\right)_{0}^{1}\left(\frac{y^{2}}{2}\right)_{1}^{2}\left(\frac{x^{2}}{2}\right)_{2}^{3}=1 \times 3 \times 5=15
$$

## Question 6

A combined trapezoidal footing of length $L$ supports two identical square columns ( $P_{1}$ and $P_{2}$ ) of size $0.5 \mathrm{~m} \times 0.5 \mathrm{~m}$, as shown in the figure. The column $P_{1}$ and $P_{2}$ carry loads of 2000 kN and 1500 kN respectively.


If the stress beneath the footing is uniform, the length of combined footing $L$ (in m , round off to two decimal places) is $\qquad$
Ans. 5.833
Sol.

C.G. of load $=\frac{A_{1} Y_{1}+A_{2} Y_{2}}{A_{1}+A_{2}}$
C.G. of load $=\frac{(2000 \times 0.25)+[1500 \times(5+0.25)]}{2000+1500}=2.393 \mathrm{~m}$

C.G. of trapezium $=\frac{2 a+b}{a+b} \times \frac{h}{3}=\frac{1.5 \times 2+5}{1.5+5} \times \frac{L}{3}=0.410 \mathrm{~L}$

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$$
\begin{aligned}
& 2.393=0.410 L \\
& L=5.833 \mathrm{~m}
\end{aligned}
$$

## Question 7

A highway designed for $80 \mathrm{~km} / \mathrm{h}$ speed has a horizontal curve section with radius 250 m . If the design lateral friction is assumed to develop fully, the required super elevation is
(A) 0.02
(B) 0.05
(C) 0.09
(D) 0.07

Ans. B
Sol. Given :
Speed of vehicle $=V=80 \mathrm{kmph}$
Radius of Curve $R=250 \mathrm{~m}$
We know that, $e+f=\frac{V^{2}}{127 R}$

$$
\begin{aligned}
& e+0.15=\frac{80^{2}}{127 \times 250} \\
& e=0.0515
\end{aligned}
$$

Hence, the correct option is (B).

## Question 8

The value of abscissa ( $x$ ) and ordinate ( $y$ ) of curve are as follows:

| $x$ | $y$ |
| :---: | :---: |
| 2.0 | 5.00 |
| 2.5 | 7.25 |
| 3.0 | 10.00 |
| 3.5 | 13.25 |
| 4.0 | 17.00 |

By Simpsons $1 / 3^{\text {rd }}$ rule, the area under the curve (round off to two decimal places) is $\qquad$
Ans. 20.67
Sol. Area $=\int_{2}^{4} f(x) d x$
Numerical Integrations, by Simson's $1 / 3{ }^{\text {rd }}$ rule

$$
\begin{aligned}
& =\frac{\mathrm{h}}{3}\left[\left(\mathrm{y}_{0}+\mathrm{y}_{4}\right)+\left(\mathrm{y}_{1}+\mathrm{y}_{3}\right)+2 \mathrm{y}_{2}\right] \\
& =\frac{1}{6}[(5+17)+4(7.25+13.25)+2 \times 10]=20.67
\end{aligned}
$$

## Question 9

On a road, the speed-density relationship of a traffic stream is given by $u=70-0.7 k$. (where speed, $u$, is in $\mathrm{km} / \mathrm{h}$ and density, $k$, is in veh/km). At capacity condition, the average time headway will be.
(A) 1.65
(B) 0.5
(C) 1.0
(D) 2.1

Ans. D
Sol. $\quad v=70-0.7 \mathrm{~K}$
$v=70\left[1-\frac{k}{\left(\frac{70}{0.7}\right)}\right]$
$v_{f}=70 \mathrm{kmph}$
$K_{I}=\frac{70}{0.7}=100 \mathrm{Veh} / \mathrm{km}$
$q_{\text {max }}=\frac{1}{4} \times v_{f} \times K_{j}=\frac{1}{4} \times 70 \times 100=1750 \mathrm{Veh} / \mathrm{hr}$
$q=\frac{3600}{t_{h}}$
$t_{h}=2.05 \mathrm{sec}$
Hence, the correct option is (D).

## Question 10

A tube-well of 20 cm diameter fully penetrates a horizontal, homogeneous and isotropic confined aquifer of infinite horizontal extent. The aquifer is of 30 m uniform thickness. A steady pumping at the rate of 40 litres $/ \mathrm{sec}$ from the well for long time results in a steady drowdown of 4 m at the well face. The subsurface flow to the well due to pumping is steady, horizontal, Darcian and the radius of influence of the well is 245 m . The hydraulic conductivity of the aquifer (in $\mathrm{m} /$ day, round off to integer) is $\qquad$
Ans. 35.884
Sol.

$Q=40 \mathrm{l} / \mathrm{s}=40 \times 10^{-3} \mathrm{~m}^{3} /$ second
$Q=\frac{2 \pi K B S_{W}}{\ln \left(\frac{R}{r}\right)}$
$40 \times 10^{-3}=\frac{2 \pi K \times 30 \times 4}{\ln \left(\frac{245}{0.1}\right)}$

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$$
\begin{aligned}
K & =\frac{40 \times 10^{-3} \times \ln \left(\frac{245}{0.1}\right)}{2 \pi \times 30 \times 4} \\
K & =0.000415 \mathrm{~m} / \mathrm{s}=35.884 \mathrm{~m} / \text { day }
\end{aligned}
$$

## Question 11

A small project has 12 activities - N, P, Q, R, S, T, U, V, W, X, Y and Z. The relationship among these activities and the duration of these activities are given in the Table.

| Activity | Duration (in weeks) | Depends upon |
| :---: | :---: | :---: |
| N | 2 | - |
| P | 5 | N |
| Q | 3 | N |
| R | 4 | P |
| S | 5 | Q |
| T | 8 | R |
| U | 7 | $\mathrm{R}, \mathrm{S}$ |
| V | 2 | U |
| W | 3 | U |
| X | 5 | $\mathrm{~T}, \mathrm{~V}$ |
| Y | 1 | W |
| Z | 3 | $\mathrm{X}, \mathrm{Y}$ |

The total float of the activity "V" (in weeks, in integer) is $\qquad$ .
Ans. 0
Sol.


Critical Path $=\mathrm{N}-\mathrm{P}-\mathrm{R}-\mathrm{D}_{1}-\mathrm{U}-\mathrm{V}-\mathrm{X}-\mathrm{Z}=28$ weeks
For any critical activity all floats are 0
So for $V$ activity total float $=0$
By formula,
Total float $=L_{j}-E_{i}-t_{i j}=20-18-2=0$

## Question 12

The direct and indirect costs estimated by a contractor for bidding a project is ₹ 160000 and ₹ 20000 respectively. If the mark up applied is $10 \%$ of the bid price, the quoted price (₹in) of the contractor is
(A) 200000
(B) 198000
(C) 196000
(D) 182000

Ans. B
Sol. Given :
Direct cost $=160000$ Rs
Indirect cost $=20000$ Rs.
Mark up rate $=10 \%$ of bid price
Total estimated cost by contractor $=160000+20000=180000$ Rs
Mark up cost $=10 \%$ of 180000 Rs $=18000$ Rs
Quoted price $=$ Total estimate cost + Markup cost $=180000+18000$ Rs $=198000$ Rs
Hence, the correct option is (B).

## Question 13

A propped cantilever beam EF is subjected to a unit moving load as shown in the figure (not to scale).
The sign convention for positive shear force at the left and right sides of any section is also shown
Unit moving loac


Sign convention for
positive shear force
The CORRECT qualitative nature of the influence line diagram for shear force at G is
(A)

(B)

(C)

(D)


Ans. A
Question 14
A cylinder ( 2.0 m diameter, 3.0 m long and 25 kN weight) is acted upon by water on one side and oil (specific gravity $=0.8$ ) on other side as shown in the figure.


The absolute ratio of the net magnitude of vertical forces to the net magnitude of horizontal forces (round off to two decimal places) is $\qquad$
Ans.
0.61

Sol.


## Given :

Diameter of cylinder $=2 \mathrm{~m}$
Length of cylinder $=3 \mathrm{~m}$
Weight $W=25 \mathrm{kN}$
To find : $\left(\frac{F_{V}}{F_{H}}\right)_{\text {net }}=$ ?

$$
\begin{aligned}
& F_{H}=\rho A g \bar{h} \\
& F_{\substack{H \text { water } \\
(\rightarrow)}}=(2 \times 3) \times 9.81 \times 2 \times 1000=11.72 \mathrm{kN}
\end{aligned}
$$

$$
{\underset{c}{H, \text { oil }}(\leftarrow)}^{F_{( }}(1 \times 3) \times 0.8 \times 1000 \times 9.81 \times 0.5=11.77 \mathrm{kN}
$$

$$
(\leftarrow)
$$

$$
\begin{equation*}
F_{H, \text { Net }}=F_{H, \text { Water }}-F_{H, \text { oil }}=105.978 \mathrm{kN} \tag{i}
\end{equation*}
$$

$$
F_{V, \text { Water }}=\rho g A \bar{h}=10^{3} \times 9.81 \times\left[\frac{\pi}{2}(1)^{2} \times 3\right]=46.228 \mathrm{kN}
$$

$$
F_{V, \text { oil }}=800 \times 9.81\left[\frac{\pi}{4}(1)^{2} \times 3\right]=18.49
$$

$$
\begin{equation*}
F_{V, \text { Net }}=F_{V, \text { Water }}+F_{V, \text { oil }}=46.228+18.49=64.7199 \mathrm{kN}(\uparrow) \tag{ii}
\end{equation*}
$$


$\therefore \quad\left(\frac{F_{V}}{F_{H}}\right)_{\text {net }}=\frac{64.7199}{105.978}=0.61$

## Question 15

The shape of cumulative distribution function of Gaussian distribution is.
(A) Horizontal line
(B) Bell shaped
(C) Straight line at 45-degree angle
(D) $S$-shaped.

Ans. B
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## Question 16

Ammonia nitrogen is present in a given waste water sample as the ammonium ion $\left(\mathrm{NH}_{4}^{+}\right)$and ammonia $\left(\mathrm{NH}_{3}\right)$. If pH is the only deciding factor for the proportion of these two constituents, which of the following is correct statement?
(A) At pH below $9.25, \mathrm{NH}_{3}$ will be predominant
(B) At $\mathrm{pH}=7, \mathrm{NH}_{4}^{+}$and $\mathrm{NH}_{3}$ will be found in equal measures.
(C) At $\mathrm{pH}=7, \mathrm{NH}_{4}^{+}$will be predominant.
(D) At pH above 9.25, only $\mathrm{NH}_{4}^{+}$will be present.

Ans. C
Sol. Ammonia exist in the form of $\mathrm{NH}_{4}{ }^{+} \& \mathrm{NH}_{3}$

- At $\mathrm{pH}<8$, all ammonia is in the form of $\mathrm{NH}_{4}{ }^{+}$
- At $\mathrm{pH}=9.5$, there are $50 \% \mathrm{NH}_{3} \& 50 \% \mathrm{NH}_{4}{ }^{+}$
- At $\mathrm{pH}>11$, all ammonia is in the form of $\mathrm{NH}_{3}$

Hence, option (C) is correct that at $\mathrm{pH}=7,+\mathrm{NH}_{4}{ }^{+}$will be predominant.
Hence, the correct option is (C).

## Question 17

An unsupported slope of height 15 m is shown in the figure (not to scale), in which the slope face makes an angle $50^{\circ}$ with the horizontal. The slope material comprises purely cohesive soil having undrained cohesion 75 kPa . A trial slip circle KLM, with a radius 25 m , passes through the crest and toe of the slope and it subtends an angle $60^{\circ}$ at its center O . The weight of the active soil mass ( W , Bounded by KLMN) is $2500 \mathrm{kN} / \mathrm{m}$, which is acting at a horizontal distance of 10 m from the toe of the slope. Consider the water table to be present at a very large depth from the ground surface.


Considering the trial slip circle KLM, the factor of safety against the failure of slope under undrained condition (round off to two decimal places) is $\qquad$
Ans. 1.96

Sol. $F O S=\frac{C \hat{L} R}{W \bar{x}}$
$\hat{L}=2 \pi \times 25 \times \frac{60}{360}$
$\hat{L}=26.18 \mathrm{~m}$
$F O S=\frac{75 \times 26.18 \times 25}{2500 \times 10}$
$F O S=1.96$

## Question 18

Which one of the following is correct?
(A) For an effluent sample of a sewage treatment plant, the ratio BOD 5 -day $20^{\circ} \mathrm{C}$ upon ultimate BOD is more than 1 .
(B) The most important type of species involved in the degradation of organic matter in the case of activated sludge process based wastewater treatment is chemoheterotrophs.
(C) The partially treated effluent from a food processing industry, containing high concentration of biodegradable organics, is being discharged into a flowing river at a point P . If the rate of degradation of the organics is higher than the rate of aeration, then dissolved oxygen of the river water will be lowest at point P .
(D) A young lake characterized by low nutrient content and low plant productivity is called eutrophic lake.
Ans. B

## Question 19

A bag house filter has to treat $12 \mathrm{~m}^{3} / \mathrm{s}$ of waste gas continuously. The baghouse is to be divided into 5 sections of equal cloth area such that one section can be shut down for cleaning and/or repairing, while the other 4 sections continue to operate. An air-to-cloth ratio of $6.0 \mathrm{~m}^{3} / \mathrm{min}-\mathrm{m}^{2}$ cloth will provide sufficient treatment to the gas. The individual bags are of 32 cm in diameter and 5 m in length. The total number of bags (in integer) required in the baghouse is $\qquad$
Ans. 30
Sol. Given :
Discharge $=12 \mathrm{~m}^{3} / \mathrm{s}$
Total surface Area with respect to discharge $=\frac{12 \mathrm{~m}^{3} / \mathrm{s}}{6 \mathrm{~m}^{3} / \mathrm{min}-\mathrm{m}^{2}}=\frac{12 \mathrm{~m}^{3} / \mathrm{s}}{\frac{6}{60} \mathrm{~m}^{3} / \mathrm{s}-\mathrm{m}^{2}}=120 \mathrm{~m}^{2}$
Area of bag $=\pi D L=\pi \times 0.32 \times 5=5.024$

- $120 \times \frac{5}{4}=150 \mathrm{~mL}$

Number of bag $=\frac{150}{5.024}=29.85 \approx 30$

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## Question 20

The cohesion (c), angle of internal friction $(\phi)$ and unit weight $(\gamma)$ of a soil are $15 \mathrm{kPa}, 20^{\circ}$ and $17.5 \mathrm{kN} / \mathrm{m}^{3}$, respectively. The maximum depth of unsupported excavation in the soil (in m , round off to two decimal places) is $\qquad$ .

Ans. 4.90
Sol. $\quad C=15 \mathrm{kPa}, \phi=20^{\circ}, \gamma_{T}=17.5 \mathrm{kN} / \mathrm{m}^{3}$

$$
H_{c}=?
$$

$$
\begin{aligned}
& H_{c}=\frac{4 C}{\gamma \sqrt{k_{a}}} \\
& k_{a}=\frac{1-\sin \phi}{1+\sin \phi}=\frac{1-\sin 20^{\circ}}{1+\sin 20^{\circ}}=0.4903 \\
& H_{c}=\frac{4 \times 15}{17.5 \sqrt{0.4903}}=4.89 \mathrm{~m} \approx 4.90 \mathrm{~m}
\end{aligned}
$$

## Question 21

A truss EFGH is shown in the figure, in which all the members have the same axial rigidity $R$. In the figure, $P$ is the magnitude of external horizontal forces acting at joints $F$ and $G$.


If $R=500 \times 10^{3} \mathrm{kN}, P=150 \mathrm{kN}$ and $L=3 \mathrm{~m}$, the magnitude of the horizontal displacement of joint $G$ (in mm round off to one decimal place)is $\qquad$ .
Ans. 0.9

## Sol. Given :

Axial rigidity $R=500 \times 10^{3} \mathrm{kN}$
$P=150 \mathrm{kN}$
$\delta_{G}=\Sigma \frac{P k l}{A E}=?$

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## Consider joint $G$ :


$\Sigma F_{x}=0$
$\Sigma F_{y}=0$
To final K, applied unit load at $G$


| Members | $\boldsymbol{P}$ | $\boldsymbol{k}$ | $\boldsymbol{L}$ | $\frac{\boldsymbol{P k L}}{\boldsymbol{A E}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $A B$ | 0 | 1 | 3 | 0 |
| $B C$ | 0 | $-\sqrt{2}$ | $3 \sqrt{2}$ | 0 |
| $A C$ | 0 | 1 | 3 | 0 |
| $B G$ | $P$ | 1 | 3 | $\frac{P(1)(3)}{A E}$ |
| $G C$ | 0 | 0 | 3 | 0 |

$\therefore \quad \delta_{G}=\frac{P(1)(3)}{A E}=\frac{150(1)(3)}{500 \times 10^{3}}=9 \times 10^{-4} \mathrm{~m}=0.9 \mathrm{~mm}$

## Question 22

A wedge $M$ and a block $N$ are subjected to forces $P$ and $Q$ as shown in the figure. If force $P$ is sufficiently large, then the block $N$ can be raised. The weights of the wedge and the block are negligible compared to the forces $P$ and $Q$. The coefficient of friction $(\mu)$ along the inclined surface between the wedges and the block is 0.2 . All other surface are frictionless. The wedge angle is $30^{\circ}$.

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Surface 3


Slock $N$ Surface $2(\mu$ 0.2)
Surface 1
The limiting force $P$, in terms of $Q$, required for impending motion of block $N$ to just move it in the upward direction is given as $P=\alpha Q$. The value of the coefficient ' $\alpha$ ' (round off to one decimal place) is
(A) 2
(B) 0.5
(C) 0.6
(D) 0.9

Ans. D
Sol. Given : Friction factor of surface $2 \mu=0.2$


Free body diagram, $N_{2} \cos 30^{\circ}=0.2 N_{2} \sin 30^{\circ}+Q$

$$
N_{2}=\frac{Q}{\cos 30^{\circ}-0.2 \sin (30)}
$$



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$$
\begin{aligned}
& P=0.2 \times 1.305 Q \cos (30)+1.305 Q Q \sin \left(30^{\circ}\right) \\
& P=0.878 Q
\end{aligned}
$$

Given condition, $P=\alpha Q$

$$
\begin{aligned}
& \alpha=0.878 \\
& \alpha \simeq 0.9
\end{aligned}
$$

Question 23
Based on drained triaxial shear tests on sands and clays, the representative variations of volumetric strain $(\Delta V / V)$ with the shear strain is shown in the figure.


Choose the CORRECT option regarding the representative behavior exhibited by Curve P and Curve Q
(A) Curve P represents loose sand and normally consolidated clay, while Curve Q represents dense sand and overconsolidated clay
(B) Curve P represents dense sand and normally consolidated clay, while Curve Q represents loose sand and overconsolidated clay
(C) Curve P represents loose sand and overconsolidated clay clay, while Curve Q represents dense sand and normally consolidated
(D) Curve P represents dense sand and overconsolidated clay, while Curve Q represents loose sand and normally consolidated clay.
Ans. D
Question 24
Traversing is carried out for a closed traverse PQRS. The internal angles at vertices $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ and S are measured as $92^{\circ}, 68,123^{\circ}$ and $77^{\circ}$ respectively. If fore bearing of a line PQ is $27^{\circ}$, fore bearing of line RS (in degrees, in integer) is $\qquad$ .
Ans. 196
Sol.

$(F B)_{R S}=$ ?
$(F B)_{P q}=27$
$(B B)_{P Q}=27^{0}+180^{\circ}=207^{\circ}$
$(F B)_{Q R}=207^{0}-68^{\circ}=139^{\circ}$
$(B B)_{Q R}=139^{\circ}+180^{\circ}=319^{0}$
$(F B)_{\text {RS }}=319-123=196^{\circ}$

## Question 25

If $P=\left[\begin{array}{ll}1 & 2 \\ 3 & 4\end{array}\right]$ and $Q=\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$ then $Q^{T} P^{T}$ is
(A) $\left[\begin{array}{ll}1 & 2 \\ 3 & 4\end{array}\right]$
(B) $\left[\begin{array}{ll}1 & 3 \\ 2 & 4\end{array}\right]$
(C) $\left[\begin{array}{ll}2 & 4 \\ 1 & 3\end{array}\right]$
(D) $\left[\begin{array}{ll}2 & 1 \\ 4 & 3\end{array}\right]$

Ans. C
Sol. $\quad(P Q)^{T}=Q^{T} P^{T}$

$$
\begin{aligned}
& P Q=\left[\begin{array}{ll}
1 & 2 \\
3 & 4
\end{array}\right]\left[\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right]=\left[\begin{array}{ll}
2 & 1 \\
4 & 3
\end{array}\right] \\
& Q^{T} P^{T}=(P Q)^{T}=\left[\begin{array}{ll}
2 & 4 \\
1 & 3
\end{array}\right]
\end{aligned}
$$

Hence, the correct option is (C).

## Question 26

The solution of the second-order differential equation $\frac{d^{2} y}{d x^{2}}+2 \frac{d y}{d x}-y=0$ with boundary conditions $y(0)=1$ and $y(1)=3$ is
(A) $e^{-x}-\left[3 e \sin \left(\frac{\pi x}{2}\right)-1\right] x e^{-x}$
(B) $e^{-x}+\left[3 e \sin \left(\frac{\pi x}{2}\right)-1\right] x e^{-x}$
(C) $e^{-x}+(3 e-1) x e^{-x}$
(D) $e^{-x}-(3 e-1) x e^{-x}$

Ans. C
Sol. Given : $\frac{d^{2} y}{d x^{2}}+\frac{2 d y}{d x}+y=0$

$$
\begin{aligned}
& y(0)=1 \\
& y(1)=3 \\
& \left(D^{2}+2 D+1\right) y=0
\end{aligned}
$$

Auxiliary equation, $m^{2}+2 m+1=0$

$$
\begin{aligned}
& (m+1)^{2}=0 \\
& m= \pm 1
\end{aligned}
$$

So,

$$
\begin{aligned}
& C F+P I=\left(C_{1}+C_{2} x\right) e^{-x} \\
& y(0)=1 \Rightarrow C_{1}=1 \\
& y(1)=3 \Rightarrow \Rightarrow C_{2}=3 e-1
\end{aligned}
$$

So, $\quad y=[1+(3 e-1) x] e^{-x}$
Hence, the correct option is (C).

## Question 27

A secondary clarifier handles a total flow of $9600 \mathrm{~m}^{3} / \mathrm{d}$ from a the aeration tank of a conventional activated-sludge treatment system. The concentration of solids in the flow from the aeration tank is 3000 $\mathrm{mg} / \mathrm{L}$. The clarifier is required to thicken the solids to $12000 \mathrm{mg} / \mathrm{L}$ and hence it is to be designed for a solid flux of $3.2 \mathrm{~kg} / \mathrm{m}^{2} \times h$. The surface area of the designed clarifier for thickening (in $\mathrm{m}^{2}$, in integer) is $\qquad$
Ans. 375
Sol. $\quad Q=9600 \mathrm{~m}^{3} /$ day
$X=3000 \mathrm{mg} / \mathrm{L}$
$X_{u}=12000 \mathrm{mg} / \mathrm{L}$
Solid flux $=3.2 \frac{\mathrm{~kg}}{\mathrm{~m}^{2} . \mathrm{h}}$
Surface area required $=\frac{9600 \times 10^{3} \times 3000}{3.2 \times 10^{6} \times 24}=375 \mathrm{~m}^{2}$

## Question 28

A column is subjected to a total load $(\mathrm{P})$ of 60 kN supported through a bracket connection as shown in the figure (not to scale).


The resultant force in bolt R (in kN , round off to one decimal place) is $\qquad$ .

Ans. 28.18
Sol.


Resultant force, $R=\sqrt{F_{1}^{2}+F_{2}^{2}+2 F_{1} F_{2} \cos \theta}$

$$
\begin{aligned}
& F_{1}=\frac{\text { Total load }(P)}{\text { Number of bolt }(n)} \\
& F_{1}=\frac{60}{6}=10 \mathrm{kN} \\
& F_{2}=\frac{P e . r_{1}}{\Sigma r_{1}^{2}} \\
& F_{2}=\frac{60 \times 100 \times 40}{\Sigma 4 \times 50^{2}+2 \times 40^{2}}=18.18 \mathrm{kN}
\end{aligned}
$$



$$
R=\sqrt{10^{2}+18.18^{2}+2 \times 10 \times 18.18 \cos 180^{0}}
$$


$R=\sqrt{10^{2}+18.18^{2}+2 \times 10 \times 18.18 \cos 0^{0}}$
$R=28.18 \mathrm{kN}$

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steps to success...

## Question 29

The equation of deformation is derived to be $y=x^{2}-x L$ for a beam shown in the figure.


The curvature of the beam at the mid-span (in units, in integer) will be $\qquad$ .
Ans. 2

## Sol. Given :


$y=x^{2}-x L$, to find curvature of beam
We know, $y=$ Deflection

$$
\begin{aligned}
& \frac{d y}{d x}=\text { Slope } \\
& \frac{d^{2} y}{d x^{2}}=\text { Curvature }=\frac{M}{E I}=\frac{1}{R} \\
& \therefore \quad \frac{d y}{d x}=2 x-1 \\
& \frac{d^{2} y}{d x^{2}}=2
\end{aligned}
$$

## Question 30

The value of $\int_{0}^{1} e^{x} d x$ using the trapezoidal rule with four equal subintervals is
(A) 2.192
(B) 2.718
(C) 1.727
(D) 1.718

Ans. C
Sol. According to trapezoidal rule, $\int_{0}^{1} e^{x} d x$


Given : $n=4, a=0, b=1, h=\frac{b-a}{x}=0.25$

| $\boldsymbol{x}$ | 0 | $1 / 4$ | $1 / 2$ | $3 / 4$ | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | 1 | $e^{1 / 4}$ | $e^{1 / 2}$ | $e^{3 / 4}$ | $e^{1}$ |

$$
I=\int_{0}^{1} e^{x} \cdot d x=\frac{h}{2}\left[y_{0}+y_{4}+2\left(y_{1}+y_{2}+y_{3}\right)\right]=1.726
$$

Hence, the correct option is (C).

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## Question 31

Employ stiffness matrix approach for the simply supported beam as shown in the figure to calculate unknown displacement/rotation. Take length, $L=8 \mathrm{~m}$ : modulus of elasticity, $E=3 \times 10^{4} \mathrm{~N} / \mathrm{mm}^{2}$; moment of inertia, $I=225 \times 10^{6} \mathrm{~mm}^{4}$.


The mid-span deflection of the beam (in mm , round off to integer) under $P=100 \mathrm{kN}$ in downward direction will be $\qquad$ .

Ans. 119
Sol. Given :
Length $=8 \mathrm{~m}$
$E=3 \times 10^{4} \mathrm{~N} / \mathrm{mm}^{2}$
$I=225 \times 10^{6} \mathrm{~mm}^{4}$
$P=100 \mathrm{kN}$


Now, convert it into conjugate beam, since its SSB then its conjugate beam remain same and $\frac{M}{E I}$ diagram becomes our loading diagram.

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Taking moment about $B$.

$$
\Sigma M_{B}=0
$$

Then,

$$
V_{A}(L)-\left[\frac{1}{2} \times \frac{P L}{8 E I} \times \frac{L}{2} \times\left(\frac{L}{2}+\frac{1}{3}\left(\frac{l}{2}\right)\right)\right]-\left[\frac{1}{2} \times \frac{P l^{2}}{4 E I} \times \frac{l}{2} \times\left(\frac{2}{3} \frac{l}{2}\right)\right]=0
$$

$$
V_{A}=\frac{P l^{2}}{48 E I}+\frac{P l^{2}}{48 E I}
$$

$$
V_{A}=\frac{P l^{2}}{24 E I}
$$

$$
\therefore \quad M_{B}=V_{A} \times \frac{L}{2}-\left[\frac{1}{2} \times \frac{P L}{8 E I} \times \frac{L}{2} \times \frac{1}{3} \times \frac{L}{2}\right]=0
$$

$$
M_{B}=\frac{P l^{2}}{24 E I} \times \frac{l}{2}-\frac{P l^{3}}{192 E I}
$$

$$
M_{B}=\frac{P l^{3}}{64 E I}
$$

$\because$ In conjugate beam where we want so find deflection, take out moment about that point.

$$
\therefore \quad \delta_{B}=\frac{P l^{3}}{64 E I}=\frac{100 \times 8^{3} \times 10^{3} \times 10^{9}}{64 \times 3 \times 10^{4} \times 225 \times 10^{6}}=118.5 \mathrm{~mm}
$$

## Question 32

Vehicular arrival at an isolated intersection follows the Poisson distribution. The mean vehicular arrival rate is 2 vehicle per minute. The probability (round off to two decimal places) that at least 2 vehicle will arrive in any given 1-minute interval is $\qquad$ -

Ans.
0.27

Sol. Given :
$\lambda=2$ Vehicle $/ \mathrm{min}=2 \mathrm{Vehical} / 60 \mathrm{sec}=\frac{1}{30} \mathrm{Veh} / \mathrm{sec}$
$n=2$ and $t=1 \mathrm{~min}$

$$
\begin{aligned}
& P(n, t)=\frac{(\lambda t)^{n} e^{-\lambda t}}{n!} \\
& P(n, t)=\frac{(2 \times 1)^{2} e^{-(2 \times 1)}}{2!}=0.2706
\end{aligned}
$$

Hence, the probability that atleast 2 vehicles will arrive in any given 1 min interval is 0.2706 .

## Question 33

A square plate O-P-Q-R of a linear elastic material with sides 1.0 m is loaded in a state of plane stress. Under a given condition, the plate deforms to a new configuration O-P'-Q'-R' as shown in the figure (not to scale). Under the given deformation the edges of the plate remain straight.


The horizontal displacement of the point ( $0.5 \mathrm{~m}, 0.5 \mathrm{~m}$ ) in the plate O-P-Q-R (in mm , round off to one decimal place) is $\qquad$
Ans. 2.5
Sol.


Horizontal displacement :


$$
\begin{aligned}
& -\frac{10}{x}=-\frac{20}{1-x} \\
& \frac{1}{2}=\frac{x}{1-x} \\
& 1-x=2 x \\
& x=\frac{1}{3} \\
& \frac{A B}{\frac{2}{3}-0.5}=\frac{20}{\frac{2}{3}} \\
& A B=5 \mathrm{~mm}
\end{aligned}
$$

So, horizontal displacement of $C=\frac{5}{2}=2.5 \mathrm{~mm}$ as OR is NOT
Moving in horizontal direction,


Vertical displacement of $C=\frac{10+5}{2}=7.5 \mathrm{~mm}$

## Question 34

A fluid flowing steadily in a circular pipe of radius $R$ has a velocity that is everywhere parallel to the axis (center line) of the pipe. The velocity distribution along the radial direction is $V_{r}=U\left(1-\frac{r^{2}}{R^{2}}\right)$, where $r$ is radial distance as measured from the pipe axis and $U$ is the maximum velocity at $r=0$. The average velocity of the fluid in the pipe is.
(A) $\frac{5}{6} U$
(B) $\frac{U}{3}$
(C) $\frac{U}{2}$
(D) $\frac{U}{4}$

Ans. C

## Sol. Given :

Velocity distribution along the radial direction $V_{r}=U\left(1-\frac{r^{2}}{R^{2}}\right)$

To find : $V_{\text {avg }}=$ ?

$$
\begin{align*}
& d A=2 \pi r \cdot d r \\
& d \dot{m}=\rho \cdot d A \cdot U \\
& \dot{m}=\int_{0}^{R} \rho \cdot d A \cdot U \\
& \dot{m}=\int_{0}^{R} \rho \cdot 2 \pi r d r \cdot U \\
& \dot{m}=\rho \cdot A \cdot V_{\text {avg }} \\
& \rho \int_{0}^{R} 2 \pi r \cdot d r U\left[1-\frac{r^{2}}{R^{2}}\right]=\rho \cdot \pi R^{2} V_{\text {avg }}  \tag{i}\\
& V_{\text {avg }}=\frac{2 U \int_{0}^{R}}{}\left[r-\frac{r^{3}}{R^{2}}\right) d r  \tag{ii}\\
& R^{2} \\
& V_{\text {avg }}=\frac{2 U}{R^{2}}\left[\frac{R^{2}}{2}-\frac{R^{2}}{4}\right]=\frac{2 U}{R^{2}}\left[\frac{R^{2}}{4}\right] \\
& V_{\text {avg }}=\frac{U}{2}
\end{align*}
$$

Hence, the correct option is (C).

## Question 35

If water is flowing at the same depth in most hydraulically efficient triangular and rectangular channel sections then the ratio of hydraulic radius of triangular section to that of rectangular section is.
(A) $\sqrt{2}$
(B) $\frac{1}{\sqrt{2}}$
(C) 1
C 4
(D) 2

Ans. B
Sol. $\frac{R_{\text {triangular }}}{R_{\text {rectangular }}}=\frac{\left(\frac{A}{P}\right)_{\text {triangular }}}{\left(\frac{A}{P}\right)_{\text {rectangular }}}$
$\frac{R_{\text {triangular }}}{R_{\text {rectangular }}}=\frac{\frac{Y}{2 \sqrt{2}}}{\frac{Y}{2}}=\frac{1}{\sqrt{2}}$
Hence, the correct option is (B).

## Question 36

Consider the limit, $\lim _{x \rightarrow 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$. The limit (correct up to one decimal place) is $\qquad$
Ans. 1/2
Sol. $\lim _{x \rightarrow 1}\left[\frac{1}{\ln (x)}-\frac{1}{x-1}\right]=\lim _{x \rightarrow 1}\left[\frac{(x-1)-\ln x}{(x-1) \ln x}\right]=\frac{1}{2}$

## Question 37

Which of the following is/are correct statement (s)?
(A) The boundary of water of a calm water pond will represent contour line.
(B) If the whole circle bearing of a line is $270^{\circ}$, its reduced bearing is $90^{\circ} \mathrm{NW}$.
(C) In the case of fixed hair stadia tachometry, the staff intercept will be larger when the staff is held nearer to the observation point.
(D) Back bearing of line is equal to Fore Bearing $\pm 180^{\circ}$

Ans. A, B, D

## Question 38

Gypsum is typically added in cement to
(A) enhance hardness
(B) increase workability
(C) decrease heat of hydration
(D) prevent quick setting

Ans. D
Sol. Gypsum is added in cement to prevent quick setting. Hence, the correct option is (D).

## Question 39

Spot speeds of vehicles observed at point on a highway are $40,55,60,65 \& 80 \mathrm{~km} / \mathrm{h}$. The space-mean speed (in km/h, round off to two decimal places) of the observed vehicles is $\qquad$ $-$
Ans. 56.99
Sol. Given :
Spot speed of vehicles on a highway are $40 \mathrm{~km} / \mathrm{h}, 55 \mathrm{~km} / \mathrm{h}, 60 \mathrm{~km} / \mathrm{h}, 65 \mathrm{~km} / \mathrm{h}$ and $85 \mathrm{~km} / \mathrm{h}$.
Space mean speed, $\frac{S}{V}=\frac{1}{V_{1}}+\frac{1}{V_{2}}+\frac{1}{V_{3}}+\frac{1}{V_{4}}+\frac{1}{V_{5}}$

$$
\frac{S}{V}=\frac{1}{40}+\frac{1}{55}+\frac{1}{60}+\frac{1}{65}+\frac{1}{80}
$$

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$$
V=56.99 \mathrm{~km} / \mathrm{hr}
$$

## Question 40

The state of stress in a deformable body is shown in the figure. Consider trAns.formation of the stress from the $x$-y coordinate system to the $X-Y$ coordinate system. The angle $\theta$, locating the $X$-axis, is assumed to be positive when measured from the axis in counter-clockwise direction.


The absolute magnitude of the shear stress component $\sigma_{x y}$ (in MPa, round off to one decimal place) in $x-y$ coordinate system is $\qquad$ .
Ans. 96.186
Sol. Given :


$$
\sigma_{x}^{\prime}=\sigma_{x} \cos ^{2} \theta+\sigma_{y} \sin ^{2} \theta+2 z_{x y} \sin \theta \cos \theta
$$

Or $\quad \sigma_{x}^{\prime}=\left(\frac{\sigma_{x}+\sigma_{y}}{2}\right)+\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right) \cos 2 \theta+z_{x y} \sin 2 \theta$

$$
\because \quad \sigma_{x}=40 \mathrm{MPa}, \sigma_{y}=35.6 \mathrm{MPa}, Q=60^{\circ} \text { (from vertical) }
$$

$$
\therefore \quad \sigma_{x}^{\prime}=120 \mathrm{MPa}
$$

$$
z_{x^{\prime} y^{\prime}}=-50
$$

$\therefore$ On substituting value in formula,

$$
\begin{array}{ll} 
& 120=\left(\frac{40+35.6}{2}\right)+\left(\frac{40-35.6}{2}\right) \cos (120)+z_{x y} \sin (120) \\
\therefore & z_{x y}=96.186 \mathrm{MPa}
\end{array}
$$

## Question 41

A 50 mL sample of industrial wastewater is taken into a silica crucible. The empty weight of the crucible is 54.352 g . The crucible with the sample is dried in a hot air oven at $104^{\circ} \mathrm{C}$ till a constant weight of 55.129 g . Thereafter, the crucible with the dried sample is fired at $600^{\circ} \mathrm{C}$ for 1 h in a muffle furnace, and the weight of the crucible along with residue is determined as 54.783 g . The concentration of total volatile solids is
(A) $6920 \mathrm{mg} / \mathrm{L}$
(B) $8620 \mathrm{mg} / \mathrm{L}$
(C) $1700 \mathrm{mg} / \mathrm{L}$
(D) $15540 \mathrm{mg} / \mathrm{L}$

Ans. A
Sol. Given :
Crucible weight $=54.352$ gms
Crucible + total solids weight $=55.129$ gms
Crucible + fixed solid weight $=54.783 \mathrm{gms}$
Volatile solids $=$ Total solids - Fixed solid $=55.129-54.783=0.346 \mathrm{gms}$
Concentration $=\frac{0.346 \times 10^{3}}{50 \times 10^{-3}} \mathrm{mg} / \mathrm{l}=6920 \mathrm{mg} / \mathrm{l}$
Hence, the correct option is (A).

## Question 42

The longitudinal section of a runway provides following data:

| End-to-end runway (m) | Gradient (\%) |
| :---: | :---: |
| 0 to 300 | 1.2 |
| 300 to 600 | -0.7 |
| 600 to 1100 | 0.6 |
| 1100 to 1400 | -0.8 |
| 1400 to 1700 | -1.0 |

The effective gradient of runway (in \%, round off to two decimal places) is $\qquad$ .
Ans. 0.317

Sol.

| End to and runway(m) | Gradient (\%) |
| :---: | :---: |
| 0 to 300 | +1.2 |
| 300 to 600 | -0.7 |
| 600 to 1100 | +0.6 |
| 1100 to 1400 | -0.8 |
| 1400 to 1700 | -1.0 |

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Effective Gradient $=\frac{4.5-(-0.9)}{1700} \times 100=0.317 \%$

## Question 43

A prismatic cantilever prestressed concrete beam of span length, $\mathrm{L}=1.5 \mathrm{~m}$ has one straight tendon placed in the cross-section as shown in the following figure (not to scale). The total prestressing force of 50 kN in the tendon is applied at $d_{c}=50 \mathrm{~mm}$ from the top in the cross-section of width, $\mathrm{b}=200 \mathrm{~mm}$ and depth, $\mathrm{d}=300 \mathrm{~mm}$.


If the concentrated load, $\mathrm{P}=5 \mathrm{kN}$, the resultant stress (in MPa , in integer) experienced at point ' Q ' will be $\qquad$ .
Ans. 0

## Sol. Given :



Let us calculate,
Bending moment $=5 \times 1.5=7.5 \mathrm{kN}-\mathrm{m}$

$$
\begin{aligned}
& \frac{P}{A}=\frac{50,000}{200 \times 300}=0.833 \\
& \frac{P e}{z}=\frac{50,000 \times 100}{\frac{200 \times 300^{2}}{6}}=1.67
\end{aligned}
$$

$$
\frac{M}{z}=\frac{7.5 \times 10^{6}}{\frac{200 \times 300^{2}}{6}}=2.5
$$

Stress at top $=0.833+1.67-2.5=3 \times 10^{-3} \mathrm{~N} / \mathrm{mm}^{2}$


Resultant stress is $Q=0$.

## Question 44

The liquid forms of particulate air pollutants are
(A) Smoke and mist
(B) Dust and mist
(C) Mist and spray
(D) Fly ash and fumes

Ans. C

## Question 45

Kinematic viscosity is dimensionally represented as
(A) $\frac{M}{L T}$
(B) $\frac{L^{2}}{T}$
(C) $\frac{M}{L^{2} T}$
(D) $\frac{T^{2}}{L}$

Ans. B
Sol. Dimension of kinematic viscosity is $\frac{L^{2}}{T}$.
Hence, the correct option is (B).

## Question 46

Contractor X is developing his bidding strategy against contractor Y . The ratio of $Y^{\prime}$ 's bid price to $X^{\prime} s$ cost for the 30 previous bids in which Contractor X has competed against Contractor Y is given in the table

| Ratio of $\boldsymbol{Y}^{\prime} \boldsymbol{s}$ bid price to $\boldsymbol{X}^{\prime} \boldsymbol{s} \boldsymbol{s}$ cost | Number of bids |
| :---: | :---: |
| 1.02 | 6 |
| 1.04 | 12 |
| 1.06 | 3 |
| 1.10 | 6 |
| 1.12 | 3 |

Based on the bidding behavior of the Contractor Y, the probability of winning against. Contractor Y at a markup of $8 \%$ for the next project is
(A) $0 \%$
(B) $100 \%$
(C) More than $0 \%$ but less then $50 \%$
(D) More than $50 \%$ but less than $100 \%$

Ans. C

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Sol.

| S. No. | Ratio of Y's bid <br> price to X's cost | Number <br> of bid | Probability of type of <br> bid from previous 30 bids |
| :---: | :---: | :---: | :---: |
| 1 | 1.02 | 6 | $\frac{6}{30}$ |
| 2 | 1.04 | 12 | $\frac{12}{30}$ |
| 3 | 1.06 | 3 | $R$ |
| 4 | 1.10 | 6 | $\frac{3}{30}$ |
| 5 | 1.12 | 3 | $\frac{6}{30}$ |

At mark up of 8\%
Bid price of contractor $X=1.08$
Contractor $X$ will with if quoted bid price of contractor $Y$ is greater than $X$ bid price,
For type 4 and type 5, $Y$ bid is higher.
So, Probability of win of contractor $X$ is $=\frac{6}{30}+\frac{3}{30}=\frac{9}{30}=0.3$
Option 3 more than $0 \%$ but less than $50 \%$.
Hence, the correct option is (C).

## Question 47

The soil profile at a construction site is shown in the figure (not to scale). Ground water table (GWT) is at 5 m below the ground level at present. An old well data shows that the ground water table was as low as 10 m below the ground level in the past. Take unit weight of water, $\gamma_{w}=9.81 \mathrm{kN} / \mathrm{m}^{3}$


The overconsolidation ratio (OCR) (round off to two decimal places) at the mid-point of the clay layer is
$\qquad$ .

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Ans. 1.22
Sol. Over consolidation ratio $=\frac{\text { Effective stress in past }\left(\bar{\sigma}_{c}\right)}{\text { Effective stress in present }\left(\bar{\sigma}_{0}\right)}$

$$
\begin{aligned}
& \bar{\sigma}_{c}=17.5 \times 10+(18.5-9.81) \times 5+(17-9.81) \times 4 \\
& \bar{\sigma}_{c}=247.21 \mathrm{kN} / \mathrm{m}^{2} \\
& \bar{\sigma}_{0}=17.5 \times 5+(18.5-9.81) \times 10+(17-9.81) \times 4 \\
& \bar{\sigma}_{0}=203.16 \mathrm{kN} / \mathrm{m}^{2} \\
& \mathrm{OCR}=\frac{\bar{\sigma}_{c}}{\bar{\sigma}_{0}}=\frac{247.21}{203.16}=1.22
\end{aligned}
$$

## Question 48

Which one the following statement is correct?
(A) Pyrolysis is an endothermic process, which takes in the place in the absence of oxygen.
(B) Combustion is an endothermic process, which takes place in the abundance of oxygen.
(C) Pyrolysis is an exothermic process, which takes in the absence of oxygen.
(D) Combustion is an exothermic process, which takes place in the absence of oxygen

Ans. A

## Question 49

Which of the following is NOT a correct statement?
(A) First reading from a level station is a 'Fore Sight'.
(B) Planimeter is used for measuring 'area'.
(C) Contours of different elevations may intersect each other in case of an overhanging cliff.
(D) Basic principle of surveying is to work from whole to parts.

Ans. A
Question 50
In an Oedometer apparatus, a specimen of fully saturated clay has been consolidated under a vertical pressure of $50 \mathrm{kN} / \mathrm{m}^{2}$ and is presently at equilibrium. The effective stress and pore water pressure immediately on increasing the vertical stress to $150 \mathrm{kN} / \mathrm{m}^{2}$, respectively are
(A) $150 \mathrm{kN} / \mathrm{m}^{2}$ and 0
(B) $50 \mathrm{kN} / \mathrm{m}^{2}$ and $100 \mathrm{kN} / \mathrm{m}^{2}$
(C) $100 \mathrm{kN} / \mathrm{m}^{2}$ and $50 \mathrm{kN} / \mathrm{m}^{2}$
(D) 0 and $150 \mathrm{kN} / \mathrm{m}^{2}$

Ans. B
Sol. Stress is increased suddenly, hence entire change will be taken by water

$$
\Delta \bar{\sigma}=\Delta U=100 \mathrm{kPa}
$$

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There will be no change in effective stress

$$
\therefore \quad \bar{\sigma}=50 \mathrm{kPa}
$$

Hence, the correct option is (B).

## Question 51

An unlined canal under regime conditions along with a silt factor of 1 has a width of flow 71.25 m . Assuming the unlined canal as a wide channel, the corresponding average depth of flow (in m , round off to two decimal places) in the canal will be $\qquad$ .
Ans. 2.92
Sol. Given :
Silt factor = 1
Width of flow $=71.25$

$$
\begin{aligned}
& A f^{2}=140\left(\frac{2}{5} f R\right)^{5 / 2} \\
& (B D) f^{2}=140\left(\frac{2}{5} f \times D\right)^{5 / 2} \\
& (71.25 \times D) \times 1=140\left(\frac{2}{5} \times 1 \times D\right)^{5 / 2} \\
& D \times 0.5089=\left(\frac{2}{5}\right)^{5 / 2} \times(D)^{5 / 2} \\
& D^{3 / 2}=5.029 \\
& D^{3 / 2}=2.94 \mathrm{~m}
\end{aligned}
$$

## Question 52

A retaining wall of height 10 m with clay backfill is shown in the figure (not to scale). Weight of the retaining wall is 5000 kN per m acting at 3.3 m from the toe of the retaining wall. The interface friction angle between base of the retaining wall and the base soil is $20^{\circ}$. The depth of clay placed in front of the retaining wall are the same. Assume that the tension crack is filled with water. Use Rankine's earth pressure theory. Take unit weight of water, $\gamma_{w}=9.81 \mathrm{kN} / \mathrm{m}^{2}$


The factor of safety (round off to two decimal places) against sliding failure of the retaining wall after ignoring the passive earth pressure will be $\qquad$ .

Ans.
3.48

Sol.

$Z_{c}=3.488$
$f=\mu N=(\tan \delta) \mathrm{W}$
$P_{w}=45.2, P_{w}=\frac{1}{2} \gamma_{w} H^{2}=490.5 \mathrm{kN}$
$F O S=\frac{\mu N}{P_{w}+P_{w}}=\frac{(\tan \delta)(5000)}{45.2+490.5}$
$F O S=\frac{(\tan 20)(5000)}{522.027}=3.48$

## Question 53

A signalized intersection operates in two phases. The lost time is 3 seconds per phase. The maximum ratios of approach flow to saturation flow for the two phases are 0.37 and 0.40 . The optimum cycle length using the Webster's method (in seconds, round off to one decimal place) is $\qquad$

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Ans. 60.87
Sol. Given :
Lost time per phase $=3$ seconds
maximum ratios of approach flow to saturation flow for the two phases are 0.37 and 0.40 .

$$
C_{0}=\frac{1.5 L+5}{1-Y}=\frac{1.5(3 \times 2)+5}{1-(0.37+0 . .40)}=60.87 \mathrm{sec}
$$

Question 54
Refer the truss a shown in the figure (not to scale).


If load, $F=10 \sqrt{3} \mathrm{kN}$, moment of inertia, $I=8.33 \times 10^{6} \mathrm{~mm}^{4}$, area of cross-section, $A=10^{4} \mathrm{~mm}^{2}$, and length, $L=2 \mathrm{~m}$ for all the members of the truss, the compressive stress (in $\mathrm{kN} / \mathrm{m}^{2}$, in integer) carried by the member $Q-R$ is $\qquad$ .
Ans. 500
Sol.


$$
V_{p}=V_{s}=5 \sqrt{3} \mathrm{kN}
$$

Consider equilibrium of LHS of section (1) - (1),


Taking moment about ( $T$ ),

$$
\begin{aligned}
& \Sigma M_{T}(\mathrm{CW})=0 \\
& (5 \sqrt{3} \times a)+f_{Q R}\left(\frac{\sqrt{3} a}{2}\right)=0 \\
& F_{Q R}=-10 \mathrm{kN} \text { or } 10 \mathrm{kN}(\mathrm{C})
\end{aligned}
$$

Compressive stress in member $\mathrm{QR}\left(\sigma_{c}\right)$

$$
\begin{aligned}
& \sigma_{c}=\frac{F_{Q R}}{2 A} \\
& \sigma_{c}=\frac{10 \mathrm{kN}}{2\left(10^{4} \times 10^{-6}\right) \mathrm{m}^{2}}=500 \mathrm{kN} / \mathrm{m}^{2}
\end{aligned}
$$

Question 55
The Rank of matrix $\left[\begin{array}{llll}1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 5 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9\end{array}\right]$ is
(A) 4
(B) 3
(C) 2
(D) 1

Ans. C
Sol. $\quad R_{2} \rightarrow R_{2} \rightarrow 3 R_{1}$

$$
R_{3} \rightarrow R_{3} \rightarrow 5 R_{1}
$$

$$
\left[\begin{array}{cccc}
1 & 2 & 2 & 3 \\
0 & -2 & -4 & -4 \\
0 & -4 & -8 & -8 \\
0 & -6 & -12 & -12
\end{array}\right]
$$

$R_{3} \rightarrow R_{3} \rightarrow 2 R_{2}$
$R_{4} \rightarrow R_{4} \rightarrow 3 R_{2}$

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| :---: | :---: | :---: |
|  | $\left[\begin{array}{cccc}1 & 2 & 2 & 3 \\ 0 & -2 & -4 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]$ |
| $p(A)=2$ |  |

Hence, the correct option is (C).


## General Aptitude

## Question 1

$\oplus$ and $\odot$ are 2 operators on numbers $p$ and $q$ such that $p \oplus q=\frac{p^{2}+q^{2}}{p q} \&$; if $x \oplus y=2 \odot 2$, then $x=$
(A) y
(B) 2 y
(C) $y / 2$
(D) $3 y / 2$

Ans. A
Sol. Given : $\mathrm{p} \oplus \mathrm{q}=\frac{\mathrm{p}^{2}+\mathrm{q}^{2}}{\mathrm{pq}}$

$$
\begin{array}{ll}
\therefore & x \oplus y=\frac{x^{2}+y^{2}}{x y} \\
\text { and } & p \odot q=\frac{p^{2}}{q} \\
\therefore & 2 \odot 2=\frac{2^{2}}{2}=2
\end{array}
$$

From equation (i) and (ii),

$$
\begin{array}{ll} 
& \frac{x^{2}+y^{2}}{x y}=2 \\
\therefore \quad & x^{2}+y^{2}-2 x y=0 \\
& (x-y)^{2}=0 \\
\therefore \quad & x=y \text { satisfy the condition. }
\end{array}
$$

Hence, the correct option is (A).

## Question 2

Four persons $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ and S are to be seated in a row, all facing the same direction, but not necessarily in the same order. P and R can not sit adjacent to each other. S should be seated to the right of Q. The number of distinct seating arrangements possible is:
(A) 4
(B) 2
(C) 8
(D) 6

Ans. D
Sol. Condition 1 : P and R can not sit adjacent to each other.
Condition 2 : $S$ should be seated to the right of $Q$,
According to this all possible cases will be

1. Q P S R
2. $\mathrm{Q} R \mathrm{~S}$ P
3. P Q S R
4. R Q S P
5. $\mathrm{P} Q \mathrm{R} \mathrm{S}$
6. R Q P S

Hence, the correct option is (D).

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steps to success...

## Question 3

Consider two rectangular sheets, sheet $M$ and sheet $N$ of dimensions $6 \mathrm{~cm} \times 4 \mathrm{~cm}$ each.
Folding operation 1: The sheet is folded into half by joining the short edges of the current shape.
Folding operation 2: The sheet is folded into half by joining the long edges of the current shape.
Folding operation 1 is carried out on Sheet M three times.
Folding operation 2 is carried out on Sheet N three times.
The ratio of perimeters of the final folded shape of Sheet N to the final folded shape of Sheet M is $\qquad$
(A) $3: 2$
(B) $7: 5$
(C) $13: 7$
(D) $5: 13$

Ans. C
Sol. According to given data, we can proceed step by step as given below,


## Question 4

A function $\lambda$ is defined by

$$
\lambda(p, q)=\left\{\begin{array}{cl}
(p-q)^{2}, & \text { if } p \geq q \\
p+q, & \text { if } p<q
\end{array}\right.
$$

The value of expression $\frac{\lambda(-(-3+2),(-2+3))}{(-(-2+1))}$ is:
(A) 0
(B) $\frac{16}{3}$
(C) 16
(D) -1

Ans. A
Sol. Given expression easily solved as,

$$
\begin{array}{ll} 
& \frac{\lambda[-(-3+2),(-2+3)]}{-(-2+1)}=\frac{\lambda(1,1)}{1}=\lambda(1,1) \\
\because & p=q=1 \\
\therefore & \lambda(1,1)=(1-1)^{2}=0
\end{array}
$$

Hence, the correct option is (A).

## Question 5



Five line segments of equal lengths, $P R, P S, Q S, Q T$ and $R T$ are used to form a star as shown in the figure above.

The value of $\theta$, in degrees, is $\qquad$
(A) 36
(B)
108 (C)
72 (D)
45

Ans. A
Sol. Here, start shape, will all sequent are equal is shown below,

$\therefore \quad \angle P=\angle T=\angle S=\angle R=\angle Q=\theta$
(Equal sides of triangle have equal angle)
In $\triangle T B R$,


$$
\operatorname{Ext} \angle B=2 \theta \quad \ldots \text { (i) } \quad(\because \text { Sum of opposite interior angle }=\text { Exterior angle })
$$

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In $\triangle Q A S$,

$\operatorname{Ext} \angle A=2 \theta$
In $\triangle P A B$,

$\because$ Sum of all interior angles in a $\Delta=180^{\circ}$.
$\therefore \quad \theta+2 \theta+2 \theta=180^{\circ}$
$\theta=36^{0}$
Hence, the correct option is (A).

## Question 6

In a company, $35 \%$ of the employees drink coffee, $40 \%$ of the employees drink tea and $10 \%$ of the employees drink both tea adn coffee. What \% of employees drink neither tea nor coffee?
(A) 25
(B) 35
(C) 40
(D) 15

Ans. B
Sol. Given : Employees drink coffee $=35 \%$
Employees drink tea $=40 \%$
Employees drink both tea and coffee $=10 \%$
So from above data we can easily sketch Venn diagram,

$25+10+30=65 \%$ employees are those who either takes coffee or tea or both.
$\therefore \quad(100-65) \%=35 \%$ are those who neither take coffee nor tea.
Hence, the correct option is (B).

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,

## Question 7

Statements: Either P marris Q or X marris Y
Among the options below, the logical NEGATION of the above statement is :
(A) Neither P marries Q nor X marries Y .
(B) P does not marry Q and X marries Y .
(C) P marries Q and X marries Y .
(D) X does not marry Y and P marries Q .

Ans. C
Sol. As we are directed to do logical negation, of given statement, situation of "either or" will becomes "Neither nor"
as: $\quad \mathrm{P}=\mathrm{Q} \quad \mathrm{X}=\mathrm{Y}$
Negation: $\quad \mathrm{P} \neq \mathrm{Q} \quad \mathrm{X} \neq \mathrm{Y}$
Hence, the correct option is (C).

## Question 8



The mirror image of the above text about the X -axis is
(A) $\perp$ BIVИDГE
(B) $\perp$ ВI $\forall$ ИСГЕ
(c) $\perp \measuredangle \mid \forall N C \Gamma E$
(D) $\perp$ В $\mid \forall И \varrho\ulcorner\exists$

Ans. B

## Question 9

Human have the ability to construct worlds entirely in their minds, which don't exist in the physical world. So far as we known, no other species possess this ability. This skill is so important that we have different words to refer to its different flavors, such as imagination, invention and innovation.
Based on above passage, which one of the following is TRUE?
(A) The terms imagination, invention and innovation refer to unrelated skills.
(B) No species possess the ability to construct worlds in their mind.
(C) Imagination, invention and innovation are unrelated to the ability to construct metal worlds.
(D) We do not know of any species other than humans. who posses the ability to construct mental worlds.

Ans. D
Sol. As given in the above passage "so far as we know, no other species posses this ability". By this we can conclude option D is correct.
Hence, the correct option is (D).

## Question 10

Getting to the top is $\qquad$ than staying on top
(A) easier
(B)
more easier
(C) much easier
(D) easiest

Ans. A
Sol. Getting to the top is easier than staying in top.
Hence, the correct option is (A).


