General Aptitude

Q.1 to Q.5 Carry one mark each

Question 1

He was not only accused of theft ____________ of conspiracy.

(A) but also (B) but even (C) rather (D) rather than

Ans. **(A)**

Question 2

The Canadian constitution requires that equal importance be given to English and French. Last year, Air Canada lost a lawsuit and had to pay a six-figure fine to a French speaking couple after they filed complaints about formal in-flight announcements in English lasting 15 seconds, as opposed to informal 5 second messages in French.

The French - speaking couple were upset at _____.

(A) the English announcements being longer than French ones.

(B) equal importance being given to English and French.

(C) the English announcements being clearer than the French ones.

(D) the in-flight announcements being made in English.

Ans. **(A)**

Sol. The French speaking couple were upset as they were not able to get the announcement properly in such a short span of five seconds and they filed complaint as the announcement in English lasted for 10 seconds. Hence the correct option is (A).

Question 3

Select the word that fits the analogy :

Explicit : Implicit : : Express : _____.

(A) Repress (B) Suppress (C) Compress (D) Impress

Ans. **(A)**

Sol. Given words in the first relation are antonyms, the correct antonym of express is repress, which means to control an emotion or to try to prevent it from being shown.

"Suppress" is synonym of repress but it means to stop something by using **force**, so it is not the correct antonym for express.

Hence, the correct option is (A).

Question 4

The untimely loss of life is a cause of serious global concern as thousands of people get killed ____ accidents every year while many other die ____ diseases like cardio vascular diseases, cancer, etc.

(A) in, of (B) during, from (C) from, of (D) from, from

Ans. **(A)**

Question 5

A superadditive function $f(\cdot)$ satisfies the following property

$$f(x_1 + x_2) \geq f(x_1) + f(x_2)$$

Which of the following functions is a superadditive function for $x > 1$?

(A) $\sqrt{x}$ (B) e^x (C) $\frac{1}{x}$ (D) e^{-x}

Ans. (B)

Sol. Given : $f(x_1 + x_2) \geq f(x_1) + f(x_2)$ $x > 1$

Consider, $x_1 = 2$ and $x_2 = 3$

Checking from options we have

For option (A),

$$\sqrt{2+3} \geq \sqrt{2} + \sqrt{3}$$

$$2.236 \geq 3.146$$

For option (B),

$$e^{2+3} \geq e^2 + e^3$$

$148.41 \geq 27.47$ it holds the inequality

For option (C),

$$\frac{1}{2+3} \geq \frac{1}{2} + \frac{1}{3}$$

$$\frac{1}{5} \geq \frac{5}{6}$$

$$0.2 \geq 0.8$$

For option (D),

$$e^{-(2+3)} \geq e^{-2} + e^{-3}$$

$$e^{-5} \geq e^{-2} + e^{-3}$$

$$6.73 \times 10^{-3} \geq 0.135 + 0.049$$

$$6.73 \times 10^{-3} \geq 0.184$$

$$0.00673 \geq 0.184$$

Hence, the correct option is (B).

Q.6 to Q.10 Carry two marks each

Question 6

a, b, c are real numbers. The quadratic equation $ax^2 - bx + c = 0$ has equal roots, which is β, then

(A) $\beta^3 = \frac{bc}{(2a^2)}$ (B) $\beta^2 = ac$ (C) $\beta = \frac{b}{a}$ (D) $b^2 \neq 4ac$

Ans. **(A)**

Sol. **Given :** a, b, c are real numbers

$$ax^2 - bx + c = 0 \quad \dots \text{(i)}$$

Given, equation (i) has equal roots, which is β.

Now, product of the roots will be

$$\beta \cdot \beta = \frac{c}{a}$$

$$\beta^2 = \frac{c}{a} \quad \dots \text{(ii)}$$

Sum of the roots will be,

$$\beta + \beta = \frac{b}{a}$$

$$2\beta = \frac{b}{a}$$

$$\beta = \frac{b}{2a} \quad \dots \text{(iii)}$$

Multiplying equation (ii) and (iii),

$$\beta^3 = \frac{bc}{2a^2}$$

Hence, the correct option is (A).

Question 7

The global financial crisis in 2008 is considered to be the most serious world-wide financial crisis, which started with the sub-prime lending crisis in USA in 2007. The sub-prime lending crisis led to the banking crisis in 2008 with the collapse of Lehman Brothers in 2008. The sub-prime lending refers to the provision of loans to those borrowers who may have difficulties in repaying loans, and it arises because of excess liquidity following the East Asian crisis. Which one of the following sequences shows the correct precedence as per the given passage?

(A) East Asian crisis → subprime lending crisis → banking crisis → global financial crisis.

(B) Banking crisis → subprime lending crisis → global financial crisis → East Asian crisis.

(C) Global financial crisis → East Asian crisis → banking crisis → subprime lending crisis.

(D) Subprime lending crisis → global financial crisis → banking crisis → East Asian crisis.

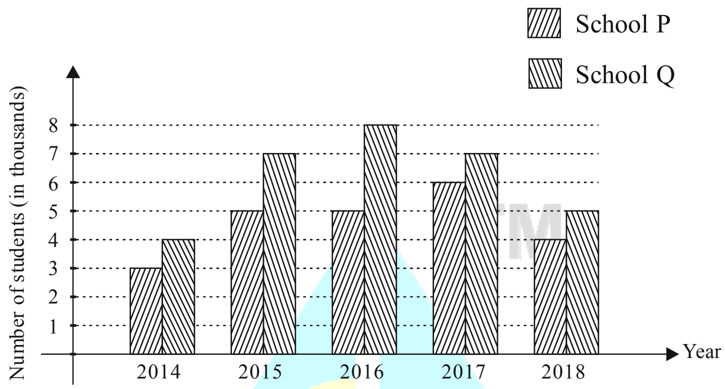
Ans. **(A)**

Sol. According to the given paragraph, because of East Asian crisis, sub-prime lending crisis occurred due to access liquidity and hence banking crisis taken place resulting in the global financial crisis.

Hence, the correct option is (A).

Question 8

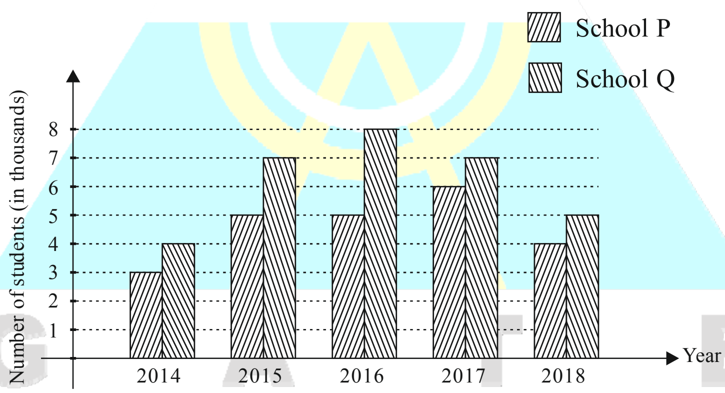
The following figure shows the data of students enrolled in 5 years (2014 to 2018) for two schools P and Q. During this period, the ratio of the average number of the students enrolled in school P to the average of the difference of the number of students enrolled in schools P and Q is



(A) 31 : 23 (B) 23 : 8 (C) 23 : 31 (D) 8 : 23

Ans. **(B)**

Sol.



Average students enrolled in

$$P = \frac{3+5+5+6+4}{5} = \frac{23}{5}$$

Average of difference of students enrolled :

School P and Q $= \frac{2+1+3+1+1}{5} = \frac{8}{5}$

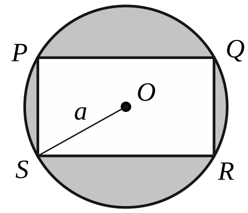
Ratio of average students enrolled in school P to the ratio of average of difference of students enrolled in school P and Q

$$= \frac{23/5}{8/5} = 23:8$$

Hence, the correct option is (B).

Question 9

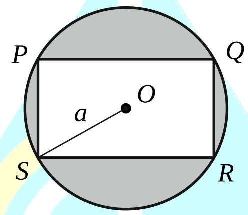
A circle with center O is shown in the figure. A rectangle $PQRS$ of maximum possible area is inscribed in the circle. If the radius of the circle is a, then the area of shaded portion is _________.



(A) $\pi a^2 - a^2$ (B) $\pi a^2 - 3a^2$ (C) $\pi a^2 - \sqrt{2}a^2$ (D) $\pi a^2 - 2a^2$

Ans. **(D)**

Sol. Let, l and b be the length and breadth of rectangle respectively.



Using Pythagoras theorem, we get

$$4a^2 = l^2 + b^2$$

$$b = \sqrt{4a^2 - l^2}$$

Area of rectangle is, $A = l \times b$ $A = l \times b = l \times \sqrt{4a^2 - l^2}$

For maxima or minima, $\dfrac{dA}{dl} = 0$

$$l \times \frac{1}{2\sqrt{4a^2 - l^2}} \times (-2l) + \sqrt{4a^2 - l^2} = 0$$

$$\sqrt{4a^2 - l^2} = \frac{l^2}{\sqrt{4a^2 - l^2}}$$

$$4a^2 - l^2 = l^2$$

$$l = \sqrt{2}a$$

$$b = \sqrt{4a^2 - 2a^2} = \sqrt{2}a$$

$\therefore$ Maximum area of rectangle $PQRS$ is,

$$\sqrt{2}a \times \sqrt{2}a = 2a^2$$

Area of shaded portion = (Area of circle) – (Area of rectangle)

$$= \pi a^2 - 2a^2$$

Hence, the correct option is (D).

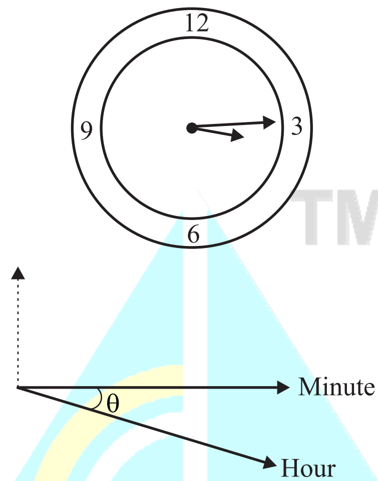
Question 10

It is quarter past three in your watch. The angle between the hour hand and the minute hand is ______.

(A) 22.5^0 (B) 15^0 (C) 7.5^0 (D) 0^0

Ans. (C)

Sol. Positions of hour and minute hands of watch is shown in figure



The displacement of hour hand during every 12 hours is 360^0.

The required angle between the hands at quarter past 3^0 is equal to the angle made by hour hand in 15 minutes as the minute hand is now at the same position where, the hour hand was present at 3'O clock.

In 12×60 min, angle made $=360^0$

In 1 min, angle made $=\left(\dfrac{360}{12\times60}\right)^0$

In 15 min, angle made $=\left(\dfrac{360}{12\times60}\times15\right)^0=7.5^0$

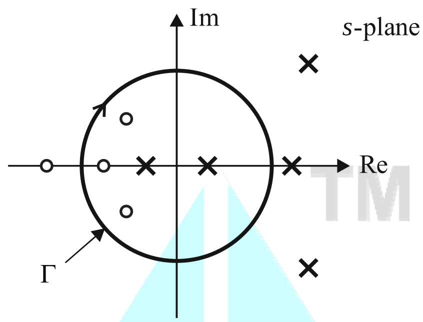
Hence, the correct option is (C).

Technical Section

Q.1 to Q.25 Carry one mark each

Question 1

The pole-zero map of a rational function $G(s)$ is shown below. When the closed contour Γ is mapped into $G(s)$-plane, then the mapping encircles



(A) the point $-1+j0$ of the $G(s)$-plane once in clockwise direction.

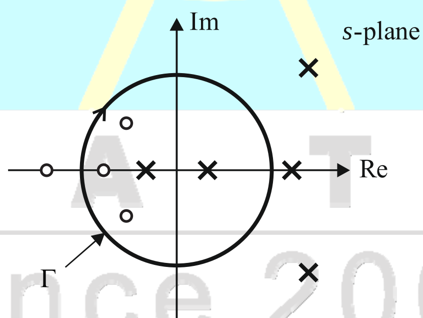
(B) the origin of $G(s)$-plane once in counter - clockwise direction.

(C) the point $-1+j0$ of the $G(s)$-plane once in counter - clockwise direction.

(D) the origin of $G(s)$-plane once in clockwise direction.

Ans. (D)

Sol. Given pole zero plot is shown in figure where the closed contour encloses 3 zeros and 2 poles in the s-plane.



From principle of argument of Nyquist plot, if a closed contour encloses P poles and Z zero's in s-plane, then the origin of $G(s)$ plane is encircled $P-Z$ times.

$$N = P - Z$$

$N =$ Number of encirclement of origin

$N =$ Positive for counter clockwise direction

Negative for clock wise direction

Here, $P = 2$, $Z = 3$

So, $N = 2 - 3 = -1$

Hence, the mapping will encircle origin of $G(s)$ plane in clockwise direction and option (D) is correct.

Question 2

If $\mathbf{v}_1, \mathbf{v}_2 \ldots \mathbf{v}_6$ are six vectors in $\mathbb{R}^4$, which one of the statements is FALSE?

(A) Any four of these vectors form a basis for $\mathbb{R}^4$.

(B) It is not necessary that these vectors span $\mathbb{R}^4$.

(C) If $\{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_5, \mathbf{v}_6\}$ spans $\mathbb{R}^4$, then it forms a basis of $\mathbb{R}^4$.

(D) These vectors are not linearly independent.

Ans. **(A)**

Sol. **Given :** $\mathbf{v}_1, \mathbf{v}_2 \ldots \mathbf{v}_6$ are six vectors in $\mathbb{R}^4$.

Four vectors that form a basis for four dimensional space must be linearly independent to each other.

Hence, the correct option is (A).

Question 3

The output $y[n]$ of a discrete - time system for an input $x[n]$ is

$$y[n] = \max_{-\infty \le k \le n} |x[k]|$$

The unit impulse response of the system is

(A) unit step signal $u[n]$.

(B) 0 for all n.

(C) unit impulse signal $\delta[n]$.

(D) 1 for all n.

Ans. **(A)**

Sol. **Given :** $y[n] = \max_{-\infty \le k \le n} |x(k)|$

To find response of the system for impulse input, i.e. impulse response , taking

$x[n] = \delta[n]$ then output

$$y[n] \cong h[n] = \max_{-\infty \le k \le n} |\delta(k)|$$

$$h[n] = 0 \text{ for } n < 0$$

$$1 \text{ for } n \ge 0$$

As impulse will occur at $k = 0$ and from the given condition $k \le n$, so impulse response will be 1 for all $n \ge 0$, which is same as the definition of unit step signal $u[n]$.

Hence the correct option is (A).

Question 4

The two sides of a *fair* coin are labelled as 0 and 1. The coin is tossed two times independently. Let M and N denote the labels corresponding to the outcomes of those tosses. For a random variable X, defined as $X = \min(M, N)$, the expected value $E[X]$ (rounded off to two decimal places) is ___________.

Ans. **(0.25)**

Sol. **Given :**

Two sides of a fair coin labelled as 0 and 1.

Either '0' is assigned for heads and '1' is assigned for tails.

Or '0' is assigned for tails and '1' is assigned for heads.

The coin is tossed two times.

So, according to question,

M and *N* denotes the corresponding outcomes of those tosses.

M	N	Random Variable $X = \min(M, N)$	Probability
0	0	$X_1 = 0$	$P(X_1) = 1/4$
0	1	$X_2 = 0$	$P(X_2) = 1/4$
1	0	$X_3 = 0$	$P(X_3) = 1/4$
1	1	$X_4 = 1$	$P(X_4) = 1/4$

$$X = \{X_1, X_2, X_3, X_4\}$$

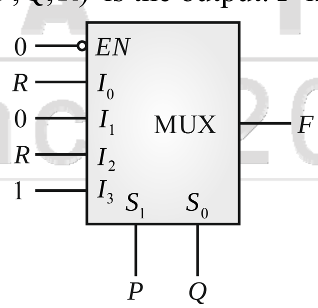
The expected value of X is given by,

$$E[X] = \sum_{i=1}^{n} x_i\, P(X = x_i)$$

$$E[X] = 0\times\frac{1}{4} + 0\times\frac{1}{4} + 0\times\frac{1}{4} + 1\times\frac{1}{4} = 0.25$$

Question 5

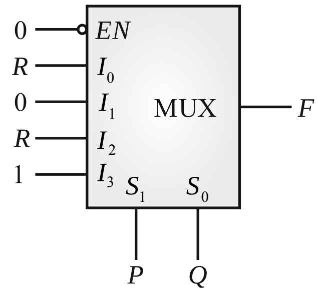
The figure below shows a multiplexer where S_1 and S_0 are the select lines, I_0 to I_3 are the input data lines, EN is the enable line, and $F(P,Q,R)$ is the output. F is



(A) $\bar{Q} + PR$ (B) $P\bar{Q}R + \bar{P}Q$ (C) $P + Q\bar{R}$ (d) $PQ + \bar{Q}R$

Ans. **(D)**

Sol. Given 4×1 multiplexer is shown in figure,



The output function will be given as,

$$F = \bar{P}\bar{Q}\cdot R + \bar{P}Q\cdot 0 + P\bar{Q}\cdot R + PQ\cdot 1$$

$$F = \bar{P}\bar{Q}\cdot R + P\bar{Q}\cdot R + PQ\cdot 1$$

$$F = \bar{Q}R(\bar{P}+P) + PQ$$

$$F = \bar{Q}R + PQ$$

Hence, the correct option is (D).

Question 6

The loop transfer function of a negative feedback system is

$$G(s)H(s) = \frac{K(s+11)}{s(s+2)(s+8)}.$$

The value of K, for which system is marginally stable, is __________.

Ans. **(160)**

Sol. **Given :** For a negative feedback system

$$G(s)H(s) = \frac{K(s+11)}{s(s+2)(s+8)}$$

Characteristic equation is given as,

$$1 + G(s)H(s) = 0$$

$$s[s^2 + 10s + 16] + Ks + 11K = 0$$

$$s^3 + 10s^2 + (16+K)s + 11K = 0$$

Method 1

Preparing RH table,

s^3	1	16+K	0
s^2	10	11 K	0
s^1	$\frac{(160+10K)-11K}{10}$	0	0
s^0	11 K	0	0

At $K = K_{\text{marginal}}$, RH table should have row of zeros other then last row.

So, $$\frac{(160+10K_{\text{marginal}}) - 11K_{\text{marginal}}}{10} = 0$$

$$160 - K_{\text{marginal}} = 0$$

$\therefore \quad K_{\text{marginal}} = 160$

Method 2

From characteristic equation,

$$s^3 + 10s^2 + (16 + K_{\text{marginal}})s + 11K_{\text{marginal}} = 0$$

At $K = K_{\text{marginal}}$,

Inner product = Outer product

$$1 \times 11K_{\text{marginal}} = 10(16 + K_{\text{marginal}})$$

$$11K_{\text{marginal}} - 10K_{\text{marginal}} = 160$$

$\therefore \quad K_{\text{marginal}} = 160$

Question 7

The general solution of $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$ is

(A) $y = C_1 e^{3x} + C_2 e^{-3x}$

(B) $y = C_1 e^{3x}$

(C) $y = (C_1 + C_2 x)e^{3x}$

(D) $y = (C_1 + C_2 x)e^{-3x}$

Ans. **(C)**

Sol. **Given :** Differential equation is

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$$

$$(D^2 - 6D + 9)y = 0$$

This is in form of a homogeneous linear differential equation

$$[f(D)]y = 0$$

The auxiliary equation is given by,

$$f(m) = 0$$

$$m^2 - 6m + 9 = 0$$

$$m^2 - 3m - 3m + 9 = 0$$

$$m(m-3) - 3(m-3) = 0$$

$$(m-3)(m-3) = 0$$

$$m = 3, 3$$

The roots are real and equal so, the complementary function is given by,

$$\text{C.F.} = (C_1 + xC_2)e^{mx}$$

$$\text{C.F.} = (C_1 + C_2 x)e^{3x}$$

Particular integral (P.I.) is '0' since it is a homogeneous equation.

The complete solution is given by,

$$y = \text{C.F.} + \text{P.I.}$$

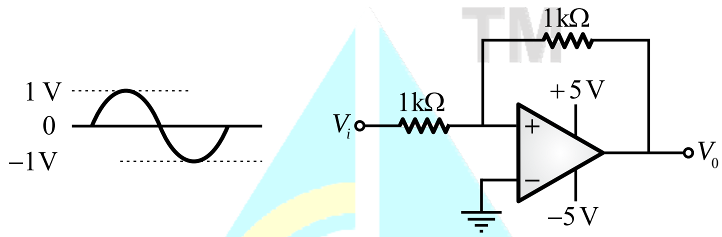
$$y = (C_1 + C_2 x)e^{3x} + 0$$

$$y = (C_1 + C_2 x)e^{3x}$$

Hence, the correct option is (C).

Question 8

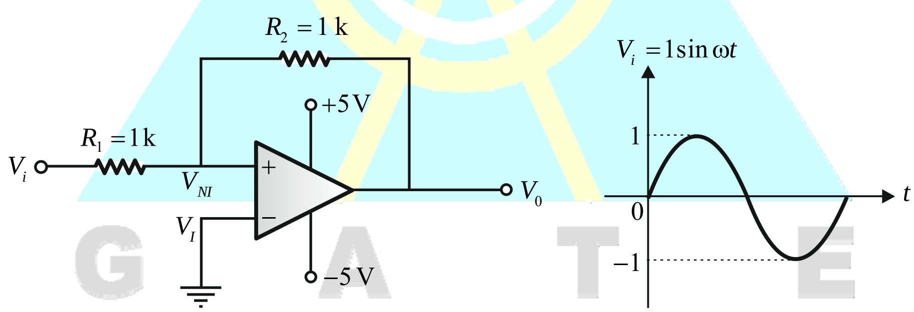
The components in the circuit shown below are ideal. If the op-amp is in positive feedback and the input voltage V_i is a sine wave of amplitude 1 V, the output voltage V_0 is



(A) a constant of either +5V or –5V.
(B) a non-inverted sine wave of 2 V amplitude.
(C) an inverted sine wave of 1 V amplitude.
(D) a square wave of 5 V amplitude.

Ans. (A)

Sol. Given circuit is an non inverting Schmitt trigger,



From given figure,

$$V_{NI} = \frac{V_i \times 1\text{k}}{1\text{k}+1\text{k}} + \frac{V_0 \times 1\text{k}}{1\text{k}+1\text{k}}$$

$$V_{NI} = \frac{V_i + V_0}{2}$$

and $V_I = 0$ V

If $V_{NI} > 0$ V then $V_0 = +V_{sat}$

If $V_{NI} < 0$ V then $V_0 = -V_{sat}$

Assuming $V_0 = +5\text{ V} = +V_{sat}$

For assumption to be true V_{NI} must be greater than V_I,

$$\frac{V_0+V_i}{2}>0\text{ V}$$

$$V_0+V_i>0$$

$$V_i>-V_0$$

$$1\sin\omega t>-5\text{ V}$$

$V_0=+V_{sat}$, assumption is true

Now assuming $V_0=-V_{sat}=-5\text{ V}$

For the assumption to be true V_{NI} must be less than V_I,

$$\frac{V_0+V_i}{2}<0\text{ V}$$

$$V_0+V_i<0\text{ V}$$

$$V_i<-V_0$$

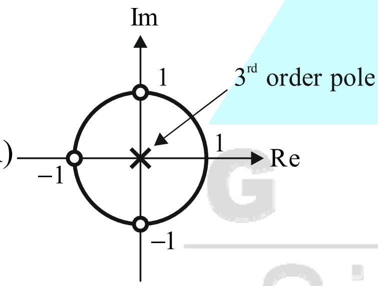
$$1\sin\omega t<+5 \qquad [\because V_0=-5\text{ V}]$$

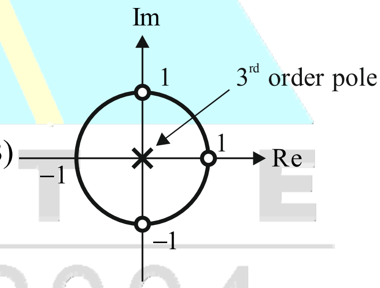
The relation holds true in this case also. So the value of output may be possibly -5 V or + 5 V as used. Hence the correct option is (A).

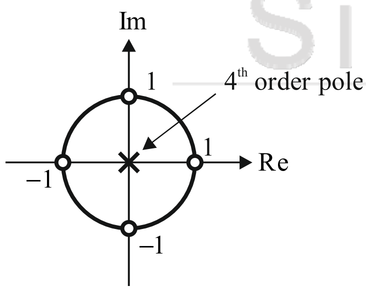
Question 9

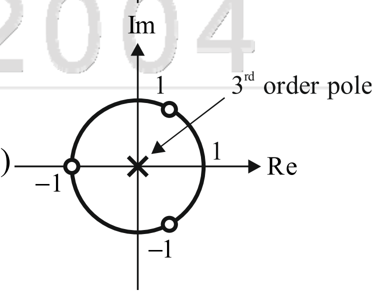
Which one of the following pole-zero plots corresponds to the transfer function of an LTI system characterized by the input-output difference equation given below?

$$y[n]=\sum_{k=0}^{3}(-1)^k x[n-k]$$

(A) 

(B) 

(C) 

(D) 

Ans. **(B)**

Sol. **Method 1**

Given : $y[n]=\sum_{k=0}^{3}(-1)^k \delta[n-k]$

To find impulse response, substitution $x[n]=\delta[n]$ in the given input output relation.

$$h[n]=\sum_{k=0}^{3}(-1)^k \delta[n-k]$$

$$h[n]=\delta[n]-\delta[n-1]+\delta[n-2]-\delta[n-3]$$

Taking z-transform both sides to find the transfer function of the system,

$$H(z)=1-z^{-1}+z^{-2}-z^{-3}$$

$$H(z)=\frac{z^3-z^2+z-1}{z^3}$$

$$H(z)\big|_{z=+1}=\frac{1-1+1-1}{1}=0$$

So, the system function has a zero at $z=+1$ that indicates that the low frequency gain of filter in zero.

So it will be a high pass filter. To check, finding $H(z)$ at $\omega=\pi$ or $z=-1$

$$H(z)\big|_{z=-1}=\frac{-1-1-1-1}{-1}=4$$

So the system behaves as a HPF and it will not have zero at $z=-1$.

The only option satisfying this is option (B).

Hence, the correct option is (B).

Method 2

Given : $y[n]=\sum_{k=0}^{3}(-1)^k x(n-k)$

$$y(n)=x(n)-x(n-1)+x(n-2)-x(n-3)$$

Taking z-transform on both sides,

$$Y(z)=(1-z^{-1}+z^{-2}-z^{-3})X(z)$$

$$\frac{Y(z)}{X(z)}=1-z^{-1}+z^{-2}-z^{-3}$$

$$H(z)=1-z^{-1}+z^{-2}-z^{-3}=\frac{z^3-z^2+z-1}{z^3}$$

$H(z)$ has a pole of order 3 at $z=0$.

To find zero's, equate numerator to zero.

$$z^3-z^2+z-1=0$$

$$(z-1)(z^2+1)=0$$

$$z=1 \text{ and } z=\pm j$$

$\therefore$ $H(z)$ has zero's at $z=1$ and $z=\pm j$

Hence, the correct option is (B).

Question 10

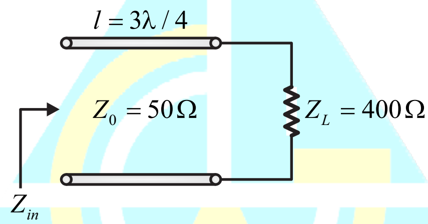
A transmission line of length $3\lambda/4$ and having a characteristic impedance of $50\ \Omega$ is terminated with a load of $400\ \Omega$. The impedance (rounded off to two decimal places) seen at the input end of the transmission line is ________ Ω.

Ans. (6.25)

Sol. **Given :** Length of transmission line, $l=\dfrac{3\lambda}{4}$

Characteristic impedance, $Z_0 = 50\Omega$

Load of transmission line, $Z_L = 400\Omega$



For a lossless transmission line the input impedance is given by,

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan\beta\ell}{Z_0 + jZ_L\tan\beta\ell}\right] \quad \ldots(i)$$

For $\dfrac{3\lambda}{4}$ transmission line,

$$\beta\ell = \frac{2\pi}{\lambda}.\frac{3\lambda}{4} = \frac{3\pi}{2} = 270^0$$

$$\tan\beta\ell = \tan 270^0 = \frac{\sin 270}{\cos 270} = \frac{-1}{0} = \infty$$

From, equation (i) $\tan\beta\ell$ will be common from numerator and denominator,

$$Z_{in} = Z_0\left[\frac{\dfrac{Z_L}{\tan\beta\ell} + jZ_0}{\dfrac{Z_0}{\tan\beta\ell} + jZ_L}\right]$$

$$Z_{in} = Z_0\left[\frac{\dfrac{Z_L}{\tan 270} + jZ_0}{\dfrac{Z_0}{\tan 270} + jZ_L}\right]$$

$$Z_{in} = Z_0 \left[\frac{0 + jZ_0}{0 + jZ_L} \right]$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

$$Z_{in} = \frac{50^2}{400} = \frac{50 \times 50}{400}$$

$$Z_{in} = 6.25\Omega$$

Hence, the impedance seen at the input end of the transmission line is 6.25Ω

Question 11

A 10 bit D/A converter is calibrated over the full range 0 to 10 V. If the input to the D/A converter is 13A (in hex), the output (rounded off to three decimal places) is _______ V.

Ans. 2.95 to 3.15 (3.069)

Sol. **Given :** Full scale reading = 10 V

$n = 10$

Digital input = $(13\text{A})_H$

Analog output = (Resolution) $\times$ (Decimal equivalent of digital input data)

$$\text{Resolution } (R) = \frac{V_r}{2^n - 1} = \frac{10}{2^{10} - 1} = 9.77 \times 10^{-3}$$

Decimal equivalent of Hexadecimal input data is,

$$(13\text{A})_{16} = 1 \times 16^2 + 3 \times 16^1 + 10 \times 16^0 = (314)_{10}$$

$$\therefore \quad V_0 = 9.77 \times 10^{-3} \times 314 = 3.069$$

Question 12

In an 8085 microprocessor, the number of address lines required to access a 16 K bytes memory bank is ______ .

Ans. (14)

Sol. 16 Kbytes memory bytes $= 16 \times 1024$

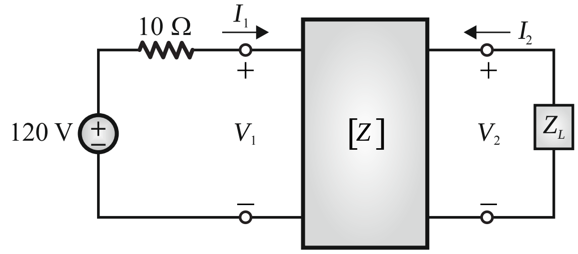
$$= (2)^4 \times (2)^{10}$$

$$= (2)^{14}$$

The number of address lines required to access a 16 K bytes memory bank $= 14$ **Ans.**

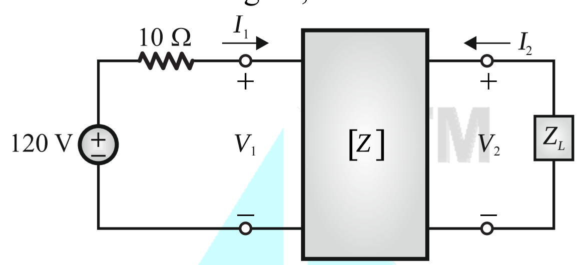
Question 13

In the given circuit, the two-port network has the impedance matrix $[Z] = \begin{bmatrix} 40 & 60 \\ 60 & 120 \end{bmatrix}$. The value of Z_L for which maximum power is transferred to the load is ______ Ω.



Ans. **(48)**

Sol. Given two port network is shown in below figure,

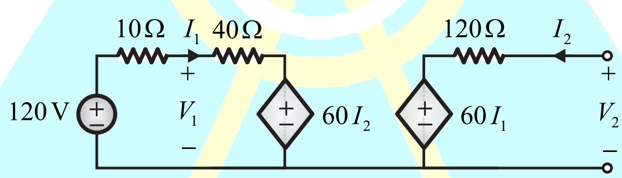


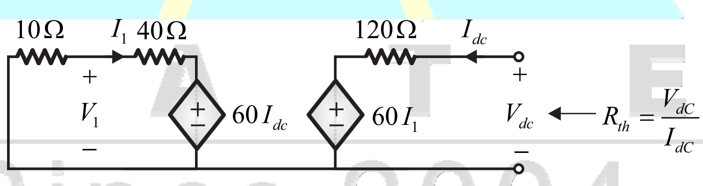
$$[Z]=\begin{bmatrix}40 & 60\\ 60 & 120\end{bmatrix}$$

Z parameter equation from matrix is,

$$V_1=40I_1+60I_2$$

$$V_2=60I_1+120I_2$$





Applying KVL in output loop,

$$-V_{dc}+120I_{dc}+60I_1=0$$

$$V_{dc}=120I_{dc}+60I_1 \qquad \ldots \text{(i)}$$

Applying KVL in input loop,

$$0+50I_1+60I_{dc}=0$$

$$I_1=\frac{-6}{5}I_{dc}$$

Substituting the value of I_1 in equation (i),

$$V_{dc}=120I_{dc}+60\left(\frac{-6}{5}\right)I_{dc}$$

$$R_{th} = \frac{V_{dc}}{I_{dc}} = 48\ \Omega$$

From maximum power transfer theorem,

$$Z_L = Z_{th}$$

$$\therefore \quad Z_L = 48\ \Omega$$

Hence, the value of Z_L for which maximum power is transferred to the load is **48 Ω**.

Question 14

Consider the recombination process via bulk traps in a forward biased *pn* homojunction diode. The maximum recombination rate is $U_{\max}$. If the electron and the hole capture cross sections are equal, which one of the following is FALSE?

(A) $U_{\max}$ occurs at the edges of the depletion region in the device.

(B) $U_{\max}$ depends exponentially on the applied bias.

(C) With all other parameters unchanged, $U_{\max}$ increases if the thermal velocity of carrier increases.

(D) With all other parameters unchanged, $U_{\max}$ decreases if the intrinsic carrier density is reduced.

Ans. **(A)**

Sol. The maximum recombination rate is given by,

$$U_{\max} = \frac{1}{2} V_{th}\ n_i \sigma_0 e^{V/2V_T} \cdot N_t$$

Where, n_i = Intrinsic concentration, σ_0 = capture cross section, V = applied voltage,

V_T = thermal voltage, V_{th} = thermal velocity, N_t = recombination centers in bulk of semiconductors

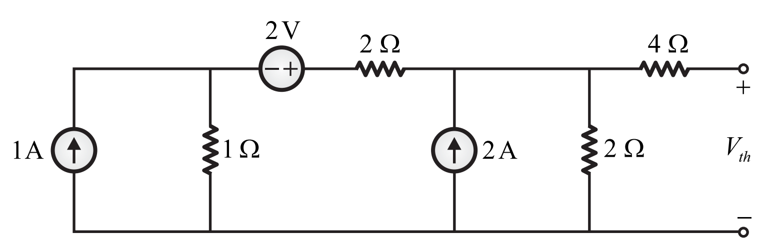
$$\sigma_n = \sigma_p = \sigma_0$$

In Homojunction *p-n* diode we find that current in a pn junction can only exist if there is recombination or generation of electron and holes somewhere throughout the structure. The ideal diode equation is a result of the recombination and generation in the quasi-neutral regions (including recombination at the contacts) whereas recombination and generation in the depletion region yield enhanced leakage or photocurrents.

Hence, the correct option is (A).

Question 15

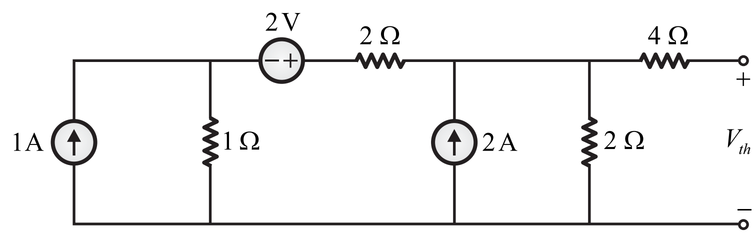
In the circuit shown below, the Thevenin voltage V_{th} is



(A) 4.5 V (B) 3.6 V (C) 2.4 V (D) 2.8 V

Ans. **(B)**

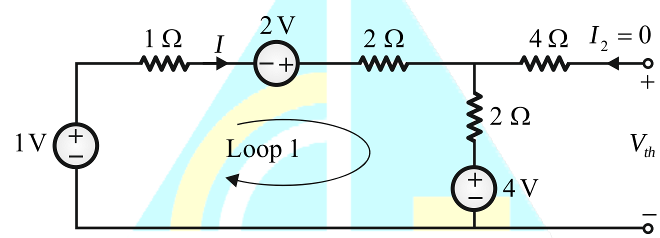
Sol. Given circuit is shown in figure,



We have to find open circuit voltage V_{th}.

Method 1

Using source transformation,



Applying KVL in loop 1,

$$1 - I(1) + 2 - 2I - 2I - 4 = 0$$

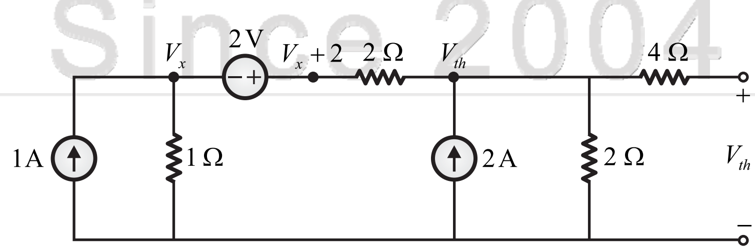
$$1 + 2 - 4 = 5I$$

$$I = -\frac{1}{5}\text{ Amp}$$

$$V_{th} = 4 + I(2) = 4 + \left(-\frac{1}{5}\right) \times 2 = 4 - 0.4 = 3.6\text{ V}$$

Method 2

By nodal method,



Applying KCL at node V_x,

$$-1 + \frac{V_x}{1} + \frac{(V_x + 2) - V_{th}}{2} = 0$$

$$1\text{ A} = V_x + \left(\frac{V_x}{2}\right) + 1 - \frac{V_{th}}{2}$$

$$\frac{3V_x}{2} = \frac{V_{th}}{2}$$

$$V_x = \frac{V_{th}}{3} \qquad \text{.... (i)}$$

Applying KCL at node V_{th},

$$2\text{ A} = \frac{V_{th} - (V_x + 2)}{2} + \frac{V_{th}}{2}$$

$$4 = V_{th} - V_x - 2 + V_{th}$$

$$4 = 2V_{th} - V_x - 2$$

$$2V_{th} - V_x = 6$$

Putting the value of V_x from equation (i),

$$2V_{th} - \frac{V_{th}}{3} = 6$$

$$\frac{6V_{th} - V_{th}}{3} = 6$$

$$\frac{5V_{th}}{3} = 6$$

$$V_{th} = \frac{6 \times 3}{5} = \frac{18}{5} = 3.6\text{ V}$$

Hence, the correct option is (B).

Question 16

A single crystal intrinsic semiconductor is at a temperature of 300 K with effective density of states for holes twice that of electrons. The thermal voltage is 26 mV. The intrinsic Fermi level is shifted from mid-bandgap energy level by

(A) 9.01 meV (B) 13.45 meV (C) 18.02 meV (D) 26.90 meV

Ans. **(A)**

Sol. **Given :** At $T = 300$ K, the effective density of states for holes is twice that of electrons i.e., $N_V = 2N_C$ and thermal voltage is 26 mV.

Location of Fermi level is given by,

$$E_F = \frac{E_C + E_V}{2} - \frac{KT}{2} \ln \frac{N_C}{N_V}$$

The difference of Fermi level from mid band gap energy level is

$$E_d = \frac{-KT}{2} \ln\left(\frac{1}{2}\right) = 9.01\text{ meV}$$

Hence, the correct option is (A).

Question 17

The random variable

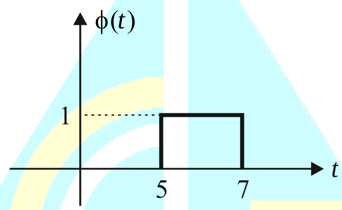
$$Y = \int_{-\infty}^{\infty} W(t)\phi(t)\,dt, \quad \text{where } \phi(t) = \begin{cases} 1, & 5 \le t \le 7 \\ 0, & \text{otherwise} \end{cases}$$

and $W(t)$ is a real white Gaussian noise process with two-sided power spectral density $S_W(f) = 3$ W/Hz, for all f. The variance of Y is ________.

Ans. **6**

Sol. **Given :** $Y = \int_{-\infty}^{\infty} W(t)\phi(t)\,dt$ where $\phi(t) = \begin{cases} 1, & 5 \le t \le 7 \\ 0, & \text{otherwise} \end{cases}$

$S_W(f) = 3$ W/Hz



As $\phi(t)$ exists only for $5 \le t \le 7$, so the product of $W(t)$ and $\phi(t)$ will also exist for this duration only.

$$\therefore \qquad Y = \int_{5}^{7} W(t)\,dt$$

Where, $W(t)$ = white Gaussian Noise with PSD $\frac{\eta}{2} = 3$ W/Hz.

So Y represents a white Gaussian noise truncated in interval 5 to 7.
As mean of WGN is zero, so the power will be equal to the variance of Y.
Variance of WGN truncated in interval T_1 to $T_1 + T$,

$$\text{Variance}\left[\int_{T_1}^{T_1+T} W(t)\,dt\right] = \frac{\eta}{2} \times T$$

$$\text{Variance}\left[\int_{5}^{5+2} W(t)\,dt\right] = \text{Variance }[Y] = 3 \times 2 = 6$$

Hence, the variance of Y is 6.

Question 18

A digital communication system transmits a block of N bits. The probability of error in decoding a bit is α. The error event of each bit is independent of the error events of the other bits. The received block is declared erroneous of at least one of its bits is decoded wrongly. The probability that the received block is erroneous, is

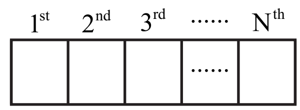
(A) $1-(1-\alpha)^N$ (B) α^N (C) $N(1-\alpha)$ (D) $1-\alpha^N$

Ans. **(A)**

Sol. Given probability of error in decoding a single bit is

$P(e) = \alpha$

Block size = N bits



The received block is declared erroneous, if at least one of bits is decoded wrongly,

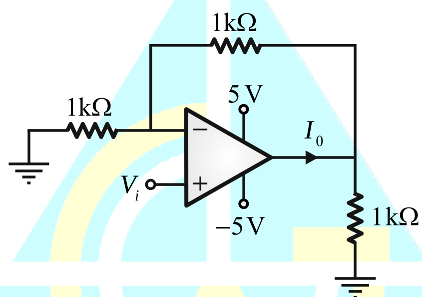
Probability of received correct block $= (1-\alpha)^N$

The probability of received block is erroneous $= 1-(1-\alpha)^N$

Hence, the correct option is (A).

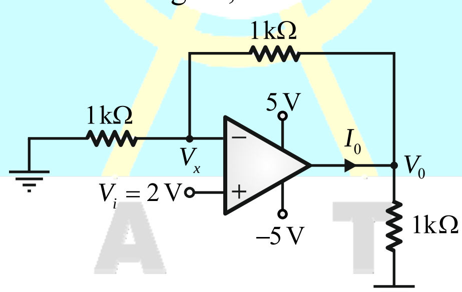
Question 19

In the circuit shown below, all the components are ideal. If V_i is $+2\text{V}$, the current I_0 sourced by the Op-Amp is _________ mA.



Ans. **(6)**

Sol. Given Op-Amp circuit is shown in below figure,



Since this is a –ve feedback Op-Amp circuit. So we can apply virtual grounded concept.

Here, $V_x = V_i = 2\text{ V}$

Applying KCL at node V_x,

$$\frac{V_x - 0}{1\text{k}} + \frac{V_x - V_0}{1\text{k}} + 0 = 0$$

$$\frac{2-0}{1\text{k}} + \frac{2-V_0}{1\text{k}} = 0$$

$$V_0 = 4\text{V}$$

Applying KCL at node V_0,

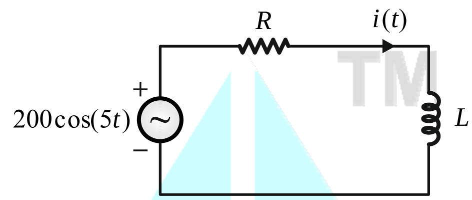
$$\frac{V_0 - V_x}{1\text{k}} + \frac{V_0 - 0}{1\text{k}} - I_0 = 0$$

$$\frac{4-2}{1\text{k}} + \frac{4}{1\text{k}} = I_0$$

$$I_0 = 6 \text{ mA}$$

Question 20

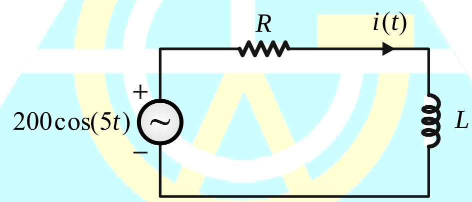
The current in the RL - circuit shown below is $i(t) = 10\cos(5t - \pi/4)$ A. The value of the inductor (rounded off to two decimal places) is ________ H.



Ans. **2.80 to 2.85 (2.828)**

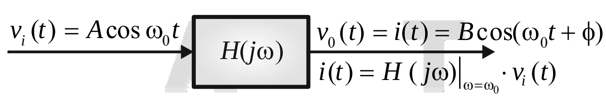
Sol. **Method 1**

Given network is shown below,



$$i(t) = 10\cos\left(5t - \frac{\pi}{4}\right)$$

Using sinusoidal response,



From the given figure,

$$A = 200,\ \omega_0 = 5 \text{ rad/sec}$$

$$B = A \cdot |H(\omega)|_{\omega=\omega_0}$$

$$\phi = \angle H(\omega)|_{\omega=\omega_0} \qquad \ldots\text{(i)}$$

$$B = 10 = A \cdot |H(\omega)|_{\omega=5} \qquad \ldots\text{(ii)}$$

$$\phi = \frac{-\pi}{4} = \angle H(\omega)|_{\omega=5}$$

The output current can be given as,

$$I(s) = \frac{V_i(s)}{R + Ls}$$

$$I(s) = V_i(s) \times H(s)$$

$$H(s) = \frac{1}{R + Ls}$$

$$H(j\omega) = \frac{1}{R + j\omega L}$$

$$|H(j\omega)| = \frac{1}{\sqrt{R^2 + j\omega^2 L^2}}$$

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i(t) = 200\cos(5t) \times \frac{1}{\sqrt{R^2 + \omega^2 L^2}} \angle -\tan\left(\frac{\omega L}{R}\right)$$

Comparing with current $i(t)$,

From equation (i),

$$\phi = -\tan^{-1}\left(\frac{\omega L}{R}\right)\Bigg|_{\omega=5} = -\frac{\pi}{4}$$

$$\frac{\omega L}{R}\Bigg|_{\omega=5} = \tan\frac{\pi}{4} = 1$$

$$5 \cdot L = R \qquad \ldots\text{(iii)}$$

From equation (ii),

$$10 = 200 \times |H(\omega)|_{\omega=5}$$

$$\frac{200}{\sqrt{R^2 + \omega^2 L^2}} = 10$$

$$400 = R^2 + 25L^2$$

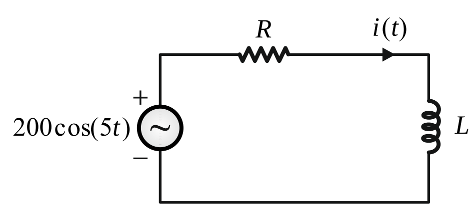
From equation (iii),

$$R = 5L$$

$\therefore$ $$400 = 25L^2 + 25L^2$$

$$L = \sqrt{\frac{400}{50}} = 2.828 \text{ H}$$

Method 2



$$Z = \frac{V}{I} = \frac{200\angle 0^0}{10\angle -45^0} = 20\angle 45^0$$

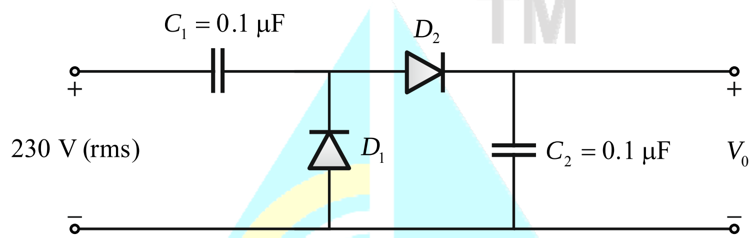
$$Z = 10\sqrt{2} + j10\sqrt{2}$$

$$X_L = 10\sqrt{2} = \omega L$$

$$L = \frac{10\sqrt{2}}{5} = 2\sqrt{2} = 2.828 \text{ H}$$

Question 21

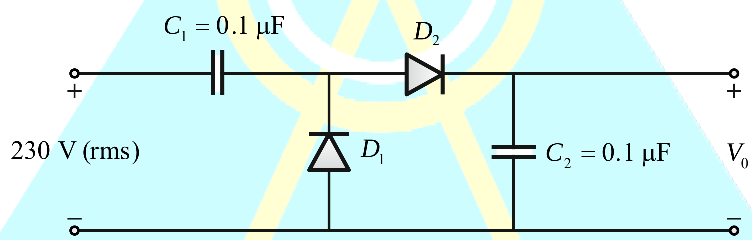
In the circuit shown below, all the components are ideal and the input voltage is sinusoidal. The magnitude of the steady - state output V_0 (rounded off to two decimal places) is ______V.



Ans. **650.538 (649.5 to 651.5)**

Sol. **Method 1**

Given circuit is shown below,



$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$V_m = \sqrt{2}\, V_{rms} = \sqrt{2} \times 230\text{V}$$

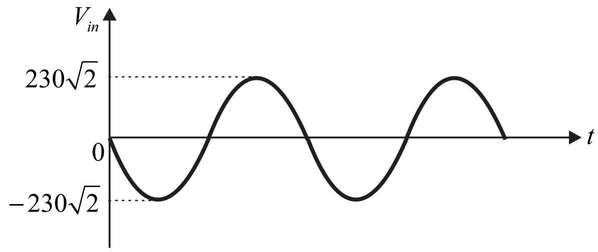
$$V_m = 230\sqrt{2}\text{ V}$$

Also given input voltage is sinusoidal.

$$V_{in} = V_m \sin \omega t$$

$$V_{in} = 230\sqrt{2}\, \sin \omega t$$

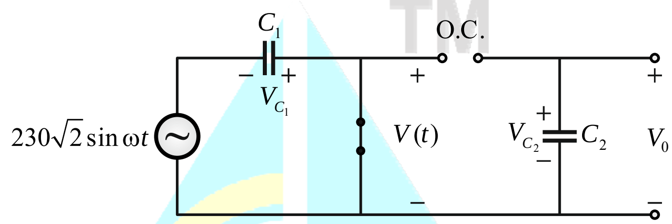
Assume that negative half cycle occurs first,



Case 1 : During 1st negative half cycle,

Diode D_1 will be forward biased and diode D_2 will be reverse biased.

Hence, charging of capacitor C_1 will start and it will charge upto maximum value of input.



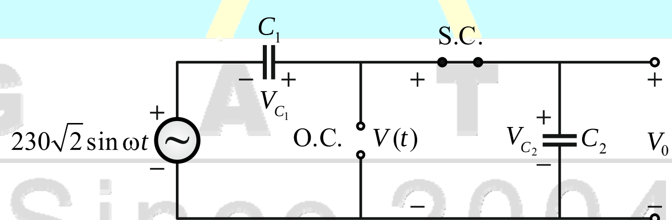
Hence, $=230\sqrt{2}\text{V}$

$$V_{C_1} = \left|230\sqrt{2}\sin\omega t\right| = 230\sqrt{2}\text{ V and } V_{C_2} = 0\text{V}$$

Case 2 : During 1st positive half cycle,

Diode D_1 will be reverse biased and diode D_2 will be forward biased.

Hence, charging of capacitor C_2 will start and capacitor C_2 will charge upto maximum value of input, which appear across capacitor C_2.



$$V_{C_2} = V_{C_1} + \left|230\sqrt{2}\sin\omega t\right|_{\max}$$

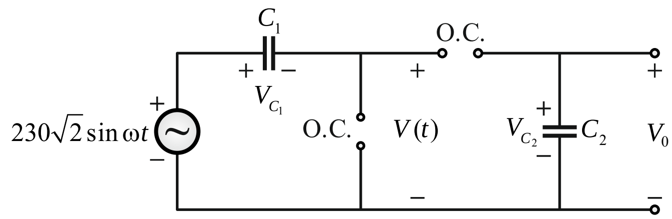
$$V_{C_2} = 230\sqrt{2} + 230\sqrt{2}$$

$$V_{C_2} = 460\sqrt{2}\text{ V}$$

Case 3 :

During 2nd positive half cycle, negative terminal of diode D_1 is at $230\sqrt{2}$ V and negative terminal of D_2 is at $460\sqrt{2}$ V. Now, diode D_1 and D_2 will be forward bias when voltage at their positive terminal will be higher than that of the negative terminal, which is never possible beyond first negative half cycle and first positive half cycle.

Hence, in this case, D_1 and D_2 will be reverse biased.



Hence, from above circuit

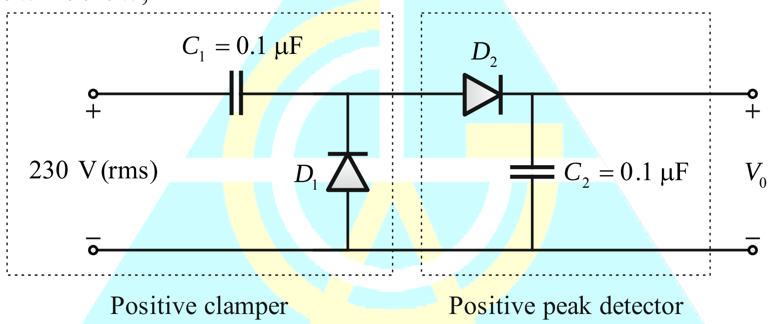
$$V_0 = V_{C_2} = 460\sqrt{2}\text{ V}$$

$$V_0 = 650.538\text{V}$$

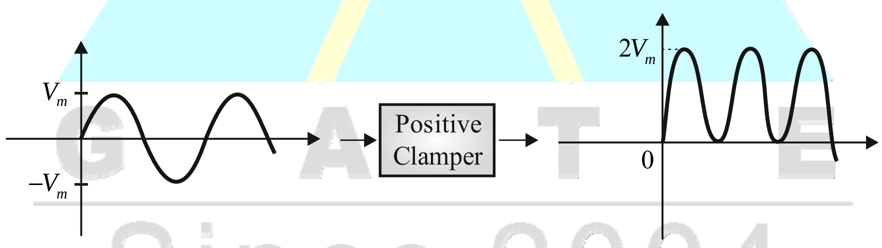
Hence, the magnitude of the steady-state output V_0 is $460\sqrt{2}$ V.

Method 2

Given circuit is shown below,



The clamper will do the shift of input voltage to $2V_m$.



Given rms value of sinusoidal input

$$\frac{V_m}{\sqrt{2}} = 230 \Rightarrow V_m = \sqrt{2}\times 230$$

Peak detector will detect the maximum value $= 2V_m = 2\times 230(\sqrt{2}) = 650.53$

Method 3

Define the circuit as a voltage doubler.

$$V_0 = 2V_m = 2\times 230\sqrt{2} = 650.4\text{ V}$$

Question 22

A binary random variable X takes the value $+2$ or -2. The probability $P(X = +2) = \alpha$. The value of α (rounded off to one decimal place), for which the entropy of X is maximum, is________.

Ans. **(0.5)**

Sol. **Given :** A random variable 'X' takes value + 2 or – 2.

Since, $P(X=+2)=\alpha$

Then $P(X=-2)=1-\alpha$

Entropy is given by,

$$H=\sum_i P(x_i)\log_2\frac{1}{P(x_i)}$$

$$H=\alpha\log_2\frac{1}{\alpha}+(1-\alpha)\log_2\left(\frac{1}{1-\alpha}\right)$$

$$H=-\alpha\log_2\alpha+(\alpha-1)\log_2(1-\alpha)$$

For maxima or minima,

$$\frac{dH}{d\alpha}=0$$

$$\frac{d}{d\alpha}[(\alpha-1)\log_2(1-\alpha)-\alpha\log_2\alpha]=0$$

$$\frac{1}{\log 2}\frac{d}{d\alpha}[(\alpha-1)\log(1-\alpha)-\alpha\log\alpha]=0$$

$$\left[(\alpha-1)\times\frac{1}{1-\alpha}\times(-1)+\log(1-\alpha)(1-0)\right]-\left(\alpha\frac{1}{\alpha}+\log\alpha\right)=0$$

$$1+\log(1-\alpha)-1-\log\alpha=0$$

$$\log(1-\alpha)=\log\alpha$$

$$1-\alpha=\alpha$$

$$2\alpha=1$$

$$\alpha=\frac{1}{2}$$

$\therefore$ Entropy of X will be maximum if $\alpha=\frac{1}{2}$.

Question 23

The partial derivative of the function

$$f(x,y,z)=e^{1-x\cos y}+xze^{\frac{-1}{(1+y^2)}}$$

with respect to x at the point $(1,0,e)$ is

(A) 1 (B) $\frac{1}{e}$ (C) 0 (D) – 1

Ans. **(C)**

Sol. **Given :** $f(x,y,z)=e^{1-x\cos y}+xze^{-1/(1+y^2)}$

Partially differentiating with respect to x,

$$\frac{\partial f}{\partial x} = e^{(1-x\cos y)}(-\cos y) + ze^{-1/(1+y^2)} \times 1$$

$$\frac{\partial f}{\partial x} = -e^{(1-x\cos y)}(-\cos y) + ze^{-1/(1+y^2)}$$

Now, finding partial derivative at point (1, 0, e),

$$\frac{\partial f}{\partial x} = -e^{(1-1)}\cos 0 + ee^{-1/(1+0)}$$

$$\frac{\partial f}{\partial x} = -1 + 1 = 0$$

Hence, the correct option is (C).

Question 24

For a vector field $\vec{A}$, which one of the following is FALSE ?

(A) $\vec{A}$ is solenoid if $\nabla \cdot \vec{A} = 0$.

(B) $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

(C) $\nabla \times \vec{A}$ is another vector field.

(D) $\vec{A}$ is irrotational if $\nabla^2 \vec{A} = 0$.

Ans. (D)

Sol. Key point :

1. $\nabla \cdot \vec{A} = 0$

   Then it is solenoidal or divergenceless.

2. $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$ [identity] referred to as vector triple product.

3. $\nabla \times \vec{A}$ is an another vector because when we have curl of vector we will get new vector.

4. For irrotational vector $\nabla \times \vec{A} = 0$.

Hence, the correct option is (D).

Question 25

The impedances $Z = jX$, for all X in the range $(-\infty, \infty)$, map to the Smith chart as

(A) a circle of radius 1 with centre at (0, 0).

(B) a line passing through the centre of the chart.

(C) a circle of radius 0.5 with centre at (0.5, 0).

(D) a point at the centre of the chart.

Ans. (A)

Sol. Given : Impedance, $Z = jX$

Normalized impedance,

$$z = \frac{Z}{Z_0} = \frac{jX}{Z_0} = r + jx$$

$$r = 0 \text{ and } x = \left(\frac{X}{Z_0}\right) \rightarrow \text{Normalized reactance}$$

We have $X \rightarrow [-\infty, \infty]$

As $r = 0 \rightarrow$ fixed, so locus point is on constants resistance circle

Centre $\left\{\frac{r}{1+r}, 0\right\} = \{0, 0\}$

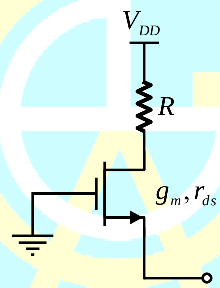
Radius $= \frac{1}{1+r} = \frac{1}{1} = 1$

Hence, the correct option is (A).

Q.26 to Q.55 Carry two marks each

Question 26

Using the incremental low frequency small - signal model of the MOS device, the Norton equivalent resistance of the following circuit is



(A) $r_{ds} + R$

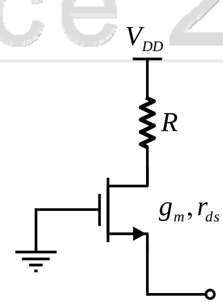
(B) $\frac{r_{ds} + R}{1 + g_m r_{ds}}$

(C) $r_{ds} + R + g_m r_{ds} R$

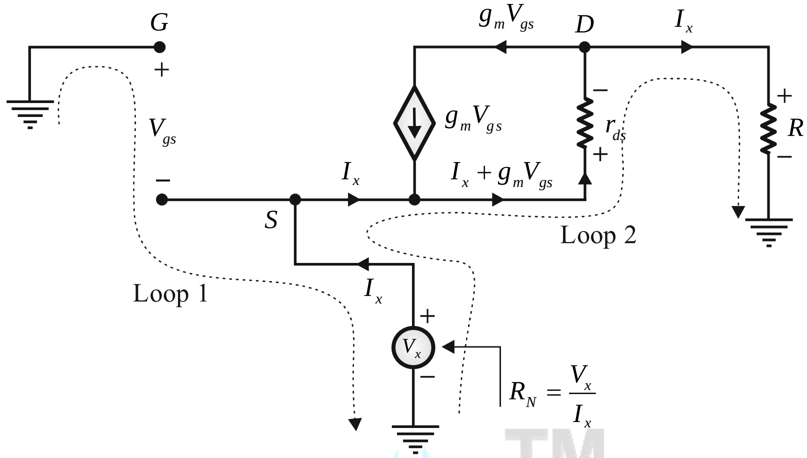
(D) $r_{ds} + \frac{1}{g_m} + R$

Ans. (B)

Sol. Given circuit is shown in figure below,



Small signal equivalent circuit is shown below,



Applying KVL in loop 1,

$$-V_{gs} - V_x = 0$$

$$V_{gs} = -V_x \quad \text{... (i)}$$

Applying KVL in loop 2,

$$V_x - r_{ds}(I_x + g_mV_{gs}) - RI_x = 0$$

From equation (i), putting $V_{gs} = -V_x$,

$$+V_x - r_{ds}I_x + g_mV_xr_{ds} - RI_x = 0$$

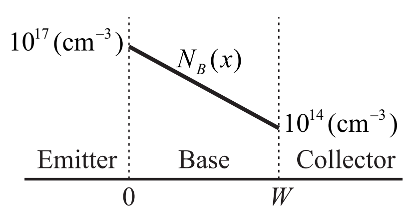
$$V_x(1 + g_mr_{ds}) = I_x(r_{ds} + R)$$

$$\frac{V_x}{I_x} = R_N = \frac{r_{ds} + R}{1 + g_mr_{ds}}$$

Hence, the correct option is (B).

Question 27

The base of an *npn* BJT $T1$ has a linear doping profile $N_B(x)$ as shown below. The base of another *npn* BJT $T2$ has a uniform doping N_B of $10^{17}\ \text{cm}^{-3}$. All other parameters are identical for both the devices. Assuming that the hole density profile is the same as that of doping, the common - emitter current gain of $T2$ is



(A) approximately 2.5 times that of $T1$.

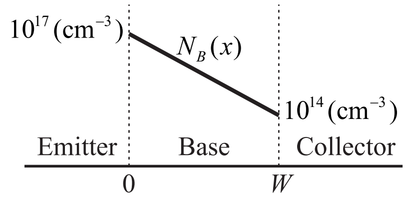
(B) approximately 2.0 times that of $T1$.

(C) approximately 3.0 times that of $T1$.

(D) approximately 0.7 times that of $T1$.

Sol. **Given :** For transistor $T2$ uniform doping $N_B(x) = 10^{17}\ \text{cm}^{-3}$

Linear doping profile $N_B(x)$ of transistor $T1$ is shown below,



Since, current gain β is given as,

$$\beta = \frac{L_E N_E D_B}{W N_B D_E}$$

Now, if other parameters are constant, only WN_B is variable, then

$$\beta \propto \frac{1}{WN_B}$$

Where, WN_B is area under the curve of doping profile.

Let, $WN_B = G_B$ then

G_{B_1} = Area under the curve for transistor $T1$ and

G_{B_2} = Area under the curve for transistor $T2$

$$G_{B_1} = \text{Area under } N_B(x) \text{ for } T1 = \int_0^W N_B(x)\,dx = \frac{(10^{17} - 10^{14})W}{2}$$

and G_{B_2} = Area under $N_{B_2} = N_B W = 10^{17} \times W$

So, the ratio of current gains of transistor $T2$ and $T1$.

$$\frac{\beta_2}{\beta_1} = \frac{G_{B_1}}{G_{B_2}} = \frac{\frac{(10^{17} - 10^{14})W}{2}}{10^{17} \times W} = 0.4995$$

Therefore, $\boldsymbol{\beta_2 \approx 0.5\beta_1}$, i.e. current gain of transistor $T2$ is approximately 0.5 times that of transistor $T1$.

IIT has declared it as MTA.

As the correct answer of the question is missing in the given options, so it is impossible to attempt the question as it is a multiple choice question having negative marking.

In the first answer key, IIT has given its answer to be option (B), i.e. current gain of transistor $T2$ is approximately 2 times that of transistor $T1$.

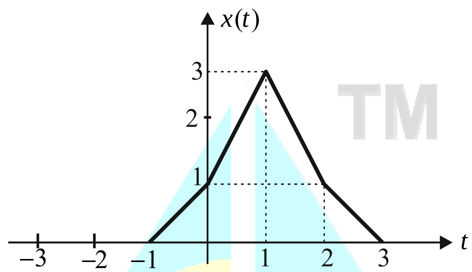
If the given answer by IIT would be option (D), i.e. approximately 0.7 times, then it can be considered that they have purposely used the word “approximately” in the statement, but as they have given the option

(B) that is approximately 2 times, it leads to the conclusion that they have taken the inverse ratio while framing the options.

So, in final answer key IIT considered this question as a **MARKS TO ALL**.

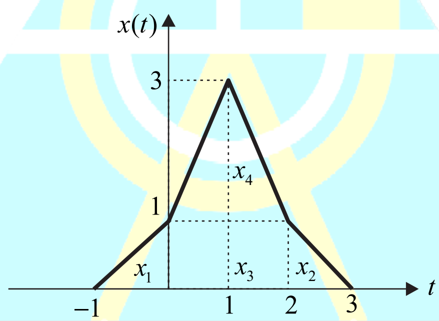
Question 28

$X(\omega)$ is the Fourier transform of $x(t)$ shown below. The value of $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$ (rounded off to two decimal places) is _________.



Ans. **58.1 to 59.1 (58.61)**

Sol. **Given :** Signal $x(t)$ is shown in figure.



Method 1

Using Rayleigh energy theorem,

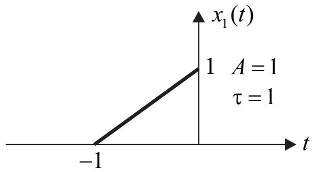
Energy of $x(t)$ is given by,

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

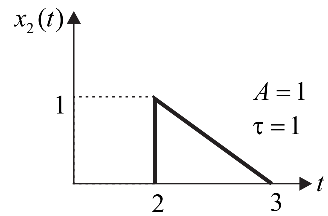
$$\therefore \quad I = \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = 2\pi E_x \quad \ldots\text{(i)}$$

To find energy of $x(t)$, we can consider $x(t)$ to be combination of signals $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$ as shown in figure. x_1 and x_2 are non overlapping with x_3 and x_4 but x_3 and x_4 are overlapping with each other.

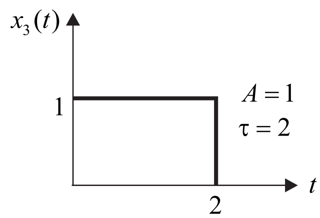
$$E_x = E[x_1(t)] + E[x_2(t)] + E[\text{Combination of } x_3(t) \text{and } x_4(t)] \quad \ldots\text{(ii)}$$



$$E_{x_1}=\frac{1}{2}\times\frac{2A^2\tau}{3}=\frac{1}{3}$$

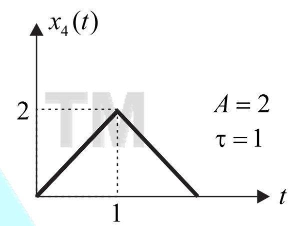


$$E_{x_2}=\frac{1}{2}\times\frac{2A^2\tau}{3}=\frac{1}{3}$$



$$E_{x_3}=A^2\tau$$

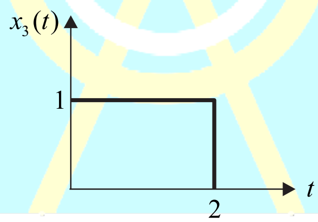
$$E_{x_3}=1\times 2=2$$

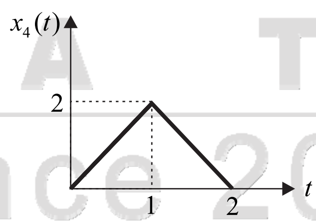


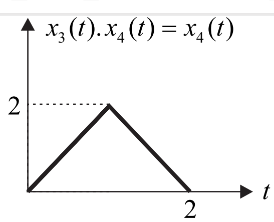
$$E_{x_4}=\frac{2A^2\tau}{3}$$

$$E_{x_4}=\frac{2\times 4\times 1}{3}=\frac{8}{3}$$

Energy in combination of $x_3(t)$ and $x_4(t)$ = $E_{x_3}+E_{x_4}+2$ [Area under function $x_3(t)$. $x_4(t)$]







From figure, $x_3(t)\cdot x_4(t)=x_4(t)$

Area $[x_3(t)\cdot x_4(t)]=\text{Area}\,[x_4(t)]=\frac{1}{2}\times 2\times 2=2$

Energy in combination of $x_3(t)$ and $x_4(t) = 2+\frac{8}{3}+(2\times 2) = 6+\frac{8}{3}$

From equation (ii), energy of $x(t)$ is

$$E_x = \frac{1}{3}+\frac{1}{3}+6+\frac{8}{3} = \frac{1+1+18+8}{3} = \frac{28}{3}$$

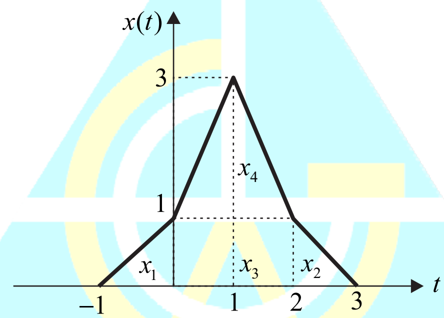
From equation (i), value of given integral

$$I = \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = 2\pi\times\frac{28}{3} = 58.64$$

Hence, the value of integral is 58.64.

Method 2

Given : Signal $x(t)$ is shown in figure,



Using Raleigh energy theorem, energy of $x(t)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

$$I = \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = 2\pi E_x = 2\pi\int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = 2\pi\times\int_{-1}^{3} |x(t)|^2 dt = 2\pi\times 2\int_{-1}^{1} |x(t)|^2 dt$$

$$E_x = 4\pi\left[\int_{-1}^{0} |x(t)|^2 dt + \int_{0}^{1} |x(t)|^2 dt\right] \quad \ldots(i)$$

Equation of line in region $(-1, 0)$ is given by,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{1-0}{0-(-1)}(x-(-1))$$

$$y = x+1 \quad \ldots(ii)$$

Equation of line in region $(0,1)$ is given by,

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-1=\frac{3-1}{1-0}(x-0)$$

$$y-1=2x$$

$$y=2x+1 \qquad \text{....(iii)}$$

Put the value from equation (ii) and (iii) in equation (i),

$$E_x=4\pi\left[\int_{-1}^{0}(x+1)^2dx+\int_{0}^{1}(2x+1)^2dx\right]$$

$$E_x=4\pi\left[\int_{-1}^{0}(x^2+2x+1)\,dx+\int_{-1}^{0}(4x^2+4x+1)\,dx\right]$$

$$E_x=4\pi\left[\left(\frac{x^3}{3}+\frac{2x^2}{2}+x\right)_{-1}^{0}+\left(\frac{4x^3}{3}+\frac{4x^2}{2}+x\right)_{0}^{1}\right]$$

$$E_x=4\pi\left[\left(\frac{1}{3}-1+1\right)+\left(\frac{4}{3}+2+1\right)\right]$$

$$E_x=4\pi\left[\frac{1}{3}+\frac{13}{3}\right]$$

$$E_x=4\pi\times\frac{14}{3}$$

$$E_x=58.64$$

$$E_x=\int_{-\infty}^{\infty}\left|x(\omega)^2\right|d\omega=58.64$$

Hence, the value of integral is 58.64.

Question 29

Consider the following system of linear equations.

$$x_1+2x_2=b_1;\ 2x_1+4x_2=b_2;\ 3x_1+7x_2=b_3;\ 3x_1+9x_2=b_4$$

Which one of the following conditions ensures that a solution exists for the above system?

(A) $b_2=2b_1$ and $3b_1-6b_3+b_4=0$

(B) $b_3=2b_1$ and $6b_1-3b_3+b_4=0$

(C) $b_2=2b_1$ and $6b_1-3b_3+b_4=0$

(D) $b_3=2b_1$ and $3b_1-6b_3+b_4=0$

Ans. (C)

Sol. **Method 1**

Given : Linear equations are

$$x_1 + 2x_2 = b_1$$

$$2x_1 + 4x_2 = b_2$$

$$3x_1 + 7x_2 = b_3$$

$$3x_1 + 9x_2 = b_4$$

Now, augmented matrix can be written as,

$$[A:B] = \begin{bmatrix} 1 & 2 & : & b_1 \\ 2 & 4 & : & b_2 \\ 3 & 7 & : & b_3 \\ 3 & 9 & : & b_4 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$

$$R_3 \to R_3 - 3R_1$$

$$R_4 \to R_4 - 3R_1$$

$$\begin{bmatrix} 1 & 2 & : & b_1 \\ 0 & 0 & : & b_2 - 2b_1 \\ 0 & 1 & : & b_3 - 3b_1 \\ 0 & 3 & : & b_4 - 3b_1 \end{bmatrix}$$

Applying $R_4 \to R_4 - 3R_3$

$$\begin{bmatrix} 1 & 2 & : & b_1 \\ 0 & 0 & : & b_2 - 2b_1 \\ 0 & 1 & : & b_3 - 3b_1 \\ 0 & 0 & : & b_4 + 6b_1 - 3b_3 \end{bmatrix}$$

For solution to exist

$$\rho(A) = \rho(A:B) = 2$$

$\therefore$ $$b_2 = 2b_1;\; 6b_1 - 3b_3 + b_4 = 0$$

Hence the correct option is (C).

Method 2

Given : Linear equations are,

$$x_1 + 2x_2 = b_1 \qquad \ldots\text{(i)}$$

$$2x_1 + 4x_2 = b_2 \quad \ldots\text{(ii)}$$

$$3x_1 + 7x_2 = b_3 \quad \ldots\text{(iii)}$$

$$3x_1 + 9x_2 = b_4 \quad \ldots\text{(iv)}$$

From option (A) :

$$b_2 = 2b_1 \text{ and } 3b_1 - 6b_3 + b_4 = 0$$

Multiplying equation (i) by 2,

$$2x_1 + 4x_2 = 2b_1 \quad \ldots\text{(v)}$$

Comparing equation (ii) and equation (v),

$$b_2 = 2b_1 \qquad \text{(satisfies the first condition)}$$

Now, justifying equation $3b_1 - 6b_3 + b_4 = 0$

$$3(x_1 + 2x_2) - 6(3x_1 + 7x_2) + 3x_1 + 9x_2 = 0$$

$$3x_1 + 6x_1 - 18x_1 - 42x_2 + 3x_1 + 9x_2 = 0$$

$$-12x_1 - 27x_2 \neq 0$$

Condition (2) is not satisfied.

Hence, option (A) is wrong.

From option (B) :

$$b_3 = 2b_1 \text{ and } 6b_1 - 3b_3 + b_a = 0$$

Multiplying equation (i) by 2,

$$2x_1 + 4x_2 = 2b_1 \quad \ldots\text{(vi)}$$

Comparing equation (iii) and equation (vi),

$$b_3 \neq 2b_1$$

Hence, option (B) is wrong.

From option (C) :

$$b_2 = 2b_1 \text{ and } 6b_1 - 3b_3 + b_4 = 0$$

Multiplying equation (i) by 2,

$$2x_1 + 4x_2 = 2b_1 \quad \ldots\text{(vii)}$$

Comparing equation (ii) and equation (vii),

$$b_2 = 2b_1 \qquad \text{(satisfied the first condition)}$$

Now, justifying equation: $6b_1 - 3b_3 + b_4 = 0$ …(viii)

Putting the value of b_1, b_3, b_4 in equation (viii),

$$6(x_1+2x_2)-3(3x_1+7x_2)+3x_1+9x_2=0$$

$$6x_1+12x_2-9x_1-21x_2+3x_1+9x_2=0$$

$$9x_1-9x_1+21x_2-21x_2=0$$

$$0=0$$

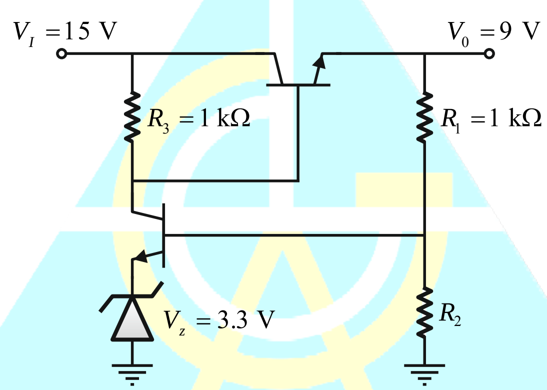
Hence, $6b_1-3b_3+b_4=0$

Both conditions are satisfied,

Hence, the correct option is (C)

Question 30

In the voltage regulator shown below, V_I is the unregulated input at 15 V. Assume $V_{BE}=0.7\text{ V}$ and the base current is negligible for both the BJTs. If the regulated output V_0 is 9 V, the value of R_2 is_____Ω.



Ans. (800)

Sol. Given for the voltage regulator

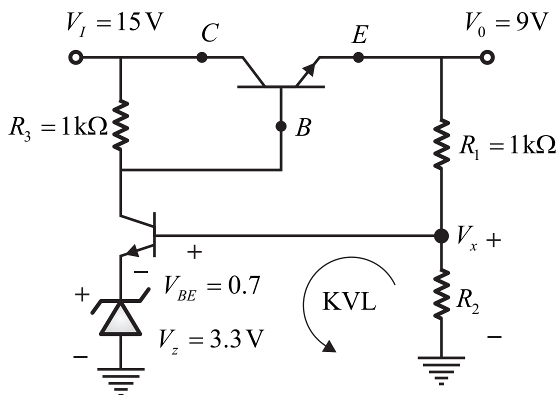
Unregulated input voltage $V_I=15$ V

Base to emitter voltage $V_{BE}=0.7$ V

Regulated output voltage $V_0=9$ V and base currents are negligible.

Method 1

Given voltage regulator circuit can be drawn as,



For finding the value of R_2, we have to find the node voltage V_x

Applying KVL in loop,

$$V_x - V_{BE} - V_z = 0$$

$$V_x = V_{BE} + V_z = 0.7 + 3.3$$

$$V_x = 4\text{V} \quad \ldots\text{(i)}$$

By voltage division rule,

$$V_x = \frac{V_0 R_2}{R_1 + R_2}$$

$$V_x = \frac{9R_2}{1000 + R_2} \quad \ldots\text{(ii)}$$

Comparing equation (i) and (ii),

$$4 = \frac{9R_2}{1000 + R_2}$$

$$4000 + 4R_2 = 9R_2$$

$$R_2 = 800\Omega$$

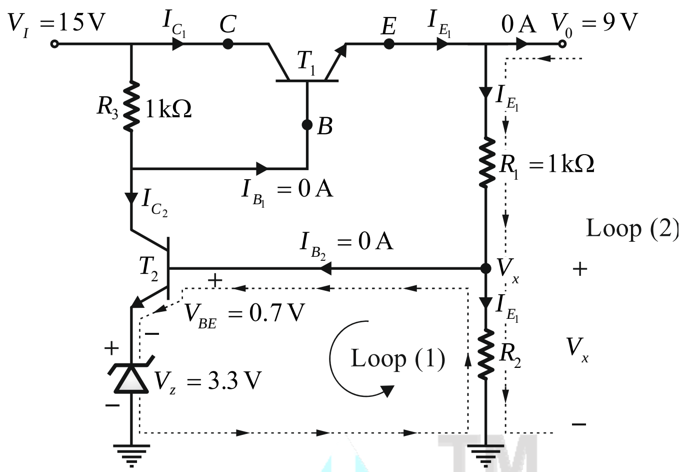
Method 2

Given that unregulated input, $V_I = 15\text{ V}$

Base to emitter voltage of transistor, $V_{BE} = 0.7\text{ V}$

Base current is negligible for both BJT, $I_{B1} = I_{B2} = 0\text{ A}$

Regulated output voltage, $V_0 = 9\text{V}$



For finding the value of R_2, we have to find the voltage across R_2

Applying KVL in loop (1),

$$-V_x + V_{BE} + V_Z = 0$$

$$V_x = V_{BE} + V_Z = 0.7 + 3.3$$

$$V_x = 4\text{V}$$

Applying KVL in loop (2),

$$-9 + I_{E_1} R_1 + V_x = 0$$

$$I_{E_1} R_1 = 9 - V_x = 9 - 4 = 5$$

$$I_{E_1} = \frac{5}{R_1} = \frac{5}{1\text{k}} = 5\,\text{mA}$$

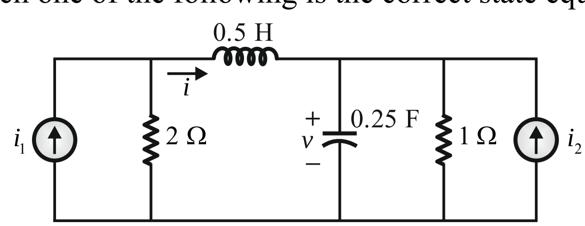
$$V_x = I_{E_1} R_2$$

$$R_2 = \frac{V_x}{I_{E_1}} = \frac{4}{5\times10^{-3}} = 0.8\times10^{3} = 800\,\Omega$$

$$R_2 = 800\,\Omega$$

Question 31

For the given circuit, which one of the following is the correct state equation?



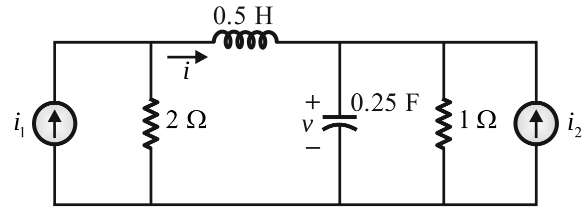
(A) $\frac{d}{dt}\begin{bmatrix} v \\ i \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

(B) $\frac{d}{dt}\begin{bmatrix} v \\ i \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

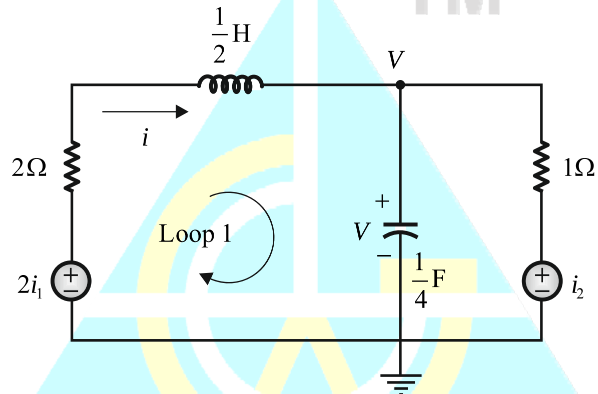
(C) $\frac{d}{dt}\begin{bmatrix} v \\ i \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$ (D) $\frac{d}{dt}\begin{bmatrix} v \\ i \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 4 & 4 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

Ans. **(B)**

Sol. Given circuit is shown below,



Converting the current source of given circuit into voltage source,



Current through the inductor and voltage across the capacitor are state variables $x_1 = i,\ x_2 = V$ then to find state equations, we have to find $\dot{x}_1 = \frac{di}{dt}$ and $\dot{x}_2 = \frac{dV}{dt}$. So we have to write current equation of capacitor and voltage equation of inductor.

Applying KCL at node V,

$$-i + C\frac{dV}{dt} + \frac{V - i_2}{1} = 0$$

$$\frac{1}{4}\frac{dV}{dt} = -V + i + i_2$$

$$\frac{dV}{dt} = -4V + 4i + 4i_2 \qquad \ldots \text{(i)}$$

Applying KVL in loop-1,

$$-2i_1 + 2i + L\frac{di}{dt} + V = 0$$

$$\frac{1}{2}\frac{di}{dt} = -V - 2i + 2i_1$$

$$\frac{di}{dt} = -2V - 4i + 4i_1 \qquad \ldots \text{(ii)}$$

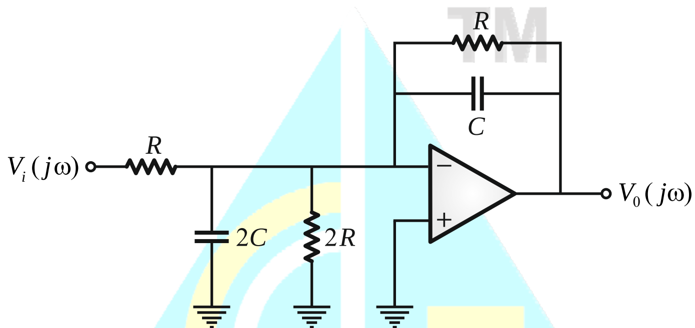
Writing equations (i) and (ii) in matrix form,

$$\frac{d}{dt}\begin{bmatrix} V \\ i \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} V \\ i \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Hence, the correct option is (B).

Question 32

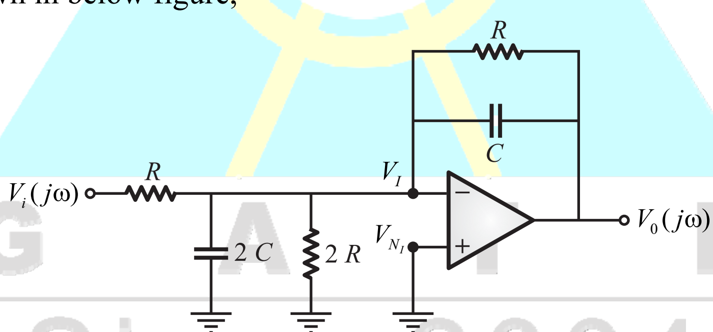
The components in the circuit given below are ideal. If $R = 2\,\text{k}\Omega$ and $C = 1\,\mu\text{F}$, the -3 dB cut-off frequency of the circuit in Hz is



(A) 79.58 (B) 14.92 (C) 59.68 (D) 34.46

Ans. **(A)**

Sol. Given circuit is shown in below figure,



This is an inverting integrator circuit.

$$V_{N_I} = 0$$

From virtual ground concept,

$$V_I = V_{N_I} = 0$$

Applying KCL at V_I,

$$\frac{V_I - 0}{2R} + \frac{V_I - 0}{\frac{1}{2sC}} + \frac{V_I - V_i}{R} + \frac{V_I - V_0}{\frac{1}{sC}} + \frac{V_I - V_0}{R} = 0$$

Putting $V_I = 0$,

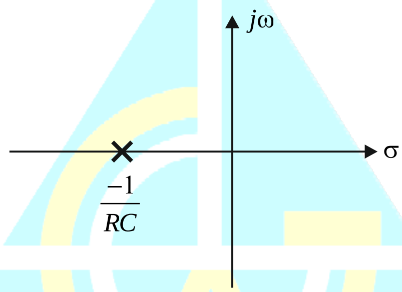
$$\frac{0-V_i}{R}+\frac{0-V_0}{\frac{1}{sC}}+\frac{0-V_0}{R}=0$$

$$\frac{-V_i}{R}=\frac{V_0(1+sRC)}{R}$$

$$V_0=\frac{-V_i}{(1+sRC)}$$

$$H(s)=\frac{-1}{1+sRC}$$

Pole at $s=\frac{-1}{RC}$,



Cutoff frequency is not necessarily $\frac{1}{RC}$ but if it equal to the magnitude of the location of pole, here pole satisfy the cutoff frequency,

$$\omega_H=\frac{1}{RC}$$

$$f_H=\frac{1}{2\pi RC}=\frac{1}{2\pi\times 2\times 10^3\times 10^{-6}}=79.58\text{ Hz}$$

Hence, the correct option is (A).

Question 33

The transfer function of a stable discrete - time LTI system is $H(z)=\frac{K(z-\alpha)}{(z+0.5)}$ where K and α are real numbers. The value of α (rounded off to one decimal place) with $|\alpha|>1$, for which magnitude response of the system is constant over all frequencies, is ________.

Ans. **(–2)**

Sol. Given transfer function for discrete time LTI system

$$H(z)=\frac{K(z-\alpha)}{(z+0.5)}$$

Where 'K' and 'α' are real numbers.

As it is asked to find value of 'α' such that the magnitude of response is constant over all frequencies, so we have to find value of 'α' for which the given LTI system will behave as an All Pass Filter.

Location of poles and zeros follows the reciprocal conjugate relationship as poles and zeros are symmetrical about the unit circle.

$$p = \frac{1}{z^*} \qquad \text{where } \begin{cases} P = \text{location of pole} \\ Z = \text{location of zero} \end{cases}$$

For given system

$$p = -0.5 \qquad z = \alpha$$

$$\therefore \quad -0.5 = \frac{1}{\alpha^*}$$

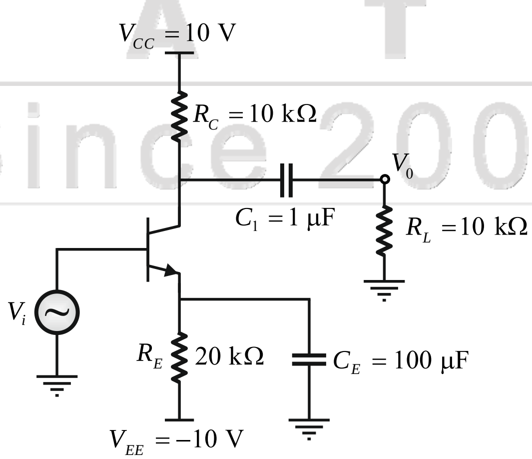
Given, 'α' is real, so, $\alpha^* = \alpha$

$$-0.5 = \frac{1}{\alpha} \qquad \therefore \ \alpha = -2$$

Hence, the value of α is **– 2**.

Question 34

For the BJT in the amplifier shown below, $V_{BE} = 0.7\,\text{V}$, $\frac{kT}{q} = 26\,\text{mV}$. Assume that the BJT output resistance (r_0) is very high and the base current is negligible. The capacitors are also assumed to be short circuited at signal frequencies. The input V_i is direct coupled. The low frequency voltage gain $\frac{V_0}{V_i}$ of the amplifier is

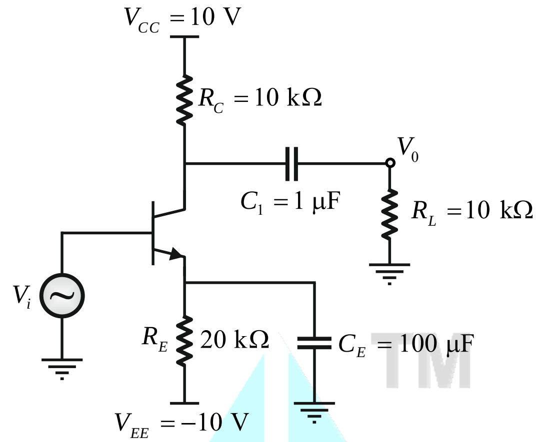


(A) −256.42 (B) −128.21

(C) −89.42 (D) −178.85

Ans. (C)

Sol. Given circuit is as shown below,

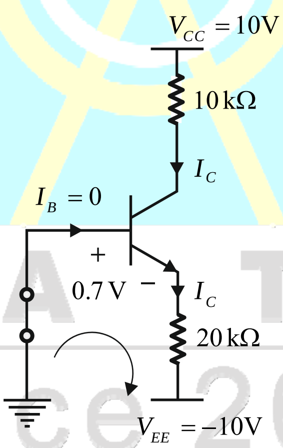


Also given, $V_{BE} = 0.7$ V and $\frac{kT}{q} = 26$ mV

r_0 is very high $\Rightarrow r_0 = \infty$ and base current is negligible $\Rightarrow I_B = 0$

D.C. Analysis :

For D.C., all capacitors will be open circuited and A.C. sources will be turned OFF. So, the circuit becomes as shown below

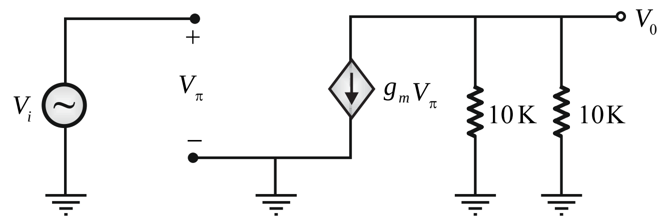


Applying KVL in the input loop,

$$0 - 0.7 - I_C \times 20\text{ k}\Omega + 10 = 0$$

$$I_C = \frac{10 - 0.7}{20\text{ k}\Omega} = 0.465\text{ mA}$$

Small signal equivalent circuit,



Voltage gain is given as,

$$A_V = \frac{V_{out}}{V_{in}} = -g_m R_L'$$

Where $R_L' = 10\ \text{k}\Omega \,||\, 10\ \text{k}\Omega = 5\ \text{k}\Omega$

$$A_V = -\left|\frac{I_C}{V_T}\right| \times R_L'$$

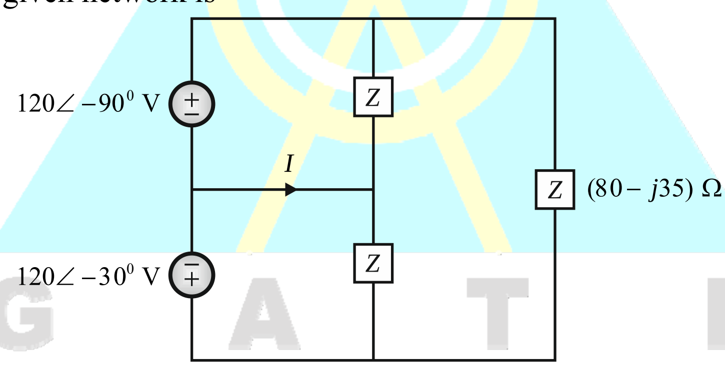
$$A_V = -\left|\frac{0.465}{26}\right| 5\ \text{k}\Omega$$

$$A_V = -89.42$$

Hence, the correct option is (C).

Question 35

The current I in the given network is



(A) $2.38\angle -96.37^0$ A.

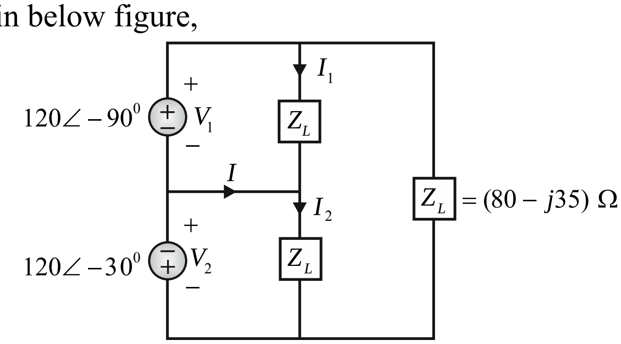
(B) 0 A.

(C) $2.38\angle -23.63^0$ A.

(D) $2.38\angle 143.63^0$ A.

Ans. **(D)**

Sol. Given circuit is shown in below figure,



$$Z_L = 80 - j35 = 87.32\angle -23.63^0$$

From figure, $-V_1 + I_1 Z_L = 0$

$$I_1 = \frac{V_1}{Z_L} = \frac{120\angle -90^0}{87.32\angle -23.63^0}$$

$$I_1 = 1.374\angle -66.37^0$$

Similarly, $-V_2 + I_2 Z_L = 0$

$$I_2 = \frac{V_2}{Z_L} = \frac{-120\angle -30^0}{87.32\angle -23.68^0}$$

$$= -1.374\angle -6.32^0$$

From circuit, $I + I_1 = I_2$

$$I = I_2 - I_1$$

$$I = (-1.374\angle -6.32^0) - (1.374\angle -66.37^0)$$

$$I = (-1.36 + 0.341j) - (0.55 - 1.258j)$$

$$I = -1.91 + 1.408j$$

$$I = 2.373\angle 143.60^0 \text{ A}$$

Hence, the correct option is (D).

Question 36

In a digital communication system, a symbol S randomly chosen from the set $\{s_1, s_2, s_3, s_4\}$ is transmitted. It is given that $s_1 = -3$, $s_2 = -1$, $s_3 = +1$ and $s_4 = +2$. The received symbol is $Y = S + W$. W is a zero mean unit - variance Gaussian random variable and is independent of S. P_i is the conditional probability of symbol error for the maximum likelihood (ML) decoding when the transmitted symbol $S = s_i$. The index i for which the conditional symbol error probability P_i is the highest is ______________.

Ans. **(3)**

Sol. **Given :** A symbol 'S' is chosen randomly from the set (S_1, S_2, S_3, S_4)

$$S_1 = -3$$

$$S_2 = -1$$

$$S_3 = +1$$

$$S_4 = +2$$

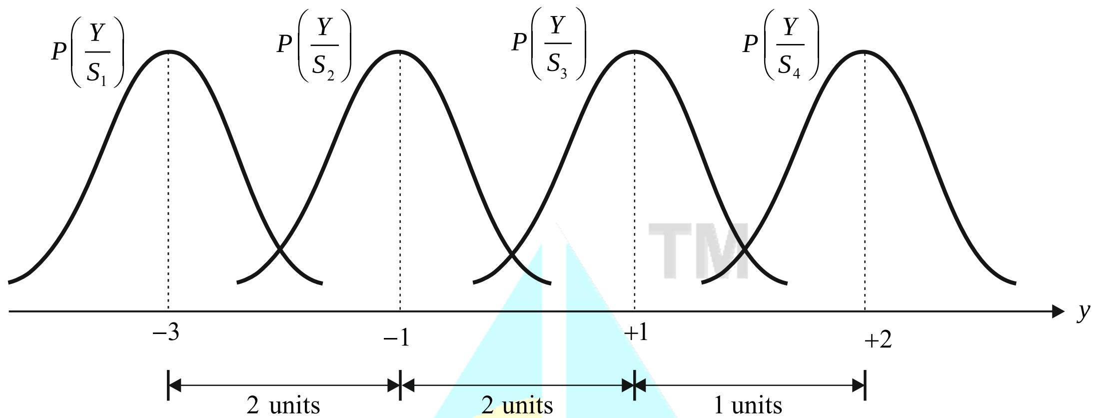
Received symbol is $Y = S + W$

Where

$W \rightarrow$ Zero mean unit variance Gaussian random variable and is independent of S.

$P_i \rightarrow$ Conditional probability of symbol error for the maximum likelihood decoding when the transmitted symbol $S = S_i$.

Conditional probability density function for the transmitted symbols.



Symbols S_1 and S_4 have only one decision boundary hence P_e will be less compared to S_2 and S_3. S_2 and S_3 are bounded by error due to 2 decision boundaries.

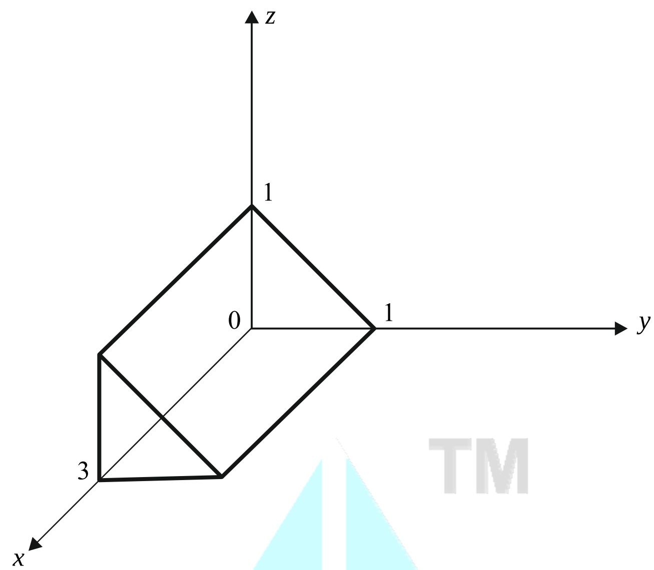
Since S_2 is comparatively distant from adjacent symbols S_1 and S_2 (2 units each respectively) it will have lower probability of error compared to S_3 which is 2 units and only 1 unit far from its adjacent symbols S_2 and S_4.

$\therefore$ S_3 has the highest probability of error.

Hence, index $i = 3$, to get highest conditional symbol error probability.

Question 37

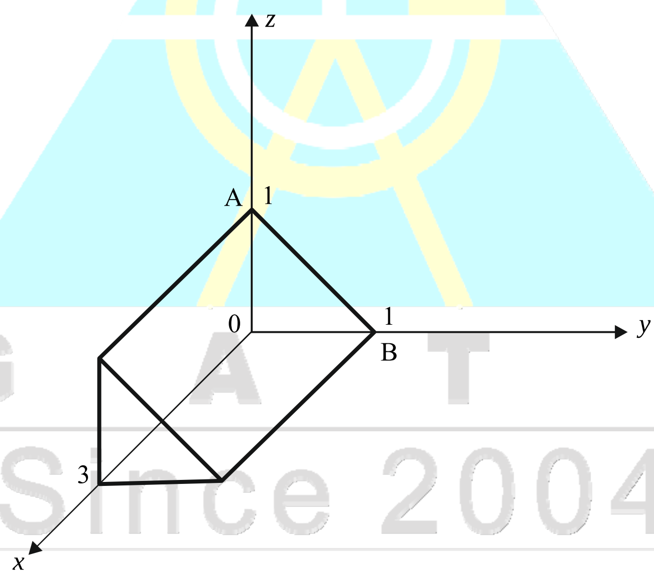
For the solid S shown below, the value of $\iiint_S x\,dx\,dy\,dz$ (rounded off to two decimal places) is ______.



Ans. **(2.25)**

Sol. **Method 1**

Given figure is shown below,



We can see that, x is varying from 0 to 3 and the yz plane consists of a triangle AOB.

Therefore, $$I = \iiint x dx dy dz = \int_{x=0}^{3} x\, dx \iint dy\, dz$$

$$I = \left(\int_{x=0}^{3} x\, dx \right) \times \text{(Area of triangle AOB)}$$

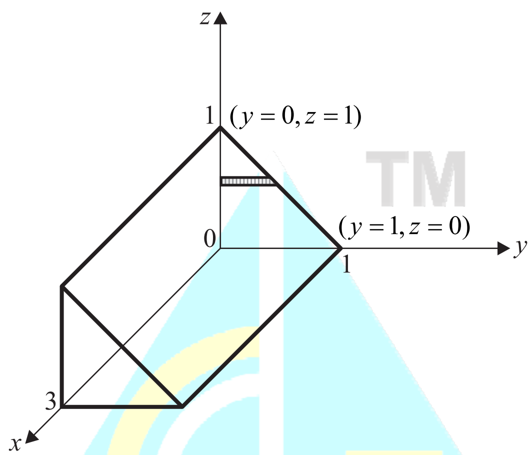
$$I = \left(\frac{x^2}{2} \right)_0^3 \times \frac{1}{2} \times 1 \times 1$$

$$I = \frac{9}{4} = 2.25$$

Hence, the value of $\iiint_S x\,dx\,dy\,dz$ is **2.25.**

Method 2

Given : The figure is shown below,



The equation of line in *yz* plane will be,

$$z - 1 = \frac{0-1}{1-0}(y-0)$$

$$z - 1 = -y$$

$$y = 1 - z$$

Therefore, $I = \iiint x dx dy dz$

We can see that, x is varying from 0 to 3 and the y is varying from 0 to (1-z) and z is varing from 0 to 1

$$x : 0 \text{ to } 3$$

$$y : 0 \text{ to } 1-z$$

$$z : 0 \text{ to } 1$$

$$I = \int_{z=0}^{1} \int_{y=0}^{1-z} \int_{x=0}^{3} x dx dy\, dz$$

$$I = \int_{z=0}^{1} \int_{y=0}^{1-z} \left[\frac{x^2}{2}\right]_0^3 dy dz = \frac{9}{2} \int_{z=0}^{1} \int_{y=0}^{1-z} dy dz$$

$$I = \frac{9}{2} \int_{z=0}^{1} [y]_0^{1-z}\, dz = \frac{9}{2} \int_{z=0}^{1} (1-z)\, dz$$

$$I = \frac{9}{2}\left[z - \frac{z^2}{2}\right]_{z=0}^{1} = \frac{9}{2}\left\{\left(1-\frac{1}{2}\right)-\left(0-\frac{0}{2}\right)\right\}$$

$$I = \frac{9}{2}\left\{\frac{1}{2}-0\right\} = \frac{9}{2}\times\frac{1}{2} = \frac{9}{4}$$

$$I = 2.25$$

Hence, the value of $\iiint_S x\,dx\,dy\,dz$ is **2.25.**

Question 38

The characteristic equation of a system is

$$s^3 + 3s^2 + (K+2)s + 3K = 0$$

In the root locus plot for the given system, as K varies from 0 to ∞, the break-away or break-in point(s) lie within

(A) $-\infty, -3$ (B) $-3, -2$ (C) $-1, 0$ (D) $-2, -1$

Ans. (C)

Sol. Given : Characteristic equation is,

$$\text{C.E} = s^3 + 3s^2 + (2+K)s + 3K = 0$$

Converting to standard form,

$$s^3 + 3s^2 + 2s + Ks + 3K = 0$$

$$s^3 + 3s^2 + 2s + K(s+3) = 0$$

$$1 + \frac{K(s+3)}{s^3 + 3s^2 + 2s} = 0$$

$$1 + \frac{K(s+3)}{s(s^2 + 3s + 2)} = 0$$

$$1 + \frac{K(s+3)}{s(s+1)(s+2)} = 0$$

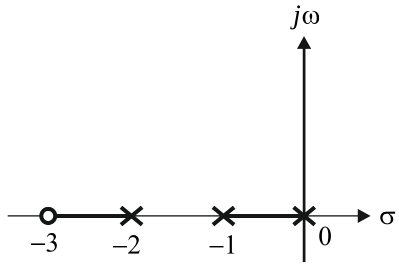
The above equation is of the standard form,

$$1 + G(s)H(s) = 0$$

Where, $G(s)H(s)$ is the open loop transfer function.

$$G(s)H(s) = \frac{K(s+3)}{s(s+1)(s+2)}$$

Drawing the pole-zero diagram,



The thick lines lies on Root Locus and thin lines does not lie on Root Locus. From the prediction of break-away or break-in point, if there are consecutively placed poles on real axis, with the section of real axis between them as a part of RL then there exists at least one break-away or break-in points between them. Since, there are poles located at –1 and 0, breakaway point will lie in between –1 and 0.

Hence, the correct option is (C).

Question 39

A system with transfer function $G(s)=\dfrac{1}{(s+1)(s+a)}$, $a>0$ is subjected to input $5\cos 3t$. The steady state output of the system is $\dfrac{1}{\sqrt{10}}\cos(3t-1.892)$. The value of a is__________.

Ans. (4)

Sol. Method 1

Given : $G(s)=\dfrac{1}{(s+1)(s+a)}$, $a>0$

Response $=\dfrac{1}{\sqrt{10}}\cos(3t-1.892^0)$

$$G(j\omega)=\frac{1}{(j\omega+1)(j\omega+a)}$$

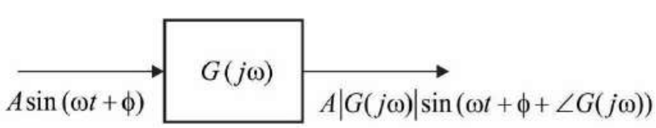
Input $=5\cos 3t$

$$\omega=3 \text{ rad/s}$$

$$G(j3)=\frac{1}{(j3+1)(j3+a)}$$

$$|G(j3)|=\frac{1}{\sqrt{(9+1)}\,\sqrt{(9+a^2)}}$$

Sinusoidal Response,



Comparing amplitudes,

$$5\times\frac{1}{\sqrt{3^2+1^2}\sqrt{9+a^2}}=\frac{1}{\sqrt{10}}$$

$$\frac{5}{\sqrt{9+a^2}}=1$$

$$25=9+a^2$$

$$a^2=16$$

$$a=\pm4 \qquad (\text{as } a>0)$$

So, $a=4$

Hence, the value of a is **4**.

Method 2

Angle criteria, $-\tan^{-1}\frac{\omega}{1}-\tan^{-1}\frac{\omega}{a}=-1.892 \qquad (\omega=3)$

$$-\tan^{-1}\frac{3}{a}=-0.643$$

$$\frac{3}{a}=\tan 0.643=0.74$$

$$a=4.004\approx 4$$

Question 40

The magnetic field of a uniform plane wave in vacuum is given by

$$\vec{H}(x,y,z,t)=(\hat{a}_x+2\hat{a}_y+b\hat{a}_z)\cos(\omega t+3x-y-z).$$

The value of b is_______________.

Ans. (1)

Sol. Given : Magnetic field of uniform plane wave is,

$$\vec{H}(x,y,z,t)=(\hat{a}_x+2\hat{a}_y+b\hat{a}_z)\cos(\omega t+3x-y-z)$$

$$\vec{H}(x,y,z,t)=(\hat{a}_x+2\hat{a}_y+b\hat{a}_z)\cos[\omega t-(-3x+y+z)]$$

Propagation vector is given by,

$$\vec{k}=-3\hat{a}_x+\hat{a}_y+\hat{a}_z$$

From Maxwell's equation,

$$\vec{k}\cdot\vec{H}=0$$

$$(-3\hat{a}_x+\hat{a}_y+\hat{a}_z)\cdot(\hat{a}_x+2\hat{a}_y+b\hat{a}_z)=0$$

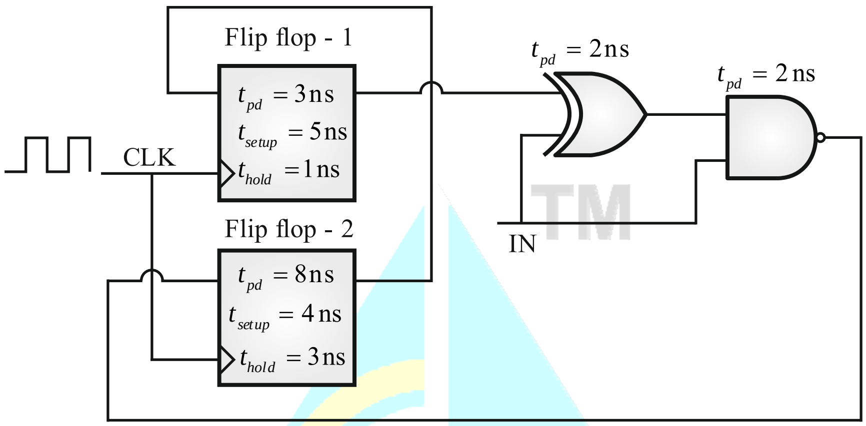
$$-3+2+b=0$$

$$b=1$$

Hence, the value of b is 1.

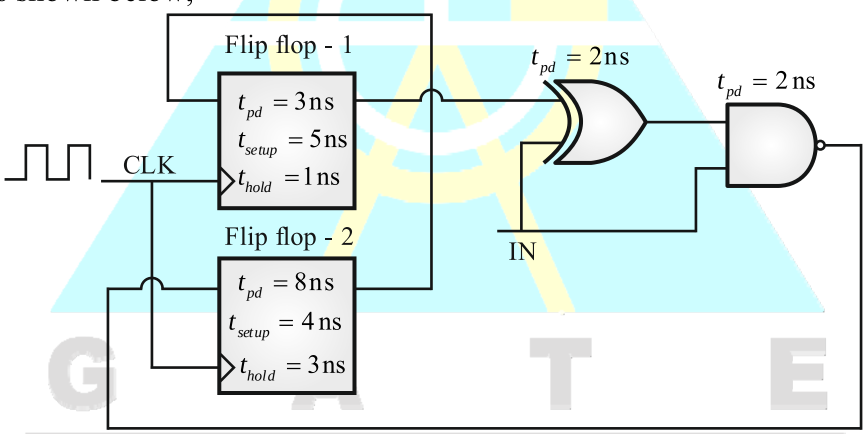
Question 41

For the components in the sequential circuit shown below, t_{pd} is the propagation delay, t_{setup} is the setup time and t_{hold} is the hold time. The maximum clock frequency (rounded off to the nearest integer), at which the given circuit can operate reliably, is ___________ MHz.



Ans. **(77)**

Sol. Given circuit is shown below,



Setup time (t_{su}) is the time required to set the input at proper level before the application of clock active edge. Hold time (t_h) is the minimum time required to maintain input at its level after the application of clock for proper output

Given for Flip flop-1,

$$t_{pd_1} = 3 \text{ ns}$$

$$t_{su_1} = 5 \text{ ns}$$

$$t_{h_1} = 1 \text{ ns}$$

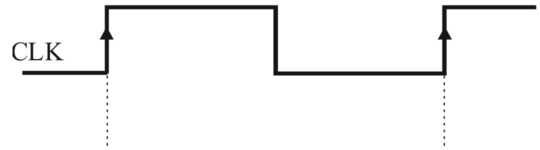
For flip flop-2,

$$t_{pd_2} = 8 \text{ ns}$$

$$t_{su_2} = 4 \text{ ns}$$

$t_{h_2} = 3$ ns

The operations that are required to perform in between two clock active edges decides the clock frequency for proper operation. It can be assumed to be divided between two stages for cross coupled flip flops.



The operations required are as follows :

If we start from flip flop-2 after $t_{pd_2} = 8$ ns of active edge of clock, output of flip flop-2 gets settled this output is feeded as input to flip flop-1, So it must be stable for at least $t_{su_1} = 5$ ns.

So the total time required during this process,

$$T_1 = t_{pd_2} + t_{su_1} = 8 + 5 = 13 \text{ ns}$$

Now starting from flip flop 1,

After $t_{pd_1} = 3$ ns of application of clock active edge, the output of flip flop-1 will get settled. So after 3 ns, the input is ready to be applied at the EXOR gate which has a propagation delay of 2 ns.

Hence after $3 + 2 = 5$ ns, the input is ready to be applied at the NAND gate which has $t_{pd} = 2$ ns.

So finally after $5 + 2 = 7$ ns of clock, input of flip flop-2 is ready but it must be stable for $t_{su_2} = 4$ ns. The total time required during this process.

$$T_2 = t_{pd_1} + (t_{pd})_{EXOR} + (t_{pd})_{NAND} + t_{su_2}$$

$$T_2 = 3 + 2 + 2 + 4 = 11 \text{ nsec}$$

For proper operation minimum difference time between active edges of clock, i.e.

$$(T_{clk})_{\min} = \text{Maximum of } (T_1 \text{ and } T_2) = 13 \text{ nsec}$$

$$(f_{clk})_{\max} = \frac{1}{13 \text{ ns}} = \frac{1000}{13} \text{ MHz} = 76.92 \text{ MHz}$$

Nearest integer = 77

Hence, the maximum clock frequency (rounded off to the nearest integer), at which the given circuit can operate reliably, is **77 MHz**.

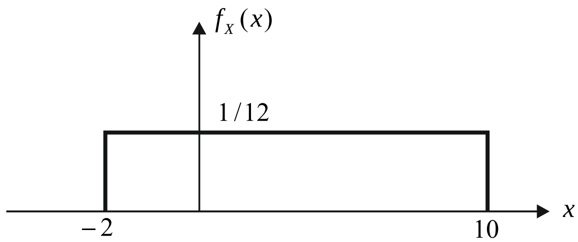
Question 42

X is a random variable with uniform probability density function in the interval $[-2, 10]$. For $Y = 2X - 6$, the conditional probability $P(Y \le 7 | X \ge 5)$ (rounded off to three decimal places) is _______.

Ans. **0.28 to 0.32 (0.3)**

Sol. **Given :** $Y = 2X - 6$

X is uniform probability density function in the interval $[-2,10]$. The probability function of X is shown below,



$$X = \frac{Y+6}{2}$$

$X_{\min}$ for Y less than 7 is 5 because X has to be greater or equal to 5.

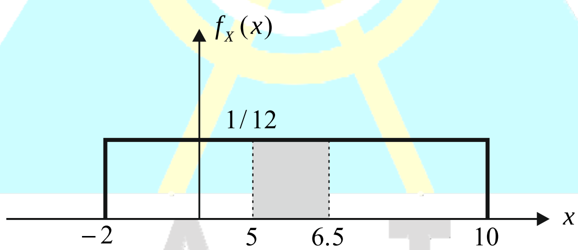
$$X_{\max} = \frac{Y_{\max}+6}{2} = \frac{7+6}{2} = 6.5$$

$$P(Y \le 7 \mid X \ge 5) = \frac{P(Y \le 7 \cap X \ge 5)}{P(X \ge 5)} = \frac{P(X \le 6.5 \cap X \ge 5)}{P(X \ge 5)} \quad \ldots \text{(i)}$$

$$P(Y \le 7 \mid X \ge 5) = \frac{P(5 \le X \le 6.5)}{P(X \ge 5)}$$

From property of PDF,

$$P[x_1 \le X \le x_2] = \int_{x_1}^{x_2} f_X(x)\,dx$$



$$P[5 \le X \le 6.5] = \int_5^{6.5} \frac{1}{12}\,dx = \frac{1.5}{12}$$

$$P(X \ge 5) = \int_5^{\infty} f_X(x)\,dx$$

$$P(X \ge 5) = \int_5^{10} \frac{1}{12}\,dx = \frac{5}{12}$$

From equation (i),

$$P(Y \le 7 \mid X \ge 5) = \frac{1.5/12}{5/12} = 0.3$$

Hence, the conditional probability is 0.3.

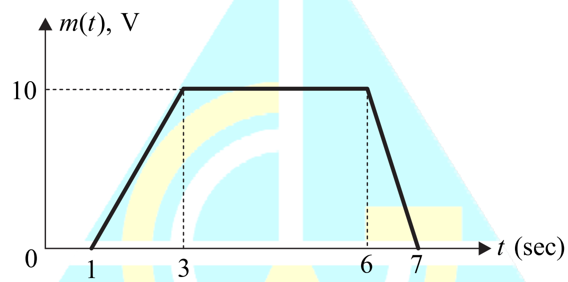
Question 43

$S_{PM}(t)$ and $S_{FM}(t)$ are defined below, are the phase modulated and the frequency modulated waveforms, respectively, corresponding to the message signal $m(t)$ shown in the figure.

$$S_{PM}(t) = \cos\left[1000\pi t + k_p\, m(t)\right]$$

$$S_{FM}(t) = \cos\left[1000\pi t + k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right]$$

Where k_p is the phase deviation constant in radians/volt and k_f is the frequency deviation constant in radians/second/volt. If the highest instantaneous frequencies of $S_{PM}(t)$ and $S_{FM}(t)$ are same, then the value of the ratio $\frac{k_p}{k_f}$ is ________seconds.



Ans. (2)

Sol. Given : Phase modulated waveform,

$$S_{PM}(t) = \cos[1000\pi t + k_p m(t)]$$

Frequency modulated waveform,

$$S_{FM}(t) = \cos\left[1000\pi t + k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right]$$

S_{PM} is phase modulated signal with instantaneous phase

$$\theta_i(t) = 1000\pi t + k_p m(t)$$

and $k_p =$ phase deviation constant (rad/volt)

$S_{FM}(t) =$ frequency modulated signal with instantaneous phase

$$\theta_i(t) = 1000\pi t + k_f \int_{-\infty}^{t} m(\tau)\, d\tau$$

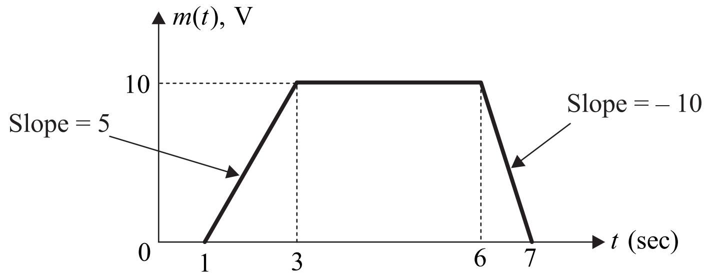
Where $k_f =$ frequency deviation constant

Given that, highest instantaneous frequency are same

$$(f_i)\big|_{PM_{\max}} = (f_i)\big|_{FM_{\max}}$$

From the relation,

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$



Instantaneous angular frequency of PM waveform,

$$(f_i)|_{PM} = \frac{1}{2\pi}\frac{d}{dt}[1000\,\pi t + k_p m(t)]$$

$$(f_i)|_{PM} = \frac{1}{2\pi}[1000\,\pi] + \frac{k_p}{2\pi}\left.\frac{dm(t)}{dt}\right|_{\max}$$

$$(f_i)|_{PM} = 500 + \frac{k_p}{2\pi}\left.\frac{dm(t)}{dt}\right|_{\max}$$

$$\left.\frac{dm(t)}{dt}\right|_{\max} = 5$$

$$f_i(t)|_{PM} = 500 + \frac{k_p}{2\pi}\times 5 \qquad \ldots \text{(i)}$$

Instantaneous angular frequency of FM waveform,

$$f_i(t)|_{FM} = \frac{1}{2\pi}\frac{d}{dt}\left[1000\,\pi t + k_f\int_{-\infty}^{t} m(\tau)\,d\tau\right]$$

$$= \frac{1}{2\pi}[1000\,\pi] + \frac{k_f}{2\pi}m(t)|_{\max}$$

$$m(t)|_{\max} = 10$$

$$f_i(t)|_{FM} = 500 + \frac{k_f}{2\pi}\times 10 \qquad \ldots \text{(ii)}$$

Equating equations (i) and (ii),

$$500 + \frac{k_p}{2\pi}\times 5 = 500 + \frac{k_f}{2\pi}\times 10$$

$$\frac{k_p}{k_f} = \frac{10}{5} = 2$$

Hence, the value of the ratio $\frac{k_p}{k_f}$ is **2**.

Question 44

A *pn* junction solar cell of area $1.0\,\text{cm}^2$, illuminated uniformly with 100 mWcm^{-2}; has the following parameter : Efficiency $=15\%$, open circuit voltage $=0.7\,\text{V}$, fill factor $=0.8$, and thickness $=200\,\mu\text{m}$. The charge of an electron is 1.6×10^{-19} C. The average optical generation rate $(\text{in}\,\text{cm}^{-3}\text{s}^{-1})$ is

(A) 0.84×10^{19} (B) 83.60×10^{19} (C) 1.04×10^{19} (D) 5.57×10^{19}

Ans. **(A)**

Sol. **Given :** Solar cell of area = $1.0\,\text{cm}^2$, Input power $P_{opt}=100\ \text{mWcm}^{-2}$,

Efficiency $\eta=15\%=0.15$, Open circuit voltage $V_{oc}=0.7\,\text{V}$, Fill factor $\text{FF}=0.8$,

and thickness $t=200\,\mu\text{m}$.

Change of electron, $q=1.6\times10^{-19}C$

Area $A=1.0\ \text{cm}^2$

We know that, $\eta=\frac{(P_{out})_{\max}}{P_{opt}}=\frac{FF\times V_{OC}\times I_{SC}}{P_{opt}}$

Also, given $P_{opt}=100\ \text{mW/cm}^2$

$$\therefore \qquad 0.15=\frac{0.8\times0.7\times I_{SC}}{100\times10^{-3}}$$

$$I_{SC}=26.78\ \text{mA}$$

$\therefore$ Optical generation rate G in $\text{cm}^{-3}\text{s}^{-1}$ is,

$$G=\frac{I_{SC}}{q\times\text{Volume}}=\frac{I_{SC}}{q\times A\times t}$$

$$G=\frac{26.78\times10^{-3}}{1.6\times10^{-19}\times1\times200\times10^{-4}}$$

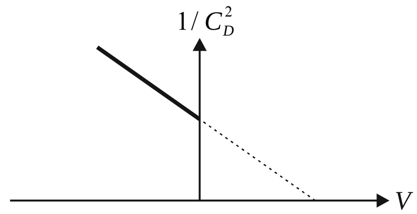
$$G=\frac{1.67\times10^{19}}{2}\text{cm}^{-3}\text{s}^{-1}$$

$$G=0.835\times10^{19}=0.84\times10^{19}\ \text{cm}^{-3}\text{s}^{-1}$$

Hence, the correct option is (A).

Question 45

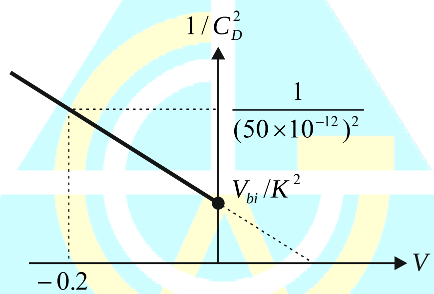
A one-sided abrupt *pn* junction diode has a depletion capacitance C_D of 50 pF at a reverse bias 0.2 V. The plot of $\frac{1}{C_D^2}$ versus the applied voltage *V* for this diode is a straight line as shown in the figure below. The slope of the plot is ___________ $\times 10^{20}\ \text{F}^{-2}\ \text{V}^{-1}$.



(A) − 1.2 (B) − 5.7 (C) − 0.4 (D) − 3.8

Ans. (B)

Sol. Given : Plot of $\frac{1}{C_D^2}$ versus reverse bias voltage is shown in figure.



For abrupt *pn* junction,

$$C_D \propto \frac{1}{\sqrt{V_j}}$$

Where, $V_j = V_R + V_{D_i}$ $\quad V_R \approx V$ (in question)

$$C_D = \frac{K}{\sqrt{V + V_{b_i}}}$$

$$\frac{1}{C_D^2} = \frac{1}{K^2}.V + \frac{1}{K^2}.V_{b_i}$$

$$\therefore \quad y \quad m\,.\,x + C \qquad \ldots(i)$$

$$\text{Slope} = \frac{1}{K^2}$$ (Coming positive as we suppose *V* to be increasing in negative direction)

It is clear from the given plot that $V_{bi} \neq 0$ as intersection with *y*-axis is at other than origin. So we cannot assume V_{bi} to be zero.

Intersection point $= \frac{1}{K^2} \cdot V_{bi}$

Actual slope $= -\frac{1}{K^2}$

Given at $V = -0.2$ V, $C_D = 50$ pF, $\frac{1}{C_D^2} = \frac{1}{2500 \times 10^{-24}}$

$$\frac{1}{2500 \times 10^{-24}} = \frac{-0.2}{K^2} + \frac{V_{bi}}{K^2}$$

$$\frac{1}{K^2}(V_{bi} - 0.2) = \frac{1 \times 10^{24}}{2500}$$

$$\frac{1}{K^2} = \frac{10000 \times 10^{20}}{2500(V_{bi} - 0.2)} \ \text{F}^{-1}\text{V}^{-1}$$

$$\frac{1}{K^2} = \frac{4}{(V_{bi} - 0.2)} \times 10^{20} \ \text{F}^{-2}\text{V}^{-1}$$

$$\left(\frac{1}{K^2}\right)_{\min} = \frac{4}{\left(V_{bi_{\max}} - 0.2\right)} \times 10^{20}$$

As $\left|V_{b_i} - 0.2\right|$ is always less than 1, so $\left(\frac{4}{V_{bi} - 0.2}\right)$ will always greater than 4

$\therefore$ $$|\text{Slope}| = \left|\frac{1}{K^2}\right| > 4 \times 10^{20}$$

The only option that satisfies the value of slope is – 5.7.

Hence, option (B) can be selected.

IIT has declared it as MTA.

In the first answer key, IIT has given its answer to be option (B), which is correct but is based on assumptions, resulting in making this question to be a wrong options eliminator type question.

The value of slope is never going to be exactly – 5.7 according to the available data but it must be predicted that the magnitude of slope will be greater than four.

So, for the question to be valid, there must be statements given as the options to identify true or false possible value of slope, but not the exact value is possible to predict from the given data.

Hence, IIT considered this question as **MARKS TO ALL** in their final answer key.

Question 46

P, Q and R are the decimal integers corresponding to the 4-bit binary number 1100 considered in signed magnitude, 1's complement and 2's complement representation, respectively. The 6-bit 2's complement representation of $(P + Q + R)$ is

(A) 111101 (B) 110010 (C) 111001 (D) 110101

Ans. **(D)**

Sol. **Given :** P, Q and R are the decimal integers corresponding to a 4-bit binary number 1100 in signed magnitude, 1's complement and 2's complement representation.

If a number in signed magnitude form is

$$(P)_{10} = 1100$$

Then MSB = 1 signifies that number is negative and other bit represents the binary of magnitude of that number

$$(P)_{10} = -(100)_2 = -4$$

If a number in 1's complement representation is

$$(Q)_{10} = 1100$$

Then MSB = 1 signifies no. is negative. To represent only negative number in 1's complement we write the positive number first and then take its 1's complement as to represent -3 in 4-bits

$$+3 = 0011$$
$$-3 = 1100$$

Hence, to find binary equivalent of any number represented in 1's complement with MSB=1.
Take 1's complement of bits other than MSB and place negative sign.

$$(Q)_{10} = 1100$$

$1 \rightarrow$ negative

$100 \rightarrow$ 1's complement of actual binary

$$(Q)_{10} = -(011)_2 = -3$$

Similarly if a number with MSB = 1 is represented in 2's complement, then take the 2's complement of bits other than MSB and place negative sign.

$$(R)_{10} = 1100$$

$1 \rightarrow$ negative

$100 \rightarrow$ 2's complement of actual number

$$(R)_{10} = -(100)_2 = -4$$

$$P+Q+R = -4-3-4 = -11$$

To represent – 11 in 2's complement in 6 bits, writing +11 first

$$+11 = 001011$$

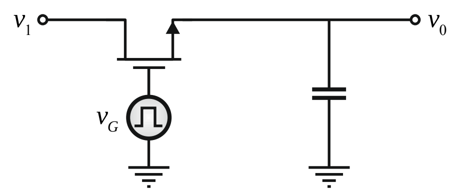
Now to represent – 11, taking 2's complement of +11

$$-11 = 110101$$

Hence, the correct option is (D).

Question 47

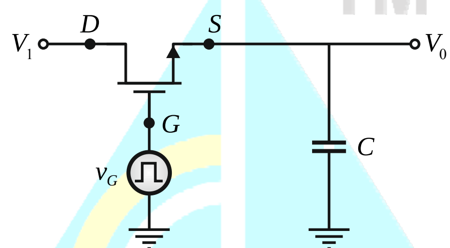
An enhancement MOSFET of threshold voltage 3 V is being used in the sample and hold circuit given below. Assume that the substrate of the MOS device is connected to -10V. If the input voltage v_1 lies between $\pm 10\text{V}$, the minimum and the maximum value of v_G required for proper sampling and holding respectively, are



(A) 10 V and – 10V
(B) 13 V and – 7V
(C) 10 V and – 13V
(D) 3 V and – 3V

Ans. (B)

Sol. The given circuit is shown in below figure,

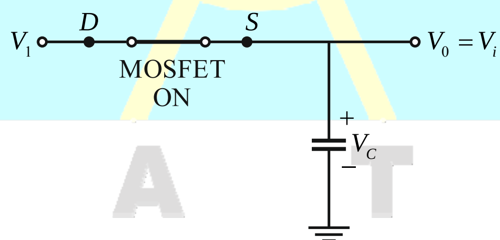


For sampling the input signal, the MOSFET must be ON.

During sampling :

$$V_G - V_1 > V_{th} \qquad \text{(MOSFET = ON)}$$

The equivalent circuit will be,



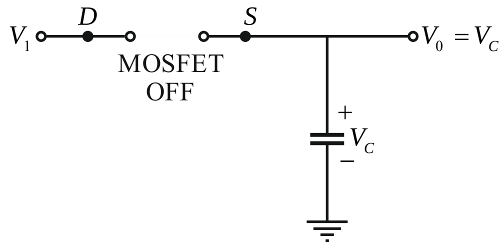
Therefore,
$$V_G - V_{i\max} > V_{th}$$
$$V_G - 10 > 3$$
$$V_G > 10 + 3$$
$$V_G > 13\text{V}$$

So, minimum value gate voltage must be 13 Volt.

For Holding :

$$V_G - V_1 < V_{th}$$

The equivalent circuit will be,



Therefore, $V_G - V_1 < V_{th}$

$V_G - V_{i\min} < V_{th}$

$V_G - (-10) < 3$

$V_G < 3 - 10$

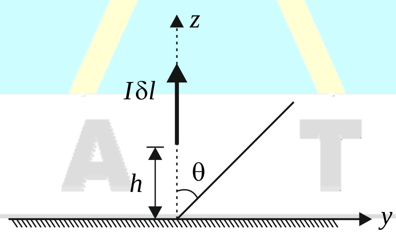
$V_G < -7 \text{ V}$

So, maximum value is -7 V.

Hence, the correct option is (B).

Question 48

For an infinitesimally small dipole in free space, the electric field E_θ in the far field is proportional to $\frac{e^{-jkr}}{r}\sin\theta$, where $k = \frac{2\pi}{\lambda}$. A vertical infinitesimally small electric dipole $(\delta l \ll \lambda)$ is placed at a distance $h(h > 0)$ above an infinite ideal conducting plane, as shown in the figure. The minimum value of *h*, for which one of the maxima in the far field radiation pattern occurs at $\theta = 60^0$, is

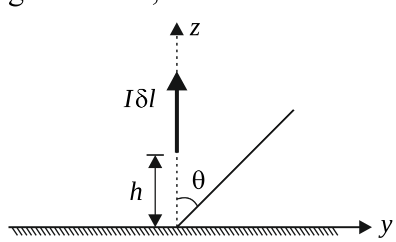


Infinite conducting plane

(A) 0.75λ (B) 0.25λ (C) 0.5λ (D) λ

Ans. **(D)**

Sol. Given arrangement is shown in figure below,



Infinite conducting plane

Method 1

From the figure,

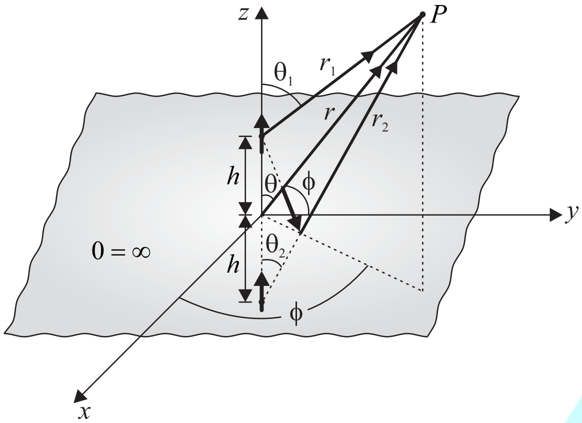


Fig. Vertical electric dipole above ground plane

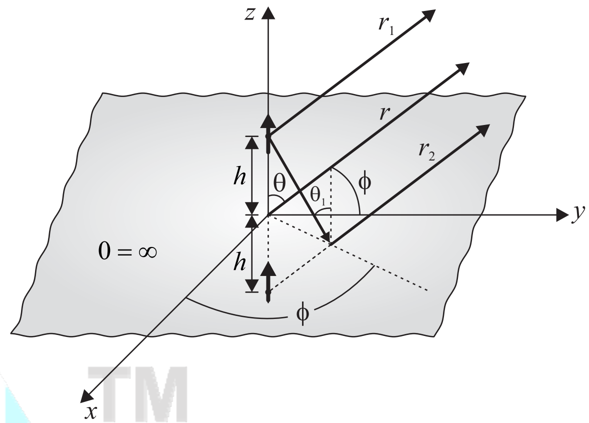


Fig. Far field observations

The far zone component of the electric field of the infinitesimal dipole of length δl, constant current I_0, is given by,

$$E_1 = \frac{\eta I_0 \delta l \sin\theta_1 e^{-j\beta r_1}}{4\pi r_1} j\beta \qquad \ldots(i)$$

The electric field of the image dipole is given by,

$$E_2 = \frac{\eta I_0 \delta l \sin\theta_2 e^{-j\beta r_2}}{4\pi r_2} j\beta \qquad \ldots(ii)$$

The total field above the interface $(z \geq 0)$ is equal to the sum of the equation (i) and (ii). Since a field cannot exist inside a perfect electric conductor, it is equal to zero below the interface.

For far-field observation $(r >> h)$, $r_1 \approx r - h\cos\theta$, $r_2 \approx r + h\cos\theta$

For far-field observation it geometrically represents parallel lines. Since amplitude variations are not as critical we can assume,

$$r_1 \approx r_2 \approx r \qquad \text{(for amplitude variation)}$$

$$\theta_1 \approx \theta_2 \approx \theta$$

Net electric field,

$$E_{total} = E_1 + E_2$$

$$E_{total} = \frac{I_0 \delta l \sin\theta e^{-j\beta(r - h\cos\theta)}}{4\pi r} j\beta + \frac{\eta I_0 \delta l \sin\theta e^{-j\beta(r + h\cos\theta)}}{4\pi r} j\beta$$

$$E_{total} = \frac{\eta I_0 \delta l \sin\theta e^{-j\beta r}}{4\pi r} j\beta[e^{j\beta h\cos\theta} + e^{-j\beta h\cos\theta}]$$

$$E_{total} = \frac{\eta I_0 \delta l \sin\theta e^{-j\beta r}}{4\pi r} j\beta 2\cos(\beta h\cos\theta)$$

It can be seen that total electric field is equal to the product of the field of a single source positioned symmetrically about origin and a factor known as array factor.

The maxima in far field radiation pattern occur $\theta = 60^0$ if

$$|\cos(\beta h\cos 60^0)| = 1$$

$$\beta h\cos 60^0 = \pi$$

$$\frac{2\pi}{\lambda} \times h \times \frac{1}{2} = \pi$$

$$h = \lambda$$

Hence, the correct option is (D).

Method 2

$$\frac{dE_{total}}{dh} = 0 \qquad \text{(for maxima)}$$

$$\frac{-\eta I_0 \delta l \sin\theta e^{-j\beta r}}{4\pi r} j\beta 2\sin(\beta h\cos\theta)\beta\cos\theta = 0$$

$$\sin(\beta h\cos\theta) = 0$$

$$\beta h\cos\theta = n\pi \qquad (n = 1, 2, 3 \ldots\ldots)$$

For $n = 1$ and $\theta = 60^0$,

$$\beta h\cos 60^0 = \pi$$

$$\frac{2\pi}{\lambda} \times h \times \frac{1}{2} = \pi$$

$$h = \lambda$$

Hence, the correct option is (D).

Note :

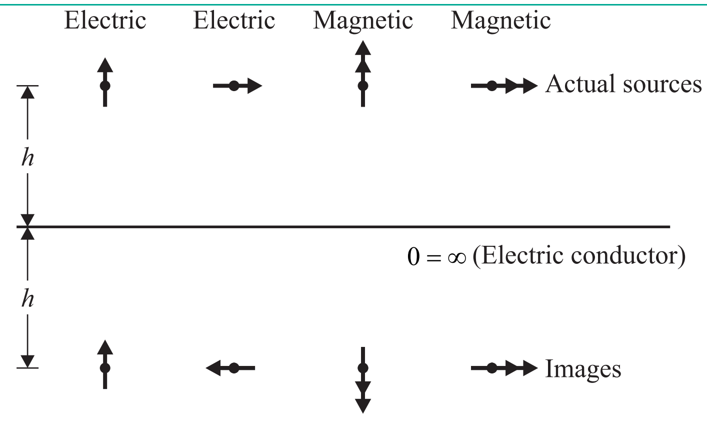


Fig. (a) Electric conductor

Question 49

Which one of the following options contains two solutions of the differential equation $\frac{dy}{dx} = (y-1)x$?

(A) $\ln|y-1| = 0.5x^2 + c$ and $y = -1$
(B) $\ln|y-1| = 2x^2 + c$ and $y = -1$
(C) $\ln|y-1| = 0.5x^2 + c$ and $y = 1$
(D) $\ln|y-1| = 2x^2 + c$ and $y = 1$

Ans. **(C)**

Sol. **Given :** $\frac{dy}{dx} = (y-1)x$

When, $y \neq 1$

By variable separate method,

$$\frac{dy}{y-1} = x\,dx$$

Integrating both side,

$$\int \frac{dy}{y-1} = \int x\,dx$$

$$\ln|y-1| = \frac{x^2}{2} + C$$

$$\ln|y-1| = 0.5x^2 + C \qquad \ldots(i)$$

When $y = 1$, $\frac{dy}{dx} = 0$

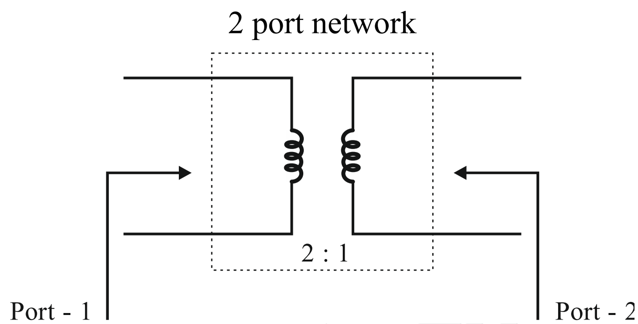
On integrating, $y = C$

This solution is valid for $y = 1$ and as we have first solution in options (A) or (C), and $y = 1$ is in option (C).

Hence, the correct option is (C).

Question 50

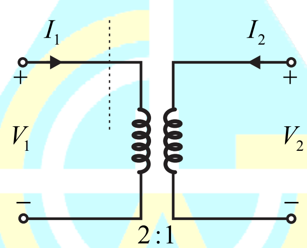
For a 2-port network consisting of an ideal lossless transformer, the parameter S_{21} (rounded off to two decimal places) for a reference impedance of $10\ \Omega$ is __________.



Ans. (0.8)

Sol. **Method 1**

Given : $Z_0 = 10\Omega$



As the given transformer is ideal,

$$\frac{V_2}{V_1} = \frac{1}{2}$$

$$V_1 = 2V_2 \qquad \ldots\text{(i)}$$

$$\frac{I_1}{I_2} = -\frac{1}{2}$$

$$I_1 = -0.5 I_2 \qquad \ldots\text{(ii)}$$

$$[T] = \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix}$$

Scattering parameter S_{21} in terms of $ABCD$ is given by,

$$S_{21} = \frac{2}{A + \frac{B}{Z_0} + \frac{C}{Z_0} + D} = \frac{2}{2+0+0+0.5} = \frac{2}{2.5} = 0.8$$

Hence, the parameter S_{21} is **0.8**.

Remember :

1. $ABCD$ parameter of an ideal transformer is given by,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

2. Relation between scattering parameter and transmission parameter is given by,

$$S_{11} = \frac{A + \frac{B}{Z_0} - CZ_0 - D}{A + \frac{B}{Z_0} + CZ_0 + D}$$

$$S_{12} = \frac{2(AD - BC)}{A + \frac{B}{Z_0} + CZ_0 + D}$$

$$S_{21} = \frac{2}{A + \frac{B}{Z_0} + CZ_0 + D}$$

$$S_{22} = \frac{-A + \frac{B}{Z_0} - CZ_0 + D}{A + \frac{B}{Z_0} + CZ_0 + D}$$

Method 2

For ideal transformer of $n:1$ the scattering matrix is,

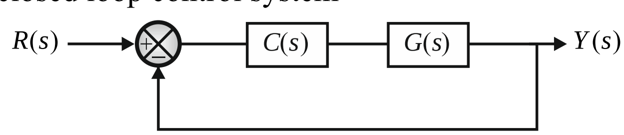
$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \frac{n^2-1}{n^2+1} & \frac{2n}{n^2+1} \\ \frac{2n}{n^2+1} & \frac{1-n^2}{1+n^2} \end{bmatrix}$$

$$S_{21} = \frac{2n}{n^2+1} = \frac{2 \times 2}{2^2+1} = \frac{4}{5} = 0.8$$

Hence, the parameter S_{21} is **0.8**.

Question 51

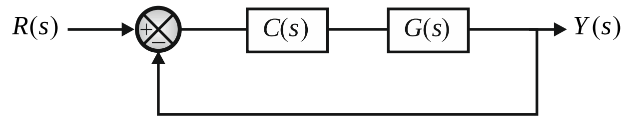
Consider the following closed loop control system



where $G(s) = \frac{1}{s(s+1)}$ and $C(s) = K\frac{s+1}{s+3}$. If the steady state error for a unit ramp input is 0.1, then the value of K is _____.

Ans. (30)

Sol. **Given :** Consider the following closed loop control system,



Where, $G(s) = \frac{1}{s(s+1)}$

$$C(s) = \frac{K(s+1)}{(s+3)}$$

Open loop transfer function is given by,

$$\text{OLTF} = G(s)C(s)$$

$$G'(s) = \frac{1}{s(s+1)}\,\frac{K(s+1)}{(s+3)} = \frac{K}{s(s+3)}$$

Given : For ramp input, $e_{ss} = 0.1$

e_{ss} for unit ramp input $= \frac{1}{K_v}$

Where, $K_v =$ Velocity error constant

$$K_v = \lim_{s \to 0} s[\text{OLTF}]$$

$$K_v = \lim_{s \to 0} s\,G'(s) = \lim_{s \to 0} s\frac{K}{s(s+3)} = \frac{K}{3}$$

Steady state error is given by,

$$e_{ss} = \frac{1}{K_v}$$

$$e_{ss} = \frac{1}{K/3} = \frac{3}{K}$$

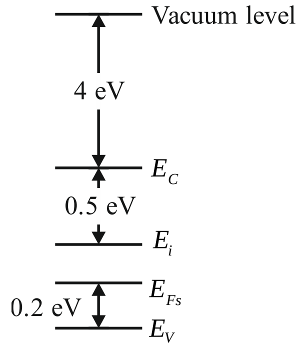
So, $\frac{3}{K} = 0.1$

$$K = 30$$

Hence, the value of K is **30**.

Question 52

The band diagram of *p*-type semiconductor with a bandgap of the 1 eV is shown. Using this semiconductor, a MOS capacitor having V_{TH} of -0.16V, C'_{ox} of 100 nF/cm^2 and metal work function of $3.87\,\text{eV}$ is fabricated. There is no charge within the oxide. If the voltage across the capacitor is V_{TH}, the magnitude of depletion charge per unit area (in C/cm^2) is

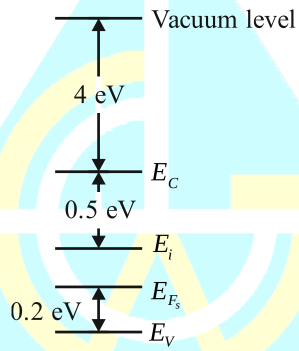


(A) 1.41×10^{-8} (B) 0.52×10^{-8} (C) 0.93×10^{-8} (D) 1.70×10^{-8}

Ans. (D)

Sol. **Given :** $V_{th} = -0.16\text{ V}$, $\phi_m = 3.879\text{ eV}$, $C_{0x} = 100\ \mu\text{F/cm}^2$ and $E_C - E_V = 1\text{ eV}$.

The band diagram is shown in below figure,



The threshold voltage is given by,

$$V_{th} = \phi_{ms} + 2\phi_{fp} - \frac{Q_d}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$$

Given that, oxide charges is zero, therefore

$$V_{th} = \phi_{ms} + 2\phi_{fp} - \frac{Q_d}{C_{ox}}$$

$$-0.16 = 3.879 - (4 + 0.5 + 0.3) + 2(E_i - E_f) - \frac{Q_d}{C_{ox}}$$

$$\frac{Q_d}{C_{ox}} = 3.879 + 0.16 + 0.6 - 4.8 = -0.161$$

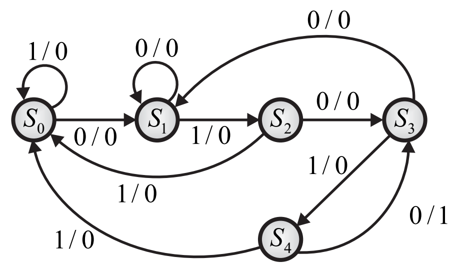
$$Q_d = -0.161\times100\times10^{-9}\ \text{C/cm}^2 = -16.1\times10^{-9}\ \text{C/cm}^2$$

$$Q_d = 1.61\times10^{-8}\ \text{C/cm}^2$$

Hence, the correct option is (D).

Question 53

The state diagram of a sequence detector is shown below. State S_0 is the initial state of the sequence detector. If the output is 1, then



(A) the sequence 01110 is detected.

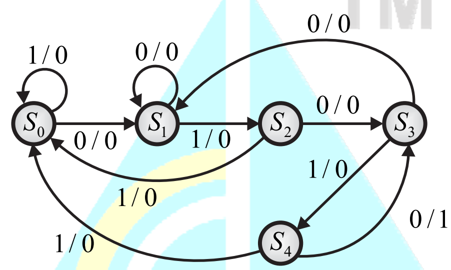
(B) the sequence 01001 is detected.

(C) the sequence 01011 is detected.

(D) the sequence 01010 is detected.

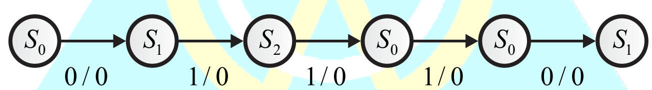
Ans. **(D)**

Sol. Given state diagram is shown in below figure,



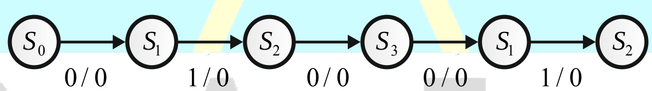
Checking from options,

(A) For 01110,



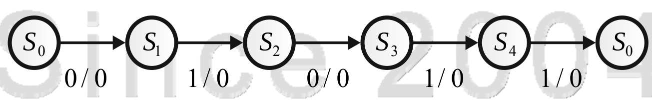
Hence final output = 0

(B) For 01001,



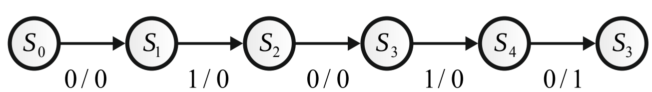
Final output = 0

(C) For 01011,



Final output = 0

(D) For 01010,



Final output = 1

Hence, the correct option is (D).

Question 54

A finite duration discrete-time signal $x[n]$ is obtained by sampling a continuous - time signal $x(t) = \cos(200\pi t)$ at sampling instants $t = \frac{n}{400}$, $n = 0, 1, ..., 7$. The 8-point discrete Fourier transform (DFT) is defined as

$$X[k] = \sum_{n=0}^{7} x[n] e^{-j\pi nk/4} \text{ for } k = 0, 1, ..., 7.$$

Which one of the following statements is TRUE?

(A) Only $X(3)$ and $X(5)$ are non-zero.

(B) Only $X(4)$ is non-zero.

(C) All $X(K)$ are non-zero.

(D) Only $X(2)$ and $X(6)$ are non-zero.

Ans. (D)

Sol. Given : $x(t) = \cos 200\pi t$ is sampled at instants $t = \frac{n}{400} \approx nT_S$

$$x(t) \xrightarrow{t \to nT_S} x[nT_S] \approx x[n]$$

$$x[n] = x(t)\Big|_{t=\frac{n}{400}} = \cos 200\pi \cdot \frac{n}{400}$$

$$x[n] = \cos n\frac{\pi}{2}$$

To calculate 8-point DFT, taking 8-point sequence $x[n]$ for $n = 0, 1, 2, \ldots 7$

$$x[n] = \{1, 0, -1, 0, 1, 0, -1, 0\}$$

If $x[n] \xleftrightarrow[DFT]{N-\text{point}} X(K)$

then by definition,

$$X(K) = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi kn}{N}} = \sum_{n=0}^{N-1} x[n] e^{\frac{-j\pi kn}{4}} \qquad (\text{for } N = 8)$$

$$X\left(\frac{N}{2}\right) = \sum_{n=0}^{N-1} x[n] \cdot e^{\frac{-j2\pi}{8} \cdot 4 \times n}$$

$$X(4) = \sum_{n=0}^{N-1} x[n](-1)^n = x(0) - x(1) + x(2) - x(3) + x(4) - x(5) + x(6) - x(7)$$

$$X(4) = 1 - 0 + (-1) - 0 + 1 - 0 + (-1) - 0 = 0$$

So, options (B) and (C) are eliminated.

So check correct option, we just have to find any of $X(2), X(6)$ or $X(3)$ and $X(5)$.

From the definition of 8-point DFT.

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5)\\ X(6)\\ X(7)\end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7\\ 1 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14}\\ 1 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21}\\ 1 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28}\\ 1 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35}\\ 1 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42}\\ 1 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49}\end{bmatrix} \begin{bmatrix} x(0)\\ x(1)\\ x(2)\\ x(3)\\ x(4)\\ x(5)\\ x(6)\\ x(7)\end{bmatrix}$$

$$X(2) = 1\cdot x(0) + W_8^2 x(1) + W_8^4 \cdot x(2) + W_8^6 \cdot x(3) + W_8^8 x(4) + W_8^{10} x(5) + W_8^{12} x(6) + W_8^{14} x(7)$$

From periodic property of $W_N^{K+N} = W_N^K$

$$X(2) = 1\cdot x(0) + W_8^2 x(1) + W_8^4 x(2) + W_8^6 x(3) + W_8^0 x(4) + W_8^2 x(5) + W_8^4 x(6) + W_8^6 x(7)$$

$$= 1\cdot 1 + 0 + W_8^4(-1) + 0 + W_8^0(1) + 0 + W_8^4\cdot(-1) + 0 = 2 - 2W_8^4$$

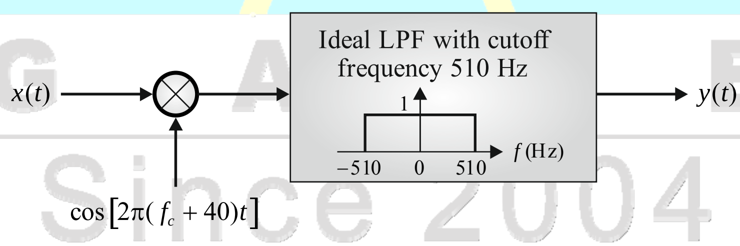
$$W_8^4 = e^{\frac{-j2\pi}{8}\times 4} = e^{-j\pi} = -1$$

$\therefore$ $X(2) = 2 - 2(-1) = 4 \neq 0$

Hence, the correct option is (D).

Question 55

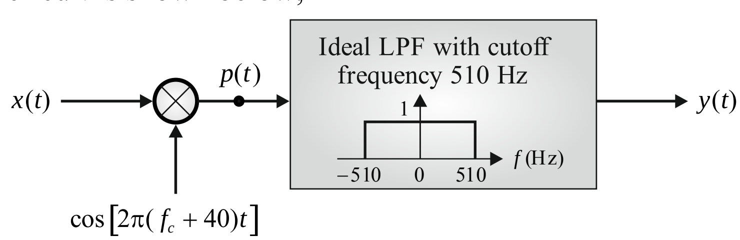
For the modulated signal $x(t) = m(t)\cos(2\pi f_c t)$, the message signal $m(t) = 4\cos(1000\pi t)$ and the carrier frequency f_c is 1 MHz. The signal $x(t)$ is passed through a demodulator, as shown in figure below. The output $y(t)$ of the demodulator is



(A) $\cos(540\pi t)$ (B) $\cos(1000\pi t)$ (C) $\cos(920\pi t)$ (D) $\cos(460\pi t)$

Ans. **(C)**

Sol. Given demodulator circuit is shown below,



Ideal LPF has cut off frequency 510 Hz

Given $x(t) = m(t)\cos(2\pi f_c t)$ where

$$m(t) = 4\cos(1000\pi t) \text{ and } f_c = 1 \text{ MHz}$$

From the figure,

$$p(t) = x(t)\cos(2\pi(f_c + 40)t)$$

$$p(t) = m(t)\cos(2\pi f_c t)\cos(2\pi f_c t + 80\pi t)$$

$$p(t) = \frac{m(t)}{2}\left[\cos(4\pi f_c t + 80\pi t) + \cos(80\pi t)\right]$$

$$p(t) = 2\cos(1000\pi t)\left[\cos(4\pi f_c t + 80\pi t) + \cos(80\pi t)\right]$$

$$p(t) = \underbrace{\cos(4\pi f_c t + 1080\pi t)}_{x_1} + \underbrace{\cos(4\pi f_c t - 920\pi t)}_{x_2} + \underbrace{\cos(1080\pi t)}_{\substack{f=540\text{ Hz} \\ x_3}} + \underbrace{\cos(920\pi t)}_{\substack{f=460\text{ Hz} \\ x_4}}$$

As LPF has cut-off frequency of 510 Hz or 1020π rad/s, so components x_1, x_2 and x_3 will be attenuated by LPF.

$$y(t) = \cos(920\pi t)$$

Hence, the correct option is (C).

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