General Aptitude

Q.1 to Q.5 Carry one mark each

Question 1

This book, including all its chapters, _____ interesting. The students as well as instructor ______ in agreement about it.

(A) is, was (B) is, are (C) were, was (D) are, are

Ans. **(B)**

Sol. 'Book' is a singular noun, so it will take singular verb 'is'

When 'as well as' is part of the sentence then the verb must agree with the noun before 'as well as'.

So "the students as well as" will take the verb 'are'

Hence the correct option is (B).

Question 2

In four-digit integer numbers from 1001 to 9999, the digit group "37" (in the same sequence) appears ________ times.

(A) 270 (B) 299 (C) 279 (D) 280

Ans. **(D)**

Sol. Number of ways by which 37 can appear in numbers 1000 to 9999 are shown below,

$$\underset{(1 \text{ to } 9)}{\overset{9 \text{ ways}}{\underline{\quad}}}\ \underset{(0 \text{ to } 9)}{\overset{10 \text{ ways}}{\underline{\quad}}}\ \underline{3}\ \underline{7} \Rightarrow 9\times 10 = 90 \quad \text{...(i)}$$

$$\underset{(1 \text{ to } 9)}{\overset{9 \text{ ways}}{\underline{\quad}}}\ \underline{3}\ \underline{7}\ \underset{(0 \text{ to } 9)}{\overset{10 \text{ ways}}{\underline{\quad}}} \Rightarrow 9\times 10 = 90 \quad \text{...(ii)}$$

$$\underline{3}\ \underline{7}\ \underset{(0 \text{ to } 9)}{\overset{10 \text{ ways}}{\underline{\quad}}}\ \underset{(0 \text{ to } 9)}{\overset{10 \text{ ways}}{\underline{\quad}}} \Rightarrow 10\times 10 = 100 \quad \text{...(iii)}$$

There is no common numbers in case (i) - case (ii) or in case (ii) - case (iii) but there exist a common number between case (iii) and case (i), that is 3737.

So, if the total number of numbers is asked in which 37 appears in this sequence, then the answer would be,

$$90+90+100-1=279$$

Which is as given in IIT answer key but the required value is asked only for repetition of 37, not the numbers which contains 37, so in the number "3737", 37 must be counted 2 times, then the correct answer would be,

$$90+90+100=280$$

IIT has changed their answer for this question from option (C) to option (D) in the final answer key, but they do not considered it as MTA.

So, students must read the statement given in the question, before going to select the correct option in such type of problems.

Question 3

Stock markets ______at the news of the coup.

(A) plugged (B) plunged (C) poised (D) probed

Ans. **(B)**

Sol. The word 'Coup' means a sudden, illegal and often violent change of government and this situation will lead to sudden 'jump'; drop or fall of shares for which the appropriate word is 'plunged'

Hence, the correct option is (B).

Question 4

People were prohibited _______ their vehicles near the entrance of the main administrative building.

(A) to park (B) to have parked (C) from parking (D) parking

Ans. **(C)**

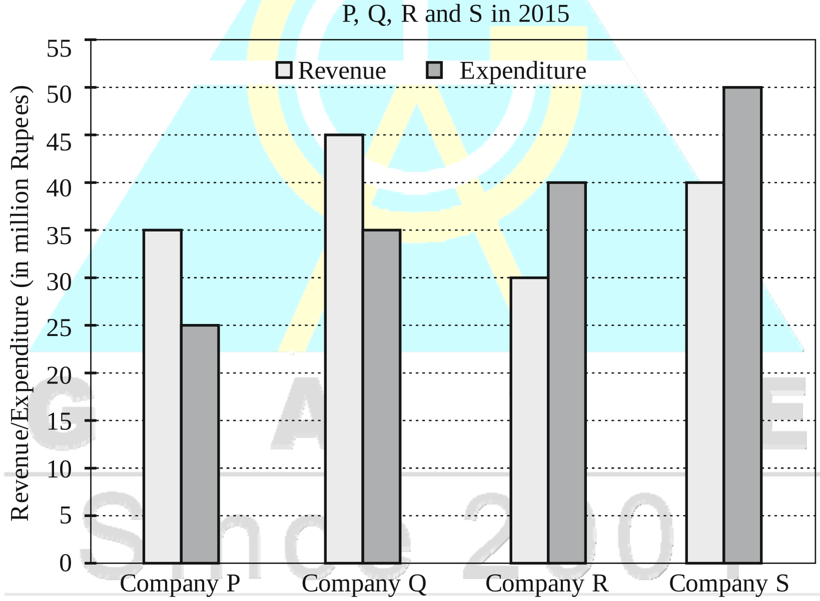
Sol. The verb 'prohibit' means to forbid or prevent, it is followed by the preposition 'form'.

Hence, the correct option is (C).

Question 5

The revenue and expenditure of four different companies P. Q, R and S in 2015 are shown in the figure. If the revenue of company Q in 2015 was 20% more than that in 2014 and company Q had earned a profit of 10% on expenditure in 2014, then its expenditure (in million rupees) in 2014 was ________.

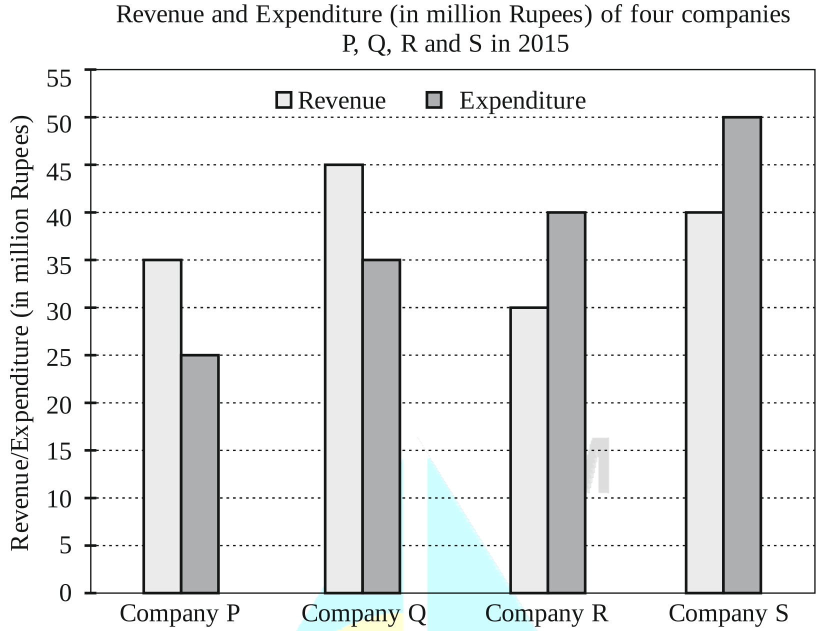
Revenue and Expenditure (in million Rupees) of four companies
P, Q, R and S in 2015



(A) 34.1 (B) 35.1 (C) 33.7 (D) 32.7

Ans. **(A)**

Sol. **Given :** Bar chart showing line revenue and expenditure of four companies P, Q, R & S is shown in figure,



Revenue of company Q in 2015 = 45 million

Given that it is 20% more than that in 2014.

Let revenue of Q in 2014 is x million

Then, $x + x \times \frac{20}{100} = 45$

$\Rightarrow \quad x + \frac{x}{5} = 45$

$\Rightarrow \quad 6x = 45 \times 5$

$x = 37.5$ million

$$\text{(Expenditure)} + \left(\text{Expenditure} \times \frac{10}{100}\right) = 37.5$$

$\Rightarrow \frac{11}{10} \times \text{Expenditure} = 37.5$

$\therefore$ Expenditure $= \frac{37.5}{1.1} = 34.09 \approx 34.1$

Hence, the correct option is (A).

Q.6 to Q.10 Carry two marks each

Question 6

Non-performing Assets (NPAs) of a bank in India is defined as an asset, which remains unpaid by a borrower for a certain period of time in terms of interest, principal, or both. Reserve Bank of India (RBI) has changed the definition of NPA thrice during 1993-2004. in terms of the holding period of loans. The holding period was reduced by one quarter each time. In 1993, the holding period was four quarters (360 days). Based on the above paragraph, the holding period of loans in 2004 after the third revision was ________ days.

(A) 90 (B) 180 (C) 45 (D) 135

Ans. **(A)**

Sol. Given Holding period was 360 days in 1993 and it has changed 3 times till 2004, reducing each time by one quarter.

In 1993, Holding period = 4 Quarter

After first amendment, holding period

$$= 4 - 1 = 3 \text{ quarter}$$

After second amendment, holding period

$$= 3 - 1 = 2 \text{ quarter}$$

$\therefore$ After third revision, holding period

$$= 2 - 1 = 1 \text{ quarter}$$

Given 1 quarter = 90 days

So, holding period after third revision in 2004 is 90 days.

Hence, the correct option is (A)

Question 7

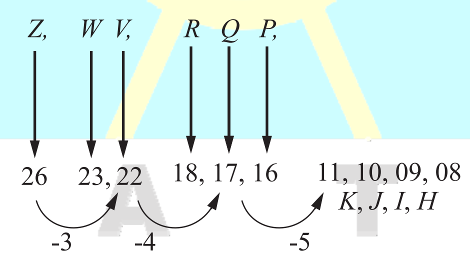
Select the next element of the series : Z, WV, RQP, _______

(A) LKJI (B) KJIH (C) NMLK (D) JIHG

Ans. **(B)**

Sol. **Method 1**

Given series



Hence, the correct option is (B).

Method 2

Hence, the correct option is (B).

Question 8

If P, Q, R, S are 4 individuals, how many teams of size exceeding one can be formed, with Q as a member?

(A) 5 (B) 7 (C) 8 (D) 6

Ans. **(B)**

Sol. No. of available players = 4

No. of teams of 2 members, including Q

$= QP, QR, QS = 03$ [No. of ways to fill 1 place by 3 members]

No. of teams of 3 members including Q

= No. of ways of filling 2 places from 3 members

$$= {}^3C_2 = \frac{3!}{2!\,1!} = \frac{3\times 2\times 1}{2\times 1} = 3$$

No. of teams of 4 members including Q

= No. of ways of filling 3 places from 3 members

$$= {}^3C_3 = \frac{3!}{3!\,0!} = 1$$

$\therefore$ Total number of teams $= 3+3+1 = 7$

Hence, the correct option is (B)

Question 9

Select the word that fits the analogy

Do : Undo : : Trust : __________

(A) Intrust (B) Untrust (C) Entrust (D) Distrust

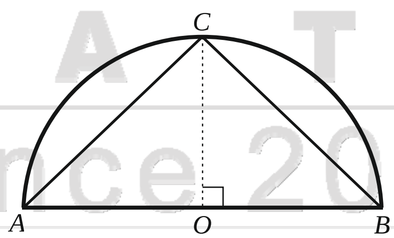
Ans. **(D)**

Sol. Given words in the first pair are antonyms, correct antonym for Trust is distrust.

Question 10

Given a semicircle with O as the centre, as shown in the figure, the ratio $\frac{\overline{AC}+\overline{CB}}{\overline{AB}}$ is _______.

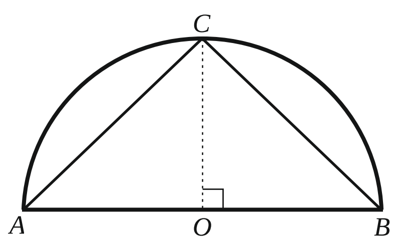
where $\overline{AC}$, $\overline{CB}$ and $\overline{AB}$ are chords.



(A) $\sqrt{3}$ (B) 3 (C) 2 (D) $\sqrt{2}$

Ans. **(D)**

Sol. Given figure is as shown below



Let ‘r’ be the radius of the semicircle

$\Rightarrow \qquad \overline{AO} = \overline{OB} = \overline{OC} = r$

Using phythogores theorem

$$\overline{AC}^2 = \overline{AO}^2 + \overline{OC}^2$$

$\Rightarrow \qquad \overline{AC}^2 = r^2 + r^2 = 2r^2$

$\Rightarrow \qquad \overline{AC} = \sqrt{2}r$

Similarly $\overline{BC} = \sqrt{2}r$

$\therefore \qquad \dfrac{\overline{AC} + \overline{CB}}{\overline{AB}} = \dfrac{\sqrt{2}r + \sqrt{2}r}{2r}$

$\Rightarrow \qquad \dfrac{\overline{AC} + \overline{CB}}{\overline{AB}} = \sqrt{2}$

Technical Section

Q.1 to Q.25 Carry one mark each

Question 1

Which of the following statements is true about the two sided Laplace transform?

(A) It exists for every signal that may or may not have a Fourier Transform.

(B) It has no poles for any bounded signal that is non-zero only inside a finite time interval.

(C) If a signal can be expressed as a weighted sum of shifted one sided exponentials, then its Laplace transform will have no poles.

(D) The number of finite poles and finite zeroes must be equal.

Ans. **(B)**

Sol. From the properties of ROC of Laplace transform :

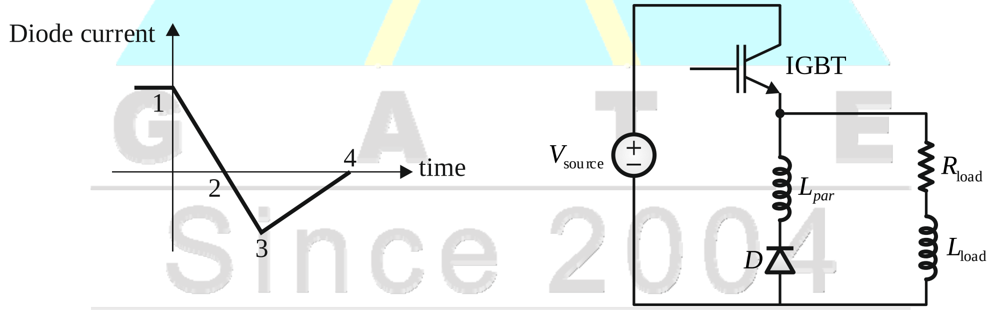
1. ROC does not contain any pole
2. ROC of transform of a bounded finite duration signal is entire S-plane

It can be said that, if a signal is bounded and exists only for finite duration, then ROC is entire s-plane, so it can not have any pole as ROC does not contain any pole.

Hence, the correct option is (B).

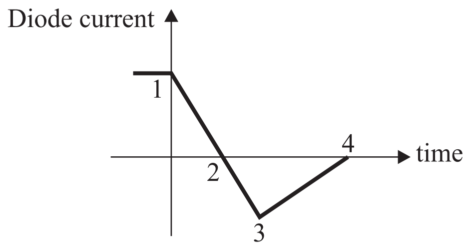
Question 2

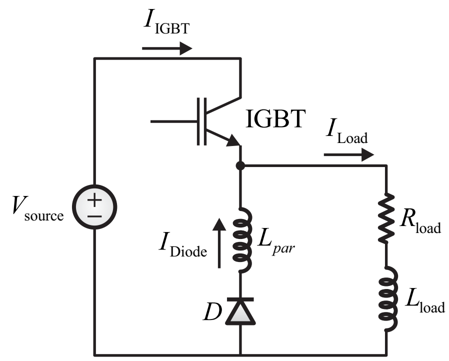
A double pulse measurement for an inductively loaded circuit controlled by the IGBT switch is carried out to evaluate the reverse recovery characteristic of the diode, D, represented approximately as a piecewise linear plot of current vs time at diode turn-off. L_{par} is a parasitic inductance due to the wiring of the circuit and is in series with the diode. The point on the plot (indicate your choice by entering 1, 2, 3 and 4) at which IGBT experiences the highest current stress is __________.



Ans. **3**

Sol. Given plot of diode current versus time and inductively loaded circuit is shown below,





Considering the load to be highly inductive

$\therefore$ Load current will be constant

Load current = Switch current + Diode current (diode current is considered to evaluate the reverse recovery characteristics of the diode)

So, $I_{IGBT} = I_{Load} - I_{Diode}$

I_{IGBT} will experience the highest current stress when the switch current will be maximum

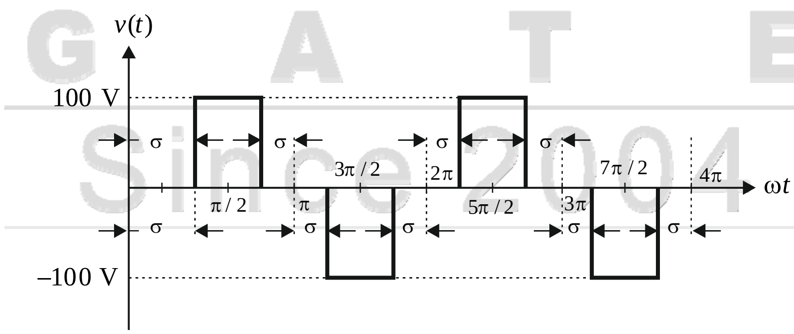
Switch current = (Load current – Diode current) to be maximum diode current has to be minimum.

Diode current is minimum at point 3.

Hence, the correct point is (3).

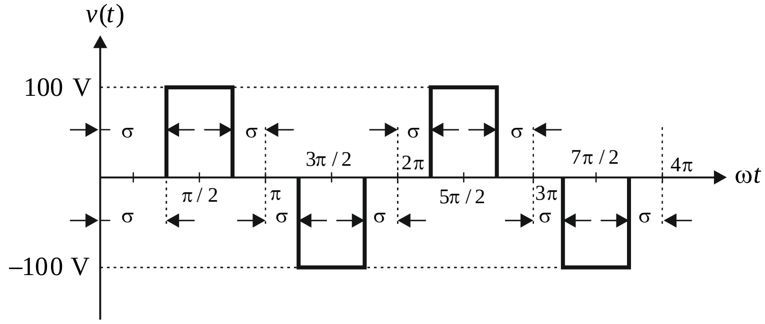
Question 3

A single-phase inverter is fed from a 100 V dc source and is controlled using a quasi square wave modulation scheme to produce an output waveform, $v(t)$ as shown. The angle σ is adjusted to entirely eliminate the 3rd harmonic component from the output voltage. Under this condition, for $v(t)$, the magnitude of the 5th harmonic component as a percentage of magnitude of the fundamental component is _______ (rounded off to two decimal places).



Ans. **20**

Sol. Given output voltage waveform is shown below,



For single phase inverter, Fourier series expression of single pulse modulation is given by,

$$v_{on}(t) = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin n\frac{\pi}{2} \sin nd \sin n\omega t$$

To eliminate third harmonics pulse width required is given by,

$$nd = 180^0$$

$$3d = 180^0 \qquad (n = \text{order of harmonics})$$

$$d = 60^0$$

$\therefore$ Fundamental harmonic component is given by,

$$I_1 = \frac{4I_0}{\pi} \sin n\frac{\pi}{2} \sin 60^0$$

$$I_1 = \frac{4I_0}{\pi} \times \frac{\sqrt{3}}{2}$$

5^{th} Harmonic component is given by,

$$I_5 = \frac{4I_0}{5\pi} \sin\left(\frac{5\pi}{2}\right) . \sin 5 \times 60^0$$

$$I_5 = \frac{4I_0}{5\pi} \times \frac{\sqrt{3}}{2}$$

$$\frac{I_5}{I_1} = 20\%$$

Question 4

Consider a signal $x[n] = \left(\frac{1}{2}\right)^n 1[n]$, where $1[n] = 0$ if $n < 0$, and $1[n] = 1$ if $n \geq 0$.

The z-transform of $x[n-k], k > 0$ is $\dfrac{z^{-k}}{1-\frac{1}{2}z^{-1}}$ with region of convergence being

(A) $|z|<2$ (B) $|z|>2$ (C) $|z|<\frac{1}{2}$ (D) $|z|>\frac{1}{2}$

Ans. **(D)**

Sol. **Given :** $x[n]=\left(\frac{1}{2}\right)^n 1[n]$

Where $1[n]=\begin{cases}1, & n\geq 0\\ 0 & n<0\end{cases}$

Comparing with definition of unit step sequence

$$1[n]=u[n]$$

$$\therefore \quad x[n]=\left(\frac{1}{2}\right)^n u[n]$$

$$\therefore \quad X(Z)=\frac{1}{1-\frac{1}{2}Z^{-1}}, \quad |Z|>\left|\frac{1}{2}\right|$$

For $x[n-k]$ transform is given using time shifting properly

If $$x[n]\longleftrightarrow\frac{1}{1-\frac{1}{2}Z^{-1}}, \quad |Z|>\frac{1}{2}$$

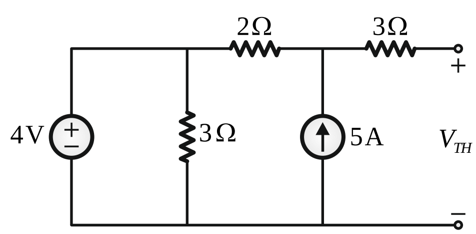
$$x[n-k]\longleftrightarrow\frac{Z^{-k}}{1-\frac{1}{2}Z^{-1}}, \quad |Z|>\frac{1}{2}$$

Given transform for $x[n-k]$, is same as obtained above having pole at $Z=\frac{1}{2}$. As the system is causal, so the ROC will be exterior of a circle outside the outer most pole.

Hence, the correct option is (D).

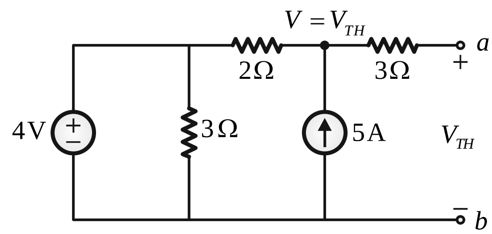
Question 5

The Thevenin equivalent voltage, V_{TH}, in V (rounded off to 2 decimal places) of the shown below network, is _______.



Ans. **14**

Sol. Given circuit is shown in figure,



To find open circuit voltage across terminals *a* and *b*, which is named as V_{TH}, applying KCL at node *V*,

$$\frac{V_{TH}-4}{2}-5=0$$

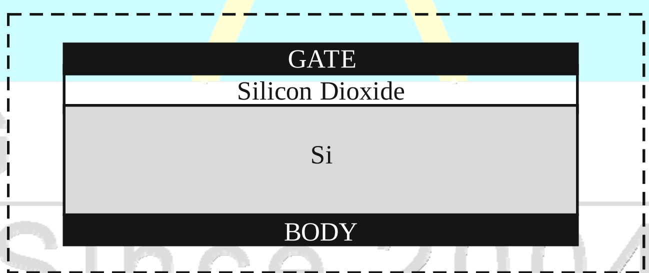
$$\frac{V_{TH}-4}{2}=5$$

$$V_{TH}-4=10$$

$$V_{TH}=14\text{ V}$$

Question 6

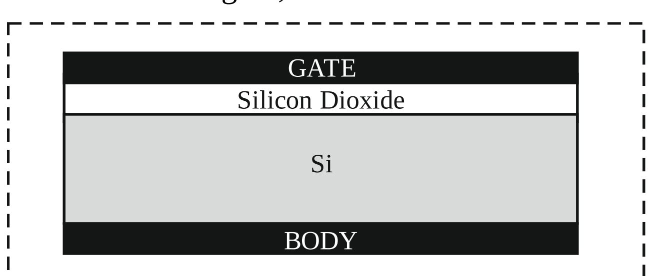
The cross-section of a metal-oxide-semiconductor structure is shown schematically. Starting from an uncharged condition, a bias o f + 3 V is applied to the gate contact with respect to the body contact. The charge inside the silicon dioxide layer is then measured to be $+Q$. The total charge contained within the dashed box shown, upon application of bias, expressed as a multiple of Q (absolute value in Coulombs, rounded off to the nearest integer) is ____________.



DASHED BOX

Ans. 0

Sol. Given cross section is shown in below figure,



DASHED BOX

The applied voltage at gate terminal can be compensated by induced charge at semiconductor surface so as a whole it is always neutral.

Question 7

Which of the following is true for all possible non-zero choices of integers $m, n; m \neq n$, or all possible non-zero choices of real numbers $p, q; p \neq q$, as applicable?

(A) $\lim_{\alpha \to \infty} \frac{1}{2a}\int_{-\alpha}^{\alpha} \sin p\theta \sin q\theta d\theta = 0$

(B) $\frac{1}{2\pi}\int_{-\pi}^{\pi} \sin p\theta \cos q\theta d\theta = 0$

(C) $\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \sin p\theta \sin q\theta d\theta = 0$

(D) $\frac{1}{\pi}\int_{0}^{\pi} \sin m\theta \sin n\theta d\theta = 0$

Ans. (A), (B) and (D)

Sol. For this question, out of given choices, more than one options are conditionally true but none of the given option is correct if we consider $m = -n$ or $p = -q$ which satisfies the given conditions of $m \neq n$ and $p \neq q$ mentioned in the question. So this question must go in the category of MTA. For any conclusion, the statement that must be given in the question should be $|m| \neq |n|$ and $|p| \neq |q|$.

IIT has given one of the conditionally true option i.e. option (B) in their final answer key by changing their previously given option (A).

Checking the options for their validity :

Given : $m, n : m \neq n \to$ Integers

$p, q : p \neq q \to$ real numbers

From option (D)

From orthogonality between sine and sine functions

$$\frac{1}{L}\int_0^L \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right)dx = \begin{Bmatrix} 0; & m \neq n \\ \frac{1}{2}; & m = n \end{Bmatrix} \quad \ldots\text{(i)}$$

$$\frac{1}{L}\int_{-L}^L \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right)dx = \begin{Bmatrix} 0; & m \neq n \\ 1; & m = n \neq 0 \end{Bmatrix} \quad \ldots\text{(ii)}$$

where m & n must be integers

For, $L = \pi$, from equation (i)

$$\frac{1}{\pi}\int_0^\pi \sin mx \sin nx dx = 0$$

So, option (D) is correct, which can be verified by evaluating the given integral as follows

$$I = \frac{1}{\pi}\int_0^\pi \sin m\theta \sin n\theta d\theta$$

$$I = \frac{1}{2\pi}\int_0^\pi -\cos(m+n)\theta\, d\theta + \int_0^\pi \cos(m-n)\theta\, d\theta$$

$$I=\frac{1}{2\pi}\left[-\frac{\sin(m+n)\theta}{(m+n)}\right]_0^{\pi}+\left[\frac{\sin(m-n)\theta}{(m-n)}\right]_0^{\pi}$$

$$I=\frac{1}{2\pi}\left[-\frac{\sin(m+n)\pi}{(m+n)}+\frac{\sin(m+n)\times 0}{(m+n)}+\frac{\sin(m-n)\pi}{(m-n)}-\frac{\sin(m-n)\times 0}{(m-n)}\right]$$

$$I=\frac{1}{2\pi}[-0+0+0-0] \qquad \text{for } |m|\neq|n| \text{ only, also } \sin m\pi=0,\ \forall m=1,\ 2,\ .....$$

$$I=0$$

From option (A), that was given as correct option in first answer key by IIT.

$$I=\lim_{\alpha\to\infty}\frac{1}{4\alpha}\left[\int_{-\alpha}^{\alpha}\cos(p-q)\theta\,d\theta-\int_{-\alpha}^{\alpha}\cos(p+q)\theta\,d\theta\right]$$

$$I=\lim_{\alpha\to\infty}\frac{1}{4\alpha}\left[\frac{\sin(p-q)\theta}{(p-q)}-\frac{\sin(p+q)\theta}{(p+q)}\right]_{-\alpha}^{\alpha}$$

$$I=\lim_{\alpha\to\infty}\frac{1}{4\alpha}\left[\frac{2\sin(p-q)\alpha}{(p-q)}-\frac{2\sin(p+q)\alpha}{(p+q)}\right]$$

$$I=\lim_{\alpha\to\infty}\frac{1}{2\alpha}\left[\frac{\sin(p-q)\alpha}{(p-q)}-\frac{\sin(p+q)\alpha}{(p+q)}\right]$$

$$I=\lim_{\alpha\to\infty}\frac{1}{\alpha}\left[\frac{\sin(p-q)\alpha}{(p-q)\alpha}-\frac{\sin(p+q)\alpha}{(p+q)\alpha}\right]$$

$$\lim_{\alpha\to\infty}\frac{\sin\theta}{\alpha}=0$$

$$\therefore \qquad I=0$$

For $|p|\neq|q|$ only.

So, option (A) is also conditionally true.

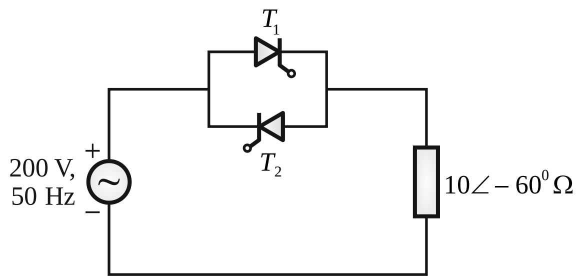
From option (B), which is given as correct option in final answer key by IIT,

$$I=\frac{1}{2\pi}\int_{-\pi}^{\pi}\sin p\theta\cos q\theta\,d\theta$$

Given integral can also conditionally give the result zero as the product of *sine* and *cos* functions will be an odd function and result of integration from negative to positive symmetric limits of an odd function is zero.

Question 8

Thyristor T_1 is triggered at an angle α (in degree) and T_2 at angle $180^0+\alpha$, in each cycle of the sinusoidal input voltage. Assume both thyistors to be ideal. To control the load power over the range 0 to 2 kW, the minimum range of variation in α is



(A) 60^0 to 120^0 (B) 0^0 to 120^0 (C) 0^0 to 60^0 (D) 60^0 to 180^0

Ans. **(MTA)**

Sol. ***This topic is out of the syllabus and the data given in the question is wrong. Hence, IIT Delhi has declared it as MTA question.***

Question 9

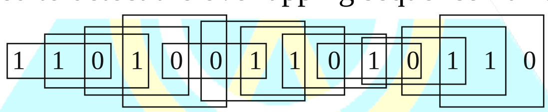
A sequence detector is designed to detect precisely 3 digital inputs, with overlapping sequences detectable. For the sequence (1,0,1) and input data (1,1,0,1,0,0,1,1,0,1,0,1,1,0), what is the output of this detector?

(A) 1,1,0,0,0,0,1,1,0,1,0,0 (B) 0,1,0,0,0,0,0,0,1,0,0,0

(C) 0,1,0,0,0,0,0,1,0,1,0,0 (D) 0,1,0,0,0,0,0,1,0,1,1,0

Ans. **(C)**

Sol. Given input date is processed to detect the overlapping sequence 101 as shown below,

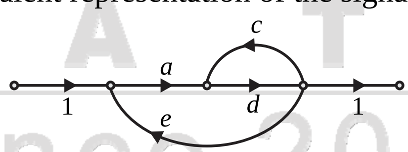


When sequence will be 101, then output will be 1 otherwise 0, so starting from first three bits, output bits for respective combination are as 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0.

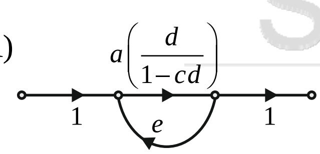
Hence, the correct option is (C).

Question 10

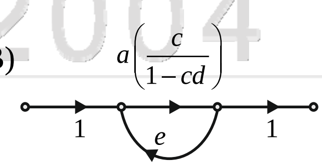
Which of the options is an equivalent representation of the signal flow graph shown here?



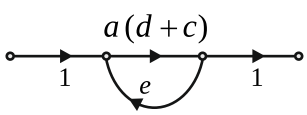
(A)



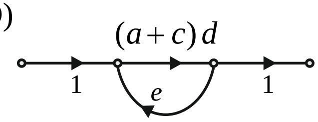
(B)



(C)

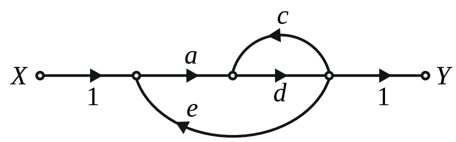


(D)



Ans. **(A)**

Sol. Given signal flow graph is shown below,



Forward path gain

$$P_1 = ad$$

Individual loop gains

$$L_1 = ade,\ \ L_2 = cd$$

Using Mason's gain formula, overall gain

$$\frac{Y}{X} = \sum_{k=1}^{i} \frac{P_K \Delta_K}{\Delta}$$

$$\Delta = 1 - \{ade + cd\}, \qquad \Delta_1 = 1 - 0 = 1$$

$$\frac{Y}{X} = \frac{ad}{1 - ade - cd}$$

From option (A) :

$$\frac{Y}{X} = \frac{\dfrac{ad}{1-cd}}{1 - \dfrac{ad\,e}{1-cd}} = \frac{ad}{1 - cd - ade}$$

Hence, the correct option is (A).

Question 11

A single-phase, full-bridge diode rectifier fed from a 230 V, 50 Hz sinusoidal source supplies a series combination of finite resistance, *R*, and a very large inductance, *L*. The two most dominant frequency components in the source current are

(A) 50 Hz, 0 Hz (B) 50 Hz, 100 Hz (C) 50 Hz, 150 Hz (D) 150 Hz, 250 Hz

Ans. (C)

Sol. Given : Single-phase, full-bridge diode rectifier

Fourier series expression of source current is given by

$$I_s(t) = \sum_{n=1,3,5}^{\infty} \frac{4I_0}{n\pi} \sin(n\omega_0 t)$$

$\therefore$ Most dominant frequency components are $\dfrac{4I_0}{\pi}, \dfrac{4I_0}{3\pi}, \dfrac{4I_0}{5\pi}$

$\therefore \qquad f = 50\,\text{Hz},\ 50\times 3\,\text{Hz},\ 50\times 5\ \text{Hz}$

Most dominant frequency components are $50\,\text{Hz},\ 150\,\text{Hz},\ 250\,\text{Hz}$.

Hence, the correct option is (C).

Question 12

Consider the initial value problem below.

The value of *y* at $x = \ln 2$, (rounded off to 3 decimal places) is ______.

$$\frac{dy}{dx} = 2x - y,\ \ y(0) = 1$$

Ans. 0.886

Sol. Given differential equation is

$$\frac{dy}{dx} = 2x - y \qquad \text{having initial condition } y(0) = 1$$

$$\Rightarrow \quad \frac{dy}{dx} + y = 2x \qquad \ldots\text{(i)}$$

(i) is of the form $\frac{dy}{dx} + p(x)y = Q(x)$

Where $p(x) = 1$ and $Q(x) = 2x$

$\therefore$ Integrating factor is

$$\text{I.F.} = e^{\int p(x)dx} = e^{\int 1dx} = e^{x}$$

$\therefore$ solution of differential equation is

$$y(\text{I.F.}) = \left[\int Q(x)(I.F.)\,dx\right] + C$$

$$\Rightarrow \quad y\,e^{x} = \int 2x\,e^{x}dx + C$$

$$\Rightarrow \quad y\,e^{x} = 2\left[x\int e^{x}dx - \int e^{x}dx\right] + C$$

$$\Rightarrow \quad y\,e^{x} = 2\left[x\,e^{x} - e^{x}\right] + C$$

$$\Rightarrow \quad y\,e^{x} = 2x\,e^{x} - 2e^{x} + C$$

$$\Rightarrow \quad y = 2x - 2 + ce^{-x}$$

At point (0, 1)

$$\Rightarrow \quad 1 = 0 - 2 + C$$

$$\Rightarrow \quad C = 3$$

$$\therefore \quad y = 2x - 2 + 3e^{x}$$

At $x = \ln 2$, solution will be

$$y = 2\ln 2 - 2 + 3e^{-\ln 2}$$

$$\Rightarrow \quad y = 2\ln 2 - 2 + \frac{3}{2}$$

$$\Rightarrow \quad y = 0.8862$$

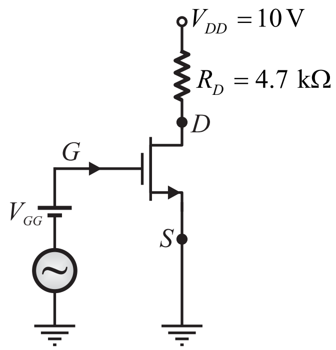
Question 13

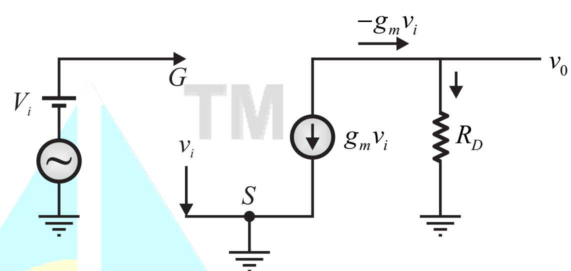
A common-source amplifier with a drain resistance, $R_D = 4.7\text{k}\Omega$, is powered using a 10 V power supply. Assuming that the trans-conductance g_m, is $520\,\mu\text{A/V}$, the voltage gain of the amplifier is closest to

(A) 2.44 (B) – 2.44 (C) 1.22 (D) – 1.22

Ans. (B)

Sol. Given for a common source amplifier,





$R_D = 4.7\text{ k}\Omega$

$V_{in} = 10\text{ V}$

$g_m = 520\ \mu\text{A/V}$

Voltage gain of common source amplifier is gives as

$$A_V = \frac{V_0}{V_i} = -g_m R_D$$

$$A_V = -520 \times 10^{-6} \times 4.7 \times 10^3$$

$$A_V = -2.44$$

Hence, the correct option is (B).

Question 14

$ax^3 + bx^2 + cx + d$ is a polynomial on real x over real coefficients a, b, c, d where in $a \neq 0$. Which of the following statements is true?

(A) No choice of coefficients can make all roots identical.

(B) a, b, c, d can be chosen to ensure that all roots are complex.

(C) d can be chosen to ensure that $x = 0$ is a root for any given set a, b, c.

(D) c alone cannot ensure that all roots are real.

Ans. (C)

Sol. **Given :** Polynomial is $f(x) = ax^3 + bx^2 + cx + d$ where x is real.

If the value of 'd' is chosen as 0, then $f(x)$ becomes

$$f(x) = ax^3 + bx^2 + cx$$

$$f(x) = x(ax^2 + bx + c)$$

$\therefore$ $x = 0$ will be one of the roots of $f(x)$ irrespective of the value of a, b, c

Hence, the correct option is (C).

Question 15

A single 50 Hz synchronous generator on droop control was delivering 100 MW power to a system. Due to increase in load, generator power had to be increased by 10 MW, as a result of which, system frequency dropped to 49.75 Hz. Further increase in load in the system resulted in a frequency of 49.25 Hz. At this condition, the power in MW supplied by the generator is_____ (rounded off to two decimal places).

Ans. **130**

Sol. **Given : Synchronous generator**

(i) $P_1 = 100$ MW

(ii) $P_2 = 110$ MW

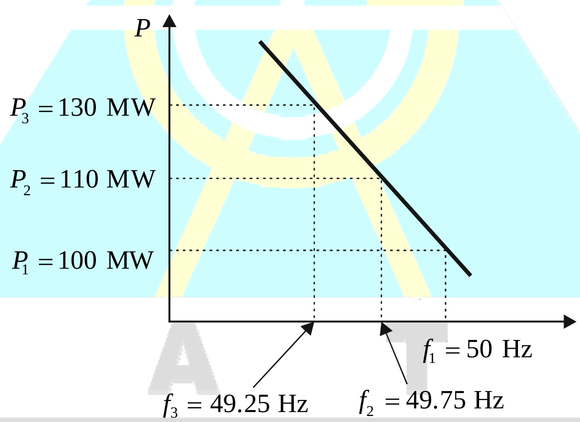
(iii) $f_1 = 50$ Hz

(iv) $f_2 = 49.75$ Hz

Method 1

Line constant :

$$Y = mX + C$$



$$m = \text{Slope} = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{110 - 100}{49.75 - 50} = -40 \frac{\text{MW}}{\text{Hz}}$$

$$100 = -40 \times 50 + C$$

$$C = 2100 \text{MW}$$

At 49.25 Hz

$$P_3 = Y = mX + C = -40 \times 49.25 + 2100$$

$$P_3 = 130 \text{ MW}$$

Method 2

We know that,

$$s_P = \frac{-\Delta P}{\Delta f} = \frac{-10 \text{ MW}}{(49.75 - 50) \text{ Hz}}$$

$\therefore$ $s_P = 40$

Now, $s_P = \dfrac{-\Delta P'}{49.25 - 49.75}$

$\Delta P' = 20$

$\therefore$ Total change $= \Delta P + \Delta P' = 30$ MW

Power delivered $= 100 + 30 = 130$ MW

Question 16

Consider a linear time-invariant system whose input $r(t)$ and output $y(t)$ are related by the following differential equation :

$$\frac{d^2y(t)}{dt^2} + 4y(t) = 6r(t)$$

The poles of this system are at

(A) $+2, -2$ (B) $+4, -4$ (C) $+2j, -2j$ (D) $+4j, -4j$

Ans. (C)

Sol. Given differential equation for LTI system is

$$\frac{d^2y(t)}{dt^2} + 4y(t) = 6r(t)$$

Taking Laplace transform both sides,

$$(s^2 + 4)Y(s) = 6R(s)$$

$$\frac{Y(s)}{R(s)} = H(s) = \frac{6}{s^2 + 4}$$

Poles $s^2 + 4 = 0$ $s^2 = -4$

$s = \pm 2j$

Hence, the correct option is (C).

Question 17

Consider a negative unity feedback system with forward path transfer function $G(s) = \dfrac{K}{(s+a)(s-b)(s+c)}$, where, K, a, b, c are positive real number. For a Nyquist path enclosing the entire imaginary axis and right half of the s-plane in the clockwise direction, the Nyquist plot of $(1+G(s))$, encircles the origin of $(1+G(s))$ plane once in a clockwise direction and never passes through this origin for a certain value of K. Then, the number of poles of $\dfrac{G(s)}{1+G(s)}$ lying in the open right half of the s-plane is ________.

Ans. **2**

Sol. Given forward path transfer function,

$$G(s) = \frac{K}{(s+a)(s-b)(s+c)}$$

$\therefore$ Number of open loop poles in right half of s-plane is $P = 1$.

Given that the Nyquist plot encircles the origin of $1+G(s)$ plane that is $(-1+j0)$ once in clockwise direction for $G(s)$ plane.

$$\therefore \qquad N = -1$$

From Nyquist stability criteria,

$$N = P - Z$$

Z is the number of poles of $\frac{G(s)}{1+G(s)}$ in right half of s-plane.

$$Z = 1-(-1) = 2$$

Question 18

A lossless transmission line with 0.2 pu reactance per phase uniformly distributed along the length of the line, connecting a generator bus to a load bus. is protected up to 80 % of its length by a distance relay placed at the generator bus. The generator terminal voltage is 1 pu. There is no generation at the load bus. The threshold pu current for operation of the distance relay for a solid three phase-to-ground fault on the transmission line is closest to

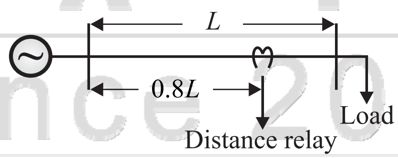
(A) 3.61 (B) 1.00 (C) 6.25 (D) 5.00

Ans. **(C)**

Sol. **Given :**

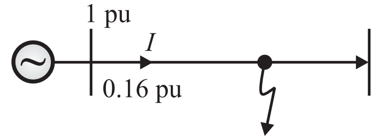
(i) $X_L = 0.2\,\text{pu}$

(ii) $V_t = 1\,\text{pu}$



The line to be protected is 80% of this it's length by a distance relay at the generator bus.

The impedance seen by the distance relay $= 0.8 \times 0.2$



$$I = \frac{1}{0.16} = 6.25\,\text{pu}$$

Hence, the correct option is (C).

Question 19

The value of the following complex integral, with C representing the unit circle centered at origin in the counterclockwise sense, is

$$\int_C \frac{z^2+1}{z^2-2z}\,dz$$

(A) $8\pi i$ (B) πi (C) $-8\pi i$ (D) $-\pi i$

Ans. (D)

Sol. Let the given integral be denoted by the function $f(z)$

$$\therefore \quad f(z)=\int_c \frac{z^2+1}{z^2-2z}\,dz$$

$$\Rightarrow \quad f(z)=\int_c \frac{z^2+1}{z(z-2)}\,dz$$

The singular points of $f(z)$ are $z=0$ and $z=2$.

Since, the curve 'c' is a circle of radius 1, the point $z=0$ will lie inside the circle and the point $z=2$ will lie outside the circle.

$\therefore$ Residue of $f(z)$ will be

$$\text{Res}[f(z)]=\lim_{z\to 0}(z-0)\frac{z^2+1}{z(z-2)}$$

$$=\frac{0+1}{0-2}=\frac{-1}{2}$$

$$\therefore \quad \int_c \frac{z^2+1}{z(z-2)}=2\pi i\,[\text{Res}(f(z))]$$ (using cauchy's residue theorem)

$$=2\pi i\times\frac{-1}{2}=-\pi i$$

Hence, the correct option is (D).

Question 20

x_R and x_A are, respectively, the rms and average values of $x(t)=x(t-T)$, and similarly, y_R and y_A are respectively, the rms and average values of $y(t)=kx(t)$. k, T are independent of t. Which of the following is true ?

(A) $y_A=kx_A$; $y_R=kx_R$ (B) $y_A\neq kx_A$; $y_R\neq kx_R$

(C) $y_A\neq kx_A$; $y_R=kx_R$ (D) $y_A=kx_A$; $y_R\neq kx_R$

Ans. (D)

Sol. **Given :** $x(t-T)=x(t)$, $x(t)$ is periodic with period T

$\therefore$ Average value of $x(t)$

$$x_A = \frac{1}{T}\int_0^T x(t)dt \qquad \ldots(i)$$

RMS value of $x(t)$

$$x_R = \sqrt{\frac{1}{T}\int_0^T |x(t)^2|\, dt} \qquad \ldots(ii)$$

Given $y(t) = k \cdot x(t)$, period of $y(t) =$ Period of $x(t) = T$

$\therefore$ Average value of $y(t)$

$$y_A = \frac{1}{T}\int_0^T k \cdot x(t)dt$$

$$y_A = k \cdot \left[\frac{1}{T}\int_0^T x(t)dt\right] = k \cdot x_A$$

RMS value of $y(t)$

$$y_R = \sqrt{\frac{1}{T}\int_0^T |kx(t)|^2\, dt}$$

$$y_R = |k|\sqrt{\frac{1}{T}\int_0^T |x(t)^2|\, dt} = |k| \cdot x_R$$

As rms value can never be negative, so irrespective of the sign of k, y_R will always be positive. So if k is a negative constant then $y_R = k \cdot x_R$ is not true.

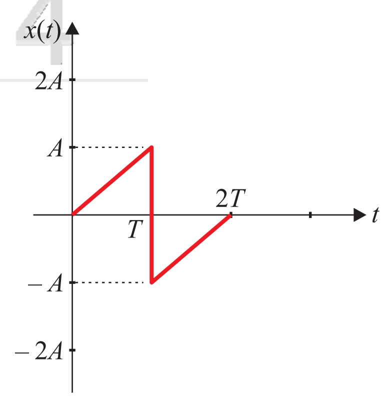
Hence, the correct option is (D).

It can be verified by a simple example, as explained below,

Consider a continuous time periodic signal $x(t)$ for which one period is shown in figure,

$$RMS = \sqrt{\text{Power}}$$

$$\text{Power} = \frac{\text{Energy in 1 period}}{\text{Time period}}$$



Energy of $x(t)$ in 1 period,

$$E_x = \frac{A^2T}{3} + \frac{A^2T}{3} = \frac{2A^2T}{3}$$

$\therefore$ Power of $x(t)$,

$$P_x = \frac{2A^2T}{3 \times 2T} = \frac{A^2}{3}$$

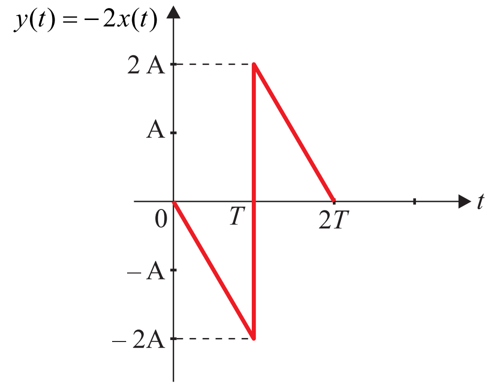
$\therefore$

$$x(t)_{rms} = x_R = \sqrt{\frac{A^2}{3}} = \frac{A}{\sqrt{3}} \qquad \ldots(i)$$

Given : $y(t) = kx(t)$

For $k = -2$, $y(t) = -2x(t)$

$y(t)$ is shown in figure,



Energy of $y(t) = \frac{4A^2T}{3} + \frac{4A^2T}{3} = \frac{8A^2T}{3}$

$\therefore$ Power of $y(t) = \frac{8A^2T}{3} \times \frac{1}{2T} = \frac{4A^2}{3}$

$\therefore$ $$y(t)_{rms} = y_R = \sqrt{\frac{4A^2}{3}} = 2 \times \frac{A}{\sqrt{3}} \qquad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$y_R \neq kx_R \text{ as } k = -2,\ k \neq 2$$

Hence, option (A) is not true for any negative value of *k*.

Given answer in IIT answer key : Option (A).

IIT should have given its correct option as option (D), but they have given option (A) only in their final answer key, which suggests that they have not considered the given relations for **negative values of *k***.

In general, option (D) is correct.

Question 21

A three-phase, 50 Hz, 4- pole induction motor runs at no-load with a slip of 1%. With full load, the slip increases to 5% . The % speed regulation of the motor (rounded off to two decimal places) is _______.

Ans. **4.21**

Sol. **Given :** Three phase induction machine

(i) $f = 50$ Hz

(ii) $P = 4$

(iii) $s_{nl} = 1\%$

(iv) $s_{fl} = 5\%$

$$\%\text{ speed regulation } = \left(\frac{N_{nL} - N_{fL}}{N_{fL}}\right) \times 100$$

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$N_{r(nl)} = N_s(1 - s_{nl})$$

$$N_{r_{(nL)}} = 1500[1 - 0.01]$$

$$N_{r(fl)} = N_s(1 - s_{fl})$$

$$N_{r_{(fL)}} = 1500[1 - 0.05]$$

$$\% \text{ speed regulation } = \left(\frac{1485-1425}{1425}\right)\times 100 = 4.21\%$$

🕮 Key Point

$$\% \text{ Speed regulation } = \left(\frac{N_{nL}-N_{fL}}{N_{fL}}\right)\times 100$$

As, $N_{\text{rated}} = N_{fl}$

$$\therefore \text{ Speed regulation} = \left(\frac{N_{nL}-N_{fL}}{N_{fL}}\right)\times 100$$

Question 22

A three-phase cylindrical rotor synchronous generator has a synchronous reactance X_S and a negligible armature resistance. The magnitude of per phase terminal voltage is V_A and the magnitude of per phase induced emf is E_A. Considering the following two statements P and Q.

P : For any three-phase balanced leading load connected across the terminals of this synchronous generator, V_A is always more than E_A.

Q : For any three-phase balanced lagging load connected across the terminals of this synchronous generator, V_A is always less than E_A.

(A) P is true and Q is false.
(B) P is true and Q is true.
(C) P is false and Q is false.
(D) P is false and Q is true.

Ans. (B)

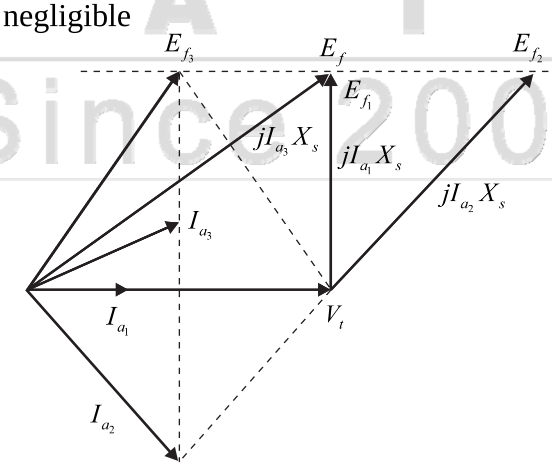
Sol. Given three-phase cylindrical rotor synchronous generator

Synchronous reactance $= X_S$

The magnitude of per phase terminal voltage = V_A

The magnitude of per phase induced emf = E_A

and armature resistance is negligible



From the above phasor diagram it can be concluded that

(i) E_{f2} is greater than V_t for lagging power factor load

(ii) E_{f3} is less than V_t for leading power factor load

Both are true.

Hence, the correct option is (B).

Question 23

A single-phase, 4 kVA, 200 V/100 V, 50 Hz transformer with laminated CRGO steel core has rated no-load loss of 450 W. When the high-voltage winding is excited with 160 V, 40 Hz sinusoidal ac supply, the no-load losses are found to be 320 W. When the high-voltage winding of the same transformer is supplied from a 100 V, 25 Hz sinusoidal ac source, the no-load losses will be ________ W (rounded off to 2 decimal places).

Ans. 162.5

Sol. Given : 4 kVA, 200 V/100 V single phase transformer

(i) $V = 200\,\text{V}$

(ii) $f = 50\,\text{Hz}$

(iii) $P_{(core)} = 450\,\text{W}$

Case-I : $V_1 = 200\,\text{V}$, $f_1 = 50\,\text{Hz}$, $P_{(core)1} = 450\,\text{W}$

$$\frac{V_1}{f_1} = \frac{200}{50} = 4$$

$$P_{(core)_1} = Af_1 + Bf_1^2$$

$$450 = 50A + 2500B \qquad \ldots\text{(i)}$$

Case-II : $V_2 = 160\,\text{V}$, $f_2 = 40\,\text{Hz}$, $P_{(core)_2} = 320\,\text{W}$

$$\frac{V_2}{f_2} = \frac{160}{40} = 4$$

$\frac{V}{f}$ is constant

$$P_{(core)2} = Af_2 + Bf_2^2$$

$$320 = 40A + 1600B \qquad \ldots\text{(ii)}$$

Solving equation (i) and (ii)

$$A = 4,\; B = 0.1$$

Case-III : $V_3 = 100\,\text{V}$, $f_3 = 25\,\text{Hz}$, $P_{(core)3} = ?$

$$P_{(core)3} = Af_3 + Bf_3^2$$

$$P_{(core)3} = 162.5\,\text{W}$$

Question 24

Out of the following options, the most relevant information needed to specify the real power (P) at the PV buses in a load flow analysis is.

(A) base power of the generator

(B) solution of economic load dispatch.

(C) rated power output of the generator

(D) rated voltage of the generator.

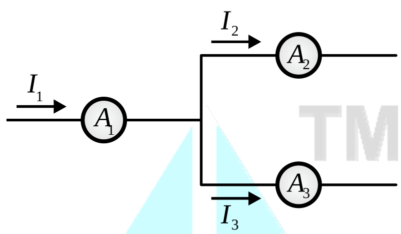
Ans. (B)

Sol. Solution of economic load dispatch is the most relevant information needed to specify the real power P at PV buses in load flow analysis.

Hence, the correct option is (B).

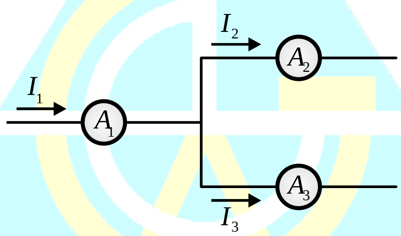
Question 25

Currents through ammeters A_2 and A_3 in the figure are $1\angle 10^0$ and $1\angle 70^0$ respectively. The reading of the ammeter A_1 (rounded off to 3 decimal places) is ____A.



Ans. 1.732

Sol. Given diagram is shown below



Applying KCL;

$$\vec{I}_1 = \vec{I}_2 + \vec{I}_3$$

$$\vec{I}_1 = 1\angle 10^0 + 1\angle 70^0$$

$$\vec{I}_1 = \cos 10^0 + j\sin 10^0 + \cos 70^0 + j\sin 70^0$$

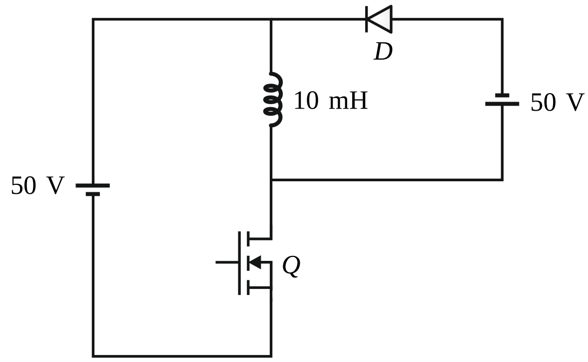
$$\vec{I}_1 = 1.3268 + j1.113$$

$$|\vec{I}_1| = 1.732 \text{ A}$$

Q.26 to Q.55 Carry two marks each

Question 26

In the dc-dc converter circuit shown, switch Q is switched at a frequency of 10 kHz with a duty ratio of 0.6. All components of circuit are ideal and the initial current in the inductor is zero. Energy stored in the inductor in mJ (rounded off to 2 decimal places) at the end of 10 complete switching cycles is ______.



Ans. 5

Sol. **Given :** Buck boost converter

(i) $f_s = 10$ kHz

(ii) $T = 0.1$ ms

(iii) $\delta = 0.6$

When the switch is ON from 0 to δT

$$\therefore \quad V_L = L\frac{di}{dt}$$

$$\frac{V_L}{L} = \frac{di}{dt}$$

$$I_L = \frac{V_L}{L}\cdot\delta T$$

$$I_L = \frac{50}{10\times10^{-3}}\times0.6\times\frac{1}{10\times10^{3}}$$

$$I_L = 0.3 \text{ A}$$

$\therefore$ When the switch is closed from δT to T

$$V_L = L\frac{di}{dt}$$

$$\Delta I_L = \frac{V_L\times(1-\delta)T}{L}$$

$$\Delta I_L = \frac{50\times(1-0.6)}{10\times10^{-3}}\times\frac{1}{10\times10^{3}}$$

$$\Delta I_L = 0.2 \text{ A}$$

$\therefore$ In first on period inductor charges from 0 A to 0.3 A, and in first of period inductor discharge from 0.3 A to 0.1 A. As it's discharged to 0.1 A it has 0.1 A as initial value for next cycle.

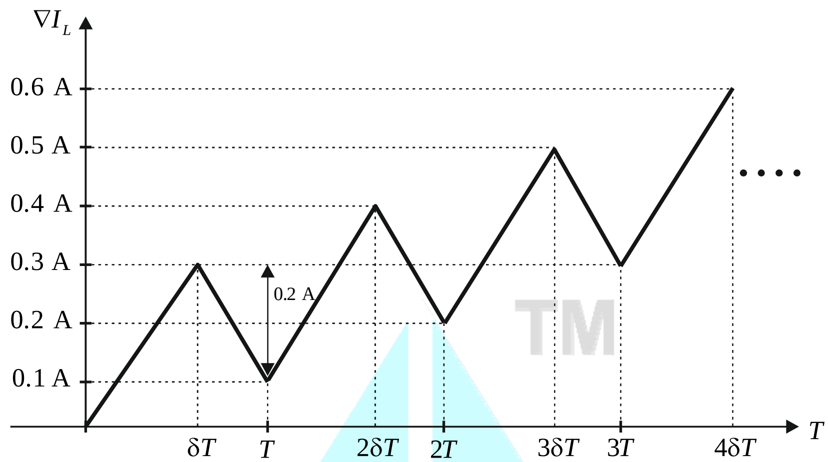
$\therefore$ In second cycle it will charge from 0.1 A to 0.4 A, (charge by 0.3 A and discharge by 0.2 A) and discharge to 0.2A from 0.4 A.

$\therefore$ After each cycle inductor current increases by 0.1 A from the value of its previous cycle.

After 10th cycle remaining inductor current will be $10\times0.1 \text{ A} = 1\text{A}$.

$\therefore$ Energy stored by inductor at the end of 10 complete switching cycles

$$=\frac{1}{2}LI^2=\frac{1}{2}\times 10\times 10^{-3}\times 1^2=5\,\text{mJ}$$



Question 27

A stable real linear time invariant system with single pole at p, has a transfer function $H(s)=\dfrac{s^2+100}{s-p}$ with a dc gain of 5. The smallest positive frequency, in rad/s, at unity gain is closest to

(A) 8.84 (B) 78.13 (C) 122.87 (D) 11.08

Ans. **(A)**

Sol. Given transfer function,

$$H(s)=\frac{s^2+100}{s-p}$$

DC gain = 5

$$H(j\omega)=\frac{-\omega^2+100}{j\omega-p}$$

$$H(j0)=\text{DC gain} \Rightarrow 5=\frac{100}{-P}$$

$$P=-20$$

$\therefore$ $$H(s)=\frac{s^2+100}{s+20}$$

$$H(j\omega)=\frac{-\omega^2+100}{j\omega+20}$$

To find unity gain frequency (let $\omega=\omega_1$),

$$|H(j\omega)|_{\omega=\omega_1}=1$$

$$1=\frac{-\omega_1^2+100}{\sqrt{\omega_1^2+400}}$$

$$\sqrt{\omega_1^2+400}=-\omega_1^2+100$$

$$\omega_1^2+400=\omega_1^4+10000-200\omega_1^2$$

$$\omega_1^4-201\omega_1^2+9600=0$$

Let $\omega_1^2=x$, $\quad x^2-201x+9600=0$

$$x=\frac{201\pm\sqrt{40401-38400}}{2}=122.86,\ 78.15$$

$$\omega_1=\pm 11.084 \quad \text{or} \quad \omega_1=\pm 8.84$$

So at $\omega=11.084$ rad/sec and $\omega=8.84$ rad/sec, the gain of the system is unity. As smallest positive frequency is asked, hence, the correct option is (A).

Question 28

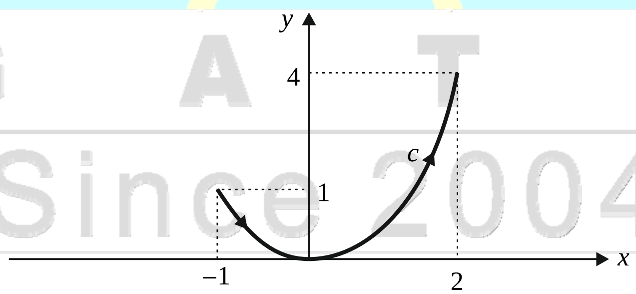
Let $\hat{a}_x$ and $\hat{a}_y$ be unit vectors along x and y directions, respectively. A vector function is given by

$$\vec{F}=\hat{a}_x y-\hat{a}_y x$$

The line integral of the above function

$$\int_c \vec{F}\cdot\vec{dl}$$

along the curve c, which follows the parabola $y=x^2$ as shown below is ______ (rounded off to 2 decimal places).



Ans. **– 3**

Sol. **Given :** $\vec{F}=\hat{a}_x y-\hat{a}_y x$

$$y=x^2$$

$$\vec{dl}=dx\,\hat{a}_x+dy\,\hat{a}_y$$

$$\int_c \vec{F}\cdot\vec{dl}=\int_c(\hat{a}_x y-\hat{a}_y x)\cdot(dx\,\hat{a}_x+dy\,\hat{a}_y)$$

$$\int_c \vec{F}\cdot\vec{dl}=\int_c(y\,dx-x\,dy)$$

Since $y = x^2$,

$dy = 2xdx$

$$\int_c \vec{F} \cdot \vec{dl} = \int_c [x^2 dx - x(2x\,dx)]$$

$$\int_c \vec{F} \cdot \vec{dl} = \int_{x=-1}^{2} -x^2\,dx = \left(-\frac{x^3}{3}\right)_{-1}^{2} = -3$$

Hence, the line integral $\int_c \vec{F} \cdot \vec{dl}$ is -3.

Question 29

An 8085 microprocessor accesses two memory locations (2001 H) and (2002 H), that contain 8-bits numbers 98 H and B1 H, respectively. The following program is executed :

```
LXI H, 2001 H
MVI A, 21 H
INX H
ADD M
INX H
MOV M, A
HLT
```

At the end of this program, the memory location 2003 H contains the number in decimal (base 10) form ___________.

Ans. 210

Sol. Given that an 8085 microprocessor accesses two memory locations (2001 H) and (2002 H), that contain 8-bits numbers 98 H and B1 H, respectively.

Traversing the given program through each instructions

LXI H, 2001 H ; Load HL pair from data available at address 2001 H

So, $[HL] = 2001$ H

MVI A, 21 H ; Move immediately 21 H to accumulator

A = 21 H

INX H ; Location pointed by HL pair is incremented by 1

$[HL] = 2002$ H

ADD M ; Add memory (content of HL pair) to accumulator

$A \leftarrow [A] + [B1]$ H

So content of accumulator is 21 H + B1 H = D2 H

INX H ; Location pointed by HL pair is incremented by 1

$[HL] = 2003$ H

MOV M, A ; Move content of accumulator to memory (HL pair)

So, data location 2003 H is D2 H.

HLT ; End

So, at the end of this program, the memory location 2003 H contains the accumulator content that is D2 H.

$$[D2]_{16} = [D \times 16 + 16^0 \times 2]_{10} = [208 + 2]_{10} = [210]_{10}$$

Hence, the decimal equivalent of content at memory location 2003 H is 210.

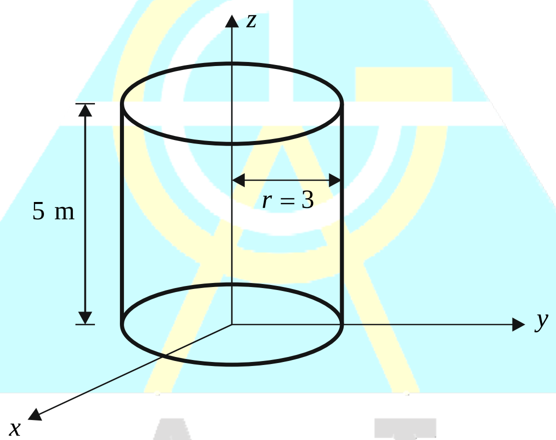
Question 30

Let $\hat{a}_r$, $\hat{a}_\phi$, $\hat{a}_z$ be unit vector along r, ϕ and z direction respectively in the cylindrical coordinate system. For the electric flux density given by $\vec{D} = (15\,\hat{a}_r + 2r\hat{a}_\phi - 3rz\,\hat{a}_z)$ C/m^2, the total electric flux, in Coulomb emanating from the volume enclosed by solid cylinder of radius 3 m and height 5 m oriented along the z-axis with its base at the origin is

(A) 54π (B) 180π (C) 90π (D) 108π

Ans. (B)

Sol. Given : $\vec{D} = 15\hat{a}_r + 2r\hat{a}_\phi - 3rz\hat{a}_z$ C/m^2



Flux passing through closed surface is given by,

$$\phi = \oint_s \vec{D} \cdot d\vec{s}$$

From divergence theorem,

$$\oint_s \vec{D} \cdot d\vec{s} = \int_v (\nabla \cdot \vec{D})\, dv$$

In cylindrical co-ordinate system,

$$\nabla \cdot \vec{D} = \frac{1}{r}\frac{\partial}{\partial r}(r D_r) + \frac{1}{r}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

$$\nabla \cdot \vec{D} = \frac{1}{r}\frac{\partial}{\partial r}(15r) + \frac{1}{r}\frac{\partial}{\partial \phi}(2r) + \frac{\partial}{\partial z}(-3rz)$$

$$\nabla \cdot \vec{D} = \frac{15}{r} + 0 - 3r = \frac{15}{r} - 3r$$

$$\oint \vec{D} \cdot d\vec{s} = \int_{z=0}^{5} \int_{\phi=0}^{2\pi} \int_{r=0}^{3} \left(\frac{15}{r} - 3r \right) r \, dr \, d\phi \, dz$$

$$\oint_s \vec{D} \cdot d\vec{s} = \int_{r=0}^{3} (15 - 3r^2) \, dr \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{5} dz$$

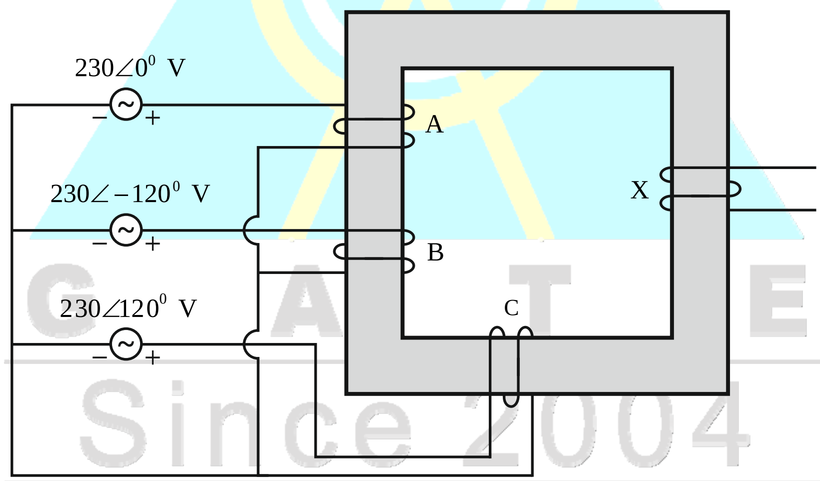
$$\oint_s \vec{D} \cdot d\vec{s} = \left[15r - r^3 \right]_0^3 (2\pi)(5) = (45 - 27)10\pi$$

$$\oint_s \vec{D} \cdot d\vec{s} = 180\pi$$

Hence, the correct option is (B).

Question 31

Windings ‘A’, ‘B’ and ‘C’ have 20 turns each and are wound on the same iron core as shown, along with winding "X' which has 2 turns. The figure shows the sense (clockwise/anti-clockwise) of each of the windings only and does not reflect the exact number of turns. If windings 'A', 'B' and 'C' are supplied with balanced 3-pbase voltages at 50 Hz and there is no core saturation, the no-load RMS voltage (in V, rounded off to 2 decimal places) across winding "X" is ________.



Ans. **46**

Sol. **Given :**

(i) $N_A = 20$

(ii) $N_B = 20$

(iii) $N_C = 20$

(iv) $N_X = 2$

$$V_A = 230\angle 0^0$$

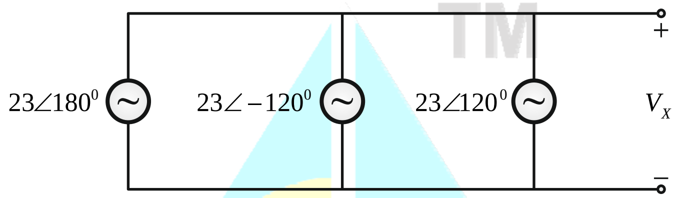
$$V_B = 230\angle -120^0$$

$$V_C = 230\angle 120^0$$

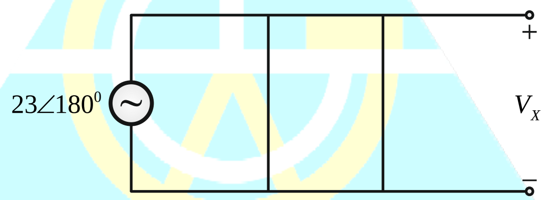
$$\text{Voltage across } = \frac{230}{10}\angle 180 + \frac{230}{10}\angle -120 + \frac{230}{10}\angle +120 = -46 \text{ Volts}$$

To obtain the no load rms voltage across winding "X" we have to apply the superposition theorem. To obtain the voltage due to single source by short circuiting other two sources the entire circuit will be short circuited hence the voltage across winding "X" will be zero.

By referring the primary side parameter to secondary side the transformer circuit will be modified as follow



Now considering the voltage induced in winding "X" due to A phase, the other two phases sources will be short circuited and the transformer circuit will be modified as follow



But here the instantaneous resultant flux due to all three phases are not considered at the same time which is wrong hence the given question is conceptually wrong.

Hence, IIT Delhi has declared it as MTA question.

Question 32

A non-ideal diode is biased with a voltage of –0.03 V and a diode current of I_1 is measured. The thermal voltage is 26 mV and the ideality factor for the diode is $15/13$. The voltage, in V, at which the measured current increases to $1.5I_1$ is closest to

(A) –4.50 (B) –0.09 (C) –1.50 (D) –0.02

Ans. (B)

Sol. Given : For non-ideal diode

$$I_D = I_0\left(e^{V/\eta V_T} - 1\right)$$

$$V_1 = -0.03 \text{ V}$$

$$V_2 = ?$$

$$I_2 = 1.5 I_1$$

$$V_T = 26 \text{ mV}$$

$$\eta = \frac{15}{13}$$

$$\frac{I_{D_2}}{I_{D_1}} = 1.5 = \frac{I_0\left(e^{V_2/\eta V_T} - 1\right)}{I_0\left(e^{V_1/\eta V_T} - 1\right)}$$

$$1.5e^{V_1/\eta V_T} - 1.5 = e^{V_2/\eta V_T} - 1$$

$$1.5e^{V_1/\eta V_T} - e^{V_2/\eta V_T} = 0.5$$

$$e^{V_2/\eta V_T} = 1.5e^{V_1/\eta V_T} - 0.5$$

$$e^{V_2/\eta V_T} = 1.5e^{\frac{-0.03}{\frac{15}{13}\times 0.026}} - 0.5$$

$$e^{V_2/\eta V_T} = 0.0518$$

$$\frac{V_2}{\eta V_T} = \ln 0.0518 = -2.9603$$

$$V_2 = -2.9603 \times \frac{15}{13} \times 0.026$$

$$V_2 = -0.088 \approx -0.09$$

Hence, the correct option is (B).

Question 33

Suppose for input $x(t)$ a linear time - invariant system with impulse response $h(t)$ produces output $y(t)$, so that $x(t) * h(t) = y(t)$. Further, if $|x(t)| * |h(t)| = z(t)$ which of the following statements is true ?

(A) For some but not all $t \in (-\infty, \infty), z(t) \le y(t)$

(B) For all $t \in (-\infty, \infty), z(t) \ge y(t)$

(C) For all $t \in (-\infty, \infty), z(t) \le y(t)$

(D) For some but not all $t \in (-\infty, \infty), z(t) \ge y(t)$

Ans. (B)

Sol. Given : $x(t) * h(t) = y(t)$

$$|x(t)| * |h(t)| = z(t)$$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) dz$$

= Area under function $x(\tau).h(t-\tau)$

= Maximum if $x(\tau)$ and $h(t-\tau)$ have same sign

$$|x(t)| * |h(t)| = \int_{-\infty}^{\infty} |x(\tau)||h(t-\tau)| dz$$

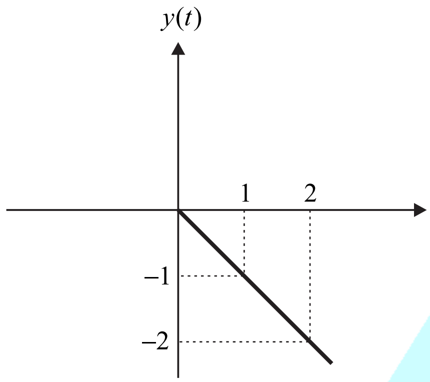
=Maximum area under $x(\tau).h(t-\tau)$

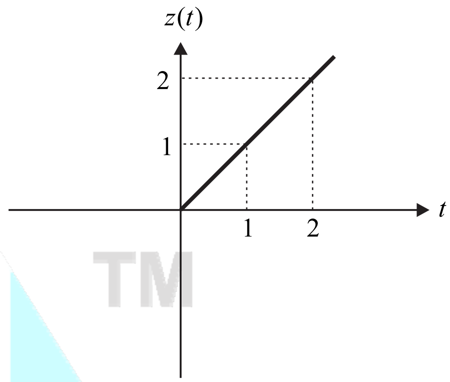
$Z(t) \ge y(t)$ for all $t \in (-\infty, \infty)$

Ex. $x(t) = -u(t) \qquad h(t) = u(t)$

$$x(t) * h(t) = -r(t) = y(t)$$

$$|x(t)| * |h(t)| = u(t) * u(t) = r(t) = Z(t)$$

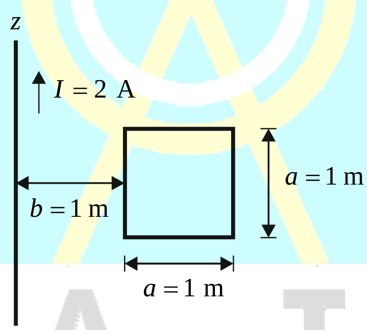




Hence, the correct option is (B).

Question 34

A conducting square loop of side length 1 m is placed at a distance of 1 m from a long straight wire carrying a current $I = 2$ A as shown below. The mutual inductance, in nH (rounded off to 2 decimal places), between conducting loop and the long wire is ___________.

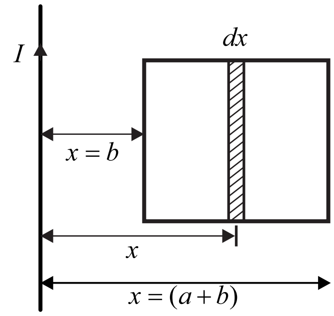


Ans. 138.63

Sol. Given : $I = 2A,\ a = 1\text{m},\ b = 1\text{m}$

Consider a strip of width dx (of the square loop) at a distance x from the wire carrying current. Magnetic field due to current carrying wire at a distance x from the wire is given by,

$$B = \frac{\mu_0 I}{2\pi x} \qquad \ldots\text{(i)}$$



Small amount of magnetic flux associated with the strip,

$$d\phi = BdA = \frac{\mu_0 I}{2\pi x}(adx) \qquad \ldots\text{(ii)}$$

Magnetic flux linked with the square loop is

$$\phi = \frac{\mu_0 Ia}{2\pi}\int_{x=b}^{a+b}\frac{dx}{x}$$

$$\phi = \frac{\mu_0 Ia}{2\pi x}\left[\ln(x)\right]_b^{a+b} = \frac{\mu_0 Ia}{2\pi}\ln\left[\frac{a+b}{b}\right]$$

Since, $\phi = MI$ where, M = Mutual Inductance

$$MI = \frac{\mu_0 Ia}{2\pi}\ln\left(\frac{a}{b}+1\right)$$

$$M = \frac{\mu_0 a}{2\pi}\ln\left(\frac{a}{b}+1\right) \qquad \ldots\text{(iii)}$$

From equation (iii),

$$M = \frac{4\pi\times 10^{-7}\times 1}{2\pi}\ln(2)$$

$$M = 138.63 \text{ nH}$$

Question 35

Consider a permanent magnet dc (PMDC) motor which is initially at rest. At t = 0, a dc voltage of 5V is applied to the motor. Its speed monotonically increases from 0 rad/s to 6.32 rad/s in 0.5s and finally settles at 10 rad/s. Assuming that the armature inductance of the motor is negligible, the transfer function for the motor is

(A) $\frac{10}{0.5s+1}$ (B) $\frac{10}{s+0.5}$ (C) $\frac{2}{s+0.5}$ (D) $\frac{2}{0.5s+1}$

Ans. (D)

Sol. **Method 1**

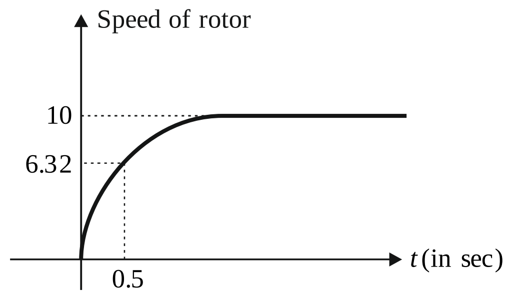
Given for a PMDC, starting from rest at t = 0,

Applied DC voltage = 5 V

Initial speed of motor at t = 0 sec is 0 rad/sec, Speed of motor at t = 0.5 sec is 6.32 rad/sec and

Final speed (in steady state) = 10 rad/sec

Speed of motor will follow the response of a first order system as shown below,



Let the transfer function,

$$H(s) = \frac{C(s)}{R(s)} = \frac{K}{1+\tau s} \quad \text{… (i)}$$

$$r(t) = \text{Applied input} = 5u(t)$$

$$R(s) = \frac{5}{s}$$

$$C(s) = \frac{5K}{\tau s\left(s+\frac{1}{\tau}\right)}$$

$$C(s) = \frac{5K}{\tau}\left[\frac{1}{\left(\frac{1}{\tau}-0\right)}\left\{\frac{1}{s}-\frac{1}{s+\frac{1}{\tau}}\right\}\right] = 5K\left[\frac{1}{s}-\frac{1}{s+\frac{1}{\tau}}\right]$$

∴ Speed of motor as a function of time is given as

$$c(t) = 5K\left[1-e^{-t/\tau}\right] \quad \text{… (ii)}$$

Given at steady state, $c(t) = 10$ rad/sec

∴ $$c(t)\big|_{t=\infty} = 10$$

$$10 = 5K(1-0) \qquad \therefore \qquad K = 2$$

Also, given that at $t = 0.5$ sec, $c(t) = 6.32$

So, from equation (ii),

$$6.32 = 2\times 5\left[1-e^{-0.5/\tau}\right]$$

$$e^{-1/2\tau} = 1-0.632 = 0.368$$

$$-\frac{1}{2\tau} = \ln 0.368 = -1$$

$$\tau = 0.5$$

So, the transfer function of the system from equation (i) is given as

$$H(s) = \frac{2}{1+0.5s}$$

Hence, the correct option is (D).

Method 2

Given that the final value for a step input of 5 V is 10.

$$\text{Find value } = \lim_{s\to 0} s(\text{T.F.}) \times \frac{5}{s}$$

$$\text{For T.F.} = \frac{10}{0.5s+1}$$

$$\text{Final value } = \lim_{s\to 0} s \times \frac{10}{0.5s+1} \times \frac{5}{s} = \frac{10\times 5}{1} = 50 \neq 10$$

Hence, the option (A) is wrong.

$$\text{For T.F.} = \frac{10}{s+0.5}$$

$$\text{Final value } = \lim_{s\to 0} s \times \frac{10}{s+0.5} \times \frac{5}{s} = \frac{10\times 5}{0.5} = 100 \neq 10$$

Hence, the option (B) is wrong.

$$\text{For T.F.} = \frac{2}{s+0.5}$$

$$\text{Final value} = \lim_{s\to 0} s \times \frac{2}{s+0.5} \times \frac{5}{s} = \frac{2\times 5}{0.5} = 20 \neq 10$$

Hence, the option (C) is wrong.

$$\text{For T.F.} = \frac{2}{0.5s+1}$$

$$\text{Final value} = \lim_{s\to 0} s \times \frac{2}{0.5s+1} \times \frac{5}{s} = \frac{2\times 5}{1} = 10$$

Hence, the option (D) is correct.

Hence, the correct option is (D).

Question 36

The causal realization of a system transfer function $H(s)$ having poles at (2, –1), (–2, 1) and zeroes at (2, 1), (–2, –1) will be

(A) Stable, real, all pass

(B) Unstable, complex, all pass

(C) Unstable, real, high pass

(D) Stable, complex, low pass

Ans. (B)

Sol. **Given :** For a causal system transfer function has poles and zeros located at

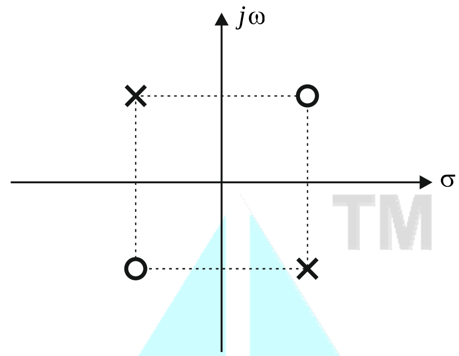
poles, $s_1 = (2, -1)$

$s_2 = (-2,1)$

Zeros $s_1 = (2,1)$

$s_2 = (-2,-1)$

Pole zero plot is shown in figure.



As the locations of poles and zeros are symmetrical about the imaginary axis, so the magnitude of transfer function $H(s)$ will be constant over all frequencies so the system represents an **all pass filter.**

As complex poles or zeros are not lying in conjugate pairs, so the system will be **complex.**

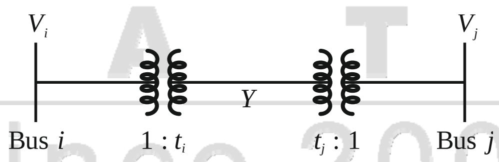
Given that the system is causal, so the ROC of system function will be right to the right most pole. Since the right most pole is having real part $\sigma = +2$ so ROC: $\text{Re}\{s\} > 2$.

As the ROC does not include $j\omega$ axis so the system is **unstable.**

Hence, the correct option is (B).

Question 37

Two buses, *i* and *j*, are connected with a transmission line of admittance *Y*, at the two ends of which there are ideal transformers with turns ratio as shown below. Bus admittance matrix for the system is



(A) $\begin{bmatrix} t_i t_j Y & -t_j^2 Y \\ -t_i^2 Y & t_i t_j Y \end{bmatrix}$

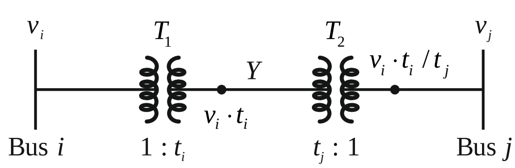
(B) $\begin{bmatrix} -t_i t_j Y & t_j^2 Y \\ t_i^2 Y & -t_i t_j Y \end{bmatrix}$

(C) $\begin{bmatrix} t_i^2 Y & -t_i t_j Y \\ -t_i t_j Y & t_j^2 Y \end{bmatrix}$

(D) $\begin{bmatrix} t_i t_j Y & -(t_i - t_j)^2 Y \\ -(t_i - t_j)^2 Y & t_i t_j Y \end{bmatrix}$

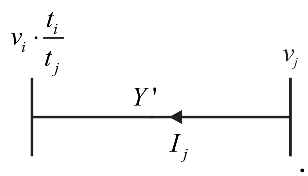
Ans. (C)

Sol.



Admittance refered to primary side of $T_{1t} = Y.t_i^2$

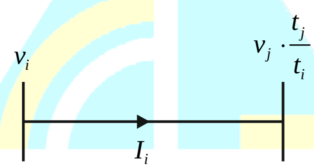
Admittance refered to secondary side of $T_2 = Y.t_j^2$



$$I_j = \left(V_j - v_i \frac{t_i}{t_j}\right) Y'$$

$$I_j = \left[V_j - V_i . \frac{v_i}{t_j}\right] Yt_j^2 = v_j.Y.t_j^2 - v_i.t_i t_j Y$$

$$I_j = -t_i t_j Y v_i + Y t_j^2 v_j$$



$$I_i = \left(v_i - v_j \frac{t_j}{t_i}\right) Y t_i^2$$

$$I_i = Y t_i^2 v_i - v_j Y t_i t_j$$

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} t_i^2 Y & -t_i t_j Y \\ -t_i t_j Y & t_j^2 Y \end{bmatrix} \begin{bmatrix} v_i \\ v_j \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} t_i^2 Y & -t_i t_j Y \\ -t_i t_j Y & t_j^2 Y \end{bmatrix}$$

Hence, the correct option is (C).

Question 38

A benchtop dc power supply acts as an ideal 4 A current source as long as its terminal voltage is below 10 V. Beyond this point, it begins to behave as an ideal 10 V voltage source for all load currents going down to 0 A, When connected to an ideal rheostat, find the load resistance value at which maximum power is transferred and the corresponding load voltage and current.

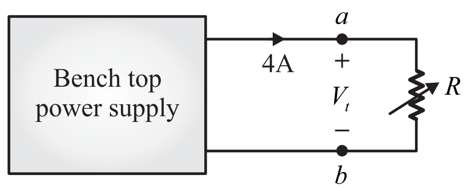
(A) 2.5 Ω, 4 A, 10 V (B) Short, ∞ A, 10 V (C) 2.5 Ω, 4 A, 5 V (D) Open, 4 A, 0 V

Ans. (A)

Sol. **Method 1**

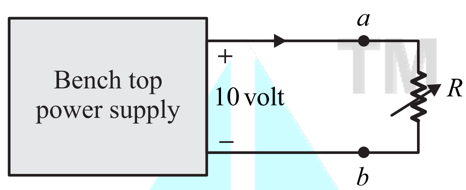
Case I :

Benchtop dc power supply will act as a ideal 4 A current source as long as it a terminal voltage is less than 10 V



Case II :

Benchtop dc power supply will act as a 10 V ideal voltage source for all load currents going down to 0 A



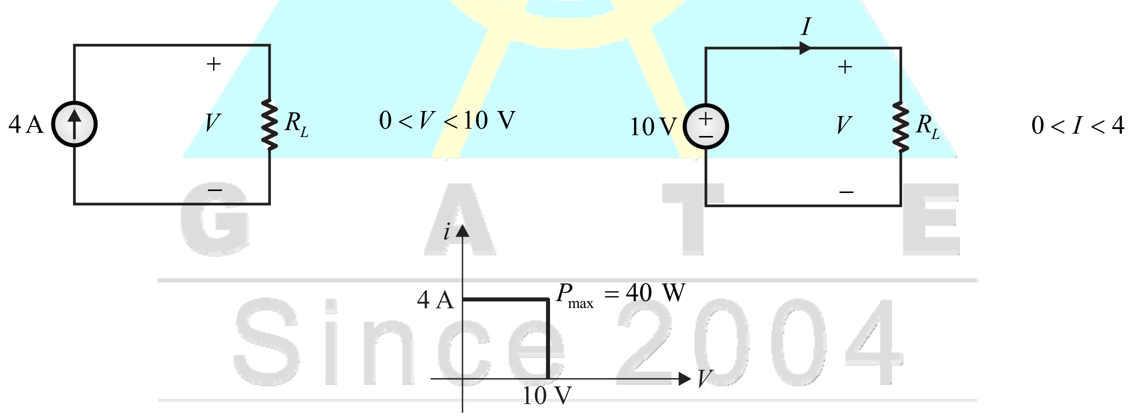
From **Case I** the maximum resistance offered by the ideal 4 A current source is $R = \frac{V}{I} = \frac{10}{4} = 2.5\Omega$

For maximum power transfer the value of load resistance should be equal to source resistance = 2.5

Current and voltage corresponding to load resistance of 2.5Ω are 4 A and 10 Volt respectively.

Hence, the correct option is (A).

Method 2



From graph, it is clear that at maximum power $V = 10$ V and $i = 4$ A.

So, $\quad 4^2 \times R_L = 40 \qquad \frac{10^2}{R_L} = 40$

$\quad R_L = 2.5\ \Omega \qquad R_L = 2.5\ \Omega$

Question 39

The vector function expressed by

$$\vec{F} = \hat{a}_x(5y - k_1 z) + \hat{a}_y(3z + k_2 x) + \hat{a}_z(k_3 y - 4x)$$

represents a conservative field, where $\hat{a}_x, \hat{a}_y, \hat{a}_z$ are unit vector along x, y and z directions, respectively. The value of constants k_1, k_2, k_3 are given by

(A) $k_1 = 3, k_2 = 3, k_3 = 7$

(B) $k_1 = 3, k_2 = 8, k_3 = 5$

(C) $k_1 = 0, k_2 = 0, k_3 = 0$

(D) $k_1 = 4, k_2 = 5, k_3 = 3$

Ans. (D)

Sol. Given : $\vec{F} = \hat{a}_x(5y - k_1 z) + \hat{a}_y(3z + k_2 x) + \hat{a}_z(k_3 y - 4x)$

Since $\vec{F}$ is conservative, $\nabla \times \vec{F} = 0$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ (5y - k_1 z) & (3z + k_2 x) & (k_3 y - 4x) \end{vmatrix} = 0$$

$$\hat{a}_x[k_3 - 3] - \hat{a}_y[-4 + k_1] + \hat{a}_z[k_2 - 5] = 0$$

Hence, $k_1 = 4,\ k_2 = 5,\ k_3 = 3$

Hence, the correct option is (D).

Question 40

A single - phase, full bridge, fully controlled thyristor rectifier feeds a load comprising a 10 Ω resistance in series with a very large inductance. The rectifier is fed from an ideal 230 V, 50 Hz sinusoidal source through cables which have negligible internal resistance and a total inductance of 2.28 mH. If the thyristors are triggered at an angle $\alpha = 45^0$, the commutation overlap angle in degree (rounded off to 2 decimal places) is ________.

Ans. 4.8

Sol. Given : Single-phase, full bridge, fully controlled thyristor rectifier

(i) $V_s = 230\text{ V}$

(ii) $R = 10\,\Omega$

(iii) $L = 2.28$ mH

(iv) $\alpha = 45^0$

(v) $f = 50$ Hz

Output voltage of Single-phase, full bridge, fully controlled thyristor rectifier with source inductance is given by

$$V_0 = \frac{2V_m}{\pi} \times \cos\alpha - 4fL_s I_0$$

$$I_0 \times R = \frac{2V_m}{\pi} \times \cos\alpha - 4fL_s I_0$$

$$I_0 \times 10 = \frac{2 \times 230\sqrt{2} \times \cos 45}{\pi} - 4 \times 50 \times 2.28 \times 10^{-3} \times I_0$$

$$I_0 = 14 \text{ A}$$

Voltage drop due to source inductance is given by,

$$2\omega L_s I_0 = V_m\left[\cos\alpha - \cos(\alpha+\mu)\right]$$

$$14 = \frac{230\sqrt{2}}{2\times 100\pi\times 2.28\times 10^{-3}}\left[\cos 45 - \cos(45+\mu)\right]$$

$$\mu = 4.8^0$$

Question 41

A cylindrical rotor synchronous generator has steady state synchronous reactance of 0.7 pu and sub transient reactance of 0.2 pu. It is operating at $(1+j0)$ pu terminal voltage with an internal emf of $(1+j0.7)$ pu. Following a 3-ϕ solid short circuit fault at the terminal of the generator, the magnitude of the subtransient internal emf (rounded off to 2 decimal places) is_______pu.

Ans. 1.02

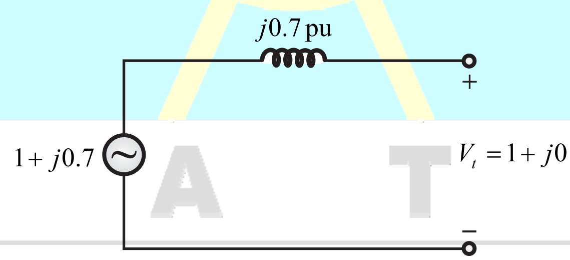
Sol. **Given :** Cylindrical rotor synchronous generator

(i) $E_g = 1 + j0.7$ pu

(ii) $X_d = j0.7$ pu

(iii) $X_d'' = j0.2$ pu

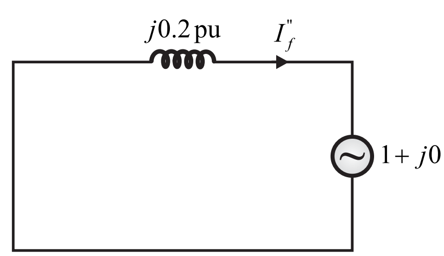
Case-I : Prefault condition



$$I_g = \frac{E_g - V_t}{X_d} = \frac{1+j0.7-1}{j0.7} = 1\angle 0^0 \text{ pu}$$

$$I_g = 1 \text{pu}$$

Case-II : Post fault



$$I_f'' = \frac{1\angle 0^0}{0.2\angle 90^0}$$

$$I_f = 5\angle -90^0$$

$$(I_g)_{total} = I_f + I_f'' = 1\angle 0^0 + 5\angle -90^0$$

$$(I_g)_{total} = 1 - j5$$

Sub transient emf $= (I_g)_{total} \times X_d''$

$$E_f'' = (1 - j5)\times j0.2$$

$$E_f'' = 1 + j0.2$$

$$|E_f''| = 1.0198\,\text{pu}$$

Question 42

A non-ideal Si-based pn junction diode is tested by sweeping the bias applied across its terminals from –5 V to +5 V. The effective thermal voltage V_T, for the diode is measured to be (29 ± 2) mV. The resolution of voltage source in the measurement range is 1mV. The percentage uncertainty (rounded off to 2 decimal places) in the measured current at a bias voltage of 0.02 V is _____.

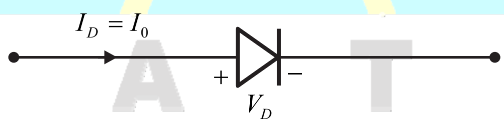
Ans. 4.75

Sol. Given : $V_D = 0.02\,\text{V}$

$$V_T = (29\pm 2)\,\text{mV} = (0.029 \pm 0.002)$$

$$I_D = I \cong I_0 e^{V_D/\eta V_T}$$

Non ideal silicon diode is as shown below



Applying log on both sides on diode current equation

$$\ln(I) = \ln(I_0) + \frac{V_D}{\eta V_T}$$

Differentiating partially with respect to V_T,

$$\frac{\partial I}{I} = 0 + \frac{V_D}{\eta}\times\left(-\frac{1}{V_T^2}\right)\partial V_T$$

$$\frac{\partial I}{I} = -\frac{IV_D}{\eta V_T^2}\times \partial V_T$$

For $[\eta = 1]$, $\quad \dfrac{\partial I}{\partial V_T} = -\dfrac{I.V_D}{V_T^2}$

The expression of uncertainty for diode current equation is given below,

$$W_{res} = W_I = \pm\sqrt{\left(\frac{\partial I}{\partial V_T}\right)^2 .W_v^2} = \pm\frac{\partial I}{\partial V_T}\times W_V$$

$$W_{res} = W_I = \pm\frac{I.V_D}{V_T^2}.W_V = \pm\frac{I\times 0.02}{(0.029)^2}\times 0.002 = \pm 0.0475I$$

$$\%\frac{W_I}{I} = \pm 0.0475\times 100 = \pm 4.75$$

Percentage uncertainty $= \pm 4.75\%$

Question 43

Which of the following options is true for a linear time - invariant discrete time system that obeys the difference equation :

$$y[n]-ay[n-1]=b_0x[n]-b_1x[n-1]$$

(A) When $x[n]=0$, $n<0$, the function $y[n]$; $n>0$ is solely determined by the function $x[n]$.

(B) The system is necessarily causal.

(C) The system impulse response is non-zero at infinitely many instants.

(D) $y[n]$ is unaffected by the values of $x[n-k]$; $k>2$.

Ans. (C)

Sol. Given for a discrete time LTI system, difference equation is

$$y[n]-ay[n-1]=b_0x[n]-b_1x[n-1]$$

$$y[n]=ay[n-1]+b_0x[n]+b_1x[n-1]$$

Given system is recursive if $a\neq 0$ and difference equation is of the standard form

$$y[n]=\sum_{K=1}^{M}a_K y[n-K]+\sum_{K=1}^{M}b_K x[n-K]$$

As recursive discrete time LTI system represents IIR filter, so the impulse response may exist for infinite instants but it depends on values of constants a, b_0 and b_1, which can be verified by taking Z-transform in equation

$$Y(z)-az^{-1}Y(z)=b_0X(z)-b_1z^{-1}X(z)$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{b_0-bz^{-1}}{1-az^{-1}}$$

$$H(z)=\frac{b_0}{1-az^{-1}}-b_1\left[\frac{1}{1-az^{-1}}\right]z^{-1} \qquad \ldots \text{(i)}$$

Taking causal inverse Z-transform,

$$h[n]=b_0.a^n u[n]-b_1\ a^{n-1}u[n-1] \qquad \ldots \text{(ii)}$$

Taking non-causal inverse Z-transform of equation (i),

$$h[n] = -b_0 a^n u[-n-1] + b_1 a^{n-1} u[-n]$$

So, a non-causal system having impulse response $h[n] = -b_0 a^n u[-n-1] + b_1 a^{n-1} u[-n]$ will also have the same transfer function as obtained in equation (i).

From equation (ii), $h[n]$ exist for infinite instants for most of the values of b_0, a and b_1 but for some values, like $b_0 = 1$, $a = 1$, $b_1 = 1$

$$h[n] = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{otherwise} \end{cases}$$

So statement (C) is conditionally true.

Hence, the correct option is (C).

Question 44

For real numbers, x and y, with $y = 3x^2 + 3x + 1$, the maximum and minimum value of y for $x \in [-2, 0]$ are respectively,

(A) -2 and $-\frac{1}{2}$ (B) 7 and 1 (C) 1 and $\frac{1}{4}$ (D) 7 and $\frac{1}{4}$

Ans. (D)

Sol. Given : $y = 3x^2 + 3x + 1$, where $x \,\varepsilon\, [-2, 0]$

For maxima or minima,

$$\frac{dy}{dx} = 0$$

$\Rightarrow$ $6x + 3 = 0$

$\Rightarrow$ $x = -\frac{1}{2}$

The second derivative of y is,

$$\frac{d^2y}{dx^2} = 6 > 0$$

$\Rightarrow$ y will have minima at $x = \frac{-1}{2} \in [-2, 0]$

$\therefore$ Minimum value of y will be

$$y_{\min} = 3\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + 1$$

$$= \frac{3}{4} - \frac{3}{2} + 1 = \frac{1}{4}$$

At $x = -2 \Rightarrow y = 3(-2)^2 + 3(-2) + 1 = 7$

And $x = 0 \Rightarrow \quad y = 3(0)^2 + 3(0) + 1 = 1$

$\therefore$ Maximum value of y occurs at $x = -2$ and is equal to 7.

And minimum value of y occurs at $x = \frac{-1}{2}$ and is equal to $\frac{1}{4}$.

Hence, the correct option is (D).

Question 45

The number of purely real elements in a lower triangular representation of the given 3×3 matrix, obtained through the given decomposition is

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}^T$$

(A) 5 (B) 8 (C) 6 (D) 9

Sol. **Given :**

$$[P] = \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix}$$

$$[Q] = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$[R]^T = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$[P] = [Q][R]^T$$

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11}^2 & a_{11}a_{12} & a_{11}a_{13} \\ a_{12}a_{11} & a_{12}^2 + a_{22}^2 & a_{12}a_{13} + a_{22}a_{23} \\ a_{13}a_{11} & a_{13}a_{12} + a_{23}a_{22} & a_{13}^2 + a_{23}^2 + a_{33}^2 \end{bmatrix}$$

Comparing the elements of matrix P with the matrix $Q.R^T$

$$a_{11}^2 = 2$$

$$a_{11} = \pm\sqrt{2}$$

$$a_{11}a_{12} = 3$$

$$a_{12}=\pm\frac{3}{\sqrt{2}}$$

$$a_{11}a_{13}=3$$

$$a_{13}=\pm\frac{3}{\sqrt{2}}$$

$$a_{12}^2+a_{22}^2=2$$

$$a_{22}^2=2-a_{12}^2=2-\left(\frac{9}{2}\right)$$

$$a_{22}^2=-\frac{5}{2}$$

$$a_{22}=\pm j\sqrt{\frac{5}{2}}$$

$$a_{12}a_{13}+a_{22}a_{23}=1$$

$$\left(\frac{3}{\sqrt{2}}\right)\left(\frac{3}{\sqrt{2}}\right)+j\sqrt{\frac{5}{2}}a_{23}=1$$

$$a_{23}j\left(\sqrt{\frac{5}{2}}\right)=1-\frac{9}{2}$$

$$a_{23}\left(j\sqrt{\frac{5}{2}}\right)=\frac{-7}{2}$$

$$a_{13}^2+a_{23}^2+a_{23}^2=7$$

$$a_{23}^2=7-\frac{9}{2}+\frac{49}{10}$$

$$a_{33}=\pm\sqrt{9.4}$$

From this the lower triangular matrix can be written as,

$$\text{Lower triangular matrix}=\begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$a_{11}=\pm\sqrt{2},\qquad a_{12}=a_{13}=\pm\frac{3}{\sqrt{2}}$$

$$a_{22}=+j\frac{\sqrt{5}}{\sqrt{2}},\quad a_{23}=\frac{j7}{\sqrt{10}}$$

$$a_{33} = \pm\sqrt{9.4}$$

In the above lower triangular matrix elements $[a_{11}, a_{13}, a_{23}, a_{33}, 0, 0, 0]$ are real elements

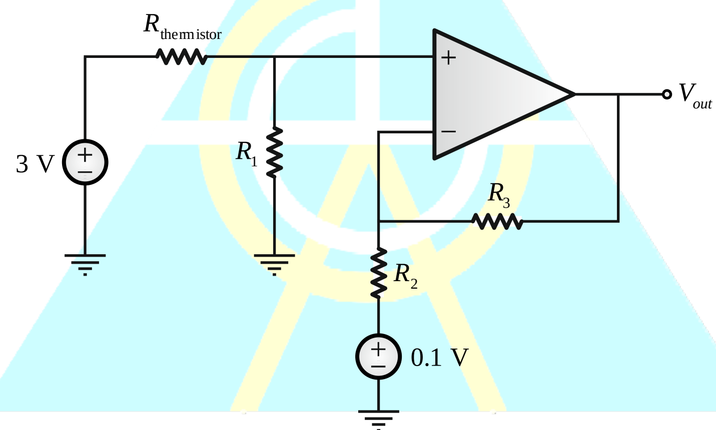
So, total number of real elements = 7

IIT has declared this questions as MTA.

As the correct answer for the question, i.e. total number of real elements equal to 7, was absent in the given options, so after facing challenge to their given option in first answer key, finally IIT has declared this question as **Marks to All.**

Question 46

The temperature of the coolant oil bath for a transformer is monitored using circuit shown. It contains a thermistor with a temperature - dependent resistance, $R_{thermistor} = 2(1+\alpha T)\ \text{k}\Omega$, where T is the temperature in ^0C. The temperature coefficient, α, is $-(4\pm0.25)\%/^0\text{C}$. Circuit parameters : $R_1 = 1\ \text{k}\Omega,\ R_2 = 1.3\ \text{k}\Omega,$ $R_3 = 2.6\ \text{k}\Omega$. The error in the output signal (in V, rounded off to 2 decimal places) at 150 ^{0}C is _______.

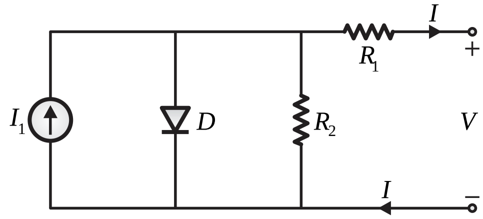


Ans. (MTA)

Sol. ***This topic is out of the syllabus and the data given in the question is wrong. Hence, IIT Delhi has declared it as MTA question.***

Question 47

Consider the diode circuit shown below. The diode D, obeys the current-voltage characteristics $I_D = I_S\left[\exp\left(\frac{V_D}{nV_T}\right)-1\right]$, where $n>1, V_T>0, V_D$ is the voltage across the diode and I_D is the current through it. The circuit is biased so that voltage, $V>0$ and current, $I<0$. If you had to design this circuit to transfer maximum power from the current source (I_1) to a resistive load (not shown) at the output, what values of R_1 and R_2 would you choose?

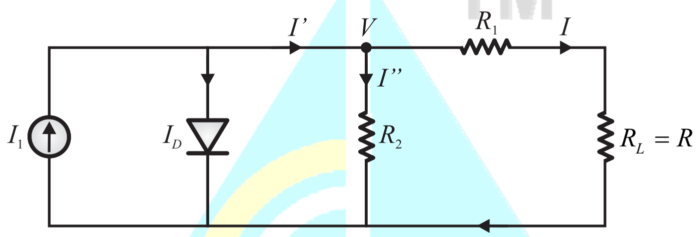


(A) Small R_1 and small R_2 (B) Small R_1 and large R_2

(C) Large R_1 and large R_2 (D) Large R_1 and small R_2

Ans. (B)

Sol. Given circuit is shown in figure,



For maximum power transfer to the resistive load, the load current must be maximum.

From KCL at node V.

$$I' = I + I'' \qquad \text{where, } I' = I_1 - I_D$$

As I' is constant, so for load current I to be maximum, I'' must be minimum.

For I'' to be minimum, R_2 must be high.

$$I'' = \frac{V}{R_2}$$

For I to be maximum R_1 must be low.

$$I = \frac{V}{R_1 + R_L}$$

Hence, the correct option is (B).

Question 48

The static electric field inside a dielectric medium with relative permittivity, $\varepsilon_r = 2.25$, expressed in cylindrical coordinate system is given by the following expression

$$\mathbf{E} = \mathbf{a}_r 2r + \mathbf{a}_\phi \left(\frac{3}{r}\right) + \mathbf{a}_z 6$$

where $\mathbf{a}_r$, $\mathbf{a}_\phi$, $\mathbf{a}_z$ are unit vectors along r, ϕ and z directions, respectively. If the above expression represents a valid electrostatic field inside the medium, then the volume charge density associated with this field in terms of free space permittivity ε_0, in SI units is given by

(A) $3\varepsilon_0$ (B) $5\varepsilon_0$

(C) $4\varepsilon_0$ (D) $9\varepsilon_0$

Ans. **(C)**

Sol. **Given :** $\mathbf{E} = \mathbf{a}_r 2r + \mathbf{a}_\phi\left(\frac{3}{r}\right) + \mathbf{a}_z 6$

From Maxwell's equation,

$$P_V = \nabla.\vec{D}$$

$$P_V = \in_0\left(\nabla.\vec{E}\right)$$

$$P = \nabla.\vec{E}$$

$$P = \frac{1}{r}\frac{\partial}{\partial r}(rE_r) + \frac{1}{r}\frac{\partial}{\partial \phi}E\phi + \frac{\partial E_z}{\partial z}$$

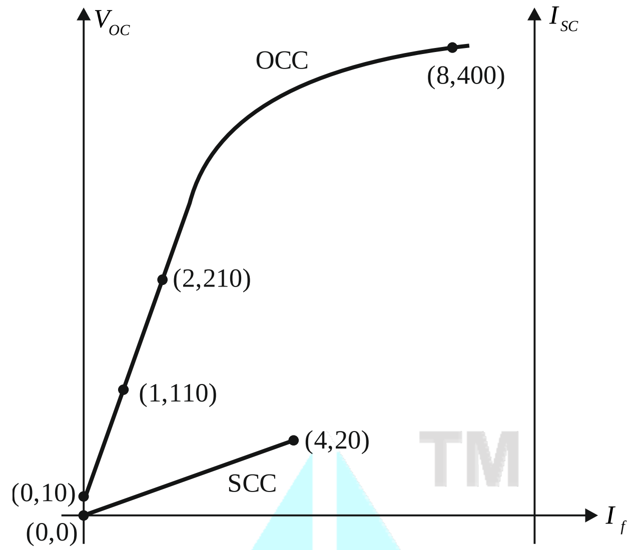
$$P = \frac{1}{r}\frac{\partial}{\partial r}\left(2r^2\right) + 0 + 0$$

$$P = \frac{4r}{r} = 4$$

$$\Rightarrow \qquad P_V = \varepsilon_0 P = 4\varepsilon_0$$

Question 49

The figure below shows the per-phase Open Circuit Characteristics (measured in V) and Short Circuit Characteristics (measured in A) of a 14 kVA, 400 V, 50 Hz, 4-pole, 3-phase, delta connected alternator, driven at 1500 rpm. The field current, I_f is measured in A. Readings taken are marked as respective (x, y) coordinates in the figure. Ratio of the unsaturated and saturated synchronous impedances $(Z_{s(\text{unsat})}/Z_{s(\text{sat})})$ of the alternator is closest to



(A) 2.025 (B) 2.000 (C) 2.100 (D) 1.000

Ans. **(C)**

Sol. **Given :** Three phase delta connected Alternator

(i) Rating $= 14\,\text{kVA}$

(ii) $V_t = 400\,\text{V}$

(iii) $f = 50\,\text{Hz}$

(iv) $P = 4$

From SCC at $I_f = 4\,\text{A}, I_{SC} = 20\,\text{A}$

$I_f = 2A, I_{SC} = 10\ \text{A}$

$I_f = 8\text{A}, I_{SC} = 40\,\text{A}$

Case I : From linear Portion

Unsaturated synchronous impedance is calculated from linear portion of OCC,

$I_f = 2\,\text{A}$

$$Z_s = \frac{V_{OC}}{I_{SC}} = \frac{210}{10} = 21\Omega$$

Case II : From nonlinear Portion

Saturated synchronous impedance is calculated from nonlinear portion of OCC,

$I_f = 8\,\text{A}$

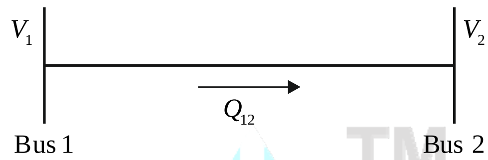
$$Z_s = \frac{V_{OC}}{I_{SC}} = \frac{400}{40} = 10\ \Omega$$

The ratio of unsaturated synchronous impedance to saturated synchronous impedance is given by

$$\frac{Z_{s(\text{unsaturated})}}{Z_{s(\text{saturated})}} = \frac{21}{10} = 2.1$$

Question 50

Bus 1 with voltage magnitude $V_1 = 1.1$ pu is sending reactive power Q_{12} towards bus 2 with voltage magnitude $V_2 = 1$ pu through a lossless transmission line of reactance X. Keeping the voltage at bus 2 fixed at 1 pu, magnitude of voltage at bus 1 is changed, so that the reactive power Q_{12} sent from bus 1 is increased by 20%. Real power flow through the line under both the condition is zero. The new value of the voltage magnitude, V_1, in pu (rounded off to 2 decimal places), at bus 1 is __________.



Ans. 1.12

Sol. Given :

(i) $V_1 = 1.1$ pu

(ii) $V_2 = 1$ pu

Reactive power, $Q_{12} = \dfrac{V_1 V_2}{X}\cos(\delta_1 - \delta_2) - \dfrac{V_2^2}{X}$

Since active power flow is zero hence, $\delta_1 = \delta_2$

$$Q_{12} = \frac{V_1 V_2}{X} - \frac{V_2^2}{X} = \frac{1.1\times 1}{X} - \frac{1}{X}$$

$$Q_{12} = \frac{0.1}{X}$$

From given condition new reactive is given by,

$$Q_{12}' = 1.2 Q_{12}$$

Let new bus 1 voltage $= V_1'$

$$1.2\times\frac{0.1}{X} = \frac{V_1' \cdot V_2}{X}\cos(\delta_1 - \delta_2) - \frac{V_2^2}{X}$$

$$\frac{0.12}{X} = \frac{V_1'}{X} - \frac{1^2}{X}$$

$$V_1' = 1 + 0.12 = 1.12 \text{ pu}$$

Question 51

A 250 V dc shunt motor has an armature resistance of 0.2 Ω and a field resistance of 100 Ω. When the motor is operated on no-load at rated voltage, it draws an armature current of 5 A and runs at 1200 rpm. When a load is coupled to the motor, it draws total line current of 50 A at rated voltage, with a 5 % reduction in the air-gap flux due to armature reaction. Voltage drop across the brushes can be taken as 1

V per brush under all operating conditions. The speed of the motor, in rpm, under this loaded condition, is closest to

(A) 900 (B) 1200 (C) 1000 (D) 1220

Ans. (D)

Sol. Given : DC shunt motor

(i) $V_t = 250\,\text{V}$

(ii) $R_a = 0.2\,\Omega$

(iii) $R_f = 100\,\Omega$

(iv) $(I_a)_{nl} = 5\,\text{A}$

(v) $N_{nl} = 1200\,\text{rpm}$

(vi) $(I_a)_l = 50\,\text{A}$

(vii) $\phi_2 = 0.95\phi_1$

(viii) $V_b = 1\text{V}$ per brush

$$I_f = \frac{V_t}{R_{sh}} = \frac{250}{100} = 2.5\,\text{A}$$

Case I : No load condition

$$V_t = (E_b)_{nl} + (I_a)_{nl} r_a + V_b \times 2$$

$$250 = (E_b)_{nl} + 5 \times 0.2 + 2 \times 1$$

$$250 = (E_b)_{nl} + 1 + 2$$

$$(E_b)_{nl} = 247\,\text{V} \quad \ldots\text{(i)}$$

Case II : Loaded condition

$$I_L = I_a + I_{sh}$$

$$I_a = 50 - 2.5 = 47.5\,\text{A}$$

$$V_t = (E_b)_l + (I_a)_l \times r_a + 2 \times V_b$$

$$250 = (E_b)_l + 47.5 \times 0.2 + 2 \times 1$$

$$(E_b)_l = 238.5\,\text{V}$$

$$\frac{(E_b)_n}{(E_b)_l} = \frac{\phi_1 \times N_{nl}}{\phi_2 \times N_l}$$

$$\frac{247}{238.5} = \frac{\phi_1 \times 1200}{0.95\phi_1 \times N_2}$$

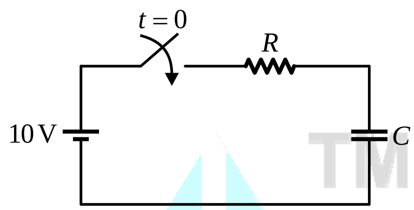
$$N_{fl} = 1220\,\text{rpm}$$

Question 52

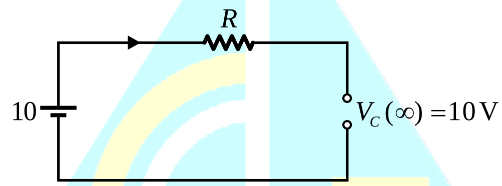
A resistor and a capacitor are connected in series to a 10 V dc supply through a switch. The switch is closed at $t=0$ and the capacitor voltage is found to cross 0 V at $t=0.4\,\tau$, where τ is circuit time constant. The absolute value of percentage change required in the initial capacitor voltage if the zero crossing has to happen at $t=0.2\tau$ is ______ (rounded off to 2 decimal places).

Ans. **54.98**

Sol. Given arrangement in the problem is shown in figure below,

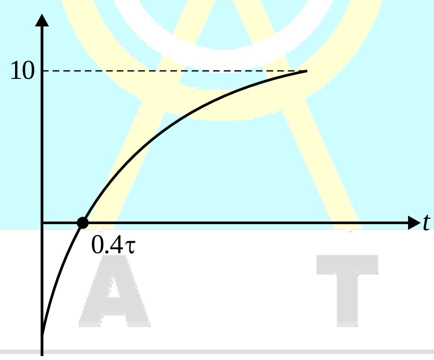


At steady state, capacitor will be open circuited. So the circuit before switching is as shown below,



From given conditions,

Case 1 : $V_c(t)=0$ at $t=0.4\tau$



$$V_c(t)=V_c(\infty)+\left\{V_c(0^+)-V_c(\infty)\right\}e^{-t/\tau}$$

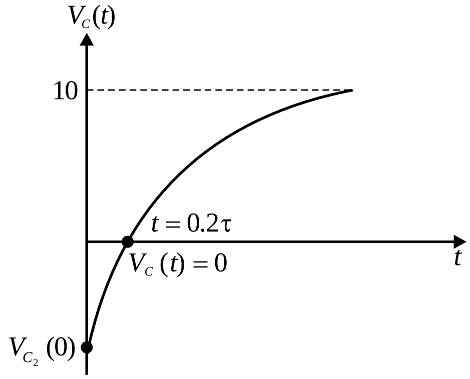
At $t=0.4\tau$, $\quad V_c(t)=0$

$$0=10+\left\{V_{c_1}(0^+)-10\right\}e^{-0.4}$$

$$V_{c_1}(0^+)=\frac{10\,e^{-0.4}-10}{e^{-0.4}}$$

$$V_{c_1}(t=0^+)=10-10e^{0.4}=-4.92\text{ V}$$

Case 2 : $V_c(t)=0$ at $t=0.2\tau$



$$V_c(t) = V_c(\infty) + \{V_c(0^+) - V_c(\infty)\}e^{-t/\tau}$$

At $t = 0.2\tau$, $V_c(t) = 0$

$$0 = 10 + \{V_{c_2}(0^+) - 10\}e^{-0.2}$$

$$V_{c_2}(0^+) = \frac{10e^{-0.2} - 10}{e^{-0.2}}$$

$$V_{c_2}(t = 0^+) = 10 - 10e^{0.2} = -2.214\,\text{V}$$

% change in initial voltage

$$= \frac{V_{c_1}(0^+) - V_{c_2}(0^+)}{V_{c_1}(0^+)} \times 100 = \frac{-4.92 - (-2.214)}{-4.92} \times 100$$

$$= 0.5499 \times 100 = 54.99\%$$

Question 53

Consider a negative unity feedback system with the forward path transfer function $\dfrac{s^2 + s + 1}{s^3 + 2s^2 + 2s + K}$ where K is positive real number. The value of K for which the system will have some of its poles on the imaginary axis, is

(A) 8 (B) 9 (C) 6 (D) 7

Ans. (A)

Sol. Given : Forward path transfer function

$$G(s) = \frac{s^2 + s + 1}{s^3 + 2s^2 + 2s + k}$$

Method 1 :

Characteristic equation is given by,

$$1 + G(s) = 0$$

$$1 + \frac{s^2 + s + 1}{s^3 + 2s^2 + 2s + k} = 0$$

$$s^3 + 3s^2 + 3s + k + 1 = 0$$

Routh Table :

s^3	1	3
s^2	3	$k+1$
s^1	$\frac{9-(k+1)}{3}$	0
s^0	$k+1$	0

When the poles will be lying on imaginary axis, then system will become marginally stable at this condition is satisfy by the row of zeros in the Routh's table, other than the last row. So

$$\frac{9-(k+1)}{3}=0$$

$$k=8$$

Hence, the correct option is (A).

Method 2 :

For characteristic equation, as^3+bs^2+cs+d

For marginal stability or for some of the poles at imaginary axis,

Inner product = Outer product

$$a\times d=b\times c$$

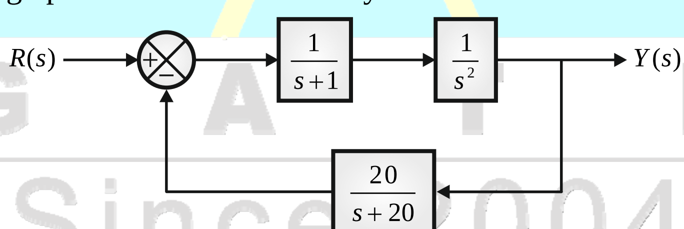
Here, $a=1$, $b=3$, $c=3$ and $d=(k+1)$

$$k+1=3\times3$$

$$k=9-1=8$$

Question 54

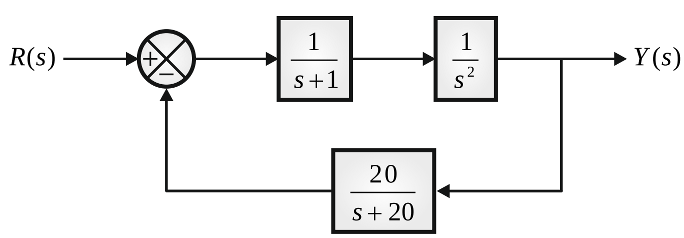
Which of the following options is correct for the system shown below?



(A) 3rd order and unstable
(B) 4th order and stable
(C) 3rd order and stable
(D) 4th order and unstable

Ans. (D)

Sol.



Characteristic equation is given by,

$$1+G(s)H(s)=0$$

$$1+\frac{1}{(s+1)}\frac{1}{s^2}\frac{20}{(s+20)}=0$$

$$s^2(s+1)(s+20)+20=0$$

$$s^2[s^2+21s+20]+20=0$$

$$s^4+21s^3+20s^2+20=0 \qquad \rightarrow 4^{th} \text{ order}$$

's' term missing hence it is unstable.

We can check using RH Rule,

s^4	1	20	20
s^3	21	0	
s^2	20	20	
s^1	–21	0	
s^0	20		

4^{th} order and unstable

Hence, the correct option is (D).

Question 55

A cylindrical rotor synchronous generator with constant real power output and constant terminal voltage is supplying 100 A current at 0.9 lagging power factor load. An ideal reactor is now connected in parallel with the load, as a result of which the total lagging reactive power requirement of the load is twice the previous value while the real power remains unchanged. The armature current is now ___________ A (rounded off to 2 decimal places).

Ans. **125.29**

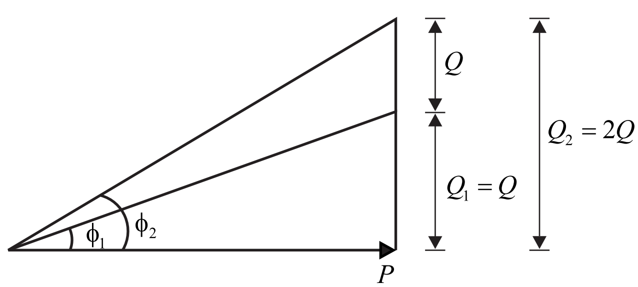
Sol. **Given :** Cylindrical rotor synchronous generator

(i) $I_a = 100\,\text{A}$

(ii) $\cos\phi_1 = 0.9\,\text{lag}$; $\phi = 25.84^0$

(iii) $Q_2 = 2Q_1$

(iv) $P_2 = P_1$



Case I : Before connecting reactor in parallel with the load

As active power is constant

$$I_1 \cos\phi_1 = I_2 \cos_2$$

Reactive power before connecting reactor in parallel with the load is given by,

$$\frac{Q_1}{P_1} = \tan\phi_1$$

$$Q_1 = P_1 \tan\phi_1 = P_1 \times \tan 25.84$$

Case II : After connecting reactor in parallel with the load

$$\tan\phi_2 = \frac{Q_2}{P_2} = \frac{Q_2}{P_1} = \frac{2Q_1}{P_1} = 2\tan\phi_1$$

$$\tan\phi_2 = 2\tan\phi_1 = 2\times\tan 25.84 = 0.9686$$

$$\cos\phi_2 = 0.7182 \,\text{lag}$$

From the given condition of constant active power

$$I_1 \cos\phi_1 = I_2 \times \cos\phi_2$$

$$100\times\cos 25.84 = I_2 \times 0.7182$$

$$I_2 = 125.29\,\text{A}$$

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