General Aptitude

Q.1 to Q.5 Carry one mark each

Question 1

He is known for his unscrupulous ways. He always sheds _______ tears to deceive people.

(A) crocodile (B) fox's (C) fox (D) crocodile's

Ans. (A)

Question 2

Jofra Archer, the England fast bowler, is _______ than accurate

(A) more fast (B) more faster (C) less fast (D) faster

Ans. (A)

Question 3

Select the word that fits the analogy

Build : Building : : Grow : _______.

(A) Growth (B) Grew (C) Growed (D) Grown

Ans. (A)

Question 4

I do not think you know the case well enough to have opinions. Having said that, I agree with your other point.

What does the phrase "having said that" mean in the given text?

(A) as opposed to what I have said

(B) despite what I have said

(C) contrary to what I have said

(D) in addition to what I have said

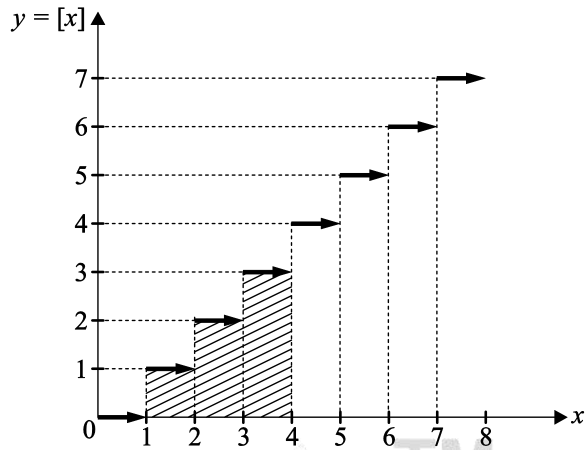
Ans. (B)

Question 5

Define $[x]$ as the greatest integer less than or equal to x for each $x \in (-\infty, \infty)$, If $y = [x]$, then area under y for $x \in [1, 4]$ is _______.

(A) 4 (B) 3 (C) 6 (D) 1

Ans. (C)



Above is the graph of $y=[x]$, where $[x]$ is the greatest integer less than or equal to x.

Area under y, for $x\in[1,\ 4]$

$$= \text{Area of shaded region}$$
$$=(1\times1)+(1+2)+(1+3)$$
$$=1+2+3=6$$

Hence, the correct option is (C).

Q.6 to Q.10 Carry two marks each

Question 6

Crowd funding deals with mobilisation of funds for a project from a large number of people, who would be willing to invest smaller amounts through web-based platforms in the project.

Based on the above paragraph, which of the following is correct about crowdfinding:

(A) Funds raised through voluntary contributions on web-based platforms.

(B) Funds raised through unwilling contributions on web-based platforms.

(C) Funds raised through large contributions on web-based platforms.

(D) Funds raised through coerced contributions on web-based platforms.

Ans. (A)

Question 7

P, Q, R and S are to be uniquely coded using α and β. If P is coded as $\alpha\alpha$ and $\alpha\beta$, then R and S respectively, can be coded as _____.

(A) $\beta\alpha$ and $\beta\beta$ (B) $\alpha\beta$ and $\beta\beta$ (C) $\beta\beta$ and $\alpha\alpha$ (D) $\beta\alpha$ and $\alpha\beta$

Ans. (A)

Given :

Sol. P is coded as $\alpha\alpha$

Q is coded as $\alpha\beta$

Since, P, Q, R and S all are uniquely coded. So, R and S cannot be coded as $\alpha\alpha$ or $\alpha\beta$.

In option (B) R is coded as $\alpha\beta$.

In option (C) S is coded as $\alpha\alpha$.

In option (D) S is coded as $\alpha\beta$.

So, these options are wrong.

Only option (A) contains unique code for R and S.

Hence, the correct option is (A).

Question 8

The sum of the first n terms in the sequence 8, 88, 888, 8888, …..is ________.

(A) $\frac{81}{80}(10^n-1)+\frac{9}{8}n$

(B) $\frac{81}{80}(10^n-1)-\frac{9}{8}n$

(C) $\frac{80}{81}(10^n-1)+\frac{8}{9}n$

(D) $\frac{80}{81}(10^n-1)-\frac{8}{9}n$

Ans. **(D)**

Sol. Given $S=8+88+888+$

Method 1 :

$$S=8+88+888+\text{..... upto } n \text{ terms}$$

$$S=8(1+11+111+......)$$

$$S=\frac{8}{9}(9+99+999+......)$$

$$S=\frac{8}{9}[(10-1)+(100-1)+(1000-1)+.....]$$

$$S=\frac{8}{9}[(10+100+1000+.....)-(1+1+1+.....n\text{ times})]$$

$$S=\frac{8}{9}[(10+100+1000+.....)-n]$$

$10+100+1000+.....$ forms G.P.

In G.P. sum of n terms,

$$S_n=\frac{a(r^n-1)}{r-1}, \qquad r>1$$

Here, $a=10$, $r=\frac{100}{10}=10$

$$\therefore \quad S=\frac{8}{9}\left[\left(\frac{10(10^n-1)}{10-1}\right)-n\right]$$

$$S=\frac{80}{81}(10^n-1)-\frac{8}{9}n$$

Hence, the correct option is (D)

Method 2 :

Sum of first 1 terms, $s_1=8$

Sum of first 2 terms, $s_2 = 8 + 88$

Sum of first 3 terms, $s_3 = 8 + 88 + 888$

So, on substituting $n = 1$ in options, one which results in 8 will be correct.

Option A :

$$= \frac{81}{80}(10-1) + \frac{9}{8}(1)$$

$$= \frac{81}{80} \times 9 + \frac{9}{8} = 10.2375$$

Option B :

$$= \frac{81}{80}(10-1) - \frac{9}{8}(1)$$

$$= \frac{81}{80} \times 9 - \frac{9}{8} = 7.9875$$

Option C :

$$= \frac{80}{81}(10-1) + \frac{8}{9}(1)$$

$$= \frac{80}{81} \times 9 + \frac{8}{9} = 9.778$$

Option D :

$$= \frac{80}{81}(10-1) - \frac{8}{9}(1)$$

$$= \frac{80}{81} \times 9 - \frac{8}{9} = 8$$

Only option (D) results to 8.

Hence, the correct option is (D).

Method 3 :

Multiply by 10 on both sides

$$10s = 80 + 880 + 8880 +$$

$$\therefore \quad -9S = 8 + 8 + 8 \text{times}$$

$$S = \frac{(8+n) \times 10 - 8n}{9}$$

$$= 80 + 800 \qquad \text{[Taking 80 common]}$$

$$S = \frac{80(10^n - 1)}{9}$$

$$\therefore \text{ Total sum } s = \frac{80(10^n - 1)}{9 \times 9} - \frac{8n}{9}$$

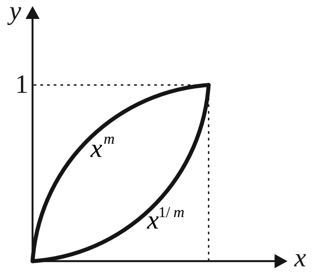
$$s = \frac{80(10^n - 1)}{81} - \frac{8n}{9}$$

Hence, the correct option is (D)

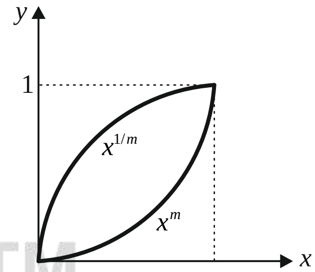
Question 9

Select the graph that schematically represents both $y = x^m$ and $y = x^{1/m}$ properly in the interval $0 \le x \le 1$, for integer values of m, where $m > 1$.

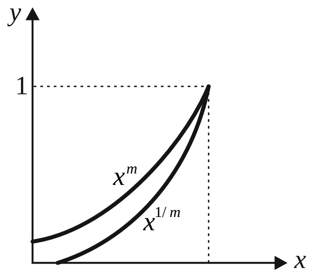
(A)



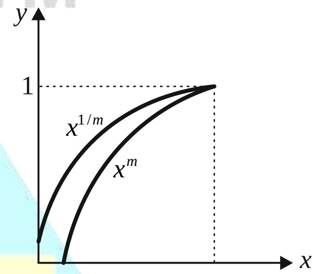
(B)



(C)



(D)



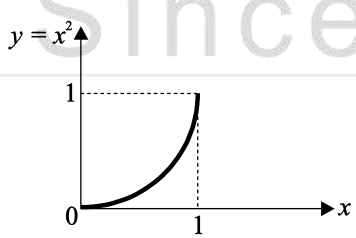
Ans. **(B)**

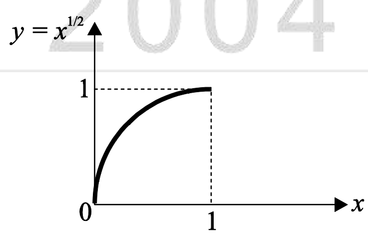
Sol. $y = x^m$ and $y = x^{1/m}$

Range of x : $0 \le x \le 1$

Let $m = 2$

x	$y = x^2$	$y = x^{1/2}$
0	0	0
0.25	0.0625	0.5
0.5	0.25	0.707
0.75	0.5625	0.8667
1	1	1



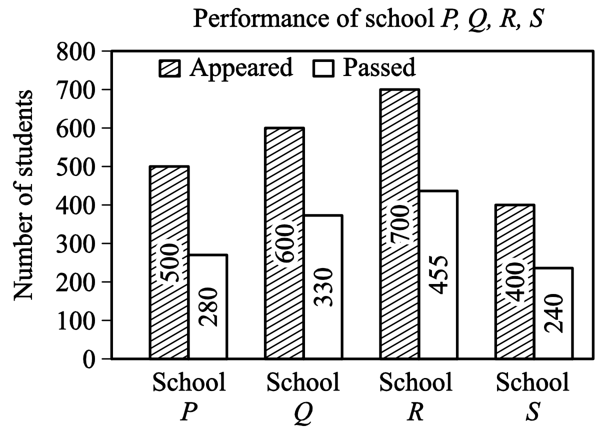


Same graph is observed for any value of m.

Hence, correct option is (B).

Question 10

The bar graph shows the data of students who appeared and passed in an examination for four schools *P*, *Q*, *R* and *S*. The average of success rate (in percentage) of these four schools is _______.



(A) 59.3 % (B) 58.8 % (C) 59.0 % (D) 58.5 %

Ans. (C)

Sol. Success rate of students of school $P = \frac{280}{500} \times 100 = 56\%$

Success rate of students of school $Q = \frac{330}{600} \times 100 = 55\%$

Success rate of students of school, $R = \frac{455}{700} \times 100 = 65\%$

Success rate of student of school, $S = \frac{240}{400} \times 100 = 60\%$

Average of success rates of these schools (in percentage)

$$= \frac{56+55+65+60}{4} = 59\%$$

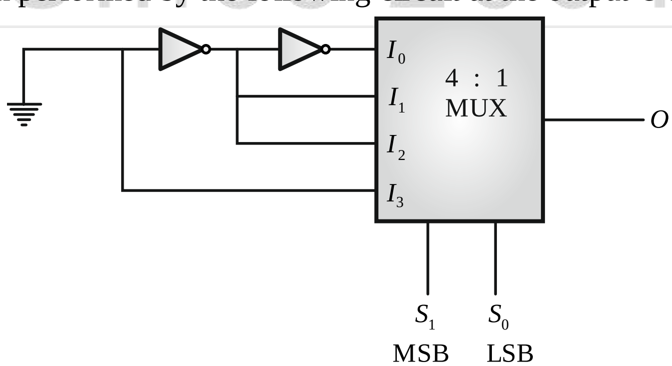
Hence, the correct option is (C).

Technical Section

Q.1 to Q.25 Carry one mark each

Question 1

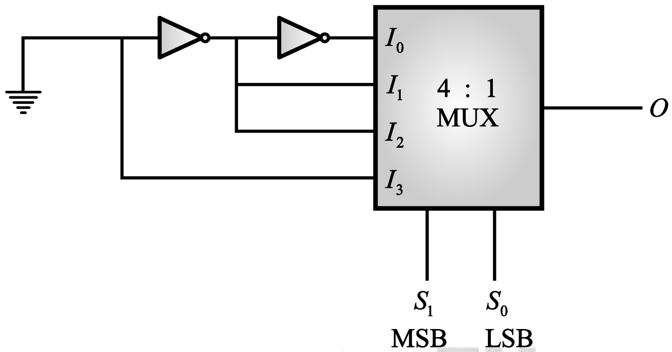
The Boolean operation performed by the following circuit at the output O is______



(A) $O = S_1 + S_0$ (B) $O = S_1 \oplus S_0$ (C) $O = S_0 \cdot \overline{S_1}$ (D) $O = S_1 \cdot \overline{S_0}$

Ans. **(B)**

Sol. Given circuit is shown in figure below,



From the given arrangement,

$$I_0 = 0$$

$$I_1 = 1$$

$$I_2 = 1$$

$$I_3 = 0$$

$$O = \bar{S}_1\bar{S}_0 I_0 + \bar{S}_1 S_0 I_1 + S_1\bar{S}_0 I_2 + S_1 S_0 I_3$$

$$O = \bar{S}_1\bar{S}_0(0) + \bar{S}_1 S_0 + S_1\bar{S}_0 + S_1 S_0(0)$$

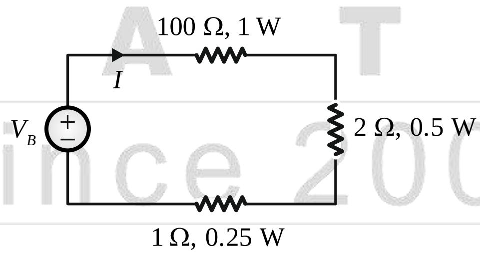
$$O = \bar{S}_1 S_0 + S_1\bar{S}_0$$

$$O = S_1 \oplus S_0$$

Hence, the correct option is (B).

Question 2

In the circuit shown below, the safe maximum value for current I is



(A) 0.1 A (B) 1.0 A (C) 0.5 A (D) 0.05 A

Ans. **(A)**

Sol. Maximum current for 100 Ω, 1 W ;

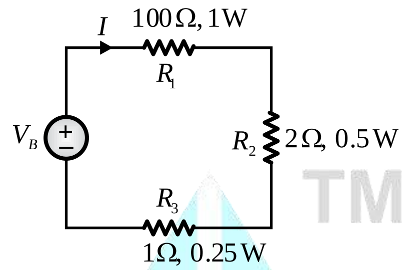
$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{1}{100}} = 0.1 \text{ A}$$

Maximum current for 2 Ω, 0.5 W ;

$$I = \sqrt{\frac{0.5}{2}} = 0.5 \text{ A}$$

Maximum current for $1\ \Omega$, 0.25 W;

$$I = \sqrt{\frac{0.25}{1}} = 0.5 \text{ A}$$

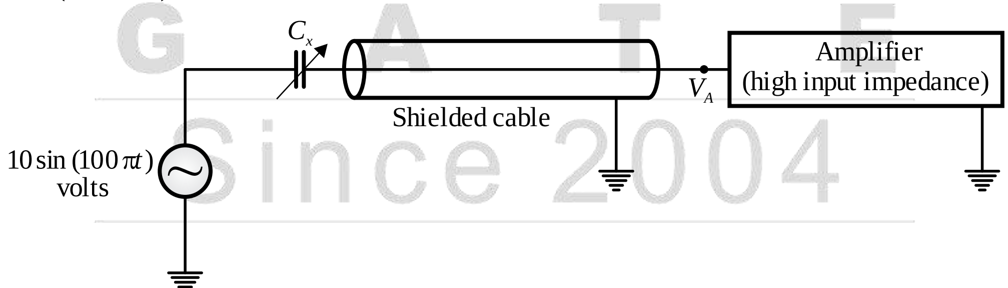


As all resistors are in series, the current through them will be same. Therefore, the maximum current will be either 0.1 A or 0.5 A. If it is 0.5 A, the resistor R_1 will be damaged as maximum allowed current through R_1 is 0.1 A. Hence the maximum current in the circuit to avoid any damage should be 0.1 A as it is less than or equal to maximum allowed currents of the individual resistors R_1, R_2 and R_3.

Hence, the correct option is (A).

Question 3

The capacitance C_x of a capacitive type sensor is $(1000\,x)$ pF, where x is the input to the sensor. As shown in the figure, the sensor is excited by a voltage $10\sin(100\,\pi t)$ V. The other terminal of the sensor is tied to the input of a high input impedance amplifier through a shielded cable, with shield connected to ground. The cable capacitance is 100 pF. The peak of the voltage V_A at the input of the amplifier when $x = 0.1$ (in volts) is _______



Ans. **(5)**

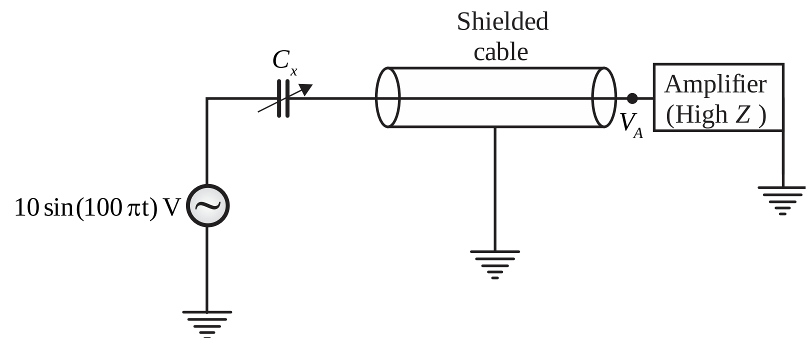
Sol. Given in question, $C_x = (1000\,x)$pF

Input voltage $= 10\sin(100\pi t)$V

$$C_{cable} = 100\,\text{pF}$$

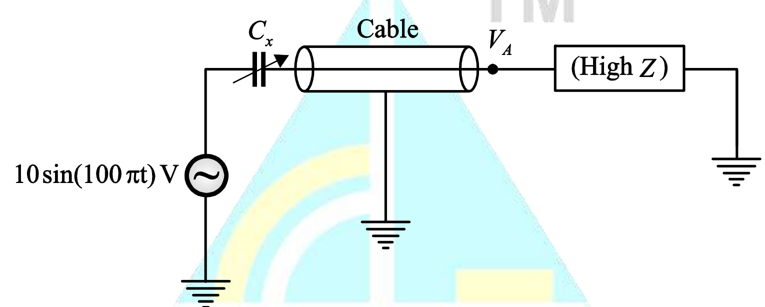
$$x = 0.1$$

To Find $V_{A_{peak}}$

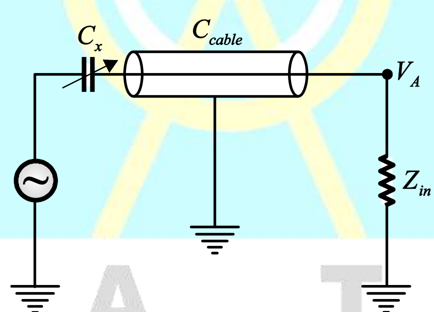


$\therefore$ $C_x = (1000 \times 0.1) = 100\,\text{pF}$

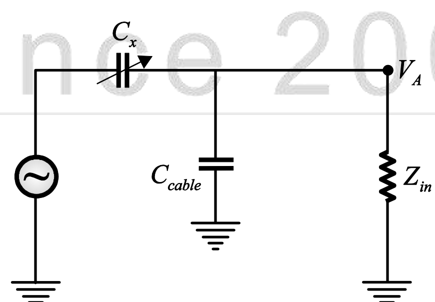
Now look at the network here



$\Rightarrow$

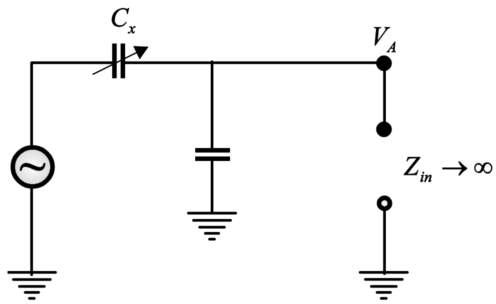


Looking at the network cable is in parallel to both V_i & Z_{in} with $C_{cable} = 100\,\text{pf}$



Here, $Z_{in} >> 1$

Since, $Z_{in} >> 1$, consider no current drawn or a practical open circuit



$\therefore$ By voltage divider rule,

$$V_{A_{peak}} = \frac{Z_{cable}}{(Z_{cable} + Z_x)} \times V_{i_{peak}}$$

$$V_{A_{peak}} = \frac{\frac{1}{j\omega C_{cable}}}{\frac{1}{j\omega C_{cable}} + \frac{1}{j\omega C_x}} \times V_{i_{peak}} = \frac{10 \times \frac{1\times 10^{12}}{j\omega \times 100}}{\frac{1}{j\omega}\left(\frac{10^{12}}{100} + \frac{10^{12}}{100}\right)}$$

$$V_{A_{peak}} = \frac{10}{2} = 5\,\text{volts}$$

$$V_{A_{peak}} = 5\,\text{V}$$

Hence, the correct answer is 5 V.

Question 4

Given $f(A,B,C,D) = \Sigma\text{m}\,(0, 1, 2, 6, 8, 9, 10, 11) + \Sigma\, d\,(3, 7, 14, 15)$ is Boolean function, where m represents min-terms and d represents don't cares. The minimal sum of product expression for f is ______

(A) $f = \bar{D} + A$ (B) $\bar{A}B + \bar{C}D$ (C) $\bar{B} + C$ (D) $A\bar{B} + CB$

Ans. **(C)**

Sol. $f(A,B,C,D) = \sum m(0,1,2,6,8,9,10,11) + \sum d(3,7,14,15)$

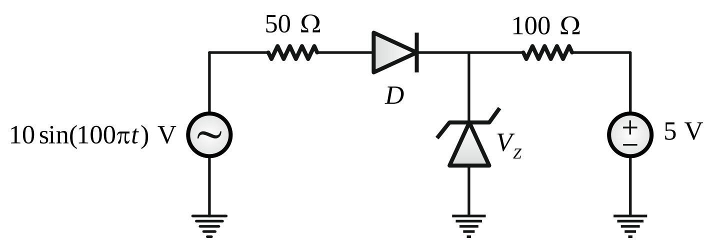
AB \ CD	00	01	11	10
00	1	1	X	1
01			X	1
11			X	X
10	1	1	1	1

$f = \bar{B} + C$

Hence, the correct option is (C).

Question 5

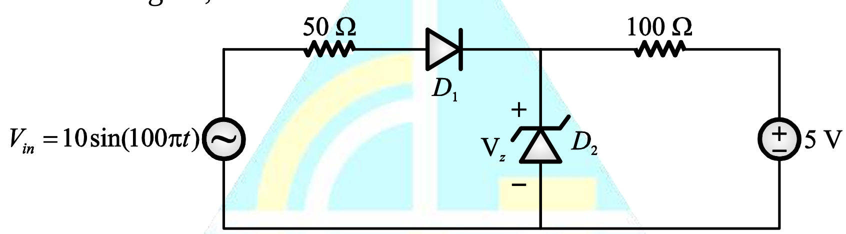
If diodes in the circuit shown are ideal and the breakdown voltage V_Z of the Zener diode is 5 V, the power dissipated in the 100 Ω resistor (in watts) is________



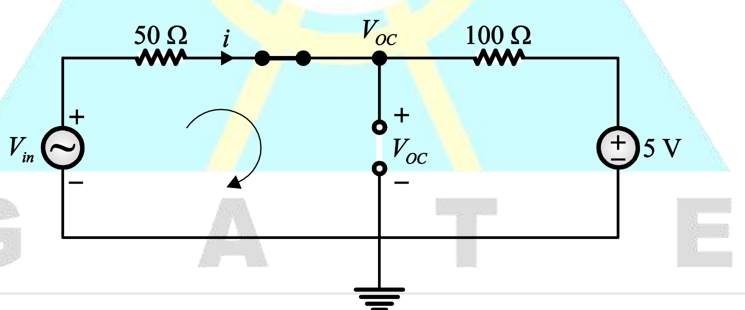
(A) 1 (B) 0 (C) $\frac{25}{100}$ (D) $\frac{225}{100}$

Ans. **(B)**

Sol. Given circuit is shown in figure,



Considering D_1 to be ON finding open circuit voltage across Zener to check whether it will go into breakdown or not,



If $V_{OC} > V_Z$ then breakdown of Zener occurs.

$$i = \frac{V_{in} - 5}{50 + 100} = \frac{10\sin 100\pi t - 5}{150} \qquad [\, i > 0 \text{ for } V_{in} > 5\,]$$

So, assumption of D_1 to be ON is correct for $V_{in} > 5$.

Applying KVL,

$$-V_{in} + 50i + V_{OC} = 0$$

$$\therefore \qquad V_{OC} = V_{in} - 50\left[\frac{10\sin 100\pi t - 5}{150}\right]$$

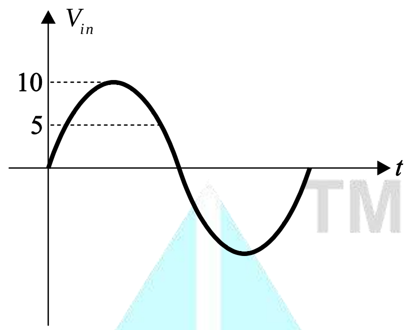
$$V_{OC} = \frac{3V_{in} - V_{in} + 5}{3} = \frac{2V_{in} + 5}{3}$$

If $V_{OC} > V_Z$ then breakdown occurs.

$$\frac{2V_{in}+5}{3} > 5$$

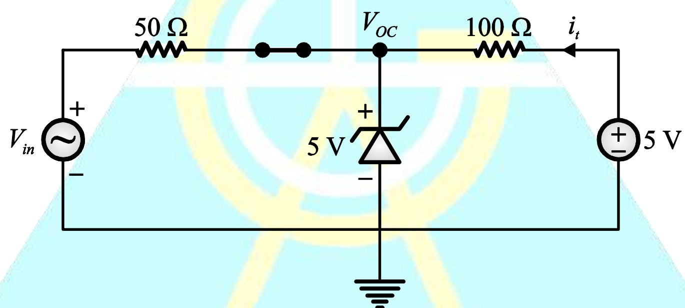
$$2V_{in} > 10\text{ V}$$

$$V_{in} > 5\text{ V}$$



Since, $V_{in} = 10\sin 100\pi t$

When input exceeds 5 V then Zener diode goes to breakdown and voltage across it is fixed at $V_Z = 5\text{V}$.



Current through 100 Ω resistor, $i_1 = \frac{5-5}{100} = 0$

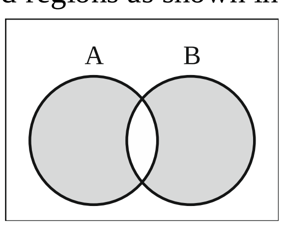
When $V_{in} < 5\text{ V}$, both diode will be OFF and current through 100 Ω remains zero.

$\therefore$ Power in 100 Ω resistor = 0 W.

Hence, the correct option is (B).

Question 6

The Boolean expression for the shaded regions as shown in the figure is

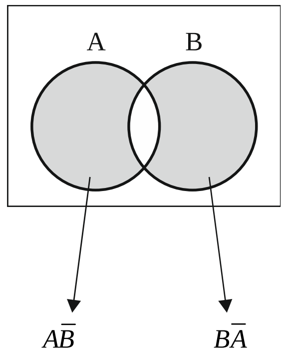


(A) $(A+\bar{B})\cdot(\bar{A}+B)$ (B) $(\bar{A}+\bar{B})\cdot(A+\bar{B})$ (C) $(\bar{A}+B)\cdot(\bar{A}+\bar{B})$ (D) $(A+B)\cdot(\bar{A}+\bar{B})$

Ans. **(D)**

Sol.

Method 1



$$Y = \Pi M(0, 3)$$

$$\overline{Y} = \overline{A}\overline{B} + AB$$

$$Y = (A + B)(\overline{A} + \overline{B})$$

$A \backslash B$	0	1
0	0	1
1	1	0

Method 2

$A + B - A \cap B$

A	B	$A+B-A\cap B$
0	0	0
0	1	1
1	0	1
1	1	0

Now cross-checking from options.

Only option D satisfies the condition.

Hence the correct option is D

Question 7

Consider the signal $x(t) = e^{-|t|}$. Let $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$ be the Fourier transform of $x(t)$. The value of $X(j0)$ is _______.

Ans. **(2)**

Sol. **Given :** $x(t) = e^{-|t|}$

Let $X(j\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

Taking Fourier transform on both sides

$$X(j\omega)=\frac{2\times 1}{\omega^2+1^2}=\frac{2}{\omega^2+1} \qquad \left[\text{Since } e^{-a|t|} \xleftrightarrow{\text{F.T.}} \frac{2a}{\omega^2+a^2}\right]$$

$$X(j0)=\frac{2}{0+1}=2$$

Hence, the correct answer is 2.

Alternate Solution :

We know that

$$\text{Area of } x(t)=X(j0)$$

$$X(j0)=\text{Area of } x(t)$$

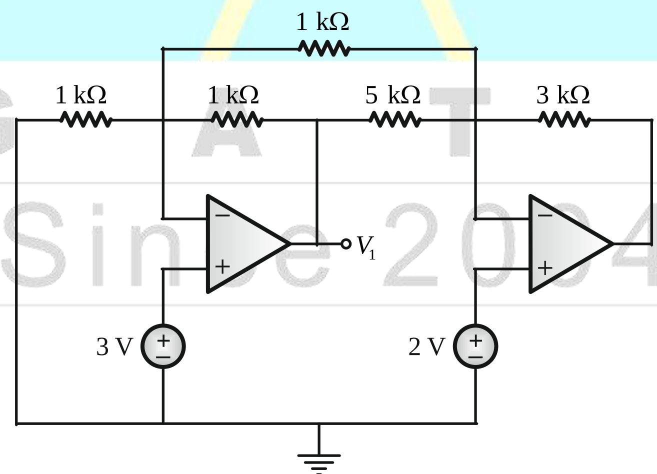
$$X(j0)=\int_{-\infty}^{\infty} x(t)\,dt$$

$$X(j0)=\int_{-\infty}^{0} e^{t}\,dt+\int_{0}^{\infty} e^{-t}\,dt$$

$$X(j0)=1+1=2$$

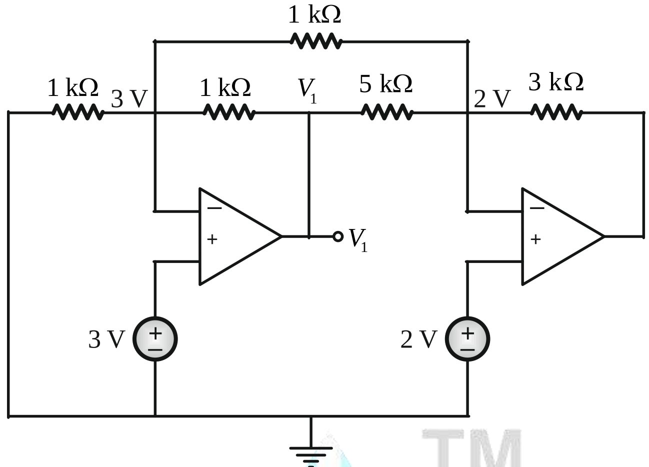
Question 8

Assuming ideal Op-Amps, the output voltage at V_1 in the figure shown (in volts) is ______



Ans. **(7)**

Sol.



Apply KCL at inverting terminal of first Op-Amp.

$$\frac{3-V_1}{1}+\frac{3-2}{1}+\frac{3-0}{1}=0$$

$\Rightarrow$ $3-V_1+1+3=0$

$\Rightarrow$ $V_1=7$ Volts

Hence, the correct answer is 7 V.

Question 9

A phase lead network has a transfer function $G(s)=\dfrac{1+0.2s}{1+0.05s}$. The angular frequency at which the maximum phase shift for network occurs is_______

(A) 200 rad/s (B) 20 rad/s (C) 10 rad/s (D) 100 rad/s

Ans. (C)

Sol. Given transfer function for lead network $G(s)=\dfrac{1+0.2s}{1+0.05s}$

General transfer function for phase lead network is,

$$G(s)=\frac{\alpha(1+sT)}{1+\alpha sT}$$

$T=0.2$

$\alpha T=0.05$

$\alpha=0.25$

Frequency in rad/sec at which maximum phase occurs is,

$$\omega_m=\frac{1}{T\sqrt{\alpha}}$$

$$\omega_m=\frac{1}{0.2\times\sqrt{0.25}}$$

$$\omega_m = 10 \text{ rad/sec}$$

Hence, the correct option is (C).

Question 10

A differentiator has transfer function whose

(A) magnitude remains constant

(B) phase increases linearly with frequency

(C) magnitude increases linearly with frequency

(D) magnitude decreases linearly with frequency

Ans. (C)

Sol. Output of differentiator,

$$v_0(t) = \frac{-RCdv_i(t)}{dt}$$

Taking Laplace transform on both sides

$$V_0(s) = -RCsV_i(s)$$

$$\frac{V_0(s)}{V_i(s)} = -RCs$$

$$T(s) = -RCs$$

$$T(j\omega) = -RC(j\omega)$$

$$|T(j\omega)| = RC\omega$$

A differentiator is a High Pass Filter and its magnitude increases with frequency.

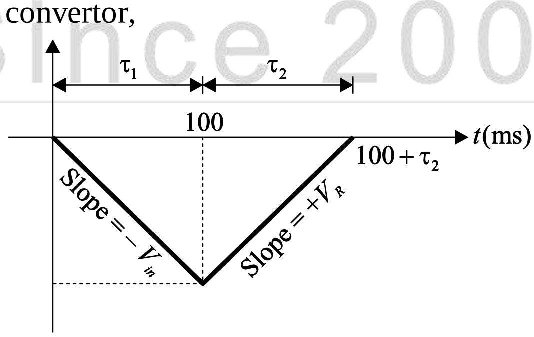
Hence, the correct option is (C).

Question 11

A 200 mV full - scale dual slope analog to digital convertor (DS-ADC) has a reference voltage of 100 mV. The first integration time is set as 100 ms. The DS-ADC is operated in the continuous conversion mode. The conversion time of the DS-ADC for an input voltage of 123.4 mV (in ms, rounded off to one decimal place) is ______

Ans. (223.4)

Sol. Given for a dual slope A/D convertor,



Full scale reading = 200 mV

Analog reference voltage $V_R = 100$ mV

Applied input analog voltage $= 123.4 \text{ mV} = V_{in}$

First integration time (fixed) $=100\text{ ms}=\tau_1$

(Input voltage × Charging time) = (Reference voltage ×Discharging time)

$$|V_{in}\times\tau_1|=|V_R\times\tau_2|$$

$$123.4\times100\times10^{-6}=100\times10^{-3}\tau_2$$

$\therefore$ $\tau_2=123.4\text{ msec}$

Total conversion time, $t_c=\tau_1+\tau_2$

$$t_c=(100+123.4)\text{ msec}=223.4\text{ msec}$$

Hence, the correct answer is 223.4 msec.

Question 12

The unit vectors along the mutually perpendicular *x*, *y* and *z* axes are $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively. Consider the plane $z=0$ and two vectors $\vec{a}$ and $\vec{b}$ on that plane such that $\vec{a}\neq\alpha\vec{b}$ for any scalar α. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ is

(A) $\hat{i}$ (B) $\hat{i}-\hat{j}$ (C) $-\hat{j}$ (D) $\hat{k}$

Ans. **(D)**

Sol. Here $z=0$

So, $\overline{a},\overline{b}$ lies in *xy* plane.

Vector perpendicular to both $\overline{a},\overline{b}$ will be in *z* axis. So it will be $\hat{k}$.

Hence, the correct answer is (D).

Question 13

A second order system has closed loop poles located at $s=-3\pm j4$. The time *t* at which the maximum value of the step response occurs (in seconds, rounded off to two decimal places) is ________.

Ans. **(0.76 to 0.81)**

Sol. Poles are located $s=-3\pm j4$

Comparing with $s=-\xi\omega_n\pm j\omega_n\sqrt{1-\xi^2}=-\xi\omega_n\pm j\omega_d$

$$\omega_d=4$$

$$\xi\omega_n=3$$

Time at which maximum value of response occur is peak time.

$$t_p=\frac{\pi}{\omega_d}=\frac{\pi}{4}$$

$$t_p=0.785\text{ sec}$$

Hence, the correct answer is 0.785 sec.

Question 14

A player throws a ball at a basket kept at a distance. The probability that the ball falls into the basket in a single attempt is 0.1. The player attempts to throw the ball twice. Considering each attempt to be independent, the probability that this player puts the ball into the basket only in the second attempt (rounded off to two decimal places) is _______

Ans. **0.09**

Sol. Probability that ball falls into basket in a single attempt $p = 0.1$.

Then probability that ball doesn't fall into basket in a single attempt, $q = 1 - p = 1 - 0.1 = 0.9$.

In order that player puts ball basket in second attempt he should fail in first attempt and success in second attempt.

$\therefore$ Probability of putting ball only in second attempt

$$= q \times p = 0.9 \times 0.1 = 0.09$$

Question 15

A set of linear equations is given in the form $Ax = b$, where A is a 2×4 matrix with real number entries and $b \neq 0$. Will it be possible to solve for x and obtain a unique solution by multiplying both left and right sides of the equation by A^T (the super script T denotes the transpose) and inverting the matrix $A^T A$? Answer is _______

(A) Yes, can obtain a unique solution provided the matrix A is well conditioned

(B) Yes, can obtain a unique solution provided the matrix $A^T A$ is well conditioned

(C) No, it is not possible to get a unique solution for any 2×4 matrix A.

(D) Yes, it is always possible to get a unique solution for any 2×4 matrix A.

Ans. **(C)**

Sol. Rank of matrix must be $\leq \min(R, C)$

Where R is row and C is Column

So, rank of matrix $A \leq 2$

Let, rank of matrix $A = n$, $n \leq 2$

Since rank of $A =$ rank of $A^T =$ rank of $A^T A$

Rank of matrix $A^T = n$

Rank of $A^T A = n$

But dimension of $A^T A$ is 4×4

So, $|A^T A| = 0$ as (Rank < Dimension)

Hence, system cannot have unique solution.

Hence, the correct option is (C).

Question 16

If I is the current flowing through a Hall effect sensor and B is the magnetic flux density perpendicular to the direction of current (in the plane of Hall effect sensor). The Hall voltage generated is

(A) Inversely proportional to both I and B

(B) Directly proportional to both I and B

(C) Inversely proportional to I and directly proportional to B

(D) Directly proportional to I and inversely proportional to B

Ans. (B)

Sol. The Hall voltage is given by

$$V_H = \frac{BIR_H}{w}$$

$$\left.\begin{aligned} V_H &\propto B \\ V_H &\propto I \end{aligned}\right\}$$

So Hall voltage is directly proportional to both B and I.

Hence, the correct option is (B).

Question 17

Consider the recursive equation $X_{n+1} = X_n - h(F(X_n) - X_n)$, with initial condition $X_0 = 1$ and $h > 0$ being a very small valued scalar. This recursion numerically solves the ordinary differential equation ________

(A) $\dot{X} = -F(X) + X,\ X(0) = 1$

(B) $\dot{X} = F(X) + X,\ X(0) = 1$

(C) $\dot{X} = F(X),\ X(0) = 1$

(D) $\dot{X} = -F(X),\ X(0) = 1$

Ans. (A)

Sol. Given, recursive equation $X_{n+1} = X_n - h[F(X_n) - X_n]$

It can also written as,

$$X_{n+1} = X_n + h[-F(X_n) + X_n] \qquad \ldots\text{(i)}$$

Forward Euler's method is given by,

$$y_n = y_{n-1} + h\,f(y_{n-1}, t_{n-1}) \qquad \ldots\text{(ii)}$$

Or $$y_n = y_{n-1} + h\,y'_{n-1}$$

Comparing equation (ii) with equation (i),

$$\frac{dX_n}{dt} = X_n - F(X_n)$$

$$\dot{X}_n = X_n - F(X_n)$$

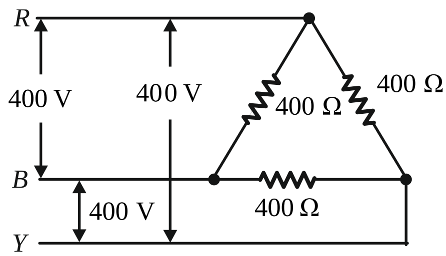
Hence, the correct option is (A).

Question 18

Three $400\ \Omega$ resistors are connected in delta and powered by a 400 V(rms) 50 Hz balanced, symmetrical $R - Y - B$ sequence, three phase three wire mains. The rms value of the line current (in amperes, rounded off to one decimal place) is ________

Ans. (1.70 to 1.80)

Sol.



$$V_P = V_L = 400 \text{ Volt}$$

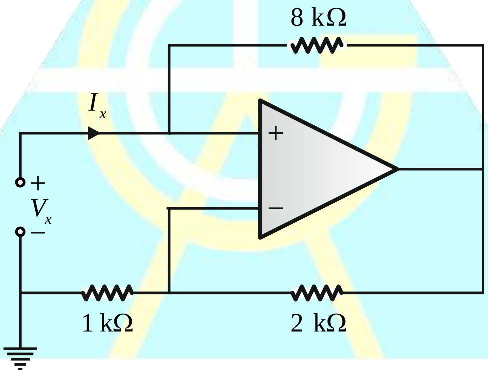
$$I_P = \frac{V_P}{400} = \frac{400}{400} = 1 \text{ A}$$

$$I_L = \sqrt{3} \times I_P = \sqrt{3} = 1.732 \text{ A}$$

Hence, the correct answer is 1.732 A.

Question 19

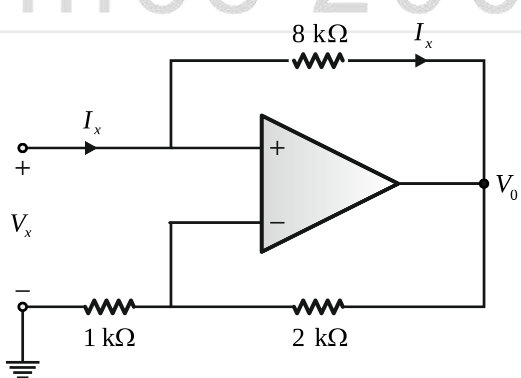
Assume that the Op-Amp in the circuit shown is ideal.



The value of $\frac{V_x}{I_x}$ (in kΩ) is ________

Ans. (– 4)

Sol.



Since, the Op-Amp is ideal, the voltage at inverting and non inverting terminals will be V_x.

Applying KCL at inverting terminal

$$\frac{V_x - 0}{1} + \frac{V_x - V_0}{2} + 0 = 0$$

$$V_0 = 3V_x$$

Apply KCL at non-inverting terminal,

$$-I_x + \frac{V_x - V_0}{8} = 0$$

$$-I_x + \frac{V_x - 3V_x}{8} = 0$$

$$\frac{V_x}{I_x} = -4 \text{ k}\Omega$$

Hence, the correct answer is $-4 \text{ k}\Omega$.

Question 20

The closed loop transfer function of control system is given by $\frac{C(s)}{R(s)} = \frac{1}{s+1}$. For the input $r(t) = \sin t$, the steady state response $c(t)$ is ______

(A) $\frac{1}{\sqrt{2}}\sin\left(t - \frac{\pi}{4}\right)$ (B) $\frac{1}{\sqrt{2}}\cos t$ (C) $\frac{1}{\sqrt{2}}\sin\left(t + \frac{\pi}{4}\right)$ (D) 1

Ans. **(A)**

Sol.

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)}$$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{1 + j\omega}$$

Given input signal, $r(t) = \sin t$, $\omega = 1$

$\therefore$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{1 + j}$$

$$\left|\frac{C(j\omega)}{R(j\omega)}\right| = \frac{1}{\sqrt{2}} = B$$

$$\phi = \angle\frac{C(j\omega)}{R(j\omega)} = -\frac{\pi}{4}$$

$$c(t) = B\sin(t + \phi)$$

$$c(t) = \frac{1}{\sqrt{2}} \cdot 1 \cdot \sin(t + \phi)$$

$$c(t) = \frac{1}{\sqrt{2}}\sin\left(t - \frac{\pi}{4}\right)$$

Hence, the correct option is (A).

Question 21

A sinusoid of 10 kHz is sampled at 15 k samples/sec. The resulting signal is passed through an ideal low pass filter (LPF) with cut-off frequency of 25 kHz. The maximum frequency component at output of LPF (in kHz) is ________

Ans. (25)

Sol. **Given :** Modulating frequency, $f_m = 10$ kHz

Sampling frequency, $f_s = 15$ k samples/sec.

Cut off frequency of LPF, $f_c = 25$ kHz

The frequency components present in sampling of single –tone modulating signal are ,

$$f_m,\ f_s \pm f_m, 2f_s \pm f_m \ldots$$

$$f_m = 10 \text{ kHz}$$

$$f_s \pm f_m = 15 \pm 10 = 25 \text{ kHz and } 5 \text{ kHz}$$

$$2f_s \pm f_m = 30 \pm 10 = 40 \text{ kHz and } 20 \text{ kHz and so on.}$$

A low-pass filter passes all frequency from zero to the cut-off frequency and blocks all the frequencies above the cut-off frequency.

Hence, at the output of LPF frequency components present are 5 kHz, 10 kHz, 20 kHz and 25 kHz.

The maximum frequency component at output of the Low Pass Filter is 25 kHz.

Hence, the correct answer is 25 kHz.

Question 22

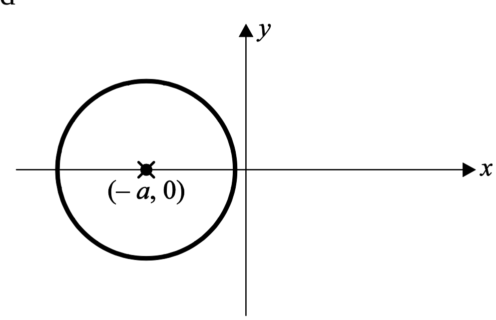
Let $f(z) = \dfrac{1}{z+a}$, $a > 0$. The value of the integral $\oint f(z)dz$ over a circle C with center $(-a, 0)$ and radius $R > 0$ evaluated in the anti-clockwise direction is________

(A) $4\pi i$ (B) $-2\pi i$ (C) 0 (D) $2\pi i$

Ans. (D)

Sol. $f(z) = \dfrac{1}{z+a}$

Centre of circle is $(-a, 0)$ and



(Complex plane)

Pole $-a$ lie inside the circle.

By residue theorem,

$$\oint_C f(z)\,dz = 2\pi i \times \left[\Sigma \text{Res}(z_i)\right]$$

Residue at $z=-a$ is given by,

$$\text{Res}(-a) = [(z+a)\times f(z)]_{z=-a}$$

$$\text{Res}(-a) = \left[(z+a)\times\frac{1}{(z+a)}\right]_{z=-a}$$

$$\text{Res}(-a) = 1$$

Now, $$\oint \frac{1}{z+a}dz = 2\pi i[\text{Res }(-a)]$$

$\therefore$ $$\oint \frac{dz}{z+a} = 2\pi i(1) = 2\pi i$$

Hence, the correct option is (D).

Question 23

A Q meter is best suited for the measurement of the

(A) Turns ratio of a transformer

(B) Quality factor of Piezoelectric sensor

(C) Quality factor of a capacitance

(D) Distributed capacitance of a coil

Ans. (D)

The Q-meter is best suited for the distributed (self) capacitor. If the values of tuning capacitor is 'C' and frequency be $'f_1'$

$\therefore$ Resonant frequency, $$f_1 = \frac{1}{2\pi\sqrt{L(C_1+C_d)}}$$

Now frequency is doubled to $'f_2'$. For bringing the resonance of the circuit let the tuning capacitor value be $'C_2'$

$\therefore$ Resonant frequency, $$f_2 = \frac{1}{2\pi\sqrt{L(C_2+C_d)}}$$

Now, $$f_2 = 2\times f_1$$

$$\frac{1}{2\pi\sqrt{L(C_2+C_d)}} = 2\times\frac{1}{2\pi\sqrt{L(C_1+C_d)}}$$

$\therefore$ Self (distributed) capacitor,

$$C_d = \frac{C_1-4C_2}{3}$$

Hence, the correct option is (D).

Question 24

Consider the signal $x[n] = \sin(2\pi n)u[n]$, where $u[n] = \begin{cases} 1, & n = 0, 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$

The period of signal $x[n]$ is

(A) 4 (B) 1 (C) 2 (D) 3

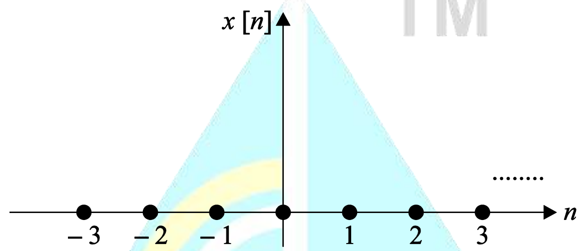
Ans. **(B)**

Sol. **Given :** $x[n] = \sin 2\pi n u[n]$, where $u[n] = 1$ for $n \geq 0$

$$x[0] = \sin 2\pi(0)u[0] = 0$$

$$x[1] = \sin 2\pi(1)u[1] = 0$$

$$x[2] = \sin 2\pi(2)u[2] = 0$$



$x[n]$ is a constant (DC) signal of zero value where it repeats it's value zero with each value of n. So, the time period = 1.

Hence, the correct option is (B).

Question 25

Two 100 Ω resister having tolerance 3% and 4% are connected in series. The effective tolerance of the series combination (in %, one decimal place) is __________

Ans. **(3.5)**

Sol. **Given:**

$$R_1 = 100\Omega$$

$$R_2 = 100\Omega$$

$$\%\varepsilon_{R_1} = 3\%$$

$$\%\varepsilon_{R_2} = 4\%$$

Method – I :

$$R_{eq} = R_1 + R_2 = 100 + 100 = 200\Omega$$

$$\%\varepsilon_{R_{eq}} = \frac{R_1}{R_{eq}} \times \%\varepsilon_{R_1} + \frac{R_2}{R_{eq}} \times \%\varepsilon_{R_2}$$

$$\therefore \quad \%\varepsilon_{R_{eq}} = \frac{100}{200} \times 3\% + \frac{100}{200} \times 4\%$$

$$\%\varepsilon_{R_{eq}} = 1.5\% + 2\%$$

$$\%\varepsilon_{R_{eq}} = 3.5\%$$

Method – II :

Absolute error, $\Delta R_1 = \frac{3}{100} \times 100 = 3\Omega$

Absolute error, $\Delta R_2 = \frac{4}{100} \times 100 = 4\Omega$

$$R_{eff} = R_1 + R_2 = 100 + 100 = 200\Omega$$

Total error $= 3 + 4 = 7\Omega$

$\therefore$ Tolerance band in expression of %

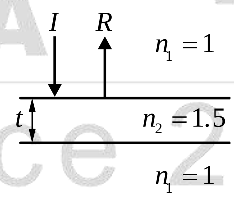
$$= \frac{7}{200} \times 100 = \pm 3.5\Omega$$

Hence, correct answer is 3.5

Q.26 to Q.55 Carry two marks each

Question 26

As shown in the figure, a slab of finite thickness t with refractive index $n_2 = 1.5$, has air $(n_1 = 1)$ above and below it. Light of free space of wavelength 600 nm is incident normally from air as shown. For a destructive interference to be observed at R, the minimum value of thickness of the slab t (in nm) is ______



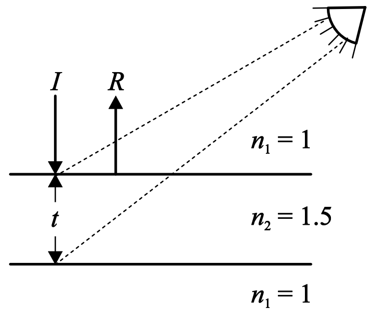
Ans. **(200)**

Sol. **Given :**

$\lambda = 600\,\text{nm}$

$n_2 = 1.5$

$n_1 = 1$



Since, light is incident normally.

For destructive interference path difference must be either $\frac{\lambda}{2}, \frac{3\lambda}{2}$ or odd multiple of 'λ' and we know that, the distance travelled $= t$

Where 't'= Thickness of the slab

$\therefore$ For minimum thickness, $\frac{\lambda}{2} = t$

But the slab has a refractive index.

$$\therefore \quad \frac{\lambda}{2n_2} = t$$

$$\frac{600}{2 \times 1.5} = t$$

$$t = \frac{600}{3} = 200\,\text{nm}$$

$$t = 200\,\text{nm}$$

Hence, correct answer is 200

Question 27

The system shown in fig. (a) has a time response $y(t)$ to an input $r(t) = 10u(t)$ as shown in fig. (b), $u(t)$ being the unit step input. Both K, τ are positive. The gain K of the system is ___________

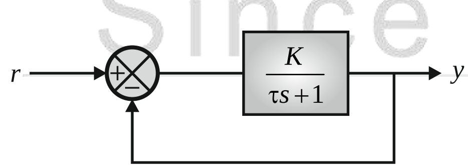


Fig. (a)

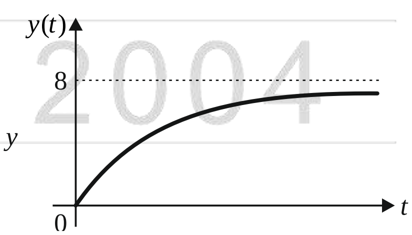


Fig. (b)

Ans. (4)

Sol. Given input, $r(t) = 10u(t)$

Steady state error is, $e_{ss} = 10 - 8 = 2$

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

$$2 = \lim_{s \to 0} \frac{s \times \frac{10}{s}}{1 + \frac{K}{\tau s + 1}}$$

$$2 = \frac{10}{1+K}$$

$$\frac{10}{1+K} = 2$$

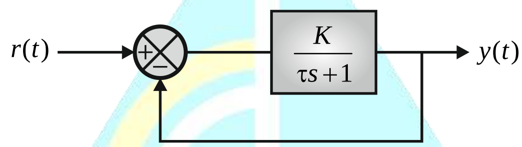
$$1 + K = 5$$

$\therefore$ $K = 4$

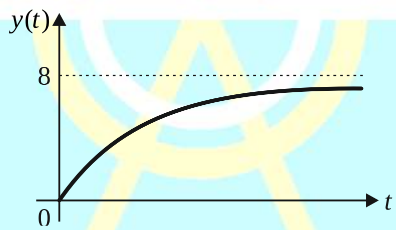
Hence, the correct answer is 4.

Alternate Solution :

Given system is as shown below,



Given waveform of $y(t)$ is



Also given $r(t) = 10u(t)$

Transfer function can be given as

$$\frac{Y(s)}{R(s)} = \frac{K}{\tau s + 1 + K}$$

But $R(s) = \frac{10}{s}$

$$Y(s) = \frac{10}{s} \cdot \frac{K}{\tau s + 1 + K}$$

From the waveform of $y(t)$, steady state value of $y(t)$ is 8.

$$\lim_{s \to 0} s \cdot \frac{10}{s} \frac{K}{\tau s + 1 + K} = 8$$

$$\frac{10K}{K+1} = 8$$

$$10K = 8K + 8$$

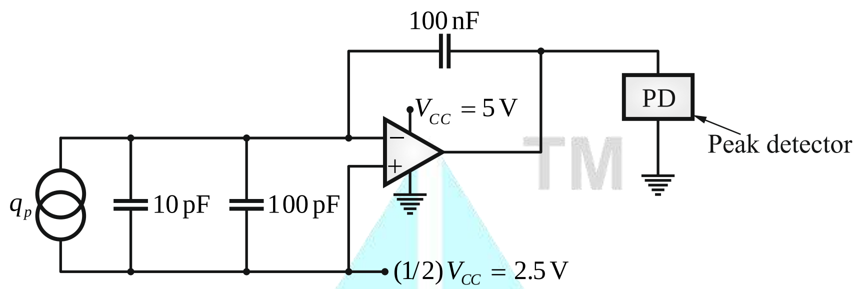
$$2K = 8$$

$$K = 4$$

Hence, the correct answer is 4.

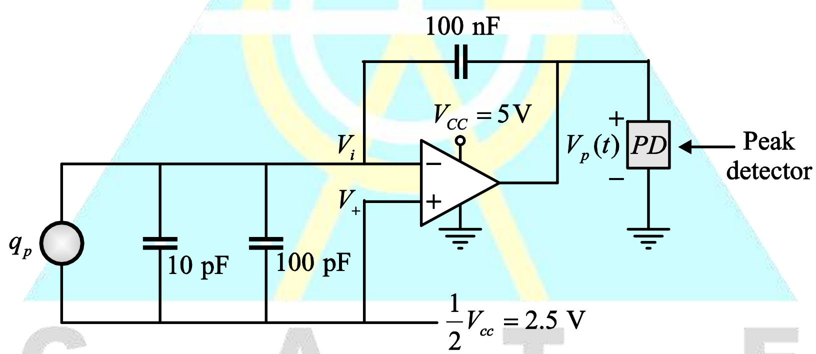
Question 28

The circuit shown uses ideal Op-Amp powered from a supply $V_{CC} = 5$ V. If the charge q_p generated by the piezoelectric sensor is of the form $q_p = 0.1\sin(10000\pi t)\,\mu C$, the peak detector output after 10 cycles of q_p (in volts, rounded off to one decimal place) is _______



Ans. **(3.4 to 3.6)**

Sol. Given op-amp circuit is shown in below figure.



Since this is a negative feedback op-amp circuit. So we can apply virtual ground concept

$$V_+ = V_- = 2.5 \text{ V}$$

Peak detector output

$$V_p(t) = 2.5 - V_c(t)$$

$$V_c(t) = 2.5 - V_p(t)$$

$$\because \quad V_c(t) = \frac{1}{C} q_p(t)$$

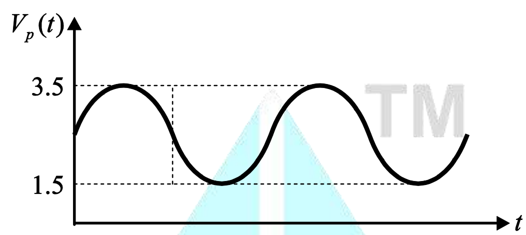
$$\therefore \quad 2.5 - V_p(t) = \frac{1}{C} q_p(t)$$

$$2.5 - V_p(t) = \frac{1}{100 \times 10^{-9}} 0.1 \sin(10^4 \pi t) \times 10^{-6}$$

$$2.5 - V_p(t) = \sin(10^4 \pi t)$$

$$V_p(t) = 2.5 - \sin(10^4 \pi t)$$

$\because \quad |\sin(10^4 \pi t)| = \pm 1$



So, peak detector output after 10 cycles of q_p is 3.5 V.

Hence, the correct answer is 3.5 V.

Question 29

The address lines $A_9 \ldots\ldots A_2$ of 10 bit, 1.023 V full - scale digital to analog converter (DAC) is connected to the data lines D_7 to D_0 of an 8-bit microprocessor with A_1 and A_0 of the DAC grounded. Now, $D_7 \ldots\ldots D_0$ is changed from 1010 1010 to 1010 1011. The corresponding change in the output of the DAC (in mV, rounded off to one decimal place) is ________

Ans. **(3.5 to 4.5)**

Sol. Given statement weight Initial final is shown in figure,

Weight	Initial	Final		DAC
2^9	1	1	D_7	A_9
2^8	0	0	D_6	A_8
2^7	1	1	D_5	A_7
2^6	0	0	D_4	A_6
2^5	1	1	D_3	A_5
2^4	0	0	D_2	A_4
2^3	1	1	D_1	A_3
2^2	0	1	D_0	A_2
2^1	0	0	'0'	A_1
2^0	0	0	'0'	A_0

Output

Here the input data is changed by 1 bit but line change in analog input will be of resolution or 1 step size.

Change in DAC o/p $= 4 \times$ step size (as the weightage of the bit changed is 2^2.)

Step size = Resolution $= \dfrac{\text{Full scale value}}{2^n - 1}$

Given 10 bit DAC, $n = 10$ Full scale value $= 1.023\,\text{V}$

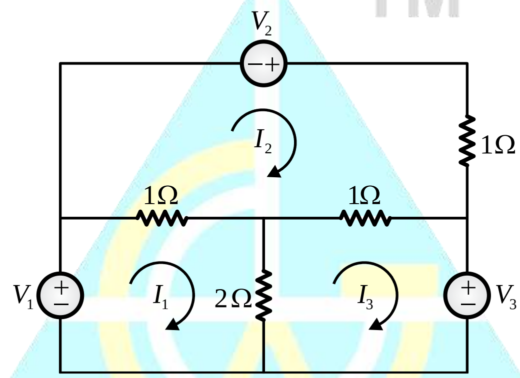
$\therefore$ Step size $= \dfrac{1.023}{2^{10}-1} = \dfrac{1.023}{1.023} = 1\,\text{mV}$

$\therefore$ Change in analog output $= 4 \times \text{step size} = 4\,\text{mV}$

Hence, the correct answer is 4 mV.

Question 30

I_1, I_2 and I_3 in the figure below are mesh currents. The correct set of mesh equations for these currents, in matrix form, is _______



(A) $\begin{bmatrix} -3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$

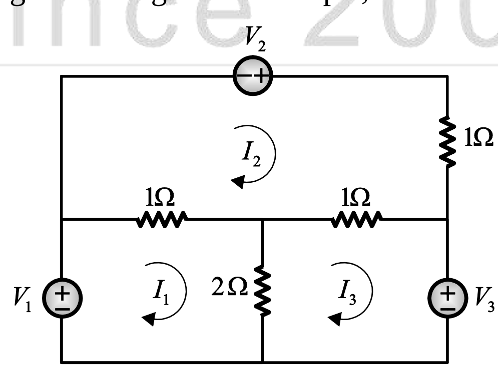
(B) $\begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & -1 \\ -2 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$

(D) $\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & -3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

Ans. **(C)**

Sol. **Given :** Circuit is shown in figure writing KVL in loop-I,



$$-V_1 + 1(I_1 - I_2) + 2(I_1 - I_3) = 0$$

$$\Rightarrow \quad V_1 = 3I_1 - I_2 - 2I_3 \quad \ldots\text{(i)}$$

Applying KVL in loop-II,

$$-V_2 + I_2 + 1(I_2 - I_3) + 1(I_2 - I_1) = 0$$

$$V_2 = -I_1 + 3I_2 - I_3 \qquad \ldots\text{(ii)}$$

Applying KVL in loop-III,

$$V_3 + 2(I_3 - I_1) + 1(I_3 - I_2) = 0$$

$$V_3 = 2I_1 + I_2 - 3I_3$$

$$-V_3 = -2I_1 - I_2 + 3I_3 \qquad \ldots\text{(iii)}$$

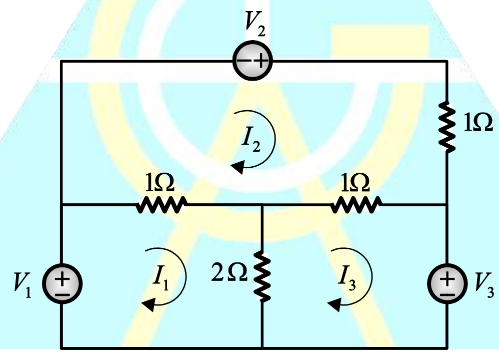
Writing equation (i), (ii) and (iii) in matrix form,

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$$

Hence, the correct option is (C).

Alternate Solution :

Given circuit is as shown below,



From ohms law we know that,

$$R \cdot I = V$$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

'R' matrix is a 3×3 matrix because there are 3 meshes present in the circuit.

R_{11}, R_{22}, R_{33} are the sum of the resistances present in mesh 1, mesh 2 and mesh 3.

$R_{12} = -$(resistance common between mesh 1 and mesh 2)

$R_{13} = -$(resistance common between mesh 1 and mesh 3)

$R_{21} = -$(resistance common between mesh 2 and mesh 1)

$R_{23} = -$(resistance common between mesh 2 and mesh 3)

$R_{31} = -$(resistance common between mesh 3 and mesh 1)

$R_{32} = -$(resistance common between mesh 3 and mesh 2)

V_1, V_2 and V_3 will be positive only if currents I_1, I_2 and I_3 are flowing from positive to negative potential through V_1, V_2 and V_3. Otherwise V_1, V_2 and V_3 will be negative.

$\therefore$ The required matrix equation will be

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Here V_3 is negative because the current is flowing from negative to positive through V_3.

Hence, the correct option is (C).

Question 31

The real power drawn by a balanced load connected to a 400 V, 50 Hz, balanced, symmetrical 3-phase, 3-wire, RYB sequence mains is measured using the two-wattmeter method. Wattmeter W_1 is connected in the R line and wattmeter W_2 is connected in the B line. The line current is measured as $\frac{1}{\sqrt{3}}$ A. If the wattmeter W_1 reads zero, the reading on W_2 (in watts) is ________

Ans. **199 to 201**

Sol. **Given:** Two wattmeter method of three phase power measurement.

$V_L = 400\,\text{V}$

$f = 50\text{ Hz}$

$I = \dfrac{1}{\sqrt{3}}\text{A}$

$W_1 = 0\,\text{W}$

Method – I :

As the reading of W_1 is zero, the entire power will be read by wattmeter 2

$W_1 = V_L I_L \cos(\phi + 30)$

$W_2 = V_L . I_L \cos(\phi - 30)$

$W_1 = 0$ when $\cos(\phi + 30) = 0$

$\phi + 30 = 90^0$

$\phi = 60^0$

$$\therefore \quad W_2 = V_L I_L \cos(60-30^0) = 400\times\frac{1}{\sqrt{3}}\times\cos 30^0$$

$$W_2 = 400\times\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2} = 200\text{ W}$$

Method – II :

As the reading of W_1 is '0' W_2 will measure the total three phase power

$$\therefore \quad = W_2 = \sqrt{3}V_L I_L\cos\phi$$

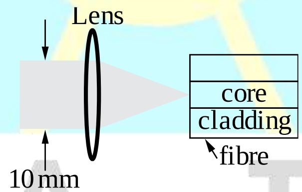
$$= \sqrt{3}\times 400\times\frac{1}{\sqrt{3}}\times\cos 60^0$$

$$= \sqrt{3}\times 400\times\frac{1}{\sqrt{3}}\times\frac{1}{2} = 200\text{ W}$$

Hence, correct answer is 200W.

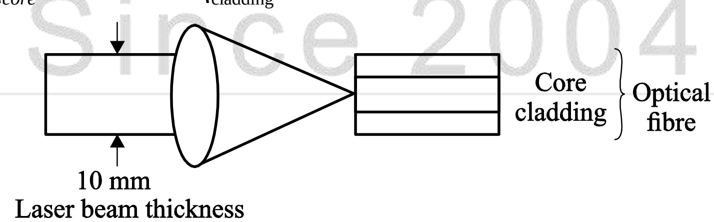
Question 32

A laser beam of 10 mm beam diameter is focused onto an optical fibre using a thin biconvex lens as shown in the figure. The refractive index of the lens is 1.5. The refractive indexes of the core and cladding of the fibre are 1.55 and 1.54 respectively. The minimum value of the focal length of the lens to attain the maximum coupling to the fibre (in mm, rounded off to one decimal place) is _______



Ans. **(27.5 to 28.5)**

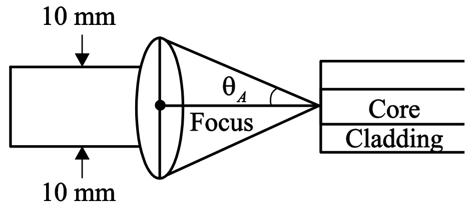
Sol. **Given :** $\eta_{lens} = 1.5$, $\eta_{core} = 1.55$ and $\eta_{cladding} = 1.54$.



Laser beam diameter $= 10\text{ mm}$

Now for the light to be completely included by the optical fibre, the light that must be entering at an angle greater than or equal to the acceptance angle of the optical fibre.

That is



By Defination Numerical aperture $= \sin\theta_A$ where θ_A = Acceptance angle

$\therefore \quad \theta_A = \sin^{-1}(N.A)$

But for optical fibre,

$$N.A = \sqrt{\frac{(n_{core})^2 - (n_{cladding})^2}{(n_{air})}}$$

$$N.A = \sqrt{(1.55)^2 - (1.54)^2}$$

$$N.A = \sqrt{(1.55)^2 - (1.54)^2}$$

$$N.A = \sqrt{2.4025 - 2.3716}$$

$$N.A = \sqrt{0.0309} = 0.1758$$

$$\therefore \quad \theta_A = \sin^{-1}(0.1758) = 10.125^0$$

Now of we assume that light strike almost at the top of the line we can assume that the height of the line $\approx 10\,\text{mm}$

$\therefore$ We can write that, $\tan\theta_A \approx \dfrac{5}{\text{Focus}} = \tan 10.125^0$

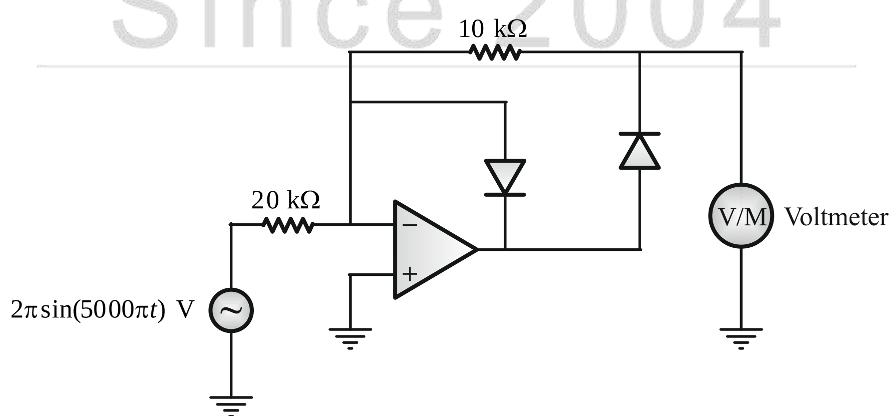
$$\frac{5}{\text{Focus}} = \tan 10.125^0 = 0.1785$$

$$\text{Focus} = 28.01\text{ mm} \simeq 28\,\text{mm}$$

Hence, the correct answer is 28 mm.

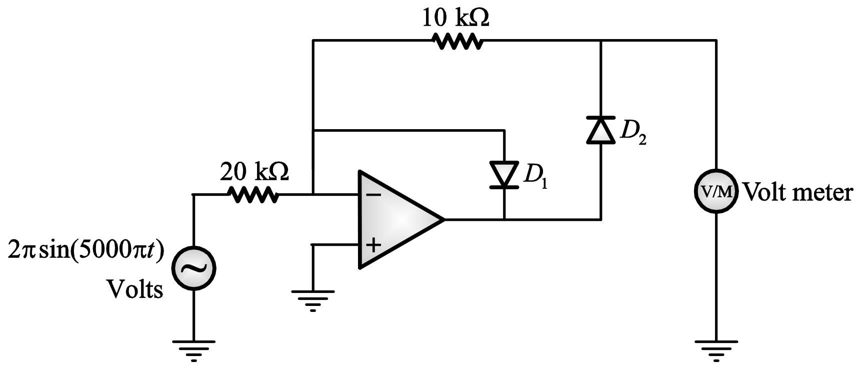
Question 33

Assuming that the Op-Amp used in the circuit shown is ideal, the reading of the 1 Hz bandwidth permanent magnet moving coil (PMMC) type voltmeter (in volts) is ________

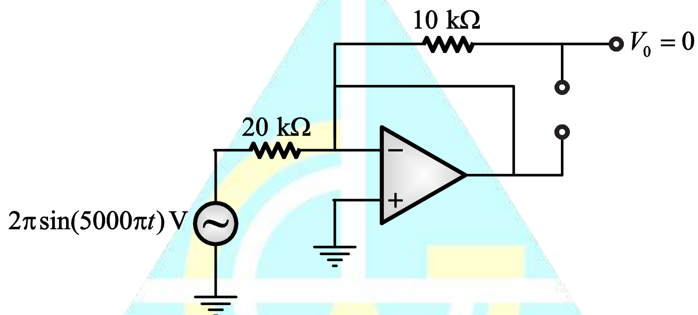


Ans. **(1)**

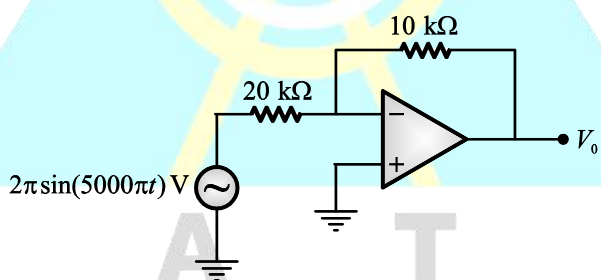
Sol.



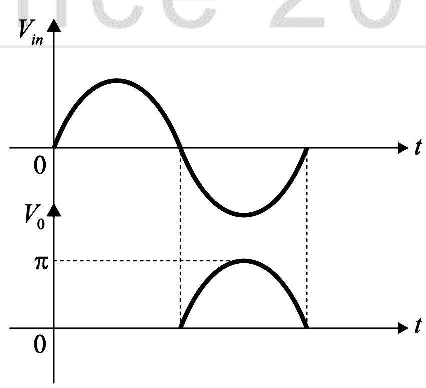
In positive half cycle of input D_1 ON and D_2 OFF.



In negative half cycle of input D_1 OFF and D_2 ON.



$$V_0 = -\frac{10}{20} \times -2\pi\sin(500\pi t) = \pi\sin(5000\pi t),$$



PMMC reading average value.

$\therefore \quad (V_0)_{avg} = \frac{\pi}{\pi} = 1\text{ V}$

Hence, the correct answer is 1.

Question 34

Consider the differential equation $\frac{dx}{dt} = \sin(x)$, with the initial condition $x(0) = 0$. The solution to this ordinary differential equation is ________

(A) $x(t) = 0$ (B) $x(t) = \sin(t)$

(C) $x(t) = \sin(t) - \cos(t)$ (D) $x(t) = \cos(t)$

Ans. **(A)**

Sol.

$$\frac{dx}{dt} = \sin(x)$$

$$\int \frac{dx}{\sin x} = \int dt$$

$$\log(\operatorname{cosec} x - \cot x) = t + C$$

$$\operatorname{cosec} x - \cot x = e^{t+C}$$

$$\frac{1 - \cos x}{\sin x} = e^{t+C}$$

$$\frac{2\sin^2 \frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = e^{t+C}$$

$$\tan\frac{x}{2} = e^{t+C}$$

$$\frac{x}{2} = \tan^{-1}[e^{t+C}]$$

At $t = 0, x = 0$

$$\therefore \quad 0 = \tan^{-1}(e^C)$$

$$e^C = \tan 0 = 0$$

$$\therefore \quad \frac{x}{2} = \tan^{-1}\left[e^t \cdot 0\right]$$

$$\frac{x}{2} = \tan^{-1} 0 = 0$$

$$x = 0$$

Hence, the correct option is (A)

Question 35

Consider the finite sequence $X = (1, 1, 1)$. The Inverse Discrete Fourier transform (IDFT) of X given as

$(x(0),\ x(1),\ x(2))$. The value of $x(2)$ is _______

Ans. (0)

Sol. Given : $X = \{1, 1, 1\}$

$$IDFT\{X\} = x(0),\ x(1),\ x(2)$$

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(k)e^{jk\left(\frac{2\pi}{N}\right)n}$$

Where $N = 3$

$$x(n) = \frac{1}{3}\sum_{K=0}^{2} X(k)e^{jk\frac{2\pi}{3}n}$$

$$x(2) = \frac{1}{3}\sum_{K=0}^{2} X(k)e^{jk\frac{4\pi}{3}}$$

$$= \frac{1}{3}\left\{X(0)e^{0} + X(1)e^{\frac{j4\pi}{3}} + X(2)e^{\frac{j8\pi}{3}}\right\}$$

$$e^{\frac{j4\pi}{3}} = \frac{-1}{2} - \frac{j\sqrt{3}}{2},\ e^{\frac{j8\pi}{3}} = \frac{-1}{2} + \frac{j\sqrt{3}}{2}$$

$$x(2) = \frac{1}{3}\left\{1 + 1\left\{\frac{-1}{2} - j\frac{\sqrt{3}}{2}\right\} + 1\left\{\frac{-1}{2} + \frac{j\sqrt{3}}{2}\right\}\right\}$$

$$= \frac{1}{3}\left\{1 - \frac{1}{2} - \frac{1}{2}\right\} = 0$$

Hence, the correct answer is 0.

Question 36

Consider the following state variable equation

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -6x_1(t) - 5x_2(t)$$

The initial condition are $x_1(0) = 0$ and $x_2(0) = 1$. At $t = 1$ second, the value of $x_2(1)$ is (rounded off to two decimal places) is ________

Ans. -0.13 to -0.11

Sol. $\dot{x}_1(t) = x_2(t)$

$$\dot{x}_2(t) = -6x_1(t) - 5x_2(t)$$

Initial conditions $x_1(0) = 0$, $x_2(0) = 1$

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -6 & -5\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

$$x(t) = \phi(s)x(0) = L^{-1}([sI - A]^{-1})x(0)$$

State transition matrix –

$$\phi(t) = e^{At} = L^{-1}[sI - A]^{-1}$$

$$\phi(s) = [sI - A]^{-1}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix}$$

$$\phi(s) = [sI - A]^{-1} = \frac{1}{s(s+5)+6}\begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}$$

$$X(s) = \phi(s)X(0)$$

$$X(s) = \frac{1}{s^2+5s+6}\begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$X(s) = \frac{1}{(s+2)(s+3)}\begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$X_1(s) = \frac{1}{(s+2)(s+3)}$$

$$X_2(s) = \frac{s}{(s+2)(s+3)}$$

$$\frac{s}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$\frac{s}{(s+2)(s+3)} = \frac{-2}{s+2} + \frac{3}{s+3}$$

$\therefore$ $$x_2(t) = ILT \text{ of } X_2(s)$$

$$x_2(t) = -2e^{-2t} + 3e^{-3t}$$

$$x_2(1) = -2e^{-2} + 3e^{-3} = -0.121$$

Hence, the correct answer is – 0.121.

Question 37

The loop transfer function of negative feedback system is $G(s)H(s) = \dfrac{2(s+1)}{s^2}$. The phase margin of the system is __________ (in degrees, rounded off to one decimal place).

Ans. **(65.3 to 65.7)**

Sol. $$G(s)H(s) = \frac{2(s+1)}{s^2}$$

$$G(j\omega)H(j\omega) = \frac{2(1+j\omega)}{(j\omega)^2}$$

$$|G(j\omega)H(j\omega)| = 1 \text{ at } \omega_{gc}$$

$$\frac{2\sqrt{1+\omega_{gc}^2}}{\omega_{gc}^2} = 1$$

$$2\left(\sqrt{1+\omega_{gc}^2}\right) = \omega_{gc}^2$$

$$4(1+\omega_{gc}^2) = \omega_{gc}^4$$

$$\omega_{gc}^4 - 4\omega_{gc}^2 - 4 = 0$$

$$\omega_{gc}^2 = \frac{4 \pm \sqrt{16+16}}{2} = \frac{4+4\sqrt{2}}{2} = 2 \pm 2\sqrt{2}$$

$$\omega_{gc}^2 = 2(1+\sqrt{2})$$

$$\omega_{gc} = 2.197$$

$$\phi = \tan^{-1}\omega_{gc} - 180^0$$

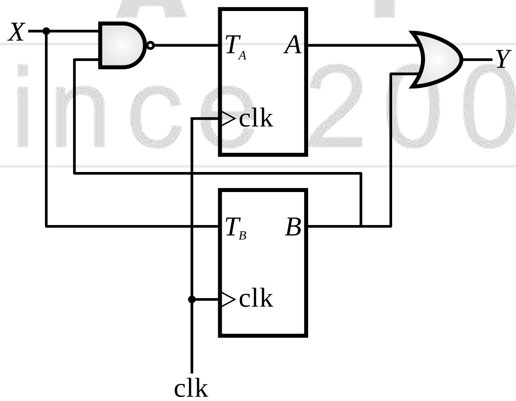
$$\text{PM} = 180^0 + \phi = 180 + \tan^{-1}\omega_{gc} - 180^0 = \tan^{-1} 2.197$$

$$\text{PM} = 65.53^0$$

Hence, the correct answer is 65.53°.

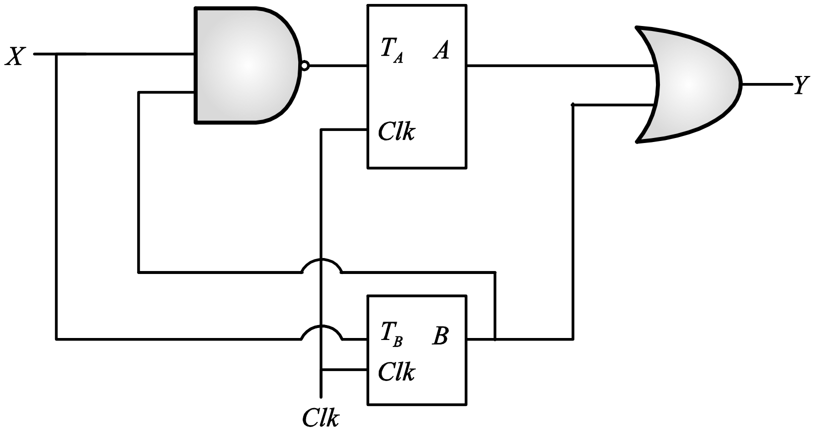
Question 38

Two T-flip flops are interconnected as shown in the figure. The present state of the flip flops are : $A = 1$, $B = 1$. The input x is given as 1, 0, 1 in the next three clock cycles. The decimal equivalent of $(ABY)_2$ with A being the MSB and Y being the LSB, after the 3rd clock cycle is ________



Ans. (7)

Sol. **Given :** Circuit is shown in figure



Both are T flip flop, so outputs A and B will toggle if T_A and T_B are 1 respectively.

$T_A = \overline{X.B} \quad T_B = X$

Initially $A=1,\ B=1$ and x takes values 1, 0, 1 in next 3 clock pulses.

Clock	X	A	B	$T_A = \overline{X.B}$	$T_B = X$	A^+	B^+	$y^+ = A^+ + B^+$
1	1	1	1	0	1	1	0	1
2	0	1	0	1	0	0	0	0
3	1	0	0	1	1	1	1	1

$\therefore$ Output $(ABY)_2$ after 3 clocks $=(111)_2$

$\therefore$ Decimal equivalent $=7$

Hence, the correct answer is 7.

Question 39

A metallic strain gauge of resistance R_x with a gauge factor of 2 is bonded to a structure made of a metal with modulus of elasticity of $200\ \text{GN/m}^2$. The value of R_x is $1\ \text{k}\Omega$ when no stress is applied. R_x is a part of a quarter bridge with three identical fixed resistors of $1\ \text{k}\Omega$ each. The bridge is excited from a DC voltage of 4 V. The structure is subjected to a stress of $100\ \text{MN/m}^2$. Magnitude of the output of the bridge (in mV, rounded off to two decimal places) is _______

Ans. **1**

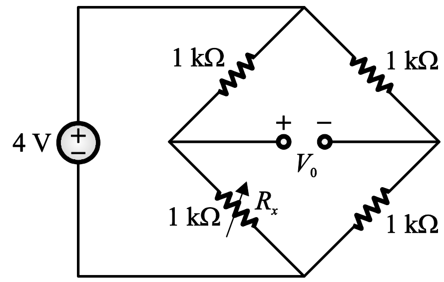
Sol. **Given :**

Guage.Factor. = 2

$\gamma = 200\times10^9 / \text{m}^2$

$V_s = 4$

$\sigma = 100\times10^6\ \text{N/m}^2$



Now we know that,

By Hooke's law, $\sigma \alpha \dfrac{\Delta L}{l}$

$$\sigma = \frac{\gamma \Delta L}{l}$$

Where $\sigma =$ Stress and $\dfrac{\Delta l}{l} =$ Strain .

$$\therefore \quad \frac{\Delta l}{l} = \frac{\sigma}{\gamma} = \frac{100 \times 10^6}{200 \times 10^9}$$

$$= 0.5 \times 10^{-3} \text{m/m} = 0.5 \text{mm/m}$$

Calculation of ΔR_x .

Now, $\quad \Delta R_x = R_x \times G.F \times \dfrac{\Delta l}{l}$

$$\therefore \quad \Delta R_x = 1000 \times 2 \times \frac{0.5}{1000} = 1\Omega$$

$$\therefore \quad \text{Output voltage} = V_s \left[\frac{R_x + \Delta R_x}{R_x + \Delta R_x + 1000} - \frac{1}{2} \right]$$

$$= 4 \left[\frac{1001}{2001} - \frac{1}{2} \right] = 0.999 \times 10^{-3}$$

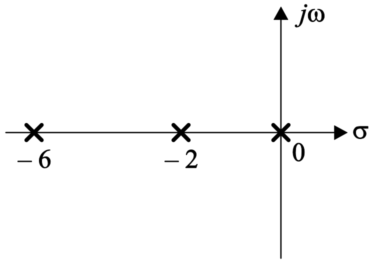
$$= 0.99 \text{mV} = 1 \text{mV}$$

Question 40

The loop transfer function of negative feedback system is given by $G(s)H(s) = \dfrac{K}{s(s+2)(s+6)}$, where $K > 0$. The value of K at the breakaway point of the root locus for the above system (rounded off to one decimal place) is __________

Ans. **(5.0 to 5.1)**

Sol. $G(s)H(s) = \dfrac{K}{s(s+2)(s+6)}$



Characteristic equation

$$1+G(s)H(s)=0$$

$$1+\frac{K}{s(s+2)(s+6)}=0$$

$$s(s+2)(s+6)+K=0$$

$$s[s^2+8s+12]+K=0$$

$$s^3+8s^2+12s+K=0$$

$$K=-(s^3+8s^2+12s)$$

$$\frac{dK}{ds}=-[3s^2+16s+12]$$

For breakaway point,

$$\frac{dK}{ds}=0$$

$$3s^2+16s+12=0$$

$$s=\frac{-16\pm\sqrt{16^2-4(3)(12)}}{6}=-0.90,-4.43$$

$s=-4.43$ does not lies on R.L.

Valid break away point is $s=-0.90$

$$G(s)H(s)=\frac{K}{s(s+2)(s+6)}$$

At break away point,

$$|G(s)H(s)|=1$$

The value of k at the breakaway point is,

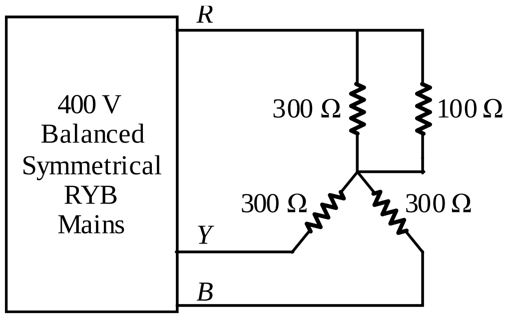
$$K=|s(s+2)(s+6)|$$

$$K=|-0.9(-0.9+2)(-0.9+6)|=5.049$$

Hence, the correct answer is 5.049.

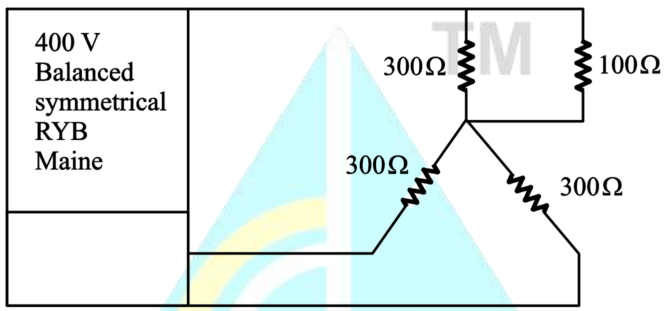
Question 41

In the circuit shown, the rms value of the voltage across the $100\ \Omega$ resistor (in volts) is _______.



Ans. **115 to 116**

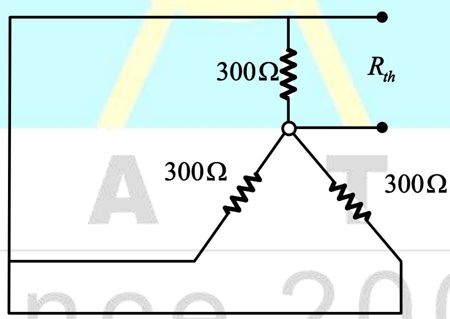
Sol. **Given:**



∴ By using Thevenin's theorem.

Remove the branch across which voltage is to be determined

To find R_{th} short circuit the voltage sources and open circuit the current sources



R_{th} can be calculated from the above figure as follows

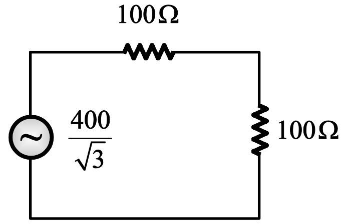
$$\therefore R_{th} = \frac{300}{3} = 100\,\Omega$$

Thevenin's voltage across $300\ \Omega$ will be the phase voltage as $300\ \Omega$ is connected in phase

V_{th} = Voltage across $300\,\Omega$,

$$\therefore \quad V_{th} = \text{phase voltage } = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}}$$

Thevenin's equivalent circuit,



$$\therefore \quad V_{100\Omega} = \frac{100}{100+100} \times \frac{400}{\sqrt{3}} = \frac{200}{\sqrt{3}} V = 115.47\text{V}$$

Hence, the correct answer is 115.47V .

Question 42

Consider the function $f(x,y) = x^2 + y^2$. The minimum value that the function attains on the line $x + y = 1$ (rounded off to one decimal place) is ________.

Ans. **(0.5)**

Sol. **Given :** $f(x,y) = x^2 + y^2$ …(i)

The given constraint is $x + y = 1$ or $y = 1 - x$ …(ii)

Putting the value of y from equation (ii) on equation (i),

$$f(x,y) = x^2 + (1-x)^2$$

Differentiating both sides with respect to x,

$$\frac{df(x,y)}{dx} = 2x + 2(1-x)(-1) = 2x - 2 + 2x$$

$$\frac{df(x,y)}{dx} = 4x - 2 \quad \text{....(iii)}$$

To determine critical point we equate the above equation with 0, we get

$$\frac{df(x,y)}{dx} = 0$$

$$4x - 2 = 0$$

$$x = \frac{1}{2}$$

From equation (ii),

$$y = 1 - x$$

$$y = \frac{1}{2}$$

To determine condition of minima, again differentiate equation (iii) with respect to x.

$$\frac{d^2 f(x,y)}{dx^2} = 4$$

Since $\dfrac{d^2 f(x,y)}{dx^2} > 0$, $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ is point of minima.

$$f(x,y) = x^2 + y^2$$

$$f\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2} = 0.5$$

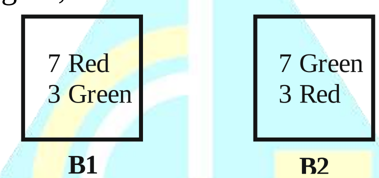
Hence, the correct answer is 0.5.

Question 43

Consider two identical bags B1 and B2 each containing 10 balls of identical shapes and sizes. Bag B1 contains 7 Red and 3 Green balls, while bag B2 contains 3 Red and 7 Green balls. A bag is picked at random and a ball is drawn from it, which found to be Red. The probability that the Red ball came from bag B1 (rounded off to one decimal place) is ________

Ans. **(0.68 to 0.72)**

Sol. Given arrangement is shown in figure,



Given that randomly drawn ball is red. We have to find probability that the red ball came from B_1.

Let, P_{B_1} = Prob. of drawn ball is from bag 1

P_{B_2} = Prob. of drawn ball is from bag 2

P_R = Prob. of drawn ball is red

Required probability $= P(B_1 / \text{R})$

$$P_{B_1} = \frac{1}{2} \text{ and } P_{B_2} = \frac{1}{2}$$

$$P(R/B_1) = \frac{7}{10}, \quad P(R/B_2) = \frac{3}{10}$$

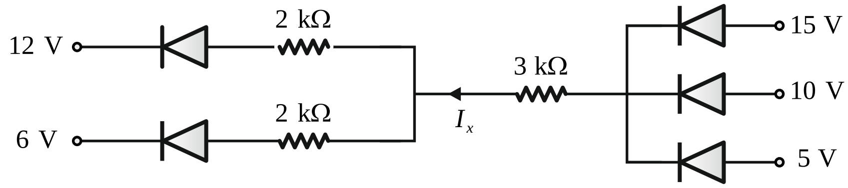
$$P(B_1/R) = \frac{P_{B_1}.P(R/B_1)}{P_{B_1}.P(R/B_1) + P_{B_2}.P(R/B_2)} \quad \text{[Using Bay's theorem]}$$

$$P(B_1/R) = \frac{0.5 \times 0.7}{0.5 \times 0.7 + 0.5 \times 0.3} = \frac{0.35}{0.35 + 0.15} = \frac{7}{10} = 0.7$$

Hence, the correct answer is 0.7.

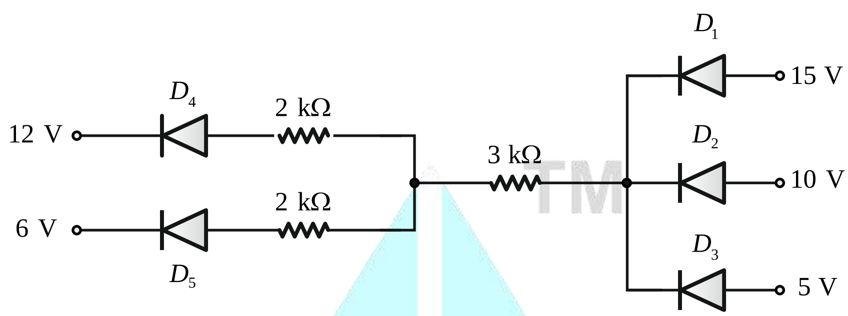
Question 44

Assume the diodes in the circuit shown are ideal. The current I_x flowing through the $3\ \text{k}\Omega$ resistor (in mA, rounded off to one decimal place) is __________

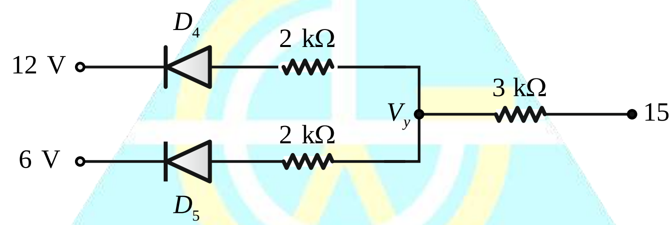


Ans. **(1.7 to 1.9)**

Sol.

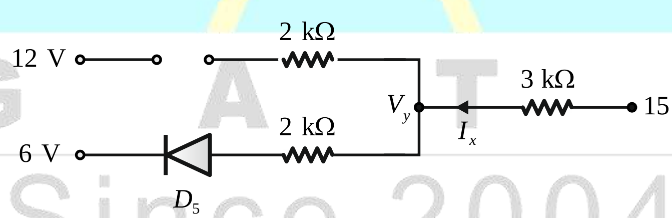


If $V_x = 15$, D_2, D_3 become off.



$$\frac{15 - V_y}{3} = \frac{V_y - 12}{2} + \frac{V_y - 6}{2}$$

$V_y = 10.5$ Hence D_4 also off



$$I_x = \frac{15 - 6}{5} = \frac{9}{5} = 1.8 \text{ mA}$$

$V_y = 9.6$ V

Hence, the correct answer is 1.8 mA.

Question 45

A $6\frac{1}{2}$ digit timer-counter is set in the 'time period' mode of operation and the range is set as 'ns'. For an input signal, the timer-counter displays 1000000. With the same input signal, the timer-counter is changed to 'frequency' mode of operation and the range is set as 'Hz'. The display will show the number ________

Ans. **(999 to 1001)**

Sol. **Given :**

$6\frac{1}{2}$ digital timer counter displays $= 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

In decimal equivalence form

$$\begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10^6 & 10^5 & 10^4 & 10^3 & 10^2 & 10^1 & 10^0 \end{array}$$

$$1\times10^6 + 0\times10^5 + 0\times10^4 + 0\times10^3 + 0\times10^2 + 0\times10^1 + 0\times10^0$$

$$T = 10^6 \text{ nsec} = 10^6 \times 10^{-9} \text{ sec}$$

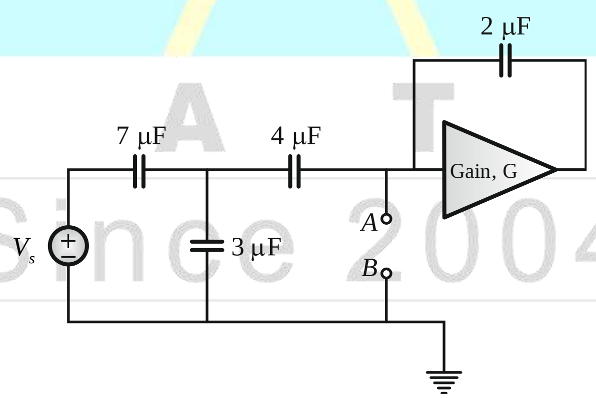
$$T = 10^{-3} \text{ sec}$$

$$\text{Frequency } = \frac{1}{T} = \frac{1}{10^{-3}} = 1000 \text{ Hz}$$

Hence, display will show the number 1000.

Question 46

A circuit consisting of capacitors, DC voltage source and an amplifier having a voltage gain $G = -5$ is shown in the figure. The effective capacitance across the nodes A and B (in μF, rounded off to one decimal place) is _______.

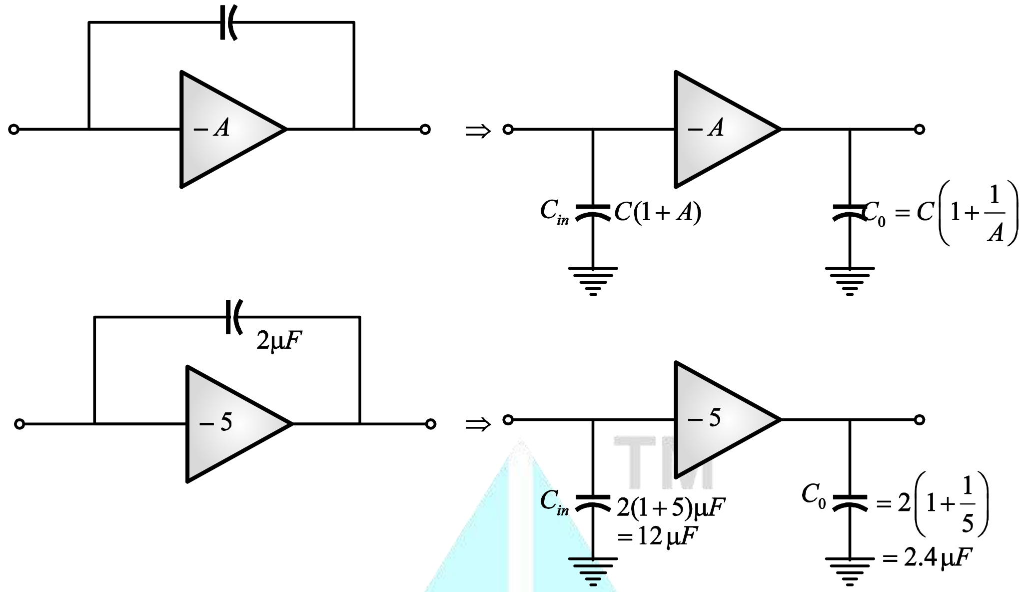


Ans. **(14.5 to 15.0)**

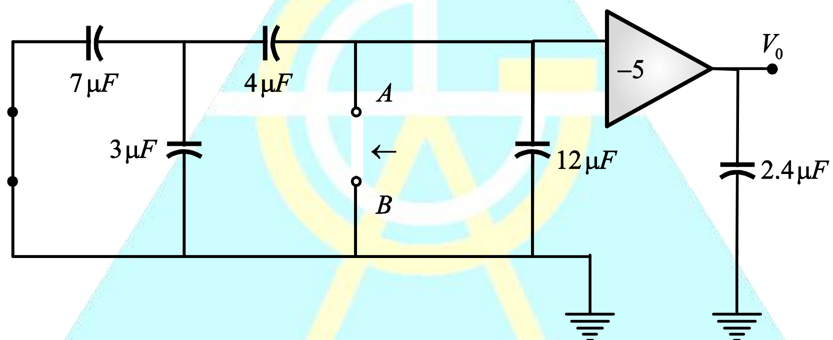
Sol. Given circuit is shown in figure,

From Miller's theorem,

To find equivalent capacitance,



$\therefore$ Equivalent circuit becomes as shown in figure,



$$C_{eq} = 12\,\mu\text{F} \parallel [4 \parallel 7 \| 3]$$

$$= 12 + \left(\frac{4\times 10}{4+10}\right) = 12 + \frac{40}{14} = \frac{208}{14} = 14.857\,\mu\text{F}$$

Hence, the correct answer is 14.857.

Question 47

Let $$g[n] = \begin{cases} 1, & n = 0 \\ 0, & n = \pm 1, \pm 2, \pm 3 \ldots \end{cases}$$

and $h[n] = \begin{cases} 1, & n = 0, 3, 6, 9 \\ 0, & \text{otherwise} \end{cases}$

Consider $y[n] = h[n] \otimes g[n]$, where $\otimes$ denotes the convolution operator. The value of $y[2]$ is________

Ans. **(0)**

Sol. **Given :** $g[n]=\begin{cases}1, & n=0\\ 0, & n=\pm1,\pm2,\ldots\end{cases}$

$\therefore$ $$g[n]=\delta[n]$$

$$h[n]=\begin{cases}1, & n=0,3,6,9\\ 0, & \text{otherwise}\end{cases}$$

$$h[n]=\delta[n]+\delta[n-3]+\delta[n-6]+\ldots$$

$$y[n]=h[n]\otimes g[n]=h[n]\otimes\delta[n]=h[n]$$

$\therefore$ $$y[n]=\{\underset{\uparrow}{1},0,0,1,0,0,1,\ldots\}$$

$\therefore$ $$y[2]=0$$

Hence, the correct answer is 0.

Question 48

The loop transfer function of a negative feedback system is $G(s)H(s)=\dfrac{1}{s(s-2)}$. The Nyquist plot for the above system ________

(A) encircles $(-1+j0)$ point once in the counterclockwise direction

(B) encircles $(-1+j0)$ point once in the clockwise direction

(C) does not encircle $(-1+j0)$ point

(D) encircles $(-1+j0)$ point twice in the counterclockwise direction

Ans. **(B)**

Sol. **Given :** $G(s)H(s)=\dfrac{1}{s(s-2)}$

Number of positive open loop poles $(P)=1$

Number of positive closed loop poles $(Z)=?$

Characteristic equation is

$$1+G(s)H(s)=0$$

$$1+\frac{1}{s(s-2)}=0$$

$$s^2-2s+1=0$$

$$\begin{array}{c|cc} s^2 & 1 & 1\\ s^1 & -2 & 0\end{array}$$

s^0	1	0

Number of positive closed loop poles

= Number of sign changes in first column in above table

$\therefore \quad Z = 2$

We know that, from Nyquist stability criteria,

$$N = P - Z$$

Where, $N =$ Number of encirclement of $(-1+j0)$ point (positive for anticlockwise and negative for clockwise).

$P =$ Number of positive open loop pole

$Z =$ Number of positive close loop pole

$$N = P - Z$$

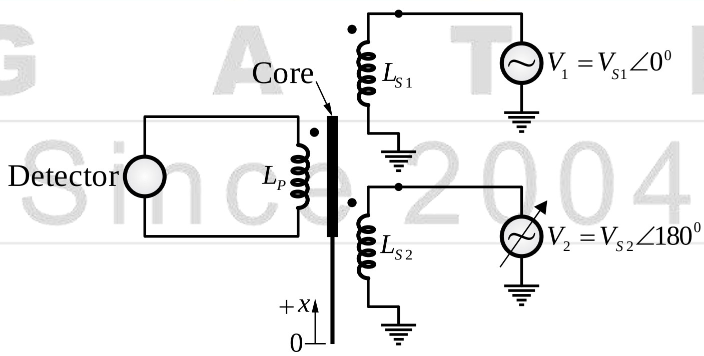
$$N = 1 - 2 = -1$$

Hence, clockwise encircle of $(-1+j0)$ point once in the clockwise direction.

Hence, the correct option is (B).

Question 49

The mutual inductances between the primary coil and the secondary coils of a linear variable differential transformer (LVDT) shown in the figure are M_1 and M_2. Assume that the self-inductances L_{S1} and L_{S2} remain constant and are independent of x. When $x = 0$, $M_1 = M_2 = M_0$. When x is in the range ± 10 mm, M_1 and M_2 change linearly with x. At $x = +10$ mm or –10 mm, the change in the magnitudes of M_1 and M_2 is 0.25 M_0. For a particular displacement $x = D$, the voltage across the detector becomes zero when $|V_2| = 1.25|V_1|$. The value of D (in mm, rounded off to one decimal place) is ________



Ans. **4.3 to 4.6**

Sol. **Given :** At $x = 0$, $M_1 = M_2 = M_0$

and M_1 & M_2 change linearly with x.

$\therefore \quad M_1 = M_0 + Ax$

$M_2 = M_0 - Bx$ (As for positive increment in x, M_1 increases and M_2 decreases)

At $x = 10$ mm,

Change in magnitude of $M_1 = 0.25M_0$

$\therefore \quad (M_0 + 0.01A) - M_0 = 0.25M_0$

$A = 25M_0$

Change in magnitude of $M_2 = 0.25M_0$

$M_0 - (M_0 - 0.01B) = 0.25M_0$

$B = 25M_0$

$\therefore \quad M_1 = M_0 + 25M_0 x$

$M_2 = M_0 - 25M_0 x$

Now, voltage across detector,

$V_D = M_1 sI_{s_1} - M_2 sI_{s_2}$ (as V_{S_1} and V_{S_2} are out of phase)

$V_{s_1} = L_{s_1} sI_{s_1}$

$sI_{s_1} = \frac{V_{s_1}}{L_{s_1}}$

$V_{s_2} = L_{s_2} sI_{s_2}$

$sI_{s_2} = \frac{V_{s_2}}{L_{s_2}}$

$\therefore \quad V_D = \frac{M_1 V_{s_1}}{L_{s_1}} - \frac{M_2 V_{s_2}}{L_{s_2}}$

At $x = D$, $\quad |V_2| = 1.25|V_1|$ and $V_D = 0$

$\therefore \quad 0 = \frac{M_1 V_{s_1}}{L_{s_1}} - \frac{M_2 1.25V_{s_1}}{L_{s_2}} \quad \ldots$(i)

As L_{s_1} and L_{s_2} are not changing with x,

$\therefore \quad L_{s_1} = L_{s_2} = L_s$

$\therefore \quad M_1 = 1.25M_2$

$M_0 + 25M_0 D = 1.25(M_0 - 25M_0 D)$

$1 + 25D = 1.25 - 31.25D$

$56.25D = 0.25$

$D = \frac{0.25}{56.25}$

$D = 4.45 \times 10^{-3} = 4.45$ mm

Hence, the correct answer is 4.45 mm.

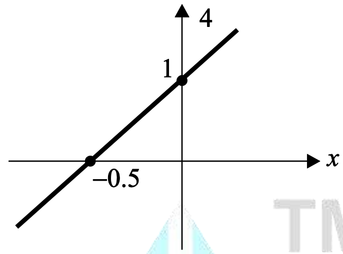
Question 50

A straight line drawn on an x-y plane intercepts the x-axis at –0.5 and the y-axis at 1. The equation that describes this line is ________

(A) $y = x - 0.5$ (B) $y = 0.5x - 1$ (C) $y = 2x + 1$ (D) $y = -0.5x + 1$

Ans. **(C)**

Sol. Given straight line intersecting x axis at -0.5 and y axis at 1 is shown in figure



Equation of straight line

$$y = mx + c$$

$$c = 1 \quad m = \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{0-(-0.5)}$$

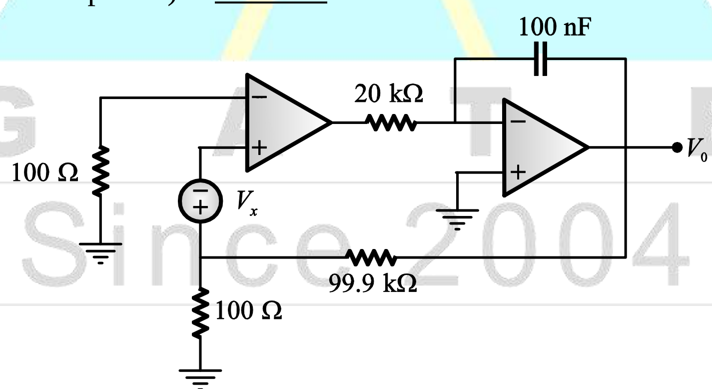
$$m = \frac{1}{0.5} = 2$$

$\therefore$ Equation of straight line is $y = 2x + 1$

Hence, the correct option is (C).

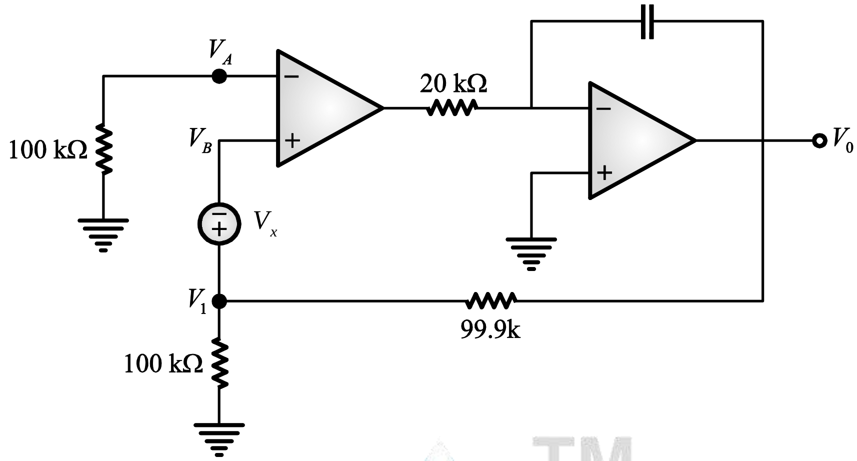
Question 51

If the Op-Amps in the circuit shown are ideal and $V_x = 0.5\text{ mV}$, the steady state value of V_0 (in volts, rounded off to two decimal places) is ________



Ans. **0.45 to 0.55**

Sol.



Steady state current means $t \to \infty$ or DC current.

For DC, $z_C = \dfrac{1}{\omega C} = \infty$

$\therefore$ Capacitor acts as open circuit.

By virtual ground, $V_A = V_B = 0$

By nodal analysis, $\dfrac{V_x}{100} + \dfrac{V_x - V_0}{99900} = 0$

$$\frac{0.5\times10^{-3}}{100} = \frac{V_0 - 0.5\times10^{-3}}{99900}$$

$\therefore$ $V_0 = +0.5\,\text{V}$

Hence, correct answer is 0.5 .

Question 52

Consider the matrix $M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$ .One of the eigen vectors of M is

(A) $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

Ans. **(C)**

Sol. **Given :** $M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$

$$[M - \lambda I] = \begin{bmatrix} 1-\lambda & -1 & 0 \\ 1 & -2-\lambda & 1 \\ 0 & -1 & 1-\lambda \end{bmatrix}$$

Characteristic equation, $|M-\lambda I|=0$

$$(1-\lambda)\{(-2-\lambda)(1-\lambda)+1\}+1\{(1-\lambda)-0\}=0$$

$\Rightarrow$ $$(1-\lambda)\{(\lambda+2)(\lambda-1)+1\}+(1-\lambda)=0$$

$\Rightarrow$ $$(1-\lambda)\{\lambda^2+\lambda-1\}+(1-\lambda)=0$$

$\Rightarrow$ $$(1-\lambda)\{\lambda^2+\lambda-1+1\}=0$$

$\Rightarrow$ $$(1-\lambda)(\lambda^2+\lambda)=0$$

$\Rightarrow$ $$\lambda(1-\lambda)(1+\lambda)=0$$

$\therefore$ $$\lambda_1=0,\ \lambda_2=1 \text{ and } \lambda_3=-1.$$

If X_i be Eigen vector corresponding to Eigen value λ_i,

$$MX_i=\lambda_i X_i$$

$$MX_i=\lambda_1 X_1$$

Let, $$X_1=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} x_1-x_2 \\ x_1-2x_2+x_3 \\ -x_2+x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1-x_2=0 \Rightarrow x_1=x_2$$

$$x_1-2x_2+x_3=0 \Rightarrow x_1-2x_1+x_3=0 \Rightarrow x_3=x_1$$

$$-x_2+x_3=0 \Rightarrow x_2=x_3$$

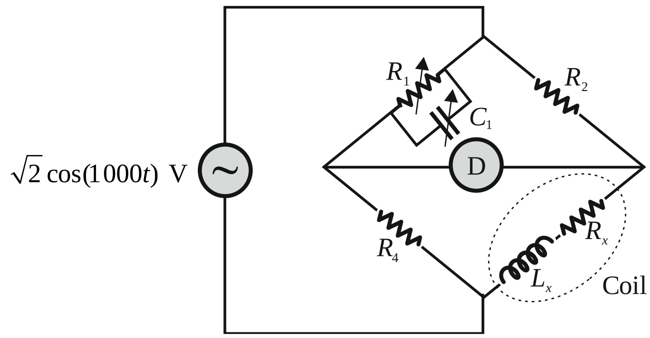
$\therefore$ $X_1=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ will be an Eigen vector

If $x_1=x_2=x_3$

Hence, line correct option is (C).

Question 53

In the Maxwell-Wien bridge shown, the detector D reads zero when C_1 = 100nF and R_1 = 100 kΩ.



The Q factor of the coil is ________

Ans. **(10 to 10)**

Sol. **Given**

$$R_1 = 100\,\text{k}\Omega$$

$$C_1 = 100\,\text{nF}$$

$$V_s = \sqrt{2}\cos(1000\,t)$$

Method – I :

Branch combinations are as follow

$$Z_1 = \frac{R_1 \times \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{1 + j\omega R_1 C_1}$$

$$Z_2 = R_2$$

$$Z_4 = R_4$$

$$Z_3 = R_x + j\omega L_x$$

According to bridge balanced condition

$$Z_1 Z_3 = Z_2 Z_4$$

$\therefore$
$$\left(\frac{R_1}{1 + j\omega R_1 C_1}\right)(R_x + j\omega L_x) = R_2 R_4$$

$$R_1(R_x + j\omega L_x) = R_2 R_4 (1 + j\omega R_1 C_1)$$

$$R_1 R_x + jR_1 \omega L_x = R_2 R_4 + j\omega R_1 R_2 R_4 C_1$$

Equating Real Parts

$$R_1 R_x = R_2 R_4$$

$$R_x = \frac{R_2 R_4}{R_1}$$

Equating Imaginary parts

$$R_1 \omega L_x = \omega R_1 R_2 R_4 C_1$$

$$L_x = R_2 R_4 C_1$$

$\therefore$ Quality Factor $Q = \frac{\omega L}{R} = \frac{\omega \times L_x}{R_x} = \frac{\omega \times R_2 R_4 C_1 R_1}{R_2 R_4}$

$$= \omega R_1 C_1$$

$$= 1000 \times 100 \times 10^3 \times 100 \times 10^{-9}$$

$$= 10^{10} \times 10^{-9}$$

$$= 10$$

Method – II :

In Maxwell – wien Bridge

$$Q = \omega C_1 R_1$$

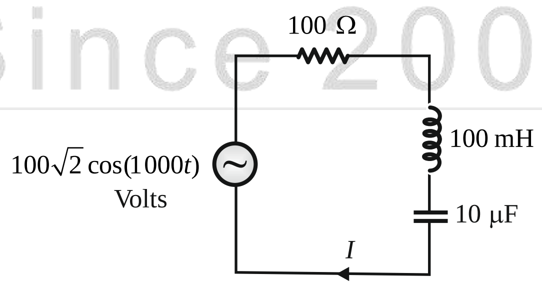
$\therefore$ $Q = 1000 \times 100 \times 10^{-9} \times 100 \times 10^3$

$$Q = 10$$

Hence correct answer is 10 .

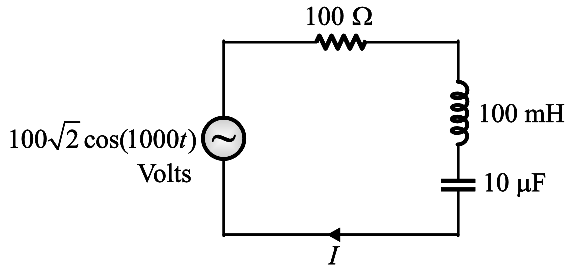
Question 54

The rms value of the phasor current I in the circuit shown (in amperes) is ________



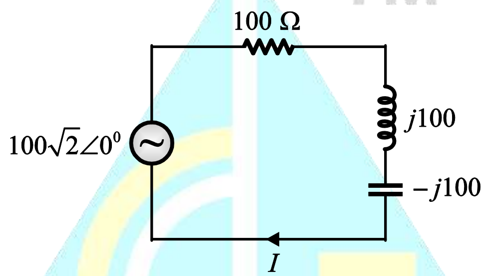
Ans. **(1)**

Sol.



$$X_L = \omega L = 1000 \times 100 \times 10^{-3} = 100$$

$$X_C = \frac{1}{\omega C} = \frac{10^6}{1000 \times 10} = 100$$



$$\therefore \quad I = \frac{100\sqrt{2}\angle 0^0}{100 + j100 - j100} = \sqrt{2}\angle 0^0$$

$$\therefore \quad I = \sqrt{2}\cos(1000t) \text{ Amperes}$$

$$I_{rms} = \frac{\sqrt{2}}{\sqrt{2}} = 1 \text{ Amperes}$$

Hence, the correct answer is 1 A.

Question 55

A 1000/1 A, 5 VA, UPF bar-primary measuring current transformer has 1000 secondary turns. The current transformer exhibits a ratio error of -0.1% and a phase error of 3.438 minutes when the primary current is 1000 A. At this operating condition, the rms value of the magnetization current of the current transformer (in amperes, rounded off to two decimal places) is ________

Ans. **0.95 to 1.05**

Sol. Given: $1000/1\text{A}, 5\text{VA}$ current transformer

$$I_1 = 1000\,\text{A}$$

$$N_2 = 1000$$

$$N_1 = 1$$

$$\delta = 0^0$$

Ratio error = – 0.1 %

Phase error = 3.438 minutes

To convert 3.438 minutes in degrees divide it by 60 $=\dfrac{3.438}{60}=0.0573^{0}$

$$\therefore \quad \text{Phase error } = \frac{I_m \cos\delta - I_e \sin\delta}{n \times I_s}$$

$$0.0573 \times \frac{\pi}{180} = \frac{I_m \times 1 - 0}{1000 \times 1}$$

$$I_m = 1\text{A}$$

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