GATE ACADEMY

## General Aptitude

## Q. 1 to Q. 5 Carry One Mark Each

Q. 1 Select the word that fits the analogy:

White : Whitening : : Light : $\qquad$ .
(A) Enlightening
(B) Lightning
(C) Lightening
(D) Lighting

Ans. (C)
Q. 2 In one of the greatest innings ever seen in 142 years of the Test history, Ben Stokes upped the tempo in a five-and-a-half hour long stay of 219 balls including 11 fours and 8 sixes that saw him finish on a 135 not out as England squared the five-match series.
Based on their connotations in the given passage, which one of the following meanings DOES NOT match?
(A) upped $=$ increase
(B) squared $=$ lost
(C) saw $=$ resulted in
(D) tempo $=$ enthusiasm

Ans. (B)
Q. 3 An Engineer measures THREE quantities $X, Y$ and $Z$ in an experiment. She finds that they follow a relationship that is represented in the figure below : (the product of X and Y linearly varies with Z ).


Then, which of the following statements is FALSE?
(A) For fixed Y ; X is proportional to Z
(B) For fixed Z ; X is proportional to Y
(C) $\mathrm{XY} / \mathrm{Z}$ is constant
(D) For fixed $\mathrm{X} ; \mathrm{Z}$ is proportional to Y

Ans. (B)
Sol. According to question $X Y \propto Z$

$$
X Y=K Z
$$

By the observation from above equation.
Q. 4 Find the missing element in the following figure.


PAGE
2
GATE 2020 [Afternoon Session] Mechanical Engineering
(A) $e$
(B) $y$
(C) $d$
(D) $w$

Ans. (C)
Sol. This is the $\log$ i.e., $(n=4)$

$$
\begin{aligned}
& 5+4=9 \\
& t=20, x=24 \\
& 20+4=24
\end{aligned}
$$

Similarly, $\quad h=8$

$$
8+4=12(l)
$$

$$
?+4=8
$$

$$
?=4(d)
$$

Q. 5 Climate change and resilience deal with two aspects-reduction of sources of non-renewable energy resources and reducing vulnerability of climate change aspects. The terms 'mitigation' and 'adaptation' are used to refer to these aspects, respectively.
Which of the following assertions is best supported by the above information?
(A) Adaptation deals with causes of climate change.
(B) Mitigation deals with consequences of climate change.
(C) Adaptation deals with actions taken to combat greenhouse gas emissions.
(D) Mitigation deals with actions taken to reduce the use of fossil fuels.

Ans. (D)

## Q. 6 to Q. 10 Carry Two Marks Each

Q. 6 There are five levels $\{P, Q, R, S, T\}$ in a linear supply chain before a product reaches customers, as shown in the figure.

$$
\mathrm{P} \Rightarrow \mathrm{Q} \Rightarrow \mathrm{R} \Rightarrow \mathrm{~S} \Rightarrow \mathrm{~T} \Rightarrow \text { Customers }
$$

At each of the five levels, the price of the product is increased by $25 \%$. If the product is produced at level P at the cost of Rs. 120 per unit, what is the price paid (in rupees) by the customers?
(A) 187.50
(B) 366.21
(C) 234.38
(D) 292.96

Ans. (B)
Sol. Initial price $=₹ 120 /$ unit
After every stage price is increasing by $25 \%$ or 1.25 and there is total 5 stage including customer stage.

$$
\therefore \quad \text { Cost }=(1.25)^{5} \times 120=366.21
$$

Q. 7 While I agree $\qquad$ his proposal this time, I do not often agree $\qquad$ him.
(A) to, with
(B) to, to
(C) with, with
(D) with, to

PAGE
3
GATE 2020 [Afternoon Session] Mechanical Engineering

GATE ACADEMY
steps to success...
Ans. (A)
Sol. to : Ideal/ Proposal with : A person
Q. 8 The recent measures to improve the output would $\qquad$ the level of production to our satisfaction
(A) equalise
(B) speed
(C) increase
(D) decrease

Ans. (C)
Q. 9 It was estimated that 52 men can complete a strip in a newly constructed highway connecting cities $P$ \& $Q$ in 10 days. Due to an emergency, 12 men were sent to another project. How many number of days, more than the original estimate, will be required to complete the strip?
(A) 3 days
(B) 10 days
(C) 5 days
(D) 13 days

Ans. (A)
Sol. $\quad 52 \times 10=40 x$
$x=13$
So, number of days required more than the original estimate $=13-10=3$.
Q. 10 The two pie-charts given below. Show the data of total students and only girls registered in different streams in a university. If the total number of students registered in the university students is 5000, and the total number of the registered girls is 1500 ; then the ratio of boys enrolled in Arts to the girls enrolled in the Management is $\qquad$ .

Percentage of students enrolled in different streams in a University


Percentage of girls enrolled in different streams

(A) $2: 1$
(B) $9: 22$
(C) $11: 9$
(D) $22: 9$

Ans. (D)
Sol. Ratio of boys enrolled in arts to the girls enrolled in management

$$
\begin{aligned}
& =\frac{\text { Total students in art }- \text { Girls in art }}{\text { Girls in management }} \\
& =\frac{5000 \times 0.20-1500 \times 0.30}{1500 \times 0.15}=22: 9
\end{aligned}
$$

## Technical Section

## Q. 1 to Q. 25 Carry One Mark Each

Q. 1 The value of enthalpies at the stator inlet and rotor outlet of a hydraulic turbomachine stage are $h_{1}$ and $h_{3}$ respectively, the enthalpy at the stator outlet (or, rotor inlet) is $h_{2}$. The condition $\left(h_{2}-h_{1}\right)=\left(h_{3}-h_{2}\right)$ indicates that the degree of reaction of this stage is
(A) $75 \%$
(B) $50 \%$
(C) Zero
(D) $100 \%$

Ans. (B)
Sol.


Degree of reaction $=\frac{h_{2}-h_{3}}{h_{1}-h_{3}}$

$$
h_{2}-h_{1}=h_{3}-h_{2}
$$

So, $h_{3}-h_{1}=2\left(h_{2}-h_{3}\right)$
D.O.R $=\frac{\left(h_{2}-h_{3}\right)}{2\left(h_{2}-h_{3}\right)}$
D.O.R $=\frac{1}{2} \Rightarrow 50 \%$
Q. 2 A bolt head has to be made at the end of a rod of diameter $d=12 \mathrm{~mm}$ by localized forging (upsetting) operation. The length of the unsupported portion of the rod is 40 mm . To avoid buckling of the rod, a closed forging operation has to be performed with a maximum die diameter of $\qquad$ mm .
Ans. 18
Sol. If $l>3 d$ then die $\operatorname{dia}=1.5 d=1.5(12)=18 \mathrm{~mm}$
Q. 3 The number of qualitatively distinct kinematic inversions possible for a Grashof chain with four revolute pairs is
(A) 2
(B) 4
(C) 1
(D) 3

Ans. (D)

PAGE
5


GATE ACADEMY'
steps to success...
Sol. They are :

1. Double crank mechanism
2. Crank-rocker mechanism
3. Double rocker mechanism.
Q. 4 Let I be a 100 dimensional identity matrix and $\mathbf{E}$ be the set of its distinct (no value appears more than once in $\mathbf{E}$ ) real eigenvalues. The number of elements in $\mathbf{E}$ is $\qquad$ .
Ans. 1
Sol. Eigen values of $I=\underbrace{1,1, \ldots ., 1}_{100 \text { times }}$
Set of distributed Eigen value, $E=(1)$
Number of elements in $E=1$.
Q. 5 Two plates, each of 6 mm thickness, are to be butt-welded. Consider the following processes and select the correct sequence in increasing order of size of the heat affected zone.
4. Arc welding
5. MIG welding
6. Laser beam welding
7. Submerged arc welding
(A) 4-3-2-1
(B) 3-2-4-1
(C) 1-4-2-3
(D) 3-4-2-1

Ans. (B)
Sol. Processes with low rate of heat input (slow heating) tend to produce high total heat constant within the metal, slow cooling rates, and large heat affected zones, high heat input process, have low total heats, fast cooling rates and small heat affected zones.
Q. 6 A closed vessel contains pure water, in thermal equilibrium with its vapour at $25^{\circ} \mathrm{C}$ (Stage\#1), as shown.


The vessel in this stage is then kept inside an isothermal oven which is having an atmosphere of hot air maintained at $80^{\circ} \mathrm{C}$. The vessel exchanges heat with the oven atmosphere and attains a new thermal equilibrium (Stag e\#2). If the valve $A$ is now opened inside the oven, what will happen immediately after opening the valve?
(A) Hot air will go inside the vessel through valve $A$
(B) Water vapor inside the vessel will come out of the valve $A$

PAGE
6
GATE 2020 [Afternoon Session] Mechanical Engineering
(C) All the vapor inside the vessel will immediately condense
(D) Nothing will happen-the vessel will continue to remain in equilibrium

Ans. (A)
Sol. Initially when water and water vapour mixture is at $25^{\circ} \mathrm{C}$ then the maximum vapour pressure that can be at $25^{\circ} \mathrm{C}$ in the saturation pressure of vapour at $25^{\circ} \mathrm{C}$.
The saturation press at $25^{\circ} \mathrm{C}$ in very less than $1 \mathrm{~atm}(101.3 \mathrm{kPa})$. It is around 3.17 kPa


Now when this vessel will be kept in oven at $80^{\circ} \mathrm{C}$ then at $80^{\circ} \mathrm{C}$ the saturation pressure of water is still less than 1 atm . It is around 47.2 kPa
The vapour pressure will reach 1 atm when temperature is $100^{\circ} \mathrm{C}$. Hence at $80^{\circ} \mathrm{C}$ also the pressure will be the than 1 atm 80 when valve is opened air will enter the valve.
Q. 7 Which one of the following statement about a phase diagram is INCORRECT?
(A) It indicates the temperature at which different phases start to melt
(B) Relative amount of different phases can be found under given equilibrium conditions
(C) It gives information on transformation rates
(D) Solid solubility limits are depicted by its

Ans. (C)
Q. 8 The process that uses a tapered horn to amplify and focus the mechanical energy for machining of glass, is
(A) electrical discharge machining
(B) electrochemical machining
(C) abrasive jet machining
(D) ultrasonic machining

Ans. (D)
Q. 9 The equation of motion of a spring - mass - damper system is given by, $\frac{d^{2} x}{d t^{2}}+3 \frac{d x}{d t}+9 x=10 \sin (5 t)$. The damping factor for the system is
(A) 0.5
(B) 0.25
(C) 3
(D) 2

Ans. (A)
Sol. Governing equation of forced damped vibration is,

$$
m \ddot{x}+c \dot{x}+k x=F_{0} \sin \omega t
$$

On comparing, $\frac{d^{2} x}{d t^{2}}+\frac{3 d x}{d t}+9 x=10 \sin 5 t$
With governing equation,
$\therefore \quad m=1, c=3, k=9$

$$
\begin{aligned}
\because \quad & \xi=\frac{c}{c_{c}}=\frac{c}{2 \sqrt{\mathrm{~km}}} \\
& \xi=\frac{3}{2 \times \sqrt{9 \times 1}}=\frac{1}{2}=0.5
\end{aligned}
$$

OR

$$
\frac{d^{2} x}{d t^{2}}+\frac{3 d x}{d t}+9 x=10 \sin (5 t)
$$

$$
\ddot{x}+2 \xi \omega_{n} \dot{x}+\omega_{n}^{2} x=10 \sin (5 t)
$$

$$
\omega_{n}^{2}=9 \Rightarrow \omega_{n}=3
$$

$$
2 \xi \omega_{n}=3
$$

$$
2 \times \xi \times 3=3
$$

$$
\xi=\frac{1}{2}=0.5
$$

Q. 10 Consider the following network of activities, with each activity named $A-L$, illustrated in the nodes of the network.


The number of hours required for each activity is shown alongside the nodes. The slack on the activity L is $\qquad$ hours.

Ans. 2
Sol.


GATE ACADEMY

Time along path $A-B-C-F-I-J-K=42$ hours
Time along path $A-B-C-E-H-L=31$ hours
Time along path $A-B-C-D-G-H-L=40$ hours
Slack for $L=42-40=2$ hours
Q. 11 Which of the following conditions is used to determine the stable equilibrium of all partially submerged floating bodies?
(A) Metacenter must be at a lower level than the centre of gravity
(B) Metacenter must be at a higher level than the centre of gravity
(C) Center of buoyancy must be below the centre of gravity
(D) Center of buoyancy must be above the centre of gravity

Ans. (B)
Q. 12 In the space above the mercury column in a barometer tube, the gauge pressure of the vapour is
(A) positive, but less than one atmosphere
(B) positive, but more than one atmosphere
(C) negative
(D) zero

Ans. (C)
Sol.

Q. 13 A machine member is subjected to fluctuating stress $\sigma=\sigma_{0} \cos (8 \pi t)$. The endurance limit of the material is 350 MPa . If the factor of safety used in the design is 3.5 then the maximum allowable value of $\sigma_{0}$ is
$\qquad$ MPa (round off to 2 decimal places).
Ans. 100
Sol. Given : $\sigma=\sigma_{0} \cos (8 \pi t)$

$$
\begin{aligned}
& \frac{d \sigma}{d t}=\sigma_{0} \times-\sin (8 \pi t) \times 8 \pi=0 \\
& \sin (8 \pi t)=0 \\
& \sin (8 \pi t)=\sin (0) \\
& \left(\frac{d^{2} \sigma}{d t^{2}}\right)_{t=0}=-\sigma_{0} \cos (8 \pi t)(8 \pi)^{2}=-64 \pi^{2} \sigma_{0}
\end{aligned}
$$

At $t=0, \sigma$ is maximum.
Also, $\quad \frac{d \sigma}{d t}=-8 \pi \sigma_{0} \sin (8 \pi t)=0$

$$
\sin (8 \pi t)=\sin \pi
$$

$$
8 \pi t=\pi
$$

$$
t=\frac{1}{8}
$$

$$
\left(\frac{d^{2} \sigma}{d t^{2}}\right)=-\sigma_{0}(8 \pi)^{2} \cos \left(8 \pi \times \frac{1}{8}\right)=\sigma_{0} 64 \pi^{2}=64 \pi^{2} \sigma_{0}
$$

$$
\sigma_{\max }=\sigma_{0}, \sigma_{a}=\sigma_{0}, \sigma_{\min }=-\sigma_{0} \text { and } \sigma_{m}=0
$$

$$
\frac{\sigma_{a}}{S_{e}}+\frac{\sigma_{m}}{S_{y t}}=\frac{1}{F O S}
$$

$$
\sigma_{a}=\frac{1}{3.5} \times 350=100 \mathrm{MPa}
$$

Q. 14 A circular disk of radius $r$ is confined to roll without slipping at $P$ and $Q$ as shown in figure.


If the plates have velocities as shown, the magnitude of the angular velocity of the disk is
(A) $\frac{2 v}{3 r}$
(B) $\frac{v}{r}$
(C) $\frac{3 v}{2 r}$
(D) $\frac{v}{2 r}$

Ans. (C)
Sol.


For pure rolling condition $P$ and $Q$ velocity should be zero. Assuming roller moves with linear velocity " $V_{0}$ ' and rotate with ' $\omega$ ' angular velocity.
$\therefore \quad$ For $P, \quad V_{0}+V=r \omega$
and,

$$
\begin{equation*}
V_{0}-r \omega=-V \tag{i}
\end{equation*}
$$

Now, for $Q, \quad V_{0}+r \omega=2 V$

From equation (i) and (ii),

$$
\begin{array}{ll} 
& V_{0}+r \omega=-V \\
& \frac{V_{0}+r \omega=2 V}{0-2 r \omega=-3 V} \\
\therefore \quad & \omega=\frac{3 V}{2 r}
\end{array}
$$

Q. 15 For an air-standard Diesel cycle,
(A) heat addition is at constant pressure and heat rejection is at constant pressure
(B) heat addition is at constant pressure and heat rejection is at constant volume
(C) heat addition is at constant volume and heat rejection is at constant volume
(D) heat addition is at constant volume and heat rejection is at constant pressure

Ans. (B)
Sol.

Q. 16 The solution of

$$
\frac{d^{2} y}{d t^{2}}-y=1
$$

which additionally satisfies $\left.y\right|_{t=0}=\left.\frac{d y}{d t}\right|_{t=0}=0$ in the Laplace s-domain is
(A) $\frac{1}{s(s-1)}$
(B) $\frac{1}{s(s+1)}$
(C) $\frac{1}{s+1}$
(D) $\frac{1}{s(s+1)(s-1)}$

Ans. (D)
Sol. $L\left\{\frac{d^{2} y}{d t^{2}}-y-1\right\}=L\{0\}$
$L\left\{\frac{d^{2} y}{d t^{2}}\right\}-L\{y\}-L\{1\}=0$
$s^{2} \bar{f}(s)-s^{1} f(0)-f^{\prime}(0)-\bar{f}(s)-\frac{1}{s}=0$
$\bar{f}(s)\left[s^{2}-1\right]-\left.s^{1} y\right|_{t=0}-\left.\frac{d y}{d t}\right|_{t=0}-\frac{1}{s}=0$
$\bar{f}(s)=\frac{1}{s\left(s^{2}-1\right)}=\frac{1}{s(s-1)(s+1)}$
Q. 17 The sum of two normally distributed random variables $X$ and $Y$ is
(A) normally distributed, only if $X$ and $Y$ have the same mean
(B) normally distributed, only if $X$ and $Y$ have the same standard deviation
(C) normally distributed, only if $X$ and $Y$ are independent
(D) always normally distributed

Ans. (D)
Sol. $\quad x_{1} \sim N\left(\mu_{1}, \sigma_{1}\right)$
and $x_{2} \sim N\left(\mu_{2}, \sigma_{2}\right)$
then $x_{1}+x_{2} \sim N\left(\mu_{1}+\mu_{2}, \sqrt{\sigma_{1}^{2}+\sigma_{2}^{2}}\right)$
Always normally distributed.
Q. 18 In Material Requirement Planning, if the inventory holding cost is very high and the setup cost is zero, which one of the following lot sizing approaches should be used ?
(A) Economic Order Quantity
(B) Lot-for-Lot
(C) Fixed Period Quantity, for 2 periods
(D) Base Stock Level

Ans. (B)
Q. 19 In a furnace the inner and outer sides of the brick wall $\left(k_{1}=2.5 \mathrm{~W} / \mathrm{mK}\right)$ are maintained at $1100^{\circ} \mathrm{C}$ and $700^{\circ} \mathrm{C}$, respectively as shown in figure


The brick wall is covered by an insulating material of thermal conductivity $k_{2}$. The thickness of insulation is $1 / 4^{\text {th }}$ of the thickness of brick wall. The outer surface of insulation is at $200^{\circ} \mathrm{C}$. The heat flux through the composite walls is $2500 \mathrm{~W} / \mathrm{m}^{2}$.

The value of $k_{2}$ is $\qquad$ W/mk. (Round off to one decimal place)

Ans. 0.5
Sol. According to question,


$$
\frac{1100-700}{\frac{L_{1}}{K_{1} A}}=\frac{700-20}{\frac{L_{2}}{K_{2} A}}
$$

$$
\frac{400}{\frac{4 L_{2}}{2.5}}=\frac{500}{\frac{L_{2}}{K_{2}}}
$$

$$
\frac{400 \times 2.5}{4 \times L_{2}}=\frac{K_{2} \times 500}{L_{2}}
$$

$$
K_{2}=0.5 \mathrm{~W} / \mathrm{mK}
$$

Q. 20 A beam of negligible mass is hinged at support P and has a roller support Q as shown in the figure.


A point load of 1200 N is applied at point R , the magnitude of reaction force at support Q is $\qquad$ N.

Ans. 1500
Sol. According question,


$$
\begin{aligned}
& R_{Q}=? \\
& 1200 \times 5-R_{Q} \times 4=0 \\
& R_{Q}=1500 \mathrm{~N}
\end{aligned}
$$

Q. 21 If a reversed carnot cycle operates between the temperature limits of $27^{\circ} \mathrm{C}$ and $-3{ }^{\circ} \mathrm{C}$, then the ratio of the COP of a refrigerator to that of a heat pump (COP of refrigerator/ COP of heat pump) based on the cycle is $\qquad$ . (round off to 2 decimal places)

GATE ACADEMY
steps to success...
Ans. 0.9
Sol. $\frac{(C O P)_{R}}{(C O P)_{H P}}=\frac{273-3}{273+27}=0.9$
Q. 22 An attempt is made to pull a roller of weight $W$ over a curb (step) by applying a horizontal force F as shown in the figure,


The coefficient of static friction between the roller and the groung (including the edge of the step) is $\mu$. Identify the correct free body diagram (FBD) of the roller when the roller is just about to climb over the step.
(A)

(B)

(C)

(D)


Ans. (C)


Weigh $=W$

## Note :

(i) When the cylinder is about to make out of the curb, it will loose its contact at point $A$, only contact will be at it $B$.
(ii) At verge of moving out of crub, Roller will be in equation under $W, F$ and contact force from $B$ and these three forces has to be concurrent so contact force from $B$ will pass through $C$
(iii) Even the surfaces are rough but there will be no friction at $B$ for the said condition FBD

Q. 23 Let $I=\int_{x=0}^{1} \int_{y=0}^{x^{2}} x y^{2} d y d x$. Then, $I$ may also be expressed as
(A) $\int_{y=0}^{1} \int_{x=\sqrt{y}}^{1} y x^{2} d x d y$
(B) $\int_{y=0}^{1} \int_{x=0}^{\sqrt{y}} y x^{2} d x d y$
(C) $\int_{y=0}^{1} \int_{x=0}^{\sqrt{y}} x y^{2} d x d y$
(D) $\int_{y=0}^{1} \int_{x=\sqrt{y}}^{1} x y^{2} d x d y$

Ans. (D)
Sol. Unit is given wrong in problem phase change,


$$
I=\int_{y=0}^{1} \int_{x=\sqrt{y}}^{1} x y^{2} d x d y
$$

Q. 24 A matrix $P$ is decomposed into its symmetric part S and skew symmetric part $V$.

If $S=\left(\begin{array}{ccc}-4 & 4 & 2 \\ 4 & 3 & 7 / 2 \\ 2 & 7 / 2 & 2\end{array}\right), V=\left(\begin{array}{ccc}0 & -2 & 3 \\ 2 & 0 & 7 / 2 \\ -3 & -7 / 2 & 0\end{array}\right)$
Then matrix $P$ is
(A) $\left(\begin{array}{ccc}-2 & 9 / 2 & -1 \\ -1 & 81 / 4 & 11 \\ -2 & 45 / 2 & 73 / 4\end{array}\right)$
(B) $\left(\begin{array}{ccc}-4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2\end{array}\right)$
(C) $\left(\begin{array}{ccc}4 & -6 & 1 \\ -2 & -3 & 0 \\ -5 & -7 & -2\end{array}\right)$
(D) $\left(\begin{array}{ccc}-4 & 6 & -1 \\ 2 & 3 & 0 \\ 5 & 7 & 2\end{array}\right)$

Ans. (B)
Sol. For matrix $P$ its symmetric part is, $S=\frac{P+P^{T}}{2}$
For matrix $P$ its skew symmetric part is, $V=\frac{P-P^{T}}{2}$

$$
P=S+V=\left(\begin{array}{ccc}
-4 & 2 & 5 \\
6 & 3 & 7 \\
-1 & 0 & 2
\end{array}\right)
$$

Q. 25 The figure below shows a symbolic representation of the surface texture in a perpendicular lay orientation with indicative values (I through VI) marking the various specifications whose definitions are listed below.
P. Maximum waviness Height (mm)
Q. Maximum Roughness Height (mm)
R. Minimum Roughness Height (mm)
S. Maximum waviness Width (mm)
T. Maximum Roughness Width (mm)
U. Roughness width cutoff (mm)


The correct match between the specifications and the symbols (I to VI) is
(A) I-U, II-S, III-Q, IV-T, V-R, VI-P
(B) I-R, II-P, III-U, IV-S, V-S, VI-Q
(C) I-Q, II-U, III-R, IV-T, V-S, VI-P
(D) I-R, II-Q, III-P, IV-S, V-U, VI-T

Ans. (D)
Sol.


I-R, II-Q, III-P, IV-S, V-U, VI-T

GATE ACADEMY

## Q. 26 to Q. 55 Carry Two Marks Each

Q. 26 For a single item inventory system, the demand is continuous, which is 10000 per year. The replenishment is instantaneous and backorders ( $S$ unit) per cycle are allowed as shown in figure


As soon as the quantity ( Q units) ordered from the supplier is received, the backordered quantity is issued to the customer. The ordering cost is Rs. 300 per order. The carrying cost Rs. 4 per unit per year.
The cost of backordering is Rs. 25 per unit per year. Based on the total cost minimization criteria, the maximum inventory reached in the system is $\qquad$ (round off to nearest integer).

## Ans. 1137.14

Sol. Given : $D=10000 /$ year , $F=$ Rs. 300/order , $C=$ Rs. 4/unit/year and $C_{s}=$ Rs. $25 /$ unit/year .

$$
\begin{aligned}
& Q=\sqrt{\frac{2 D F}{C} \times \frac{C+C_{s}}{C_{s}}} \\
& Q=\sqrt{\frac{2 \times 10000 \times 300 \times 29}{4 \times 25}}=1319.09 \text { units } \\
& S=\frac{Q C}{C+C_{s}} \\
& S=\frac{1319 \times 4}{29}=181.94 \text { units } \\
& I_{m}=Q-S=1137.14 \text { units }
\end{aligned}
$$

Q. 27 The spectral distribution of radiation from a black body at $T_{1}=3000 \mathrm{~K}$ has a maximum at wavelength $\lambda_{\text {max }}$ . The body cools down to a temperature $T_{2}$. If the wavelength corresponding to the maximum of the spectral distribution at $T_{2}$ is 1.2 times of the original wavelength $\lambda_{\max }$ then the temperature $T_{2}$ is $\qquad$ K (round off to nearest integer).

Ans. 2500
Sol. Given : $T_{1}=3000 \mathrm{~K}$.

$$
\lambda T=C
$$

$$
\begin{aligned}
& \lambda_{1}=\lambda \\
& \lambda_{2}=1.2 \lambda \\
& \lambda_{1} T_{1}=\lambda_{2} T_{2} \\
& \lambda \times 3000=1.2 \lambda T_{2} \\
& T_{2}=2500 \mathrm{~K}
\end{aligned}
$$

Q. 28 Moist air at $105 \mathrm{kPa}, 30^{\circ} \mathrm{C}$ and $80 \%$ relative humidity flows over a cooling coil in an insulated airconditioning duct. Saturated air exists the duct at 100 kPa and $15^{\circ} \mathrm{C}$. The saturation pressure of water at $30^{\circ} \mathrm{C}$ and $15^{\circ} \mathrm{C}$ are 4.24 kPa and 1.7 kPa respectively. Molecular weight of water is $18 \mathrm{~g} / \mathrm{mol}$ and that of air is $28.94 \mathrm{~g} / \mathrm{mol}$. The mass of water condensing out from the duct is $\qquad$ $\mathrm{g} / \mathrm{kg}$ of dry air (round off to nearest integer)
Ans. 10
Sol. Given : $P_{1}=105 \mathrm{kPa}, P_{2}=100 \mathrm{kPa}, T_{1}=30^{\circ} \mathrm{C}, T_{2}=15^{\circ} \mathrm{C}, R H=0.8, R H=1$ (saturated air), $P_{s_{1}}=4.24 \mathrm{kPa}, P_{s_{2}}=1.7 \mathrm{kPa}, M_{w}=18 \mathrm{~g} / \mathrm{mol}$ and $M_{a}=28.94 \mathrm{~g} / \mathrm{mol}$.

Now,

$$
R H=\frac{P_{V}}{P_{s}}
$$

$$
0.8=\frac{P_{V_{1}}}{4.24}
$$

$$
P_{V_{1}}=3.392 \mathrm{kPa}
$$

$$
\therefore \quad w_{1}=0.622 \times \frac{P_{V_{1}}}{P_{a t m}-P_{V_{1}}}
$$

$$
w_{1}=0.622 \times \frac{3.392}{105-3.392}=20.764 \mathrm{~g} / \mathrm{kg} \text { of air }
$$

Also,

$$
\begin{aligned}
& w_{2}=0.622 \times \frac{P_{V_{2}}}{P_{a t m}-P_{V_{2}}} \quad\left(P_{V_{2}}=P_{s_{2}}, \text { as saturated air }\right) \\
& w_{2}=0.622 \times \frac{1.7}{100-1.7}=10.756 \mathrm{~g} / \mathrm{kg} \text { of air }
\end{aligned}
$$

$\therefore \quad$ Mass of water condensed $=20.764-10.756=10.007 \mathrm{~g} / \mathrm{kg}$ of air.
Q. 29 Water flows through a tube of 3 cm internal diameter and length 20 m . The outside surface of the tube is heated electrically so that it is subjected to uniform heat flux circumferentially and axially. The mean inlet and exit temperatures of water are $10^{\circ} \mathrm{C}$ and $70^{\circ} \mathrm{C}$ respectively. The mass flow rate of the water is $720 \mathrm{~kg} / \mathrm{h}$. Disregard the thermal resistance of the tube wall. The internal heat transfer coefficient is 1697 $\mathrm{W} / \mathrm{m}^{2} \mathrm{~K}$. Take specific heat $\boldsymbol{c}_{\boldsymbol{p}}$ of water as $4.179 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$. The inner surface temperature at the exist section of the tube is $\qquad$ ${ }^{0} \mathrm{C}$ (round of to one decimal place)

Ans. 85.7
Sol. Water flows through 3 cm in a dia.,

Q. 30 The directional derivative of $f(x, y, z)=x y z$ at point $(-1,1,3)$ in the direction of vector $\hat{i}-2 \hat{j}+2 \hat{k}$ is
(A) $3 \hat{i}-3 \hat{j}-\hat{k}$
(B) $\frac{-7}{3}$
(C) $\frac{7}{3}$
(D) 7

Ans. (C)
Sol. $f=x y z, \vec{a}=\hat{i}-2 \hat{j}+2 \hat{k}$

$$
\begin{array}{ll}
\therefore \quad & \nabla f=\hat{i} \frac{\partial f}{\partial x}+\hat{j} \frac{\partial f}{\partial y}+\hat{k} \frac{\partial f}{\partial z} \\
& \nabla f=y z \hat{i}+z x \hat{j}+x y \hat{k} \\
& (\nabla f)_{-1,1,3}=3 \hat{i}-3 \hat{j}-\hat{k}
\end{array}
$$

$$
D . D=(\nabla f)_{-1,1,3} \frac{\vec{a}}{|\vec{a}|}
$$

$$
D . D=(3 \hat{i}-3 j-\hat{k}) \times \frac{(\hat{i}-2 \hat{j}+2 \hat{k})}{\sqrt{1^{2}+(-2)^{2}+2^{2}}}
$$

$$
D . D=\frac{3+6-2}{\sqrt{9}}=\frac{7}{3}
$$

Q. 31 A fair coin is tossed 20 times. The probability that 'head' will appear exactly 4 times in the first ten tosses, and 'tail' will appear exactly 4 times in the next ten tosses is $\qquad$ (round off to 3 decimal places)
Ans. 0.042

Sol. Given : $P_{1}=\frac{1}{2}, a_{1}=\frac{1}{2}, P_{2}=\frac{1}{2}, a_{2}=\frac{1}{2}$
Probability $={ }^{10} C_{4}\left(P_{1}\right)^{4}\left(a_{1}\right)^{(10-4)} \times{ }^{10} C_{4}\left(P_{2}\right)^{4}(a)^{(10-4)}$
Probability $={ }^{10} C_{4}\left(\frac{1}{2}\right)^{10} \times{ }^{10} C_{4}\left(\frac{1}{2}\right)^{10}=0.042$
Q. 32 There are two identical shaping machines $S_{1}$ and $S_{2}$. In machine $S_{2}$, the width of the work piece is increased by $10 \%$ and the feed is decreased by $10 \%$, with respect to that of $S_{1}$. If all other conditions remain the same then the ratio of total time per pass in $S_{1}$ and $S_{2}$ will be $\qquad$ (round of to one decimal place).
Ans. 0.82
Sol.

$$
\begin{array}{|c|c|}
\hline S_{1} & S_{2} \\
W_{1}=W=\text { Width of work piece } & W_{2}=1.1 W \\
f_{1}=f & f_{2}=0.9 f \\
\hline T_{m}=\left(T_{C}+T_{R}\right) \times \frac{W}{f} \\
& T_{m} \propto \frac{W}{f} \\
& \frac{\left(T_{m}\right)_{S_{1}}}{\left(T_{m}\right)_{S_{2}}}=\frac{W_{1}}{W_{2}} \times \frac{f_{2}}{f_{1}} \\
\frac{\left(T_{m}\right)_{S_{1}}}{\left(T_{m}\right)_{S_{2}}}=\frac{1}{1.1} \times \frac{0.9}{1}=0.818 \approx 0.82
\end{array}
$$

Q. 33 For the integral $\int_{0}^{\pi / 2}(8+4 \cos x) d x$, the absolute percentage error in numerical evaluation with the

Trapezoidal rule, using only the end points, is $\qquad$ (round off to one decimal place).
Ans. 5.2
Sol. $\int_{0}^{\pi / 2}(8+4 \cos x) d x$
$f(x)=8+4 \cos x$

| $\boldsymbol{x}$ | 0 | $\pi / 2$ |
| :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | 12 | 8 |

Using trapezoidal method,

$$
\begin{aligned}
& \int_{0}^{\pi / 2}(8+4 \cos x) d x=\frac{h}{2}(12+8)=\frac{\pi}{4} \times 20 \\
& \int_{0}^{\pi / 2}(8+4 \cos x) d x=\pi \times 5=15.70
\end{aligned}
$$

Now,

$$
\begin{aligned}
& \int_{0}^{\pi / 2}(8+4 \cos x) d x=[8 x+4 \sin x]_{0}^{\pi / 2} \\
& \int_{0}^{\pi / 2}(8+4 \cos x) d x=\left[8 \times \frac{\pi}{2}-8 \times 0\right]+\left[4\left(\sin \frac{\pi}{2}-\sin 0\right)\right] \\
& \int_{0}^{\pi / 2}(8+4 \cos x) d x=4 \pi+4=16.566
\end{aligned}
$$

$\therefore \quad$ Absolute $\%$ error $=\frac{16.57-15.70}{15.70} \times 100=5.51 \%$
Q. 34 Water (density $=1000 \mathrm{~kg} / \mathrm{m}^{3}$ ) flow through an inclined pipe of uniform diameter. The velocity, pressure and elevation at section $A$ are $V_{A}=3.2 \mathrm{~m} / \mathrm{s}, p_{A}=186 \mathrm{kPa}$ and $z_{A}=24.5 \mathrm{~m}$, respectively, and those at section $B$ are $V_{B}=3.2 \mathrm{~m} / \mathrm{s}, P_{B}=260 \mathrm{kPa}$ and $z_{B}=9.1 \mathrm{~m}$, respectively. If acceleration due to gravity is $10 \mathrm{~m} / \mathrm{s}^{2}$ then the head lost due to friction is $\qquad$ m (round off to one decimal place).
Ans. 8
Sol.

Q. 35 Consider a flow through nozzle, as shown in the figure below.


The air flow is steady, incompressible and inviscid. The density of air is $1.23 \mathrm{~kg} / \mathrm{m}^{3}$. The pressure difference $\left(p_{1}-p_{\text {atm }}\right)$ is $\qquad$ kPa . (round off to 2 decimal place)

Ans. 1.52
Sol. Given : $A_{1}=0.2 \mathrm{~m}^{2}, P_{2}=P_{\text {atm }}, V_{2}=50 \mathrm{~m} / \mathrm{s}, A_{2}=0.02 \mathrm{~m}^{2}, \rho_{\text {air }}=1.23 \mathrm{~kg} / \mathrm{m}^{3}$

$$
\begin{aligned}
& p_{1} \\
& P_{1} A_{1} V_{1}=P_{2} A_{2} V_{2} \\
& 0.2 \times V_{1}=0.02 \times 50 \\
& V_{1}=\frac{0.02 \times 50}{0.2}=\frac{1}{0.2}=5 \mathrm{~m} / \mathrm{s} \\
& \frac{P_{1}}{\rho g}+\frac{V_{1}^{2}}{2 g}+Z_{1}=\frac{P_{2}}{\rho g}+\frac{V_{2}^{2}}{2 g}+Z_{2} \\
& \frac{P_{1}-P_{\text {atm }}}{1.23}=\frac{V_{2}^{2}-V_{1}^{2}}{2}=\frac{50^{2}-5^{2}}{2} \\
& P_{1}-P_{\text {atm }}=1522.125 \mathrm{~m} / \mathrm{s} \\
& A_{2}=1.522 \mathrm{kPa}
\end{aligned}
$$

Q. 36 The function $f(z)$ of complex variable $z=x+i y$, where $i=\sqrt{-1}$, is given as $f(z)=\left(x^{3}-3 x y^{2}\right)+i v(x, y)$. For this, function to be analytic $v(x, y)$ should be
(A) $\left(3 x y^{2}-y^{3}\right)+$ constant
(B) $\left(x^{3}-3 x^{2} y\right)+$ constant
(C) $\left(3 x^{2} y^{2}-y^{3}\right)+$ constant
(D) $\left(3 x^{2} y-y^{3}\right)+$ constant

Ans. (D)
Sol. $\quad f(z)=u+i v$
Where, $u=x^{3}-3 x y^{2}$ and $v=$ ?
C-R equation, $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$
$\therefore \quad \frac{\partial u}{\partial x}=3 x^{2}-3 y^{2}$

$$
\begin{align*}
& \frac{\partial v}{\partial y}=3 x^{2}-3 y^{2} \\
& v=3 x^{2} y-y^{3}+f(x) \tag{i}
\end{align*}
$$

and,

$$
\begin{aligned}
& \frac{\partial v}{\partial x}=6 x y+f^{\prime}(x) \\
& -\frac{\partial u}{\partial y}=6 x y+f^{\prime}(x) \\
& -(-6 x y)=6 x y+f^{\prime}(x) \\
& f^{\prime}(x)=0
\end{aligned}
$$

$$
\therefore \quad f(x)=\text { Constant }
$$

From equation (i), $v=3 x^{2} y-y^{3}+$ Constant.
Q. 37 Bars of 250 mm length and 25 mm diameter are to be turned on a lathe with a feed of $0.2 \mathrm{~mm} / \mathrm{rev}$. Each regrinding of tool cost Rs. 20. The time required for each tool change is 1 min . Tool life equation is given as $V T^{0.2}=24$ (where cutting speed V is in $\mathrm{m} / \mathrm{min}$ and tool life T is in min). The optimum tool cost per piece for maximum production rate is Rs. $\qquad$ (round off to 2 decimal places)
Ans. 26.99
Sol. Optimum tool life $\left(T_{0}\right)=\frac{T_{c}(1-n)}{n}=\frac{1(1-0.2)}{0.2}=4 \mathrm{~min}$

$$
\begin{aligned}
& V_{0} T_{0}^{n}=C \text { or } V_{0}=\frac{C}{T_{0}^{n}}=\frac{24}{4^{0.2}}=18.19 \mathrm{~m} / \mathrm{min} \\
& v=D N \\
\Rightarrow & 18.19=\pi \times 0.025 \times N \\
\Rightarrow & N=231.6 \mathrm{rpm}
\end{aligned}
$$

Maching time per piece

$$
\left(T_{m}\right)=\frac{L}{f N}=\frac{250}{0.2 \times 231.6}=5.397 \mathrm{~min}
$$

Number of tool needed per piece work

$$
=\frac{5.397}{4} \text { piece }
$$

$\therefore$ The optimum tool cost per piece

$$
=\frac{5.397}{4} \times 20=26.985
$$

Q. 38 A helical spring has spring constant $k$. If the wire diameter, spring diameter and the number of coils are all doubled then the spring constant of the new spring becomes
(A) $\frac{k}{2}$
(B) $k$
(C) $8 k$
(D) $16 k$

Ans. (B)
Sol. $\delta=\frac{8 P D^{3} N}{G d^{4}}$
$k=\frac{P}{\delta}=\frac{P \times G d^{4}}{8 P D^{3} N}=\frac{G d^{4}}{8 D^{3} N}$
$k^{\prime}=\frac{G \times 2^{4} d^{4}}{8 \times 2^{3} D^{3} \times 2 N}$
$k^{\prime}=k$
Q. 39 A hollow spherical ball of radius 20 cm floats in still water, with half of it's volume submerged. Taking the density of water as $1000 \mathrm{~kg} / \mathrm{m}^{3}$, and the acceleration due to gravity as $10 \mathrm{~m} / \mathrm{s}^{2}$, the natural frequency of small oscillations of the ball, normal to the water surface is $\qquad$ $\mathrm{rad} / \mathrm{s}$ (round off to 2 decimal place).
Ans. 8.66
Sol.

$m \ddot{x}+V_{\text {extra }} \cdot \rho \cdot g=0$
$\rho \cdot \frac{V_{s}}{2} \ddot{x}+\left(\pi R^{2} \cdot x\right) \rho \cdot g=0$

$$
\begin{aligned}
& \quad \frac{4}{3} \frac{\pi R^{3}}{2} \hat{x}+\pi R^{2} x g=0 \\
& \ddot{x}+\frac{3 g}{2 R} x=0
\end{aligned}
$$

$\omega_{n}=\sqrt{\frac{3 g}{2 R}}=\sqrt{\frac{3 \times 10}{2 \times 0.20}}=\sqrt{\frac{30}{0.4}}$
$\omega_{n}=\sqrt{\frac{300}{4}}=\sqrt{75}=8.66 \mathrm{rad} / \mathrm{s}$
Q. 40 One kg of air in a closed system undergoes an irreversible process from an initial state of $p_{1}=1$ bar (absolute) and $T_{1}=27^{\circ} \mathrm{C}$, to a final state of $p_{2}=3 \mathrm{bar}$ (absolute) and $T_{2}=127^{\circ} \mathrm{C}$. If the gas constant of air is $287 \mathrm{~J} / \mathrm{kg} \mathrm{K}$ and the ratio of specific heats $\gamma=1.4$, then the change in the specific entropy (in $\mathrm{J} / \mathrm{kg} \mathrm{K}$ ) of the air in the process is
(A) 172.0
(B) indeterminate, as the process is irreversible
(C) 28.4
(D) -26.3

Ans. (D)
Sol. Given : $T_{1}=300 \mathrm{~K}, T_{2}=400 \mathrm{~K}, p_{2}=3 \mathrm{bar}, p_{1}=1 \mathrm{bar}$

$$
\begin{aligned}
& \Delta S=C_{p} \ln \left(\frac{T_{2}}{T_{1}}\right)-R \ln \left(\frac{p_{2}}{p_{1}}\right) \\
& \Delta S=1005 \ln \left(\frac{400}{300}\right)-287 \ln \left(\frac{3}{1}\right)=-26.18 \mathrm{~J} / \mathrm{kgK}
\end{aligned}
$$

Q. 41 The turning moment diagram of a flywheel fitted to a fictitious engine is shown in the figure.


The mean turning moment is $2000 \mathrm{~N}-\mathrm{m}$. The average engine speed is 1000 rpm . For fluctuation in speed to be within $\pm 2 \%$ of the average speed, the mass moment of inertia of the flywheel is $\qquad$ $\mathrm{kg}-\mathrm{m}^{2}$.
Ans. 3.58
Sol. Given : $T_{\text {mean }}=2000 \mathrm{Nm}, N=1000 \mathrm{rpm}, C_{\mathrm{s}}= \pm 2 \%$ and $I=$ ?


$$
\begin{aligned}
& \Delta E=1000 \times \frac{\pi}{2}=500 \pi \\
& 500 \pi=I\left(\frac{2 \pi \times 1000}{60}\right)^{2} \times 0.04 \\
& I=3.58 \mathrm{kgm}^{2}
\end{aligned}
$$

Q. 42 A cylindrical bar with 200 mm diameter is being turned with a tool having geometry $0^{0}-9^{0}-7^{0}-8^{0}$ $-15^{0}-30^{0}-0.05$ inch (Coordinate system, ASA) in a cutting force $F_{c_{1}}$. If the tool geometry is changed to $0^{0}-9^{0}-7^{0}-8^{0}-15^{0}-0^{0}-0.05$ inch (Coordinate system, ASA) and all other parameter remain unchanged, the cutting force changes to $F_{c_{2}}$. Specific cutting energy in $\left(\mathrm{J} / \mathrm{mm}^{3}\right)$ is $U_{C}=U_{0}\left(t_{1}\right)^{-0.4}$, where $U_{0}$ is specific energy coefficient, and $t_{1}$ is the uncut thickness in mm . The value of percentage change in cutting force $F_{c_{2}}$, i.e., $\left(\frac{F_{c_{2}}-F_{c_{1}}}{F_{c_{1}}}\right) \times 100$ is___ (round off to one decimal place).
Ans. - 5.6
Sol. $\quad C_{s_{1}}=30^{\circ}$

$$
\begin{array}{ll}
\therefore & \lambda_{1}=90-30=60^{\circ} \\
& C_{s_{2}}=0^{0} \\
\therefore & \lambda_{2}=90-0=90^{\circ}
\end{array}
$$

We know that specific energy consumption,

$$
\begin{array}{ll} 
& U_{c}=\frac{F_{c}}{1000 f d}=U_{0}\left(t_{1}\right)^{-0.4} \text { (given) } \\
\therefore & F_{c}=U_{0}(f \sin \lambda)^{-0.4} \times 1000 f d \\
\therefore & F_{c} \propto(\sin \lambda)^{-0.4} \\
\therefore & \frac{F_{c_{2}}-F_{c_{1}}}{F_{c_{1}}} \times 100=\left[\left(\frac{\sin \lambda_{2}}{\sin \lambda_{1}}\right)^{-0.4}-1\right] \times 100 \\
& \frac{F_{c_{2}}-F_{c_{1}}}{F_{c_{1}}} \times 100=\left[\left(\frac{\sin 90^{0}}{\sin 60^{0}}\right)^{-0.4}-1\right] \times 100=-5.59
\end{array}
$$

Q. 43 A mould cavity of $1200 \mathrm{~cm}^{3}$ volume has to be filled through a sprue of 10 cm length feeding a horizontal runner. Corss-sectional area at the base of the sprue is $2 \mathrm{~cm}^{2}$. Consider acceleration due to gravity as $9.81 \mathrm{~m} / \mathrm{s}^{2}$. Neglecting frictional losses due to molten metal flow, the time taken to fill the mold cavity is
$\qquad$ seconds (round off to 2 decimal places).

## Ans. 4.28

Sol. Given : $V_{m}=1200 \mathrm{~cm}^{3}, A_{g}=2 \mathrm{~cm}^{2}$ and $T_{\text {Тор }}=$ ?


$$
\begin{aligned}
& T_{\text {Top }}=\frac{V_{m}}{A_{g} \sqrt{2 g h_{t}}} \\
& T_{\text {Top }}=\frac{1200}{2 \sqrt{2 \times 9.81 \times 100 \times 10}}=4.28 \mathrm{sec}
\end{aligned}
$$

Q. 44 Uniaxial compression test data for a solid metal bar of length 1 m is shown in the figure.


The bar material has a linear elastic response from 0 to P followed by a nonlinear response. The point $P$ represents the yield point of material. The rod is pinned at both ends. The minimum diameter of bar so that it does not buckle under axial loadings before reaching the yield point is $\qquad$ mm (round off to one decimal place).
Ans. 56.94
Sol. Given : $\sigma=100 \mathrm{MPa}$ and $L=1 \mathrm{~m}$.

$$
\begin{aligned}
& E=\frac{100 \times 10^{6}}{0.002}=50 \times 10^{9} \\
& I=\frac{\pi}{64} d^{4} \\
& \sigma(\mathrm{MPa})
\end{aligned}
$$

$$
\begin{aligned}
& \sigma_{c r}=\frac{P_{c r}}{A}=\frac{\pi^{2} E I}{A L^{2}} \\
& 100 \times 10^{6}=\frac{\pi^{2} \times 50 \times 10^{9}}{\frac{\pi}{4} d^{2} L^{2}} \times \frac{\pi d^{4}}{64} \\
& \frac{100 \times 10^{6} \times 64}{4 \times \pi^{2} \times 50 \times 10^{3}}=d^{2} \\
& d=0.0569 \mathrm{~m}=56.94 \mathrm{~mm}
\end{aligned}
$$

Q. 45 A cantilever of length $l$, and flexural rigidity EI, stiffened by a spring of stiffness $k$, is loaded by a transverse force $P$, as shown.


The transverse deflection under the load is,
(A) $\frac{P l^{3}}{3 E I}\left(\frac{6 E I-K l^{3}}{6 E I}\right)$
(B) $\frac{P l^{3}}{3 E I}\left(\frac{3 E I}{3 E I+2 K l^{3}}\right)$
(C) $\frac{P l^{3}}{3 E I}\left(\frac{3 E I-K l^{3}}{3 E I}\right)$
(D) $\frac{P l^{3}}{3 E I}\left(\frac{3 E I}{3 E I+K l^{3}}\right)$

Ans. (D)
Sol. According to question,

$$
\begin{aligned}
& \delta=\frac{P l^{3}}{3 E I+K l^{3}} \\
& R=K x
\end{aligned}
$$

$$
x=\frac{R}{K}
$$

$$
\frac{P l^{3}}{3 E I}-\frac{R l^{3}}{3 E I}=x
$$

$$
\frac{-P l^{3}}{3 E I}+\frac{R l^{3}}{3 E I}+\frac{R}{K}=0
$$

$$
\frac{-P l^{3}}{3 E I}+\frac{K x l^{3}}{3 E I}+x=0
$$

$$
\begin{aligned}
& x\left(\frac{K l^{3}}{3 E I}+1\right)=\frac{P l^{3}}{3 E I} \\
& x\left(\frac{K l^{3}+3 E I}{3 E I}\right)=\frac{P l^{3}}{3 E I} \\
& x=\frac{P l^{3}}{K l^{3}+3 E I}=\frac{P l^{3}}{3 E I}\left(\frac{3 E I}{3 E I+K l^{3}}\right)
\end{aligned}
$$

Q. 46 A steel spur pinion has a module (m) of $1.25 \mathrm{~mm}, 20$ teeth and $20^{\circ}$ pressure angle. The pinion rotates at 1200 rpm and transmits power to a 60 teeth gear. The face width $(\mathrm{F})$ is 50 mm , Lewis form factor $Y=0.322$ and a dynamic factor, $K_{V}=1.26$. The bending stress $(\sigma)$, induced in a tooth can be calculated by using the Lewis formula given below. If the maximum bending stress experienced by the pinion is 400 MPa , the power transmitted is $\qquad$ kW . (round off to one decimal place)

Lewis formula : $\sigma=\frac{K_{V} W^{t}}{F m Y}$, Where $W^{t}$ is the tangential load acting on the pinion.
Ans. 10.035
Sol. Given : Module $(m)=1.25 \mathrm{~mm}$, Number of tooth $\left(Z_{p}\right)=20$ teeth,
Pressure angle $(\phi)=20^{\circ}, \operatorname{Rpm}(N)=1200 \mathrm{rpm}$, Face width $(F)=50 \mathrm{~mm}$ Lewis form factor $(Y)=0.322$ and Dynamic factor $\left(K_{V}\right)$.

$$
\begin{aligned}
& \sigma=\frac{K_{V} W^{t}}{F m Y} \\
& 400=\frac{1.26 \times\left(W^{t}\right)}{50 \times 1.25 \times 0.322}
\end{aligned}
$$

$$
W^{t}=6388.88 \mathrm{~N}
$$

Power $(P)=\frac{\left(M_{t}\right) 2 \pi N}{60}=\frac{\left(W^{t} \times R\right) 2 \pi N}{60}$

$$
\begin{aligned}
& =\frac{6388.88 \times\left(\frac{m Z_{p}}{2}\right) \times 10^{-3} \times 2 \pi 1200}{60} \\
& =\frac{6388.88 \times\left(\frac{1.25 \times 20}{2}\right) \times 2 \pi 1200}{60 \times 1000} \\
& =10035.63 \mathrm{~W}=10.035 \mathrm{~kW}
\end{aligned}
$$

Q. 47 Keeping all other parameters identical, the Compression Ratio ( $C, R$ ) of an air standard diesel cycle is increased from 15 to 21 . Take ratio of specific heats $=1.3$ and cutoff ratio of the cycle $r_{c}=2$. The difference between the new and the old efficiency values, in percentage,

$$
\left(\left.\eta_{\text {new }}\right|_{C R=21}\right)-\left(\left.\eta_{\text {old }}\right|_{C R=15}\right)=\ldots \quad \text { (round off to one decimal place). }
$$

Ans. 4.8
Sol. According to question, keeping all the parameter identical,

$$
\begin{aligned}
& \left.\quad \eta_{\text {new }}\right|_{C R=21}=1-\frac{1}{(r)^{\gamma-1}} \times\left(\frac{\rho^{r}-1}{\rho-1}\right) \times \frac{1}{r} \\
& \\
& \left.\eta_{\text {new }}\right|_{C R=21}=1-\frac{1}{(C R)^{\gamma-1}} \times\left(\frac{\rho^{\gamma}-1}{\rho-1}\right) \times \frac{1}{\gamma} \\
& \\
& \left.\eta_{\text {new }}\right|_{C R=21}=1-\frac{1}{(21)^{1.3-1}} \times\left(\frac{2^{1.3}-1}{2-1}\right) \times \frac{1}{1.3}=0.5487 \\
& \\
& \left.\eta_{\text {old }}\right|_{C R=15}=1-\frac{1}{(C R)^{\gamma-1}} \times\left(\frac{\rho^{\gamma}-1}{\rho-1}\right) \times \frac{1}{\gamma} \\
& \left.\therefore \quad \eta_{\text {old }}\right|_{C R=15}=1-\frac{1}{(15)^{1.3-1}} \times\left(\frac{2^{1.3}-1}{2-1}\right) \times \frac{1}{1.3}=0.5008 \\
& \\
& \left(\eta_{\text {new }}\right)_{C R=21}-\left(\eta_{\text {old }}\right)_{C R=15}=0.5487-0.5008=0.0479 \\
& \\
& \left(\eta_{\text {new }}\right)_{C R=21}-\left(\eta_{\text {old }}\right)_{C R=15}=4.79 \%
\end{aligned}
$$

Q. 48 A thin-walled cylinder of radius $r$ and thickness $t$ is open at both ends, and fits snugly between two rigid walls under ambient conditions, as shown in figure.


The material of the cylinder has Young's modulus E, Poisson's ratio $v$ and coefficient of thermal expansion $\alpha$. What is minimum rise in temperature $\Delta T$ of the cylinder (assume uniform cylinder temperature with no buckling of the cylinder) required to prevent gas leakage if the cylinder has to store the gas at an internal pressure of $p$ above the atmosphere?
(A) $\Delta T=\frac{v p r}{\alpha t E}$
(B) $\Delta T=\frac{3 v p r}{2 \alpha t E}$
(C) $\Delta T=\left(v-\frac{1}{4}\right) \frac{p \mathrm{r}}{\alpha t E}$
(D) $\Delta T=\left(v+\frac{1}{2}\right) \frac{p r}{\alpha t E}$

Ans. (A)
Sol. Since cylinder is open at both end

$$
\therefore \quad \sigma_{L}=0
$$

For no leakage, $\varepsilon_{L_{p_{r}}}=\varepsilon_{L_{\text {enp }}}$

$$
\begin{aligned}
\frac{v \sigma_{n}}{E} & =\alpha \Delta T \\
\frac{v \operatorname{Pr}}{t E} & =\alpha \Delta T \\
\Delta T & =\frac{v p r}{\alpha t E}
\end{aligned}
$$

Q. 49 Air is contained in a frictionless piston-cylinder arrangement as shown in the figure.


The atmospheric pressure is 100 kPa and the initial pressure of air in the cylinder is 105 kPa . The area of piston is $300 \mathrm{~cm}^{2}$. Heat is now added and the piston moves slowly from its initial position until it reaches the stops. The spring constant of the linear spring is $12.5 \mathrm{~N} / \mathrm{mm}$. Considering the air inside the cylinder as the system, the work interaction is $\qquad$ J (round off to the nearest integer).
Ans. 544
Sol. Given : $p_{1}=105 \mathrm{kPa}, p_{\text {atm }}=100 \mathrm{kPa}, A=300 \mathrm{~cm}^{2}$ and $K=12.5 \mathrm{~N} / \mathrm{mm}$.


Initially up to 8 cm .


$$
W=p d V=105 \times 300 \times 8 \times 10^{-6}=252
$$

Finally,


$$
\begin{aligned}
& p_{2}=p_{a t m}+\frac{W}{A}+\frac{F_{s}}{A} \\
& p_{2}=p_{1}+\frac{k x}{A}
\end{aligned}
$$

$$
p_{2}=105+\frac{12.5 \times 8 \times 10^{-2}}{300 \times 10^{-4}}=138.333 \mathrm{kPa}
$$



$$
\begin{aligned}
& W=\frac{1}{2} \times(138.33+105) \times 300 \times 10^{-4} \times 8 \times 10^{-2} \\
& W=292000 \times 10^{3} \times 10^{-6}=292 \mathrm{~J}
\end{aligned}
$$

Total work $=252+292=544 \mathrm{~J}$
OR
$W_{\text {piston }}=105 \times(300 \times 16) \times 10^{-6}=504 \mathrm{~J}$
$W_{\text {spring }}=\frac{1}{2} k x^{2}$
$W_{\text {spring }}=\frac{1}{2} \times 12.5 \times 64 \times 10^{-4} \times 10^{3}=\frac{1}{2} \times 12.5 \times 6.4=40$
$W_{\text {total }}=504+40=544 \mathrm{~J}$
Q. 50 The forecast for the monthly demand of a product is given in the table below.

| Month | Forecast | Actual sales |
| :---: | :---: | :---: |
| 1 | 32.00 | 30.00 |
| 2 | 31.80 | 32.00 |
| 3 | 31.82 | 30.00 |

The forecast is made by using exponential smoothing method. The exponential smoothing coefficient used in forecasting the demand is
(A) 0.50
(B) 0.40
(C) 0.10
(D) 1.0

GATE ACADEMY
steps to success...
Ans. (C)
Sol. $\quad 31.82=\alpha D+(1-\alpha) F$
$31.82=32 \alpha+(1-\alpha) 31.8=32 \alpha+31.8-31.8 \alpha$
$31.82-31.8=(32-31.8) \alpha$
$\alpha=0.1$
Q. 51 The sun (S) and planet (P) of an epicyclic gear train shown in the figure have identical number of teeth.


If the sun $(S)$ and the outer ring $(R)$ gears are rotated in the same direction with angular speed $\omega_{s}$ and $\omega_{R}$, respectively, then the angular speed of the arm $A B$ is
(A) $\frac{3}{4} \omega_{S}+\frac{1}{4} \omega_{R}$
(B) $\frac{1}{2} \omega_{R}-\frac{1}{2} \omega_{S}$
(C) $\frac{3}{4} \omega_{R}+\frac{1}{4} \omega_{S}$
(D) $\frac{3}{4} \omega_{R}-\frac{1}{4} \omega_{S}$

Ans. (C)
Sol. $\quad 3 T_{A}=T_{R}$
$T_{A}+2 T_{R}=T_{R}$
$T_{A}=T_{B}$
$\frac{m T_{A}}{2}+m T_{B}=\frac{m T_{R}}{2}$
$T_{A}=T, T_{B}=T, T_{R}=3 T$


| Arm | SA | $\boldsymbol{B}$ | $\boldsymbol{R}$ |
| :---: | :---: | :---: | :---: |
| 0 | $x$ | $-x \times \frac{T}{T}$ | $-x \times \frac{T}{3 T}$ |
| $y$ | $y+x$ | $y-x$ | $y-\frac{x}{3}$ |

$$
y+x=\omega_{S}
$$

$$
y-\frac{x}{3}=\omega_{R}
$$

$$
4 \frac{x}{3}=\omega_{S}-\omega_{R}
$$

$$
x=\frac{3\left(\omega_{S}-\omega_{R}\right)}{4}
$$

and

$$
y=\omega_{S}-\frac{3}{4} \omega_{S}+\frac{3}{4} \omega_{R}=\frac{3}{4} \omega_{R}+\frac{1}{4} \omega_{S}
$$

Ans.
Q. 52 In a steam power plant, superheated steam at 10 MPa and $500^{\circ} \mathrm{C}$, is expanded isentropically in a turbine until it becomes a saturated vapour. It is then reheated at constant pressure to $500^{\circ} \mathrm{C}$. The steam is next expanded isentropically in another turbine until it reaches the condenser pressure of 20 kPa . Relevant properties of steam are given in the following two tables. The work done by both the turbines together is
$\qquad$ $\mathrm{kJ} / \mathrm{kg}$. (round off to the nearest integer)
Superheated steam table :

| Pressure, $\boldsymbol{p}$ (MPa) | Temperature, $\boldsymbol{T}\left({ }^{\mathbf{0}} \mathbf{C}\right)$ | Enthalpy, $\boldsymbol{h}(\mathbf{k J} / \mathbf{k g})$ | Entropy, $\boldsymbol{s}(\mathbf{k J} / \mathbf{k g . K})$ |
| :---: | :---: | :---: | :---: |
| 10 | 500 | 3373.6 | 6.5965 |
| 1 | 500 | 3478.4 | 7.7621 |

Saturated steam table :

| Pressure, $\boldsymbol{p}$ (MPa) | Temperature, $\boldsymbol{T}\left({ }^{\mathbf{0}} \mathbf{C}\right)$ | Enthalpy, $\boldsymbol{h}(\mathbf{k J} / \mathbf{k g})$ |  | Entropy, $\boldsymbol{s}(\mathbf{k J} / \mathbf{k g} . K)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\boldsymbol{h}_{\boldsymbol{f}}$ | $\boldsymbol{h}_{\boldsymbol{g}}$ | $\boldsymbol{s}_{\boldsymbol{f}}$ | $\boldsymbol{s}_{\boldsymbol{g}}$ |
| 1 | 179.91 | 762.9 | 2778.1 | 2.1386 | 6.5965 |
| 20 | 60.06 | 251.38 | 2609.7 | 0.8319 | 7.9085 |

Ans. 1513
Sol.

$S_{4}=S_{3}=S_{f}+x\left(S_{g}-\mathrm{S}_{f}\right)$
$7.7621=0.8319+x(7.9085-0.8319)$
$x=0.9793$

$$
\begin{aligned}
h_{4} & =h_{f}+x\left(h_{g}-h_{f}\right) \\
& =251.38+0.9793(2609.7-251.38) \\
h_{4} & =2560.91 \\
W_{T} & =W_{T_{1}}+W_{T_{2}} \\
& =(3373.6-2778.1)+(3478.4-2560.91) \\
W_{T} & =1512.98 \frac{\mathrm{~kJ}}{\mathrm{~kg}}
\end{aligned}
$$

Q. 53 A rigid block of mass $m_{1}=10 \mathrm{~kg}$ having velocity $v_{0}=2 \mathrm{~m} / \mathrm{s}$ strikes a stationary block of ma $m_{2}=30 \mathrm{~kg}$ after travelling 1 m along a frictionless horizontal surface as shown in the figure.


The two masses stick together and jointly move by a distance of 0.25 m further along the same frictionless surface, before they touch the mass - less buffer that is connected to the rigid vertical wall by means of a linear spring having a spring constant $k=10^{5} \mathrm{~N} / \mathrm{m}$. the maximum deflection of the spring is $\qquad$ cm (round off to 2 decimal places).
Ans. 1
Sol. Collision Theory
Conservation of momentum

$$
\begin{aligned}
& m_{1} \times v_{0}+m_{2} \times 0=\left(m_{1}+m_{2}\right) \times v \\
& 10 \times 2=(10+30) v \\
& 20=40 \mathrm{v} \\
& v=0.5 \mathrm{~m} / \mathrm{s} \\
& \text { Now, } \frac{1}{2}\left(m_{1}+m_{2}\right) v^{2}=\frac{1}{2} k x^{2} \\
& \frac{1}{2} \times 40 \times(0.5)^{2}=\frac{1}{2} \times 10^{5} \times x^{2} \\
& 10=10^{5} \times x^{2} \\
& \Rightarrow \quad x^{2}=\frac{1}{10^{4}} \\
& x=\frac{1}{100} \mathrm{~m}=1 \mathrm{~cm}
\end{aligned}
$$

Q. 54 Two rollers of diameters $D_{1}$ (in mm ) and $D_{2}$ (in mm ) are used to measure the internal taper angle in the $V$-groove of a machined component. The heights $H_{1}$ (in mm ) are measured by using a height gauge after inserting the rollers into the same $V$ - groove as shown in the figure.


$$
D_{1}>D_{2}
$$

Which one of the following is the correct relationship to evaluate the angle $\alpha$ as shown in the figure ?
(A) $\sin \alpha=\frac{\left(H_{1}-H_{2}\right)}{\left(D_{1}-D_{2}\right)}$
(B) $\operatorname{cosec} \alpha=\frac{\left(H_{1}-H_{2}\right)-\left(D_{1}-D_{2}\right)}{2\left(D_{1}-D_{2}\right)}$
(C) $\sin \alpha=\frac{\left(D_{1}-D_{2}\right)}{2\left(H_{1}-H_{2}\right)-\left(D_{1}-D_{2}\right)}$
(D) $\cos \alpha=\frac{\left(D_{1}-D_{2}\right)}{2\left(H_{1}-H_{2}\right)-2\left(D_{1}-D_{2}\right)}$

Ans. (C)
Sol.

$O B=\left(H_{1}-\frac{D_{1}}{2}\right)-\left(H_{2}-\frac{D_{2}}{2}\right)=\left(H_{1}-H_{2}\right)-\left(\frac{D_{1}-D_{2}}{2}\right)$
$O A=\frac{D_{1}-D_{2}}{2}$
$\sin \alpha=\frac{\frac{\left(D_{1}-D_{2}\right)}{2}}{\left(H_{1}-H_{2}\right)-\frac{\left(D_{1}-D_{2}\right)}{2}}$
$\sin \alpha=\frac{\left(D_{1}-D_{2}\right)}{2\left(H_{1}-H_{2}\right)-\left(D_{1}-D_{2}\right)}$
Q. 55 A point ' P ' on a CNC controlled XY-stage is moved to another point ' Q ' using the coordinate system shown in the figure below and rapid positioning command (G00).


A pair of stepping motors with maximum speed of 800 rpm , controlling both the X and Y motion of the stage, are directly coupled to a pair of lead screws, each with a uniform pitch of 0.5 mm . The time needed to position the point ' P ' to the point ' Q ' is $\qquad$ minutes (round off to 2 decimal places).
Ans. 1.5
Sol. $\quad N=800 \mathrm{rpm}, P=0.5 \mathrm{~mm} / \mathrm{rev}$
$V=N \times P=\mathrm{rev} / \mathrm{min} \times \mathrm{mm} / \mathrm{rev}=400 \mathrm{~mm} / \mathrm{min}$
$\Delta t_{x}=\frac{600}{400}=1.5 \mathrm{~min}$
$\Delta t_{y}=\frac{300}{400}=0.75 \mathrm{~min}$
There are two stepper motor so both will work till 0.75 min then $y$ axis motor will stop then only $x$ axis motor will run for 0.75 more, so total time will be 1.5 min


