General Aptitude

Q.1 to Q.5 Carry One Mark Each.

Question 1

Writing too many things on the ________ while teaching could make the students get _________.

(A) bored / board (B) board / bored (C) board / board (D) bored / bored

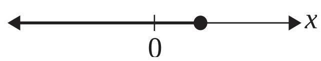
Ans. **(B)**

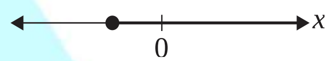
Sol. Writing too many things on the board while teaching could make the students get bored.

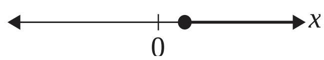
Hence, the correct option is (B).

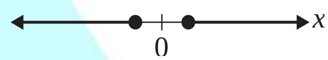
Question 2

Which one of the following is a representation (not to scale and in bold) of all values of x satisfying the inequality $2-5x \le -\dfrac{6x-5}{3}$ on the real number line?

(A) 

(B) 

(C) 

(D) 

Ans. **(C)**

Sol.

$$2-5x \le -\frac{6x-5}{3}$$
$$=3(2-5x) \le -(6x-5)$$
$$=6-15x \le -6x+5$$
$$=1 \le 9x$$
$$x \ge \frac{1}{9}$$

Hence, the correct option is (C).

Question 3

If $(x) = 2\ln\left(\sqrt{e^x}\right)$, what is the area bounded by $f(x)$ for the interval [0, 2] on the x-axis?

(A) $\dfrac{1}{2}$ (B) 1 (C) 2 (D) 4

Ans. **(C)**

Sol.

Sol. **Given :** Function,

$$f(x) = 2\ln(\sqrt{e^x})$$

Area bounded by $f(x)$ for the interval $[0,2]$ on the x-axis,

$$\text{Area } = \int_{x=a}^{x=b} f(x)\,dx$$

$$=\int_0^2 2\ln(\sqrt{e^x})\,dx$$

$$=2\int_0^2 \ln(\sqrt{e^x})\,dx$$

$$=2\int_0^2 \ln(e^x)^{1/2}\,dx$$

$$=2\int_0^2 \frac{1}{2}\ln(e^x)\,dx$$

$$=2\times\frac{1}{2}\int_0^2 x\,dx$$

$$=\left[\frac{x^2}{2}\right]_0^2=\left[\frac{4}{2}-0\right]$$

$$=2$$

Therefore, area will be 2.

Hence, the correct option is (C).

Question 4

A person was born on the fifth Monday of February in a particular year. Which one of the following statements is correct based on the above information?

(A) The 2nd February of that year is a Tuesday

(B) There will be five Sundays in the month of February in that year

(C) The 1st February of that year is a Sunday

(D) All Mondays of February in that year have even dates

Ans. **(A)**

Sol. **Given :**

A person was born on the fifth Monday of February in a particular year.

Then,

That year should be a leap year, as in a leap year February month is of 29 days.

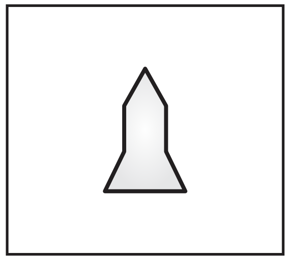
If 1st February is Monday then next Monday is on 8th, 15th, 22nd and 29th of the month.

So 1st and last day of the month February will be same.

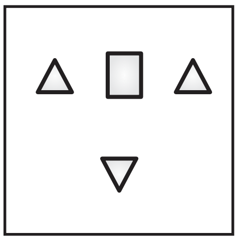
Here, the 2nd February of that year is a Tuesday.

Hence, the correct option is (A).

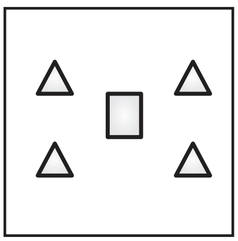
Question 5



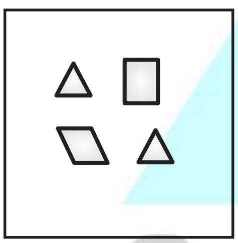
Which one of the groups given below can be assembled to get the shape that is shown above using each piece only once without overlapping with each other? (rotation and translation operations may be used).



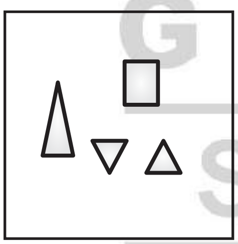
(A)



(B)



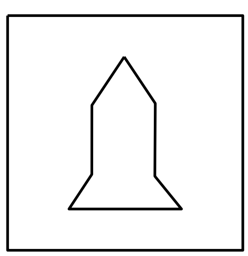
(C)



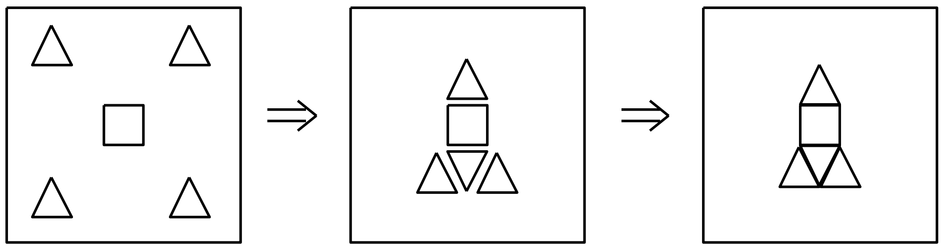
(D)

Ans. **(B)**

Sol. Given :



The group that can be assembled to get the shape that is shown above is, according to option (B).



Hence, the correct option is (B).

Q. 6 to Q. 10 Carry Two Marks Each.

Question 6

Fish belonging to species S in the deep sea have skins that are extremely black (ultra-black skin). This helps them not only to avoid predators but also sneakily attack their prey. However, having this extra layer of black pigment results in lower collagen on their skin, making their skin more fragile.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

(A) Having ultra-black skin is only advantageous to species S

(B) Species S with lower collagen in their skin are at an advantage because it helps them avoid predators

(C) Having ultra-black skin has both advantages and disadvantages to species S

(D) Having ultra-black skin is only disadvantageous to species S but advantageous only to their predators

Ans. **(C)**

Sol. Option (C) can be inferred, as having ultra black skin has both advantages and disadvantages to species S.

Hence, the correct option is (C).

Question 7

For the past m days, the average daily production at a company was 100 units per day.

If today's production of 180 units changes the average to 110 units per day, what is the value of m?

(A) 18 (B) 10 (C) 7 (D) 5

Ans. **(C)**

Sol. Average production $= \dfrac{\text{all production till to day}}{\text{m}} = 100$

According today, average production $= \dfrac{\text{all production till to day} + 180}{\text{m} + 1}$

$$= 100 = \frac{100\text{m} + 180}{\text{m} + 1}$$

$$= 110\text{m} + 110 = 100\text{m} + 180$$

$$= 10\text{m} : 70$$

$\therefore$ $\text{m} = 7$

Hence, the correct option is (C).

Question 8

Consider the following functions for non-zero positive integers, p and q.

$$f(p,q) = \underbrace{p \times p \times p \times \ldots \ldots \ldots \times p}_{q \text{ terms}} = p^q;\ f(p,1) = p$$

$$g(p,q) = p^{p^{p^{p^{p^{\cdot^{\cdot^{\cdot^{(\text{up to } q \text{ terms})}}}}}}}} \qquad ; g(p,1) = p$$

Which one of the following options is correct based on the above?

(A) $f(2,2) = g(2,2)$

(B) $f(g(2,2), 2) < f(2, g(2,2))$

(C) $g(2,1) \neq f(2,1)$

(D) $f(3,2) > g(3,2)$

Ans. **(A)**

Sol. **Given:**

$$f(p,q) = p^q; \qquad f(p,1) = p$$

$$g(p,q) = p^{p^{p^{p^{p^{\cdot^{\cdot^{\cdot^{(\text{up to } q \text{ terms})}}}}}}}} \qquad ; g(p,1) = p$$

(A) $f(2,2) = 2^2 = 4$

$g(2,2) = 2^2 = 4$

$f(2,2) = g(2,2)$

$\therefore$ option (A) is correct

(B) $f(4,2) = f(2,4)$

$4^2 = 2^4$

$\therefore$ option (B) is wrong

(C) $g(2,1) = 2;\ f(2,1) = 2$

$\therefore$ option (C) is wrong

(D) $f(3,2) = 3^2 = 9$

$g(3,2) = 3^3 = 27$

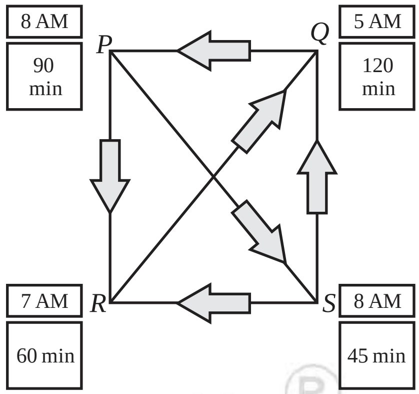
$\therefore$ option (D) is wrong

Hence, the correct option is (A).

Question 9

Four cities P, Q, R and S are connected through one-way routes as shown in the figure. The travel time between any two connected cities is one hour. The boxes beside each city name describe the starting time of first train of the day and their frequency of operation. For example, from city P, the first trains of the day start at 8 AM with a frequency of 90 minutes to each of R and S. A person does not spend additional time at any city other than the waiting time for the next connecting train.

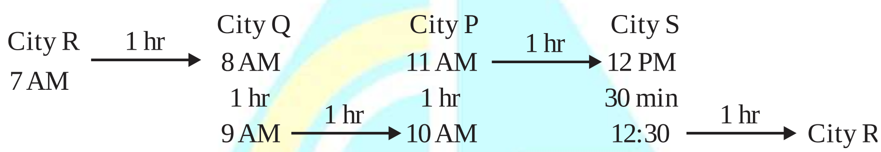
If the person starts from R at 7 AM and is required to visit S and return to R, what is the minimum time required?



(A) 6 hours 30 minutes
(B) 3 hours 45 minutes
(C) 4 hours 30 minutes
(D) 5 hours 15 minutes

Ans. **(A)**

Sol.



Travel time between two city is 1 hr.

Hence, the total Journey time

R to Q = 1 hr

Wait at Q = 1 hr

Q to P = 1 hr

Wait at P = 1 hr

P to S = 1 hr

Wait at S = 30 min

S to R = 1 hr

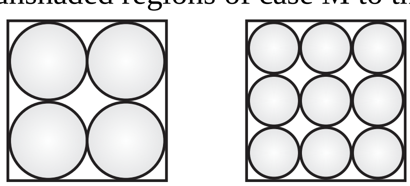
Total Journey hours from R to R is 6 hr 30 m.

Hence, the correct option is (A).

Question 10

Equal sized circular regions are shaded in a square sheet of paper of 1 cm side length. Two cases, case M and case N, are considered as shown in the figures below. In the case M, four circles are shaded in the square sheet and in the case N, nine circles are shaded in the square sheet as shown.

What is the ratio of the areas of unshaded regions of case M to that of case N?

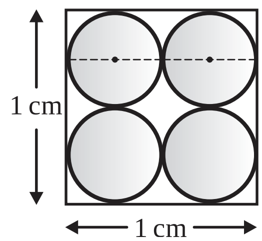


Case *M* Case *N*

(A) 2 : 3
(B) 1 : 1
(C) 3 : 2
(D) 2 : 1

Ans. **(B)**

Sol. **Case M :**



$\because$ Diameter of a shaded circle $= \frac{1}{2}$ cm

$\therefore$ Radius of shaded circle $= \frac{1}{4}$ cm

Area $= \frac{\pi}{16}$ cm^2

So, Area of unshaded region $=$ Area of square sheet – Area of 4 shaded circular region.

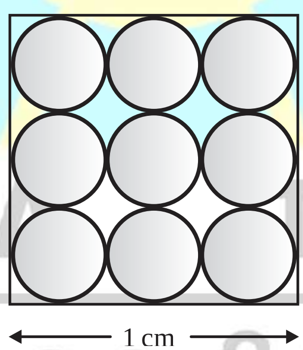
$\therefore$ Area of unshaded region

$$= \left[1 - 4 \times \frac{\pi}{16}\right] \text{cm}^2$$

$$= \left[1 - \frac{22}{28}\right] \text{cm}^2$$

$$= \frac{6}{28} \text{cm}^2$$

Case N :



diameter $= \frac{1}{3}$ and radius $= \frac{1}{6}$

$\therefore$ Area of unshaded region of case N = Area of square sheet – Area of shaded circular region.

$$= \left[1 - 9 \times \frac{\pi}{36}\right] \text{cm}^2$$

$$= \left[1 - \frac{22}{28}\right] \text{cm}^2$$

$$= \frac{6}{28} \text{cm}^2$$

$\therefore$ The ratio of the areas of unshaded region of case M to that of case N

$$\frac{6}{28}\text{ cm}^2 : \frac{6}{28}\text{ cm}^2$$
$$=1:1$$

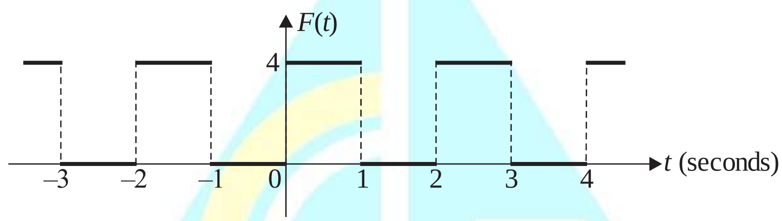
Hence, the correct option is (B).

Technical Section

Q.1 to 25 Carry One Mark Each.

Question 1

$F(t)$ is a periodic square wave function as shown. It takes only two values, 4 and 0, and stays at each of these values for 1 second before changing. What is the constant term in the Fourier series expansion of $F(t)$?



(A) 1 (B) 2 (C) 3 (D) 4

Ans. **(B)**

Sol. **Given :**

$F(t)$ is a periodic square wave function with time period $T=2$ second.

Complete Fourier series is given by,

$$F(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nt+\sum_{n=1}^{\infty}\text{b}_n\sin nt$$

Constant term in Fourier is given by $\frac{a_0}{2}$

$$a_0=\frac{1}{T}\int_0^T F(t)dt$$
$$a_0=\frac{1}{2}\int_0^2 4dt$$
$$a_0=\frac{1}{2}\times4\times[t]_0^2$$
$$a_0=\frac{1}{2}\times4\times[2-0]$$
$$a_0=4$$

Thus, constant term, $\frac{a_0}{2}=\frac{4}{2}=2$

Hence, the correct option is (B).

Question 2

Consider a cube of unit edge length and sides parallel to co-ordinate axes, with its centroid at the point (1, 2, 3). The surface integral $\int_A \vec{F}.\, d\vec{A}$ of a vector field $\vec{F} = 3x\hat{i} + 5y\hat{j} + 6z\hat{k}$ over the entire surface A of the cube is ______.

(A) 14 (B) 27 (C) 28 (D) 31

Ans. **(A)**

Sol. **Given :** Vector field, $\vec{F} = 3x\,\hat{i} + 5y\,\hat{j} + 6z\,\hat{k}$

Surface integral, $I = \int_A \vec{F} \cdot \overrightarrow{dA} = ?$

where, A is entire surface A of the cube.

By divergence theorem,

$$\oiint_S \vec{F}\,\hat{n}\,ds = \iiint_V \vec{\nabla} \cdot \vec{F}\,dv$$

$$I = \iiint_V \vec{\nabla} \cdot \vec{F}\,dv \qquad \ldots\text{(i)}$$

$$\text{Div}(\vec{F}) = \vec{\nabla} \cdot \vec{F} = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right) \cdot (3x\,\hat{i} + 5y\,\hat{j} + 6z\,\hat{k})$$

$$= \vec{\nabla} \cdot \vec{F} = 3 + 5 + 6 = 14$$

By equation (i),

$$I = \iiint_V 14\,dv$$

$$I = 14\iiint_V dv$$

$$I = 14V$$

Volume of cube, $V = a^3$ (with $a = 1$ unit edge length)

$$I = 14 \times a^3$$

$$I = 14 \times 1^3$$

$$I = 14$$

Hence, the correct option is (A).

Question 3

Consider the definite integral

$$\int_1^2 \left(4x^2 + 2x + 6\right) dx.$$

Let I_e be the exact value of the integral. If the same integral is estimated using Simpson's rule with 10 equal subintervals, the value is I_s. The percentage error is defined as $e = 100 \times (I_e - I_s)/I_e$. The value of e is

(A) 2.5 (B) 3.5 (C) 1.2 (D) 0

Ans. **(D)**

Sol. **Given :** Definite integral, $I = \int_1^2 (4x^2 + 2x + 6)\, dx$

Percentage error, $e = 100 \times \dfrac{I_e - I_s}{I_e}$

where, I_e be the exact value of integral and I_s be the value which is evaluated using Simpson's $\dfrac{1}{3}$ rule with 10 equal subintervals.

The given $f(x) = 4x^2 + 2x + 6$ is a quadratic equation, for which exact value (I_e) is always equal to approximated value (I_s).

Therefore, $I_e = I_s$

$$e = 100 \times \frac{0-0}{I_e}$$

$$e = 0\%$$

Hence, the correct option is (D).

Question 4

Given $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$.

If a and b are positive integers, the value of $\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx$ is ________.

(A) $\sqrt{\pi a}$ (B) $\sqrt{\dfrac{\pi}{a}}$ (C) $b\sqrt{\pi a}$ (D) $b\sqrt{\dfrac{\pi}{a}}$

Ans. **(B)**

Sol. **Given :**

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi} \qquad \ldots(i)$$

We need to find the value of $\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx$

where, a and b are positive integers.

$$I = \int_{-\infty}^{\infty} e^{-a(x+b)^2} dx$$

Let $x + b = t$

$dx = dt$

For $x = -\infty \Rightarrow t = -\infty$

and $x = \infty \Rightarrow t = \infty$

$$I = \int_{-\infty}^{\infty} e^{-at^2} dt$$

Let $at^2 = p^2$

$$2at\,dt = 2p\,dp$$

$$dt = \frac{p}{at}dp$$

$$dt = \frac{p}{a \times \frac{p}{\sqrt{a}}}dp$$

$$dt = \frac{1}{\sqrt{a}}dp$$

For $t = -\infty \Rightarrow p = -\infty$

and $t = \infty \Rightarrow p = \infty$

$$I = \int_{-\infty}^{\infty} e^{-p^2} \frac{1}{\sqrt{a}}\, dp$$

$$I = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} e^{-p^2} dp$$

From equation (i),

$$I = \frac{1}{\sqrt{a}} \times \sqrt{\pi}$$

$$I = \sqrt{\frac{\pi}{a}}$$

Hence, the correct option is (B).

Question 5

A polynomial $\varphi(s) = a_n s^n + a_{n-1}s^{n-1} + ... + a_1 s + a_0$ of degree $n > 3$ with constant real coefficients $a_n, a_{n-1}, ...a_0$ has triple roots at $s = -\sigma$. Which one of the following conditions must be satisfied?

(A) $\varphi(s) = 0$ at all the three values of s satisfying $s^3 + \sigma^3 = 0$

(B) $\varphi(s) = 0, \dfrac{d\varphi(s)}{ds} = 0,$ and $\dfrac{d^2\varphi(s)}{ds^2} = 0$ at $s = -\sigma$

(C) $\varphi(s) = 0, \dfrac{d^2\varphi(s)}{ds^2} = 0$, and $\dfrac{d^4\varphi(s)}{ds^4} = 0$ at $s = -\sigma$

(D) $\varphi(s) = 0$, and $\dfrac{d^3\varphi(s)}{ds^3} = 0$ at $s = -\sigma$

Ans. **(B)**

Sol. The given polynomial

$\phi(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0$

For stationary point, $\dfrac{d\phi(s)}{ds} = 0$

Since, $\phi(s)$ has triple roots at $s = -\sigma$, $(s+\sigma)^3$ will be one of it factors and $\phi(-\sigma) = 0$.

There will be an inflection point on the curve of $\phi(S)$. Therefore, the first order $\dfrac{d}{ds}\phi(s)$ and second order derivative $\dfrac{d^2}{ds^2}\phi(s)$ at $s = -\sigma$ will be zero.

$\therefore \quad \phi(s) = 0, \dfrac{d}{ds}\phi(s) = 0$ and $\dfrac{d^2}{ds^2}\phi(s) = 0$ at $s = -\sigma$

Hence, the correct option is (B).

Question 6

Which one of the following is the definition of ultimate tensile strength (UTS) obtained from a stress-strain test on a metal specimen?

(A) Stress value where the stress-strain curve transitions from elastic to plastic behavior

(B) The maximum load attained divided by the original cross-sectional area

(C) The maximum load attained divided by the corresponding instantaneous cross-sectional area

(D) Stress where the specimen fractures

Ans. **(B)**

Sol. The definition of ultimate tensile strength (UTS) obtained from a stress-strain test on a metal specimen is the maximum load attained divided by the original cross-sectional area.

i.e. $\sigma_{ult} = \dfrac{P_{max}}{A_0}$

Where, σ_{ult} = ultimate tensile strength,

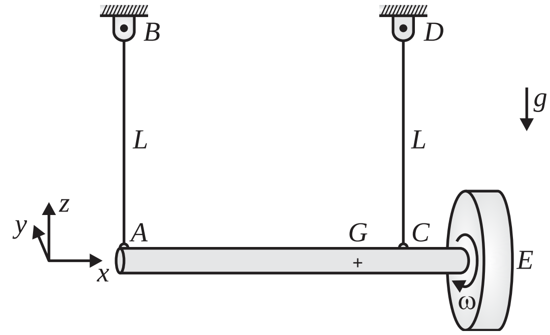
P_{max} = maximum load obtained in a tensile test,

A_0 = original cross-sectional area before load applied

Hence, the correct option is (B).

Question 7

A massive uniform rigid circular disc is mounted on a frictionless bearing at the end E of a massive uniform rigid shaft AE which is suspended horizontally in a uniform gravitational field by two identical light inextensible strings AB and CD as shown, where G is the center of mass of the shaft-disc assembly and g is the acceleration due to gravity. The disc is then given a rapid spin ω about its axis in the positive x – axis direction as shown, while the shaft remains at rest. The direction of rotation is defined by using the right-hand thumb rule. If the string AB is suddenly cut, assuming negligible energy dissipation, the shaft AE will



(A) rotate slowly (compared to ω) about the negative z-axis direction

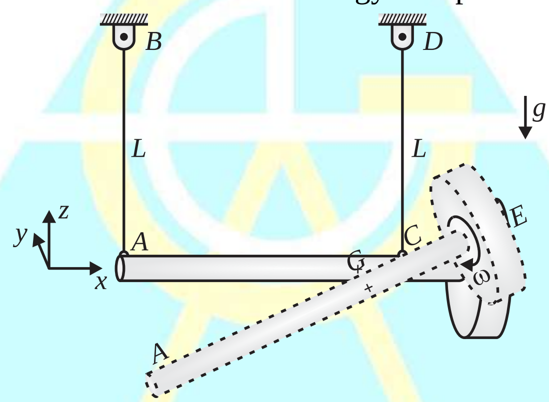
(B) rotate slowly (compared to ω) about the positive z-axis direction

(C) rotate slowly (compared to ω) about the positive y-axis direction

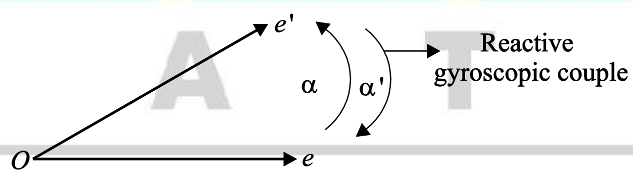
(D) rotate slowly (compared to ω) about the negative y-axis direction

Ans. **(A)**

Sol. It is given that when view from rear disc rotates clock-wise, when string AB cut the point A goes down because $C.G.$ of shaft away from C and towards A. So gyroscopic effect will be



Angular velocity vector



Rotate slowly (compared to ω) direction about the negative z-axis.

Hence, the correct option is (A).

Question 8

A structural member under loading has a uniform state of plane stress which in usual notations is given by $\sigma_x = 3P, \sigma_y = -2P$ and $\tau_{xy} = \sqrt{2}P$, where $P > 0$. The yield strength of the material is 350 MPa. If the member is designed using the maximum distortion energy theory, then the value of P at which yielding starts (according to the maximum distortion energy theory) is

(A) 70 MPa (B) 90 MPa (C) 120 MPa (D) 75 MPa

Ans. **(A)**

Sol. For the maximum distortion energy theory,

$$U_d = \frac{1}{6G}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] \qquad \ldots \text{(i)}$$

For actual condition,

$\sigma_3 = 0$ (because 2D condition)

Radius of Mohr's circle $(R) = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$

$$R = \sqrt{\left(\frac{3P + 2P}{2}\right)^2 + (\sqrt{2}P)^2}$$

$$R = \sqrt{\frac{25P^2}{4} + 2P^2}$$

$$= P\sqrt{\frac{25+8}{4}}$$

$$= P\sqrt{\frac{33}{4}}$$

$$R = 2.8722P$$

Now, principal stresses,

$$\sigma_{1,2} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm R$$

$$= \frac{3P - 2P}{2} \pm 2.8722P$$

$$\sigma_{1,2} = \frac{P}{2} \pm 2.8722P$$

$$\sigma_1 = \frac{P}{2} + 2.8722P$$

$$\sigma_1 = \frac{6.7444P}{2}$$

$$\sigma_1 = 3.3722P$$

And $\sigma_2 = \frac{P}{2} - 2.8722P$

$$= -\frac{4.7444P}{2}$$

$$\sigma_2 = -2.3722P$$

By putting in equation (i),

$$U_d = \frac{1}{6G}[(3.3722P)^2 + (-2.3722P)^2 + 0 - (-3.3722P \times 2.3722P) + 0 + 0]$$

$$= \frac{1}{6G}[11.37P^2 + 5.627P^2 + 7.999P^2]$$

$$= \frac{1}{6G}[16.997P^2 + 7.999P^2]$$

$$U_d = \frac{1}{6G}[24.996P^2] \qquad \text{… (ii)}$$

During tensile test failure,

$$\sigma_1 = \sigma_f,\ \sigma_2 = 0,\ \sigma_3 = 0$$

Where, σ_f is tensile yield strength = 350 MPa.

So, $(U_d)_{\text{failure}} = \dfrac{1}{6G}[\sigma_f^2 + 0 + 0 - (\sigma_f \times 0 + 0 + 0)]$

$$(U_d)_f = \frac{\sigma_f^2}{6G} \qquad \text{… (iii)}$$

Equating equation (ii) and (iii),

$$U_d = (U_d)_f$$

$$\frac{1}{6G} \times 24.996P^2 = \frac{\sigma_f^2}{6G}$$

$$24.996P^2 = \sigma_f^2$$

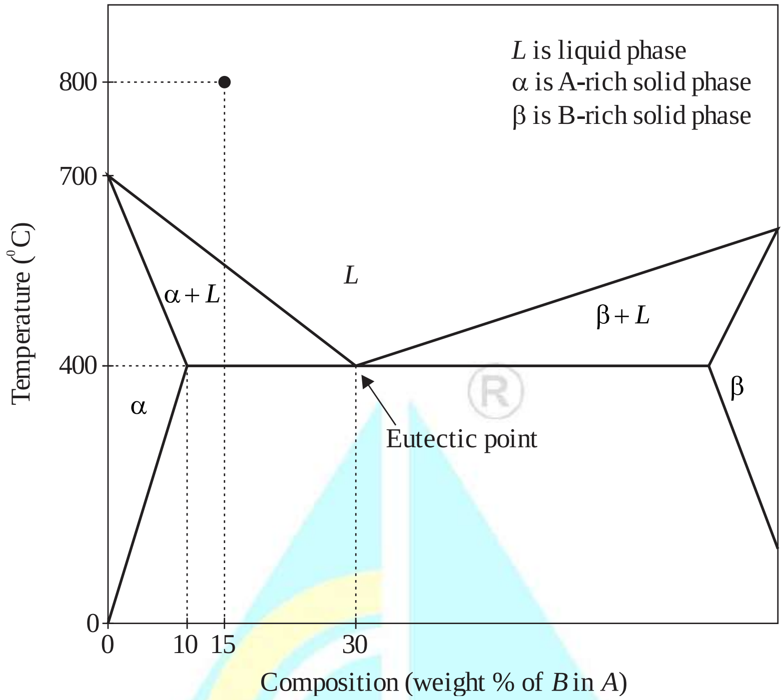
$$24.996P^2 = (350)^2 \qquad (\because\ \sigma_f = 350\text{ MPa})$$

$$P = 70\text{ MPa}$$

Hence, the correct option is (A).

Question 9

Fluidity of a molten alloy during sand casting depends on its solidification range. The phase diagram of a hypothetical binary alloy of components A and B is shown in the figure with its eutectic composition and temperature. All the lines in this phase diagram, including the solidus and liquidus lines, are straight lines. If this binary alloy with 15 weight % of B is poured into a mould at a pouring temperature of 800 ºC, then the solidification range is



(A) 400 ºC (B) 250 ºC (C) 800 ºC (D) 150 ºC

Ans. **(D)**

Sol. Since, $MN = 300$, $XY = 15$, $NO = 30$ and $XN =$ solidification range = ?

By linear interpolation,

In similar triangles ΔMXY and ΔMNO,

$$\frac{MX}{XY} = \frac{MN}{NO}$$

$$\frac{MX}{15} = \frac{300}{30}$$

$$MX = 150$$

$\therefore$ For solidification range, $XN = MN - MX$

$$XN = 300 - 150$$

Solidification range, $XN = 150^0\text{C}$

Hence, the correct option is (D).

Question 10

A shaft of diameter $25^{-0.04}_{-0.07}$ mm is assembled in a hole of diameter $25^{+0.02}_{-0.00}$ mm. Match the allowance and limit parameter in Column I with its corresponding quantitative value in Column II for this shaft-hole assembly.

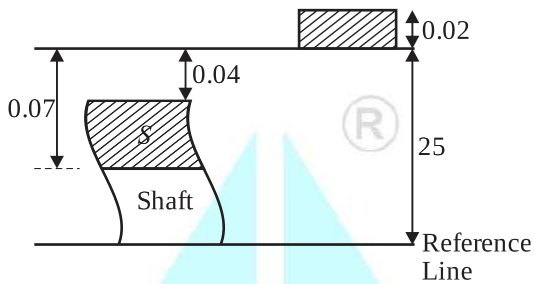
Allowance and limit parameter (Column I)		Quantitative value (Column II)	
P	Allowance	1	0.09 mm
Q	Maximum clearance	2	24.96 mm
R	Maximum material limit for hole	3	0.04 mm

		4	25.0 mm

(A) P-3, Q-1, R-4
(B) P-1, Q-3, R-2
(C) P-1, Q-3, R-4
(D) P-3, Q-1, R-2

Ans. (A)

Sol.



1. Allowance – 0.04 mm (due to minimum clearance)
2. Maximum clearance $\rightarrow$ $(0.07+0.02)=0.09$ mm
3. Maximum material limit for hole $\rightarrow$ lower limit g hole $=25$ mm

Hence, the correct option is (A).

Question 11

Match the additive manufacturing technique in Column I with its corresponding input material in Column II.

Additive manufacturing technique (Column I)		Input material (Column II)	
P	Fused deposition modelling	1	Photo sensitive liquid resin
Q	Laminated object manufacturing	2	Heat fusible powder
R	Selective laser sintering	3	Filament of polymer
		4	Sheet of thermoplastic or green compacted metal sheet

(A) P-3, Q-4, R-2
(B) P-1, Q-2, R-4
(C) P-2, Q-3, R-1
(D) P-4, Q-1, R-4

Ans. (A)

Sol. P. Fused deposition modelling – Filament of polymer

Q. Laminated object manufacturing – Sheet of thermoplastic or green compacted metal sheet

R. Selective laser sintering – Heat fusible powder

Hence, the correct option is (A).

Question 12

Which one of the following **CANNOT** impart linear motion in a CNC machine?

(A) Linear motor
(B) Ball screw
(C) Lead screw
(D) Chain and sprocket

Ans. **(D)**

Sol. Chain and sprocket cannot impart linear motion in a CNC machine.

Hence, the correct option is (D).

Question 13

Which one of the following is an intensive property of a thermodynamic system?

(A) Mass (B) Density

(C) Energy (D) Volume

Ans. **(B)**

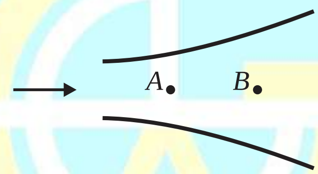
Sol. Density is an intensive property of a thermodynamic system.

Hence, the correct option is (B).

Question 14

Consider a steady flow through a horizontal divergent channel, as shown in the figure, with supersonic flow at the inlet. The direction of flow is from left to right.

Pressure at location *B* is observed to be higher than that at an upstream location *A*. Which among the following options can be the reason?



(A) Since volume flow rate is constant, velocity at *B* is lower than velocity at *A*

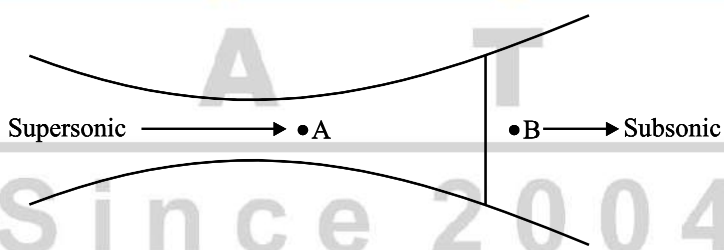
(B) Normal shock

(C) Viscous effect

(D) Boundary layer separation

Ans. **(B)**

Sol.



Within the distance of a mean free path, the flow passes from a supersonic to a subsonic state, the velocity decreases suddenly and the pressure rises sharply. A normal shock is said to have occurred if there is an abrupt reduction of velocity in the downstream in course of a supersonic flow in a passage or around a body.

Since, volume flow rate is constant, velocity at *B* is lower than velocity at *A* and pressure at location *B* is observed to be higher than that at an upstream location *A*. Therefore, it indicates the normal shock.

Hence, the correct option is (B).

Question 15

Which of the following non-dimensional terms is an estimate of Nusselt number?

(A) Ratio of internal thermal resistance of a solid to the boundary layer thermal resistance

(B) Ratio of the rate at which internal energy is advected to the rate of conduction heat transfer

(C) Non-dimensional temperature gradient

(D) Non-dimensional velocity gradient multiplied by Prandtl number

Ans. **(C)**

Sol. Nusselt number equal to the dimensionless temperature gradient at the surface, and it provides a measure of the convective heat transfer occurring at the surface.

Nusselt number, $Nu = \dfrac{hD}{k_{fluid}}$

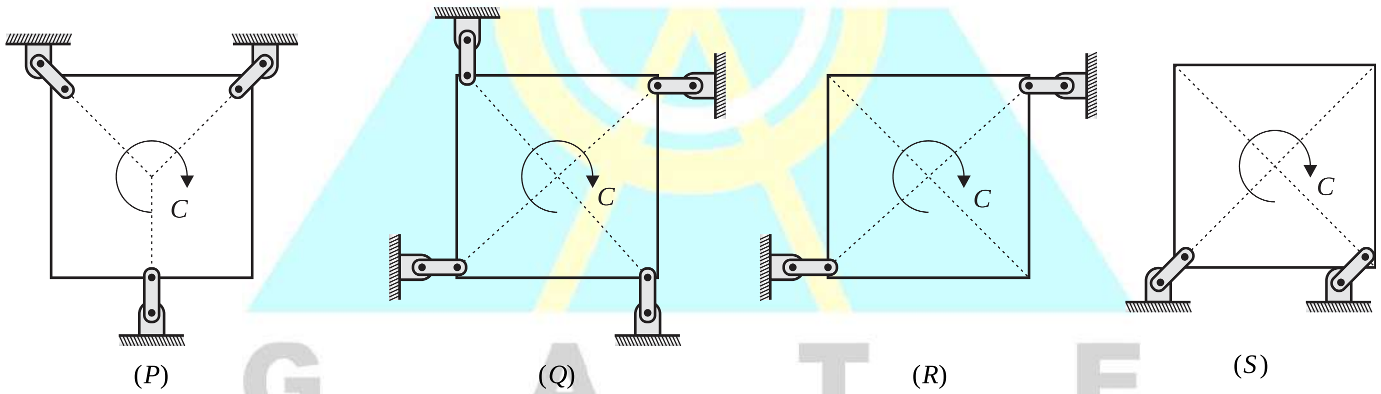
Where, k_{fluid} = thermal conductivity of fluid

$$Nu = \frac{Q_{conv.}}{Q_{cond.}} = \frac{\text{Condution thermal resistance offered by fluid if it were stationary}}{\text{Surface convection thermal resistance}}$$

Hence, the correct option is (C).

Question 16

A square plate is supported in four different ways (configurations (P) to (S) as shown in the figure). A couple moment C is applied on the plate. Assume all the members to be rigid and mass-less, and all joints to be frictionless. All support links of the plate are identical.



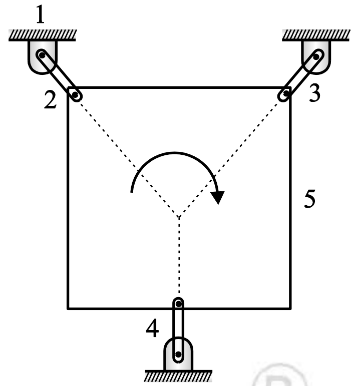
The square plate can remain in equilibrium in its initial state for which one or more of the following support configurations?

(A) Configuration (P) (B) Configuration (Q)

(C) Configuration (R) (D) Configuration (S)

Ans. **(B,C,D)**

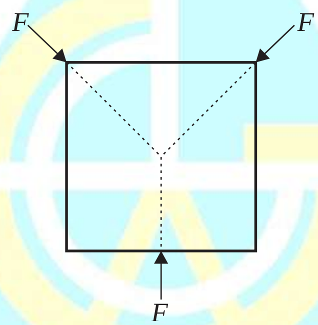
Sol. Configuration (P) :



Therefore, it is structure and stable system will be in equilibrium.

But, we know equilibrium is a state of the body where all the forces are of the same magnitude making the net resultant zero. This is called static concurrent equilibrium.

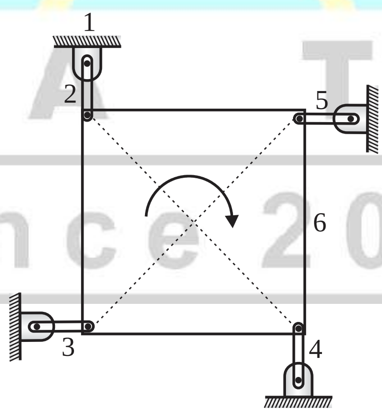
i.e.



But, the given square plate has couple moment c which will be the disturbing action for the concurrent forces and the square plate (configurations P) due to this action cannot be remain in equilibrium.

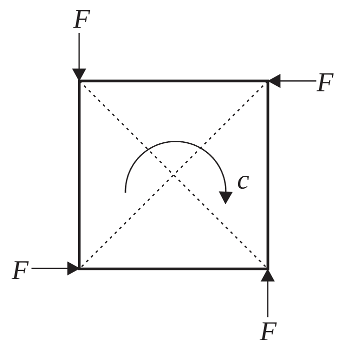
Hence, cannot be remain in equilibrium.

Configuration (Q) :

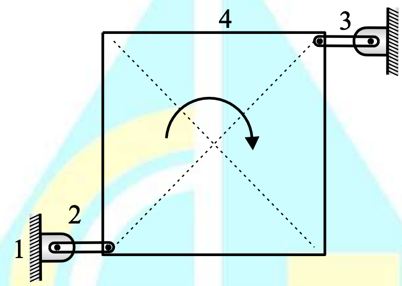


Here, the forces acting on the square plate is countering the action of couple moment c. Due to this counter action of forces (F) to resist the couple moment c, the square plate will remain in equilibrium.

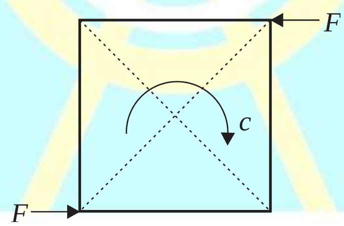
i.e.



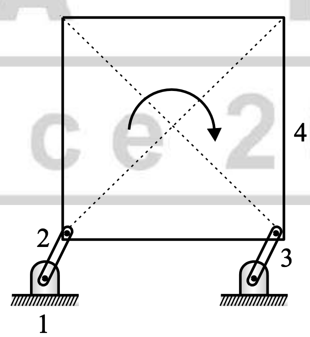
Forces (F) giving counter clockwise moment and couple moment c giving clockwise moment.
Hence, will remain in equilibrium.
Configuration (R) :



Similarly as we discussed above, here also the forces (F) resisting the couple moment c which is in clockwise direction by providing the anticlockwise moment.
i.e.

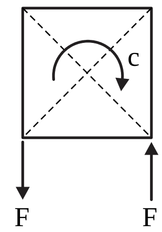


Hence, will remain in equilibrium.
Configuration (S) :



In the above arrangement, due to couple moment c, the reactions forces will try to counter the clockwise rotation which in result bring the arrangement in equilibrium or stable position

i.e.



Hence, will remain in equilibrium.

Hence the correct option is (B), (C), (D).

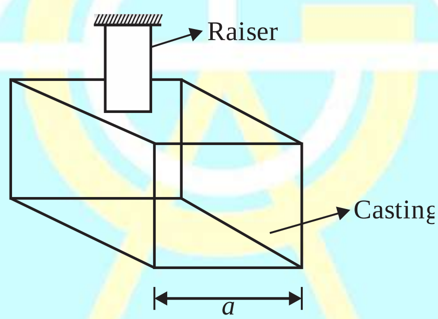
Question 17

Consider sand casting of a cube of edge length a. A cylindrical riser is placed at the top of the casting. Assume solidification time, $t_s \propto V/A$, where V is the volume and A is the total surface area dissipating heat. If the top of the riser is insulated, which of the following radius/radii of riser is/are acceptable?

(A) $\frac{a}{3}$ (B) $\frac{a}{2}$ (C) $\frac{a}{4}$ (D) $\frac{a}{6}$

Ans. **(A,B)**

Sol.



Here,

$$t_s \propto \left(\frac{V}{A}\right)$$

We know that,

$$(t_s)_{riser} \geq (t_s)_{casting}$$

$\Rightarrow$ Volume of riser $\geq$ Volume of casting

$$\frac{\frac{\pi}{4}d^2h}{\pi dh} \geq \frac{a}{6}$$

$$\Rightarrow \frac{2r}{4} \geq \frac{a}{6}$$

$$\Rightarrow \frac{r}{2} \geq \frac{a}{6}$$

$$r = \frac{a}{3}$$

Hence, the correct option is (A, B).

Question 18

Which of these processes involve(s) melting in metallic workpieces?

(A) Electrochemical machining
(B) Electric discharge machining
(C) Laser beam machining
(D) Electron beam machining

Ans. **(B,C,D)**

Sol. In, electron beam machining, electrical discharge machining and Laser beam machining, involves melting in metallic workpiece.

Hence, the correct option is (B,C,D).

Question 19

The velocity field in a fluid is given to be $\vec{V} = (4xy)\hat{i} + 2(x^2 - y^2)\hat{j}$.

Which of the following statement(s) is/are correct?

(A) The velocity field is one-dimensional.
(B) The flow is incompressible.
(C) The flow is irrotational.
(D) The acceleration experienced by a fluid particle is zero at (x = 0, y = 0).

Ans. **(B,C,D)**

Sol.

$$\vec{V} = (4xy)\hat{i} + 2(x^2 - y^2)\hat{j}$$

$$\because \quad \vec{\nabla}\cdot\vec{V} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial(4xy)}{\partial x} + \frac{\partial[2(x^2 - y^2)]}{\partial y} = 0 \qquad \left(\because \frac{\partial w}{\partial z} = 0\right)$$

$$4y - 4y = 0$$

$$0 = 0 \text{ (Flow is incompressible)}$$

Now, checking irrotational,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left(\frac{\partial[2(x^2 - y^2)]}{\partial x} - \frac{\partial(4xy)}{\partial y}\right)$$

$$= \frac{1}{2}(4x - 4x)$$

$$\omega_z = 0 \text{ (Flow is irrotational)}$$

Now, checking the acceleration experienced by a fluid particle is zero at $(x = 0,\ y = 0)$,

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + 0 + 0$$

$$= 4xy\frac{\partial}{\partial x}4xy + (2x^2 - 2y^2)\frac{\partial}{\partial y}(2x^2 - 2y^2)$$

$$= 16xy^2 + (2x^2 - 2y^2)\times(-4y)$$

$$a_x = 16xy^2 - 8x^2y + 8y^3$$

At $(x=0,\ y=0)$:

$$a_x = 0$$

And $a_y = u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + 0 + 0$

$$= 4xy\frac{\partial}{\partial x}(2x^2 - 2y^2) + (2x^2 - 2y^2)\frac{\partial}{\partial y}(2x^2 - 2y^2)$$

$$= 4xy\times 4x + (2x^2 - 2y^2)\times(-4y)$$

$$a_y = 16x^2y - 8x^2y + 8y^3$$

At $(x=0,\ y=0)$:

$$a_y = 0$$

$\therefore$ Total acceleration, $a_{\text{total}} = \sqrt{a_x^2 + a_y^2}$

$$a_{\text{total}} = \sqrt{0+0}$$

$a_{\text{total}} = 0$ (Acceleration experienced by a fluid particle is zero at $x=0,\ y=0$).

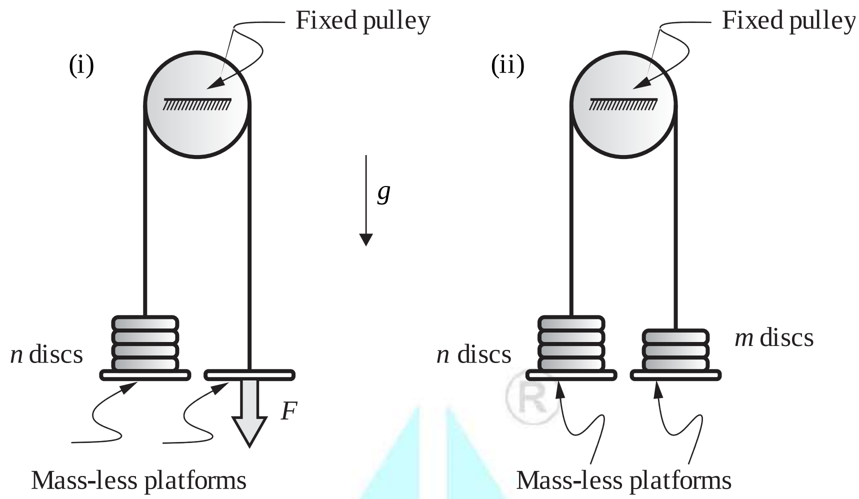
Since, $\vec{V} = (4xy)\hat{i} + 2(x^2 - y^2)\hat{j}$ shows $4xy$ in $\hat{i}$ direction and $2(x^2 - y^2)$ in $\hat{j}$ direction i.e. velocity field is two dimensional.

Hence, the correct option is (B), (C) and (D).

Question 20

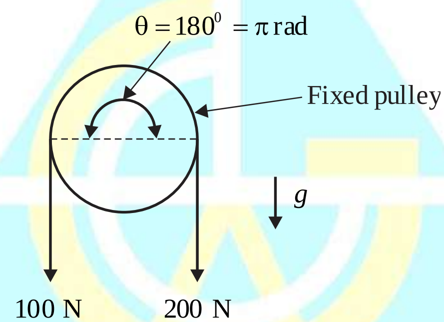
A rope with two mass-less platforms at its two ends passes over a fixed pulley as shown in the figure. Discs with narrow slots and having equal weight of 20 N each can be placed on the platforms. The number of discs placed on the left side platform is n and that on the right side platform is m.

It is found that for n = 5 and m = 0, a force F= 200 N (refer to part (i) of the figure) is just sufficient to initiate upward motion of the left side platform. If the force F is removed then the minimum value of m (refer to part (ii) of the figure) required to prevent downward motion of the left side platform is______ (*in integer*).



Ans. 3 to 3

Sol. Condition - I



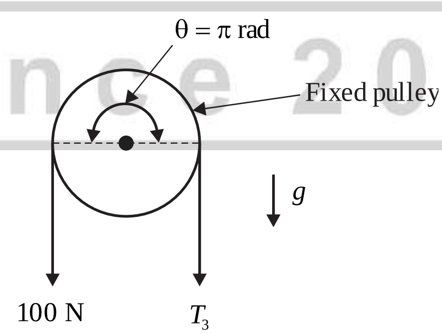
$$\frac{T_1}{T_2} = e^{\mu\,\theta}$$

Where angle of lap $(\theta) = \pi$

$$\frac{200}{100} = e^{\mu\theta}$$

$$2 = e^{\mu\theta} \quad \ldots(i)$$

Condition-II



$$\frac{T_1}{T_3} = e^{\mu\,\pi}$$

$$\frac{200}{T_3} = e^{\mu\pi} \qquad \ldots(ii)$$

On equating equation (i) and (ii)

$T_3 = 50\text{ N}$

For 50 N number of discs $= \frac{50}{20} = 2.5$

Hence minimum value of m required to prevent downward motion of the left side of platform is 3.

Question 21

For a dynamical system governed by the equation,

$$\ddot{x}(t) + 2\zeta\omega\dot{x}(t) + \omega_n^2 x(t) = 0,$$

the damping ratio ζ is equal to $\frac{1}{2\pi}\log_e 2$. The displacement x of this system is measured during a hammer test. A displacement peak in the positive displacement direction is measured to be 4 mm. Neglecting higher powers (>1) of the damping ratio, the displacement at the next peak in the positive direction will be ___________ mm (*in integer*).

Ans. **2**

Sol. We know that,

Displacement ratio, $\frac{x_0}{x_1} = e^{\delta} \qquad \ldots$ (i)

Where, δ is logarithmic decrement.

$$\because \quad \delta = \frac{2x\xi}{\sqrt{1-\xi^2}}$$

$$\delta = \frac{2\pi \times 0.1103}{\sqrt{1-(0.1103)^2}} \qquad \left(\because \xi = \frac{1}{2\pi}\log_e 2,\ \xi = 0.1103\right)$$

$$\delta = 0.69728$$

By putting in equation (i),

$$\frac{x_0}{x_1} = e^{0.69728}$$

$$\frac{4}{x_1} = e^{0.69728} \qquad (\because\ x_0 = 4\text{ mm})$$

$$x_1 = 2\text{ mm}$$

Hence, the displacement at the next peak in the positive direction will be 2 mm.

Question 22

An electric car manufacturer underestimated the January sales of car by 20 units, while the actual sales was 120 units. If the manufacturer uses exponential smoothing method with a smoothing constant of α = 0.2, then the sales forecast for the month of February of the same year is ___________units (*in integer*).

Ans. **104**

Sol. Forecast of January, $F_{Jan} = D_{Jan} - 20$ $\quad(\because 20$ units is under estimated in January$)$

$$F_{Jan} = 120 - 20 = 100$$

Forecast for the month of February,

$$F_{Feb} = F_{Jan} + \alpha[D_{Jan} - F_{Jan}]$$

$$F_{Feb} = 100 + 0.2[120 - 100]$$

$$F_{Feb} = 104$$

Hence, the sales forecast for the month of February of the same year is 104 units.

Question 23

The demand of a certain part is 1000 parts/year and its cost is Rs. 1000/part. The orders are placed based on the economic order quantity (EOQ). The cost of ordering is Rs. 100/order and the lead time for receiving the orders is 5 days. If the holding cost is Rs. 20/part/year, the inventory level for placing the orders is _________ parts (*round off to the nearest integer*).

Ans. **14**

Sol. Given :

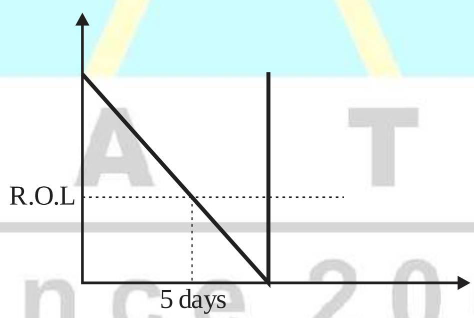
Demand of certain part $(D) = 1000$ parts/year

Cost $(C) = \text{Rs.}\ 1000/\text{part}$

Ordering cost $(C_0) = \text{Rs.}\ 100/\text{order}$

Lead time $(L_T) = 5$ days

Holding cost $(C_h) = \text{Rs.}\ 20/\text{part/year}$



Where, R.O.L = Re-order level

$\because$ $\quad \text{R.O.L} = L_T \times d$ $\quad \ldots$ (i)

Where, $d =$ consumption per day

$\because$ $\quad d = \dfrac{1000}{365}$ unit/day

From equation (i),

$$\text{R.O.L} = 5 \times \frac{1000}{365}$$

$$R.O.L = 13.69 \simeq 14 \text{ parts}$$

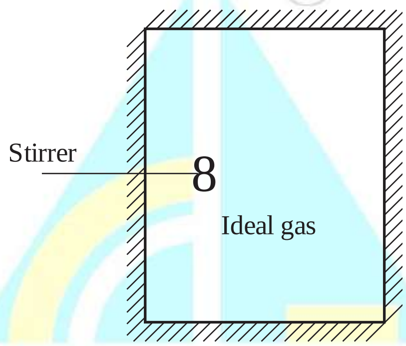
Hence, the inventory level for placing the orders is 14 parts.

Question 24

Consider 1 kg of an ideal gas at 1 bar and 300 K contained in a rigid and perfectly insulated container. The specific heat of the gas at constant volume c_v is equal to 750 $\text{J.kg}^{-1}.\text{K}^{-1}$. A stirrer performs 225 kJ of work on the gas. Assume that the container does not participate in the thermodynamic interaction. The final pressure of the gas will be ___________ bar (*in integer*).

Ans. 2

Sol.



From first law,

$$dQ = dv + (dw)_{disp} + (dw)_{stirrer}$$

$$dV = -(dw)_{stirrer} \qquad (\because (dw)_{disp} = 0 \text{ and } dQ = 0)$$

$$mc_v(T_F - T:) = -(225)$$

$$1 \times 0.750(T_F - 300) = 225$$

$$T_F = 300 + \frac{225}{0.75}$$

$$T_F = 600 \text{ K}$$

Since, rigid container (volume = constant),

$$\therefore \quad p \propto T$$

$$p_2 = 1 \times \frac{600}{300}$$

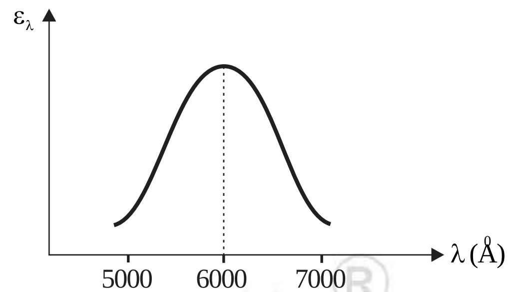
$$\frac{p_2}{p_1} = \frac{T_2}{T_1}$$

$$p_2 = 2 \text{ bar}$$

Hence, the final pressure of the gas will be 2 bar.

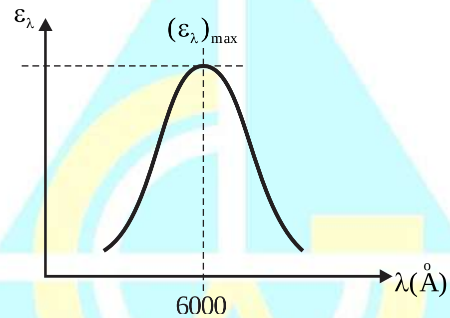
Question 25

Wien's law is stated as follows: $\lambda_m T = C$, where C is 2898 μm·K and λ_m is the wave length at which the emissive power of a black body is maximum for a given temperature T. The spectral hemispherical emissivity (ε_λ) of a surface is shown in the figure below (1Å = 10^{-10} m). The temperature at which the total hemispherical emissivity will be highest is __________ K (*round off to the nearest integer*).



Ans. **4830**

Sol.



$\because \quad \lambda_m T = C$

$$(6000\times10^{-10})\times T_{\max} = 2898\times10^{-6}$$

$$T_{\max} = 4830 \text{ K}$$

Hence, the temperature at which the total hemispherical emissivity will be highest is 4830 K.

Q.26 to 25 Carry Two Marks Each.

Question 26

For the exact differential equation,

$$\frac{du}{dx} = \frac{-xu^2}{2+x^2u},$$

which one of the following is the solution?

(A) $u^2 + 2x^2 = \text{constant}$ (B) $xu^2 + u = \text{constant}$

(C) $\frac{1}{2}x^2u^2 + 2u = \text{constant}$ (D) $\frac{1}{2}ux^2 + 2x = \text{constant}$

Ans. **(C)**

Sol. **Given :** The exact differential equation,

$$\frac{du}{dx} = \frac{-xu^2}{2+x^2u}$$

$$(2+x^2u)\,du = -xu^2 dx$$

$$(2+x^2u)\,du + xu^2 dx = 0$$

We know that,

A ordinary differential equation $M\,du + N\,dx = 0$ is to be exact.

If $\dfrac{\delta M}{\delta x} = \dfrac{\delta N}{\delta u}$

That mean, $M = 2 + x^2u,\quad N = xu^2$

$$\frac{\delta M}{\delta x} = 0 + 2xu,\quad \frac{\delta N}{\delta u} = 2xu$$

$$\frac{\delta M}{\delta x} = 2xu$$

Here, $\dfrac{\delta M}{\delta x} = \dfrac{\delta N}{\delta u}$ [Exactness condition]

The given differential equation is exact (already given in question).

Solution for exact differential equation is given by,

$$\int M\,du + \int N\,dx = \text{Constant}$$ [where, N is not containing x terms]

$$\int (2+x^2u)\,du + \int 0\,dx = \text{Constant}$$

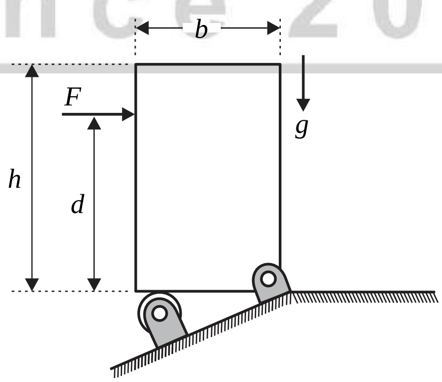
$$2u + \frac{x^2u^2}{2} = \text{Constant}$$

$$\frac{1}{2}x^2u^2 + 2u = \text{Constant}$$

Hence, the correct option is (C).

Question 27

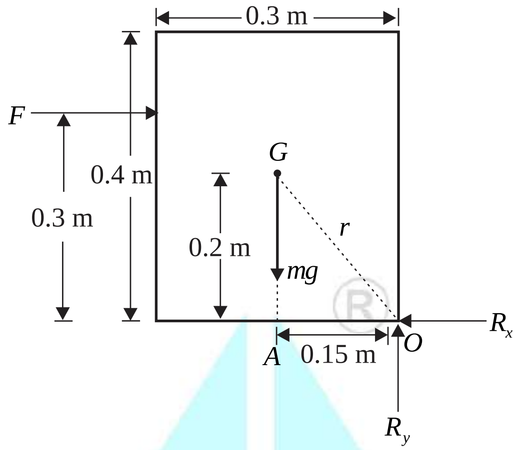
A rigid homogeneous uniform block of mass 1 kg, height h = 0.4 m and width b = 0.3 m is pinned at one corner and placed upright in a uniform gravitational field (g = 9.81 m/s^2), supported by a roller in the configuration shown in the figure. A short duration (impulsive) force F, producing an impulse I_F, is applied at a height of d = 0.3 m from the bottom as shown. Assume all joints to be frictionless. The minimum value of I_F required to topple the block is



(A) 0.953 Ns (B) 1.403 Ns (C) 0.814 Ns (D) 1.172 Ns

Ans. **(A)**

Sol.



Where, $G=$ centre of gravity point

For ΔAOG:

$$OG^2 = AG^2 + AO^2$$

$$r^2 = (0.2)^2 + (0.15)^2$$

$$r = 0.25 \text{ m}$$

Moment of intertial about point O :

Applying parallel axis theorem,

$$I_0 = I_G + m(r)^2 \quad \dots \text{(i)}$$

Where, $I_G =$ moment of inertia about its centre,

$m=$ mass of block

We know, moment of inertia of rectangular block,

$$I_G = \frac{m}{12}(h^2+b^2)$$

$$= \frac{1}{12}(0.4^2+0.3^2)$$

$$I_G = 0.0208 \text{ kgm}^2$$

By putting in equation (i),

$$I_0 = 0.0208 + 1(0.25)^2$$

$$I_0 = 0.0833 \text{ kgm}^2$$

Now, net moment $(\Sigma M_0) = I_0\alpha$

$$F\times 0.3 - mg\times 0.15 = I_0 \times \frac{d\omega}{dt}$$

$$(F\times 0.3 - 1\times 9.81\times 0.15)dt = I_0\times d\omega$$

$$(F\times 0.3 - 1.4715)dt = I_0\times d\omega$$

Integrating the equation by taking limit from initial position 1 (not topple) to final position 2 (toppled).

$$0.3\int_1^2 F dt - 1.4715\int_1^2 dt = \int_1^2 I_0 d\omega$$

Here, $\int_1^2 F dt$ representing impulse force, (I_F).

$$\therefore \quad 0.3\times I_F - 1.4715\int_1^2 dt = \int_1^2 I_0 d\omega$$

Since, the impulsive force is acting for short duration, therefore neglecting, $dt = 0$

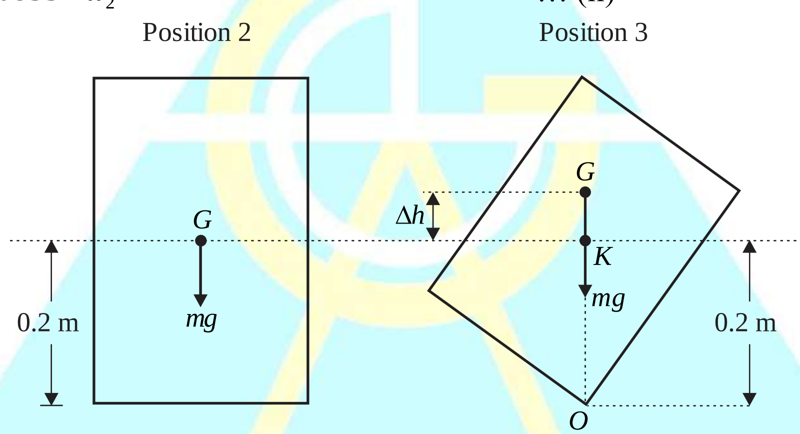
Now,

$$0.3\times I_F - 0 = \int_1^2 I_0 d\omega$$

$$0.3\times I_F = I_0(\omega_2 - \omega_1)$$

$$0.3\times I_F = 0.0833(\omega_2 - 0) \quad (\because \ \omega_1 \text{ is at initial position during not topple will be zero})$$

$$0.3\times I_F = 0.0833\times\omega_2 \qquad \ldots \text{(ii)}$$



Then, at position 2 the block will come in angular motion with ω_2 but after shifting of centroid point i.e. G along the axis of point 'O' let say position 3, at position 3, ω_3 will become zero because at this point angular rotation will be due to shifting of centroid of block and eccentric distance will produce torque that will be the actual reason of rotation only.

$\because \quad OG = r = 0.25$ m, $OK = 0.2$ m

$\therefore \quad \Delta h = OG - OK$

$\Delta h = 0.25 - 0.2$

$\Delta h = 0.05$ m

Balancing energy conservation for position (2) and position (3) :

$$(K.E.)_2 = (K.E.)_3 + (P.E.)_3$$

$$\frac{1}{2}I_0\omega_2^2 = \frac{1}{2}I_0\omega_3^2 + mg\times\Delta h$$

$$\frac{1}{2}I_0\omega_2^2 = 0 + mg\times\Delta h \qquad (\because \ \omega_3 = 0)$$

$$\frac{1}{2}\times 0.0833\times\omega_2^2 = 0 + 1\times 9.81\times 0.05$$

$$0.04165\times\omega_2^2 = 0.4905$$

$$\omega_2 = \sqrt{11.777}$$

$$\omega_2 = 3.4317 \text{ rad/sec}$$

By putting in equation (ii),

$$0.3\times I_F = 0.0833\times 3.4317$$

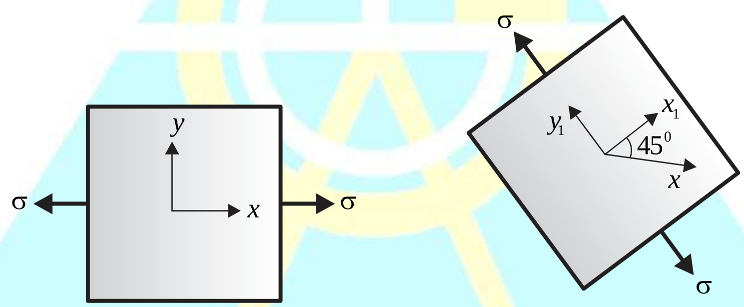
$$I_F = 0.9528 \text{ N-s} \simeq 0.953 \text{ N-s}$$

Impulse, $I_F = 0.953$ N-s

Hence, the correct option is (A).

Question 28

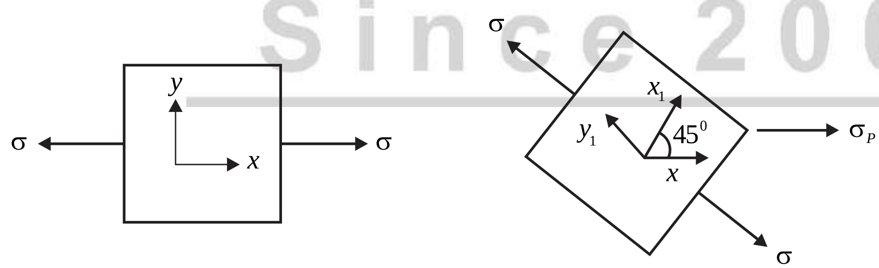
A linear elastic structure under plane stress condition is subjected to two sets of loading, I and II. The resulting states of stress at a point corresponding to these two loadings are as shown in the figure below. If these two sets of loading are applied simultaneously, then the net normal component of stress σ_{xx} is ________.



(A) $\dfrac{3\sigma}{2}$ (B) $\sigma\left(1+\dfrac{1}{\sqrt{2}}\right)$ (C) $\dfrac{\sigma}{2}$ (D) $\sigma\left(1-\dfrac{1}{\sqrt{2}}\right)$

Ans. **(A)**

Sol.



$$\sigma_{xx} = \sigma + \sigma_p \qquad \ldots\text{(i)}$$

For loading II

$\because \quad \sigma_p = \left[\dfrac{\sigma_x + \sigma_y}{2}\right] + \left[\dfrac{\sigma_x - \sigma_y}{2}\right]\cos 2\theta + \tau_{xy}\sin 2\theta$

$\sigma_x = 0,\ \sigma_y = \sigma,\ \tau_{xy} = 0, \quad \theta = -45^0$

(negative sign of angle represents that the angle is measured with reference to x_1 axis)

$$\sigma_p = \left[\frac{0+6}{2}\right] + \left[\frac{0-\sigma}{2}\right]\cos 2(-45) + 0$$

$$\sigma_p = \frac{\sigma}{2} + \left(\frac{-\sigma}{2}\right) \times 0$$

$$\sigma_p = \frac{\sigma}{2} \quad \ldots\text{(ii)}$$

From equation (i)

$$\sigma_{xx} = \sigma + \sigma_p$$

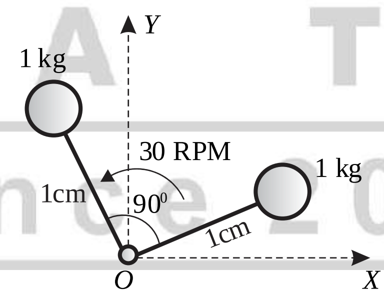
$$\sigma_{xx} = \sigma + \frac{\sigma}{2}$$

$$\sigma_{xx} = \frac{3}{2}\sigma$$

Hence, the correct option is (A).

Question 29

A rigid body in the **X-Y** plane consists of two point masses (1 kg each) attached to the ends of two massless rods, each of 1 cm length, as shown in the figure. It rotates at 30 RPM counter-clockwise about the **Z**-axis passing through point *O*. *A* point mass of $\sqrt{2}$ kg, attached to one end of a third massless rod, is used for balancing the body by attaching the free end of the rod to point *O*. The length of the third rod is ________ cm.



(A) 1 (B) $\sqrt{2}$ (C) $\dfrac{1}{\sqrt{2}}$ (D) $\dfrac{1}{2\sqrt{2}}$

Ans. **(A)**

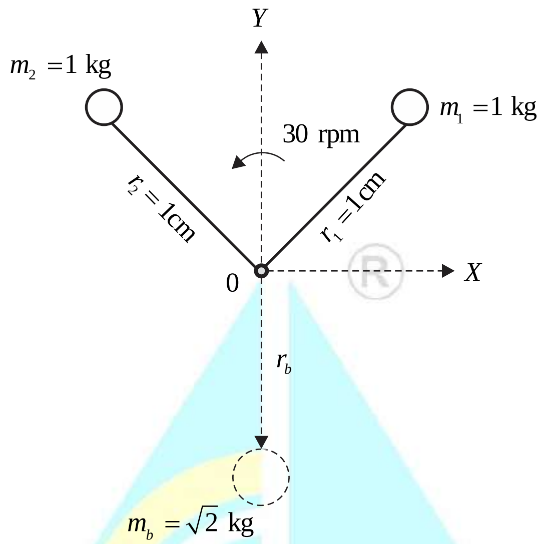
Sol. Given :

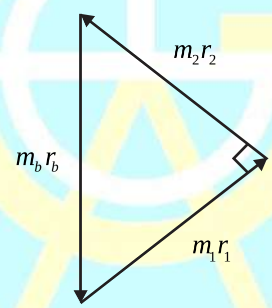
$m_1 = 1$ kg

$m_2 = 1$ kg

$r_1 = 1$ cm

$r_2 = 1$ cm





Balancing resultant force,

$$(m_b r_b)^2 = (m_1 r_1)^2 + (m_2 r_2)^2$$

$$\left(\sqrt{2}.r_b\right)^2 = (1\times1)^2 + (1\times1)^2$$

$$\sqrt{2}.r_b = \sqrt{1+1}$$

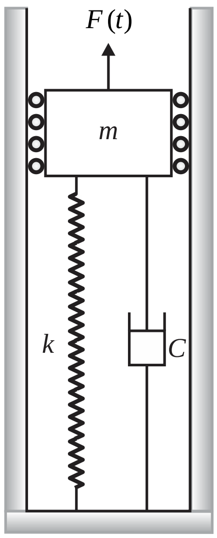
$$r_b = \frac{\sqrt{2}}{\sqrt{2}}$$

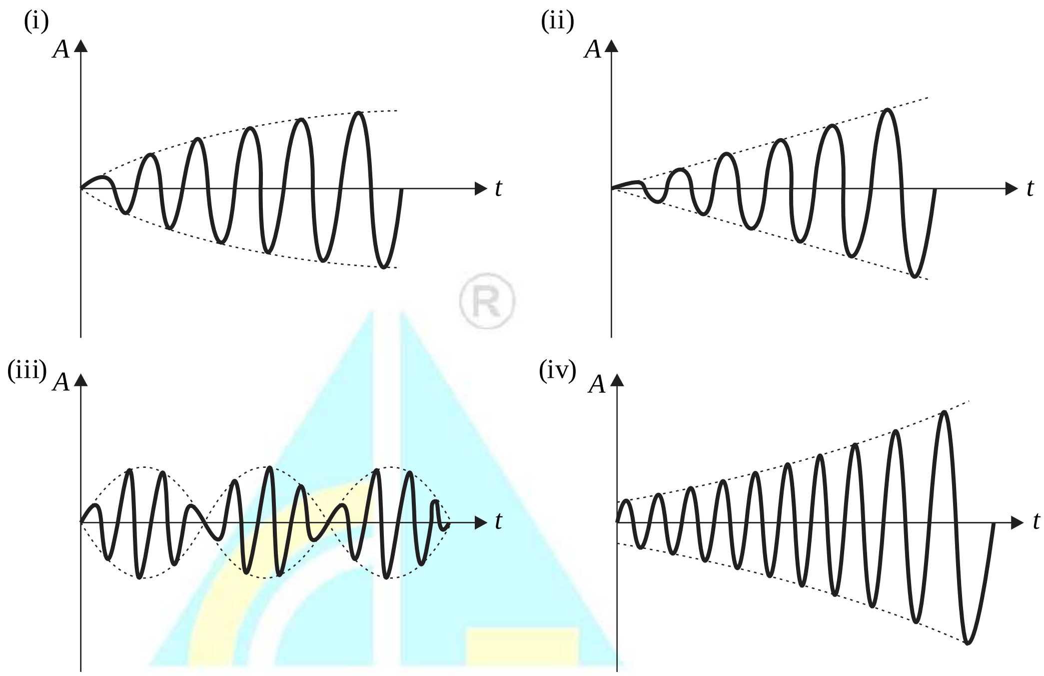
$$r_b = 1 \text{ cm}$$

Hence, the correct option is (A).

Question 30

A spring mass damper system (mass m, stiffness k, and damping coefficient c) excited by a force $(t) = B \sin \omega t$, where B, ω and t are the amplitude, frequency and time, respectively, is shown in the figure. Four different responses of the system (marked as (i) to (iv)) are shown just to the right of the system figure. In the figures of the responses, A is the amplitude of response shown in red color and the dashed lines indicate its envelope. The responses represent only the qualitative trend and those are not drawn to any specific scale.





Four different parameter and forcing conditions are mentioned below.

(P) $c > 0$ and $\omega = \sqrt{k/m}$ (Q) $c < 0$ and $\omega \neq 0$

(R) $c = 0$ and $\omega = \sqrt{k/m}$ (S) $c = 0$ and $\omega \cong \sqrt{k/m}$

Which one of the following options gives correct match (indicated by arrow $\rightarrow$) of the parameter and forcing conditions to the responses?

(A) (P) $\rightarrow$ (i), (Q) $\rightarrow$ (iii), (R) $\rightarrow$ (iv), (S) $\rightarrow$ (ii)

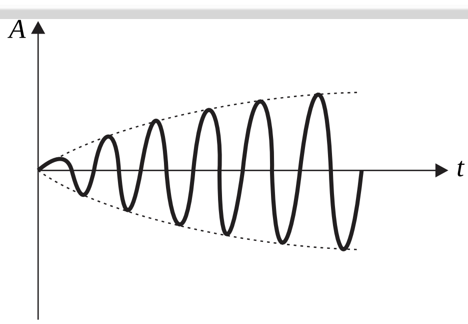
(B) (P) $\rightarrow$ (ii), (Q) $\rightarrow$ (iii), (R) $\rightarrow$ (iv), (S) $\rightarrow$ (i)

(C) (P) $\rightarrow$ (i), (Q) $\rightarrow$ (iv), (R) $\rightarrow$ (ii), (S) $\rightarrow$ (iii)

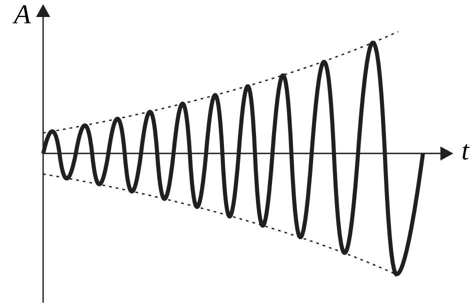
(D) (P) $\rightarrow$ (iii), (Q) $\rightarrow$ (iv), (R) $\rightarrow$ (ii), (S) $\rightarrow$ (i)

Ans. **(C)**

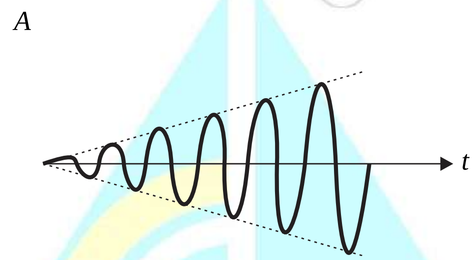
Sol. (P) $c > 0$ and $\omega = \sqrt{\dfrac{K}{m}}$



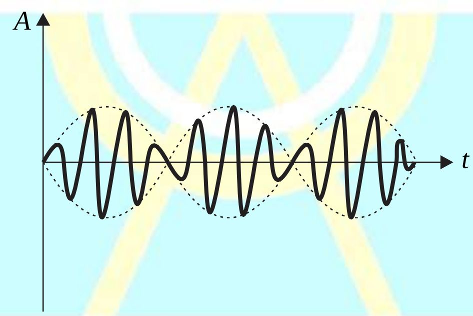
(Q) $c < 0$ and $\omega \neq 0$



(R) $c = 0$ and $\omega = \sqrt{\dfrac{K}{m}}$



(S) $c = 0$ and $\omega \cong \sqrt{\dfrac{K}{m}}$



Hence, the correct option is (C).

Question 31

Parts P1-P7 are machined first on a milling machine and then polished at a separate machine. Using the information in the following table, the minimum total completion time required for carrying out both the operations for all 7 parts is ________ hours.

Part	Milling (hours)	Polishing (hours)
P1	8	6
P2	3	2
P3	3	4
P4	4	6
P5	5	7
P6	6	4
P7	2	1

(A) 31 (B) 33 (C) 30 (D) 32

Ans. (B)

Sol.

Part	Milling (hours)	Polishing (hours)
P1	8	6
P2	3	2
P3	3	4
P4	4	6
P5	5	7
P6	6	4
P7	2	1

According to Johnson's algorithm, optimum job sequence is,

$P3 \rightarrow P4 \rightarrow P5 \rightarrow P1 \rightarrow P6 \rightarrow P2 \rightarrow P7$

Minimum total completion time:-

Part	Milling (hours)			Polishing (hours)		
	In time	Processing time	Out time	In time	Processing time	Out time
$P3$	0	3	3	3	4	7
$P4$	3	4	7	7	6	13
$P5$	7	5	12	13	7	20
$P1$	12	8	20	20	6	26
$P6$	20	6	26	26	4	30
$P2$	26	3	29	30	2	32
$P7$	29	2	31	32	1	33

The minimum total completion time required for carrying out both the operations for all 7 parts is 33 hours.

Hence, the correct option is (B).

Question 32

A manufacturing unit produces two products P1 and P2. For each piece of P1 and P2, the table below provides quantities of materials M1, M2, and M3 required, and also the profit earned. The maximum quantity available per day for M1, M2 and M3 is also provided. The maximum possible profit per day is Rs. ___________.

	M1	M2	M3	Profit per piece (Rs.)
P1	2	2	0	150
P2	3	1	2	100
Maximum quantity available per day	70	50	40	

(A) 5000 (B) 4000 (C) 3000 (D) 6000

Ans. **(B)**

Sol.

	$M1$	$M2$	$M3$	Profit per piece (Rs.)
$P1$	2	2	0	150
$P2$	3	1	2	100
Maximum quantity available per day	70	50	40	

$2x+3y \le 70$ ...(i)

$2x+y \le 50$ ...(ii)

$2y \le 40$ ...(iii)

$Z = 150x+100y$ ...(iv)

From equation (i),

When $x=0$, then $y=\dfrac{70}{3} \Rightarrow$ $(0, 70/3)$

$y=0$, then $x=35 \Rightarrow (35, 0)$

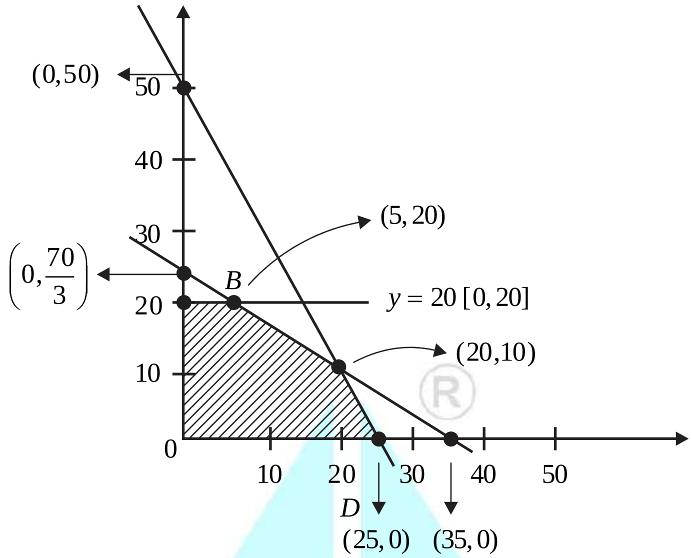
From equation (ii)

When $x=0$ then $y=50 \Rightarrow (0, 50)$

$y=0$, then $x=25 \Rightarrow (25, 0)$

From equation (iii)

$y=40$



$0 \le 70$

$0 \le 50$

$0 \le 40$

Point B: Intersection of

$2x + 3y \le 70$

$2y \le 40$

$x = 5$ and $y = 20$

Point C: Intersection of

$2x + 3y \le 70$

$2x + y \le 50$

After solving, we get $x = 20$ and $y = 10$

From equation (iv)

$Z = 150x + 100y$

$Z_A = 150(0) + 100(20) = 2000$

$Z_B = 150(5) + 100(20) = 2750$

$Z_C = 150(20) + 100(10) = 4000$

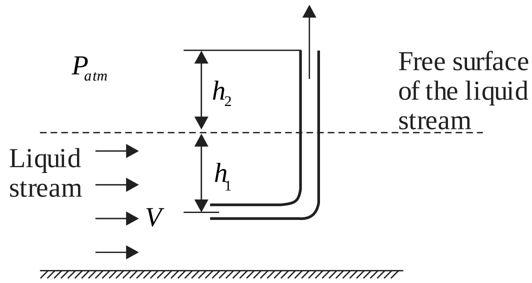
$Z_D = 150(25) + 100(0) = 3750$

$Z_{max} = 4000$ at $x = 20$ and $y = 0$

Hence, the correct option is (B).

Question 33

A tube of uniform diameter D is immersed in a steady flowing inviscid liquid stream of velocity V, as shown in the figure. Gravitational acceleration is represented by g. The volume flow rate through the tube is ______.



(A) $\frac{\pi}{4}D^2V$

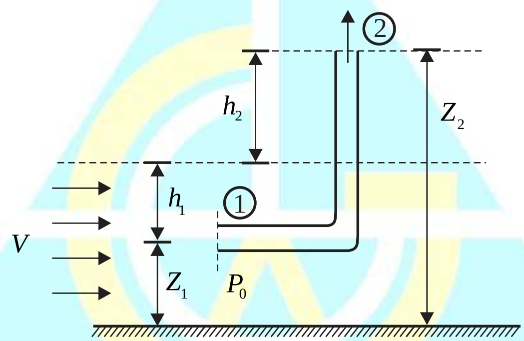
(B) $\frac{\pi}{4}D^2\sqrt{2gh_2}$

(C) $\frac{\pi}{4}D^2\sqrt{2g(h_1+h_2)}$

(D) $\frac{\pi}{4}D^2\sqrt{V^2-2gh_2}$

Ans. **(D)**

Sol.



From Bernoulli's equation (Point 1 to 2)

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$\{\because A_1 = A_2 \,\&\, V_1 = V_2\}$

$$\frac{P_1}{\rho g}+Z_1=0+Z_2$$

$$P_1=\rho g(Z_2-Z_1)$$

$$=\rho g(h_1+h_2)$$

$\because$ from diagram, it is clear that $(Z_2-Z_1)=(h_2+h_1)$

$$P_0=\rho gh_1$$

$$\frac{P_0}{\rho g}+\frac{V^2}{2g}+Z_0=\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1$$

$\{\because \quad Z_0 = Z_1\}$

$$\frac{\rho gh_1}{\rho g}+\frac{V^2}{2g}=\frac{\rho g(h_1+h_2)}{\rho g}+\frac{V_1^2}{2g}$$

$$h_1+\frac{V^2}{2g}=h_1+h_2+\frac{V_1^2}{2g}$$

$$\frac{V_1^2}{2g}=h_1-h_1-h_2+\frac{V^2}{2g}$$

$$\frac{V_1^2}{2g}=\frac{V^2}{2g}-h_2$$

$$V_1=\sqrt{V^2-2gh_2}$$

Now,

$$Q=AV=\frac{\pi}{4}D^2\sqrt{V^2-2gh_2}$$

$\{\because A=\text{Area}\}$

Hence, the correct option is (D).

Question 34

The steady velocity field in an inviscid fluid of density 1.5 is given to be $\vec{V}=\left(y^2-x^2\right)\hat{i}+\left(2xy\right)\hat{j}$. Neglecting body forces, the pressure gradient at (x = 1, y = 1) is __________ .

(A) $10\,\hat{j}$ (B) $20\,\hat{i}$ (C) $-6\,\hat{i}-6\,\hat{j}$ (D) $-4\,\hat{i}-4\,\hat{j}$

Ans. **(C)**

Sol. $\vec{V}=(y^2-x^2)\hat{i}+(2xy)\hat{j}$

$\because$ $\vec{F}_g+\vec{F}_v+\vec{F}_p=m\times a_{total}$

$$0+0+\vec{F}_p=m\times a_{total}$$

$$-\nabla P+\rho\, g=\rho\left(\frac{dv}{dt}\right)$$

$$-\nabla_P=\rho\left[u.\frac{\partial\vec{V}}{\partial x}\hat{i}+v\frac{\partial\vec{V}}{\partial y}\hat{j}\right] \quad \ldots\text{(i)}$$

At $x=1, y=1$

$$a_x=u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+0+0$$

$$a_x=(y^2-x^2)(-2x)+2xy(2y)$$

$$a_x=4\hat{i} \quad \ldots\text{(ii)}$$

And $a_y=u\dfrac{\partial v}{\partial x}+v\dfrac{\partial v}{\partial y}+0+0$

$$a_y=(y^2-x^2)(2y)+2xy(2x)$$

$a_y = 2\hat{j}$ ...(iii)

From equation (i), (ii) and (iii)

$$-\nabla P = \rho\left[u\frac{\partial\vec{V}}{\partial x}\hat{i} + v\frac{\partial\vec{V}}{\partial y}\hat{j}\right]$$

$$-\nabla P = \rho\left[4\hat{i} + 4\hat{j}\right]$$

$$-\nabla P = 1.5\left[4\hat{i} + 4\hat{j}\right]$$

$$\nabla P = -6\hat{i} - 6\hat{j}$$

Hence, the correct option is (C).

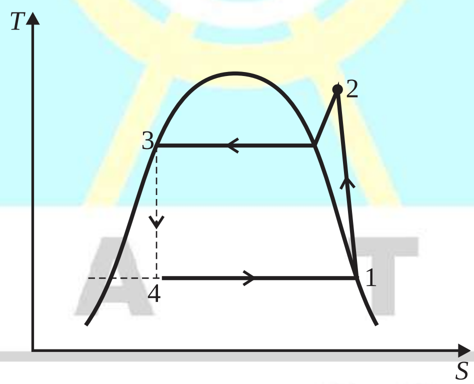
Question 35

In a vapour compression refrigeration cycle, the refrigerant enters the compressor in saturated vapour state at evaporator pressure, with specific enthalpy equal to 250 kJ/kg. The exit of the compressor is superheated at condenser pressure with specific enthalpy equal to 300 kJ/kg. At the condenser exit, the refrigerant is throttled to the evaporator pressure. The coefficient of performance (COP) of the cycle is 3. If the specific enthalpy of the saturated liquid at evaporator pressure is 50 kJ/kg, then the dryness fraction of the refrigerant at entry to evaporator is _________.

(A) 0.2 (B) 0.25 (C) 0.3 (D) 0.35

Ans. (B)

Sol.



$$h_4 = h_f + x(h_g - h_f) \quad \text{... (i)}$$

$$\because \quad COP = \frac{h_1 - h_4}{h_2 - h_1} = \frac{250 - h_4}{300 - 250}$$

$$3 = \frac{250 - h_4}{300 - 250}$$

$$h_4 = 100 \text{ kJ/kg}$$

From equation (i),

$$100 = 50 + x(250 - 50)$$

$$x = 0.25$$

Hence, the correct option is (B).

Question 36

$\boldsymbol{A}$ is a 3×5 real matrix of rank 2. For the set of homogeneous equations $\boldsymbol{Ax} = \boldsymbol{0}$, where $\boldsymbol{0}$ is a zero vector and $\boldsymbol{x}$ is a vector of unknown variables, which of the following is/are true?

(A) The given set of equations will have a unique solution.

(B) The given set of equations will be satisfied by a zero vector of appropriate size.

(C) The given set of equations will have infinitely many solutions.

(D) The given set of equations will have many but a finite number of solutions.

Ans. **(B,C)**

Sol. **Given :** A is a (3×5) real matrix,

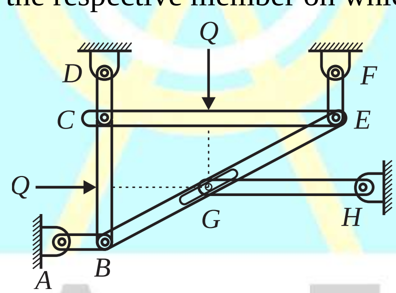
$$[A]_{3\times5} \Rightarrow \rho(A) \le \min(3,5) \Rightarrow \rho(A) \le 3$$

Rank of matrix $\rho(A)=2$, that means there must be two linearly independent rows, also $|A|_{3\times3}=0$. It shows $[A]_{3\times3}$ will have non-trivial solution or infinitely many solutions. ($AX=0,$ set of homogenous equation)

Hence, the correct option are (B) and (C).

Question 37

The lengths of members BC and CE in the frame shown in the figure are equal. All the members are rigid and lightweight, and the friction at the joints is negligible. Two forces of magnitude $Q > 0$ are applied as shown, each at the mid-length of the respective member on which it acts.



Which one or more of the following members do not carry any load (force)?

(A) AB (B) CD (C) EF (D) GH

Ans. **(B,D)**

Sol.

Question 38

If the sum and product of eigenvalues of a 2×2 real matrix $\begin{bmatrix} 3 & p \\ p & q \end{bmatrix}$ are 4 and −1 respectively, then $|p|$ is ______ (*in integer*).

Ans. **2**

Sol. **Given :** Matrix of order, $[A]=\begin{bmatrix} 3 & p \\ p & q \end{bmatrix}_{2\times2}$

Sum of eigen values $=4$

Product of eigen values $=-1$

As we know that,

Sum of eigen values = Trace of matrix

and Product of eigen values = Determinant of matrix

$$\lambda_1+\lambda_2=3+q=4$$
$$q=4-3$$
$$q=1$$
$$\lambda_1\cdot\lambda_2=3q-p^2=-1$$
$$3\times1-p^2=-1$$
$$3-p^2=-1$$
$$p^2=4$$
$$p=\pm2$$

then, $|p|=+2$

Hence, the $|p|$ is 2.

Question 39

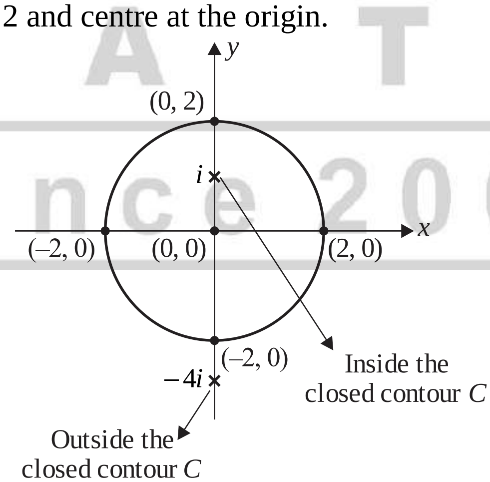
Given $z = x + iy$, $i = \sqrt{-1}$. C is a circle of radius 2 with the centre at the origin. If the contour C is traversed anticlockwise, then the value of the integral $\dfrac{1}{2\pi}\int_C \dfrac{1}{(z-i)(z+4i)}dz$ is _______ (*round off to one decimal place*).

Ans. **0.2**

Sol. **Given :** Complex integration,

$$I=\frac{1}{2\pi}\int_C \frac{1}{(z-i)(z+4i)}dz$$

where, C is a circle of radius 2 and centre at the origin.



Complex function,

$$f(z)=\frac{1}{(z-i)(z+4i)}$$

Poles are $z=i,\ z=-4i$

We know that,

$$I=2\pi i\times[\text{Sum of all residues ww.r.t all the poles inside or on the closed contour}]$$

$$\text{Res }(z=a)=\lim_{z\to a} f(z)(z-a)$$

$$\text{Res }(z=-4i)=0$$

$$\text{Res }(z=i)=\lim_{z\to i}\frac{1}{(z-i)(z+4i)}\times(z-i)=\lim_{z\to i}\frac{1}{z+4i}$$

$$\text{Res }(z=i)=\frac{1}{i+4i}=\frac{1}{5i}$$

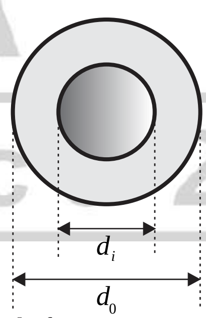
$$I=\frac{1}{2\pi}\times 2\pi i\times\frac{1}{5i}$$

$$I=\frac{1}{5}=0.2$$

Hence, the value of the integral is 0.2.

Question 40

A shaft of length L is made of two materials, one in the inner core and the other in the outer rim, and the two are perfectly joined together (no slip at the interface) along the entire length of the shaft. The diameter of the inner core is d_i and the external diameter of the rim is d_o , as shown in the figure. The modulus of rigidity of the core and rim materials are G_i and G_o, respectively. It is given that $d_o=2d_i$ and $G_i=3G_o$. When the shaft is twisted by application of a torque along the shaft axis, the maximum shear stress developed in the outer rim and the inner core turn out to be τ_o and τ_i, respectively. All the deformations are in the elastic range and stress strain relations are linear. Then the ratio $\frac{\tau_i}{\tau_o}$ is ______ (*round off to 2 decimal places*).



Shaft cross-section

Ans. **1.5**

Sol. **Given:**

$d_0=2d_j$

$G_i=3G_0$

Core and Rim are perfectly joined (no slip)

$L = L_i = L_0$ and $\theta = \theta_i = \theta_0$

From Torsion equation,

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

$$\tau_{\max} = \frac{G\theta}{L} \times r_{\max} \qquad \ldots\text{(i)}$$

For core,

$$\tau_i = (\tau_{\max})_{core} = \frac{G_i\theta_i}{L_i} \times \frac{d_i}{2}$$

$$= \frac{G_i\theta_i d_i}{2L_i} \qquad \ldots\text{(ii)}$$

$$\tau_0 = (\tau_{\max})_{Rin} = \frac{G_0\theta_0 d_0}{2L_0} \qquad \ldots\text{(ii)}$$

From equation (i) & (ii)

$$\frac{\tau_i}{\tau_0} = \frac{\frac{G_i\theta_i d_i}{2L_i}}{\frac{G_0\theta_0 d_0}{2L_0}}$$

$$\frac{\tau_i}{\tau_0} = \frac{G_i\theta_i d_i}{2L_i} \times \frac{2L_0}{G_0\theta_0 d_0}$$

$$\frac{\tau_i}{\tau_0} = \frac{3G_0 \times \theta\, d_i}{2L} \times \frac{2L}{G_0 \times \theta \times 2d_i}$$

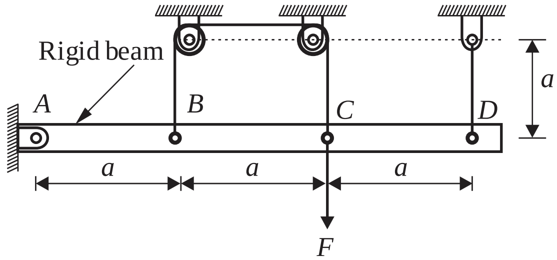
$$\frac{\tau_i}{\tau_0} = \frac{3\times 2}{2\times 2} = \frac{6}{4}$$

$$\frac{\tau_i}{\tau_0} = 1.5$$

Hence, the ratio $\frac{\tau_i}{\tau_o}$ is 1.5.

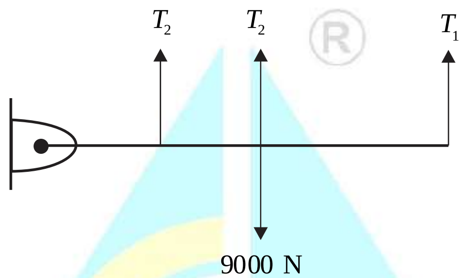
Question 41

A rigid beam *AD* of length 3*a* = 6 m is hinged at frictionless pin joint *A* and supported by two strings as shown in the figure. String *BC* passes over two small frictionless pulleys of negligible radius. All the strings are made of the same material and have equal cross-sectional area. A force *F* = 9 kN is applied at C and the resulting stresses in the strings are within linear elastic limit. The self-weight of the beam is negligible with respect to the applied load. Assuming small deflections, the tension developed in the string at *C* is _________ kN (*round off to 2 decimal places*).



Ans. 1.5

Sol.

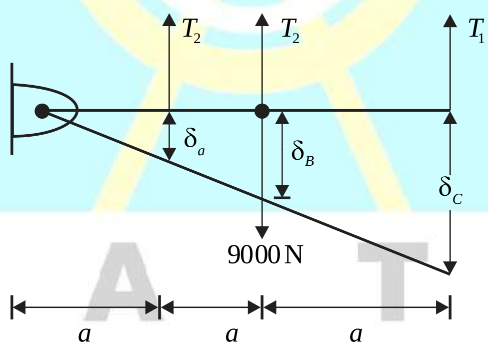


$$\Sigma M_0 = 0$$

$$T_1 \times 3a + T_2 \times 2a + T_2 \times a - 9000 \times 2a = 0$$

$$3T_1 + 3T_2 - 1800 = 0$$

$$T_1 + T_2 = 6000 \qquad \ldots(i)$$



$$\frac{\delta_C}{3a} = \frac{\delta_B}{2a} = \frac{\delta_a}{a}$$

$$\delta_C = 3\delta_a$$

$$\delta_B = 2\delta_a$$

$$\delta_1 = \delta_C$$

$$\delta_2 = \delta_a + \delta_B \qquad \left[\because \ \delta = \frac{PL}{AE}\right]$$

$$\frac{T_2 \times 3a}{AE} = \delta_a + 2\delta_a$$

$$\frac{T_2 \times 3a}{AE} = 3\delta_a$$

$$\delta_a = \frac{T_2 a}{AE} \qquad \ldots\text{(ii)}$$

We know,

$$\delta_1 = \delta_C$$

$$\frac{T_1 a}{AE} = 3\delta_a$$

$$\delta_a = \frac{T_1 a}{3AE} \qquad \ldots\text{(iii)}$$

Equating eq. (2) & (3)

$$\frac{T_2 a}{AE} = \frac{T_1 a}{3AE}$$

$$T_1 = 3T_2$$

Putting the value of T_1 in equation (i)

$$T_1 + T_2 = 6000$$

$$3T_2 + T_2 = 6000$$

$$4T_2 = 6000$$

$$T_2 = \frac{6000}{4}$$

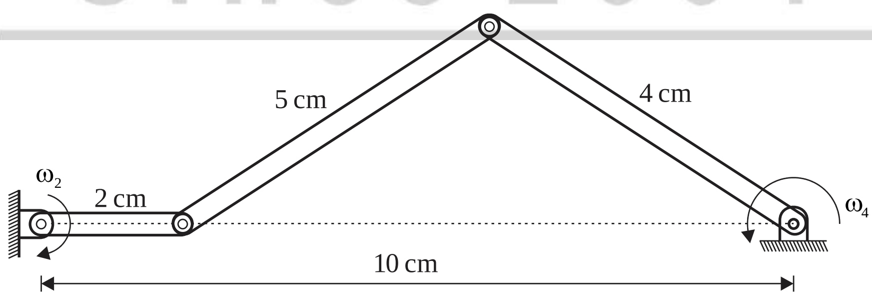
$$T_2 = 1500 \text{ N}$$

$$T_2 = 1.5 \text{ kN}$$

Hence, the tension developed in the string at C is 1.5 kN.

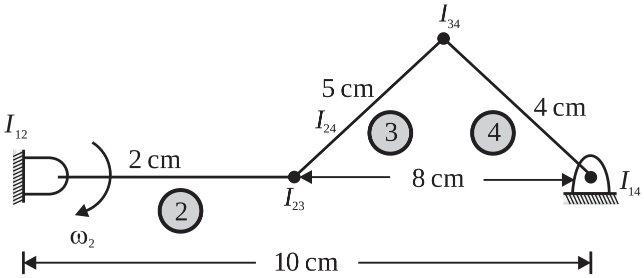
Question 42

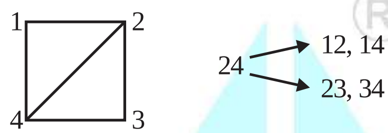
In the configuration of the planar four-bar mechanism at a certain instant as shown in the figure, the angular velocity of the 2 cm long link is $\omega_2 = 5$ rad/s. Given the dimensions as shown, the magnitude of the angular velocity ω_4 of the 4 cm long link is given by _____ rad/s (*round off to 2 decimal places*).



Ans. 1.25

Sol.





$$\because \quad \omega_4(I_{24}I_{14}) = \omega_2(I_{24}I_{12})$$

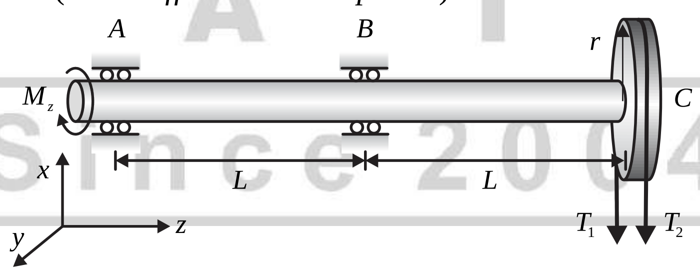
$$\omega_4 \times 8 = 5 \times 2$$

$$\omega_4 = 1.25 \text{ rad/s}$$

Hence, the magnitude of the angular velocity ω_4 of the 4 cm long link is given by 1.25 rad/sec.

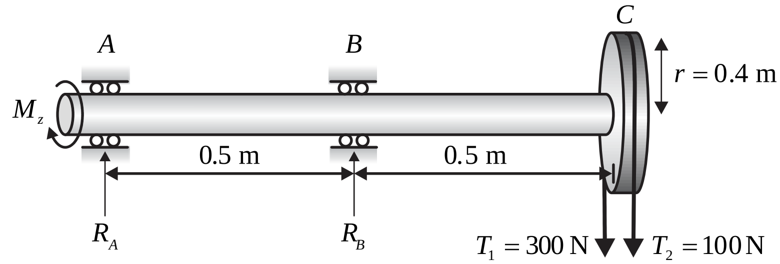
Question 43

A shaft *AC* rotating at a constant speed carries a thin pulley of radius $r = 0.4$ m at the end *C* which drives a belt. *A* motor is coupled at the end *A* of the shaft such that it applies a torque M_z about the shaft axis without causing any bending moment. The shaft is mounted on narrow frictionless bearings at *A* and *B* where $AB = BC = L = 0.5$ m. The taut and slack side tensions of the belt are $T_1 = 300$ N and $T_2 = 100$ N, respectively. The allowable shear stress for the shaft material is 80 MPa. The self-weights of the pulley and the shaft are negligible. Use the value of π available in the *on-screen virtual calculator*. Neglecting shock and fatigue loading and assuming maximum shear stress theory, the minimum required shaft diameter is ________ mm (*round off to 2 decimal places*).



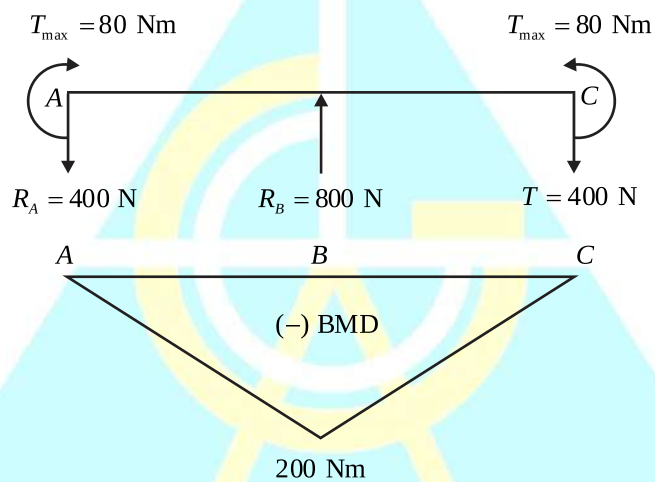
Ans. **23.60 to 24.20**

Sol.



Twisting moment is given by,

$$M_z = T_{max} = (T_1 - T_2)r$$
$$= (300 - 100) \times 0.4$$
$$= 80 \text{ Nm} = 80 \times 10^3 \text{ N.mm}$$



Taking moment about point A

$\Sigma M_A = 0$

$$R_B(0.5) = 400 \times 2 \times 0.5$$
$$R_B = 800 \text{ N}(\uparrow)$$
$$\Sigma V = 0$$
$$R_A = 400(\uparrow)$$

According to maximum shear stress theory,

$$\frac{16}{\pi d^3}\sqrt{M_{max}^2 + T_{max}^2} = \frac{S_{ys}}{FOS} \qquad [\because S_{ys} = 80 \text{ MPa}]$$

$$\frac{16}{\pi d^3}\sqrt{(200\times10^3)^2 + (80\times10^3)^2} = \frac{80}{1}$$

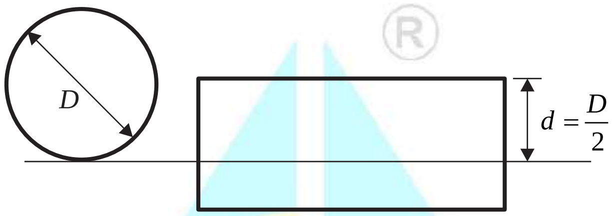
$$d = 23.9357 \text{ mm}$$

Hence, the minimum required shaft diameter is 23.9357 mm.

Question 44

A straight-teeth horizontal slab milling cutter is shown in the figure. It has 4 teeth and diameter (D) of 200 mm. The rotational speed of the cutter is 100 rpm and the linear feed given to the workpiece is 1000 mm/minute. The width of the workpiece (w) is 100 mm, and the entire width is milled in a single pass of the cutter. The cutting force/tooth is given by $F = Kt_c w$, where specific cutting force K = 10 N/mm^2, w is the width of cut, and t_c is the uncut chip thickness.

The depth of cut (d) is D/2, and hence the assumption of $\frac{d}{D} << 1$ is invalid. The maximum cutting force required is ________ kN (*round off to one decimal place*).



Ans. **2.3 to 2.7**

Sol. **Given:**

Number of teeths $(n) = 4$

Diameter of cutter $(D) = 200$ mm

Rotational speed $(N) = 100$ rpm

Width of workpiece $(w) = 100$ mm

Depth of cut $(d) = \dfrac{D}{2}$

Feed $(f) = \dfrac{1000}{100} = 10$ mm/rev

Maximum uncut chip thickness is given by,

$$(t_c)_{\max} = \frac{2f}{n}\sqrt{\frac{d}{D}\left(1 - \frac{d}{D}\right)}$$

$$= \frac{2\times 10}{4}\sqrt{\frac{D/2}{D}\left(1 - \frac{D/2}{D}\right)}$$

$$= 5\sqrt{\frac{1}{2}\left(1 - \frac{1}{2}\right)}$$

$$= 5\times\frac{1}{2}$$

$$= 2.5\,\text{mm}$$

Maximum force is given by, $F_{max} = k\left(t_c\right)_{max} w$

$$= 10 \times 2.5 \times 100$$
$$= 2500 \text{ N}$$
$$= 2.5 \text{ kN}$$

Hence, the maximum cutting force required is 2.5 kN.

Question 45

In an orthogonal machining operation, the cutting and thrust forces are equal in magnitude. The uncut chip thickness is 0.5 mm and the shear angle is 15°. The orthogonal rake angle of the tool is 0° and the width of cut is 2 mm. The workpiece material is perfectly plastic and its yield shear strength is 500 MPa. The cutting force is _________ N (*round off to the nearest integer*).

Ans. **2732.05**

Sol. **Given :**

Thickness of chip $(t) = 0.5$ mm

Shear angle $(\phi) = 15^0$

rake angle $(\alpha) = 0^0$

Width of cut $(b) = 2$ mm

Yield shear strength $(\tau_s) = 500$ MPa

$\because$ Cutting force = thrust force

then

$$F_S = \frac{\tau bt}{\sin\phi} = F_C\cos(\phi) - F_T\sin(\phi)$$

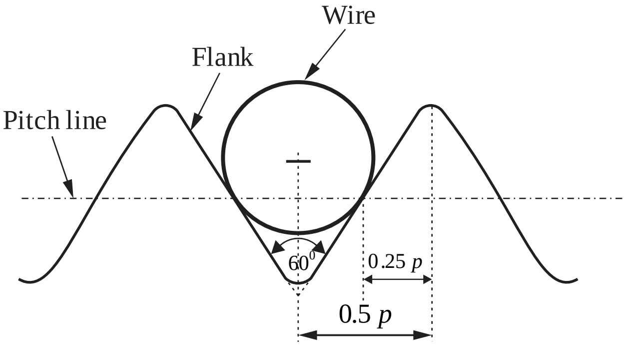
$$F_C = \frac{\tau bt}{\sin\phi(\cos\phi - \sin\phi)}$$

$$= \frac{500 \times 2 \times 0.5}{\sin 15^0(\cos 15^0 - \sin 15^0)} = 2732.05 \text{ N}$$

Hence, the cutting force is 2732.05 N.

Question 46

The best size wire is fitted in a groove of a metric screw such that the wire touches the flanks of the thread on the pitch line as shown in the figure. The pitch (p) and included angle of the thread are 4 mm and 60°, respectively. The diameter of the best size wire is ___________ mm (*round off to 2 decimal places*).



Ans. 2.31

Sol. **Given :**

Pitch $(P) = 4$ mm

Included angle $= (2\alpha) = 60^0$

So, the best wire size

$$d\omega = \frac{P}{2}\sec\alpha$$

$$d\omega = \frac{4}{2}\sec(30)$$

$$d\omega = 2.31 \text{ mm}$$

Hence, the diameter of the best size wire is 2.31 mm.

Question 47

In a direct current arc welding process, the power source has an open circuit voltage of 100 V and short circuit current of 1000 A. Assume a linear relationship between voltage and current. The arc voltage (V) varies with the arc length (l) as $V = 10 + 5l$, where V is in volts and l is in mm. The maximum available arc power during the process is ________ kVA (*in integer*).

Ans. 25

Sol. **Given :**

Open Circuit Voltage $(V_{OC}) = 100$ V

Short Circuit Current $(I_{SC}) = 1000$ A

Here,

$$V = 10 + 5l$$

For maximum power

$$V_{Arc} = \frac{V_{OC}}{2}$$

$$I_{arc} = \frac{I_{SC}}{2}$$

So,

Maximum Power P,

$$P = V_{arc} \times I_{arc}$$
$$= \frac{V_{OC}}{2} \times \frac{I_{OC}}{2}$$
$$= \frac{100}{2} \times \frac{1000}{2}$$
$$= 50 \times 500$$
$$= 25000 \text{ VA}$$
$$= 25 \text{ kVA}$$

Hence, the maximum available power during the process is 25 kVA.

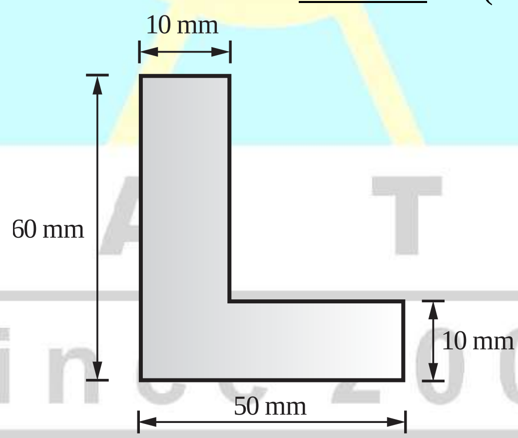
Question 48

A cylindrical billet of 100 mm diameter and 100 mm length is extruded by a direct extrusion process to produce a bar of *L*-section. The cross sectional dimensions of this *L*-section bar are shown in the figure. The total extrusion pressure (p) in MPa for the above process is related to extrusion ratio (r) as

$$p = K_s \sigma_m \left[0.8 + 1.5 \ln(r) + \frac{2l}{d_0} \right],$$

where σ_m is the mean flow strength of the billet material in MPa, l is the portion of the billet length remaining to be extruded in mm, d_0 is the initial diameter of the billet in mm, and K_s is the die shape factor.

If the mean flow strength of the billet material is 50 MPa and the die shape factor is 1.05, then the maximum force required at the start of extrusion is ________ kN (*round off to one decimal place*).



Ans. **2429.201**

Sol. $A_f = 60 \times 10 = 1000 \text{ mm}^2$

$$A_D = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 100^2 = 7853.98 \text{ mm}^2$$

$$r = \frac{A_O}{A_f} = 7.853$$

$$p = 1.05 \times 50 \left[0.8 + 1.5 \times \ln(7.853) + 2 \times \frac{100}{100} \right]$$

$$f = p \times A_o = 309.53 \times \frac{\pi}{4} \times 100^2$$

$$= 2429201.415 \text{ N}$$

$$f = 2429.201 \text{ kN}$$

Hence, the maximum force required at the start of extrusion is 2429.201 kN.

Question 49

A project consists of five activities (*A*, *B*, *C*, *D* and *E*). The duration of each activity follows beta distribution. The three time estimates (in weeks) of each activity and immediate predecessor(s) are listed in the table. The expected time of the project completion is __________ weeks (*in integer*).

Activity	Time estimates (in weeks)			Immediate predecessor(s)
	Optimistic time	Most likely time	Pessimistic time	
A	4	5	6	None
B	1	3	5	*A*
C	1	2	3	*A*
D	2	4	6	*C*
E	3	4	5	*B, D*

Ans. **15**

Sol.

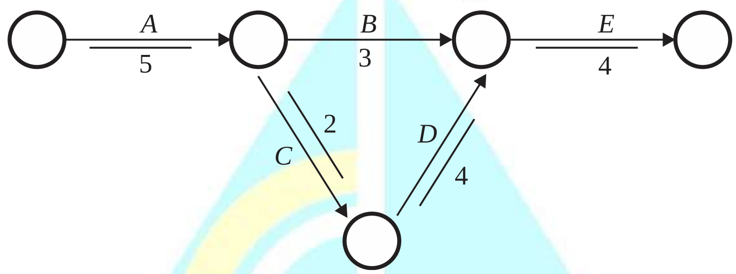
Activity	**Time estimates (in weeks)**			**Immediate predecessor(s)**
	Optimistic time (t_o)	**Most likely time (t_m)**	**Pessimistic time (t_p)**	
A	4	5	6	None
B	1	3	5	A
C	1	2	3	A
D	2	4	6	C
E	3	4	5	B, D

We know that,

$$\text{Expected time, } T_E = \frac{t_o + 4t_m + t_p}{6}$$

Therefore,

Activity	Time estimates (in weeks)			Immediate predecessor(s)	T_E
	(t_o)	(t_m)	(t_p)		
A	4	5	6	None	5
B	1	3	5	A	3
C	1	2	3	A	2
D	2	4	6	C	4
E	3	4	5	B, D	4



Projected completion time $= 5+2+4+4$

$= 15$

Hence, the expected time of the project completion is 15 weeks.

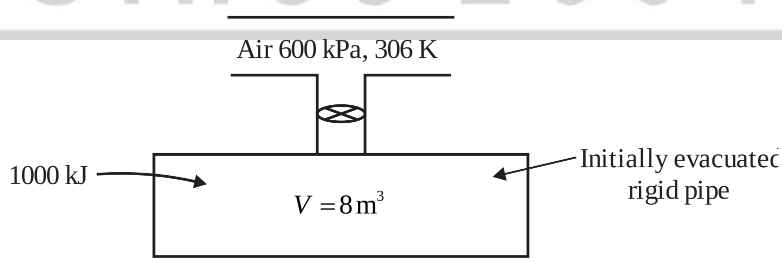
Question 50

A rigid tank of volume of 8 m^3 is being filled up with air from a pipeline connected through a valve. Initially the valve is closed and the tank is assumed to be completely evacuated. The air pressure and temperature inside the pipeline are maintained at 600 kPa and 306 K, respectively. The filling of the tank begins by opening the valve and the process ends when the tank pressure is equal to the pipeline pressure. During the filling process, heat loss to the surrounding is 1000 kJ. The specific heats of air at constant pressure and at constant volume are 1.005 kJ/kg.K and 0.718 kJ/kg.K, respectively. Neglect changes in kinetic energy and potential energy.

The final temperature of the tank after the completion of the filling process is ________ K (*round off to the nearest integer*).

Ans. **395.39**

Sol.



Unsteady flow energy equation :

$$\frac{dU}{dt} = \dot{m}_i h_i - \dot{m}_e h_e + \dot{Q} - \dot{W}$$

$\because \quad (\dot{m}_e = 0 \text{ and } \dot{W} = 0)$

We know, $\dfrac{dm}{dt} = \dot{m}_i - \dot{m}_e$

$$\frac{dm}{dt} = \dot{m}_i \qquad (\because \dot{m}_e = 0)$$

$$\therefore \quad \frac{dU}{dt} = \frac{dm}{dt}h_i + (-1000)$$

$$m_2u_2 - m_1u_1 = (m_2 - m_1)h_i - 1000$$

$$m_2u_2 = m_2h_i - 1000 \qquad (m_1 = 0, \text{because initially evacuated})$$

$$u_2 = h_i - \frac{1000}{m_2}$$

$$C_VT_F = C_PT_i - \frac{1000}{m_2}$$

$$\because \quad m_2 = \frac{P_2V_2}{RT_2}$$

$$\therefore \quad C_VT_F = C_PT_i - \frac{1000}{P_2V_2} \times RT_2$$

$$0.718T_F = 1.005 \times 306 - \frac{1000}{600 \times 8} \times 0.287 \times T_F \qquad (\because T_2 = T_F)$$

$$T_F = 395.389 \text{ K} \simeq 395.39 \text{ K}$$

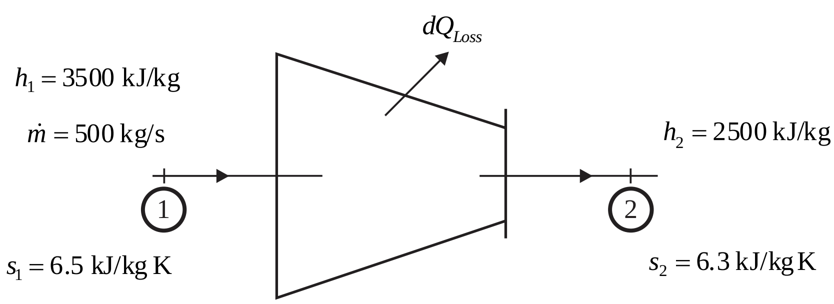
Hence, the final temperature of the tank after the completion of the filling process is 395.39 K.

Question 51

At steady state, 500 kg/s of steam enters a turbine with specific enthalpy equal to 3500 kJ/kg and specific entropy equal to 6.5 kJ·kg^{-1}·K^{-1}. It expands reversibly in the turbine to the condenser pressure. Heat loss occurs reversibly in the turbine at a temperature of 500 K. If the exit specific enthalpy and specific entropy are 2500 kJ/kg and 6.3 kJ·kg^{-1}·K^{-1}, respectively, the work output from the turbine is ________ MW (*in integer*).

Ans. 450

Sol.



Applying steady flow energy equation,

$$h_1 + \frac{v_1^2}{2000} + \frac{gz_1}{1000} + dQ = h_2 + \frac{v_2^2}{2000} + \frac{gz_2}{1000} + dw$$

$$dw = (h_1 - h_2) + dQ \quad \text{... (i)}$$

$$(\because v_1 = 0, v_2 = 0, \ z_1 = z_2)$$

We know,

Power $(P) = \dot{m}(dw)$

$$P = \dot{m}\left[(h_1 - h_2) + dQ\right] \quad \text{... (ii)}$$

Since, reversible heat transfer,

$$(s_2 - s_1) = \frac{dQ}{T}$$

$$dQ = T(s_2 - s_1)$$

$$dQ = 500(6.3 - 6.5)$$

$$dQ = -100 \text{ kJ/kg}$$

By putting in equation (ii),

$$P = 500\left[(3500 - 2500) - 100\right]$$

$$P = 450000 \text{ kW}$$

$$P = 450 \text{ MW}$$

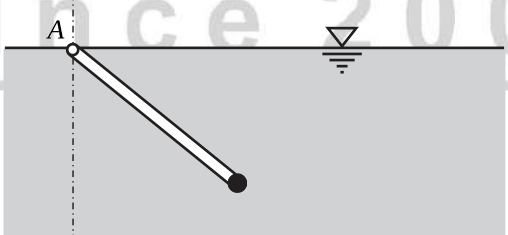
Hence, the work output from the turbine is 450 MW.

Question 52

A uniform wooden rod (specific gravity = 0.6, diameter = 4 cm and length = 8 m) is immersed in the water and is hinged without friction at point *A* on the waterline as shown in the figure. A solid spherical ball made of lead (specific gravity = 11.4) is attached to the free end of the rod to keep the assembly in static equilibrium inside the water. For simplicity, assume that the radius of the ball is much smaller than the length of the rod.

Assume density of water = 10^3 kg/m^3 and $\pi = 3.14$.

Radius of the ball is ________ cm (*round off to 2 decimal places*).



Ans. **3.61**

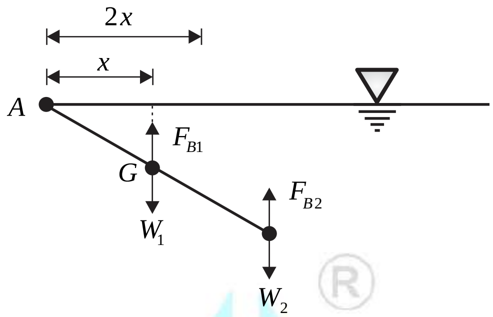
Sol. Given

Specific gravity of wooden rod $(S_1) = 0.6$

Diameter of the rod $(D_1) = 4 \text{ cm} \Rightarrow r_1 = 2 \text{ cm}$

Length of rod $(l) = 8\text{m}$

Specific gravity of spherical ball $(S_2) = 11.4$



Weight of the rod $W_1 = \rho_1 g \pi r_1^2 l$

Weight of the sphere ball $W_2 = \rho_2 g \dfrac{4}{3}\pi r_2^3$

Where,

$r_1 =$ radius of rod

$r_2 =$ radius of sphere

Buoyant force $= \rho_F g V_{Fd} = \rho_F g\left[\pi r_1^2 l + \dfrac{4}{3}\pi r_2^3\right]$

$\sum M_A = 0$ For equilibrium

$$W_1 x + W_2(2x) = F_{B_1} \times x + F_{B_2} \times 2x$$

$$\rho_1 g \pi r_1^2 l + 2\rho_2 g \frac{4}{3}\pi r_2^3 = \rho_W g\left[\pi r_1^2 l + \frac{4}{3}\pi r_2^3 \times 2\right]$$

$$S_1 \pi l r_1^2 + 2S_2 g \frac{4}{3}\pi r_2^3 = \pi r_1^2 l + \frac{4}{3}\pi r_2^3 \times 2$$

$$2S_2 g \frac{4}{3}\pi r_2^3 - 2\times\frac{4}{3}\pi r_2^3 = \pi r_1^2 l - S_1 \pi l r_1^2 \qquad \left[S_1 = \frac{\rho_1}{\rho_w} \text{ and } S_2 = \frac{\rho_2}{\rho_w}\right]$$

$$(2S_2 - 2)\frac{4}{3} r_2^3 = (1 - S_1) r_1^2 l$$

$$(2\times 11.2 - 2)\frac{4}{3} r_2^3 = (1 - 0.6)\times 8 \times (0.02)^2$$

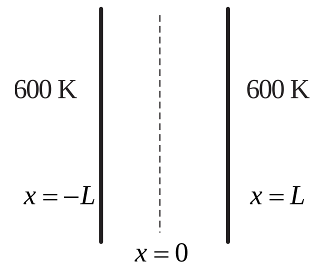
$$r_2 = 0.03610 \text{ m}$$

$$r_2 = 3.61 \text{ cm}$$

Hence, the radius of the ball is 3.61 cm.

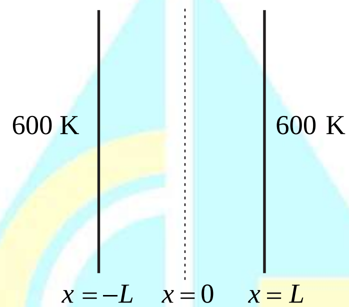
Question 53

Consider steady state, one-dimensional heat conduction in an infinite slab of thickness $2L$ (L = 1 m) as shown in the figure. The conductivity (k) of the material varies with temperature as $k = CT$, where T is the temperature in K, and C is a constant equal to 2 W·m^{-1}·K^{-2}. There is a uniform heat generation of 1280 kW/m^3 in the slab. If both faces of the slab are maintained at 600 K, then the temperature at x = 0 is ________ K (*in integer*).



Ans. **990 to 1010**

Sol. Given data



Thickness of slab 2 L (L = 1 m)

$k = CT,$

Where

$C = 2\ \text{Wm}^{-1}\text{K}^{-2}$

Uniform heat generation $(q) = 1280\ \text{kW/m}^3$

Temperature of the slab $(T) = 600$ K

Steady state, one dimensional heat conduction

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = -q$$

On integrating

$$k\frac{dT}{dx} = -qx + C_1$$

$$2T\frac{dT}{dx} = -qx + C_1$$

$$TdT = \left[\frac{-qx + C_1}{2}\right]dx$$

On integrating,

$$\frac{T^2}{2} = \frac{1}{2}\left[-\frac{qx^2}{2} + C_1 x\right] + C_2 \qquad \ldots \text{(i)}$$

Boundary condition:

At $x = \text{L} = 1$ m $\qquad \Rightarrow \text{T} = 600$ K

Put the value in equations (i)

$$600^2 = \frac{-1280\times10^3\times1^2}{2} + C_1\times1 + C_2$$

$$\Rightarrow C_1 + C_2 = 10^6 \qquad \ldots \text{(ii)}$$

At $x = -L = -1 \qquad \Rightarrow T = 600\text{ K}$

$$600^2 = \frac{-1280\times10^3\times(-1)^2}{2} + C_1\times(-1) + C_2$$

$$\Rightarrow -C_1 + C_2 = 10^6 \qquad \ldots \text{(iii)}$$

By equations (ii) and (iii) we get,

$C_1 = 0 \qquad C_2 = 10^6$

By so equations (i) we get

$$T^2 = \frac{-qx^2}{2} + C_2$$

At $x = 0$

$$[T(0)]^2 = C_2$$

$$[T(0)] = \sqrt{C_2} = \sqrt{10^6}$$

$$T = 10^3 = 1000\text{ K}$$

$$T_{x=0} = 1000\text{ K}$$

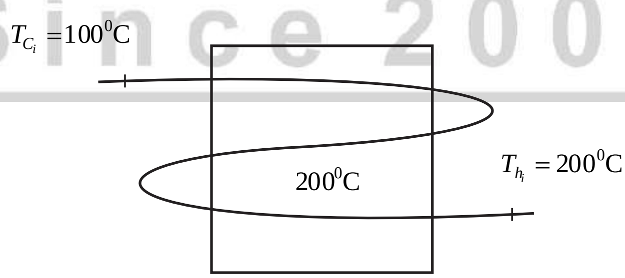
Hence, the temperature at $x = 0$ is 1000 K.

Question 54

Saturated vapor at $200\,^0\text{C}$ condenses to saturated liquid at the rate of 150 kg/s on the shell side of a heat exchanger (enthalpy of condensation h_{fg} = 2400 kJ/kg). A fluid with c_p = 4 kJ·kg^{-1}·K^{-1} enters at $100\,^0\text{C}$ on the tube side. If the effectiveness of the heat exchanger is 0.9, then the mass flow rate of the fluid in the tube side is ________ kg/s (*in integer*).

Ans. 1000

Sol.



$$\varepsilon = 0.9$$

$$\dot{m}_{cond} = 150\text{ kg/sec}$$

$$h_{fg} = \text{ Latent heat of vapourisation } = 2400\text{ kJ/kg}$$

Heat released $= \dot{m}_{cond} \times h_{fg}$

Heat released $= 150 \times 2400$

$\because$ Heat released $=$ Heat absorbed by cold fluid.

$$150 \times 2400 = m_{CF} \times (c_p)_{CF} \times (T_F - T_i) \qquad \ldots \text{(i)}$$

We know that, for condensation :

ΔT for condensing fluid $\rightarrow 0$

$$\dot{m}c \rightarrow \infty$$

Therefore, for cold fluid $\rightarrow (\dot{m}C)$ will be minimum.

$$\therefore \quad 0.9 = \frac{T_{C_e} - T_{C_i}}{T_{h_i} - T_{C_i}}$$

$$0.9 = \frac{T_{C_e} - 100}{200 - 100}$$

$$T_{C_e} = 190$$

From equation (i),

$$150 \times 2400 = \dot{m}_{CF} \times 4 \times (190 - 100) \qquad \left(\because T_F = T_{C_e} \text{ and } T_i = T_{C_i}\right)$$

$$\dot{m}_{CF} = 1000 \text{ kg/sec}$$

Hence, the mass flow rate of the fluid in the tube side is 1000 kg/s.

Question 55

Consider a hydrodynamically and thermally fully-developed, steady fluid flow of 1 kg/s in a uniformly heated pipe with diameter of 0.1 m and length of 40 m. A constant heat flux of magnitude 15000 W/m^2 is imposed on the outer surface of the pipe. The bulk-mean temperature of the fluid at the entrance to the pipe is 200 ^{0}C. The Reynolds number (*Re*) of the flow is 85000, and the Prandtl number (*Pr*) of the fluid is 5. The thermal conductivity and the specific heat of the fluid are 0.08 W·m^{-1}·K^{-1}and 2600 J·kg^{-1}·K^{-1}, respectively. The correlation $Nu = 0.023\ Re^{0.8}\ Pr^{0.4}$ is applicable, where the Nusselt Number (*Nu*) is defined on the basis of the pipe diameter. The pipe surface temperature at the exit is ________ ^{0}C (*round off to the nearest integer*).

Ans. **321.267**

Sol. Given

Entrance temp (T_b inlet) 200^0C

Reynold number $(R_e) = 85000$

Prandtl number $(p_r) = 5$

Thermal conductivity $(K) = 0.08 \text{ Wm}^{-1}\text{K} - 1$

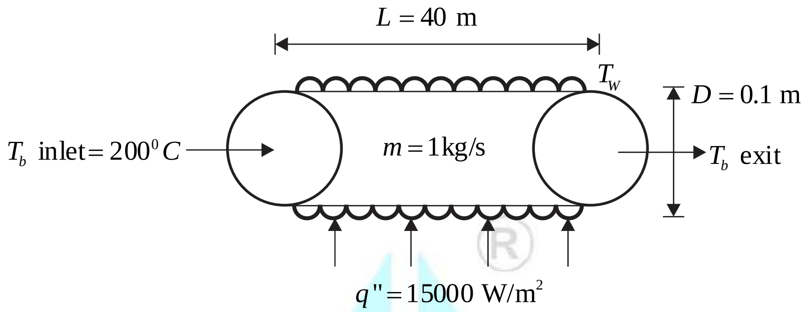
Specific heat of the fluid $(C_p) = 2600\,\text{J.kg}^{-1}\text{K}^{-1}$

Heat flux $(q'') = 15000 \text{ W/m}^2$

Diameter of pipe $(D) = 0.1\text{m}$

Length of the pipe $(L) = 40$ m

Fluid flow of mass $(m) = 1$ kg/s



To get T_b exit

$$q'' \times \pi DL = m_f \times C_p \quad (T_b \text{ exit} - T_b \text{ inlet})$$

$$15000 \times \pi \times 0.1 \times 40 \quad = 1 \times 2600 \times (T_b \text{ exit} - 200)$$

$$T_b \text{ exit} = 272.498^0 C$$

Nusselt number, $Nu = \dfrac{hD}{k} = 0.023 \text{Re}^{0.8} Pr^{0.4}$

So,

$$\frac{h \times 0.1}{0.08} = 0.023 \times (85000)^{0.8} \times 5^{0.4}$$

$$h = 307.5678 \text{ W/m}^2\text{K}$$

Now for T_W wall temperature

By newton's law of cooling

$$q'' = h(\text{T}_W \text{ exit } - T_b \text{exit})$$

$$15000 = 307.567(T_W \text{ exit} - 272.498)$$

$$T_W \text{ exit} = \frac{15000}{307.567} + 272.498$$

$$T_W \text{ exit} = 321.267^0 C$$

Hence, the pipe surface temperature at the exit is 321.267^0C.

❖❖❖❖