General Aptitude

Q.1 to Q.5 Carry ONE Mark Each

Question 1

Mr. X speaks ______ Japanese ________Chinese.

(A) either/nor (B) also/but

(C) neither/nor (D) neither/or

Ans. C

Sol. "either" is used with "or"

"neither" is used with "nor"

Because of these fixed combinations option (A) and option (D) are eliminated.

Option (B), in this option we are not getting contrast tone in given filler that's why this option is also eliminated.

Hence, the correct option is (C).

Question 2

A sum of money is to be distributed among *P*, *Q*, *R* and *S* in the proportion 5 : 2 : 4 : 3, respectively.

If *R* get Rs.1000 more than *S*, what is the share of *Q* (in Rs.)?

(A) 1000 (B) 1500

(C) 2000 (D) 500

Ans. C

Sol. **Given :** A sum of money is to be distributed among *P*, *Q*, *R* and *S* is $5x$, $2x$, $4x$ and $3x$ respectively.

Where x is a common multiplication factor.

As *R* gets Rs.1000 more than *S*.

Then, $4x - 3x = 1000$

$$x = 1000$$

Now, share of *Q* is $2x = 2 \times 1000$ = Rs.2000

Share of *Q* is Rs.2000.

Hence, the correct option is (C).

Question 3

A trapezium has vertices marked as *P*, *Q*, *R* and *S* (in that order anticlockwise). The side *PQ* is parallel to side *SR*. Further, it is given that, *PQ* = 11 cm, *QR* = 4 cm, *RS* = 6 cm and *SP* = 3 cm.

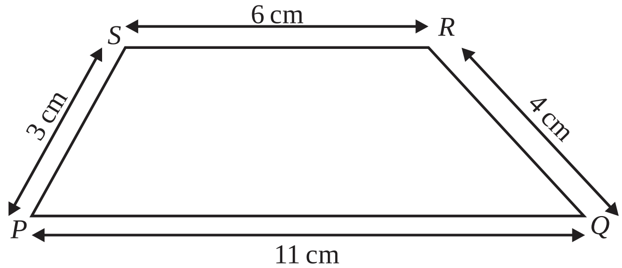
What is shortest distance between *PQ* and *SR* (in cm)?

(A) 4.20 (B) 1.80

(C) 5.76 (D) 2.40

Ans. D

Sol. **Given :**



There is a Trapezium *PQRS*,

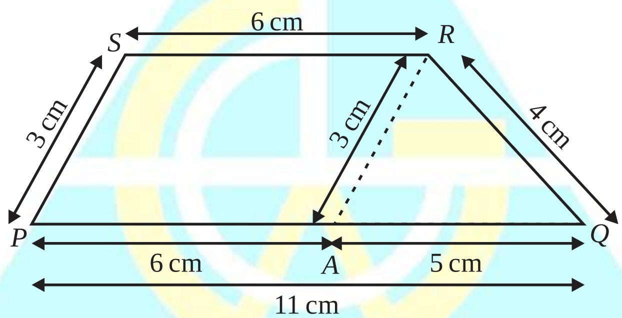
Length of *PQ* = 11 cm

Length of *QR* = 4 cm

Length of *PS* = 3 cm

Length of *RS* = 6 cm

Let us consider, a line segment *RA* which is parallel to *PS*.



Now, length of side of *AQ* = 5 cm

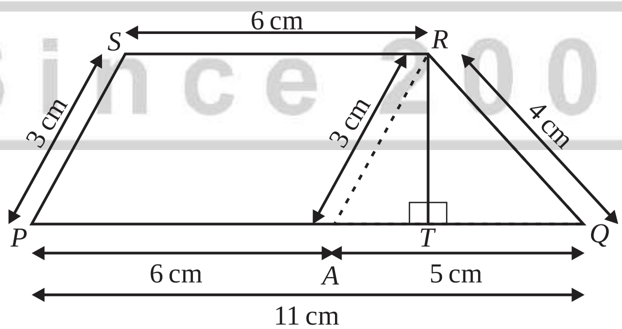
$$PA \| SR,\ PS \| AR$$

So, length of side *AR* = 3 cm

Length of side *RQ* = 4 cm

Length of side $AQ = 5$ cm (Sides of ΔARQ follows the property of Pythagoras theorem)

Let us consider a perpendicular *RT* which is shortest path of *PQ* and *RS*



Area of Triangle $= \frac{1}{2}(\text{Base} \times \text{Height})$

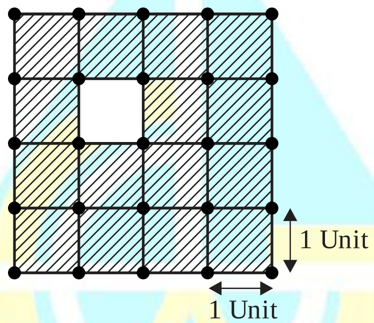
Area of $\Delta ARQ = \frac{1}{2} \times 3 \times 4$ sq. cm and also, area of $\Delta ARQ = \frac{1}{2} \times AQ \times RT$

$$\frac{1}{2} \times AQ \times RT = \frac{1}{2} \times 3 \times 4 \text{ sq. cm}$$
$$AQ \times RT = 12 \text{ sq. cm}$$
$$5 \times RT = 12 \text{ sq. cm}$$
$$RT = 2.4 \text{ cm}$$

Hence, the correct option is (D).

Question 4

The figure shows a grid formed by a collection of the unit squares. The unshaded unit square in the grid represents a hole.



What is the maximum number of squares without a "hole in the interior" that can be formed within the 4×4 grid using the unit squares as building blocks?

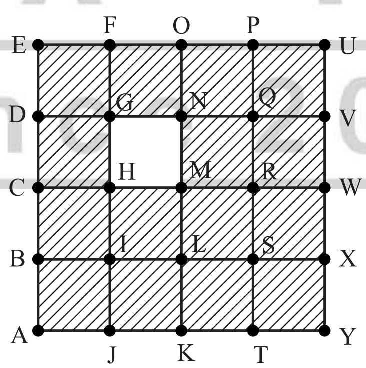
(A) 26 (B) 21

(C) 15 (D) 20

Ans. **D**

Sol. **Given :** The figure shows a grid formed by a collection of the unit square. The unshaded unit square in the grid represents a hole.

The maximum number of square without a "hole in the interior" that can be formed within the 4×4 grid can be counted as.



(ABIJ, BCHI, CDGH, DEFG, FONG, HMLI, ILKJ, KLST, LMRS, MNQR, NOPQ, PUVQ, QVWR, RWSX, SXYT, ACMK, JHRT, KMWY, LNVX, MOUW)

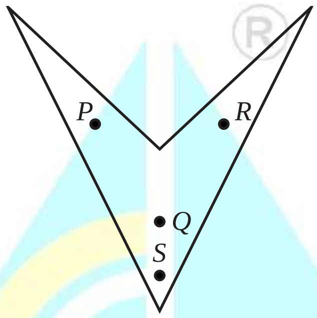
Total number of squares are 20.

Hence, the correct option is (D).

Question 5

An art gallery engages a security guard to ensure that the items displayed are protected. The diagram below represents the plan of the gallery where the boundary walls are opaque. The location the security guard posted is identified such that all the inner space (shaded region in the plan) of the gallery is within the line of sight of the security guard.

If the security guard does not move around the posted location and has a 360° view, which one of the following correctly represents the set of ALL possible locations among the locations P, Q, R and S, where the security guard can be posted to watch over the entire inner space of the gallery.

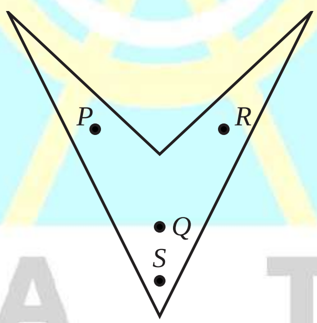


(A) *P* and *Q*   (B) *Q* and *S*

(C) *R* and *S*   (D) *Q*

Ans. **B**

Sol. **Given :** The diagram below represents the plan of the gallery where the boundary wall are opaque.



The art gallery engages a security guard to ensure that the items displayed are protected.

The location where the security guard posted is identified such that all the inner space of the gallery is within the line of sight of the security guard.

The condition is if the security guard does not move around the posted location and has 360^{0} view,

The set of all possible location among the locations P, Q, R and S are Q and S from where the security guard can be posted to watch over the entire inner space of the gallery

Hence, the current option is (B).

Q.6 to Q.10 Carry TWO Marks Each

Question 6

Mosquitoes pose a threat to human health. Controlling mosquitoes using chemicals may have undesired consequences. In Florida, authorities have used genetically modified mosquitoes to control the overall mosquito population. It remains to be seen if this novel approach has unforeseen consequences.

Which one of the following is the correct logical inference based on the information in the above passage?

(A) Using genetically modified mosquitoes is better than using chemicals to kill mosquitoes because they do not have any side effects.

(B) Using chemicals to kill mosquitoes may have undesired consequences but it is not clear if using genetically modified mosquitoes has any negative consequence.

(C) Using chemicals to kill mosquitoes is better than using genetically modified mosquitoes because genetic engineering is dangerous.

(D) Both using genetically modified mosquitoes and chemicals have undesired consequences and can be dangerous.

Ans. **B**

Sol. Option (A), in this option, it is mentioned that "genetically modified mosquitoes are better than using chemical to kill mosquitoes" which is not mentioned in the given passage and that's why this option is eliminated.

Option (B), in this option, it is clearly mentioned in given passage that "using chemical to kill mosquitoes may have undesired consequences but it is not clear if using genetically modified mosquitoes has any negative consequence" so this option is correct.

Option (C), in this option, it is mentioned "genetic engineering is dangerous" which is not mentioned so this option is also eliminated.

Option (D), in this option, it is mentioned that "both using genetically modified mosquitoes and chemicals have undesired consequence and can be dangerous" which is not related to given passage so this option is also eliminated.

Hence, the correct option is (B).

Question 7

Consider the following inequalities.

(i) $2x-1>7$ (ii) $2x-9<1$

Which one of the following expressions below satisfies the above two inequalities?

(A) $x \le -4$ (B) $-4<x\le 4$

(C) $x \ge 5$ (D) $4<x<5$

Ans. **D**

Sol. **Given :** First Inequality, $2x-1>7$

$$2x-1>7$$
$$2x>8$$
$$x>4$$

And second Inequality, $2x-9<1$

$$2x-9<1$$
$$2x<10$$
$$x<5$$

By these two Inequalities we have value of x as $4 < x < 5$.

Hence, the correct option is (D).

Question 8

Four points $P(0, 1)$, $Q(0, -3)$, $R(-2, -1)$ and $S(2, -1)$ represent the vertices of a quadrilateral. What is the area enclosed by the quadrilateral?

(A) $8\sqrt{2}$ (B) 4

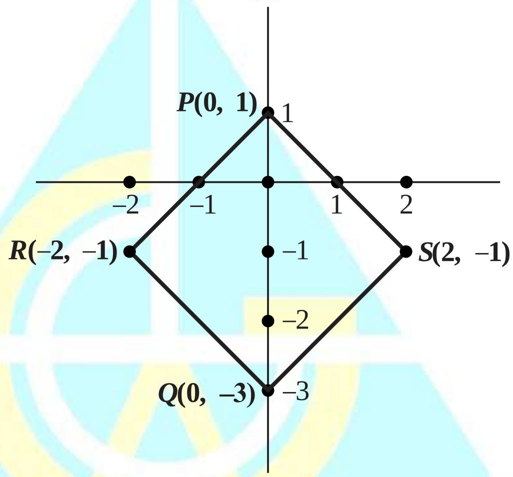
(C) 8 (D) $4\sqrt{2}$

Ans. **C**

Sol. **Given :** The vertices of a Quadrilateral have four points ***P*(0, 1), *Q*(0, –3), *R*(–2, –1)** and ***S*(2 , –1)**.

On plotting points on graph,

According to the given data a Quadrilateral ***PSQR*** will form as shown in below figure,



The distance formula between two points $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Length of side $\overline{PS}=\sqrt{(2-0)^2+(-1-1)^2}=\sqrt{4+4}=\sqrt{8}$

Length of side $\overline{QS}=\sqrt{(2-0)^2+(-1+3)^2}=\sqrt{4+4}=\sqrt{8}$

Length of side $\overline{QR}=\sqrt{(0-(-2))^2+\sqrt{(-3-(-1))^2}}=\sqrt{4+4}=\sqrt{8}$

Length of side $\overline{PR}=\sqrt{(0-(-2))^2+\sqrt{(1-(-1))^2}}=\sqrt{4+4}=\sqrt{8}$

Length of side $\boldsymbol{\overline{PS}=\overline{QS}=\overline{QR}=\overline{PR}}$

and, length of diagonal $\overline{PQ}=\sqrt{(0-0)^2+(1-(-3))^2}=4$

Length of diagonal $\overline{RS}=\sqrt{(2-(-2))^2+(-1-(-1))^2}=4$

We can see clearly,

Length of diagonal $\boldsymbol{\overline{PQ}=\overline{RS}}$

So, Quadrilateral *PSQR* follows property of square.

Area of Quadrilateral *PSQR* = Side^2

Area of Quadrilateral $PSQR=(\sqrt{8})^2=8$ Sq. unit and also by,

Area of Rhombus = $\frac{1}{2}$(product of both diagonals)

$$=\frac{1}{2}(4\times4)\text{ Sq. unit} = 8\text{ Sq. unit}$$

Hence, the correct option is (C).

Question 9

In a class of five students P, Q, R, S and T, only one student is known to have copied in the exam. The disciplinary committee has investigated the situation and recorded the statements from the students as given below.

Statement of P : R has copied in the exam.

Statement of Q : S has copied in the exam.

Statement of R : P did not copy in the exam.

Statement of S : Only one of us is telling the truth.

Statement of T : R is telling the truth.

The investigating team had authentic information that S never lies.

Based on the information given above, the person who has copied in the exam is

(A) R (B) P

(C) Q (D) T

Ans. **B**

Sol. **Given :** In a class of five students P, Q, R, S and T only student is known to have copied in the exam.

The recorded statement from students

Statement of P : R has copied in the exam.

Statement of Q : S has copied in the exam.

Statement of R : P did not copy in the exam.

Statement of S : only one of us is telling the truth.

Statement of T : R is telling the truth.

The investigating team have authentic information that S never lies.

S says only one of us is tilling the truth.

Hence, statement of all other students will be wrong.

Statement of P: R copied in the exam.

Statement of Q : S copied in the exam.

Statement of R : P copied in the exam.

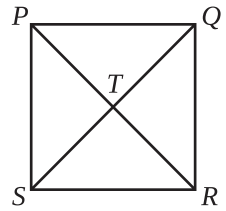
Statement of T : P copied in the exam.

So, P copied in the exam.

Hence, the correct option is (B).

Question 10

Consider the following square with the four corners and the center marked as P, Q, R, S and T respectively.



Let X, Y and Z represent the following operations:

X: rotation of the square by 180 degree with respect to the S-Q axis.

Y: rotation of the square by 180 degree with respect to the P-R axis.

Z: rotation of the square by 90 degree clockwise with respect to the axis perpendicular, going into the screen and passing through the point T.

Consider the following three distinct sequences of operation (which are applied in the left to right order).

(1) XYZZ

(2) XY

(3) ZZZZ

Which one of the following statements is correct as per the information provided above?

(A) The sequence of operations 1, 2 and 3 are equivalent

(B) The sequence of operations 2 and 3 are equivalent

(C) The sequence of operations 1 and 3 are equivalent

(D) The sequence of operations 1 and 2 are equivalent

Ans. **C**

Sol. **Given :** A square with the four corners and the center marked as P, Q, R, S and T respectively and XYZ represent following operation.

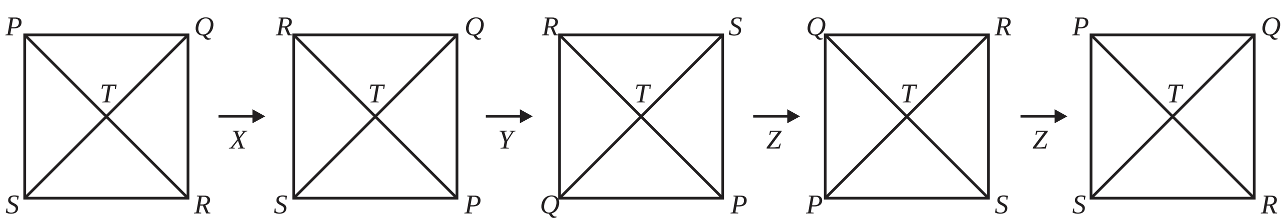
X : Rotation of the square by 180 degree with respect to the S-Q axis

Y : Rotation of the square by 180 degree with respect to the P-R axis

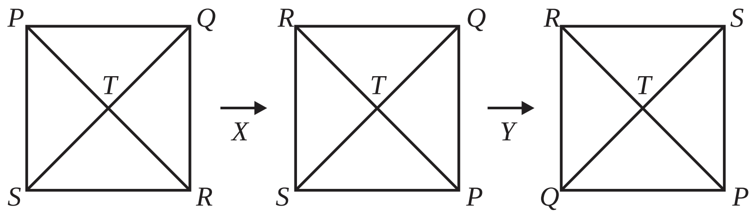
Z : Rotation of the square by 90 degree clockwise with respect to the axis perpendicular going into the screen and passing through the point T.

Following three distinct sequences of operation are as follows.

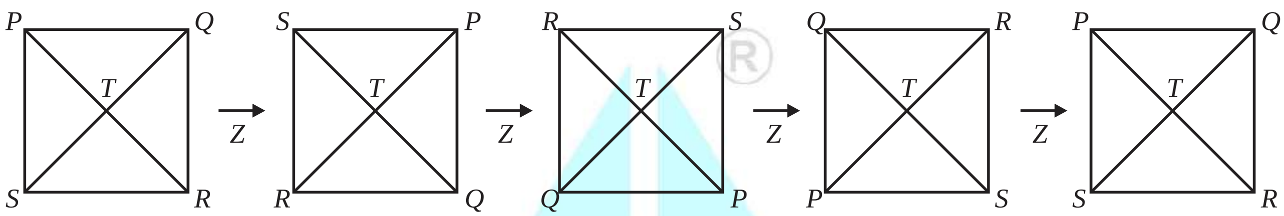
1. $XYZZ : X \rightarrow Y \rightarrow Z \rightarrow Z$



2. XY

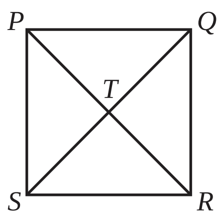


3. $ZZZZ$

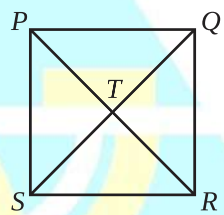


As per the information and operations sequence of operation 1 and 3 are equivalent.

1. $XYZZ$



3. $ZZZZ$



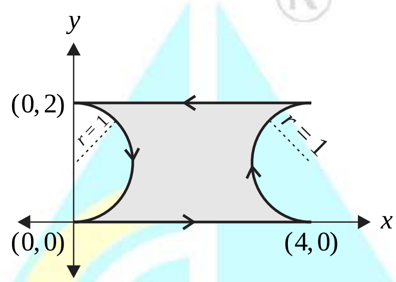
Hence, the correct option is (C).

Technical Section

Q.11 to Q.35 Carry ONE Mark Each

Question 11 **Mathematics**

Consider the two-dimensional vector field $\vec{F}(x, y) = x\hat{i} + y\hat{j}$, where $\hat{i}$ and $\hat{j}$ denote the unit vectors along the x-axis and the y-axis, respectively. A contour C in the x-y plane, as shown in the figure, is composed of two horizontal lines connected at the two ends by two semicircular arcs of unit radius. The contour is traversed in the counter-clockwise sense. The value of the closed path integral $\oint_C \vec{F}(x, y) \cdot (dx\hat{i} + dy\hat{j})$ is

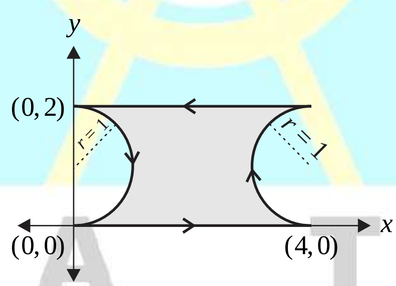


(A) 0 (B) 1

(C) $8 + 2\pi$ (D) -1

Ans. **A**

Sol. Given figure shown in below,



Method 1

We have a closed contour so we will use stokes theorem i.e.,

$$\oint \vec{F} \cdot dl = \iint (\nabla \times \vec{F}) \cdot ds$$

Given : $\vec{F}(x, y) = x\hat{i} + y\hat{j}$

Here, $F_x = x, F_y = y, F_z = 0$.

$$\text{Thus, } \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\nabla\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x & y & 0\end{vmatrix}$$

$$\nabla\times\vec{F}=\hat{i}(0-0)-\hat{j}(0-0)+\hat{k}(0-0)=0$$

$$\iint(\nabla\times\vec{F})\cdot ds=0$$

Hence, the correct option is (A).

Method 2

Given : $\oint_C \vec{F}(x,y)\cdot(dx\,\hat{i}+dy\,\hat{j})$

$$\vec{F}(x,\ y)=x\hat{i}+y\hat{j}$$

$$\oint(x\,\hat{i}+y\,\hat{j})\cdot(dx\,\hat{i}+dy\,\hat{j})$$

$$\oint(xdx+ydy) \qquad (\because\ \hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=1) \qquad \ldots\text{(i)}$$

Applying green's theorem,

$$\oint(Mdx+Ndy)=\iint\left(\frac{\partial N}{\partial y}-\frac{\partial M}{\partial x}\right)dxdy$$

$$M=x,\ N=y$$

$$\frac{\partial M}{\partial x}=1,\ \frac{\partial N}{\partial y}=1$$

From equation (i),

$$\oint(xdx+ydy)=\iint(1-1)\,dxdy=0$$

Hence, the correct option is (A).

Question 12 **Mathematics**

Consider a system of linear equations $Ax=b$, where

$$A=\begin{bmatrix}1 & -\sqrt{2} & 3\\ -1 & \sqrt{2} & -3\end{bmatrix},\ b=\begin{bmatrix}1\\3\end{bmatrix}$$

This system of equations admits

(A) a unique solution for x

(B) No solutions for x

(C) exactly two solutions for x

(D) infinitely many solutions for x

Ans. B

Sol. System of linear equation, $Ax = b$

Given matrix, $A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix}_{2\times 3}$, $b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Augmented matrix, $C = [A:b] = \begin{bmatrix} 1 & -\sqrt{2} & 3 & : & 1 \\ -1 & \sqrt{2} & -3 & : & 3 \end{bmatrix}_{2\times 4}$

$R_2 \to R_2 + R_1$,

$C = \begin{bmatrix} 1 & -\sqrt{2} & 3 & : & 1 \\ 0 & 0 & 0 & : & 4 \end{bmatrix}_{2\times 4}$

We can see that, of the above augmented matrix.

Matrix A has only one linearly independent row, $\rho(A) = 1$

and matrix $[A:b]$ has two linearly independent rows

$\rho(A:b) = 2$

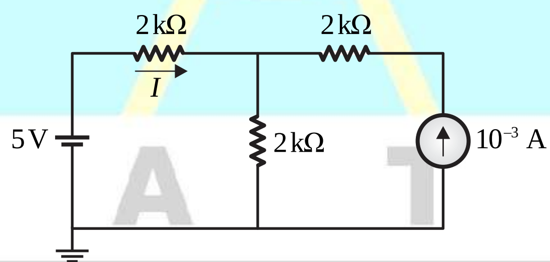
Here, $\rho(A) \neq \rho(A:b)$

So, system has no solution for x.

Hence, the correct option is (B).

Question 13 **Network Theory**

The current I in the circuit shown is

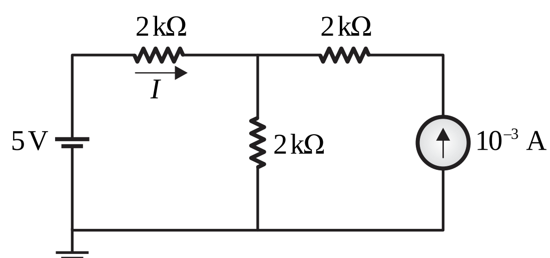


(A) 1.16×10^{-3} A (B) 0.75×10^{-3} A

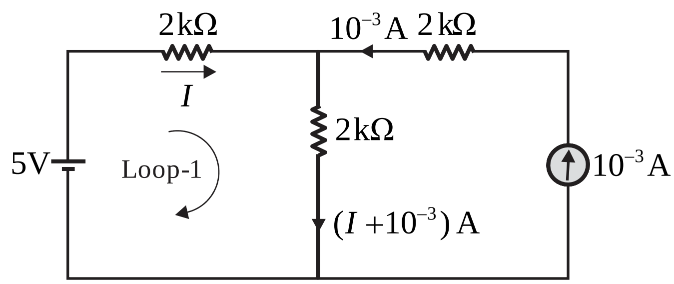
(C) -0.5×10^{-3} A (D) 1.25×10^{-3} A

Ans. **B**

Sol. Given circuit is shown below,



Method 1



Applying KVL in loop-1 shown in above figure,

$$-5+2000I+2000(I+10^{-3})=0$$

$$5=2000I+2000(I+10^{-3})$$

$$5=2000I+2000I+2$$

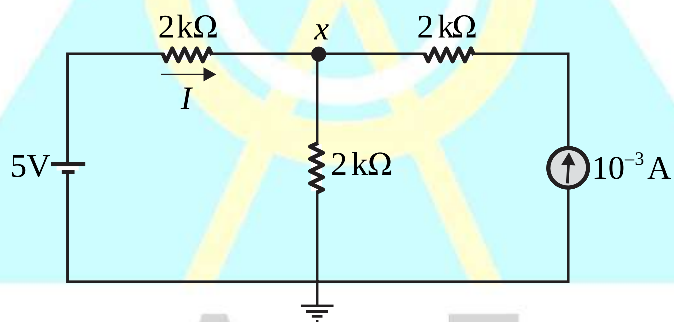
$$4000I=5-2$$

$$I=\frac{3}{4000}=0.75\times10^{-3}\text{ A}$$

Hence, the correct option is (B).

Method 2

Assuming voltage V at node x as shown below,



Applying KCL at node x,

$$\frac{V-5}{2000}+\frac{V}{2000}-10^{-3}=0$$

$$\frac{V-5}{2000}+\frac{V}{2000}=10^{-3}$$

$$2V-5=2$$

$$2V=5+2=7$$

$$V=3.5\text{V}$$

From figure,

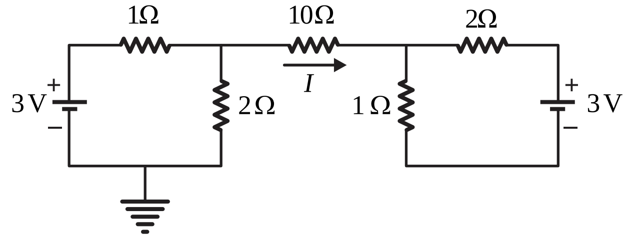
$$I=\frac{5-V}{2000}=\frac{5-3.5}{2000}$$

$$I=0.75\times10^{-3}\text{A}$$

Hence, the correct option is (B).

Question 14 **Network Theory**

Consider the circuit shown in the figure. The current I flowing through the 10 Ω resistor is

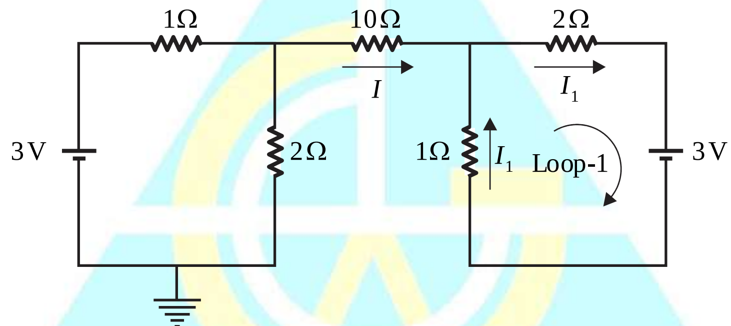


(A) 1 A (B) – 0.1 A

(C) 0 A (D) 0.1 A

Ans. C

Sol. Given circuit is shown below



Apply KVL in loop-1,

$2I_1 + 3 + I_1 = 0$

$I_1 = -1 \text{ A}$

I_1 can flow in loop-1 only, so current I need to be zero. Otherwise it violets the KCL.

So, the value of $I = 0$.

Hence, the correct option is (C).

Key Point

Since, there is no return path for current I. Hence the value of $I = 0$.

Question 15 **Signals & Systems**

The Fourier transform $X(j\omega)$ of the signal $x(t) = \dfrac{t}{(1+t^2)^2}$ is

(A) $\dfrac{\pi}{2}\omega e^{-|\omega|}$ (B) $\dfrac{\pi}{2j}\omega e^{-|\omega|}$

(C) $\dfrac{\pi}{2j}e^{-|\omega|}$ (D) $\dfrac{\pi}{2}e^{-|\omega|}$

Ans. B

Sol. **Given :** Signal $x(t) = \dfrac{t}{(1+t^2)^2}$

We know that,

$$e^{-a|t|} \xleftrightarrow{FT} \frac{2a}{a^2+\omega^2}$$

Put $a=1$ on both side,

$$e^{-|t|} \xleftrightarrow{FT} \frac{2}{1+\omega^2}$$

$$te^{-|t|} \xleftrightarrow{FT} j\frac{d}{d\omega}\left(\frac{2}{1+\omega^2}\right)$$

$$te^{-|t|} \xleftrightarrow{FT} \frac{j2[-2\omega]}{[1+\omega^2]^2}$$

From multiplication by $'t'$ property of Fourier transform,

$$te^{-|t|} \xleftrightarrow{FT} \frac{-4j\omega}{(1+\omega^2)^2}$$

From duality property,

$$\frac{-4jt}{(1+t^2)^2} \xleftrightarrow{FT} -2\pi\omega e^{-|\omega|}$$

$$\frac{t}{(1+t^2)^2} \xleftrightarrow{FT} \frac{-2\pi\omega e^{-|\omega|}}{-4j}$$

$$\frac{t}{(1+t^2)^2} \xleftrightarrow{FT} \frac{\pi\omega}{2j}e^{-|\omega|}$$

Hence, the correct option is (B).

Key Point

According to duality property,

$$e^{-a|t|} \xleftrightarrow{FT} \frac{2a}{a^2+\omega^2}$$

$$\frac{2a}{a^2+t^2} \xleftrightarrow{FT} 2\pi e^{-a|\omega|}$$

Question 16 **Electronic Devices**

Consider a long rectangular bar of direct bandgap p-type semiconductor. The equilibrium hole density is $10^{17}\,\text{cm}^{-3}$ the intrinsic carrier concentration is $10^{10}\,\text{cm}^{-3}$. Electron and hole diffusion lengths are $2\,\mu\text{m}$ and $1\,\mu\text{m}$, respectively.

The left side of the bar $(x=0)$ is uniformly illuminated with a laser having photon energy greater than the bandgap of the semiconductor. Excess electron-hole pairs are generated ONLY at $x=0$ because of the laser. The steady state electron density at $x=0$ is 10^{14}cm^{-3} due to laser illumination. Under these conditions and ignoring electric field, the closest approximation (among the given options) of the steady state electron density at $x=2\mu\text{m}$, is

(A) $0.63\times10^{13}\text{cm}^{-3}$ (B) $3.7\times10^{14}\text{cm}^{-3}$

(C) $0.37\times10^{14}\text{cm}^{-3}$ (D) 10^{3}cm^{-3}

Ans. C

Sol. **Given :**

(i) p-type semiconductor

(ii) Equilibrium hole density, $p_0=10^{17}\text{ cm}^{-3}$

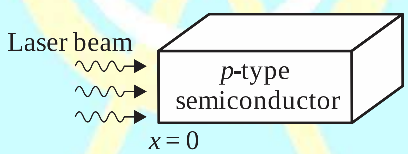
(iii) Intrinsic carrier concentration, $n_i=10^{10}\text{ cm}^{-3}$

(iv) Electron diffusion length, $L_n=2\ \mu\text{m}$

(v) Hole diffusion length, $L_p=1\ \mu\text{m}$

(vi) Excess electron density at $x=0$, $n_p(0)=10^{14}\text{ cm}^{-3}$

Consider a bar of direct bandgap p-type semiconductor with light is illuminated at left side of the bar at $x=0$.

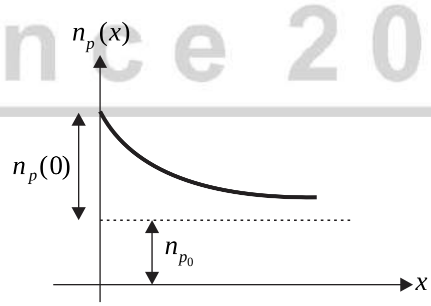


From **Mass action law**, $np=n_i^2$

Electron or minority carrier concentration is given by,

$$n_{p_0}=\frac{n_i^2}{p_0}=\frac{(10^{10})^2}{10^{17}}=\frac{10^{20}}{10^{17}}$$

$$n_{p_0}=10^{3}\text{ cm}^{-3}$$



Electron concentration at any distance 'x' is given by,

$$n_p(x)=n_{p_0}+n_p(0)e^{-x/L_n}$$

$$n_p(x)=10^{3}+10^{14}e^{-x/L_n}$$

Electron concentration (density) at $x=2\ \mu m$ is,

$$n_p(x=2)=10^3+10^{14}e^{-2/2}$$

$$n_p(x=2)=10^3+10^{14}e^{-1}$$

$$n_p(x=2)=10^3+10^{14}\times 0.37$$

$$n_p(x=2)\cong 0.37\times 10^{14}\ \text{cm}^{-3}$$

Hence, the correct option is (C).

Question 17 **Electronic Devices**

In a non-degenerate bulk semiconductor with electron density $n=10^{16}\,\text{cm}^{-3}$, the value of $E_C-E_{Fn}=200\,\text{meV}$, where E_C and E_{Fn} denote the bottom of the conduction band energy and electron Fermi level energy, respectively. Assume thermal voltage as 26 meV and the intrinsic carrier concentration is $10^{10}\ \text{cm}^{-3}$. For $n=0.5\times 10^{16}\ \text{cm}^{-3}$, the closest approximation of the value of (E_C-E_{Fn}), among the given options, is

(A) 174 meV (B) 182 meV

(C) 226 meV (D) 218 meV

Ans. **D**

Sol. **Given :**

(i) Electron density, $n_1=10^{16}\ \text{cm}^{-3}$

(ii) $(E_c-E_{Fn})_1=200\ \text{meV}$

(iii) Intrinsic carrier concentration, $n_i=10^{10}\ \text{cm}^{-3}$

(iv) We have to find $(E_c-E_{Fn})_2$ for electron density, $n_2=0.5\times 10^{16}\ \text{cm}^{-3}$

Since, electron density for $n-$type semiconductor is given by,

$$n=N_c e^{\frac{-(E_c-E_{Fn})}{kT}}$$

Where, $n=$ Electron density, $N_c=$ Effective density of states at the edge of conduction band.

$$n_1=N_c e^{\frac{-(E_C-E_{Fn})_1}{kT}} \quad \ldots \text{(i)}$$

$$n_2=N_c e^{\frac{-(E_c-E_{Fn})_2}{kT}} \quad \ldots \text{(ii)}$$

Dividing equations (i) and (ii),

$$\frac{n_1}{n_2}=\frac{N_c e^{\frac{-(E_c-E_{Fn})_1}{kT}}}{N_c e^{\frac{-(E_c-E_{Fn})_2}{kT}}}$$

$$\frac{10^{16}}{0.5\times 10^{16}}=e^{\frac{(E_c-E_{Fn})_2-(E_c-E_{Fn})_1}{kT}}$$

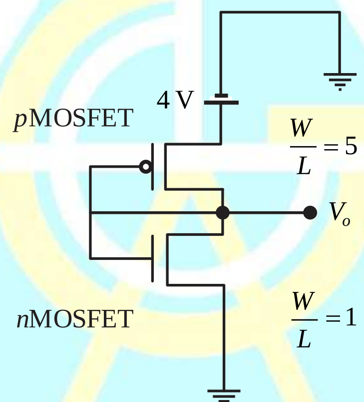
$$2=e^{\frac{(E_c-E_{Fn})-200\times 10^{-3}}{26\times 10^{-3}}}$$

$$(E_c - E_{Fn})_2 - 200\times10^{-3} = 26\ln 2$$
$$(E_c - E_{Fn})_2 = 26\ln 2 + 200$$
$$(E_c - E_{Fn})_2 = 218\text{ meV}$$

Hence, the correct option is (D).

Question 18 **Electronic Devices**

Consider the CMOS circuit shown in the figure (substrates are connected to their respective sources). The gate width (W) to gate length (L) ratios $\left(\frac{W}{L}\right)$ of the transistors are as shown. Both the transistors have the same gate oxide capacitance per unit area. For the p MOSFET, the threshold voltage is –1 V and the mobility of holes is $40\,\frac{\text{cm}^2}{\text{V-s}}$. For the n MOSFET, the threshold voltage is 1 V the mobility of electrons is $300\,\frac{\text{cm}^2}{\text{V-s}}$. The steady state output voltage V_0 is



(A) Equal to 0 V
(B) Equal to 2 V
(C) More than 2 V
(D) Less than 2 V

Ans. **D**

Sol. **Given :**

(i) Both transistors have same gate oxide capacitance

(ii) $V_{Th_p} = -1$ V

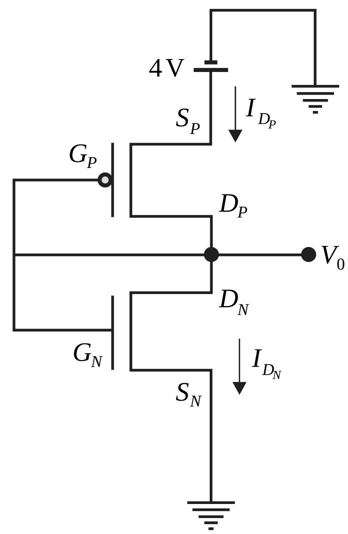
(iii) $V_{Th_N} = 1$ V

(iv) $\mu_p = 40\,\frac{\text{cm}^2}{\text{V-s}}$

(v) $\mu_n = 40\,\frac{\text{cm}^2}{\text{V-s}}$

(vi) $\left(\frac{W}{L}\right)_p = 5$

(vii) $\left(\frac{W}{L}\right)_n = 1$



For *n* MOSFET :

Condition for saturation is given by,

$$V_{DS_N} \geq V_{GS_N} - V_{Th_N}$$

$$V_{D_N} - V_{S_N} \geq V_{G_N} - V_{S_N} - V_{Th_N}$$

$$V_{D_N} \geq V_{G_N} - V_{Th_N}$$

$$V_{D_N} - V_{G_N} \geq -V_{Th_N}$$

If this inequality is satisfied then *n* MOSFET will be operate in saturation region.

From given circuit, $V_{G_N} = V_{D_N}$

So, $V_{D_N} - V_{G_N} \geq -V_{Th_N}$

$$0 \geq -1$$

Therefore *n* MOSFET is operated in saturation region.

For *p* MOSFET :

Condition for saturation is given by

$$V_{SD_p} \geq V_{SG_P} - |V_{Th_P}|$$

$$V_{S_p} - V_{D_p} \geq V_{S_p} - V_{G_p} - |V_{Th_p}|$$

$$V_{G_p} - V_{D_p} \geq -|V_{Th_p}|$$

If this inequality is satisfied then p MOSFET will be operate in saturation region.

From given circuit, $V_{G_P} = V_{D_P}$

Hence, $V_{G_P} \geq -|V_{Th_P}|$

$$0 \geq -1$$

Therefore *p* MOSFET is operated in saturation region.

We can say that, whenever gate and drain terminal of *n* MOSFET and *p* MOSFET connected together, both MOSFET will be operated in saturation region.

Since, current flow through gate terminal of MOSFETs is zero, hence both MOSFET will have same drain current.

$$I_{D_N} = I_{D_P}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{GS} - V_{Th_N})^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_{SG} - |V_{Th_P}|)^2$$

$$\frac{1}{2}\times 300\times C_{ox}\times 1(V_0 - 1) = \frac{1}{2}\times 40\times C_{ox}\times 5(4 - V_0 - 1)^2$$

$$300(V_0 - 1)^2 = 40\times 5(4 - V_0 - 1)^2$$

$$3(V_0 - 1)^2 = 2(3 - V_0)^2$$

$$3V_0^2 + 3 - 6V_0 = 18 + 2V_0^2 - 12V_0$$

$$V_0^2 + 6V_0 - 15 = 0$$

$$V_0 = \frac{-6 \pm \sqrt{36+60}}{2}$$

$$V_0 = 1.89\text{V}, \ -7.89 \text{ V}$$

Taking negative value of V_0.

$$V_0 = -7.89 \text{ V}$$

For *n* MOSFET :

$$V_0 = V_{G_N} = -7.89 \text{ V}$$

$$V_{S_N} = 0\text{V}$$

$$V_{GS_N} = V_{G_N} - V_{S_N} = V_{GS_N} = -7.89\text{V} - 0\text{V} = -7.89 \text{ V}$$

$$V_{Th_N} = 1 \text{ V}$$

$$V_{GS_N} < V_{Th_N}$$

Hence, *n* MOSFET will be OFF and drain current $I_{D_N} = 0$.

Since, drain current through *n* MOSFET is zero so we do not need to check whether *p* MOSFET is ON or OFF.

Taking positive value of V_0.

$$V_0 = 1.89 \text{ V}$$

For *n* MOSFET :

$$V_0 = V_{G_N} = 1.89 \text{ V}$$

$$V_{D_N} = 1.89 \text{ V}$$

$$V_{GS_N} = V_{G_N} - V_{S_N} = 1.89\text{V} - 0\text{V}$$

$$V_{GS_N} = 1.89 \text{ V}$$

$$V_{Th_N} = 1\text{V}$$

$$V_{GS_N} > V_{Th_N}$$

Hence, *n* MOSFET will be ON for $V_0 = 1.89\text{V}$

Now we will check n MOSFET is operate either in saturation or linear region

$$V_{G_N} = 1.89 \text{ V}$$

$$V_{D_N} = 1.89 \text{ V}$$

$$V_{S_N} = 0 \text{ V}$$

$$V_{DS_N} = V_{D_N} - V_{S_N} = 1.89\text{V} - 0\text{V} = 1.89\text{V}$$

$$V_{GS_N} = V_{G_N} - V_{S_N} = 1.89\text{V} - 0\text{V} = 1.89\text{V}$$

The condition for saturation is given by,

$$V_{DS_N} \geq V_{GS_N} - V_{Th_N}$$

$$V_{DS_N} = 1.89 \text{ V}$$

$V_{GS_N} = 1.89\text{ V}$

$1.89\text{V} \geq 1.89 - 1\text{V}$

$1.89 \geq 0.89$

Therefore, n MOSFET is operates in saturation region for $V_0 = 1.89\text{ V}$

For *p* MOSFET :

$V_0 = V_{G_P} = 1.89\text{ V}$

$V_{S_p} = 4\text{ V}$

$V_{SG_p} = V_{S_p} - V_{G_p} = 4\text{V} - 1.89\text{ V} = 2.11\text{ V}$

$V_{SG_p} = 2.11\text{ V}$

$V_{Th_p} = -1\text{ V}$

$|V_{Th_p}| = 1\text{ V}$

$V_{SF_p} > |V_{Th_p}|$

Hence, p MOSFET will be ON for $V_0 = 1.89\text{ V}$

Now we will check p MOSFET is operate either in saturation or in linear region. $V_0 = 1.89\text{ V}$

$V_{G_P} = 1.89\text{ V}$

$V_{D_P} = 1.89\text{ V}$

$V_{S_P} = 4\text{ V}$

$|V_{Th_P}| = 1\text{ V}$

$V_{SG_p} = V_{S_p} - V_{G_p} = 4\text{V} - 1.89\text{ V} = 2.11\text{ V}$

$V_{SD_p} = V_{S_p} - V_{D_p} = 4\text{V} - 1.89\text{V}$

The conditions for saturation is given by,

$V_{SD_P} \geq V_{SG_P} - |V_{Th_P}|$

$2.11\text{V} \geq 2.11\text{V} - 1\text{ V}$

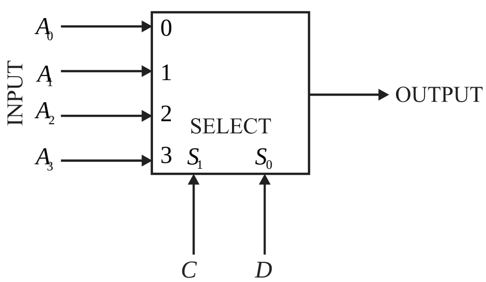
$2.11 \geq 1.11$

Therefore, p MOSFET is operates in saturation region for $V_0 = 1.89\text{ V}$.

Hence, the correct option is (D).

Question 19 **Digital Electronics**

Consider the 2-bit multiplexer (MUX) shown in the figure. For OUTPUT to be the XOR of C and D, the values for A_0, A_1, A_2, and A_3 are

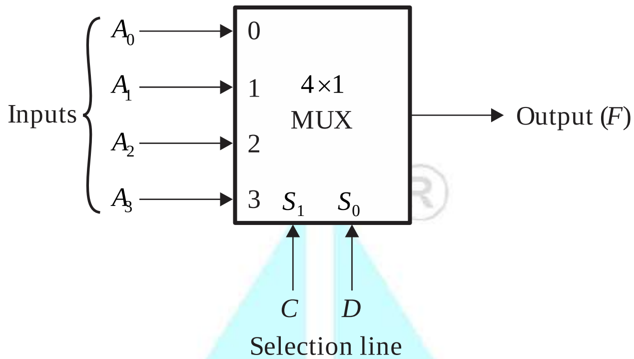


(A) $A_0 = 0,\ A_1 = 0,\ A_2 = 1,\ A_3 = 1$ (B) $A_0 = 1,\ A_1 = 1,\ A_2 = 0,\ A_3 = 0$

(C) $A_0 = 0,\ A_1 = 1,\ A_2 = 1,\ A_3 = 0$ (D) $A_0 = 1,\ A_1 = 0,\ A_2 = 1,\ A_3 = 0$

Ans. C

Sol. **Given :** A 4×1 MUX is shown below



Output (F) of given 4×1 MUX is,

$$F = \bar{S}_1\bar{S}_0A_0 + \bar{S}_1S_0A_1 + S_1\bar{S}_0A_2 + S_1S_0A_3 \quad \ldots\text{(i)}$$

Here, $S_1 = C$ and $S_0 = D$

$$F = \bar{C}\bar{D}A_0 + \bar{C}DA_1 + C\bar{D}A_2 + CDA_3$$

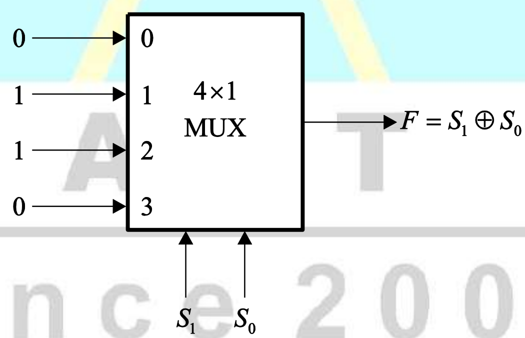
To make $F = C\oplus D = \bar{C}D + C\bar{D}$ we have to take

$A_0 = 0$, $A_1 = 1$, $A_2 = 1$ and $A_3 = 0$

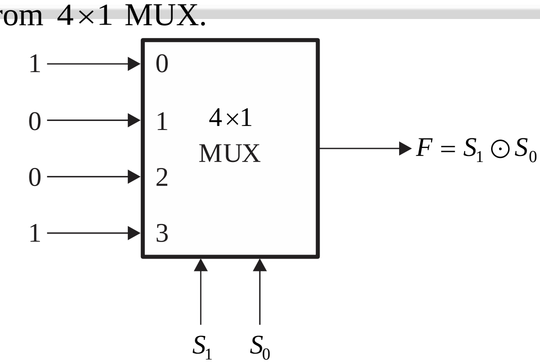
Hence, the correct option is (C).

Key Point

- To make Ex-OR GATE from 4×1 MUX

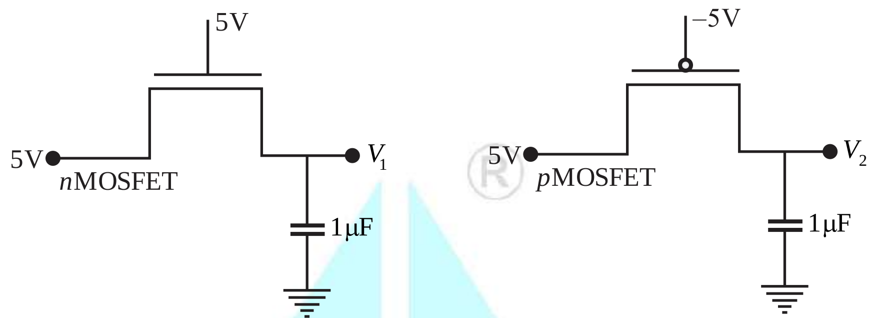


- To make Ex-OR GATE from 4×1 MUX.



Question 20 **Electronic Devices**

The ideal long channel *n* MOSFET and *p* MOSFET devices shown in the circuits have threshold voltages of 1 V and −1 V, respectively. The MOSFET substrates are connected to their respective sources. Ignore leakage currents and assume that the capacitors are initially discharged. For the applied voltages as shown, the steady state voltages are



(A) $V_1 = 5\text{V}, V_2 = 4\text{V}$ (B) $V_1 = 4\text{V}, V_2 = 5\text{V}$

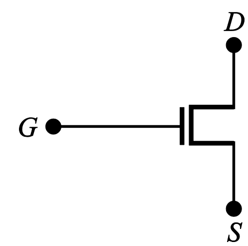
(C) $V_1 = 4\text{V}, V_2 = -5\text{V}$ (D) $V_1 = 5\text{V}, V_2 = 5\text{V}$

Ans. **B**

Sol. **Given :**

(i) $V_{Tn} = 1\text{V}$

(ii) $V_{Tp} = -1\text{V}$



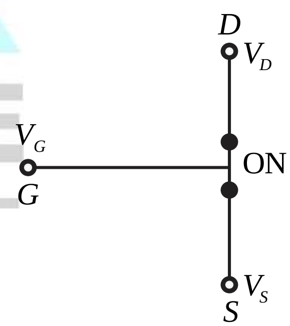
In NMOS transistor, gate voltage (V_G) work as a controlled input. When NMOS work as a switch then,

If $V_G = 0\,\text{V}$, then NMOS will be OFF and if $V_G = +5\,\text{V}$, then NMOS will be ON.

(i) For $V_{DS} < V_{GS} - V_{Tn}$

$V_D - V_S < V_G - V_S - V_{Tn}$

$V_D < V_G - V_{Tn}$



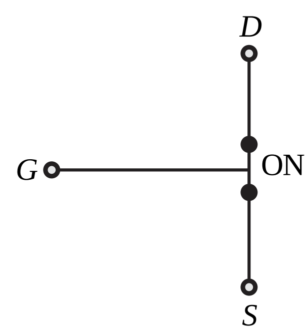
When NMOS Will satisfy above equation, then NMOS will be in linear region and hence it will be ON.

Therefore, $V_D = V_S$

(ii) For $V_{DS} \geq V_{GS} - V_{Tn}$

$V_D - V_S \geq V_G - V_S - V_{Tn}$

$V_D \geq V_G - V_{Tn}$



When NMOS will satisfy above equation, then NMOS will be in saturation region and hence it will be ON.

Therefore, $V_D = V_S = V_G - V_{Tn}$

$V_s = V_G - V_{Tn}$

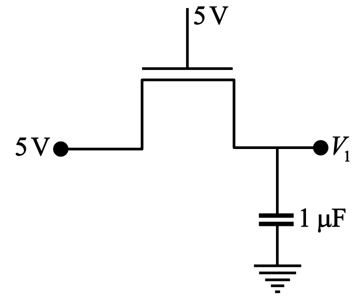
For NMOS pass transistor logic,

If $V_D - V_G \geq -V_{Tn}$

Then, $V_G - V_S = V_{Tn}$

If $V_D - V_G < -V_{Tn}$

Then, $V_S = V_D$



$V_D = 5$ V

$V_G = 5$ V

$V_S = V_1$

$V_D - V_G = 5 - 5 = 0$

$V_D - V_G \geq -V_{Tn}$

$0 \geq -1$ [True]

Hence, $V_G - V_S = V_{Tn}$

$5 - V_1 = 1$

$5 - 1 = V_1$

$V_1 = 4$ V

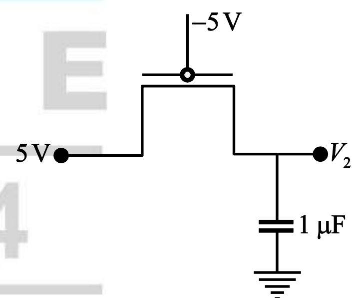
For PMOS pass transistor logic,

If $V_G - V_D \geq -|V_{Tp}|$

Then, $V_S - V_G = -|V_{Tp}|$

If $V_G - V_D < -|V_{Tp}|$

Then, $V_S = V_D$



$V_G = -5$ V

$V_D = 5$ V

$V_S = V_2$

$V_G - V_D = -5 - 5 = -10$

$V_G - V_D < -|V_{Tp}|$

$-10 < -1$ [True]

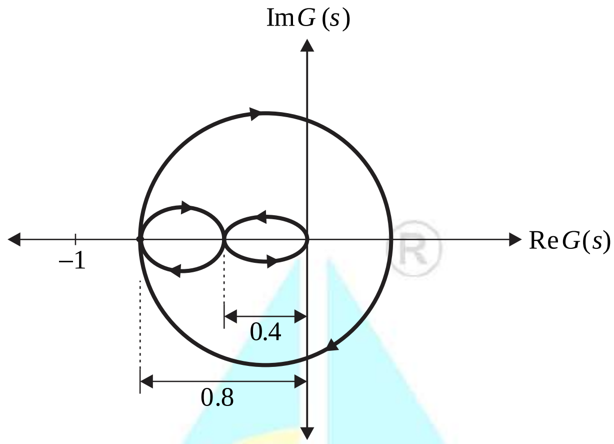
Hence, $V_S = V_D$

$V_2 = 5$ V

Hence, the correct option is (B).

Question 21 **Control Systems**

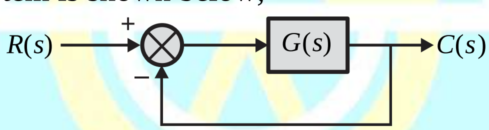
Consider a closed-loop control system with unity negative feedback and $KG(s)$ in the forward path, where the gain $K = 2$. The complete Nyquist plot of the transfer function $G(s)$ is shown in the figure. Note that the Nyquist contour has been chosen to have the clockwise sense. Assume $G(s)$ has no poles on the closed right-half of the complex plane. The number of poles of the closed-loop transfer function in the closed right-half of the complex plane is



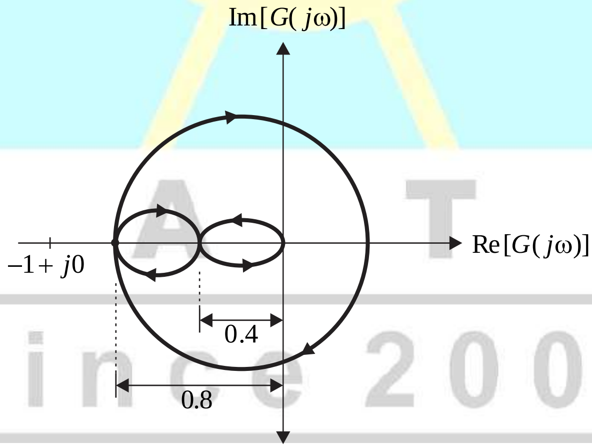
(A) 1 (B) 3
(C) 2 (D) 0

Ans. C

Sol. Given closed loop control system is shown below,

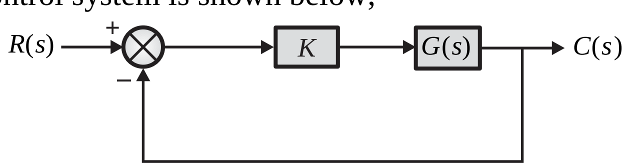


Nyquist plot of $G(s)$ is shown below,



Given, $G(s)$ has no poles in the right half of s – plane i.e. $P = 0$.

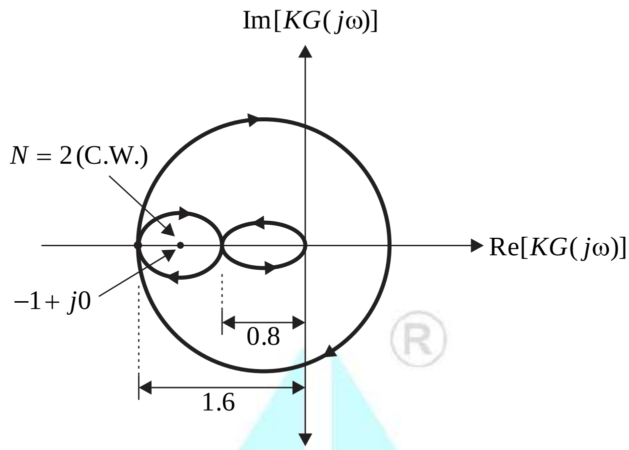
Modified closed loop control system is shown below,



Modified open loop transfer function is $KG(s)$

$K = 2$ (given)

The Nyquist plot for $K = 2$ is,



Number of anticlockwise encirclement about critical point $(-1+j0)$

$N = -2$

Number of right hand poles of open loop transfer function $P = 0$.

Nyquist stability criteria is given by,

$N = P - Z$ (A.C.W.)

$-2 = 0 - Z$

$Z = 2$

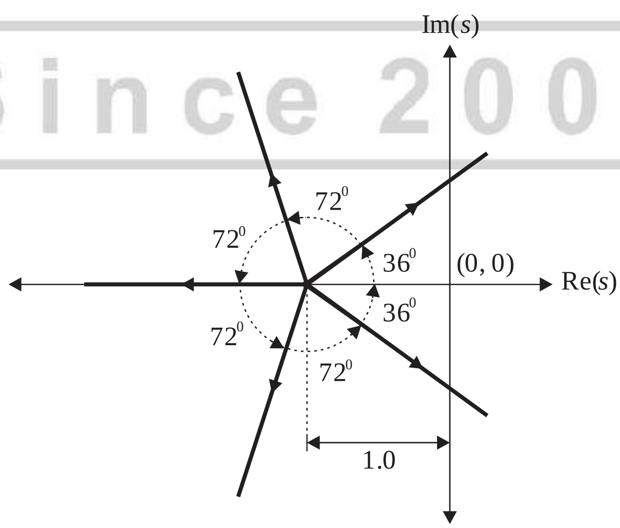
Hence, the number of closed loop poles in the right half of complex plane is 2.

Hence, the correct option is (C).

Question 22 **Control System**

The root-locus plot of a closed-loop system with unity negative feedback and transfer function $KG(s)$ in the forward path is shown in the figure. Note that K is varied from 0 to ∞.

Select the transfer function $G(s)$ that results in the root-locus plot of the closed-loop system as shown in the figure.

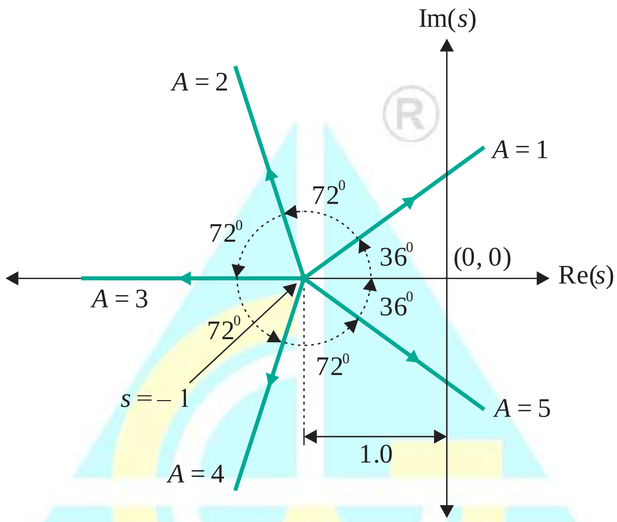


(A) $G(s)=\dfrac{s+1}{s^6+1}$ (B) $G(s)=\dfrac{s-1}{(s+1)^6}$

(C) $G(s)=\dfrac{1}{s^5+1}$ (D) $G(s)=\dfrac{1}{(s+1)^5}$

Ans. **D**

Sol. Given root locus diagram is shown below,



Form above figure, at $s=-1$, five root locus branches are originating following the asymptotic angles $36^0, 108^0, 180^0, 252^0, 324^0$.

Note : Angle of asymptotes for root locus, $\angle A=\dfrac{(2\alpha+1)\times 180^0}{5}$

$\alpha=0, 1, 2, 3, 4$

$\angle A=36^0(2\alpha+1)=36^0, 108^0, 180^0, 252^0, 324^0$

Hence, there will be five poles at $s=-1$

Since, all the five root locus branches are terminating at infinity, hence there will be five virtual zeros i.e. no physical zero.

Therefore OLTF is given by,

$$G(s)=\frac{1}{(s+1)^5}$$

Hence, the correct option is (D).

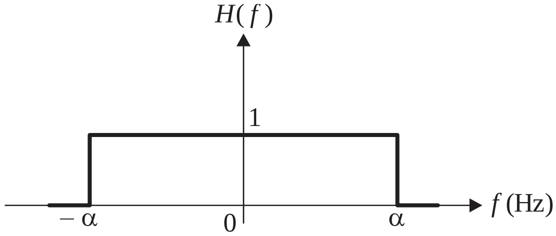
Question 23 **Communication**

The frequency response $H(f)$ of a linear time-invariant system has magnitude as shown in the figure.

Statement I : The system is necessarily a pure delay system for inputs which are bandlimited to $-\alpha \le f \le \alpha$.

Statement II : For any wide-sense stationary input process with power spectral density $S_X(f)$, the output power spectral density $S_Y(f)$ obeys $S_Y(f)=S_X(f)$ for $-\alpha \le f \le \alpha$.

Which one of the following combinations is true?



(A) Statement I is incorrect, Statement II is incorrect

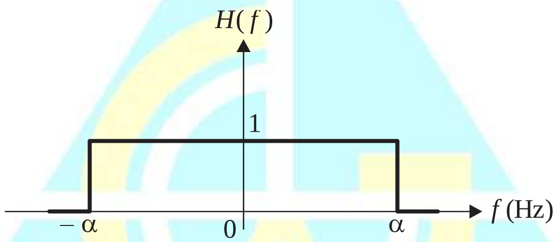
(B) Statement I is incorrect, Statement II is correct

(C) Statement I is correct, Statement II is incorrect

(D) Statement I is correct, Statement II is correct

Ans. **B**

Sol. Given frequency response is shown below



Let $x(t)$ be the input and $y(t)$ be the output of a purely delay system.

So, $y(t) = x(t - t_0)$ where t_0 is the time delay.

Taking Fourier transform on both sides we get.

$$Y(\omega) = e^{-j\omega t_0} X(\omega)$$

$$\frac{Y(\omega)}{X(\omega)} = H(\omega) = e^{-j\omega t_0}$$

$$|H(\omega)| = |e^{-j\omega t_0}| = 1$$

$|H(f)| = 1$ for all frequencies $\quad \ldots$(i)

From the given frequency response it can be concluded that

$|H(f)| = 1$ for $-\alpha \le f \le \alpha \quad \ldots$(ii)

$\quad = 0$ otherwise

On comparing (i) and (ii) we can conclude that the given system will not act as a purely delay system.

Thus, statement 1 is incorrect.

The output power spectral density of an L.T.I. system is related to input power spectral density as

$$S_Y(f) = |H(f)|^2 S_X(f)$$

But $|H(f)|=1$ for $-\alpha \le f \le \alpha$

$S_Y(f)=(1)^2 \times S_X(f)$ for $-\alpha \le f \le \alpha$

$S_Y(f)=S_X(f)$ for $-\alpha \le f \le \alpha$

So, statement 2 is correct.

Hence, the correct option is (B).

Question 24 **Electromagnetic Theory**

In a circuit, there is a series connection of an ideal resistor and an ideal capacitor. The conduction current (in Amperes) through the resistor is $2\sin\left(t+\frac{\pi}{2}\right)$. The displacement current (in Amperes) through the capacitor is

(A) 0 (B) $2\sin(t)$

(C) $2\sin(t+\pi)$ (D) $2\sin\left(t+\frac{\pi}{2}\right)$

Ans. **D**

Sol. **Given :** $I_R(t)=2\sin\left(t+\frac{\pi}{2}\right)$

Total current in capacitor, $I=I_C+I_D$

Where, outside capacitor plates we have only conduction current I_C and no displacement current. On the other hand inside the capacitor there is no conduction current i.e. $I_C=0$ and there is only displacement current.

So, $I_{conduction}=I_{displacement}$

$\because$ $I_D=\varepsilon\frac{d\phi_E}{dt}$ and $\phi_E=EA$

$I_D=\varepsilon A\frac{dE}{dt}$

$\because$ $E=\frac{\sigma}{\varepsilon_0}$ [For parallel plate capacitor]

$I_D=A\frac{d\sigma}{dt}$ $\because$ $\sigma=\frac{Q}{A}$

$I_D=\frac{dQ}{dt}$

$I_D=I_{resistor}$

So, they will be in-phase, resistor and capacitor are in series so both current will be equal.

Hence, the correct option is (D).

Question 25 **(MSQ)** **Mathematics**

Consider the following partial differential equation (PDE)

$$a\frac{\partial^2 f(x,y)}{\partial x^2}+b\frac{\partial^2 f(x,y)}{\partial y^2}=f(x,y),$$

where a and b are distinct positive real numbers. Select the combination(s) of values of real parameters ε and η such that $f(x,y)=e^{(\varepsilon x+\eta y)}$ is a solution of the given PDE.

(A) $\varepsilon=\frac{1}{\sqrt{a}},\ \eta=0$

(B) $\varepsilon=0,\ \eta=0$

(C) $\varepsilon=\frac{1}{\sqrt{2a}},\ \eta=\frac{1}{\sqrt{2b}}$

(D) $\varepsilon=\frac{1}{\sqrt{a}},\ \eta=\frac{1}{\sqrt{b}}$

Ans. **A, C**

Sol. Given partial differential equation (PDE) is,

$$a\frac{\partial^2 f(x,y)}{\partial x^2}+b\frac{\partial^2 f(x,y)}{\partial y^2}=f(x,y) \qquad \ldots\text{(i)}$$

$$f(x,y)=e^{(\varepsilon x+\eta y)} \qquad \ldots\text{(ii)}$$

Differentiating two-times equation (ii) with respect to x,

$$\frac{\partial f(x,\ y)}{\partial x}=\varepsilon e^{(\varepsilon x+\eta y)}$$

$$\frac{\partial^2 f(x,\ y)}{\partial x^2}=\varepsilon^2 e^{(\varepsilon x+\eta y)}$$

Multiply a on both sides,

$$a\frac{\partial^2 f(x,\ y)}{\partial x^2}=a\varepsilon^2 e^{(\varepsilon x+\eta y)} \qquad \ldots\text{(iii)}$$

Differentiating two-times equation (ii) with respect to y,

$$\frac{\partial f(x,\ y)}{\partial y}=\eta e^{(\varepsilon x+\eta y)}$$

$$\frac{\partial^2 f(x,\ y)}{\partial y^2}=\eta^2 e^{(\varepsilon x+\eta y)}$$

Multiply b on both sides,

$$b\frac{\partial^2 f(x,\ y)}{\partial y^2}=b\eta^2 e^{(\varepsilon x+\eta y)} \qquad \ldots\text{(iv)}$$

Adding equation (iii) and (iv),

$$a\frac{\partial^2 f(x,y)}{\partial x^2}+b\frac{\partial^2 f(x,y)}{\partial y^2}=(a\varepsilon^2+b\eta^2)e^{(\varepsilon x+\eta y)}$$

$$a\frac{\partial^2 f(x,y)}{\partial x^2}+b\frac{\partial^2 f(x,y)}{\partial y^2}=(a\varepsilon^2+b\eta^2)f(x,\ y) \qquad \ldots\text{(v)}$$

Compare equation (i) and (v),

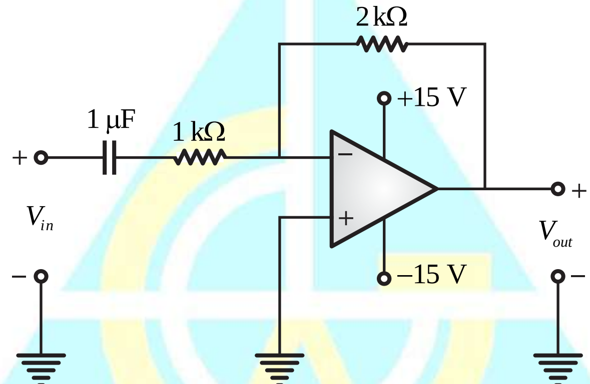
$$a\varepsilon^2 + b\eta^2 = 1 \qquad \ldots(vi)$$

Equation (vi) must be satisfied if $\varepsilon = \frac{1}{\sqrt{a}}, \eta = 0$ and $\varepsilon = \frac{1}{\sqrt{2a}}, \eta = \frac{1}{\sqrt{2b}}$.

Hence, the correct options are (A) & (C).

Question 26 (MSQ) **Analog Electronics**

An ideal OPAMP circuit with a sinusoidal input is shown in the figure. The 3 dB frequency is the frequency at which the magnitude of the voltage gain decreases by 3 dB from the maximum value. Which of the options is/are correct?



(A) The circuit is a low pass filter

(B) The 3-dB frequency is $\frac{1000}{3}$ rad/s

(C) The 3-dB frequency is 1000 rad/s

(D) The circuit is a high pass filter

Ans. **C, D**

Sol. Given circuit is shown below,

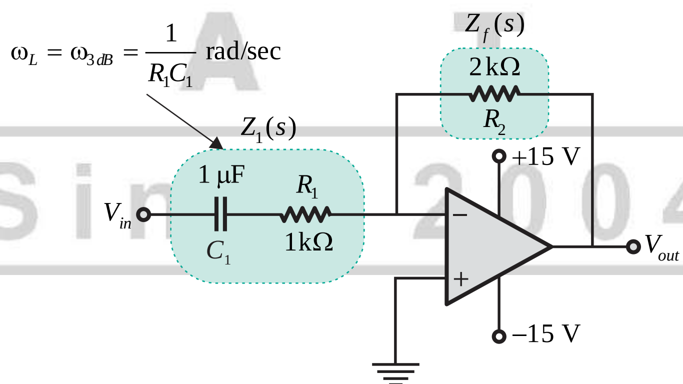


Fig. Practical differentiator circuit

The transfer function of above circuit,

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{-Z_f(s)}{Z_1(s)} = \frac{-R_2}{R_1 + \frac{1}{C_1 s}} = \frac{-sC_1R_2}{1 + sC_1R_1}$$

$$T(s)=\left|\frac{V_{out}(s)}{V_{in}(s)}\right|=\frac{Z_f(s)}{Z_1(s)}=\frac{sC_1R_2}{1+sC_1R_1}$$

Put $s=j\omega$,

$$T(j\omega)=\frac{j\omega C_1R_2}{1+j\omega C_1R_1}$$

At $\omega=0,\ T(0)=0$

At $\omega=\infty,\ T(\infty)=\frac{R_2}{R_1}$

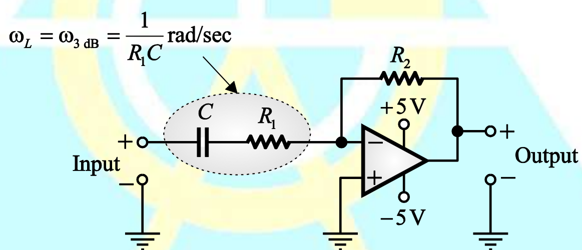
So, circuit behave as HPF, because it passes only high frequency and block low frequency.

The 3 dB frequency of practical differentiator circuit is,

$$\omega_{3\text{ dB}}=\frac{1}{R_1C_1}=\frac{1}{1000\times 1\mu\text{F}}=1000\text{ rad/sec}$$

Hence, the correct options are (C) & (D).

Key Point



This circuit represent practical differentiator circuit which will act as a high pass filter.

Question 27 (MSQ) **Digital Electronics**

Select the Boolean function(s) equivalent to $x+yz$, where $x, y,$ and z are Boolean variables, and '$+$' denotes logical OR operation.

(A) $x+xz+xy$ (B) $x+z+xy$

(C) $(x+y)(x+z)$ (D) $x+xy+yz$

Ans. **C, D**

Sol. **Given :** $(x+yz)$

Method 1

Choosing from options :

From option (A) :

$$x+xz+xy$$

$$x(1+z+y) \qquad (\because 1+\text{Anything}=1)$$

So, option (A) is incorrect.

From option (B) :

$$x+z+xy$$

$$x(1+y)+z \qquad (\because 1+\text{Anything}=1)$$

$$x+z$$

So, option (B) is incorrect.

From option (C) :

$$(x+y)(x+z)$$

$$x+xz+yx+yz$$

$$x(1+z+y) \qquad (\because 1+\text{Anything}=1)$$

$$x+yz$$

So, option (C) is correct.

From option (D) :

$$x+xy+yz$$

$$x(1+y)+yz$$

$$x+yz \qquad (\because 1+\text{Anything}=1)$$

So, option (D) is correct.

Hence, the correct options are (C) & (D).

Method 2

Given : $(x+yz)$ ...(i)

From Boolean algebra, $A+BC=(A+B)(A+C)$

So, equation (i) can write as,

$$x+yz=(x+y)(x+z)$$

Again from Boolean algebra, $A+BC=A+AB+BC$

So, equation (i) can write as,

$$x+yz=x+xy+yz$$

Hence, the correct options are (C) & (D).

Question 28 (MSQ) **Electronic Devices**

Select the correct statement(s) regarding CMOS implementation of NOT gates.

(A) Noise Margin High (NM_H) is always equal to the Noise Margin Low (NM_L) irrespective of the sizing of transistors.

(B) Mobility of electrons never influences the switching speed of the NOT gate.

(C) For a logical high input under steady state, the n MOSFET is in the linear regime of operation.

(D) Dynamic power consumption during switching is zero.

Ans. C

Sol. **Given :**

For option (A) :

Noise margin high (NM_H) and Noise margin low (NM_L) is depends on the following parameters of MOSFET.

(i) Size of n MOSFET and p MOSFET

(ii) Threshold voltage of n MOSFET and p MOSFET.

Noise Margin High is equal to Noise Margin Low if

(i) $\left(\frac{W}{L}\right)_n = \left(\frac{W}{L}\right)_p$

(ii) $V_{Tn} = V_{Tp}$

If

(i) $\left(\frac{W}{L}\right)_n \neq \left(\frac{W}{L}\right)_p$

(ii) $V_{Tn} \neq V_{Tp}$ then $NM_H \neq NM_L$.

So, option (A) is incorrect.

For option (B) :

Mobility is the ability of charge carrier to move freely or be easily moved.

The mobility of electrons influences the switching speed because propagation delay (τ) depends on mobility.

Propagation delay, $\tau = \frac{\tau_{PLH} + \tau_{PHL}}{2}$

Where, $\tau_{PLH} = \frac{C_L V_{DD}}{\mu_p C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{Tp})^2}$

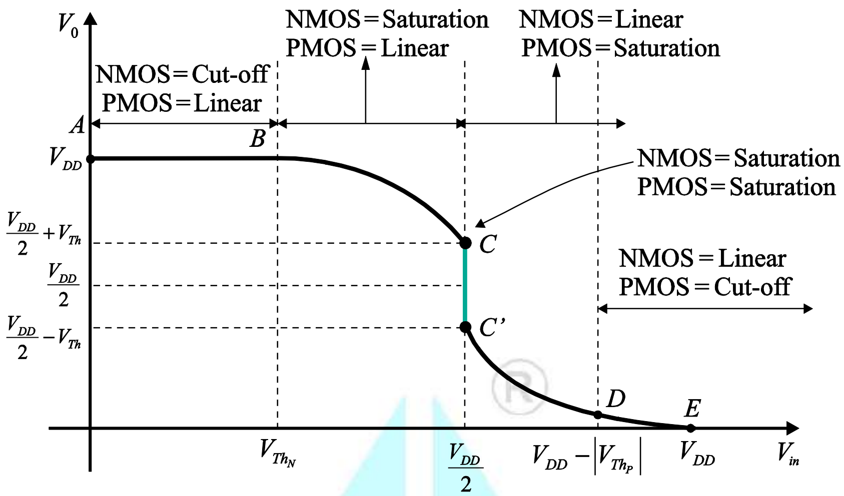
$$\tau_{PHL} = \frac{C_L V_{DD}}{\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{Tn})^2}$$

If mobility of electrons (μ_n) increases, τ_{PHL} decreases. Therefore propagation delay (τ) decrease.

So, option (B) is incorrect.

For option (C) :

The transfer characteristic of practical CMOS inverter is given by,



Region	NMOS	PMOS	V_{in}	V_0
AB	Cut-off	Linear	$0<V_{in}<V_{Th_N}$	$V_0=V_{DD}$
BC	Saturation	Linear	$V_{Th_N}<V_{in}<\frac{V_{DD}}{2}$	$\frac{V_{DD}}{2}<V_0<V_{DD}$
CC'	Saturation	Saturation	$V_{in}=\frac{V_{DD}}{2}$	$V_0=\frac{V_{DD}}{2}$
C'D	Linear	Saturation	$\frac{V_{DD}}{2}<V_{in}<V_{DD}-\lvert V_{Th_p}\rvert$	$V_0<\frac{V_{DD}}{2}$
DE	Linear	Cut-off	$V_{in}>V_{DD}-\lvert V_{Th_p}\rvert$	$V_0\approx 0\text{ V}$

For a logical high input under steady state, the *n* MOSFET is in the linear regime of operation.

So, option (C) is correct.

For option (D) :

Power dissipation in CMOS inverter :

(i) Static power :

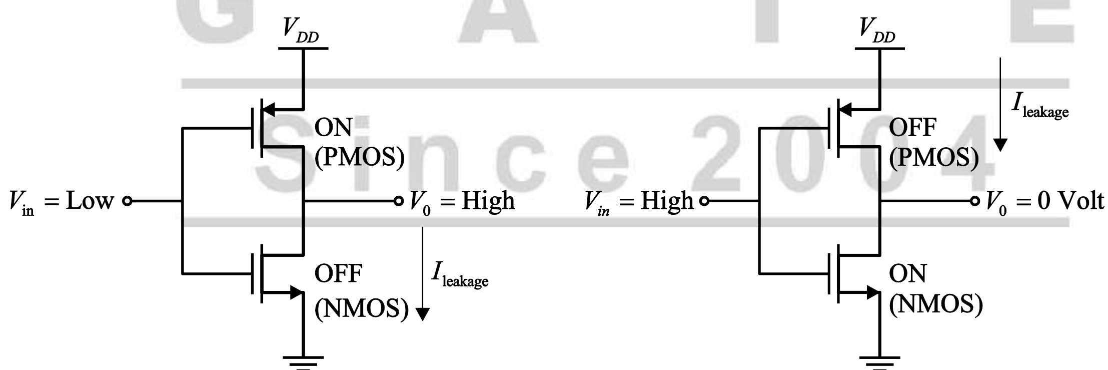


Fig. (a) **Fig. (b)**

(a) Static power exists due to leakage current in stable state or steady state of CMOS inverter.

(b) $P_{D(\text{static})}=V_{DD}\times I_{\text{leakage}}$

where, $I_{\text{leakage}}=I_{\text{leakage(NMOS)}}+I_{\text{leakage(PMOS)}}$

(c) Static power is the power consumed by the MOSFET during stable state i.e. it is the power consumed by MOSFET during continuously ON at logic 1 and continuously OFF at logic 0.

(ii) Dynamic capacitive power :

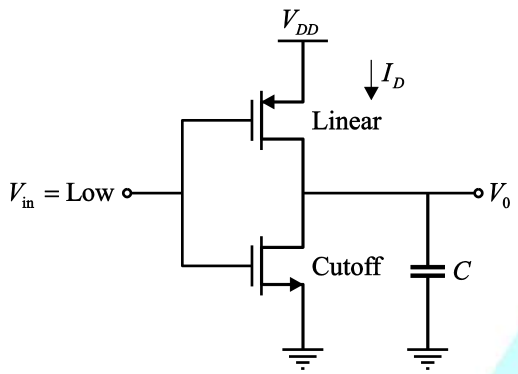


Fig. (a)

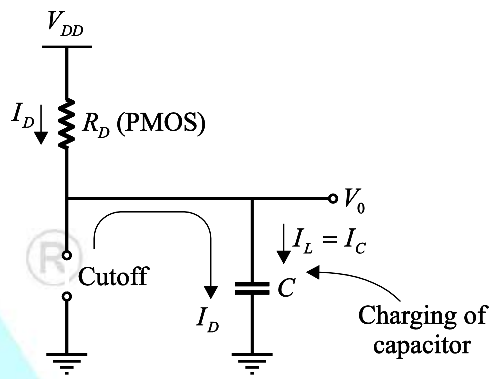


Fig. (b)

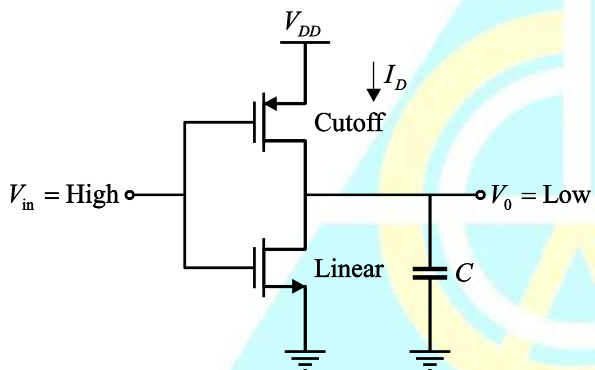


Fig. (c)

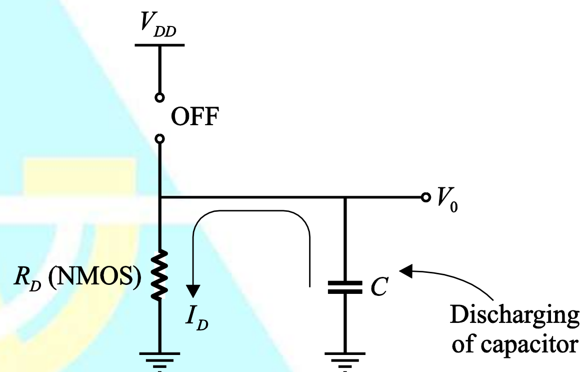


Fig. (d)

(a) It is the power consumed by the MOSFET during its transition state i.e. power consumed by the MOSFET during logic 1 to logic 0 or logic 0 to logic 1.

(b) It depends on charging and discharging of capacitive load.

(c) It is also referred as switching power dissipation.

(d) Energy stored in PMOS, $E_{D(\text{PMOS})} = \frac{1}{2} C V_{DD}^2$

Energy stored in NMOS, $E_{D(\text{NMOS})} = \frac{1}{2} C V_{DD}^2$

Energy stored in CMOS, $E_{D(\text{CMOS})} = E_{D(\text{NMOS})} + E_{D(\text{PMOS})} = C V_{DD}^2$

Dynamic power dissipation of CMOS inverter,

$$P_{\text{dynamic(CMOS)}} = \frac{E_{D(\text{CMOS})}}{T}$$

$$P_{\text{dynamic(CMOS)}} = f_{SW} C V_{DD}^2$$

f_{SW} = Switching frequency

$f_{SW} = \alpha f$

Where, $\alpha =$ Activity factor $(0 < \alpha < 1)$ and $f =$ operating frequency.

If α is not mentioned then we assume $\alpha = 1$.
Therefore, dynamic power consumption during switching is non-zero.
So, option (D) is incorrect.
Hence, the correct option is (C).

Question 29 (MSQ) **Communication**

Let $H(X)$ denote the entropy of a discrete random variable X taking K possible distinct real values. Which of the following statements is/are necessarily true?

(A) $H(X) \le \log_2 K$ bits (B) $H(X) \le H(2^X)$

(C) $H(X) \le H(2X)$ (D) $H(X) \le H(X^2)$

Ans. **A, B, C**

Sol. **Given :**

$H(X)$ is entropy of a discrete random variable X taking K possible distinct real values.

Let variable X is taking values as x_i so set of possible values is $\{x_1, x_2, x_3 .... x_k\}$.

The entropy will be, $H(X) = \sum_{i=1}^{k} P(x_i) \log \frac{1}{P(x_i)}$

Note : Base 2 of log gives the unit of "bits" or "Shannon's", base e gives natural unit "nat" and base 10 gives unit of "dits" or "Hartley's".

Case-1 : When all values are equiprobable i.e. $P(x_i) = \frac{1}{K}$ for each distinct 'K' values, then entropy will be given as,

$$H(X) = \frac{1}{K}\log K + \frac{1}{K}\log K .... \ K \text{ times}$$

$$H(X) = \frac{K}{K}\log K$$

$$H(X) = \log K$$

When we choose base as 2,

$$H(X) = \log_2 K \text{ bits}$$

We know that, in case of equal probability the entropy will be highest,

So, $H(X) \le \log_2 K$ is always true for any base value.

Hence, option **(A)** is correct.

Case-2 : Assuming a new random variable $Y = 2X$ which is mapped by random variable X as shown below,

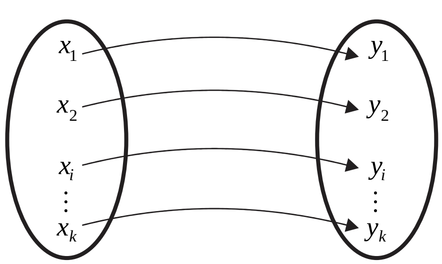


Figure: Mapping for $Y = 2X$

So, mapping will be one to one as we have linear relation between X and Y.

The random variable $X=\{x_1, x_2, x_3 ....x_k\}$ in the same way random variable Y $Y=\{y_1=2x_1, y_2=2x_2, ... y_K=2x_K\}$

Along with values the probability of occurring each value will also be mapped and hence x_i and corresponding y_i will have identical probability, i. e. $P(x_i)=P(y_i)$

We can say that $P(x_1)=P(y_1), P(x_2)=P(y_2) .... P(x_K)=P(y_K)$

Probability of random variable X and Y are same, because one to one mapping therefore, entropy of random variable X and Y are same. i. e. $H(Y=2X)=H(X)$

So we will never have situation of $H(X)<H(2X)$

Hence, option **(B)** is correct

Case-3 : Assume new random variable $Y=2^X$, This $Y=2^X$ will give one to one mapping because different values of X provide different values of Y so that probability of random variable Y is same as probabilities of random variable X so that entropy

$H(Y=2^X)=H(X)$

So, $H(X)<H(2^X)$ never occur

So, option **(C)** is also correct.

Case-4 : Assuming new random variable $Y=X^2$.

Possibility-01 : Suppose we have 3 positive values of random variable X is $x_1=1$, $x_2=2$, $x_3=3$ and assuming corresponding probabilities of x_1, x_2, x_3 are $\frac{1}{5}, \frac{1}{2}, \frac{3}{10}$ respectively

i.e. $X=\{1, \quad 2, \quad 3\}$

Probability, $P(X)=\left\{\frac{1}{5}, \quad \frac{1}{2}, \quad \frac{3}{10}\right\}$

Thus, random variable, $Y=X^2=\{1,4,9\}$

Here we can say different values of X gives different values of Y, it shows one to one mapping is possible between X and Y.

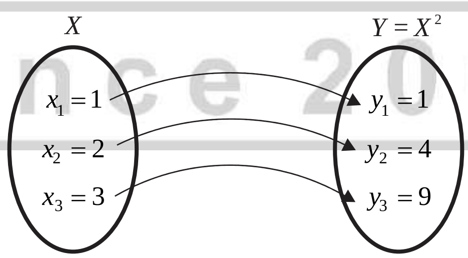


Fig. One to one mapping

So, probabilities of random variable Y is same as probability of random variable X.

It means, $P(Y)=\left\{\frac{1}{5}, \quad \frac{1}{2}, \quad \frac{3}{10}\right\}=P(X)$

Thus we can say that entropy of $H(X)=H(Y=X^2)$

Possibility-02 : Suppose we have 3 negative values of random variable X is $x_1 = -1$, $x_2 = -2$, $x_3 = -3$ and assuming corresponding probabilities of x_1, x_2, x_3 are $\frac{1}{5}, \frac{1}{2}, \frac{3}{10}$ respectively

i.e. $X = \{-1, \quad -2, \quad -3\}$

Probability, $P(X) = \left\{\frac{1}{5}, \quad \frac{1}{2}, \quad \frac{3}{10}\right\}$

Thus, random variable, $Y = X^2 = \{1, 4, 9\}$

Here we can say different values of *X* gives different values of *Y*, it shows one to one mapping is possible between *X* and *Y*.

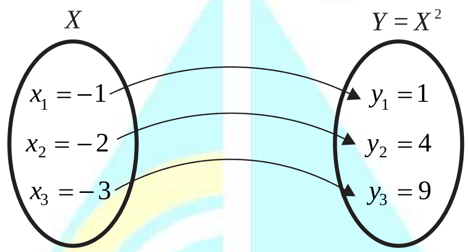


Fig. One to one mapping $Y = X^2$

So, probabilities of random variable *Y* is same as probability of random variable *X*,

It means, $P(Y) = \left\{\frac{1}{5}, \quad \frac{1}{2}, \quad \frac{3}{10}\right\} = P(X)$

Thus we can say that entropy of $H(X) = H(Y = X^2)$.

Possibility-03 : Suppose we have both positive and negative values of random varible X is $x_1 = -1$, $x_2 = 1$, $x_3 = 2$ and assuming corresponding probabilities of x_1, x_2 and x_3 are $\frac{1}{5}$, $\frac{1}{2}$ and $\frac{3}{10}$.

i.e. $X = \{-1, \quad 1, \quad 2\}$

Probability of *X*, $P(X) = \left\{\frac{1}{5}, \quad \frac{1}{2}, \quad \frac{3}{10}\right\}$

Thus random variable, $Y = X^2 = \{1, \quad 1, \quad 4\}$

Here we see different values of X gives same value of Y, it show one to one mapping is not possible here.

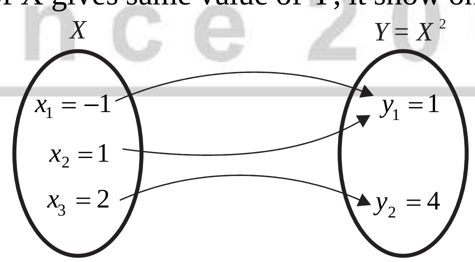


Fig. Mapping of $Y = X^2$

So, random variable *Y* have only 2-values i.e. $Y = [1, 4]$

So, probability of $P(y_1 = 1)$ is sum of probability of $P(x_1 = -1)$ and $P(x_2 = 1)$ so, $P(y_1 = 1) = \frac{1}{5} + \frac{1}{2} = \frac{7}{10}$

Probability of $P(y_2=4)$ remains same as probability of $P(x_3=2)$.

i.e. $P(y_2=4)=P(x_3=2)=\frac{3}{10}$

So entropy of $H(Y)$ is

$$H(Y)=P(y_1=1)\log_2\frac{1}{P(y_1=1)}+P(y_2=4)\log_2\frac{1}{P(y_2=4)}$$

$$=\frac{7}{10}\log_2\frac{10}{7}+\frac{3}{10}\log_2\frac{10}{3}=0.265$$

Entropy of $H(X)$ is,

$$=P(x_1=-1)\log_2\frac{1}{P(x_1=-1)}+P(x_2=1)\log_2\frac{1}{P(x_2=1)}+P(x_3=2)\log_2\frac{1}{P(x_3=2)}$$

$$=\frac{1}{5}\log_2 5+\frac{1}{2}\log_2 2+\frac{3}{10}\log_2\frac{10}{3}$$

$$=0.4643+0.5+0.521089$$

$$=1.4854$$

From Case-4, it is clear that $H(X)=H(X)^2$ is possible only when random variable X have all positive or negative values and X can take combination of positive and negative values then $H(X)>H(X^2)$ that why option (D) is incorrect.

Hence, the correct options are (A), (B) & (C).

Question 30 (MSQ) **Electromagnetic Theory**

Consider the following the wave equation,

$$\frac{\partial^2 f(x,t)}{\partial t^2}=10000\frac{\partial^2 f(x,t)}{\partial x^2}$$

Which of the given options is/are solution(s) to the given wave equation?

(A) $f(x,t)=e^{-(x-100t)^2}+e^{-(x+100t)^2}$ (B) $f(x,t)=e^{-(x-100t)}+\sin(x+100t)$

(C) $f(x,t)=e^{-(x-100t)}+0.5e^{-(x+1000t)}$ (D) $f(x,t)=e^{j100\pi(-100x+t)}+e^{j100\pi(100x+t)}$

Ans. **A, B**

Sol. Given differential form of wave equation,

$$\frac{\partial^2 f(x,t)}{\partial t^2}=10000\frac{\partial^2 f(x,t)}{\partial x^2}$$

where $f(x,t)$ is associated electric or magnetic field associated with wave $E(x,t)$ or $H(x,t)$ and wave is traveling in x direction.

Given options are the wave equation in general solution form, would satisfy its partial differential equation.

Choosing from options :

From Option (A) : $\dfrac{\partial^2 f(x,t)}{\partial t^2} = 10000 \dfrac{\partial^2 f}{\partial x^2}$

From Option (B) : $\dfrac{\partial^2 f(x,t)}{\partial t^2} = 1000 \dfrac{\partial^2 f}{\partial x^2}$

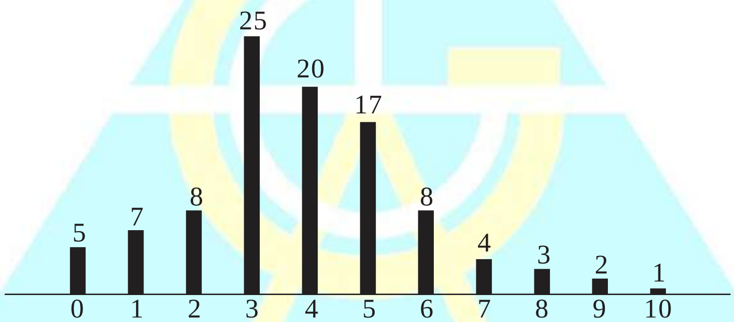
From Option (C) : $\dfrac{\partial^2 f(x,t)}{\partial t^2} \neq 1000 \dfrac{\partial^2 f}{\partial x^2}$

From Option (D) : $\dfrac{\partial^2 f(x,t)}{\partial t^2} \neq 1000 \dfrac{\partial^2 f}{\partial x^2}$

Hence, the correct options are (A) & (B).

Question 31 **Mathematics**

The bar graph shows the frequency of the number of wickets taken in a match by a bowler in her career. For example, in 17 of her matches, the bowler has taken 5 wickets each. The median number of wickets taken by the bowler in a match is __________ (rounded off to one decimal place).



Number of wickets in a match

Ans. **4 (Range : 4.0 to 4.0)**

Sol. **Given :**

Number of wickets in ascending order :

0, 0, 0, 0, 0, 1, 1 .... 9, 9, 10

Total number of data = 100

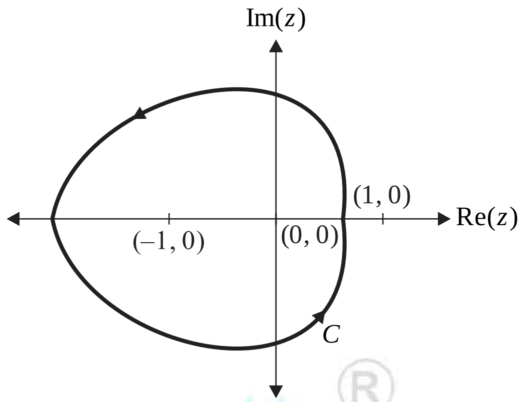
$$\text{Median} = \frac{50^{th}\text{ data} + 51^{th}\text{ data}}{2}$$

$$\text{Median} = \frac{4+4}{2} = 4$$

Hence, the correct answer is 4.

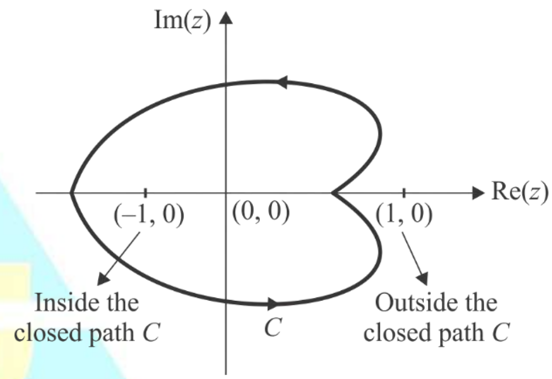
Question 32 **Mathematics**

A simple closed path C in the complex plane is shown in figure. If $\oint_C \dfrac{2^z}{z^2-1} dz = -i\pi A$, where $i = \sqrt{-1}$, then the value of A is ________. (rounded off to two decimal places)



Ans. **0.5 (Range : 0.50 to 0.50)**

Sol. **Given :** Complex integral,



$$I=\oint_C \frac{2^z}{(z^2-1)}dz=-i\pi A$$

$$I=\oint_C \frac{2^z}{(z-1)(z+1)}dz=-i\pi A$$

Poles are, $(z-1)(z+1)=0$

$$z=1,\ z=-1$$

Only $z=-1$ is enclosed.

$$\text{Re}(z=a)=\lim_{z\to a} f(z)(z-a)$$

$$\text{Re}(z=-1)=\lim_{z\to -1}\frac{2^z}{(z-1)(z+1)}(z+1)$$

$$\text{Re}(z=-1)=\lim_{z\to -1}\frac{2^z}{z-1}=\frac{2^{-1}}{-1-1}=\frac{1/2}{-2}=-\frac{1}{4}$$

$$I=2\pi i\times \text{Re}(z=-1)=-i\pi A$$

$$I=2\pi i\times\left(-\frac{1}{4}\right)=-i\pi A$$

So, $A=\frac{1}{2}=0.5$

Hence, the correct answer is 0.5.

Question 33 **Signals & Systems**

Let $x_1(t)=e^{-t}u(t)$ and $x_2(t)=u(t)-u(t-2)$, where $u(\cdot)$ denotes the unit step function.

If $y(t)$ denotes the convolution of $x_1(t)$ and $x_2(t)$, then $\lim_{t\to\infty} y(t)=$________. (rounded off to one decimal place)

Ans. **0 (Range : 0.0 to 0.0)**

Sol. **Given :**

Signal $x_1(t) = e^{-t}u(t)$

$x_2(t) = u(t) - u(t-2)$

Method 1

$y(t) = x_1(t) \otimes x_2(t)$

Taking Laplace transform on both sides,

$$X_1(s) = \frac{1}{s+1}; \qquad \text{Re}(s) > 0$$

$$X_2(s) = \frac{1-e^{-2s}}{s}; \qquad \text{Re}(s) > \text{Entire s-plane}$$

$$Y(s) = X_1(s)X_2(s); \qquad \text{Re}(s) > 0$$

$$Y(s) = \frac{1}{s+1} \times \frac{1-e^{-2s}}{s}$$

$$Y(s) = \frac{1-e^{-2s}}{s(s+1)}$$

Applying final value theorem,

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$$

$$\lim_{t\to\infty} y(t) = \frac{s(1-e^{-2s})}{s(s+1)}$$

$$\lim_{t\to\infty} y(t) = \frac{1-1}{1} = 0$$

Hence, the correct answer is 0.

Key Point

If $x_1(t) \otimes x_2(t) \xrightarrow{LT} X_1(s)X_2(s)$ only when common ROC will be exist.

Method 2

$x_1(t) = e^{-t}u(t)$

$x_2(t) = u(t) - u(t-2)$

$y(t) = x_1(t) \otimes x_2(t)$

$y(t) = e^{-t}u(t) \otimes [u(t) - u(t-2)]$

$y(t) = e^{-t}u(t) \otimes u(t) - e^{-t}u(t) \otimes u(t-2)$

$$y(t) = \frac{(1-e^{-t})}{1}u(t) - \frac{(1-e^{-(t-2)})}{1}u(t-2)$$

$$\lim_{t\to\infty} y(t) = (1-e^{-\infty})u(\infty) - (1-e^{-(\infty-2)})u(\infty-2)$$

$$\lim_{t\to\infty} y(t) = (1-0)\times 1 - (1-0)\times 1 = 1-1 = 0$$

Hence, the correct answer is 0.

Key Point

If $a > 0$, then

(i) $e^{-at}u(t) \otimes u(t) = \dfrac{(1-e^{-at})}{a} u(t)$

(ii) $e^{-at}u(t) \otimes u(t-t_0) = \dfrac{(1-e^{-a(t-t_0)})}{a} u(t-t_0)$.

Question 34 **Electronic Devices**

An ideal MOS capacitor (p-type semiconductor) is shown in the figure. The MOS capacitor is under strong inversion with $V_G = 2\text{ V}$. The corresponding inversion charge density (Q_{IN}) is $2.2\,\mu\text{C/cm}^2$. Assume oxide capacitance per unit area as $C_{OX} = 1.7\,\mu\text{F/cm}^2$. For $V_G = 4\text{ V}$, the value of Q_{IN} is________ $\mu\text{C/cm}^2$. (rounded off to one decimal place)

Ans. **5.6 (Range : 5.5 to 5.7)**

Sol. **Given :**

(i) $V_{G1} = 2\text{ V}$

(ii) Inversion charge density $(Q_{IN})_1 = 2.2\ \mu\text{C/cm}^2$

(iii) $V_{G2} = 4\text{ V}$

(iv) Oxide capacitance per unit area, $C_{ox} = 1.7\ \mu\text{F/cm}^2$

(v) We have to find inversion charge density $(Q_{IN})_2$ at $V_{G2} = 4\text{ V}$

The magnitude of inversion charge density is given by,

$$|Q_{IN}| = C_{ox}(V_G - V_T)$$

$$(Q_{IN})_1 = C_{ox}(V_{G_1} - V_T)$$

$$2.2 = 1.7(2 - V_T)$$

$$V_T = 2 - \frac{2.2}{1.7}$$

$$V_T = 0.70\text{ V}$$

Inversion charge density at $V_{G2} = 4$ V is given by,

$$(Q_{IN})_2 = C_{ox}(V_{G_2} - V_T)$$
$$(Q_{IN})_2 = 1.7(4-0.70)$$
$$(Q_{IN})_2 = 5.6\ \mu\text{F/cm}^2$$

Hence, the correct answer is 5.6.

Question 35 **Communication System**

A symbol stream contains alternate QPSK and 16-QAM symbols. If symbols from this stream are transmitted at the rate of 1 mega-symbols per second, the raw (uncoded) data rate is _______ mega-bits per second (rounded off to one decimal place).

Ans. **3 (Range : 2.99 to 3.01)**

Sol. **Given :** Symbol rate $= 1$ mega symbol per second $= 10^6$ symbols per second

$$\text{Symbol rate} = \frac{\text{data rate}}{\log_2 M} \qquad (\because\ M \text{ is the number of symbols})$$

For QPSK, $M = 4$

$$10^6 = \frac{\text{data rate}}{\log_2 4}$$

$$\text{data rate} = 10^6 \times \log_2 4$$

$$\text{data rate} = 2\times 10^6 \text{bps}.$$

For 16 QAM, $M = 16$

$$10^6 = \frac{\text{data rate}}{\log_2 16}$$

$$\text{data rate} = 10^6 \times \log_2 16$$

$$\text{data rate} = 4\times 10^6 \text{bps}$$

As the symbol stream contains alternate QPSK and 16 QAM symbol, the overall data rate will be

$$(\text{data rate})_{overall} = \frac{2\times 10^6 + 4\times 10^6}{2}$$

$$(\text{data rate})_{overall} = 3\times 10^6 = 3\text{Mbps}$$

Hence, the correct answer is 3.

Q.36 to Q.65 Carry TWO Marks Each

Question 36 **Mathematics**

The function $f(x) = 8\log_e x - x^2 + 3$ attains its minimum over the interval $[1, e]$ at $x =$

(Here $\log_e x$ is the natural logarithm of x)

(A) e (B) 2

(C) $\frac{(1+e)}{2}$ (D) 1

Ans. **D**

Sol. **Method 1**

Given : Function $f(x)=8\log_e x-x^2+3$

Differentiate above function with respect to x,

$$f'(x)=\frac{8}{x}-2x$$

For maxima or minima,

$$f'(x)=0$$

$$\frac{8}{x}-2x=0$$

$$\frac{8}{x}=2x$$

$$x^2=4$$

$$x=\pm 2$$

So, we take $x=2\in[1,e]$

$$f''(x)=\frac{-8}{x^2}-2$$

$$f''(x=2)=-\frac{8}{4}-2=-4$$

$$f''(2)<0$$

At $x=2$, function $f(x)$ is maximum. Therefore, function $f(x)$ is attains minimum in closed intervals $[1, e]$.

$$f(x)_{\min}=\min\{f(1),\ f(e)\}$$

At $x=1$, $\quad f(1)=8\log_e 1-1+3=0-1+3=2$

At $x=e$, $\quad f(e)=8\log_e e-e^2+3$

$$f(e)=8+3-e^2$$

$$f(e)=11-7.38=3.61$$

$$f(x)_{\min}=\min\{2, 3.61\}=2$$

We can see, that minimum value occur at point $x=1$.

Hence, the correct option is (D).

Method 2

Given function, $f(x)=8\log_e x-x^2+3,\quad x\in[1,e]$ as closed interval is given.

Checking options,

Option (A) : $x=e$,

$$f(x=e)=8\log_e(e)-e^2+3$$

$$f(x=e)=8-e^2+3=3.61$$

Option (B) : $x=2$,

$$f(x=2)=8\log_e(2)-2^2+3$$

$$f(x=2)=4.545$$

Option (C) : $x=\frac{1+e}{2}$,

$$f\left(x=\frac{1+e}{2}\right)=4.504$$

Option (D) : $x=1$,

$$f(x=1)=8\log_e(1)-1^2+3$$

$$f(x=1)=8\times 0-1+3=2$$

At $x=1$ it shows minimum value.

Hence, the correct option is (D).

Question 37 **Mathematics**

Let α, β be two non-zero real numbers and v_1, v_2 be two non-zero real vectors of size 3×1. Suppose that v_1 and v_2 satisfy $v_1^T v_2=0$, $v_1^T v_1=1$ and $v_2^T v_2=1$. Let A be the 3×3 matrix given by $A=\alpha v_1 v_1^T+\beta v_2 v_2^T$. The eigenvalues of A are

(A) 0, 0, $\sqrt{\alpha^2+\beta^2}$

(B) 0, $\frac{\alpha+\beta}{2}$, $\sqrt{\alpha\beta}$

(C) 0, $\alpha+\beta$, $\alpha-\beta$

(D) 0, α, β

Ans. **D**

Sol. **Given :** $\alpha,\beta\to$ Two non-zero real numbers.

$v_1,v_2\to$ Two non-zero real numbers of size 3×1

$$v_1^T.v_2=0,\ v_1^T.v_1=1,\ v_2^T.v_2=1$$

$$A=\alpha v_1 v^T+\beta v_2 v_2^T \qquad \ldots\text{(i)}$$

Every square matrix satisfies the following equation

$$[A-\lambda I][X]=[0]$$

$$Ax=\lambda x \qquad \ldots\text{(ii)}$$

By equation (i) and (ii),

$$Av_1=\alpha v_1 v_1 v_1^T+\beta v_2 v_1 v_2^T$$

$$Av_1=\alpha v_1+0$$

$$Av_1=\alpha v_1$$

$\Rightarrow$ $A=\alpha$ is Eigen value of matrix A.

By equation (i) and (ii),

$$Av_2 = \alpha v_1 v_2 v_1^T + \beta v_2 v_2 v_2^T$$

$$Av_2 = 0 + \beta v_2$$

$$Av_2 = \beta v_2$$

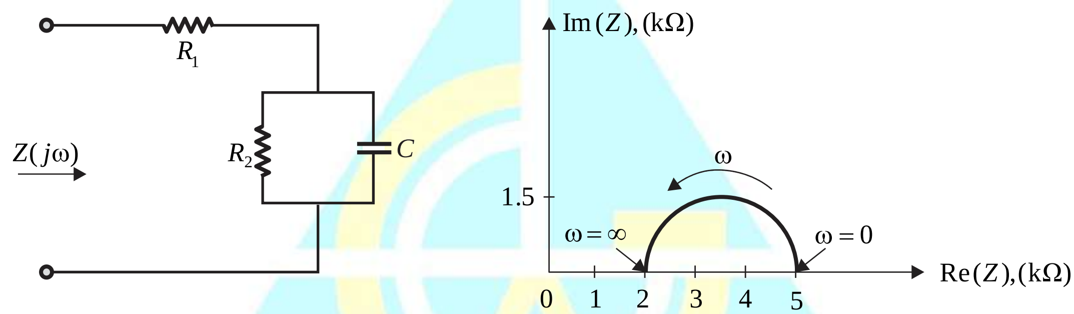
$\Rightarrow \quad A = \beta$ is eigen value of matrix A

Only option (D) satisfies.

Hence, the correct option is (D).

Question 38 **Network Theory**

For the circuit shown, the locus of the impedance $Z(j\omega)$ is plotted as ω increases from zero to infinity. The values of R_1 and R_2 are :

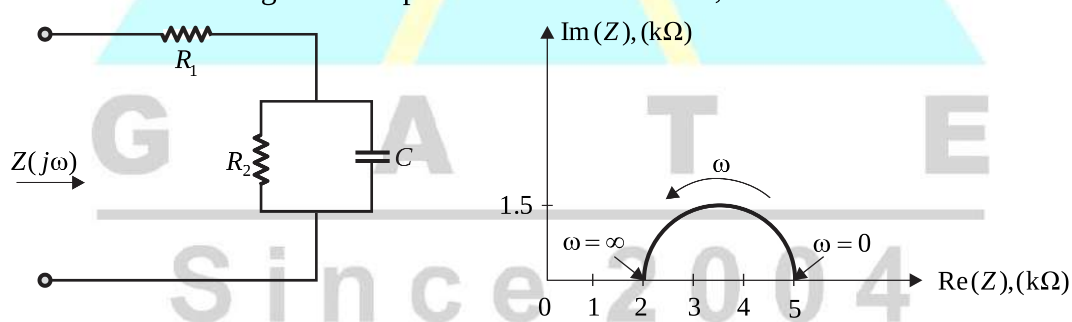


(A) $R_1 = 2\,\text{k}\Omega,\ R_2 = 5\,\text{k}\Omega$ (B) $R_1 = 5\,\text{k}\Omega,\ R_2 = 2.5\,\text{k}\Omega$

(C) $R_1 = 2\,\text{k}\Omega,\ R_2 = 3\,\text{k}\Omega$ (D) $R_1 = 5\,\text{k}\Omega,\ R_2 = 2\,\text{k}\Omega$

Ans. **C**

Sol. Given circuit and locus diagram of impedance is shown below,



Method 1

Impedance of the given circuit is,

$$Z(j\omega) = R_1 + \left[R_2 \,||\, \frac{1}{j\omega C} \right]$$

$$Z(j\omega) = R_1 + \left[\frac{\dfrac{R_2}{j\omega C}}{R_2 + \dfrac{1}{j\omega C}} \right] = R_1 + \left[\frac{R_2}{1 + j\omega R_2 C} \right] \qquad \ldots \text{(i)}$$

At $\omega = 0$ rad/sec, $Z(j0) = R_1 + R_2$

From the given locus diagram, $Z(j0) = 5\text{k}\Omega$

Therefore, $R_1 + R_2 = 5\text{k}\Omega$ ...(ii)

At $\omega = \infty$ rad/sec

From equation (i),

$$Z(j\infty) = R_1$$

From the given locus diagram, $Z(j\infty) = 2\text{k}\Omega$

Therefore, $R_1 = 2\,\text{k}\Omega$ ...(iii)

Substituting the value of R_1 from equation (iii) in equation (ii),

$$2\,\text{k}\Omega + R_2 = 5\,\text{k}\Omega$$

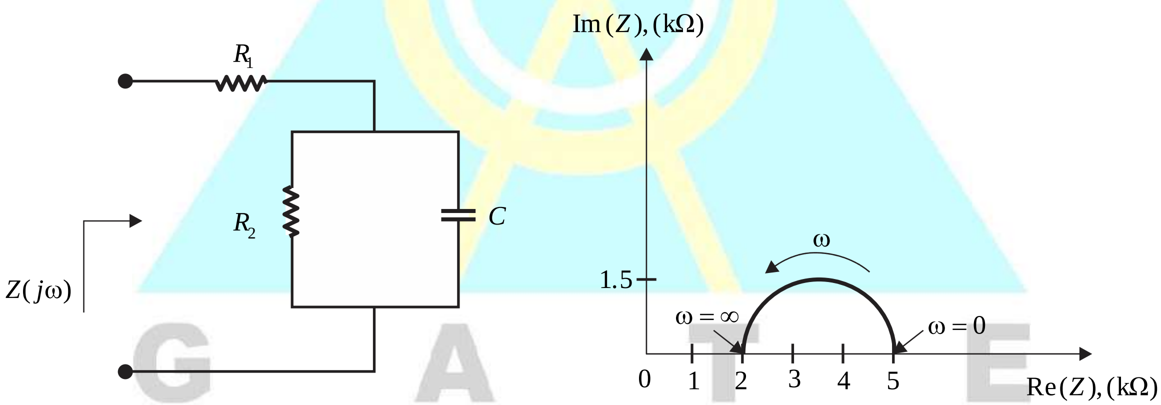
$$R_2 = 3\text{k}\Omega$$

Therefore, $R_1 = 2\,\text{k}\Omega$ and $R_2 = 3\text{k}\Omega$

Hence, the correct option is (C).

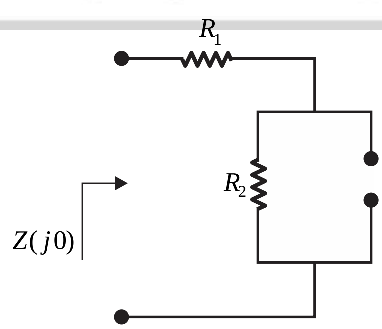
Method 2

Given circuit and locus diagram of impedance is shown below,



At $\omega = 0$ rad/sec, capacitive impedance $X_c = \dfrac{1}{j\omega C} = \infty$

Therefore, capacitor behaves as open circuit and given circuit at $\omega = 0$ rad/sec becomes as,



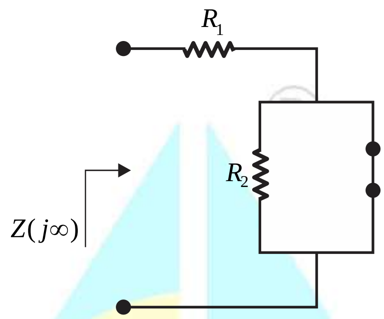
From the above circuit, $Z(j0) = R_1 + R_2$

From the given locus diagram, $Z(j0) = 5\text{k}\Omega$

Therefore, $R_1 + R_2 = 5\text{k}\Omega$ ...(i)

At $\omega = \infty$ rad/sec, $X_c = \frac{1}{\infty} = 0$

Therefore, capacitor behaves as short circuit and given circuit at $\omega = \infty$ rad/sec becomes,



From the above circuit, $Z(j\infty) = R_1$

From the given locus diagram, $Z(j\infty) = 2\text{k}\Omega$

Therefore, $R_1 = 2\text{k}\Omega$ ...(ii)

Substituting the value of R_1 from equation (ii) in equation (i),

$2\text{k}\Omega + R_2 = 5\text{k}\Omega$

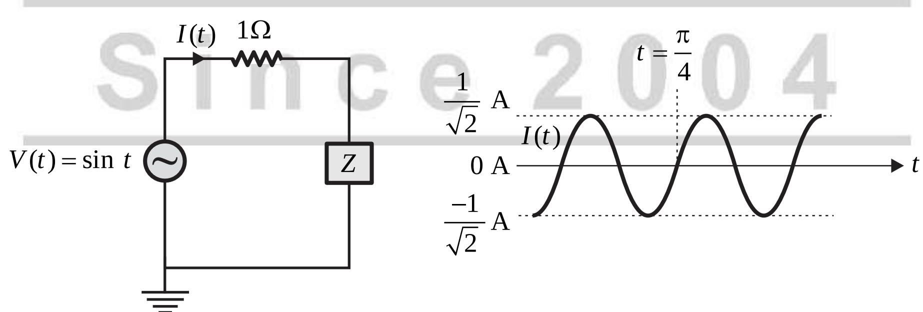
$R_2 = 3\text{k}\Omega$

Therefore, $R_1 = 2\text{k}\Omega$ and $R_2 = 3\text{k}\Omega$

Hence, the correct option is (C).

Question 39 **Network Theory**

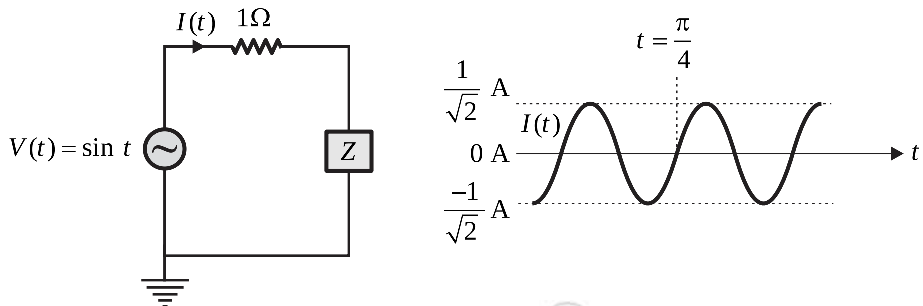
Consider the circuit shown in the figure with input $V(t)$ in volts. The sinusoidal steady state current $I(t)$ flowing through the circuit is shown graphically (where t is in seconds). The circuit element Z can be



(A) a capacitor of 1 F
(B) an inductor of $\sqrt{3}$ H
(C) a capacitor of $\sqrt{3}$ F
(D) an inductor of 1 H

Ans. D

Sol. Given circuit and the waveform of sinusoidal steady state current $I(t)$ is shown below,



Method 1

From the waveform of current $I(t)$, the current is lagging with respect to supply voltage $V(t)$, because at $t=0$, $V(t)$ is 0 and $t=\frac{\pi}{4}$, $I(t)$ is 0.

Hence, the circuit element Z is inductor.

Thus, $Z = jX_L$

Peak value of current $I(t)=\frac{1}{\sqrt{2}}\text{A}$

Peak value of voltage $V(t)=1\text{V}$

Therefore impedance of circuit, $(R+jX_L)=\frac{1}{\frac{1}{\sqrt{2}}}=\sqrt{2}\,\Omega$

$$|R+jX_L|=\sqrt{2}\ \Omega$$

$$\sqrt{R^2+X_L^2}=\sqrt{2}$$

$$\sqrt{1+X_L^2}=\sqrt{2}$$

$$1+X_L^2=2$$

$$X_L^2=1$$

$$X_L=1\Omega$$

Since, $X_L=\omega L$

$$\omega L=1$$

Here $\omega=1\text{rad/sec}$

$$L=1\text{H}$$

Hence, the correct option is (D).

Key Point

From the given current $I(t)$ and voltage $V(t)$ waveform it is clear that,

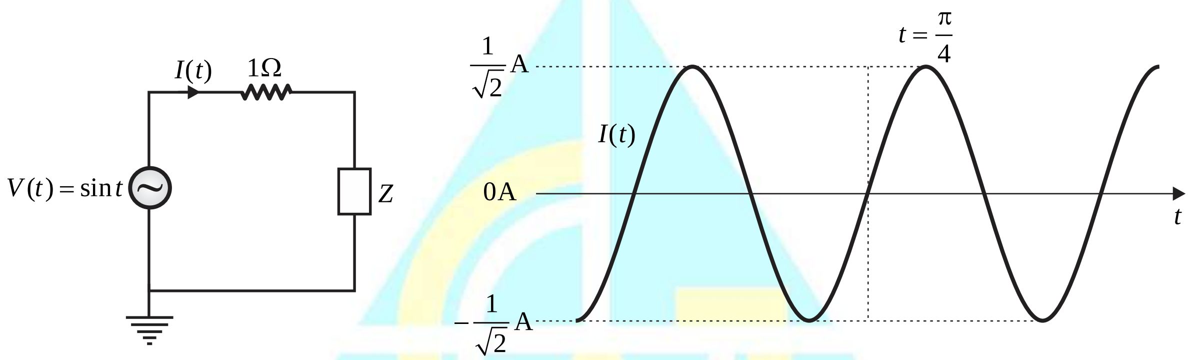
$V(t=0)=0$

$I\left(t=\frac{\pi}{4}\right)=0$

It shows current $I(t)$ is lagging from voltage $V(t)$.

Method 2

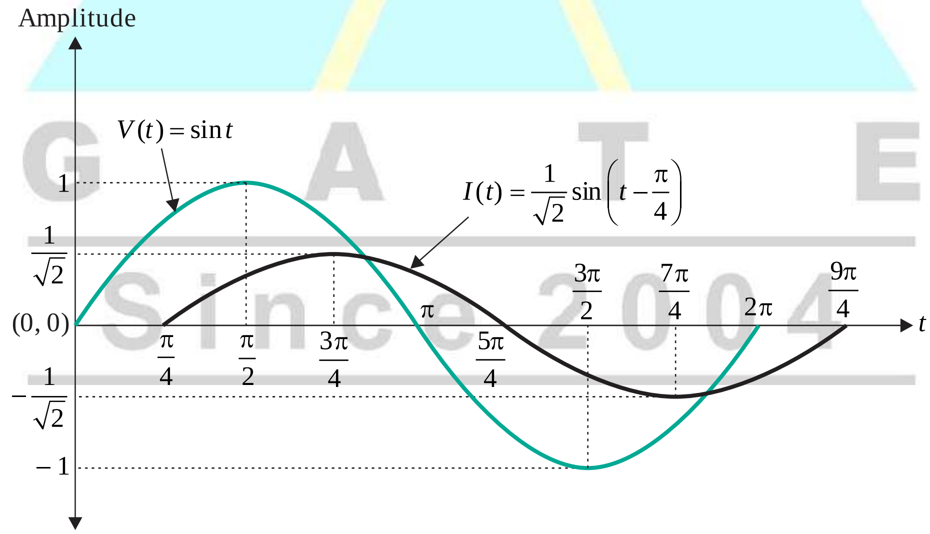
Using sinusoidal steady state analysis :



Given : $V(t)=\sin t$, $\omega=1$ rad/sec

From circuit, $I(t)=\dfrac{V(t)}{Z+1}=\dfrac{\sin t}{Z+1}$ ...(i)

The given waveform of current $I(t)$ and voltage $V(t)$ is shown below,



From above figure it is clear that current, $I(t)=\frac{1}{\sqrt{2}}\sin\left(t-\frac{\pi}{4}\right)$, it shows $I(t)$ lags from voltage $V(t)$ by an angle of $\frac{\pi}{4}$ it mean given circuit offer lagging power factor that's why Z must be inductive reactance.

Thus, $Z = j\omega L$ so equation (i) becomes as,

$$I(t) = \frac{\sin t}{j\omega L + 1}$$

If $L = 1\text{ H}$, then and only then,

$$I(t) = \frac{\sin t}{j1+1} = \frac{\sin t}{\sqrt{2}\angle 45^0}$$

$$I(t) = \frac{1}{\sqrt{2}}\sin(t - 45^0)$$

Hence, the correct option is (D).

Question 40 **Electronic Devices**

Consider an ideal long channel n MOSFET (enhancement mode) with gate length $10\,\mu\text{m}$ and width $100\,\mu\text{m}$. The product of electron mobility (μ_n) and oxide capacitance per unit area (C_{OX}) is $\mu_n C_{OX} = 1\text{mA/V}^2$. The threshold voltage of the transistor is 1 V. For a gate-to-source voltage $V_{GS} = [2 - \sin(2t)]\text{V}$ and drain-to-source voltage $V_{DS} = 1\text{V}$ (substrate connected to the source), the maximum value of the drain-to-source current is

(A) 15 mA (B) 20 mA

(C) 5 mA (D) 40 mA

Ans. **A**

Sol. **Given :**

(i) n-channel MOSFET

(ii) Threshold voltage, $V_{Th_N} = 1\text{V}$

(iii) Drain to source voltage, $V_{DS} = 1\text{V}$

(iv) Gate to source voltage, $V_{GS} = [2 - \sin(2t)]\text{V}$

(v) $\mu_n C_{ox} = 1\text{mA/V}^2$

V_{GS} minimum exist at positive value of $\sin 2t$

$$V_{GS(\min)} = [2-1] = 1\text{ V}$$

Over-drive voltage is given by,

$$V_{OV} = V_{GS(\min)} - V_{ThN}$$

$$V_{OV} = 1 - 1 = 0$$

Since over-drive voltage is zero at $V_{GS(\min)}$, so no current flow in the n MOSFET.

V_{GS} maximum exist at negative value of $\sin 2t$

$$\therefore \quad V_{GS(\max)} = [2 - (-1)] = 3\text{ V}$$

Over-drive voltage is given by,

$$V_{OV} = V_{GS(\max)} - V_{Th_N}$$

$$V_{OV} = 3 - 1 = 2 \text{ V}$$

$$V_{DS} = 1 \text{ V}$$

$$V_{DS} < V_{OV}$$

Hence, n MOSFET operate in linear region.

For n MOSFET current in linear region is given by,

$$I_D = \mu_n C_{ox} \frac{W}{L}\left\{(V_{GS} - V_{Th_N})V_{DS} - \frac{V_{DS}^2}{2}\right\}$$

$$I_D = \frac{1 \text{ mA}}{V^2} \times \frac{100}{10}\left\{(3-1)\times 1 - \frac{1}{2}\right\}$$

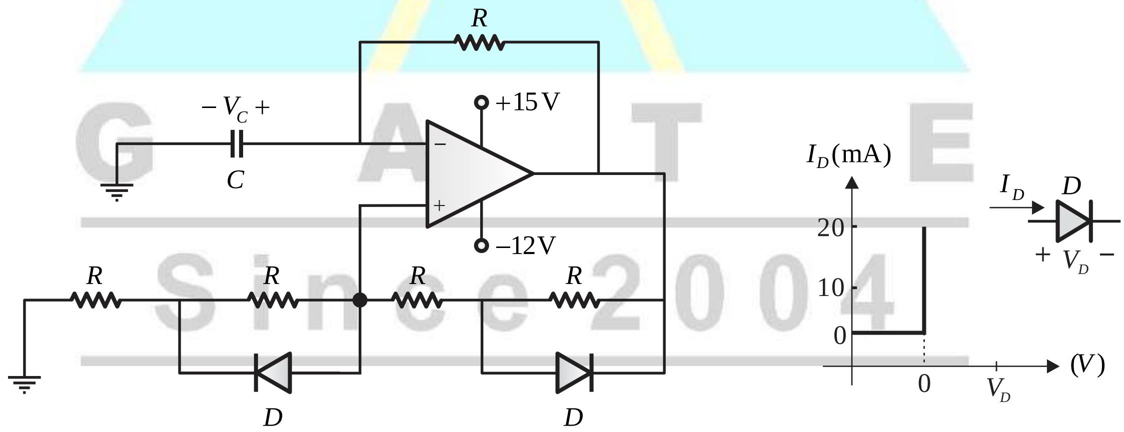
$$I_D = 10 \times \left(2 - \frac{1}{2}\right)$$

$$I_D = 10 \times \frac{3}{2} = 15 \text{ mA}$$

Hence, the correct option is (A).

Question 41 **Analog Electronics**

For the following circuit with an ideal OPAMP, the difference between the maximum and the minimum values of the capacitor voltage (V_C) is



(A) 15 V (B) 27 V

(C) 14 V (D) 13 V

Ans. **D**

Sol. The given circuit is an astable multivibrator/free running oscillator.

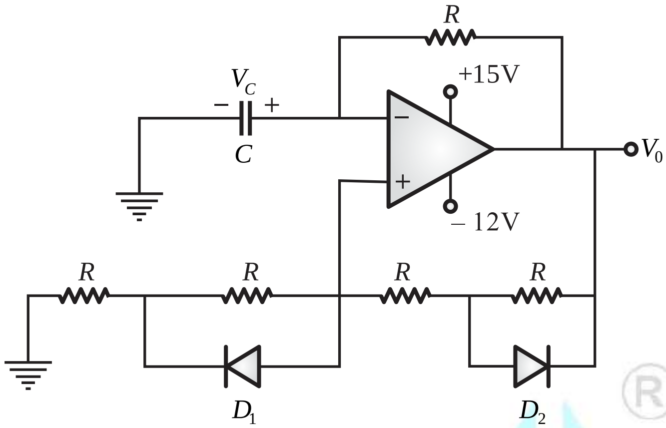


Fig. Astable multivibrator

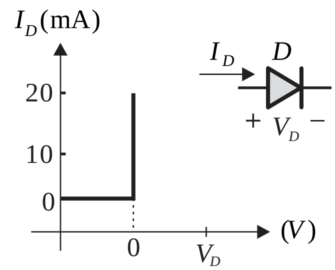
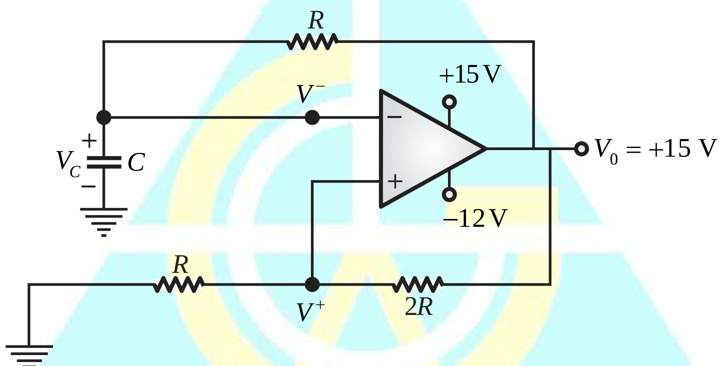


Fig. Ideal characteristic of diode

Case 1 : When $V_0 = 15\text{ V}$, D_1 is ON and D_2 is OFF then above circuit becomes as,



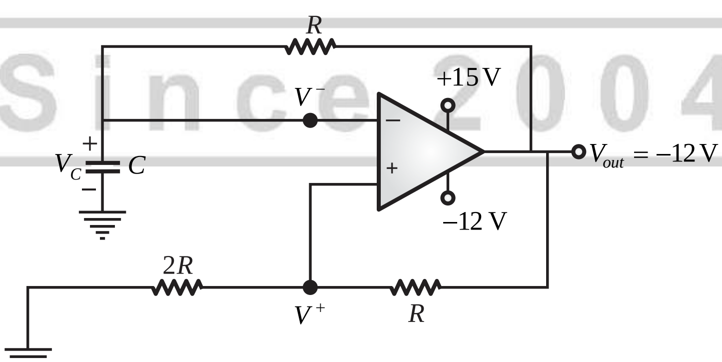
Apply voltage division rule to calculate V^+,

$$V^+ = \frac{R \times V_0}{2R+R} = \frac{15\times R}{3R} = \frac{15}{3} = 5\text{ V}$$

Maximum voltage across capacitor is (i.e. capacitor charged upto $V_{UTP} = V^+$),

$[V_c(t)]_{\max} = 5\text{ V} = V_{UTP}$

Case 2 : When $V_0 = -12\text{ V}$, D_1 is OFF and D_2 is ON, then above circuit becomes as,



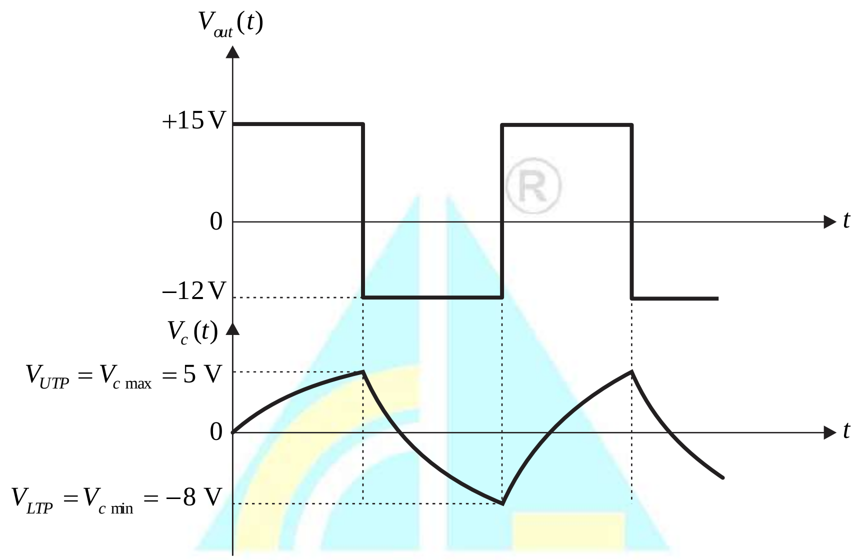
Applying voltage division rule to calculate V^+,

So, $$V^+ = V_{out} \times \frac{2R}{2R+R} = \frac{-12}{3}\times 2 = -8\text{ V}$$

Minimum capacitor voltage is (i.e. capacitor charged upto $V_{LTP} = V^+$),

$[V_c(t)]_{\min} = -8 \text{ V} = V_{LTP}$

The waveform of output voltage $V_{out}(t)$ and capacitor voltage $V_c(t)$ is,



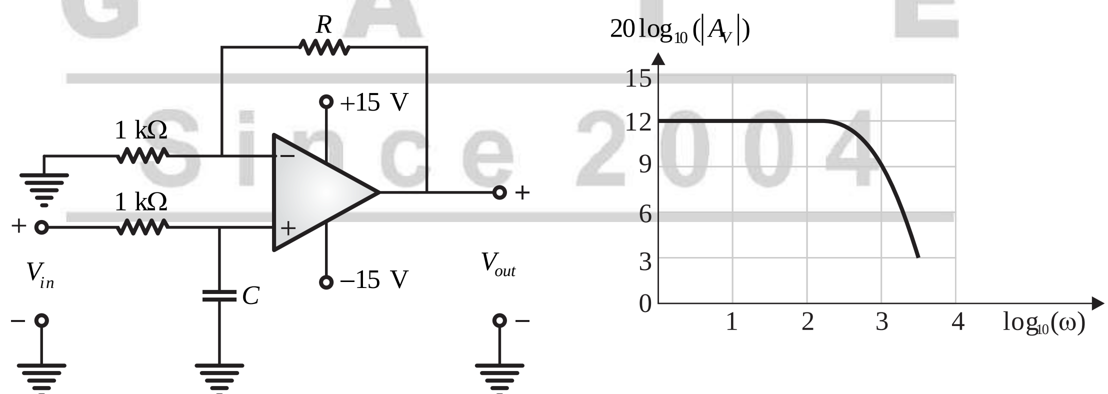
Thus, $V_{c\max} - V_{c\min} = 5 - (-8) = 13 \text{ V}$

Hence, the correct option is (D).

Question 42 **Analog Electronics**

A circuit with an ideal OPAMP is shown. The Bode plot for the magnitude (in dB) of the gain transfer function $\left(A_V(j\omega) = \dfrac{V_{out}(j\omega)}{V_{in}(j\omega)} \right)$ of the circuit is also provided (here, ω is the angular frequency in rad/sec).

The values of R and C are



(A) $R = 3\text{k}\Omega, C = 1\mu\text{F}$ (B) $R = 3\text{k}\Omega, C = 2\mu\text{F}$

(C) $R = 1\text{k}\Omega, C = 3\mu\text{F}$ (D) $R = 4\text{k}\Omega, C = 1\mu\text{F}$

Ans. A

Sol. The given Op-Amp circuit is shown below,

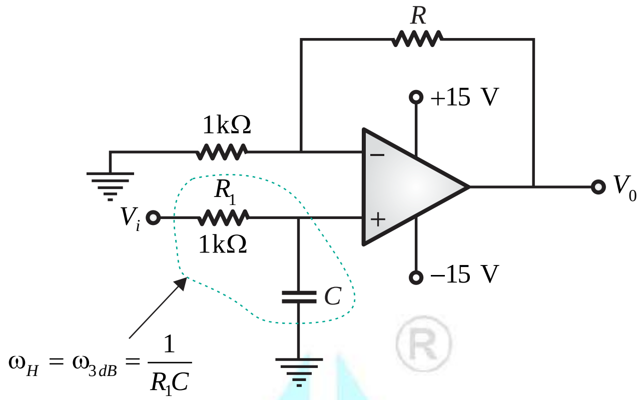
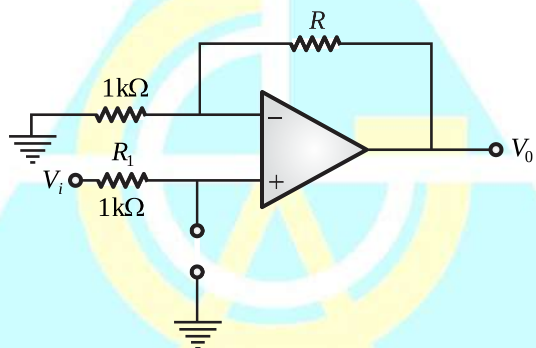


Fig. Active low pass filter

The given Op-Amp circuit is first order active low pass filter, so low frequency $(\omega = 0)$ gain of given low pass filter is,



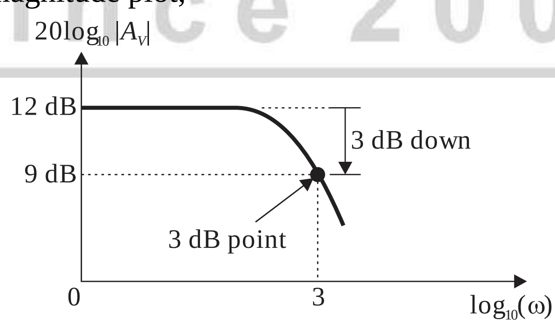
The above Op-Amp circuit becomes as non-inverting closed loop Op-Amp, so gain (A_V) is,

$$A_V = \left[1 + \frac{R}{1}\right] = 1 + R \qquad \ldots\text{(i)}$$

The 3-dB cut-off frequency of given low pass filter circuit is,

$$\omega_{3dB} = \frac{1}{R_1C} \text{ rad/sec} \qquad \ldots\text{(ii)}$$

According to given bode magnitude plot,



Low frequency $(\omega = 0)$ gain from bode magnitude plot is,

$$12\,\text{dB} = 20\log_{10}(A_V)$$

$$A_V = \text{Antilog}\left[\frac{12}{20}\right]$$

$$1 + R = 4$$

$$R = 3\ \text{k}\Omega$$

The 3-dB frequency from bode magnitude plot,

$$\log_{10}\omega_{3dB} = 3$$

$$\omega_{3dB} = \text{Antilog}\ (3)$$

$$\omega_{3dB} = 1000\ \text{rad/sec}$$

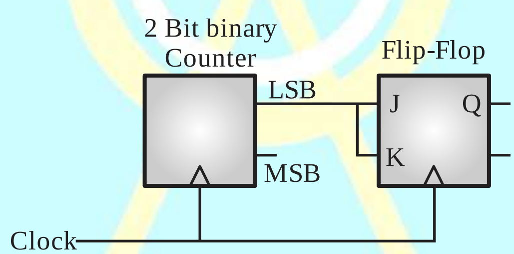
Thus, $\omega_{3dB} = \dfrac{1}{R_1 C} = 1000$

$$C = \frac{1}{1000R_1} = \frac{1}{1000 \times 1000} = 1\ \mu\text{F}$$

Hence, the correct option is (A).

Question 43 **Digital Electronics**

For the circuit shown, the clock frequency is f_0 and the duty cycle is 25%. For the signal at the Q output of the Flip-Flop.



(A) Frequency is $f_0/2$ and duty cycle is 50%.

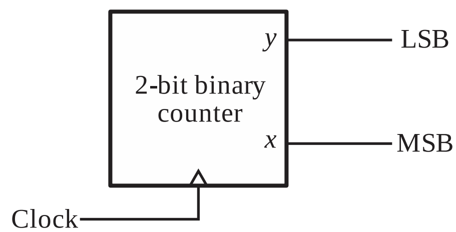
(B) Frequency is $f_0/4$ and duty cycle is 50%.

(C) Frequency is $f_0/4$ and duty cycle is 25%.

(D) Frequency is f_0 and duty cycle is 25%.

Ans. B

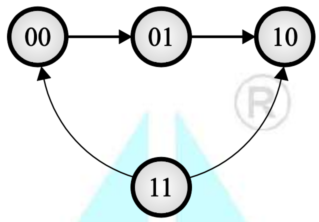
Sol. Given : 2-bit binary counter is shown below,



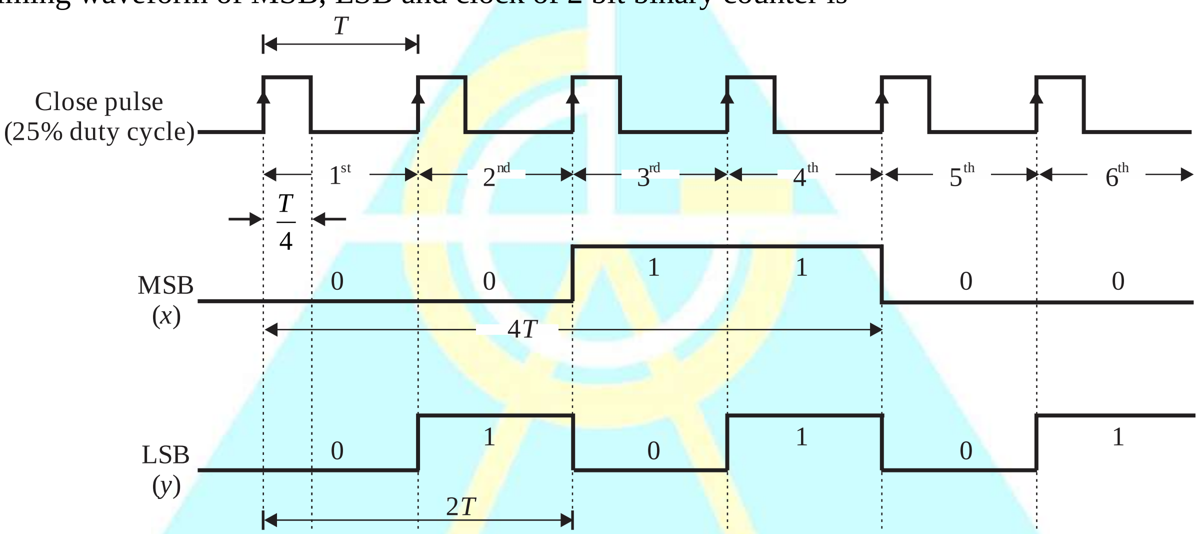
Frequency of clock is f_0 so, time period (T) of clock is $T = \dfrac{1}{f_0}$

Counting sequence of 2-bit binary counter is

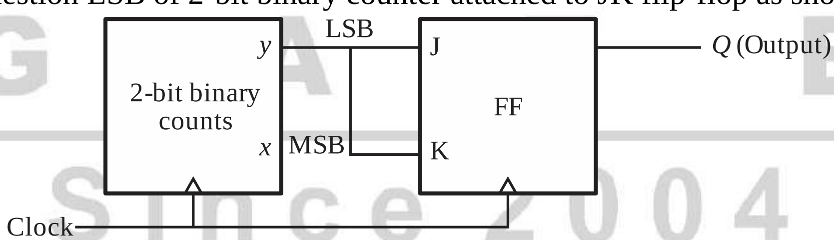
Clock	MSB(x)	LSB(y)
0	0	0
1st	0	1
2nd	1	0
3rd	1	1



Timing waveform of MSB, LSB and clock of 2 bit binary counter is



According to question LSB of 2-bit binary counter attached to JK flip-flop as shown below

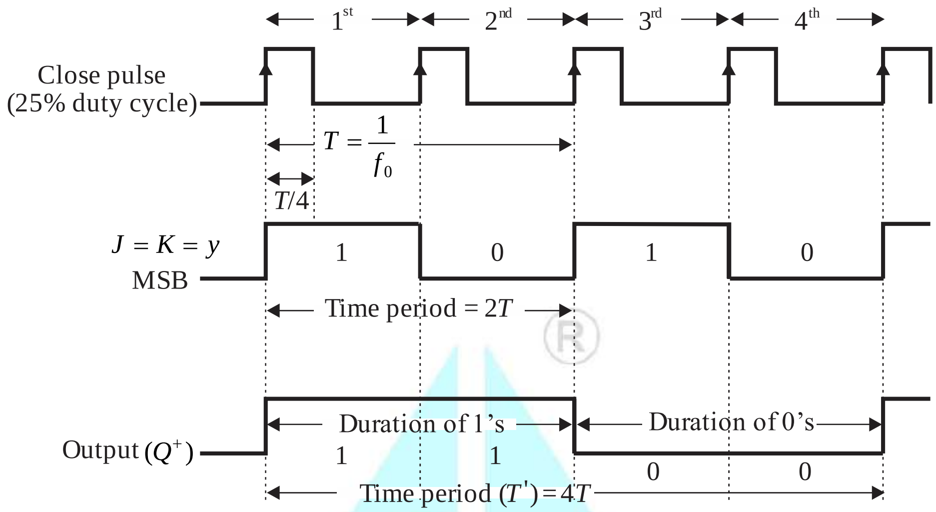


Here input $J = K = LSB(\text{y})$

Output of JK flip-flip is,

Clock	$J = y$	$K = y$	Q^+
0	0	0	0
1st	1	1	1
2nd	0	0	1
3rd	1	1	0
4th	0	0	0

Output (Q^+) waveform of given circuit is



Here, time period of output (Q^+) is,

$T' = 4T$ $\qquad\qquad\qquad\qquad (\because\ T = 1/f_0)$

$$\frac{1}{f_0'} = 4 \times \frac{1}{f_0}$$

$$f_0' = \frac{f_0}{4}$$

In output waveform of Q^+ duration of 1's is equal to duration of 0's. Thus output waveform shows 50% duty cycle.

Hence, the correct option is (B).

Note : In question examiner did not mentioned about counter whether it is up or down so we assume counter is up. If anyone wants to take it down counter then answer remains same.

Question 44 **Control System**

Consider an even polynomial $p(s)$ given by

$$p(s) = s^4 + 5s^2 + 4 + K$$

where K is an unknown real parameter. The complete range of K for which $p(s)$ has all its roots on the imaginary axis is

(A) $-3 \le K \le \frac{9}{2}$ (B) $-5 \le K \le 0$

(C) $-4 \le K \le \frac{9}{4}$ (D) $-6 \le K \le \frac{5}{4}$

Ans. **C**

Sol. The given even polynomial $p(s)$ is given by,

$$p(s) = s^4 + 5s^2 + 4 + K$$

Method 1

Routh tabulatoin :

s^4	1	5	$4+K$
s^3	0	0	
s^2			
s^1			
s^0			

Since, a row of zeros appears, we form auxiliary equation using coefficient of s^4 row,

$$A(s) = s^4 + 5s^2 + 4 + K$$

The derivative of $A(s)$ with respect to s is.

$$\frac{dA(s)}{ds} = 4s^3 + 10s$$

From which, the coefficients 4 and 10 replace the zeros in s^3 row of the original tabulation.

Routh tabulatoion

s^4	1	5	$4+K$
s^3	4	10	0
s^2	2.5	$4+K$	0
s^1	$\frac{9-4K}{2.5}$	0	0
s^0	$4+K$		

For $p(s)$ has all its roots on the imaginary axis

$$\frac{9-4K}{2.5} \geq 0$$

$$9-4K \geq 0$$

$$K \leq \frac{9}{4} \qquad \ldots \text{(i)}$$

And

$$4+K \geq 0$$

$$K \geq -4 \qquad \ldots \text{(ii)}$$

Combining equation (i) and (ii),

$$-4 \leq K \leq \frac{9}{4}$$

Hence, the correct option is (C).

Method 2

The even polynomial $p(s)$ is given by

$$p(s) = s^4 + 5s^2 + 4 + K$$

Put $s^2 = x$

$$x^2 + 5x + 4 + K = 0$$

Root of the above quadratic equation

$$x = \frac{-5 \pm \sqrt{25 - 16 - 4K}}{2} \quad \ldots \text{(i)}$$

For root of $p(s)$ on imaginary axis, the value of x must be negative [since $s = \sqrt{x}$].

For x to be negative, there are two possibilities.

Checking for option (C) :

Case 1 : Let $K = \frac{9}{4}$

From equation (i)

$$x = \frac{-5 \pm \sqrt{25 - 16 - 4\left(\frac{9}{4}\right)}}{2}$$

$$x = \frac{-5 \pm \sqrt{25 - 25}}{2}$$

$$x = \frac{-5}{2}$$

At $x = \frac{-5}{2}$

$$s = \sqrt{x} = \sqrt{\frac{-5}{2}} = \pm j1.58$$

Hence, root of $p(s)$ are on img axis for the value of $K = \frac{9}{4}$

Case 2 : Let $K = -4$

$$x = \frac{-5 \pm \sqrt{25 - 16 - 4(-4)}}{2}$$

$$x = \frac{-5 \pm \sqrt{25}}{5} = \frac{-5 \pm 5}{2}$$

Then $x = -5$ or $x = 0$.

For $x = -5$,

$$s = \sqrt{x} = \sqrt{-5} = \pm j2.23$$

Hence, the roots of $p(s)$ on img axis for $K = -4$.

Hence, combining the two cases the range of K for all the roots of $p(s)$ on img axis is,

$$-4 \le K \le \frac{9}{4}$$

Hence, the correct option is (C).

Question 45 (MSQ) **Mathematics**

Consider the following series :

$$\sum_{n=1}^{\infty} \frac{n^d}{c^n}$$

For which of the following combinations of c, d values does this series converge?

(A) $c=1,\ d=-1$

(B) $c=2,\ d=1$

(C) $c=0.5,\ d=-10$

(D) $c=1,\ d=-2$

Ans. **B, D**

Sol. **Given :** Series $\sum_{n=1}^{\infty} \frac{n^d}{c^n}$

Let $$a_n = \frac{n^d}{c^n}$$

Checking from options :

From Options (A) : $c=1,\ d=1$

$$\sum a_n = \sum_{n=1}^{\infty} \frac{n^d}{c^n} = \sum_{n=1}^{\infty} \frac{n^{-1}}{(1)^n}$$

$$\sum a_n = \sum_{n=1}^{\infty} \frac{1}{n}$$

It is clear that, it is divergent by P-test.

From Options (B) : $c=2,\ d=1$

$$\sum a_n = \sum_{n=1}^{\infty} \frac{n^d}{c^n} = \sum_{n=1}^{\infty} \frac{n^1}{2^n} = \sum_{n=1}^{\infty} \frac{n}{2^n}$$

Performing ratio test,

$$\lim_{n\to\infty} \frac{a_n+1}{a_n} = \lim_{n\to\infty} \frac{n+1}{2^{n+1}} \times \frac{2^n}{n} = \frac{1}{2}$$

So, that $$\frac{1}{2} < 1$$

by ratio test, it is clear that $\sum a_n$ is convergent.

From Options (C) : $c=0.5,\ d=-10$

$$\sum a_n = \sum_{n=1}^{\infty} \frac{n^{-10}}{0.5^n}$$

Performing ratio test,

$$\lim_{n\to\infty}\frac{a_n+1}{a_n}=\lim_{n\to\infty}\frac{(n+1)^{-10}}{(0.5)^{n+1}}\times\frac{(0.5)^n}{n^{10}}=\frac{1}{0.5}=2$$

So, that $\quad 2>1$

It is clear that, it is divergent by ratio test.

From Options (D) : $c=1,\ d=-2$

$$\sum a_n=\sum_{n=1}^{\infty}\frac{n^{-2}}{(1)^n}=\sum_{n=1}^{\infty}\frac{1}{n^2}$$

It is clear that, it is convergent by P-test.

Hence, the correct option is (B) and (D).

Question 46 (MSQ) **Signals & Systems**

The outputs of four systems (S_1, S_2, S_3 and S_4) corresponding to the input signal $\sin(t)$, for all time t, are shown in the figure. Based on the given information, which of the four system is/are definitely NOT LTI (linear and time-invariant)?

$$\sin(t)\longrightarrow\boxed{S_1}\longrightarrow\sin(-t)$$

$$\sin(t)\longrightarrow\boxed{S_2}\longrightarrow\sin(t+1)$$

$$\sin(t)\longrightarrow\boxed{S_3}\longrightarrow\sin(2t)$$

$$\sin(t)\longrightarrow\boxed{S_4}\longrightarrow\sin^2(t)$$

(A) S_4 (B) S_1

(C) S_3 (D) S_2

Ans. **A, C**

Sol. Assume a system (S) having input $x(t)$ and output $y(t)$ as shown below

$$x(t)\longrightarrow\boxed{\begin{array}{c}S\\ \text{system}\end{array}}\longrightarrow y(t)$$

According to question, four systems (S_1, S_2, S_3, S_4) are given with input signal $\sin t$.

Method 1

For system (S_1) :

$$x(t)=\sin(t)\longrightarrow\boxed{S_1}\longrightarrow y(t)=\sin(-t)=-\sin t$$

Here, $y(t)=-\sin t=-x(t)$

Thus, $y(t)\propto x(t)$

S_1 directly follow proportional relationship between input $x(t)$ and output $y(t)$ it means S_1 follow both homogeneity and additivity property. Hence S_1 is linear.

For S_1, $y(t) = -x(t)$, it shows no change in time instant, it means system S_1 is time invariant.

Hence, system S_1 is linear and time invariant.

For system (S_2) :

$$x(t) = \sin(t) \longrightarrow \boxed{S_2} \longrightarrow y(t) = \sin(t+1)$$

Here, $y(t) = \sin(t+1) = x(t+1)$

Thus, $y(t) \propto x(t+1)$

S_2 directly follow proportional relationship between input $x(t)$ and output $y(t)$ it means S_2 follow both homogeneity and additivity property. Hence S_2 is linear.

For S_2, $y(t) = x(t+1)$

Delayed response for S_2, $y(t-t_0) = x(t-t_0+1)$

Response to delayed input $x(t-t_0)$ in system S_2, $y(t,t_0) = x(t-t_0+1)$

Thus, delayed response $y(t-t_0)$ and response to delayed input $y(t,t_0)$ are same for system S_2, that is why system S_2 is time invariant.

Hence, system S_2 is linear and time invariant.

For system (S_3) :

$$x(t) = \sin(t) \longrightarrow \boxed{S_3} \longrightarrow y(t) = \sin(2t)$$

Here, $y(t) = \sin 2t = x(2t)$

Thus, $y(t) \propto x(2t)$

S_3 directly follow proportional relationship between input $x(t)$ and output $y(t)$ it means S_3 follow both homogeneity and additivity property. Hence S_3 is linear.

For S_3, $y(t) = x(2t)$

Delayed response for S_3, $y(t-t_0) = x(2t-t_0)$

Response to delayed input $x(t-t_0)$ in system S_3, $y(t,t_0) = x(2t-2t_0)$

Thus, delayed response $y(t-t_0)$ and response to delayed input $y(t,t_0)$ are same for system S_3, that is why system S_3 is time variant.

Hence, system S_3 is linear but time variant.

For system (S_4) :

$$x(t)=\sin(t) \longrightarrow \boxed{S_4} \longrightarrow y(t)=\sin^2(t)$$

Here, $y(t)=\sin^2 t = x^2(t)$

Thus, $y(t) \propto$ square of $x(t)$

S_4 directly follow square relationship between input $x(t)$ and output $y(t)$ it shows S_4 gives non-linear relationship between input $x(t)$ and output $y(t)$. Hence S_4 is non-linear.

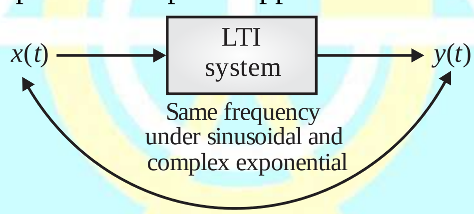
For S_4, $y(t)=x^2(t)$, it shows no change in time instant t, it means system S_4 is time invariant.

Hence, system S_4 is non-linear and time invariant.

Hence, the correct option is (A) and (C).

Method 2

For LTI system, the response to sinusoidal or complex exponential input is a sinusoidal or complex exponential output at same frequency as the Input. It means LTI system cannot change output frequency when sinusoidal or complex exponential input is applied.



According to question, all four system (S_1, S_2, S_3, S_4) have sinusoidal input $\sin t$,

For system (S_1) :

$$x(t)=\sin(t) \longrightarrow \boxed{S_1} \longrightarrow y(t)=\sin(-t)=-\sin t$$

Thus, $x(t)=\sin t$, frequency of input is 1 rad/sec.

$y(t)=\sin t$, frequency of output is 1 rad/sec.

So, frequency of input $x(t)$ and output $y(t)$ is not changed.

Hence S_1 is definitely LTI.

For system (S_2) :

$$x(t)=\sin(t) \longrightarrow \boxed{S_2} \longrightarrow y(t)=\sin(t+1)$$

Thus, $x(t)=\sin t$, frequency of input is 1 rad/sec.

$y(t)=\sin(t+1)$, frequency of output is 1 rad/sec.

So, frequency of input $x(t)$ and output $y(t)$ is not changed.

Hence S_2 is definitely LTI.

For system (S_3) :

$$x(t)=\sin(t) \longrightarrow \boxed{S_3} \longrightarrow y(t)=\sin(2t)$$

Thus, $x(t)=\sin t$, frequency of input is 1 rad/sec.

$y(t)=\sin(2t)$, frequency of output is 2 rad/sec.

So, frequency of input $x(t)$ and output $y(t)$ is not same.

Hence S_3 is not definitely LTI.

For system (S_4) :

$$x(t)=\sin(t) \longrightarrow \boxed{S_4} \longrightarrow y(t)=\sin^2(t)$$

Thus, $x(t)=\sin t$, frequency of input is 1 rad/sec.

$y(t)=\sin^2(t)=\dfrac{1+\cos 2t}{2}$, frequency of output is 2 rad/sec.

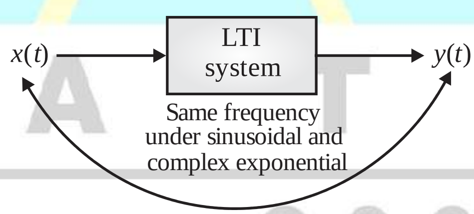
So, frequency of input $x(t)$ and output $y(t)$ is not same.

Hence S_4 is not definitely LTI.

Hence, the correct option is (A) and (C).

Key Point

For LTI system, the response to sinusoidal or complex exponential input is a sinusoidal or complex exponential output at same frequency as the Input. It means LTI system cannot change output frequency when sinusoidal or complex exponential input is applied.



Question 47 **(MSQ)** **Electronic Devices**

Select the CORRECT statement(s) regarding semiconductor devices

(A) Mobility of electrons always increases with temperature in Silicon beyond 300 K.

(B) Electrons and holes are of equal density in an intrinsic semiconductor at equilibrium.

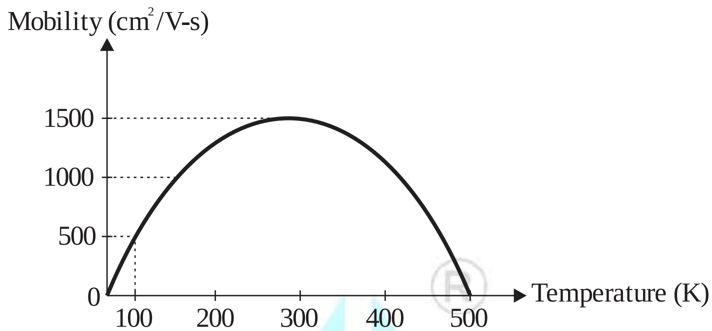
(C) Total current is spatially constant in a two terminal electronic device in dark under steady state condition.

(D) Collector region is generally more heavily doped than Base region in a BJT.

Ans. **B, C**

Sol. **From option (A) :**

The mobility verses temperature diagram is given by,



The mobility of electrons increases upto 300 K temperature. If we increases temperature beyond 300 K then mobility of electrons start decreasing.

Hence, option (A) is incorrect.

From option (B) :

Electrons and holes are of equal density in an intrinsic semiconductor at equilibrium

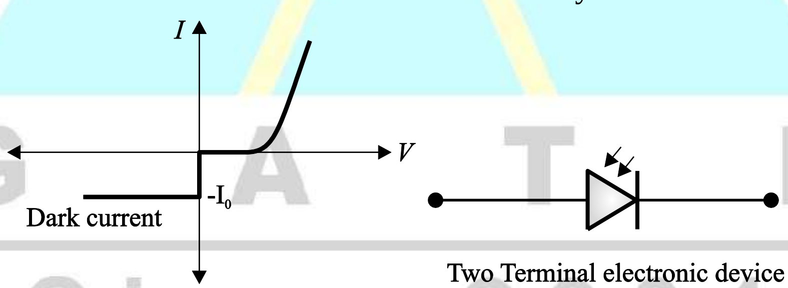
i.e. $n = p = n_i$

Hence, option (B) is correct.

Form option (C) :

Under dark: When there is no external illumination of light in two terminal electronic devices and in such condition only current flow due to minority charge carriers.

$V - I$ characteristic of two terminal device in dark under steady state condition is given by.



In under dark, $\frac{dI_0}{dV} = 0$

The total current is constant in a two terminal electronic device in dark under steady state condition.

Hence, option (C) is correct.

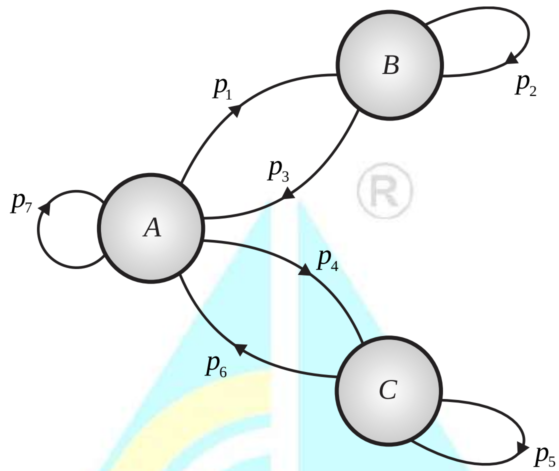
Form option (D) : In BJT the emitter region has the highest doping, the collector region is doped the lowest, the doping level in the base region is smaller than that in the emitter region but higher than that in the collector region.

$N_E >> N_B >> N_C$

Hence, the correct options are (B) and (C).

Question 48 (MSQ) **Digital Electronics**

A state transition diagram with states A, B, and C, and transition probabilities $p_1, p_2, \ldots, p_7$ is shown in the figure (e.g., p_1 denotes the probability of transition from state A to B). For this state diagram, select the statement(s) which is/are universally true.



(A) $p_2 + p_5 + p_7 = 1$

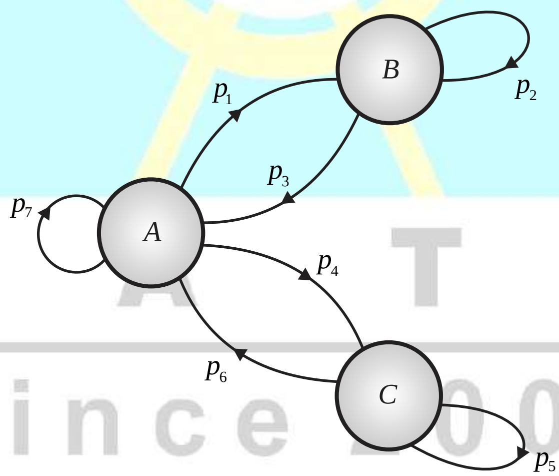
(B) $p_1 + p_4 + p_7 = 1$

(C) $p_1 + p_3 = p_4 + p_6$

(D) $p_2 + p_3 = p_5 + p_6$

Ans. **B, D**

Sol. Given state transition diagram is shown below,



Case - 1 : If Present State $(P.S.) = \text{A}$

Next state $(N.S.)$ is either A or B or C

So, transition probabilities of A, B and C is,

$$p_1 + p_4 + p_7 = 1$$

Case - 2 : If present state then next state will be A or B.

So, transition probabilities of A and B is,

$$p_2 + p_3 = 1$$

Case - 3 : If present state is C, then next state is *A* or *C*.

So, present state to next state probability is, $p_5 + p_6 = 1$

Then, $p_2 + p_3 = p_5 + p_6$

Hence, the correct options are (B) & (D).

Question 49 (MSQ) **Digital Electronics**

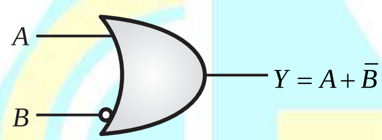
Consider a Boolean gate (D) where the output *Y* is related to the inputs *A* and *B* as, $Y = A + \bar{B}$, where '+' denotes logical OR operation. The Boolean inputs '0' and '1' are also available separately. Using instances of only D gates and inputs '0' and '1', (select the correct option(s)).

(A) AND logic cannot be implemented (B) NOR logic can be implemented

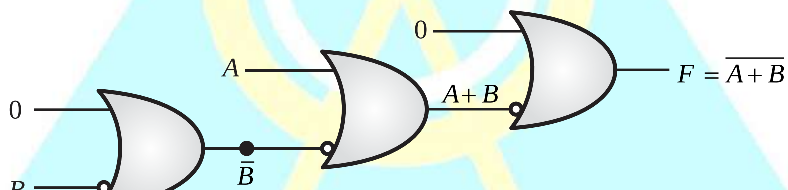
(C) OR logic cannot be implemented (D) NAND logic can be implemented

Ans. **B, D**

Sol. **Given :** $Y = A + \bar{B}$



Implementing NOR logic using $Y = A + \bar{B}$ as shown below,



We have implemented NOR gate by given logic $Y = A + \bar{B}$, which is universal gate, so we can implemented any logic given by logic i.e. $Y = A + \bar{B}$.

Hence, the correct options are (B) & (D).

Question 50 (MSQ) **Control System**

Two linear time-invariant systems with transfer functions

$$G_1(s) = \frac{10}{s^2 + s + 1} \text{ and } G_2(s) = \frac{10}{s^2 + s\sqrt{10} + 10}$$

have unit step responses $y_1(t)$ and $y_2(t)$, respectively. Which of the following statements is/are true?

(A) $y_1(t)$ and $y_2(t)$ have the same percentage peak overshoot.

(B) $y_1(t)$ and $y_2(t)$ have the same steady-state value.

(C) $y_1(t)$ and $y_2(t)$ have the same 2% settling time.

(D) $y_1(t)$ and $y_2(t)$ have the same damped frequency of oscillation.

Ans. **A**

Sol. Given transfer functions of two linear time invariant system.

$$G_1(s) = \frac{10}{s^2+s+1} \text{ and } G_2(s) = \frac{10}{s^2+\sqrt{10}s+10}$$

For transfer function $G_1(s)$:

$$G_1(s) = \frac{10}{s^2+s+1}$$

Characteristic equation of $G_1(s)$ is,

$$1+G_1(s) = 0$$

$$s^2+s+1=0 \qquad \ldots \text{(i)}$$

Standard characteristic equation for second order system is given by,

$$s^2+2\xi\omega_n s+\omega_n^2 = 0 \qquad \ldots \text{(ii)}$$

On comparing equation (i) and (ii),

$$2\xi\omega_n = 1,\ \omega_n^2 = 1$$

$$\omega_n = 1 \text{ rad/sec},\ \xi = 0.5$$

Maximum Peak Overshoot (MPO) is given by,

$$\text{MPO} = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$\text{MPO} = e^{-\pi(0.5)/\sqrt{1-(0.5)^2}} = e^{-\pi(0.5)/0.86} = 0.1630$$

$$\%\text{MPO} = 16.30\%$$

Settling time for 2% tolerance band is given by,

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.5} = 8 \text{ sec}$$

Damped frequency of oscillation is given by,

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 1\times\sqrt{1-(0.5)^2} = 0.86 \text{ rad/sec}$$

Steady state value of $y_1(t)$ is,

$$Y_1(s) = G_1(s)U(s) \qquad \left(\because\ U(s) = \frac{1}{s}\right)$$

$$Y_1(s) = \frac{1}{s}G_1(s) = \frac{1}{s}\left(\frac{10}{s^2+s+1}\right)$$

Apply final value theorem,

$$y_1(\infty) = \lim_{s\to 0} sY_1(s)$$

$$y_1(\infty) = \lim_{s\to 0} s\frac{\left(\frac{1}{s}\right)\times 10}{s^2+s+1} = \frac{10}{1} = 10 = 10$$

For transfer function $G_2(s)$:

$$G_2(s)=\frac{10}{s^2+\sqrt{10}s+10}$$

Characteristic equation of $G_2(s)$ is, $1+G_2(s)=0$

$$s^2+\sqrt{10}s+10=0 \quad \dots \text{(iii)}$$

Standard characteristic equation for second order system is given by,

$$s^2+2\xi\omega_n s+\omega_n^2=0 \quad \dots \text{(iv)}$$

Comparing equations (iii) and (iv),

$$2\xi\omega_n=\sqrt{10}, \qquad \omega_n^2=10$$

$$\omega_n=\sqrt{10}\text{ rad/sec}, \qquad \xi=\frac{\sqrt{10}}{2\sqrt{10}}=\frac{1}{2}=0.5$$

Maximum Peak Overshoot (MPO) is given by,

$$\text{MPO}=e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$\text{MPO}=e^{-\pi(0.5)/\sqrt{1-(0.5)^2}}=0.1630$$

$$\%\text{MPO}=16.30\%$$

Settling time for 2% tolerance band is given by,

$$t_s=\frac{4}{\xi\omega_n}=\frac{4}{0.5\sqrt{10}}=\frac{4}{1.58}$$

$$t_s=2.53\text{ sec}$$

Damped frequency of oscillation is given by

$$\omega_d=\omega_n\sqrt{1-\xi^2}$$

$$\omega_d=\sqrt{10}\times\sqrt{1-(0.5)^2}$$

$$\omega_d=2.73\text{ rad/sec}$$

Steady state value of $y_2(t)$ is,

$$Y_2(s)=G_2(s)U(s) \qquad \left(\because U(s)=\frac{1}{s}\right)$$

$$Y_2(s)=\frac{1}{s}G_2(s)$$

Apply final value theorem,

$$y_2(\infty)=\lim_{s\to 0} sY_2(s)$$

$$y_2(\infty)=\lim_{s\to 0} s\frac{\left(\frac{1}{s}\right)\times 10}{s^2+\sqrt{10}s+10}=\frac{10}{10}=1$$

So, $y_1(t)$ and $y_2(t)$ have the same percentage overshoot.

Hence, the correct option is (A).

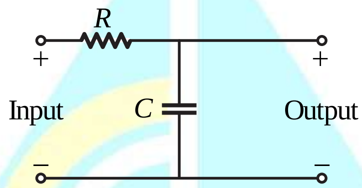
Note : Maximum Peak Overshoot (MPO) depends on the value of ξ. For same value of ξ, Maximum Peak Overshoot (MPO) will be same.

Question 51 (MSQ) **Communication**

Consider an FM broadcast that employs the pre-emphasis filter with frequency response.

$$H_{pe}(\omega) = 1 + \frac{j\omega}{\omega_0}, \qquad \text{where } \omega_0 = 10^4 \text{ rad/sec}$$

For the network shown in the figure to act as a corresponding de-emphasis filter, the appropriate pair(s) of (R, C) values is/are



(A) $R = 2\text{ k}\Omega, C = 1\ \mu\text{F}$ (B) $R = 1\text{ k}\Omega, C = 0.1\ \mu\text{F}$

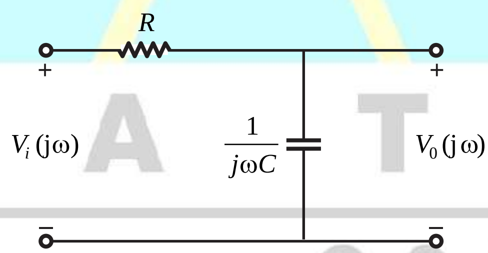
(C) $R = 1\text{ k}\Omega, C = 2\ \mu\text{F}$ (D) $R = 2\text{ k}\Omega, C = 0.5\ \mu\text{F}$

Ans. **B**

Sol. **Given :** Frequency response of pre-emphasis filter is,

$$H_{pre}(\omega) = 1 + j\frac{\omega}{\omega_0} \text{ and } \omega_0 = 10^4 \text{ rad/sec}$$

The R-C circuit as shown below



Transfer function of the above R-C circuit will be

$$H(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)} = \frac{1}{1 + j\omega RC}$$

The given R-C circuit to act as de-emphasis filter

$$|H(j\omega)| = \frac{1}{|H_{pre}(\omega)|}$$

$$\frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$1+(\omega RC)^2 = 1+\left(\frac{\omega}{\omega_0}\right)^2$$

$$\omega^2 (RC)^2 = \frac{\omega^2}{\omega_0^2}$$

$$\omega_0^2 = \frac{1}{(RC)^2}$$

$$\omega_0 = \frac{1}{RC} = 10^4 \text{ rad/sec}$$

Thus, $\omega_0 = 10^4$ rad/sec only possible if we choose $R = 1\text{k}\Omega$ and $C = 0.1\mu\text{F}$ from options.

Hence, the correct option is (B).

Question 52 (MSQ) **Electromagnetic Theory**

A waveguide consists of two infinite parallel plates (perfect conductors) at a separation of 10^{-4} cm, with air as the dielectric. Assume the speed of light in air to be 3×10^8 m/s. The frequency/frequencies of TM waves which can propagate in this waveguide is/are

(A) 0.5×10^{12} Hz (B) 6×10^{15} Hz

(C) 8×10^{14} Hz (D) 1×10^{13} Hz

Ans. A, B, C, D

Sol. For parallel plate wave guide

It passes all of the frequency components.

Hence, the correct options are (A), (B), (C) and (D).

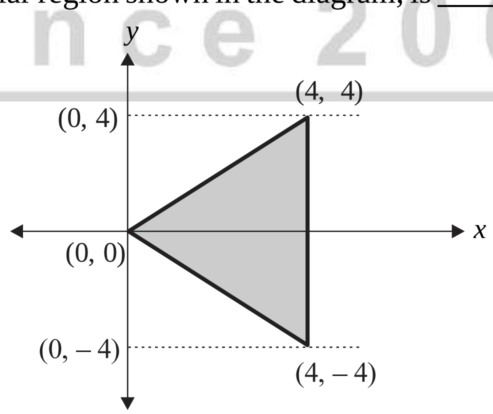
Note : All options of this Questions are correct, but IIT Kharagpur provides MTA for this Question.

Question 53 **Mathematics**

The value of the integral

$$\iint_D 3(x^2+y^2)\,dx\,dy,$$

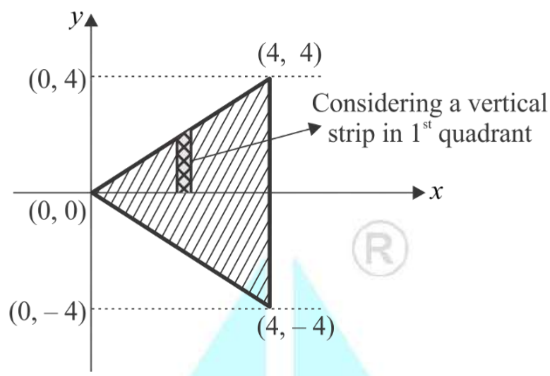
where D is the shaded triangular region shown in the diagram, is ______. (round off to the nearest integer)



Ans. 512 (Range : 512 to 512)

Sol. **Given :** $I = \iint_D 3(x^2+y^2)\,dxdy$

Consider a vertical strip in 1st quadrant as shown below,



$$I = 2\times\int_{x=0}^{x=4}\int_{y=0}^{y=x} 3(x^2+y^2)\,dydx$$

$$I = 6\int_0^4\int_0^x (x^2+y^2)\,dydx = 6\int_0^4\left[x^2y+\frac{y^3}{3}\right]_0^x dx$$

$$I = 6\int_0^4\left[\left(x^3+\frac{x^3}{3}\right)-(0+0)\right]dx$$

$$I = 6\int_0^4\left(\frac{4x^3}{3}\right)dx = \frac{6\times 4}{3}\left[\frac{x^4}{4}\right]_0^4$$

$$I = 512$$

Hence, the correct answer is 512.

Question 54 **Network Theory**

A linear **2**-port network is shown in Fig. (a). An ideal DC voltage source of **10 V** is connected across Port **1**. A variable resistance R is connected across Port **2**. As R is varied, the measured voltage and current at Port **2** is shown in Fig. (b) as a V_2 versus $-I_2$ plot. Note that for $V_2 = 5$ V, $I_2 = 0$ mA, and for $V_2 = 4$ V, $I_2 = -4$ mA. When the variable resistance R at Port **2** is replaced by the load shown in Fig. (c), the current I_2 is _______ mA (round off to one decimal place).

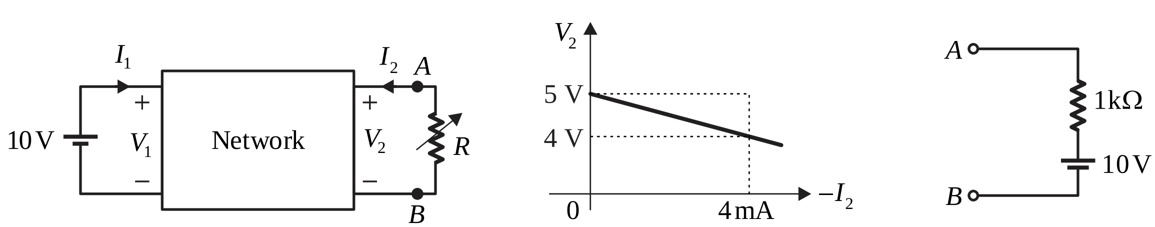


Fig. (a) **Fig. (b)** **Fig. (c)**

Ans. **4 (Range : 3.9 to 4.1)**

Sol. Given linear two port network, its voltage and current plot at port 2 [V_2 versus $-I_2$] and load circuit is shown below,

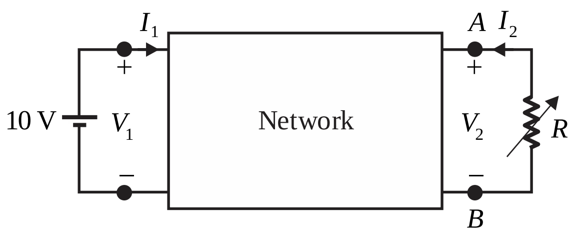


Figure (a)

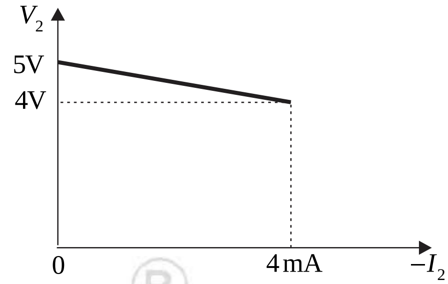


Figure (b)

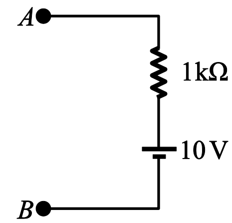


Figure (c)

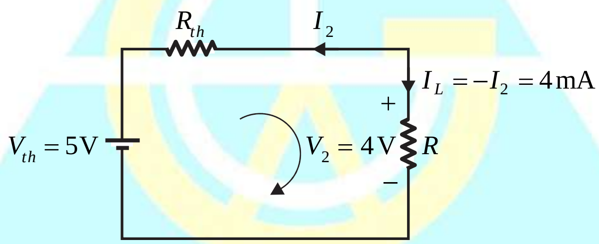
Open circuit voltage across terminal AB,

$V_{OC} = V_{th} = 5\text{V}$ [From V_2 and $-I_2$ plot where $I_2 = 0$]

From V_2 and $-I_2$ plot,

For $V_2 = 4\text{V}, I_2 = -4\,\text{mA}$

Thus, figure (a) can be replaced by thevenin's equivalent circuit as,



Applying KVL in loop shown,

$$-5+4+R_{th}(4\times10^{-3})=0$$

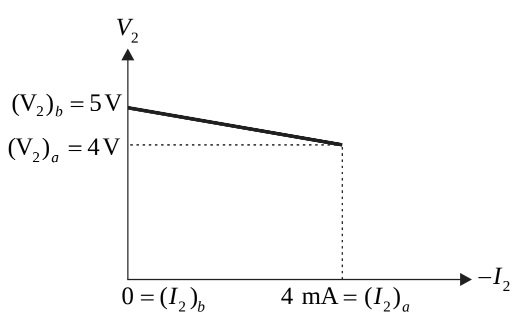
$$1=R_{th}(4\times10^{-3})$$

$$R_{th}=\frac{1}{4\times10^{-3}}=250\,\Omega$$

Alternate method to calculate R_{th} :

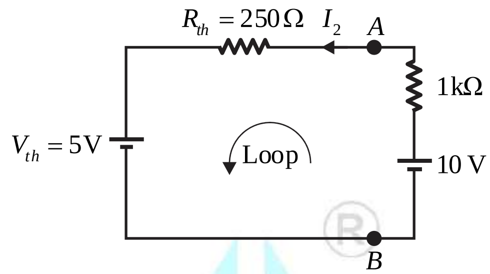
Here, V_2 Vs $-I_2$ characteristic is given, so slope of characteristics gives R_{th} across port-2.

$$R_{th}=\frac{\Delta V_2}{\Delta I_2}=\frac{(V_2)_a-(V_2)_b}{(I_2)_a-(I_2)_b}$$



$$R_{th} = \frac{4-5}{-4\times10^{-3}-0} = 250\ \Omega$$

Replace the resistance R with the load circuit given in figure (c) in thevenin's equivalent circuit.



Applying KVL in loop shown,

$$-10+1000I_2+250I_2+5=0$$

$$10-5=(1000+250)I_2$$

$$I_2 = \frac{5}{1250}$$

$$I_2 = 4\,\text{mA}$$

Hence, the value of current I_2 is 4.

Question 55 **Signals & Systems**

For a vector $\bar{x} = \lfloor x[0],\ x[1],......x[7] \rfloor$, the 8-point discrete Fourier transform (DFT) is denoted by

$\overline{X} = \text{DFT}(\bar{x}) = [X[0],\ X[1],....,\ X[7]]$, where

$$X[k] = \sum_{n=0}^{7} x[n]\exp\left(-j\frac{2\pi}{8}nk\right)$$

Here, $j=\sqrt{-1}$. If $\bar{x} = [1,0,0,0,2,0,0,0]$ and $\bar{y} = \text{DFT}(\text{DFT}(\bar{x}))$, then the value of $y[0]$ is ________ (round off to one decimal place).

Ans. **8 (Range : 7.9 to 8.1)**

Sol. **Given :** Vector $\bar{x} = [1,\ 0,\ 0,\ 0,\ 2,\ 0,\ 0,\ 0]$

$$\text{DFT}\,[\bar{x}] = \sum_{n=0}^{7}\bar{x}(n)e^{\frac{-j2\pi kn}{8}} = \bar{x}(0)+\bar{x}(4)e^{-j\pi k} = 1+2(-1)^k$$

As k is varying from 0 on to 7.

$$\text{DFT}\,[\bar{x}] = [3,\ -1,\ 3,\ -1,\ 3,\ -1,\ 3,\ -1]$$

Also given, $y = \text{DFT}\,(\text{DFT}\,(\bar{x}))$

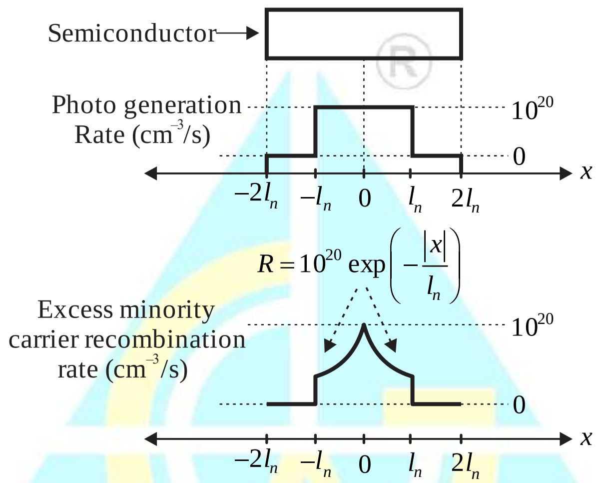
As the value of DFT at origin is sum of the samples of the original sequence.

$$y(0) = 3-1+3-1+3-1+3-1 = 8$$

Hence, the correct answer is 8.

Question 56 **Electronic Devices**

A p-type semiconductor with zero electric field is under illumination (low level injection) in steady state condition. Excess minority carrier density is zero at $x=\pm 2l_n$, where $l_n=10^{-4}$ cm is the diffusion length of electrons. Assume electronic charge, $q=-1.6\times10^{-19}$ C. The profiles of photo-generation rate of carriers and the recombination rate of excess minority carriers (R) are shown. Under these conditions, the magnitude of the current density due to the photo-generated electrons at $x=+2l_n$ is _________ mA/cm^2 (round off to two decimal places).



Ans. **0.58 (Range : 0.57 to 0.61)**

Sol. **Given :**

(i) p- Type semiconductor

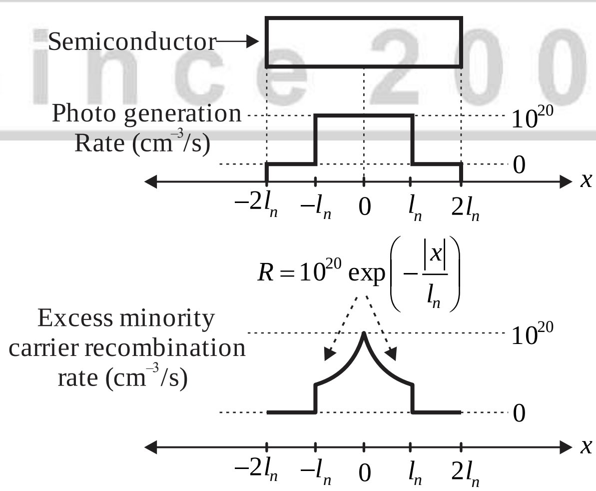
(ii) Diffusion length of electrons, $l_n=10^{-4}$ cm

(iii) Photo generation exist only between $-l_n$ to l_n

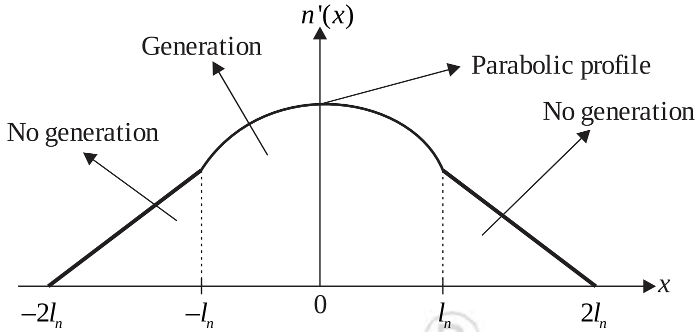
(iv) Rate of recombination is exponential decay.

(v) No recombination and no generation exist between l_n to $2l_n$ and $-l_n$ to $-2l_n$

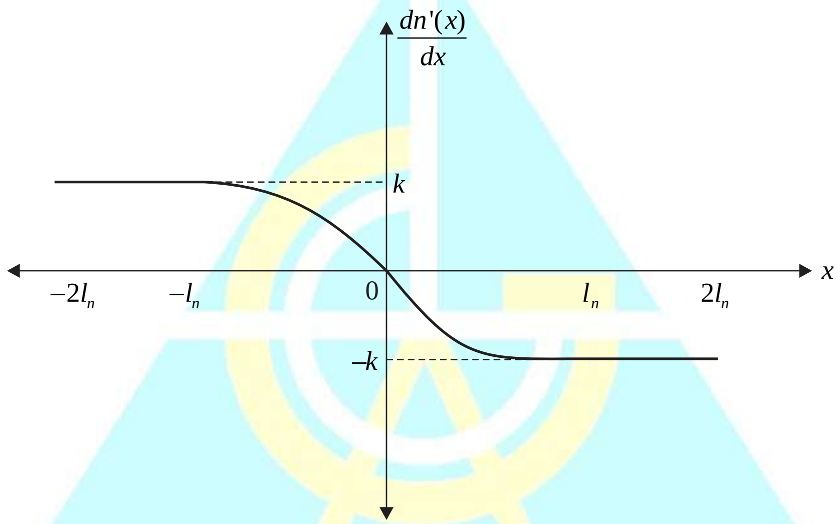
Given diagram is shown below.



Excess minority carrier concentration is given by,



Differentiating excess minority carrier concentration $n'(x)$ profile.



Here, $\frac{dn'(x)}{dx}$ at $x = 0$ is zero.

Electron current density at $x = 2l_n$ is given by,

$$J_{n(drift)} = \left.\frac{qD_n dn'(x)}{dx}\right|_{x=2l_n}$$

$$J_{n(drift)} = qD_n(-k)$$

From continuity equation,

$$\frac{dn(t)}{dt} = D_n \frac{d^2n'(x)}{dx^2} + (G - R) \quad \text{...(i)}$$

At steady state, $\frac{dn(t)}{dt} = 0$

Hence, equations (i) becomes,

$$D_n \frac{d^2n'(x)}{dx^2} + (G - R) = 0$$

$$D_n \frac{d^2n'(x)}{dx^2} = -(G - R) \quad \text{... (ii)}$$

Where, $G =$ Photo generation rate $(\text{cm}^{-3}/\text{s})$, $R =$ Excess minority carrier recombination rate $(\text{cm}^{-3}/\text{s})$.

In the range, $-l_n < x < 0$

$$G = 10^{20}\,\text{cm}^{-3}/\text{s}$$

$$R = 10^{20} e^{+\frac{x}{l_n}}$$

From equation (ii),

$$\frac{D_n d^2 n'(x)}{dx^2} = -\left(10^{20} - 10^{20} e^{x/\ln}\right)$$

$$\frac{D_n d^2 n'(x)}{dx^2} = -10^{20} + 10^{20} e^{x/l_n}$$

Integrating both sides,

$$\frac{D_n dn'(x)}{dx} = -10^{20} x + 10^{20} l_n e^{x/l_n} + C \qquad \dots \text{(iii)}$$

Since at $x = 0$, $\frac{dn'(x)}{dx} = 0$

Putting $x = 0$ and $\frac{dn'(x)}{dx} = 0$ in equations (iii).

$$-10^{20} \times 0 + 10^{20} l_n e^{x/l_n} + C = D_n \times 0$$

$$10^{20} l_n + C = 0$$

$$C = -10^{20} l_n$$

Putting $C = -10^{20} l_n$ in equation (iii).

From excess minority carrier concentration profile,

$\frac{dn'(x)}{dx}$ at $x = -l_n$ is given by, $\frac{dn'(x)}{dx} = +k$

From equation (iii),

$$D_n k = 10^{20} l_n + 10^{20} l_n e^{-l_n/l_n} - 10^{20} l_n$$

$$D_n k = 10^{20} l_n e^{-1}$$

Electron current density at $x = 2l_n$ is given by,

$$J_{n(drift)} = -qD_n k$$

$$J_{n(drift)} = -1.6 \times 10^{-19} \times 10^{20} l_n e^{-1}$$

$$J_{n(drift)} = -1.6 \times 10 \times 10^{-4} \times e^{-1}$$

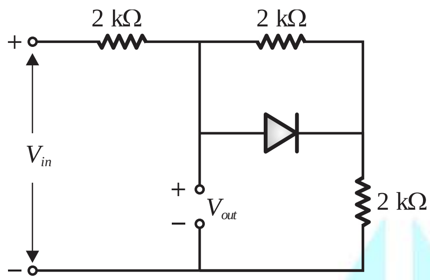
$$J_{n(drift)} = -0.588\,\text{mA/cm}^2$$

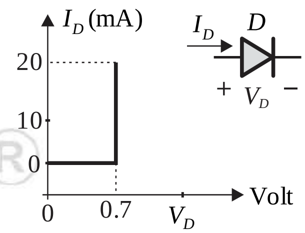
$$\left|J_{n(drift)}\right| = 0.588\ \text{mA/cm}^2$$

Hence, the correct answer is 0.58.

Question 57 **Analog Electronics**

A circuit and the characteristics of the diode (D) in it are shown. The ratio of the minimum to the maximum small signal voltage gain $\frac{\partial V_{out}}{\partial V_{in}}$ is ________. (round off to two decimal places)





Ans. **0.75 (Range : 0.70 to 0.80)**

Sol. Given circuit shown below,

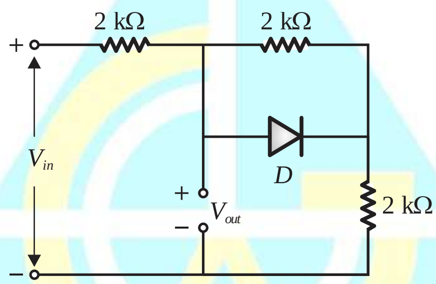


Fig. (a)

From the given I_D Vs. V_D characteristics,

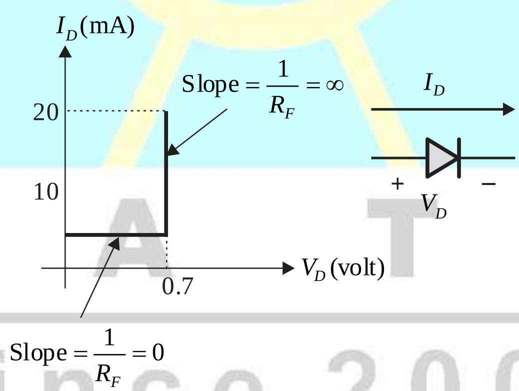
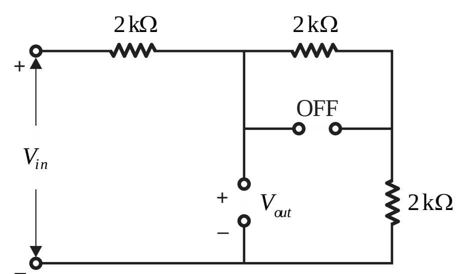


Fig. (b)

Slope of I_D Vs. V_D characteristics shows inverse of forward resistance of diode.

Case 1 : If $V_{in} < 0.7$ V, then diode is OFF so that R_F is ∞ (open circuit), then given circuit becomes,



Applying voltage division rule,

$$V_{out} = \frac{V_{in} \times 4}{2+2+2} = \frac{2V_{in}}{3}$$

$$V_{out} = \frac{2}{3} V_{in}$$

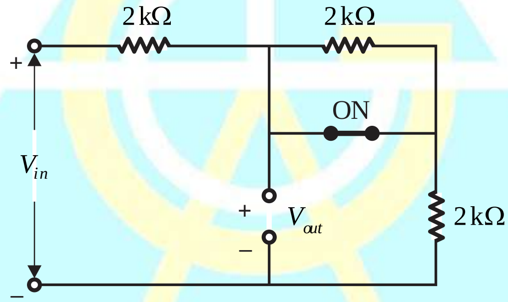
$$\frac{V_{out}}{V_{in}} = \frac{2}{3} \qquad \text{(Constant)}$$

Thus,

$$\left.\frac{\partial V_{out}}{\partial V_{in}}\right|_{max} = \frac{2}{3} \qquad \ldots\text{(i)}$$

Here, $\frac{\partial V_{out}}{\partial V_{in}}$ is become maximum because $V_{in} < 0.7\text{ V}$.

Case 2 : If $V_{in} > 0.7\,V$, then diode is ON so, that R_F is $0\ \Omega$ (short circuit) so, given circuit becomes,



Applying voltage division rule,

$$V_{out} = \frac{2k}{4k} V_{in} = \frac{1}{2} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2} \qquad \text{(Constant)}$$

Thus, $$\left.\frac{\partial V_{out}}{\partial V_{in}}\right|_{min} = \frac{1}{2} \qquad \ldots\text{(ii)}$$

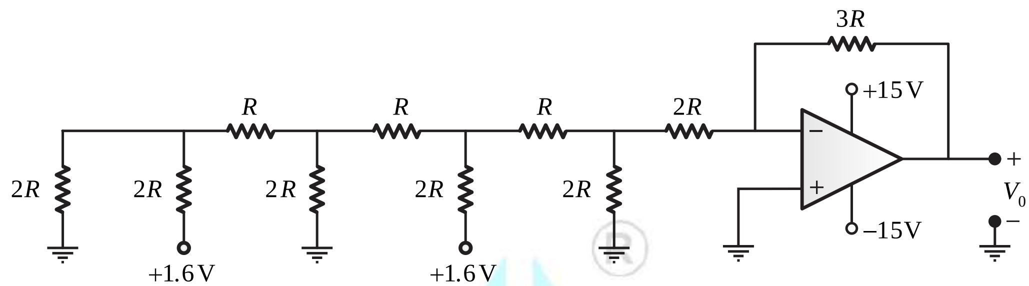
Here, $\frac{\partial V_{out}}{\partial V_{in}}$ is minimum because $V_{in} > 0.7\text{ V}$.

Thus, $$\frac{\left.\frac{\partial V_{out}}{\partial V_{in}}\right|_{min}}{\left.\frac{\partial V_{out}}{\partial V_{in}}\right|_{max}} = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{3}{4} = 0.75$$

Hence, the correct answer is 0.75.

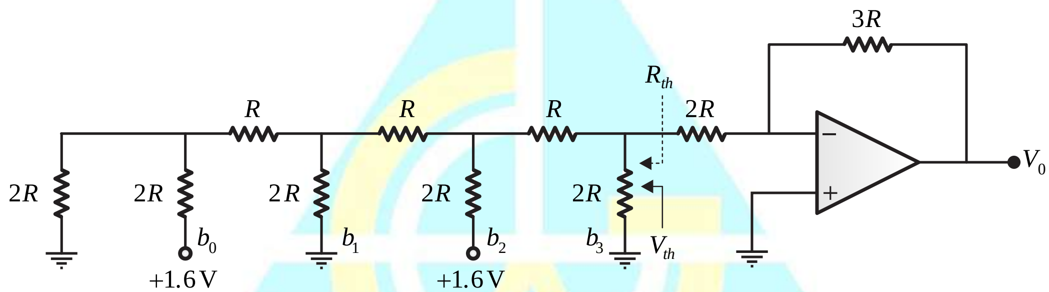
Question 58 **Analog Electronics**

Consider the circuit shown with an ideal OPAMP. The output voltage V_0 is _______V. (round off to two decimal places) $R = 1\text{ k}\Omega$



Ans. – 0.5 (Range : – 0.55 to – 0.45)

Sol. Given Op-Amp circuit is shown below,



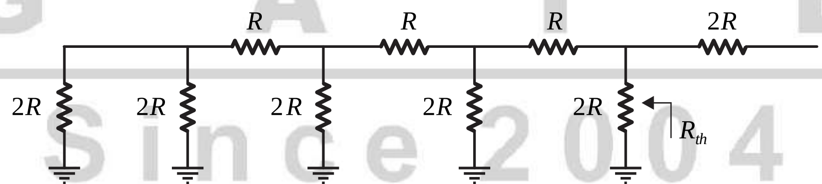
Calculation of V_{th} :

Generalized form of V_{th} is,

$$V_{th} = \frac{2^0 b_0 V_{CC} + 2^1 b_1 V_{CC} + 2^2 b_2 V_{CC} + 2^3 b_3 V_{CC}}{2^4}$$

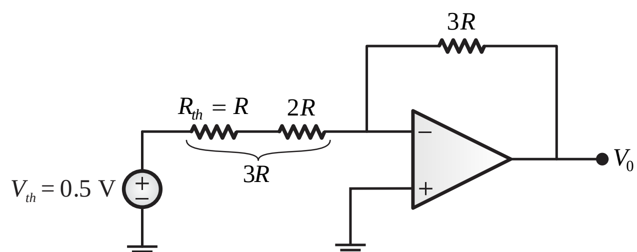
$$V_{th} = \frac{(1\times 1.6\text{ V}) + (4\times 1.6\text{ V})}{16} = 0.5\text{ V}$$

Calculation of R_{th} :



$$R_{th} = \left\{\left[\left\{\left[\{[(2R \,||\, 2R) + R] \,||\, 2R\} + R\right] || \, 2R\right\} + R\right] || \, 2R\right\} = R$$

Equivalent circuit will be,



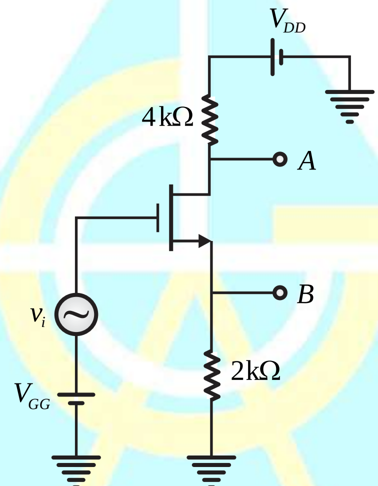
Output of above inverting Op-Amp is,

$$V_0 = \frac{-3R}{3R} \times 0.5 = -0.5 \text{ V}$$

Hence, the correct answer is – 0.5 V.

Question 59 **Electronic Devices**

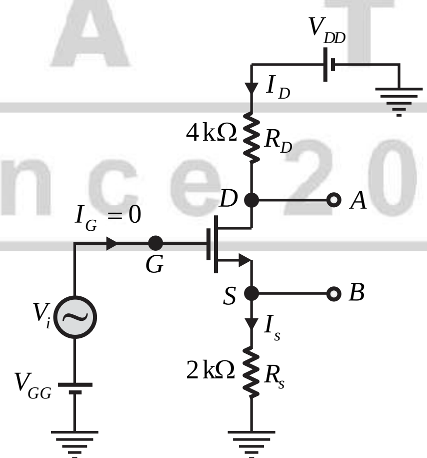
Consider the circuit shown with an ideal long channel nMOSFET (enhancement-mode, substrate is connected to the source). The transistor is appropriately biased in the saturation region with V_{GG} and V_{DD} such that it acts as a linear amplifier. v_i is the small-signal ac input voltage. v_A and v_B represent the small-signal voltages at the nodes A and B, respectively. The value of $\frac{v_A}{v_B}$ is _______. (round off to one decimal place)



Ans. **– 2 (Range : – 2.1 to – 1.9)**

Sol. **Method 1**

Given circuit is shown below,



From above figure,

$$I_G = 0$$

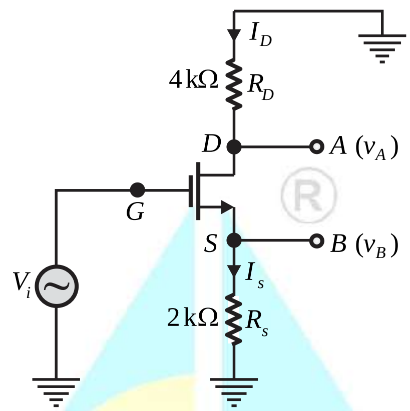
So, $I_D = I_S$

AC equivalent circuit :

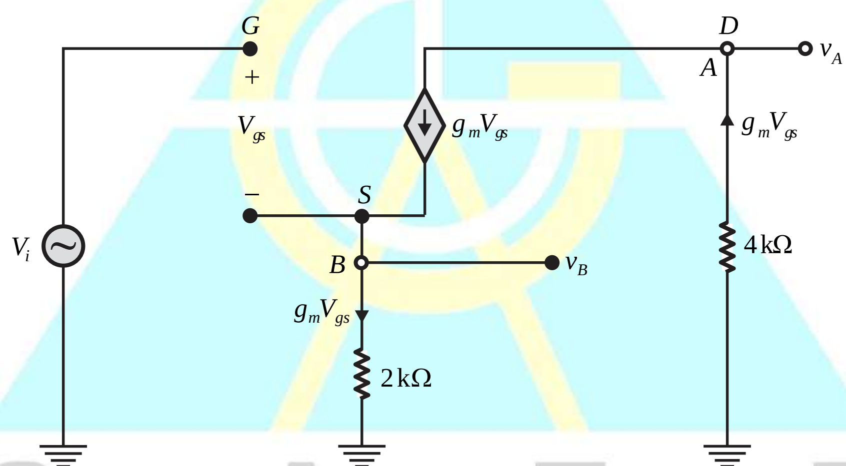
(i) All capacitors are short circuited.

(ii) All DC voltage sources are replaced by short circuit.

(iii) All DC current sources are replaced by open circuit.



Small signal equivalent circuit is shown below,



Voltage at A, $v_A = -R_D g_m V_{gs}$ …(i)

Voltage at B, $v_B = R_s g_m V_{gs}$ …(ii)

Thus, $\dfrac{v_A}{v_B} = \dfrac{-R_D g_m V_{gs}}{R_s g_m V_{gs}} = \dfrac{-R_D}{R_s} = \dfrac{-4}{2} = -2$

Hence, the correct answer is – 2.

Method 2

Case 1 : Consider output at point-A only i.e. v_A, so given circuit behave as common source amplifier whose small signal equivalent circuit is shown below,

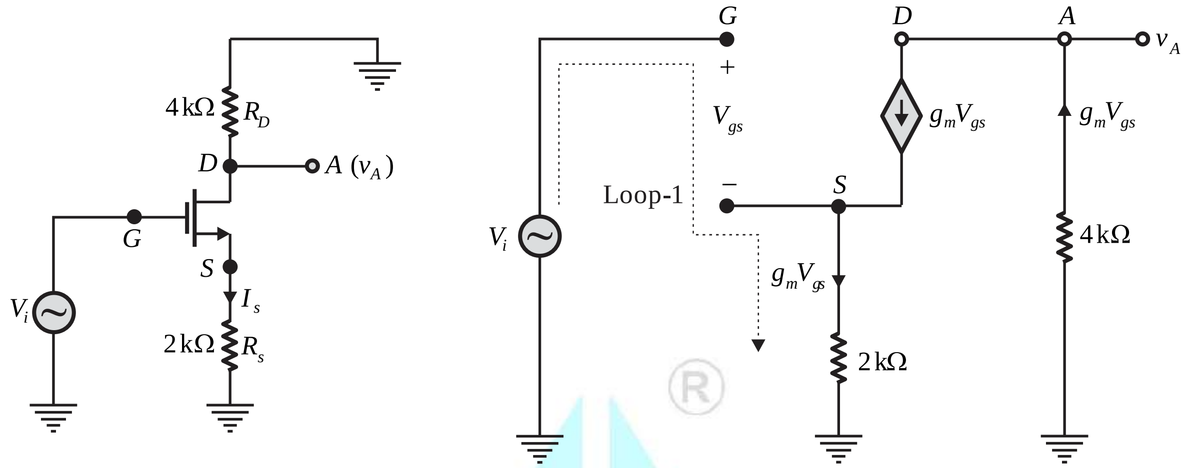


Fig. Common source amplifier

Apply KVL in loop-1,

$$-V_i + V_{gs} + R_s g_m V_{gs} = 0$$

$$V_i = (1 + R_s g_m)V_{gs} \qquad \ldots\text{(i)}$$

Voltage at point-A is,

$$v_A = -R_D g_m V_{gs} \qquad \ldots\text{(ii)}$$

Thus, $$\frac{v_A}{V_i} = \frac{-R_D g_m}{1 + R_s g_m} \qquad \ldots\text{(iii)}$$

Case 2 : Consider output at point-B only i.e. v_B so given circuit behave as common drain amplifier, whose small signal equivalent circuit is shown below,

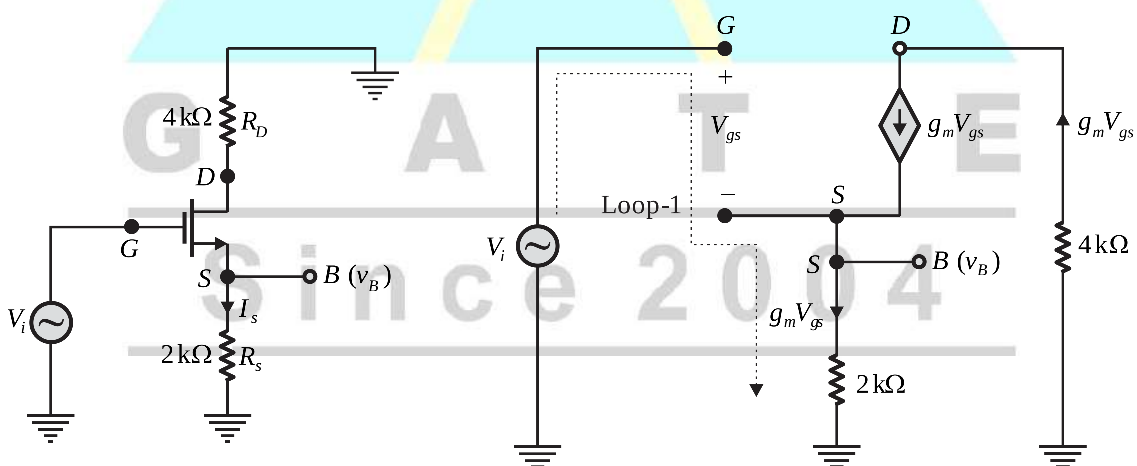


Fig. Common drain amplifier

Apply KVL is loop-1,

$$-V_i + V_{gs} + R_s g_m V_{gs} = 0$$

$$V_i = (1 + R_s g_m)V_{gs} \qquad \ldots\text{(iv)}$$

Voltage at point-B,

$$v_B = R_s g_m V_{gs} \quad \ldots(v)$$

Thus, $$\frac{v_B}{V_i} = \frac{R_s g_m}{1+R_s g_m} \quad \ldots(vi)$$

From equation (iii) and (vi),

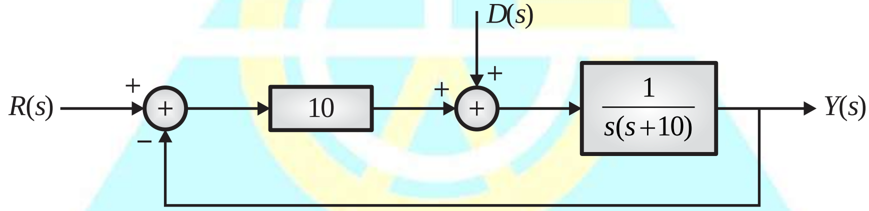
$$\frac{v_A}{V_i} \times \frac{V_i}{v_B} = \frac{-R_D g_m}{1+R_s g_m} \times \frac{1+R_s g_m}{R_s g_m}$$

$$\frac{v_A}{v_B} = \frac{-R_D}{R_s} = \frac{-4}{2} = -2$$

Hence, the correct answer is – 2.

Question 60 **Control System**

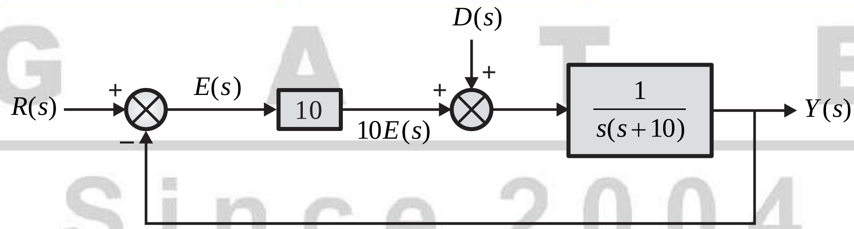
The block diagram of a closed-loop control system is shown in the figure. $R(s)$, $Y(s)$, and $D(s)$ are the Laplace transforms of the time-domain signals $r(t)$, $y(t)$, and $d(t)$, respectively. Let the error signal be defined as $e(t) = r(t) - y(t)$. Assuming the reference input $r(t) = 0$ for all t, the steady-state error $e(\infty)$, due to a unit step disturbance $d(t)$, is ________ (round off to two decimal places).



Ans. **– 0.1 (Range : – 0.11 to – 0.09)**

Sol. **Method 1**

Given block diagram of a closed loop control system is shown below,



Given : $r(t) = 0$ and $d(t) = u(t)$

Error is given by

$$e(t) = r(t) - y(t)$$

Taking Laplace transform of above equation

$$E(s) = R(s) - Y(s)$$

According to the equation

Reference input $r(t) = 0$

i.e. $R(s) = 0$

Substituting the value of $R(s)=0$, in the above equation of $E(s)$

$$E(s)=R(s)-Y(s)$$

$$R(s)=0$$

$$E(s)=-Y(s) \quad \ldots \text{(i)}$$

From the block diagram

$$Y(s)=[10E(s)+D(s)]\frac{1}{s(s+10)} \quad \ldots \text{(ii)}$$

Substituting the value of $Y(s)$ from equation (ii) in equation (i),

$$E(s)=-Y(s)$$

$$-E(s)=[10E(s)+D(s)]\frac{1}{s(s+10)}$$

$$-E(s)=\frac{10}{s(s+10)}E(s)+\frac{D(s)}{s(s+10)}$$

$$-E(s)-\frac{10E(s)}{s(s+10)}=\frac{D(s)}{s(s+10)}$$

$$-E(s)\left[1+\frac{10}{s(s+10)}\right]=\frac{D(s)}{s(s+10)}$$

$$-E(s)\left[\frac{s^2+10s+10}{s(s+10)}\right]=\frac{D(s)}{s(s+10)}$$

$$-E(s)[s^2+10s+10]=D(s)$$

$$E(s)=\frac{-D(s)}{s^2+10s+10} \quad \ldots \text{(iii)}$$

Given $d(t)=u(t)$

Hence, $D(s)=\frac{1}{s}$

Substituting the value of $D(s)=\frac{1}{s}$ in equation (iii)

$$E(s)=\frac{-1/s}{s^2+10s+10}$$

Steady state error is given by,

$$e_{ss}=e(\infty)=\lim_{s\to 0} s\,E(s)$$

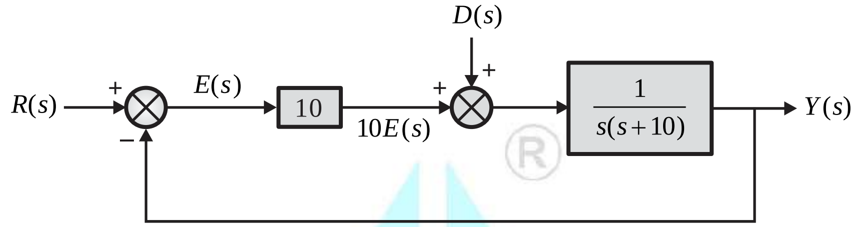
$$e_{ss}=e(\infty)=\lim_{s\to 0} s\left(\frac{\frac{-1}{s}}{s^2+10s+10}\right)$$

$$e(\infty) = \lim_{s \to 0} \frac{-1}{s^2 + 10s + 10} = \frac{-1}{10} = -0.1$$

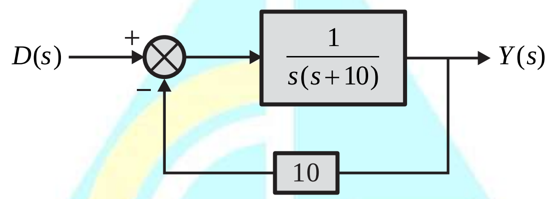
Hence, the steady state error $e(\infty)$ due to unit step disturbance $d(t)$ is -0.1.

Method 2

Given block diagram of a closed loop control system is shown below,



Here, $R(s) = 0$, so above block diagrams can reduced as,



Thus, transfer function of above block diagram is,

$$\frac{Y(s)}{D(s)} = \frac{1}{s(s+10)+10} \qquad \ldots\text{(i)}$$

According to question,

Error signal, $e(t) = r(t) - y(t)$

$$e(t) = -y(t) \qquad [\because r(t) = 0]$$

$$E(s) = -Y(s)$$

So, equation (i) becomes as,

$$-E(s) = \frac{D(s)}{s(s+10)+10}$$

Steady-state error $e(\infty)$ under unit step disturbance $\left[D(s) = \frac{1}{s}\right]$ is,

$$e(\infty) = \lim_{s \to 0} sE(s)$$

$$e(\infty) = \lim_{s \to 0} s\left[\frac{-D(s)}{s(s+10)+10}\right] \qquad \left[\because D(s) = \frac{1}{s}\right]$$

$$e(\infty) = \lim_{s \to 0} s\left[\frac{-\frac{1}{s}}{s(s+10)+10}\right]$$

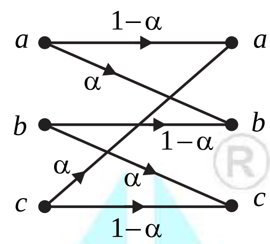
$$e(\infty) = -\frac{1}{10} = -0.1$$

Hence, the steady state error $e(\infty)$ due to unit step disturbance $d(t)$ is -0.1.

Question 61 **Communication**

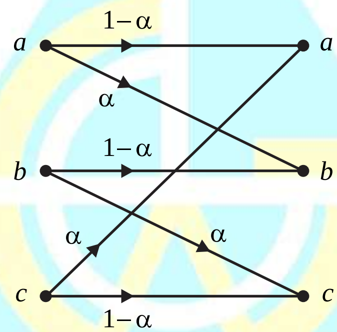
The transition diagram of a discrete memoryless channel with three input symbols and three output symbols is shown in the figure. The transition probabilities are as marked.

The parameter α lies in the interval [0.25, 1]. The value of α for which the capacity of this channel is maximized is ________ (round off to two decimal places).



Ans. **1 (Range : 1.00 to 1.00)**

Sol. Given channel is a symmetric channel as shown below,



For a symmetric channel having N inputs, the channel capacity is,

$$C = \log_2 N + P\log_2 P + (1-P)\log_2(1-P)$$

Where, P is the probability. In this problem $N = 3$ and $P = \alpha$.

$\therefore$ Channel capacity will be

$$C = \log_2 3 + \alpha\log_2 \alpha + (1-\alpha)\log_2(1-\alpha)$$

Differentiating both sides with respect to α, we get

$$\frac{dC}{d\alpha} = \frac{1}{\log 2}\left[\alpha \times \frac{1}{\alpha} + \log\alpha + 1 - \alpha \times \frac{1}{1-\alpha} \times -1 + \log(1-\alpha) \times -1\right]$$

$$\frac{dC}{d\alpha} = \frac{1}{\log 2}[1 + \log\alpha - 1 - \log(1-\alpha)]$$

$$\frac{dC}{d\alpha} = \frac{1}{\log 2}[\log\alpha - \log(1-\alpha)]$$

For maxima or minima $\dfrac{dC}{d\alpha} = 0$

$$\log\alpha = \log(1-\alpha)$$

$$\alpha = 1-\alpha$$

$$2\alpha = 1$$

$$\alpha = \frac{1}{2}$$

$$\frac{d^2C}{d\alpha^2} = \frac{1}{\log 2}\left[\frac{1}{\alpha} + \frac{1}{1-\alpha}\right]$$

$$\left.\frac{d^2C}{d\alpha^2}\right|_{\alpha=\frac{1}{2}} = \frac{1}{\log 2}[2+2] = \frac{4}{\log 2} > 0$$

$\therefore$ Channel capacity will be minimum if $\alpha = \frac{1}{2}$

The minimum value of channel capacity will be

$$C_{\min} = \log_2 3 + \frac{1}{2}\log_2 \frac{1}{2} + \frac{1}{2}\log_2 \frac{1}{2}$$

$$C_{\min} = \log_2 3 - \frac{1}{2}\log_2 2 - \frac{1}{2}\log_2 2$$

$$C_{\min} = \log_2 - \frac{1}{2} - \frac{1}{2}$$

$$C_{\min} = \log_2 3 - 1$$

$$C_{\min} = 0.584$$

If $\alpha = 1$ then

$$C = \log_2 3 + 1\log_2 1 + 0\log_2 0$$

$$C = \log_2 3 + 0 = \log_2 3 = 1.584$$

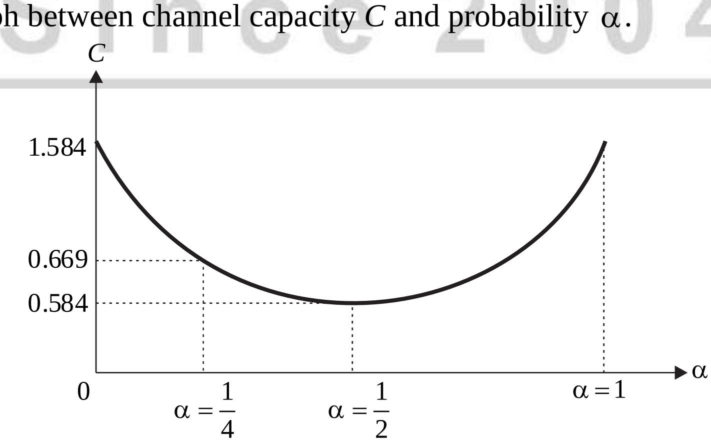
If $\alpha = 0.25 = \frac{1}{4}$, then

$$C = \log_2 3 + \frac{1}{4}\log_2 \frac{1}{4} + \frac{3}{4}\log_2 \frac{3}{4}$$

$$C = \log_2 3 - 0.5 - 0.415$$

$$C = 1.584 - 0.5 - 0.415 = 0.669$$

On plotting the graph between channel capacity C and probability α.



$\therefore$ We can conclude that if $\alpha = 1$, then the channel capacity will be maximum.

Hence, the correct answer is 1.

Key Point

For a symmetric channel having N inputs the channel capacity is

$C = \log_2 N + P\log_2 P + (1-P)\log_2(1-P)$ where P is the probability.

Question 62 **Communication**

Consider communication over a memoryless binary symmetric channel using a (7, 4) Hamming code. Each transmitted bit is received correctly with probability $(1-\varepsilon)$, and flipped with probability ε. For each codeword transmission, the receiver performs minimum Hamming distance decoding, and correctly decodes the message bits if and only if the channel introduces at most one bit error. For $\varepsilon = 0.1$, the probability that a transmitted codeword is decoded correctly is _________ (round off to two decimal places).

Ans. 0.850 (Range : 0.84 to 0.86)

Sol. Given that the bits are transmitted using (7, 4) hamming code

$\therefore$ Total number of bits $n = 7$.

Given that the transmitted bit are received correctly with probability $1-\varepsilon$.

So, probability of receiving the bits wrongly will be $1-(1-\varepsilon) = \varepsilon$

Let, $p = \varepsilon$ and $q = 1-\varepsilon$

$p = 0.1$ and $q = 1-0.1 = 0.9$ $\qquad (\because \varepsilon = 0.1)$

If X is the event of transmitting the bits, then the probability of at most one bit error will be,

$$P(X \le 1) = P(X = 0) + P(X = 1)$$

$$P(X \le 1) = {}^7C_0\, p^0 q^{7-0} + {}^7C_1 p^1 q^{7-1}$$

$$P(X \le 1) = {}^7C_0 p^0 q^7 + {}^7C_1 p^1 q^6$$

$$P(X \le 1) = q^7 + 7pq^6$$

$$P(X \le 1) = (0.9)^7 + 7 \times 0.1 \times (0.9)^6$$

$$P(X \le 1) = 0.4782 + 0.3720 = 0.8502$$

Hence, the correct answer is 0.8502.

Question 63 **Communication**

Consider a channel over which either symbol x_A or symbol x_B is transmitted. Let the output of the channel Y be the input to a maximum likelihood (ML) detector at the receiver. The conditional probability density functions for Y given x_A and x_B are:

$$f_{Y|x_A}(y) = e^{-(y+1)}u(y+1)$$

$$f_{Y|x_B}(y) = e^{(y-1)}[1-u(y-1)]$$

where $u(\cdot)$ is the standard unit step function. The probability of symbol error for this system is _________ (round off to two decimal places).

Ans. 0.23 (Range : 0.22 to 0.25)

Sol. Given that X_A and X_B are transmitted through a channel and given as input to a ML detector

$$\therefore \quad P(X_A) = P(X_B) = \frac{1}{2}$$

Given conditional density functions are

$$f_{Y/X_A}(y) = e^{-(y+1)}u(y+1)$$

$$f_{Y/X_B}(y) = e^{(y-1)}[1-u(y-1)]$$

The point of intersection of the above two conditional density functions gives optimum threshold.

Let, y_{th} be the optimum threshold.

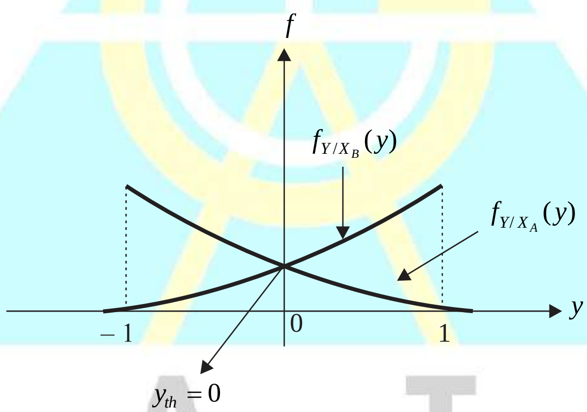
$\therefore$ Point of intersection can be found out as shown below.

$$e^{-(y_{th}+1)}u(y_{th}+1) = e^{(y_{th}-1)}[1-u(y_{th}-1)]$$

Only $y_{th} = 0$ satisfies the above equation.

$\therefore \quad y_{th} = 0$ is the optimum threshold.

Diagrammatically, the optimum threshold can be represented as shown below.



$$P_{Y/X_A} = \int_0^1 f_{Y/X_A}(y)\,dy = \int_0^1 e^{-(y+1)}dy$$

$$P_{Y/X_A} = e^{-1}\int_0^1 e^{-y}dy = e^{-1}(-e^{-y})_0^1 = e^{-1}(-e^{-1}+1)$$

$$P_{Y/X_A} = e^{-1}(-e^{-1}+1)$$

$$P_{Y/X_B} = \int_{-1}^0 f_{Y/X_B}(y)\,dy = \int_{-1}^0 e^{(y-1)}dy$$

$$P_{Y/X_B} = e^{-1}\int_{-1}^0 e^{y}dy = e^{-1}(e^{y})_{-1}^0$$

$$P_{Y/X_B} = e^{-1}(e^0 - e^{-1}) = e^{-1}(1-e^{-1})$$

$\therefore$ Probability of error will be,

$$P_e = P(X_A)P_{Y/X_A} + P(X_B)P_{Y/X_B}$$

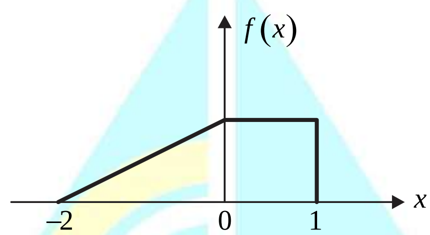
$$P_e = \frac{1}{2}\times e^{-1}(-e^{-1}+1) + \frac{1}{2}\times e^{-1}(1-e^{-1})$$

$$P_e = e^{-1} - e^{-2} = 0.23$$

Hence, the correct answer is 0.23.

Question 64 **Communication**

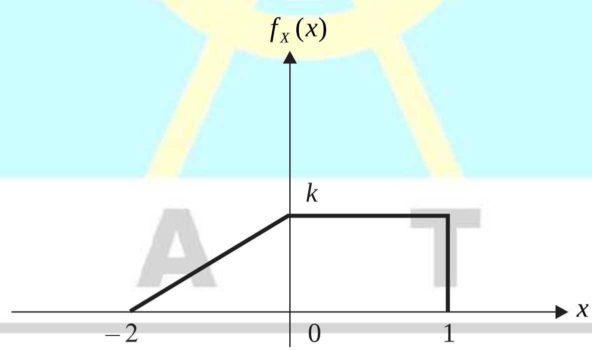
Consider a real valued source whose samples are independent and identically distributed random variables with the probability density function, $f(x)$, as shown in the figure.



Consider a 1-bit quantizer that maps positive samples to value α and others to value β. If α^* and β^* are the respective choices for α and β that minimize the mean square quantization error, then $(\alpha^* - \beta^*) =$ _____. (round off to two decimal places)

Ans. **1.167 (Range : 1.15 to 1.18)**

Sol. Given probability density function can be drawn as shown below,



As area under probability density function is 1.

$$\frac{1}{2}\times 2\times k + 1\times k = 1$$

$$k = \frac{1}{2}$$

Function $f_X(x)$ can defined as,

$$f_X(x) = \begin{cases} \frac{1}{4}(x+2), & -2 \le x \le 0 \\ \frac{1}{2}, & 0 \le x \le 1 \end{cases}$$

Given quantized values are,

$q_1 = \alpha$ for $0 \le x \le 1$

$q_2 = \beta$ for $-2 \le x \le 0$

For $0 \le x \le 1$,

Mean square quantization error will be,

$$N_{q_1} = E[X - q_1)^2]$$

$$N_{q_1} = E[(X - \alpha)^2]$$

$$N_{q_1} = \int_0^1 (x-\alpha)^2 \times \frac{1}{2}\, dx$$

$$N_{q_1} = \frac{1}{2}\int_0^1 (x-\alpha)^2\, dx$$

$$N_{q_1} = \frac{1}{2}\left(\frac{(x-\alpha)^3}{3}\right)_0^1$$

$$N_{q_1} = \frac{1}{2}\left[\frac{(1-\alpha)^3}{3} + \frac{\alpha^3}{3}\right]$$

$$N_{q_1} = \frac{(1-\alpha)^3 + \alpha^3}{6}$$

For N_{q_1} to be minimum, $\alpha = \alpha^*$ and $\frac{dN_{q_1}}{d\alpha^*} = 0$

$$3(\alpha^*)^2 = 3(1-\alpha^*)^2$$

$$(\alpha^*)^2 = (1-\alpha^*)^2$$

$$(\alpha^*) = 1 - \alpha^*$$

$$2\alpha^* = 1$$

$$\alpha^* = \frac{1}{2} = 0.5$$

For $-2 \le x \le 0$,

Mean square quantization error will be,

$$N_{q_2} = E[(X - q_2)^2]$$

$$N_{q_2} = E[(X - \beta)^2]$$

$$N_{q_2} = \int_{-2}^0 (x-\beta)^2 \times \frac{1}{4}(x+2)\, dx$$

$$N_{q_2} = \frac{1}{4}\int_{-2}^0 (x^2 + \beta^2 - 2x\beta)(x+2)\, dx$$

$$N_{q_2} = \frac{1}{4}\int_{-2}^{0}(x^3 + 2x^2 + \beta^2 x + 2\beta^2 - 2x^2\beta - 4x\beta)\,dx$$

$$N_{q_2} = \frac{1}{4}\int_{-2}^{0}(x^3 + (2-2\beta)x^2 + (\beta^2 - 4\beta)x + 2\beta^2)dx$$

$$N_{q_2} = \frac{1}{4}\left[\frac{x^4}{4} + \frac{(2-2\beta)x^3}{3} + \frac{(\beta^2-4\beta)x^2}{2} + 2\beta^2 x\right]_{-2}^{0}$$

$$N_{q_2} = \frac{1}{4}\left[(0) - \left(\frac{16}{4} - \frac{(2-2\beta)\times 8}{3} + 2(\beta^2 - 4\beta) - 4\beta^2\right)\right]$$

$$N_{q_2} = \frac{1}{4}\left[0 - 4 + \frac{8}{3}(2-2\beta) - 2(\beta^2 - 4\beta) + 4\beta^2\right]$$

$$N_{q_2} = \beta^2 - \frac{1}{2}(\beta^2 - 4\beta) + \frac{2}{3}(2-2\beta) - 1$$

For N_{q_2} to be minimum, $\beta = \beta^*$ and $\frac{dN_{q_2}}{d\beta^*} = 0$

$$2\beta^* - \frac{1}{2}(2\beta^* - 4) + \frac{2}{3}(0-2) = 0$$

$$2\beta^* - \beta^* + 2 - \frac{4}{3} = 0$$

$$\beta^* = \frac{4}{3} - 2$$

$$\beta^* = -\frac{2}{3} = -0.66$$

$$\therefore \quad \alpha^* - \beta^* = 0.5 + 0.66 = 1.16$$

Hence, the correct answer is 1.16.

Question 65 **Electromagnetic Theory**

In a electrostatic field, the electric displacement density vector, $\vec{D}$, is given by

$$\vec{D}(x,y,z) = (x^3\,\vec{i} + y^3\,\vec{j} + xy^2\,\vec{k})\ \text{C/m}^2,$$

where $\vec{i}$, $\vec{j}$, $\vec{k}$ are the unit vectors along x-axis, y-axis, and z-axis, respectively. Consider a cubical region R centered at the origin with each side of length 1 m, and vertices at (± 0.5 m, ± 0.5 m, ± 0.5 m). The electric charge enclosed within R is _________ C (round off to two decimal places).

Ans. 0.5 (Range : 0.48 to 0.52)

Sol. **Given :** $\vec{D}(x,y,z) = (x^3\,\vec{i} + y^3\,\vec{j} + xy^2\,\vec{k})$ C/m^2

From Gauss law in point from, we have

$$\nabla \cdot D = \rho_v$$

$$\rho_v = \frac{d}{dx}x^3 + \frac{d}{dy}y^3 + 0$$

$$\rho_v = 3x^2 + 3y^2$$

We know that $\frac{dQ}{dV} = \rho_v$

$$Q = \iiint \rho_v dV$$

$$Q = \iiint (3x^2 + 3y^2) dx\, dy\, dz$$

$$Q = 3\left[\frac{x^3}{3}\right]_{-0.5}^{0.5} + 3\left[\frac{y^3}{3}\right]_{-0.5}^{0.5}$$

$$Q = 2\times 0.125 + 2\times 0.125$$

$$Q = 0.5\ \text{C}$$

Hence, the correct answer is 0.5.

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