General Aptitude

Q.1 to Q.5 Carry ONE Mark Each

Question 1

As you grow older, an injury to your ______ may take longer to ______.

(A) heal/heal (B) heel/heel
(C) heel/heal (D) heal/heel

Ans. C

Sol. Heal : to become healthy again.

Heel : the back part of your foot.

In this question 'an injury to your' is given that means we need a body part that means 'heel' is correct.

So option (A) and (D) is eliminated and we need 'heal' in second filler.

Hence, the correct option is (C).

Question 2

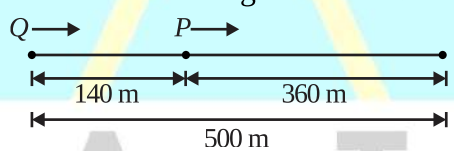
In a 500 m race, P and Q have speeds in the ratio of 3 : 4. Q starts the race when P has already covered 140 m. What is the distance between P and Q (in m) when P wins the race?

(A) 20 (B) 140
(C) 40 (D) 60

Ans. A

Sol. **Given :** P and Q have speed $3x$ and $4x$ respectively.

P has already covered 140 m at the time of starting of the race.



Distance = Speed × Time

Distance covered by P is $3x \times t = 360$ m

$xt = 120$ m

Distance covered by Q is $4x \times t = 4 \times 120 = 480$ m

So, the distance between P and Q, when P wins the race = 500 m – 480 m = 20 m

Hence, the correct option is (A).

Question 3

Three bells P, Q and R are rung periodically in a school. P is rung every 20 minutes, Q is rung every 30 minutes and R is rung every 50 minutes. If all the three bells are rung at 12:00 PM, when will the three bells ring together again the next time?

(A) 6:30 PM (B) 5:00 PM
(C) 6:00 PM (D) 5:30 PM

Ans. B

Sol. **Given :** Three bells P, Q and R rings periodically in a school.

P rings in every 20 minutes, Q rings in every 30 minutes and R rings in every 50 minutes.

Bells, P, Q and R will ring after the respective Least Common Multiple of ringing time intervals of them.

Least Common Multiple of 20 min, 30 min and 50 min is 300 min.

Where 300 minutes is equals to 5 hours.

As, all the three bells rangs at 12:00 PM then after 12 PM, the three bells ring together again, after 5 hours at 5 PM.

Hence, the correct option is (B).

Question 4

Given below are two statements and four conclusions drawn based on the statements.

Statement 1 : Some bottles are cups.

Statement 2 : All cups are knives.

Conclusion I : Some bottles are knives.

Conclusion II : Some knives are cups.

Conclusion III : All cups are bottles.

Conclusion IV : All knives are cups.

Which one of the following options can be logically inferred?

(A) Only conclusion II and conclusion III are correct

(B) Only conclusion II and conclusion IV are correct

(C) Only conclusion III and conclusion IV are correct

(D) Only conclusion I and conclusion II are correct

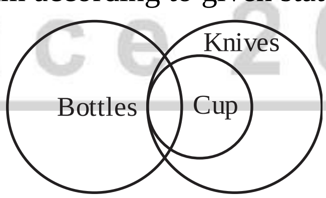
Ans. **D**

Sol. **Given :**

Statement I : Some bottles are cups.

Statement II : All cups are knives.

Below figure shows the venn diagram according to given statements,



Conclusion I : Some bottles are knives, is correct as we can see in the figure some part of the bottle is intersect with knives.

Conclusion II : Some knives are cups, is correct as we can see in the figure all part of the cup is intersect with knives.

Conclusion III : All cups are bottles, is not correct as we can see in the figure some part of the cup is intersect with bottle and we have no definite information about remaining part of cup.

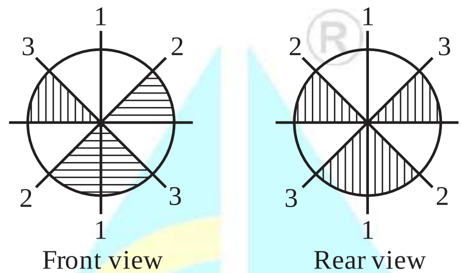
Conclusion IV : All knives are cups, is not correct as we can see in the figure some part of the knives is intersect with cup and we have no definite information about remaining part of knives.

Hence, the correct option is (D).

Question 5

The figure below shows the front and rear view of a disc, which is shaded with identical patterns. The disc is flipped once with respect to any one of the fixed axes 1-1, 2-2 or 3-3 chosen uniformly at random.

What is the probability that the disc DOES NOT retain the same front and rear views after the flipping operation?



(A) 1 (B) 0

(C) $\frac{2}{3}$ (D) $\frac{1}{3}$

Ans. **C**

Sol. If we rotate the disc along 1-1 axis,

We get front as rear and rear as front

That means this is not our favorable outcome.

If we rotated the disc along 2-2 axis,

We do not get front as rear and rear as front.

That means disc does not retain the same and this is our favorable outcome.

If we rotated the disc along 3-3 axis,

We do not get front as rear and rear as front.

That means disc does not retain the same and this is also our favorable outcome.

After the flipping operation the probability that the disc does not retain the same front and rear view is

$$P(E) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{2}{3}$$

Hence, the correct option is (C).

Q.6 to Q.10 Carry TWO Marks Each

Question 6

Altruism is the human concern for the wellbeing of others. Altruism has been shown to be motivated more by social bonding, familiarity and identification of belongingness to a group. The notion that altruism may be attributed to empathy or guilt has now been rejected.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

(A) Humans engage in altruism due to guilt but not empathy.
(B) Humans engage in altruism due to group identification but not empathy.
(C) Human engage in altruism due to empathy but not guilt.
(D) Human engage in altruism due to empathy but not familiarity.

Ans. **B**

Sol. Option (A), can not be inferred as here guilty and empathy are used in opposite tone but in passage they are used in similar tone.

Option (B), can be inferred as it is clearly mentioned the term 'due to group identification but not empathy'.

Option (C), can not be inferred as in this option again empathy and guilty are used in different tone.

Option (D), can not be inferred as in this option familiarity is used in negative tone and in passage it is given in positive tone.

Hence, the correct option is (B).

Question 7

There are two identical dice with a single letter on each of the faces. The following six letters : Q, R, S, T, U and V, one on each of the faces. Any of the six outcomes are equally likely.

The two dice are thrown once independently at random.

What is the probability that the outcomes on the dice were composed only of any combination of the following possible outcomes : Q, U and V?

(A) $\frac{1}{6}$ (B) $\frac{3}{4}$

(C) $\frac{5}{36}$ (D) $\frac{1}{4}$

Ans. **D**

Sol. **Given :** Two identical dice with a single letter on each of the faces.

The following six letters : Q, R, S, T, U and V one on each of the faces.

Any of the six outcomes are equally likely.

P(Event) = P(getting outcomes from dice one) . P(getting outcomes from dice two)

For favorable outcome we need any one of them Q, U, V

$$P(E) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

$$P(E) = \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$$

Hence, the correct option is (D).

Question 8

The price of an item is 10% cheaper in an online store S compared to the price at another online store M. Store S charges ₹ 150 for delivery. There are no delivery charges for orders from the store M. A person bought the item from the store S and saved ₹ 100.

What is the price of the item at the online store S (in ₹) if there are no other charges than what is described above?

(A) 1500 (B) 2500
(C) 1750 (D) 2250

Ans. D

Sol. **Given :** The price of an item is 10% cheaper in an online store S compared to the price at another online store M.

Delivery charges of an item at the online store S and M is ₹150 and 0 respectively.

Cost of an item from the online store S is ₹100 less than the item from online store M.

Let, the price of an item at the online store M is ₹x.

Then price of an item at the online store S is $(x - 0.1x) = 0.9x$

So, $x - (0.9x + 150) = 100$

$0.1x - 150 = 100$

$0.1x = 250$

$x = 250$

The price of the item at the online store S (in ₹) without any delivery charges is,

$0.9x = 0.9 \times 250 = 2250$

Hence, the correct option is (D).

Question 9

The letters P, Q, R, S, T and U are to be placed one per vertex on a regular convex hexagon, but not necessarily in the same order.

Consider the following statements :

- The line segment joining R and S is longer than the line segment joining P and Q.
- The line segment joining R and S is perpendicular to the line segment joining P and Q.
- The line segment joining R and U is parallel to the line segment joining T and Q.

Based on the above statements, which one of the following options is CORRECT?

(A) The line segment joining T and Q is parallel to the line joining P and U.

(B) The line segment joining Q and S is perpendicular to the line segment joining R and P.

(C) The line segment joining R and T is parallel to the line segment joining Q and S.

(D) The line segment joining R and P is perpendicular to the line segment joining U and Q.

Ans. C

Sol. **Given :**

The letters P, Q, R, S, T and U are to be placed one per vertex on regular convex hexagon.

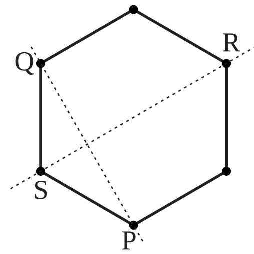
Given conditions are as follows,

The line segment joining R and S is longer then the line segment joining P and Q.

$RS > PQ$

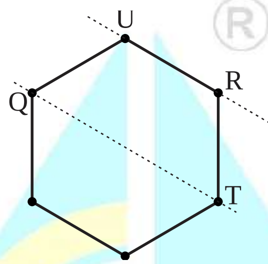
The line segment joining R and S is perpendicular to the line segment joining P and Q.

$RS \perp PQ$

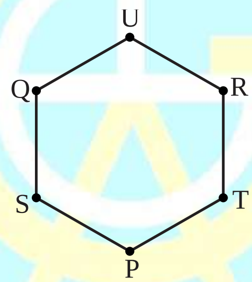


The line segment joining R and U is parallel to the line segment joining T and Q.

$RS \parallel TQ$



According to given conditions we can draw a diagram shown below



Option (A) is eliminated, as line TQ is not parallel to line PQ.

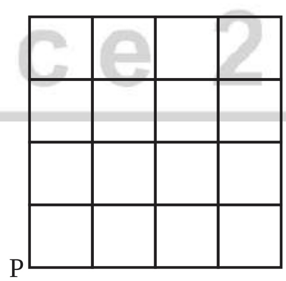
Option (B) is eliminated, as line QS is not perpendicular to line RP.

Option (C) is correct as line RT is parallel to line QS.

Option (D) is eliminated, as line RP is not perpendicular to line UQ.

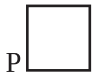
Hence, the correct option is (C).

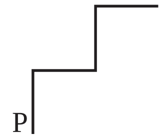
Question 10

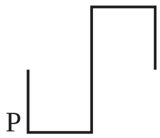


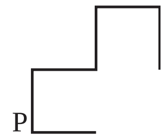
An ant is at the bottom-left corner of a grid (point P) as shown above. It aims to move to the top right corner of the grid. The ant moves only along the lines marked in the grid such that the current distance to the top-right corner strictly decreases.

Which one of the following is a part of a possible trajectory of the ant during the movement?

(A) 

(B) 

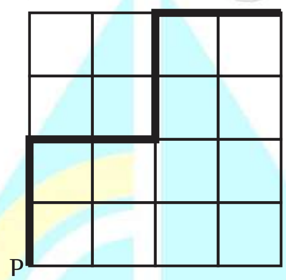
(C) 

(D) 

Ans. **B**

Sol. **Given :**

An ant is at the bottom left corner of a grid (point p)



Ant aims to move to the top right corner of the grid.

If the ant moves only along the lines marked in the grid so that the current distance to the top right corner strictly decreases.

Ant has to move according to the path given in option (B).

Hence, the correct option is (B).

Technical Section

Q.11 to Q.35 Carry ONE Mark Each

Question 11 **Control System**

The transfer function of a real system, $H(s)$, is given as :

$$H(s)=\frac{As+B}{s^2+Cs+D}$$

where A, B, C and D are positive constants. This system cannot operate as

(A) band pass filter (B) low pass filter

(C) an integrator (D) high pass filter

Ans. **C or D**

Sol. **Given :**

Transfer function of a real system $H(s)$ is given by,

$$H(s)=\frac{As+B}{s^2+Cs+D} \qquad \ldots\text{(i)}$$

Put $s=j\omega$ in equation (i),

$$H(j\omega)=\frac{A(j\omega)+B}{(j\omega)^2+j\omega C+D}$$

Magnitude, $$|H(\omega)|=\frac{\sqrt{(A\omega)^2+B^2}}{\sqrt{(D-\omega)^2+(\omega C)^2}} \qquad \ldots\text{(ii)}$$

Case 1 : At low frequency $(\omega=0)$, equation (ii) can be approximated as,

$$|H(\omega=0)|=\frac{B}{D}$$

It means, low frequency passed by given real system $H(s)$.

At high frequency $(\omega=\infty)$, equation (ii) can be approximated as,

$$|H(\omega=\infty)|=0$$

It means, high frequency blocked by given real system $H(s)$.

Thus, given real system $H(s)$ work as low pass filter but not high pass filter.

Case 2 : Assuming B is very less positive valued constant as compare to values of positive constant A, C and D then $H(s)$ can be approximated as,

$$H(s)=\frac{As}{s^2+Cs+D}$$

Put $s=j\omega$ in above equation,

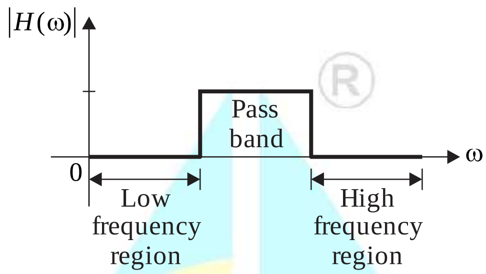
$$H(j\omega)=\frac{A(j\omega)}{(j\omega)^2+j\omega C+D}$$

Magnitude, $|H(\omega)| = \dfrac{(A\omega)^2}{\sqrt{(D-\omega)^2+(\omega C)^2}}$

At low frequency $(\omega = 0)$, $|H(\omega=0)| = 0$; it means block low frequency.

At high frequency $(\omega=\infty)$, $|H(\omega=\infty)| = 0$; it means block high frequency.

Magnitude plot of $|H(\omega)|$ is,



It is given $H(s)$ pass frequency between low frequency and high frequency.

Thus, real system $H(s)$ can work as band pass filter also.

Hence, the correct options is (C) or (D).

Note : This questions is MCQ type but IIT KGP provide two answers for this question, even the real system $H(s)$ can approximate into integrator as shown below :

Under very high value of frequency $(\omega \to \infty)$, the $H(s)$ can be approximated as,

$$H(s) \approx \frac{As}{s^2} \approx \frac{A}{s}$$

Thus, real system $H(s)$ can work as integrator.

Question 12 **Analog Electronics**

For an ideal MOSFET biased in saturation, the magnitude of the small signal current gain for a common drain amplifier is

(A) infinite (B) 1

(C) 100 (D) 0

Ans. **A**

Sol. **Given :**

(i) MOSFET is ideal

(ii) Biased under saturation

(iii) Ideal common drain amplifier

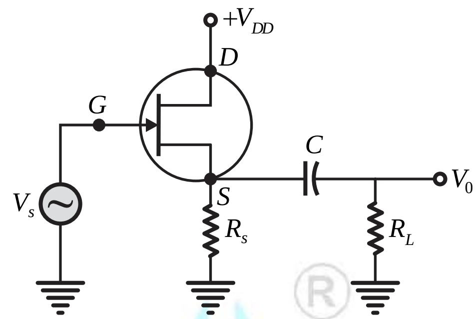
Thus current gain (A_I) of ideal common drain amplifier

$$A_I = \infty$$

Hence, the correct option is (A).

🕮 Key Point

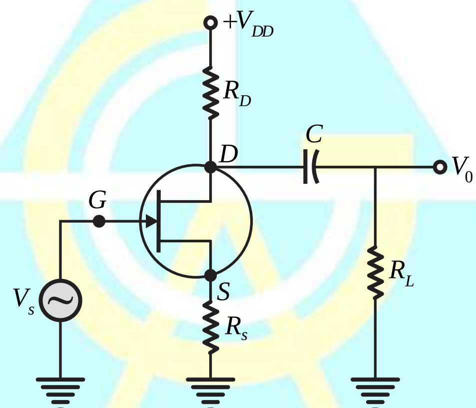
- Ideal MOSFET under common drain amplifier :



(i) Voltage gain (A_V) is 1.

(ii) Current gain (A_I) is ∞ because $I_G = 0$.

- Ideal MOSFET under common source amplifier :



(i) Voltage gain (A_V) is

$$A_V = \frac{-\mu R_L'}{R_L' + r_0 + R_s(1+\mu)}$$

$(R_L' = R_D \,||\, R_L)$

(ii) Current gain (A_I) is ∞ because $I_G = 0$.

Question 13 **Power System**

The most commonly used relay, for the protection of an alternator against loss of excitation, is

(A) differential relay (B) offset Mho relay.

(C) Buchholz relay (D) over current relay.

Ans. **B**

Sol. 1. Buchholz's relay is used for the protection of transformer

2. Differential relay is used for alternator winding protection

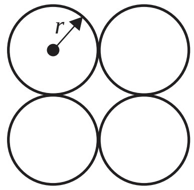
3. Over-current relay is used for location where there is a chance of high current during fault.

4. Offset mho relay is used for the protection of an alternator against loss of excitation.

Hence, the correct option is (B).

Question 14 **Power Systems**

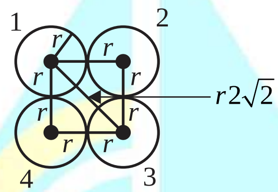
The geometric mean radius of a conductor, having four equal strands with each strand of radius '*r*', as shown in the figure below, is



(A) 1.723 *r* (B) 2 *r*
(C) 4 *r* (D) 1.414 *r*

Ans. **A**

Sol. Given diagram is



To calculate GMR, we multiply all the distances including self-distance, i.e. r'

Geometric mean radius $(\text{GMR}) = 4\sqrt{D_{11}\, D_{12}\, D_{13}\, D_{14}}$

$$D_{11} = r' = 0.7788r$$
$$D_{12} = 2r$$
$$D_{13} = r2\sqrt{2}$$
$$D_{14} = 2r$$
$$\text{GMR} = \left(0.7788r \times 2r \times r2\sqrt{2} \times 2r\right)^{1/4}$$
$$\text{GMR} = 1.723r$$

Hence, the correct option is (A).

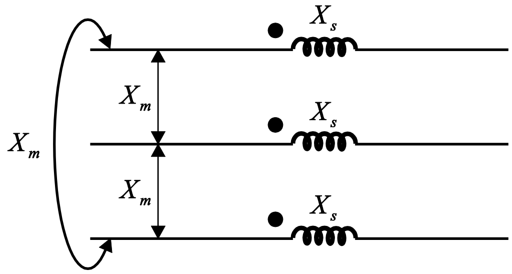
Question 15 **Power System**

The valid positive, negative and zero sequence impedance (in p.u.), respectively, for a 220 kV, fully transposed three-phase transmission line, from the given choices are

(A) 0.15, 0.15 and 0.35 (B) 0.2, 0.2 and 0.2
(C) 1.1, 0.15 and 0.08 (D) 0.1, 0.3 and 0.1

Ans. **A**

Sol.



In case of transposed transmission lines

Zero-sequence impedance, $X_0 = X_s + 2X_m$

Positive-sequence impedance, $X_1 = X_s - X_m$

Negative-sequence impedance, $X_2 = X_s - X_m$

So for transmission line, $X_1 = X_2 < X_0$

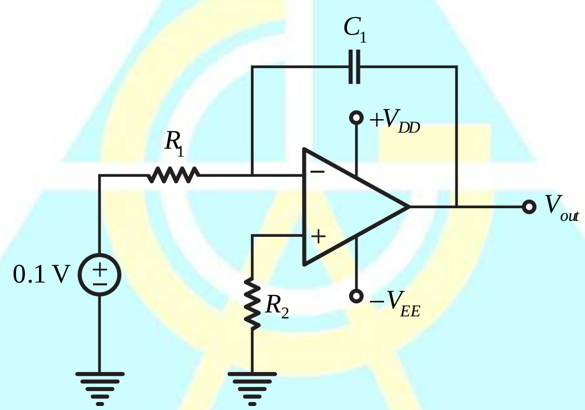
In option (A) only have, $X_1 = X_2 = 0.15$

$X_0 = 0.35$

Hence, the correct option is (A).

Question 16 **Analog Electronics**

The steady state output (V_{out}), of the circuit shown below, will

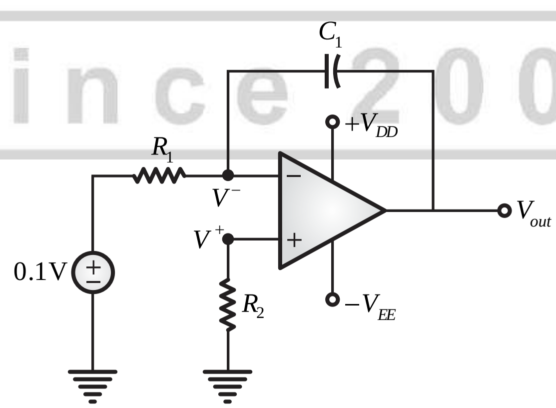


(A) saturate to $-V_{EE}$ (B) saturate to $+V_{DD}$

(C) become equal to -0.1 V (D) become equal to 0.1 V

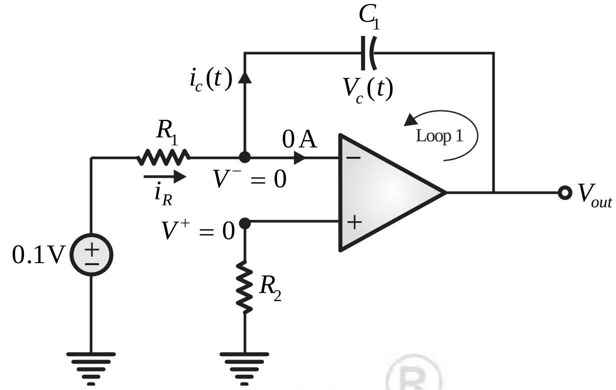
Ans. **A**

Sol. Given Op-Amp circuit is shown below,



Assuming Op-Amp is ideal so that virtual ground concept is applicable.

Thus, $V^+ = V^- = 0$



Applying KVL in loop-1,

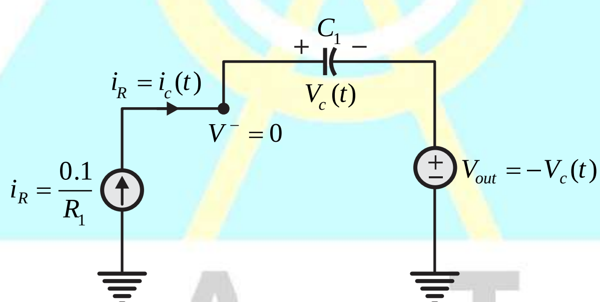
$$0 + V_c(t) + V_{out} = 0$$

$$V_{out} = -V_c(t) \qquad \ldots(i)$$

Applying KCL at node V^-,

$$i_R = i_c(t) = \frac{0.1 - 0}{R_1} = \frac{0.1}{R_1} \qquad \ldots(ii)$$

As current flow through capacitor C_1 is constant of value $\frac{0.1}{R_1}$ A, it shows the linear charging of capacitor C_1,

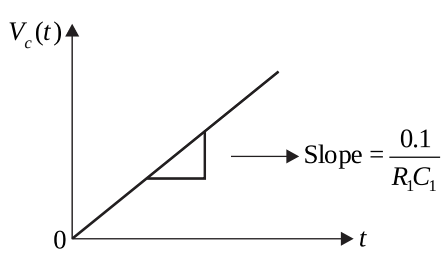


The voltage across the capacitor $V_c(t)$ under linear charging is,

$$V_c(t) = \frac{1}{C_1}\int_0^t i_c(t)dt$$

$$V_c(t) = \frac{1}{C_1}\int_0^t \frac{0.1}{R_1}dt = \frac{0.1}{R_1 C_1}t$$

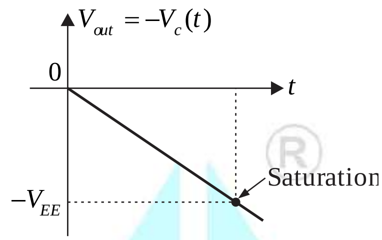
So, graph between $V_c(t)$ verses t, is a linear line as shown below



From above graph it is clear that, capacitor charged continuously upto infinite time. Thus from equation (i),

$$V_{out} = -V_c(t) = -\frac{0.1}{R_1 C_1} t$$

The graph of V_{out} is shown below,



Under steady state $(t = \infty)$, $V_{out} = -V_c(\infty) = -\infty$

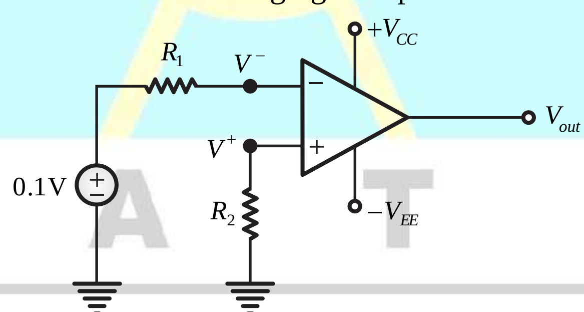
Thus, Op-Amp does not allow V_{out} to be $-\infty$ so, Op-Amp will saturate to $-V_{EE}$ so that

$$V_{out} = -V_{EE}$$

Hence, the correct option is (A).

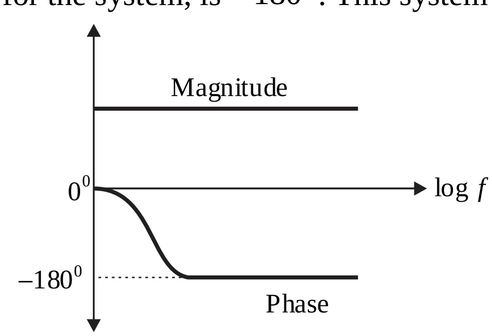
☒ Avoid this Mistake :

- Don't make capacitor open circuit under steady-state condition because constant current is flowing through capacitor so it is the case of linear charging of capacitor.
- Many aspirant try to think that under steady state capacitor is open circuit and Op-Amp behave as comparator and $V^+ < V^-$ in given circuit due to this, you will get answer $V_{out} = -V_{EE}$ but conceptually it is wrong because it is the case of linear charging of capacitor.



Question 17 **Control Systems**

The Bode magnitude plot of a first order stable system is constant with frequency. The asymptotic value of the high frequency phase, for the system, is -180^0. This system has



(A) one LHP pole and one RHP zero at the same frequency.

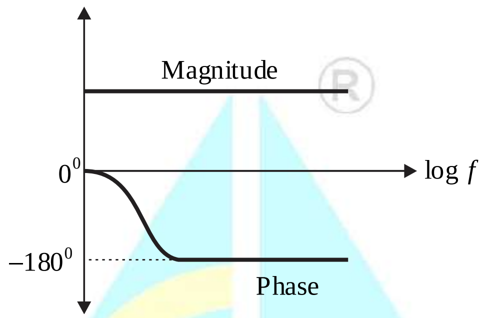
(B) one LHP pole and one LHP zero at the same frequency.

(C) two LHP poles and one RHP zero.

(D) two RHP poles and one LHP zero.

Ans. **A**

Sol. Given bode magnitude and phase plot of a first order stable system is shown below.



Flat constant magnitude response for all frequency of system shows that it is an all pass system.

In all pass system, poles and zeros are symmetrical about $j\omega$ axis.

The possible transfer functions all pass system are

$$T_1(s) = \frac{s-1}{s+1}$$

$$T_2(s) = \frac{1-s}{1+s}$$

From the phase plot as $\omega \to \infty$ shows $\phi = -180^0$

For $T_1(s)$:

$$T_1(s) = \frac{s-1}{s+1}$$

Put $s = j\omega$

$$T_1(j\omega) = \frac{j\omega - 1}{j\omega + 1}$$

$$\angle T_1(j\omega) = 180^0 - \tan^{-1}\omega - \tan^{-1}\omega = 180^0 - 2\tan^{-1}\omega$$

At $\omega = \infty$,

$$\angle T_1(j\infty) = 180^0 - 2\tan^{-1}\infty$$

$$\angle T_1(j\infty) = 0^0$$

For $T_2(s)$:

$$T_2(s) = \frac{1-s}{1+s}$$

Put $s = j\omega$,

$$T_2(j\omega) = \frac{1 - j\omega}{1 + j\omega}$$

$$\angle T_2(j\omega) = -\tan^{-1}\omega - \tan^{-1}\omega = -2\tan^{-1}\omega$$

At $\omega = \infty$,

$$\angle T_2(j\omega) = -2\tan^{-1}\infty$$

$$\angle T_2(j\omega) = -180^0$$

Hence, the transfer function of given all pass filter.

$$T(s) = \frac{1 - s}{1 + s}$$

Hence, the system has one LHP pole and one RHP zero at the same frequency.

Hence, the correct option is (A).

Question 18 **Measurement**

A balanced Wheatstone bridge *ABCD* has the following arm resistances : $R_{AB} = 1\text{ k}\Omega \pm 2.1\%$; $R_{BC} = 100\ \Omega \pm 0.5\%$; R_{CD} is an unknown resistance; $R_{DA} = 300\ \Omega \pm 0.4\%$. The value of R_{CD} and its accuracy is

(A) $3000\ \Omega \pm 90\ \Omega$ (B) $3000\ \Omega \pm 3\ \Omega$

(C) $30\ \Omega \pm 0.9\ \Omega$ (D) $30\ \Omega \pm 3\ \Omega$

Ans. **C**

Sol. **Given :**

Balanced Wheatstone bridge ABCD, has the following arm's resistance are,

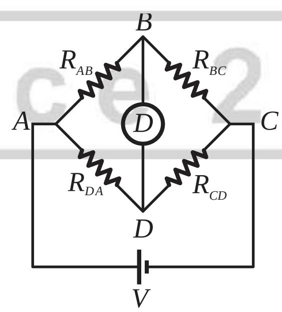
$$R_{AB} = 1\text{ k}\Omega \pm 2.1\%$$

$$R_{BC} = 100\ \Omega \pm 0.5\%$$

$$R_{DA} = 300\ \Omega \pm 0.4\%$$

Calculate :

The value of R_{CD} and its accuracy is,



$$R_{CD} = \frac{R_{BC} \times R_{DA}}{R_{AB}} = \frac{100 \times 300}{1000} = 30\ \Omega$$

$$\%\varepsilon_r = \pm[2.1 + 0.5 + 0.4] = \pm 3\%$$

$$R_{CD} = 30 \pm 3\%$$

Now, %RLE error is defined as the formula,

$$\%\varepsilon = \frac{\delta R_{CD}}{R_{CD}} \times 100$$

$$\delta R_{CD} = \text{Absolute error}$$

$$\delta R_{CD} = \%\varepsilon \times R_{CD}$$

$$\delta R_{CD} = \frac{3}{100} \times 30 = \frac{9}{10} = 0.9\ \Omega$$

$$R_{CD} = 30\ \Omega \pm 0.9\ \Omega$$

Hence, the correct option is (C).

Key Point

(i) Absolute error : It is defined it is the difference between measured value and true value.

$$\delta A = A_m - A_T$$

Where, $\delta A =$ Absolute error, $A_m =$ Measured value and $A_T =$ True value.

(ii) %REL is defined, it is the ratio of absolute error to the true value.

$$\%RLE = \%\varepsilon = \frac{\delta A}{A_T} \times 100$$

Question 19 **Control Systems**

The open loop transfer function of a unity gain negative feedback system is given by,

$$G(s) = \frac{K}{s^2 + 4s - 5}$$

The range of K for which the system is stable, is

(A) $K > 5$ (B) $K < 5$

(C) $K < 3$ (D) $K > 3$

Ans. **A**

Sol. **Given :**

$$G(s) = \frac{K}{s^2 + 4s - 5}$$

The characteristic equation is given by,

$$1 + G(s) = 0$$

$$1 + \frac{K}{s^2 + 4s - 5} = 0$$

$$s^2 + 4s - 5 + K = 0$$

$$s^2 + 4s + K - 5 = 0$$

Routh tabulation :

s^2	1	$K-5$
s^1	4	
s^0	$K-5$	

For the system to be stable, all the roots must be in the left half of s – plane, thus all the coefficients in the first column of Routh tabulation must have the same sign. Therefore, the coefficient of first column of the Routh's table should be positive.

Therefore for stability

$K-5>0$

$K>5$

Hence, the correct option is (A).

Key Point

For stability of second order system all coefficient of characteristic equation should be positive.

Question 20 **Mathematics**

Consider a 3×3 matrix A whose $(i,\ j)$-th element, $a_{i,j}=(i-j)^3$. Then the matrix A will be

(A) null.
(B) symmetric.
(C) skew-symmetric.
(D) unitary.

Ans. **C**

Sol. **Given :** $A=[aij]_{3\times3}$, $aij=(i-j)^3\ \forall\ i$ and j

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$$

$$A=\begin{bmatrix} (1-1)^3 & (1-2)^3 & (1-3)^3 \\ (2-1)^3 & (2-2)^3 & (2-3)^3 \\ (3-1)^3 & (3-2)^3 & (3-3)^3 \end{bmatrix}_{3\times3}$$

$$A=\begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & -1 \\ 8 & 1 & 0 \end{bmatrix}_{3\times3}$$

If we take transpose of matrix A.

$$A^T=\begin{bmatrix} 0 & 1 & 8 \\ -1 & 0 & 1 \\ -8 & -1 & 0 \end{bmatrix}_{3\times3}$$

$$-A^T = \begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & -1 \\ 8 & 1 & 0 \end{bmatrix}_{3\times 3}$$

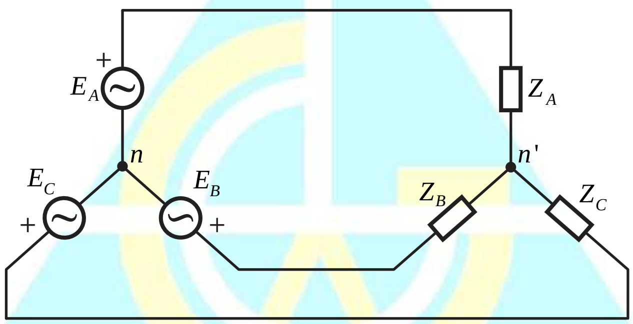
We can see that, $A = -A^T$

So, matrix A is skew symmetric matrix.

Hence, the correct option is (C).

Question 21 **Network Theory**

In the circuit shown below, a three-phase star-connected unbalanced load is connected to a balanced three-phase supply of $100\sqrt{3}$ V with phase sequence ABC. The star connected load has $Z_A = 10\,\Omega$ and $Z_B = 20\angle 60^0\,\Omega$. The value of Z_C in Ω, for which the voltage difference across the nodes n and n' is zero, is

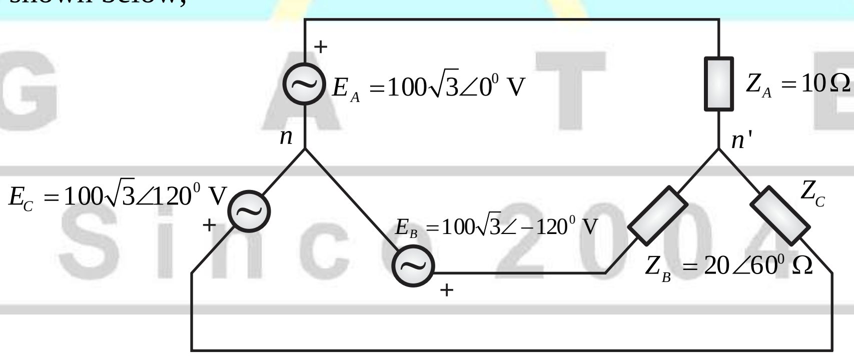


(A) $20\angle -30^0$ (B) $20\angle 30^0$

(C) $20\angle 60^0$ (D) $20\angle -60^0$

Ans. **D**

Sol. Given circuit is shown below,



Here,

$$E_A = 100\sqrt{3}\angle 0^0 \text{ V}$$

$$E_B = 100\sqrt{3}\angle -120^0 \text{ V}$$

$$E_C = 100\sqrt{3}\angle 120^0 \text{ V}$$

$$Z_A = 10\,\Omega$$

$$Z_B = 20\angle 60^0\,\Omega$$

In case of balance system, $V_n = 0\,\text{V}$

By varying Z_C we can make $V_n' = 0\,\text{V}$

n and n' are at same potential

Applying KCL at node n'

$$\frac{0-100\sqrt{3}\angle 0^0}{10}+\frac{0-100\sqrt{3}\angle -120^0}{20\angle 60^0}+\frac{0-100\sqrt{3}\angle 120^0}{Z_C}=0$$

$$10\sqrt{3}+5\sqrt{3}\angle -180^0+\frac{100\sqrt{3}\angle 120^0}{Z_C}=0$$

$$5\sqrt{3}\angle 0^0+\frac{100\sqrt{3}\angle 120^0}{Z_C}=0$$

$$5\sqrt{3}\angle 0^0=\frac{-100\sqrt{3}\angle 120^0}{Z_C}$$

$$Z_C=\frac{-100\sqrt{3}\angle 120^0}{5\sqrt{3}\angle 0^0}$$

$$Z_C=20\angle -60^0$$

Hence, the correct option is (D).

Question 22 **Power Electronics**

A charger supplies 100 W at 20 V for charging the battery of a laptop. The power devices, used in the converter inside the charger, operate at a switching frequency of 200 kHz. Which power device is best suited for this purpose?

(A) Thyristor (B) MOSFET

(C) BJT (D) IGBT

Ans. B

Sol. For high switching frequency and low voltage operation, the best suitable device is MOSFET.

Hence, the correct option is (B).

Question 23 **Electromagnetic Field**

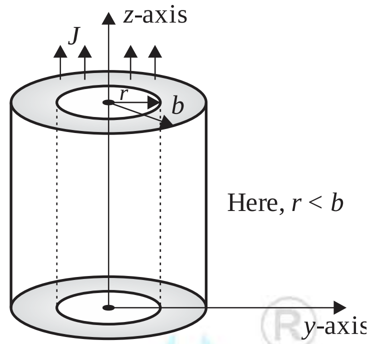
A long conducting cylinder having a radius 'b' is placed along the z axis. The current density is $\mathbf{J} = J_a r^3 \hat{z}$ for the region $r < b$ where r is the distance in the radial direction. The magnetic field intensity (**H**) for the region inside the conductor (i.e. for $r < b$) is

(A) $\frac{J_a}{3} r^3$ (B) $\frac{J_a}{4} r^4$

(C) $J_a r^3$ (D) $\frac{J_a}{5} r^4$

Ans. D

Sol. **Given :** Current density $\mathbf{J} = J_a r^3 \hat{z}$



Here, consider only ϕ- direction component of $\bar{H}$ and $d\bar{l}$, because we have to find $\bar{H}$ inside the conductor region.

From Ampere's circulal law, $\oint \bar{H} \cdot d\bar{l} = I_{enclosed}$

$$\oint H_\phi \hat{a}_\phi \cdot d\bar{l} = I_{enclosed}$$

$$\because \quad d\bar{l} = rd\phi \hat{a}_\phi$$

$$\oint H_\phi \hat{a}_\phi \cdot rd\phi \hat{a}_\phi = I_{enclosed} \qquad (\because \hat{a}_\phi \cdot \hat{a}_\phi = 1)$$

$$H_\phi \oint rd\phi = I_{enclosed}$$

$$H_\phi r \oint d\phi = I_{enclosed}$$

$$H_\phi r \times 2\pi = I_{enclosed}$$

$$2\pi r H_\phi = I_{enclosed} \qquad \ldots(i)$$

Current (I) in terms of current density (J),

$$I = \oint_S J \cdot d\bar{s}$$

Here, $\bar{J}$ is in $\hat{a}_z$ direction, so as must be in $\hat{a}_z$ direction. So $d\bar{s} = rdrd\phi \hat{a}_z$.

$$I = \oint_S (J_0 r^3 \hat{a}_z) \cdot (rdrd\phi)\hat{a}_z$$

$$I = J_0 \int_0^r r^4 dr \int_{\phi=0}^{2\pi} d\phi$$

$$I = J_0 \frac{r^5}{5}(2\pi) \qquad (\because \hat{a}_z \cdot \hat{a}_z = 1)$$

$$I = J_0 \frac{2\pi}{5}(r^5) \qquad \ldots(ii)$$

Compare equation (i) and (ii),

$$H_\phi(2\pi r) = J_0 \frac{2\pi}{5}(r^5)$$

$$H_\phi = J_0\left(\frac{r^4}{5}\right)$$

Thus, $\bar{H} = H_\phi \hat{a}_\phi = J_0\left(\frac{r^4}{5}\right)\hat{a}_\phi$

Hence, the correct option is (D).

Question 24 **Electrical Machine**

The type of single-phase induction motor, expected to have the maximum power factor during steady state running condition, is

(A) capacitor start (B) split phase (resistance start)

(C) capacitor start, capacitor run (D) shaded pole

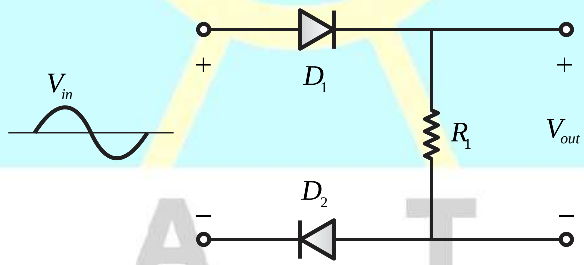
Ans. **C**

Sol. Under steady state running condition power factor of capacitor start capacitor run (CSCR) induction motor is high because one capacitor is permanently connected to the circuit, which improves the running power factor.

Hence, the correct option is (C).

Question 25 **Analog Electronics**

For the circuit shown below with ideal diodes, the output will be

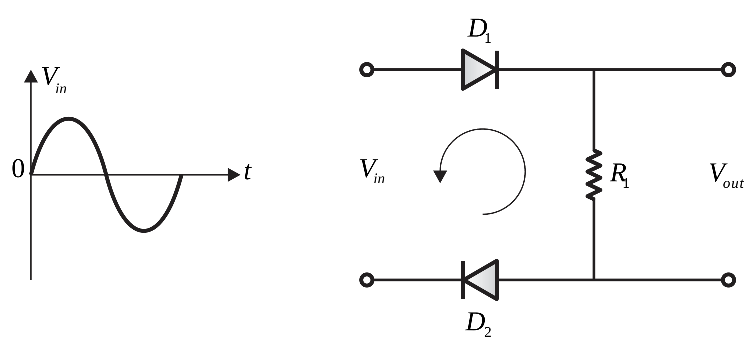


(A) $V_{out} = V_{in}$ for $V_{in} > 0$ (B) $V_{out} = V_{in}$ for $V_{in} < 0$

(C) $V_{out} = -V_{in}$ for $V_{in} > 0$ (D) $V_{out} = -V_{in}$ for $V_{in} < 0$

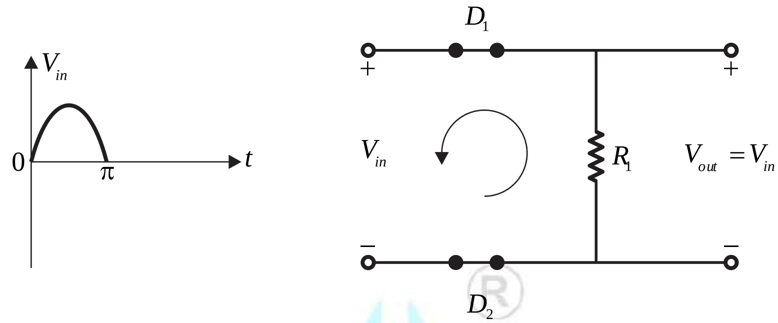
Ans. **A**

Sol. Given circuit with ideal diodes is shown below,



For Positive half cycle :

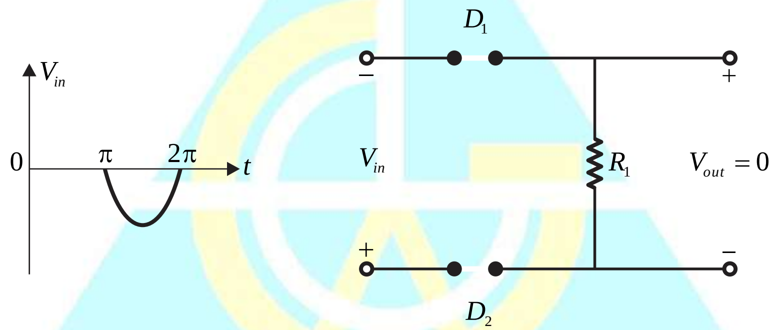
D_1 and D_2 will be ON, and replace by short circuit. So circuit becomes as,



Here, $V_{out} = V_{in}$

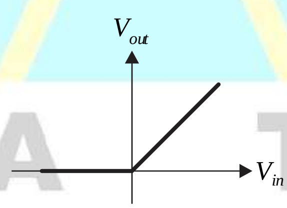
For Negative half cycle :

D_1 and D_2 will be OFF, and replace by open circuit. So circuit becomes as,



Here, $V_{out} = 0$

Thus output V_{out},



$$V_{out} = \begin{cases} V_{in}; & V_{in} > 0 \\ 0; & V_{in} < 0 \end{cases}$$

Hence, the correct option is (A).

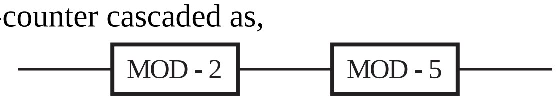
Question 26 **Digital Electronics**

A MOD 2 and a MOD 5 up-counter when cascaded together results in a MOD _____ counter. (in integer)

Ans. **10 (Range : 10 to 10)**

Sol. **Given :**

MOD 2 and a MOD 5, up-counter cascaded as,



Overall MOD of a cascaded counter $= 2 \times 5 = 10$

Hence, the correct answer is 10.

Question 27 **Network Theory**

An inductor having a Q-factor of 60 is connected in series with a capacitor having a Q-factor of 240. The overall Q-factor of the circuit is ______. (rounded off to nearest integer)

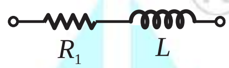
Ans. **48 (Range : 48 to 48)**

Sol. **Given :**

Quality factor of inductor $(Q_L) = 60$

Quality factor of capacitor $(Q_C) = 240$

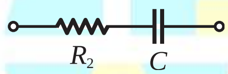
Lossy inductor,



Quality factor $(Q_L) = \dfrac{V_L}{V}$ (At resonance $V = V_R$ and $\omega = \omega_0$)

$$\text{Quality factor } (Q_L) = \frac{V_L}{V_R} = \frac{IX_L}{IR_1} = \frac{X_L}{R_1} = \frac{\omega_0 L}{R_1}$$

Lossy capacitor,



Quality factor $(Q_C) = \dfrac{V_C}{V}$ (At resonance $V = V_R$ and $\omega = \omega_0$)

$$\text{Quality factor } (Q_C) = \frac{V_C}{V_R} = \frac{IX_C}{IR_2} = \frac{X_C}{R_2} = \frac{1}{\omega_0 C R_2}$$

At resonance, $\omega = \omega_0$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_C = \frac{1}{\omega_0 C R_2} = \frac{\omega_0}{\omega_0^2 C R_2} = \frac{\omega_0}{\frac{1}{LC} C R_2}$$

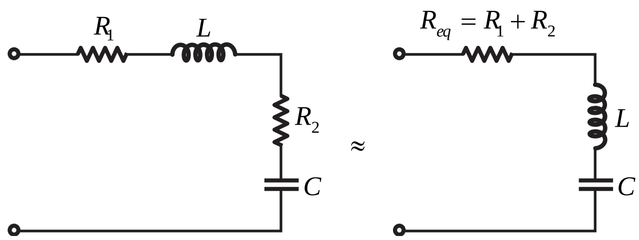
$$Q_C = \frac{\omega_0 L}{R_2} \qquad \ldots\text{(i)}$$

$$Q_L = \frac{\omega_0 L}{R_1} = \frac{\omega_0^2 L}{\omega_0 R_1} = \frac{\frac{1}{LC} L}{\omega_0 R_1}$$

$$Q_L = \frac{1}{\omega_0 C R_1} \qquad \ldots\text{(ii)}$$

Method 1

Overall Q-factor of series combination of inductor and capacitor is given by,



$$Q_{overall} = \frac{\omega_0 L}{R_{eq}} = \frac{\omega_0 L}{R_1 + R_2}$$

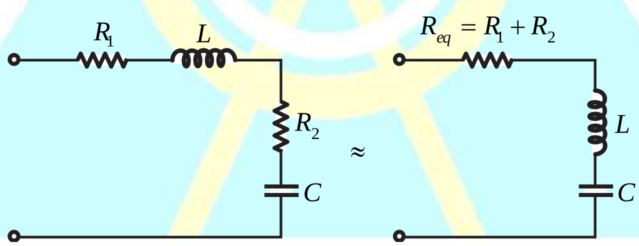
$$Q_{overall} = \frac{1}{\frac{R_1}{\omega_0 L} + \frac{R_2}{\omega_0 L}} = \frac{1}{\frac{1}{Q_L} + \frac{1}{Q_C}} \qquad \left[\because Q_C = \frac{\omega_0 L}{R_2} \text{ and } Q_L = \frac{1}{\omega_0 C R_1}\right]$$

$$Q_{overall} = \frac{Q_L Q_C}{Q_L + Q_C} = \frac{60 \times 240}{60 + 240} = 48$$

Hence, the overall Q-factor of series combination of inductor and capacitor is 48.

Method 2

Overall Q-factor of series combination of inductor and capacitor is given by,



$$Q_{overall} = \frac{1}{\omega_0 C R_{eq}} = \frac{1}{\omega_0 C (R_1 + R_2)}$$

$$Q_{overall} = \frac{1}{\omega_0 C R_1 + \omega_0 C R_2}$$

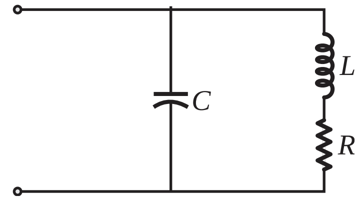
$$Q_{overall} = \frac{1}{\frac{1}{Q_L} + \frac{1}{Q_C}} \qquad \left[\because Q_C = \frac{1}{\omega_0 C R_2} \text{ and } Q_L = \frac{1}{\omega_0 C R_1}\right]$$

$$Q_{overall} = \frac{Q_L Q_C}{Q_L + Q_C} = \frac{60 \times 240}{60 + 240} = 48$$

Hence, the overall Q-factor of series combination of inductor and capacitor is 48.

Question 28 **Network Theory**

The network shown below has a resonant frequency of 150 kHz and a bandwidth of 600 Hz. The Q-factor of the network is ______. (round off to nearest integer)



Ans. 250 (Range : 250 to 250)

Sol. **Given :** Resonant frequency $f_0 = 150$ kHz

Bandwidth $\Delta f = 600$ Hz

Q factor of RLC circuit in terms of resonant frequency and bandwidth is given by,

$$Q = \frac{\text{Resonant frequency}}{\text{Bandwidth}} = \frac{f_0}{\Delta f}$$

$$Q = \frac{150 \times 10^3}{600}$$

$$Q = 250$$

Hence, the value of Q factor of circuit is 250.

Question 29 **Digital Electronics**

The maximum clock frequency in MHz of a 4-stage ripple counter, utilizing flip-flops, with each flip-flop having a propagation delay of 20 ns, is _______. (round off to one decimal place).

Ans. 12.5 (Range : 12.3 to 12.7)

Sol. **Given :**

(i) 4-stage ripple counter

(ii) Number of bits $n = 4$

(iii) Propagation delay of each flip-flop $t_{pd} = 20$ nsec

(iv) Maximum clock frequency of n-state ripple counter,

$$f_{CLK} = \frac{1}{nt_{pd}} = \frac{1}{4 \times 20 \times 10^{-9}} = 12.5 \text{ MHz}$$

Hence, the correct answer is 12.5.

Question 30 **Power System**

If only 5% of the supplied power to a cable reaches the output terminal, the power loss in the cable, in decibels, is _______. (round off to nearest integer)

Ans. 13 (Range : 13 to 13)

Sol. **Given :**

Power reaches at output terminal $(P_{out}) = 5\%$ of the supplied power (P_{in}).

Power loss in decibels (dB), in cable $= 10 \log\left(\frac{P_{in}}{P_{out}}\right) = 10 \log P_{in} - 10 \log P_{out}$

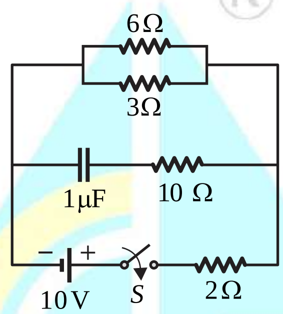
Power loss in decibels (dB), in cable $= 10\log\dfrac{P_{in}}{0.05\,P_{in}} = 10\log 20$

Power loss in decibels (dB), in cable $= 13.01\text{ dB} \approx 13\text{ dB}$

Hence, the power loss in the cable, in decibels is 13 dB.

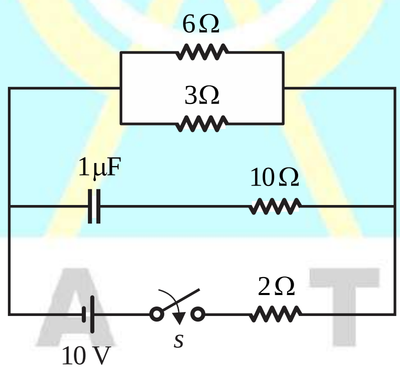
Question 31 **Network Theory**

In the circuit shown below, the switch S is closed at $t = 0$. The magnitude of the steady state voltage, in volts, across the $6\,\Omega$ resistor is _______. (round off to two decimal places).



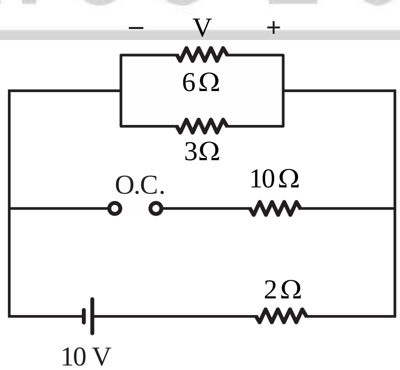
Ans. 5 (Range : 4.95 to 5.50)

Sol. Given circuit is shown below,

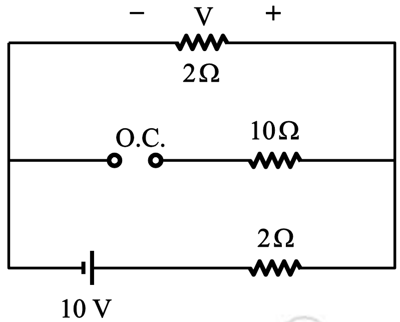


At steady state : Switch s is closed.

In steady state capacitor behaves as open circuit.



After simplification,



Applying voltage division rule, $V = \frac{2\times 10}{2+2} = 5\text{ V}$

Hence, the magnitude of steady state voltage across $6\,\Omega$ resistor is 5 V.

Question 32 **Power Electronics**

A single-phase full-bridge diode rectifier feeds a resistive load of $50\ \Omega$ from a 200 V, 50 Hz single phase AC supply. If the diodes are ideal, then the active power, in watts, drawn by the load is ______. (round off to nearest integer)

Ans. **800 (Range : 795 to 805)**

Sol. **Given :**

(i) 1-ϕ full bridge rectifier.

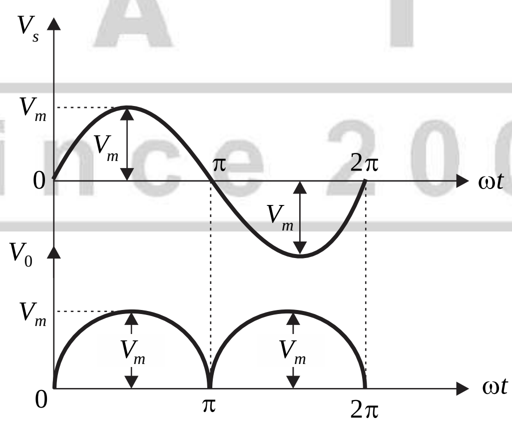
(ii) Resistive load, $R = 50\ \Omega$

(iii) Supply voltage, $V_{s(rms)} = 200$ V

(iv) Frequency, $f = 50$ Hz

Peak voltage, $V_m = V_{s(rms)} \times \sqrt{2} = 200\sqrt{2}$ Volts

The active power drawn by the load in watts.



We know, average output power for the resistive load is given by,

$$P_{0(avg)} = \frac{V_{0(rms)}^2}{R}$$

For 1-ϕ full bridge diode rectifier, in case of resistive load,

$$V_{0(rms)} = V_{s(rms)} = 200 \text{ V}$$

$$P_0(\text{avg}) = \frac{200^2}{50} = 800 \text{ W}$$

Hence, the correct answer is 800 W.

Question 33 **Power Electronics**

The voltage at the input of an AC-DC rectifier is given by $v(t) = 230\sqrt{2}\sin\omega t$ where $\omega = 2\pi\times 50$ rad/sec. The input current drawn by the rectifier is given by

$$i(t) = 10\sin\left(\omega t - \frac{\pi}{3}\right) + 4\sin\left(3\omega t - \frac{\pi}{6}\right) + 3\sin\left(5\omega t - \frac{\pi}{3}\right).$$

The input power factor, (rounded off to two decimal places), is, ________ lag.

Ans. 0.4473 (Range : 0.43 to 0.47)

Sol. Given :

(i) $V(t) = 230\sqrt{2}\sin(\omega t)$

(ii) $i(t) = 10\sin\left(\omega t - \frac{\pi}{3}\right) + 4\sin\left(3\omega t - \frac{\pi}{6}\right) + 3\sin\left(5\omega t - \frac{\pi}{3}\right)$

(iii) $f = 50$ Hz

(iv) $\omega = 2\pi\times 50$ rad/sec

Input power factor (IPF),

$$V(t)|_{RMS} = 230 \text{ V},\ i_1(t)|_{RMS} = \frac{10}{\sqrt{2}} \text{ A}$$

Fundamental displacement factor (FDF) $= \cos\phi_1 = \cos\frac{\pi}{3} = \frac{1}{2}$

Where, ϕ_1 is the angle between voltage and fundamental component of current.

$$i(t)|_{RMS} = \sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = 7.905 \text{ A}$$

We know, IPF $=$ Distortion factor $\times$ Fundamental displacement factor

IPF $= g \times \text{FDF}$

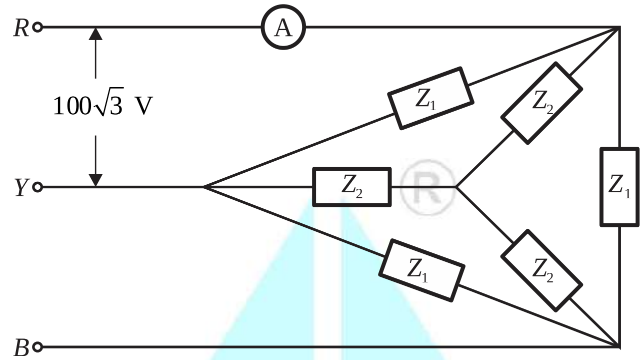
Also, $g = \frac{i_1(t)|_{RMS}}{i(t)|_{RMS}} = \frac{10/\sqrt{2}}{7.905} = 0.8946$

Therefore, IPF $= 0.8946 \times 0.5 = 0.4473$

Hence, the correct answer is 0.4473.

Question 34 **Power System**

Two balanced three-phase loads, as shown in the figure, are connected to a $100\sqrt{3}$ V, three phase, 50 Hz main supply. Given : $Z_1 = (18 + j24)\ \Omega$ and $Z_2 = (6 + j8)\ \Omega$. The ammeter reading, in amperes, is ______. (round off to nearest integer)



Ans. **20 (Range : 20 to 20)**

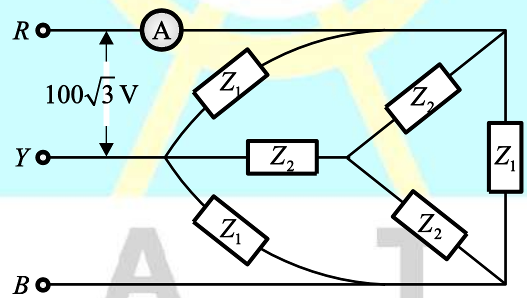
Sol. **Given :**

(i) Line to line voltage, $V_{LL} = 100\sqrt{3}$ V

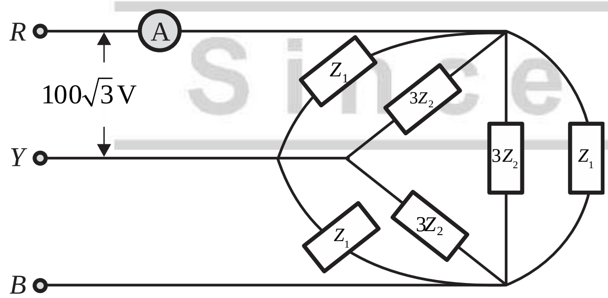
(ii) 3-ϕ, 50 Hz

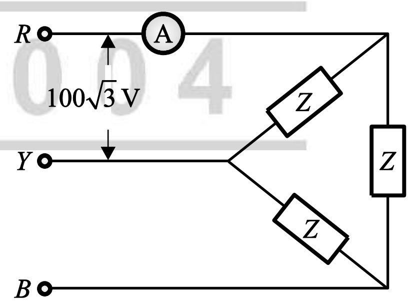
(iii) $Z_1 = (18 + j24)\Omega$

(iv) $Z_2 = (6 + j8)\Omega$



From above given figure, converting inner-star into delta.





Per-phase equivalent impedance is,

$$Z = Z_{ph} = Z_1 \| 3Z_2$$

$$Z_{ph} = \frac{3Z_1 Z_2}{Z_1 + 3Z_2} = \frac{3(18 + j24)(6 + j8)}{18 + j24 + 3(6 + j8)}$$

$$Z_{ph} = \frac{3(18+j24)(6+j8)}{18+j24+18+j24}$$

$$Z_{ph} = 15\angle 53.10\Omega$$

In case of delta connection,

$$V_{ph} = V_{LL} \text{ and } I_{LL} = \sqrt{3} I_{ph}$$

Ammeter reading in ampere is,

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{100\sqrt{3}}{15\angle 53.10}$$

$$I_{ph} = 11.547\angle -53.10$$

Magnitude ammeter reading,

$$I_{LL} = \sqrt{3} I_{ph} = \sqrt{3} \times 11.547$$

$$I_{LL} = 20\,\text{A}$$

Hence, the correct answer is 20.

Question 35 **Electrical Machine**

The frequencies of the stator and rotor currents flowing in a three-phase, 8-pole induction motor are 40 Hz and 1 Hz, respectively. The motor speed, in rpm, is _______. (round off to nearest integer)

Ans. **585 (Range : 580 to 590)**

Sol. **Given :**

(i) 3-ϕ Induction motor

(ii) Pole, $P = 8$

(iii) Rotor current frequency, $f_r = 1\text{Hz}$

(iv) Stator current frequency, $f_s = 40$ Hz

Motor speed, N_r (in rpm)

Rotor current frequency, $f_r = s f_s$

Slip, $s = \frac{f_r}{f_s} = \frac{1}{40}$

Synchronous speed, $N_s = \frac{120 \times f_s}{P} = \frac{120 \times 40}{8} = 600$ rpm

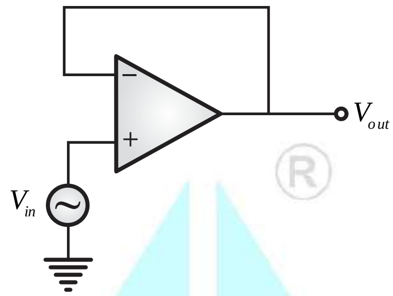
Motor speed, $N_r = (1-s)N_s = (1-0.025) \times 600 = 585$ rpm

Hence, the correct answer is 585 rpm.

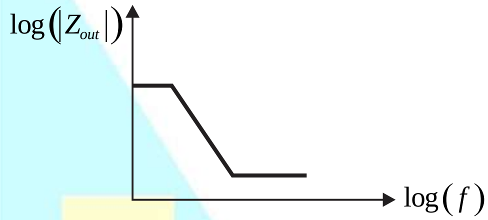
Q.36 to Q.65 Carry TWO Marks Each

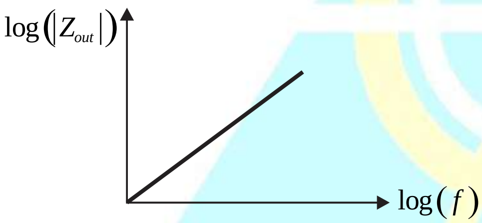
Question 36 **Analog Electronics**

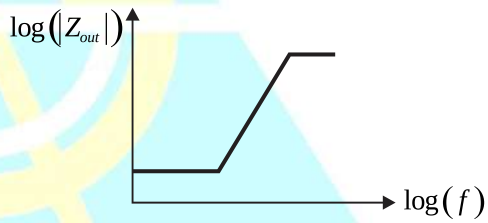
The output impedance of a non-ideal operational amplifier is denoted by Z_{out}. The variation in the magnitude of Z_{out} with increasing frequency, f, in the circuit shown below, is best represented by



(A) 

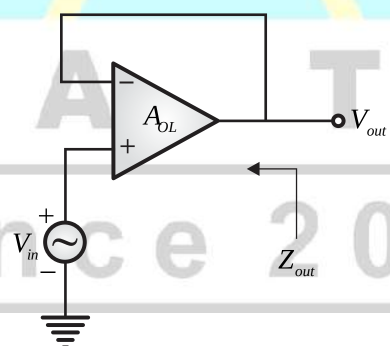
(B) 

(C) 

(D) 

Ans. **D**

Sol. Given circuit is shown below,



This is non-ideal Op-Amp circuit.

Hence, $$A_{CL} = \frac{V_{out}}{V_{in}} = \frac{A_{OL}}{1 + A_{OL}\beta}$$

Where, A_{OL} : Open loop gain, β : Feedback factor.

Here $\beta = 1 \quad \Rightarrow \quad A_{CL} = \frac{A_{OL}}{1 + A_{OL}}$

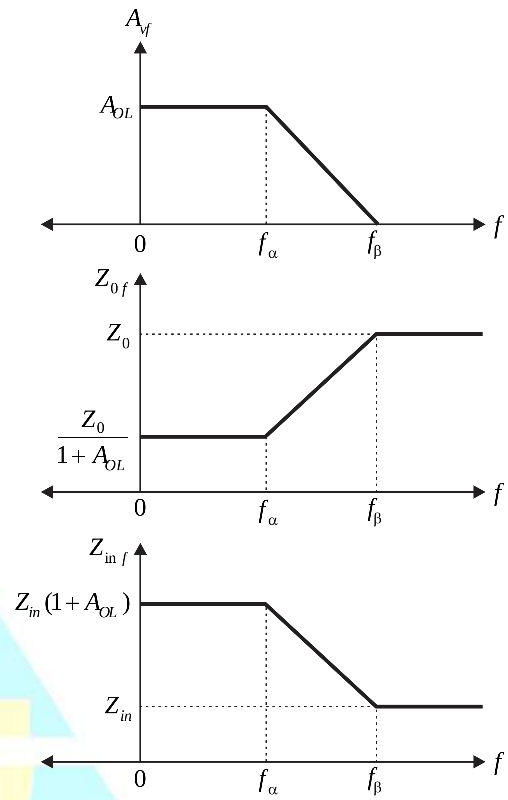
Now, its voltage series feedback amplifier,

Z_{in} = Input impedance without feedback

Z_0 = Output impedance without feedback

Z_{0f} = Output impedance with feedback

$Z_{in\,f}$ = Input impedance with feedback



As it is series shunt (Voltage series) feedback

$$Z_{out} = \frac{Z_0}{1+A_{OL}\beta}$$

$$Z_{in\,f} = Z_{in}(1+A_{OL}\beta)$$

Here, $\beta = 1 \quad \Rightarrow \quad Z_{0f} = \frac{Z_0}{1+A_{OL}}$

Now,

(i) 0 to f_α (Low frequency)

A_{OL} : Constant

Z_{0f} : Constant

(ii) $f > f_\alpha$ and $f < f_\beta$

A_{OL} will decrease

$$\uparrow Z_{0f} = \frac{Z_0}{1+A_{OL}\downarrow}$$

Z_{0f} will increase

(iii) $f > f_\beta$ (High frequency)

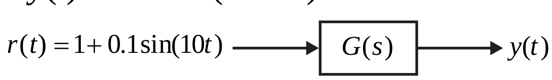
$$A_{OL} = 0$$

$$Z_{0f} = Z_0 \text{ (Constant)}$$

Hence, the correct option is (D).

Question 37 **Network Theory**

An LTI system is shown in the figure where $G(s) = \frac{100}{s^2 + 0.1s + 100}$. The steady state output of the system, to the input $r(t)$, is given as $y(t) = a + b\sin(10t + \theta)$. The values of '$a$' and '$b$' will be



(A) a = 100, b = 1 (B) a = 10, b = 1

(C) a = 1, b = 10 (D) a = 1, b = 100

Ans. C

Sol. **Given :** $G(s)=\dfrac{100}{s^2+0.1s+100}$

$$y(t)=a+b\sin(10t+\theta)$$

$$r(t)=1+0.1\sin 10t$$

$$r(t)=1+0.1\sin 10t \longrightarrow \boxed{\dfrac{100}{s^2+0.1s+100}} \longrightarrow y(t)=a+b\sin(10t+\theta)$$

Using super position theorem,

Case 1 : $r(t)=1$, $\omega=0$ rad/sec

The steady state response,

$$y_1(t)=\frac{100}{s^2+0.1s+100}\times 1$$

$$y_1(t)=\frac{100}{(j\omega)^2+0.1j\omega+100}$$

$$y_1(t)=\frac{100}{100}=1$$

$$y_1(t)\Big|_{\omega=0}=\frac{100}{100}=1$$

Case 2 : $r(t)=0.1\sin 10t$, $\omega=10$ rad/sec

The steady state response,

$$y_2(t)=\frac{100}{(j\omega)^2+j0.1\omega+100}\times 0.1\sin 10t$$

$$y_2(t)=\frac{10\sin 10t}{-100+j+100}=10\sin\left(10t-90^0\right)$$

Total steady state response,

$$y(t)=y_1(t)+y_2(t)$$

$$y(t)=1+10\sin\left(10t-90^0\right) \quad \ldots\text{(i)}$$

Given response, $y(t)=a+b\sin(10t+\theta)$ ...(ii)

Compare equation (i) and (ii),

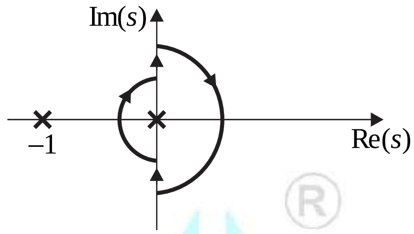
$a=1$ and $b=10$

Hence, the correct option is (C).

Question 38 **Control System**

The open loop transfer function of a unity gain negative feedback system is given as $G(s)=\dfrac{1}{s(s+1)}$.

The Nyquist contour in the s-plane encloses the entire right half plane and a small neighborhood around the origin in the left half plane, as shown in figure below. The number of encirclements of the point $(-1+j0)$ by the Nyquist plot of $G(s)$, corresponding to the Nyquist contour, is denoted as N. Then N equals to



(A) 3 (B) 2

(C) 0 (D) 1

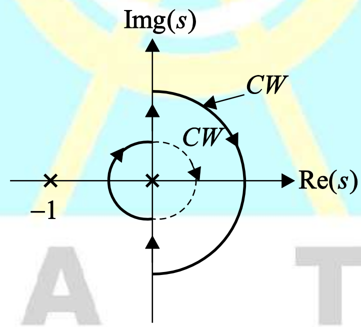
Ans. D

Sol. **Method 1**

Given open loop transfer function is,

$$G(s)=\frac{1}{s(s+1)}$$

The Nyquist contour in the s-plane encloses the entire right half plane and a small neighborhood around the origin as shown in figure below



In the nyquist contour, the encirclement about origin is clockwise

$N=P-Z$ [ACW]

$P=$ Number of right hand poles of open loop transfer function including the poles at origin

$N=$ Number of anticlockwise encirclement about the critical point $(-1+j0)$ in counter clockwise direction

$Z=$ Number of right hand poles of closed loop transfer function

From the open loop transfer function

$P=1$ [Since $G(s)$ has one pole at origin]

Characteristic equation,

$$1+G(s)=0$$

$$s^2+s+1=0$$

Since, all the coefficient of characteristic equation of second order system are positive. System is stable i.e. no-poles in RHP.

$Z = 0$

$N = P - Z$ [A.C.W.]

$N = 1 - 0$

$N = 1$

Hence, the number of encirclement around the critical points $(-1 + j0)$ is 1.

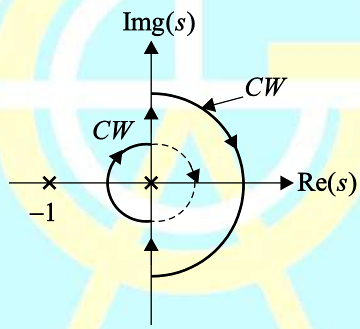
Hence, the correct option is (D).

Method 2

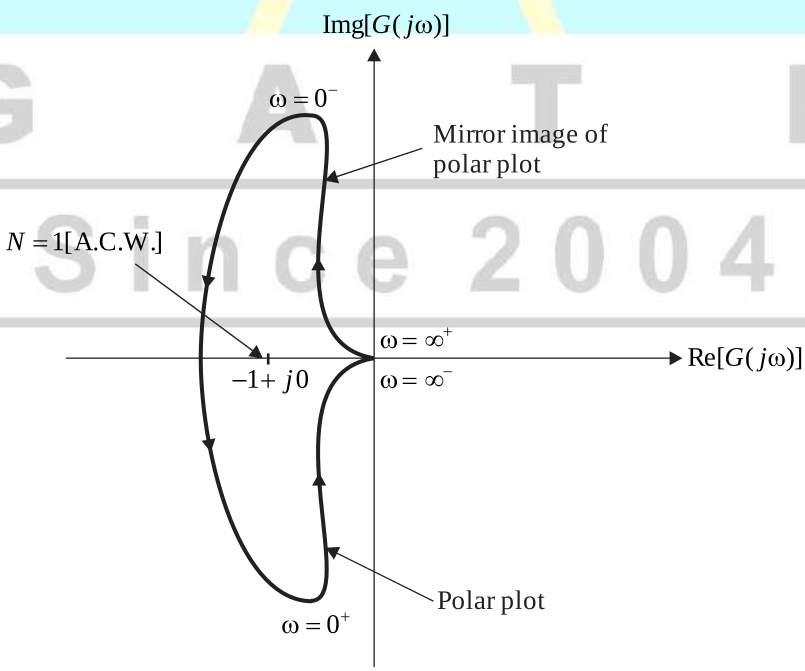
Given open loop transfer function is,

$$G(s) = \frac{1}{s(s+1)}$$

The Nyquist contour in the s-plane encloses the entire right half plane and a small neighborhood around the origin as shown in figure below



Corresponding to the Nyquist contour the nyquist plot is shown below.



Note : In nyquist plot ACW movement from $\omega = 0^-$ to $\omega = 0^+$
Since, in nyquist contour CW movement around the origin.

Hence, the number of encirclement around the critical point $(-1 + j0)$ in nyquist plot is 1.

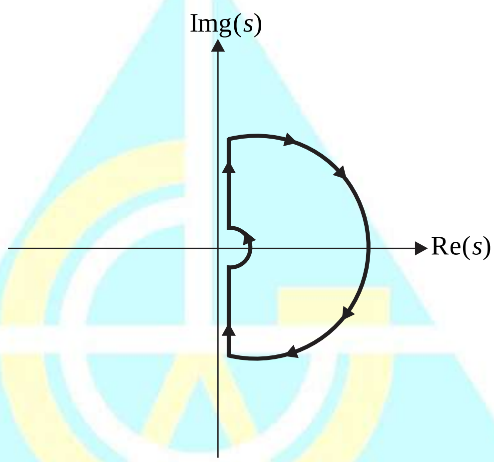
Hence, the correct option is (D).

☒ Avoid this Mistake :

Mistake 1 :

Given open loop transfer function, $G(s) = \dfrac{1}{s(s+1)}$

In general, the nyquist contour in the *s*-plane encloses the entire right half plane as shown in figure below



For above nyquist contour, $N = P - Z$ (Anticlockwise)

Where, $N =$ Number of encirclement about critical
Point $(-1 + j0)$ in counter clockwise direction
$P =$ Number of right hand poles of open loop transfer function i.e. $G(s)$
$Z =$ Number of right hand poles of closed loop transfer function

From the open loop transfer function $P = 0$

Characteristic equation is given by,

$$1 + G(s) = 0$$

$$s^2 + s + 1 = 0$$

Since, all the coefficient of characteristic equation of second order system are positive. System is stable i.e. no poles in RHP.

$$Z = 0$$

$$N = P - Z$$

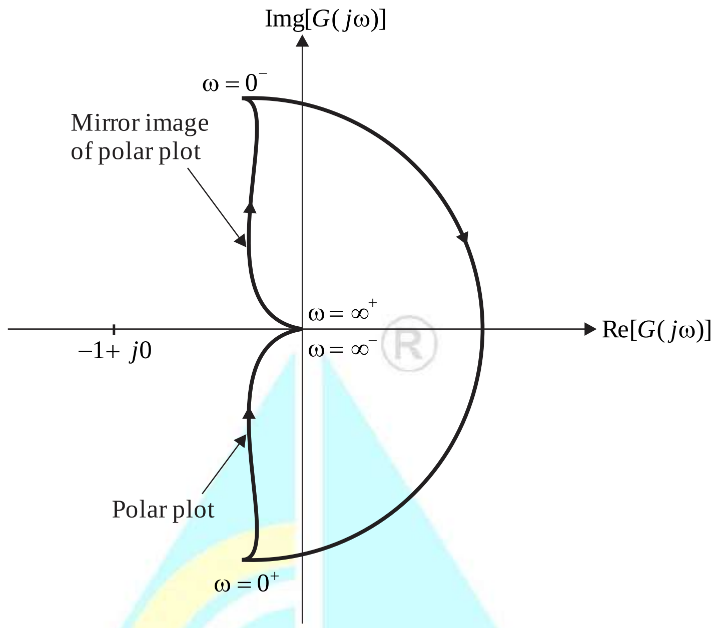
$$N = 0$$

The number of encirclement about the point $(-1 + j0)$ in the Nyquist polt of $G(s)$ is zero.

Mistake 2 :

Given open loop transfer function, $G(s) = \dfrac{1}{s(s+1)}$

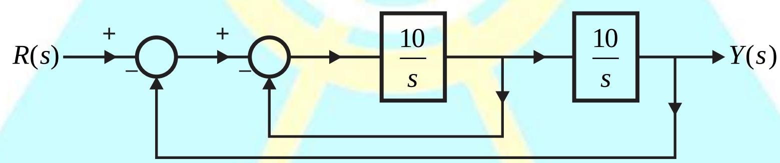
Nyquist plot is,



Number of encirclement about the critical point $N=0$.

Question 39 **Control System**

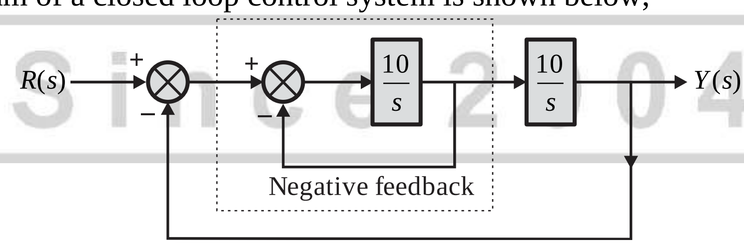
The damping ratio and undamped natural frequency of a closed loop system as shown in the figure, are denoted as ξ and ω_n, respectively. The values of ξ and ω_n are



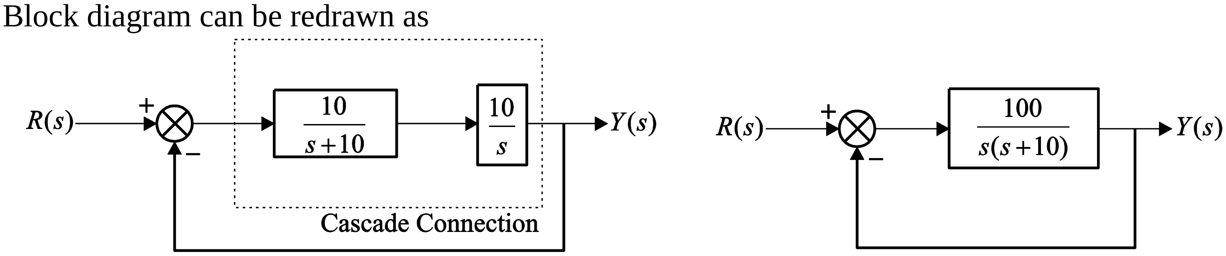
(A) $\xi = 0.1$ and $\omega_n = 10$ rad/s
(B) $\xi = 0.5$ and $\omega_n = 10$ rad/s
(C) $\xi = 0.707$ and $\omega_n = 10$ rad/s
(D) $\xi = 0.707$ and $\omega_n = 100$ rad/s

Ans. **B**

Sol. Given block diagram of a closed loop control system is shown below,



Block diagram can be redrawn as



Transfer function of above block diagrams is,

$$\frac{Y(s)}{R(s)} = \frac{\frac{100}{s(s+10)}}{1+\frac{100}{s(s+10)}}$$

$$\frac{Y(s)}{R(s)} = \frac{100}{s^2+10s+100} \qquad \ldots \text{(i)}$$

Transfer function for standard second order system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \qquad \ldots \text{(ii)}$$

Where, $\xi =$ Damping ratio, $\omega_n =$ Natural angular frequency.

On comparing equation (i) and equation (ii),

$\omega_n^2 = 100$; $\omega_n = 10$ rad/sec

$2\xi\omega_n = 10$

$$\xi = \frac{10}{2(10)} = \frac{1}{2} = 0.5$$

Hence, the correct option is (B).

Question 40 **Mathematics**

e^A denotes the exponential of a square matrix A. Suppose λ is an eigenvalue and v is the corresponding eigen-vector of matrix A.

Consider the following two statements :

Statement 1 : e^{λ} is an eigenvalue of e^A.

Statement 2 : v is an eigen-vector of e^A.

Which one of the following options is correct?

(A) Statement 1 is true and statement 2 is false.

(B) Statement 1 is false and statement 2 is true.

(C) Both the statements are correct.

(D) Both the statements are false.

Ans. **C**

Sol. **Given :** Exponential series,

$$e^A = 1+\frac{A}{1!}+\frac{A^2}{2!}+\frac{A^3}{3!}+\ldots\ldots.\infty$$

$$e^A = 1+\frac{A}{1}+\frac{A^2}{2}+\frac{A^3}{6}+\ldots\ldots.\infty$$

Polynomial function of infinite series.

where, $A_{n\times n} =$ Matrix, $\lambda_A =$ Eigen value of A and $V =$ Eigen vector corresponding to λ_A.

Any operation on matrix, same operation are performed on eigen values, so e^{λ} will the eigen value of e^{A}, but eigen vector remains same.

So, V will be an eigen vector of e^{A}.

That means, both statements are correct.

Hence, the correct option is (C).

Question 41 **Mathematics**

Let $f(x)=\int_0^x e^t(t-1)(t-2)\,dt$. Then $f(x)$ decreases in the interval

(A) $x\in(2,3)$ (B) $x\in(1,2)$

(C) $x\in(0,1)$ (D) $x\in(0.5,1)$

Ans. **B**

Sol. **Given :** $f(x)=\int_0^x e^t(t-1)(t-2)\,dt$

$f(x)$ decreases, that means slope of $f(x)$ means $(f'(x))$ will be negative.

i.e., $f'(x)<0$ [Slope $=\dfrac{d}{dx}f(x)=f'(x)$]

We need to find derivative of $f(x)$.

By Leibnitz rule,

$$f'(x)=\frac{d}{dx}\int_0^x e^t(t-1)(t-2)\,dt \qquad f'(x)<0$$

$$\left[e^x(x^2-3x+2)(1)-e^0(0^2+0+2)(0)\right]<0$$

$$(x^2-3x+2)<0$$

So, $1<x<2$

Hence, the correct option is (B).

Question 42 **Mathematics**

Consider a matrix $A=\begin{bmatrix}1&0&0\\0&4&-2\\0&1&1\end{bmatrix}$.

The matrix A satisfies the equation $6A^{-1}=A^2+cA+dI$, where c and d are scalars and I is the identity matrix. Then $(c+d)$ is equal to

(A) 5 (B) 17

(C) 11 (D) – 6

Ans. **A**

Sol. **Given :** Matrix $A=\begin{bmatrix}1&0&0\\0&4&-2\\0&1&1\end{bmatrix}_{3\times3}$

Method 1

Characteristics equation,

$$|A-\lambda I|=0$$

$$\begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 4-\lambda & -2 \\ 0 & 1 & 1-\lambda \end{bmatrix}=0$$

$$(1-\lambda)\left[(4-\lambda)(1-\lambda)+2\right]-0+0=0$$

$$(1-\lambda)\left[(4-\lambda)(1-\lambda)+2\right]=0$$

$$(1-\lambda)\left[4-5\lambda+\lambda^2+2\right]=0$$

$$(1-\lambda)(\lambda^2-5\lambda+6)=0 \qquad \text{[Characteristics equation of matrix } A\text{]}$$

Root of characteristic equation,

$$\lambda=1,2,3 \qquad \text{[Eigen values of matrix } A\text{]}$$

$$(1-\lambda)(\lambda^2-5\lambda+6)=0$$

$$\lambda^2-5\lambda+6-\lambda^3+5\lambda-6\lambda=0$$

$$-\lambda^3+\lambda^2-6\lambda+6=0$$

$$\lambda^3-\lambda^2+6\lambda-6=0$$

According to Cayley Hamilton's Theorem, $A=\lambda$

$$A^3-A^2+6A-6I=0$$

Multiplying A^{-1} both sides.

$$A^{-1}A^3-A^{-1}A^2+6A^{-1}A-6A^{-1}=0$$

$$A^2-A+6I=6A^{-1}$$

$$6A^{-1}=A^2-A+6I \qquad \ldots\text{(i)}$$

As given in question,

$$6A^{-1}=A^2+CA+DI \qquad \ldots\text{(ii)}$$

From equation (i) and (ii),

We get, $C=-1,\ D=6$

So, $C+D=-1+6=5$

Hence, the correct option is (A).

Method 2

By Cayley Hamilton's Theorem,

Every square matrix satisfies its characteristics equation $(A=\lambda)$.

$$6A^{-1}=A^2+CA+dI \qquad \ldots\text{(i)}$$

Characteristics equation is given by,

$$\lambda^3 - (tr)\lambda^2 + \sum(\text{Principle co-factor})\lambda - |A| = 0 \quad \ldots(ii)$$

$$tr(A) = 1+4+1 = 6$$

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{vmatrix} = 1(4+2) - 0 + 0 = 6$$

$$\sum(\text{Principle cofactor}) = 6+1+4 = 11$$

By equation (ii),

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$A^3 - 6A^2 + 11A\lambda - 6I = 0$$

Multiplying A^{-1} both sides.

$$A^{-1}A^3 - A^{-1}A^2 + 6A^{-1}A - 6A^{-1} = 0$$

$$A^2 - A + 6I = 6A^{-1}$$

$$6A^{-1} = A^2 - A + 6I \quad \ldots(iii)$$

As given in question,

$$6A^{-1} = A^2 + CA + DI \quad \ldots(iv)$$

From equation (iii) and (iv),

We get, $C = -1$, $D = 6$

So, $C + D = -1 + 6 = 5$

Hence, the correct option is (A).

Question 43 **Power System**

The fuel cost functions in rupees/hour for two 600 MW thermal power plants are given by

Plant 1 : $C_1 = 350 + 6P_1 + 0.004P_1^2$

Plant 2 : $C_2 = 450 + aP_2 + 0.003P_2^2$

where P_1 and P_2 are power generated by plant 1 and plant 2, respectively, in MW and a is constant. The incremental cost of power (λ) is 8 rupees per MWh. The two thermal power plants together meet a total power demand of 550 MW. The optimal generation of plant 1 and plant 2 in MW, respectively, are

(A) 200, 350 (B) 350, 200

(C) 325, 225 (D) 250, 300

Ans. **D**

Sol. **Given :** Fuel cost functions

Plant 1 : $C_1 = 350 + 6P_1 + 0.004P_1^2$ Rs/hr

Plant 2 : $C_2 = 450 + aP_1 + 0.003P_2^2$ Rs/hr

For optimal generator, to share total load of 550 MW, incremental cost (I.C.) of units in operation must be same.

$$(IC)_1 = \frac{dC_1}{dP_1} = 6 + 0.008P_1 \text{ Rs/MWhr}$$

$$(IC)_2 = \frac{dC_2}{dP_2} = a + 0.006P_2 \text{ Rs/MWhr}$$

For optimal generator, $(IC)_1 = (IC)_2 = \lambda$

$$6 + 0.008P_1 = a + 0.006P_2 = 8\,\text{MW}$$

$$P_1 = \frac{8-6}{0.008} = 250\,\text{MW}$$

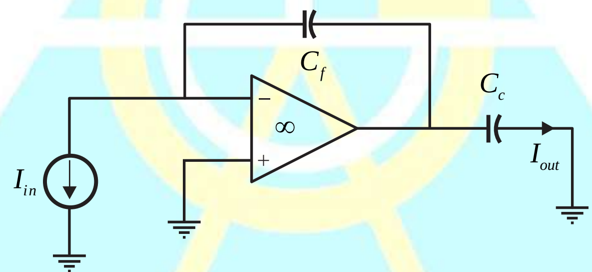
$$P_2 = (550 - 250)\,\text{MW} = 300\,\text{MW}$$

$$P_1 = 250\,\text{MW} \text{ and } P_2 = 300\,\text{MW}$$

Hence, the correct option is (D).

Question 44 **Analog Electronics**

The current gain (I_{out} / I_{in}) in the circuit with an ideal current amplifier given below is



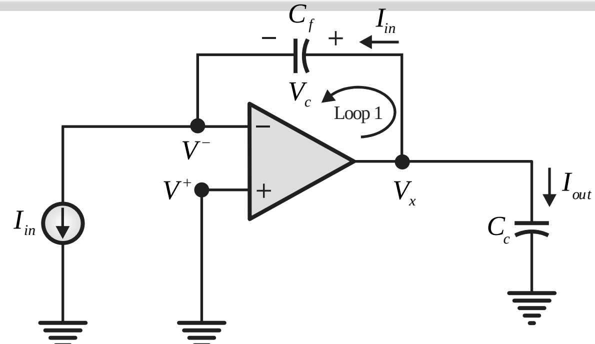
(A) $-\frac{C_c}{C_f}$ (B) $-\frac{C_f}{C_c}$

(C) $\frac{C_c}{C_f}$ (D) $\frac{C_f}{C_c}$

Ans. **C**

Sol. **Method 1**

Given circuit is shown below,



From virtual ground concept, $V^- = V^+ = 0$

Let us assume voltage across C_f is V_c

Applying KVL in loop-1,

$$-V_x + V_c + V^- = 0$$

$$V_x = V_c \qquad [\because V^- = 0]$$

Capacitor voltage V_c can calculated as,

$$V_c = \frac{1}{C_f}\int_0^t I_{in}\,dt$$

$$V_c = \frac{I_{in}}{C_f}\int_0^t dt \qquad [\because I_{in} \text{ is constant}]$$

$$V_c = \frac{I_{in}}{C_f}\times t$$

$$V_c = \frac{t}{C_f} I_{in} \qquad [\because V_c = V_x] \qquad \ldots\text{(i)}$$

Here V_x is also a voltage across capacitor C_c so it can calculated as,

$$V_x = \frac{1}{C_c}\int_0^t I_{out}\,dt$$

$$V_x = \frac{I_{out}}{C_c}\int_0^t dt \qquad [\because I_{out} \text{ is constant}]$$

$$V_x = \frac{I_{out}}{C_c}\times t$$

$$V_x = \frac{t}{C_c} I_{out} \qquad \ldots\text{(ii)}$$

Compare equation (i) and (ii),

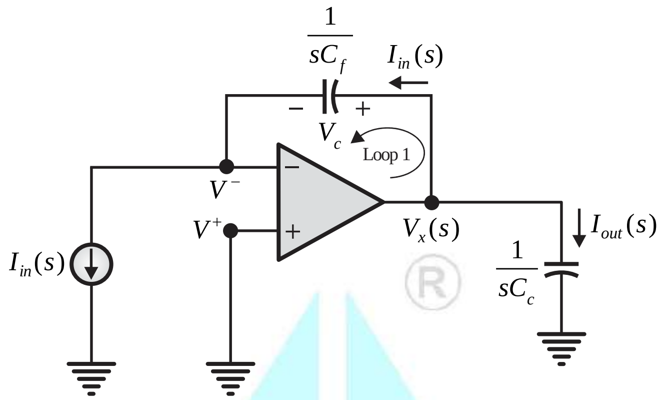
$$\frac{t}{C_f} I_{in} = \frac{t}{C_c} I_{out}$$

$$\frac{I_{out}}{I_{in}} = \frac{C_c}{C_f}$$

Hence, the correct option is (C).

Method 2

Given circuit is shown below in the form of Laplace transform,



Due to virtual ground concept, $V^- = V^+ = 0$

Applying KVL in loop-1,

$$V_x(s) = \frac{1}{sC_f} I_{in}(s) \quad \text{...(i)}$$

In the same way,

$$V_x(s) = \frac{1}{sC_c} I_{out}(s) \quad \text{...(ii)}$$

Compare equations (i) and (ii),

$$\frac{1}{sC_f} I_{in}(s) = \frac{1}{sC_c} I_{out}(s)$$

$$\frac{I_{out}(s)}{I_{in}(s)} = \frac{C_c}{C_f}$$

Taking inverse Laplace transform, $\frac{I_{out}}{I_{in}} = \frac{C_c}{C_f}$

Hence, the correct option is (C).

Question 45 **Electromagnetic Field**

If the magnetic field intensity (**H**) in a conducting region is given by the expression, $\mathbf{H} = x^2\hat{i} + x^2y^2\hat{j} + x^2y^2z^2\hat{k}$ A/m. The magnitude of the current density, in A/m², at x = 1 m, y = 2 m and z = 1 m, is

(A) 8 (B) 12

(C) 16 (D) 20

Ans. **B**

Sol. **Given :** Magnetic field intensity $\vec{H} = x^2\hat{i} + x^2y^2\hat{j} + x^2y^2z^2\hat{k}$ A/m

$$H_x = x^2,\ H_y = x^2y^2,\ H_z = x^2y^2z^2$$

From ampere's law, $\bar{J} = \nabla \times \bar{H}$

Magnitude of current density,

$$|\bar{J}| = |\nabla \times \bar{H}|$$

$$\bar{J} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$\bar{J} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & x^2 y & x^2 y^2 z^2 \end{vmatrix}$$

$$\bar{J} = 2x^2 yz^2 \hat{i} - 2xy^2 z^2 \hat{j} + 2xy^2 \hat{k}$$

$\vec{J}$ at point $(x=1,\ y=2,\ z=1)$ is,

$\therefore$ $$\bar{J}\Big|_{(1,2,1)} = 4\hat{i} - 8\hat{j} + 8\hat{k}$$

$$|\bar{J}| = 12 \text{ A/m}^2$$

Hence, the correct option is (B).

Question 46 **Signals & Systems**

Let a causal LTI system be governed by the following differential equation $y(t) + \frac{1}{4}\frac{dy}{dt} = 2x(t)$, where $x(t)$ and $y(t)$ are the input and output respectively. Its impulse response is

(A) $8e^{-4t}u(t)$ (B) $8e^{-\frac{1}{4}t}u(t)$

(C) $2e^{-4t}u(t)$ (D) $2e^{-\frac{1}{4}t}u(t)$

Ans. **A**

Sol. Given differential equation is, $y(t) + \frac{1}{4}\frac{dy}{dt} = 2x(t)$

Taking Laplace transform on both sides, we get

$$Y(s) + \frac{1}{4}sY(s) = 2X(s)$$

$$\left(\frac{s}{4} + 1\right)Y(s) = 2X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{2}{\frac{s}{4} + 1}$$

$$\frac{Y(s)}{X(s)} = \frac{8}{s+4}$$ which is the transfer function of this system.

$\therefore$ Impulse response will be, $h(t) = L^{-1}\left[\frac{8}{s+4}\right] = 8e^{-4t}u(t)$

Hence, the correct option is (A).

Question 47 **Signals & Systems**

Let an input $x(t) = 2\sin(10\pi t) + 5\cos(15\pi t) + 7\sin(42\pi t) + 4\cos(45\pi t)$ is passed through an LTI system having an impulse response,

$$h(t) = 2\left(\frac{\sin(10\pi t)}{\pi t}\right)\cos(40\pi t).$$

The output of the system is

(A) $2\sin(10\pi t) + 5\cos(15\pi t)$

(B) $2\sin(10\pi t) + 4\cos(45\pi t)$

(C) $7\sin(42\pi t) + 4\cos(45\pi t)$

(D) $5\cos(15\pi t) + 7\sin(42\pi t)$

Ans. **C**

Sol. Given impulse response is,

$$h(t) = 2\left(\frac{\sin 10\pi t}{\pi t}\right)\cos(40\pi t)$$

$$h(t) = \frac{\sin 50\pi t - \sin 30\pi t}{\pi t}$$

$$h(t) = \frac{\sin 50\pi t}{\pi t} - \frac{\sin 30\pi t}{\pi t}$$

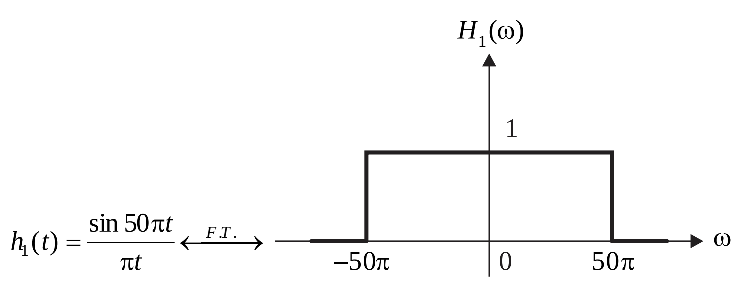
$$h(t) = h_1(t) - h_2(t)$$

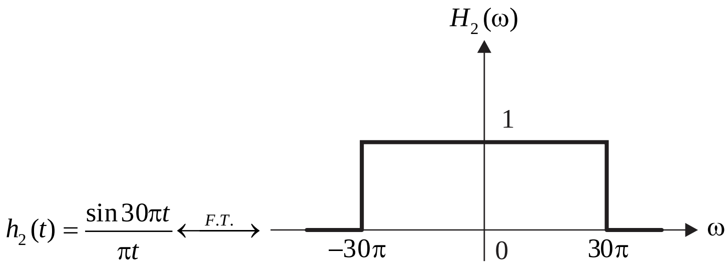
Where, $h_1(t) = \frac{\sin 50\pi t}{\pi t}$ and $h_2(t) = \frac{\sin 30\pi t}{\pi t}$

The Fourier transform of $h(t)$ is,

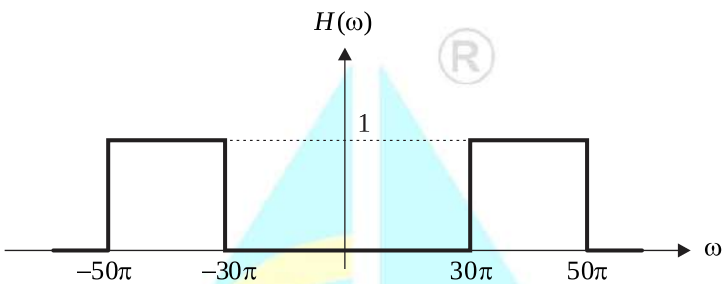
$$h(t) \xleftrightarrow{FT} H(\omega) = H_1(\omega) - H_2(\omega)$$

$$H(\omega) = H_1(\omega) - H_2(\omega)$$





$\therefore$ $H(\omega) = H_1(\omega) - H_2(\omega)$ can be plotted as shown below,



Given system is a bandpass filter with cut off frequencies 30π rad/sec and 50π rad/sec.

Given that, input $x(t) = 2\sin(10\pi t) + 5\cos(15\pi t) + 7\sin(42\pi t) + 4\cos(45\pi t)$

The frequency components of $x(t)$ are,

$\omega_1 = 10\pi$ rad/sec, $\omega_2 = 15\pi$ rad/sec

$\omega_3 = 42\pi$ rad/sec and $\omega_4 = 45\pi$ rad/sec

$\therefore$ Only the frequency components ω_3 and ω_4 will be passed through this system as these frequency components lie in the range $30\pi \le |\omega| \le 50\pi$ which is the range of frequencies of $H(\omega)$.

As $|H(\omega)| = 1$, the amplitude of the frequency components ω_3 and ω_4 will be unchanged.

$\therefore$ The output of the system will be $y(t) = 7\sin(42\pi t) + 4\cos(45\pi t)$

Hence, the correct option is (C).

Question 48 **Signals & Systems**

Consider the system as shown below

where $y(t) = x(e^t)$. The system is

(A) non-linear and causal.
(B) linear and non-causal.
(C) non-linear and non-causal.
(D) linear and causal.

Ans. **B**

Sol. **Given :** $y(t) = x(e^t)$

For input $x_1(e^t)$, output will be $y_1(t)$

$$y_1(t) = x_1(e^t)$$

Multiplying both sides by a scalar quantity 'a'

$$a y_1(t) = a x_1(e^t) \quad \ldots(i)$$

For input $x_2(e^t)$, output will $y_2(t)$

$$y_2(t) = x_2(e^t)$$

Multiplying both sides by a scalar quantity 'b'

$$b\, y_2(t) = b\, x_2(e^t) \quad \ldots(ii)$$

Adding equation (i) and (ii) we get

$$a\, y_1(t) + b\, y_2(t) = a\, x_1(e^t) + b\, x_2(e^t) \quad \ldots(iii)$$

Let for input $a\, x_1(e^t) + b\, x_2(e^t)$, output is $y_3(t)$

$$y_3(t) = a\, x_1(e^t) + b\, x_2(e^t)$$

But $a\, x_1(e^t) + b\, x_2(e^t) = a\, y_1(t) + b\, y_2(t)$

$\therefore$ $$y_3(t) = a\, y_1(t) + b\, y_2(t)$$

Hence, system satisfies both additivity and homogeneity law.

$\therefore$ The system is a linear system.

From the given system $y(t) = x(e^t)$

$$y(0) = x(e^0) = x(1)$$

$$y(1) = x(e^1) = x(e) = x(2.71)$$

So, present value of output depends on future value of input.

$\therefore$ The given system is non causal.

Hence, the correct option is (B).

Question 49 **Signals & Systems**

The discrete time Fourier series representation of a signal $x[n]$ with period N is written as $x[n] = \sum_{k=0}^{N-1} a_k e^{j(2kn\pi/N)}$. A discrete time periodic signal with period $N = 3$, has the non-zero Fourier series coefficients : $a_{-3} = 2$ and $a_4 = 1$. The signal is

(A) $2 + 2e^{-\left(j\frac{2\pi}{6}n\right)} \cos\left(\frac{2\pi}{6}n\right)$

(B) $2 + 2e^{\left(j\frac{2\pi}{6}n\right)} \cos\left(\frac{2\pi}{6}n\right)$

(C) $1 + 2e^{\left(j\frac{2\pi}{6}n\right)} \cos\left(\frac{2\pi}{6}n\right)$

(D) $1 + 2e^{\left(j\frac{2\pi}{3}n\right)} \cos\left(\frac{2\pi}{6}n\right)$

Ans. **C**

Sol. **Given :** $x[n]=\sum_{k=0}^{N-1} a_k e^{j\left(\frac{2\pi kn}{N}\right)}$

Also given that $N=3$, $a_{-3}=2$ and $a_4=1$

As the Fourier series coefficients are periodic in nature, so

$a_{N+k}=a_k$

$a_{N-k}=a_{-k}$ (N is period of the signal)

For $k=3$,

$a_{N-3}=a_{-3}$

$a_{3-3}=a_{-3}$

$a_0=a_{-3}=2$

For $k=4$,

$a_{N+1}=a_1$

$a_{3+1}=a_1$ $(\because \quad N=3)$

$a_4=a_1=1$

Thus, $a_0=2,\ a_1=1$.

On expanding $x(n)$, we get

$$x[n]=a_0+a_1 e^{\frac{j2\pi n}{3}}$$

$$x[n]=2+e^{\frac{j2\pi n}{3}}$$

$$x[n]=1+1+e^{\frac{j2\pi n}{3}}$$

$$x[n]=1+e^{\frac{j\pi n}{3}}\left[e^{-j\frac{\pi}{3}n}+e^{j\frac{\pi}{3}n}\right]$$

$$x[n]=1+e^{\frac{j\pi n}{3}}\cdot 2\cos\frac{\pi n}{3}$$

$$x[n]=1+2e^{\frac{j\pi n}{3}}\cdot\cos\frac{2\pi n}{6}$$

Hence, the correct option is (C).

Key Point

For a periodic signal $x[n]$ has period N, the discrete time Fourier series coefficient a_k is also periodic in nature $a_{k+N}=a_k$ and $a_{-k+N}=a_{-k}$.

Question 50 **Mathematics**

Let, $f(x, y, z) = 4x^2 + 7xy + 3xz^2$. The direction in which the function $f(x, y, z)$ increases most rapidly at point $P = (1, 0, 2)$ is

(A) $20\hat{i} + 12\hat{k}$ (B) $20\hat{i} + 7\hat{j} + 12k$

(C) $20\hat{i} + 7\hat{j}$ (D) $20\hat{i}$

Ans. **B**

Sol. Given, scalar point function,

$$f(x, y, z) = 4x^2 + 7xy + 3xz^2$$

Gradient gives the direction in which the directional derivative will be maximum.

$$\text{Grad}(f) = \nabla \cdot f$$

$$\nabla \cdot f = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(4x^2 + 7xy + 3xz^2)$$

$$\nabla \cdot f = (8x + 7y + 3z^2)\hat{i} + (7x)\hat{j} + (6xz)\hat{k}$$

At point $P(1, 0, 2)$,

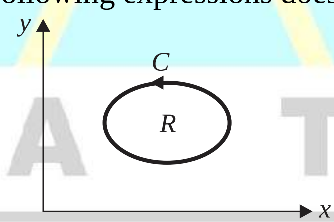
$$\nabla \cdot f = (8\times1 + 7\times0 + 3\times2^2)\hat{i} + (7\times1)\hat{j} + (6\times1\times2)\hat{k}$$

$$\nabla \cdot f = 20\hat{i} + 7\hat{j} + 12\hat{k}$$

Hence, the correct option is (B).

Question 51 **Mathematics**

Let R be a region in the first quadrant of the xy plane enclosed by a closed curve C considered in counter-clockwise direction. Which of the following expressions does not represent the area of the region R?

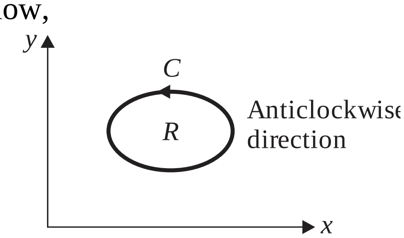


(A) $\oint_C x\,dy$ (B) $\frac{1}{2}\oint_C (x\,dy - y\,dx)$

(C) $\iint_R dx\,dy$ (D) $\oint_C y\,dx$

Ans. **D**

Sol. Given closed curve is shown below,



Green's theorem, $\oint_C \phi dx + \psi dy = \iint_S \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y} \right) dx\, dy$

Checking from options,

From option (A) :

$$\oint_C x dy = \iint_S (1-0)\, dx\, dy = \iint_S dx\, dy \qquad [\phi = 0,\ \psi = x]$$

[Area of region R in anticlockwise direction]

From option (B) :

$$\frac{1}{2}\oint_C x dy - y dy = \frac{1}{2}\iint_S (1-(-1)) dx\, dy \qquad [\phi = -y,\ \psi = x]$$

$$= \frac{1}{2}\iint_S 2 dx dy = \iint_S dx dy$$

[Area of region R in anticlockwise direction]

From option (C) :

$$\oint_C dx dy = \text{ Area of region } R$$

From option (D) :

$$\oint_C y dx = \iint_S (0-(1))\, dx\, dy \qquad [\phi = y,\ \psi = 0]$$

$$= \iint -1 dx\, dy = -\iint dx\, dy \qquad \text{[Area of region } R \text{ in clockwise direction]}$$

Hence, the correct option is (D).

Question 52 **Mathematics**

Let $\vec{E}(x, y, z) = 2x^2 \hat{i} + 5y \hat{j} + 3z \hat{k}$. The value of $\iiint_V (\vec{\nabla} \cdot \vec{E}) dV$, where V is the volume enclosed by the unit cube defined by $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$, is

(A) 3 (B) 8

(C) 10 (D) 5

Ans. C

Sol. Given vector, $\vec{E} = 2x^2 \hat{i} + 5y \hat{j} + 3z \hat{k}$

From above equation $E_x = 2x^2$, $E_y = 5y$, $E_z = 3z$.

Divergence of $\vec{E}(x, y, z)$,

$$\nabla \cdot \vec{E} = \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right)$$

$$\nabla \cdot \vec{E} = 4x + 8$$

$$\iiint_V (\vec{\nabla}\cdot\vec{E})\,dV = \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1}(4x+8)\cdot dx\,dy\,dz \qquad (\because \; dV = dxdydz \text{ according to question})$$

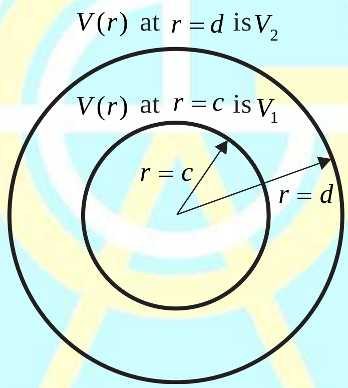
$$\iiint_V (\vec{\nabla}\cdot\vec{E})\,dV = 4\frac{x^2}{2}\Big|_0^1 \, y\Big|_0^1 \, z\Big|_0^1 + 8x\Big|_0^1 \, y\Big|_0^1 \, z\Big|_0^1$$

$$\iiint_V (\vec{\nabla}\cdot\vec{E})\,dV = 4\left(\frac{1}{2}\right)(1)(1)+8(1)(1)(1) = 10$$

Hence, the correct option is (C).

Question 53 **Electromagnetic Field**

As shown in the figure below, two concentric conducting spherical shells, centered at $r=0$ and having radii $r=c$ and $r=d$ are maintained at potentials such that the potential $V(r)$ at $r=c$ is V_1 and $V(r)$ at $r=d$ is V_2. Assume that $V(r)$ depends only on r, where r is the radial distance. The expression for $V(r)$ in the region between $r=c$ and $r=d$ is

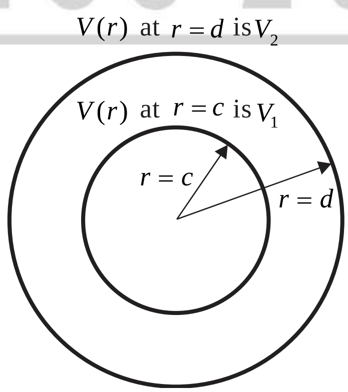


(A) $V(r)=\dfrac{cd(V_2-V_1)}{(d-c)r}-\dfrac{V_1c+V_2d-2V_1d}{d-c}$ (B) $V(r)=\dfrac{cd(V_1-V_2)}{(d-c)r}+\dfrac{V_2d-V_1c}{d-c}$

(C) $V(r)=\dfrac{cd(V_1-V_2)}{(d-c)r}-\dfrac{V_1c-V_2c}{d-c}$ (D) $V(r)=\dfrac{cd(V_2-V_1)}{(d-c)r}-\dfrac{V_2c-V_1c}{d-c}$

Ans. **B**

Sol. Given figure shown in below,



There is no charge present in side conducting spherical shell $(\rho_V = 0)$, so according to Laplace equation

$$\nabla^2 V(r) = 0$$

Laplacian formula, (only consider *r*-direction component)

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V(r)}{\partial r}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V(r)}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2 V(r)}{\partial\phi^2}\right)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V(r)}{\partial r}\right) = 0$$

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial V(r)}{\partial r}\right) = 0$$

Integrate the above equation on both side with respect to *r*,

$$r^2 \frac{\partial V(r)}{\partial r} = k_1 \qquad (\because\ k_1 \text{ is arbitrary constant})$$

$$\frac{\partial V(r)}{\partial r} = \frac{k_1}{r_2}$$

Again integrate above equation on both side with respect to *r*,

$$\int \partial V(r) = \int \frac{k_1}{r^2}\partial r$$

$$V(r) = -\frac{k_1}{r} + k_2 \qquad \ldots \text{(i)} \quad (\because\ k_2 \text{ is arbitrary constant})$$

Given at $r = c$; $V(r = c) = V_1$

$$V_1 = \frac{-k_1}{c} + k_2 \qquad \ldots\text{(ii)}$$

Given at $r = d$; $V(r = d) = V_2$

$$\frac{-k_1}{d} + k_2 = V_2 \qquad \ldots\text{(iii)}$$

From equation (ii) and (iii),

$$\frac{-k_1}{d} + \frac{k_1}{c} = V_2 - V_1$$

$$k_1 = \frac{(V_2 - V_1)}{d - \dfrac{c}{cd}}$$

$$k_1 = \frac{(V_2 - V_1)cd}{d - c}$$

Put the value of k_1 in equation (ii),

$$k_2 = V_1 + \frac{k_1}{c}$$

$$k_2 = V_1 + \frac{(V_2 - V_1)cd}{(d-c)c}$$

$$k_2 = \frac{-V_1 c + V_2 d}{d-c}$$

Put the value of k_1 and k_2 in equation (i),

So, $V(r) = \dfrac{cd(V_1 - V_2)}{(d-c)r} + \dfrac{(V_2 d - V_1 c)}{d-c}$

Hence, the correct option is (B).

Question 54 **Mathematics**

Let the probability density function of a random variable x be given as $f(x) = ae^{-2|x|}$. The value of 'a' is ______.

Ans. **1 (Range : 0.99 to 1.01)**

Sol. **Given :** $f(x) = ae^{-2|x|}, \; x \in (-\infty, \infty)$

Probability Density Function (P.D.F.) for a valid PDF.

Area under the complete curve is unity.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} a e^{-2|x|}\,dx = 1 \qquad \left[\because |x| = \begin{cases} +x\,; & x \ge 0 \\ -x\,; & x < 0 \end{cases}\right]$$

$$2\int_{0}^{\infty} a e^{-2x}\,dx = 1$$

$$2a\int_{0}^{\infty} e^{-2x}\,dx = 1$$

$$2a\left[\frac{e^{-2x}}{-2}\right]_0^{\infty} = 1$$

$$\frac{2a}{-2}\left[e^{-\infty} - e^{0}\right] = 1$$

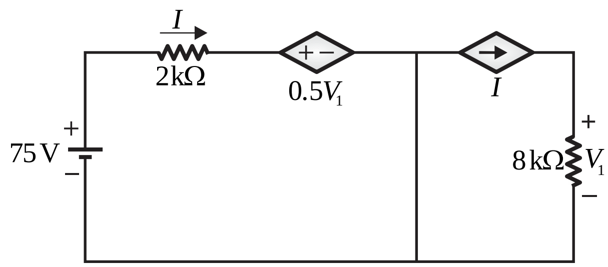
$$-a(0-1) = 1$$

$$a = 1$$

Hence, the correct answer is 1.

Question 55 **Network Theory**

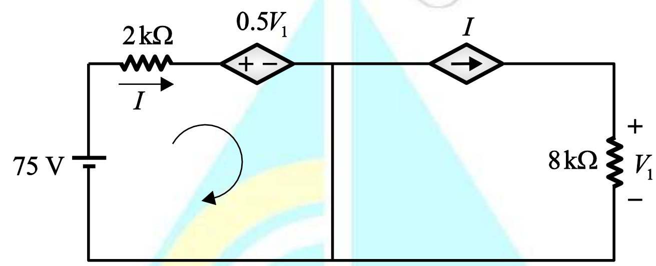
In the circuit shown below, the magnitude of the voltage V_1 in volts, across the $8\ \text{k}\Omega$ resistor is ______. (round off to nearest integer)



Ans. 100 (Range : 98 to 102)

Sol. **Method 1**

Given circuit is shown below



Applying KVL in loop shown,

$$-75+2000I+0.5V_1=0$$

$$75=2000I+0.5V_1 \qquad \ldots\text{(i)}$$

And from given circuit, $V_1=8000I$

$$I=\frac{V_1}{8000} \qquad \ldots\text{(ii)}$$

Substituting the value of I from equation (ii) in equation (i),

$$75=2000\left(\frac{V_1}{8000}\right)+0.5V_1$$

$$75=0.25V_1+0.5V_1$$

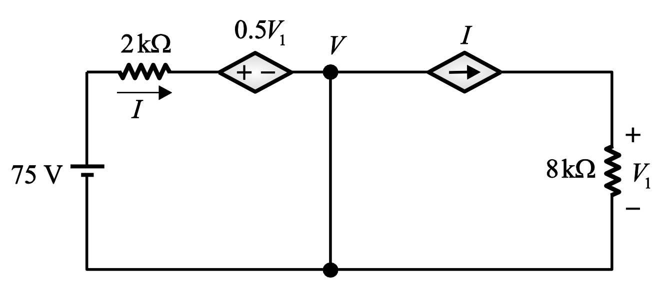
$$75=0.75V_1$$

$$V_1=100\text{ V}$$

Hence, the value of voltage across $8\text{k}\Omega$ resistance is 100 V.

Method 2

Given circuit is shown below



Applying KCL at node V,

$$\frac{V-75+0.5V_1}{2000}+I=0$$

Form the given circuit $V=0\text{V}$

$$\frac{0-75+0.5V_1}{2000}+I=0$$

$$-75+0.5V_1+2000I=0 \quad \ldots\text{(i)}$$

From the given circuit, $V_1=8000I$

$$I=\frac{V_1}{8000} \quad \ldots\text{(ii)}$$

Substituting the value of I from equation (ii) in equation (i),

$$-75+0.5V_1+2000\left[\frac{V_1}{8000}\right]=0$$

$$-75+0.5V_1+0.25V_1=0$$

$$0.75V_1=75$$

$$V_1=100\text{ V}$$

Hence, the value of voltage across $8\text{k}\Omega$ resistance is 100 V.

Question 56 **Electrical Machine**

Two generating units rated for 250 MW and 400 MW have governor speed regulations of 6% and 6.4%, respectively, from no load to full load. Both the generating units are operating in parallel to share a load of 500 MW. Assuming free governor action, the load shared in MW, by the 250 MW generating unit is ______. (round off to nearest integer)

Ans. **200 (Range : 188 to 192 or 198 to 202)**

Sol. **Given :**

(i) **Generating unit 1 :**

$P_{1r}=250\text{ MW}$

Speed regulations $=6\%$

Generating unit 2 :

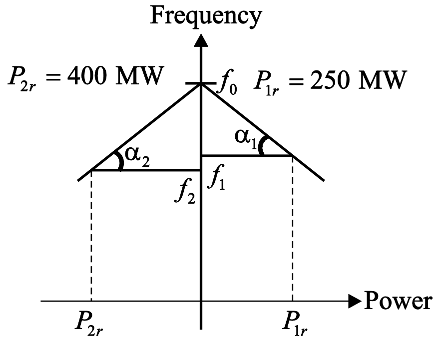
$P_{2r}=400\text{ MW}$

Speed regulation $=6.4\%$

(ii) Load $=500\text{ MW}$

$$P_1+P_2=500\text{ MW} \quad \ldots\text{(i)}$$

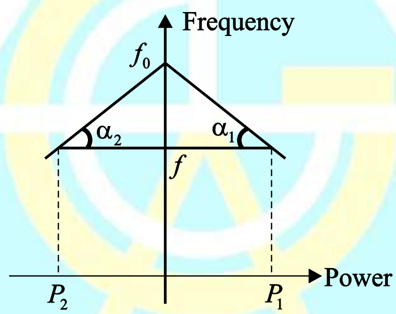
Load shared by 250 MW generating unit (P_1).



$$\tan\alpha_1 = \frac{f_0 - f_1}{250} = \frac{0.06 f_0}{250}$$

$$\tan\alpha_2 = \frac{f_0 - f_1}{400} = \frac{0.064 f_0}{400}$$

When sharing, $P_L = 500$ MW



$$\tan\alpha_1 = \frac{\Delta f}{P_1} = \frac{0.06 f_0}{250} \quad \ldots \text{(ii)}$$

$$\tan\alpha_2 = \frac{\Delta f}{P_2} = \frac{0.064 f_0}{400} \quad \ldots \text{(iii)}$$

Equating equation (ii) and (iii),

$$\frac{0.06 f_0 P_1}{250} = \frac{0.064 f_0 P_2}{400}$$

$$\Rightarrow P_1 = 0.66\, P_2 \quad \ldots\text{(iv)}$$

and $P_1 + P_2 = 500$

From equation (i) and (iv), we get

$$\Rightarrow P_1 = 199.9 \text{ MW} \approx 200 \text{ MW}$$

Hence, the correct answer is 200 MW.

Question 57 **Power System**

A 20 MVA, 11.2 kV, 4-pole, 50 Hz alternator has an inertia constant of 15 MJ/MVA. If the input and output powers of the alternator are 15 MW and 10 MW, respectively, the angular acceleration in mechanical degree/s^2 is _______. (round off to nearest integer)

Ans. **75 (Range : 74 to 76)**

Sol. **Given :**

(i) $S = 20\,\text{MVA}$

(ii) Voltage $(V) = 11.2\,\text{kV}$

(iii) Number of poles, $P = 4$

(iv) Inertia constant, $H = 15\,\text{MJ/MVA}$

(v) Input power, $P_{in} = 15\,\text{MW} = P_m$

(vi) Output power, $P_e = 10\,\text{MW}$

Angular acceleration _______ mech.degree/ s^2

Swing equation, $M\dfrac{d^2\delta}{dt^2} = P_m - P_e$

$$\alpha = \frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M} \text{ elect.degree/ } s^2 \quad \ldots\text{(i)}$$

Note :

We know, $\theta_e = \dfrac{P}{2}\theta_m$

$$\frac{d\theta_e}{dt} = \frac{P}{2}\frac{d\theta_m}{dt}$$

$$\omega_e = \frac{P}{2}\omega_m$$

$$\alpha_e = \frac{P}{2}\alpha_m$$

$$\alpha_m = \frac{2}{P}\alpha_e$$

Angular acceleration in mech.deg./ $s^2 = \dfrac{2}{P} \times$ angular acceleration in elect.deg./ s^2.

So equation (i),

$$\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M} \text{ elect.deg./ } s^2$$

$$\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M} \times \frac{2}{P} \text{ mech.deg./ } s^2$$

$$\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{\frac{GH}{\pi f}} \times \frac{2}{P} \text{mech.deg./ } s^2$$

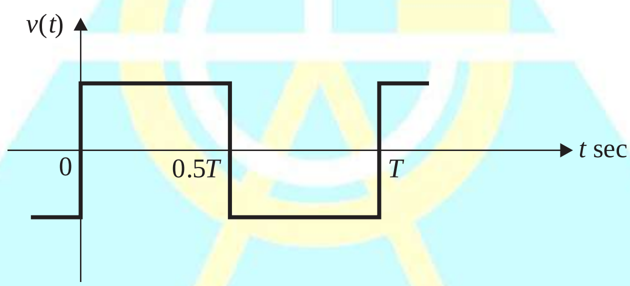
$$\frac{d^2\delta}{dt^2} = \frac{(15-10)\times 180 \times 50}{20\times 15} \times \frac{2}{4} \text{ mech.deg/} s^2$$

$$\frac{d^2\delta}{dt^2} = 75 \text{ mech.deg/} s^2$$

Hence, the correct answer is 75.

Question 58 **Power Electronics**

Consider an ideal full-bridge single-phase DC-AC inverter with a DC bus voltage magnitude of 1000 V. The inverter output voltage $v(t)$ shown below, is obtained when diagonal switches of the inverter are switched with 50% duty cycle. The inverter feeds a load with a sinusoidal current given by, $i(t) = 10\sin\left(\omega t - \frac{\pi}{3}\right)$ A, where $\omega = \frac{2\pi}{T}$. The active power, in watts, delivered to the load is ______. (round off to nearest integer)



Ans. **3183 (Range : 3170 to 3190)**

Sol. **Given :**

(i) DC bus voltage magnitude $V_{dc} = 1000$ V

(ii) $i(t) = 10\sin\left(\omega_0 t - \frac{\pi}{3}\right)$, where $\omega = \frac{2\pi}{T}$

The Fourier series expansion of the given voltage $v(t)$ is,

$$v(t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{n\pi} \sin(n\omega_0 t) \qquad (\because \ v(t) \text{ has half wave symmetry})$$

$$\therefore \quad v(t)\big|_{n=1} = \frac{4\times 1000}{\pi} \sin(\omega_0 t)$$

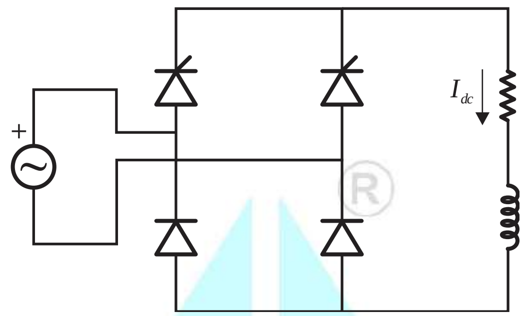
The active power delivered to load in watts,

$$\text{Power, } P = v(t)\big|_{n=1}(\text{rms}) \times i(t)\big|(\text{rms}) \times \cos\phi = \frac{4\times 1000}{\pi \times \sqrt{2}} \times \frac{10}{\sqrt{2}} \times \left(\cos\frac{\pi}{3}\right) = 3183\,\text{W}$$

Hence, the correct answer is 3183.

Question 59 **Power Electronics**

For the ideal AC-DC rectifier circuit shown in the figure below, the load current magnitude is $I_{dc} = 15$ A and is ripple free. The thyristors are fired with a delay angle of 45^0. The amplitude of the fundamental component of the source current, in amperes, is ______. (round off to two decimal places)



Ans. **17.65 (Range : 17.30 to 18)**

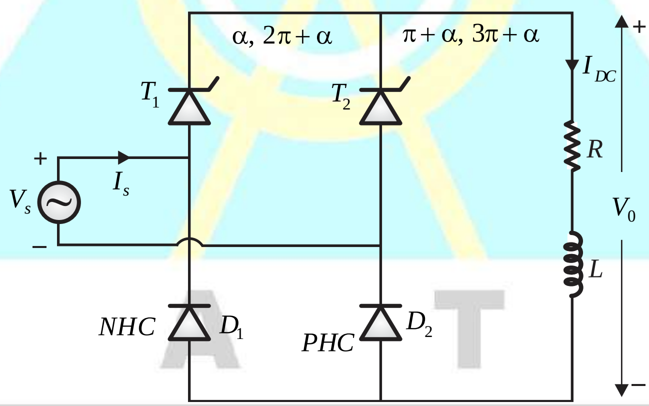
Sol. **Given :**

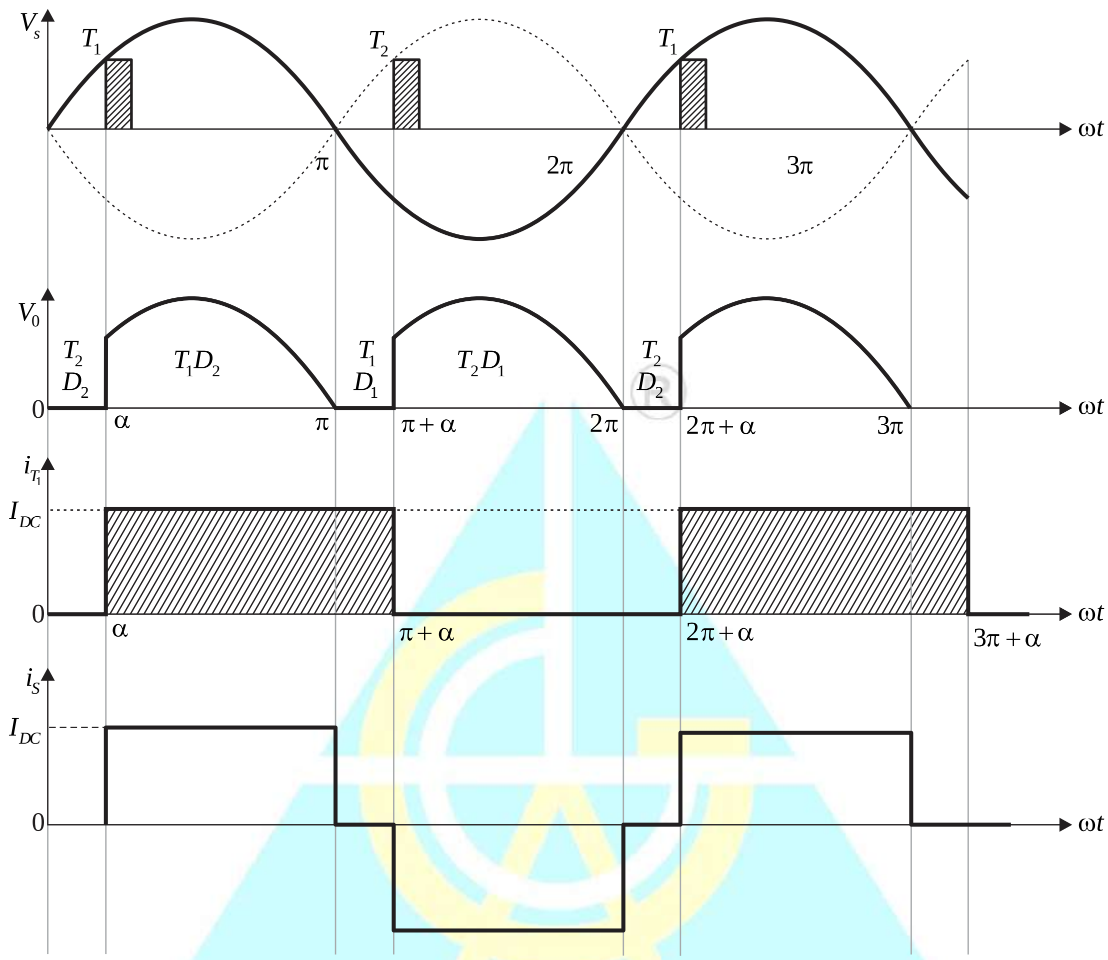
Single phase symmetrical semi-converter

(i) Load current magnitude, $I_{DC} = 15\,\text{A}$

(ii) Firing angle, $\alpha = 45^0$

A symmetrical single phase semi converter is shown below,





The Fourier series representation of supply current is given by,

$$I_s(t) = \sum_{n=1,3,5.....}^{\infty} \frac{4I_{DC}}{n\pi} \cos\frac{n\pi}{2} \sin\left(n\,\omega t - \frac{n\alpha}{2}\right)$$

As firing angle $\alpha = 45^0$

Hence the amplitude of the fundamental component of a source current is given by.

$$(I_{SI})_{peak} = \frac{4I_{DC}}{\pi} \cos\left(\frac{\alpha}{2}\right) = \frac{4\times 15}{\pi} \times \cos\left(\frac{45^0}{2}\right) = 17.64 \text{ A}$$

Hence, the correct answer is 17.65.

Question 60 **Power Electronics**

A 3-phase grid-connected voltage source converter with DC link voltage of 1000 V is switched using sinusoidal Pulse Width Modulation (PWM) technique. If the grid phase current is 10 A and the 3-phase complex power supplied by the converter is given by (– 4000 – *j*3000) VA, then the modulation index used in sinusoidal PWM is _______. (round off to two decimal places)

Ans. **0.471 (Range : 0.46 to 0.48)**

Sol. **Given :**

(i) 3-phase grid connected voltage source converter with DC link.

$V_{dc} = 1000$ V using sinusoidal pulse with modulation (PWM)

(ii) Grid phase current $I_L = 10$ A

(iii) $S = (-4000 - j3000)$ VA

The modulation index.

The expression of fundamental phase voltage of 3-phase sinusoidal PWM.

$$(v_{ph})_{01} = m\frac{V_{dc}}{2}\sin(\omega t) \qquad \left[v_{L_{01(\max)}} = \frac{\sqrt{3}\times m\times V_{dc}}{2}\right]$$

$$\sqrt{3}v_{L_{01(rms)}}I_L = S$$

Where, $m =$ Modulation index.

$$\therefore \quad \sqrt{3}v_{L_{01(rms)}}I_L = S$$

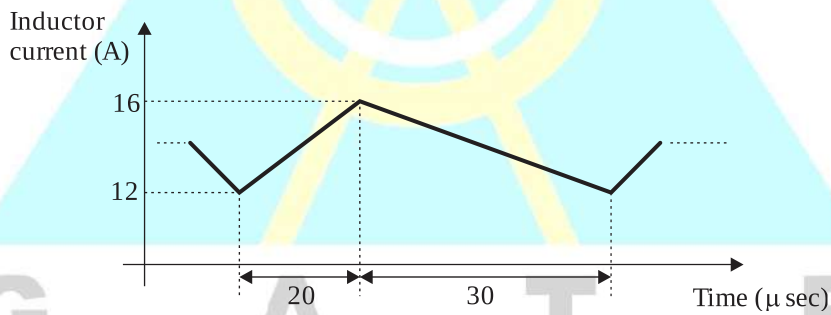
$$\sqrt{3}\times\frac{\sqrt{3}m\times V_{dc}}{2\sqrt{2}}\times 10 = \sqrt{4000^2 + 3000^2}$$

$$\therefore \quad m = 0.471$$

Hence, the correct answer is 0.471.

Question 61 **Power Electronics**

The steady state current flowing through the inductor of a DC-DC buck-boost converter is given in the figure below. If the peak-to-peak ripple in the output voltage of the converter is 1 V, then the value of the output capacitor, in μF, is _____. (round off to nearest integer)



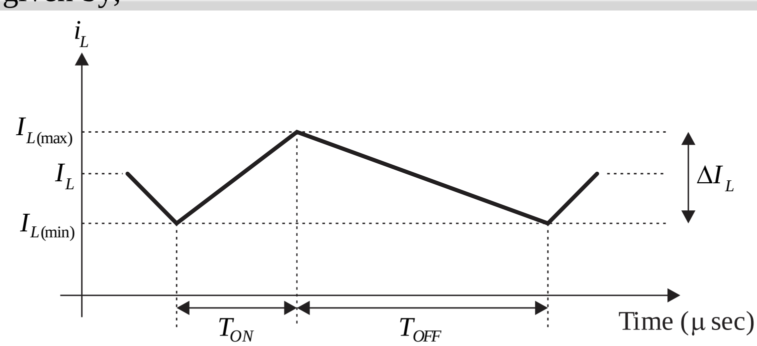
Ans. 168 (Range : 165 to 171)

Sol. Given :

A DC-DC buck-boost converter.

(i) Peak to peak ripple in output voltage, $\Delta V_0 = \Delta V_C = 1$V

Inductor current is given by,



Form figure are can see that,

(ii) $I_{L(\max)} = 16$ A

(iii) $I_{L(\min)} = 12$ A

(v) $T_{ON} = DT = 20$ μs

(v) $T = T_{ON} + T_{OFF} = 20 + 30 = 50$ μs

Now, $D =$ Duty ratio $= \frac{T_{ON}}{T} = \frac{20}{50} = \frac{2}{5}$

We know, $I_{L(avg)} = \frac{I_{L(\max)} + I_{L(\min)}}{2} = \frac{12+16}{2} = 14$ A

Also $I_{L(avg)} = \frac{I_0}{1-D} = \frac{I_0}{1-\frac{2}{5}}$

$\therefore \quad I_0 = 14 \times \frac{2}{5} = 8.4$ A

For buck boost converter,

The ripple in capacitor voltage $=$ The ripple in output voltage

$\Delta V_0 = \Delta V_C = 1\text{V}$

As $\Delta V_C = \frac{DI_0}{fC} = \frac{\frac{2}{5} \times \frac{14 \times 3}{5}}{\frac{1}{50 \times 10^{-6} \times C}}$

$\therefore \quad 1 = \frac{\frac{2}{5} \times \frac{14 \times 3}{5} \times 50 \times 10^{-6}}{C}$

$C = 168$ μF

Hence, the correct answer is 168 μF.

Question 62 **Electrical Machine**

A 280 V, separately excited DC motor with armature resistance of $1\ \Omega$ and constant field excitation drives a load. The load torque is proportional to the speed. The motor draws a current of 30 A when running at a speed of 1000 rpm. Neglect frictional losses in the motor. The speed, in rpm, at which the motor will run, if an additional resistance of value $10\ \Omega$ is connected in series with the armature, is ______. (round off to nearest integer)

Ans. **483 (Range : 480 to 485)**

Sol. **Given :**

(i) Separately excited DC motor

(ii) Terminal voltage, $V_t = 280$ V

(iii)Armature resistance, $R_a = 1\ \Omega$

(iv)Constant field excitation i.e. $\phi =$ Constant

(v) Load torque is proportional to speed i.e $T \propto N$

(vi) $I_{a1} = 30$ A at $N_1 = 1000$ rpm

(vii) Additional resistance $R_{ext} = 10\ \Omega$

Motor speed in rpm (N_2) when additional resistance is connected in series with armature.

Back emf, $E_b = K\phi\omega$

$E_b \propto N$ $\quad\quad (\because \phi = \text{Constant})$

Where, ω is angular speed in rad/sec and N is speed in rpm.

Load Torque,

$T = K\phi I_a$

$T \propto I_a$ $\quad\quad (\because \phi = \text{Constant})$

Where, I_a is armature current.

Also $T \propto N$ (Given)

$N \propto I_a$

Now back EMF at 1000 rpm when $I_{a1} = 30$ A is,

$E_{b1} = V_t - I_{a1}R_a = 280 - 30 \times 1$

$E_{b1} = 250$ V

Armature current when external resistance is added in series

$\because \quad N \propto I$

$$\frac{N_1}{N_2} = \frac{I_{a1}}{I_{a2}}$$

$$I_{a2} = \frac{N_2}{N_1} \times I_{a1} = \frac{30}{1000} N_2$$

$$I_{a2} = 0.03\ N_2 \quad \ldots \text{(i)}$$

Back emf after adding external armature resistance is,

$E_{b2} = V_t - I_{a2}(R_a + R_{ext})$

From equation (i),

$E_{b2} = 280 - 0.03N_2(1+10)$

$$E_{b2} = 280 - 0.33N_2 \quad \ldots \text{(ii)}$$

Now as $E_b \propto N$

So, $\dfrac{E_{b2}}{E_{b1}} = \dfrac{N_2}{N_1}$

$$\frac{280 - 0.33N_2}{250} = \frac{N_2}{1000}$$

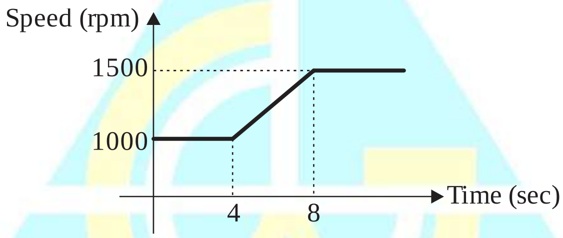
$$N_2 = 4 \times (280 - 0.33N_2)$$

$$N_2 = 482.75 \text{ rpm}$$

Hence, the correct answer is 482.75 rpm.

Question 63 **Electrical Machine**

A 4-pole induction motor with inertia of 0.1 kg-m^2 drives a constant load torque of 2 Nm. The speed of the motor is increased linearly from 1000 rpm to 1500 rpm in 4 seconds as shown in the figure below. Neglect losses in the motor. The energy, in joules, consumed by the motor during the speed change is ______. (round off to nearest integer)



Ans. 1732.5 (Range : 675 to 700 or 1725 to 1740)

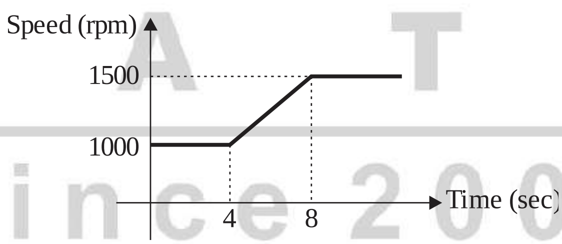
Sol. Given :

(i) Induction motor

(ii) Pole, $P = 4$

(iii) Inertia, $J = 0.1 \text{ kgm}^2$

(iv) Constant load torque T_L = Load torque constant = 2 Nm



Energy consumed by motor = Energy stored in rotor + Energy delivered to load

Energy stored in rotor $= \frac{1}{2} I\omega_{m_1}^2 - \frac{1}{2} I\omega_{m_2}^2 = \frac{1}{2} I[\omega_{m_1}^2 - \omega_{m_2}^2]$

$$= \frac{1}{2} \times 0.1 \times \left\{ \left(\frac{2\pi \times 1500}{60} \right)^2 - \left(\frac{2\pi \times 1000}{60} \right)^2 \right\} = 685.4 \text{ Joules}$$

Energy delivered to load $= \int P dt = \int T\omega dt$

Energy delivered to load $= T \int \omega dt$

Energy delivered to load $= T$ (Area under the curve)

Energy delivered to load $= T \times \left\{ \left(4 \times 1000 \times \frac{2\pi}{60} \right) + \left(\frac{1}{2} \times 4 \times (1500 - 1000) \times \frac{2\pi}{60} \right) \right\} = 1047.2$ Joules

Total energy consumed by motor $= 685.4 + 1047.2 = 1732.5$ Joules

Hence, the correct answer is 1732.5.

Question 64 **Electrical Machine**

A star-connected 3-phase, 400 V, 50 kVA, 50 Hz synchronous motor has a synchronous reactance of 1 ohm per phase with negligible armature resistance. The shaft load on the motor is 10 kW while the power factor is 0.8 leading. The loss in the motor is 2 kW. The magnitude of the per phase excitation emf of the motor, in volts, is ______ (round off to nearest integer).

Ans. 244.5 (Range : 240 to 248)

Sol. Given :

(i) $3-\phi$, star connected synchronous motor.

(ii) Terminal voltage, $V_L = 400$ V

(iii)Rated KVA $S = 50$ KVA

(iv)Frequency, $f = 50$ Hz

(v) Synchronous reactance, $X_s = 1\ \Omega/\text{ph}$

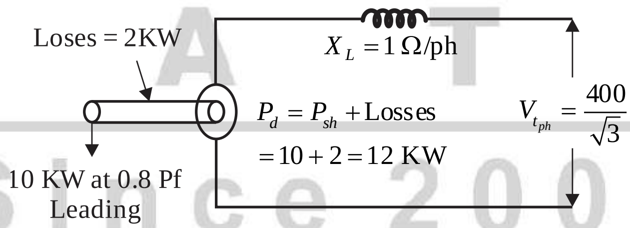
(vi)Negligible armature resistance

(vii) Shaft load, $P_{sh} = 10$ kW

(viii) Load power factor, $\cos\phi = 0.8$ leading

(ix)Losses $= 2$ kW

Magnitude of the per phase excitation of the motor (in volts),



Developed power, $P_d = P_{sh} + \text{Losses} = 10 + 2 = 12$ kW ... (i)

As armature resistance is negligible of synchronous motor,

So, $P_d = P_{input}$ and $P_{input} = \sqrt{3} V_L I_L \cos\phi$

$$\sqrt{3} V_L I_L \cos\phi = 12 \text{ kW}$$

$$I_L = \frac{12000}{\sqrt{3} \times 400 \times 0.8} = 21.65 \text{ A}$$

$$I_a = I_L = 21.65 \text{ A}$$

Now for synchronous motor in case of leading power factor.

$$E_{ph}^2 = (V_{ph}\cos\phi - I_a R_a)^2 + (V_{ph}\sin\phi + I_a X_a)^2$$

$$E_{ph}^2 = \left(\frac{400}{\sqrt{3}}\times 0.8 - 0\right)^2 + \left(\frac{400}{\sqrt{3}}\times 0.6 + 21.65\times 1\right)^2$$

$$E_{ph} = 244.54 \text{ V}$$

Hence, the correct answer is 244.5 V.

Question 65 **Electrical Machine**

A 3-phase, 415 V, 4-pole, 50 Hz induction motor draws 5 times the rated current at rated voltage at starting. It is required to bring down the starting current from the supply to 2 times of the rated current using a 3-phase autotransformer. If the magnetizing impedance of the induction motor and no load current of the autotransformer is neglected, then the transformation ratio of the autotransformer is given by ______. (round off to two decimal places)

Ans. 0.63 (Range : 0.61 to 0.65)

Sol. Given :

(i) $3\text{-}\phi$, 415 V, 4-pole, 50 Hz Induction motor.

(ii) At Rated Voltage, at Starting (DOL), $I_{st} = I_{sc} = 5I_{fl}$ … (i)

(iii) By using auto-transformer $I_{st} = x^2 I_{sc} = 2I_{fl}$ … (ii)

Where x is transformation ratio.

Transformation ration $'x'$ of auto transformer

From equation (i) and (ii),

$$x^2(I_{sc}) = 2I_{fl}$$

$$x^2(5I_{fl}) = 2I_{fl}$$

$$x^2 = \frac{2}{5}$$

$$x = \sqrt{\frac{2}{5}} = 0.632$$

Hence the correct answer is 0.632.

❖❖❖❖