General Aptitude

Q.1 to Q.5 Carry ONE Mark Each

Question 1

If '→' denotes increasing order of intensity, then the meaning of the words [simmer → seethe → smolder] is analogous to [break → raze → ______].

Which one of the given options is appropriate to fill the blank? **[Verbal Ability, 1]**

(A) obfuscate (B) obliterate

(C) fracture (D) fissure

Ans. (B)

Sol. Given :

[simmer → seethe → smolder]

'→' denotes increasing order of intensity

[break → raze →]

Meaning of the given words are

simmer : to cook gently in a liquid that is almost boling.

seethe : to be very angry

smolder : to bure slowly without a flame

break : to separate or make something separate, into two or more pieces

raze : to completely destroy a building, town, etc so that nothing is left.

From options

(A) obfuscate : to make something unclear and more difficult to understand

(B) obliterate : to remove all signs of something by destroying or covering it completely

(C) fracture : a fracture is a partial or complete break in the bone

(D) fissure : a narrow opening or and depth usually occurring from some breaking or parting

From the above options and there meaning we see options (A), (C) and (D) are not appropriate for gives analogy and option (B) is logical in the analogy.

Hence, the correct option is (B).

Question 2

In a locality, the houses are numbered in the following way:

The house-numbers on one side of a road are consecutive odd integers starting from 301, while the house-numbers on the other side of the road are consecutive even numbers starting from 302. The total number of houses is the same on both sides of the road.

If the difference of the sum of the house-numbers between the two sides of the road is 27, then the number of houses on each side of the road is **[Numerical Ability, 1]**

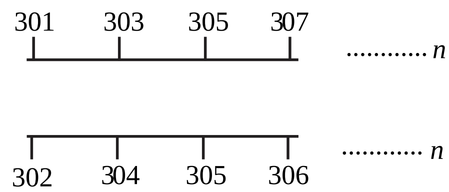
(A) 27 (B) 52

(C) 54 (D) 26

Ans. (A)

Sol. **Given :** House no. on one side of road are consecutive odd integers starting from 301.

House no. on another side of road are consecutive even integers starting from 302.



Difference of the sum of the house number between the two sides of the road = 27

Let the no. of houses on each side of road be n.

Sum of odd numbered houses

$= 301 + 303 + 305 + \ldots\ldots\ldots\ldots\ldots n$

$= \frac{n}{2}[2 \times 301 + (n-1)2]$

$= \frac{n}{2}[2n + 600] = n[n + 300]$ ...(i)

Sum of even numbered houses

$= 302 + 304 + 306\ldots\ldots\ldots\ldots\ldots n$

$= \frac{n}{2}[2 \times 302 + (n-1)2]$

$= \frac{n}{2}[2n + 602] = n[n + 301]$ ...(ii)

According to question,

Difference of the sum of the house numbers between the two sides of the road is 27

i.e., $n[n + 301] - n[n + 300] = 27$

$n^2 + 301n - n^2 - 300n = 27$

$n = 27$

Hence, the number of houses on each side of the road is 27.

Hence, the correct option is (A).

Question 3

For positive integers p and q, with $\frac{p}{q} \neq 1$, $\left(\frac{p}{q}\right)^{\frac{p}{q}} = p^{\left(\frac{p}{q}-1\right)}$. Then **[Logical Reasoning, 3]**

(A) $q^p = p^q$

(B) $q^p = p^{2q}$

(C) $\sqrt{q} = \sqrt{p}$

(D) $\sqrt[p]{q} = \sqrt[q]{p}$

Ans. (A)

Sol. **Given :** Positive integers p and q,

$\frac{p}{q} \neq 1$

and $\left(\frac{p}{q}\right)^{\frac{p}{q}} = p^{\left(\frac{p}{q}-1\right)}$ $\left[\because \left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}, (xy)^m = x^m . y^m\right]$

$\frac{p^{p/q}}{q^{p/q}} = p^{p/q}p^{-1}$

$\frac{p^{p/q}}{q^{p/q}} = \frac{p^{p/q}}{p}$

$pp^{p/q} = q^{p/q}p^{p/q}$

$p = q^{p/q}$

$p = (q^{1/q})^p$ $[\because (x^m)^n = x^{mn}]$

Taking $\frac{1}{p}$ in exponent on both sides

$p^{1/q} = q^{1/q}$

Taking pq in exponent on both sides

$(p^{1/p})^{pq} = (q^{1/q})^{pq}$ $[\because (x^m)^n = x^{mn}]$

$p^{pq/p} = q^{pq/q}$

$p^q = q^p$

Hence, the correct option is (A).

Question 4

Which one of the given options is a possible value of *x* in the following sequence?

3, 7, 15, *x*, 63, 127, 255 **[Numerical Ability, 1]**

(A) 35 (B) 40

(C) 45 (D) 31

Ans. (D)

Sol. **Given :** Sequence of number

3, 7, 15, *x*, 63, 127, 255

After analyzing the given sequence. We can observe that each number is the multiplication of previous number with 2 and adding one to the result obtained.

i.e.,

$1 \times 2 + 1 = 3$

$3 \times 2 + 1 = 7$

$7 \times 2 + 1 = 15$

$15 \times 2 + 1 = \boxed{31}$

$31 \times 2 + 1 = 63$

$63 \times 2 + 1 = 127$

$127 \times 2 + 1 = 255$

Hence, the possible value of *x* in the given sequence is 31.

Hence, the correct option is (D).

Question 5

On a given day, how many times will the second-hand and the minute-hand of a clock cross each other during the clock time 12:05:00 hours to 12:55:00 hours? **[Numerical Ability, 1]**

(A) 51 (B) 49

(C) 50 (D) 55

Ans. (C)

Sol. There is 50 min duration between 12:05:00 to 12:55:00 hours we know that in every minute, minute hand crosses second hand. So, during these 50 min. they will cross each other 50 times.

Hence, the correct option is (C).

Q.6 to Q.10 Carry TWO Marks Each

Question 6

In the given text, the blanks are numbered (i)-(iv). Select the best match for all the blanks.

From the ancient Athenian arena to the modern Olympic stadiums, athletics __(i)__ the potential for a spectacle. The crowd _(ii)_ with bated breath as the Olympian artist twists his body, stretching the javelin behind him. Twelve strides in, he begins to cross-step. Six cross-steps (iii) in an abrupt stop on his left foot. As his body (iv) like a door turning on a hinge, the javelin is launched skyward at a precise angle.

[Logical Reasoning, 4]

(A) (i) hold (ii) waits (iii) culminates (iv) pivot
(B) (i) holds (ii) wait (iii) culminates (iv) pivot
(C) (i) hold (ii) wait (iii) culminate (iv) pivots
(D) (i) holds (ii) waits (iii) culminate (iv) pivots

Ans. (D)

Sol. In the context, the word choices should (D) align with the tense and subject-verb agreement the correct option is (D).

Where "holds" (i) matches the singular subject "athletics". "waits" (ii) agrees with the singular "crowd", "culminate" (iii) fits the sequence, and "pivots" (iv) corresponds to the action of the body.

Hence, the correct option is (D).

Question 7

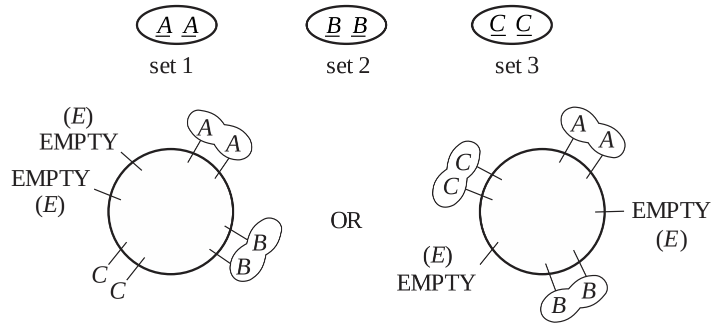
Three distinct sets of indistinguishable twins are to be seated at a circular table that has 8 identical chairs. Unique seating arrangements are defined by the relative positions of the people.
How many unique seating arrangements are possible such that each person is sitting next to their twin?

[Numerical Ability, 4]

(A) 12 (B) 14
(C) 10 (D) 28

Ans. (A)

Sol.



Set 1 Set 2 Set 3 E E ≡ Total 5 units

Circular arrangement of 5 units out of which 2 are alike (E and E)

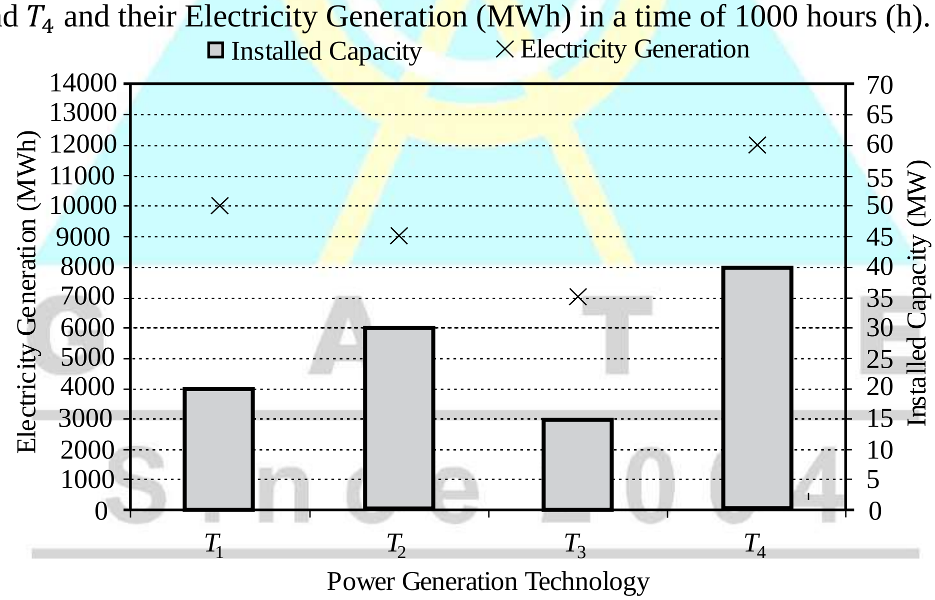
$$= \frac{(5-1)!}{2!} = \frac{4!}{2!} = \frac{4\times3\times2\times1}{2\times1} = 12$$

So, number of unique sitting arrangemet Possible = 12

Hence, the correct option is (A).

Question 8

The chart given below compares the Installed Capacity (MW) of four power generation technologies, T_1, T_2, T_3 and T_4 and their Electricity Generation (MWh) in a time of 1000 hours (h).



The Capacity Factor of a power generation technology is:

$$\text{Capacity Factor} = \frac{\text{Electricity Generation (MWh)}}{\text{Installed Capacity (MW)}\times 1000(h)}$$

Which one of the given technologies has the highest Capacity Factor? **[Logical Reasoning, 1]**

(A) T_1 (B) T_2

(C) T_3 (D) T_4

Ans. (A)

Sol. **Given :**

$$\text{Capacity factor} = \frac{\text{Electricity Generation (Mwh)}}{\text{Installed capacity(Mw)}\times 1000(h)}$$

From option (A) : The capacity factor of a power Generation technology (T_1) is

$$\text{Capacity factor} = \frac{10{,}000}{20\times 1000}$$

$$\text{Capacity factor} = 0.5$$

From option (B) : The capacity of a power Generation technology (T_2) is

$$\text{Capacity factor} = \frac{9{,}000}{30\times 1000} = \frac{9}{30} = 0.3$$

From option (C) : The capacity of a power Generation technology (T_3) is

$$\text{Capacity factor} = \frac{7{,}000}{15\times 1000} = \frac{7}{15} = 0.466$$

From option (D) : The capacity of a power generation technology (T_4) is

$$\text{Capacity factor} = \frac{12{,}000}{40\times 1000} = 0.3$$

The highest capacity factor of a power generation is T_1.

Hence the correct option is (A).

Question 9

In the 4 × 4 array shown below, each cell of the first three columns has either a cross (X) or a number, as per the given rule.

1	1	2	
2	×	3	
2	×	4	
1	2	×	

Rule: The number in a cell represents the count of crosses around its immediate neighboring cells (left, right, top, bottom, diagonals).

As per this rule, the **maximum** number of crosses possible in the empty column is

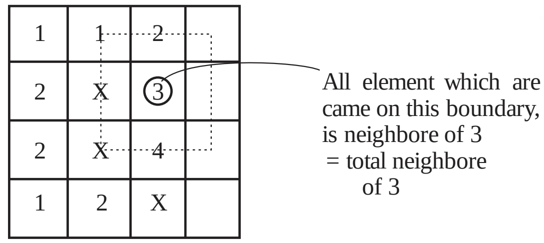
[Logical Reasoning, 3]

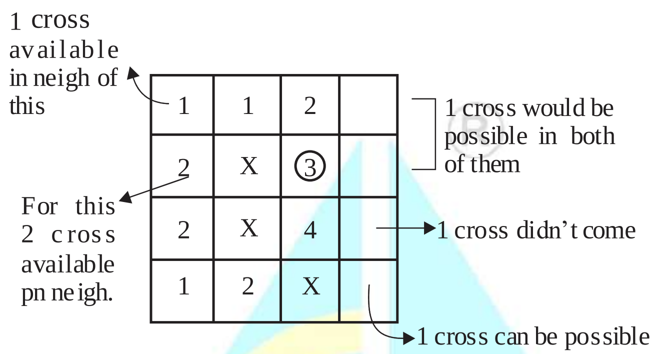
(A) 0 (B) 1

(C) 2 (D) 3

Ans. **(C)**

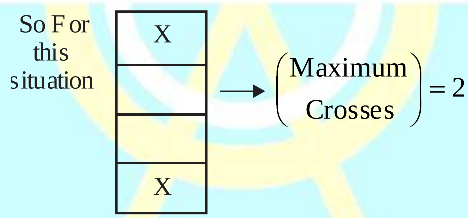
Sol.





But we wants to required maximum number of crosses.

So for this situation



Hence, the correct option is (A).

Question 10

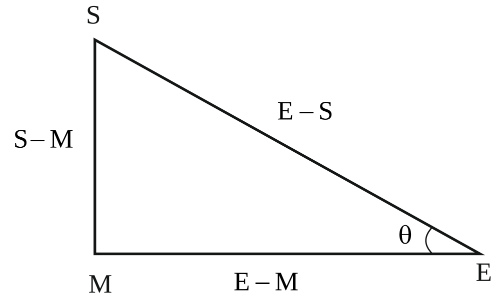
During a half-moon phase, the Earth-Moon-Sun form a right triangle. If the Moon-Earth-Sun angle at this half-moon phase is measured to be 89.85^0, the ratio of the Earth-Sun and Earth-Moon distances is closest to **[Numerical Ability, 3]**

(A) 328 (B) 382

(C) 238 (D) 283

Ans. (B)

Sol.



$\angle \text{Moon} - \text{earth} - \text{sun} = 89.85^0$

$\theta = 89.85^0$

$\frac{\text{Earth -sun}}{\text{Earth -Moon}} = \sec\theta = \sec(89 \cdot 85°)$

$\frac{E-S}{E-M} = 381.97 \simeq 382$

Hence, the correct option is (B).

Technical Section

Q.11 to Q.35 Carry ONE Mark Each

Question 11

The first non-zero term in the Taylor series expansion of $(1-x)-e^{-x}$ about $x=0$ is

[Engineering Mathematics, Limit and Series Expansion](1 Mark)

(A) 1 (B) –1

(C) $\frac{x^2}{2}$ (D) $-\frac{x^2}{2}$

Ans. **(D)**

Sol. **Given :**

$f(x) = (1-x) - e^{-x}$

The Taylor series of function $f(x)$ is given by,

$f(x) = f(a) + (x-a)f'(a) + (x-a)^2 \frac{f''(a)}{2!} + \ldots\ldots$

Taylor series expansion around $x = 0$ implies a = 0

So,

$f(x) = f(0) + xf'(0) + x^2\frac{f''(0)}{2!} + x^3\frac{f'''(0)}{3!} + \ldots\ldots$ ...(i)

$f(x) = (1-x) - e^{-x}$ ...(ii)

$f(0) = 1 - e^{-0}$

$= 1 - 1 = 0$

Differentiating equation (ii) with respect to x

$$f'(x) = (0-1) + e^{-x} = -1 + e^{-x} \quad \ldots\text{(iii)}$$

$$f'(0) = -1 + e^{0}$$

$$= -1 + 1 = 0$$

Differentiating equation (iii) with respect to x

$$f''(x) = 0 - e^{-x}$$

$$f''(x) = -e^{-x} \quad \ldots\text{(iv)}$$

$$f''(0) = -e^{0} = -1$$

Differentiating equation (iv) with respect to x

$$f'''(x) = e^{-x} \quad \ldots\text{(v)}$$

$$f'''(0) = e^{-0}$$

$$f'''(0) = 1$$

Substituting all the values of $f(0), f'(0), f''(0)$ and $f'''(0)$ in equation (i)

$$f(x) = f(0) + xf'(0) + x^2 \frac{f''(0)}{2!} + x^3 \frac{f'''(0)}{3!} + \ldots\ldots$$

$$f(x) = 0 + x \cdot 0 + \frac{x^2(-1)}{2!} + \frac{x^3(1)}{3!} + \ldots\ldots$$

$$f(x) = 0 + 0 - \frac{x^2}{2} + \frac{x^3}{6} + \ldots\ldots$$

First non zero term in the Taylor series expansion of $(1-x) - e^{-x}$ about $x = 0$ is $\frac{-x^2}{2}$

Hence, the correct option is (D).

Question 12

Consider the normal probability distribution function

$$f(x) = \frac{4}{\sqrt{2\pi}} e^{-8(x+3)^2}$$

If μ and σ are the mean and standard deviation of $f(x)$ respectively, then the ordered pair (μ, σ) is

[Engineering Mathematics, Probability] (1 Mark)

(A) $\left(3, \frac{1}{4}\right)$ (B) $\left(-3, \frac{1}{4}\right)$

(C) (3, 4) (D) (–3, 4)

Ans. (B)

Sol. Given : $f(x) = \frac{4}{\sqrt{2\pi}} e^{-8(x+3)^2} \quad \ldots\text{(i)}$

Standard equation of normal distribution.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad \ldots\text{(ii)}$$

After Comparing.

Standard deviation $(\sigma) = \frac{1}{4}$

Mean $(\mu) = -3$

Hence the correct option is (B).

Question 13

If $z_1 = -1 + i$ and $z_2 = 2i$, where $i = \sqrt{-1}$, then $\text{Arg}(z_1/z_2)$ is **(1 Mark)**

[Engineering Mathematics, Complex Variable]

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{3}$

Ans. (B)

Sol. Given :

$$z_1 = -1 + i$$
$$z_2 = 2i$$

Argument of z_1 is

$$\theta_1 = 180^0 - tan^{-1}\left(\frac{1}{1}\right)$$
$$\theta_1 = 180^0 - tan^{-1} 1$$
$$\theta_1 = 180^0 - 45^0$$
$$\theta_1 = 135^0$$

Argument of z_2 is

$$\theta_2 = tan^{-1}\left(\frac{2i}{0}\right) = tan^{-1}(\infty)$$
$$\theta_2 = 90^0$$

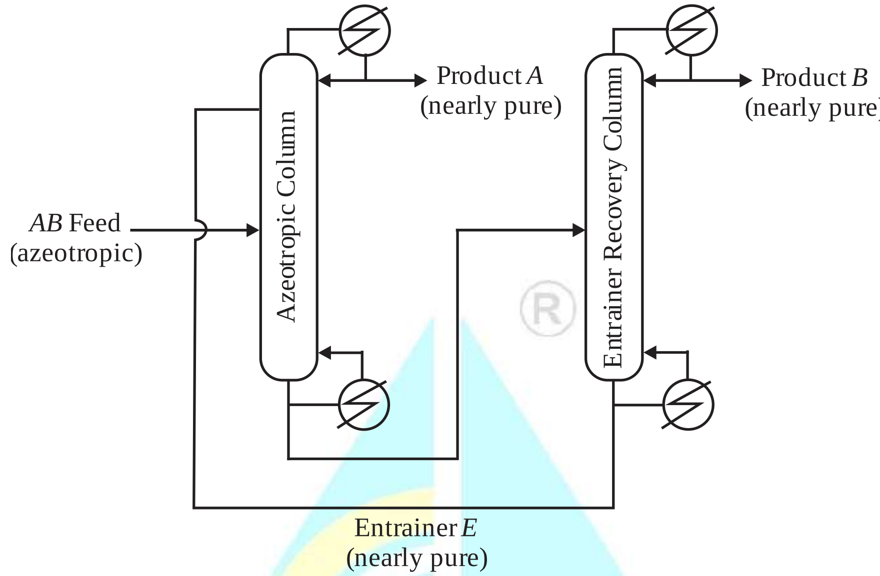
By the properties of complex numbers.

$$arg\left[\frac{z_1}{z_2}\right] = arg(z_1) - arg(z_2)$$
$$arg\left[\frac{z_1}{z_2}\right] = \theta_1 - \theta_2$$
$$arg\left[\frac{z_1}{z_2}\right] = 135^0 - 90^0$$
$$arg\left[\frac{z_1}{z_2}\right] = 45^0$$
$$arg\left[\frac{z_1}{z_2}\right] = \frac{\pi}{4} \qquad \left(\because 45^0 = \frac{\pi}{4}\right)$$

Hence, the correct option is (B).

Question 14

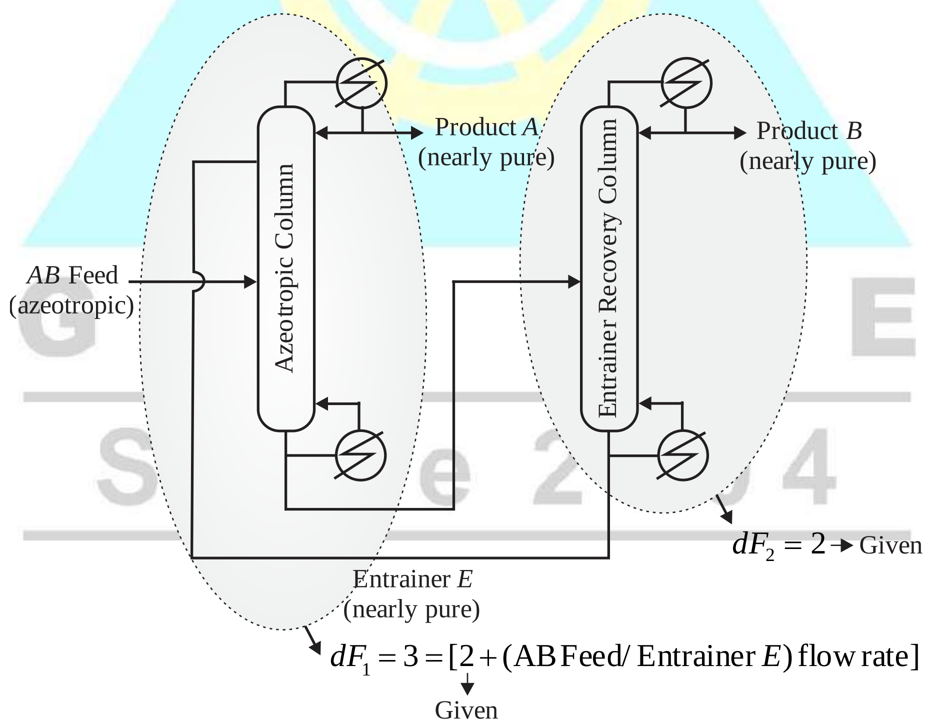
A homogeneous azeotropic distillation process separates an azeotropic *AB* binary feed using a heavy entrainer, *E*, as shown in the figure. The loss of E in the two product streams is negligible so that *E* circulates around the process in a closed-circuit. For a distillation column with fully specified feed(s), given operating pressure, a single distillate stream and a single bottoms stream, the steady-state degrees of freedom equals 2. For the process in the figure with a fully specified *AB* feed stream and given column operating pressures, the steady-state degrees of freedom equals **[Mass Transfer, 1 Marks]**



(A) 3
(B) 4
(C) 5
(D) 6

Ans. (C)

Sol.



Given : For a distillation column with fully specified feed(*s*), given operating pressure, a single distillate stream and a single bottoms stream, the steady - state degrees of freedom equals to 2.

Degree of freedom basically tells number of variable which needs to define to define our system.

$\therefore \quad d_F = d_{F_1} + d_{F_2}$

$dF_2 = 2$

$dF_1 = 3$

$d_F = 3 + 2$

$d_F = 5$

Question 15

An infinitely long cylindrical water filament of radius R is surrounded by air. Assume water and air to be static. The pressure outside the filament is P_{out} and the pressure inside is P_{in}. If γ is the surface tension of the water-air interface, then $P_{in} - P_{out}$ is **[Fluid Mechanics, 1 Marks]**

(A) $\frac{2\gamma}{R}$ (B) 0

(C) $\frac{\gamma}{R}$ (D) $\frac{4\gamma}{R}$

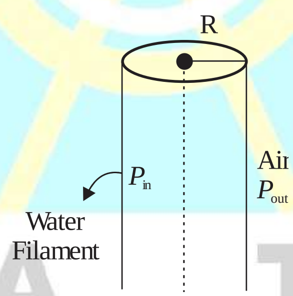
Ans. (C)

Sol. **Given ;** An infinitely long cylindrical water filament of radius R is surrounded by air.

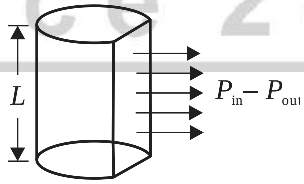
Let γ be surface tension and Pi and P_{out} is Pressure inside and outside, assume a Length of L water film.

At Equilibrium,

Pressure Force = surface tension force



$$(P_{in} - P_{out}) \times \underbrace{(DL)}_{\text{Projected area of filament}} = \underbrace{(2)\gamma L}_{\text{Both side of film}}$$



$$(\Delta P_e) = \frac{2\gamma}{d} = \frac{2\gamma}{2R}$$

$$\Delta P_e = P_{in} - P_{out} = \frac{\gamma}{R}$$

Question 16

The velocity field in an incompressible flow is $v = \alpha xy\hat{\imath} + v_y\hat{\jmath} + \beta\hat{k}$, where $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are unit-vectors in the (x, y, z) Cartesian coordinate system. Given that α and β are constants, and at $y = 0$, the correct expression for v_y is **[Fluid Mechanics, 1 Marks]**

(A) $\frac{-\alpha xy}{2}$ (B) $\frac{-\alpha y^2}{2}$

(C) $\frac{\alpha y^2}{2}$ (D) $\frac{\alpha xy}{2}$

Ans. (B)

Sol. **Given** $\vec{v} = \alpha xy\hat{\imath} + V_y\hat{\jmath} + \beta\hat{k}$

For incompressible Flow; $\frac{\partial v}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$ i.e $\nabla . v = 0$

$$\frac{\partial}{\partial x}(\alpha xy) + \frac{\partial}{\partial y}(v_y) + \frac{\partial}{\partial z}(\beta) = 0$$

$$\alpha y + \frac{\partial}{\partial y}(v_y) = 0$$

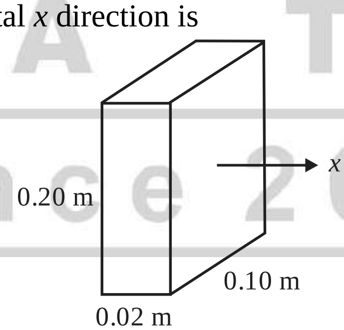
$$v_y = \frac{-\alpha y^2}{2} + c$$

B.C at $y = 0$ $v_y = 0 \Rightarrow c = 0$

$\therefore$ $v_y = \frac{-\alpha y^2}{2}$

Question 17

Consider the steady, uni-directional diffusion of a binary mixture of A and B across a vertical slab of dimensions 0.2 m × 0.1 m × 0.02 m as shown in the figure. The total molar concentration of A and B is constant at 100 molm^{-3}. The mole fraction of A on the left and right faces of the slab are maintained at 0.8 and 0.2, respectively. If the binary diffusion coefficient $D_{AB} = 1 \times 10^{-5}$ m$^2 s^{-1}$, the molar flow rate of A in mol s^{-1}, along the horizontal x direction is **[Mass Transfer, 1 Marks]**



(A) 6×10^{-4} (B) 6×10^{-6}

(C) 3×10^{-6} (D) 3×10^{-4}

Ans. (A)

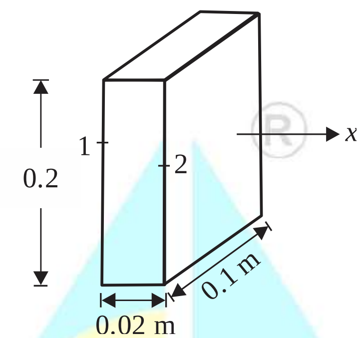
Sol. **Given,**

$D_{AB} = 1 \times 10^{-5} \frac{m^2}{\sec}$

$x_{A1} = 0.8$

$x_{A2} = 0.2$

C = 100 molm^{-3}.



The general fick's law equation can be written for binary mixture of A and B as follow

$$N_A = \frac{D_{AB}C(x_{A1}-x_{A2})}{z_2-z_1}$$

$$N_A = \frac{10^{-5}\times 10^2(0.8-0.2)}{0.02}$$

$$N_A = \frac{0.6}{0.02}\times 10^{-3}\frac{\text{mol}}{m^2\text{sec}}$$

Since In question it is asked molar flow rate not Molar flux So have to multiply by area also

$W_A = N_A \times A \rightarrow$ Area for diffusive flux = 0.2× 0.1

$$W_A = \frac{0.6}{0.02}\times 10^{-3}\times 0.02$$

$$W_A = 6\times 10^{-4}\text{mol/sec}$$

Question 18

Consider a vapour-liquid mixture of components *A* and *B* that obeys Raoult's law. The vapour pressure of *A* is half that of *B*. The vapour phase concentrations of *A* and *B* are 3 molm^{-3}, and 6 molm^{-3}, respectively. At equilibrium, the ratio of the liquid phase concentration of *A* to that of *B* is

[Mass Transfer, 1 Marks]

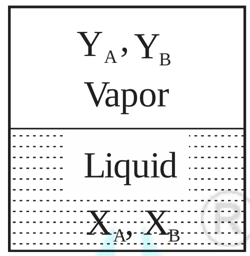
(A) 1.0 (B) 0.5

(C) 2.0 (D) 1.5

Ans. (A)

Sol. Given: $P_A^v = \frac{1}{2}P_B^v$

Vapour phase concentration: $C_A^V = 3\,\text{mol/m}^3$, $C_B^V = 6\,\text{mol/m}^3$



Raoult's Law state that Partial pressure of component A in vapor phase is directly proportional to mole fraction of that component in Liquid phase.

$$P_A \propto x_A$$

$$P^T y_A = x_A P_A^V, \qquad y_B P^T = x_A P_B^{sat},$$

$$\frac{y_A}{y_B} = \frac{x_A}{x_B}\frac{P_A^{sat}}{P_B^{sat}}$$

$$\frac{C_A^V / C^T}{C_B^V / C^T} = \frac{C_A^L / C^T}{C_B^L / C^T} \times 0.5$$

$$0.5 = \frac{3}{6} = \frac{C_A^L}{C_B^L} \times 0.5$$

$$\frac{C_A^L}{C_B^L} = 1$$

Question 19

The ratio of the activation energy of a chemical reaction to the universal gas constant is 1000 K. The temperature-dependence of the reaction rate constant follows the collision theory. The ratio of the rate constant at 600 K to that at 400 K is **[Chemical reaction Engineering, 1 Marks]**

(A) 2.818 (B) 4.323

(C) 1.502 (D) 1.000

Ans. (A)

Sol. **Given**: $\frac{Ea}{R} = 1000K$ and asked $\frac{K_{600}}{K_{400}} = ?$

From collision theory: -

Rate of reaction $=$ (Fraction of molecule which have Energy $> Ea$) $\times$ (No. of collision)

$$k = k_0 T^{\frac{1}{2}} e^{\frac{-Ea}{RT}}$$

$$ln\left(\frac{k_2}{k_1}\right) = \frac{1}{2} ln\left(\frac{T_2}{T_1}\right) - \frac{E_a}{RT}$$

$$ln\left(\frac{k_2}{k_1}\right) = \frac{1}{2} ln\left(\frac{T_2}{T_1}\right) + \frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$ln\left(\frac{k_2}{k_1}\right) = \frac{1}{2} \times ln\left(\frac{600}{400}\right) + 1000 \times \left(\frac{1}{400} - \frac{1}{600}\right)$$

$$\frac{k_{600}}{k_{400}} = 2.818$$

Remark:

(1) Always convert Temp in kelvin while using temperature dependency formula.

(2) All Temp dependency theory are applicable at const concentration

(3) $E_{arr} > E_{coll} > E_{tran}$

(4) $K_{tran} > K_{coll} > K_{arr}$

Question 20

The rate of a reaction $A \rightarrow B$ is 0.2 mol$m^{-3}s^{-1}$ at a particular concentration C_{A1}. The rate constant of the reaction at a given temperature is 0.1 $\text{m}^3\text{mol}^{-1}s^{-1}$. If the reactant concentration is increased to $10C_{A1}$ at the same temperature, the reaction rate, in mol$m^{-3}s^{-1}$, is

[Chemical reaction Engineering, 1 Marks]

(A) 20 (B) 10

(C) 100 (D) 50

Ans. (A)

Sol. Given: $A \xrightarrow{k} B$

$-r_{A_1} = 0.2\text{mol/m}^3\text{sec at } C_{A_1}$

$k = 0.1m^3\text{/mol.sec, find } r_{A_2} \text{ at } 10C_{A_1}$

From the rate constant unit, we can come to know order is 2

$$K \text{ unit} = \left(\frac{\text{mol}}{\text{lit}}\right)^{1-n} \text{sec}^{-1}$$

$1 - n = -1, \; n = 2$

$\therefore \; -r_{A_1} = 0.1C_A^2$

$$\frac{-r_{A_2}}{-r_{A_1}} = \frac{0.1CA_2^2}{0.1CA_1^2}$$ {K remain same as the Temp remain same}

$$C_{A_2} = 10C_{A1}$$

$$\frac{-r_{A_2}}{0.2} = \left(\frac{10C_{A_1}}{C_{A_1}}\right)^2$$

$$-r_{A_2} = 20\text{mol/m}^3\text{sec}$$

Question 21

Two parallel first-order liquid phase reactions $A \xrightarrow{k_1} B$ and $A \xrightarrow{k_2} C$ are carried out in a well-mixed isothermal batch reactor. The initial concentration of A in the reactor is 1 kmolm^{-3}, while that of B and C is zero. After 2 hours, the concentration of A reduces to half its initial value, and the concentration of B is twice that of C. The rate constants k_1 and k_2, in h^{-1}, are, respectively

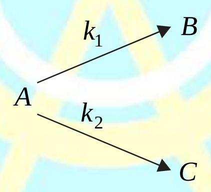
[Chemical reaction Engineering, 1 Marks]

(A) 0.40, 0.20 (B) 0.23, 0.12

(C) 0.50, 0.25 (D) 0.36, 0.18

Ans. (B)

Sol.



Given:

- 1^{st} order reaction
- Liquid phase reaction So $E_a = 0$
- Reaction Carried out in a batch.

$$C_{A_0} = 1\frac{K\text{mol}}{m^3} \qquad C_{B_0} = C_{C_0} = 0$$

After 2 hr $C_A = \frac{C_{A_0}}{2} = 0.5\frac{K\text{mol}}{m^3}$ and $C_B = 2C_C$

$$-r_A = (k_1 + k_2)C_A \qquad \ldots\text{(i)}$$

$$+r_B = k_1 C_A \qquad \ldots\text{(ii)}$$

$$+r_C = k_2 C_A \qquad \ldots\text{(iii)}$$

Isothermal batch reactor performance equation

$$\frac{-dc_A}{dt} = (k_1 + k_2)c_A$$

$$\frac{-dC_A}{C_A} = (k_1 + k_2)dt$$

$$C_A = C_{A_0} e^{-(K_1+K_2)t} \quad ....(iv)$$

$$-\ln\left(\frac{C_A}{C_{A_0}}\right) = -(K_1 + K_2) \times 2$$

$$K_1 + K_2 = \frac{\ln 2}{2} \quad ....(v)$$

From Equation (2) $r_B = \frac{dc_B}{dt} = k_1 C_A$

From Equation (3) $r_C = \frac{dc_B}{dt} = k_2 C_A$

Dividing Equation (2) and Equation (3)

$$\frac{r_B}{r_C} = \frac{dc_B}{dc_C} = \frac{k_1}{k_2}$$

$$\frac{C_B - 0}{C_C - 0} = \frac{k_1}{k_2}$$

$$\frac{k_1}{k_2} = 2$$

$$k_1 = 2k_2 \quad ....(vi)$$

From Equation (v)

$$K_1 + K_2 = \frac{\ln 2}{2}$$

$$K_1 + K_2 = 0.3465$$

$$2K_2 + K_2 = 0.3465$$

$$3K_2 = 0.3465$$

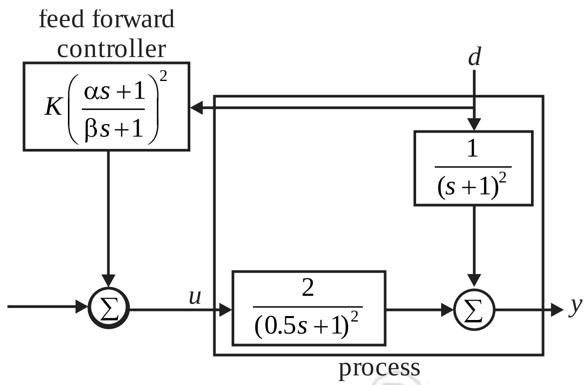
$$K_2 = 0.12$$

$$K_1 = 0.23$$

Question 22

Consider the block diagram in the figure with control input u, disturbance d and output y. For the feedforward controller, the ordered pair $(K, \alpha/\beta)$ is

[Instrumentation and Process Control, 1 Marks]



(A) (0.5, 2)
(B) (–0.5, 0.5)
(C) (–2, 2)
(D) (2, 0.5)

Ans. (B)

Sol. **Given:** control input u, disturbance d and output y in given block diagram.

For Perfect feed Forward, controller should be able to cancel the effect of disturbance change on process output or coefficient of d(s) in the transfer function should be equal to zero

$$\alpha \frac{1}{(s+1)^2} + \alpha k\left(\frac{\alpha s+1}{\beta s+1}\right)^2 \times \frac{2}{(0.55+1)^2} = y$$

Coefficient of d(s) = 0

$$\frac{1}{(s+1)^2} + k\left(\frac{\alpha s+1}{\beta s+1}\right)^2 \times \frac{2}{(0.55+1)^2} = 0$$

$$k\frac{(\alpha s+1)^2}{(\beta s+1)^2} = \frac{-1}{2} \times \frac{(0.55+1)^2}{(s+1)^2}$$

Compare both side

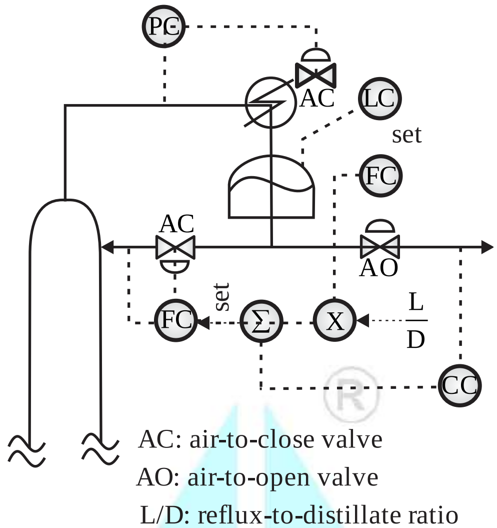
$$k = -0.5, \alpha = 0.5, \beta = 1$$

$$\frac{\alpha}{\beta} = 0.5$$

$\therefore$ $\left(k, \frac{\alpha}{\beta}\right) = (-0.5, 0.5)$

Question 23

Consider the control structure for the overhead section of a distillation column shown in the figure. The composition controller (CC) controls the heavy key impurity in the distillate by adjusting the setpoint of the reflux flow controller in a cascade arrangement. The sign of the controller gain for the pressure controller (PC) and that for the composition controller (CC) are, respectively,



[Instrumentation and Process Control, 1 Marks]

(A) negative, negative
(B) negative, positive
(C) positive, positive
(D) positive, negative

Ans. (D)

Sol. Given :

PC : Pressure Controller

LC : Level Controller

FC : Flow Controller

CC : Composition Controller

AC : air to close valve are valve which are held open by a valve spring and required air pressure to move them toward closed position. The valve closes progressively as the air pressure increases.

AO : air to open valves open progressively as air pressure increases.

PC : when column pressure increases, relative volatility between component will decrease. This will increase the reflux ratio and required increased condenser and reboiler duty.

∴ Positive PC gain is recommended.

CC : The reflux flow controller gets a combined input from CC (Composition Controller). A positive deviation from heavy key concentration set point warrant a negative gain to adjust the reflux FC.

∴ Negative Composition Controller is recommended.

Question 24

Which one of the given statements is correct with reference to gas-liquid contactors for mass transfer applications?

(A) A tray tower is more suitable for foaming systems than a packed tower.
(B) Tray towers are preferred over packed towers for systems requiring frequent cleaning.

(C) For a given liquid flow rate, the gas flow rate in the loading region is greater than that in the flooding region.

(D) Flooding can never occur for counter-current contact.

[Mass Transfer, 1 Marks]

Ans. (B)

Sol. **Given:** for given gas liquid contractors.

Tray tower is preferred over packed tower for system requiring frequent cleaning.

Because for such fouling fluid, if it is passed over packed section than pore at packed section might be clogged that result in poor redistribution of liquid and result in channeling in the tower so interfacial contact got decrease and mass transfer also got decreased.

Question 25

In an ammonia manufacturing facility, the necessary hydrogen is generated from methane. The facility consists of the following process units –

P : Methanator, Q: CO shift convertor, R : CO2 stripper, S : Reformer, T: Ammonia convertor

The correct order of these units, starting from methane feed is **[Chemical Technology, 1 Marks]**

(A) S, Q, R, P, T (B) P, Q, R, S, T

(C) S, P, Q, R, T (D) P, S, T, Q, R

Ans. (A)

Sol. Steam Reformer → CO shift convertor → CO_2 stripper →methanator → ammonia convertor

Steam reformer → $CH_{4(g)} + H_2O_{(g)} \rightarrow CO_{2(g)} + 3H_{2(g)}$

CO Shift Convertor :

Water gas shift reaction: $CO_{(g)} + H_2O_{(g)} \rightarrow CO_{2(g)} + H_{2(g)}$

CO Stripper: CO_2 is undesirable in Final ammonia product

This unit removes CO_2 from syn gas stream using solvent (ethylamine)

Methanator : A small amount of unreacted methane may be present after reforming. The methanator convert this remaining methane any CO that might have slipped through CO shift reactor back into CH_4 and CO_2

Ammonia Convertor :

Haber bosch process → $N_{2(g)} + 3H_{2(g)} \rightarrow 2NH_{3(g)}$

Question 26 [MSQ]

Consider a linear homogeneous system of equations $Ax = 0$, where A is an $n \times n$ matrix, x is an $n \times 1$ vector and 0 is an $n \times 1$ null vector. Let r be the rank of A. For a non-trivial solution to exist, which of the following conditions is/are satisfied? **[Engineering Mathematics, Linear Algebra] (1 Mark)**

(A) Determinant of $A = 0$ (B) $r = n$

(C) $r < n$ (D) Determinant of $A \neq 0$

Ans. (A, C)

Sol. **Given :** System of homogeneous equation

$Ax = 0$

Where, A is an $n \times n$ matrix,

x is an $n \times 1$ vector

0 is an $n \times 1$ null vector

For non trival solution to exist

$\rho(A) < n$

i.e. $|A| = 0$

Hence rank of A, $r < n$

and determinant of A, $|A| = 0$

Hence, the correct options are (A) and (C).

Question 27

If the Prandtl number Pr = 0.01, which of the following statements is/are correct?

(A) The momentum diffusivity is much larger than the thermal diffusivity.

(B) The thickness of the momentum boundary layer is much smaller than that of the thermal boundary layer.

(C) The thickness of the momentum boundary layer is much larger than that of the thermal boundary layer.

(D) The momentum diffusivity is much smaller than the thermal diffusivity.

[Heat Transfer, 1 Marks]

Ans. (B, D)

Sol. **Given:** $Pr = 0.01$

$$\text{Prandtl Number} = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$$

as $Pr = 0.01$

So momentum diffusivity is much smaller than thermal diffusivity → (D)

$$= \frac{\text{thickness momentum } b.l}{\text{thickness of thermal } b.l} = \frac{\delta_k}{\delta_t} = (\text{Pr})^{1/3}$$

as $Pr = 0.01$ So $\delta_h < \delta_t \rightarrow (B)$

Question 28

For the electrolytic cell in a chlor-alkali plant, which of the following statements is/are correct?

(A) A membrane cell operates at a higher brine concentration than a diaphragm cell.

(B) Chlorine gas is produced at the cathode.

(C) Hydrogen gas is produced at the cathode.

(D) The caustic product stream exits the cathode compartment. **[Chemical Technology, 1 Marks]**

Ans. (A, C, D)

Sol. Option (A) : Correct

In a membrane cell the electrolysis of brine (NaCl) solution occur according to following equation

$$2NaCl + 2H_2O \longrightarrow Cl_2 \uparrow + H_2 \uparrow + 2NaOH$$

at anode: $$2Cl^- \rightarrow Cl_2 + 2e$$

at cathode: $$2H_2O + 2e \rightarrow H_2 + 2OH^-$$

a higher concentration of brine may be used in a membrane cell, because the membrane prevents mixing of the product allowing for better control for production of H_2,Cl_2 and NaOH.

Option (B) : Incorrect

Cl_2 gas is produced at anode.

Option (C) : Correct

H_2gas is produced at cathode.

Option (D) : Correct

In both membrane and diaphragm cells, the caustic (NaOH) product stream exit the cathode compartment.

Question 29

Which of the following statements with reference to the petroleum/petrochemical industry is/are correct?

(A) Catalytic hydrocracking converts heavier hydrocarbons to lighter hydrocarbons.

(B) Catalytic reforming converts straight-chain hydrocarbons to aromatics.

(C) Cumene is manufactured by the catalytic alkylation of benzene with propylene.

(D) Vinyl acetate is manufactured by reacting methane with acetic acid over a palladium catalyst.

[Chemical Technology, 1 Marks]

Ans. (A, B, C)

Sol. Option (A) : Correct

Catalytic hydrocracking uses a catalyst and hydrogen to breakdown larger hydrocarbon molecule (found in heavy oil) into smaller, more useful one, like gasoline and diesel.

$$C_{10}H_{22}(\text{Heavyhydrocarbon}) + H_2 \rightarrow C_6H_{14}(\text{Lighter}) + \text{Otherproduct}$$

Option (B) : Correct

Catalytic reforming utilizes a catalyst to rearrange hydrocarbon molecules, particularly converting linear alkane into aromatic compounds which are valuable component in gasoline.

$$C_6H_{16} \rightarrow C_6H_6 + H_2$$

Option (C) : Correct

cumene (isopropyl benzene) is produced via alkylation of benzene by Friedel – Craft alkylation, using Lewis acid catalyst like aluminum chloride ($AlCl_3$) is the typical process for this reaction.

Option (C) : Incorrect

Vinyl acetate is not commercially produced by direct reaction of methane and acetic acid. It is usually synthesized from ethylene and acetic acid using palladium catalyst in a different process.

Question 30

Consider a matrix $A = \begin{bmatrix} -5 & a \\ -2 & -2 \end{bmatrix}$, where a is a constant. If the eigenvalues of A are -1 and -6, then the value of a, rounded off to the nearest integer, is ______.

[Engineering Mathematics, Linear Algebra] (1 Mark)

Ans. -2 to -2

Sol. Given :

$A = \begin{bmatrix} -5 & a \\ -2 & -2 \end{bmatrix}$, where a is a constant

Eigen values of A are –1 and –6

$\lambda_1 = -1, \lambda_2 = -6$

Since, the product of eigen values of a matrix A is equal determinant of matrix.

Determinant of matrix A = Product of eigen values

$|A| = \lambda_1 \lambda_2$

$(10 + 2a) = (-1)(-6)$

$10 + 2a = 6$

$2a = 6 - 10$

$2a = -4$

$a = -2$

Hence, the value of a is –2.

Question 31

Consider the reaction $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$ in a continuous flow reactor under steady-state conditions. The component flow rates at the reactor inlet are $F^0_{N_2} = 100\text{mol}s^{-1}, F^0_{H_2} = 1\text{mol}s^{-1}, F^0_{inert} = 1\text{mol}s^{-1}$. If the fractional conversion of H_2 is 0.60, the outlet flow rate of N_2, in $mols^{-1}$, rounded off to the nearest integer, is _______ . **[Chemical Reaction Engineering, 1 Marks]**

Ans. 40 to 40

Sol. **Given :** $t = 0$ $\quad N_2 \quad + \quad 3H_2 \quad \rightleftharpoons \quad 2NH_3$

100mol/sec $\quad$ 300mol/sec $\quad$ Inert = 1 mol/sec

$\because$ 3 mol H_2 react with = 1 mol N_2

$\therefore$ 300×0.6 mol H_2 react with $= \frac{1}{3} \times 300 \times 0.6$

$= 60$ mol N_2 reacts

remaining moles of $N_2 = 100 - 60 = 40$ mole N_2

Question 32

Consider a binary mixture of components A and B at temperature T and pressure P. Let $\bar{V}_A$ and $\bar{V}_B$ be the partial molar volumes of A and B, respectively. At a certain mole fraction of A, x_A

$$\left(\frac{\partial \bar{V}_A}{\partial x_A}\right)_{T,P} = 22\text{cm}^3\text{mol}^{-1} \text{ and } \left(\frac{\partial \bar{V}_B}{\partial x_A}\right)_{T,P} = -18\text{cm}^3\text{mol}^{-1}$$

The value of x_A, rounded off to 2 decimal places, is _______. **[Thermodynamics, 1 Marks]**

Ans. 0.44 to 0.46

Sol. **Given**: binary mixture A and B and $\left(\frac{\partial \bar{V}_A}{\partial x_A}\right)_{T,P} = 22\text{cm}^3\text{mol}^{-1}$, $\left(\frac{\partial \bar{V}_B}{\partial x_A}\right)_{T,P} = -18\text{cm}^3\text{mol}^{-1}$

From gibb's duhem equation at const T, P

$\sum x_i d\bar{m}_i = 0$ $\qquad m$ = molar property

$\therefore$ $x_A d\bar{v}_A + x_B d\bar{v}_B = 0$

Dividing by dx_A

$$x_A \frac{\partial \bar{v}_A}{\partial x_A} + x_B \frac{\partial \bar{v}_B}{\partial x_A} = 0 \rightarrow \text{For consistant system}$$

$x_A \times 22 + (1 - x_A)(-18) = 0$

$40x_A = 18$

$x_A = 0.45$

Question 33

Consider the steady, uni-directional, fully-developed, pressure-driven laminar flow of an incompressible Newtonian fluid through a circular pipe of inner radius 5.0 cm. the magnitude of shear stress at the inner wall of the pipe is $0.1Nm^{-2}$. At a radial distance of 1.0 cm from the pipe axis, the magnitude of the shear stress, in Nm^{-2}, rounded off to 3 decimal places, is ______. **[Fluid Mechanics, 1 Marks]**

Ans. 0.019 to 0.021

Sol. **Given :** Laminar flow

$\tau_w = 0.1 \text{ N/m}^2$

Fully developed Newtonian fluid.

For laminar fully developed flow

$$\frac{dp}{dx}+\frac{2\tau}{r}=0$$

$$\frac{\tau(r)}{\tau_w}=\frac{r}{R}$$

$$\frac{\tau(r)}{0.1}=\frac{1}{5}$$

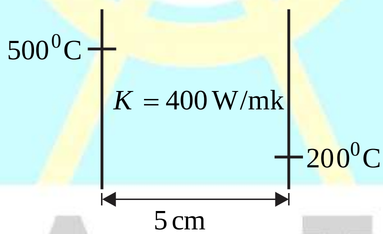
$$\tau(r=1)=\frac{0.1}{5}=0.020\ \text{N/m}^2$$

Question 34

The opposite faces of a metal slab of thickness 5 cm and thermal conductivity $400 Wm^{-1}{}^0C^{-1}$ are maintained at 500^0C and 200^0C. The area of each face is $0.02 m^2$. Assume that the heat transfer is steady and occurs only in the direction perpendicular to the faces. The magnitude of the heat transfer rate, in kW, rounded off to the nearest integer, is ________. **[Heat Transfer, 1 Marks]**

Ans. 48 to 48

Sol. **Given:**



From Fourier law of heat conduction

$$Q=\frac{\Delta T}{\left(\frac{L}{KA}\right)}$$

$$\Delta T=500-200=300^0C$$

$$L=0.05\ \text{m}$$

$$A=0.02\ \text{m}^2$$

$$K=400\ \text{W/m}^0C$$

$$Q=\frac{300}{\left(\frac{0.05}{400\times0.02}\right)}=48000=48\ \text{kW}$$

Question 35

The capital cost of a distillation column is Rs. 90 lakhs. The cost is to be fully depreciated (salvage value is zero) using the double-declining balance method over 10 years. At the end of two years of continuous operation, the book-value of the column, in lakhs of rupees, rounded off to 1 decimal place, is ________.

[Plant Design and Economic, 1 Marks]

Ans. 57.5 to 57.7

Sol. Given :

$v_0 = 90$ Lakh = initial value

$v_s = 0$ = salvage value

n = 10 years = service time

$V_2 = ?$

From double declining balance method book value for a[th] year

$v_a = v_0(1-f)^a$

Where $f = \frac{2}{n}$

$\therefore \quad f = \frac{2}{10} = 0.2$

And $v_a = v_0(1-f)^a$

So $v_2 = 90(1-0.2)^2$

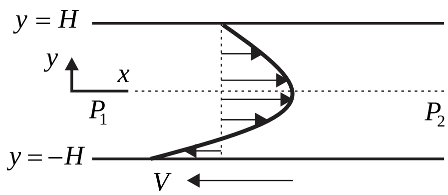
$v_2 = 57.6$ Lakhs

Q.36 to Q.65 Carry TWO Marks Each

Question 36

Consider a steady, fully-developed, uni-directional laminar flow of an incompressible Newtonian fluid (viscosity μ) between two infinitely long horizontal plates separated by a distance $2H$ as shown in the figure. The flow is driven by the combined action of a pressure gradient and the motion of the bottom plate at $y = -H$ in the negative x direction. Given that $\frac{\Delta P}{L} = \frac{(P_1 - P_2)}{L} > 0$, where P_1 and P_2 are the pressures at two x locations separated by a distance L. The bottom plate has a velocity of magnitude V with respect to the stationary top plate at $y = H$. Which one of the following represents the x −component of the fluid velocity vector ?

[Fluid Mechanics, 2 Marks]



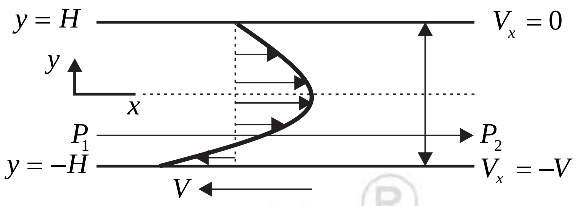
(A) $\frac{\Delta P}{L}\frac{H^2}{2\mu}\left(1 - \frac{y^2}{H^2}\right) + \frac{V}{2}\left(\frac{y}{H} - 1\right)$

(B) $\frac{\Delta P}{L}\frac{H^2}{2\mu}\left(\frac{y^2}{H^2} - 1\right) + \frac{V}{2}\left(\frac{y}{H} - 1\right)$

(C) $\frac{\Delta P}{L}\frac{H^2}{2\mu}\left(\frac{y^2}{H^2}-1\right)-\frac{V}{2}\left(\frac{y}{H}-1\right)$ (D) $\frac{\Delta P}{L}\frac{H^2}{2\mu}\left(1-\frac{y^2}{H^2}\right)-\frac{V}{2}\left(\frac{y}{H}-1\right)$

Ans. (A)

Sol.



Given :

Steady, fully developed, incompressible, unidirectional, laminar flow of Newtonian fluid.

Step 1 :

A differential momentum balance leads to following differential equation for momentum flux.

$$\frac{d\tau_{xz}}{dx}=\frac{P_0-P_L}{L}\Rightarrow\frac{d\tau_{yx}}{dy}=\frac{P_1-P_2}{L}$$

For Newtonian Fluid,

$$\tau_{yx}=-\mu\frac{dV_x}{dy}$$

$$\frac{d\tau_{yx}}{dy}=\frac{P_1-P_2}{L}$$

or, $\frac{d}{dy}\left(-\mu\frac{dV_x}{dy}\right)=\frac{P_1-P_2}{L}$

or, $-\mu\frac{d^2V_x}{dy^2}=\frac{\Delta P}{L}$ (Given $\frac{P_1-P_2}{L}=\frac{\Delta P}{L}$)

or, $\int\frac{d^2V_x}{dy^2}=-\int\frac{1}{\mu}\left(\frac{\Delta P}{L}\right)+C_1$

or, $\frac{dV_x}{dy}=-\frac{1}{\mu}\left(\frac{\Delta P}{L}\right)y+C_1$

Integrating again,

$$V_x=-\frac{1}{\mu}\left(\frac{\Delta P}{L}\right)\frac{y^2}{2}+C_1y+C_2 \quad .....(1)$$

Boundary condition,

B.C (1) at $Y=H$ $V_x=0$ (No slip condition) .....(2)

B.C (2) at $Y=-H$ $V_x=-V$ (Plate moving with velocity V in negative x direction) .....(3)

Step 2 :

Using B.C (1)

$$0 = \frac{-1}{\mu}\left(\frac{\Delta P}{L}\right)\left(\frac{H^2}{2}\right) + C_1 H + C_2 \qquad \ldots..(4)$$

Using B.C (2)

$$-V = \frac{-1}{\mu}\left(\frac{\Delta P}{L}\right)\left[\frac{(-H)^2}{2}\right] + C_1(-H) + C_2$$

$$-V = \frac{-1}{\mu}\left(\frac{\Delta P}{L}\right)\left[\frac{(H)^2}{2}\right] + C_2(-H) + C_2 \qquad \ldots..(5)$$

Equation (4) – Equation (5)

$$0 - (-V) = C_1 H - C_1(-H) = 2C_1 H$$

$$C_1 = \frac{V}{2H} \qquad \ldots..(6)$$

From equation (4) and equation (6)

$$0 = \frac{-1}{\mu}\left(\frac{\Delta P}{L}\right)\left(\frac{H^2}{2}\right) + \left(\frac{V}{2H}\right) H + C_2$$

or, $$C_2 = \frac{1}{\mu}\left(\frac{\Delta P}{L}\right)\left(\frac{H^2}{2}\right) - \frac{V}{2} \qquad \ldots..(7)$$

Equation (1), equation (6) and equation (7)

$$V_x = \frac{-1}{\mu}\left(\frac{\Delta P}{L}\right)\frac{y^2}{2} + C_1 y + C_2 \qquad \ldots..(1)$$

$\therefore$ $$V_x = \frac{-1}{\mu}\left(\frac{\Delta P}{L}\right)\frac{y^2}{2} + \frac{V}{2H}y + \frac{1}{\mu}\left(\frac{\Delta P}{L}\right)\frac{H^2}{2} - \frac{V}{2}$$

$$V_x = \left(\frac{\Delta P}{L}\right)\left(\frac{1}{2\mu}\right)(H^2 - y^2) + \frac{V}{2}\left[\frac{y}{H} - 1\right]$$

or, $$V_x = \left(\frac{\Delta P}{L}\right)\left(\frac{H^2}{2\mu}\right)\left(1 - \frac{y^2}{H^2}\right) + \frac{V}{2}\left[\frac{y}{H} - 1\right]$$

Question 37

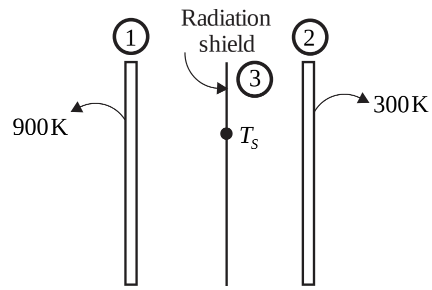
The temperatures of two large parallel plates of equal emissivity are 900 K and 300 K. a reflection radiation shield of low emissivity and negligible conductive resistance is placed parallelly between them. The steady-state temperature of the shield, in K, is **[Heat Transfer, 2 Marks]**

(A) 759 (B) 559

(C) 659 (D) 859

Ans. (A)

Sol. Given:



Since the shield does not deliver or remove heat from the system, the heat transfer between plate (1) and shield mush be same as that between shield and plate (2)

$$\left(\frac{q}{A}\right)_{1-3} = \left(\frac{q}{A}\right)_{3-2}$$

$$\frac{\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1} = \frac{\sigma(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1}$$

$\varepsilon_1 = \varepsilon_2 = \varepsilon_3$ So denominator cancelled

$$T_1^4 - T_3^4 = T_3^4 - T_2^4$$

$$T_3^4 = \frac{1}{2}(T_1^4 + T_2^4)$$

$$T_3^4 = \frac{1}{2}[(900)^4 + (300)^4]$$

$$T_3 = 759.14K$$

Remark: Always Put temp in kelvin

Question 38

Hot oil at 110^0C heats water from 30^0C to 70^0C in a counter-current double-pipe heat exchanger. The flow rates of water and oil are $50kg\,min^{-1}$ and 100kgmin^{-1}, respectively and their specific heat capacities are $4.2\text{kJkg}^{-1}{}^0C^{-1}$ and $2.0\text{kJkg}^{-1}{}^0C^{-1}$, respectively. assume the heat exchanger is at steady state. If the overall heat transfer coefficient is $200Wm^{-2}{}^0C^{-1}$, the heat transfer area in m^2 is

[Heat transfer, 2 Marks]

(A) 17.9 (B) 1.1

(C) 5.2 (D) 35.2

Ans. (A)

Sol. Given:

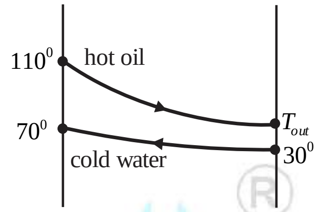
$$\dot{m}_w = 50\text{kg/min}$$

$$\dot{m}_{oil} = 100\text{kg/min}$$

$C_{pwater} = 4.2\ \text{kJ/kg}^0C$

$C_{poil} = 2\ \text{kJ/kg}^0C$

$U = 200\ \text{w/m}^{2\,0}C$ Calculate A =?



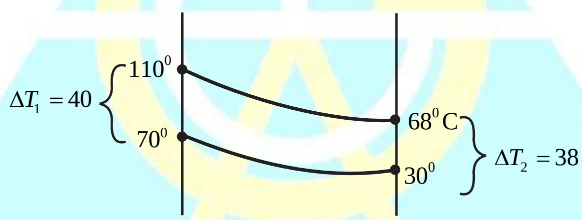
At steady state, apply heat balance

heat transfer by hot fluid = heat gained by cold fluid

$$m_0 c_{p_{oil}}(T_{h_{in}} - T_{h_{out}}) = m_c c_{p_c}(T_{c_{out}} - T_{c_{in}})$$

$$100 \times 2 \times (110 - T_{h_{out}}) = 50 \times 4.2 \times (70 - 30)$$

$$T_{h_{out}} = 68^0C$$



$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

$$\Delta T_{lm} = 38.99$$

Now, $Q = UA\Delta T_{lm}$

$$\frac{50}{60} \times 4.2 \times 10^3 \times (70 - 30) = 200 \times A \times 38.99$$

$$A = 17.95\ \text{m}^2$$

Question 39

A solid slab of thickness H_1 is initially at a uniform temperature T_0. At time $t = 0$, the temperature of the top surface at $y = H_1$ is increased to T_1, while the bottom surface at $y = 0$ is maintained at T_0 for $t \geq 0$. Assume heat transfer occurs only in the y −direction, and all thermal properties of the slab are constant. The time required for the temperature at $y = H_1/2$ to reach 99% of its final steady value is τ_1. If the thickness of the slab is doubled to $H_2 = 2H_1$, and the time required for the temperature at $y = H_2/2$ to reach 99% of its final steady value is τ_2, then τ_2/τ_1 is

[Heat transfer, 2 Marks]

(A) 2 (B) $\frac{1}{4}$

(C) 4 (D) $\frac{1}{2}$

Ans. (C)

Sol. Given: Slab thickness $= H_1$

Heat transfer occurs in y direction only

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \{\text{assuming heat generation} = 0\}$$

$$T(0,t) = T_0$$

$$T(y_T,0) = T$$

$$T(\infty,t) = T_1$$

$$\frac{T(y,t)-T_0}{T_1-T_0} = erf\left(\frac{y}{2\sqrt{\alpha t}}\right)$$

In both condition temp reaches to 99% of steady state value

$$T(y,t) = 0.99T_1$$

$$\therefore \quad erf\left(\frac{y}{2\sqrt{\alpha t}}\right)_1 = erf\left(\frac{y}{2\sqrt{\alpha t}}\right)_2$$

$$\frac{y_1}{\sqrt{t_1}} = \frac{y_2}{\sqrt{t_2}}$$

Given $y_1 = \frac{H_1}{2}$, $y_2 = \frac{H_2}{2} = \frac{2H_1}{2}$, $y_2 = H_1$

$$\frac{\frac{H_1}{2}}{\sqrt{t_1}} = \frac{H_1}{\sqrt{t_2}}$$

$$\frac{t_2}{t_1} = \left(\frac{1}{4}\right)^{-1} \Rightarrow \frac{t_2}{t_1} = 4$$

Question 40

A gas stream containing 95 mol% CO_2 and 5 mol% ethanol is to be scrubbed with pure water in a counter-current, isothermal absorption column to remove ethanol. The desired composition of ethanol in the exit gas stream is 0.5 mol%. the equilibrium mole fraction of ethanol in the gas phase, $y*$, is related to that in the liquid phase, x, as $y* = 2x$. Assume CO_2 is insoluble in water and neglect evaporation of water. If the water flow rate is twice the minimum, the mole fraction of ethanol in the spent water is

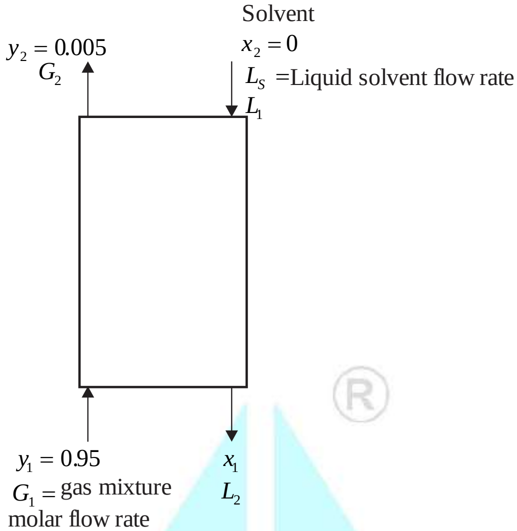
[Mass transfer, 2 Marks]

(A) 0.0225 (B) 0.0126

(C) 0.428 (D) 0.0316

Ans. (B)

Sol. Given:



y_1 = mole fraction of solute in gas mixture in inlet

y_2 = mole fraction of solute in gas mixture at outlet

x_1 = mole fraction at solute on liquid at inlet.

given $y = 2x$

and $L_s = 2L_{smin}$

Assume $(L_1 = L_2)$ and $G_1 = G_2 = G_s$

Convert and calculate in solute free basis

Convert mole fraction in mole ratio

and $Y_1 = \frac{y_1}{1-y_1}, Y_2 = \frac{y_2}{1-y_2}$, $\quad Y_1 = \frac{1}{19} \quad Y_2 = \frac{1}{199}$

Applying material balance in the section.

$$L_1x_2 + G_1y_1 = G_2y_2 + L_2x_1 \qquad \ldots (1)$$

as $L_1 = L_2 = L_s$ and $G_1 = G_2 = G_s$ (solute free basis)

From (1)

$$\frac{L_s}{G_s} = \frac{Y_1 - Y_2}{X_1 - X_2}$$

∵ We are applying equilibrium relation to calculate y and x so we will Find $\frac{L_{smin}}{G_s}$ because at equilibrium we require min solvent.

So $$\frac{L_{smin}}{G_s} = \frac{Y_1 - Y_2}{\left(\frac{Y_{1max}}{2}\right)}$$

$X_2 = X_0 = 0$ because we take solute free solvent

$$\frac{L_{smin}}{G_s} = \frac{\frac{1}{19}-\frac{1}{199}}{\frac{1}{19\times2}-0}$$

$$\frac{L_{smin}}{G_s}=\frac{0.0476}{0.0263}=1.8098$$

given $\quad L_s = 2L_{smin}$

So $\frac{L_s}{G_s} = 2\times\left(\frac{L_{smin}}{G_s}\right)$

$$\frac{L_s}{G_s} = 2\times1.8098 = 3.6196$$

Now,

$$\frac{L_s}{G_s} = \frac{Y_1-Y_2}{X_1-X_2}$$

$$3.6196 = \frac{\frac{1}{19}-\frac{1}{199}}{X_2}$$

$$X_2 = 0.0132$$

It is mole ratio, and we are required to find mole fraction

So $X_n = X_2 = \frac{x_2}{1-x_2} = 0.0132$

$$x_2 = 0.013$$

Question 41

Sulfur dioxide (SO_2) gas diffuses through a stagnant air-film of thickness 2 mm at 1 bar and 30^0C. The diffusion coefficient of SO_2 in air is $1\times10^{-5}m^2s^{-1}$. The SO_2 partial pressures at the opposite sides of the film are 0.15 bar and 0.05 bar. The universal gas constant is $8.314J\text{mol}^{-1}K^{-1}$. Assuming ideal gas behavior, the steady-state flux of SO_2 in $molm^{-2}s^{-1}$ through the air-film is

[Mass transfer, 2 Marks]

(A) 0.077 (B) 0.022

(C) 0.085 (D) 0.057

Ans. (B)

Sol. Given :

SO_2 is diffusing through a stagnant air film

thickness $= z_2 - z_1 = 2\times10^{-3}$

total pressure $= P_T = 10^5$ Pa

Temperature $T = 30^0 = 303$ K

at Location (1) → $z_1 \quad P_{A_1} = 0.15$ bar $\quad P_{B_1} = P_T - P_{A_1} = 0.85$ bar at Location (2) → z_2 $P_{A_2} = 0.05$ bar $P_{B_2} = P_T - P_{A_2} = 0.95$ bar

Steady state flux of SO_2 can be given by

$$N_A|_{SO_2} = \frac{D_{AB}P_T}{RT}\frac{(P_{A_1}-P_{A_2})}{P_{Blm}} \times \frac{1}{z_2-z_1}$$

$$P_{Blm} = \frac{P_{B_1}-P_{B_2}}{ln\left(\frac{P_{B_1}}{P_{B_2}}\right)} = \frac{0.10}{ln\left(\frac{0.95}{0.85}\right)}$$

$$N_A|_{SO_2} = \frac{10^{-3}\times 10^5}{8.314\times 303\times 2\times 10^{-4}} \times \frac{0.10}{ln\left(\frac{0.95}{0.85}\right)}$$

$$N_A|_{SO_2} = 0.022 \text{ mol/m}^2\text{sec}$$

Question 42

A simple distillation column separates a binary mixture of A and B. The relative volatility of A with respect to B is 2. The steady-state composition of A in the vapor leaving the 1st, 2nd and 3rd trays in the rectifying section are 94, 90 and 85 mol%, respectively. for ideal trays and constant molal overflow, the reflux-to-distillate ratio is

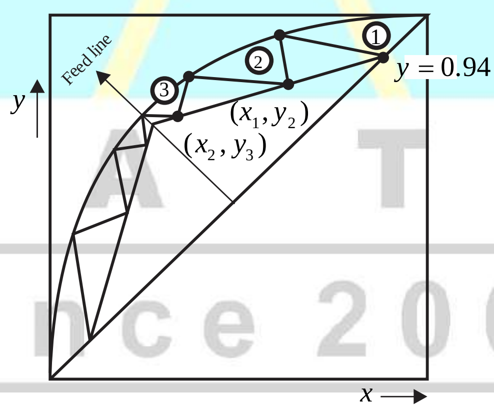
[Mass transfer, 2 Marks]

(A) 1.9 (B) 2.7

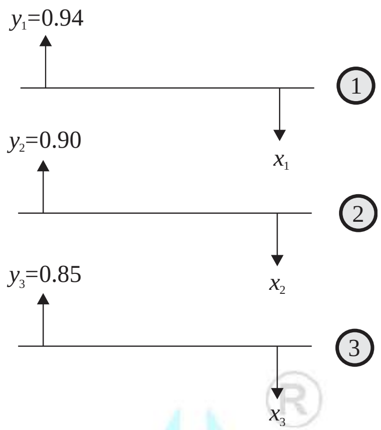
(C) 1.2 (D) 1.1

Ans. (B)

Sol. Given:



$\alpha_{AB} = 2$ $\frac{R}{D} = ?$



→ x_1 and y_1 are at equilibrium curve.

→ x_2 and y_2 are at equilibrium curve.

Note :

Stream lies on same side lies on operating line.

Stream lie on opposite side lies on equilibrium line.

$$y_1 = \frac{\alpha x_1}{1+(\alpha-1)x_1} \qquad y_2 = \frac{\alpha x_2}{1+(\alpha-1)x_2}$$

$$0.94 = \frac{2x_1}{1+x_1} \qquad 0.9 = \frac{2x_2}{1+x_2}$$

$$x_1 = \frac{0.94}{1.06} \qquad x_2 = \frac{0.9}{1.1}$$

Enriching section operating line can be written as :

$$y = \frac{R}{R+1}x + \frac{x_D}{R+1}$$

Slope $= \frac{R}{R+1}$ =Slope between (x_2, y_3) and (x_1, y_2)

$$\frac{R}{R+1} = \frac{0.9-0.85}{\frac{0.94}{1.06} - \frac{0.9}{1.1}}$$

$$R = 2.68$$

$$R = 2.7 = \frac{L}{D}$$

Question 43

Alumina particles with an initial moisture content of 5 kg per kg dry solid are dried in a batch dryer. For the first two hours, the measured drying rate is constant at $2\text{kg}m^{-2}h^{-1}$. Thereafter, in the falling-rate period, the rate decreases linearly with the moisture content. The equilibrium moisture content is 0.05 kg per dry solid and the drying area of the particles is $0.5m^2$perkgdry solid. The total drying time, in h, to reduce the moisture content to half its initial value is **[Mass Transfer, 2 Marks]**

(A) 4.13 (B) 2.55

(C) 3.22 (D) 5.13

Ans. (B)

Sol. **Given:** Initial moisture content $(x_1) = \frac{5 \text{ kg } H_2O}{1 \text{ kg dry solid}}$

Constant rate drying time $(t_c) = 2$ hour

$$N_C = 2\frac{\text{kg}}{m^2h}$$

Equilibrium moisture content $x^* = \frac{0.05 \text{ kg } H_2O}{1\text{kg dry solid}}$

Drying area $= \frac{0.5m^2}{1\text{kg dry solid}}$

$$x_F = \frac{x_1}{2} = 2.5$$

Total drying time $(t) = \frac{w_S}{AN_C}(x_1 - x_c) + (x_c - x_e^*)\, ln\left(\frac{x_C - x_e^*}{x_F - x_e^*}\right)$

$$t = t_c + \frac{W_S}{AN_C}(x_c - x_e^*)\, ln\left(\frac{x_C - x_e^*}{x_F - x_e^*}\right)$$

$\because$ $t_c = 2 = \frac{W_S}{AN_C}(x_1 - x_c)$

$$2 = \frac{1}{0.5\times 2}(5 - x_c)$$

$$x_C = 3$$

$$t = 2 + \frac{1}{0.5\times 2}(3 - 0.05)\, ln\left(\frac{3 - 0.05}{2.5 - 0.05}\right)$$

$$t_{Total} = 2.547 = 2.55 \text{ hours}$$

Question 44

A first-order heterogeneous reaction $A \rightarrow B$ is carried out using a porous spherical catalyst. Assume isothermal conditions, and that intraphase diffusion controls the reaction rate. At a bulk A concentration of $0.3\text{mol}L^{-1}$, the observed reaction rate in a 3 mm diameter catalyst particle is $0.2\text{mol}s^{-1}L^{-1}$ catalyst volume. At a bulk A concentration of $0.1\text{mol}L^{-1}$, the observed reaction rate, in $\text{mol}s^{-1}L^{-1}$catalyst volume, in a 6 mm diameter catalyst particle, is **[Chemical reaction engineering, 2 Marks]**

(A) 0.011 (B) 0.033

(C) 0.022 (D) 0.005

Ans. (B)

Sol.

Given:

$$A \xrightarrow{n=1} B$$

Porous spherical catalyst diffusion control reaction rate at

$C_{A_1} = 0.3\frac{\text{mol}}{L}$, $d_{A_1} = 3\text{mm}$, $-r_A = 0.2\frac{\text{mol}}{\text{sec L}}$

$C_{A_2} = 0.1\frac{\text{mol}}{L}$, $d_{A_2} = 6\text{mm}$, $-r_A = ?$

When diffusion control regime (strong pore diffusion regime)

$$\eta = \frac{(-r_A)_{obs}}{(-r_A)} = \frac{1}{\phi}$$

$$(-r_A)_{ob} = \frac{1}{mL}(-r_A) = \frac{KC_A}{mL}$$

$$\frac{(-r_A)_{ob_2}}{(-r_A)_{ob_1}} = \frac{R_1 C_{A_2}}{R_2 C_{A_1}}$$

$$\frac{(-r_A)_{obs_2}}{0.2} = \frac{3}{6} \times \frac{0.1}{0.3}$$

$$(-r_A)_{obs_2} = 0.033\frac{\text{mol}}{L\text{ sec}}$$

Question 45

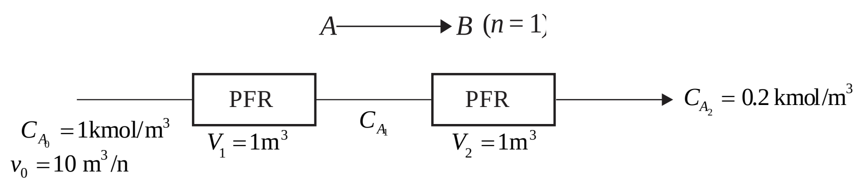
A first-order liquid phase reaction $A \rightarrow B$ is carried out in two isothermal plug flow reactors (PFRs) of volume $1m^3$ each, connected in series. The feed flow rate and concentration of A to the first reactor are $10m^3h^{-1}$ and $1\text{kmol}m^{-3}$, respectively. At steady-state, the concentration of A at the exit of the second reactor is $0.2\text{kmol}m^{-3}$. If the two PFRs are replaced by two equal-volume continuously stirred tank reactors (CSTRs) to achieve the same overall steady-state conversion, the volume of each CSTR, in m^3, is **[Chemical reaction engineering, 2 Marks]**

(A) 1.54 (B) 3.84

(C) 7.28 (D) 1.98

Ans. (A)

Sol. **Given:**



Overall conversion,

$$X = 1 - \frac{0.2}{1} = 0.8$$

For $PFR_1 \rightarrow C_{A_1} = C_{A_0} e^{-k\tau_1}$

$$\tau_1 = \frac{V_1}{v_0} = 0.1$$

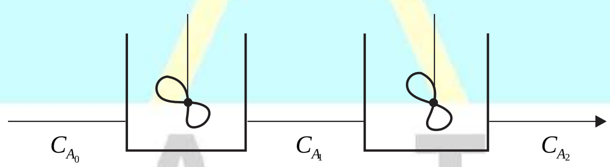
For $overall \rightarrow C_{A_2} = C_{A_0} e^{-k(\tau_1+\tau_2)}$

$$\tau_2 = \frac{V_2}{v_0} = 0.1$$

$$0.2 = 1e^{-k(0.2)}$$

$$k = 5\, ln\, 5 = 8.047$$

Now both PFR are replaced by 2 CSTR of equal volume



$$\frac{C_{A_1}}{C_{A_0}} = \frac{1}{1+K\tau_1}, \frac{C_{A_2}}{C_{A_1}} = \frac{1}{1+K\tau_2}$$

$$\frac{C_{A_2}}{C_{A_0}} = \frac{1}{(1+K\tau)^2} \quad \{\text{because both } \tau \text{ are same}\}$$

$$\frac{C_{A_2}}{C_{A_0}} = \frac{0.2}{1} = \frac{1}{(1+8.047\tau)^2}$$

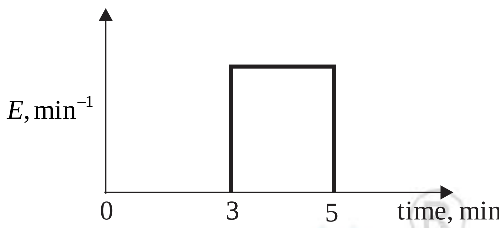
$$\tau = 0.15316 = \frac{V_1}{v_0}$$

$$V_1 = 1.5316 \text{ m}^3$$

Question 46

The residence time distribution, E, for a non-ideal flow reactor is given in the figure. A first-order liquid phase reaction with a rate constant $0.2\,min^{-1}$ is carried out in the reactor. For an inlet reactant concentration of $2 \text{mol} L^{-1}$, the reactant concentration (in molL^{-1}) in the exit stream is

[Chemical reaction engineering, 2 Marks]

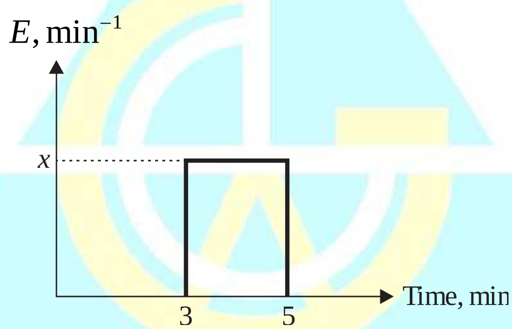


(A) 0.905 (B) 0.452

(C) 1.902 (D) 0.502

Ans. (A)

Sol.



Given: Rate constant $k = 0.2\,min^{-1}$

inlet concentration $(C_{A_0}) = 2$ mol/L

From segregation model

$$X_A = \int_0^\infty X_A E(t)dt$$

or $$\frac{\bar{C}_A}{C_{A_0}} = \int_0^\infty \frac{C_A}{C_{A_0}} E(t)dt$$

From graph $\int_0^\infty E(t)dt = 1$

$$2 \times x = 1$$

$$x = \frac{1}{2}$$

$$E(t) = \begin{cases} \frac{1}{2}, & \text{when } 3 \le t \le 5 \\ 0, & \text{otherwise} \end{cases}$$

For First order system

$$\frac{C_A}{C_{A_0}} = e^{-kt} = e^{-0.2t}$$

$$\therefore \quad \frac{\bar{C}_A}{C_{A_0}} = \int_0^{\infty} e^{-0.2t} \times 0.5dt$$

$$= \int_3^5 e^{-0.2t} \times 0.5dt$$

$$= 0.5\frac{[e^{-0.2t}]_3^5}{-0.2}$$

$$= -2.5[e^{-1} - e^{-0.6}]$$

$$\frac{\bar{C}_A}{C_{A_0}} = 0.4523$$

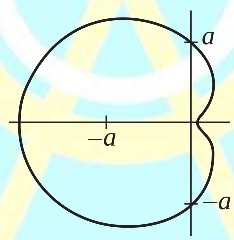
$$\therefore \quad \bar{C}_A = 0.4523 \times 2$$

$$\bar{C}_A = 0.9046 \approx 0.905$$

Question 47

Let r and θ be the polar coordinates defined by $x = r\cos\theta$ and $y = r\sin\theta$. The area of the cardioid $r = a(1-\cos\theta), 0 \le \theta \le 2\pi$, is

[Engineering Mathematics, Integral and Differential Calculus] (2 Marks)



(A) $\frac{3\pi a^2}{2}$ (B) $\frac{2\pi a^2}{3}$

(C) $3\pi a^2$ (D) $2\pi a^2$

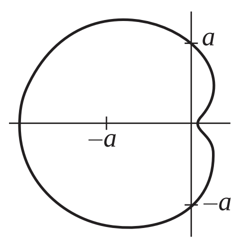
Ans. (A)

Sol. Given :

$x = r\cos\theta$ and

$y = r\sin\theta$

$r = a(1-\cos\theta), \quad 0 \le \theta \le 2\pi$



The Area of the Region bounded by a Curve $y = f(x)$ is given by

$$\text{Area} = \int_a^b f(x)dx$$

In polar Coordinates the Area bounded by a Curve $r = f(\theta)$ is given by

$$\text{Area} = \int_\alpha^\beta \frac{r^2}{2} d\theta$$

Area of cardioid

$$\int_0^{2\pi} \frac{r^2}{2} d\theta = \int_0^{2\pi} \frac{a^2(1-\cos\theta)^2}{2} d\theta$$

$$= 2\int_0^{\pi} \frac{a^2(1-\cos\theta)^2}{2} d\theta$$

$$= a^2 \int_0^{\pi} (1+\cos^2\theta - 2\cos\theta)d\theta$$

$$= a^2 \int_0^{\pi} \left[1 - 2\cos\theta + \frac{(1+\cos 2\theta)}{2}\right] d\theta$$

$$= a^2 \int_0^{\pi} \left[\frac{3}{2} - 2\cos\theta + \frac{\cos 2\theta}{2}\right] d\theta$$

$$= a^2 \left[\frac{3}{2}\theta - 2\sin\theta + \frac{\sin 2\theta}{4}\right]_0^{\pi}$$

$$= a^2\left[\frac{3}{2}\pi\right] = \frac{3}{2}a^2\pi$$

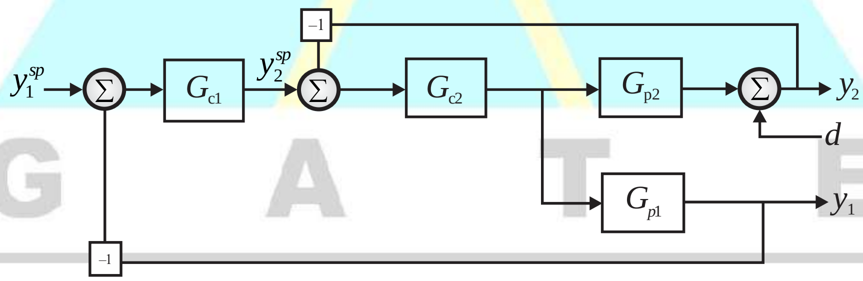
Area of the cardioid $= \frac{3}{2}a^2(\pi)$

Hence the correct option is (A).

Question 48

For the block diagram shown in the figure, the correct expression for the transfer function $G_d = \frac{y_1(s)}{d(s)}$ is

[Instrumentation and process control 2 marks]



(A) $\frac{-G_{p1}G_{c2}}{(1+G_{c1}G_{c2}G_{p1})(1+G_{c2}G_{p2})}$

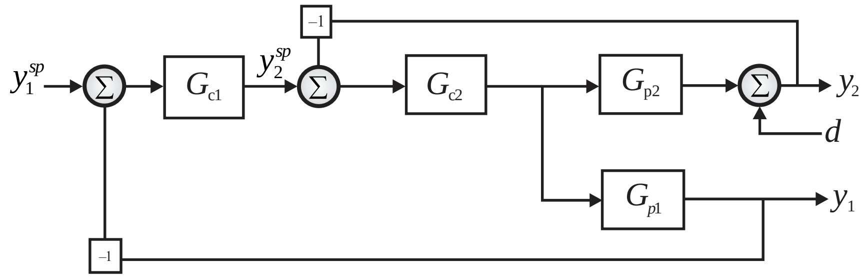
(B) $\frac{-G_{p1}G_{c2}}{(1+G_{c2}G_{p2}+G_{c1}G_{c2}G_{p1}G_{p2})}$

(C) $\frac{-G_{p1}G_{c2}}{(1+G_{c2}G_{p2}+G_{c1}G_{c2}G_{p1})}$

(D) $\frac{1}{(1+G_{c2}G_{p2}+G_{c1}G_{c2}G_{p1}G_{p2})}$

Ans. (A)

Sol. Given:



$G_d = \frac{y_1(s)}{d(s)}$

$(y_1^{sp} - y_1)G_{c_1} = y_2^{sp}$ … (1) (at point 1)

$\therefore$ $(y_2^{sp} - y_2)G_{c_2}G_{p_1} = y_1$ … (2)

and $(y_2^{sp} - y_2)G_{c_2}G_{p_2} + d = y_2$ … (3)

$y_2^{sp}G_{c_2}G_{p_2} + d = y_2(1 + G_{c_2}G_{p_2})$ … (4)

$\because$ $(y_1^{sp} - y_1)G_{c_1} = y_2^{sp}$

We have to calculate $\frac{y_1}{d}$ so change in set point will be zero $y_1^{sp} = 0$

$\therefore$ From equation (1) $\rightarrow$ $(0 - y_1G_{c_1}) = y_2^{sp}$ … (5)

From equation (4) and (5)

$(-y_1G_{c_1} - y_2)G_{c_2}G_{p_2} + d = y_2$

$-y_1G_{c_1}G_{c_2}G_{p_2} + d = y_2(1 + G_{c_2}G_{p_2})$ … (6)

From equation (2) and (3)

$\frac{y_1}{G_{p_1}} \times G_{p_2} + d = y_2$ … (7)

On comparing equation (6) and (7)

$-y_1G_{c_1}G_{c_2}G_{p_2} + d = \left[\frac{y_1G_{p_2}}{G_{p_1}} + d\right](1 + G_{c_2}G_{p_2})$

$-y_1G_{c_1}G_{c_2}G_{p_1}G_{p_2} + dG_{p_1} = (y_1G_{p_2} + dG_{p_1})(1 + G_{c_2}G_{p_2})$

$-y_1G_{c_1}G_{c_2}G_{p_1}G_{p_2} + dG_{p_1} = y_1G_{p_2} + y_1G_{p_2}^2G_{c_2} + dG_{p_1} + dG_{p_1}G_{c_2}G_{p_2}$

$y_1[G_{p_2} + G_{p_2}^2G_{c_2} + G_{c_1}G_{c_2}G_{p_1}G_{p_2}] = -dG_{p_1}G_{c_2}G_{p_2}$

$\frac{y_1}{d} = \frac{-G_{p_1}G_{c_2}}{1+G_{p_2}G_{c_2}+G_{c_1}G_{c_2}G_{p_1}}$

Question 49

For purchasing a batch reactor, three alternatives P, Q and R have emerged, as summarized in the table. For a compound interest rate of 10% per annum, choose the correct option that arranges the alternatives, in order, from the least expensive to the most expensive.

[Plant design and economic 2 marks]

	P	Q	R
Installed Cost (lakh rupees)	15	25	35
Equipment Life (years)	3	5	7
Maintenance Cost (lakh rupees per year)	4	3	2

(A) P, Q, R

(B) R, P, Q

(C) R, Q, P

(D) Q, R, P

Ans. (C)

Sol. Given:

	P	Q	R
Installed cost (Lakh)	15	25	35
Equipment cost (Year)	3	5	7
Maintenance cost $\left(\frac{\text{Lakh}}{\text{year}}\right)$	4	3	2

alternative $P \rightarrow$

total capitalized cost $= c_0 + \frac{c_0 - c_s}{(1+i)^n - 1} + \frac{c_m}{i}$

where c_0 = installed cost

i = interest per year

i = equipment life

c_m = maintainence cost or annual expense

c_s = Salvage value = 0

$$(TCC)_P = 15 + \frac{15-0}{(1\cdot1)^3 - 1} + \frac{4}{0.1} = 100.317 \text{ Lakh}$$

(alternative Q):-

$$(TCC)_Q = 25 + \frac{25-0}{(1\cdot1)^5 - 1} + \frac{3}{0.1} = 95.944 \text{ Lakh}$$

(alternative *R*) :-

$$(TCC)_R = 35 + \frac{35-0}{(1\cdot1)^7-1} + \frac{2}{0.1} = 91.89 \text{ Lakh}$$

Concept: whose total Capitalized cost is minimum, that method is least expensive

$\because \quad (TCC)_R < (TCC)_Q < (TCC)_P$

Least to most expensive *R*, *Q*, *P*

Question 50

The Newton-Raphson method is used to solve $f(x) = 0$, where $f(x) = e^x - 5x$. If the initial guess $x^{(0)} = 1.0$, the value of the next iterate, $x^{(1)}$, rounded off to 2 decimal places, is ______.

[Engineering Mathematics, Numerical Methods] (2 Marks)

Ans. -0.01 to 0.01

Sol. Given :

$$f(x) = e^x - 5x$$

Initial guess $x^{(0)} = x_0 = 1$

Differentiating function $f(x)$ with respect to x,

$$f'(x) = e^x - 5$$

By Newton Raphson's iterative method,

First iteration is given by,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = x_0 - \frac{e^{x_0} - 5x_0}{e^{x_0} - 5}$$

$$x_1 = \frac{x_0(e^{x_0} - 5) - e^{x_0} + 5x_0}{e^{x_0} - 5}$$

$$x_1 = \frac{x_0 e^{x_0} - 5x_0 - e^{x_0} + 5x_0}{e^{x_0} - 5}$$

$$x_1 = \frac{x_0 e^{x_0} - e^{x_0}}{e^{x_0} - 5}$$

Putting the value of $x_0 = 1$

$$x_1 = \frac{e^1 - e^1}{e^1 - 5} = \frac{0}{e^1 - 5}$$

$$x_1 = 0$$

Hence the value of next iterate $x^{(1)}$ is zero.

Question 51

Consider the line integral $\int_C F(r).dr$, with $F(r) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, where $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are unit vectors in the (x, y, z) Cartesian coordinate system. The path C is given by $r(t) = cos(t)\,\hat{\imath} + sin(t)\,\hat{\jmath} + t\hat{k}$, where $0 \le t \le \pi$. The value of the integral, rounded off to 2 decimal places, is _________.

[Engineering Mathematics, Vector Calculus] (2 Marks)

Ans. **4.91 to 4.95**

Sol. **Given :**

(i) $F(r) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Where $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit vectors in the (x, y, z) Cartesian coordinate system.

(ii) The path C is given by

$r(t) = cos(t)\hat{\imath} + sin(t)\hat{\jmath} + t\hat{k}$

The line integral

$$\int_c F(r) \cdot dr = \int_c (x\hat{\imath} + y\hat{\jmath} + z\hat{k}) \cdot (dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k})$$

$$= \int_c (xdx + ydy + zdz)$$

Parameters are

$$x = cos\, t, y = sin\, t, z = t$$

$$dx = -sin\, t\, dt$$

$$dy = cos\, t\, dt$$

$$dz = dt$$

$$= \int_0^\pi cos\, t\,(-sin\, t)dt + cos\, t\,(sin\, t)dt + tdt$$

$$\int_0^\pi tdt = \frac{t^2}{2}\Big|_0^\pi = \frac{\pi^2}{2} = 4.93$$

Hence, the value of integral is 4.93.

Question 52

Consider the ordinary differential equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = 0$, with the boundary conditions $y(x = 1) = 2$ and $y(x = 2) = 17/2$. The solution $y(x)$ at $x = 3/2$, rounded off to 2 decimal places, is ______.

[Engineering Mathematics, Differential Equation] **(2 Marks)**

Ans. **4.00 to 4.08**

Sol. **Given :**

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = 0 \quad \ldots\text{(i)}$$

Boundary Conditions is

$$y(x = 1) = 2 \text{ and } y(x = 2) = \frac{17}{2}$$

This is in the form of Cauchy linear differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Put $x = e^z$ and substituting,

$$x^2\frac{d^2y}{dx^2} = D(D-1)y$$

and $$x\frac{dy}{dx} = Dy$$

Equation (i) can be written as

$$D(D-1)y - Dy - 3y = 0$$
$$[D^2 - D - D - 3]y = 0$$
$$[D^2 - 2D - 3]y = 0$$

This is in form of homogenous linear differential equation

$$[f(D)]y = 0$$

The auxiliary equation is given by,

$$f(m) = 0$$
$$m^2 - 2m - 3 = 0$$
$$m^2 - 3m + m - 3 = 0$$
$$m(m-3) + (m-3) = 0$$
$$(m+1)(m-3) = 0$$
$$m_1 = -1, m_2 = 3$$

The roots are real and distinct

The complementary function is given by,

$$\text{C.F.} = C_1 e^{m_1 z} + C_2 e^{m_2 z}$$
$$\text{C.F.} = C_1 e^{-z} + C_2 e^{3z}$$

Particular integral (P.I) is 0, since it is a homogenous equation,

The complete solution is given by,

$$y = \text{C.F.} + \text{P.I.}$$
$$y = C_1 e^{-z} + C_2 e^{3z}$$

Put $z = log\, x$,

$$y = C_1 e^{-\log x} + C_2 e^{3 \log x}$$
$$y = C_1 e^{\log\left(\frac{1}{x}\right)} + C_2 e^{\log x^3}$$
$$y = C_1 \frac{1}{x} + C_2 x^3 \qquad \ldots\text{(ii)}$$

Using Boundary Conditions :

(i) When $x = 1, y = 2$

$$2 = C_1 + C_2$$

(ii) When $x = 2, y = \frac{17}{2}$

$$\frac{17}{2} = \frac{C_1}{2} + C_1(2)^3$$
$$\frac{17}{2} = \frac{C_1}{2} + 8C_2$$
$$17 = C_1 + 16C_2$$

Solving for C_1 and C_2

$$C_1 = 1 \text{ and } C_2 = 1$$

Put the value of C_1 and C_2 in equation (ii)

$$y = \frac{1}{x} + x^3$$

At $x = \frac{3}{2}$

$$y = \frac{1}{x} + x^3$$

$$y = \frac{1}{\left(\frac{3}{2}\right)} + \left(\frac{3}{2}\right)^3$$

$$y = \frac{2}{3} + \left(\frac{3}{2}\right)^3$$

$$y = 0.66 + 3.37$$

$$y = 4.03$$

Hence, the value of $y(x)$ at $x = \frac{3}{2}$ is 4.03.

Question 53

Consider the function $f(x, y, z) = x^4 + 2y^3 + z^2$. The directional derivative of the function at the point $P(-1,1,-1)$ along $(\hat{\imath} + \hat{\jmath})$, where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors in the x and y directions, respectively, rounded off to 2 decimal places, is ________.

[Engineering Mathematics, Vector Calculus] (2 Marks)

Ans. 1.39 to 1.43

Sol. Given :

$$f(x, y, z) = x^4 + 2y^3 + z^2$$

$$\vec{a} = \hat{\imath} + \hat{\jmath}$$

Gradient of function f is given by,

$$\nabla f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$$

$$\nabla f = 4x^3\hat{\imath} + 6y^2\hat{\jmath} + 2z\hat{k}$$

At point (–1, 1, –1)

$$\nabla f|_{(-1,1,-1)} = -4\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$$

Unit normal vector is given by,

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

Directional derivative of f along $(\vec{a})$ is given by,

$$DD = \nabla \cdot f|_P \cdot \hat{a}$$

Given $\vec{a} = \hat{\imath} + \hat{\jmath}$

$$|\vec{a}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\hat{a} = \frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}$$

$$DD = (-4\hat{\imath} + 6\hat{\jmath} - 2\hat{k}) \cdot \left(\frac{\hat{\imath}}{\sqrt{2}} + \frac{\hat{\jmath}}{\sqrt{2}}\right)$$

$DD = \frac{-4}{\sqrt{2}} + \frac{6}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} = 1.414$

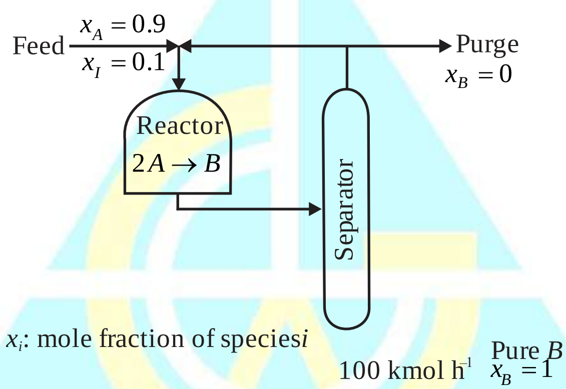
Hence, the directional derivative of the function at the point $P(-1,1,-1)$ along $(\hat{\imath} + \hat{\jmath})$ is 1.41

Question 54

Consider the process in the figure for manufacturing B. The feed to the process is 90 mol% A and a close-boiling inert component I. At a particular steady-state : **[Process calculation 2 marks]**

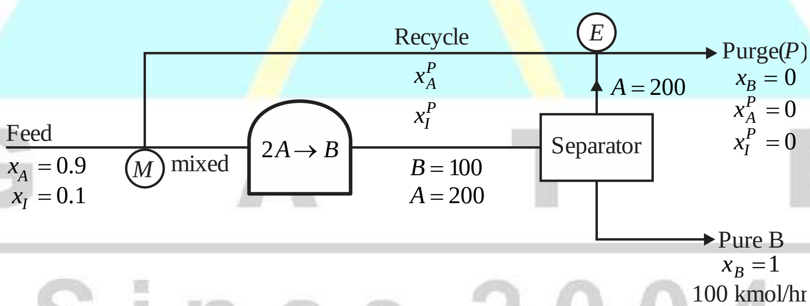
- B product rate is $100 \text{kmol} h^{-1}$
- Single-pass conversion of A in the reactor is 50%
- Recycle-to-purge stream flow ratio is 10

The flow rate of A in the purge stream in $\text{kmol} h^{-1}$, rounded off to 1 decimal place, is _______.



Ans. 18.1 to 18.3

Sol. Given:



Applying overall material balance

$$F = P + 100 \qquad \ldots(i)$$

Applying inert balance

$$F x_I^F = P x_I^P$$

$$F \times 0.1 = P x_I^P \qquad \ldots(ii)$$

Given $\frac{R}{P} = 10$

Single pass Conversion = 0.5

∵ 100 Kmol/hr B react with 200 Kmol/hr A

But SPC (Single pass Conversion) = 0.5

∴ A mole in feed $= \frac{200}{0.5} = 400\frac{\text{Kmol}}{\text{hr}}$

At point m, applying A balance

$F \times 0.9 + 10P(1 - x_I^P) = 400$

$0.9F + 10P - 10Px_I^P = 400$

$0.9F + 10P - 10 \times 0.1F = 400$

$10P - 0.1F = 400$ …(iii)

$P - F = -100$ Equation (i) × 10

$10P - 10F = -1000$ …(iv)

Equation (iii) – Equation (iv)

9.9 F = 1400

$F = \frac{1400}{9.9} = 141.414$

From Equation (i) F = P +10

∴ $P = 41.44\text{Kmol/hr}$

Applying balance on point E(on A Species)

$200 = Rx_A^P + Px_A^P$

$200 = 11Px_A^P$

$Px_A^P = 18.18$

Question 55

Methane combusts with air in a furnace as $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$. The heat of reaction $\Delta H_{rxn} = -880kJ\text{permol}CH_4$ and is assumed to be constant. The furnace is well-insulated and no other side reactions occur. All components behave s ideal gases with a constant molar heat capacity of $44Jmol^{-1}{}^0C^{-1}$. Air may be considered as 20 mol% O_2 and 80 mol% N_2. The air-fuel mixture enters the furnace at 50^0C. The methane conversion X varies with the air-to-methane mole ratio, r, as

$X = 1 - 0.1e^{-2(r-r_s)}$ with $0.9r_s \leq r \leq 1.1r_s$

where r_s is the stoichiometric air-to-methane mole ratio. For $r = 1.05r_s$, the exit flue gas temperature in $0C$, rounded off to 1 decimal place, is ________. **[Chemical reaction engineering 2 marks]**

Ans. 1719.0 to 1730.0

Sol. $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2$

Given : $\Delta H_{rxn} = -880\text{KJ/molCH}_4$

$C_{P_{CH_4}} = C_{P_{O_2}} = C_{P_{CO_2}} = C_{P_{H_2}} = 44\text{J/mol}^0C$

air → 20% O_2, 80% N_2

$X = 1 - 0.1e^{-2(r-rs)}$,

$0.9r_s \le r \le 1.1r_s$

Where r_s = air to methane mole ratio

For $r = 1.05r_s$, Exit flue gas temp = ?

Let $n_{CH_4} = 1\text{mol}$

$\therefore \quad n_{O_2} = 2mol = 0.2n_{air}$

$\therefore \quad r_s = \frac{\frac{2}{0.2}}{1} = 10$

$X = 1 - 0.1e^{-2(1.05r_s - r_s)}$

$X = 1 - 0.1e^{-2\times 0.05\times 10}$

$X = 1 - 0.1e^{-1} = 0.9632$

Applying Energy balance on Furnace

$\dot{Q} - \dot{W}_S = \sum nihi/_{exit} - \sum nihi|/_{exit} + \mathcal{E}\Delta H_{rxn}$

$\because$ Furnaces is well insulated

$\therefore \quad Q = w_s = 0$

$\therefore \quad 0 = \sum niC_{Pi}(T_{out} - T_{ref}) - \sum niC_{P_i}(T_{in} - T_{ref}) + \Delta H_{rxn}\mathcal{E}$

Let $T_{ref} = T_{out}$

$\therefore \quad 0 = 0 - \sum niC_{pi}(T_{in} - T_0) + \Delta H_{rxn}\mathcal{E}$

$\mathcal{E}$ =Extent of reaction

$\therefore \quad \sum niC_{P_i}(T_i - T_0) = \Delta H \times \mathcal{E}$

$C_{P_i} = 44\text{J/mol}^0C$

$(\sum n_i)C_{p_i}(T_{in} - T_{out}) = \Delta H_{rxn}\mathcal{E}$

$\sum n_i$ =total number of moles at input

= 1 + air moles

↑ ↑

CH_4 $r = 1.05 r_s$

$r = 1.05 \times 10 = 10.5$ air/methane

$\therefore$ $(moles)_{\text{air}} = 10.5\text{moles}$

$(moles)_{CH_4} = 1\text{mole}$

$\sum n_i = 11.5$

$\varepsilon = \frac{\text{moles of methane reacted}}{\text{moles of methane enter}} = \frac{0.9632}{1}$

$\therefore$ $44(50 - T_0) \times 11.5 = -880 \times 10^3 \times 0.9632$

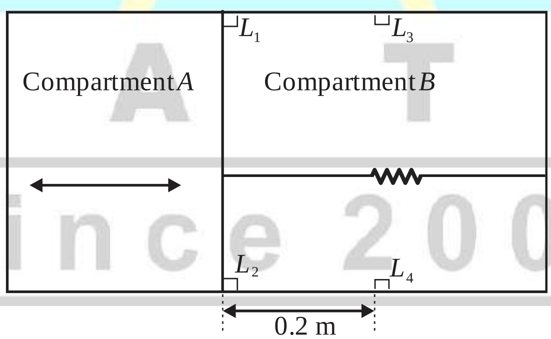
$\therefore$ $50 - T_0 = -1675.15$

$T_0 = 1725.15^0 C$

Question 56

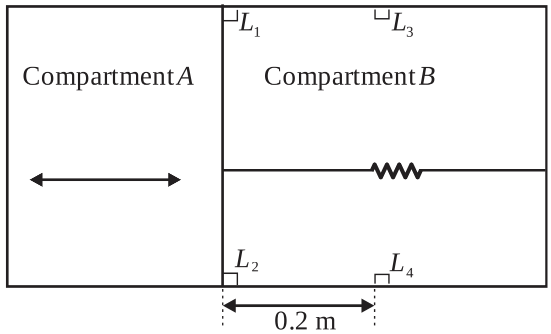
An isolated system consists of two perfectly sealed cuboidal compartments A and B separated by a movable rigid wall of cross-sectional area $0.1 m^2$ as shown in the figure. Initially, the movable wall is held in place by latches L_1 and L_2 such that the volume of compartment A is $0.1 m^3$. Compartment Acontains a monoatomic ideal gas at 5 bar and 400 K. compartment B is perfectly evacuated and contains a massless Hookean spring of force constant $0.3 Nm^{-1}$ at its equilibrium length (stored elastic energy is zero). The latches L_1 and $L_2$are released, the wall moves to the right by 0.2 m, where it is held at the new position by latches L_3 and L_4. Assume all the walls and latches are massless. The final equilibrium temperature, in K, of the gas in compartment A, rounded off to 1 decimal place, is _______.

[Thermodynamics 2 marks]



Ans. **399.9**

Sol.



$\Delta \cup = \Delta Q + \Delta W$

For I solated system, $\Delta Q = 0$

$\Delta \cup = \dfrac{0.3}{0.2} nR\Delta T$ [For Monoatomic Ideal gas]

$PV = nRT$

$n = \dfrac{5\times 10^5 \times 0.1}{R\times 400}$

As compartment moves right hence spring will exert force towards left hence work is done on the system.

$F = -Rx$

$P = \dfrac{F}{A}$

$P = \dfrac{-Rx}{A}$

$P = \dfrac{-0.3x}{0.1}$

$W = \int pdv$

As dv = Adx

Thus

$W = \int_0^{0.2} -3x \times 0.1 dx$

$W = -\dfrac{0.3}{2}[x]_0^{0.2}$

$$W = -\frac{0.3}{2} \times (0.2)^2$$

Thus

$$\frac{3}{2} \times \frac{R \times 5 \times 10^5 \times 0.1}{R \times 400}(T_f - 400) = \frac{-0.3}{2} \times (0.2)^2$$

$$T_f = 399.9$$

Question 57

Ethylene obeys the truncated virial equation-of-state

$$\frac{PV}{RT} = 1 + \frac{BP}{RT}$$

Where P is the pressure, V is the molar volume, T is the absolute temperature and B is the second virial coefficient. The universal gas constant $R = 83.14 \text{bar} m^3 \text{mol}^{-1} K^{-1}$. At 340 K, the slope of the compressibility factor vs. pressure curve is $-3.538 \times 10^{-3} \text{bar}^{-1}$. Let G^R denote the molar residual Gibbs free energy. At these conditions, the value of $\left(\frac{\partial G^R}{\partial P}\right)_T$, in $\text{cm}^3\text{mol}^{-1}$, rounded off to 1 decimal place, is ___________. [Thermodynamics 2 marks]

Ans. -101.0 to -99.0

Sol. $\frac{PV}{RT} = 1 + \frac{BP}{RT}$

Given: R = 83.14 bar $\text{cm}^3/\text{mol}K$

At 340K, $\frac{\partial z}{\partial p} = -3.53 \times 10^{-3} \text{bar}^{-1}$

$$\left.\frac{\partial G^R}{\partial P}\right|_T = ?$$

$\because \quad z = \frac{PV}{RT} = 1 + \frac{BP}{RT}$

$\therefore \quad \text{Slope} = \left.\frac{\partial z}{\partial p}\right|_T = \frac{B}{RT} = -3.538 \times 10^{-3}$

$$\frac{B}{R \times 340} = -3.538 \times 10^{-3} \text{bar}^{-1}$$

$$B = -3.538 \times 10^{-3} \times 340 \text{bar}^{-1} \times 83.14$$

$$B = -100 \text{cm}^3/\text{mol}$$

$\because \quad \frac{G^R}{RT} = \int_0^P (z-1)\frac{dp}{P}$

$$\left\{z - 1 = \frac{BP}{RT}\right\}$$

$$\frac{G^R}{RT} = \frac{BP}{RT}$$

$$\left.\frac{G^R}{\partial P}\right|_T = B = -100\ \text{cm}^3/\text{mol}$$

Question 58

A metallic spherical particle of density $7001\text{kg}m^{-3}$ and diameter $1mm$ is settling steadily due to gravity in a stagnant gas of density $1kgm^{-3}$ and viscosity $10^{-5}\text{kg}m^{-1}s^{-1}$. Take $g = 9.8ms^{-2}$. Assume that the settling occurs in the regime where the drag coefficient C_D is independent of the Reynolds number, and equals 0.44. The terminal settling velocity of the particle, in ms^{-1}, rounded off to 2 decimal places, is ________. **[Mechanical operation 2 marks]**

Ans. 14.30 to 14.60

Sol. Given:

Spherical Particle density $(\rho_p) = 7001\text{kg/m}^3$

Spherical particle diameter $(dp) = 10^{-3}m$

Fluid (gas) density $(\rho_f) = 1\ kg/m^3$.

Gas viscosity $(\mu) = 10^{-5}\text{kg/m.sec}$

C_D is independent on Reynold number = 0.44

Which means settling occur in Newton's Law region

Terminal settling velocity in newton law regime

$$u_t^2 = \frac{3.03g(\rho_p - \rho_F)dp}{\rho_F}$$

Or

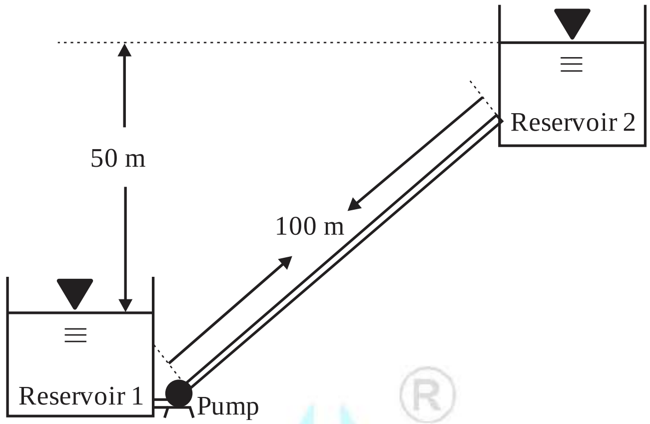
$$u_t = 1.75\sqrt{\left(\frac{\rho_p - \rho_f}{\rho_F}\right) \times gdp}$$

So $u_t^2 = 3.03 \times 9.8\frac{(7001-1)}{1} \times 10^{-3}$

$$u_t = 14.417\ \text{m/s}$$

Question 59

Water of density $1000\text{kg}m^{-3}$ is pumped at a volumetric flow rate of $3.14 \times 10^{-2}m^3s^{-1}$, through a pipe of inner diameter 10 cm and length 100 m, from a large Reservoir 1 to another large Reservoir 2 at a height 50 m above Reservoir 1, as shown in the figure. The flow in the pipe is in the turbulent regime with a Darcy friction factor $f = 0.06$, and a kinetic energy correction factor $\alpha = 1$. Take $g = 9.8ms^{-2}$. If all minor losses are negligible, and the pump efficiency is 100%, the pump power, in kW, rounded off to 2 decimal places, is __________. [Fluid mechanics 2 marks]



Ans. 30.40 to 30.50

Sol. Given :

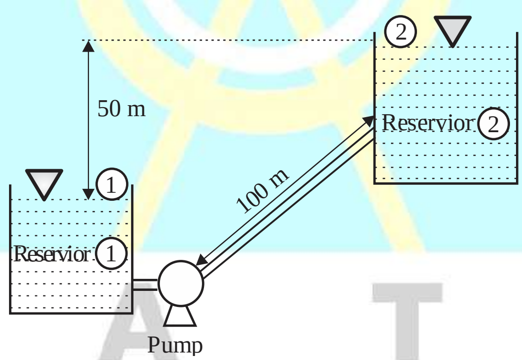
Water density $(\rho_w) = 1000 \text{ kg/m}^3$

Volumetric Flow rate $(Q) = 3.14 \times 10^{-2} m^3/\text{sec}$

Pipe inner diameter $(d_i) = 10\, cm = 0.1 m$

Pipe length (L) =100 m

Since two Large reservoirs are there, so velocity at top of reservoir will be negligible



Applying Bernoulli equation at point (i) and (ii)

$$\frac{P_1}{\rho_g} + \frac{\alpha_1 v_1^2}{2g} + h_1 + \eta h_p = \frac{P_2}{\rho_g} + \frac{\alpha_2 V_1^2}{2_g} + h_2 + h_f$$

$P_1 = P_2 = 1\text{atm}$

$\alpha_1 = \alpha_2 = 1$ $\quad\quad$ $\eta = 1$ (given)

$v_1 = v_2 = 0$

$h_1 = 0, h_2 = 50m$

$h_f = \frac{fLv^2}{2gd}$, $\quad$ f = Darcy friction Factor

$v = \frac{Q}{A} = \frac{3.14\times10^{-2}}{\frac{\pi}{4}\times(0.1)^2}$

$v = 4\text{m/s}$

∵ L = 100 given

f = 0.06 given

∴ $h_F = \frac{0.06\times100\times(4)^2}{2\times0.1\times9.8} = 48.97$

From Bernoulli equation:

$\frac{P}{\rho g} + 0 + 0 + 1 \times h_w = \frac{P}{\rho_g} + 0 + 50 + 48.97$

$h_w = 98.9795\ m$

Head developed by pump

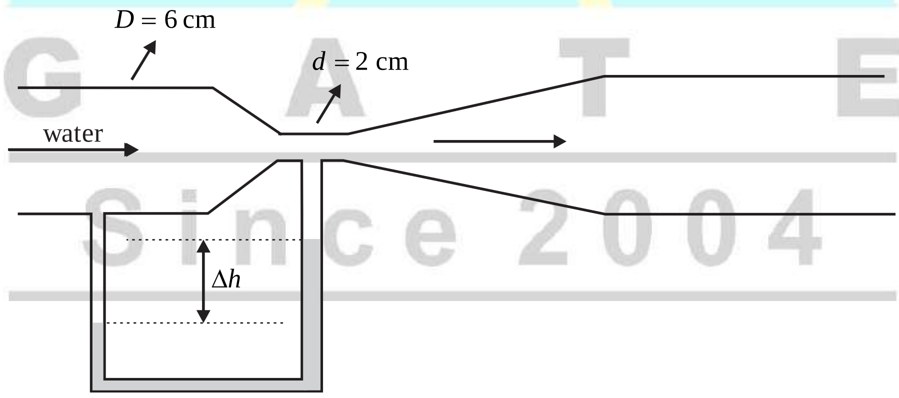
Pump power $P = \rho Q g h_w$

$P = 10^3 \times 3.14 \times 10^{-2} \times 9.8 \times 98.97$

P = 30.45 kW

Question 60

A Venturi meter with a throat diameter $d = 2cm$ measures the flow rate in a pipe of diameter $D = 6cm$, as shown in the figure. A U-tube manometer is connected to measure the pressure drop. Assume the discharge coefficient is independent of the Reynolds number and geometric ratios. If the volumetric flow rate through the pipe is doubled $Q_2 = 2Q_1$, the corresponding ratio of the manometer readings $\Delta h_2/\Delta h_1$, rounded off to the nearest integer, is ________. [Fluid mechanics 2 marks]



Ans. 4 to 4

Sol. Given :

Diameter of pipe = 6cm

Diameter of throat = 2cm

$$Q_2 = 2Q_1, \quad \frac{\Delta h_2}{\Delta h_1} = ?$$

Discharge through venturi meter :-

$$Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{\frac{2\Delta P}{\rho}}$$

Where, A_1 = Area of pipe

A_2 = Area of throat

As,

$$Q \propto \sqrt{\Delta P}$$

$$Q \propto \sqrt{\Delta h}$$

$$\frac{Q_2}{Q_1} = \sqrt{\frac{\Delta h_2}{\Delta h_1}}$$

$$(2)^2 = \frac{\Delta h_2}{\Delta h_1}$$

$$\frac{\Delta h_2}{\Delta h_1} = 4$$

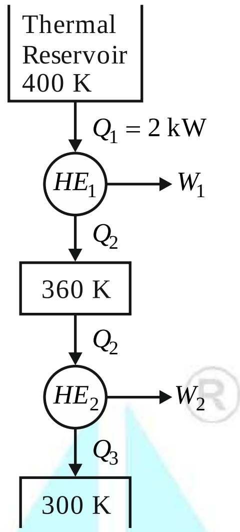
Question 61

Heat is available at a rate of 2 kW from a thermal reservoir at 400 K. A two-stage process harnesses this heat to produce power. Stages 1 and 2 reject heat at 360 K and 300 K, respectively. Stage 2 is driven by the heat rejected by Stage 1. If the overall process efficiency is 50% of the corresponding Carnot efficiency, the power delivered by the process, in kW, rounded off to 2 decimal places, is ________.

[Thermodynamics 2 marks]

Ans. 0.24 to 0.26

Sol.



$Q_1 = W_1 + Q_2$ ……(1)

$Q_2 = W_2 + Q_3$ ……(2)

Given :

Overall process efficiency = 50% corresponding carnot efficiency.

$\eta_1 = 1 - \frac{360}{400} = \frac{40}{400}$

$\eta_{P_1} = 0.5\eta_1 = \frac{W_1}{Q_1}$ $\left(\eta_C = 1 - \frac{T_L}{T_H}\right)$

$0.5 \times \frac{40}{400} = \frac{W_1}{2}$ $\left[W_1 = \frac{1}{10}\right]$

From equation (1),

$Q_1 = W_1 + Q_2$

$Q_2 = 2 - 0.1 = 1.9$

$\eta_{p_2} = 0.5\eta_2$

$0.5\left(1 - \frac{300}{360}\right) = \frac{W_2}{Q_2}$

$W_2 = 0.5 \times \frac{60}{360} \times 1.9 = 0.1583$

∴ Power produced $= W_1 + W_2$

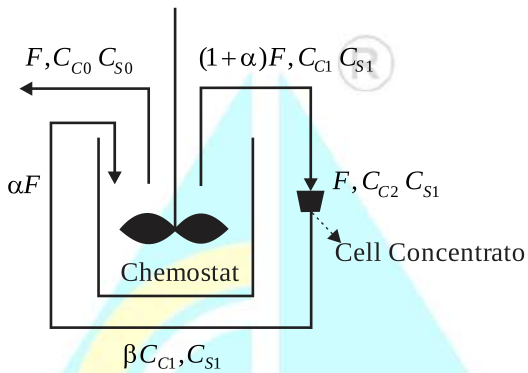
$= 0.1 + 0.1583$

$= 0.2583$

≈ 0.26

Question 62

A chemostat with cell recycle is shown in the figure. The feed flow rate and culture volume are $F = 75Lh^{-1}$ and $V = 200L$, respectively. The glucose concentration in the feed $C_{S0} = 15gL^{-1}$. Assume Monod kinetics with specific cell growth rate $\mu_g = \frac{1}{C_C}\frac{dC_C}{dt} = \frac{\mu_m C_S}{K_S + C_S}$, where $\mu_m = 0.25h^{-1}$ and $K_S = 1gL^{-1}$. Assume maintenance and death rates to be zero, input feed to sterile ($C_{C0} = 0$) and steady-state operation. The glucose concentration in the recycle stream, C_{S1}, in gL^{-1}, rounded off to 1 decimal place, is _________. **[Chemical reaction engineering 2 marks]**



F: Volumetric flow rate

C_C: Cell concentration

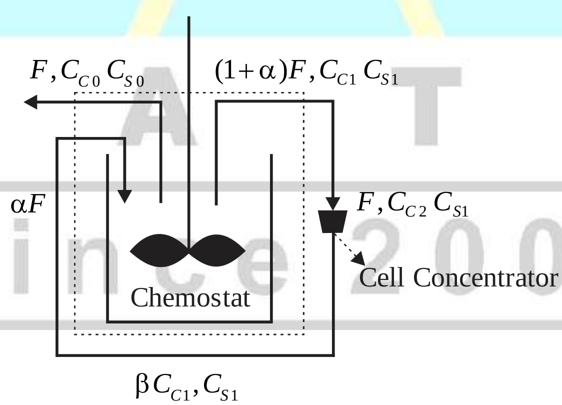
C_S: Substrate (glucose) concentration

α : Recycle ratio ($\alpha = 0.5$)

β : Concentration factor ($\beta = 2.0$)

Ans. 2.9 to 3.1

Sol.



Given :

$F = 75\ \text{Lh}^{-1}$ $\quad$ $\mu_m = 0.25\ \text{h}^{-1}$

$C_{C_0} = 0\text{g/L}$ $\quad$ $K_S = 1g\ \text{L}^{-1}$

$C_{S0} = 15\ \text{g/L}$

$$\mu_g = \frac{1}{C_C}\frac{dC_C}{dt} = \frac{\mu_m C_S}{K_S + C_S}$$

$\therefore \quad \frac{dC_C}{dt} = \mu_g C_C = \frac{\mu_m C_S C_C}{K_S + C_S}$ ….. monod equation

where, $\mu_m = 0.25\ \text{h}^{-1}$

$K_s = 1$ g/L

Material balance on cell biomass (around dotted line boundary)

$C_C \rightarrow$ cell concentration

Step I :

Rate of accumulation = Input – Output + Generation

$$\frac{VdC_{C_1}}{dt} = \left[\underbrace{\underbrace{FC_{C_0}}_{\text{From Feed}} + (\alpha F)\underbrace{(\beta C C_1)}_{\text{From recycle}}}_{\text{Input}}\right] - \boxed{\underset{0}{\text{consumption}}} - \left[\underbrace{(1+\alpha)FC_{C_1}}_{\text{cell output from chemostat}}\right] + \left[\underbrace{V\mu_g C C_1}_{\text{cell generation (monodkinetics)}}\right]$$

Step II :

At Steady state $\frac{dC_{C_1}}{dt} = 0$

Given, $C_{C_0} = 0$ (Input feed is sterile i.e., no active cell presents in feed)

$$V\overset{0}{\boxed{\frac{dC_{C_1}}{dt}}} = \left[F\overset{0}{\boxed{C_{C_0}}} + (\alpha F)(\beta C_{C_1})\right] - \left[(1+\alpha)FC_{C_1}\right] + \left[V\mu_g C_{C_1}\right]$$

$\therefore \quad 0 = \alpha F\beta C_{C_1} - (1+\alpha)FC_{C_1} + V\mu_g C_{C_1}$

$0 = C_{C_1}\left[\alpha F\beta - (1+\alpha)F + V\mu_g\right]$

where, C_{C_1} is steady state cell concentration in chemostate and recycle stream

$\therefore \quad \alpha F\beta - (1+\alpha)F + V\mu_g = 0$

$\mu_g = \frac{F}{V}(1+\alpha-\alpha\beta) = D(1+\alpha-\alpha\beta)$

where, $D = \frac{F}{V}$ =dilution ratio

Given $\alpha = 0.5 \quad \beta = 0.5$

$F = 75$L/H and $V = 200$L

$\therefore \quad \mu_g = \frac{75}{200}(1+0.5-0.5\times 2.0)$

$\mu_g = \frac{75}{200}(0.5) = 0.1875\,\text{h}^{-1}$

Step III :

Given monod equation

$$\mu_g = \frac{\mu_m C_{S_1}}{1+C_{S_1}}$$

$\mu_m = 0.25\,\text{h}^{-1}$

$k_s = 1\,\text{g/l}$

$$0.1875 = \frac{0.25C_{S_1}}{1+C_{S_1}}$$

$0.1875(1+C_{S_1}) = 0.25C_{S_1}$

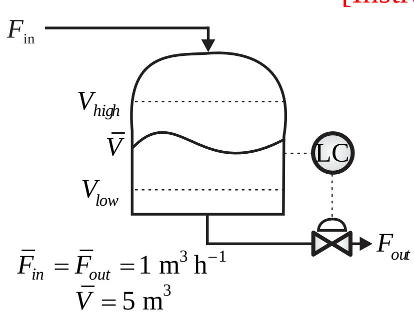
$C_{S_1}(0.25-0.1875) = 0.1875$

$$\therefore \quad C_{S_1} = \frac{0.1875}{0.0625} = 3\,\text{g/l}$$

Question 63

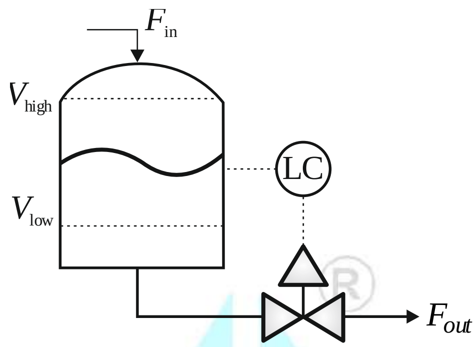
Consider the surge drum in the figure. Initially the system is at steady-state with a hold-up $\bar{V} = 5m^3$, which is 50% of full tank capacity, V_{full}, and volumetric flow rates $\bar{F}_{in} = \bar{F}_{out} = 1m^3h^{-1}$. The high hold-up alarm limit $V_{high} = 0.8V_{full}$ while the low hold-up alarm $V_{low} = 0.2V_{full}$. A proportional (P –only) controller manipulates the outflow to regulate the hold-up V as $F_{out} = K_C(V-\bar{V}) + \bar{F}_{out}$. At $t = 0$, F_{in} increases as a step from $1m^3h^{-1}$ to $2m^3h^{-1}$. Assume linear control valves and instantaneous valve dynamics. Let K_C^{min} be the minimum controller gain that ensures V never exceeds V_{high}. The value of K_C^{min}, in h^{-1}, rounded off to 2 decimal places, is ________.

[Instrumentation and process control 2 marks]



Ans. 0.33 (0.32 to 0.34)

Sol.



Given :

Steady state hold up, $V_S = 5m^3$

0.5 (tank capacity) $= V_S$

$\therefore \quad V_{Full\ Capacity} = \frac{V_S}{0.5} = \frac{5}{0.5} = 10m^3$

$\overline{F_{in}} = \overline{F_{out}} = \frac{1m^3}{h}$

$V_{low} = 0.2V_{\text{Full}}$ (Low holdup alarm)

$V_{high} = 0.8V_{\text{Full}}$ (High holdup alarm)

$\therefore \quad V_{low} = 2m^3$

$V_{high} = 8m^3$

$F_{out} = K_C(V - \bar{V}) + F_{out}$

Step changes of magnitude $1m^3$/hr (Given)

Value of K_C^{min} when $V \leq V_{high}$

Apply material balance,

$\rho F_{in} - \rho F_{out} = \frac{d(\rho v)}{dt}$

$F_{in} - F_{out} = \frac{dv}{dt}$ .....(1)

At steady state,

$F_{in,s} - F_{out,s} = \frac{dV_s}{dt}$ .....(2)

Equation (1) – Equation (2)

$$(F_{in} - F_{in,s}) - (F_{out} - F_{out,s}) = \frac{d}{dt}(V - V_s)$$

$$\overline{F_{in}}(S) - K_C \bar{V}(S) = S\bar{V}(S)$$

$$\overline{F_{in}}(S) = (S + K_C)\bar{V}(S)$$

$$\frac{\bar{V}(S)}{\overline{F_{in}}(S)} = \frac{1}{S+K_C} = \frac{\frac{1}{K_C}}{\left(\frac{1}{K_C}\right)S+1}$$

$$\overline{F_{in}}(S) = \frac{M}{S} = \frac{1}{S}$$ (M = magnitude of step input change)

$$\overline{F_{in}} = \frac{M}{S} = \frac{1}{S}$$

∴ $$\bar{V}(t) = \frac{1}{K_C}(1 - e^{-K_C t})$$

$$V(t) - V_{SS} = \frac{1}{K_C}(1 - e^{-K_C t})$$

When, $t \to \infty$

$$V(t \to \infty) = V_S + \frac{1}{K_C}$$ $(V \leq V_{high})$

∴ $$V = V_{high}$$ (Boundary condition)

$$V_S + \frac{1}{K_C} = 8$$

$$5 + \frac{1}{K_C} = 8$$

Kc = 0.33

Question 64

A PD controller with transfer function G_c is used to stabilize an open-loop unstable process with transfer function G_p, where

$$G_c = K_c \frac{\tau_D s+1}{\left(\frac{\tau_D}{20}\right)s+1}, G_p = \frac{1}{(s-1)(10s+1)}$$

and time is in minutes. From the necessary conditions for closed-loop stability, the maximum feasible value of τ_D, in minutes, rounded off to 1 decimal place, is ___________.

[Instrumentation and process control 2 marks]

Ans. 22.2 (22.1 to 22.3)

Sol. Given:

$$G_C = \frac{K_C(\tau_D S+1)}{\left(\frac{\tau_D}{20}S+1\right)}, \quad G_P = \frac{1}{(S-1)(10S+1)}$$

Using Routh stability criteria

First write characteristic equation

$$1+G_{OL}=0$$

$$\therefore \quad 1+G_C G_P=0$$

$$1+\frac{20K_C(\tau_D S+1)}{(\tau_D S+20)}\times\frac{1}{(10s^2-9s-1)}=0$$

$$(10\tau_D)S^3+S^2(200-9\tau_D)+S(20K_C\tau_D-\tau_D-180)+(20K_C-20)=0$$

$$\begin{array}{c|cc} S^3 & 10\tau_D & 20K_C\tau_D-\tau_D-180 \\ S^2 & 200-9\tau_D & 20K_C-20 \\ S^1 & b_1 & b_2 \\ S^0 & c_1 & c_2 \end{array}$$

$$b_1=\frac{10\tau_D(20K_C-20)-(200-9\tau_D)\times(20K_C\tau_D-\tau_D-180)}{200-9\tau_D}$$

For closed loop stability, first column should be +ve.

$$\therefore \quad 200-9\tau_D>0$$

$$\tau_D<\frac{200}{9}$$

$$\tau_D<22.22$$

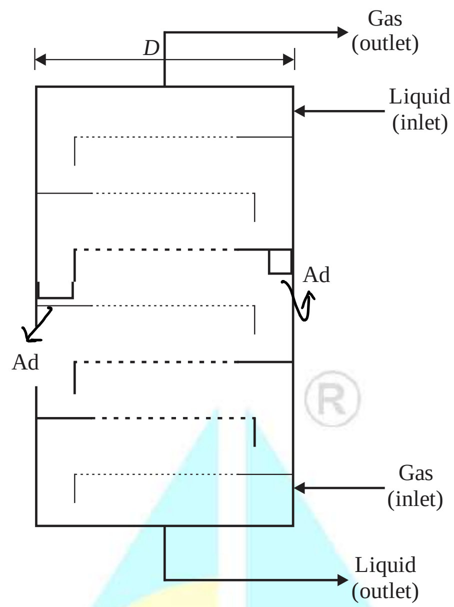
Maximum feasible value of $\tau_D = 22.22$

Question 65

Consider a tray-column of diameter 120 cm. Each downcomer has a cross-sectional area of 575cm^2. For a tray, the percentage column cross-sectional area not available for vapor flow due to the downcomers, rounded off to 1 decimal place, is ________. [Mass transfer 2 marks]

Ans. 10.0 to 10.3

Sol. Given:



A_d =Down comer area

A_T =Column area

$A_T = \frac{\pi}{4} D_T^2 = \frac{\pi}{4}(1.2)^2 = 11309.73\text{cm}^2$

$A_d = \frac{\pi}{4} D_d^2 = 575\text{cm}^2$

∵ Down comer, only liquid allow to flow, NO VAPOR flow in downcomer.

∴ $\frac{A_d}{A_T} = \frac{\text{downcomer area available onTray}}{\text{Tray column area}}$

$= \frac{2\times575}{11309.73} \times 100$

$= 10.16$

❖❖❖❖