GATE 2023 [Afternoon Session]
Civil Engineering

General Aptitude Part

Q.1 to Q.5 Carry One Mark Each

Question 1 **[Verbal Ability : Sentence Completion]**

The line ran _____ the page, right through the centre, and divided the page into two.

(A) across (B) of

(C) between (D) about

Ans. (A)

Sol. Given :

The line ran **<u>across</u>** the page, right through the centre, and divided the page into two.

Hence, the correct option is (A).

Question 2 **[Verbal Ability : Synonyms & Antonyms]**

Kind: _____ : : Often : Seldom

(By word meaning)

(A) Cruel (B) Variety

(C) Type (D) Kindred

Ans. (A)

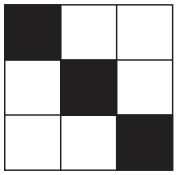
Sol. Given :

Often is related to seldom in special manner as they are opposite to each other in meaning. In the same way of relationship kind will be related to cruel as they are opposite to each other in meaning.

Hence, the correct option is (A).

Question 3 **[Numerical Ability : Permutation & Combination]**

In how many ways can cells in a 3×3 grid be shaded, such that each row and each column have exactly one shaded cell? An example of one valid shading is shown.



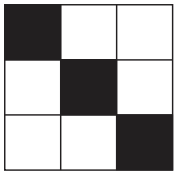
(A) 2 (B) 9

(C) 3 (D) 6

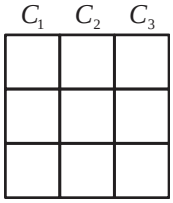
Ans. (D)

Sol. Given :

Cells of 3×3 grid are to be shaded such that each row and each column have exactly one shaded cell. An example of one valid shading is given below,



On the basis of the example, let's take a 3×3 grid.



In column C_1 there are 3 ways to shaded a cell and with reference to the condition, for column C_2 there will be 2 ways to shaded a cell and in column C_3 there will be only 1 way to shaded a cell.

Hence, total number of ways will be $3\times2\times1=6$.

Such that each row and each column have exactly one shaded cell.

Hence, the correct option is (D).

Question 4 **[Numerical Ability : Permutation & Combination]**

There are 4 red, 5 green, and 6 blue balls inside a box. If N number of balls are picked simultaneously, what is the smallest value of N that guarantees there will be at least two balls of the same colour?

One cannot see the colour of the balls until they are picked.

(A) 4 (B) 15

(C) 5 (D) 2

Ans. (A)

Sol. **Given :** 4 Red, 5 Green and 6 Blue

We select three balls in worst case

1 Red, 1 Green and 1 Blue

If we select fourth ball then we found two balls are of same colour.

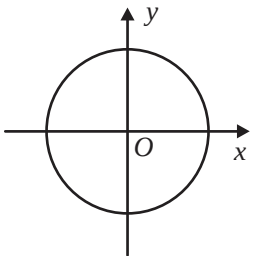
Question 5 **[Logical Reasoning : Data Interpretation]**

Consider a circle with its centre at the origin (O), as shown. Two operations are allowed on the circle.

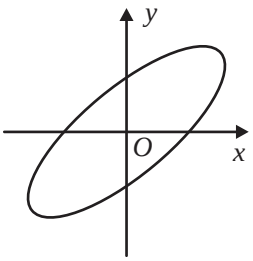
Operation 1: Scale independently along the x and y axes.

Operation 2: Rotation in any direction about the origin.

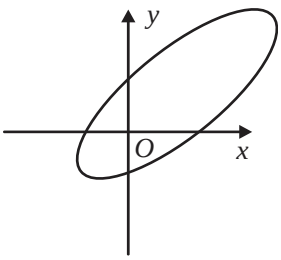
Which figure among the options can be achieved through a combination of these two operations on the given circle?

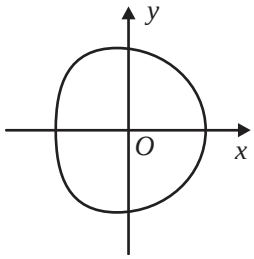


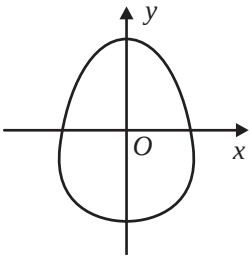
(A)



(B)



(C) 

(D) 

Ans. (A)

Q.6 to Q.10 Carry Two Marks Each

Question 6

Elvesland is a country that has peculiar beliefs and practices. They express almost all their emotions by gifting flowers. For instance, if anyone gifts a white flower to someone, then it is always taken to be a declaration of one's love for that person. In a similar manner, the gifting of a yellow flower to someone often means that one is angry with that person.

Based only on the information provided above, which one of the following sets of statement(s) can be logically inferred with *certainty*?

(i) In Elvesland, one always declares one's love by gifting a white flower.

(ii) In Elvesland, all emotions are declared by gifting flowers.

(iii) In Elvesland, sometimes one expresses one's anger by gifting a flower that is not yellow.

(iv) In Elvesland, sometimes one expresses one's love by gifting a white flower.

(A) only (ii) (B) (i), (ii) and (iii)

(C) (i), (iii) and (iv) (D) only (iv)

Ans. (D)

Sol. **Given :**

Elvesland is a country that has peculiar beliefs and practices. They express almost all their emotions by gifting flowers. For instance, if anyone gifts a white flower to someone, then it is always taken to be a declaration of one's love for that person. In a similar manner, the gifting of a yellow flower to someone often means that one is angry with that person.

(i) In Elvesland, one always declares one's love by gifting a white flower cannot be inferred with certainty, as gifting white flower is sign of declaration of ones love for the person. It is not declares by a person.

(ii) In Elvesland, all emotions are declared by gifting flowers cannot be inferred with certainty, as they express "almost all" their emotions by gifting flowers. Therefore "All" (definite) cannot be true.

(iii) In Elvesland, sometimes one expresses one's anger by gifting a flower that is not yellow is definitely cannot inferred as the gifting of a yellow flower to someone often means that one is angry with that person.

(iv) In Elvesland, sometimes one expresses one's love by gifting a white flower can be inferred with certainty, as if anyone gifts a white flower to someone, then it is always taken to be a declaration of one's love for that person.

Hence, the correct option is (D).

Question 7 **[Numerical Ability : Permutation & Combination]**

Three husband-wife pairs are to be seated at a circular table that has six identical chairs. Seating arrangements are defined only by the relative position of the people. How many seating arrangements are possible such that every husband sits next to his wife?

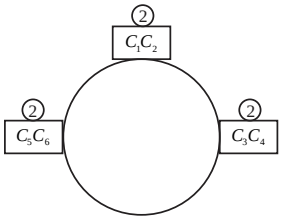
(A) 16 (B) 4

(C) 120 (D) 720

Ans. (A)

Sol. **Given :** $n = 3$

$(n-1)! = 2!$



$= 2 \times 2 \times 2 \times 2 = 16$

Hence, the correct option is (A).

Question 8

Based only on the following passage, which one of the options can be inferred with *certainty*?

When the congregation sang together, Apenyo would also join, though her little screams were not quite audible because of the group singing. But whenever there was a special number, trouble would begin; Apenyo would try singing along, much to the embarrassment of her mother. After two or three such mortifying Sunday evenings, the mother stopped going to church altogether until Apenyo became older and learnt to behave.

At home too, Apenyo never kept quiet; she hummed or made up silly songs to sing by herself, which annoyed her mother at times but most often made her become pensive. She was by now convinced that her daughter had inherited her love of singing from her father who had died unexpectedly away from home.

[Excerpt from *These Hills Called Home* by Temsula Ao]

(A) The mother was embarrassed about her daughter's singing at home.

(B) The mother's feelings about her daughter's singing at home were only of annoyance.

(C) The mother was not sure if Apenyo had inherited her love of singing from her father.

(D) When Apenyo hummed at home, her mother tended to become thoughtful

Ans. (D)

Sol. **Given :**

When the congregation sang together, Apenyo would also join, though her little screams were not quite audible because of the group singing. But whenever there was a special number, trouble would begin; Apenyo would try singing along, much to the embarrassment of her mother. After two or three such mortifying Sunday evenings, the mother stopped going to church altogether until Apenyo became older and learnt to behave.

At home too, Apenyo never kept quiet; she hummed or made up silly songs to sing by herself, which annoyed her mother at times but most often made her become pensive. She was by now convinced that her daughter had inherited her love of singing from her father who had died unexpectedly away from home.

Option (A) cannot be inferred as her mother was embarrassed about her daughter's singing at church.

Option (B) cannot be inferred as the mothers feelings about her daughter's singing at home were not only of annoyance but also she become pensive.

Option (C) cannot be inferred as she was convinced that her daughter had inherited her love of singing from her father.

Option (D) can be inferred as when open go hummed at home, her mother tended to become thought-full and then convinced.

Hence, the correct option is (D).

Question 9

If x satisfies the equation $4^{8^x} = 256$, then x is equal to _____.

(A) $\frac{1}{2}$ (B) $\log_{16} 8$

(C) $\frac{2}{3}$ (D) $\log_4 8$

Ans. (C)

Sol. $4^{8x} = 256$

$\Rightarrow 4^{8x} = 4^4$

$\Rightarrow 8^x = 4$

$\Rightarrow \left(2^3\right)^x = 2^2$

So, $x = \frac{2}{3}$

Hence, the correct option is (C).

Question 10 **[Logical Reasoning : Miscellaneous]**

Consider a spherical globe rotating about an axis passing through its poles. There are three points P, Q, and R situated respectively on the equator, the north pole, and midway between the equator and the north pole in the northern hemisphere. Let P, Q, and R move with speeds v_P, v_Q, and v_R, respectively.

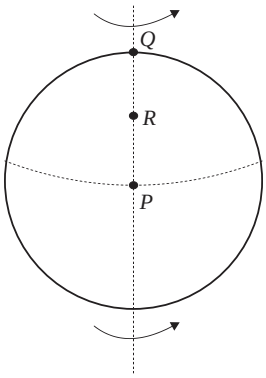
Which one of the following options is CORRECT?

(A) $v_P < v_R < v_Q$ (B) $v_P < v_Q < v_R$

(C) $v_P > v_R > v_Q$ (D) $v_P = v_R \neq v_Q$

Ans. (C)

Sol.



Spherical globe

We know velocity $(V) = \omega r$, where $\omega = \text{Constant}$

$\therefore \quad V \propto r$

Hence, more is the distance from the axis of rotation more will be the velocity.

So, $V_P > V_R > V_Q$

Hence, the correct option is (C).

Technical Part

Q.11 to Q.35 Carry One Mark Each

Question 11 **[Engineering Mathematics : Vector Calculus]**

Let ϕ be a scalar field, and $\boldsymbol{u}$ be a vector field. Which of the following identities is true for div($\phi\boldsymbol{u}$)?

(A) div($\phi\boldsymbol{u}$) = ϕdiv($\boldsymbol{u}$) + $\boldsymbol{u}$ · grad(ϕ)
(B) div($\phi\boldsymbol{u}$) = ϕdiv($\boldsymbol{u}$) + $\boldsymbol{u}$ × grad(ϕ)
(C) div($\phi\boldsymbol{u}$) = ϕgrad($\boldsymbol{u}$) + $\boldsymbol{u}$ · grad(ϕ)
(D) div($\phi\boldsymbol{u}$) = ϕgrad($\boldsymbol{u}$) + $\boldsymbol{u}$ × grad(ϕ)

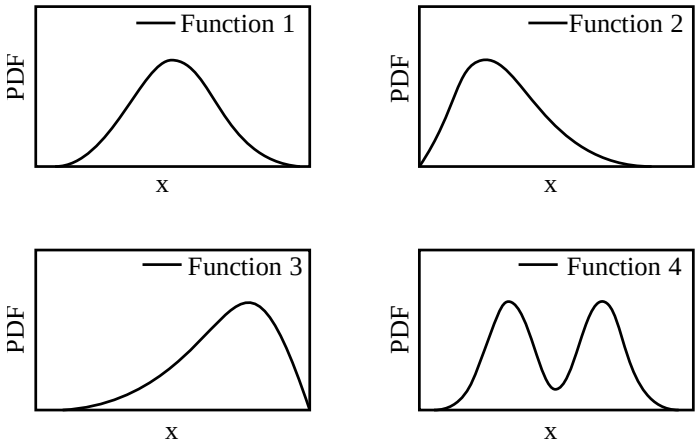
Ans. (A)

Sol. $div(\phi u) = \phi\, div(u) + u.grad(\phi)$

Hence, the correct option is (A).

Question 12

Which of the following probability distribution functions (PDFs) has the mean greater than the median?



(A) Function 1 (B) Function 2
(C) Function 3 (D) Function 4

Ans. (B)

Question 13 **[Engineering Mathematics : Probability & Statistics]**

A remote village has exactly 1000 vehicles with sequential registration numbers starting from 1000. Out of the total vehicles, 30% are without pollution clearance certificate. Further, even- and odd-numbered vehicles are operated on even- and odd-numbered dates, respectively.

If 100 vehicles are chosen at random on an even-numbered date, the number of vehicles expected without pollution clearance certificate is_____.

(A) 15 (B) 30
(C) 50 (D) 70

Ans. (B)

Sol. **Given :** 30% of total vehicles are without pollution clearance certificate.

Randomly choosen vehicle = 100

Probability of selecting even vehicles on even numbered dates =1

$\therefore$ The number of vehicles expected without pollution clearance certificate = $1000\times1\times0.3=30$

Hence, the correct option is (B).

Question 14 **[Strength of Materials : Torsion of Shafts and Pressure Vessels]**

A circular solid shaft of span L = 5 m is fixed at one end and free at the other end. A torque T = 100 kN.m is applied at the free end. The shear modulus and polar moment of inertia of the section are denoted as G and J, respectively. The torsional rigidity GJ is $50{,}000\ \text{kN.m}^2/\text{rad}$. The following are reported for this shaft:

Statement i) The rotation at the free end is 0.01 rad

Statement ii) The torsional strain energy is 1.0 kN.m

With reference to the above statements, which of the following is true?

(A) Both the statements are correct
(B) Statement i) is correct, but Statement ii) is wrong
(C) Statement i) is wrong, but Statement ii) is correct
(D) Both the statements are wrong

Ans. (B)

Sol. (i) We know, $\frac{T}{J}=\frac{G\theta}{L}$

Rotation at the free end is given as

$$\Rightarrow \theta=\frac{TL}{GJ}=\frac{100\times10^3\times5}{50000\times10^3}=\frac{500}{50000}=0.01$$

$$\therefore \quad \theta=0.01 \text{ rad}$$

(ii) $$\text{Strain energy}=\frac{1}{2}\times T\times\theta=\frac{T^2L}{2GJ}=\frac{(100)^2\times5}{2\times50000}=0.5 \text{ kN-m}$$

Thus, statement I is true and Statement II is False.

Hence, the correct option is (B).

Question 15 **[Construction Materials & Management]**

M20 concrete as per IS 456: 2000 refers to concrete with a design mix having_____.

(A) an average cube strength of 20 MPa
(B) an average cylinder strength of 20 MPa
(C) a 5-percentile cube strength of 20 MPa
(D) a 5-percentile cylinder strength of 20 MPa

Ans. (C)

Question 16 **[Strength of Materials : Deflection of Beams]**

When a simply-supported elastic beam of span L and flexural rigidity EI (E is the modulus of elasticity and I is the moment of inertia of the section) is loaded with a uniformly distributed load w per unit length, the deflection at the mid-span is

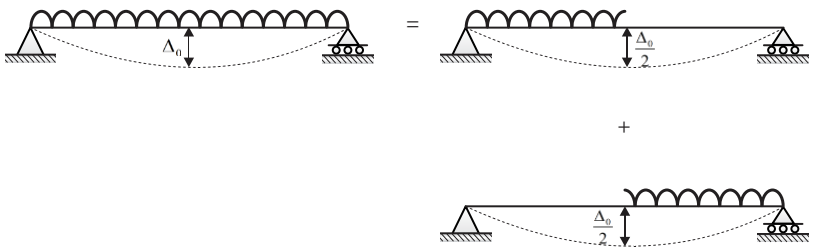
$$\Delta_0 = \frac{5}{384}\frac{wL^4}{EI}$$

If the load on one half of the span is now removed, the mid-span deflection_____.

(A) Reduces to $\Delta_0/2$
(B) Reduces to a value less than $\Delta_0/2$
(C) Reduces to a value greater than $\Delta_0/2$
(D) Remains unchanged at Δ_0

Ans. (A)

Sol. **Given :** $\Delta_0 = \frac{5}{384}\frac{wL^4}{EI}$



From above, when load on one half of span is removed, the mid span deflection reduced to $\frac{\Delta_0}{2}$.

Hence, the correct option is (A).

Question 17 **[Structural Analysis : Influence Line Diagram and Rolling Loads]**

Muller-Breslau principle is used in analysis of structures for _____.

(A) Drawing an influence line diagram for any force response in the structure
(B) Writing the virtual work expression to get the equilibrium equation
(C) Superposing the load effects to get the total force response in the structure
(D) Relating the deflection between two points in a member with the curvature diagram in-between

Ans. (A)

Question 18 **[Geotechnical Engineering : Shallow Foundation]**

A standard penetration test (SPT) was carried out at a location by using a manually operated hammer dropping system with 50% efficiency. The recorded SPT value at a particular depth is 28. If an automatic hammer dropping system with 70% efficiency is used at the same location, the recorded SPT value will be_____.

(A) 28 (B) 20

(C) 40 (D) 25

Ans. (B)

Sol. We know, $\eta_{h_1} \times N_1 = \eta_{h_2} \times N_2$

where, $\eta_{h_1} = 0.5$

$N_1 = (\text{Recorded SPT}) = 28$

$\eta_{h_2} = 0.7$

$\eta_{h_1} \times N_1 = \eta_{h_2} \times N_2$

$\Rightarrow \quad 0.5 \times 28 = 0.7 \times N_2$

$\Rightarrow \quad N_2 = 20$

Hence, the correct option is (B).

Question 19 **[Geotechnical Engineering : Effective Stress & Permeability]**

A vertical sheet pile wall is installed in an anisotropic soil having coefficient of horizontal permeability, k_H and coefficient of vertical permeability, k_V. In order to draw the flow net for the isotropic condition, the embedment depth of the wall should be scaled by a factor of_____ , without changing the horizontal scale.

(A) $\sqrt{\frac{k_H}{k_V}}$ (B) $\sqrt{\frac{k_V}{k_H}}$

(C) 1.0 (D) $\frac{k_H}{k_V}$

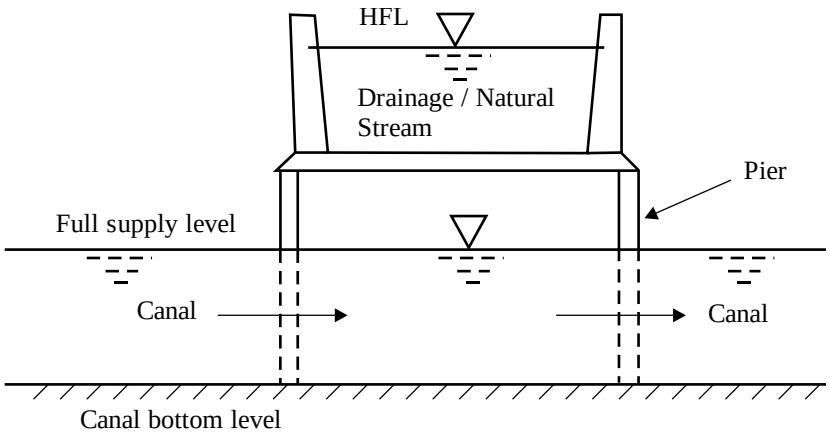
Ans. (A)

Sol. In order to draw the flow net for the isotropic condition, the embedment depth of the wall should be scaled by a factor of $\sqrt{\frac{k_H}{k_V}}$.

Hence, the correct option is (A).

Question 20

Identify the cross-drainage work in the figure.



(A) Super passage
(B) Aqueduct
(C) Siphon aqueduct
(D) Level crossing

Ans. (A)

Question 21

Which one of the following options provides the correct match of the terms listed in Column-1 and Column-2?

Column-1	Column-2
P:Horton equation	I: Precipitation
Q: Muskingum method	II: Flood frequency
R: Penman method	III: Evapotranspiration
	IV: Infiltration
	V: Channel routing

(A) P-IV, Q-V, R-III
(B) P-III, Q-IV, R-I
(C) P-IV, Q-III, R-II
(D) P-III, Q-I, R-IV

Ans. (A)

Sol. Horton equation – Infiltration
Muskingum method – Channel routing
Penman method – Evapotranspiration
Hence, the correct option is (A).

Question 22 **[Environmental Engineering : Disposal of Sewage Effluent]**

In the context of Municipal Solid Waste Management, 'Haul' in 'Hauled Container System operated in conventional mode' includes the_____.

(A) Time spent by the transport truck at the disposal site

(B) Time spent by the transport truck in traveling between a pickup point and the disposal site with a loaded container

(C) Time spent by the transport truck in picking up a loaded container at a pickup point

(D) Time spent by the transport truck in driving from the depot to the first pickup point

Ans. **(B)**

Sol. **Haul :** Haul in a hauled container system operated in conventional mode include the time required to reach the disposal site, starting after a container whose contents are to be emptied has been loaded on the truck plus the time after leaving the disposal site until the truck arrives at the location where the empty container is to be redeposited.

Hence, the correct option is (B).

Question 23 **[Transportation Engineering : Geometrical Design of Highway]**

Which of the following is equal to the stopping sight distance?

(A) (Braking distance required to come to stop) + (Distance travelled during the perception-reaction time)

(B) (Braking distance required to come to stop) – (Distance travelled during the perception-reaction time)

(C) (Braking distance required to come to stop)

(D) (Distance travelled during the perception-reaction time)

Ans. **(A)**

Sol. $SSD = \text{Lag distance} + \text{Braking distance}$

$$= 0.278\text{Vt} + \frac{V^2}{253(nf \pm s)}$$

Hence, the correct option is (A).

Question 24 **[Geomatics Engineering : Traversing]**

The magnetic bearing of the sun for a location at noon is 183° 30′. If the sun is exactly on the geographic meridian at noon, the magnetic declination of the location is_____ .

(A) 3° 30′ W (B) 3° 30′ E

(C) 93° 30′ W (D) 93° 30′ E

Ans. **(A)**

Sol. **Given :** Magnetic Bearing of sun = $183^0 30'$

We know, true bearing of sun at noon = 180^o

True Bearing = magnetic Bearing ± declination

Declination $= 180 - 183^0 30' = -3^\circ 30' = 3^0 30'$ W

Hence, the correct option is (A).

Question 25 **[Engineering Mathematics : Linear Algebra]**

For the matrix

$$[A]=\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}$$

Which of the following statements is/are TRUE?

(A) $[A]\{x\} = \{b\}$ has a unique solution

(B) $[A]\{x\} = \{b\}$ does not have a unique solution

(C) $[A]$ has three linearly independent eigenvectors

(D) $[A]$ is a positive definite matrix

Ans. **(B), (C)**

Sol. **Given :** $[A]=\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}_{3\times3}$

Characteristic equation, $|A-\lambda I|=0$

$$\begin{bmatrix}1-\lambda & -1 & 0\\ -1 & 2-\lambda & -1\\ 0 & -1 & 1-\lambda\end{bmatrix}=0$$

$$(1-\lambda)[\lambda^2-3\lambda+2-1]+1(\lambda-1)=0$$

$$(1-\lambda)[\lambda^2-3\lambda+2-1-1]=0$$

$$(1-\lambda)(\lambda^2-3\lambda)=0$$

$$\lambda^2-3\lambda-\lambda^3+3\lambda^2=0$$

$$-\lambda^3+4\lambda^2-3\lambda=0$$

$$(-\lambda)(\lambda^2-4\lambda+3)=0$$

$$\lambda=0,1,3$$

Since there are three Eigen values so number of linearly independent Eigen vectors are 3.

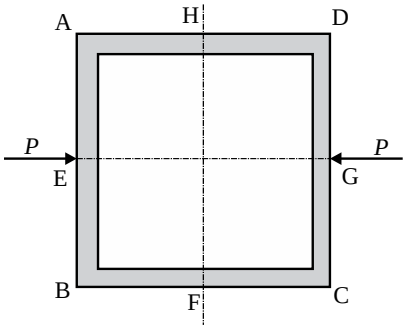
Here one Eigen value $\lambda=0$,so [A] is not a positive definite matrix.

Also, $[A]\{x\}=\{b\}$, does not have a unique solution.

Hence, correct options are (B) and (C).

Question 26 **[Strength of Materials : Shear Force and Bending Moment]**

In the frame shown in the figure (not to scale), all four members (AB, BC, CD, and AD) have the same length and same constant flexural rigidity. All the joints *A*, *B*, *C*, and *D* are rigid joints. The midpoints of AB, BC, CD, and AD, are denoted by *E*, *F*, *G*, and *H*, respectively. The frame is in unstable equilibrium under the shown forces of magnitude P acting at *E* and *G*. Which of the following statements is/are TRUE?



(A) Shear forces at H and F are zero
(B) Horizontal displacements at H and F are zero
(C) Vertical displacements at H and F are zero
(D) Slopes at E, F, G, and H are zero

Ans. **(A), (B), (D)**

Sol. Member AD and BC are subjected to pure bending, so shear force at H and F are zero.

$$V = \frac{dM}{dx}$$

V = 0 (Zero)

Horizontal displacement of H and F are zero.

Vertical displacement at H and F are not zero.

Slopes at E, F, G and H are zero because at axis of symmetry slope must be zero for symmetrical deflection profile.

Hence, correct options are (A), (B) and (D).

Question 27 **[RCC Structures : Shear, Bond, Torsion, Anchorage & Development Length]**

With regard to the shear design of RCC beams, which of the following statements is/are TRUE?

(A) Excessive shear reinforcement can lead to compression failure in concrete
(B) Beams without shear reinforcement, even if adequately designed for flexure, can have brittle failure
(C) The main (longitudinal) reinforcement plays no role in the shear resistance of beam
(D) As per IS456:2000, the nominal shear stress in the beams of varying depth depends on both the design shear force as well as the design bending moment

Ans. **(A), (B), (D)**

Sol. In case of excessive shear reinforcement, concrete become stronger in diagonal on failure and compression Failure may occur before the shear reinforcement has yielded.

The main reinforcement increase shear resistance of beam.

Nominal shear stress for beam with varying depth, $\tau_v = \dfrac{V_u \pm \dfrac{M_u}{d}(\tan\beta)}{bd}$

Where, τ_v depend on both design shear force as well as the design bending moment.

Hence, correct options are (A), (B) and (D).

Question 28 **[Geotechnical Engineering : Compressibility & Consolidation]**

The reason(s) of the non-uniform elastic settlement profile below a flexible footing, resting on a cohesionless soil while subjected to uniform loading, is/are:

(A) Variation of friction angle along the width of the footing

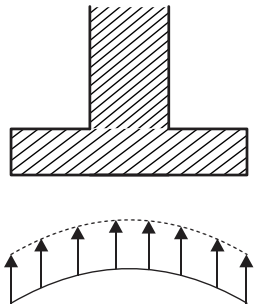
(B) Variation of soil stiffness along the width of the footing

(C) Variation of friction angle along the depth of the footing

(D) Variation of soil stiffness along the depth of the footing

Ans. **(B)**

Sol.



If a flexible footing is resting on a cohesionless soil, there is non uniform elastic settlement profile below footing because modulus of elasticity varies with width of footing so variation in soil stiffness along the width of footing.

Hence, the correct option is (B).

Question 29 **[Environmental Engineering : Treatment of Water]**

Which of the following is/are NOT active disinfectant(s) in water treatment?

(A) OH (hydroxyl radical)

(B) O_3 (ozone)

(C) OCl^- (hypochlorite ion)

(D) Cl^- (chloride ion)

Ans. **(D)**

Sol. Chloride ion $\left(Cl^-\right)$ is not active disinfectant in water treatment while ozone, OCl^- and •OH are considered to be the active disinfectants in water treatment.

Hence, the correct option is (D).

Question 30 **[Transportation Engineering : Geometrical Design of Highway]**

As per the Indian Roads Congress guidelines (IRC 86: 2018), extra widening depends on which of the following parameters?

(A) Horizontal curve radius

(B) Superelevation

(C) Number of lanes

(D) Longitudinal gradient

Ans. **(A), (C)**

Sol. Extra widening = Mechanical widening + Psychological widening

$$W_E = \frac{nl^2}{2R} + \frac{V}{9.5\sqrt{R}}$$

$$W_E \propto \frac{1}{R} \propto \text{Number of lanes}$$

Hence, correct options are (A) and (C).

Question 31

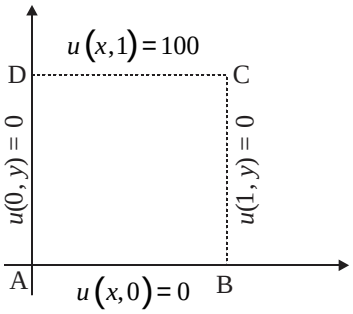
The steady-state temperature distribution in a square plate ABCD is governed by the 2-dimensional Laplace equation. The side AB is kept at a temperature of 100 °C and the other three sides are kept at a temperature of 0 °C. Ignoring the effect of discontinuities in the boundary conditions at the corners, the steady-state temperature at the center of the plate is obtained as $T_0{}^{\circ}C$. Due to symmetry, the steady-state temperature at the center will be same ($T_0{}^{\circ}C$), when any one side of the square is kept at a temperature of 100 °C and the remaining three sides are kept at a temperature of 0 °C. Using the principle of superposition, the value of T_0 is_____.(rounded off to two decimal places).

Ans. 25.1 (24.90 to 25.10)

Sol. Laplace equation is $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ ...(i)

Solution of Laplace equation

$$u(x,y)=(C_1\cos\lambda x+C_2\sin\lambda x)(C_3e^{\lambda y}+C_4e^{-\lambda y}) \quad ...(ii)$$



Assume, AB = CD = 1

Use $u(x,0)=0$

$\therefore \quad C_3=-C_4$

Use $(0,y)=0$

$\therefore \quad C_1=0$

Substituting C_1 and C_4 values in equation (ii)

$$u(x,y)=(0+C_2\sin\lambda x)(C_3e^{\lambda y}-C_3e^{-\lambda y})$$

$$u(x,y)=a_n\sin\lambda x(e^{\lambda y}-e^{-\lambda y})$$

From above figure, $u(1,y)=0$

$\sin\lambda=0\Rightarrow\lambda=n\pi$

$\therefore \quad u(x,y)=\sum a_n\sin(n\pi x)\left[e^{n\pi y}-e^{-n\pi y}\right]$ ...(iii)

$$u(x,1)=100$$

$$\therefore \quad a_n=\frac{100}{\sin n\pi x\left[e^{n\pi}-e^{-n\pi}\right]}$$

From equation (iii),

$$u(x,y)=\sum_{n=1}^{\infty}\frac{100}{e^{n\pi}-e^{-n\pi}}\left(e^{n\pi y}-e^{-n\pi y}\right)$$

At mid-point, $u\left(\frac{1}{2},\frac{1}{2}\right)=T_0$

Hence, $$T_0=\sum_{n=1}^{\infty}\frac{100}{\left(e^{n\pi}-e^{-n\pi}\right)}\left(e^{\frac{n\pi}{2}}-e^{-\frac{n\pi}{2}}\right)$$

$$=100\left[\left(\frac{e^{\frac{\pi}{2}}-e^{-\frac{\pi}{2}}}{e^{\pi}-e^{-\pi}}\right)+\left(\frac{e^{\pi}-e^{-\pi}}{e^{2\pi}-e^{-2\pi}}\right)+\frac{e^{\frac{3\pi}{2}}-e^{\frac{-3\pi}{2}}}{e^{3\pi}-e^{-3\pi}}+.....\right]$$

$$=100\left[\left(\frac{1}{e^{\frac{\pi}{2}}-e^{-\frac{\pi}{2}}}\right)+\left(\frac{1}{e^{\pi}-e^{-\pi}}\right)+\frac{1}{e^{\frac{3\pi}{2}}-e^{\frac{-3\pi}{2}}}+.....\right]$$

$$=50\left[\frac{1}{\cosh\left(\frac{\pi}{2}\right)}+\frac{1}{\cosh(\pi)}+\frac{1}{\cosh\left(\frac{3\pi}{2}\right)}+.....\right]$$

$$=50\left[\frac{1}{2.509}+\frac{1}{11.59}+\frac{1}{55.66}+.....\right]$$

$$= 19.928 + 4.314 + 0.903 + \ldots\ldots$$

$$= 25.145 \approx 25.1$$

$$\therefore \quad T_0=25.1^{\circ}C$$

Hence, the correct answer is 25.1.

Question 32 **[Geotechnical Engineering : Shear Strength of Soil]**

An unconfined compression strength test was conducted on a cohesive soil. The test specimen failed at an axial stress of 76 kPa. The undrained cohesion (in kPa, in integer) of the soil is_____ .

Ans. 38 (38 to 38)

Sol. **Given :** $\sigma_3=0$ (Unconfined compression strength test), $\sigma_1=76$ kPa

Cohesion, C = ?

$$\sigma_1=\sigma_3\tan^2\left(45+\frac{\phi}{2}\right)+2C\tan\left(45+\frac{\phi}{2}\right)$$

Since $\sigma_3=0$ and $\phi=0$

$\therefore \quad \sigma_1 = 2C$

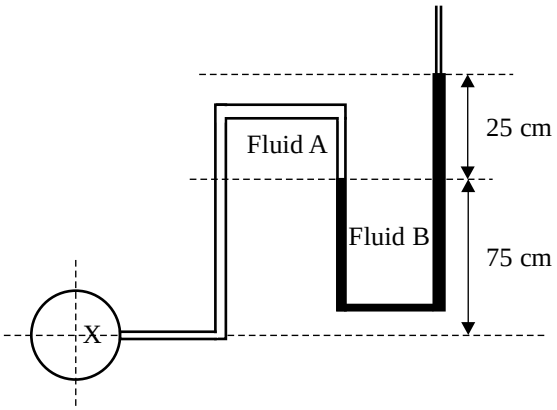
$\Rightarrow \quad 76 = 2C$

$\therefore \quad C = 38 \text{ kPa}$

Hence, the correct answer is 38.

Question 33

The pressure in a pipe at X is to be measured by an open manometer as shown in figure. Fluid A is oil with a specific gravity of 0.8 and Fluid B is mercury with a specific gravity of 13.6. The absolute pressure at X is_____ kN/m^2 (Round off to one decimal place). [Assume density of water as 1000 kg/m^3 and acceleration due to gravity as 9.81 m/s^2 and atmospheric pressure as 101.3 kN/m^2]



[Note : Figure is not to scale]

Ans. 140.54 (140.0 to 141.0)

Sol. $P_x - (800 \times 9.81 \times 0.75) - (13600 \times 9.81 \times 0.25) = P_{atm} \qquad (P_{atm} = 101.3 \times 10^3 \text{ N/m}^2)$

$\Rightarrow \quad P_x = (101.3 \times 10^3) + (800 \times 9.81 \times 0.75) + (13600 \times 9.81 \times 0.25)$

$\therefore \quad P_x = 140.54 \text{ kN/m}^2$

Hence, the correct answer is 140.54.

Question 34

For the elevation and temperature data given in the table, the existing lapse rate in the environment is_____ °C/100 m (round off to two decimal places).

Elevation from ground level (m)	Temperature (^0C)
5	14.2
325	16.9

Ans. 0.84 (0.84 to 0.85)

Sol. Existing lapse rate, in environment is given as

$$\text{Lapse rate} = \frac{\text{Difference in temperature}}{\text{Difference in height}}$$

$$\lambda = \frac{\Delta T}{\Delta H} = \frac{16.9 - 14.2}{325 - 5}$$

$$\lambda' = \frac{\lambda \times 100}{100} = 8.4375 \times 10^{-3} = 0.84°C/100\ m$$

Hence, the correct answer is 0.84.

Question 35 **[Geomatics Engineering : Field Astronomy & Photogrammetric Surveying]**

If the size of the ground area is 6 km × 3 km and the corresponding photo size in the aerial photograph is 30 cm × 15 cm, then the scale of the photograph is 1: _____. (in integer).

Ans. **20000 (20000 to 20000)**

Sol. **Given :** Ground area = $6\ km \times 3\ km$

Map area = $30\ cm \times 15\ cm$

We know, scale = $\sqrt{\frac{\text{Map area}}{\text{Ground area}}} = \sqrt{\frac{30 \times 15}{6 \times 10^5 \times 3 \times 10^5}} = \frac{1}{20000}$

$\therefore$ Scale of photograph is 1:20000.

Hence, the correct answer is 1 : 20000.

Q.36 to Q.65 Carry Two Marks Each

Question 36 **[Engineering Mathematics : Differential Equations]**

The solution of the differential equation

$$\frac{d^3y}{dx^3} - 5.5\frac{d^2y}{dx^2} + 9.5\frac{dy}{dx} - 5y = 0$$

is expressed as $y = C_1e^{2.5x} + C_2e^{\alpha x} + C_3e^{\beta x}$, where C_1, C_2, C_3, α and β are constants, with α and β being distinct and not equal to 2.5. Which of the following options is correct for the values of α and β?

(A) 1 and 2 (B) −1 and −2

(C) 2 and 3 (D) −2 and −3

Ans. **(A)**

Sol. **Given :** $\frac{d^3y}{dx^3} - 5.5\frac{d^2y}{dx^2} + 9.5\frac{dy}{dx} - 5y = 0$

The differential equation can be written as

$$D^3 - 5.5D^2 + 9.5D - 5 = 0$$

By solving above equation we get,

$$D = 2,\ 1,\ 2.5$$

Hence the solution is given as

$$y = c_1e^{2.5x} + c_2e^{1x} + c_3e^{2x}$$

So, α and β are 1 and 2 respectively.

Hence, the correct option is (A).

Question 37 **[Engineering Mathematics : Linear Algebra]**

Two vectors $[2 \quad 1 \quad 0 \quad 3]^T$ and $[1 \quad 0 \quad 1 \quad 2]^T$ belong to the null space of a 4 × 4 matrix of rank 2. Which one of the following vectors also belongs to the null space?

(A) $[1 \quad 1 \quad -1 \quad 1]^T$ (B) $[2 \quad 0 \quad 1 \quad 2]^T$

(C) $[0 \quad -2 \quad 1 \quad -1]^T$ (D) $[3 \quad 1 \quad 1 \quad 2]^T$

Ans. **(A)**

Sol. **Given :** Matrix is 4×4

Rank of matrix = 2

$N(A)$ = Number of column – Rank of matrix = 4 – 2 = 2

We can say that null space of A will consists only two linearly independent vectors which is given as x and y.

Eigen vectors of Matrix A, $\begin{bmatrix}2\\1\\0\\3\end{bmatrix}$ and $\begin{bmatrix}1\\0\\1\\2\end{bmatrix}$

Therefore, remaining eigen vectors must be linearly dependent.

Hence, $X - Y = \begin{bmatrix}1\\1\\-1\\1\end{bmatrix}$

Hence, the correct option is (A).

Question 38 **[Engineering Mathematics : Linear Algebra]**

Cholesky decomposition is carried out on the following square matrix [A].

$$[A] = \begin{bmatrix}8 & -5\\ -5 & a_{22}\end{bmatrix}$$

Let i_{ij} and a_{ij} be the $(i, j)^{\text{th}}$ elements of matrices [L] and [A], respectively. If the element l_{22} of the decomposed lower triangular matrix [L] is 1.968, what is the value (rounded off to the nearest integer) of the element a_{22} ?

(A) 5 (B) 7

(C) 9 (D) 11

Ans. **(B)**

Sol. Cholesky decomposition, $A = LL^T$

$$\begin{bmatrix}8 & -5\\ -5 & a_{22}\end{bmatrix} = \begin{bmatrix}L_{11} & 0\\ L_{21} & L_{22}\end{bmatrix}\begin{bmatrix}L_{11} & L_{21}\\ 0 & L_{22}\end{bmatrix}$$

On compare both side

We get, $L_{11} = 2\sqrt{2}$

$$L_{21} = \frac{-5}{2\sqrt{2}}$$

$$L_{21}^2 + L_{22}^2 = a_{22}$$

$$\left(\frac{-5}{2\sqrt{2}}\right)^2 + 1.968 = a_{22}$$

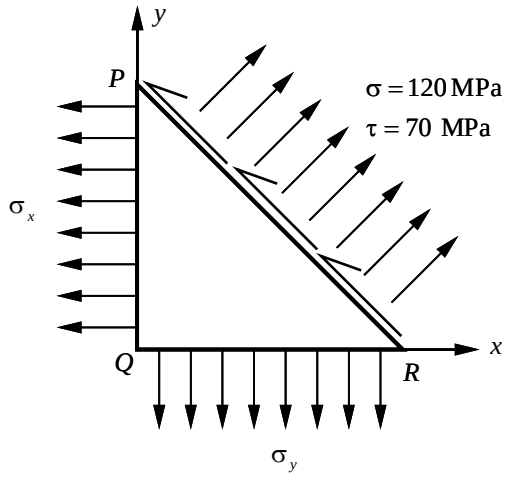
$$a_{22} = 6.99$$

$$a_{22} \approx 7$$

Hence, the correct option is (B).

Question 39 **[Strength of Materials : Principal Stress and Principal Strain]**

In a two-dimensional stress analysis, the state of stress at a point is shown in the figure. The values of length of PQ, QR, and RP are 4, 3, and 5 units, respectively. The principal stresses are_____. (Round off to one decimal place)



(A) $\sigma_x = 26.7\,\text{MPa}, \sigma_y = 172.5\,\text{MPa}$

(B) $\sigma_x = 54.0\,\text{MPa}, \sigma_y = 128.5\,\text{MPa}$

(C) $\sigma_x = 67.5\,\text{MPa}, \sigma_y = 213.3\,\text{MPa}$

(D) $\sigma_x = 16.0\,\text{MPa}, \sigma_y = 138.5\,\text{MPa}$

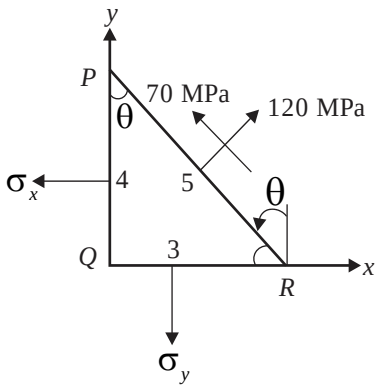
Ans. **(C)**

Sol. $\tau_{xy} = 0$

$$\cos\theta = \frac{4}{5}$$

$$\sin\theta = \frac{3}{5}$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \quad \text{...(i)}$$



$$\tau = \left(\frac{\sigma_y - \sigma_x}{2}\right)\sin(2\theta) + \tau_{xy}\cos 2\theta \qquad ...(ii)$$

From equation (i),

$$120 = \sigma_x\left(\frac{4}{5}\right)^2 + \sigma_y\left(\frac{3}{5}\right)^2 \qquad ...(iii)$$

From equation (ii),

$$70 = \left(\frac{\sigma_y - \sigma_x}{2}\right)2\times\frac{4}{5}\times\frac{3}{5} \qquad ...(iv)$$

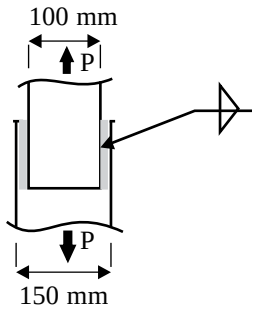
By solving equation (iii) and (iv),

We get, $\sigma_x = 67.5$ MPa, $\sigma_y = 213.33$ MPa

Hence, the correct option is (C).

Question 40 **[Steel Structures : Design of Tension Member]**

Two plates are connected by fillet welds of size 10 mm and subjected to tension, as shown in the figure. The thickness of each plate is 12 mm. The yield stress and the ultimate stress of steel under tension are 250 MPa and 410 MPa, respectively. The welding is done in the workshop (partial safety factor, $\gamma_{mw} = 1.25$). As per the Limit State Method of IS 800: 2007, what is the minimum length (in mm, rounded off to the nearest higher multiple of 5 mm) required of each weld to transmit a factored force P equal to 275 kN?



(A) 100 (B) 105
(C) 110 (D) 115

Ans. (B)

Sol. **Given :** $f_u = 410$ MPa , $f_y = 250$ MPa , $\gamma_{mw} = 1.25$ and t = 12 mm , Size of weld, S = 10 mm .

Factored force P = 275 kN

$$P = \frac{f_u}{\sqrt{3}\gamma_{mw}}\times L_w \times 0.7 \times S \qquad ...(A)$$

Assume minimum length of weld required on each size = l

$\therefore \quad L_w = 2\times l$

Substitute P and L_w in equation (A),

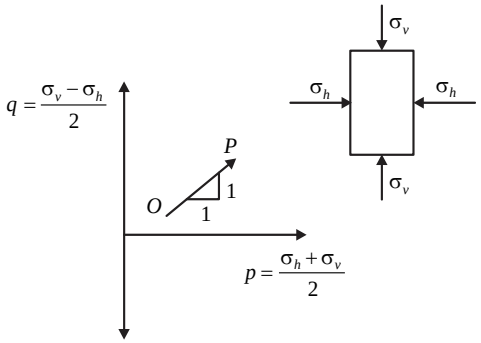
$$275\times10^3=\frac{410}{\sqrt{3}\times1.25}\times2l\times0.7\times10$$

$$l=103.25\text{ mm}\approx105\text{ mm}$$

Hence, the correct option is (B).

Question 41 **[Geotechnical Engineering : Shear Strength of Soil]**

In the given figure, Point O indicates the stress point of a soil element at initial non-hydrostatic stress condition. For the stress path (OP), which of the following loading conditions is correct?



(A) σ_v is increasing and σ_h is constant

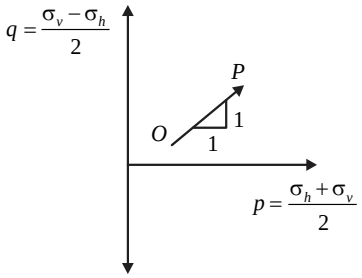
(B) σ_v is constant and σ_h is increasing

(C) σ_v is increasing and σ_h is decreasing

(D) σ_v is decreasing and σ_h is increasing

Ans. (A)

Sol.



Here, σ_v = Major principal stress and σ_h = Minor principal stress.

Given slope is 1:1

$$\therefore\quad \frac{dq}{dp}=1$$

$$\frac{d\sigma_v-d\sigma_h}{d\sigma_v+d\sigma_h}=1$$

$$d\sigma_h=0$$

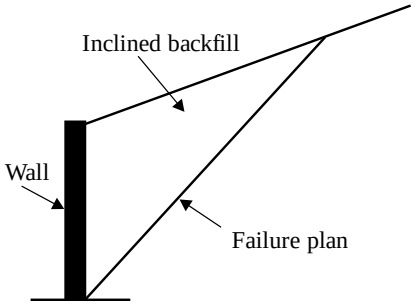
$\therefore$ σ_h is constant and σ_v is increasing.

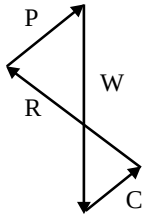
Hence, the correct option is (A).

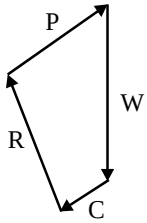
Question 42 **[Geotechnical Engineering : Retaining Wall & Earth Pressure]**

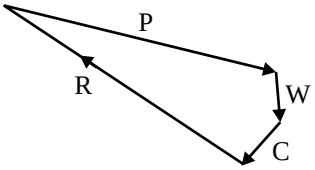
The figure shows a vertical retaining wall with backfill consisting of cohesive-frictional soil and a failure plane developed due to passive earth pressure. The forces acting on the failure wedge are: P as

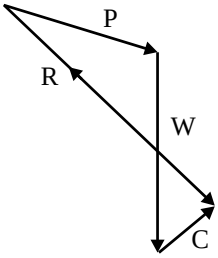
the reaction force between the wall and the soil, R as the reaction force on the failure plane, C as the cohesive force along the failure plane and W as the weight of the failure wedge. Assuming that there is no adhesion between the wall and the wedge, identify the most appropriate force polygon for the wedge.



(A)


(B)


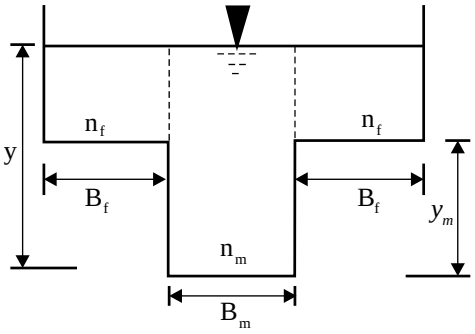
(C)


(D)


Ans. (C)

Question 43 **[Fluid Mechanics : Open Channel Flow]**

A compound symmetrical open channel section as shown in the figure has a maximum of_____ critical depth(s).

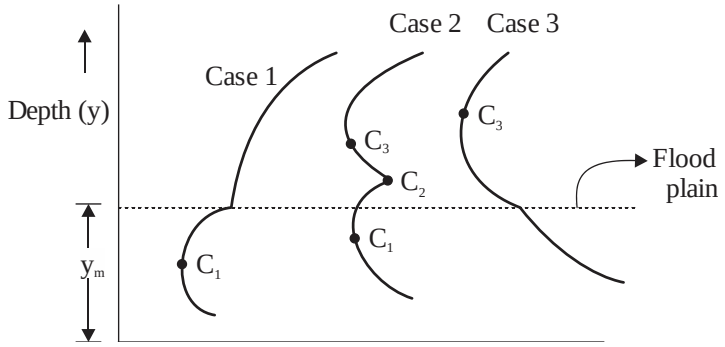


B_m-Bottom width of main channel
B_f-Bottom width of flood channel
y_m -Depth of main channel
y -Total depth of channel
n_m-Manning's roughness of the main channel
n_f -Manning's roughness of the flood channel

(A) 3
(B) 2
(C) 1
(D) 4

Ans. (A)

Sol. For given compound symmetric open channel section has maximum of three critical depth.



Hence, the correct option is (A).

Question 44 **[Fluid Mechanics : Open Channel Flow]**

The critical flow condition in a channel is given by _____.

[Note: α – kinetic energy correction factor; Q – discharge; A_c – cross-sectional area of flow at critical flow condition; T_c – top width of flow at critical flow condition; g – acceleration due to gravity]

(A) $\frac{\alpha Q^2}{g}=\frac{A_c^3}{T_c}$ (B) $\frac{\alpha Q}{g}=\frac{A_c^3}{T_c^2}$

(C) $\frac{\alpha Q^2}{g}=\frac{A_c^3}{T_c^2}$ (D) $\frac{\alpha Q}{g}=\frac{A_c^3}{T_c}$

Ans. **(A)**

Sol. In critical flow condition, $F_r=1$

$$\therefore \quad \frac{Q^2T_c}{gA_c^3}=1$$

$$\Rightarrow \quad \frac{Q^2}{g}=\frac{A_c^3}{T_c}$$

If we consider kinetic energy correction factor then $\frac{\alpha Q^2}{g}=\frac{A_c^3}{T_c}$

Hence, the correct option is (A).

Question 45

Match the following air pollutants with the most appropriate adverse health effects:

Air pollutant	Health effect to human and/or test animal
(P) Aromatic hydrocarbons	(I) Reduce the capacity of the blood of carry oxygen
(Q) Carbon monoxide	(II) Bronchitis and pulmonary emphysema
(R) Sulfur oxides	(III) Damage of chromosomes
(S) Ozone	(IV) Carcinogenic effect

(A) (P) – (II), (Q) – (I), (R) – (IV), (S) – (III) (B) (P) – (IV), (Q) – (I), (R) – (III), (S) – (II)

(C) (P) – (III), (Q) – (I), (R) – (II), (S) – (IV) (D) (P) – (IV), (Q) – (I), (R) – (II), (S) – (III)

Ans. **(D)**

Sol. **Aromatic hydrocarbons :**

- Aromatic hydrocarbons are responsible for cancer.
- It is considered as carcinogenic agents.

Carbon monoxide :

- Carbon monoxide binds with hemoglobin of blood to form COHb.
- Due to COHb, less oxygen gets transported to body tissues.

Sulphur dioxide :

- Sulphur dioxide leads to bronchial spasms.

Ozone :

- Ozone can damage the tissues of the respiratory which cause inflammation and irritation and results in coughing and asthma symptoms.

Hence, the correct option is (D).

Question 46 **[Geomatics Engineering : Traversing]**

A delivery agent is at a location R. To deliver the order, she is instructed to travel to location P along straight-line paths of RC, CA, AB and BP of 5 km each. The direction of each path is given in the table below as whole circle bearings. Assume that the latitude (L) and departure (D) of R is (0, 0) km. What is the latitude and departure of P (in km, rounded off to one decimal place)?

Paths	RC	CA	AB	BP
Directions (in degrees)	120	0	90	240

(A) L = 2.5; D = 5.0
(B) L = 0.0; D = 5.0
(C) L = 5.0; D = 2.5
(D) L = 0.0; D = 0.0

Ans. **(B)**

Sol. Given each path length (l) = 5 km

Path (l)	**Direction (θ)**	**Latitude $(l\cos\theta)$**	**Departure $(l\sin\theta)$**
RC	120^0	-2.5	4.33
CA	0^0	5	0
AB	90^0	0	5
BP	240^0	-2.5	-4.33

$\therefore$ Latitude of p in km = $\Sigma\text{latitude} = -2.5+5-2.5=0$

and Departure of p in km = $\Sigma\text{departure} = 4.33+5-4.33=5$

Hence, Latitude (L) = 0 and Departure (D) = 5

Hence, the correct option is (B).

Question 47

Which of the following statements is/are TRUE?

(A) The thickness of a turbulent boundary layer on a flat plate kept parallel to the flow direction is proportional to the square root of the distance from the leading edge

(B) If the streamlines and equipotential lines of a source are interchanged with each other, the resulting flow will be a sink

(C) For a curved surface immersed in a stationary liquid, the vertical component of the force on the curved surface is equal to the weight of the liquid above it

(D) For flow through circular pipes, the momentum correction factor for laminar flow is larger than that for turbulent flow

Ans. **(C), (D)**

Sol. In turbulent flow, thickness of boundary layer,

$$\frac{\delta}{x} = \frac{0.376}{\text{Re}_x^{1/5}}$$

$$\delta \propto x^{415}$$

Momentum correction factor (β)

Flow	(β)
Laminar flow between circular pipe	$\frac{4}{3}$
Turbulent flow	1.015

Hence, the correct options are (C) and (D).

Question 48 **[Environmental Engineering : Quality Parameters of Water]**

In the context of water and wastewater treatments, the correct statements are:

(A) Particulate matter may shield microorganisms during disinfection

(B) Ammonia decreases chlorine demand

(C) Phosphorous stimulates algal and aquatic growth

(D) Calcium and magnesium increase hardness and total dissolved solids

Ans. **(A), (C), (D)**

Sol. Ammonia increase chlorine demand because the ammonia is converted into nitrogen gas and chlorine becomes chloride.

Hence, the correct option is (B).

Question 49 **[Environmental Engineering : Disposal of Sewage Effluent]**

Which of the following statements is/are TRUE for the aerobic composting of sewage sludge?

(A) Bulking agent is added during the composting process to reduce the porosity of the solid mixture

(B) Leachate can be generated during composting

(C) Actinomycetes are involved in the process

(D) In-vessel composting systems cannot be operated in the plug-flow mode

Ans. **(B), (C)**

Sol. Bulking agent is added during the composition process to increase the porosity of the soil mixture.

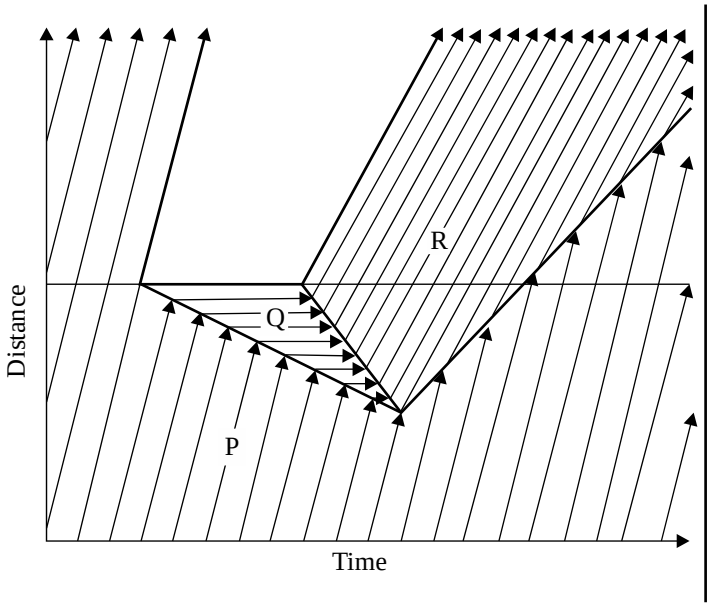
In-vessel composting system can be divided into two modes :

(i) Plug flow modes (ii) Agitated bed modes

Hence, the correct options are (A) and (D).

Question 50 **[Transportation Engineering : Traffic Engineering]**

The figure presents the time-space diagram for when the traffic on a highway is suddenly stopped for a certain time and then released. Which of the following statements are true?



(A) Speed is higher in Region R than in Region P

(B) Volume is lower in Region Q than in Region P

(C) Volume is higher in Region R than in Region P

(D) Density is higher in Region Q than in Region R

Ans. **(B), (C), (D)**

Sol.

- Distance time graph slope of region P is more than region R so speed is higher in region P than in region R. Hence option 'a' is incorrect.
- Volume is lower in region Q than in region P because velocity of vehicle is zero in region Q and hence volume is zero.
- Volume is higher in region 'R' than in Region 'P'.
- Region 'Q' are practically at jam density or maximum density, so density is higher in region Q than in region R.

Hence, the correct options are (B), (C) and (D).

Question 51 **[Transportation Engineering : Highway Materials]**

Consider the Marshall method of mix design for bituminous mix. With the increase in bitumen content, which of the following statements is/are TRUE?

(A) The Stability decreases initially and then increases

(B) The Flow increases monotonically

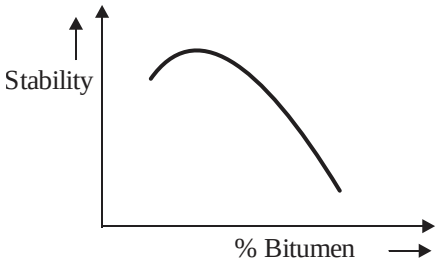
(C) The air voids (VA) increases initially and then decreases

(D) The voids filled with bitumen (VFB) increases monotonically

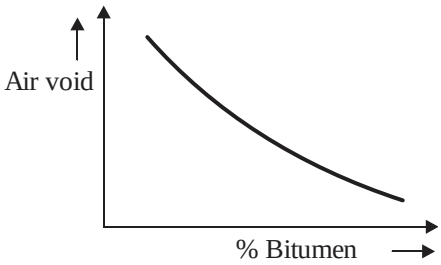
Ans. **(B), (D)**

Sol.

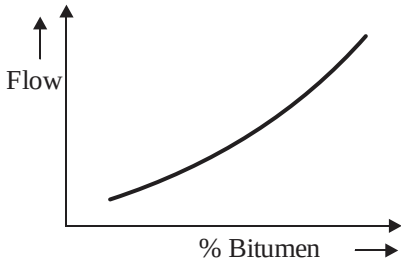
- Option (A) is incorrect because stability increases initially and then decreases.



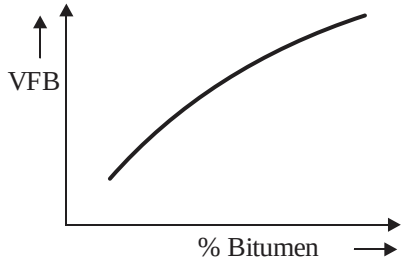
- Option (C) is incorrect because air voids decreases continuously with the increase of bituminous mix.



- Flow increases monotonically.



- With the increase in bitumen content, the VFB increases monotonically.



Hence, correct options are (B) and (D).

Question 52 **[Strength of Materials : Properties of Metals, Stress and Strain]**

A 5 cm long metal rod *AB* was initially at a uniform temperature of T_0 °C. Thereafter, temperature at both the ends are maintained at 0°C. Neglecting the heat transfer from the lateral surface of the rod, the heat transfer in the rod is governed by the one-dimensional diffusion equation $\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}$, where D is the thermal diffusivity of the metal, given as 1.0 cm^2/s.

The temperature distribution in the rod is obtained as $T(x,T)=\sum_{n=1,3,5...}^{\infty} C_n \sin\frac{n\pi x}{5} e^{-\beta n^2 t}$,

Where, x is in cm measured from A to B with x = 0 at A, t is in s, C_n are constants in °C, T is in °C, and β is in s^{-1}.

The value of β (in s^{-1}, rounded off to three decimal places) is _____.

Ans. 0.394 (0.394 to 0.396)

Sol. $T(x,t)=\sum_{n=1,3,5}^{\infty} c_n \sin\left(\frac{n\pi x}{5}\right)e^{-\beta n^2 t}$

$$T(x,t)=\sum_{n=1}^{\infty}\beta_n \sin\left(\frac{n\pi x}{l}\right)e^{\frac{-n^2\pi^2\alpha^2 t}{l^2}}$$

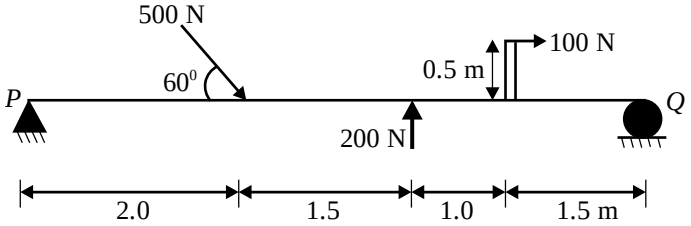
$$\alpha^2 = D = 1 \qquad (l=5)$$

$$\therefore\ \beta=\frac{\pi^2\alpha^2}{l^2}=\frac{\pi^2(1)}{25}=\frac{\pi^2}{25}=\frac{(3.14)^2}{25}=0.394$$

Hence, the correct answer is 0.394.

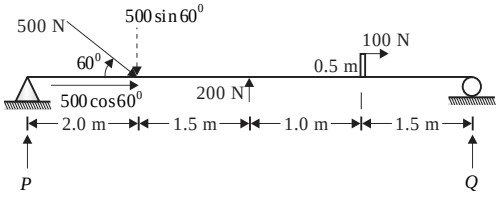
Question 53 [Strength of Materials : Shear Force and Bending Moment]

A beam is subjected to a system of coplanar forces as shown in the figure. The magnitude of vertical reaction at Support P is _____ N (Round off to one decimal place).



Ans. 197 (195.0 to 200.0)

Sol.



Taking moment about hinge support Q,

$$\Sigma M_Q = 0$$

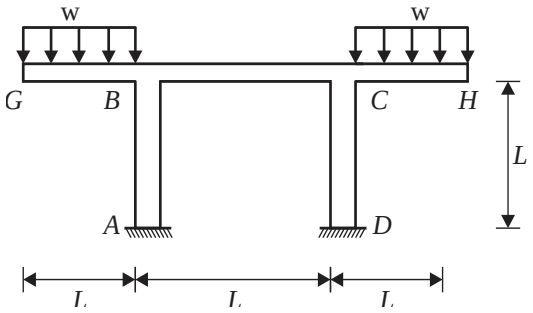
$$\Rightarrow \quad P\times(2+1.5+1+1.5)+(100\times0.5)+(200\times2.5)-500\sin60\times4=0$$

$$\therefore \quad P=197\ \text{N}$$

Hence, the correct answer is 197.

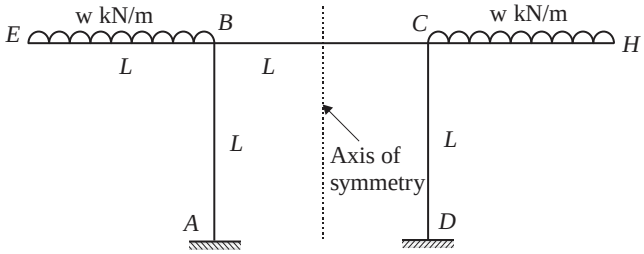
Question 54 [Structural Analysis : Displacement Method of Analysis]

For the frame shown in the figure (not to scale), all members (*AB*, *BC*, *CD*, *GB*, and *CH*) have the same length, *L* and flexural rigidity, *EI*. The joints at *B* and *C* are rigid joints, and the supports *A* and *D* are fixed supports. Beams *GB* and *CH* carry uniformly distributed loads of *w* per unit length. The magnitude of the moment reaction at *A* is wL^2/k. What is the value of *k* (in integer)? _____.

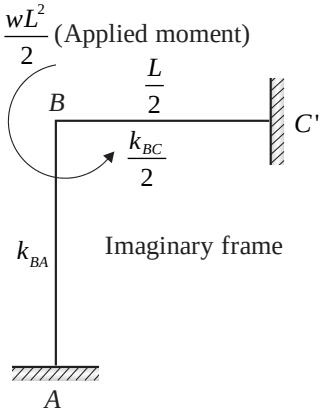


Ans. 6 (6 to 6)

Sol.



Distribution factors of member BA and BC



Joint	Member	k	$\sum k$	D.F $= \frac{k}{\sum k}$
B	BA	$\frac{I}{L}$	$\frac{3}{2}\left(\frac{I}{L}\right)$	$\frac{2}{3}$
	BC'	$\frac{1}{2}\left(\frac{I}{L}\right)$		$\frac{1}{3}$

Distribution of end moment $\rightarrow$

Joint	A	B	C
D.F.	0	$\frac{2}{3}$	$\frac{1}{3}$
Balance	0	$\frac{-wL^2}{3}$	$\frac{-wL^2}{6}$
C.O.M.	$\frac{-wL^2}{6}$		–
Final end moment	$\frac{-wL^2}{6}$	$\frac{-wL^2}{3}$	$\frac{-wL^2}{6}$ –

From above, moment at A $= \frac{wL^2}{6}$ ...(i)

Given moment at A $= \frac{wL^2}{k}$ ...(ii)

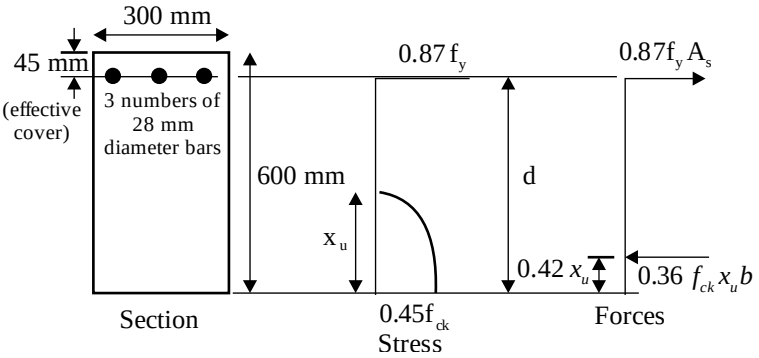
From equation (i) and (ii), $\alpha = 6$

Hence, the correct answer is 6.

Question 55 **[RCC Structures : Design & Analysis of Beam and Slab]**

Consider the singly reinforced section of a cantilever concrete beam under bending, as shown in the figure (M25 grade concrete, Fe415 grade steel). The stress block parameters for the section at ultimate limit state, as per IS 456: 2000 notations, are given. The ultimate moment of resistance for the section by the Limit State Method is _____ kN.m (Round off to one decimal place).

[**Note :** Here, A_s is the total area of tension steel bars, b is the width of the section, d is the effective depth of the bars, f_{ck} is the characteristic compressive cube strength of concrete, f_y is the yield stress of steel, and x_u is the depth of neutral axis.]



Ans. **300.9 (295.0 to 305.0)**

Sol. **Given :** Width (B) = 300 mm

Effective depth (d) = 600 – 45 = 555 mm

Area of steel $= 3 \times \frac{\pi}{4} \times 28^2 = 1847.256 \text{ mm}^2$

Depth of N.A.:

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$0.36 \times 25 \times 300 \times x_u = 0.87 \times 415 \times 1847.256$$

$$x_u = \frac{0.87 \times 415 \times 1847.256}{0.36 \times 25 \times 300} = 247.02 \text{ mm}$$

For Fe415 grade steel, $x_{u\lim} = 0.48d = 0.48 \times 555 = 266.4 \text{ mm} > x_u$ (= 247.02 mm)

Hence, the section is under reinforced.

For Under reinforced section, moment of resistance is given as

$$M.R. = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$= 0.36 \times 25 \times 300 \times 247.02 (555 - 0.42 \times 247.02) = 300964059.6 \text{ N-mm}$$

$\therefore$ $M.R. = 300.96$ kN-m

Hence, the correct answer is 300.96.

Question 56 **[Strength of Materials : Properties of Metals, Stress and Strain]**

A 2D thin plate with modulus of elasticity, E = 1.0 N/m^2, and Poisson's ratio, $\mu = 0.5$, is in plane stress condition. The displacement field in the plate is given by $u = Cx^2y$ and v = 0, where u and v are displacements (in m) along the X and Y directions, respectively, and C is a constant (in m^{-2}). The distances x and y along X and Y, respectively, are in m. The stress in the X direction is $\sigma_{XX} = 40xy$ N/m^2, and the shear stress is $\tau_{XY} = \alpha x^2$ N/m^2. What is the value of α (in N/m^4, in integer)? _____

Ans. 5 (5 to 5)

Sol. **Given :** $E = 1.0$ N/m^2, μ = 0.5

Displacement field in the plate is

$$u = Cx^2y$$

$$v = 0$$

$\therefore$ $$\varepsilon_x = \frac{\partial u}{\partial x} = 2Cxy$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = 0$$

. Stress in x direction , $\sigma_{xx} = 40xy \left(\frac{\text{N}}{\text{m}^2}\right)$

Shear stress, $\tau_{xy} = \alpha x^2 \left(\frac{\text{N}}{\text{m}^2}\right)$

We know, shear modulus $G = \frac{\tau_{xy}}{\gamma_{xy}}$ ...(A)

Stress in x direction; $\sigma_x = \left(\frac{E}{1-\mu^2}\right)(\varepsilon_x + \mu\varepsilon_y)$ ...(i)

Substitute all known value in equation (i) we get,

$$40xy = \frac{1}{\left(1-0.5^2\right)}\left(2Cxy + 0.5\times 0\right)$$

$\therefore$ C = 15

Shear strain $\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0 + Cx^2$

$$\gamma_{xy} = Cx^2$$

Substitute γ_{xy} and C in equation (A),

$$G = \frac{\tau_{xy}}{\gamma_{xy}}$$

$$\frac{E}{2(1+\mu)} = \frac{\tau_{xy}}{Cx^2}$$

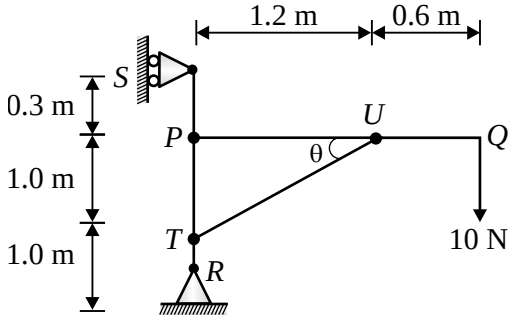
$$\tau_{xy} = \frac{15x^2 \times 1}{2(1+0.5)} = 5x^2$$

By comparing τ_{xy}, we get $\alpha = 5$

Hence, the correct answer is 5.

Question 57 **[Structural Analysis : Force and Energy Methods]**

An idealised frame supports a load as shown in the figure. The horizontal component of the force transferred from the horizontal member *PQ* to the vertical member *RS* at *P* is _____ N (round off to one decimal place).

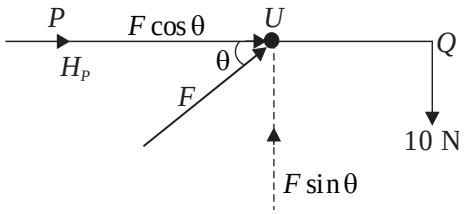


Ans. **18 (17.5 to 18.5)**

Sol. $\tan\theta = \frac{1}{1.2}$

$$\theta = \tan^{-1}\left(\frac{1}{1.2}\right) = 39.80°$$

FBD :



Taking moment about P = 0

$\Rightarrow \quad \Sigma M_p = 0$

$\Rightarrow \quad F\sin\theta \times 1.2 = 10\times(1.2+0.6)$

$\Rightarrow \quad F = 23.43$ N

Horizontal component of force transferred from PQ to member RS at P

$= F\cos\theta = 23.43\cos(39.805) = 18$ N

Hence, the correct answer is 18.

Question 58 **[Geotechnical Engineering : Shallow Foundation]**

A square footing is to be designed to carry a column load of 500 kN which is resting on a soil stratum having the following average properties: bulk unit weight = 19 kN/m^3; angle of internal friction = 0^o and cohesion = 25 kPa. Considering the depth of the footing as 1 m and adopting Meyerhof's bearing capacity theory with a factor of safety of 3, the width of the footing (in m) is _____ (Round off to one decimal place)

[Assume the applicable shape and depth factor values as unity; ground water level at greater depth.]

Ans. **3.4 (3.0 to 3.5)**

Sol. Load from column $(Q_s) = 500$ kN, $\gamma = 19$ kN/m^3, $\phi = 0^0$, $C = 25$ kPa, $D_f = 1$ m, $FOS = 3$.

As per Meyerhof Analysis, $q_u = CN_CS_Cd_Ci_C + \bar{\sigma}N_qS_qd_qi_q + \frac{1}{2}B\gamma N_\gamma S_\gamma d_\gamma i_\gamma$ …(i)

For $\phi = 0^0$, $N_C = 5.14$, $N_q = 1$ and $N_\gamma = 0$

Substitute N_C, N_q and N_γ in equation (i),

$\Rightarrow \quad q_u = 25\times5.14 + (19)\times1 + 0$

$\therefore \quad q_u = 147.5$ kPa

We know $q_{nu} = q_u - \gamma D_f$

$q_{nu} = 147.5 - 19$

$q_{nu} = 128.5$ kPa

$q_{ns} = \frac{q_{nu}}{FOS}$

$\therefore \quad q_{ns} = \frac{128.5}{3} = 42.83$ kPa

Given : $Q_{ns} = 500$ kN

We can write as, $Q_{ns} = q_{ns} \times \text{Area}$

$Q_{ns} = q_{ns} \times \text{B}^2$

$500 = 43.83 \times \text{B}^2$

B = 3.41 m

Hence, the correct answer is 3.41.

Question 59 **[Geotechnical Engineering : Shallow Foundation]**

A circular pile of diameter 0.6 m and length 8 m was constructed in a cohesive soil stratum having the following properties: bulk unit weight = 19 kN/m^3; angle of internal friction = 0^o and cohesion = 25 kPa. The allowable load the pile can carry with a factor of safety of 3 is _____ kN (round off to one decimal place).

[Adopt: Adhesion factor, α = 1.0 and Bearing capacity factor, $\text{N}_c = 9.0$]

Ans. 146.8 (145.0 to 149.0)

Sol. **Given :** $D = 0.6 \text{ m}, L = 8 \text{ m}, \gamma = 19 \text{ kN/m}^3, \phi = 0, C = 25 \text{ kPa}$

$\text{FOS} = 3, \alpha = 1, N_C = 9$

Ultimate pile capacity = End bearing capacity + Skin friction capacity $= CN_C A_b + \alpha \bar{C} A_s$

$$Q_{up} = 9C\left(\frac{\pi}{4} \times D^2\right) + \alpha \bar{C}(\pi DL)$$

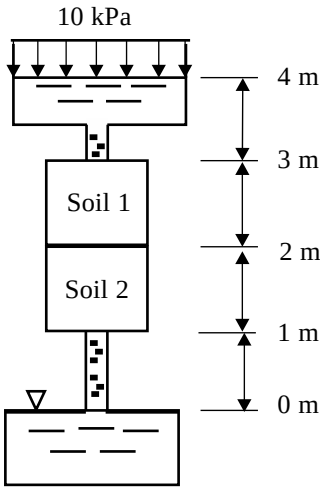
$$= 9 \times 25 \times \frac{\pi}{4} \times 0.6^2 + 1 \times 25 \times \pi \times 0.6 \times 8 = 63.585 + 377 = 440.57$$

Allowable load, $Q_{ap} = \dfrac{Q_{up}}{FOS} = \dfrac{440.57}{3} = 146.85 \text{ kN}$

Hence, the correct answer is 146.85.

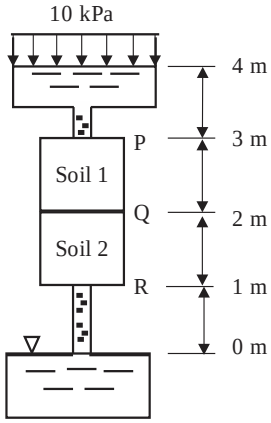
Question 60 **[Geotechnical Engineering : Shallow Foundation]**

For the flow setup shown in the figure (not to scale), the hydraulic conductivities of the two soil samples, Soil 1 and Soil 2, are 10 mm/s and 1 mm/s, respectively. Assume the unit weight of water as 10 kN/m^3 and ignore the velocity head. At steady state, what is the total head (in m, rounded off to two decimal places) at any point located at the junction of the two samples? _____.



Ans. **4.54 (4.50 to 4.60)**

Sol. Head due to external pressure of 10 kPa



$$\text{Head } (h_w) = \frac{10 \text{ kPa}}{10\left(\text{kN/m}^3\right)}$$

$$h_w = 1 \text{ m}$$

Total head at P is = Elevation head + Pressure head

$$(TH)_P = 3 + 1 + h_w = 3 + 1 + 1 = 5 \text{ m}$$

Now, total head at junction Q,

$$(TH)_Q = (TH)_P - h_{L_1} \qquad \ldots(\text{A})$$

Discharge, $Q = k_{eq} \times i \times A$

$$Q = \left(\frac{L_1 + L_2}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}\right) \times \frac{h_L}{L} \times A = \left[\frac{1+1}{\left(\frac{1}{10} + \frac{1}{1}\right)}\right] \times \frac{5}{2} \times A$$

$$Q = 4.545\,A \qquad \ldots(i)$$

Discharge through soil 1 is given as

$$Q = k_L \times i_1 \times A = 10 \times \left(\frac{h_{L_1}}{L_1}\right) \times A$$

$$Q = 10 \times \left(\frac{h_{L_1}}{1}\right) \times A \qquad \ldots(\text{ii})$$

Substitute Q from equation (i) and (ii),

$$4.545A = 10 \times \frac{h_{L_1}}{1} \times A$$

$$h_{L_1} = 0.4545$$

From equation (A),

$(TH)_Q = (TH)_P - h_{L_1} = 5 - 0.4545$

$(TH)_Q = 4.54 \text{ m}$

Hence, the correct answer is 4.54.

Question 61 **[Geotechnical Engineering : Compressibility & Consolidation]**

A consolidated drained (CD) triaxial test was carried out on a sand sample with the known effective shear strength parameters, $c' = 0$ and $\phi' = 30°$. In the test, prior to the failure, when the sample was undergoing axial compression under constant cell pressure, the drainage valve was accidentally closed. At the failure, 360 kPa deviatoric stress was recorded along with 70 kPa pore water pressure. If the test is repeated without such error, and no back pressure is applied in either of the tests, what is the deviatoric stress (in kPa in integer) at the failure _____.

Ans. **500 (500 to 500)**

Sol. $\overline{\sigma_1} = \overline{\sigma_3}\tan^2\left(45 + \frac{\phi}{2}\right) + 2c'\tan\left(45 + \frac{\phi}{2}\right) \quad ...(A)$

$\therefore \quad \overline{\sigma_3} = \sigma_3 - 70$

and $\overline{\sigma_1} = \sigma_1 - 70 = \sigma_3 + \sigma_d - 70 = (\sigma_3 + 360 - 70)$

Substitute $\overline{\sigma_1}$ and $\overline{\sigma_3}$ in equation (A), we get

$(A) \Rightarrow (\sigma_3 + 360 - 70) = (\sigma_3 - 70)\tan^2\left(45 + \frac{30}{2}\right) \quad [\because c = 0]$

$\Rightarrow \quad \sigma_3 = 250 \text{ kPa}$

When drainage is open,

$$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right) \quad [\because c = 0]$$

$$\Rightarrow \quad (\sigma_3 + \sigma_d) = \sigma_3 \tan^2\left(45 + \frac{30}{2}\right)$$

$$\Rightarrow \quad (250 + \sigma_d) = 250 \times 3$$

$$\Rightarrow \quad \sigma_d = 500 \text{ kPa}$$

Hence, deviatoric stress is 500 kPa.

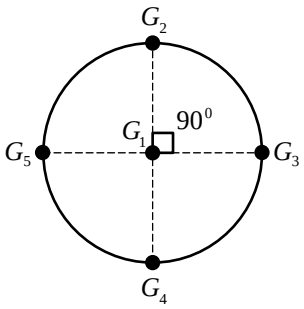
Question 62 **[Engineering Hydrology : Precipitation & Frequency of Rainfall Data]**

A catchment may be idealized as a circle of radius 30 km. There are five rain gauges, one at the center of the catchment and four on the boundary (equi-spaced), as shown in the figure (not to scale).

The annual rainfall recorded at these gauges in a particular year are given below.

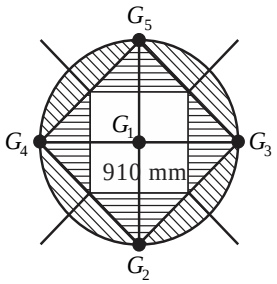
Gauge	G_1	G_2	G_3	G_4	G_5
Rainfall (mm)	910	930	925	895	905

Using the Thiessen polygon method, what is the average rainfall (in mm, rounded off to two decimal places) over the catchment in that year?



Ans. 912.55 (912.28 to 912.82)

Sol.



Given : $R = 30$ km, $G_1 = 910$ mm, $G_2 = 930$ mm, $G_3 = 925$ mm, $G_4 = 895$ mm, $G_5 = 905$ mm

Area under station G_1; $A_1 = 30 \times 30 = 900$ km^2

Area under influence of stations G_2, G_3, G_4, G_5,

$$A_2 = A_3 = A_4 = A_5 = \frac{\pi \times 30^2 - 900}{4} = 481.86 \text{ km}^2$$

Average rainfall = $\overline{P} = \dfrac{G_1A_1 + G_2A_2 + G_3A_3 + G_4A_4 + G_5A_5}{A}$

$$\Rightarrow \quad P_{Avg} = \frac{910 \times 900 + 481.86[930 + 925 + 895 + 905]}{\pi \times 30^2}$$

$$\therefore \quad P_{Avg} = 912.556 \text{ mm}$$

Hence, the average rainfall is 912.55 mm.

Question 63 **[Fluid Mechanics : Open Channel Flow]**

The cross-section of a small river is sub-divided into seven segments of width 1.5 m each. The average depth, and velocity at different depths were measured during a field campaign at the middle of each

segment width. The discharge computed by the velocity area method for the given data is _____ m^3/s (round off to one decimal place).

Segment	Average depth (D) (m)	Velocity (m/s) at different depths		
		0.2D	0.6D	0.8 D
1	0.40	--	0.40	--
2	0.70	0.76	--	0.70
3	1.20	1.19	--	1.13
4	1.40	1.25	--	1.10
5	1.10	1.13	--	1.09
6	0.80	0.69	--	0.65
7	0.45	--	0.42	--

Ans. **8.5 (8.4 to 8.6)**

Sol. Average width for 1st section is given as,

$$\overline{W_1} = \frac{\left(w_1 + \frac{w_2}{2}\right)^2}{2w_1} = \frac{\left(1.5 + \frac{1.5}{2}\right)^2}{2\times 1.5} = 1.6875$$

Segment	Average Width (m)	Average Depth (m)	Average Velocity (m/s)	Segmental Discharge $\Delta Q\ (m^3/s)$
1	1.6875	0.4	0.4	0.27
2	1.5	0.7	0.73	0.7665
3	1.5	1.2	1.16	2.088
4	1.5	1.4	1.175	2.4675
5	1.5	1.1	1.11	1.8315
6	1.5	0.8	0.67	0.804
7	1.6875	0.45	0.42	0.3189375
				8.5464375

$\therefore$ Discharge = 8.54 m^3/s

Hence, the correct answer is 8.54.

Question 64 **[Environmental Engineering : Waste Water Characteristics]**

The theoretical aerobic oxidation of biomass $(C_5H_7O_2N)$ is given below:

$$C_5H_7O_2N + 5O_2 \rightarrow 5CO_2 + NH_3 + 2H_2O$$

The biochemical oxidation of biomass is assumed as a first-order reaction with a rate constant of 0.23/d at 20°C (logarithm to base *e*). Neglecting the second-stage oxygen demand from its biochemical

oxidation, the ratio of BOD_5 at 20°C to total organic carbon (TOC) of biomass is _____ (round off to two decimal places).

[Consider the atomic weights of *C*, *H*, *O* and N as 12 g/mol, 1 g/mol, 16 g/mol and 14 g/mol, respectively]

Ans. 1.82 (1.80 to 2.00)

Sol. **Given :** $C_5H_7O_2N+5O_2 \rightarrow 5CO_2+NH_3+2H_2O$

Molar mass of $C_5H_7O_2N+5O_2 = 113\text{g}$

Total organic carbon $= 5\times12 = 60\,\text{g}$

Ultimate BOD $= 5\times32 = 160\,\text{g}$

We know, $BOD_5 = (BOD)_{ultimate}\left(1-e^{-kt}\right)$

$\text{BOD}_5\,(20°\text{C}) = 160\left(1-e^{-0.23\times5}\right)$

$\text{BOD}_5\,(20°\text{C}) = 109.34\,\text{g}$

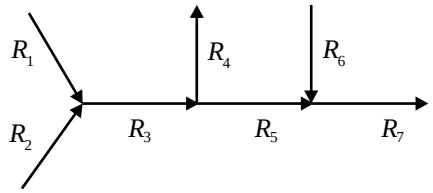
Ratio of BOD to TOC is given as,

$$\frac{\text{BOD}_5\,(20°\text{C})}{TOC} = \frac{109.34}{60} = 1.822$$

Hence, the correct answer is 1.82.

Question 65 **[Environmental Engineering : Disposal of Sewage Effluent]**

A system of seven river segments is shown in the schematic diagram. The R_i's, Q_i's, and C_i's (i = 1 to 7) are the river segments, their corresponding flow rates, and concentrations of a conservative pollutant, respectively. Assume complete mixing at the intersections, no additional water loss or gain in the system, and steady state condition. Given: Q_1 = 5 m^3/s; Q_2 = 15 m^3/s; Q_4 = 3 m^3/s; Q_6 = 8 m^3/s; C_1 = 8 kg/m^3; C_2 = 12 kg/m^3; C_6 = 10 kg/m^3. What is the steady state concentration (in kg/m^3) (Rounded off to two decimal places) of the pollutant in the river segment 7 _____.



Ans. 10.68 (10.58 to 10.78)

Sol. **Given :** $Q_1 = 5\,\text{m}^3/\text{s}$, $Q_2 = 15\,\text{m}^3/\text{s}$, $Q_4 = 3\,\text{m}^3/\text{s}$, $Q_6 = 8\,\text{m}^3/\text{s}$, $C_1 = 8\,\text{kg/m}^3$, $C_2 = 12\,\text{kg/m}^3$, $C_6 = 10\,\text{kg/m}^3$

From diagram, $Q_3 = Q_1 + Q_2 = 5+15 = 20\,\text{m}^3/\text{s}$

$$C_3 = \frac{Q_1C_1 + Q_2C_2}{Q_1+Q_2}$$

$$C_3 = \frac{(5)(8)+(15)(12)}{5+15} = 11\,\text{kg/m}^3$$

$$Q_5 = Q_3 - Q_4 = 20-3 = 17\,\text{m}^3/\text{s}$$

$$C_5 = C_3 = 11\,\text{kg/m}^3$$

$$Q_7 = Q_5 + Q_6 = 17+8 = 25\,\text{m}^3/\text{s}$$

$$C_7 = \frac{Q_5C_5 + Q_6C_6}{Q_5 + Q_6}$$

$$C_7 = \frac{(17)(11)+(8)(10)}{25} = 10.68\,\text{kg/m}^3$$

Hence, steady state concentration of the pollutant in river segment 7 is 10.68 kg/m^3.

❑❑❑