General Aptitude Part

Q.1 to Q.5 Carry One Mark Each

Question 1 **General Aptitude : Verbal Ability**

"I cannot support this proposal. My______ will not permit it."

(A) conscious (B) consensus

(C) conscience (D) consent

Ans. (C)

Sol. Given :

I cannot support this proposal. My **conscience** will not permit it.

Conscious : Aware of and responding to one's surroundings awake.

Consensus : A general agreement

Conscience : An inner feeling or voice, viewed as acting as a guide to the rightness or wrongness of one's behavior.

Consent : Permission for something to happen or agreement to do something as per the meanings of all wards the correct word which could be used is conscience by which the statement will be meaningful.

Hence, the correct option is (C).

Question 2 **General Aptitude : Verbal Ability**

Courts : _______ : : Parliament : Legislature.

(by word meaning)

(A) Judiciary (B) Executive

(C) Governmental (D) Legal

Ans. (A)

Sol. Given :

Parliament is related to legislature in a special manner as parliament is legislative body. In the same way courts is related to judiciary as courts are judiciary body.

Hence, the correct option is (A).

Question 3 **General Aptitude : Numerical Ability**

What is the smallest number with distinct digits whose digits adds up to 45?

(A) 123555789 (B) 123457869

(C) 123456789 (D) 99999

Ans. (C)

Sol. Given :

The smallest number with distinct digits whose digits adds up to 45

(A) 99999, the digits of this number are not distinct, so it is not correct.

(B) 123457869, the digits of this number are distinct, so it is correct.

(C) 123456789, the digits of this number are distinct, so it is correct.

(D) 123555789, the digits of this number are not distinct, so it is not correct.

As we need the smallest number with distinct digits whose digits adds upto 45,
Option (B) and (C) both followed this condition.
Here, the number in option (C) is less than the number in option (B).
Hence, the correct option is (C).

Question 4 **General Aptitude : Numerical Ability**

In a class of 100 students

(i) there are 30 students who neither like romantic movies nor comedy movies,

(ii) the number of students who like romantic movies is twice the number of students who like comedy movies, and

(iii) the number of students who like both romantic movies and comedy movies is 20.

How many students in the class like romantic movies?

(A) 40 (B) 20

(C) 60 (D) 30

Ans. (C)

Sol. Given :

There are 100 students from which so students who neither like romantic movies nor comedy movies.

Therefore, $100-30=70$

70 students will like either romantic or comedy movies.

The number of students who like romantic movie is twice the number of students who like comedy movies.

Let, number of students like comedy movie is x, then number of students like romantic movie is $2x$. the number of students who like both romantic movies and comedy movies is 20.

The number of students in the class like romantic movies,

$$2x+x-20=70$$

$$3x=90$$

$$x=30$$

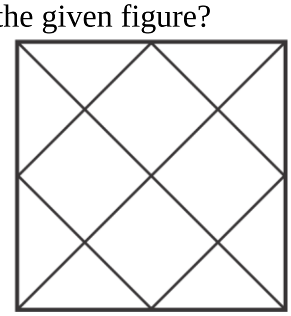
So, $2x=2\times 30=60$

Therefore, the number of students who like romantic movies are 60.

Hence, the correct option is (C).

Question 5 **General Aptitude : Verbal Ability**

How many rectangles are present in the given figure?

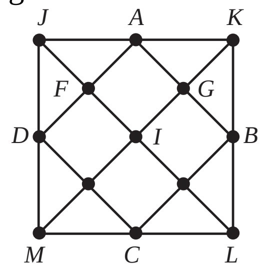


(A) 8 (B) 9

(C) 10 (D) 12

Ans. (C)

Sol. In rectangle, $ABCD$ there are 9 rectangles. $AGIF$, $GBEI$, $IECH$, $FIHD$, $ABEF$, $FECD$, $AGHD$, $GBCH$ and $GBCH$. One more rectangles is $JKLM$. Therefore, there are total 10 rectangles.



Hence, the correct option is (C).

Q.6 to Q.10 Carry Two Marks Each

Question 6 **General Aptitude : Verbal Ability**

Forestland is a planet inhabited by different kinds of creatures. Among other creatures, it is populated by animals all of whom are ferocious. There are also creatures that have claws, and some that do not. All creatures that have claws are ferocious.

Based only on the information provided above, which one of the following options can be logically inferred with certainty?

(A) All creatures with claws are animals.
(B) Some creatures with claws are non-ferocious.
(C) Some non-ferocious creatures have claws.
(D) Some ferocious creatures are creatures with claws.

Ans. (D)

Sol. Given :

Forestland is a planet inhabited by different kinds of creatures. Among other creatures, it is populated by animals all of whom are ferocious. There are also creatures that have claws, and some that do not. All creatures that have claws are ferocious.

Option (A) cannot be inferred as, it is not necessary condition that all creature with claws are animals.

Option (B) cannot be inferred as, all creatures that have claws are ferocious is given in the passage.

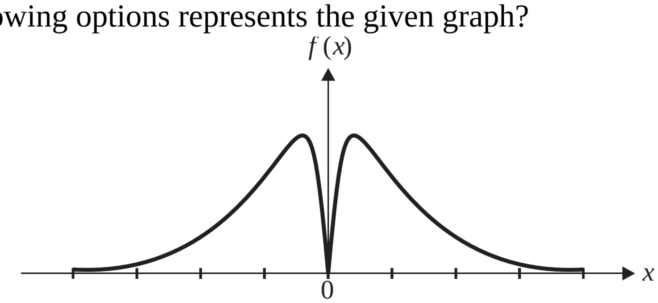
Option (C) can also not be inferred as, all creatures that have claws are ferocious is given in the passage.

Option (D) can be inferred certainly as, “There are also creatures that have claws, and some that do not. All creatures that have claws are ferocious.” Is mention in the passage. From this we can conclude some ferocious creatures are creatures with claws.

Hence, the correct option is (D).

Question 7 **General Aptitude : Numerical Ability**

Which one of the following options represents the given graph?

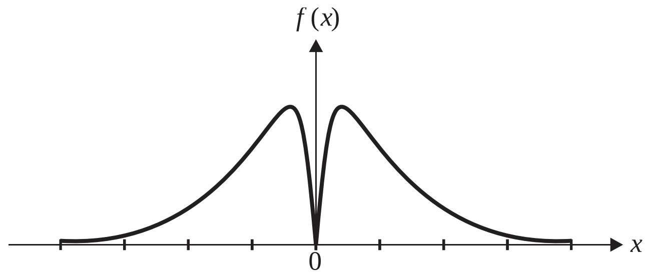


(A) $f(x) = x^2 2^{-|x|}$ (B) $f(x) = x2^{-|x|}$

(C) $f(x) = |x| 2^{-x}$ (D) $f(x) = x2^{-x}$

Ans. (A)

Sol. Given : A graph represents $f(x)$ versus x,



The whole graph is above x-axis i.e. positive for all 'x'.

The given graph is also symmetrical about y-axis.

These two conditions satisfies in only even power of x, i.e. $f(x) = x^2 2^{-|x|}$

Hence, the correct option is (A).

Question 8 **General Aptitude : Verbal Ability**

Which one of the following options can be inferred from the given passage alone?

When I was a kid, I was partial to stories about other worlds and interplanetary travel. I used to imagine that I could just gaze off into space and be whisked to another planet.

Which option can be inferred from the given passage above?

[Excerpt from *The Truth about Stories* by T. King]

(A) It is a child's description of what he or she likes.

(B) It is an adult's memory of what he or she liked as a child.

(C) The child in the passage read stories about interplanetary travel only in parts.

(D) It teaches us that stories are good for children.

Ans. (B)

Sol. Given :

When I was a kid, I was partial to stories about other worlds and interplanetary travel. I used to imagine that I could just gaze off into space and be whisked to another planet.

As mentioned in the given passage "when I was a kit" it clearly indicates that it is an adult's memory of what he or she liked as child by which option (B) can be inferred.

Hence, the correct option is (B).

Question 9 **General Aptitude : Numerical Ability**

Out of 1000 individuals in the town, 100 unidentified individuals are covid positive. Due to lack of adequate covid-testing kits, the health authorities of the town devised a strategy to identify these covid positive individuals. The strategy is to :

(i) Collect saliva sample from all 1000 individual and randomly group them into sets of 5.

(ii) Mix the samples within each set and test the mixed sample for covid.

(iii) If the test done in (ii) gives a negative result, then declare all the 5 individual to be covid negative.

(iv) If the test done in (ii) give positive result, then all the 5 individuals are separately tested for covid.

Given this strategy, no more than ________ testing kits will be required to identify all the 100 covid positive individuals irrespective of how they are grouped.

(A) 700 (B) 600

(C) 800 (D) 1000

Ans. (A)

Sol. Given :

Number of individuals in town is, 1000

Number of unidentified covid individuals in town is, 100

Due to lack of adequate covid-testing kits, the health authorities of the town devised a strategy to identify these covid positive individuals. The strategy is to :

(i) Collect saliva sample from all 1000 individual and randomly group them into sets of 5.

The number of total set required will be, $1000 \div 5 = 200$,

(ii) Mix the samples within each set and test the mixed sample for covid.

For the testing of one set of 5 people, 1 testing kit required

So, for 200 set total 200 testing kit will required

(iii) If the test done in (ii) gives a negative result, then declare all the 5 individual to be covid negative.

(iv) If the test done in (ii) give positive result, then all the 5 individuals are separately tested for covid.

The maximum number of 100 unidentified individuals are covid positive.

Let, all unidentified covid positive are in different set, then the number of kits required will be,

Number of people in one set × Number of set,

$5 \times 100 = 500$ kits

Maximum number of kits required will be,

$200 + 500 = 700$ kits

no more than **700** testing kits will be required to identify all the 100 covid positive individuals irrespective of how they are grouped.

Hence, the correct option is (A).

Question 10 **General Aptitude : Verbal Ability**

A $100\text{ cm} \times 32\text{ cm}$ rectangular sheet is folded 5 times. Each time the sheet is folded, the long edge aligns with its opposite side. Eventually, the folded sheet is a rectangle of dimensions $100\text{ cm} \times 1\text{ cm}$.

The total number of creases visible when the sheet is unfolded is ______.

(A) 32 (B) 5

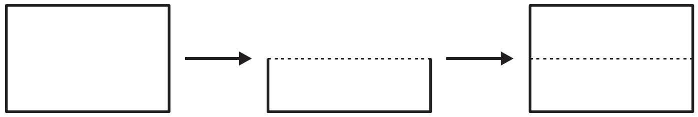
(C) 31 (D) 63

Ans. (C)

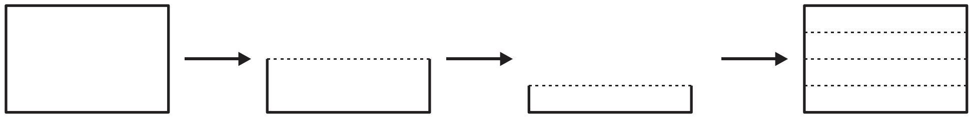
Sol. Given :

A $100\text{ cm} \times 32\text{ cm}$ rectangular sheet is folded 5 times. Each time the sheet is folded, the long edge aligns with its opposite side. Eventually, the folded sheet is a rectangle of dimensions $100\text{ cm} \times 1\text{ cm}$.

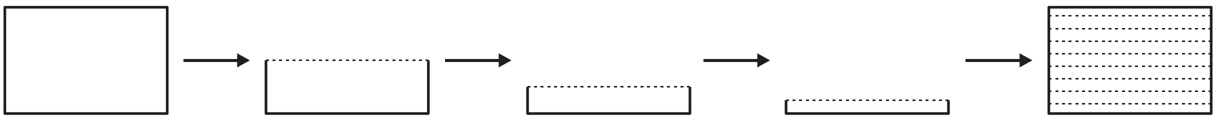
The total number of creases visible when sheet is unfolded can be find by fallowing given procedure, lets take a sheet of paper and fold it for one time and open it, total number of creases visible will be one.



Now, fold the paper for two times and unfold it the number of lines visible will be three.



Now, fold the paper for three times and unfold it, the number of lines visible will be eight.



By doing this we can observe, number of creases visible is equals to $(2^n - 1)$,

Here, n = Number of folds.

$2^5 - 1 = 32 - 1 = 31$

The total number of creases visible when sheet is unfolded is 31.

Hence, the correct option is (C).

Technical Part

Q.11 to Q.35 Carry One Mark Each

Question 11 **Mathematics : Vector Calculus**

Let $V_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $V_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ be two vectors. The value of the coefficient α in the expression $V_1 = \alpha V_2 + e$ which minimizes the length of the error vector e, is

(A) $\frac{7}{2}$

(B) $-\frac{2}{7}$

(C) $\frac{2}{7}$

(D) $-\frac{7}{2}$

Ans. **(C)**

Sol. **Given :** $V_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} V_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

Method 1

$e = V_1 - \alpha V_2$

$e = (i + 2j + 0k) - \alpha(2i + j + 3k)$

$\hat{e} = (1 - 2\alpha)\hat{i} + (2 - \alpha)\hat{j} + (0 - 3\alpha)\hat{k}$

$$|\hat{e}| = \sqrt{(1-2\alpha)^2 + (2-\alpha)^2 + (-3\alpha)^2}$$

$|\hat{e}|^2 = 5 + 14\alpha^2 - 8\alpha$ to be minimum $\dfrac{de^2}{d\alpha} = 28\alpha - 8 = 0$

$\therefore \quad \alpha = \dfrac{2}{7}$ is the stationary point.

Hence, the correct option is (C).

Method 2

$$V_1 = \alpha V_2 + e$$

$$e = V_1 - \alpha V_2$$

$$e = \begin{bmatrix} 1-2\alpha \\ 2-\alpha \\ -3\alpha \end{bmatrix}$$

Length $\|e\| = \sqrt{(1-2\alpha)^2 + (2-\alpha)^2 + (-3\alpha)^2}$

$$\|e\| = \sqrt{14\alpha^2 - 8\alpha + 5}$$

For minimum length of e,

$$\frac{d}{d\alpha}\|e\| = 0$$

$$\frac{28\alpha - 8}{2\sqrt{14\alpha^2 - 8\alpha + 5}} = 0$$

$$\alpha = \frac{2}{7}$$

Hence, the correct option is (C).

Question 12 **Mathematics : Vector Calculus**

The rate of increase, of a scalar field $f(x, y, z) = xyz$, in the direction $v = (2, 1, 2)$ at a point (0, 2, 1) is

(A) $\dfrac{2}{3}$ (B) $\dfrac{4}{3}$

(C) 2 (D) 4

Ans. **(B)**

Sol. **Given :** $f(x, y, z) = xyz$

$$\nabla f = yz\hat{i} + zx\hat{j} + xy\hat{k}$$

$$\nabla f_{(0,2,1)} = 2\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{v} = 2\hat{i} + \hat{j} + 2\hat{k}$$

Directional derivative, $DD = \nabla f \cdot \dfrac{\vec{v}}{|\vec{v}|}$

$$DD = (2\hat{i} + 0\hat{j} + 0\hat{k}).\frac{(2\hat{i} + \hat{j} + 2\hat{k})}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{4}{\sqrt{9}} = \frac{4}{3}$$

Hence, the correct option is (B).

Question 13 **Signals & Systems : DTFT & DFT**

Let $w^4 = 16j$. Which of the following cannot be a value of w?

(A) $2e^{j2\pi/8}$ (B) $2e^{j\pi/8}$

(C) $2e^{j5\pi/8}$ (D) $2e^{j9\pi/8}$

Ans. (A)

Sol. Given : $w^4 = 16j$

$$w = (2)j^{1/4}$$

$$w = 2(0 + j)^{1/4}$$

$$w = 2[e^{j(2n+1)\pi/2}]^{1/4} = 2\left[e^{\frac{j(2n+1)\pi}{8}}\right]$$

For $n = 0$, $w = 2e^{j\pi/8}$

For $n = 2$, $w = 2e^{j5\pi/8}$

For $n = 4$, $w = 2e^{j9\pi/8}$

So, only option (A) cannot be the value of w.

Hence, the correct option is (A).

Question 14 **Mathematics : Complex Variable**

The value of the contour integral, $\oint_c \left(\frac{z+2}{z^2+2z+2}\right)dz$, where the contour C is $\left\{z : \left|z + 1 - \frac{3}{2}j\right| = 1\right\}$, taken in the counter clockwise direction, is

(A) $-\pi(1+j)$ (B) $\pi(1+j)$

(C) $\pi(1-j)$ (D) $-\pi(1-j)$

Ans. (B)

Sol. Given integral is, $I = \int_c \left(\frac{z+2}{z^2+2z+2}\right)dz$

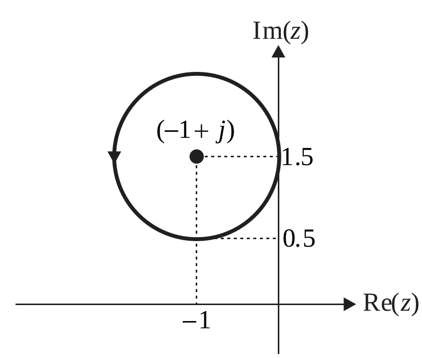
Let $f(z) = \dfrac{z+2}{z^2+2z+2}$

Singular points of $f(z)$ can be obtained by equating denominator to zero,

$$z^2 + 2z + 2 = 0$$

$$z = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$z = \frac{-2 \pm 2j}{2} = -1 \pm j$$



Given curve is, $\left|z+1-\frac{3}{2}j\right|=1$

If $z=-1+j$ then $\left|z+1-\frac{3}{2}j\right|=\left|-1+j+1-\frac{3}{2}j\right|=0.5<1$

So, $z=-1+j$ lies inside the curve

If $z=-1-j$ then $\left|z+1-\frac{3}{2}j\right|=\left|-1-j+1-\frac{3}{2}j\right|=2.5>1$

So, $z=-1-j$ lies outside the curve

$\therefore$ Residue at $z=-1-j$ will be 0

Residue at $z=-1+j$ will be,

$$R=\lim_{z\to -1+j}\frac{(z+2)(z+1-j)}{z^2+2z+2}$$

$$=\lim_{z\to -1+j}\frac{(z+2)(z+1-j)}{(z+1-j)(z+1+j)}=\lim_{z\to -1+j}\frac{z+2}{z+1+j}$$

$$=\frac{-1+j+2}{-1+j+1+j}=\frac{1+j}{2j}$$

$\therefore$ From Cauchy's residue theorem,

$$\oint_c f(z)dz=2\pi jR=2\pi j\times\frac{(1+j)}{2j}=\pi(1+j)$$

Hence, the correct option is (B).

Question 15 **Mathematics : Linear Algebra**

Let the sets of eigenvalues and eigenvectors of a matrix B be $\{\lambda_k \,|\, 1\le k\le n\}$ and $\{V_k \,|\, 1\le k\le n\}$, respectively. For any invertible matrix P, the sets of eigenvalues and eigenvectors of the matrix A, where $B=P^{-1}AP$, respectively are

(A) $\{\lambda_k \det(A) \,|\, 1\le k\le n\}$ and $\{PV_k \,|\, 1\le k\le n\}$

(B) $\{\lambda_k \,|\, 1\le k\le n\}$ and $\{V_k \,|\, 1\le k\le n\}$

(C) $\{\lambda_k \,|\, 1\le k\le n\}$ and $\{PV_k \,|\, 1\le k\le n\}$

(D) $\{\lambda_k \,|\, 1\le k\le n\}$ and $\{P^{-1}V_k \,|\, \le 1\le k\le n\}$

Ans. **(C)**

Sol. **Given :** Matrix B has Eigen value λ_k and Eigen vector V_k.

$\therefore$ $BV_k=\lambda_k V_k$

Also given $B=P^{-1}AP$

$(P^{-1}AP)V_k=\lambda_k V_k$

Multiplying with P on both sides, we get

$A(PV_k)=\lambda_k(PV_k)$

$\therefore$ Eigen value of $A=\lambda_k$

Eigen vector of matrix $A=PV_k$

Hence, the correct option is (C).

Question 16 **Electronic Devices & Circuits : Basic Semiconductor Physics**

In a semiconductor, if the Fermi energy level lies in the conduction band, then the semiconductor is known as

(A) degenerate n type (B) degenerate p-type

(C) non-degenerate n type (D) non-degenerate p-type

Ans. **(A)**

Sol. Given that the fermi level lies in the conduction band. So, it will be a degenerate n- type semiconductor.

Hence, the correct option is (A).

Question 17 **Electronic Devices & Circuits : Basic Semiconductor Physics**

For an intrinsic semiconductor at $T = 0$ K, which of the following statement is true?

(A) All energy states in the valence band are filled with electrons and all energy states in the conduction band are empty of electrons.

(B) All energy states in the valence band are empty of electrons and all energy states in the conduction band are filled with electrons.

(C) All energy states in the valence and conduction band are filled with holes.

(D) All energy states in the valence and conduction band are filled with electrons.

Ans. **(A)**

Sol. For an intrinsic semi-conductor at $T = 0^0\text{K}$, the valence band will be filled with electrons and conduction band will be empty.

Hence, the correct option is (A).

Question 18 **Network Theory : Resonance**

A series RLC circuit has a quality factor Q of 1000 at a center frequency of 10^6 rad/sec. The possible value of R, L and C are

(A) $R = 1\Omega, L = 1\mu\text{H}$ and $C = 1\mu\text{F}$ (B) $R = 0.1\Omega, L = 1\mu\text{H}$ and $C = 1\mu\text{F}$

(C) $R = 0.01\Omega, L = 1\mu\text{H}$ and $C = 1\mu\text{F}$ (D) $R = 0.001\Omega, L = 1\mu\text{H}$ and $C = 1\mu\text{F}$

Ans. **(D)**

Sol. **Given :** $Q = 1000,\ \omega_0 = 10^6$ rad/sec

Quality factor and resonant frequency of series RLC circuit is given by,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \text{ and } \omega_0 = \frac{1}{\sqrt{LC}}$$

Checking from options for options :

For option (A) :

$$L = 1\ \mu\text{H}, C = 1\ \mu\text{F},\ \ R = 1\ \Omega$$

$$Q = \frac{1}{1}\sqrt{\frac{10^{-6}}{10^{-6}}}$$

$$Q = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-12}}} = 10^6 \text{ rad/sec}$$

For option (B) :

$L = 1\ \mu\text{H}, C = 1\ \mu\text{F},\ \ R = 0.1\ \Omega$

$$Q = \frac{1}{0.1}\sqrt{\frac{10^{-6}}{10^{-6}}} = 10$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6}\times 10^{-6}}} = 10^6\ \text{rad/sec}$$

For option (C) :

$R = 0.01\ \Omega,\ L = 1\ \mu\text{H},\ C = 1\ \mu\text{F}$

$$Q = \frac{1}{0.01}\sqrt{\frac{10^{-6}}{10^{-6}}} = 100$$

$$\omega_0 = \frac{1}{\sqrt{10^{-6}\times 10^{-6}}} = 10^6\ \text{rad/sec}$$

For option (D) :

$R = 0.001\ \Omega,\ L = 1\mu\text{H},\ C = 1\ \mu\text{F}$

$$Q = \frac{1}{0.001}\sqrt{\frac{10^{-6}}{10^{-6}}} = 1000$$

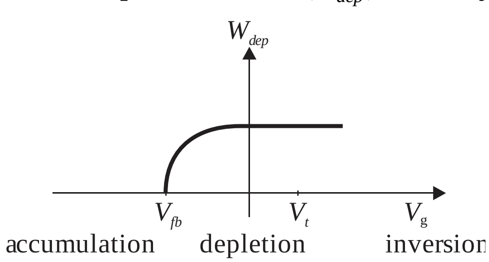
$$\omega_0 = \frac{1}{\sqrt{10^{-6}\times 10^{-6}}} = 10^6\ \text{rad/sec}$$

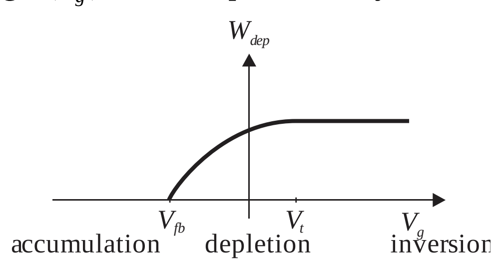
So, option (D) satisfies the condition given in the question.

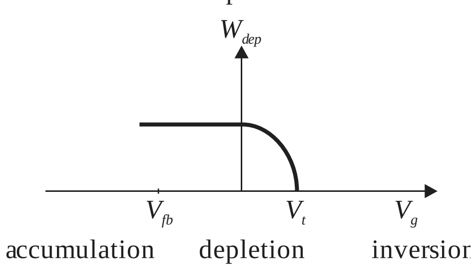
Hence, the correct option is (D).

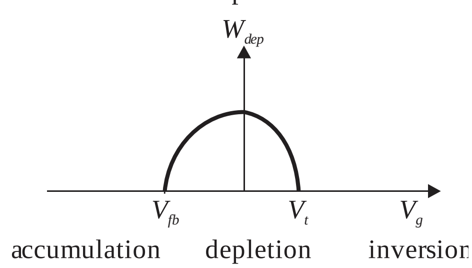
Question 19 **Electronic Devices & Circuits : MOS Capacitor**

For a MOS capacitor V_{fb} and V_t are the flat-band voltage and threshold voltage, respectively. The variation of depletion width (W_{dep}) for varying gate voltage (V_g) is best represented by

(A) 

(B) 

(C) 

(D) 

Ans. (B)

Sol. (i) For $V_G < V_{FB}$

Accumulation mode

$\downarrow$

In this mode only accumulation of charges occurs

$\downarrow$

So, no depletion charges present

$\downarrow$

So this, no depletion width $W_d = 0$

(ii) For $V_{FB} < V_G < V_T$

It is depletion mode

$\downarrow$

Here depletion width is available and nature of depletion width (W_d) is of increasing with V_G

(iii) For $V_G > V_T$

It is in inversion mode

$\downarrow$

No further depletion of charges possible

$\downarrow$

Depletion width $(W_d) \equiv$ constant

Only option (B) satisfies the above conditions.

Hence, the correct option is (B).

Question 20 **Electromagnetic Theory : Plane Wave Propagation**

Consider a narrow band signal, propagating in a lossless dielectric medium $(\varepsilon_r = 4,\ \mu_r = 1)$, with phase velocity v_p and group velocity v_g. Which of following statement is true? (c is the velocity of light in vacuum)

(A) $v_p > c,\ v_g > c$

(B) $v_p < c, v_g > c$

(C) $v_P > c,\ v_g < c$

(D) $v_P < c,\ v_g < c$

Ans. **(D)**

Sol. Phase velocity,

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\in}}$$

$$v_p = \frac{1}{\sqrt{\mu\in}} = \frac{c}{\sqrt{\mu_r \in_r}}$$

Group velocity,

$$v_g = \frac{d\omega}{d\beta} = \frac{v_p}{1 - \left(\frac{\omega}{v_p}\right)\left(\frac{dv_p}{d\omega}\right)}$$

$\because$ Here $v_p \neq f(\omega)$

$$v_p = v_g$$

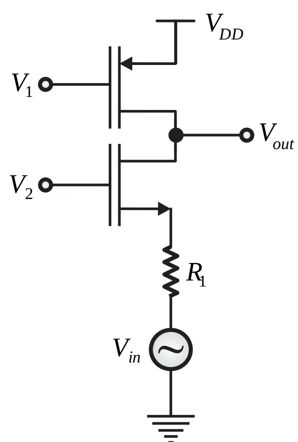
So, $v_p < c$

$$v_g < c$$

Hence, the correct option is (D).

Question 21 **Analog Electronics : JFET and MOSFET Amplifier with Biasing**

In the circuit shown below, V_1 and V_2 are bias voltages. Based on input and output impedances, the circuit behaves as a



(A) voltage controlled voltage source (B) voltage controlled current source

(C) current controlled voltage source (D) current controlled current source

Ans. **(D)**

Sol. Given circuit acts as a common gate amplifier. Common gate amplifier has low input impedance and high output impedance

$\therefore$ The given circuit acts as a current controlled current source.

Hence, the correct option is (D).

Question 22 **Analog Electronics : Feedback Amplifiers**

A cascade of common source amplifiers in a unity gain feedback configuration oscillates when

(A) the closed loop gain is less than 1 and the phase shift is less than 180^0

(B) the closed loop gain is greater than 1 and the phase shift is less than 180^0

(C) the closed loop gain is less than 1 and the phase shift is greater than 180^0

(D) the closed loop gain is greater than 1 and the phase shift is greater than 180^0

Ans. **(D)**

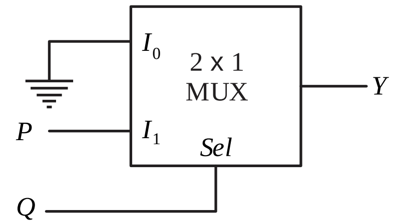
Sol. When cascaded amplifier acts as oscillator, then increasing oscillations will be observed.

$\therefore$ Closed loop gain >1 and phase shift $=360^0$

Hence, the correct option is (D).

Question 23 **Digital Electronics : Combinational Circuits**

In the circuit shown below, P and Q are the inputs. The logical function realized by the circuit shown below is



(A) $Y = PQ$

(B) $Y = P + Q$

(C) $Y = \overline{PQ}$

(D) $Y = \overline{P+Q}$

Ans. **(A)**

Sol. The output of MUX is given as, $Y = \bar{S}I_0 + SI_1$

Given : $S = Q,\ I_0 = 0,\ I_1 = P$

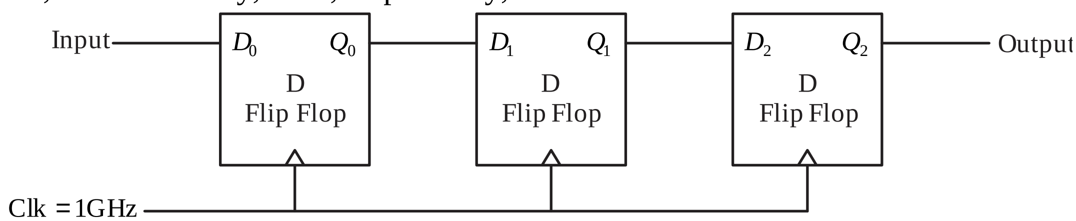
$Y = \overline{Q}.0 + Q.P$

$Y = PQ$

Hence, the correct option is (A).

Question 24 **Digital Electronics : Sequential Circuits**

The synchronous sequential circuit shown below works at a clock frequency of 1 GHz. The throughput, in Mbits/s, and the latency, in ns, respectively, are



(A) 1000, 3

(B) 333.33, 1

(C) 2000, 3

(D) 333.33, 3

Ans. **(A)**

Sol. Given circuit is Serial In Serial Out shift register,

$f_{clk} = 1\text{GHz}$

$$T_{clk} = \frac{1}{f_{clk}} = \frac{1}{10^9} = 10^{-9}\text{ sec}$$

$T_{clk} = 1\text{n sec}$

For Serial In Serial Out shift register,

Latency is given by NT_{clk}, where N is number of Flip-Flops.

Latency $= NT_{\text{clk}} = 3\times 1\text{ns} = 3\text{ns}$

Throughput is given by number of bits per second.

In this circuit, we get 1 bit of output every $T_{\text{clk}} (=1\text{ns})$

So, we get 1 bit every 1 nsec.

$\Rightarrow 10^9$ bits every second

$\Rightarrow 10^3$ Mbits per second

Hence, the correct option is (A).

Question 25 **Control Systems : Polar Plot**

The open loop transfer function of a unity negative feedback system is $G(s)=\dfrac{K}{s(1+sT_1)(1+sT_2)}$, where K, T_1 and T_2 are positive constants. The phase cross-over frequency, in rad/sec, is

(A) $\dfrac{1}{\sqrt{T_1T_2}}$ (B) $\dfrac{1}{T_1T_2}$

(C) $\dfrac{1}{T_1\sqrt{T_2}}$ (D) $\dfrac{1}{T_2\sqrt{T_1}}$

Ans. (A)

Sol. Given open loop transfer function is, $G(s)=\dfrac{K}{s(1+sT_1)(1+sT_2)}$

At phase cross over frequency, $\angle G(j\omega) = -180^0$

$$\angle G(j\omega) = -90^0 - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$$

$$\angle G(j\omega) = -180^0 \text{ at } \omega=\omega_{pc}$$

$$-90^0 - \tan^{-1}\omega_{pc}T_1 - \tan^{-1}\omega_{pc}T_2 = -180^0$$

$$90^0 = \tan^{-1}\omega_{pc}T_1 + \tan^{-1}\omega_{pc}T_2$$

$$90^0 = \tan^{-1}\left[\frac{\omega_{pc}T_1+\omega_{pc}T_2}{1-\omega_{pc}^2T_1T_2}\right]$$

$$\tan 90^0 = \frac{\omega_{pc}T_1+\omega_{pc}T_2}{1-\omega_{pc}^2T_1T_2}$$

$$1-\omega_{pc}^2T_1T_2 = 0$$

$$\omega_{pc}^2T_1T_2 = 1$$

$$\omega_{pc} = \frac{1}{\sqrt{T_1T_2}} \text{ rad/sec}$$

Hence, the correct option is (A).

Question 26 **Signals & Systems : Classification of Systems**

Consider a system with input $x(t)$ and output $y(t) = x(e^t)$. The system is

(A) Causal & time invariant
(B) Non-causal & time varying
(C) Causal & time varying
(D) Non-causal & time invariant

Ans. (B)

Sol. **Given :** $y(t) = x(e^t)$

To check time invariance,

$$y(t,t_0) = T\{x(t-t_0)\} = x(e^t - t_0) \quad \ldots\text{(i)}$$

$$y(t-t_0) = x(e^{t-t_0}) \quad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$y(t,t_0) \neq y(t-t_0)$$

So, given system is time variant

Let $t = 3$ sec

$$y(3) = x(e^3)$$

According to above equation, output depends on future inputs.

So, given system is non-causal.

Hence, the correct option is (B).

Question 27 **Signals & Systems : Continuous Time Fourier Transform**

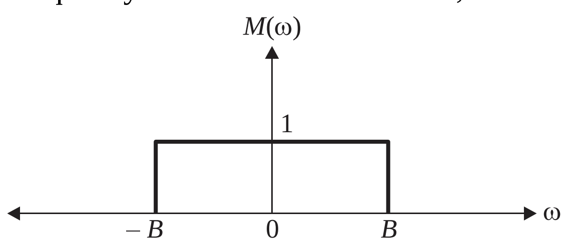
Let $m(t)$ be a strictly band limited signal with bandwidth B and energy E. Assuming $\omega_0 = 10B$, the energy in the signal $m(t)\cos\omega_0 t$ is

(A) $\dfrac{E}{4}$
(B) $\dfrac{E}{2}$
(C) E
(D) $2E$

Ans. (B)

Sol. Given that $m(t)$ is a bandlimited signal with bandwidth B and energy E.

Let $m(t)$ be represented in frequency domain as shown below,



Energy of $m(t)$ will be,

$$E = \frac{1}{2\pi}\int_{-B}^{B} 1^2\, d\omega = \frac{2B}{2\pi} = \frac{B}{\pi} \quad \ldots\text{(i)}$$

Let $y(t) = m(t)\cos\omega_0 t$

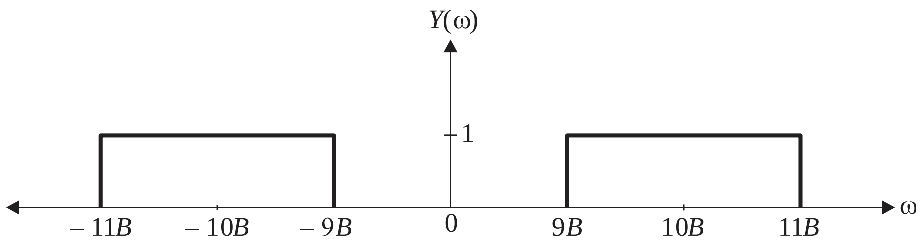
Given : $\omega_0 = 10B$

So, $y(t) = m(t)\cos 10Bt$

Taking Fourier transform on both sides, we get

$$Y(\omega) = \frac{1}{2}[M(\omega - 10B) + M(\omega + 10B)]$$

$Y(\omega)$ can be plotted as shown below,



Now, energy of $y(t)$ will be,

$$E_y = 2\left[\frac{1}{2\pi}\int_{9B}^{11B} 1^2\, d\omega\right]$$

$$E_y = \frac{1}{\pi}[11B - 9B] = \frac{2B}{\pi} \qquad \ldots\text{(ii)}$$

So, from equation (i) and (ii),

$$E_y = \frac{E}{2}$$

Hence, the correct option is (B).

Question 28 **Signals & Systems : Continuous Time Fourier Transform**

The Fourier transform $X(\omega)$ of $x(t) = e^{-t^2}$ is

Note : $\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$

(A) $\sqrt{\pi}\, e^{\frac{\omega^2}{2}}$

(B) $\dfrac{e^{\frac{-\omega^2}{4}}}{2\sqrt{\pi}}$

(C) $\sqrt{\pi}\, e^{\frac{-\omega^2}{4}}$

(D) $\sqrt{\pi}\, e^{\frac{-\omega^2}{2}}$

Ans. **(C)**

Sol. We know that, $e^{-\pi t^2} \longleftrightarrow e^{-\pi f^2} = e^{-\pi\left(\frac{\omega}{2\pi}\right)^2} = e^{\frac{-\omega^2}{4\pi}}$

$$e^{-\pi t^2} \longleftrightarrow e^{\frac{-\omega^2}{4\pi}}$$

Put $t = \dfrac{t}{\sqrt{\pi}}$

$$e^{-\pi\left(\frac{t}{\sqrt{\pi}}\right)^2} \longleftrightarrow \sqrt{\pi} e^{\frac{-\omega^2(\sqrt{\pi})^2}{4\pi}}$$

$$\left[x(at) \longleftrightarrow \frac{1}{|a|} \times \left(\frac{\omega}{a}\right)\right]$$

$$e^{-t^2} \longleftrightarrow \sqrt{\pi}e^{\frac{-\omega^2}{4}}$$

Hence, the correct option is (C).

Question 29 **Signals & Systems : Continuous Time Fourier Transform**

In the table shown below, match the signal type with its spectral characteristics.

	Signal type		Spectral characteristics
(i)	Continuous, aperiodic	(a)	Continuous, aperiodic
(ii)	Continuous, periodic	(b)	Continuous, periodic
(iii)	Discrete, aperiodic	(c)	Discrete, aperiodic
(iv)	Discrete, periodic	(d)	Discrete, periodic

(A) (i)-(a), (ii)-(b), (iii)-(c), (iv)-(d)
(B) (i)-(a), (ii)-(c), (iii)-(b), (iv)-(d)
(C) (i)-(d), (ii)-(b), (iii)-(c), (iv)-(a)
(D) (i)-(a), (ii)-(c), (iii)-(d), (iv)-(b)

Ans. (B)

Sol. (1) A continuous and aperiodic signal has continuous and aperiodic spectrum (Fourier Transform).

(2) A continuous and periodic signal has discrete and aperiodic spectrum (Continuous Time Fourier Series).

(3) A discrete and aperiodic signal has continuous and periodic spectrum (Discrete Time Fourier Transform).

(4) A discrete and periodic signal has discrete and periodic spectrum (Discrete Fourier Transform/Discrete Time Fourier Series).

Hence, the correct option is (B).

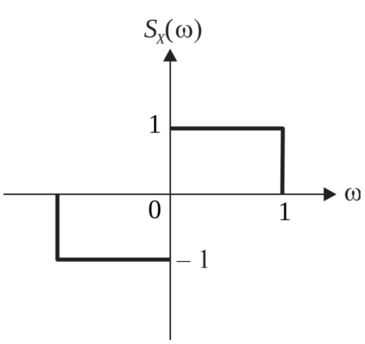
Question 30 **Communication Systems : Random Variables and Random Processes**

For a real signal, which of the following is/are valid power spectral density/densities?

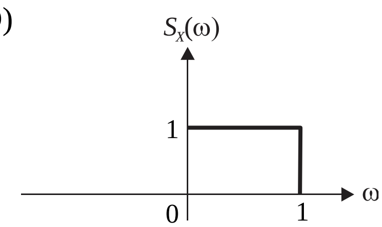
(A) $S_X(\omega) = \dfrac{2}{9+\omega^2}$

(B) $S_X(\omega) = e^{-\omega^2}\cos^2\omega$

(C)



(D)



Ans. (A), (B)

Sol. From the properties of power spectral density $S_X(-\omega) = S_X(\omega) \forall \omega$ and $S_X(\omega) \geq 0 \forall \omega$.

Consider option (A) :

$$S_X(\omega) = \frac{2}{9+\omega^2}$$

For any value of ω,

$S_X(\omega)$ is positive. So, $S_X(\omega) > 0$

$$S_X(-\omega) = \frac{2}{9+(-\omega)^2} = \frac{2}{9+\omega^2} = S_X(\omega)$$

So, option (A) is a valid power spectral density.

Consider option (B) :

$$S_X(\omega) = e^{-\omega^2}\cos^2\omega$$

At $\omega = \frac{\pi}{2}$, $S_X\left(\frac{\pi}{2}\right) = e^{-\left(\frac{\pi}{2}\right)^2}\cos^2\frac{\pi}{2} = 0$

For any other value of ω,

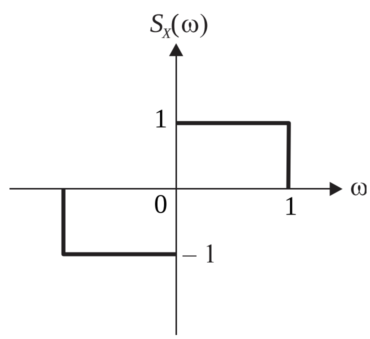
$S_X(\omega)$ is positive. So, $S_X(\omega) \ge 0$

$$S_X(-\omega) = e^{-(-\omega)^2}\cos^2(\omega) = e^{-\omega^2}\cos^2\omega$$

$$S_X(-\omega) = S_X(\omega)$$

So, option (B) is a valid power spectral density.

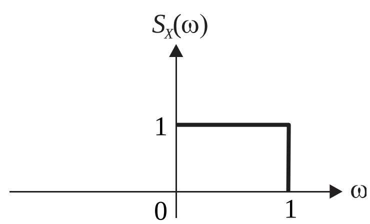
Consider option (C) :



From the above plot, $S_X(\omega)$ is negative for some values of ω and odd function.

So, option (C) is not a valid power spectral density.

Consider option (D) :



From the above plot, $S_X(\omega)$ is positive but not an even function, that is $S_X(\omega) \ge 0$ but $S_X(-\omega) \ne S_X(\omega)$

So, option (D) is not a valid power spectral density.

Hence, the correct options are (A) & (B).

Question 31 **Digital Electronics : ADC and DAC**

The signal-to-noise ratio (SNR) of an ADC with a full-scale sinusoidal input is given to be 61.96 dB. The resolution of the ADC is _______ bits. (rounded off to nearest integer)

Ans. 10 (10 to 10)

Sol. Given :

SNR of ADC $= 61.96\,\text{dB}$

SNR $= 1.76 + 6.02n$ dB

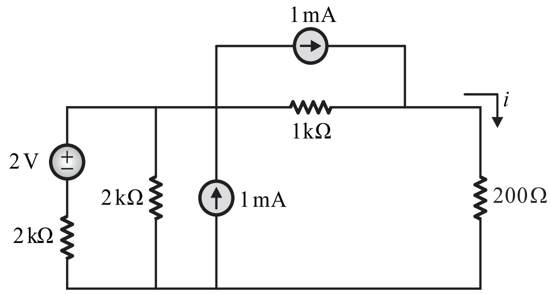
$61.96 = 1.76 + 6.02n$

$n = 10$

Hence, the correct answer is 10.

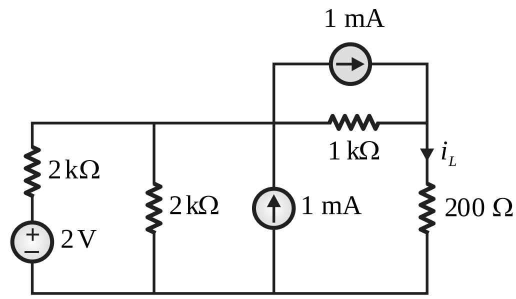
Question 32 **Network Theory : Basic Concepts of Networks**

In the circuit shown below, the current i flowing through $200\,\Omega$ resistor is ________ mA (rounded off to two decimal places).

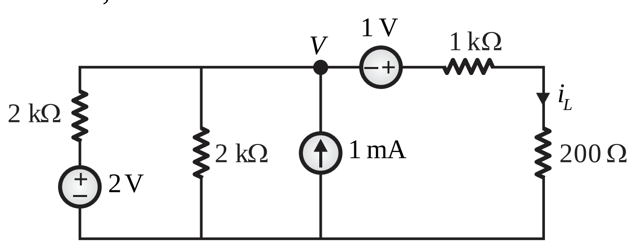


Ans. 1.36 (1.30 to 1.40)

Sol. Given circuit is,



Applying source transformation,



Applying nodal analysis at node V,

$$\frac{V-2}{2000} + \frac{V}{2000} - 10^{-3} + \frac{V+1}{1200} = 0$$

$V = 0.637\text{V}$

Current through 200Ω is,

$$i_L = \frac{V+1}{1200} = \frac{0.637+1}{1200}$$

$$i_L = 1.36\,\text{mA}$$

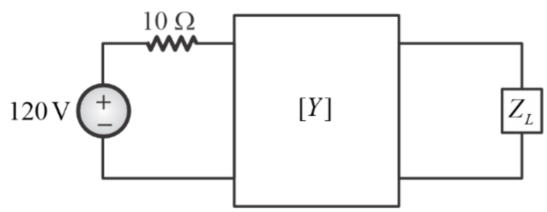
Hence, the correct answer is 1.36.

Question 33 **Network Theory : Two port Networks**

For the two port network below, the [Y]-parameters is given as

$$[\text{Y}] = \frac{1}{100}\begin{bmatrix} 2 & -1 \\ -1 & \frac{4}{3} \end{bmatrix}\text{S}$$

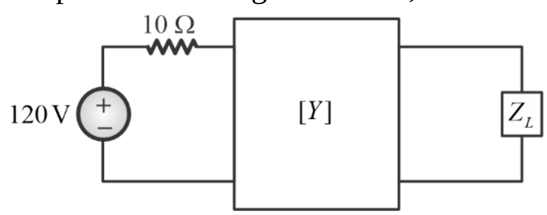
The value of load impedance Z_L (in Ω), for maximum power transfer will be _______ (rounded off to the nearest integer).



Ans. **80 (80 to 80)**

Sol. **Method 1**

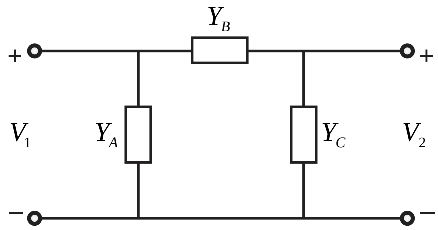
Given two-port network and Y-parameters are given below,



$$[Y] = \frac{1}{100}\begin{bmatrix} 2 & -1 \\ -1 & \frac{4}{3} \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{2}{100} & \frac{-1}{100} \\ \frac{-1}{100} & \frac{4}{300} \end{bmatrix}$$

Obtaining π-network from given $[Y]$ parameters



$$Y_{11} = Y_A + Y_B = \frac{2}{100}$$

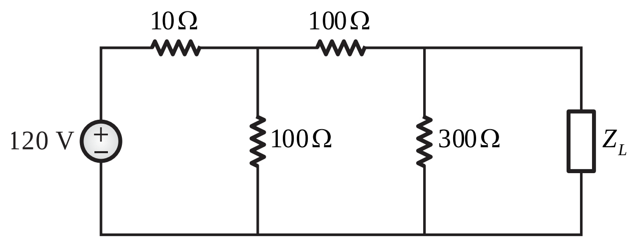
$$Y_{12} = Y_{21} = -Y_B = \frac{-1}{100}$$

$$Y_{22} = Y_B + Y_C = \frac{4}{300}$$

From the above equations,

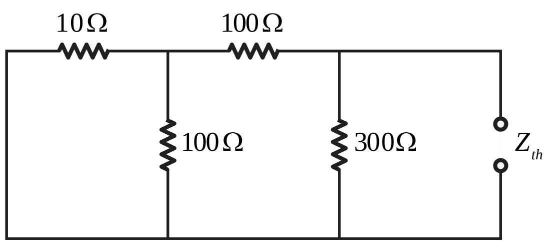
$$Y_A = \frac{1}{100} S,\ Y_B = \frac{1}{100} S,\ Y_C = \frac{1}{300} S$$

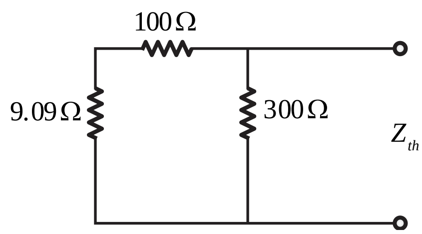
Now replacing this pie network in two port network.



For maximum power transfer, $Z_L = Z_{th}$.

For Z_{th}, circuit becomes.





$$Z_{th} = 300 \| 109.09 = 79.99\Omega$$

$$Z_{th} = 80\Omega$$

For maximum power transfer

$$Z_L = Z_{th} = 80\Omega$$

Hence, the correct answer is 80.

Method 2

Given : $[Y] = \frac{1}{100}\begin{bmatrix} 2 & -1 \\ -1 & \frac{4}{3} \end{bmatrix}$ S

To find the value of Z_L for maximum power transfer, all the independent sources will be disabled and the given 2-port network can be drawn as shown below,

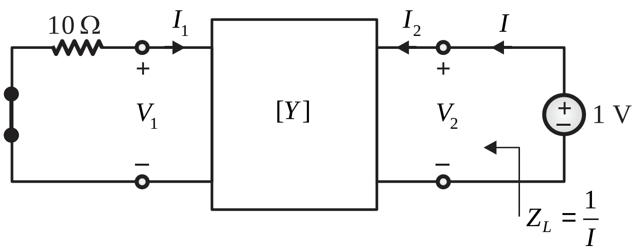


Figure (a) : Two-port network for maximum power transfer to Z_L

Equations for Y-parameter's are,

$$I_1 = \frac{2}{100}V_1 - \frac{1}{100}V_2 \quad \text{...(i)}$$

$$I_2 = \frac{-1}{100}V_1 - \frac{4}{300}V_2 \quad \text{...(ii)}$$

From figure (A),

$$V_1 = -10I_1 \Rightarrow I_1 = \frac{-V_1}{10} \quad \text{...(iii)}$$

$$V_2 = 1 \quad \text{...(iv)}$$

$$I_2 = I \quad \text{...(v)}$$

Substituting equation (iii) and (iv) in equation (i), we get

$$-\frac{V_1}{10} = \frac{2}{100}V_1 - \frac{1}{100}$$

$$\frac{12V_1}{100} = \frac{1}{100}$$

$$12V_1 = 1$$

$$V_1 = \frac{1}{12} \quad \text{...(vi)}$$

Substituting equation (v) and (vi) in equation (ii), we get

$$I = -\frac{1}{100}\times\frac{1}{12} + \frac{4}{300}$$

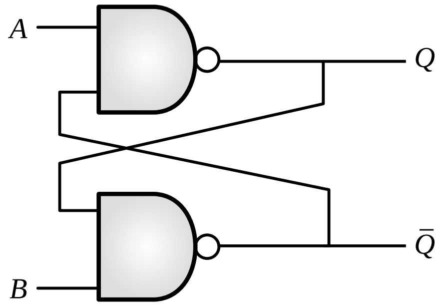
$$I = \frac{16-1}{1200} = \frac{15}{1200}$$

$$\therefore \quad Z_L = \frac{1}{I} = \frac{1200}{15} = 80$$

Hence, the correct answer is 80.

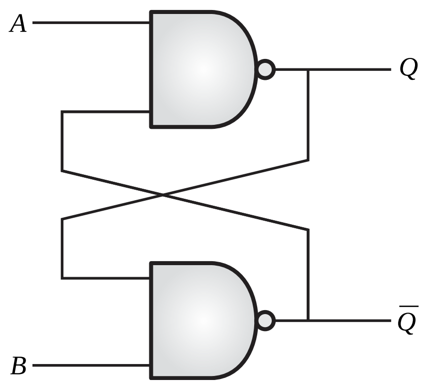
Question 34 **Digital Electronics : Logic Gates**

For the circuit shown below, the propagation delay of each NAND gate is 1 ns. The critical path delay, in ns, is _______ (rounded off to the nearest integer).

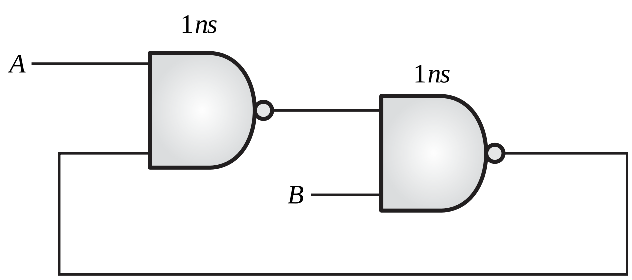


Ans. **2 (02 to 02)**

Sol. Given circuit is



Redrawing the circuit as

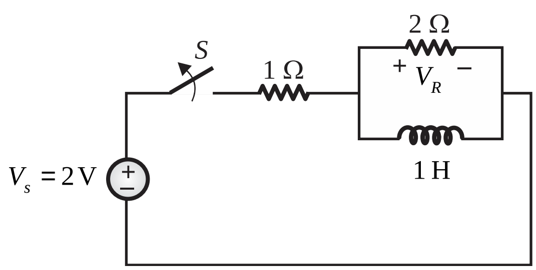


Critical path delay $= 1 + 1 = 2\,\text{ns}$

Hence, the correct answer is 2.

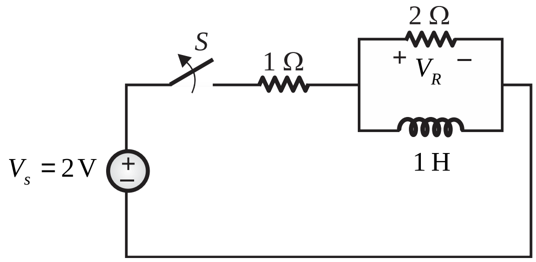
Question 35 **Network Theory : Transient Analysis**

In the circuit shown below, switch S was closed for a long time. If switch is opened at $t = 0$, the maximum magnitude of the voltage V_R, in volts, is ______ (rounded off to the nearest integer).

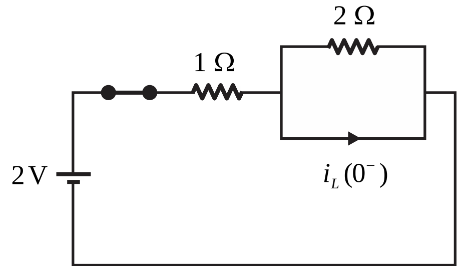


Ans. **4 (04 to 04)**

Sol. Given circuit is

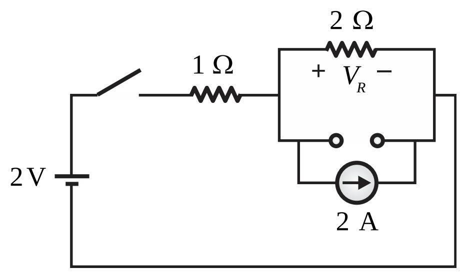


At $t = 0^-$, switch is closed circuit is in steady state and inductor acts as S.C.



$i_L(0^-) = 2$ A

At $t = 0^+$, switch is open and inductor acts as O.C.



$V_R = 2\times -2 = -4V$

Magnitude of $V_R = |-4|$

$|V_R| = 4V$

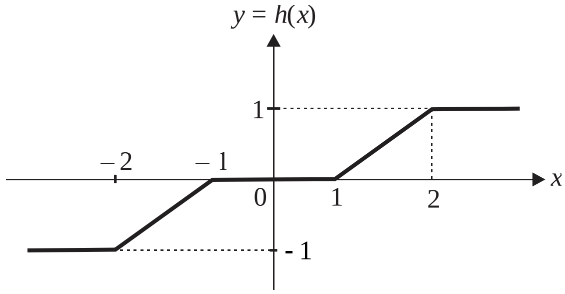
Hence, the correct answer is 4.

Q.36 to Q.65 Carry Two Marks Each

Question 36 **Communication Systems : Random Variables & Random Processes**

A random variable X, distributed normally as $N(0, 1)$, undergoes the transformation $Y = h(X)$, given in the figure. The form of probability density function of Y is

(In the options given below, a, b, c non-zero constant and $g(y)$ is piecewise continuous function)



(A) $a\delta[y-1]+b\delta[y+1]+g[y]$ (B) $a\delta[y+1]+b\delta[y]+c\delta[y-1]+g[y]$

(C) $a\delta[y+2]+b\delta[y]+c\delta[y-2]+g[y]$ (D) $a\delta[y+2]+b\delta[y-2]+g[y]$

Ans. **(B)**

Sol. Given that the random variable *X*, undergoes the transformation $Y=h(X)$.

From the given figure, it can be concluded that, *Y* takes the discrete set of values {– 1, 0, 1}.

So, probability density function of *Y* will consist of impulses at $y=-1$, $y=0$ and $y=1$.

$\therefore$ The probability density function of *Y* can be represented as,

$$f(y)=a\delta[y+1]+b\delta[y]+c\delta[y-1]+g[y]$$

Hence, the correct option is (B).

Question 37 **Mathematics : Integral & Differential Calculus**

The value of the line integral $\int_P^Q (z^2dx+3y^2dy+2xz\,dz)$ along the straight line joining the points $P(1,1,2)$ and $Q(2,3,1)$ is

(A) 20 (B) 24

(C) 29 (D) -5

Ans. **(B)**

Sol. **Method 1**

Given integral is, $I=\int_P^Q (z^2dx+3y^2dy+2xzdz)$

Also given points $P(1,1,2)$ and $Q(2,3,1)$

Equation of line joining the points P and Q is,

$$\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{-1}$$

Let $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{-1}=t$

$x=t+1\Rightarrow dx=dt$

$y=2t+1\Rightarrow dy=2dt$

$z=2-t\Rightarrow dz=-dt$

t limits $t=0$ to $t=1$

$$I=\int_{t=0}^{1}[(2-t)^2+3\times 2(2t+1)-2(t+1)(2-t)]dt$$

$$I = \int_{t=0}^{1} (27t^2 + 18t + 6)dt$$

$$I = \left[\frac{27t^3}{3} + \frac{18t^2}{2} + 6t\right]_0^1$$

$$I = \frac{27}{3} + \frac{18}{2} + 6 = 24$$

Hence, the correct option is (B).

Method 2

Given integral is, $I = \int_P^Q (z^2 dx + 3y^2 dy + 2xzdz)$

$$I = \int_P^Q (d(xz^2) + 3y^2 dy) = \int_{x=1,z=2}^{x=2,z=1} d(xz^2) + \int_{y=1}^{y=3} 3y^2 dy$$

$$I = (xz^2)_{x=1,z=2}^{x=2,z=1} + 3\left(\frac{y^3}{3}\right)_1^3 = 2(1)^2 - 1(2)^2 + 3\left[\frac{27}{3} - \frac{1}{3}\right]$$

$$I = 2 - 4 + 26 = 24$$

Hence, the correct option is (B).

Question 38 **Mathematics : Linear Algebra**

Let x be an $n \times 1$ real column vector with length $l = \sqrt{x^T x}$. The trace of matrix $P = xx^T$ is

(A) l^2

(B) $\frac{l^2}{4}$

(C) l

(D) $\frac{l^2}{2}$

Ans. **(A)**

Sol. Given that x is a $n \times 1$ column vector.

Let, $x = \begin{bmatrix} x_1 \\ x_2 \\ : \\ x_n \end{bmatrix}$

$$x^T = [x_1 x_2 .... x_n]$$

Given length of vector x.

$$l = \sqrt{x^T x}$$

$$l^2 = x^T x$$

$$l^2 = [x_1 x_2 \ldots\ldots\ldots\ldots x_n]\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$l^2 = x_1^2 + x_2^2 + \ldots\ldots + x_n^2 \qquad \ldots(i)$$

Now we have to find trace of matrix, $P = xx^T$

$$P = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}[x_1 x_2 \ldots\ldots\ldots x_n]$$

$$P = \begin{bmatrix} x_1^2 & x_1x_2 \ldots\ldots\ldots x_1x_n \\ x_2x_1 & x_2^2 \ldots\ldots\ldots\ldots x_2x_n \\ \vdots & \\ x_nx_1 & x_nx_2 \ldots\ldots\ldots\ldots x_n^2 \end{bmatrix}$$

$\therefore$ Trace of matrix P will be, $tr(P) = x_1^2 + x_2^2 + \ldots\ldots + x_n^2 \qquad \ldots(ii)$

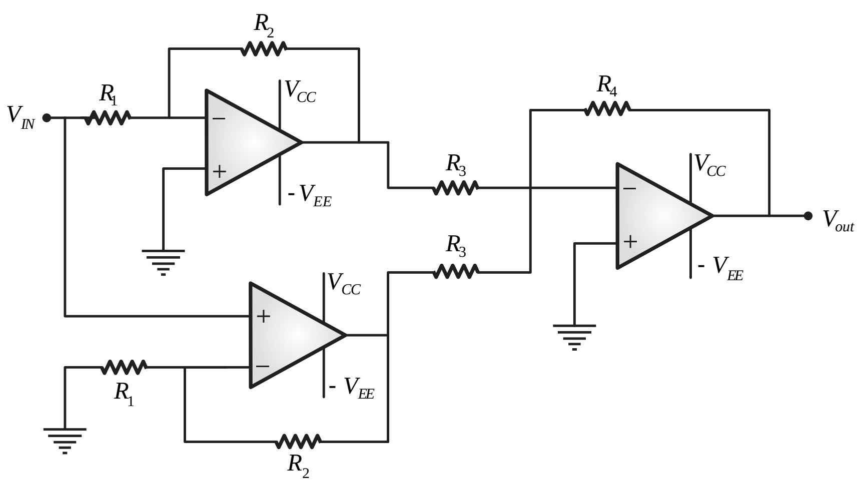
From equation (i) and (ii),

$tr(P) = l^2$

Hence, the correct option is (A).

Question 39 **Analog Electronics : Operational Amplifiers**

The $\dfrac{V_{out}}{V_{in}}$ of the circuit shown below is



(A) $-\dfrac{R_4}{R_3}$

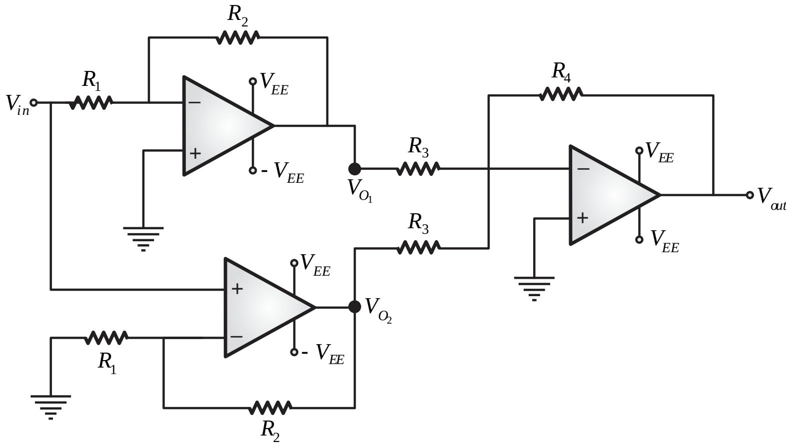
(B) $\dfrac{R_4}{R_3}$

(C) $1+\dfrac{R_4}{R_3}$

(D) $1-\dfrac{R_4}{R_3}$

Ans. (A)

Sol. Given circuit can be drawn as shown below



From the above figure, $V_{01} = -\frac{R_2}{R_1} \times V_{in}$ ...(i)

$$V_{02} = \left(1 + \frac{R_2}{R_1}\right) V_{in} \quad \text{...(ii)}$$

$$\therefore \quad V_{out} = -\frac{R_4}{R_3} V_{01} - \frac{R_4}{R_3} V_{02}$$

$$V_{out} = \frac{-R_4}{R_3}\left[V_{01} + V_{02}\right] \quad \text{...(iii)}$$

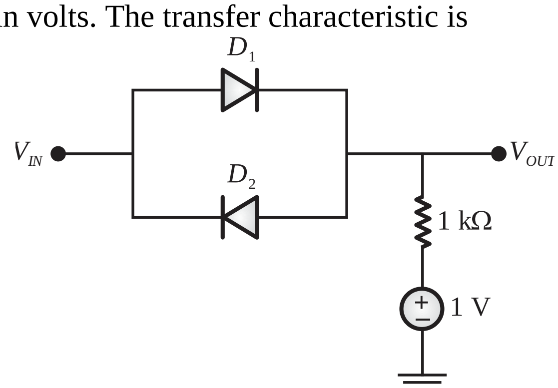
Substituting equations (i) and (ii) in equation (iii), we get

$$V_{out} = \frac{-R_4}{R_3}\left[\frac{-R_2}{R_1} V_{in} + V_{in} + \frac{R_2}{R_1} V_{in}\right] = \frac{-R_4}{R_3} V_{in} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{-R_4}{R_3}$$

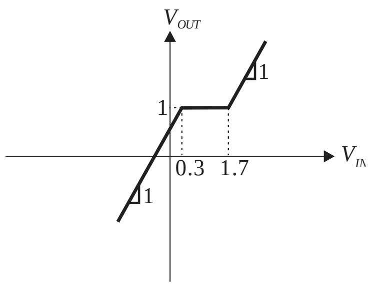
Hence, the correct option is (A).

Question 40 **Analog Electronics : Diode Circuits & Applications**

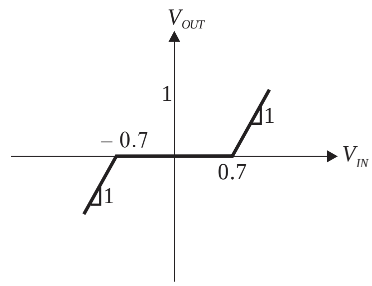
In the circuit shown below, D_1 and D_2 are silicon diodes with cut-in voltage of 0.7 V. V_{IN} and V_{OUT} are input and output voltages in volts. The transfer characteristic is



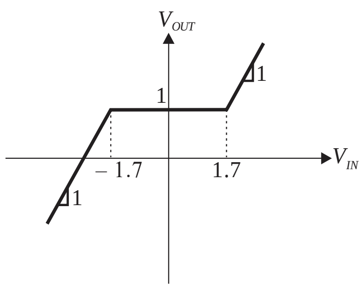
(A)



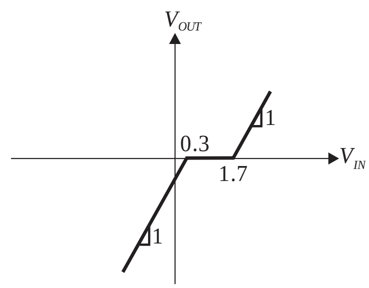
(B)



(C)

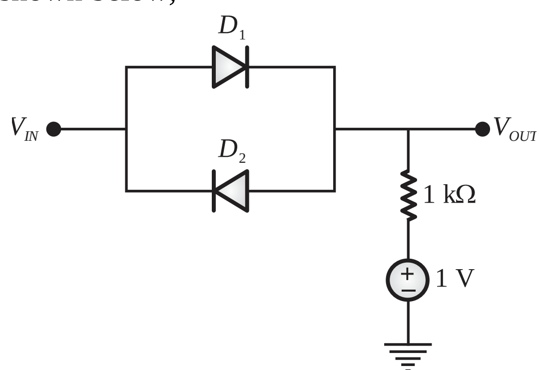


(D)

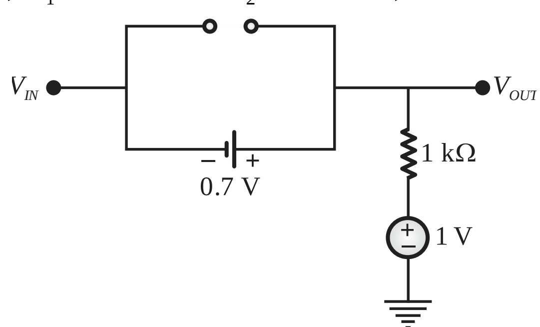


Ans. (A)

Sol. Given : Given circuit is as shown below,

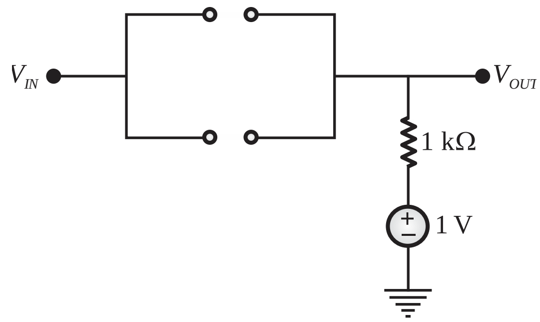


Case 1 : When $V_{IN} < 0.3\text{ V}$, D_1 is OFF and D_2 is ON. So, the circuit becomes,



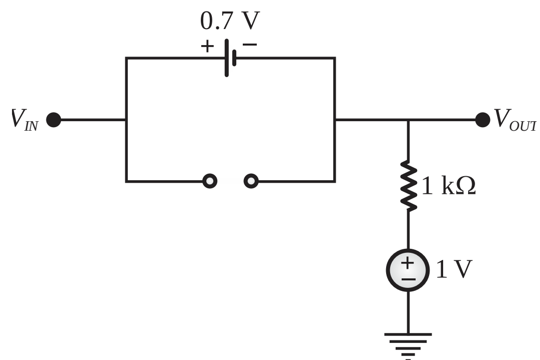
$$\therefore \quad V_{out} = V_{IN} + 0.7$$

Case 2 : When $0.3\text{ V} < V_{IN} < 1.7\text{ V}$, both D_1 and D_2 are OFF. So, the circuit becomes.



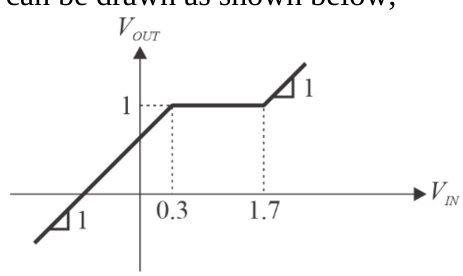
$\therefore \quad V_{out} = 1\text{ V}$

Case 3 : When $V_{IN} > 1.7\text{ V}$, D_1 is ON and D_2 is OFF. So, the circuit becomes,



$\therefore \quad V_{out} = V_{IN} - 0.7$

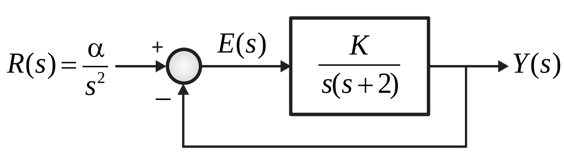
So, the transfer characteristics can be drawn as shown below,



Hence, the correct option is (A).

Question 41 **Control Systems : Time Response Analysis**

A closed loop system is shown in the figure where $K > 0$ and $\alpha > 0$. The steady state error due to a ramp input $\left(R(s) = \frac{\alpha}{s^2} \right)$ is given by



(A) $\frac{2\alpha}{K}$

(B) $\frac{\alpha}{K}$

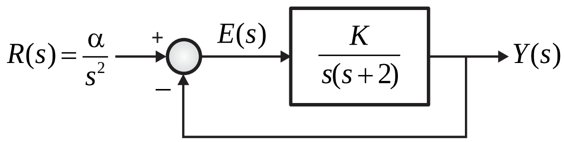
(C) $\frac{\alpha}{2K}$

(D) $\frac{\alpha}{4K}$

Ans. (A)

Sol. Given :

$$R(s)=\frac{\alpha}{s^2}$$



$$G(s)=\frac{K}{s(s+2)}$$

Steady state error for ramp input is given by

$$e_{ss}=\frac{\alpha}{K_v}$$

$$K_v=\lim_{s\to 0} sG(s)=\lim_{s\to 0}\frac{K}{(s+2)}$$

$$K_v=\frac{K}{2}$$

$$e_{ss}=\frac{\alpha}{K/2}$$

$$e_{ss}=\frac{2\alpha}{K}$$

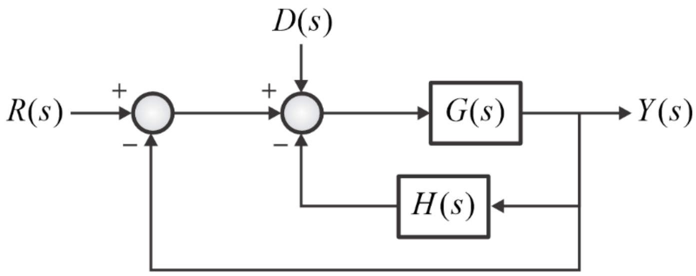
Hence, the correct option is (A).

Question 42 **Control Systems : Block Diagram & Signal Flow Graph**

In the following block diagram, $R(s)$ and $D(s)$ are two inputs. The output $Y(s)$ is expressed as

$$Y(s)=G_1(s)R(s)+G_2(s)D(s).$$

$G_1(s)$ and $G_2(s)$ are given by



(A) $G_1(s)=\frac{G(s)}{1+G(s)+G(s)H(s)}$ and $G_2(s)=\frac{G(s)}{1+G(s)+G(s)H(s)}$

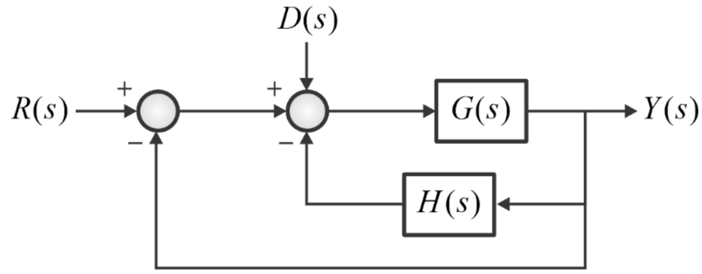
(B) $G_1(s)=\frac{G(s)}{1+G(s)+H(s)}$ and $G_2(s)=\frac{G(s)}{1+G(s)+H(s)}$

(C) $G_1(s)=\frac{G(s)}{1+G(s)+H(s)}$ and $G_2(s)=\frac{G(s)}{1+G(s)+G(s)H(s)}$

(D) $G_1(s)=\frac{G(s)}{1+G(s)+G(s)H(s)}$ and $G_2(s)=\frac{G(s)}{1+G(s)+H(s)}$

Ans. **(A)**

Sol. Given :

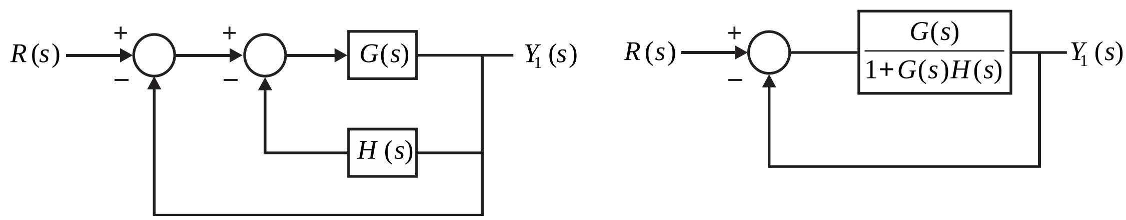


and $Y(s) = G_1(s)R(s) + G_2(s)D(s)$

Let $Y(s) = Y_1(s) + Y_2(s)$

Where, $Y_1(s) =$ Output considering only $R(s)$, $Y_2(s) =$ Output considering only $D(s)$.

When only $R(s)$ is present.

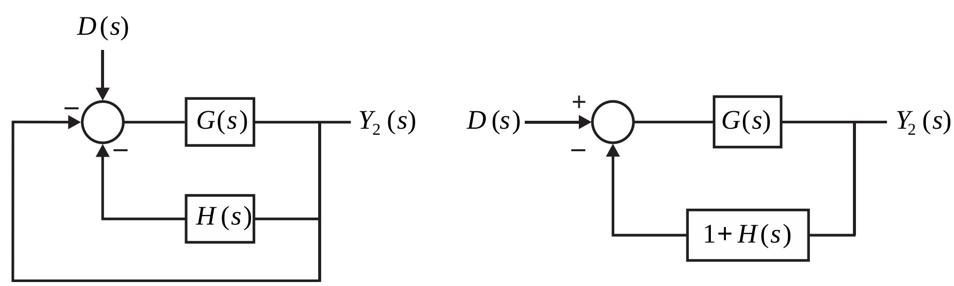


$$\frac{Y_1(s)}{R(s)} = \frac{\dfrac{G(s)}{1+G(s)H(s)}}{1+\dfrac{G(s)}{1+G(s)H(s)}}$$

$$Y_1(s) = \left[\frac{G(s)}{1+G(s)+G(s)H(s)}\right]R(s)$$

Hence, $G_1(s) = \dfrac{G(s)}{1+G(s)+G(s)H(s)}$

When only $D(s)$ is present.



$$\frac{Y_2(s)}{D(s)} = \frac{G(s)}{1+G(s)[1+H(s)]}$$

$$Y_2(s) = \left[\frac{G(s)}{1+G(s)[1+H(s)]}\right] D(s)$$

$$\frac{Y_2(s)}{D(s)} = \frac{G(s)}{1+G(s)+G(s)H(s)}$$

$$Y_2(s) = \frac{G(s)}{1+G(s)+G(s)H(s)} D(s)$$

$$G_2(s) = \frac{G(s)}{1+G(s)+G(s)H(s)}$$

Hence, the correct option is (A).

Question 43 **Mathematics : Linear Algebra**

The state equation of a second order system is $\dot{x}(t) = Ax(t)$, $x(0)$ is the initial condition.

Suppose λ_1 and λ_2 are two distinct Eigen values of A and V_1 and V_2 are the corresponding Eigen vectors.

For constant α_1 and α_2, the solution, $x(t)$ of the state equation is

(A) $\sum_{i=1}^{2} \alpha_i e^{\lambda_i t} V_i$ (B) $\sum_{i=1}^{2} \alpha_i e^{2\lambda_i t} V_i$

(C) $\sum_{i=1}^{2} \alpha_i e^{3\lambda_i t} V_i$ (D) $\sum_{i=1}^{2} \alpha_i e^{4\lambda_i t} V_i$

Ans. (A)

Sol. Given : $\dot{x}(t) = Ax(t)$

If λ is the eigen value of matrix A then $\dot{x}(t) = \lambda x(t)$

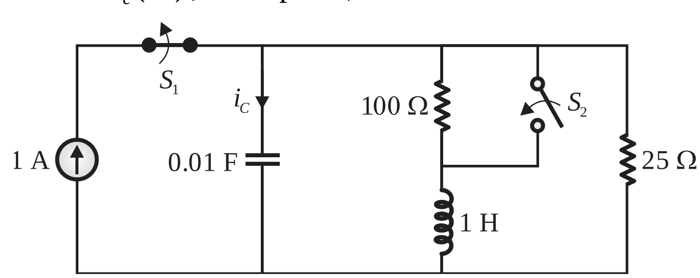
As there are 2 eigen values λ_1 and λ_2 of matrix A, the solution of state equation will be,

$$x(t) = \sum_{i=1}^{2} \alpha_i e^{\lambda_i t} V_i$$

Hence, the correct option is (A).

Question 44 **Network Theory : Transient Analysis**

The switch S_1 was closed and S_2 was open for a long time. At $t = 0$, switch S_1 is opened and S_2 is closed, simultaneously. The value of $i_c(0^+)$, in amperes, is

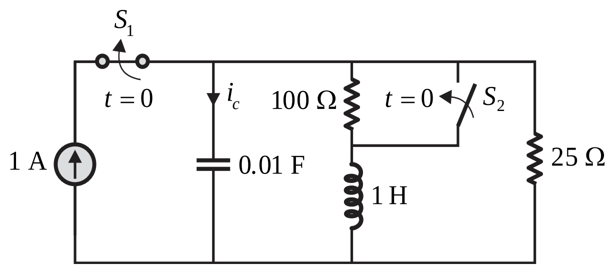


(A) 1 (B) – 1

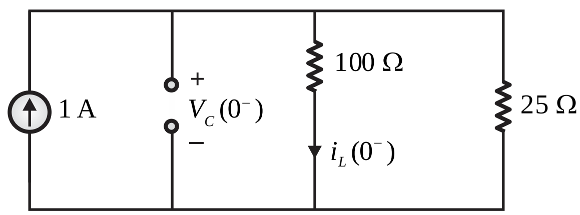
(C) 0.2 (D) 0.8

Ans. **(B)**

Sol. Given circuit is,



At $t=0^-, S_1$ is closed and S_2 is opened, at steady state $L \rightarrow$ S.C., $C \rightarrow$ O.C.

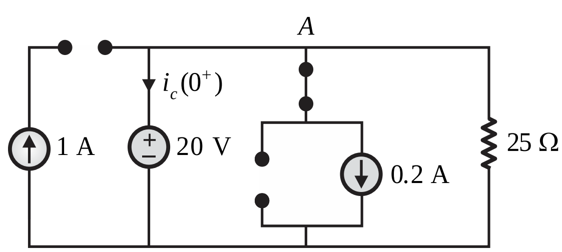


$$i_L(0^-) = \frac{1 \times 25}{100+25} \text{ (C.D.R)}$$

$$i_L(0^-) = 0.2 \text{ A}$$

$$V_C(0^-) = 100 \times 0.2 = 20 \text{V}$$

At $t=0^+, S_1$ opened S_2 is closed $L \rightarrow$ O.C., $C \rightarrow$ S.C.



Applying KCL at node A,

$$i_C(0^+) + 0.2 + \frac{20}{25} = 0$$

$$i_C(0^+) = -0.8 - 0.2$$

$$i_C(0^+) = -1\text{A}$$

Hence, the correct option is (B).

Question 45 **Communication Systems : Anagle Modulation**

Let a frequency modulated (FM) signal $x(t) = A\cos[\omega_c t + k_f \int_{-\infty}^{t} m(\lambda) d\lambda]$, where $m(t)$ is the message signal of bandwidth W. It is passed through a non-linear system with output $y(t) = 2x(t) + 5[x(t)]^2$. Let B_T denote the FM bandwidth. The minimum value of ω_c required to recover $x(t)$ from $y(t)$ is

(A) $B_T + W$ (B) $\frac{3}{2}B_T$

(D) $2B_T + W$ (D) $\frac{5}{2}B_T$

Ans. (B)

Sol. Given : $x(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\lambda)d\lambda\right)$

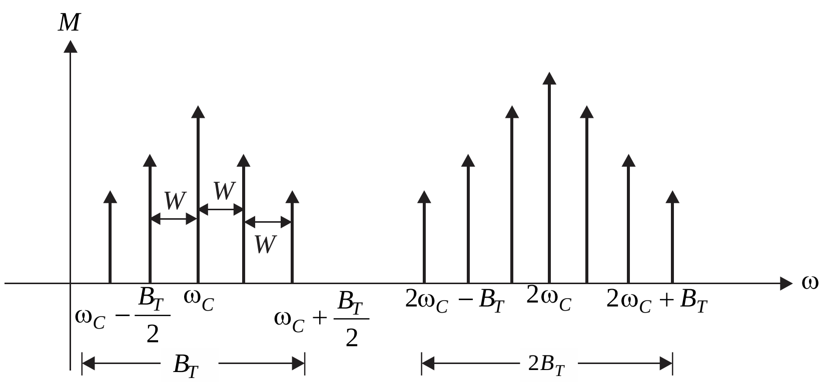
Also, $y(t) = 2x(t) + 5x^2(t)$

$$y(t) = 2A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\lambda)d\lambda\right) + 5A^2 cos^2\left(\omega_c t + k_f \int_{-\infty}^{t} m(\lambda)d\lambda\right)$$

$$y(t) = 2A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\lambda)d\lambda\right) + 5A^2\left(\frac{1+\cos\left(2\omega_c t + 2k_f \int_{-\infty}^{t} m(\lambda)d\lambda\right)}{2}\right)$$

$$y(t) = 2A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\lambda)d\lambda\right) + \frac{5A^2}{2}\cos\left(2\omega_c t + k_f \int_{-\infty}^{t} m(\lambda)d\lambda\right) + \frac{5A^2}{2}$$

Om plotting the frequency spectrum of $y(t)$



To recover $x(t)$ from $y(t)$

$$2\omega_c - B_T > \omega_c + \frac{B_T}{2}$$

$$\omega_c > \frac{3B_T}{2}$$

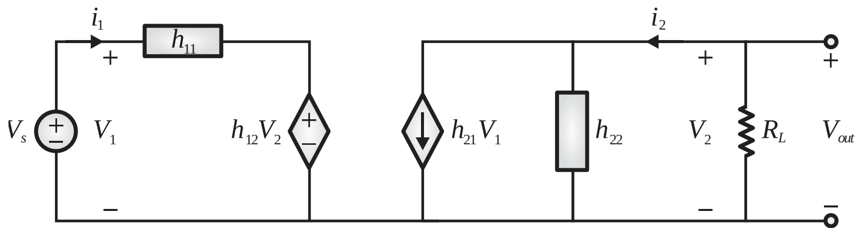
$\therefore$ Minimum value of ω_c will be

$$(\omega_c)_{\min} = \frac{3B_T}{2}$$

Hence, the correct option is (B).

Question 46 **Network Theory : Two port Networks**

The h-parameters of a two-port network are shown below. The condition for the maximum small signal voltage gain $\frac{V_{out}}{V_s}$ is



(A) $h_{11}=0,\ h_{12}=0,\ h_{21}=$ very high and $h_{22}=0$

(B) $h_{11}=$ very high, $h_{12}=0, h_{21}=$ very high and $h_{22}=0$

(C) $h_{11}=0, h_{12}=$ very high, $h_{21}=$ very high and $h_{22}=0$

(D) $h_{11}=0, h_{12}=0, h_{21}=$ very high and $h_{22}=$ very high

Ans. (A) [MTA]

Sol. $$\frac{V_0}{V_s}=\frac{-h_{21}\times R_L}{h_{11}}$$

For $\frac{V_0}{V_s}=\frac{-h_{21}\times R_L}{h_{11}}$ to be maximum, $h_{11}=0$ and h_{21} should be very high. Ideally h_{12} and h_{22} should be zero.

In the figure of the given question the current source is modelled as $\mathbf{h_{21}V_1}$. But according to the h-parameter model, it should be $\mathbf{h_{21}I_1}$. Hence, it should be marks to all question (MTA). If $\mathbf{h_{21}I_1}$ had been given instead of $\mathbf{h_{21}V_1}$, then option (A) would have been correct.

Question 47 **Signals & Systems : Continuous & Discrete Time Fourier Series**

Consider a discrete time periodic signal with period $N=5$. Let the discrete time Fourier series (DTFS) representation be $x[n]=\sum_{k=0}^{4}a_k e^{j\frac{k2\pi n}{5}}$, where, $a_0=1,\ a_1=3j, a_2=2j,\ a_3=-2j$ and $a_4=-3j$. The value of the sum $\sum_{n=0}^{4}x[n]\sin\frac{4\pi n}{5}$ is

(A) –10 (B) 10

(C) –2 (D) 2

Ans. (A)

Sol. **Given :** $x[n]=\sum_{k=0}^{4}a_k e^{j\frac{k2\pi n}{5}}$

where, $a_0=1,\ a_1=3j, a_2=2j,\ a_3=-2j$ and $a_4=-3j$

$$\therefore \quad \sum_{n=0}^{4} x(n)\sin\frac{4\pi n}{5} = \sum_{n=0}^{4} x(n)\left[\frac{e^{\frac{j4\pi n}{5}} - e^{\frac{-j4\pi n}{5}}}{2j}\right]$$

$$\sum_{n=0}^{4} x(n)\sin\frac{4\pi n}{5} = \frac{1}{2j}\left[\sum_{n=0}^{4} x(n)e^{\frac{j2\pi(2)n}{5}} - \sum_{n=0}^{4} x(n)e^{\frac{-j2\pi(2)n}{5}}\right] \quad \ldots\text{(i)}$$

DTFS coefficient is given by,

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{\frac{-j2\pi kn}{N}}$$

Given that time period of $x(n)$ is $N = 5$ sec

$$a_k = \frac{1}{5}\sum_{n=0}^{4} x(n)e^{\frac{-j2\pi kn}{5}}$$

$$\therefore \quad \sum_{n=0}^{4} x(n)e^{\frac{-j2\pi kn}{5}} = 5a_k$$

If $k = -2$ then $\sum_{n=0}^{4} x(n)e^{\frac{j2\pi(2)n}{5}} = 5a_{-2}$ $\ldots$(ii)

If $k = 2$ then $\sum_{n=0}^{4} x(n)e^{\frac{-j2\pi(2)n}{5}} = 5a_{2}$ $\ldots$(iii)

Substituting equation (ii) and (iii) in equation (i), we get

$$\sum_{n=0}^{4} x(n)\sin\frac{4\pi n}{5} = \frac{1}{2j}[5a_{-2} - 5a_2] \quad \ldots\text{(iv)}$$

From the property of discrete Fourier series.

$a_k = a_{k+N}$, where N is the time period.

$$\therefore \quad a_{-2} = a_{-2+5} = a_3$$

So, equation (iv) is becomes,

$$\sum_{n=0}^{4} x(n)\sin\frac{4\pi n}{5} = \frac{1}{2j}[5a_3 - 5a_2] = \frac{1}{2j}[5\times(-2j) - 5\times 2j] = -10$$

Hence, the correct option is (A).

Question 48 **Signals & Systems : DTFT and DFT**

Let an input $x[n]$ having discrete time Fourier transform $X(e^{j\Omega}) = 1 - e^{-j\Omega} + 2e^{-3j\Omega}$ be passed through an LTI system. The frequency response of the LTI system is $H(e^{j\Omega}) = 1 - \frac{1}{2}e^{-j2\Omega}$. The output $y[n]$ of the system is

(A) $\delta[n] + \delta(n-1) - \frac{1}{2}\delta[n-2] - \frac{5}{2}\delta[n-3] + \delta[n-5]$

(B) $\delta[n]-\delta(n-1)-\frac{1}{2}\delta[n-2]-\frac{5}{2}\delta[n-3]+\delta[n-5]$

(C) $\delta[n]-\delta(n-1)-\frac{1}{2}\delta[n-2]+\frac{5}{2}\delta[n-3]-\delta[n-5]$

(D) $\delta[n]+\delta(n-1)+\frac{1}{2}\delta[n-2]+\frac{5}{2}\delta[n-3]+\delta[n-5]$

Ans. (C)

Sol. **Method 1**

Given : $X(e^{j\Omega})=1-e^{-j\Omega}+2e^{-3j\Omega}$

$$H(e^{j\Omega})=1-\frac{1}{2}e^{-j2\Omega}$$

We know that, $y(n)=x(n)\otimes h(n)$

Apply DTFT on both sides,

$$Y(e^{j\Omega})=X(e^{j\Omega})H(e^{j\Omega})$$

$$Y(e^{j\Omega})=(1-e^{-j\Omega}+2e^{-3j\Omega})\left(1-\frac{1}{2}e^{-j2\Omega}\right)$$

$$=1-\frac{1}{2}e^{-j2\Omega}-e^{-j\Omega}+\frac{1}{2}e^{-j3\Omega}+2e^{-j3\Omega}-e^{-j5\Omega}$$

$$Y(e^{j\Omega})=1-e^{-j\Omega}-\frac{1}{2}e^{-j2\Omega}+\frac{5}{2}e^{-j3\Omega}-e^{-j5\Omega}$$

$$\therefore \quad y(n)=\delta(n)-\delta(n-1)-\frac{1}{2}\delta(n-2)+\frac{5}{2}\delta(n-3)-\delta(n-5)$$

Hence, the correct option is (C).

Method 2

Given : $X(e^{j\Omega})=1-e^{-j\Omega}+2e^{-3j\Omega}$

Taking inverse Fourier transform on both sides, we get

$$x(n)=\delta(n)-\delta(n-1)+2\delta(n-3)$$

Also, given $H(e^{j\Omega})=1-\frac{1}{2}e^{-j2\Omega}$

Taking inverse Fourier transform on both sides, we get

$$h(n)=\delta(n)-\frac{1}{2}\delta(n-2)$$

$\therefore$ Output $y(n)=x(n)\otimes h(n)$

$$y(n)=[\delta(n)-\delta(n-1)+2\delta(n-3)]\otimes\left[\delta(n)-\frac{1}{2}\delta(n-2)\right]$$

We know, that $x(n)\otimes\delta(n-n_0)=x(n-n_0)$

$$y(n)=\delta(n)-\frac{1}{2}\delta(n-2)-\delta(n-1)+\frac{1}{2}\delta(n-3)+2\delta(n-3)-\delta(n-5)$$

$$y(n)=\delta(n)-\delta(n-1)-\frac{1}{2}\delta(n-2)+\frac{5}{2}\delta(n-3)-\delta(n-5)$$

Hence, the correct option is (C).

Question 49 **Signals & Systems : Classification of Systems**

Let $x(t)=10\cos(10.5Wt)$ is passed through an LTI system having impulse response

$h(t)=\pi\left(\frac{\sin Wt}{\pi t}\right)^2\cos 10Wt$. The output of the system is

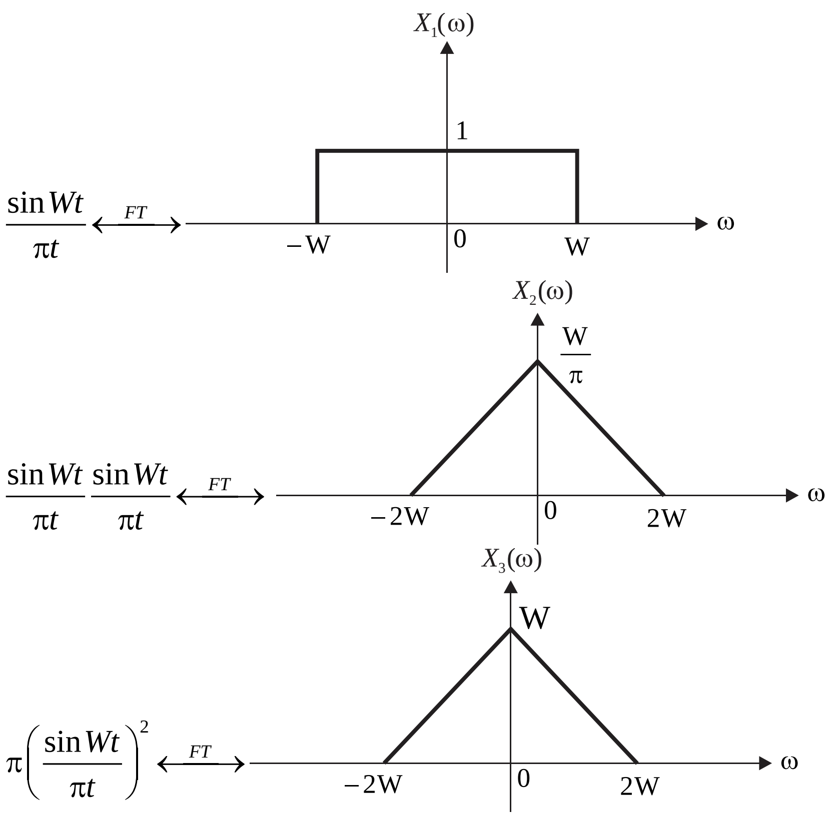
(A) $\left(\frac{15W}{4}\right)\cos(10.5Wt)$ (B) $\left(\frac{15W}{2}\right)\cos(10.5Wt)$

(C) $\left(\frac{15W}{8}\right)\cos(10.5Wt)$ (D) $(15W)\cos(10.5Wt)$

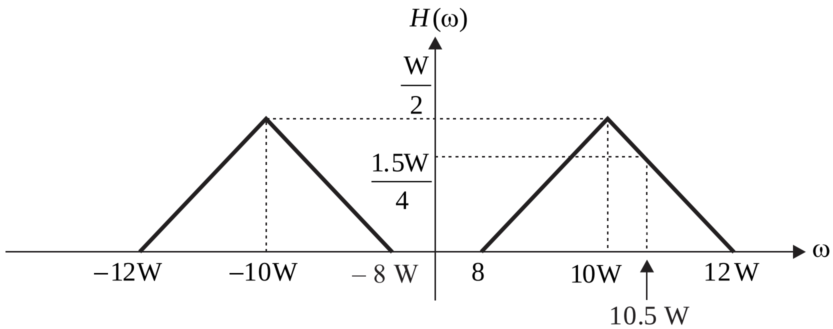
Ans. **(A)**

Sol. **Given :** $x(t)=10\cos(10.5\text{Wt})$

$$h(t)=\pi\left(\frac{\sin Wt}{\pi t}\right)^2\cos(10\,\text{Wt})$$



$$\pi\left(\frac{\sin Wt}{\pi t}\right)^2 \cos(10Wt) \xleftrightarrow{FT}$$



$$|H(10.5\,\mathrm{W})| = \frac{1.5\,\mathrm{W}}{4} \text{ and } \angle H(10.5\,\mathrm{W}) = 0^0$$

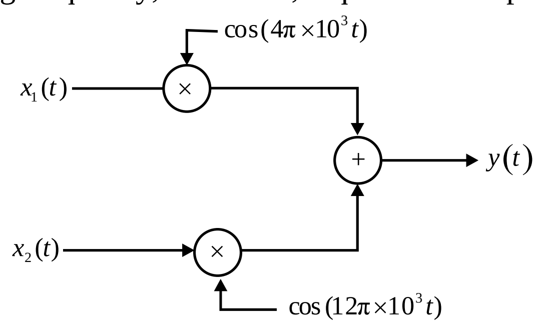
$$\therefore \quad y(t) = |H(10.5\,\mathrm{W})|\, x(t)$$

$$y(t) = \frac{1.5\,\mathrm{W}}{4} \times 10\cos(10.5\mathrm{W}t) = \frac{15\,\mathrm{W}}{4}\cos(10.5\mathrm{W}t)$$

Hence, the correct option is (A).

Question 50 **Communication Systems : Baseband Transmission**

Let $x_1(t)$ and $x_2(t)$ are two band limited signal having bandwidth $B = 4\pi\times10^3$ rad/sec each. In the figure below, the Nyquist sampling frequency, in rad/sec, required to sample $y(t)$ is



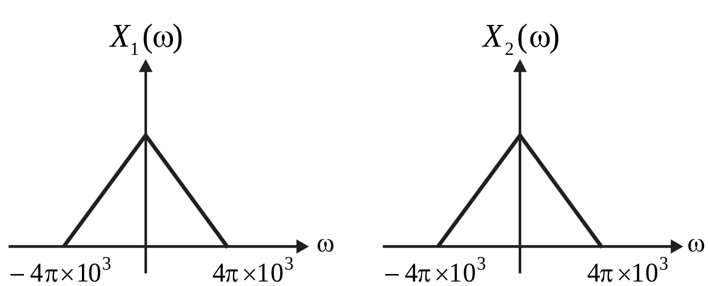
(A) $20\pi\times10^3$ (B) $40\pi\times10^3$

(C) $8\pi\times10^3$ (D) $32\pi\times10^3$

Ans. **(D)**

Sol. Given that $x_1(t)$ and $x_2(t)$ are bandlimited to $4\pi\times10^3$ rad/sec.

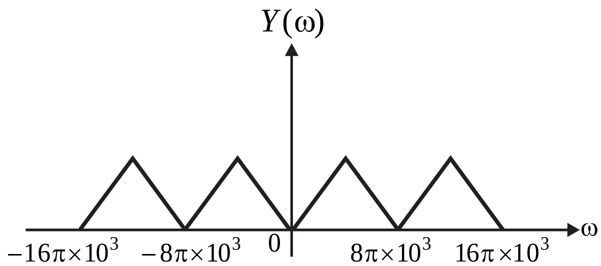
$x_1(t)$ and $x_2(t)$ can be represented in frequency domain as shown below,



From the given figure,

$$y(t) = x_1(t)\cos(12\pi\times10^3 t) + x_2(t)\cos(4\pi\times10^3 t)$$

The spectrum of $y(t)$ can be drawn as shown below



$\therefore$ Maximum frequency of $Y(\omega)$ is

$$\omega_m = 16\pi\times10^3 \text{ rad/sec}$$

Nyquist sampling rate will be

$$\omega_s = 2\omega_m = 32\pi\times10^3 \text{ rad/sec}$$

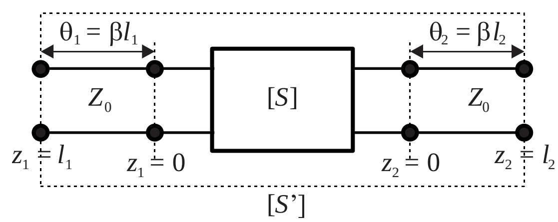
Hence, the correct option is (D).

Question 51 **Electromagnetic Theory : Transmission Lines**

The S-parameters of a two port network is given as

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

with reference to Z_0. Two lossless transmission line sections of electrical lengths $\theta_1 = \beta l_1$ and $\theta_2 = \beta l_2$ are added to input and output ports for measurement purposes, respectively. The S parameters [S'] of the resultant two port network is



(A) $\begin{bmatrix} S_{11}e^{-j2\theta_1} & S_{12}e^{-j(\theta_1+\theta_2)} \\ S_{21}e^{-j(\theta_1+\theta_2)} & S_{22}e^{-j2\theta_2} \end{bmatrix}$

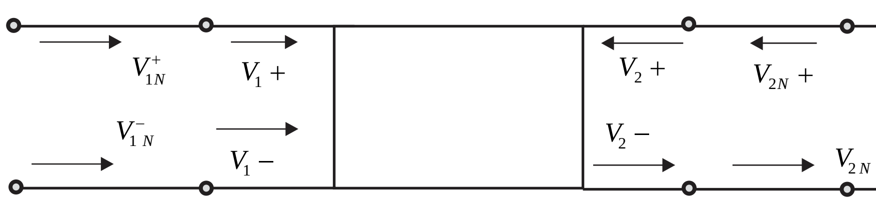
(B) $\begin{bmatrix} S_{11}e^{j2\theta_1} & S_{12}e^{-j(\theta_1+\theta_2)} \\ S_{21}e^{-j(\theta_1+\theta_2)} & S_{22}e^{j2\theta_2} \end{bmatrix}$

(C) $\begin{bmatrix} S_{11}e^{j2\theta_1} & S_{12}e^{j(\theta_1+\theta_2)} \\ S_{21}e^{j(\theta_1+\theta_2)} & S_{22}e^{j2\theta_2} \end{bmatrix}$

(D) $\begin{bmatrix} S_{11}e^{-j2\theta_1} & S_{12}e^{j(\theta_1+\theta_2)} \\ S_{21}e^{j(\theta_1+\theta_2)} & S_{22}e^{-j2\theta_2} \end{bmatrix}$

Ans. (A)

Sol.



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} \quad \text{....(i)}$$

To be calculated,

$$\begin{bmatrix} V_{1N}^- \\ V_{2N}^- \end{bmatrix} = \begin{bmatrix} S_{11N} & S_{12N} \\ S_{21N} & S_{22N} \end{bmatrix} \begin{bmatrix} V_{1N}^+ \\ V_{2N}^+ \end{bmatrix}$$

$$V_1^+ = V_{1N}^+ e^{-j\theta_1}$$

$$V_{1N}^- = V_1^- e^{-j\theta_1} = V_1^- = V_{1N}^- e^{j\theta_1}$$

$$V_2^+ = V_{2N}^+ e^{-j\theta_2}$$

$$V_{2N}^- = V_2^- e^{-j\theta_2} = V_2^- = V_{2N}^- e^{j\theta_2}$$

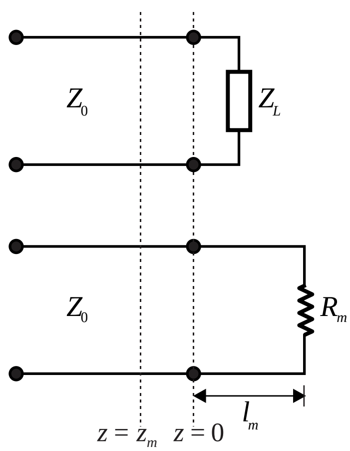
Feeding the data in equation (i),

$$\begin{bmatrix} V_{1N}^- \\ V_{2N}^- \end{bmatrix} = \begin{bmatrix} S_{11N} e^{-j(2\theta_1)} & S_{12N} e^{-j(\theta_1+\theta_2)} \\ S_{21N} e^{-j(\theta_1+\theta_2)} & S_{22N} e^{-j2(\theta_2)} \end{bmatrix}$$

Hence, the correct option is (A).

Question 52 **Electromagnetic Theory : Transmission Lines**

The standing wave ratio on a 50 Ω lossless transmission line terminated in an unknown load impedance is found to be 2.0. The distance between successive voltage minima is 30 cm and first minimum is located at 10 cm from the load. Z_L can be replaced by an equivalent length l_m and terminating resistance R_m of same line. The value of R_m and l_m, respectively, are



(A) $R_m = 100\,\Omega, l_m = 20\,\text{cm}$

(B) $R_m = 25\,\Omega, l_m = 20\,\text{cm}$

(C) $R_m = 100\,\Omega, l_m = 5\,\text{cm}$

(D) $R_m = 25\,\Omega, l_m = 5\,\text{cm}$

Ans. **(B), (C)**

Sol. **Given :** $s = 2 \Rightarrow |\Gamma| = \dfrac{5-1}{5+1} = \dfrac{1}{3}$

Also, $\dfrac{\lambda}{2} = 30 \Rightarrow \lambda = 60$ cm

First minima occurs at $Z_{\min} = 10$ cm

$\therefore \quad 2\beta \times Z_{\min} = \pi + \theta_\Gamma$

$$\Rightarrow \frac{2\times 2\pi}{60}\times 10=\pi+\theta_\Gamma$$

$$\Rightarrow \theta_\Gamma=\frac{-\pi}{3}$$

$$\therefore \quad \Gamma=\frac{1}{3}\angle -60^0$$

$$\Rightarrow Z_L=Z_0\left[\frac{1+\Gamma}{1-\Gamma}\right]=67.97\angle -32.67^0$$

Now if Z_L is replaced by an equivalent of length ℓ_m and terminating resistance R_m then

$$Z_L=Z_0\left[\frac{R_m+jZ_0\tan\beta\ell_m}{Z_0+jR_m\tan\beta\ell_m}\right]$$

$$\Rightarrow 67.97\underline{|-32.67^0}=50\left[\frac{R_m+j50\tan\beta\ell_m}{50+jR_m\tan\beta\ell_m}\right]$$

Only options B and C satisfies the condition (i).

Hence, the correct options are (B) and (C).

Question 53 **Electromagnetic Theory : Polarization**

The electric field of a plane electromagnetic wave is $\vec{E}=\hat{a}_xC_{1x}\cos(\omega t-\beta z)+\hat{a}_yC_{1y}\cos(\omega t-\beta z+\theta)$ V/m.

Which of the following combination(s) will give rise to a left handed elliptically polarized (LHEP) wave?

(A) $C_{1x}=1,\ C_{1y}=1,\ \theta=\frac{\pi}{4}$

(B) $C_{1x}=2,\ C_{1y}=1,\ \theta=\frac{\pi}{2}$

(C) $C_{1x}=1,\ C_{1y}=2,\ \theta=\frac{3\pi}{2}$

(D) $C_{1x}=2,\ C_{1y}=1,\ \theta=\frac{3\pi}{4}$

Ans. **(A), (B), (D)**

Sol. Given electromagnetic wave will be left elliptically polarized if

(i) y component leads x component

(ii) $C_{1x}\neq C_{1y},\theta\neq\frac{\pi}{2}$

(iii) $C_{1x}\neq C_{1y},\theta=\frac{\pi}{2}$

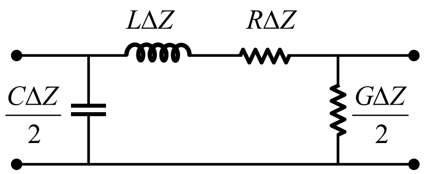
(iv) $C_{1x}=C_{1y},\theta\neq\frac{\pi}{2}$

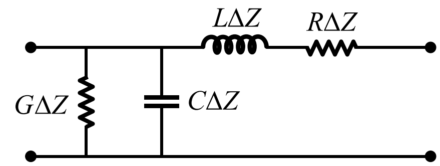
Only option (A), (B) and (D) satisfies the above conditions.

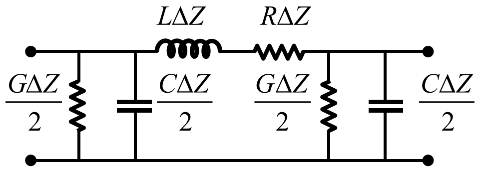
Hence, the correct options are (A), (B) and (D).

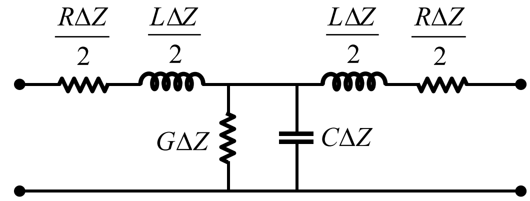
Question 54 **Electromagnetic Theory : Transmission Lines**

The following circuit(s) representing a lumped element equivalent of an infinitesimal section of a transmission line is/are

(A)


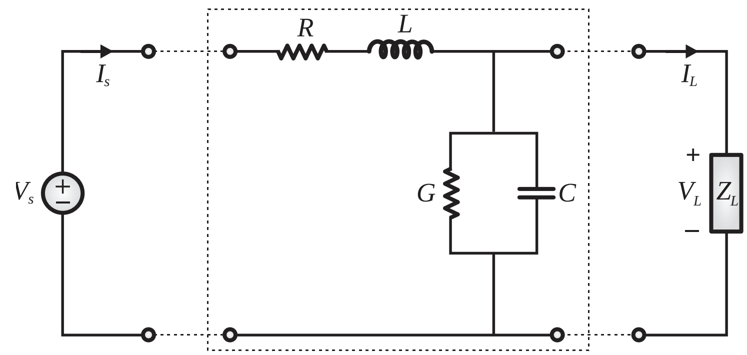
(B)


(C)


(D)


Ans. **(B), (C) & (D)**

Sol. Any transmission line can be represented in terms of primary constants R, L, G and C as shown below,



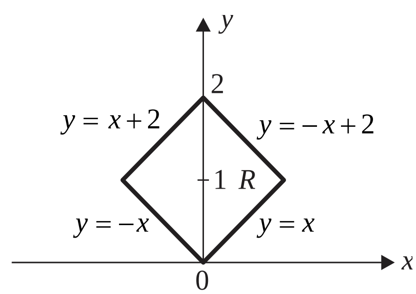
It can be seen from the above representation that R and L are in series, G and C are in parallel.

With this logic, options (B), (C) and (D) will be the appropriate answer.

Hence, the correct options are (B), (C) & (D).

Question 55 **Mathematics : Integral & Differential Calculus**

The value of the integral $\iint_R xy\,dx\,dy$ over the region R, given in the figure, is ________ (rounded off to the nearest integer).



Ans. **0 (0 to 0)**

Sol. Method 1

From the given region R,

$$y = x + 2$$

$$y - x = 2 \qquad \ldots\text{(i)}$$

$$y = -x + 2$$

$$y + x = 2 \qquad \ldots\text{(ii)}$$

$y=-x$

$y+x=0$ ...(iii)

$y=x$

$y-x=0$ ...(iv)

Let $u=y-x$ (5) and $v=y+x$ (6)

Jacobian $J\left(\frac{u,v}{x,y}\right)=\begin{vmatrix} u_x & u_y \\ V_x & V_y \end{vmatrix}=\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$

$$J\left(\frac{u,v}{x,y}\right)=-1-1=-2$$

So, $J\left(\frac{x,y}{u,v}\right)=-\frac{1}{2}$

$$\left|J\left(\frac{x,y}{u,v}\right)\right|=\left|-\frac{1}{2}\right|=\frac{1}{2}$$

u limits : $u=0$ to $u=2$

v limits : $v=0$ to $v=2$

From equations (v) and (vi),

$$x=\frac{v-u}{2}$$

$$y=\frac{v+u}{2}$$

$$\therefore \quad \iint xy\,dx\,dy=\int_0^2\int_0^2\left(\frac{v-u}{2}\right)\left(\frac{v+u}{2}\right)|J|\,du\,dv$$

$$=\int_0^2\int_0^2\frac{v^2-u^2}{4}\times\frac{1}{2}du\,dv=\frac{1}{8}\int_0^2\int_0^2(v^2-u^2)du\,dv$$

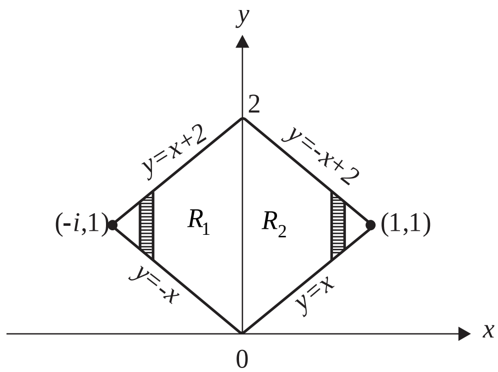
$$=\frac{1}{8}\int_0^2\left(uv^2-\frac{u^3}{3}\right)_0^2 dv=\frac{1}{8}\int_0^2\left(2v^2-\frac{8}{3}\right)dv$$

$$=\frac{1}{8}\left(\frac{2v^3}{3}-\frac{8}{3}\right)_0^2=\frac{1}{8}\left[\frac{16}{3}-\frac{16}{3}\right]=0$$

Hence, the correct answer is 0.

Method 2

Given region R can be drawn as shown below



$$\therefore \quad \iint_R xy\,dx\,dy = \iint_{R_1} xy\,dx\,dy + \iint_{R_2} xy\,dx\,dy$$

$$= \int_{x=-1}^{0}\int_{y=-x}^{x+2} xydydx + \int_{x=0}^{1}\int_{y=x}^{-x+2} xydydx$$

$$= \int_{x=-1}^{0}\left(\frac{xy^2}{2}\right)_{-x}^{2+x} dx + \int_{x=0}^{1}\left(\frac{xy^2}{2}\right)_{x}^{2-x} dx$$

$$= \int_{x=-1}^{0}\left(\frac{x(2+x)^2}{2} - \frac{x^3}{2}\right)dx + \int_{x=0}^{1}\left(\frac{x(2-x)^2}{2} - \frac{x^3}{2}\right)dx$$

$$= \int_{-1}^{0}\frac{x}{2}\left[(x+2)^2 - (x)^2\right]dx + \int_{0}^{1}\frac{x}{2}\left[(2-x)^2 - x^2\right]dx$$

$$= \int_{-1}^{0}\frac{x}{2}(x^2+4x+4-x^2)\,dx + \int_{0}^{1}\frac{x}{2}(x^2-4x+4-x^2)\,dx$$

$$= \int_{-1}^{0}\frac{x}{2}(4x+4)\,dx + \int_{0}^{1}\frac{x}{2}(4-4x)\,dx$$

$$= \frac{1}{2}\int_{-1}^{0}(4x^2+4x)\,dx + \frac{1}{2}\int_{0}^{1}(4x-4x^2)\,dx$$

$$= \frac{1}{2}\left(\frac{4x^3}{3}+\frac{4x^2}{2}\right)\Bigg|_{-1}^{0} + \frac{1}{2}\left(\frac{4x^2}{2}-\frac{4x^3}{3}\right)\Bigg|_{0}^{1}$$

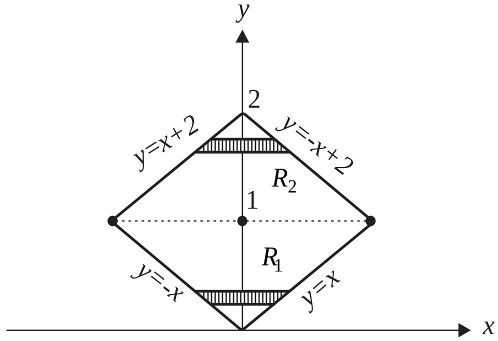
$$= \frac{1}{2}\left[(0+0) - \left(-\frac{4}{3}+\frac{4}{2}\right)\right] + \frac{1}{2}\left(\frac{4}{2}-\frac{4}{3}\right)$$

$$= \frac{1}{2}\left(\frac{4}{3}-\frac{4}{2}\right) + \frac{1}{2}\left(\frac{4}{2}-\frac{4}{3}\right) = \frac{1}{2}\left(\frac{4}{3}-\frac{4}{2}+\frac{4}{2}-\frac{4}{3}\right) = 0$$

Hence, the correct answer is 0.

Method 3

Given region R can be drawn as shown below,



$$\iint_R xy\,dx\,dy = \int_{y=0}^{1}\int_{x=-y}^{y} xydxdy + \int_{y=1}^{2}\int_{x=y-2}^{2-y} xydxdy$$

$$= \int_{y=0}^{1} y\left(\frac{x^2}{2}\right)_{-y}^{y} dy + \int_{y=1}^{2} y\left(\frac{x^2}{2}\right)_{y-2}^{2-y} dy = 0+0=0$$

Hence, the correct answer is 0.

Question 56 **Electronic Devices & Circuits : Basic Semiconductor Physics**

In an extrinsic semiconductor, the hole concentration is given to be $1.5n_i$, where n_i is intrinsic carrier concentration of $1\times10^{10}\ \text{cm}^{-3}$. The ratio of electron to hole mobility for equal hole and electron drift current is given as ______ (rounded off to two decimal places).

Ans. **2.25 (2.20 to 2.30)**

Sol. **Given :** Hole concentration $p = 1.5n_i$

Intrinsic concentration $n_i = 1.5\times10^{10}\,cm^{-3}$

Also given, $I_n = I_p$

$\Rightarrow J_n \times A = J_p \times A$

$\Rightarrow J_n = J_p$

$\Rightarrow nq\mu_n E = pq\mu_p E$

$\Rightarrow \dfrac{\mu_n}{\mu_p} = \dfrac{p}{n}$

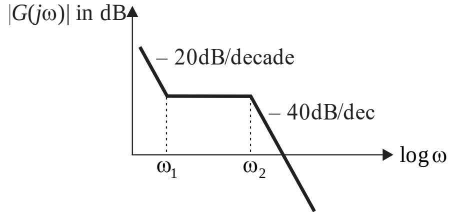
$\Rightarrow \dfrac{\mu_n}{\mu_p} = \dfrac{p^2}{n_i^2} = \left(\dfrac{p}{n_i}\right)^2$

$\Rightarrow \dfrac{\mu_n}{\mu_p} = \left(\dfrac{1.5n_i}{n_i}\right)^2 = 2.25$

Hence, the correct answer is 2.25.

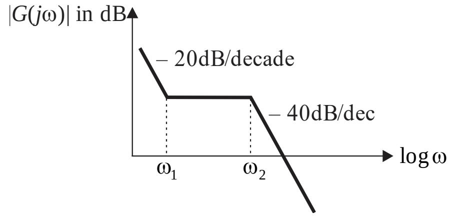
Question 57 **Control Systems : Bode Plot**

The asymptotic magnitude Bode plot of a minimum phase system is shown in the figure. The transfer function of the system is $G(s)=\frac{K(s+z)^a}{s^b(s+p)^c}$, where *K*, *z*, *p*, *a*, *b* and *c* are positive constants. The value of $(a+b+c)$ is _______ (rounded off to the nearest integer).



Ans. **4 (4 to 4)**

Sol. Given bode plot is,



and $G(s)=\frac{k(s+z)^a}{s^b(s+p)^c}$

Initial slope is $-20\,\text{dB/dec}$

From bode plot, it is clear that there is one pole at origin.

Hence, $b=1$

At $\omega=\omega_1$, slope becomes 0 dB/dec, hence there is a zero at $\omega=\omega_1$.

Hence, $a=1$

At $\omega=\omega_1$, slope becomes $-40\,\text{dB/dec}$, hence there is two pole at $\omega=\omega_2$

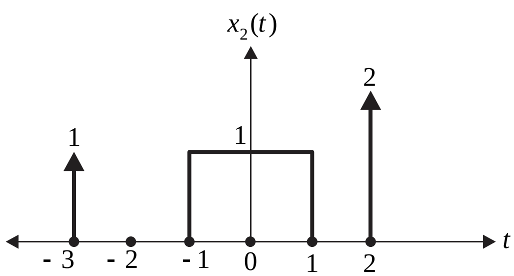
Hence, $c=2$

$a+b+c=1+1+2=4$

Hence, the correct answer is 4.

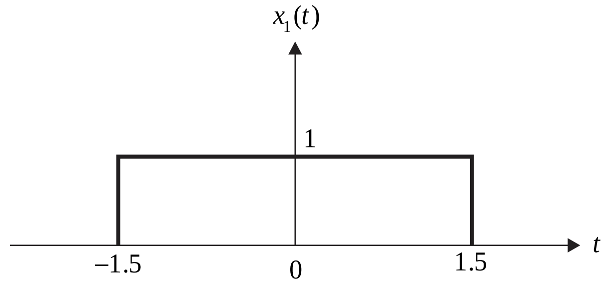
Question 58 **Signals & Systems : Continuous Time Convolution**

Let $x_1(t)=u(t+1.5)-u(t-1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t)=x_1(t)*x_2(t)$, the $\int_{-\infty}^{\infty} y(t)\,dt$ is _________ (rounded off to the nearest integer).



Ans. **15**

Sol. Given : $x_1(t) = u(t+1.5) - u(t-1.5)$



$$y(t) = x_1(t) \otimes x_2(t)$$

$$\int_{-\infty}^{\infty} y(t)\,dt = \int_{-\infty}^{\infty} x_1(t)\,dt \times \int_{-\infty}^{\infty} x_2(t)\,dt = 3\times1\times(2\times1+3) = 3\times1\times5 = 15$$

Hence, the correct answer is 15.

Key Point :

If $x(t) = x_1(t) \otimes x_2(t)$, then

Area of $x(t) =$ Area of $x_1(t)$ $\times$ Area of $x_2(t)$

$$= \int_{-\infty}^{\infty} x(t)dt = \int_{-\infty}^{\infty} x_1(t)dt \times \int_{-\infty}^{\infty} x_2(t)dt$$

Question 59 **Communication Systems : Random Variables & Random Processes**

Let $X(t)$ be a white Gaussian noise with power spectral density $\frac{1}{2}$ W/Hz. If $X(t)$ is input to an LTI system with impulse response $e^{-t}u(t)$. The average power of the system output is ________ W (rounded off to two decimal places).

Ans. 0.25 (0.24 to 0.26)

Sol. Given input power spectral density, $S_x(f) = \frac{1}{2}$ W/Hz

Output power spectral density will be,

$$S_Y(f) = |H(f)|^2 \times S_X(f)$$

$$S_Y(f) = |H(f)|^2 \times \frac{1}{2}$$

Output power,

$$P_0 = \int_{-\infty}^{\infty} S_Y(f)df = \int_{-\infty}^{\infty} |H(f)|^2 \times \frac{1}{2} df$$

$$P_0 = \frac{1}{2}\int_{-\infty}^{\infty} |H(f)|^2 df = \frac{1}{2} \times \text{Energy of } h(t)$$

$$P_0 = \frac{1}{2} \times \int_{-\infty}^{\infty} [e^{-t} u(t)]^2 \, dt$$

$$P_0 = \frac{1}{2} \times \int_0^{\infty} e^{-2t} dt = \frac{1}{2} \times \left(\frac{e^{-2t}}{-2} \right)_0^{\infty} = 0.25 \text{ W}$$

Hence, the correct answer is 0.25.

Question 60 **Electromagnetic Theory : Plane Wave Propagation**

A transparent dielectric coating is applied to glass $(\varepsilon_r = 4, \mu_r = 1)$ to eliminate the reflection of red light $(\lambda_0 = 0.75 \mu\text{m})$. The minimum thickness of the dielectric coating, in μm, that can be used is _______ (rounded off to two decimal places).

Ans. 0.133 (0.12 to 0.14)

Sol. **Given :** $\varepsilon_r = 4, \mu_r = 1$ and $\lambda_0 = 0.75 \ \mu\text{m}$.

For no reflection to occur, impedance of glass and dielectric must be matched.

$$\therefore \quad \eta_{\text{dielectric}} = \sqrt{\eta_{glass} \times \eta_0}$$

$$\frac{120\pi}{\sqrt{\in_{\text{rdielectric}}}} = \sqrt{\frac{120\pi}{\sqrt{4}} \times 120\pi}$$

$$\Rightarrow \frac{120\pi}{\sqrt{\in_{\text{rdielectric}}}} = \frac{120\pi}{\sqrt{2}}$$

$$\Rightarrow \in_{\text{rdielectric}} = 2$$

Also impedance matching will be seen if thickness of dielectric is

$$t = \frac{\lambda}{4}$$

$$\Rightarrow t = \frac{\lambda_0}{4\sqrt{\in_{\text{rdielectric}}}}$$

$$\Rightarrow t = \frac{0.75 \times 10^{-6}}{4 \times \sqrt{2}} = 0.133 \ \mu\text{m}$$

Hence, the correct answer is 0.133.

Question 61 **Electronic Devices & Circuits : Basic Semiconductor Physics**

In a semiconductor device, the Fermi-energy level is 0.35 eV above the valence band energy. The effective density of states in the valence band at $T = 300\text{K}$ is $1 \times 10^{19} \text{cm}^{-3}$. The thermal equilibrium hole concentrate in silicon at 400 K is ______ $\times 10^{13} \text{cm}^{-3}$ (rounded off to two decimal places).

Given : kT at 300 K is 0.026 eV.

Ans. 69.87

Sol. **Given :** $E_F - E_V = 0.35$ eV

$$N_V (T = 300^0 K) = 1 \times 10^{19} \text{cm}^{-3}$$

We know that, $V_T = \dfrac{T}{11{,}600}$

$$\Rightarrow \frac{V_{T1}}{V_{T2}} = \frac{T_1}{T_2}$$

$$\Rightarrow \frac{0.026}{V_{T2}} = \frac{300}{400}$$

$$\Rightarrow V_{T2} = \frac{0.026 \times 4}{3} = 0.035\,\text{V}$$

$$\Rightarrow kT_2 = 0.035\text{ eV}$$

Also $N_V \propto T^{\frac{3}{2}}$

$$\Rightarrow \frac{N_{V_1}}{N_{V_2}} = \left(\frac{T_1}{T_2}\right)^{\frac{3}{2}} = \left(\frac{3}{4}\right)^{\frac{3}{2}}$$

$$\Rightarrow \frac{N_{V_1}}{N_{V_2}} = 0.6495$$

$$\Rightarrow N_{V_2} = \frac{N_{V_1}}{0.6495} = \frac{1 \times 10^{19}}{0.6495}$$

$$\Rightarrow N_V (T = 400^0 K) = 1.539 \times 10^{19}$$

$\therefore$ Hole concentration at 400^0K will be

$$P = N_{V_2} e^{-\left(\frac{E_F - E_V}{kT_2}\right)} = 1.539 \times 10^{19} e^{-\frac{0.35}{0.035}} = 69.87 \times 10^{13}\,\text{cm}^{-3}$$

Hence, the correct answer is 69.87.

Question 62 **Analog Electronics : Operational Amplifiers**

A sample and hold circuit is implemented using a resistive switch and a capacitor with a time constant of $1\ \mu\text{s}$. The time for the sampling switch to stay closed to charge a capacitor adequately to a full-scale voltage of 1V with 12-bit accuracy is ________ μs (rounded off to two decimal places).

Ans. 8.317 (8.30 to 8.34)

Sol. **Given :** $V_{FS} = 1\text{V}$, $n = 12$

Time constant $\tau = 1\ \mu\text{sec}$

$$e^{-\frac{t}{\tau}} = \frac{V_{FS}}{2^n}$$

$$\frac{-t}{\tau} = \ln\left(\frac{V_{FS}}{2^n}\right)$$

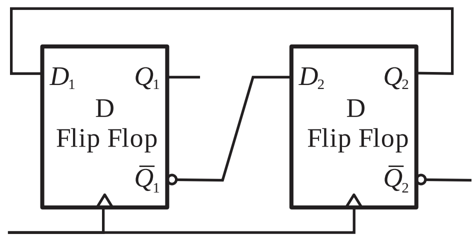
$$\frac{t}{\tau} = \ln\left(\frac{2^n}{V_{FS}}\right)$$

$$t = \tau \ln\left(\frac{2^n}{V_{FS}}\right) = 10^{-6} \ln\left(\frac{2^{12}}{1}\right) = 8.317\,\mu\text{sec}$$

Hence, the correct answer is 8.317.

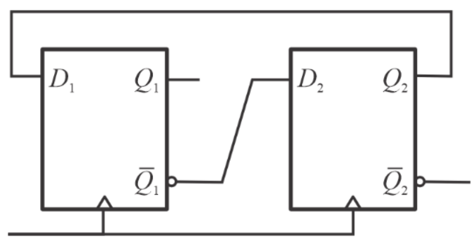
Question 63 **Digital Electronics : Sequential Circuits**

In a given sequential circuit, initial states are $Q_1 = 1$ and $Q_2 = 0$. For a clock frequency of 1 MHz, the frequency of signal at Q_2 in kHz, is _________ (rounded off to the nearest integer).



Ans. **250 (250 to 250)**

Sol. Given circuit is



$D_1 = Q_2$, $D_2 = \bar{Q}_1$

Initially $Q_1 = 1$, $Q_2 = 0$

CLK	$D_1 = Q_2$	$D_2 = \bar{Q}_1$	Q_1	Q_2	
Initially			1	0	
1	0	0	0	0	
2	0	1	0	1	
3	1	1	1	1	
4	1	0	1	0 $\rightarrow$	repeat

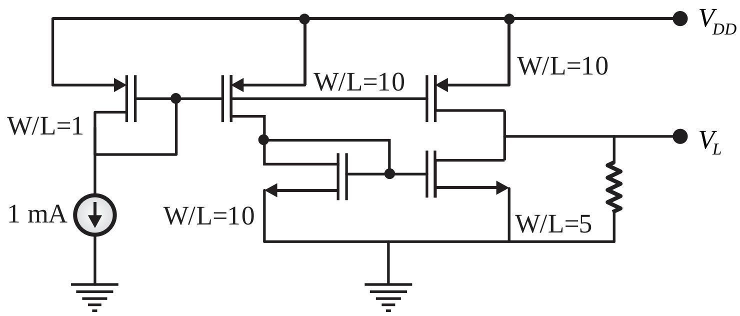
Hence, given counter having MOD-4 output frequency,

$$f_0 = \frac{f_i}{4} = \frac{1000\,k}{4} = 250\,\text{kHz}$$

Hence, the correct answer is 250.

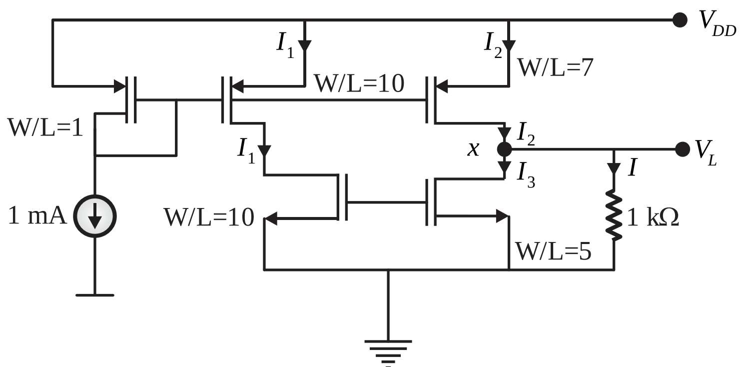
Question 64 **Electronic Devices & Circuits : MOSFET**

In the circuit below, the voltage V_L is _______ V (rounded off to two decimal places).



Ans. **2 (2.00 to 2.00)**

Sol. Given circuit can be drawn as shown below



From the above figure,

$$I_1 = \frac{10}{1} \times 1\text{ mA} = 10\text{ mA}$$

$$I_2 = 7 \times 1\text{ mA} = 7\text{ mA}$$

$$I_3 = \frac{5}{10} \times 10\text{ mA} = 5\text{ mA}$$

KCL at node $x \Rightarrow I = I_2 - I_3 \;\Rightarrow\; I = 7\text{ mA} - 5\text{ mA} = 2\text{ mA}$

$\therefore\;\; V_L = 2\text{ mA} \times 1\text{ k}\Omega = 2\text{V}$

Hence, the correct answer is 2.

Question 65 **Communication Systems : Information Theory & Error Correction**

The frequency of occurrence of 8 symbols (a-h) is shown in the table below. A symbol is chosen and it is determined by asking a series of "yes/no" questions which are assumed to be truthfully answered. The average number of questions when asked in the most efficient sequence, to determine the chosen symbol, is ________ (rounded off to two decimal places).

Symbols	a	b	c	d	e	f	g	h
Frequency of occurrence	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{128}$

Ans. **1.98 (1.97 to 1.99)**

Sol. Given data is as shown below,

Symbols	a	b	c	d	e	f	g	h

Frequency of occurrence	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{128}$

Average number of questions asked in the most efficient sequence will be equal to the entropy $H(x)$

$$H(x) = \sum_{K=1}^{8} P_K \log_2 \frac{1}{P_K}$$

$$H(x) = \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{16}\log_2 16 + \frac{1}{32}\log_2 32 + \frac{1}{64}\log_2 64 + \frac{1}{128}\log_2 128 + \frac{1}{128}\log_2 128$$

$$H(x) = 1.98$$

Hence, the correct answer is 1.98.

❑❑❑